MATH 102, Spring 2023 **Name:** ____________________

In class work 9 **Row and Seat**: ____________________

Mistakes are a fact of life. It is the response to the error that counts. NIKKI GIOVANNI

In class work 9 has questions 1 through 5 with a total of 12 points. This assignment is due at the end of the class period (9:55 AM). This assignment is printed on **both** sides of the paper.

1. Find the solution set to $\frac{2x+3}{4x+1} \le 1$ by following these steps.

$\boxed{1}$ (a) Use algebra tools to find an equivalent inequality of the form $\frac{P(x)}{Q(x)} \le 0$, where P and Q are polynomials.

Solution: We have

$$\begin{aligned}\left[\frac{2x+3}{4x+1} \le 1\right] &= \left[\frac{2x+3}{4x+1} - 1 \le 0\right], && \text{(subtract 1)}\\ &= \left[\frac{2x+3}{4x+1} - \frac{4x+1}{4x+1} \le 0\right], && \text{(make a common denominator)}\\ &= \left[\frac{-2x+2}{4x+1} \le 0\right]. && \text{(combine numerators)}\end{aligned}$$

$\boxed{1}$ (b) Find all x-intercepts and all VAs for $\frac{P(x)}{Q(x)}$.

Solution: To find the x-intercepts we set the numerator to zero and solve:

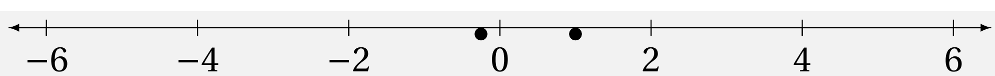
$$[-2x+2=0] = [x=1].$$

To find the VA we set the denominator to zero and solve:

$$[4x+1=0] = \left[x = -\frac{1}{4}\right]. \tag{1}$$

$\boxed{1}$ (c) Put all x-intercepts and VAs on to a number line.

Solution:



$\boxed{1}$ (d) Build the chart with columns for the interval, the test number, evaluation at the test number, and the true/false value.

Solution:

Interval	Test	$\frac{2x+3}{4x+1} \leq 1$	True or False
$(-\infty, -1/4)$	-1	$-\frac{1}{3} \leq 1$	True
$(-1/4, 1)$	0	$3 \leq 1$	False
$(1, \infty)$	2	$\frac{7}{9} \leq 1$	True

$\boxed{1}$ (e) Test each interval endpoint for inclusion or exclusion into the solution set.

Solution:

Endpoint	$\frac{2x+3}{4x+1} \leq 1$	True or False
$-1/4$	dne	False
1	$1 \leq 1$	True

$\boxed{1}$ (f) Express the solution set in either interval notation, pictorially, or set builder notation.

Solution: In interval notation the solution set is $(-\infty, -\frac{1}{4}) \cup [1, \infty)$. In set builder notation, it is $\{x | x < -\frac{1}{4} \text{ or } x \geq 1\}$.

2. Find the vertex of each parabola.

$\boxed{1}$ (a) $y - 2 = 5(x+1)^2$.

Solution: The easy way is to match to $y - k = a(x-h)^2$. The match is $k = 2$, $a = 5$, and $h = -1$. The vertex is the point $(x = -1, y = 2)$.

$\boxed{1}$ (b) $y = 3x^2 + 2x + 9$

Solution: This time we match to $y = ax^2 + bx + c$. The match is $a = 3, b = 2$ and $c = 9$. The vertex is the point

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right) = (x = -\frac{1}{3}, y = \frac{26}{3}).$$

$\boxed{1}$ (c) $y = x(1-x)$

Solution: The easy way is to remember that the x-coordinate of the vertex is the midpoint of the x-intercepts. The x-intercepts are 0 and 1, so the x-coordinate of the vertex is $x = 1/2$. To find the y-coordinate of the vertex, paste $x = 1/2$ into $y = x(1-x)$. So the vertex is $(x = 1/2, y = 1/4)$.

3. Morwenna grows and sells organic mustard greens. The number q of bunches of greens she can sell in a day is related to the selling price of p dollars per bunch by $q = 20 - 2p$.

$\boxed{1}$ (a) Express the *revenue* R she gets for selling q bunches of greens for p dollars per bunch as a function of the selling price.

Solution:

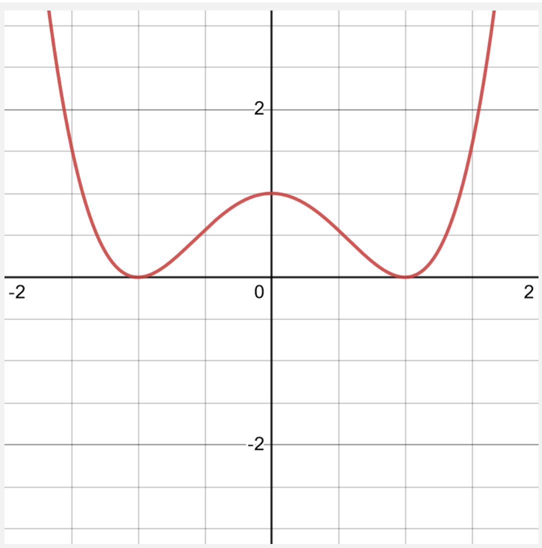
$$R = pq = p(20 - 2p). \tag{2}$$

$\boxed{1}$ (b) Find the selling price p that will maximize Morwenna's daily revenue.

Solution: The graph of revenue is a downward facing parabola with intercepts $p = 0$ and $p = 10$. To maximize the revenue, Morwenna should sell each bunch of mustard greens for 5 dollars (the midpoint of 0 and 10).

4. Sketch a pretty good graph of $y = (x-1)^2(x+1)^2$.

Solution:



$\boxed{1}$ 5. Given that P is a third degree polynomial such that (a) P has a zero with multiplicity of 2 at 5; (b) P has a zero with multiplicity 1 at -2; and $P(0) = 1$, find an equation for P.

Solution: Using the data about the degree, zeros and their multiplicities, the polynomial is $P(x) = a(x-5)^2(x+2)$, where a is some number. Using $P(0) = 1$, gives $a = 1/50$. So $P(x) = \frac{1}{50}(x-5)^2(x+2)$,