

In class work 8 has questions 1 through 2 with a total of 5 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 9 November 13:15 PM*.

- 5 1. Show that among all rectangles with a given perimeter, a square has the greatest area.

To do this, let the lengths of the two perpendicular sides of the rectangle be  $x$  and  $y$  and let  $L$  be the perimeter of the rectangle. That makes  $L = 2x + 2y$  a constraint. The other constraints are  $0 \leq x$  and  $0 \leq y$ . We have  $A = xy$ . Your task is to maximize  $A$  subject to the constraints  $L = 2x + 2y$ ,  $0 \leq x$ , and  $0 \leq y$  with  $L$  given.

2. Show that among all isosceles triangles with a given perimeter, the equilateral triangle has the greatest area.

To do this, let the lengths of the sides of the triangle be  $x, x$ , and  $y$  and let  $L$  be the perimeter. That makes  $L = 2x + y$  a constraint. The other constraints are  $0 \leq x$  and  $0 \leq y$ . The area  $A$  of the triangle is (this is a specialization of the wonderful formula for the area of a triangle that is due to Hero of Alexandria 10 AD – c. 70 AD)

$$A = (x + y) \sqrt{2y(x + 2y)}$$

Solve the constraint  $L = 2x + y$  for  $y$  and paste that result in the formula for  $A$ . Now do some calculus. If the algebra seems daunting to you, set  $L = 3$  and work that specialization.