

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	\emptyset	integers	\mathbf{Z}
real numbers	\mathbf{R}	positive integers	$\mathbf{Z}_{>0}$
ordered pairs	\mathbf{R}^2	positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol	Meaning	Symbol
is a member	\in	union	\cup
subset	\subset	complement	superscript \complement
intersection	\cap	set minus	\setminus

Logic symbols

Meaning	Symbol	Meaning	Symbol
negation	\neg	equivalent	\equiv
and	\wedge	iff	\iff
or	\vee	for all	\forall
implies	\implies	there exists	\exists

Arithmetic properties of \mathbf{R}

$(\forall a, b \in \mathbf{R})(a + b = b + a)$	commutativity
$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$	associative
$(\forall a, b \in \mathbf{R})(ab = ba)$	commutativity
$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$	associative
$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$	distributive

Intervals

For numbers a and b , we define the intervals

$$\begin{aligned}(a, b) &= \{x \in \mathbf{R} \mid a < x < b\} \\ [a, b] &= \{x \in \mathbf{R} \mid a \leq x \leq b\} \\ (a, b] &= \{x \in \mathbf{R} \mid a < x \leq b\} \\ [a, b] &= \{x \in \mathbf{R} \mid a \leq x \leq b\}\end{aligned}$$

Distance & Midpoint

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The midpoint is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Exponents

For $a, b > 0$ and m, n real:

$$\begin{aligned}a^0 &= 1, & 0^a &= 0 \\ 1^a &= 1, & a^n a^m &= a^{n+m} \\ a^n / a^m &= a^{n-m}, & (a^n)^m &= a^{n \cdot m} \\ a^{-m} &= 1/a^m, & (a/b)^m &= a^m / b^m\end{aligned}$$

Radicals

$$\begin{aligned}\sqrt[n]{a} &= a^{1/n} \\ \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} \quad (\text{provided } a, b \geq 0) \\ \sqrt[m]{\sqrt[n]{a}} &= \sqrt[mn]{a} \\ \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\ \sqrt[n]{a^n} &= \begin{cases} a & n \text{ odd} \\ |a| & n \text{ even} \end{cases}\end{aligned}$$

Identities

$$\begin{aligned}a(b + c) &= ab + ac \\ ((a + b)(c + d)) &= ac + ad + bc + bd \\ \frac{ab + ac}{a} &= b + c \quad (\text{provided } a \neq 0) \\ \frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{ad}{bc} \quad (\text{provided } b, d \neq 0) \\ \sqrt{ab} &= \sqrt{a}\sqrt{b} \quad (\text{provided } a \geq 0, b \geq 0) \\ \ln(ab) &= \ln(a) + \ln(b) \quad (\text{provided } a \geq 0, b \geq 0)\end{aligned}$$

Solution of Equations

Algebraic

Suppose X, Y, P , and Q possibly depend on the unknown x ; and suppose a, b , and c do not depend on the unknown.

$$\begin{aligned}[XY = 0] &\equiv [X = 0 \text{ or } Y = 0] \\ [X^2 = Y^2] &\equiv [X = Y \text{ or } X = -Y] \\ \left[\frac{X}{Y} = 0\right] &\equiv [X = 0 \text{ and } Y \neq 0] \\ \left[\frac{X}{Y} = \frac{P}{Q}\right] &\equiv [XQ = YP \text{ and } Y \neq 0 \text{ and } Q \neq 0] \\ [|X| = |Y|] &\equiv [X = Y \text{ or } X = -Y] \\ [\sqrt{X} = Y] &\equiv [X = Y^2 \text{ and } Y \geq 0]\end{aligned}$$

For $a \neq 0$,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

Logarithmic and Exponential

$$\begin{aligned}[\ln(X) = 0] &\equiv [X = 1] \\ [e^X = 1] &\equiv [X = 0] \\ [\log_a(X) = b] &\equiv [X = a^b] \\ [a^X = a^Y] &\equiv [X = Y] \\ [\log_a(X) = \log_a(Y)] &\equiv [X = Y]\end{aligned}$$

Logarithms

For $x > 0$ and $y > 0$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\log_a(y) + \log_a(x) = \log_a(xy)$$

$$\log(x^z) = z \log(x)$$

Graph Translations

For the graph of $F(x, y) = 0$

- The graph of $F(x-h, y) = 0$ is the graph of $F(x, y) = 0$ translated h units to the right.
- The graph of $F(x, y-k) = 0$ is the graph of $F(x, y) = 0$ translated k units up.
- The graph of $F(x/c, y) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c horizontally.
- The graph of $F(x, y/c) = 0$ is the graph of $F(x, y) = 0$ stretched a factor of c vertically.

Circles

Equation of circle centered at (h, k) with radius r is

$$(x-h)^2 + (y-k)^2 = r^2.$$

Expanded the equation is

$$x^2 - 2hx + y^2 - 2ky = r^2 - h^2 - k^2.$$

Parabolas & Lines

The vertex of the parabola $ax^2 + bx + c = y$ is

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right).$$

An equation of the line that contains the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The number $\frac{y_2 - y_1}{x_2 - x_1}$ is the slope.

Function notation

$\text{dom}(F)$	domain of function F
$\text{range}(F)$	range of function F

Domains, Ranges, and Zeros

Function	Domain	Range	Zeros
ln, log	$(0, \infty)$	$(-\infty, \infty)$	1
exp	$(-\infty, \infty)$	$(0, \infty)$	\emptyset
abs	$(-\infty, \infty)$	$(0, \infty)$	0
$\sqrt{}$	$(0, \infty)$	$(0, \infty)$	0
$\sqrt[3]{}$	$(-\infty, \infty)$	$(-\infty, \infty)$	0
floor	$(-\infty, \infty)$	\mathbf{Z}	$[0, 1)$
ceiling	$(-\infty, \infty)$	\mathbf{Z}	$(-1, 0]$

Compound Interest

Interest rate r compounded n times per year

$$A = P(1 + r/n)^{nt}$$

Continuous compounding:

$$A = Pe^{rt}$$

Exponential Growth

The exponential function that contains the points $(t = t_o, y = y_o)$ and $(t = t_1, y = y_1)$ is

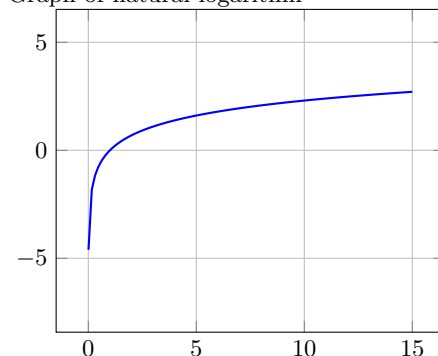
$$y = y_o \left(\frac{y_1}{y_o}\right)^{\frac{t-t_o}{t_1-t_o}}.$$

Common Errors

Error	Correct or Example
$x/0 = 0$ or x	$x/0$ is undefined
$-x^2 = x^2$	$-x^2 = -(x^2)$
$a/(b+c) = a/b + a/c$	$1/(1+1) \neq 1/1 + 1/1$
$a+bx/a = 1 + bx$	$a+bx/a = 1 + bx/a$
$(a+b)^2 = a^2 + b^2$	$(a+b)^2 = a^2 + 2ab + b^2$
$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$	$\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$

Graphs

Graph of natural logarithm



Graph of natural exponential

