Review for Exam I

1. Larry claims that it is true that $\lim_{x\to\sqrt{7}} \lfloor x \rfloor = \lfloor \sqrt{7} \rfloor$, but Larry can't remember the justification for this calculation. Explain to Larry what function property justifies this calculation. Write your answer in sentence form.
2. Find the value of $\lim_{x\to\pi} (5\lfloor x \rfloor - \lfloor 5x \rfloor)$.
3. Define a function $A(x) = x^2|x|$. Use the definition of the derivative as a limit of a Newton quotient to find the value of $A'(0)$.
4. Find the value of $\lim_{x\to 2^{(-)}} \lfloor x \rfloor$.
5. The *domain* of the natural exponential function is _____.
6. The *range* of the natural exponential function is _____.
7. The *domain* of the natural logarithm function is _____.
8. The *range* of the natural logarithm function is _____.
9. Find an equation of the tangent line (TL) to the curve $y = x(x-4)$. The point of tangency is $(x = 5, y = 5)$.
10. Find an equation of the tangent line (TL) to the curve $y = \mathrm{e}^x$. The point of tangency is $(x = 0, y = 1)$.
11. Find the *natural domain* of the function whose formula is $W(x) = \frac{5}{x} - \frac{x}{5}$.
12. Find the *natural domain* of the function whose formula is $Q(x) = \frac{5}{1-\frac{1}{x}}$.
13. Find each derivative
    (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{107}\right]$
    (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[2x^2 + 31x + 107\right]$
    (c) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{2}x - \sqrt{2x}\right]$
    (d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-5)(x-7)\right]$
    (e) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x-1}{x}\right]$
    (f) $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x+6)(x+8)\right]$
    (g) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x+6}{x+8}\right]$
    (h) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\mathrm{e}^x\right]$

(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2+1}{x^2-1}\right]$

14. Sketch a graph of $y = \begin{cases} x/20 & x < 20 \\ 1 & x \geq 20 \end{cases}$.

    Find a formula for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

15. In the year 1969 at age 11, child actress Eve Plumb purchased a Malibu beach house for \$55,000. Forty-seven years later she sold it for \$3.9 million. Her annual percent yield r on this investment is given by the solution to

    $$3,900,000 = 55,000 \times (1+r)^{47}.$$

    Find Eve Plumb's return on this investment. You will need to solve the given equation for r.

16. After graduation, suppose your starting salary is \$64,000. Further, suppose that you expect to earn a 3.5% pay rise each year you work. What is your salary for your 40^{th} year of work? **Hint:** Your salary for your 3^{rd} year of work is $\$64,000 \times 1.035^2$.

17. Define $Q(x) = x^3 + 1$ and $\mathrm{dom}(Q) = (-\infty, \infty)$. Find the formula and the domain of Q^{-1}.

18. Find the *natural domain* of the function F whose formula is $F(x) = \frac{1}{5+\frac{1}{x}}$

19. Find the value of each limit:

    (a) $\lim\limits_{x\to 0} \dfrac{x|x|}{x}$.

    (b) $\lim\limits_{x\to 1^{(-)}} \begin{cases} 3 & x < 1 \\ x & 1 \leq x \end{cases}$

    (c) $\lim\limits_{x\to 1^{(+)}} \begin{cases} 3 & x < 1 \\ x & 1 \leq x \end{cases}$

    (d) $\lim\limits_{x\to 1} \begin{cases} 3 & x < 1 \\ x & 1 \leq x \end{cases}$

    (e) $\lim\limits_{x\to 1} \begin{cases} 3 & x < 10 \\ \ln(x^x+1)\sin(1/x) & 10 \leq x \end{cases}$

    (f) $\lim\limits_{x\to 5} \dfrac{\sqrt{x+2}-\sqrt{7}}{x-5}$

(g) $\lim_{x\to\pi} \dfrac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

(h) $\lim_{x\to 3} \dfrac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

(i) $\lim_{x\to\sqrt{107}} \dfrac{x}{|x|}$

(j) $\lim_{x\to-\sqrt{107}} \dfrac{x}{|x|}$

20. Find each of the following limits. Use the rules

Rule #0 (constant) $\lim_{x\to c}(a) = a$.

Rule #1 (linearity) $\lim_{x\to c}(aF(x)+bG(x)) = a\lim_{x\to c}(F(x)) + b\lim_{x\to c}(G(x))$.

Rule #2 (product) $\lim_{x\to c}(F(x)G(x)) = \lim_{x\to c}(F(x)) \times \lim_{x\to c}(G(x))$.

Rule #3 (quotient) Provided $\lim_{x\to c}(G(x)) \neq 0$, we have $\lim_{x\to c}\dfrac{F(x)}{G(x)} = \dfrac{\lim_{x\to c}(F(x))}{\lim_{x\to c}(G(x))}$.

Rule #4 (power) $\lim_{x\to c} F(x)^n = \left(\lim_{x\to c} F(x)\right)^n$.

Rule #5 (root) Provided $\left(\lim_{x\to c} F(x)\right)^{1/n}$ is real, $\lim_{x\to c} F(x)^{1/n} = \left(\lim_{x\to c} F(x)\right)^{1/n}$.

Rule #6 (polynomial) Provided F is a polynomial, we have $\lim_{x\to c} F(x) = F(c)$

Rule #7 (rational) Provided F is a rational function and $c \in \mathrm{dom}(F)$, we have $\lim_{x\to c} F(x) = F(c)$.

to justify each of your steps by referencing one of our rules numbered zero through seven.

2 (a) $\lim_{x\to\pi}\left(x^3+x\right)$

2 (b) $\lim_{x\to\sqrt{2}} \sqrt{x+1}$

2 (c) $\lim_{x\to\sqrt{2}} \dfrac{x+1}{x-1}$