MATH 115 **Name:** ______________________

In class work 3, Fall 2022 **Row and Seat:** ______________________

In class work 3 has questions 1 through 2 with a total of 10 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 7 September 13:15* PM.

1. Find each of the following limits. Justify each of your steps by referencing one of our rules numbered zero through seven.

$\boxed{2}$ (a) $\lim_{x \to \pi} \left(x^3 + x\right)$

Solution: Since $x^3 + x$ is a polynomial, we can use Rule 6; thus

$$\lim_{x \to \pi} \left(x^3 + x\right) = \pi^3 + \pi. \qquad \text{(Rule 6)}$$

Alternatively, we could first use Rule 1 (linearity) followed by Rule 4; thus

$$\begin{aligned} \lim_{x \to \pi} \left(x^3 + x\right) &= \lim_{x \to \pi} (x^3) + \lim_{x \to \pi} (x), && \text{(Rule 1)} \\ &= \pi^3 + \pi. && \text{(Rule 4, twice)} \end{aligned}$$

To apply Rule 4 to $\lim_{x \to \pi} (x)$, match to the algebraically equivalent $\lim_{x \to \pi} (x^1)$.

$\boxed{2}$ (b) $\lim_{x \to \sqrt{2}} \sqrt{x+1}$

Solution: We have

$$\begin{aligned} \lim_{x \to \sqrt{2}} \sqrt{x+1} &= \sqrt{\lim_{x \to \sqrt{2}} (x+1)}, && \text{(Rule 5)} \\ &= \sqrt{1+\sqrt{2}}. && \text{(Rule 6)} \end{aligned}$$

Since $x+1$ is a polynomial, using Rule 6 in the second step is OK.

$\boxed{2}$ (c) $\lim_{x \to \sqrt{2}} \frac{x+1}{x-1}$

Solution: We have

$$\begin{aligned} \lim_{x \to \sqrt{2}} \frac{x+1}{x-1} &= \frac{\sqrt{2}+1}{\sqrt{2}-1}, && \text{(Rule 7)} \\ &= 3 + 2\sqrt{2}. && \text{(simplification)} \end{aligned}$$

The simplification step is done using a multiply by one trick:

$$\begin{aligned}
\frac{\sqrt{2}+1}{\sqrt{2}-1} &= \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}, && \text{(multiply by one)}\\
&= \frac{(\sqrt{2}+1)^2}{2+\sqrt{2}-\sqrt{2}-1}, && \text{(distribute denominator)}\\
&= \frac{(\sqrt{2}+1)^2}{1}, && \text{(collect like terms)}\\
&= 2+2\sqrt{2}+1, && \text{(divisor of one)}\\
&= 3+2\sqrt{2}. && \text{(collect like terms)}
\end{aligned}$$

To earn full credit with our online homework system, generally removing radicals from denominator is required.

$\boxed{2}$ (d) $\lim_{x\to 35} \sqrt{12-2\sqrt{x}}$

Solution: We have

$$\begin{aligned}
\lim_{x\to 35} \sqrt{12-2\sqrt{x}} &= \sqrt{\lim_{x\to 35}(12-2\sqrt{x})}, && \text{(Rule 5)}\\
&= \sqrt{\lim_{x\to 35}(12) - 2\lim_{x\to 35}\sqrt{x}}, && \text{(Rule 1)}\\
&= \sqrt{12-2\sqrt{35}}, && \text{(Rule 0, Rule 5)}\\
&= \sqrt{7}-\sqrt{5}. && \text{(simplification)}
\end{aligned}$$

The simplification in the last step is an application of the square root denesting identity

$$(\forall a, b \in \mathbf{R}_{\geq 0})\left(\sqrt{a+b-2\sqrt{ab}} = \sqrt{\max(a,b)} - \sqrt{\min(a,b)}\right).$$

And no, such simplifications are *not* required (but it is cute).

You'll be happy to know that denesting radicals is one of my hobbies. Some time ago, as a challenge from a friend, I denested

$$2028 \times (2 \times 3^{5/2}\mathrm{i}+35)^{1/3} - (2\times 3^{5/2}\mathrm{i}+35)^{2/3} \times (8 \times 3^{7/2}\mathrm{i} - 420) + 10140.$$

to the integer 24336. To do this, in part, I used an algorithm that was invented by the mathematician turned artist Helaman Ferguson `https://en.wikipedia.org/wiki/Helaman_Ferguson`

2. A graph of a function Q is shown. Using the graph, as best you can find the numerical value of each limit.

$\boxed{1}$ (a) $\lim_{x \to 2} Q(x)$

Solution: Looking at the graph, it appears that when the input is close to 2, the output is close to 2. Moving closer to two from either the left or the right, the output grows ever closer to two. Thus, visual evidence suggests that

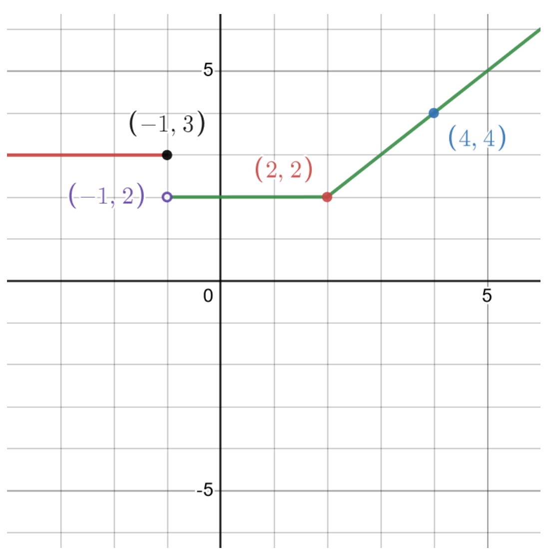
$$\lim_{x \to 2} Q(x) = 2.$$

The visual evidence **isn't** a proof. Maybe if we zoomed in on the graph magnifying by a factor of trillions, that maybe the graph will wiggle around in some crazy way near the limit point.

$\boxed{1}$ (b) $\lim_{x \to -1^{(+)}} Q(x)$

Solution: Remember that the special notation $-1^{(+)}$ means that the limit point is -1, but we only consider inputs that are *larger* (or to the right) of -1. Immediately to the right of the limit point, the output of the Q appears to be a constant 2. Thus, the visual evidence is that $\lim_{x \to -1^{(+)}} Q(x) = 2$.

Once again, visual evidence **isn't** a proof.



Suppose functions F and G have limits toward c and suppose $a, b \in \mathbf{R}$ and n is a positive integer. Then

Rule #0 (constant) $\lim_{x\to c}(a) = a$.

Rule #1 (linearity) $\lim_{x\to c}(aF(x) + bG(x)) = a\lim_{x\to c}(F(x)) + b\lim_{x\to c}(G(x))$.

Rule #2 (product) $\lim_{x\to c}(F(x)G(x)) = \lim_{x\to c}(F(x)) \times \lim_{x\to c}(G(x))$.

Rule #3 (quotient) Provided $\lim_{x\to c}(G(x)) \neq 0$, we have $\lim_{x\to c}\dfrac{F(x)}{G(x)} = \dfrac{\lim_{x\to c}(F(x))}{\lim_{x\to c}(G(x))}$.

Rule #4 (power) $\lim_{x\to c} F(x)^n = \left(\lim_{x\to c} F(x)\right)^n$.

Rule #5 (root) Provided $\left(\lim_{x\to c} F(x)\right)^{1/n}$ is real, $\lim_{x\to c} F(x)^{1/n} = \left(\lim_{x\to c} F(x)\right)^{1/n}$.

Rule #6 (polynomial) Provided F is a polynomial, we have $\lim_{x\to c} F(x) = F(c)$

Rule #7 (rational) Provided F is a rational function and $c \in \operatorname{dom}(F)$, we have $\lim_{x\to c} F(x) = F(c)$.