MATH 115 **Name:** ____________________

In class work 13, Fall 2022 **Row and Seat**:____________________

In class work 13 has questions 1 through 2 with a total of 9 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 16 November 13:15* PM.

1. Find a formula for each antiderivative.

$\boxed{1}$ (a) $\int (6x+3)(x+1)\,\mathrm{d}x =$

Solution: The integrand is a product. To use the additive property, our first step will be to expand the integrand. We have

$$\begin{aligned}
\int (6x+3)(x+1)\,\mathrm{d}x &= \int 6x^2+9x+3\,\mathrm{d}x, && \text{(expand)}\\
&= \int 6x^2\,\mathrm{d}x + \int 9x\,\mathrm{d}x + \int 3\,\mathrm{d}x, && \text{(additive property)}\\
&= 6\int x^2\,\mathrm{d}x + 9\int x\,\mathrm{d}x + 3\int 1\,\mathrm{d}x, && \text{(outative property)}\\
&= 6\times\frac{1}{3}x^3 + 9\times\frac{1}{2}x^2 + 3x + C, && \text{(power rule)}\\
&= 2x^3 + \frac{9x^2}{2} + 3x + C. && \text{(simplification)}
\end{aligned}$$

$\boxed{1}$ (b) $\int (x-1)(x+2)\,\mathrm{d}x =$

Solution: Our process is much the same as for the first problem. Here is the answer without the steps:

$$\int (x-1)(x+2)\,\mathrm{d}x = \frac{x^3}{3} + \frac{x^2}{2} - 2x + C.$$

$\boxed{1}$ (c) $\int \frac{7}{x} + \frac{x}{7}\,\mathrm{d}x =$

Solution: For the interval $(-\infty, 0)$, we have

$$\int \frac{7}{x} + \frac{x}{7}\,\mathrm{d}x = 7\ln(-x) + \frac{x^2}{14} + C.$$

For the interval $(0,\infty)$, we have

$$\int \frac{7}{x} + \frac{x}{7}\,\mathrm{d}x = 7\ln(x) + \frac{x^2}{14} + C.$$

And for either the interval $(-\infty, 0)$ or $(0, \infty)$, we have

$$\int \frac{7}{x} + \frac{x}{7}\,\mathrm{d}x = 7\ln(|x|) + \frac{x^2}{14} + C.$$

We do need to be careful with the result expressed this way. Since the natural logarithm function is not continuous at zero, this antiderivative is not valid on any interval that contains zero.

$\boxed{1}$ (d) $\int \frac{x+1}{\sqrt{x}}\,\mathrm{d}x =$

Solution:

$$\begin{aligned}\int \frac{x+1}{\sqrt{x}}\,\mathrm{d}x, &= \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,\mathrm{d}x,\\ &= \frac{2x^{\frac{3}{2}}}{3} + 2\sqrt{x} + C.\end{aligned}$$

$\boxed{1}$ (e) $\int \cos(23\pi x)\,\mathrm{d}x =$

Solution: For $a \in \mathbf{R}_{\neq 0}$, we have the rule

$$\int \cos(ax)\,\mathrm{d}x = \frac{1}{a}\sin(ax) + C.$$

Matching to this rule, we have

$$\int \cos(23\pi x)\,\mathrm{d}x = \frac{1}{23\pi}\sin(23\pi x) + C.$$

$\boxed{1}$ (f) $\int \cos(\pi x)^2 + \sin(\pi x)^2\,\mathrm{d}x =$

Solution: Ha! This one is a trick! The integrand simplifies to one. Thus

$$\int \cos(\pi x)^2 + \sin(\pi x)^2\,\mathrm{d}x = \int 1\,\mathrm{d}x = x + C.$$

$\boxed{1}$ (g) $\int 5\mathrm{d}x =$

Solution:

$$\int 5\mathrm{d}x = \frac{5}{2}x^2 + C.$$

Yes, it really is that easy–it was intended to test your confidence. Problem sets are generally arranged from easy to challenging–presented the other way, we can be thrown off. But life isn't always arranged from easy to hard–we need to build our confidence.

$\boxed{1}$ (h) $\int e^{5x} dx =$

Solution: For $a \in \mathbf{R}_{\neq 0}$, we have the rule

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C.$$

Matching to this rule, we have

$$\int e^{5x} dx = \frac{1}{5} e^{5x} + C.$$

$\boxed{1}$ 2. Find numbers a and b such that $\int x e^x \, dx = (a + bx)e^x + C$ is correct. Do this by requiring that $\frac{d}{dx}((a + bx)e^x) = xe^x$ be an identity.

Solution: We need to find numbers a and b such that

$$\frac{d}{dx}\left((a + bx)e^x\right) = xe^x$$

is an identity. Evaluating the derivative on the left yields

$$bx\, e^x + b\, e^x + a\, e^x = x\, e^x.$$

Subtracting left form right and rearranging gives

$$(b - 1)\, x\, e^x + (b + a)\, e^x = 0.$$

Dividing this by e^x (which is nonzero) gives

$$(b - 1)\, x + (b + a) = 0$$

This is supposed to be an *identity*. It is **not** an equation that we are supposed to solve for x. Since it is supposed to be an identity, it means that for *every* real number x, we have $(b - 1)\, x + (b + a) = 0$ How can this be? We *must* choose the numbers a and b such that $b - 1 = 0$ and $a + b = 0$. Thus $b = 1$ and $a = -1$. Thus we are claiming that

$$\int x e^x \, dx = (x - 1)e^x + C.$$

Checking this by differentiation shows that it is correct.