

Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	\emptyset
real numbers	\mathbf{R}
ordered pairs	\mathbf{R}^2
integers	\mathbf{Z}
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	\in
subset	\subset
intersection	\cap
union	\cup
set minus	\setminus

Intervals

For numbers a and b , we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

Logic Symbols

Meaning	Symbol
negation	\neg
and	\wedge
or	\vee
implies	\Rightarrow
equivalent	\equiv
iff	\Leftrightarrow
for all	\forall
there exists	\exists

Exponents

For $a, b > 0$, $x \in \mathbf{R}$, and m, n real:

$$a^0 = 1, \quad 0^a = 0$$

$$1^a = 1, \quad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \quad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \quad (a/b)^m = a^m / b^m$$

$$\sqrt{x^2} = |x|$$

Trigonometric Identities

$$\cos(x)^2 + \sin(x)^2 = 1$$

$$2 \cos(x)^2 = 1 + \cos(2x)$$

$$2 \sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\operatorname{arccot}(x) = \pi/2 - \arctan(x)$$

$$\operatorname{arccsc}(x) = \pi/2 + \arcsin(1/x)$$

$$\operatorname{arcsec}(x) = \arccos(1/x)$$

Limits

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

$$\lim_{x \rightarrow \infty} e^x = \infty \quad \lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} \ln(x) = \infty \quad \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x) \tan(x)$
$\csc(x)$	$-\cot(x) \csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\operatorname{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\operatorname{arsinh}(x)$	$1/\sqrt{1+x^2}$
$\operatorname{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1}(x)$	$1/f'(f^{-1}(x))$

Antiderivatives

$$\int a \, dx = ax$$

$$\int x^a \, dx = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x|$$

$$\int \cos(x) \, dx = \sin(x)$$

$$\int \sin(x) \, dx = -\cos(x)$$

$$\int \tan(x) \, dx = \ln|\sec(x)|$$

$$\int \sec(x) \, dx = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, dx = \ln|\sin(x)|$$

$$\int 2|x| \, dx = x|x|$$

Sums

For $k, m, n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

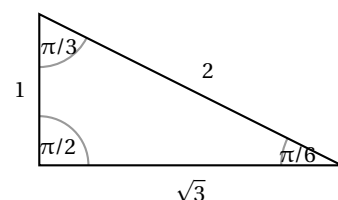
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

Logarithms

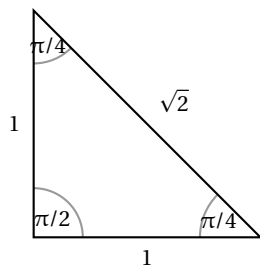
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Famous Triangles

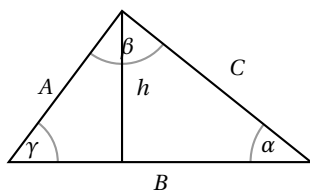
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine: $C^2 = A^2 + B^2 - 2AB \cos(\gamma)$

Law of sines: $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

Area: $\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB \sin(\gamma)$

Hyperbolic Functions

$$2 \cosh(x) = \exp(x) + \exp(-x)$$

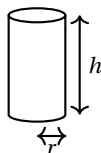
$$2 \sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = \cosh(x)/\sinh(x)$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

Volumes

Right Circular Cylinder

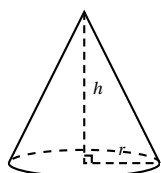


Volume: $V = \pi r^2 h$

Area: (not including circular ends)

$$A = 2\pi r h$$

Cone



Volume: $V = \frac{1}{3}\pi r^2 h$

Area (not including circular base)

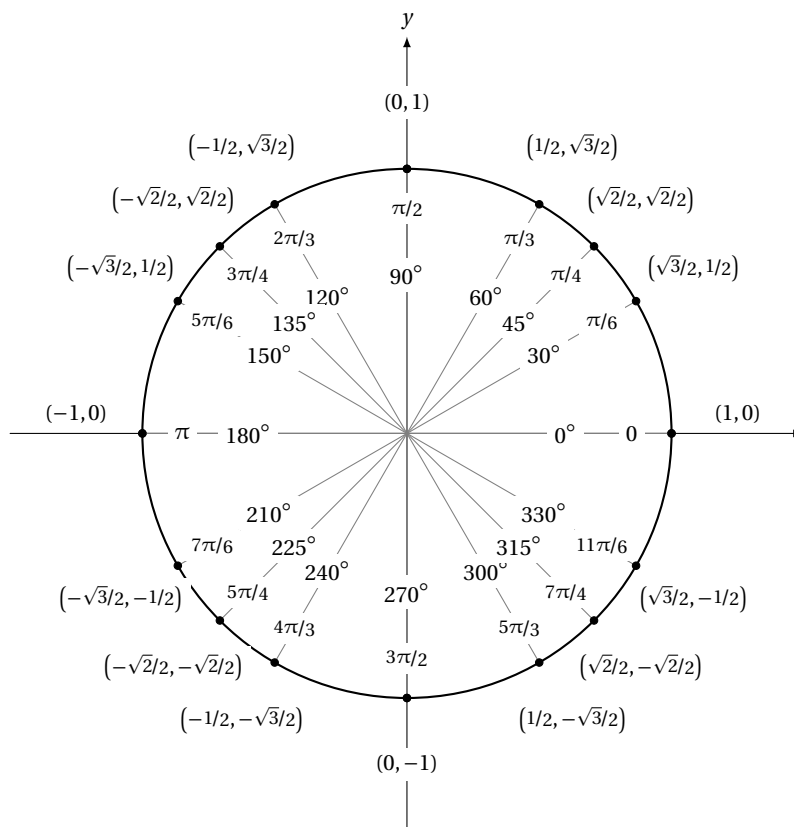
$$A = \pi r \sqrt{r^2 + h^2}$$

Sphere

Area: $A = 4\pi r^2$

Volume: $V = \frac{4\pi}{3}r^3$

Unit Circle



Applications

Arclength of curve $y = f(x)$ with $a \leq x \leq b$

$$= \int_a^b \sqrt{1 + f'(x)^2} dx$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(x) \leq y \leq g(x) \wedge a \leq x \leq b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x) dx$$

Assuming $0 \leq f(x)$ and rotating about the x -axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2 dx$$

Assuming $0 \leq a < b$ and rotating about the y -axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x)) dx$$

Centroid

$$\text{Area}(Q) \times \bar{x} = \int_a^b x(g(x) - f(x)) dx$$

$$\text{Area}(Q) \times \bar{y} = \frac{1}{2} \int_a^b (g(x)^2 - f(x)^2) dx$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(y) \leq x \leq g(y) \wedge a \leq y \leq b\},$$

interchange x and y in *all* the previous formulas.

Revised February 18, 2023. Barton Willis is the author of this work. This work is licensed under Attribution 4.0 International (CC BY 4.0) For the current version of this document, visit