MATH 115 **Name:** ____________________

In class work 2, Fall 2022 **Row and Seat:** ____________________

In class work 2 has questions 1 through 3 with a total of 15 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 31 August at 13:15* PM.

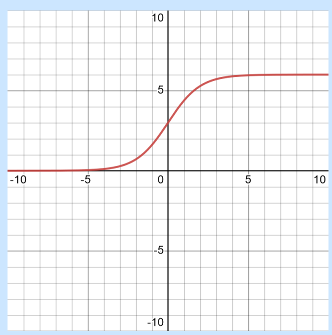
$\boxed{5}$ 1. After graduation, suppose your starting salary is \$46,000. Further, suppose that you expect to earn a 4.1% pay rise each year you work. What is your salary for your 40^{th} year of work? **Hint:** Your salary for your 3^{rd} year of work is $\$46,000 \times 1.041^2$.

Solution: In your 40^{th} year of work, you will have earned 39 pay rises each of 4.1%. Rounded to the nearest penny, your salary for your 40^{th} year of work is

$$46,000 \times 1.041^{39} = 220,463.29.$$

$\boxed{5}$ 2. Let $Q(x) = \frac{6}{1+\exp(-x)}$. As best you can, reproduce the graph here. Using the graph, find range(Q). Be careful: Is zero in the range? What is the solution set to $0 = \frac{6}{1+\exp(-x)}$? Is six in the range? What is the solution set to $6 = \frac{6}{1+\exp(-x)}$?

Solution: Here is the graph:



From the picture, it's not entirely clear if 6 is in the range. Maybe the graph of $y = Q(x)$ stays below the horizontal line $y = 6$ or maybe it touches it, but the picture isn't decisive. To decide, you might try numerical evaluation–say rounding to about 15 decimal places, the number used by most calculators, we have

$$Q(45) = \frac{6}{1+\exp(-45)} = 6.00000000000000.$$

So you might conclude that indeed 6 is in the range. But rounding to 42 decimal places, the story is different:

$$Q(45) = \frac{6}{1+\exp(-45)} = 5.99999999999999999982824888516703638133671.$$

Now what do you think?

To decide if 6 is in the range, let's solve the equation:

$$\begin{aligned}\left[6 = \frac{6}{1+\exp(-x)}\right] &= \left[1+\exp(-x) = 1\right], && \text{(cross multiply)}\\ &= \left[\exp(-x) = 0\right], && \text{(subtract 1)}\\ &= \varnothing && \text{(zero not in range of exp)}\end{aligned}$$

Since the solution set is empty, the number $6 \notin \text{range}(Q)$. Let's determine if 0 is in the range:

$$\begin{aligned}\left[0 = \frac{6}{1+\exp(-x)}\right] &= [0 = 6], && \text{(cross multiply)}\\ &= \varnothing\end{aligned}$$

From the graph and the above calculations, almost surely we have $\text{range}(Q) = (0, 6)$.

$\boxed{5}$ 3. Define $Q(x) = (x-1)^2 + 1$ and $\mathrm{dom}(Q) = [1, \infty)$. Find the formula and the domain of Q^{-1}. Use desmos to graph both Q and Q^{-1}. As best you can, reproduce your graphs here.

Solution: First, let's find the formula for the inverse function. We need to solve

$$\begin{aligned} \left[\left(y = (x-1)^2 + 1\right)\right] &= \left[y - 1 = (x-1)^2\right], && \text{(subtract 1)} \\ &= \left[\left(x - 1 = -\sqrt{y-1}\right) \vee \left(x - 1 = -\sqrt{y-1}\right)\right], \\ &= \left[\left(x = 1 - \sqrt{y-1}\right) \vee \left(x = 1 + \sqrt{y-1}\right)\right]. && \text{(add 1)} \end{aligned}$$

Yikes! There are two solutions–that means the function isn't one-to-one. But wait! Remember the condition $1 \leq x$? The solution $x = 1 - \sqrt{y-1}$ gives a value for x that is *less* than one. So the solution $x = 1 - \sqrt{y-1}$ is rubbish, leaving exactly one solution.

To find the domain of Q^{-1} we need $1 + \sqrt{y-1}$ to be real–that tells us that $y \geq 1$. Putting all together, we have $Q^{-1}(y) = 1 + \sqrt{y-1}$ and $\mathrm{dom}(Q^{-1}) = [1, \infty)$. Here are my graphs: