

In class work 8 has questions 1 through 3 with a total of 4 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 11 October at 13:15 P.M.*

1. The quantities  $x$  and  $y$  both depend on time  $t$ . Suppose that  $y = \tan(x)$ . Further, suppose that at some moment, we have

$$x = \frac{\pi}{3} \text{ and } \frac{dy}{dt} = 42.$$

At this moment, find  $y$  and  $\frac{dx}{dt}$ .

**Solution:** We have

$$\frac{d}{dt} [y = \tan(x)] = \left[ \frac{dy}{dt} = \sec(x)^2 \frac{dx}{dt} \right].$$

Pasting in the data into the undifferentiated and differentiated equations, gives

$$y = \tan\left(\frac{\pi}{3}\right), \quad 42 = \sec\left(\frac{\pi}{3}\right)^2 \frac{dx}{dt}.$$

So

$$y = \sqrt{3} \text{ and } \frac{dx}{dt} = \frac{21}{2}.$$

2. Assume a circular pancake. Buckwheat pancake batter is spreading out on a hot cast iron skillet. At some moment, the radius of the pancake is 4 inches and the rate of change of its radius is  $1/20$  inch per second. At this moment, find the rate of change of the area of the buckwheat pancake.

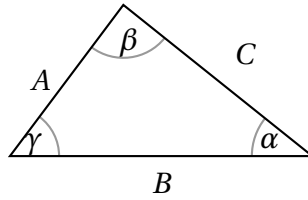
**Solution:** Let  $A$  be the area. We have  $A = \pi r^2$ . So

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Pasting in the data yields

$$\frac{dA}{dt} = 2\pi \times 4 \times \frac{1}{20} = \frac{2\pi}{5} \text{ in}^2/\text{sec}.$$

3. For the labeled triangle shown below, assume that the quantities  $A, B, C, \alpha, \beta$ , and  $\gamma$  all depend on time  $t$ .



At some moment, suppose

$$A = 1, \quad B = 1, \quad C = 1, \quad \frac{dA}{dt} = 8, \quad \frac{dB}{dt} = 3, \quad \frac{dC}{dt} = 2.$$

- 1 (a) At this moment, find  $\frac{d\alpha}{dt} = 2$ .
- 1 (b) At this moment, find the rate of change of the *area* of the triangle.

**Solution:** Let's use the law of cosines:

$$A^2 = B^2 + C^2 - 2BC \cos(\alpha).$$

Differentiating gives

$$2AA' = 2BB' + 2CC' - 2B'C \cos(\alpha) - 2BC' \cos(\alpha) + 2BC \sin(\alpha) \alpha'.$$

For the given data, the triangle is equilateral. So  $\alpha = \pi/3$ . Pasting in this data and solving for  $\alpha'$  gives  $\alpha' = \frac{11}{\sqrt{3}}$

Also Area =  $\frac{1}{2}BC \sin(\alpha)$ . So

$$\frac{d\text{Area}}{dt} = \frac{1}{2}B'C \sin(\alpha) + \frac{1}{2}BC' \sin(\alpha) + \frac{1}{2}BC \cos(\alpha) \alpha'.$$

Pasting in the data gives

$$\frac{d\text{Area}}{dt} = \frac{13}{2\sqrt{3}}.$$