

## Homework 3, Spring 2023

“Mathematics is common sense.”

ERRETT BISHOP (1928–1983)

Errett Bishop was born in Newton, Kansas and was the son of a Wichita State mathematics professor. He is best known for his book *Foundations of Constructive Analysis*. Constructive mathematics rejects the Law of excluded middle.

I have neither given nor received unauthorized assistance on this assignment.

Homework 3 has questions 1 through 2 with a total of 10 points. For this assignment, use Overleaf to complete the assignment and upload the pdf to Canvas.

- 5 1. Show that for all positive integers  $p$  and  $q$  that  $3q^2 \neq p^2$ .

To do this, use the fact that every positive integer can be uniquely expressed in the form  $3^e n$ , where  $e$  is a nonnegative integer and  $n$  is not divisible by 3. For example,  $24 = 3^1 \times 8$  (and 8 is not divisible by 3); and  $963 = 3^2 \times 107$  (and 107 is not divisible by 3). Uniqueness of this representation means that

$$\left[ 3^e n = 3^{e'} n' \right] \equiv \left[ (e = e') \wedge (n = n') \right].$$

Equivalently

$$\left[ 3^e n \neq 3^{e'} n' \right] \equiv \left[ (e \neq e') \vee (n \neq n') \right].$$

**Solution:**

- 5 2. For all positive real numbers  $x$  and  $y$  with  $x \neq y$ , show that

$$\frac{x}{y} + \frac{y}{x} > 2.$$

To do this, assume that  $\frac{x}{y} + \frac{y}{x} \leq 2$ . Use some algebra facts to derive a contradiction.