

# College Algebra

## MATH 102-01

### Spring 2023

**Instructor:** Barton Willis, PhD, Professor of Mathematics

**Office:** Discovery Hall, Room 368

**☎:** 308-865-8868

**✉:** willisb@unk.edu

**Zoom for classes:** For Zoom class meetings, use the Meeting ID: 616 568 5706.

**Office Hours:** Monday, Wednesday, and Friday 10:00AM-11:00AM, Tuesday and Thursday 12:00 noon-2:00PM, and by appointment.

### Class meeting time and place

This class meets Monday, Wednesday, and Friday in DSCH 116 from 9:05 AM to 9:55 AM.

### Course Resources

Our textbook is *College Algebra*, 11<sup>th</sup> edition, by Michael Sullivan with access to MyLab Math; ISBN: 9780136483151

### Important Dates

<b>First Homework due</b>	<b>February 4</b>
<b>Exam 1</b>	<b>February 24</b>
<b>Exam 2</b>	<b>March 31</b>
<b>Exam 3</b>	<b>April 21</b>
<b>Final exam</b>	<b>May 17, 8:00 AM–10:00 AM</b>

### Grading

Your course grade will be based on online homework, in class work, three midterm exams, and a comprehensive final exam; specifically:

<b>In class work</b> 14 ten point assignments	140 (total)
<b>Online homework</b> 31 five point assignments	155 (total)
<b>Mid-term exams 1,2, and 3</b> 100 points each	300 (total)
<b>Comprehensive Final exam</b>	150 (total)

If it is necessary to adjust the number of homework assignments, your homework point total will be scaled to a total of 165. For example, if we have only ten homework sets, your homework score will be scaled by a factor of 165/150. The same holds for class participation.

The following table shows the *minimum* number of points (out of 675) that are required for each of the twelve letter grades D- through A+. For example, a point total of 581 points will earn you a grade of B+, and a point total of 608 points will earn you a grade of A-. A point total of 404 or less earns you a failing course grade.

D- .....	405	B- .....	540
D .....	425	B .....	560
D+ .....	446	B+ .....	581
C- .....	473	A- .....	608
C .....	493	A .....	628
C+ .....	513	A+ .....	662

## **In class work**

Each week, for a portion of class time we will do in class work. Either in a small group or individually, you will complete a worksheet in class. Generally, it will be due at the end of the class period.

Most weeks, we'll do in class work on Wednesday.

## **Prerequisite**

The prerequisite for this class is either an earned passing grade (D- or higher) in MATH 101 or Math ACT score of 20 or greater.

## **Catalog description**

**College Algebra (3 credit hours)** A college level algebra course which includes a study of linear equations and inequalities, relations and functions, graphing of linear and quadratic functions, polynomial and rational functions, logarithmic and exponential functions, systems of equations, matrices, sequences and series, and other selected topics all of which are necessary for the study of calculus.

## **LOPER 4 Learning Outcomes**

College Algebra is a LOPER 4 General Studies class. The Learning Outcomes for all LOPER 4 classes are

- (a) Can describe problems using mathematical, statistical, or programming language.
- (b) Can solve problems using mathematical, statistical, or programming techniques.
- (c) Can construct logical arguments using mathematical, statistical, or programming concepts.
- (d) Can interpret and express numerical data or graphical information using mathematical, statistical, or programming concepts and methods.

## **College Algebra specific learning outcomes**

In this course students will learn the concepts of relations and functions, properties and graphing of various types functions, and other topics such as systems of equations, matrices, and/or sequences and series. Students will investigate applied problems from various disciplines and algebraic principles used in the study of calculus, linear programming, and other areas of mathematics. Through this course, students will be able to

- (a) Solve algebraic equations and inequalities in one or two variables and illustrate these solutions graphically on the real number line and/or the rectangular coordinate system.
- (b) Use the Pythagorean Theorem to develop distance computations and circular relationships in two variables on the coordinate system.
- (c) Understand the concepts of slope and the linear relationship, using graphic, algebraic, and verbal descriptions of this relationship.
- (d) Apply the definition of the function, function notation, and the connections between graphic, algebraic, and verbal descriptions of functions.
- (e) Understand the domain and the range of a function, how to determine domain using equations and graphs of functions, and to determine range of a function by graph analysis. The students will be able to recognize other properties of several types of functions.
- (f) Understand the concepts of composite functions, one-to-one functions and inverse functions, and their importance in mathematics.

- (g) Sketch and analyze graphs of functions and use transformations. The students will be able to analyze the relationships between the parent and the resulting functions using analytic and graphical techniques. These function types include general functions, linear functions, piecewise-defined functions, quadratic functions and quadratic inequalities, rational functions, polynomial functions, exponential functions, and logarithmic functions.
- (h) Develop an understanding of end behaviors of quadratic, rational, polynomial, exponential, and logarithmic functions.
- (i) Use functions algebraically, numerically, and graphical to model real life applications.
- (j) Set up and use systems of equations and inequalities to solve equations simultaneously.
- (k) Develop an understanding of sequences and series; be able to differentiate between geometric and arithmetic.
- (l) Appreciate other conic sections including the ellipse and hyperbola and their practical applications.

The LOPER 4 Learning Outcome 'a' is broadly met mathematically in this course. This includes developing the concepts of distance in the rectangular coordinate system (2), timing of projectiles (7, 8), population and natural phenomena modeling (3, 5, 6, 7, 8, 9), and modeling economic and financial criteria using functions and mathematical reasoning (4, 5, 7). These are assessed via homework, quizzes, and exams where a portion of the grade for a solution is based on the mathematical set up, describing the problem, and defending the concluding results.

The LOPER 4 Learning Outcomes 'b' and 'c' are achieved throughout the course through assigned homework, quizzes, exams, collaborative work and/or other assignments that are assessed by logically defending their solutions (1, 9, 10), and producing graphical, numerical, and/or algebraic support of their work (1, 2, 3, 4, 7, 8, 10, 11, 12).

The LOPER 4 Learning Outcome 'd' is met directly (2, 3, 4, 7, 8, 10, 11, 12) and indirectly (5, 6, 9) in this course. It is assessed by homework, quizzes, exams, and/or other work where a portion of the grade is based on the accuracy of their work and graphs and their interpretation of the data presented in the problems.

## Course Calendar

Generally, we'll adhere to the scheduled exam dates even if we are ahead or behind with course work. When we are ahead or behind, the topics on the exams will be appropriately adjusted.

## Notices:

- (a) *Exams will be given on the **Friday** of the week they are assigned.*
- (b) Online homework (**OL**) will be due at midnight on Saturday of the week they are assigned.
- (c) In class work (**IC**) will be given on Wednesday of the week it is assigned.
- (d)

Week	Week Starting	Section(s)	Topic(s)	Assessment
1	January 23	\$2.1, \$2.2	Distance, midpoint, graphs	IC 1
2	January 30	\$2.3, \$2.4, \$3.1	Lines, circles, functions	IC 2, OL 1
3	February 6	\$3.2, \$3.3, \$3.4	Graphs, library of functions, piecewise functions	IC 3, OL 2
4	February 13	\$3.4, \$3.5	Library of functions, Transformations	IC 4, OL 3
5	February 20	\$4.1	Linear functions & models	Exam 1, IC 5, OL 4
6	February 27	\$4.3, \$4.4	Quadratic functions & models	IC 6, OL 5
7	March 6	\$4.5, \$5.1	Quadratic inequalities; Polynomials	IC 7, OL 6
8	March 20	\$5.2, \$5.3, \$5.4	Rational functions; Graphs of polynomials	IC 8
9	March 27	\$5.4	Graphs of rational functions	Exam 2, IC 9, OL 7
10	April 3	\$6.1, \$6.2	Function composition; Inverse functions	IC 10, OL 8
11	April 10	\$6.3, \$6.4, \$6.5	Exponential & logarithmic functions	IC 11, OL 9
12	April 17	\$6.6, \$6.7	Logarithmic & exponential equations; Financial models	Exam 3, IC 12, OL 10
13	April 24	\$6.8	Exponential growth and decay	IC 13, OL 11
14	May 1	\$8.1, \$8.4,	Linear equations; Matrices	IC 14, OL 12
15	May 8	\$9.1, \$9.2, \$9.3	Sequences and series	OL 13
16	May 15			Final Exam

## Policies

For the university academic integrity policy, please read <https://catalog.unk.edu/undergraduate/academics/academic-regulations/academic-integrity-policy/>

Specially, our course policies are:

1. Regular in person class attendance is required. If you are ill or need to miss class due to athletics, please let me know ahead of time and I will make an effort to put the class on Zoom. Our classroom technology often doesn't work, so do not rely on watching recorded classes.
2. There is no explicit grade penalty for not attending class. But if you choose to not attend class for reasons other than illness or athletics, I reserve the right to not be all that helpful in giving you assistance on homework or helping you learn missed material.
3. All examinations, including the final exam, must be taken in person.
4. For examinations and in class assignments, show your work. *No credit will be given for multistep problems without the necessary work. Your solution must contain enough detail so that I am convinced that you could correctly work any similar problem.* Also erase or clearly mark any work you want me to ignore; otherwise, I'll grade it.
5. The work you turn in is expected to be *accurate, complete, concise, neat, and well-organized. You will not earn full credit on work that falls short of these expectations.*
6. Class cancellations due to weather, illness, or other unplanned circumstances may require that we make adjustments to the course calendar, exam dates, due dates, or specifics for course assessments.
7. Extra credit is not allowed.
8. For examinations, you may use a teacher provided quick reference sheet, but no other reference materials. You may also use a pencil, eraser, and a scientific calculator. For examinations, your phone and all such devices must be turned off and *out of sight*.

*Using unauthorized materials on an exam will earn you a grade of zero on that assessment.*

9. Generally, if you are ill or absent for any reason (including athletics), you must turn in your in class work on time. Permission to turn in work late must be made before the due date, otherwise late in class work will count zero points.
10. During class time, please refrain from using electronic devices. If your device usage distracts your classmates, I will ask you to put it away. If it's my impression that you are often not paying attention in class, I reserve the right to decline to help you during office hours.
11. The final examination will be *comprehensive* and it will be given during the time scheduled by the University. Except for *extraordinary circumstances* you must take the exam at this time.
12. If you have questions about how your work has been graded, make an appointment with me immediately.
13. Please regularly check Canvas to verify that your scores have been recorded correctly. If I made a mistake in recording one of your grades, I'll correct it provided you saved your paper.
14. The course calendar might be modified. It is your responsibility to attend class and to keep up to date on modifications to the course calendar.

### **Reporting Student Sexual Harassment, Sexual Violence or Sexual Assault**

Reporting allegations of rape, domestic violence, dating violence, sexual assault, sexual harassment, and stalking enables the University to promptly provide support to the impacted student(s), and to take appropriate action to prevent a recurrence of such sexual misconduct and protect the campus community. Confidentiality will be respected to the greatest degree possible. Any student who believes they may be the victim of sexual misconduct is encouraged to report to one or more of the following resources:

**Local Domestic Violence, Sexual Assault Advocacy Agency** 308-237-2599

**Campus Police (or Security)** 308-865-8911

**Title IX Coordinator** 308-865-8655

Retaliation against the student making the report, whether by students or University employees, will not be tolerated.

### **Students with Disabilities**

It is the policy of the University of Nebraska at Kearney to provide flexible and individualized reasonable accommodation to students with documented disabilities. To receive accommodation services for a disability, students must be registered with the UNK Disabilities Services for Students (DSS) office, 175 Memorial Student Affairs Building, 308-865-8214 or by email [unkdso@unk.edu](mailto:unkdso@unk.edu)

### **Students Who are Pregnant**

It is the policy of the University of Nebraska at Kearney to provide flexible and individualized reasonable accommodation to students who are pregnant. To receive accommodation services due to pregnancy, students must contact the Student Health office at 308-865-8218. The following links provide information for students and faculty regarding pregnancy rights:

1. <https://thepregnantscholar.org/title-ix-basics/>
2. <https://nwlc.org/resource/faq-pregnant-and-parenting-college-graduate-students-rights/UNKStatementInclusion>

## **UNK Statement of Diversity & Inclusion**

UNK stands in solidarity and unity with our students of color, our Latinx and international students, our LGBTQIA+ students and students from other marginalized groups in opposition to racism and prejudice in any form, wherever it may exist. It is the job of institutions of higher education, indeed their duty, to provide a haven for the safe and meaningful exchange of ideas and to support peaceful disagreement and discussion. In our classes, we strive to maintain a positive learning environment based upon open communication and mutual respect. UNK does not discriminate on the basis of race, color, national origin, age, religion, sex, gender, sexual orientation, disability or political affiliation. Respect for the diversity of our backgrounds and varied life experiences is essential to learning from our similarities as well as our differences. The following link provides resources and other information regarding D&I: <https://www.unk.edu/about/equity-access-diversity.php>

**Barton Willis, PhD**

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**Spring 2023**; ☎: 308-865-8868; ✉: willisb@unk.edu

Blocks marked "Appointments" (light yellow highlight) means usually available to make appointments.

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00	<b>MATH 102</b> 9:05-9:55 DSCH 116	<b>Unavailable</b> 8:00-12:00	<b>MATH 102</b> 9:05-9:55 DSCH 116	<b>Unavailable</b> 8:00-12:00	<b>MATH 102</b> 9:05-9:55 DSCH 116
10:00	<b>Office Hours</b> 10:00-11:00 DSCH 368		<b>Office Hours</b> 10:00-11:00 DSCH 368		<b>Office Hours</b> 10:00-11:00 DSCH 368
11:00	<b>Lunch</b> 11:00-12:00 Off campus	<b>Lunch</b> 11:00-12:00 Off campus	<b>Lunch</b> 11:00-12:00 Off campus	<b>Lunch</b> 11:00-12:00 Off campus	<b>Lunch</b> 11:00-12:00 Off campus
12:00	<b>Appointments</b> 12:00-13:20 DSCH 368	<b>Office Hours</b> 12:00-14:00 DSCH 368	<b>Appointments</b> 12:00-13:20 DSCH 368	<b>Office Hours</b> 12:00-14:00 DSCH 368	<b>Appointments</b> 12:00-13:20 DSCH 368
13:00					
13:30	<b>MATH 365-01</b> 13:25-14:15 DSCH 386		<b>MATH 365-01</b> 13:25-14:15 DSCH 386		<b>MATH 365-01</b> 13:25-14:15 DSCH 386
14:00		<b>Meeting</b> 14:00-15:00		<b>Meetings</b> 14:00-15:00	
14:15					
14:30	<b>MATH 250-01</b> 14:30-15:20 DSCH 386		<b>MATH 250-01</b> 14:30-15:20 DSCH 386		<b>MATH 250-01</b> 14:30-15:20 DSCH 386
14:45					
15:00	<b>Unavailable</b> 15:30-16:30	<b>Appointments</b> 15:00-16:30 DSCH 368	<b>Unavailable</b> 15:30-16:30	<b>Appointments</b> 15:00-16:30 DSCH 368	<b>Unavailable</b> 15:30-16:30
15:15					
15:30					
15:45					
16:00					

## Greek characters

Name	Symbol	Typical use(s)
alpha	$\alpha$	angle, constant
beta	$\beta$	angle, constant
gamma	$\gamma$	angle, constant
delta	$\delta$	limit definition
epsilon	$\epsilon$ or $\varepsilon$	limit definition
theta	$\theta$ or $\vartheta$	angle
pi	$\pi$ or $\pi$	circular constant
phi	$\phi$ or $\varphi$	angle, constant

## Named sets

empty set	$\emptyset$	integers	$\mathbf{Z}$
real numbers	$\mathbf{R}$	positive integers	$\mathbf{Z}_{>0}$
ordered pairs	$\mathbf{R}^2$	positive reals	$\mathbf{R}_{>0}$

## Set symbols

Meaning	Symbol	Meaning	Symbol
is a member	$\in$	union	$\cup$
subset	$\subset$	complement	superscript $^C$
intersection	$\cap$	set minus	$\setminus$

## Logic symbols

Meaning	Symbol	Meaning	Symbol
negation	$\neg$	equivalent	$\equiv$
and	$\wedge$	iff	$\iff$
or	$\vee$	for all	$\forall$
implies	$\implies$	there exists	$\exists$

## Arithmetic properties of R

$(\forall a, b \in \mathbf{R})(a + b = b + a)$	commutivity
$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$	associative
$(\forall a, b \in \mathbf{R})(ab = ba)$	commutivity
$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$	associative
$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$	distributive

## Intervals

For numbers  $a$  and  $b$ , we define the intervals

$$\begin{aligned}(a, b) &= \{x \in \mathbf{R} \mid a < x < b\} \\ [a, b) &= \{x \in \mathbf{R} \mid a \leq x < b\} \\ (a, b] &= \{x \in \mathbf{R} \mid a < x \leq b\} \\ [a, b] &= \{x \in \mathbf{R} \mid a \leq x \leq b\}\end{aligned}$$

## Distance & Midpoint

The distance between the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The midpoint is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

## Exponents

For  $a, b > 0$  and  $m, n$  real:

$$\begin{aligned}a^0 &= 1, & 0^a &= 0 \\ 1^a &= 1, & a^n a^m &= a^{n+m} \\ a^n / a^m &= a^{n-m}, & (a^n)^m &= a^{n \cdot m} \\ a^{-m} &= 1/a^m, & (a/b)^m &= a^m / b^m\end{aligned}$$

## Radicals

$$\begin{aligned}\sqrt[n]{a} &= a^{1/n} \\ \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} \quad (\text{provided } a, b \geq 0) \\ \sqrt[m]{\sqrt[n]{a}} &= \sqrt[mn]{a} \\ \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\ \sqrt[n]{a^n} &= \begin{cases} a & n \text{ odd} \\ |a| & n \text{ even} \end{cases}\end{aligned}$$

## Identities

$$\begin{aligned}a(b + c) &= ab + ac \\ ((a + b)(c + d)) &= ac + ad + bc + bd \\ \frac{ab + ac}{a} &= b + c \quad (\text{provided } a \neq 0) \\ \frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{ad}{bc} \quad (\text{provided } b, d \neq 0) \\ \sqrt{ab} &= \sqrt{a}\sqrt{b} \quad (\text{provided } a \geq 0, b \geq 0) \\ \ln(ab) &= \ln(a) + \ln(b) \quad (\text{provided } a \geq 0, b \geq 0)\end{aligned}$$

## Solution of Equations

### Algebraic

$$\begin{aligned}[ab = 0] &\equiv [a = 0 \text{ or } b = 0] \\ [a^2 = b^2] &\equiv [a = b \text{ or } a = -b] \\ [\frac{a}{b} = 0] &\equiv [a = 0 \text{ and } b \neq 0] \\ [\frac{a}{b} = \frac{c}{d}] &\equiv [ad = bc \text{ and } b \neq 0 \text{ and } d \neq 0] \\ [|a| = |b|] &\equiv [a = b \text{ or } a = -b] \\ [\sqrt{a} = b] &\equiv [a = b^2 \text{ and } b \geq 0]\end{aligned}$$

For  $a \neq 0$ ,

$$[ax^2 + bx + c = 0] \equiv \left[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

### Exponential

$$\begin{aligned}[\ln(a) = 0] &\equiv [a = 1] \\ [e^a = 1] &\equiv [a = 0] \\ [\ln(a) = b] &\equiv [a = e^b]\end{aligned}$$

## Logarithms

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$



## Graph Translations

For the graph of  $F(x, y) = 0$

- The graph of  $F(x - h, y) = 0$  is the graph of  $F(x, y) = 0$  translated  $h$  units to the right.
- The graph of  $F(x, y - k) = 0$  is the graph of  $F(x, y) = 0$  translated  $k$  units up.
- The graph of  $F(x/c, y) = 0$  is the graph of  $F(x, y) = 0$  stretched a factor of  $c$  horizontally.
- The graph of  $F(x, y/c) = 0$  is the graph of  $F(x, y) = 0$  stretched a factor of  $c$  vertically.

## Circles

Equation of circle centered at  $(h, k)$  with radius  $r$  is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Expanded the equation is

$$x^2 - 2hx + y^2 - 2ky = r^2 - h^2 - k^2.$$

## Parabolas & Lines

The vertex of the parabola  $ax^2 + bx + c = y$  is

$$\left(x = -\frac{b}{2a}, y = c - \frac{b^2}{4a}\right).$$

An equation of the line that contains the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The number  $\frac{y_2 - y_1}{x_2 - x_1}$  is the slope.

## Function notation

$\text{dom}(F)$	domain of function $F$
$\text{range}(F)$	range of function $F$

## Domains, Ranges, and Zeros

Function	Domain	Range	Zeros
$\ln, \log$	$(0, \infty)$	$(-\infty, \infty)$	1
$\exp$	$(-\infty, \infty)$	$(0, \infty)$	$\emptyset$
$\text{abs}$	$(-\infty, \infty)$	$(0, \infty)$	0
$\sqrt{\phantom{x}}$	$(0, \infty)$	$(0, \infty)$	0
$\sqrt[3]{\phantom{x}}$	$(-\infty, \infty)$	$(-\infty, \infty)$	0
floor	$(-\infty, \infty)$	$\mathbf{Z}$	$[0, 1)$
ceiling	$(-\infty, \infty)$	$\mathbf{Z}$	$(-1, 0]$

## Compound Interest

Interest rate  $r$  compounded  $n$  times per year

$$A = P(1 + r/n)^{nt}$$

Continuous compounding:

$$A = Pe^{rt}$$

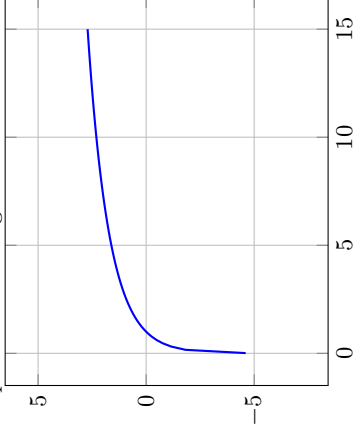
## Exponential Growth

The exponential function that contains the points  $(t = t_o, y = y_o)$  and  $(t = t_1, y = y_1)$  is

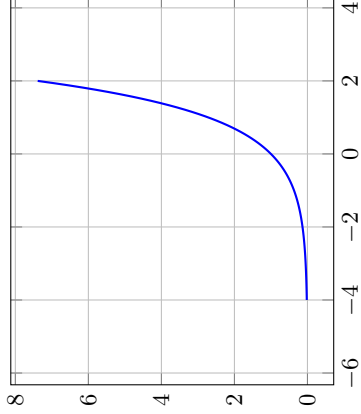
$$y = y_o \left(\frac{y_1}{y_o}\right)^{\frac{t - t_o}{t_1 - t_o}}.$$

## Graphs

Graph of natural logarithm



Graph of natural exponential



## Common Errors

Error	Correct or Example
$x/0 = 0$ or $x^{-2} = x^2$	$x/0$ is undefined $-x^2 = -(x^2)$
$a/(b+c) = a/b + a/c$ $a+bx/a = 1 + bx$	$1/(1+1) \neq 1/1 + 1/1$ $a+bx/a = 1 + bx/a$
$(a+b)^2 = a^2 + b^2$	$(a+b)^2 = a^2 + 2ab + b^2$
$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$	$\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$