Week 3 Friday Work

1. Find the value of each limit:

(a) $\lim\limits_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

Solution: We're looking at the limit from the left toward 1. That allows us to simplify $\begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ to 3. Thus

$$\lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} = \lim_{x\to 1^{(-)}} 3 \qquad \text{(simplification)}$$
$$= 3 \qquad \text{(limit of constant)}$$

(b) $\lim\limits_{x\to 1^{(+)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

Solution: We're looking at the limit from the right toward 1. That allows us to simplify $\begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ to x. Thus

$$\lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} = \lim_{x\to 1^{(-)}} x \qquad \text{(simplification)}$$
$$= 1 \qquad \text{(limit of constant))}$$

(c) $\lim\limits_{x\to 1} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

Solution: From parts 'a' and 'b', we have $\lim\limits_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} \neq \lim\limits_{x\to 1^{(+)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$, so $\lim\limits_{x\to 1} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ does not exist (aka dne)

(d) $\lim\limits_{x\to 1} \begin{cases} 3 & x<10 \\ \ln(x^x+1)\sin(1/x) & 10\le x \end{cases}$

Solution: The limit point is 1. For x near the limit point, we can simplify $\lim_{x\to 1}\begin{cases}3 & x<10\\ \ln(x^x+1)\sin(1/x) & 10\le x\end{cases}$ to 3. Thus

$$\lim_{x\to 1}\begin{cases}3 & x<10\\ \ln(x^x+1)\sin(1/x) & 10\le x\end{cases}=\lim_{x\to 1}3=3.$$

(e) $\lim_{x\to 5}\dfrac{\sqrt{x+2}-\sqrt{7}}{x-5}$

Solution: Direct substitution is not an option. To start, let's do some tricky algebra:

$$\begin{aligned}\frac{\sqrt{x+2}-\sqrt{7}}{x-5}&=\frac{\sqrt{x+2}-\sqrt{7}}{x-5}\times\frac{\sqrt{x+2}+\sqrt{7}}{\sqrt{x+2}+\sqrt{7}},\\&=\frac{x+2-7}{(x-5)(\sqrt{x+2}+\sqrt{7})},\\&=\frac{1}{\sqrt{x+2}+\sqrt{7}}.\end{aligned}$$

(f) $\lim_{x\to\pi}\dfrac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

Solution:

(g) $\lim_{x\to 3}\dfrac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

Solution:

(h) $\lim_{x\to\sqrt{107}}\dfrac{x}{|x|}$

Solution:

(i) $\lim_{x\to-\sqrt{107}}\dfrac{x}{|x|}$

Solution: