MATH 115 **Name:** _____

In class work 8, Fall 2022 **Row and Seat:** _____

In class work 8 has questions 1 through 3 with a total of 4 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 11 October at 13:15* PM.

[1] 1. The quantities x and y both depend on time t. Suppose that $y = \tan(x)$. Further, suppose that at some moment, we have

$$x = \frac{\pi}{3} \text{ and } \frac{\mathrm{d}y}{\mathrm{d}t} = 42.$$

At this moment, find y and $\frac{\mathrm{d}x}{\mathrm{d}t}$.

Solution: We have

$$\frac{\mathrm{d}}{\mathrm{d}}\left[y = \tan(x)\right] = \left[\frac{\mathrm{d}y}{\mathrm{d}t} = \sec(x)^2 \frac{\mathrm{d}x}{\mathrm{d}t}\right].$$

Pasting in the data into the undifferentiated and differentiated equations, gives

$$y = \tan\left(\frac{\pi}{3}\right), \quad 42 = \sec\left(\frac{\pi}{3}\right)^2 \frac{\mathrm{d}x}{\mathrm{d}t}.$$

So

$$y = \sqrt{3} \text{ and } \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{21}{2}.$$

[1] 2. Assume a circular pancake. Buckwheat pancake batter is spreading out on a hot cast iron skillet. At some moment, the radius of the pancake is 4 inches and the rate of change of its radius is 1/20 inch per second. At this moment, find the rate of change of the area of the buckwheat pancake.

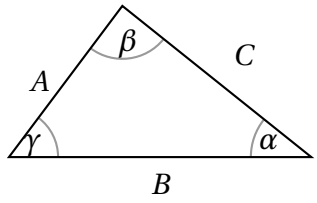
Solution: Let A be the area. We have $A = \pi r^2$. So

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi r \frac{\mathrm{d}r}{\mathrm{d}t}$$

Pasting in the data yields

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi \times 4 \times \frac{1}{20} = \frac{2\pi}{5} \text{in}^2/\text{sec}.$$

3. For the labeled triangle shown below, assume that the quantities A, B, C, α, β, and γ all depend on time t.



At some moment, suppose

$$A = 1, \quad B = 1, \quad C = 1, \quad \frac{\mathrm{d}A}{\mathrm{d}t} = 8, \quad \frac{\mathrm{d}B}{\mathrm{d}t} = 3, \quad \frac{\mathrm{d}C}{\mathrm{d}t} = 2.$$

[1] (a) At this moment, find $\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 2$.

[1] (b) At this moment, find the rate of change of the *area* of the triangle.

Solution: Let's use the law of cosines:

$$A^2 = B^2 + C^2 - 2BC\cos(\alpha).$$

Differentiating gives

$$2AA' = 2BB' + 2CC' - 2B'C\cos(\alpha) - 2BC'\cos(\alpha) + 2BC\sin(\alpha)\alpha'.$$

For the given data, the triangle is equilateral. So $\alpha = \pi/3$. Pasting in this data and solving for α' gives $\alpha' = \frac{11}{\sqrt{3}}$

Also Area $= \frac{1}{2}BC\sin(\alpha)$. So

$$\frac{\mathrm{dArea}}{\mathrm{d}t} = \frac{1}{2}B'C\sin(\alpha) + \frac{1}{2}BC'\sin(\alpha) + \frac{1}{2}BC\cos(\alpha)\alpha'.$$

Pasting in the data gives

$$\frac{d\mathrm{Area}}{\mathrm{d}t} = \frac{13}{2\sqrt{3}}.$$