Calculus I with Analytic Geometry

MATH 115–02

Fall 2022

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Office Hours: Monday, Wednesday, and Friday 10:00 AM – 11:00 AM, Tuesday and Thursday 9:30 AM– 11:00 AM, and by appointment.

Important Dates

First Online Homework due ..... 3 September
Exam 1 ..... 23 September
Exam 2 ..... 4 November
Exam 3 ..... 2 December
Exam 4 ..... 12 December, from 1:00 PM – 3:00 PM
Final exam ..... 15 December, from 10:30 AM – 12:30 PM

Class meeting times

This class meets This class meets Monday, Wednesday, and Friday from 12:20 PM – 1:10 PM and Tuesday and Thursday from 12:30 PM – 1:10 PM in Discovery Hall, room 383.

Grading

Your course grade will be based on weekly in class work, online homework, four midterm exams, and a comprehensive final exam; specifically:

In class work *15 ten point assignments* ..... 150 (total)
Online homework *39 four point assignments* ..... 156 (total)
Mid-term Exams 1,2, and 3 *100 points each* ..... 300 (total)
Exam 4 *25 points* ..... 25 (total)
Comprehensive Final exam ..... 150 (total)

Exam 4, worth 25 points covers the final two weeks of the course and it is given during final exam week. The comprehensive final exam, worth 150 points, is also given during final exam week. If we end the term with slightly fewer or more in class assignments, your in class work score will be proportionally scaled to 150 points. The same scaling for online homework.

The following table shows the *minimum* number of points (out of 781) that are required for each of the twelve letter grades D- through A+. For example, a point total of 677 points will earn you a grade of B+, and a point total of 703 points will earn you a grade of A-. If you earn a point total of 468 or less, you a failing course grade.

D-	469	B-	625
D	494	B	651
D+	521	B+	677
C-	547	A-	703
C	572	A	729
C+	599	A+	765

Course Resources

1. Our textbook is *University Calculus: Early Transcendentals, Single Variable*, 4th Edition, by Joel R. Hass, Christopher E Heil, Przemyslaw Bogacki, Maurice D. Weir, and George B. Thomas, Jr.

2. We will be using the online homework system Pearson MyLab Math. Your online homework grade is a substantial part of your course grade (about 19%). You *must* sign up for the homework system in the first week of the term. If you purchase a used book without an access code, you will may need to purchase access to the online homework system separately.

3. A computer or tablet (not a phone) with an Internet connection to use the online homework.

4. For exams, you will need a scientific calculator (includes trigonometric, logarithmic, and exponential functions). You do not need anything more fancy than that. You *may* use a graphing calculator, but it will not be of any great advantage.

5. You will need a (functioning) camera on your phone or some other device for scanning a document and turning it electronically.

6. The UNK Learning Commons[1] provides peer tutoring for this class.

7. Pencils, erasers, notebook for note taking. Colored pens or pencils are nice for note taking.

8. Other resources include Desmos[2].

Course Calendar

Generally, we'll adhere to the scheduled exam dates even if we are ahead or behind with coursework. When we are ahead or behind, the topics on the exams will be appropriately adjusted.
Notices:

(a) *Exams will be given on the last class day (generally **Friday**) of the week they are assigned.*

(b) Homework (labeled **HW**) will be due one minute before midnight on Saturday of the week they are assigned.

(c) In class work will generally be on Wednesday the week it is assigned.

(d) The final exam will be given on 15 December, from 10:30 AM – 12:30 PM.

[1] `https://www.unk.edu/offices/learning_commons/`
[2] `https://www.desmos.com/`

Week	Week of	Section	Topic & Assessment
1	22 August	§1.1	Functions and Their Graphs
		§1.2	Combining Functions; Shifting and Scaling Graphs
		§1.3	Trigonometric Functions
		§1.4	Graphing with Software **In-class 1**
2	29 August	§1.5	Exponential Functions
		§1.6	Inverse Functions and Logarithms
		§2.1	Rates of Change and Tangent Lines to Curves **HW, In-class 2**
3	5 September	§2.2	Limit of a Function and Limit Laws
		§2.3	The Precise Definition of a Limit
		§2.4	One-Sided Limits **HW, In-class 3**
4	12 September	§2.5	Continuity
		§2.6	Limits Involving Infinity; Asymptotes **Exam 1, HW, In-class 4**
5	19 September	§3.1	Tangent Lines and the Derivative at a Point
		§3.2	The Derivative as a Function
		§3.3	Differentiation Rules **HW, In-class 5**
6	26 September	§3.4	The Derivative as a Rate of Change
		§3.5	Derivatives of the Trigonometric Functions
		§3.6	The Chain Rule **HW, In-class 6**
7	3 October	§3.7	Implicit Differentiation
		§3.8	Derivatives of Inverse Functions and Logarithms
		§3.9	Inverse Trigonometric Functions **HW, In-class 7**
8	10 October	§3.10	Related Rates
		§3.11	Linearization and Differentials **Exam 2, HW, In-class 8**
9	17 October	§4.1	Extreme Values of Functions on Closed Intervals **HW, In-class 9**
10	24 October	§4.2	The Mean Value Theorem
		§4.3	Monotonic Functions and the First Derivative Test **HW, In-class 10**
11	31 October	§4.4	Concavity and Curve Sketching
		§4.5	Indeterminate Forms and L'Hôpital's Rule
		§4.6	Applied Optimization **HW, In-class 11**
12	7 November	§4.6	Applied Optimization (continued)
		§4.7	Newton's Method
		§4.8	Antiderivatives **HW, In-class 12**
13	14 November	§5.1	Area and Estimating with Finite Sums
		§5.2	Sigma Notation and Limits of Finite Sums **Exam 3, HW, In-class 13**
14	21 November	§5.3	The Definite Integral
		§5.4	The Fundamental Theorem of Calculus
		§5.5	Indefinite Integrals and the Substitution Method **HW, In-class 14**
15	28 November	§5.6	Area Between curves
		§6.1	Volumes Using Cross-Sections **HW, In-class 15**
16	5 December	§6.2	Volumes Using Cylindrical Shells
17			**Exam Exam 4** (50 points) Monday 14 December 2020, 13:00 – 15:00
			Comprehensive Final Exam Tuesday 15 December 2020, 10:30 – 12:30

University Policies

For the UNK's Policies and statements on: Attendance Policy, Academic Honesty Policy, Reporting Student Sexual Harassment, Sexual Violence or Sexual Assault, Students with Disabilities, Students Who are Pregnant, and UNK Statement of Diversity & Inclusion, you *must read* `https://www.unk.edu/academic_affairs/asa_forms/course-policies-and-resources.php`.

Class Policies

1. Regular in person class attendance is required. If you are ill or need to miss class due to athletics, please let me know ahead of time, and I will make an effort to put the class on Zoom.

2. If you are ill and unable to attend for an exam, you *must* notify me before 10:00 AM the day of the assessment. Generally if you are ill and must stay home for in class work, you will need to complete the assignment and turn it in at the due date. If you do not inform me *before* 10:00 AM the day of the assessment generally you will earn a grade of zero on that assessment.

3. If you must miss an exam due to athletics, you must inform me five days before the exam to arrange to make up the exam.

4. For examinations and in class assignments, show your work. *No credit will be given for multi-step problems without the necessary work. Your solution must contain enough detail so that I am convinced that you could correctly work any similar problem.* Also erase or clearly mark any work you want me to ignore; otherwise, I'll grade it.

5. Most weeks, online homework is due one minute before midnight on the Saturday of the week it was assigned. I will allow at most two extensions on the online homework. After two extensions, late online homework will earn a grade of zero.

6. The work you turn in is expected to be *accurate, complete, concise, neat*, and *well-organized. You will not earn full credit on work that falls short of these expectations.*

7. Class cancellations due to weather, illness, or other unplanned circumstances may require that we make adjustments to the course calendar, exam dates, and due dates or specifics for course assessments.

8. Extra credit is not allowed. Retaking an exam in an effort to earn a higher grade is not allowed.

9. For examinations, you may use a teacher provided quick reference sheet, but no other reference materials or scratch paper. You may also use a pencil, eraser, and a scientific calculator. For examinations, your phone and all such devices must be turned off and *out of sight*.

10. During class time, please refrain from using electronic devices. If your device usage distracts your classmates, I will ask you to put it away. If it's my impression that you are often not paying attention in class, I reserve the right to decline to help you during office hours.

11. The final examination will be *comprehensive* and it will be given during the time scheduled by the University. Except for *extraordinary circumstances* you must take the exam at this time.

12. If you have questions about how your work has been graded, make an appointment with me immediately.

13. Please regularly check Canvas to verify that your scores have been recorded correctly. If I made a mistake in recording one of your grades, I'll correct it provided you saved your paper.

Prerequisite

The prerequisite for this class is either a passing grade (D- or higher) in MATH 103 or a Math ACT score of 23 or above. It is *suggested* (but not required) that if you qualify by your Math ACT score that you have successfully completed four years of high school math, including two years of algebra, one year of geometry, and a senior level pre-calculus class.

Course Objectives

Students will learn the concepts of continuity, the limit, the derivative, and the indefinite and definite integrals. Students will apply these concepts to problems involving the sciences and to applied problems of mathematics including geometry and the extreme values of functions.

Learning Commons

UNK provides assistance to help you improve your academic performance. The Learning Commons, located on the Second floor of the Calvin T. Ryan Library, centralizes several academic services in one convenient place: Language Learning Support, Library Services, Subject Tutoring, Success Coaching, Supplemental Instruction, and the Writing Center are all offered in a casual, collaborative environment. Most services are facilitated by fellow UNK students, which means you will be able to learn and practice more effective study skills, problem-solving techniques, and writing strategies with people who have been there and done that! Statistics indicate that students who come to the LC regularly are more likely to succeed in their classes–so come early and come often. For more information about schedules and services, contact the Learning Commons at 865-8905 or visit them online at `www.unk.edu/lc`.

Catalog description

MATH 115 – Calculus I with Analytic Geometry (5 credit hours) Limits and continuity, differentiation of algebraic and trigonometric functions, elementary integration (with applications) of algebraic and trigonometric functions.

Calculus specific learning outcomes (CSLO)

Students will learn the concepts of continuity, the limit, the derivative, and the indefinite and definite integrals. Students will apply these concepts to problems involving the sciences and to applied problems of mathematics including geometry and the extreme values of functions. Specifically, students will:

1. understand and compute limits and directional limits of functions in one variable and be able to determine (directional) continuity of such functions using the limit point definition of continuity.
2. be able to discuss asymptotic behavior in terms of limits.
3. understand the definition of derivative and its geometric interpretations.
4. compute derivatives using both the definition of derivative and derivative rules.
5. use and apply the concept of the derivative as a rate of change to related rates, linearization, and differential problems.
6. apply differentiation concepts through the mean value theorem and to curve sketching using concavity and intervals of increase/decrease, and to optimization and root finding (Newton's method).
7. understand the basics of anti-derivatives and integration.
8. develop an deeper understanding of summation notation and the relationship to integration and Riemann sums.
9. compute basic definite integrals using Riemann sums and the fundamental theorem of calculus.
10. compute basic integrals using integration by substitution techniques.
11. Be able to set up and find the area between curves using integration.

General Studies and Learning Outcomes

This class satisfies the General Studies (GS) program in LOPER 4 (Mathematics, Statistics, and Quantitative Reasoning) requirement. The purpose of GS courses is:

> The UNK LOPERs General Studies Program helps students to develop core academic skills in collecting and using information, communications in speech and writing, and quantitative reasoning (LOPERs 1-4); to acquire broad knowledge ina variety of disciplines across the arts, humanities, social sciences, and natural sciences (LOPERs 5-8); and to instill dispositions that prepare students to lead responsible and productive lives in a democratic, multicultural society (LOPERs 9-11).

Specifically LOPER 4 courses are "designed for students to develop core academic skills in collecting and using information, communications in speech and writing, and quantitative reasoning." The learning outcomes for the LOPER 4 requirement are:

1. Can describe problems using mathematical, statistical, or programming language.
2. Can solve problems using mathematical, statistical, or programming techniques.
3. Can construct logical arguments using mathematical, statistical, or programming concepts.
4. Can interpret and express numerical data or graphical information using mathematical, statistical, or programming concepts and methods.

Learning outcome 1 is met most directly by the mathematical set up and preparation of solutions to problems involving related rates and optimization in the calculus specific learning outcomes (CSLO) 5 and 6. (The CSLO are found below.) It is also met directly in discussion of asymptotic behavior CSLO 2. It is met indirectly through word problems related to many of the other CSLO. It is assessed by the grading of problems related to these CSLO on homework, quizzes, exams, and/or projects where a portion of the grade for a solution is based on the set up and defend of the submitted work. Learning outcomes 2 and 3 are closely related in this course and are met in solving problems related to every single CSLO. These are assessed via homework, quizzes, exams, and/or projects where a portion of the grade for a solution is based on the validity of the submitted solution's logical reasoning and the accuracy of answers. Learning outcome 4 is met most directly by CSLO 3, 6, and 11 as well as through reading and creating graphs and/or tabular data related all of the CSLO. It is assessed by homework, quizzes, exams, and/or projects where a portion of the grade is based on the accuracy of the graphs and tables in the submitted solutions and/or the accuracy of the interpretation of such data from the assigned problem.

Practice Problems

I *strongly* encourage you to work as many of these practice problems as you can. These problems are *not* graded, but problems similar to these will appear in other graded work. And of course, if you have questions about these practice problems, either ask me in class or seek the assistance of the Learning Commons.

§1.1	1–31 (odd)
§1.2	1–31 (odd)
§1.3	7–29 (odd)
§1.4	1–5
§1.5	1–21 (odd)
§1.6	1–31 (odd)
§2.1	1–25 (odd)
§2.2	1–49 (odd)
§2.3	1–7
§2.4	1–21 (odd)
§2.5	1–29 (odd), 41, 43
§2.6	1–51 (odd)
§3.1	1–31 (odd)
§3.2	1–35 (odd)
§3.3	1–51 (odd)
§3.4	xxx
§3.4	1–9 (odd), 15
§3.5	1–55 (odd)
§3.6	1–37 (odd), 51–77 (odd)
§3.7	1–17 (odd), 21,23,43
§3.8	1–47 (odd), 107

§3.9	1–43 (odd)
§3.10	1–11 (odd), 21, 25, 27
§3.11	1–13 (odd)
§4.1	1–7 (odd), 21, 25, 29
§4.2	1, 13, 21
§4.3	1–17, 19–27 (odd)
§4.4	1–7, 9–17 (odd)
§4.5	1–51 (odd)
§4.6	1–11 (odd)
§4.7	1–7 (odd)
§4.8	1–41, 71–77 (odd)
§5.1	1–7 (odd)
§5.2	1–25, 37–41 (odd)
§5.3	1–49 (odd)
§5.4	1–55 (odd)
§5.5	1–45 (odd)
§5.6	1–41 (odd)
§3.5	1–55 (odd)
§6.1	1–11 (odd)
§6.2	1–21 (odd)
§6.3	1–7 (odd)

Additional learning resources

MyLab | Math has many learning resources. In the left column, look for a link to "eText Contents." From there, you will find a link to the textbook. You can navigate to a specific page in the book by entering the page number in the box in the top middle. Also in the left column, look for a link to "Video Resource Library." From there, selecting a chapter and section and clicking on "video" gives a list of videos to watch. Instead, selecting "Multimedia Textbook," gives us the texbook.

The lectures of Professor Leonard follow our course fairly well. For a link to all of his calculus videos, start

here: `https://www.youtube.com/channel/UCoHhuummRZaIVX7bD4t2czg`

Here is a listing of our sections with a link to a corresponding Professor Leonard video. For some sections, there doesn't seem to be a matching video from Professor Leonard; for these, please use the resources from our textbook. '

Section	Alternative lecture
§1.1	https://www.youtube.com/watch?v=1EGFSefe5II&list=PLF797E961509B4EB5&index=3&t=0
§1.2	https://www.youtube.com/watch?v=f-_UsIP5jyA&list=PLF797E961509B4EB5&index=4
§1.3	https://www.youtube.com/watch?v=SzLF-wLZF_I&list=PLF797E961509B4EB5&index=4&t=0
§1.5	(our textbook)
§1.6	(our textbook)
§2.1 & §2.2	https://www.youtube.com/watch?v=54_XRjHhZzI&list=PLF797E961509B4EB5&index=5
§2.2	https://www.youtube.com/watch?v=VSqOZNULRjQ
§2.3	(our textbook)
§2.4	(our textbook)
§2.5	https://www.youtube.com/watch?v=OEE5-M4aY4k&list=PLF797E961509B4EB5&index=8&t=0
§2.6	https://www.youtube.com/watch?v=-PYebK8DKPc&list=PLF797E961509B4EB5&index=21
§3.1 & §3.2	https://www.youtube.com/watch?v=962lLfW-8Jo&list=PLF797E961509B4EB5&index=9
§3.3	https://www.youtube.com/watch?v=EY6FHX6asU0&list=PLF797E961509B4EB5&index=10
§3.3	https://www.youtube.com/watch?v=AvCQQ3X4Nuc&list=PLF797E961509B4EB5&index=11
§3.4	https://www.youtube.com/watch?v=qr1WXiq3S3k&list=PLF797E961509B4EB5&index=12
§3.5	https://www.youtube.com/watch?v=RJJSiNz5oto&list=PLF797E961509B4EB5&index=13
§3.6	https://www.youtube.com/watch?v=8dr1dZjfhmc&list=PLF797E961509B4EB5&index=14
§3.7	https://www.youtube.com/watch?v=RUS4mKo9tBk&list=PLF797E961509B4EB5&index=15
§3.8	(our textbook)
§3.9	(our textbook)
§3.10	https://www.youtube.com/watch?v=43Qt6wc44To&list=PLF797E961509B4EB5&index=16
§3.11	(our textbook)
§4.1	https://www.youtube.com/watch?v=Mx39JbbzEAo&list=PLF797E961509B4EB5&index=17
§4.2	https://www.youtube.com/watch?v=qW89xdGfSzw&list=PLF797E961509B4EB5&index=18
§4.3	https://www.youtube.com/watch?v=nQ6tOORDQ3I&list=PLF797E961509B4EB5&index=19
§4.4	https://www.youtube.com/watch?v=8u6woY05aL0&list=PLF797E961509B4EB5&index=22
§4.5	(our textbook)
§4.6	https://www.youtube.com/watch?v=SWZcq_biZLw&list=PLF797E961509B4EB5&index=23
§4.7	(our textbook)
§4.8	https://www.youtube.com/watch?v=b2ZFpE_yrLg&list=PLF797E961509B4EB5&index=24
§5.1 & §5.2	https://www.youtube.com/watch?v=F0uuW-I6icY&list=PLF797E961509B4EB5&index=26
§5.3	https://www.youtube.com/watch?v=K0ORDCt5Ig0&list=PLF797E961509B4EB5&index=27
§5.4	https://www.youtube.com/watch?v=xjtEfS0vY2o&list=PLF797E961509B4EB5&index=28
§5.5	https://www.youtube.com/watch?v=aiBD9aI69C8&list=PLF797E961509B4EB5&index=25
§5.6	https://www.youtube.com/watch?v=c7wur9Lixb0&list=PLF797E961509B4EB5&index=29
§6.1	https://www.youtube.com/watch?v=GJOJl47l2_4&list=PLF797E961509B4EB5&index=30
§6.2	https://www.youtube.com/watch?v=BDmlottZVd4&list=PLF797E961509B4EB5&index=31
§6.3	https://www.youtube.com/watch?v=5Yuw1jCBq-0&list=PLF797E961509B4EB5&index=32

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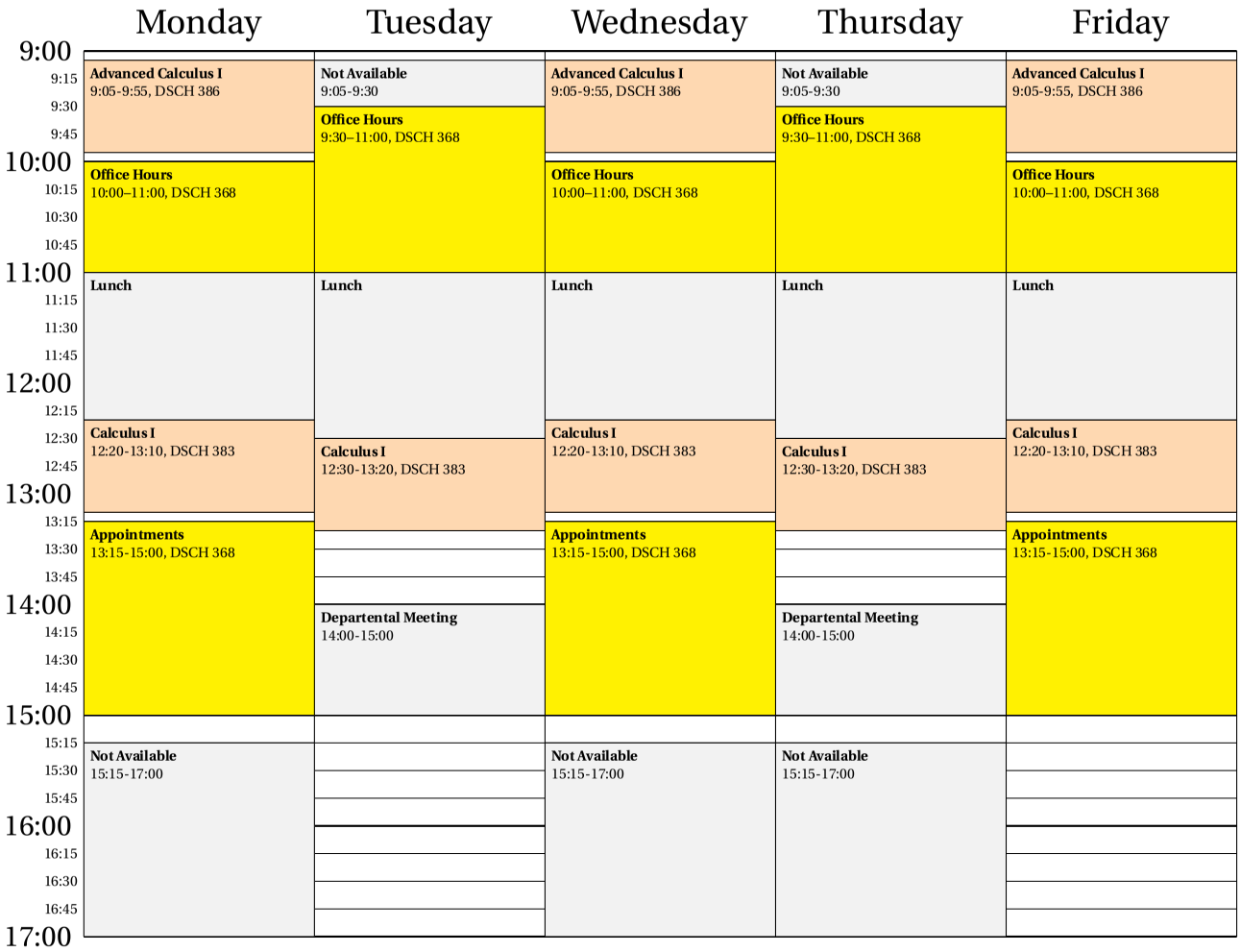
Blocks marked “Appointments” (yellow highlight) means *usually* available to make appointments. Blocks with gray highlights mean not available.

Fall 2022



Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

Intervals

For numbers a and b, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a,b) = \{x \in \mathbf{R} \mid a \le x < b\}$$
$$(a,b] = \{x \in \mathbf{R} \mid a < x \le b\}$$
$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Exponents

For $a, b > 0, x \in \mathbf{R}$, and m, n real:

$$a^0 = 1, \qquad 0^a = 0$$
$$1^a = 1, \qquad a^n a^m = a^{n+m}$$
$$a^n / a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$
$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m / b^m$$
$$\sqrt{x^2} = |x|$$

Trigonometric Identities

$$\cos(x)^2 + \sin(x)^2 = 1$$
$$2\cos(x)^2 = 1 + \cos(2x)$$
$$2\sin(x)^2 = 1 - \cos(2x)$$
$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\operatorname{arccot}(x) = \arctan(1/x)$$
$$\operatorname{arccsc}(x) = \arcsin(1/x)$$
$$\operatorname{arcsec}(x) = \arccos(1/x)$$

Limits

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0$$
$$\lim_{x \to \infty} e^x = \infty \qquad \lim_{x \to -\infty} e^x = 0$$
$$\lim_{x \to \infty} \ln(x) = \infty \qquad \lim_{x \to 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\operatorname{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\operatorname{arcsinh}(x)$	$1/\sqrt{1+x^2}$
$\operatorname{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1\prime}(x)$	$1/f'(f^{-1}(x))$

Antiderivatives

$$\int a\,dx = ax$$
$$\int x^a\,dx = \frac{1}{1+a}x^{a+1}, \quad \text{if } a \neq -1$$
$$\int \frac{1}{x}\,dx = \ln|x|$$
$$\int \cos(x)\,dx = \sin(x)$$
$$\int \sin(x)\,dx = -\cos(x)$$
$$\int \tan(x)\,dx = \ln|\sec(x)|$$
$$\int \sec(x)\,dx = \ln|\tan(x) + \sec(x)|$$
$$\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)|$$
$$\int \cot(x)\,dx = \ln|\sin(x)|$$
$$\int 2|x|\,dx = x|x|$$

Sums

For $k, m, n \in \mathbf{Z}_{>0}$

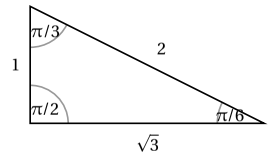
$$\sum_{k=0}^{n-1} 1 = n$$
$$\sum_{k=0}^{n-1} k = \frac{(n-1)\,n}{2}$$
$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)\,n\,(2n-1)}{6}$$
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

Logarithms

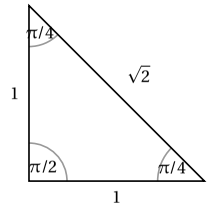
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Famous Triangles

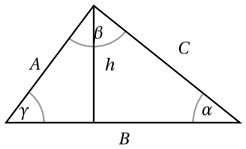
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine: $C^2 = A^2 + B^2 - 2AB\cos(\gamma)$

Law of sines: $\dfrac{\sin(\alpha)}{A} = \dfrac{\sin(\beta)}{B} = \dfrac{\sin(\gamma)}{C}$

Area: $\text{Area} = \dfrac{1}{2}hB = \dfrac{1}{2}AB\sin(\gamma)$

Hyperbolic Functions

$$2\cosh(x) = \exp(x) + \exp(-x)$$

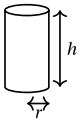
$$2\sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = \cosh(x)/\sinh(x)$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

Volumes

Right Circular Cylinder

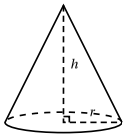


Volume: $\text{V} = \pi r^2 h$

Area: (not including circular ends)

$\text{A} = 2\pi r h$

Cone



Volume: $\text{V} = \dfrac{1}{3}\pi r^2 h$

Area (not including circular base)

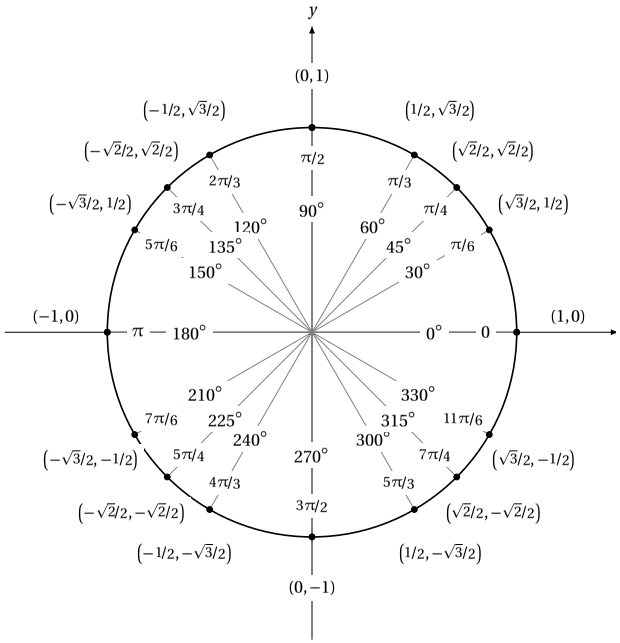
$\text{A} = \pi r\sqrt{r^2 + h^2}$

Sphere

Area: $A = 4\pi r^2$

Volume: $V = \dfrac{4\pi}{3}r^3$

Unit Circle



Applications

Arclength of curve $y = f(x)$ with $a \le x \le b$

$$= \int_a^b \sqrt{1 + f'(x)^2}\,\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(x) \le y \le g(x) \wedge a \le x \le b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x)\,\mathrm{d}x$$

Assuming $0 \le f(x)$ and rotating about the x-axis

$$\text{Vol}(Q) = \pi\int_a^b g(x)^2 - f(x)^2\,\mathrm{d}x$$

Assuming $0 \le a < b$ and rotating about the y-axis

$$\text{Vol}(Q) = 2\pi\int_a^b x(g(x) - f(x))\,\mathrm{d}x$$

Centroid

$$\text{Area}(Q) \times \overline{x} = \int_a^b x\left(g(x) - f(x)\right)\mathrm{d}x$$

$$\text{Area}(Q) \times \overline{y} = \frac{1}{2}\int_a^b \left(g(x)^2 - f(x)^2\right)\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(y) \le x \le g(y) \wedge a \le y \le b\},$$

interchange x and y in *all* the previous formulas.