FAO PENMAN-MONTEITH EQUATION: CALCULATIONS AND OPTIONS FOR SUBSTITUTIONS FOR MISSING DATA

The FAO Penman-Monteith equation allows for data that is missing to be estimated or calculated using other available data. In some cases there is more than one option of how to calculate or estimate the missing data. This section describes all the options available. Appendix B focuses on the calculations required to calculate ET_o using only temperature data.

Summarized from “Crop evapotranspiration, Guidelines for computing crop water requirements” (FAO paper 56, 1998)

$$ET_o = \left[\frac{0.408\,\Delta\,(R_n - G) + \gamma \left(\frac{900}{T + 273} \right) u_2\ (VPD)}{\Delta + \gamma\,(1 + 0.34\,u_2)} \right]$$

Where:
ET_o = daily reference ET [mm/d], for longer periods 900 becomes 37
T = air temperature at 2 m high [°C]
VPD = vapor pressure deficit [kPa]
u_2 = wind speed at 2 m high [m/s] = 2 m/s
R_n = net radiation at the crop surface [MJ m^{-2} d^{-1}]
Δ = slope vapour pressure curve [kPa $°C^{-1}$]
γ = psychometric constant [kPa $°C^{-1}$]
G = soil heat flux density [MJ m^{-2} d^{-1}]

TEMPERATURE, T

The mean of the maximum and the minimum recorded temperatures.

$$T = \left(\frac{T_{min} + T_{max}}{2} \right)$$

Where
T_{max} = maximum temperature [°C]
T_{min} = minimum temperature [°C]

Adjust temperatures of non-reference climate station, where T_{dew} is available
Climate stations used to make observations for reference ET_o should be surrounded by a well watered crop. As this is not the case with most climate stations in BC. Therefore, the temperatures should be adjusted using the dew point temperature and a correction factor, K_o. K_o will be 2 °C for all stations since the temperatures will be taken from a non-reference station.

$$\left(T_{\max}\right)_{cor} = (T_{\max})_{obs} - \left(\frac{\Delta\mathrm{T} - \mathrm{K_o}}{2}\right)$$

$$\left(T_{\min}\right)_{cor} = (T_{\min})_{obs} - \left(\frac{\Delta\mathrm{T} - \mathrm{K_o}}{2}\right)$$

Where:

K_o = 2 °C if the non-reference station is not compared to a reference station
$\Delta T = T_{min} - T_{dew}$

K_o = 0 °C if ΔT is calculated using reference station data
$\Delta T = (T_{min} - T_{dew})_{n/ref} - (T_{min} - T_{dew})_{ref}$

T_{min} = minimum daily temperature [°C]
T_{max} = maximum daily temperature [°C]
T_{dew} = dew point temperature [°C]
cor = corrected value
obs = observed value

VAPOUR PRESSURE DEFICIT, VPD [kPa]

$$VPD = (e_s - e_a)$$

$$e_T = 0.6108 \exp\left(\frac{17.27\,T}{T + 237.3}\right)$$

SATURATED VAPOUR PRESSURE, e_S [kPa]

Saturated vapour pressure is given by

however, when calculating for 24 hour time periods minimum and maximum daily temperatures should be used:

$$e_s = \frac{e_{(T_{\max})} + e_{(T_{\min})}}{2}$$

Where:

$$e_{(T_{\min})} = 0.6108 \exp\left(\frac{17.27\, T_{\min}}{T_{\min} + 237.3}\right)$$

$$e_{(T_{\max})} = 0.6108 \exp\left(\frac{17.27\, T_{\max}}{T_{\max} + 237.3}\right)$$

Actual vapour pressure where Tdew is available, ea [kPa]

$$e_a = 0.6108 \exp\left(\frac{17.27\, T_{dew}}{T_{dew} + 237.3}\right)$$

Actual vapour pressure where RH is available, ea [kPa]

$$e_a = \frac{RH_{mean}}{100}\left[\frac{e_{(T_{\max})} + e_{(T_{\min})}}{2}\right]$$

Where:
RH_{mean} = mean relative humidity [%]

Actual vapour pressure where only Tmin is available, e_a [kPa]

The equation assumes that the dew point temperature (T_{dew}) is near the minimum temperature (T_{min}). This assumes that at sunrise when the air is close to Tmin, the air is nearly saturated with water vapour and relative humidity is near 100%.

This assumption holds for locations where the cover crop is well watered.
For arid regions the air may not be saturated when the temperature is Tmin. Subtracting 2 to 3 °C from Tmin would better approximate the value used for Tmin in the equation. Replace T_{dew} with (T_{min}- K_o).

$$e_a = 0.6108 \exp\left[\frac{17.27\,(T_{\min} - K_o)}{(T_{\min} - K_o) + 237.3}\right]$$

Where:
K_o = 0 °C for humid and sub-humid climates
K_o = 2 °C for arid and semi-arid climates.

WIND SPEED

Where wind speed data is available, u_2 [m/s]
Meteorological anemometers record information at 10m from the ground surface.

$$u_2 = u_y \left[\frac{4.87}{\ln(67.8\, y - 5.42)} \right]$$

Agrometerological anemometers are located 2-3 m above the ground. The wind information can be adjusted if the height of the instrument is known. (FAO, p55-56)

Where:
y = height of instrument above ground [m]
u_y = measured wind speed at y meters above ground level [m s^{-1}]
u_2 = wind speed at 2 m above ground [m s^{-1}]

Where <u>wind speed data is not available,</u> u_2 [m/s]
Where wind data in not available, use a value that is greater than or equal to 0.5 m/s in the ET_o equation.

The average worldwide value used as a temporary estimate is **2 m/s**.

<u>NET RADIATION, RN [MJ M^{-2} D^{-1}]</u>

$$R_n = R_{ns} - R_{nl}$$

Where:
R_{ns} = net solar radiation [MJ m^{-2} d^{-1}]
R_{nl} = net long wave radiation [MJ m^{-2} d^{-1}]

Net solar radiation, R_{ns} [MJ m^{-2} d^{-1}]

$$R_{ns} = (1 - \alpha)\, R_s$$

Where:
R_s = solar radiation [MJ m^{-2} d^{-1}], see section below
α = albedo or canopy reflection coefficient which is 0.23 for the hypothetical reference crop.

Solar Radiation if <u>sunshine hours, n, is not available</u>, R_s [MJ m^{-2} d^{-1}] (p60)
Because the amount of radiation reaching the earth is reflected in the minimum and maximum temperatures the following equation can be use to estimate solar radiation. It is not appropriate to import radiation data from other stations because to the difference in exposures due to the mountainous and coastal areas.
For island conditions using only temperature to estimate Rs is not appropriate due to the effects of the surrounding water body.

$$R_s = K_{Rs} \sqrt{(T_{max} - T_{min})}\, R_a$$

Where:
R_s = Solar radiation [MJ m^{-2} d^{-1}],
R_a = extraterrestrial radiation [MJ m^{-2} d^{-1}], see section below
K_{Rs} = 0.16 for interior locations
K_{Rs} = 0.19 for coastal locations

Solar Radiation <u>if sunshine hours, n, is available</u>, R_s [MJ m^{-2} d^{-1}] (p50)

$$R_s = \left(0.25 + 0.50\,\frac{n}{N}\right) Ra$$

Where:
n = sunshine hours recorded [hour]
R_a = extra terrestrial radiation, [MJ m^{-2} d^{-1}]
N = available duration of sunshine hours [hour]

There are lookup tables referenced by month and degrees latitude (FAO 1998, p220) to determine N. Interpolation of the table is required.

Or calculate:

$$N = \frac{24}{\pi}\,\omega_s$$

Where:
ω_s = sunset hour angle, see below for complete calculation [rad]
R_a = extra terrestrial radiation, [MJ m^{-2} d^{-1}]

Extra terrestrial radiation, R_a [MJ m^{-2} d^{-1}]

Use lookup table referenced by month and degrees latitude (FAO 1998p219), to determine extraterrestrial. Interpolation of the table is required.

Or calculate:

$$R_a = \frac{24 \times 60}{\pi}\,G_{sc}\,d_r\,\left[\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)\right]$$

Where:
G_{sc} = 0.082 [MJ m^{-2} min^{-1}], solar constant
d_r = inverse relative distance Earth-Sun

$$d_r = 1 + 0.033\cos\left(\frac{2\,\pi}{365}\,J\right)$$

ω_s = sunset hour angle [rad]

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right]$$

φ = latitude [rad]

$$\varphi(rad) = \frac{\pi}{180}\, lat(°)$$

δ = solar declination [rad]

$$\delta = 0.409 \sin\left(\frac{2\,\pi}{365}\, J - 1.39\right)$$

J = Julian day

$$J = \text{int}\left[275\left(\frac{M}{9}\right) - 30 + D\right] - 2$$

Where:
M = month
D = day

(note: 24 x 60 x G_{sc}/3.14 = 37.6)
(note: 2 x 3.14/365 = 0.0172)

Clear Sky Solar Radiation, R_{so} [MJ m^{-2} d^{-1}]

$$R_{so} = \left[0.75 + 2 \times 10^{-5}\, z\right] R_a$$

Where:
z = elevation of climate station above sea level [m]

Net longwave radiation, R_{nl} [MJ m^{-2} d^{-1}]

$$R_{nl} = \sigma\, \frac{(T_{\max} + 273.16)^4 + (T_{\min} + 273.16)^4}{2} \left(0.34 - 0.14\sqrt{e_a}\right)\left[1.35\, \frac{Rs}{R_{so}} - 0.35\right]$$

Where all variables are calculated above or known.
σ = 4.903x10^{-9} MJ$K^{-4}$$m^{-2}$$day^{-1}$

Slope vapour pressure curve, Δ [kPa °C^{-1}]

$$\Delta = 4098 \left[\frac{0.6108 \exp\left(\frac{17.27\,T}{T + 237.3} \right)}{(T + 237.13)^2} \right]$$

Where:
T = mean temperature, [°C]

PSYCHOMETRIC CONSTANT, γ [kPa/°C]

$$\gamma = 0.665 \times 10^{-3}\ P$$

Where:
P = atmospheric pressure, [kPa]

$$P = 101.3 \left(\frac{293 - 0.0065\,z}{293} \right)^{5.26}$$

Where:
z = elevation above sea level, [m]

SOIL HEAT FLUX DENSITY , G

Ignored for daily calculations

PROCEDURE FOR CALCULATING ET_O USING FAO PENMAN-MONTEITH WITH ONLY MINIMUM AND MAXIMUM TEMPERATURE

DATA REQUIRED:

Elevation, metres [m]
Latitude, degrees [^{o}]
Minimum Temperature, degree Celsius [°C]
Maximum Temperature, degree Celsius [°C]
Classification as Coastal or Interior
Classification as Arid of Humid
Julian day

DATA ASSUMED OR CONSTANTS:

Wind speed	2 m/s
Albedo or canopy reflection coefficient, α	0.23
Solar constant, G_{sc}	0.082 $MJ^{-2}min^{-1}$
Interior and Coastal coefficients, K_{Rs}	0.16 for interior locations
	0.19 for coastal locations
Humid and arid region coefficients, K_o	0 °C for humid / sub-humid climates
	2 °C for arid / semi-arid climates

PROCEDURE:

1. Calculate mean air temperature, T [°C]

$$T = \left(\frac{T_{\min} + T_{\max}}{2} \right)$$

2. Calculate actual vapour pressure, e_a [kPa]
   Use minimum temperature and adjustment factor depending on climate classification humid or semi-arid.

$$e_a = 0.6108 \ \exp \left[\frac{17.27\ (T_{\min} - K_o)}{(T_{\min} - K_o) + 237.3} \right]$$

where:

K_o = 0 °C for humid and sub-humid climates
K_o = 2 °C for arid and semi-arid climates

Stations are classified as coastal and interior, interior stations are considered semi-arid, while coastal stations are considered to be humid.

3. Calculate saturated vapour pressure for T_{max}, $e_{(T_{max})}$ [kPa]

$$e_{(T_{max})} = 0.6108 \ \exp\left[\frac{17.27 \ T_{max}}{T_{max} + 237.3}\right]$$

4. Calculate saturated vapour pressure for T_{min}, $e_{(T_{min})}$ [kPa]

$$e_{(T_{min})} = 0.6108 \ \exp\left[\frac{17.27 \, T_{min}}{T_{min} + 237.3}\right]$$

5. Calculate saturated vapour pressure, e_s [kPa]

$$e_s = \left(\frac{e_{(T_{min})} + e_{(T_{max})}}{2}\right)$$

where:

$e_{(T_{max})}$ = Step 3
$e_{(T_{min})}$ = Step 4

6. Calculate inverse relative distance Earth-Sun, d_r [rad]

$$d_r = 1 + 0.033 \ \cos\left(\frac{2\,\pi}{365} \ J\right)$$

where:

J = Julian day

7. Convert latitude to radians, φ [rad]

$$\varphi(rad) = \frac{\pi}{180} \, lat(°)$$

where:

lat = latitude of station in degrees

8. Calculate solar declination, δ [rad]

$$\delta = 0.409 \sin\left(\frac{2\,\pi}{365} \ J - 1.39\right)$$

where:

J = Julian day

9. Calculate sunset hour angle, ω_s [rad]

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right]$$

where:
 δ = Step 7
 φ = Step 8

10. Calculate extraterrestrial radiation, R_a [$MJm^{-2}\ day^{-1}$]

$$R_a = \frac{24 \times 60}{\pi} G_{sc}\ d_r \left[\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)\right]$$

where:
 d_r = Step 6
 φ = Step 7
 δ = Step 8
 ω_s = Step 9
 G_{sc} = solar constant = 0.082 $MJm^{-2}\ min^{-1}$

11. Calculate clear sky solar radiation, R_{so} [$MJm^{-2}\ day^{-1}$]

$$R_{so} = \left(0.75 + 2 \times 10^{-5} z\right) R_a$$

where:
 Z = elevation of climate station above sea level [m]
 R_a = Step 10

12. Calculate solar radiation, R_s [$MJm^{-2}\ day^{-1}$]
Use adjustment factor K_{Rs} depending on station location, coastal or interior

$$R_s = K_{Rs}\ \sqrt{(T_{\max} - T_{\min})}\ R_a$$

where:
 K_{Rs} = 0.16 for interior locations
 K_{Rs} = 0.19 for coastal locations

13. Calculate net longwave radiation, R_{nl} [$MJm^{-2}\ day^{-1}$]

$$R_{nl} = \sigma\ \frac{(T_{\max} + 273.16)^4 + (T_{\min} + 273.16)^4}{2} \left(0.34 - 0.14\sqrt{e_a}\right)\left(1.35\frac{R_s}{R_{so}} - 0.35\right)$$

where:
 e_a = Step 2
 R_s = Step 12
 R_{so} = Step 11
 σ = 4.903 x 10^{-9} $MJK^{-4}m^{-2}day^{-1}$

14. Calculate net solar radiation, R_{ns} [MJm^{-2} day^{-1}]

$$R_{ns} = (1-\alpha)\ R_s$$

where:

R_s = Step 12
α = 0.23

15. Calculate net radiation, R_n [MJm^{-2} day^{-1}]

$$R_n = R_{ns} - R_{nl}$$

where:

R_{ns} = Step 14
R_{nl} = Step 13

16. Calculate slope vapour pressure, Δ [kPa $°C^{-1}$]

$$\Delta = \frac{2504\ \exp\left(\frac{17.27\,T}{T+237.3}\right)}{\left(T+237.3\right)^2}$$

17. Calculate atmospheric pressure, P [kPa]

$$P = 101.3\ \left(\frac{293-0.0065\,z}{293}\right)^{5.26}$$

where:

z = elevation above sea level [m]

18. Calculate psychometric constant, γ [kPa $°C^{-1}$]

$$\gamma = 0.665\times10^{-3}\,P$$

where:

P = Step 17

19. Calculate evapotranspiration, ET_o

$$ET_o = \left[\frac{0.408\,\Delta\,R_n + \gamma\ \left(\frac{900}{T+273}\right)\ u_2\ (e_s - e_a)}{\Delta + \gamma\ \ (1+0.34\,u_2)}\right]$$

PROCEDURE FOR CALCULATING ETO USING FAO PENMAN-MONTEITH WITH MINIMUM AND MAXIMUM TEMPERATURE AND SUNSHINE HOURS

DATA REQUIRED:

Elevation, metres [m]
Latitude, degrees [^{o}]
Minimum Temperature, degree Celsius [°C]
Maximum Temperature, degree Celsius [°C]
Sunshine hours, hour [hr]
Classification as Coastal or Interior
Classification as Arid of Humid
Julian day

DATA ASSUMED OR CONSTANTS:

Wind Speed	2 m/s
Albedo or canopy reflection coefficient, α	0.23
Solar constant, G_{sc}	0.082 $MJ^{-2}min^{-1}$
Humid and arid region coefficients, K_o	0 °C for humid / sub-humid climates
	2 °C for arid / semi-arid climates

Use the Same Procedure as out lined for Calculating ET using only temperature data. Step 12, the calculation of solar radiation, with the following calculation.

12. Calculate solar radiation, R_s [MJm^{-2} day^{-1}]

$$R_s = \left(0.25 + 0.50\frac{n}{N}\right)R_a$$

Where:
n = sunshine hours recorded [hour]
R_a = extra terrestrial radiation, [MJ m^{-2} d^{-1}]
N = available duration of sunshine hours [hour]

There are lookup tables referenced by month and degrees latitude (FAO 1998, p220) to determine N. Interpolation of the table is required.

Or calculate:

$$N = \frac{24}{\pi} \times \omega_s$$

Where:
ω_s = sunset hour angle, see below for complete calculation [rad]
R_a = extra terrestrial radiation, [MJ m^{-2} d^{-1}]