MATH 115 **Name:** ____________________

In class work 7, Fall 2022 **Row and Seat:** ____________________

In class work 7 has questions 1 through 3 with a total of 6 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 5 October at 13:15* PM.

1. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ evaluated at $(x = 1/\sqrt{2}, y = 1/\sqrt{2})$ given $x^2 + y^2 = 1$.

Solution:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x^2 + y^2 = 1\right] = \left[2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0\right].$$

Solving this for $\frac{\mathrm{d}y}{\mathrm{d}x}$ gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}.$$

Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives $\frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=1/\sqrt{2}y=1/\sqrt{2}} = -1.$

To find the second derivative, we differentiate $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$. This gives

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left[-\frac{x}{y}\right] = -\frac{y - xy'}{y^2}.$$

Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\Big|_{x=1/\sqrt{2}y=1/\sqrt{2}} = -\frac{1/\sqrt{2} + 1/\sqrt{2}}{1/2} = -2\sqrt{2}.$$

$\boxed{1}$ 2. The equation $xy = y - 1 + \mathrm{e}^{-y}$ defines[1] y as a function of x. Find a formula for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[1] This problem is motivated by an unpublished mathematical model of hemoglobin glycation.

Solution: We have

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[xy = y - 1 + \mathrm{e}^{-y}\right] = \left[y + xy' = y' - y'\mathrm{e}^{-y}\right],$$
$$= \left[y = (1 - x - \mathrm{e}^{-y})y'\right],$$
$$= \left[y' = \frac{y}{1 - x - \mathrm{e}^{-y}}\right].$$

3. Find a formula for each derivative

$\boxed{1}$ (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x(x-1))\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x-1}{(x-1)\,x} = \frac{1}{x} + \frac{1}{x-1}$$

$\boxed{1}$ (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}(x^2))\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{x^4+1}.$$

$\boxed{1}$ (c) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\csc^{-1}(1/x^2))\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{\sqrt{1-x^4}}$$

$\boxed{1}$ (d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\tan^{-1}(x)\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan^{-1}(x) + \frac{x}{x^2+1}$$

$\boxed{1}$ (e) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\cot^{-1}(x) + \tan^{-1}(x)\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{1+x^2} + \frac{1}{1+x^2} = 0.$$