

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	\emptyset	integers	\mathbf{Z}
real numbers	\mathbf{R}	positive integers	$\mathbf{Z}_{>0}$
ordered pairs	\mathbf{R}^2	positive reals	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol	Meaning	Symbol
is a member	\in	union	\cup
subset	\subset	complement	superscript \complement
intersection	\cap	set minus	\setminus

Logic Symbols

Meaning	Symbol	Meaning	Symbol
negation	\neg	equivalent	\equiv
and	\wedge	iff	\iff
or	\vee	for all	\forall
implies	\implies	there exists	\exists

Function Notation

$\text{dom}(F)$	domain of function F
$\text{range}(F)$	range of function F
C_A	set of continuous functions on set A
C_A^1	set of differentiable functions on set A
$A \rightarrow B$	set of functions from A to B

Magnitude & Conjugate

For all $a, b \in \mathbf{R}$

$$|a + ib| = \sqrt{a^2 + b^2}$$

$$\overline{a + ib} = a - ib$$

For all $x, y, z \in \mathbf{C}$, we have

$$|xy| = |x||y|$$

$$|x + y| \leq |x| + |y|$$

$$||x| - |y|| \leq |x - y|$$

$$\overline{\overline{xy}} = xy$$

$$\frac{1}{z} = \frac{\bar{z}}{|z|^2} \text{ (for } z \neq 0\text{)}$$

$$\overline{\left(\frac{x}{y}\right)} = \frac{\bar{x}}{\bar{y}}$$

Complex Exponential

For $x, y \in \mathbf{R}$

$$e^{iy} = \cos(y) + i \sin(y)$$

$$e^{x+iy} = e^x (\cos(y) + i \sin(y))$$

For all $z_1, z_2 \in \mathbf{C}$,

$$e^{z_1+z_2} = e^{z_1} e^{z_2}$$

$$[e^{z_1} = e^{z_2}] \equiv [z_1 - z_2 = 2\pi n, n \in \mathbf{Z}]$$

Argument & Polar form

For all $z \in \mathbf{C}_{\neq 0}$, there is $\theta \in \mathbf{R}$ such that

$$z = |z|(\cos(\theta) + i \sin(\theta))$$

$$\arg(z) = \theta$$

We have

$$\sqrt{z} = \sqrt{|z|}(\cos(\theta/2) + i \sin(\theta/2))$$

$$z^a = |z|^a(\cos(\theta/a) + i \sin(\theta/a)) \text{ (for } a \in \mathbf{R}\text{)}$$

$$\log(z) = \log(|z|) + i \arg(z)$$

When $\theta \in (-\pi, \pi]$, we say θ is the principle argument $\text{Arg}(z)$; further

$$\text{Arg}(z) = \arg(z) - 2\pi \left\lfloor \frac{\arg(z) - \pi}{2\pi} \right\rfloor.$$

Topology & Disks

For $a \in \mathbf{C}$ and $r \in \mathbf{R}_{>0}$, we define

$$D[a, r] = \{z \in \mathbf{C} \mid |z - a| < r\},$$

$$D'[a, r] = \{z \in \mathbf{C} \mid 0 < |z - a| < r\},$$

$$C[a, r] = \{z \in \mathbf{C} \mid |z - a| = r\}.$$

Let G be a subset of \mathbf{C} . We say

- z is an *interior point* of G provided $(\exists r \in \mathbf{R}_{>0})(D[z, r] \subset G)$.
- z is a *boundary point* of G provided $(\forall r \in \mathbf{R}_{>0})(D[z, r] \cap G \neq \emptyset \wedge D[z, r] \cap G^c \neq \emptyset)$.
- z is an *accumulation point* of G provided $(\forall r \in \mathbf{R}_{>0})(D'[z, r] \cap G \neq \emptyset)$.
- G is *open* provided every member of G is an interior point of G .
- G is *bounded* provided $(\exists r \in \mathbf{R}_{>0})(G \subset D[0, r])$.

Power Series

For $z \in D[0, 1]$, we have

$$\sum_{k=0}^{\infty} z^k = \frac{1}{1-z}$$

For all $z \in \mathbf{C}$, we have

$$\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

Limits

Let G be a subset of \mathbf{C} , z_o be an accumulation point of G , and $F \in G \rightarrow \mathbf{C}$. We say F has a limit towards z_o provided

- $\exists L \in \mathbf{C}$
- $\forall \varepsilon \in \mathbf{R}_{>0}$
- $\exists \delta \in \mathbf{R}_{>0}$
- $\forall z \in D[z_o, \delta]$
- we have $|F(z) - L| < \varepsilon$.

Continuity

Let G be a subset of \mathbf{C} and let $z_o \in \text{dom}F$. We say F is *continuous at z_o* provided

(a) $\forall \varepsilon \in \mathbf{R}_{>0}$

(b) $\exists \delta \in \mathbf{R}_{>0}$

(c) $\forall z \in D[z_o, \delta]$

(d) we have $|F(z) - F(z_o)| < \varepsilon$.

Derivatives

Let G be a subset of \mathbf{C} and let z_o be an interior point of G . We say F is *differentiable at z_o* provided

$$\lim_{z \rightarrow z_o} \frac{F(z) - F(z_o)}{z - z_o}$$

exists.

Cauchy Reimann

Let $F(x + iy) = u(x, y) + iv(x, y)$. The function F is differentiable on a region G provided for all $x + iy \in G$, we have

$$\begin{bmatrix} u \\ v \end{bmatrix}_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}_y$$