MATH 102, Spring 2023 **Name:** ______________________

In class work 14 **Row and Seat**:______________________

"Sometimes it's a little better to travel than to arrive." ROBERT M. PIRSIG

In class work 14 has questions 1 through 3 with a total of 8 points. This assignment is due at the end of the class period (9:55 AM). This assignment is printed on **both** sides of the paper.

1. The human population P of Long Pine, Nebraska is an exponential function of the years T after the year 2000. Specifically, the population in the years 2000 and 2010 are given in the table

Year	T	P
2000	0	341
2010	10	305

Figure 1: Human population of Long Pine, Nebraska for the years 2000 and 2010.

$\boxed{2}$ (a) Find the exponential function that matches the given data.

Solution: We just need to match to the general result in the QRS. That gives

$$P = 341 \times \left(\frac{305}{341}\right)^{T/10}.$$

Converting this to the form $P = 341 \times \mathrm{e}^{-0.011157\cdots \times T}$ *isn't* required by the problem statement; so LIB. And converting this to the form $P = 341 \times 0.98890\cdots^{T}$, *isn't* required by the problem statement; so again LIB.

$\boxed{2}$ (b) Using your exponential function from part 'a,' when will the population of Long Pine be 280?

Solution: We need to solve $280 = 341 \times \left(\frac{305}{341}\right)^{T/10}$ for T. Some (many) students will be more comfortable using decimal approximations to various quotients as they go. That's OK.

$$\left[280 = 341 \times \left(\frac{305}{341}\right)^{T/10}\right] = \left[\frac{280}{341} = \left(\frac{305}{341}\right)^{T/10}\right] \qquad (\div\ 341)$$
$$= \left[\log(\frac{280}{341}) = \log\left(\left(\frac{305}{341}\right)^{T/10}\right)\right] \qquad \text{(log of left and right)}$$
$$= \left[\log\left(\frac{280}{341}\right) = \frac{T}{10}\log\left(\frac{305}{341}\right)\right] \qquad \text{(log property)}$$
$$= \left[T = 10 \times \frac{\log\left(\frac{280}{341}\right)}{\log\left(\frac{305}{341}\right)}\right], \qquad \text{(divide)}$$
$$= \left[T \approx 17.7\text{years}\right].$$

$\boxed{2}$ 2. Find the solution to the linear equations

$$5x + 8y = 14,$$
$$2x - 2y = 3.$$

Solution: Let's use our matrix method. Ordering the unknowns as x (first column) and y (second column), our system in matrix form is

$$\begin{bmatrix} 5 & 8 & 14 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} \\ 2 \quad R_2 \leftarrow 2R_1 - 5R_2 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & 26 & 13 \end{bmatrix}$$

The second equation is $26y = 13$. So $y = \frac{1}{2}$. Pasting that into the first equation gives $5x + 4 = 14$. So $x = 2$.

Should we check our work? Sure.

$$[5x + 8y = 14] = \left[5 \times 2 + 8 \times \frac{1}{2} = 14\right] = [10 + 4 = 14] = \text{True!}$$

And one more time

$$[2x - 2y = 3] = \left[2 \times 2 - 2 \times \frac{1}{2} = 3\right] = [4 - 1 = 3] = \text{True!}$$

$\boxed{2}$ 3. Find the solution to the linear equations

$$x + y + z = 14,$$
$$y - z = 10,$$
$$2z = 8.$$

Solution: Ha! Let's be lazy and start at the bottom and work up! That gives $z = 4$; so $y - 4 = 10$. So $y = 14$. And finally $x + 14 + 4 = 14$. So $x = -4$.