

In class work 7 has questions 1 through 3 with a total of 6 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 5 October at 13:15 PM*.

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ evaluated at $(x = 1/\sqrt{2}, y = 1/\sqrt{2})$ given $x^2 + y^2 = 1$.

Solution:

$$\begin{aligned}\frac{d}{dx} [x^2 + y^2 = 1] &= \left[2x + 2y \frac{dy}{dx} = 0 \right], && \text{(additive and chain rules)} \\ &= \left[\frac{dy}{dx} = -\frac{x}{y} \right]. && \text{(algebra)}\end{aligned}$$

Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives $\frac{dy}{dx} \Big|_{x=1/\sqrt{2}, y=1/\sqrt{2}} = -1$.

To find the second derivative, we differentiate $\frac{dy}{dx} = -\frac{x}{y}$. This gives

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[-\frac{x}{y} \right] = -\frac{y - xy'}{y^2}.$$

Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives

$$\frac{d^2y}{dx^2} \Big|_{x=1/\sqrt{2}, y=1/\sqrt{2}} = -\frac{1/\sqrt{2} + 1/\sqrt{2}}{1/2} = -2\sqrt{2}.$$

- 1 2. The equation $xy = y - 1 + e^{-y}$ defines¹ y as a function of x . Find a formula for $\frac{dy}{dx}$.

Solution: We have

$$\begin{aligned}\frac{d}{dx} [xy = y - 1 + e^{-y}] &= [y + xy' = y' - y'e^{-y}], \\ &= [y = (1 - x - e^{-y})y'], \\ &= \left[y' = \frac{y}{1 - x - e^{-y}} \right].\end{aligned}$$

¹This problem is motivated by an unpublished mathematical model of hemoglobin glycation.

FYI It's easy to solve the equation $xy = y - 1 + e^{-y}$ for x . The solution is $x = \frac{y-1+e^{-y}}{y}$. And asking our favorite graphing tool to graph this shows that when x is the dependent variable and y the independent variable, the graph is one-to-one. The one-to-one property tells us that if we solved $xy = y - 1 + e^{-y}$ for y , there would be only one solution for each x . Which means that indeed the equation $xy = y - 1 + e^{-y}$ defines y as a function of x .

But solving $xy = y - 1 + e^{-y}$ for y involves an obscure trick, so we won't go there.

3. Find a formula for each derivative

1 (a) $\frac{d}{dx} [\ln(x(x-1))]$

Solution:

$$\frac{dy}{dx} = \frac{(x(x-1))'}{x(x-1)} = \frac{2x-1}{(x-1)x} = \frac{1}{x} + \frac{1}{x-1}$$

1 (b) $\frac{d}{dx} [\tan^{-1}(x^2)]$

Solution:

$$\frac{dy}{dx} = \frac{2x}{1+(x^2)^2} = \frac{2x}{x^4+1}.$$

1 (c) $\frac{d}{dx} [\csc^{-1}(1/x^2)]$

Solution:

$$\frac{dy}{dx} = -\frac{(1/x^2)'}{\sqrt{(1/x^2)^2-1} |1/x^2|} = \frac{2x}{\sqrt{1-x^4}}$$

1 (d) $\frac{d}{dx} [x \tan^{-1}(x)]$

Solution:

$$\frac{dy}{dx} = \tan^{-1}(x) + \frac{x}{x^2+1}$$

1 (e) $\frac{d}{dx} [\cot^{-1}(x) + \tan^{-1}(x)]$

Solution:

$$\frac{dy}{dx} = -\frac{1}{1+x^2} + \frac{1}{1+x^2} = 0.$$