MATH 115 **Name:** ______________________

In class work 6, Fall 2022 **Row and Seat**:______________________

In class work 6 has questions 1 through 2 with a total of 10 points. Turn in your work at the end of class *on paper*. This assignment is due *Wednesday 27 September at 13:15* PM.

1. As a function of time t (seconds), the position s (feet) of a 22 horsepower 1952 Farmall® Super H tractor moving along a flat piece of Floyd Creek Road is given by $s = 2t^{3/2}$. This relation holds for $1 \leq t \leq 9$. Since our friends in the Physics department might be watching, we should append the correct units to all answers: for example, $\frac{\mathrm{d}s}{\mathrm{d}t} = 46$ ft/sec and $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = 107$ ft/sec^2, and **not** $\frac{\mathrm{d}s}{\mathrm{d}t} = 46$ and $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = 107$.

$\boxed{1}$ (a) Find the *displacement* of the tractor on the time interval $[1,9]$.

Solution:

$$s|_{t\leftarrow 9} - s|_{t\leftarrow 1} = (54-2)\text{ feet} = 52\text{ feet}.$$

$\boxed{1}$ (b) Find the *average velocity* of the tractor on the time interval $[1,9]$.

Solution:

$$\frac{s|_{t\leftarrow 9} - s|_{t\leftarrow 1}}{9-1} = \frac{52}{8} = \frac{13}{2}\frac{\text{ft}}{\text{sec}}.$$

$\boxed{1}$ (c) Find the *velocity* of the tractor at the time $t = 1$. That is, find $\left.\frac{\mathrm{d}s}{\mathrm{d}t}\right|_{t=1}$.

Solution: First we find the velocity. After that, we paste in $t \leftarrow 1$. We have $\frac{\mathrm{d}s}{\mathrm{d}t} = 3t^{1/2}$. So $\left.\frac{\mathrm{d}s}{\mathrm{d}t}\right|_{t\leftarrow 1} = 3\frac{\text{feet}}{\text{sec}}$.

$\boxed{1}$ (d) Find the *velocity* of the tractor at the time $t = 9$. That is, find $\left.\frac{\mathrm{d}s}{\mathrm{d}t}\right|_{t=9}$.

Solution: We have $\left.\frac{\mathrm{d}s}{\mathrm{d}t}\right|_{t\leftarrow 9} = 9\frac{\text{feet}}{\text{sec}}$

$\boxed{1}$ (e) Find the *acceleration* of the tractor at the time $t = 2$. That is, find $\left.\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}\right|_{t=2}$.

Solution: We have

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left[3t^{1/2}\right] = \frac{3}{2}t^{-1/2}.$$

Pasting in $t \leftarrow 2$ gives $\left.\dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}\right|_{t\leftarrow 2} = \dfrac{3}{2\sqrt{2}}$.

$\boxed{1}$ (f) Show that av, where v is the velocity and a is the acceleration of the tractor is a constant[1] for times t in the interval $[1,9]$.

Solution: We have

$$av = \frac{3}{2}t^{-1/2} \times 3t^{1/2} = \frac{9}{2}\frac{\mathrm{ft}^2}{\mathrm{sec}^3}.$$

And that's a constant (that is doesn't depend on t)

[1] For acceleration with a constant power, av is constant. For the most part, internal combustion engines deliver constant power at any speed, making acceleration of an automobile different from acceleration with a constant force. But starting from a stop, initially a vehicle with an internal combustion engine accelerates with a constant force before it transitions to acceleration with a constant power. Physics texts are full of problems involving acceleration with a constant force, but problems involving acceleration with a constant power are obscure. Challenge: Look up the data about a Farmall tractor (weight and power). You'll discover that $s \approx 1.7t^{3/2}$. To simplify the arithmetic, I rounded that to two.

2. The position s of my pet American Fuzzy Lop rabbit Wilber moving along a line as a function of time t is $s = \frac{1}{2}t^2 - 2t + 4$, where we consider positive values of s to be to the right and negative to the left.

$\boxed{1}$ (a) When is Wilber moving to the right? That is, when is $\frac{\mathrm{d}s}{\mathrm{d}t} > 0$?

Solution: First, we have $\frac{\mathrm{d}s}{\mathrm{d}t} = t - 2$. So solving $\frac{\mathrm{d}s}{\mathrm{d}t} > 0$ gives $t > 2$.

$\boxed{1}$ (b) When is Wilber moving to the left? That is, when is $\frac{\mathrm{d}s}{\mathrm{d}t} < 0$?

Solution: This time we solve $\frac{\mathrm{d}s}{\mathrm{d}t} > 0$. The solution is $t < 2$.

$\boxed{1}$ (c) Find Wilber's *speed* when $t = 2$.

Solution: We have

$$\left|\frac{\mathrm{d}s}{\mathrm{d}t}\right|\Bigg|_{t \leftarrow 2} = |2-2| = 0\frac{\text{ft}}{\text{sec}}$$

$\boxed{1}$ (d) When is Wilber's *speed* zero?

Solution: We need To solve $|t-2| = 0$. The solution is $t = 2$.

Addendum In Leibniz notation, it might seem peculiar that the derivative order is a superscript of d in the numerator but a superscript of the variable in the denominator. For example, we write $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$ and **not** $\frac{\mathrm{d}^3 y}{\mathrm{d}^3 x}$.

This notation might seem less peculiar if we were to define second and higher order derivatives by a generalized Newton quotient. That would be a fun thing to do–let's save that for another day.

Another way to make the positioning of the derivative orders seem less peculiar is to think about the physical units of the derivative. If we denote (special notation I just made up) the physical unit of a quantity q by $\langle q \rangle$, we have the rule

$$\left\langle \frac{\mathrm{d}^n y}{\mathrm{d}x^n} \right\rangle = \frac{\langle y \rangle}{\langle x \rangle^n}.$$

The fact that the numerator of the units is $\langle y \rangle$, not $\langle y \rangle^n$ and the denominator of the units is $\langle x \rangle^n$, not $\langle x \rangle$ makes the positioning of the superscripts seem more natural, I think. Here is an example of this rule: if $\langle y \rangle = \text{ft}$ and $\langle t \rangle = \text{sec}$, we have $\left\langle \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} \right\rangle = \frac{\text{ft}}{\text{sec}^2}$.

Finally, for those of you taking CHEM 160: if you learn how to simply multiply and divide physical quantities to make the units come out correct, you'll be able to solve some multiple choice questions without understanding anything about chemistry. Using physical units is a powerful way to check your work, but let's not substitute it for a proper understanding.