MATH 102, Spring 2023 **Name:** ______________________________

In class work 11 **Row and Seat:**______________________________

> *"Study hard what interests you the most in the most undisciplined, irreverent and original manner possible."* RICHARD FEYNMANN

In class work 11 has questions 1 through 4 with a total of 10 points. This assignment is due at the end of the class period (9:55 AM). This assignment is printed on **both** sides of the paper.

$\boxed{2}$ 1. Given that E is an exponential function and that $E(0) = 9$ and $E(2) = 11$, find a formula for E.

Solution: The formula for E has the form $E(x) = Ca^x$, where C is the initial value and a is the growth rate. The initial value is $E(0)$, so $C = 9$. We have to work a bit harder to find the growth rate. We have

$$\frac{11}{9} = \frac{E(2)}{E(0)} = \frac{Ca^2}{Ca^0} = a^2.$$

Solving this for a gives $a = \pm\frac{\sqrt{11}}{3}$. But the growth rate is always a positive number, so $a = \frac{\sqrt{11}}{3}$. Gathering all this up, we have

$$E(x) = 9\left(\frac{\sqrt{11}}{3}\right)^x.$$

The parenthesis surrounding $\frac{\sqrt{11}}{3}$ in $9\left(\frac{\sqrt{11}}{3}\right)^x$ makes it clear that the entire fraction is raised to the power x. Without the parenthesis, the meaning of the expression is unclear.

$\boxed{2}$ 2. Given that H is an exponential function with initial value of 8 and that

$$\frac{H(4)}{H(3)} = \frac{2}{3},$$

find a formula for H.

Solution: For any exponential function H, the growth rate is the quotient $\frac{H(x+1)}{H(x)}$, where x is any real number. Specializing this to $x = 3$, we see that the growth rate is $\frac{H(3+1)}{H(3)} = \frac{H(4)}{H(3)} = \frac{2}{3}$. Since the initial value is 8, we have

$$H(x) = 8 \times \left(\frac{2}{3}\right)^x.$$

$\boxed{2}$ 3. At 6 AM, Louisa has 340 mg of caffeine circulating in her blood. After T hours, the amount of caffeine C in her blood is $C = 340 \times 0.9^T$. When Louisa goes to bed at 10 PM, how much caffeine is still in circulation?

Solution: We have

$$C = 340 \times 0.9^{16} = 63.0\,\text{mg}.$$

I rounded the value to the nearest tenth—given the context, that is reasonable.

4. Intense physical exercise can temporarily raise the amount of creatinine in the blood above its normal level.[1] After intense exercise, Martin's blood creatinine level C is

$$C = 0.9 + 0.2 \times \left(\frac{1}{2}\right)^{T/4},$$

where T is the number of hours after exercise.

$\boxed{2}$ (a) Make a table of Martin's creatine levels after 2, 4, 8, and 16 hours.

Solution: This problem is a calculator exercise—be sure you know how to use your own calculator to evaluate exponential functions.

Time (hours)	Creatinine (mg/dL)
2	1.04
4	1.00
8	0.95
16	0.91

I rounded these values to the nearest hundredth—given that the problem involved medical data, that seemed reasonable. The problem statement

[1] I suggest that you *not* take medical advice from a mathematician, but if you are scheduled for a kidney function test, skipping rope for 60 minutes followed by 20 minutes of burpees the day before might lead to worry and additional medical tests.

didn't include the units on the creatinine—if you need to know, it's mg/dL, or milligrams per deciliter of blood.

$\boxed{2}$ (b) Many many hours after intense exercise, what is Martin's blood creatinine level? Specifically, what is the horizontal asymptote toward infinity to the equation $C = 0.9 + 0.2 \times \left(\frac{1}{2}\right)^{T/4}$?

Solution: For large T, the term $0.2 \times \left(\frac{1}{2}\right)^{T/4}$ is close to zero. So for very large T, we have $C \approx 0.9$.