Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

Intervals

For numbers a and b, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a,b) = \{x \in \mathbf{R} \mid a \le x < b\}$$

$$(a,b] = \{x \in \mathbf{R} \mid a < x \le b\}$$

$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Exponents

For $a, b > 0, x \in \mathbf{R}$, and m, n real:

$$a^0 = 1, \qquad 0^a = 0$$

$$1^a = 1, \qquad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m / b^m$$

$$\sqrt{x^2} = |x|$$

Trigonometric Identities

$$\cos(x)^2 + \sin(x)^2 = 1$$

$$2\cos(x)^2 = 1 + \cos(2x)$$

$$2\sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\operatorname{arccot}(x) = \pi/2 - \arctan(x)$$

$$\operatorname{arccsc}(x) = \pi/2 + \arcsin(1/x)$$

$$\operatorname{arcsec}(x) = \arccos(1/x)$$

Limits

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x \to 0} \frac{1-\cos(x)}{x} = 0$$

$$\lim_{x \to \infty} e^x = \infty \qquad \lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to \infty} \ln(x) = \infty \qquad \lim_{x \to 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/\left(x^2+1\right)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\operatorname{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\operatorname{arcsinh}(x)$	$1/\sqrt{1+x^2}$
$\operatorname{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'\left(g(x)\right)$
$f^{-1\prime}(x)$	$1/f'(f^{-1}(x))$

Antiderivatives

$$\int a \, \mathrm{d}x = ax$$

$$\int x^a \, \mathrm{d}x = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln|x|$$

$$\int \cos(x) \, \mathrm{d}x = \sin(x)$$

$$\int \sin(x) \, \mathrm{d}x = -\cos(x)$$

$$\int \tan(x) \, \mathrm{d}x = \ln|\sec(x)|$$

$$\int \sec(x) \, \mathrm{d}x = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, \mathrm{d}x = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, \mathrm{d}x = \ln|\sin(x)|$$

$$\int 2|x| \, \mathrm{d}x = x|x|$$

Sums

For $k, m, n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)\,n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)\,n\,(2n-1)}{6}$$

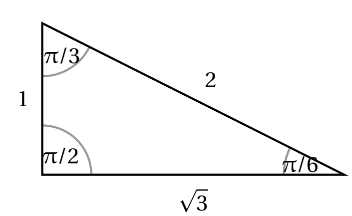
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

Logarithms

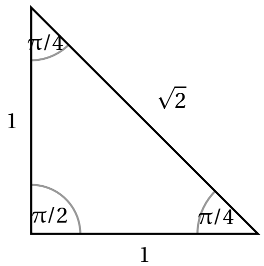
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Famous Triangles

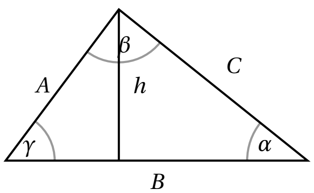
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine: $C^2 = A^2 + B^2 - 2AB\cos(\gamma)$

Law of sines: $\dfrac{\sin(\alpha)}{A} = \dfrac{\sin(\beta)}{B} = \dfrac{\sin(\gamma)}{C}$

Area: $\text{Area} = \dfrac{1}{2}hB = \dfrac{1}{2}AB\sin(\gamma)$

Hyperbolic Functions

$$2\cosh(x) = \exp(x) + \exp(-x)$$

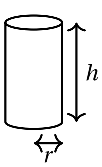
$$2\sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = {\cosh(x)}/{\sinh(x)}$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

Volumes

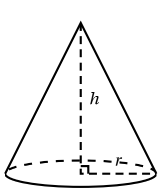
Right Circular Cylinder



Volume: $V = \pi r^2 h$

Area: (not including circular ends) $A = 2\pi r h$

Cone



Volume: $V = \dfrac{1}{3}\pi r^2 h$

Area (not including circular base)

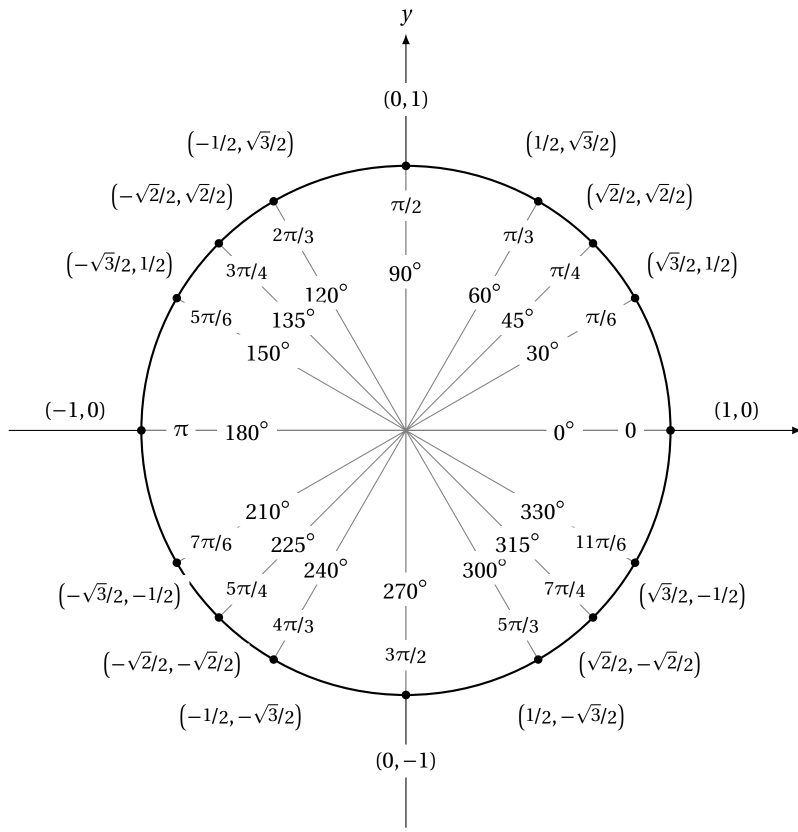
$A = \pi r\sqrt{r^2 + h^2}$

Sphere

Area: $A = 4\pi r^2$

Volume: $V = \dfrac{4\pi}{3} r^3$

Unit Circle



Applications

Arclength of curve $y = f(x)$ with $a \le x \le b$

$$= \int_a^b \sqrt{1 + f'(x)^2}\,\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(x) \le y \le g(x) \wedge a \le x \le b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x)\,\mathrm{d}x$$

Assuming $0 \le f(x)$ and rotating about the x-axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2\,\mathrm{d}x$$

Assuming $0 \le a < b$ and rotating about the y-axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x))\,\mathrm{d}x$$

Centroid

$$\text{Area}(Q) \times \overline{x} = \int_a^b x\left(g(x) - f(x)\right)\mathrm{d}x$$

$$\text{Area}(Q) \times \overline{y} = \frac{1}{2}\int_a^b \left(g(x)^2 - f(x)^2\right)\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(y) \le x \le g(y) \wedge a \le y \le b\},$$

interchange x and y in *all* the previous formulas.