MATH 102, Spring 2023 **Name:** _____
Review for Exam III **Row and Seat**: _____

Exam III has questions 1 through 9 with a total of 18 points.

1. **Show all of your work.** Do not expect to earn full credit for a correct answer without the needed work.
2. Divine intervention is *not* a substitute for showing your work.
3. If your answer is wrong, but your work shows me that you know the major steps in solving a problem, you will likely earn some partial credit.
4. Your work should convince me that not only could you correctly solve the given problem, but you could also solve any related problem.
5. If a question asks for a sentence, write your answer as an English sentence.
6. No talking, no sharing calculators, and no scratch paper.
7. Turn your phone off and put it out of sight.
8. Clear your desk of everything, except a pencil, eraser, and a calculator.
9. If you never make a mistake, you may use ink; otherwise use a pencil.
10. Do not unstaple the pages of your exam.
11. We'll all start at the same time–it's the polite thing to do.
12. Write your answers in the space provided.
13. If you do not want something graded, erase it or clearly cross it out.
14. You may stare at your feet, your paper, or the ceiling, but nowhere else.
15. If you wear a baseball cap, wear it backwards so I can see your eyes.
16. Work each problem correctly.
17. When you are finished, collect your things, place your exam paper in the folder on the front desk, and quietly leave the room.
18. After you turn in your paper, I will not answer questions about the test until after it is graded.
19. Not knowing the rules is not a valid excuse for not following them.
20. Read all directions and problems carefully.

1. Find the inverse of the function $f(x) = \frac{2x+1}{x-1}$, $x \neq 1$.

2. Sketch a pretty good graph of the equation $y = 2 + \left(\frac{1}{2}\right)^x$.

3. Given that $f(x) = 2x + 3$ and $g(x) = 1 - x$, find the *numerical value* of each of the following

2 (a) $f \circ g(2)$

2 (b) $g \circ f(2)$

2 (c) $g \circ g(1)$

2 4. Given that $f(x) = 2x - 3$ and $g(x) = 1 + x$, find a formula for $f \circ g$.

5. A table of values for functions f and g are

x	$f(x)$
0	3
1	2
2	1
3	0

x	$g(x)$
0	1
1	3
2	0
3	2

Find the *numerical values* of

2 (a) $f \circ g(1)$

2 (b) $g \circ f(1)$

2 (c) Using only the values from the table, find the solution to the equation $g \circ g(x) = 2$.

2 6. Given that E is an exponential function and that $E(0) = 9$ and $E(1) = 10$, find a formula for E.

2 7. At 6 AM, Louisa has 300 mg of caffeine circulating in her blood. After T hours, the amount of caffeine C in her blood is $C = 300 \times 0.8^T$. When Louisa goes to bed at 10 PM, how much caffeine is still in circulation?

8. At age 18, Larry saves invests \$1,000 in a bank stock headquartered in Crooked Creek, Nevada. After T years, he expects the account value V to be $V = 1000 \times 1.082^T$. When Larry retires at age 98, what is the account value?

9. Find each solution set.

(a) $2^x = 2^{1-x}$

(b) $\log(x+1) = \log(2x-4)$

(c) $2^x = -5$

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or Π	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript^C
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Arithmetic properties of $\mathbf{R}$

$(\forall a, b \in \mathbf{R})(a + b = b + a)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$ associative

$(\forall a, b \in \mathbf{R})(ab = ba)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$ associative

$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$ distributive

Intervals

For numbers a and b, we define the intervals

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a, b) = \{x \in \mathbf{R} \mid a \le x < b\}$$
$$(a, b] = \{x \in \mathbf{R} \mid a < x \le b\}$$
$$[a, b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Distance & Midpoint

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The midpoint is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Exponents

For $a, b > 0$ and m, n real:

$$a^0 = 1, \qquad 0^a = 0$$
$$1^a = 1, \qquad a^n a^m = a^{n+m}$$
$$a^n/a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$
$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m/b^m$$

Radicals

$$\sqrt[n]{a} = a^{1/n}$$
$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \quad \text{(provided } a, b \ge 0\text{)}$$
$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$\sqrt[n]{a^n} = \begin{cases} a & n \text{ odd} \\ |a| & n \text{ even} \end{cases}$$

Identities

$$a(b + c) = ab + ac$$
$$((a + b)(c + d)) = ac + ad + bc + bd$$
$$\frac{ab + ac}{a} = b + c \quad \text{(provided } a \ne 0\text{)}$$
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \quad \text{(provided } b, d \ne 0\text{)}$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \text{(provided } a \ge 0, b \ge 0\text{)}$$
$$\ln(ab) = \ln(a) + \ln(b) \quad \text{(provided } a \ge 0, b \ge 0\text{)}$$

Solution of Equations

Algebraic

$$[ab = 0] \equiv [a = 0 \textbf{ or } b = 0]$$
$$[a^2 = b^2] \equiv [a = b \textbf{ or } a = -b]$$
$$\left[\frac{a}{b} = 0\right] \equiv [a = 0 \textbf{ and } b \ne 0]$$
$$\left[\frac{a}{b} = \frac{c}{d}\right] \equiv [ad = bc \textbf{ and } b \ne 0 \textbf{ and } d \ne 0]$$
$$[|a| = |b|] \equiv [a = b \textbf{ or } a = -b]$$
$$[\sqrt{a} = b] \equiv [a = b^2 \textbf{ and } b \ge 0]$$

For $a \ne 0$,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

Exponential

$$[\ln(a) = 0] \equiv [a = 1]$$
$$[e^a = 1] \equiv [a = 0]$$
$$[\ln(a) = b] \equiv [a = e^b]$$

Logarithms

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Graph Translations

For the graph of $F(x, y) = 0$

- The graph of $F(x - h, y) = 0$ is the graph of $F(x, y) = 0$ translated h units to the right.
- The graph of $F(x, y - k) = 0$ is the graph of $F(x, y) = 0$ translated k units up.
- The graph of $F(x/c, y) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c horizontally.
- The graph of $F(x, y/c) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c vertically.

Circles

Equation of circle centered at (h, k) with radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Expanded the equation is

$$x^2 - 2hx + y^2 - 2ky = r^2 - h^2 - k^2.$$

Parabolas & Lines

The vertex of the parabola $ax^2 + bx + c = y$ is

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right).$$

An equation of the line that contains the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The number $\frac{y_2 - y_1}{x_2 - x_1}$ is the slope.

Function notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F

Domains, Ranges, and Zeros

Function	Domain	Range	Zeros
$\ln, \log$	$(0, \infty)$	$(-\infty, \infty)$	1
exp	$(-\infty, \infty)$	$(0, \infty)$	$\varnothing$
abs	$(-\infty, \infty)$	$(0, \infty)$	0
$\sqrt{}$	$(0, \infty)$	$(0, \infty)$	0
$\sqrt[3]{}$	$(-\infty, \infty)$	$(-\infty, \infty)$	0
floor	$(-\infty, \infty)$	$\mathbf{Z}$	$[0, 1)$
ceiling	$(-\infty, \infty)$	$\mathbf{Z}$	$(-1, 0]$

Compound Interest

Interest rate r compounded n times per year

$$A = P(1 + r/n)^{nt}$$

Continuous compounding:

$$A = P\mathrm{e}^{rt}$$

[c]

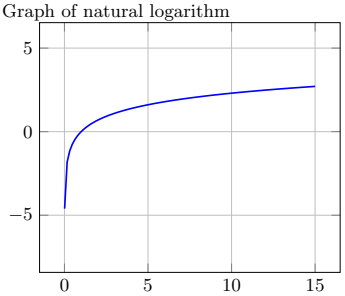
Exponential Growth

The exponential function that contains the points $(t = t_o, y = y_o)$ and $(t = t_1, y = y_1)$ is

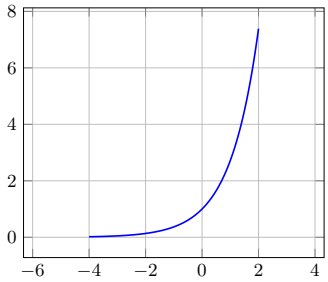
$$y = y_o\left(\frac{y_1}{y_o}\right)^{\frac{t - t_o}{t_1 - t_o}}.$$

Graphs

Graph of natural logarithm



Graph of natural exponential



Common Errors

Error	Correct or Example
$x/0 = 0$ or x	$x/0$ is undefined
$-x^2 = x^2$	$-x^2 = -(x^2)$
$a/(b+c) = a/b + a/c$	$1/(1+1) \neq 1/1 + 1/1$
$\frac{a+bx}{a} = 1 + bx$	$\frac{a+bx}{a} = 1 + \frac{bx}{a}$
$(a+b)^2 = a^2 + b^2$	$(a+b)^2 = a^2 + 2ab + b^2$
$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$	$\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$