Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript^C
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Arithmetic properties of R

$$(\forall a, b \in \mathbf{R})(a + b = b + a) \qquad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c) \qquad \text{associative}$$
$$(\forall a, b \in \mathbf{R})(ab = ba) \qquad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c) \qquad \text{associative}$$
$$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac) \qquad \text{distributive}$$

Intervals

For numbers a and b, we define the intervals

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a, b) = \{x \in \mathbf{R} \mid a \le x < b\}$$
$$(a, b] = \{x \in \mathbf{R} \mid a < x \le b\}$$
$$[a, b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Distance & Midpoint

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The midpoint is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Exponents

For $a, b > 0$ and m, n real:

$$a^0 = 1, \qquad 0^a = 0$$
$$1^a = 1, \qquad a^n a^m = a^{n+m}$$
$$a^n/a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$
$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m/b^m$$

Radicals

$$\sqrt[n]{a} = a^{1/n}$$
$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} \quad (\text{provided } a, b \ge 0)$$
$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$\sqrt[n]{a^n} = \begin{cases} a & n \text{ odd} \\ |a| & n \text{ even} \end{cases}$$

Identities

$$a(b + c) = ab + ac$$
$$((a + b)(c + d)) = ac + ad + bc + bd$$
$$\frac{ab + ac}{a} = b + c \quad (\text{provided } a \ne 0)$$
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \quad (\text{provided } b, d \ne 0)$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (\text{provided } a \ge 0, b \ge 0)$$
$$\ln(ab) = \ln(a) + \ln(b) \quad (\text{provided } a \ge 0, b \ge 0)$$

Solution of Equations

Algebraic

Suppose $X, Y, P,$ and Q possibly depend on the unknown x; and suppose $a, b,$ and c do not depend on the unknown.

$$[XY = 0] \equiv [X = 0 \textbf{ or } Y = 0]$$
$$[X^2 = Y^2] \equiv [X = Y \textbf{ or } X = -Y]$$
$$\left[\frac{X}{Y} = 0\right] \equiv [X = 0 \textbf{ and } Y \ne 0]$$
$$\left[\frac{X}{Y} = \frac{P}{Q}\right] \equiv [XQ = YP \textbf{ and } Y \ne 0 \textbf{ and } Q \ne 0]$$
$$[|X| = |Y|] \equiv [X = Y \textbf{ or } X = -Y]$$
$$[\sqrt{X} = Y] \equiv [X = Y^2 \textbf{ and } Y \ge 0]$$

For $a \ne 0$,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

Logarithmic and Exponential

$$[\ln(X) = 0] \equiv [X = 1]$$
$$[e^X = 1] \equiv [X = 0]$$
$$[\log_a(X) = b] \equiv [X = a^b]$$
$$[a^X = a^Y] \equiv [X = Y]$$
$$[\log_a(X) = \log_a(Y)] \equiv [X = Y \textbf{ and } X > 0]$$

Logarithms

For $x > 0$ and $y > 0$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$
$$\log_a(y) + \log_a(y) = \log_a(xy)$$
$$\log(x^z) = z\log(x)$$

Graph Translations

For the graph of $F(x, y) = 0$

- The graph of $F(x-h, y) = 0$ is the graph of $F(x, y) = 0$ translated h units to the right.
- The graph of $F(x, y-k) = 0$ is the graph of $F(x, y) = 0$ translated k units up.
- The graph of $F(x/c, y) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c horizontally.
- The graph of $F(x, y/c) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c vertically.

Circles

Equation of circle centered at (h, k) with radius r is

$$(x-h)^2 + (y-k)^2 = r^2.$$

Expanded the equation is

$$x^2 - 2hx + y^2 - 2ky = r^2 - h^2 - k^2.$$

Parabolas & Lines

The vertex of the parabola $ax^2 + bx + c = y$ is

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right).$$

An equation of the line that contains the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The number $\frac{y_2 - y_1}{x_2 - x_1}$ is the slope.

Function notation

$\operatorname{dom}(F)$	domain of function F
$\operatorname{range}(F)$	range of function F

Domains, Ranges, and Zeros

Function	Domain	Range	Zeros
ln, log	$(0, \infty)$	$(-\infty, \infty)$	1
exp	$(-\infty, \infty)$	$(0, \infty)$	$\varnothing$
abs	$(-\infty, \infty)$	$(0, \infty)$	0
$\sqrt{\ }$	$(0, \infty)$	$(0, \infty)$	0
$\sqrt[3]{\ }$	$(-\infty, \infty)$	$(-\infty, \infty)$	0
floor	$(-\infty, \infty)$	$\mathbf{Z}$	$[0, 1)$
ceiling	$(-\infty, \infty)$	$\mathbf{Z}$	$(-1, 0]$

Compound Interest

Current value A, principal P, APY r, time t, then $A = P(1+r)^t$

Exponential Growth

The exponential function that contains the points $(t = t_o, y = y_o)$ and $(t = t_1, y = y_1)$ is

$$y = y_o\left(\frac{y_1}{y_o}\right)^{\frac{t-t_o}{t_1-t_o}}.$$

Common Errors

Error	**Correct or Example**
${}^{x}/_{0} = 0$ or x	${}^{x}/_{0}$ is undefined
$-x^2 = x^2$	$-x^2 = -(x^2)$
${}^{a}/_{(b+c)} = {}^{a}/_{b} + {}^{a}/_{c}$	${}^{1}/_{(1+1)} \neq {}^{1}/_{1} + {}^{1}/_{1}$
${}^{a+bx}/_{a} = 1 + bx$	${}^{a+bx}/_{a} = 1 + {}^{bx}/_{a}$
$(a+b)^2 = a^2 + b^2$	$(a+b)^2 = a^2 + 2ab + b^2$
$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$	$\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$

Summations

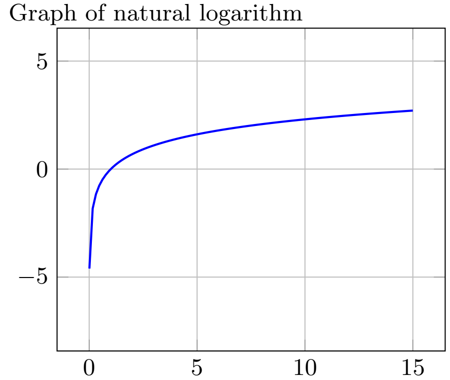
$$\sum_{k=1}^{n} 1 = n$$
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} z^k = \frac{z^{n+1} - z}{z - 1}, z \neq 1$$

Sequences

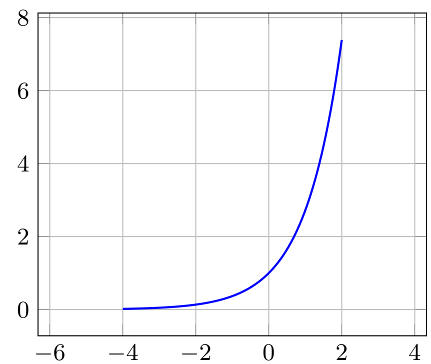
A sequence is *arithmetic* if $f_n = an + b$; it is *geometric* if $f_n = ca^n$ where a, b, c are real numbers.

Graphs

Graph of natural logarithm



Graph of natural exponential



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