MATH 250   **Name:**
Homework 10, Spring 2023

> "Study without desire spoils the memory, and it retains nothing that it takes in."
>   LEONARDO DA VINCI

I have neither given nor received unauthorized assistance on this assignment.

Homework 10 has questions 1 through 3 with a total of 12 points. For this assignment, use Overleaf to typeset your work and upload a pdf to Canvas. This assignment is due Saturday 15 April at 11:59 P.M.

1. A four-digit integer has the form $d_1 d_2 d_3 d_4$, where $d_1, d_2 \ldots, d_4 \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $d_1 \neq 0$. Thus 1225 and 9876 are four-digit numbers, but 0123 isn't a four-digit number (it is a three digit number).

   In each of the following cases, determine the number of four-digit integers that satisfy the given condition. Clearly explain your work with a few sentences.

   [2]   (a) No additional restrictions on the digits.

   > **Solution:**

   [2]   (b) Every digit is even.

   > **Solution:**

   [2]   (c) There are no repeated digits. Thus 1078 is admissible, but 1040 is not admissible.

   > **Solution:**

   [2]   (d) No two consecutive digits are equal. Thus 1313 is admissible, but 1331 is not admissible.

   > **Solution:**

[2] 2. From a litter of six border collies named Patsy, George, Buddy, Larry, Joey, and Clover, I'm going to adopt three of them. How many distinct sets of three puppies can I adopt? Clearly explain your work with a few sentences.

Solution:

2   3. The prime factorization of $220870604912736225056311099$ is $19^3 \times 37^4 \times 107^8$. Every divisor of $220870604912736225056311099$ has the form $19^\ell \times 37^m \times 107^n$, where $\ell \in 0, 1, 2, 3,$ $m \in 0, 1, 2, 3, 4,$ and $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. How many divisors of $220870604912736225056311099$ are there?

Solution: