MATH 365 **Name:**______________________________
Review for Exam 1

1. In the context of complex numbers, know the definitions of a *neighborhood, open set, closed set, connected set, bounded set.*

2. Find the *rectangluar form* of $\sqrt{1+\mathrm{i}}$.

3. Explain why the solution set of $\sqrt{z} = -2019 + \mathrm{i}$ is empty.

4. Using the *limit of a Newton quotient*, find the derivative of $z \in \mathbf{C} \mapsto z^2$ at i.

5. Use the Cauchy Riemann (CR) equations to show that the function $z \in \mathbf{C} \mapsto z + \overline{z}$ is not differentiable anywhere on $\mathbf{C}$.

6. Find the *rectangular representation* for each complex number; that is express each complex number *explicitly* in the form $a + ib$, where $a, b \in \mathbf{R}$.
   (a) $(5+\mathrm{i})(2+\mathrm{i}) =$
   (b) $\frac{2+\mathrm{i}}{2-\mathrm{i}} =$
   (c) $(2+\mathrm{i})\overline{(3+\mathrm{i})} =$

7. Using the definition of the limit, show that $\lim_{z\to\mathrm{i}} z^2 = -1$.

8. Find the *rectangluar form* for each square root
   (a) $\sqrt{\frac{5\sqrt{3}\,\mathrm{i}}{2} + \frac{5}{2}}$
   (b) $\sqrt{-\mathrm{i}}$

9. Show that the function $z \in \mathbf{C} \mapsto \overline{z}$ is *continuous* at 0.

10. Use the Cauchy Riemann (CR) equations to show that the function $z \in \mathbf{C} \mapsto z - \overline{z}$ is not differentiable anywhere on $\mathbf{C}$.

11. Using the exponential form for the trigonometric functions, show that

$$\cos(x)\sin(x) = \frac{\sin(2x)}{2}.$$

    The exponential form for cosine is $\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$.

12. Use the definition of continuity (from the classnotes–the one with the ε and δ in it) to show that the function $z \in \mathbf{C} \mapsto z^2$ is continuous at i.

13. Show that $\sqrt{zw} = \sqrt{z}\sqrt{w}$ is *not* an identity for complex z and w. Here the square root is the principal square root.

14. Know everything about complex number arithmetic; know what the commutative and associative properties are; know how to find the rectangular form of sums, products, differences, and quotients of complex numbers.

15. Know what the conjugate is and know its properties.

16. Know the definition of the argument of a complex number.

17. Know how to use $\exp(ix) = \cos(x) + i\sin(x)$ along with the rule of exponents to derive trigonometric identities (mostly classnotes).

18. Know how the principal square root of a complex number is defined;

19. Know the triangle inequality.

20. Know the derivation of the CR equations

21. Know the defintion of continuity for a $\mathbf{C} \to \mathbf{C}$ function.

22. Use $\exp(ix) = \cos(x) + i\sin(x)$ to show that

$$\cos(x)^3 - 3\cos(x)\sin(x)^2 = \cos(3x)$$

is an identity. More esthetically, the identity is $4\cos(x)^3 - 3\cos(x) = \cos(3x)$.

23. Find the *polar* representation for the number $-1+i$. Use the polar form to find the *polar* representation for $\sqrt{-1+i}$.

24. Using the rule of exponents along with $\exp(ix) = \cos(x) + i\sin(x)$ to show that $\cos(2x) = \cos^2(x) - \sin^2(x)$ is an identity.

25. Know the triangle inequality.

26. For $x \in \mathbf{R}$, we have the identities $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$ and $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$. Use these facts along with facts about the complex exponential function to show that

$$\sin(x)^3 = -\frac{\sin(3x) - 3\sin(x)}{4}$$

is an identity.

27. Find the *rectangular* form for $\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)^{107}$.