Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ϵ	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	Ø
real numbers	R
ordered pairs	\mathbf{R}^2
integers	Z
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	€
subset	C
intersection	Λ
union	U
set minus	\

Intervals

For numbers *a* and *b*, we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \le x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \le b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic Symbols

Meaning	Symbol
negation	7
and	٨
or	V
implies	\Rightarrow
equivalent	≡
iff	\iff
for all	A
there exists	Э

Exponents

For a, b > 0 and m, n real:

$$a^{0} = 1,$$
 $0^{a} = 0$
 $1^{a} = 1,$ $a^{n} a^{m} = a^{n+m}$
 $a^{n}/a^{m} = a^{n-m},$ $(a^{n})^{m} = a^{n \cdot m}$
 $a^{-m} = 1/a^{m},$ $(a/b)^{m} = a^{m}/b^{m}$

Trigonometric Identities

$$\sin(x)^{2} + \cos(x)^{2} = 1$$

$$2\cos(x)^{2} = 1 + \cos(2x)$$

$$2\sin(x)^{2} = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\operatorname{arccot}(x) = \arctan(1/x)$$

$$\operatorname{arccsc}(x) = \arcsin(1/x)$$

$$\operatorname{arccsc}(x) = \arcsin(1/x)$$

Limits

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0$$

$$\lim_{x \to \infty} e^x = \infty \qquad \lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to \infty} \ln(x) = \infty \qquad \lim_{x \to 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

F(x)	F'(x)
$\cos(x)$	$-\sin(x)$
sin(x)	$\cos(x)$
tan(x)	$sec(x)^2$
sec(x)	sec(x) tan(x)
$\csc(x)$	$-\cot(x)\csc(x)$
cot(x)	$-\csc(x)^2$
arccos(x)	$-1/\sqrt{1-x^2}$
arcsin(x)	$1/\sqrt{1-x^2}$
arctan(x)	$1/(x^2+1)$
$\exp(x)$	$\exp(x)$
ln(x)	1/ <i>x</i>

General Cases

F(x)	F'(x)
af(x) + bg(x)	af'(x) + bg'(x)
f(x)g(x)	f'(x)g(x) + f(x)g'(x)
1/g(x)	$-g'(x)/g(x)^2$
f(x)/g(x)	$(g(x)f'(x)-f(x)g'(x))/g(x)^2$
f(g(x))	g'(x)f'(g(x))
$f^{-1\prime}(x)$	$1/f'(f^{-1}(x))$

Antiderivatives

$$\int a \, dx = ax$$

$$\int x^a \, dx = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x|$$

$$\int \cos(x) \, dx = \sin(x)$$

$$\int \sin(x) \, dx = -\cos(x)$$

$$\int \tan(x) \, dx = \ln|\sec(x)|$$

$$\int \sec(x) \, dx = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, dx = \ln|\sin(x)|$$

$$\int 2|x| \, dx = x|x|$$

$$\int 2|x| \, dx = (2x-1)|x| - |x|^2$$

$$\int 2|x| \, dx = (2x+1)|x| - |x|^2$$

Sums

For $k, m, n \in \mathbb{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

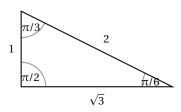
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

Logarithms

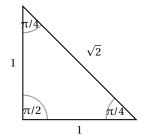
$$\log_a(x) = \frac{1}{\ln(a)} \ln(x)$$

Famous Triangles

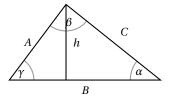
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine: $C^2 = A^2 + B^2 - 2AB\cos(\gamma)$ Law of sines: $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

Area: Area = $\frac{1}{2}hB = \frac{1}{2}AB\sin(\gamma)$

Volumes

Right Circular Cylinder



Cone



Sphere

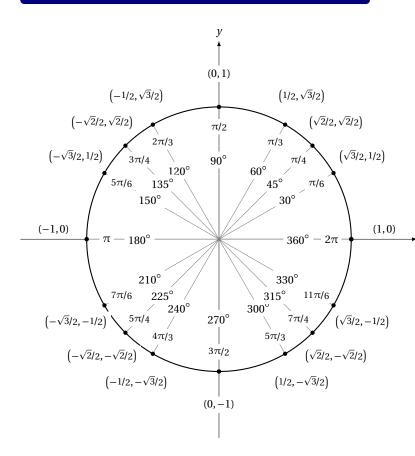
Volume: $V = \pi r^2 h$

Area: (not including circular ends) $A = 2\pi r h$

Volume: $V = \frac{1}{3}\pi r^2 h$ **Area** (not including circular base) $A = \pi r \left(r + \sqrt{r^2 + h^2} \right)$

Area: $A = 4\pi r^2$ Volume: $V = \frac{4\pi}{3}r^3$

Unit Circle



Applications

Arclength of curve y = f(x) with $a \le x \le b$

$$= \int_a^b \sqrt{1 + f'(x)^2} \, \mathrm{d}x$$

For the region *Q* of the xy plane given by

$$Q = \{(x,y) \mid f(x) \le y \le g(x) \land a \le x \le b\},$$

we have

Area(Q) =
$$\int_{a}^{b} g(x) - f(x) dx$$

Assuming $0 \le f(x)$ and rotating about the x-axis

$$Vol(Q) = \pi \int_{a}^{b} g(x)^{2} - f(x)^{2} dx$$

Assuming $0 \le a < b$ and rotating about the y-axis

$$Vol(Q) = 2\pi \int_{a}^{b} x(g(x) - f(x)) dx$$

Centroid

Area(Q)
$$\times \overline{x} = \int_{a}^{b} x (g(x) - f(x)) dx$$

Area(Q) ×
$$\overline{y} = \frac{1}{2} \int_{a}^{b} \left(g(x)^{2} - f(x)^{2} \right) dx$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(y) \le x \le g(y) \land a \le y \le b\},\$$

interchange x and y in *all* the previous formulas.

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