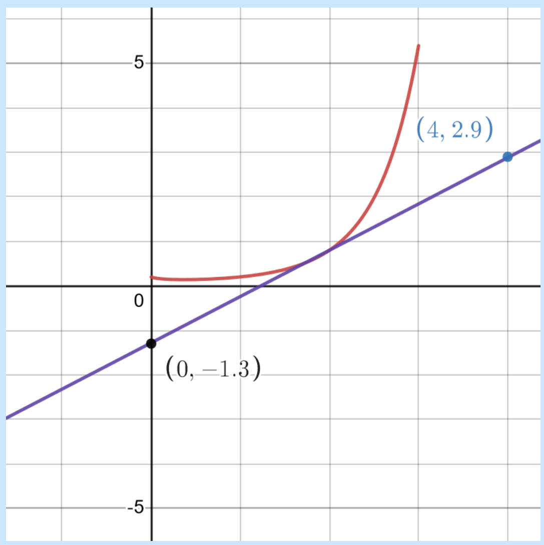
MATH 115 **Name:** ____________________

In class work 4, Fall 2022 **Row and Seat:** ____________________

In class work 4 has questions 1 through 3 with a total of 8 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 14 September 13:15* PM.

$\boxed{1}$ 1. Shown below is a graph of a function Q. Use the graph to estimate the numerical value of $Q'(2)$.

Solution: Your TL might look different–here is mine



Again, your estimated dots on the TL might be different, but using the two labeled dots on the TL, the slope of the TL is

$$\frac{2.9+1.3}{4-0} \approx 1.0.$$

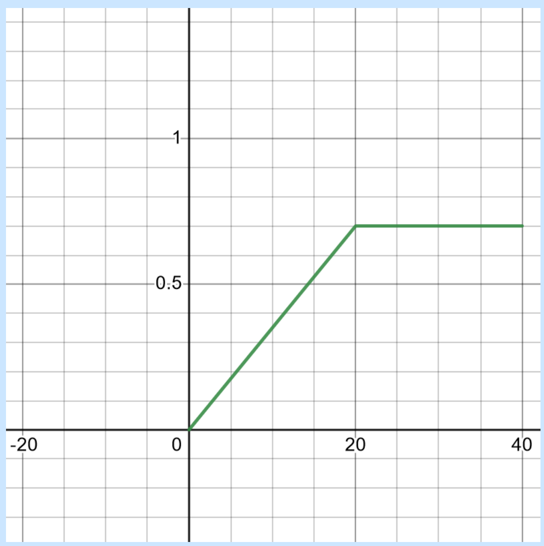
1 2. When a Nebraska judge resigns, the judge receives a lifetime pension of P dollars per year that depends on their ending salary S and their years of service Y. The formula for the pension is

$$P = S\begin{cases} \frac{35}{1000}Y & Y \leq 20 \\ \frac{7}{10} & Y > 20 \end{cases}. \tag{1}$$

1 (a) Using $S = 1$, draw a graph of this function. That is, draw a graph of $P = \begin{cases} \frac{35}{1000}Y & Y \leq 20 \\ \frac{7}{10} & Y > 20 \end{cases}$.

Solution:

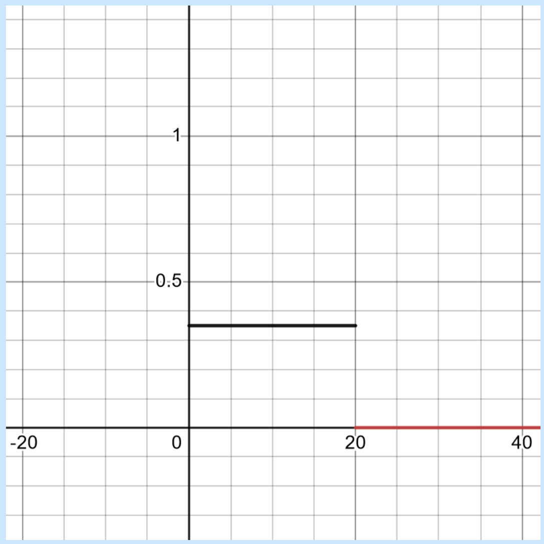
Here is my graph:



$\boxed{1}$ (b) Using the graph, draw a graph of $\frac{\mathrm{d}P}{\mathrm{d}Y}$. Is the function differentiable at $Y = 20$?

Solution:

For $0 < Y < 20$, the graph is a line with slope 0.035. So for the interval $0 < Y < 20$, we have $\frac{\mathrm{d}P}{\mathrm{d}Y} = 0.035$. For $20 < Y$, the graph is a horizontal line with slope 0. So for the interval $0 < Y < 20$, we have $\frac{\mathrm{d}P}{\mathrm{d}Y} = 0$. At $Y = 20$, the graph doesn't have a TL, so the derivative at 20 doesn't exist. A graph of the derivative is



3. Using our list of rules for computing derivatives, find each derivative. *Justify each step by stating which rule you used.*

$\boxed{1}$ (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\cos(2)x^2 + \sin(2)x + 46\right]$

Solution:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left[\cos(2)x^2 + \sin(2)x + 46\right] &= \cos(2)\frac{\mathrm{d}}{\mathrm{d}x}\left[x^2\right] + \sin(2)\frac{\mathrm{d}}{\mathrm{d}x}[x] + \frac{\mathrm{d}}{\mathrm{d}x}[46] && \text{(Rule 1)}\\
&= \cos(2)(2x) + \sin(2) + 0 && \text{(Rule 2)}\\
&= 2\cos(2)x + \sin(2) && \text{(algebra)}
\end{aligned}$$

Since cos(2) and sin(2) are both real numbers, we can use what I call the outative property on them. There is a **huge** difference between $\frac{\mathrm{d}}{\mathrm{d}x}[\cos(x)]$ and $\frac{\mathrm{d}}{\mathrm{d}x}[\cos(2)]$. So be careful with that!

$\boxed{1}$ (b) $\dfrac{d}{dx}\left[\sin(3)x^2 + \ln(107)x\right]$

Solution:

$$\begin{aligned}\frac{d}{dx}\left[\sin(3)x^2 + \ln(107)x\right] &= \sin(3)\frac{d}{dx}\left[x^2\right] + \ln(107)\frac{d}{dx}[x] && \text{(Rule 1)}\\ &= \sin(3)(2x) + \ln(107) && \text{(Rule 2)}\\ &= 2\sin(3)x + \ln(107) && \text{(algebra)}\end{aligned}$$

$\boxed{1}$ (c) $\dfrac{d}{dx}\left[\dfrac{x^2+1}{x}\right]$

Solution: Yikes! We need the derivative of a quotient, but none of our rules match a quotient. What to do? We need to use some algebra to change the expression into something that matches one of our rules:

$$\begin{aligned}\frac{d}{dx}\left[\frac{x^2+1}{x}\right] &= \frac{d}{dx}\left[x + x^{-1}\right] && \text{(algebra)}\\ &= 1 - x^{-2} && \text{(Rule 2)}\end{aligned}$$

$\boxed{1}$ (d) $\dfrac{d}{dx}\left[x^e - e^x\right]$

Solution:

$$\frac{d}{dx}\left[x^e - e^x\right] = ex^{e-1} - e^x. \qquad \text{(Rules 2 and 3)}$$

For any real number a and any real number n, we have

Rule #0 (constant) $\dfrac{d}{dx}[a] = 0.$

Rule #1 (linearity) $\dfrac{d}{dx}[aF(x) + bG(x)] = aF'(x) + bG'(x).$

Rule #2 (power) $\dfrac{d}{dx}\left[x^n\right] = nx^{n-1}$

Rule #3 (natural exponential) $\dfrac{d}{dx}\left[e^x\right] = e^x$