

## Greek Characters

Name	Symbol	Typical use(s)
alpha	$\alpha$	angle, constant
beta	$\beta$	angle, constant
gamma	$\gamma$	angle, constant
delta	$\delta$	limit definition
epsilon	$\epsilon$ or $\varepsilon$	limit definition
theta	$\theta$ or $\vartheta$	angle
pi	$\pi$ or $\pi$	circular constant
phi	$\phi$ or $\varphi$	angle, constant

## Named Sets

empty set	$\emptyset$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

## Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

## Intervals

For numbers  $a$  and  $b$ , we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

## Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\Rightarrow$
equivalent	$\equiv$
iff	$\Leftrightarrow$
for all	$\forall$
there exists	$\exists$

## Exponents

For  $a, b > 0$ ,  $x \in \mathbf{R}$ , and  $m, n$  real:

$$a^0 = 1, \quad 0^a = 0$$

$$1^a = 1, \quad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \quad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \quad (a/b)^m = a^m / b^m$$

$$\sqrt{x^2} = |x|$$

## Trigonometric Identities

$$(\cos(x))^2 + (\sin(x))^2 = 1$$

$$2(\cos(x))^2 = 1 + \cos(2x)$$

$$2(\sin(x))^2 = 1 - \cos(2x)$$

$$(\cos(x))^2 - (\sin(x))^2 = \cos(2x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\operatorname{arccot}(x) = \pi/2 - \arctan(x), \quad \operatorname{dom}(\operatorname{arccot}) = (0, \pi)$$

$$\operatorname{arccsc}(x) = \pi/2 + \arcsin(1/x)$$

$$\operatorname{arcsec}(x) = \arccos(1/x)$$

$$\arcsin(x) + \arccos(x) = \pi/2$$

$$\arctan(x) + \operatorname{arccot}(x) = \pi/2$$

## Limits

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

$$\lim_{x \rightarrow \infty} e^x = \infty \quad \lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} \ln(x) = \infty \quad \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

## Derivatives

### Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\operatorname{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\operatorname{arsinh}(x)$	$1/\sqrt{1+x^2}$
$\operatorname{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

## General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1}(x)$	$1/f'(f^{-1}(x))$

## Antiderivatives

$$\int a \, dx = ax$$

$$\int x^a \, dx = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x|$$

$$\int \cos(x) \, dx = \sin(x)$$

$$\int \sin(x) \, dx = -\cos(x)$$

$$\int \tan(x) \, dx = \ln|\sec(x)|$$

$$\int \sec(x) \, dx = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, dx = \ln|\sin(x)|$$

$$\int 2|x| \, dx = x|x|$$

## Sums

For  $n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

$$\sum_{k=0}^{n-1} k^3 = \frac{(n-1)^2 n^2}{4}$$

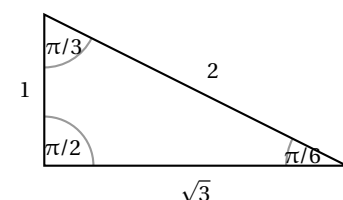
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

## Logarithms

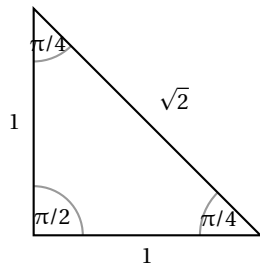
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

## Famous Triangles

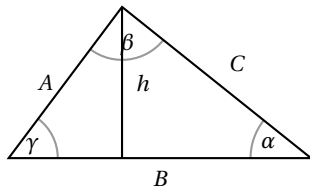
### The 30-60-90 triangle



## The 45-45-90 triangle



## Laws of Cosine & Sine



**Law of cosine:**  $C^2 = A^2 + B^2 - 2AB \cos(\gamma)$

**Law of sines:**  $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

**Area:**  $\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB \sin(\gamma)$

## Hyperbolic Functions

$$2 \cosh(x) = \exp(x) + \exp(-x)$$

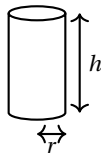
$$2 \sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = \cosh(x)/\sinh(x)$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

## Volumes

### Right Circular Cylinder

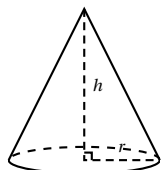


**Volume:**  $V = \pi r^2 h$

**Area:** (not including circular ends)

$$A = 2\pi r h$$

### Cone



**Volume:**  $V = \frac{1}{3}\pi r^2 h$

**Area** (not including circular base)

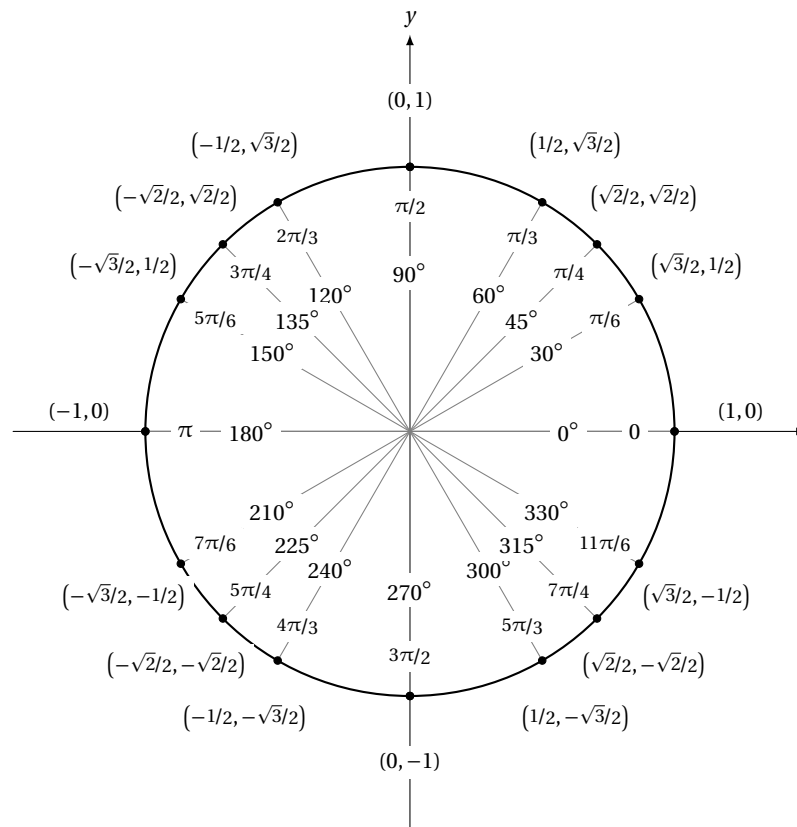
$$A = \pi r \sqrt{r^2 + h^2}$$

### Sphere

**Area:**  $A = 4\pi r^2$

**Volume:**  $V = \frac{4\pi}{3}r^3$

## Unit Circle



## Applications

Arclength of curve  $y = f(x)$  with  $a \leq x \leq b$

$$= \int_a^b \sqrt{1 + f'(x)^2} dx$$

For the region  $Q$  of the  $xy$  plane given by

$$Q = \{(x, y) \mid f(x) \leq y \leq g(x) \wedge a \leq x \leq b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x) dx$$

Assuming  $0 \leq f(x)$  and rotating about the  $x$ -axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2 dx$$

Assuming  $0 \leq a < b$  and rotating about the  $y$ -axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x)) dx$$

Centroid

$$\text{Area}(Q) \times \bar{x} = \int_a^b x(g(x) - f(x)) dx$$

$$\text{Area}(Q) \times \bar{y} = \frac{1}{2} \int_a^b (g(x)^2 - f(x)^2) dx$$

For the region  $Q$  of the  $xy$  plane given by

$$Q = \{(x, y) \mid f(y) \leq x \leq g(y) \wedge a \leq y \leq b\},$$

interchange  $x$  and  $y$  in *all* the previous formulas.

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