MATH 115 **Name:** ______________________

In class work 7, Fall 2022 **Row and Seat**: ______________________

In class work 7 has questions 1 through 3 with a total of 6 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 5 October at 13:15* PM.

1. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ evaluated at $(x = 1/\sqrt{2}, y = 1/\sqrt{2})$ given $x^2 + y^2 = 1$.

Solution:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x^2 + y^2 = 1\right] = \left[2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0\right], \qquad \text{(additive and chain rules)}$$
$$= \left[\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}\right]. \qquad \text{(algebra)}$$

Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1/\sqrt{2} y=1/\sqrt{2}} = -1.$

To find the second derivative, we differentiate $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$. The quotient rule gives

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left[-\frac{x}{y}\right] = -\frac{y - xy'}{y^2}.$$

To save some space, I used y' instead of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Pasting in the data $x \leftarrow 1/\sqrt{2}, y \leftarrow 1/\sqrt{2}$ gives

$$\left.\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right|_{x=1/\sqrt{2} y=1/\sqrt{2}} = -\frac{1/\sqrt{2} + 1/\sqrt{2}}{1/2} = -2\sqrt{2}.$$

$\boxed{1}$ 2. The equation $xy = y - 1 + \mathrm{e}^{-y}$ defines[1] y as a function of x. Find a formula for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Solution: Again, let's use y' instead of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. We have

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[xy = y - 1 + \mathrm{e}^{-y}\right] = \left[y + xy' = y' - y'\mathrm{e}^{-y}\right],$$
$$= \left[y = (1 - x - \mathrm{e}^{-y})y'\right],$$
$$= \left[y' = \frac{y}{1 - x - \mathrm{e}^{-y}}\right].$$

[1]This problem is motivated by an unpublished mathematical model of hemoglobin glycation.

FYI It's easy to solve the equation $xy = y-1+e^{-y}$ for x. The solution is $x = \frac{y-1+e^{-y}}{y}$. And asking our favorite graphing tool to graph this shows that when x is the dependent variable and y the independent variable, the graph is one-to-one. The one-to-one property tells us that if we solved $xy = y-1+e^{-y}$ for y, there would be only one solution for each x. Which means that indeed the equation $xy = y-1+e^{-y}$ defines y as a function of x.

But solving $xy = y-1+e^{-y}$ for y involves an obscure trick, so we won't go there.

3. Find a formula for each derivative

$\boxed{1}$ (a) $\dfrac{d}{dx}[\ln(x(x-1))]$

Solution:

$$\frac{dy}{dx} = \frac{(x(x-1)')}{x(x-1)} = \frac{2x-1}{(x-1)\,x} = \frac{1}{x} + \frac{1}{x-1}.$$

The optional final answer of $\frac{1}{x} + \frac{1}{x-1}$ is a partial fraction decomposision (pfd). And the pdf is one of my happy places.

$\boxed{1}$ (b) $\dfrac{d}{dx}\left[\tan^{-1}(x^2)\right]$

Solution:

$$\frac{dy}{dx} = \frac{2x}{1+(x^2)^2} = \frac{2x}{x^4+1}.$$

$\boxed{1}$ (c) $\dfrac{d}{dx}\left[\csc^{-1}(1/x^2)\right]$

Solution:

$$\frac{dy}{dx} = -\frac{(1/x^2)'}{\sqrt{(1/x^2)^2-1}\,|1/x^2|} = -\frac{-2/x^3}{1/x^2\sqrt{1/x^4-1}} = \frac{2x}{\sqrt{1-x^4}}$$

$\boxed{1}$ (d) $\dfrac{d}{dx}\left[x\tan^{-1}(x)\right]$

Solution:

$$\frac{dy}{dx} = \tan^{-1}(x) + \frac{x}{x^2+1}$$

$\boxed{1}$ (e) $\dfrac{d}{dx}\left[\cot^{-1}(x) + \tan^{-1}(x)\right]$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{1+x^2} + \frac{1}{1+x^2} = 0.$$