

Greek Characters

| Name | Symbol | Typical use(s) |
|---------|-----------------------------|-------------------|
| alpha | α | angle, constant |
| beta | β | angle, constant |
| gamma | γ | angle, constant |
| delta | δ | limit definition |
| epsilon | ϵ or ε | limit definition |
| theta | θ or ϑ | angle |
| pi | π or π | circular constant |
| phi | ϕ or φ | angle, constant |

Named Sets

| | |
|-----------------------|-------------------|
| empty set | \emptyset |
| real numbers | \mathbf{R} |
| ordered pairs | \mathbf{R}^2 |
| integers | \mathbf{Z} |
| positive integers | $\mathbf{Z}_{>0}$ |
| positive real numbers | $\mathbf{R}_{>0}$ |

Set Symbols

| Meaning | Symbol |
|--------------|-------------|
| is a member | \in |
| subset | \subset |
| intersection | \cap |
| union | \cup |
| set minus | \setminus |

Intervals

For numbers a and b , we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

Logic Symbols

| Meaning | Symbol |
|--------------|-------------------|
| negation | \neg |
| and | \wedge |
| or | \vee |
| implies | \Rightarrow |
| equivalent | \equiv |
| iff | \Leftrightarrow |
| for all | \forall |
| there exists | \exists |

Exponents

For $a, b > 0$, $x \in \mathbf{R}$, and m, n real:

$$a^0 = 1, \quad 0^a = 0$$

$$1^a = 1, \quad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \quad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \quad (a/b)^m = a^m / b^m$$

$$\sqrt{x^2} = |x|$$

Trigonometric Identities

$$\cos(x)^2 + \sin(x)^2 = 1$$

$$2 \cos(x)^2 = 1 + \cos(2x)$$

$$2 \sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\operatorname{arccot}(x) = \arctan(1/x)$$

$$\operatorname{arccsc}(x) = \arcsin(1/x)$$

$$\operatorname{arcsec}(x) = \arccos(1/x)$$

Limits

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

$$\lim_{x \rightarrow \infty} e^x = \infty \quad \lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} \ln(x) = \infty \quad \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

| $F(x)$ | $F'(x)$ |
|-----------------------------|--------------------|
| $\cos(x)$ | $-\sin(x)$ |
| $\sin(x)$ | $\cos(x)$ |
| $\tan(x)$ | $\sec(x)^2$ |
| $\sec(x)$ | $\sec(x) \tan(x)$ |
| $\csc(x)$ | $-\cot(x) \csc(x)$ |
| $\cot(x)$ | $-\csc(x)^2$ |
| $\arccos(x)$ | $-1/\sqrt{1-x^2}$ |
| $\arcsin(x)$ | $1/\sqrt{1-x^2}$ |
| $\arctan(x)$ | $1/(x^2+1)$ |
| $\cosh(x)$ | $\sinh(x)$ |
| $\sinh(x)$ | $\cosh(x)$ |
| $\tanh(x)$ | $1/\cosh(x)^2$ |
| $\operatorname{arccosh}(x)$ | $1/\sqrt{x^2-1}$ |
| $\operatorname{arsinh}(x)$ | $1/\sqrt{1+x^2}$ |
| $\operatorname{arctanh}(x)$ | $1/(1-x^2)$ |
| $\exp(x)$ | $\exp(x)$ |
| $\ln(x)$ | $1/x$ |

General Cases

| $F(x)$ | $F'(x)$ |
|-----------------|----------------------------------|
| $af(x) + bg(x)$ | $af'(x) + bg'(x)$ |
| $f(x)g(x)$ | $f'(x)g(x) + f(x)g'(x)$ |
| $1/g(x)$ | $-g'(x)/g(x)^2$ |
| $f(x)/g(x)$ | $(g(x)f'(x) - f(x)g'(x))/g(x)^2$ |
| $f(g(x))$ | $g'(x)f'(g(x))$ |
| $f^{-1}(x)$ | $1/f'(f^{-1}(x))$ |

Antiderivatives

$$\int a \, dx = ax$$

$$\int x^a \, dx = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x|$$

$$\int \cos(x) \, dx = \sin(x)$$

$$\int \sin(x) \, dx = -\cos(x)$$

$$\int \tan(x) \, dx = \ln|\sec(x)|$$

$$\int \sec(x) \, dx = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, dx = \ln|\sin(x)|$$

$$\int 2|x| \, dx = x|x|$$

Sums

For $k, m, n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

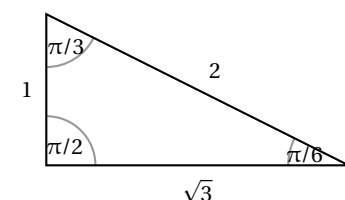
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

Logarithms

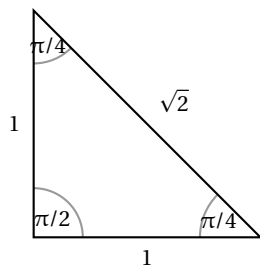
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Famous Triangles

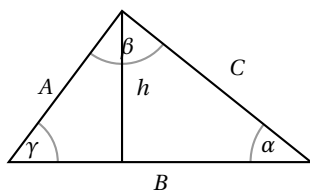
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine: $C^2 = A^2 + B^2 - 2AB \cos(\gamma)$

Law of sines: $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

Area: $\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB \sin(\gamma)$

Hyperbolic Functions

$$2 \cosh(x) = \exp(x) + \exp(-x)$$

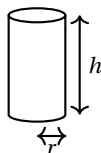
$$2 \sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = \cosh(x)/\sinh(x)$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

Volumes

Right Circular Cylinder

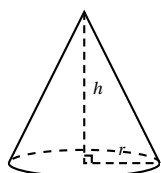


Volume: $V = \pi r^2 h$

Area: (not including circular ends)

$$A = 2\pi r h$$

Cone



Volume: $V = \frac{1}{3}\pi r^2 h$

Area (not including circular base)

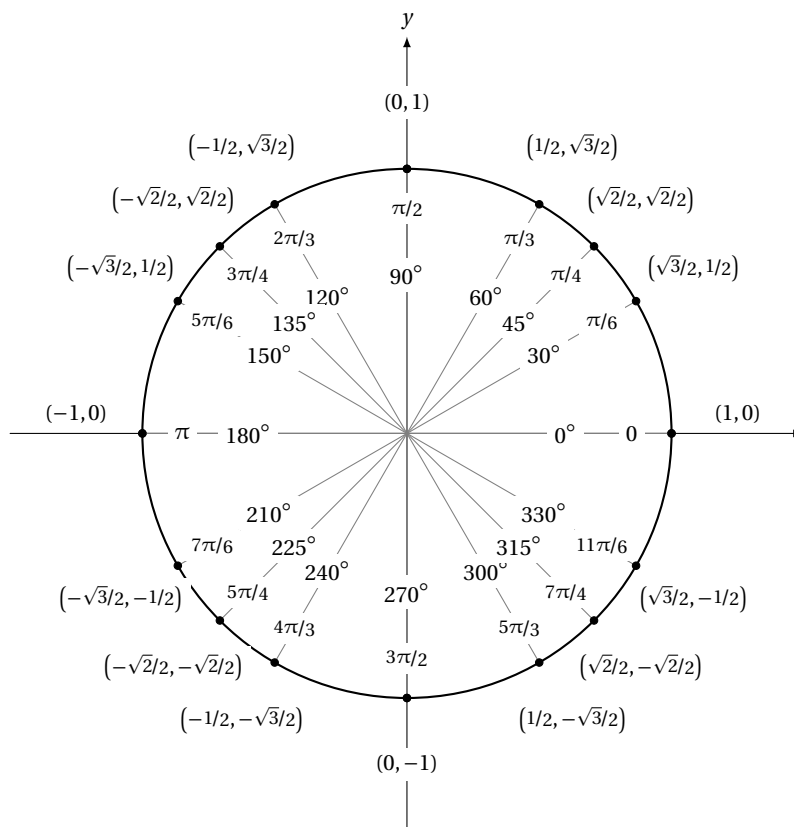
$$A = \pi r \sqrt{r^2 + h^2}$$

Sphere

Area: $A = 4\pi r^2$

Volume: $V = \frac{4\pi}{3}r^3$

Unit Circle



Applications

Arclength of curve $y = f(x)$ with $a \leq x \leq b$

$$= \int_a^b \sqrt{1 + f'(x)^2} dx$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(x) \leq y \leq g(x) \wedge a \leq x \leq b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x) dx$$

Assuming $0 \leq f(x)$ and rotating about the x -axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2 dx$$

Assuming $0 \leq a < b$ and rotating about the y -axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x)) dx$$

Centroid

$$\text{Area}(Q) \times \bar{x} = \int_a^b x(g(x) - f(x)) dx$$

$$\text{Area}(Q) \times \bar{y} = \frac{1}{2} \int_a^b (g(x)^2 - f(x)^2) dx$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(y) \leq x \leq g(y) \wedge a \leq y \leq b\},$$

interchange x and y in *all* the previous formulas.

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