Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Function Notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F
C_A	set of continuous functions on set A
C^1_A	set of differentiable functions on set A
$A \to B$	set of functions from A to B

Magnitude & Conjugate

For all $a, b \in \mathbf{R}$

$$|a + \mathrm{i}b| = \sqrt{a^2 + b^2}$$
$$\overline{a + \mathrm{i}b} = a - \mathrm{i}b$$

For all $x, y, z \in \mathbf{C}$, we have

$$|xy| = |x||y|$$
$$|x + y| \leq |x| + |y|$$
$$\big||x| - |y|\big| \leq |x - y|$$
$$\overline{xy} = \overline{x}\overline{y}$$
$$\frac{1}{z} = \frac{\overline{z}}{|z|^2} (\text{ for } z \neq 0)$$
$$\overline{\left(\frac{x}{y}\right)} = \frac{\overline{x}}{\overline{y}}$$

Complex Exponential

For $x, y \in \mathbf{R}$

$$\mathrm{e}^{\mathrm{i}y} = \cos(y) + \mathrm{i}\sin(y)$$
$$\mathrm{e}^{x+\mathrm{i}y} = \mathrm{e}^x\,(\cos(y) + \mathrm{i}\sin(y))$$

For all $z_1, z_2 \in \mathbf{C}$,

$$\mathrm{e}^{z_1+z_2} = \mathrm{e}^{z_1}\mathrm{e}^{z_2}$$
$$[\mathrm{e}^{z_1} = \mathrm{e}^{z_2}] \equiv [z_1 - z_2 = 2\pi n, n \in \mathbf{Z}]$$

Argument & Polar form

For all $z \in \mathbf{C}_{\neq 0}$, there is $\theta \in \mathbf{R}$ such that

$$z = |z|(\cos(\theta) + \mathrm{i}\sin(\theta))$$
$$\arg(z) = \theta$$

We have

$$\sqrt{z} = \sqrt{|z|}(\cos(\theta/2) + \mathrm{i}\sin(\theta/2))$$
$$z^a = |z|^a(\cos(\theta/a) + \mathrm{i}\sin(\theta/a)) \quad (\text{for } a \in \mathbf{R})$$
$$\log(z) = \log(|z|) + \mathrm{i}\arg(z)$$

When $\theta \in (-\pi, \pi]$, we say θ is the principle argument $\mathrm{Arg}(z)$; further

$$\mathrm{Arg}(z) = \arg(z) - 2\pi\left\lceil\frac{\arg(z) - \pi}{2\pi}\right\rceil.$$

Principal Logarithm

$$\mathrm{Ln}(z) = \ln(|z|) + \mathrm{i}\mathrm{Arg}(z).$$

The function Ln is holomorphic on $\mathbf{C} \setminus (-\infty, 0]$ and its domain is $\mathbf{C}_{\neq 0}$. Its derivative is

$$\mathrm{Ln}'(z) = \frac{1}{z}, \quad z \in \mathbf{C} \setminus (-\infty, 0].$$

Topology & Disks

For $a \in \mathbf{C}$ and $r \in \mathbf{R}_{>0}$, we define

$$D[a, r] = \{z \in \mathbf{C} \mid |z - a| < r\},$$
$$D'[a, r] = \{z \in \mathbf{C} \mid 0 < |z - a| < r\},$$
$$C[a, r] = \{z \in \mathbf{C} \mid |z - a| = r\}.$$

Let G be a subset of $\mathbf{C}$. We say

- z is an *interior point of* G provided $(\exists r \in \mathbf{R}_{>0})(D[z, r] \subset G)$.
- z is a *boundary point of* G provided $(\forall r \in \mathbf{R}_{>0})(D[z, r] \cap G \neq \varnothing \wedge D[z, r] \cap G^C \neq \varnothing\}$.
- z is an *accumulation point of* G provided $(\forall r \in \mathbf{R}_{>0})(D'[z, r] \cap G \neq \varnothing\}$.
- G is *open* provided every member of G is an interior point of G.
- G is *bounded* provided $(\exists r \in \mathbf{R}_{>0})(G \subset D[0, r])$.

Power Series

For $z \in D[0, 1]$, we have

$$\sum_{k=0}^{\infty} z^k = \frac{1}{1 - z}$$

For all $z \in \mathbf{C}$, we have

$$\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

Limits

Let G be a subset of $\mathbf{C}$, z_o be an accumulation point of G, and $F \in G \to \mathbf{C}$. We say F *has a limit towards* z_o provided

(a) $\exists L \in \mathbf{C}$

(b) $\forall \varepsilon \in \mathbf{R}_{>0}$

(c) $\exists \delta \in \mathbf{R}_{>0}$

(d) $\forall z \in D[z_o, \delta]$

(e) we have $|F(z) - L| < \varepsilon$.

Continuity

Let G be a subset of $\mathbf{C}$ and let $z_o \in \text{dom} F$. We say F *is continuous at* z_o provided

(a) $\forall \varepsilon \in \mathbf{R}_{>0}$

(b) $\exists \delta \in \mathbf{R}_{>0}$

(c) $\forall z \in D[z_o, \delta]$

(d) we have $|F(z) - F(z_o)| < \varepsilon$.

Derivatives

Let G be a subset of $\mathbf{C}$ and let z_o be an interior point of G. We say F *is differentiable at* z_o provided

$$\lim_{z \to z_o} \frac{F(z) - F(z_o)}{z - z_o}$$

exists.

Cauchy Reimann

Let $F(x + \mathrm{i}y) = u(x, y) + \mathrm{i}v(x, y)$. The function F is differentiable on a region G provided for all $x + \mathrm{i}y \in G$, we have

$$\begin{bmatrix} u \\ v \end{bmatrix}_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}_y$$