Week 3 Friday Work

1. Find the value of each limit:

   (a) $\lim\limits_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

   Solution: We're looking at the limit from the *left* toward 1. That makes $x<1$, so we can simplify $\begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ to 3. Thus

   $$\begin{aligned} \lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} &= \lim_{x\to 1^{(-)}} 3 && \text{(simplification)} \\ &= 3 && \text{(limit of constant)} \end{aligned}$$

   (b) $\lim\limits_{x\to 1^{(+)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

   Solution: We're looking at the limit from the *right* toward 1. That allows us to simplify $\begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ to x. Thus

   $$\begin{aligned} \lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} &= \lim_{x\to 1^{(-)}} x && \text{(simplification)} \\ &= 1 && \text{(limit of constant))} \end{aligned}$$

   A few of you correctly simplified, but then failed to find the limit–something like

   $$\lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} = x.$$

   OK–there is a pending limit to evaluate. Simplifying is just the first step, but we still need to evaluate the limit.

   (c) $\lim\limits_{x\to 1} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$

Solution: From parts 'a' and 'b', we have $\lim_{x\to 1^{(-)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases} \neq \lim_{x\to 1^{(+)}} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$, so $\lim_{x\to 1} \begin{cases} 3 & x<1 \\ x & 1\le x \end{cases}$ does not exist (aka dne).

(d) $\lim_{x\to 1} \begin{cases} 3 & x<10 \\ \ln(x^x+1)\sin(1/x) & 10\le x \end{cases}$

Solution: The limit point is 1. For x near the limit point, we can simplify $\lim_{x\to 1} \begin{cases} 3 & x<10 \\ \ln(x^x+1)\sin(1/x) & 10\le x \end{cases}$ to 3. The ugly case of $10 \le x$ just "simplifies away" and causes us no trouble! Thus

$$\lim_{x\to 1} \begin{cases} 3 & x<10 \\ \ln(x^x+1)\sin(1/x) & 10\le x \end{cases} = \lim_{x\to 1} 3 = 3.$$

(e) $\lim_{x\to 5} \frac{\sqrt{x+2}-\sqrt{7}}{x-5}$

Solution: Direct substitution is not an option. To start, let's do some tricky algebra:

$$\begin{aligned} \frac{\sqrt{x+2}-\sqrt{7}}{x-5} &= \frac{\sqrt{x+2}-\sqrt{7}}{x-5} \times \frac{\sqrt{x+2}+\sqrt{7}}{\sqrt{x+2}+\sqrt{7}}, \\ &= \frac{x+2-7}{(x-5)(\sqrt{x+2}+\sqrt{7})}, \\ &= \frac{1}{\sqrt{x+2}+\sqrt{7}}. \end{aligned}$$

Now the evaluating the limit, is DS:

$$\lim_{x\to 5} \frac{\sqrt{x+2}-\sqrt{7}}{x-5} = \lim_{x\to 5} \frac{1}{\sqrt{x+2}+\sqrt{7}} = \frac{1}{2\sqrt{7}} = \frac{\sqrt{7}}{14}$$

(f) $\lim_{x\to \pi} \frac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

Solution: This one is not that much different from the previous problem:

$$\lim_{x\to \pi} \frac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi} = \lim_{x\to \pi} \frac{1}{\sqrt{x+\pi}+\sqrt{2\pi}} = \frac{1}{2\sqrt{2\pi}}$$

(g) $\lim_{x\to 3} \frac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi}$

Solution: Ha! This one isn't similar the previous–DS is just fine!

$$\lim_{x\to 3} \frac{\sqrt{x+\pi}-\sqrt{2\pi}}{x-\pi} = \frac{\sqrt{3+\pi}-\sqrt{2\pi}}{3-\pi}.$$

(h) $\lim_{x\to\sqrt{107}} \frac{x}{|x|}$

Solution: Near $\sqrt{107}$, we can simplify $\frac{x}{|x|}$ to 1. So

$$\lim_{x\to\sqrt{107}} \frac{x}{|x|} = \lim_{x\to\sqrt{107}} 1 = 1.$$

(i) $\lim_{x\to -\sqrt{107}} \frac{x}{|x|}$

Solution: Near $-\sqrt{107}$, we can simplify $\frac{x}{|x|}$ to -1. So

$$\lim_{x\to\sqrt{107}} \frac{x}{|x|} = \lim_{x\to\sqrt{107}} (-1) = -1.$$