MATH 102, Spring 2023 **Name:** _____
In class work 11 **Row and Seat:** _____

"Study hard what interests you the most in the most undisciplined, irreverent and original manner possible." RICHARD FEYNMANN

In class work 11 has questions 1 through 4 with a total of 10 points. This assignment is due at the end of the class period (9:55 AM). This assignment is printed on **both** sides of the paper.

2 1. Given that E is an exponential function and that $E(0) = 9$ and $E(2) = 11$, find a formula for E.

Solution: The formula for E has the form $E(x) = Ca^x$, where C is the initial value and a is the growth rate. The initial value C is $E(0)$, so $C = 9$. We have to work a bit harder to find the growth rate. We have

$$\frac{11}{9} = \frac{E(2)}{E(0)} = \frac{Ca^2}{Ca^0} = a^2.$$

Solving this for a gives $a = \pm\frac{\sqrt{11}}{3}$. But the growth rate is always a positive number, so $a = \frac{\sqrt{11}}{3}$. Gathering all this up, we have

$$E(x) = 9\left(\frac{\sqrt{11}}{3}\right)^x.$$

2 2. Given that H is an exponential function with initial value of 8 and that

$$\frac{H(4)}{H(3)} = \frac{2}{3},$$

find a formula for H.

Solution: For any exponential function H, the growth rate is the quotient $\frac{H(x+1)}{H(x)}$. Specializing this to $x = 3$, we see that the growth rate is $\frac{2}{3}$. Since the initial value is 8, we have

$$H(x) = 8 \times \left(\frac{2}{3}\right)^x.$$

2 3. At 6 AM, Louisa has 340 mg of caffeine circulating in her blood. After T hours, the amount of caffeine C in her blood is $C = 340 \times 0.9^T$. When Louisa goes to bed at 10 PM, how much caffeine is still in circulation?

Solution: We have

$$C = 340 \times 0.9^{16} = 63.0\text{mg}.$$

4. Intense physical exercise can temporarily raise the amount of creatine in the blood above its normal level.[1] After intense exercise, Martin's blood creatine level C is

$$C = 0.9 + 0.2 \times \left(\frac{1}{2}\right)^{T/4},$$

where T is the number of hours after exercise.

2 (a) Make a table of Martin's creatine levels after 2,4,8, and 16 hours.

Solution:

T	Creatine
2	1.04
4	1.00
8	0.95
16	0.91

2 (b) Many many hours after intense exercise, what is Martin's blood creatine level? Specifically, what is the horizontal asymptote toward infinity to the equation $C = 0.9 + 0.2 \times \left(\frac{1}{2}\right)^{T/4}$?

Solution: For large T, the term $0.2 \times \left(\frac{1}{2}\right)^{T/4}$ is close to zero. So for very large T, we have $C \approx 0.9$.

[1] I suggest that you *not* take medical advice from a mathematician, but if you are scheduled for a kidney function test, skipping rope for 60 minutes followed by 20 minutes of burpees the day before might lead to worry and additional medical tests.