MATH 115 **Name:** ______________________

In class work 12, Fall 2022 **Row and Seat**:______________________

In class work 12 has questions 1 through 2 with a total of 10 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 9 November 13:15* PM.

$\boxed{5}$ 1. Show that among all rectangles with a given perimeter, a square has the greatest area.

To do this, let the lengths of the two perpendicular sides of the rectangle be x and y and let L be the perimeter of the rectangle. That makes $L = 2x + 2y$ a constraint. The other constraints are $0 \le x$ and $0 \le y$. We have $A = xy$. Your task is to maximize A subject to the constraints $L = 2x + 2y, 0 \le x$, and $0 \le y$ with L given.

Solution: Let's solve the constraint $L = 2x + 2y$ for y; the solution is $y = \frac{L-2x}{2}$. Pasting this into $A = xy$ gives $A = x(\frac{L-2x}{2})$. The graph of A as a function of x is a downward facing parabola that intersects the x-axis at 0 and at $L/2$. The x-coordinate of the vertex of the parabola is halfway between the x-intercepts; thus the x-coordinate of the vertex is $x = L/4$. Using $y = \frac{L-2x}{2}$, the y-coordinate of the vertex is $y = L/4$.

So to maximize the area of the rectangle, we need $x = L/4$ and $y = L/4$. And that's a square.

$\boxed{5}$ 2. Show that among all isosceles triangles with a given perimeter, the equilateral triangle has the greatest area.

To do this, let the lengths of the sides of the triangle be x, x, and y and let L be the perimeter. That makes $L = 2x + y$ a constraint. The other constraints are $0 \leq x$ and $0 \leq y$. The area A of the triangle is (this is a specialization of the wonderful formula for the area of a triangle that is due to Hero of Alexandria 10 AD – c. 70 AD)

$$16A^2 = 4x^2 y^2 - y^4.$$

Solve the constraint $L = 2x + y$ for y and paste that result in the formula for $16A^2$. Now do some calculus. **Hint:** Maximizing $16A^2$ also maximizes A. Thus alternatively, maximize the value of Q where

$$Q = 4x^2 y^2 - y^4.$$

You don't have to use this hint, but it's the easy way, I think. If the algebra seems daunting to you, set $L = 3$ and work that specialization.

Solution: Let's begin by solving the constraint $L = 2x + y$ for y; thus $y = L - 2x$. Paste this into Q. We have

$$\begin{aligned} Q &= 4x^2 y^2 - y^4, \\ &= 4(L-2x)^2 x^2 - (L-2x)^4, \\ &= L(2x-L)^2 (4x-L). \end{aligned}$$

Now find the derivative of Q. We have

$$\begin{aligned} \frac{dQ}{dx} &= 4L(2x-L)^2 + 4L(2x-L)(4x-L), \\ &= 8L(2x-L)(3x-L). \end{aligned}$$

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