Week 8 FLO (Friday Learning Opportunity)

1. Find a TL to $y = \sqrt[3]{x}$ with a point of tangency of $(x = 1000, y = 10)$.

Solution: We have $\dfrac{dy}{dx} = \dfrac{1}{3}x^{-2/3}$. So $\left.\dfrac{dy}{dx}\right|_{x=1000} = \frac{1}{300}$. So

$$L(x) = 10 + \frac{1}{300}(x - 1000).$$

2. Use the TL from Question 1 to estimate $\sqrt[3]{1007}$. Compare this to is true value.

Solution:

$$\sqrt[3]{1007} \approx L(1007) = 10 + \frac{1}{300}(1007 - 1000) = \frac{3007}{300} \approx 10.023.$$

The true value is approximately $10.02327909963426271929128421643233544331097738794160816 26$. So 10.023 is a pretty good estimate.

3. Assume an elliptical pancake. As a cornmeal pancake expands on the skillet, its semi-major axis remains three times its semi-minor axis. At the moment the surface area of the pancake is 12 square inches, the rate of change of its area is $1/2$ square inches per second. At this moment, find the rate of change of its semi-minor axis.

Solution: Let the semi-major axis be a and the semi-minor axis be b. We are given that $a = 3b$. The area A is $A = \pi ab = 3\pi b^2$. So

$$\frac{dA}{dt} = 6\pi b \frac{db}{dt}. \tag{1}$$

Pasting in the data gives

$$12 = 3\pi b^2 \qquad \frac{1}{2} = 6\pi b \frac{db}{dt}. \tag{2}$$

Solving for b and $\frac{db}{dt}$ gives $\frac{db}{dt} = \frac{1}{24\sqrt{\pi}}$.

4. A monarch butterfly is flying directly south and a goldfinch is flying directly east. As some moment, the monarch is 3 miles south of the UNK bell tower and the goldfinch is five miles east. Also at this moment, the speed of the monarch is 8 mph and the speed of the goldfinch is 47 mph. At this moment, what is the rate of change of the distance between the monarch and the goldfinch?

Solution: Let the position of the monarch be y, the position of the goldfinch be x, and distance in between them be R. Differentiating with respect to t gives

$$2R\frac{dR}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}. \tag{3}$$

Pasting in the data gives the equations

$$\begin{aligned} R^2 &= 34, \\ 2R\frac{\mathrm{d}R}{\mathrm{d}t} &= 2 \times 5 \times 47 + 2 \times 3 \times 8. \end{aligned}$$

So

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{259}{\sqrt{34}} \text{ mph}$$