

1. Find the solution set for the equation $\frac{107}{1+\exp(x)} = 0$.

Solution: Remember that $\left[\frac{a}{b} = 0\right] = [(a = 0) \wedge (b \neq 0)]$. Using this fact, we have

$$\begin{aligned}\left[\frac{107}{1+\exp(x)} = 0\right] &= [(107 = 0) \wedge (1 + \exp(x) \neq 0)] \\ &= \emptyset\end{aligned}$$

2. Inflation is causing the cost of chicken eggs to increase. The cost (in dollars) of a dozen chicken eggs is $C = 1.83 \times 1.052^T$, where T is the number of years after 1 January, 2022. When will chicken eggs cost \$2.00 per dozen?

Solution:

$$\begin{aligned}[2 = 1.83 \times 1.052^T] &= \left[\frac{2}{1.83} = 1.052^T\right], \\ &= \left[\ln\left(\frac{2}{1.83}\right) = T \ln(1.052)\right], \\ &= \left[T = \frac{\ln(\frac{2}{1.83})}{\ln(1.052)}\right], \\ &= [T \approx 1.75 \text{ years}].\end{aligned}$$

3. Find the inverse of the function $W(x) = 5x + 1$ and $\text{dom}(W) = \mathbf{R}$.

Solution:

$$\begin{aligned}[y = 5x + 1, -\infty < x < \infty] &= \left[x = \frac{y-1}{5}, -\infty < \frac{y-1}{5} < \infty\right], \\ &= \left[x = \frac{y-1}{5}, -\infty < y < \infty\right]\end{aligned}$$

So $W^{-1}(y) = \frac{y-1}{5}$ and $\text{dom}(W^{-1}) = \mathbf{R}$.

If y is a real number, so is $\frac{y-1}{5}$. Thus the solution set of $-\infty < \frac{y-1}{5} < \infty$ is $-\infty < y < \infty$.

4. Define $Z(t) = t - 2t^2$. Find

(a) $\text{ARC}_{[1,1.1]}(Z) =$

Solution:

$$\text{ARC}_{[1,1.1]}(Z) = \frac{Z(1.1) - Z(1)}{1.1 - 1} = -3.2$$

(b) $\text{ARC}_{[1,1.01]}(Z) =$

Solution:

$$\text{ARC}_{[1,1.01]}(Z) = \frac{Z(1.01) - Z(1)}{1.01 - 1} = -3.02$$

(c) $\text{ARC}_{[1,1.001]}(Z) =$

Solution:

$$\text{ARC}_{[1,1.001]}(Z) = \frac{Z(1.001) - Z(1)}{1.001 - 1} = -3.002$$

(d) a simplified formula for $\text{ARC}_{[1,b]}(Z) =$

Solution:

We have $(-2b^2 + b + 1) \div (b - 1) = -2b - 1$. Using this easy formula, parts 'a' through 'c' are easy.

If you need a refresher on polynomial division, see
https://www.youtube.com/watch?v=RPXMBIFG_W4