Week 4 Friday Work

1. Find the value of each limit. Fully justify your work.

   (a) $\lim_{x \to \pi} 4\lfloor x \rfloor$

   (b) $\lim_{x \to 2/3} \dfrac{x^2+1}{x^2-10}$

   (c) $\lim_{x \to 23} \cos(x)$

   (d) $\lim_{x \to 0} \left(\cos(1/x)^2 + \sin(1/x)^2\right)$

(e) $\lim_{x \to 2} \left(\cos(x^2 + x + 1)\right)$

2. Find each derivative

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[2x^2 + 31x + 107\right]$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{2}x - \sqrt{2x}\right]$

(c) $\frac{\mathrm{d}}{\mathrm{d}x}[(x-5)(x-7)]$

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x-1}{x}\right]$

3. Sketch a graph of $y = \begin{cases} x/20 & x < 20 \\ 1 & x \geq 20 \end{cases}$.

   Find a formula for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Rule #0 (polynomial) Every polynomial is continuous everywhere.

Rule #1 (rational) Every rational function is continuous everywhere it's defined.

Rule #2 Each of the following functions are continuous everywhere they are defined: power (both integer and noninteger powers), trigonometric, inverse trigonometric, exponential, and logarithmic.

Rule #3 Let F and G be functions that are continuous at c and let a, b be numbers and let n be a positive integer; each of the following are continuous at c:

$aF + bG$

FG

F/G (provided $G(c) \neq 0$)

F^n

$F^{1/n}$ (provided $F1/n$ is defined on a neighborhood of c)

Rule #4 Let G be continuous at c and let F be continuous at $F(c)$. Then $F \circ G$ is continuous at c.