MATH 102, Spring 2023 **Name:** _____
In class work 15 **Row and Seat**: _____

"Self-education is, I firmly believe, the only kind of education there is." ISAAC ASIMOV

1. Given that F is a sequence and that $F_1 = 5, F_2 = 8$, and $F_3 = 42$. Is F an *arithmetic* sequence? Explain.

2. Given that Q is a sequence and that $Q_1 = 2, Q_2 = 8$, and $Q_3 = 42$. Is Q an *geometric* sequence? Explain.

3. Given that G is an *arithmetic* sequence and that $G_2 = 8$ and $G_4 = 9$, find a *formula* for G.

4. Given that H is a *geometric* sequence and that $H_2 = 8$ and $H_4 = 9$, find a *formula* for G.

5. Find the *numerical value* of each sum

   (a) $\sum_{k=1}^{46} 1$

   (b) $\sum_{k=1}^{107} (2k+3)$

6. Given that W is an *arithmetic sequence* and that $W_1 = 2$ and $W_2 = 3$, find the numerical value of $\sum_{k=1}^{42} W_k$.

7. After graduation from UNK, you expect a first year salary of $\$70,000$ and you expect a raise of 4% for each of the 42 years that you will work. Your salary for your n^{th} year of work S_n is

$$S_n = 70000 \times 1.04^{n-1}$$

(a) Find the numerical value of S_{42}. This is your salary the year you retire.

(b) Over your career, your total earnings will be

$$\sum_{k-1}^{42} 70000 \times 1.04^{k-1}.$$

Find the numerical value of your total earnings.

8. The human population P of Floyd, Virginia is an exponential function of the years T after the year 2000. Specifically, the population in the years 2000 and 2010 are given in the table

Year	T	P
2000	0	432
2010	10	425

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Figure 1: Human population of Floyd for the years 2000 and 2010.

2 (a) Find the exponential function that matches the given data.

2 (b) Using your exponential function from part 'a,' when will the population of Floyd be 250?

2 9. Find the solution to the linear equations

$$5x + 8y = 14,$$
$$x - y = 3.$$

2 10. Find the solution to the linear equations

$$\begin{aligned} x + y + z &= 6, \\ y + z &= 10, \\ y - z &= 20. \end{aligned}$$

2 11. Larry is saving her money to purchase a 1968 Fender Stratocaster Sunburst guitar. To save for the guitar, he invests \$5,000 into a bank CD with an APY of 5.0%. How long will Larry need to wait until he can purchase the Stratocaster that costs \$7,700?

2 12. In April 2004, Ms Oro purchases one ounce of gold for $647. Today (that is, twenty years later), Ms Oro sells her gold for $1,995. Find the APY for this investment.

13. To save for the purchase of a new 781 Porsche Boxster, Morweena purchases a bond with a value of $80,000 when it matures in 20 years.

2 (a) At the time of purchase, the 30 year APY is 5.0%. Find the *purchase price* of the bond.

2 (b) After five years, Morweena decides to sell her bond and to use the proceeds to purchase an organic vanilla bean farm in Hawaii. At the time of sale, the 30 year APY is 4.0%. Find the sale price of the bond.

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or Π	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Arithmetic properties of $\mathbf{R}$

$(\forall a, b \in \mathbf{R})(a + b = b + a)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$ associative

$(\forall a, b \in \mathbf{R})(ab = ba)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$ associative

$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$ distributive

Intervals

For numbers a and b, we define the intervals

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a, b) = \{x \in \mathbf{R} \mid a \le x < b\}$$
$$(a, b] = \{x \in \mathbf{R} \mid a < x \le b\}$$
$$[a, b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Distance & Midpoint

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The midpoint is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Exponents

For $a, b > 0$ and m, n real:

$$a^0 = 1, \qquad 0^a = 0$$
$$1^a = 1, \qquad a^n a^m = a^{n+m}$$
$$a^n/a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$
$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m/b^m$$

Radicals

$$\sqrt[n]{a} = a^{1/n}$$
$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \quad \text{(provided } a, b \ge 0\text{)}$$
$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$\sqrt[n]{a^n} = \begin{cases} a & n \text{ odd} \\ |a| & n \text{ even} \end{cases}$$

Identities

$$a(b + c) = ab + ac$$
$$((a + b)(c + d)) = ac + ad + bc + bd$$
$$\frac{ab + ac}{a} = b + c \quad \text{(provided } a \ne 0\text{)}$$
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \quad \text{(provided } b, d \ne 0\text{)}$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \text{(provided } a \ge 0, b \ge 0\text{)}$$
$$\ln(ab) = \ln(a) + \ln(b) \quad \text{(provided } a \ge 0, b \ge 0\text{)}$$

Solution of Equations

Algebraic

Suppose X, Y, P, and Q possibly depend on the unknown x; and suppose a, b, and c do not depend on the unknown.

$$[XY = 0] \equiv [X = 0 \textbf{ or } Y = 0]$$
$$[X^2 = Y^2] \equiv [X = Y \textbf{ or } X = -Y]$$
$$\left[\frac{X}{Y} = 0\right] \equiv [X = 0 \textbf{ and } Y \ne 0]$$
$$\left[\frac{X}{Y} = \frac{P}{Q}\right] \equiv [XQ = YP \textbf{ and } Y \ne 0 \textbf{ and } Q \ne 0]$$
$$[|X| = |Y|] \equiv [X = Y \textbf{ or } X = -Y]$$
$$[\sqrt{X} = Y] \equiv [X = Y^2 \textbf{ and } Y \ge 0]$$

For $a \ne 0$,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

Logarithmic and Exponential

$$[\ln(X) = 0] \equiv [X = 1]$$
$$[\mathrm{e}^X = 1] \equiv [X = 0]$$
$$[\log_a(X) = b] \equiv [X = a^b]$$
$$[a^X = a^Y] \equiv [X = Y]$$
$$[\log_a(X) = \log_a(Y)] \equiv [X = Y \textbf{ and } X > 0]$$

Logarithms

For $x > 0$ and $y > 0$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$
$$\log_a(y) + \log_a(y) = \log_a(xy)$$
$$\log(x^z) = z\log(x)$$

Graph Translations

For the graph of $F(x, y) = 0$

- The graph of $F(x - h, y) = 0$ is the graph of $F(x, y) = 0$ translated h units to the right.
- The graph of $F(x, y - k) = 0$ is the graph of $F(x, y) = 0$ translated k units up.
- The graph of $F(x/c, y) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c horizontally.
- The graph of $F(x, y/c) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c vertically.

Circles

Equation of circle centered at (h, k) with radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Expanded the equation is

$$x^2 - 2hx + y^2 - 2ky = r^2 - h^2 - k^2.$$

Parabolas & Lines

The vertex of the parabola $ax^2 + bx + c = y$ is

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right).$$

An equation of the line that contains the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The number $\frac{y_2 - y_1}{x_2 - x_1}$ is the slope.

Function notation

$\text{dom}(F)$	domain of function F
$\text{range}(F)$	range of function F

Domains, Ranges, and Zeros

Function	Domain	Range	Zeros
$\ln, \log$	$(0, \infty)$	$(-\infty, \infty)$	1
exp	$(-\infty, \infty)$	$(0, \infty)$	$\varnothing$
abs	$(-\infty, \infty)$	$(0, \infty)$	0
$\sqrt{}$	$(0, \infty)$	$(0, \infty)$	0
$\sqrt[3]{}$	$(-\infty, \infty)$	$(-\infty, \infty)$	0
floor	$(-\infty, \infty)$	$\mathbf{Z}$	$[0, 1)$
ceiling	$(-\infty, \infty)$	$\mathbf{Z}$	$(-1, 0]$

Compound Interest

Current value A, principal P, APY r, time t, then $A = P(1 + r)^t$

Exponential Growth

The exponential function that contains the points $(t = t_o, y = y_o)$ and $(t = t_1, y = y_1)$ is

$$y = y_o\left(\frac{y_1}{y_o}\right)^{\frac{t - t_o}{t_1 - t_o}}.$$

Common Errors

Error	**Correct or Example**
$x/0 = 0$ or x	$x/0$ is undefined
$-x^2 = x^2$	$-x^2 = -(x^2)$
$a/(b+c) = a/b + a/c$	$1/(1+1) \neq 1/1 + 1/1$
$\frac{a+bx}{a} = 1 + bx$	$\frac{a+bx}{a} = 1 + \frac{bx}{a}$
$(a + b)^2 = a^2 + b^2$	$(a + b)^2 = a^2 + 2ab + b^2$
$\sqrt{a + b} = \sqrt{a} + \sqrt{b}$	$\sqrt{1 + 1} \neq \sqrt{1} + \sqrt{1}$

Summations

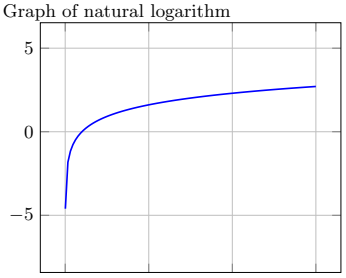
$$\sum_{k=1}^{n} 1 = n$$
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} z^k = \frac{z^{n+1} - z}{z - 1}, z \neq 1$$

Sequences

A sequence is *arithmetic* if $f_n = an + b$; it is *geometric* if $f_n = ca^n$ where a, b, c are real numbers.

Graphs

Graph of natural logarithm



Graph of natural exponential