MATH 365 **Name:**

In class work 5, Spring 2023

1. Sketch the parametrically defined curve $x = \frac{t^2}{1+t^2}$, $y = -\frac{t}{1+t^2}$, where $t \in \mathbf{R}$. Suggestion: Use Desmos.
2. Algebraically show that the curve from part one is a circle with one point missing.
3. Solve the equation $w = \frac{z}{z+\mathrm{i}}$ for z.
4. Show that the function $z \in \mathbf{C}_{\neq -\mathrm{i}} \mapsto \frac{z}{z+\mathrm{i}}$ is one-to-one.
5. Find the inverse to the function $z \in \mathbf{C}_{\neq -\mathrm{i}} \mapsto \frac{z}{z+\mathrm{i}}$.
6. For a complex variable w, say $w = a + \mathrm{i}b$, find the imaginary part of $\frac{\mathrm{i}w}{1-w}$.
7. In the a, b plane, sketch a graph of the equation $a^2 - a + b^2 = 0$.
8. Find the range of function $x \in \mathbf{R} \mapsto \frac{x}{x+\mathrm{i}}$.