MATH 365 **Name:**____________________________________

Review for Exam 3 **Row and Seat:**______________________________

1. Find the radius of convergence for each series

$\boxed{10}$ (a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{1+k^2} x^k$

$\boxed{10}$ (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(k!)^2}{(3k)!} x^k$

$\boxed{10}$ 2. Use the bound

$$\left|\int_\gamma F(z)\,\mathrm{d}z\right| \le \max_{z\in\gamma}|F(z)| \times \text{length}(\gamma)$$

to find an *upper bound* for

$$\int_L \frac{1}{x-\mathrm{i}}\,\mathrm{d}x,$$

where L is the line segment from $-1-5\mathrm{i}$ to $1+5\mathrm{i}$.

3. Write a critique of Billy's work

$$\int_0^{2\pi} \frac{1}{4+\cos(x)}\,\mathrm{d}x = \left.\frac{2\arctan\left(\frac{3\sin(x)}{\sqrt{15}(\cos(x)+1)}\right)}{\sqrt{15}}\right|_{x=0}^{x=2\pi}$$
$$= 0 - 0$$
$$= 0.$$

Fact: Everywhere $\frac{2\arctan\left(\frac{3\sin(x)}{\sqrt{15}(\cos(x)+1)}\right)}{\sqrt{15}}$ is differentiable, its derivative is the integrand. Here is a graph of the antiderivative.

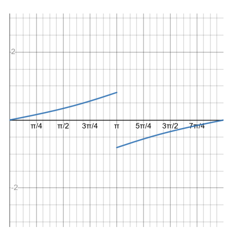

Figure 1: Graph of antiderivative on the interval $[0,2\pi]$.

4. Find the *numerical value* of each sum.

$\boxed{10}$ (a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)^{k+1}$

$\boxed{10}$ (b) $\displaystyle\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^{k+1}$

5. Find the *numerical value* for each contour integral, where C is the unit circle.

$\boxed{10}$ (a) $\displaystyle\int_C \frac{1}{z-5\mathrm{i}}\,\mathrm{d}z.$

$\boxed{10}$ (b) $\displaystyle\int_C \frac{1}{z - \mathrm{i}/5}\,\mathrm{d}z.$

$\boxed{10}$ 6. For $w > 0$, find the value of

$$\int_{-\infty}^{\infty} \frac{1}{4 + x^2} \mathrm{e}^{\mathrm{i}wx}\,\mathrm{d}x. \tag{1}$$

$\boxed{10}$ 7. My friend Pauline claims that since the function $x \in \mathbf{C} \mapsto e^{e^{ix}}$ is entire, we have

$$\int_0^{2\pi} e^{e^{ix}}\, dx = 0.$$

Pauline is correct that the function $x \mapsto e^{e^{ix}}$ is entire. But is Pauline correct about the value of the integral? If not, find the correct value for this definite integral.