Foundations of Mathematics

MATH 250–01

Spring 2023

Instructor: Barton Willis, PhD, Professor of Mathematics
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Zoom for classes: For Zoom class meetings, use the Meeting ID: 616 568 5706.
Office Hours: Monday, Wednesday, and Friday 10:00AM-11:00AM, Tuesday and Thursday 12:00 noon-2:00PM, and by appointment.

Class meeting time and place

This class meets Monday, Wednesday, and Friday in Discovery Hall, room 386 from 2:30PM to 3:20PM.

Course Resources

Our textbook is *Introduction to Mathematical Structures and Proofs*, 2nd edition, by Larry Gerstein. Some homework assignments for this course will need to be typeset. To do this, you will need to create a *no cost* account on Overleaf (`https://www.overleaf.com/`). For tutorial for using Overleaf, see `https://www.overleaf.com/tutorial`.

Important Dates

First Homework due ..... **January 28**
Exam 1 ..... **February 17**
Exam 2 ..... **March 24**
Exam 3 ..... **April 21**
Final exam ..... **May 15, 3:30 PM– 5:30 PM**

Grading

Your course grade will be based on weekly homework sets, three midterm exams, and a comprehensive final exam; specifically:

Weekly Homework *11 fifteen point assignments* ..... 165 (total)
Mid-term exams 1,2, and 3 *100 points each* ..... 300 (total)
Comprehensive Final exam ..... 150 (total)

If it is necessary to adjust the number of homework assignments, your homework point total will be scaled to a total of 165. For example, if we have only ten homework sets, your homework score will be scaled by a factor of 165/150.

The following table shows the *minimum* number of points (out of 615) that are required for each of the twelve letter grades D- through A+. For example, a point total of 529 points will earn you a grade of B+, and a point total of 554 points will earn you a grade of A-. A point total of 368 or less earns you a failing course grade.

D-	369	B-	492
D	387	B	510
D+	406	B+	529
C-	431	A-	554
C	449	A	572
C+	467	A+	603

Prerequisite

The prerequisite for MATH 250 is an earned grade of D- or higher in either MATH 115 or MATH 123.

Catalog description

Foundations of Mathematics (3 credit hours) Topics of sets and symbolic logic are studied with the objective of using them in the detailed study of the nature of different types of proofs used in mathematics. Also, the processes of problem solving are studied for developing strategies of problem solving.

Learning Outcomes

On completion of this course, students will

(a) gain an understanding of naïve set theory.

(b) gain an understanding of symbolic logic, quantifiers, and functions.

(c) gain an understanding of direct proofs, proofs by contradiction, proofs by contrapositive, and proofs by induction.

(d) gain the ability to read and understand mathematical proofs.

(e) gain the problem solving skills that are needed to create a mathematical proof.

Course Calendar

Generally, we'll adhere to the scheduled exam dates even if we are ahead or behind with course work. When we are ahead or behind, the topics on the exams will be appropriately adjusted.

Notices:

(a) *Exams will be given on the **Friday** of the week they are assigned.*

(b) Homework (**HW**) will be due at midnight on Saturday of the week they are assigned.

Week	Week Starting	Section(s)	Topic(s)	Assessment
1	January 23	§1.1, §1.2	Logical connectives; Truth tables	**HW 1**
2	January 30	§1.3	Introduction to Overleaf; Conditional statements	**HW 2**
3	February 6	§1.4, §1.5	Proof structures; Logical equivalence	**HW 3**
4	February 13	§2.1, §2.2	Sets; Russell's paradox	**Exam 1**
5	February 20	§2.3, §2.4	Quantifiers; Set inclusion	**HW 4**
6	February 27	§2.5, §2.6	Union, intersection, complements; Indexed sets	**HW 5**
7	March 6	§2.7, §2.8	Power sets; Ordered pairs	**HW 6**
8	March 20	§2.9	Set decompositions	**Exam 2**
9	March 27	§2.10	Induction	**HW 7**
10	April 3	§3.1	Functions	**HW 8**
11	April 10	§3.2, §3.3	One-to-one and onto functions; Function composition	**HW 9**
12	April 17	§4.1, §4.2	Cardinality; Finite and infinite sets	**Exam 3**
13	April 24	§4.3	Countable and uncountable sets	**HW 10**
14	May 1	§5.1, §5.2	Combinatorial problems; Addition and product rules	**HW 11**
15	May 8	§5.3, §5.4	Permutations; Geometric symmetry	
16	May 15			**Final Exam**

Policies

Unless an assessment is *explicitly* stated to be a group project, *all work you turn in for a grade must be your own.* If you need assistance in completing a homework assignment, you may ask me for help. Googling for answers, seeking help from the Learning Commons or other faculty members, or using solution keys from previous terms (either from UNK or other universities) is also prohibited. Violation of these rules will result in earning a grade of zero on the assessment. Each homework assignment you turn in for a grade must include the statement:

> I have neither given nor received unauthorized assistance on this assignment.

If two assignments are so similar that only collaboration could explain their similarities, both assignments will receive a grade of zero. Using unauthorized materials or communication devices (cell phone, for example) while taking a test will earn you a grade of zero on that assessment. For the university academic integrity policy, please read `https://catalog.unk.edu/undergraduate/academics/academic-regulations/academic-integrity-policy/`

Specially, our course policies are:

1. Regular in person class attendance is required. If you are ill or need to miss class due to athletics, please let me know ahead of time and I will make an effort to put the class on Zoom. Our classroom technology often doesn't work, so do not rely on watching recorded classes.
2. There is no explicit grade penalty for not attending class. But if you choose to not attend class for reasons other than illness or athletics, I reserve the right to not be all that helpful in giving you assistance on homework or helping you learn missed material.
3. All examinations, including the final exam, must be taken in person.
4. For examinations and in class assignments, show your work. *No credit will be given for multistep problems without the necessary work. Your solution must contain enough detail so that I am convinced that you could correctly work any similar problem.* Also erase or clearly mark any work you want me to ignore; otherwise, I'll grade it.
5. The work you turn in is expected to be *accurate, complete, concise, neat*, and *well-organized. You will not earn full credit on work that falls short of these expectations.*
6. Class cancellations due to weather, illness, or other unplanned circumstances may require that we make adjustments to the course calendar, exam dates, due dates, or specifics for course assessments.
7. Extra credit is not allowed.
8. For examinations, you may use a teacher provided quick reference sheet, but no other reference materials. You may also use a pencil, eraser, and a scientific calculator. For examinations, your phone and all such devices must be turned off and *out of sight.*
9. Generally, if you are ill or absent for any reason (including athletics), you must turn in your in class work on time. Permission to turn in work late must be made before the due date, otherwise late in class work will count zero points.
10. During class time, please refrain from using electronic devices. If your device usage distracts your classmates, I will ask you to put it away. If it's my impression that you are often not paying attention in class, I reserve the right to decline to help you during office hours.
11. The final examination will be *comprehensive* and it will be given during the time scheduled by the University. Except for *extraordinary circumstances* you must take the exam at this time.
12. If you have questions about how your work has been graded, make an appointment with me immediately.
13. Please regularly check Canvas to verify that your scores have been recorded correctly. If I made a mistake in recording one of your grades, I'll correct it provided you saved your paper.

Reporting Student Sexual Harassment, Sexual Violence or Sexual Assault

Reporting allegations of rape, domestic violence, dating violence, sexual assault, sexual harassment, and stalking enables the University to promptly provide support to the impacted student(s), and to take appropriate action to prevent a recurrence of such sexual misconduct and protect the campus community. Confidentiality will be respected to the greatest degree possible. Any student who believes they may be the victim of sexual misconduct is encouraged to report to one or more of the following resources:
Local Domestic Violence, Sexual Assault Advocacy Agency 308-237-2599
Campus Police (or Security) 308-865-8911
Title IX Coordinator 308-865-8655
Retaliation against the student making the report, whether by students or University employees, will not be tolerated.

Students with Disabilities

It is the policy of the University of Nebraska at Kearney to provide flexible and individualized reasonable accommodation to students with documented disabilities. To receive accommodation services for a disability, students must be registered with the UNK Disabilities Services for Students (DSS) office, 175 Memorial Student Affairs Building, 308-865-8214 or by email unkdso@unk.edu

Students Who are Pregnant

It is the policy of the University of Nebraska at Kearney to provide flexible and individualized reasonable accommodation to students who are pregnant. To receive accommodation services due to pregnancy, students must contact the Student Health office at 308-865-8218. The following links provide information for students and faculty regarding pregnancy rights:

1. `https://thepregnantscholar.org/title-ix-basics/`
2. `https://nwlc.org/resource/faq-pregnant-and-parenting-college-graduate-students-rights/UNKStatemen`
   `Inclusion`

UNK Statement of Diversity & Inclusion

UNK stands in solidarity and unity with our students of color, our Latinx and international students, our LGBTQIA+ students and students from other marginalized groups in opposition to racism and prejudice in any form, wherever it may exist. It is the job of institutions of higher education, indeed their duty, to provide a haven for the safe and meaningful exchange of ideas and to support peaceful disagreement and discussion. In our classes, we strive to maintain a positive learning environment based upon open communication and mutual respect. UNK does not discriminate on the basis of race, color, national origin, age, religion, sex, gender, sexual orientation, disability or political affiliation. Respect for the diversity of our backgrounds and varied life experiences is essential to learning from our similarities as well as our differences. The following link provides resources and other information regarding D&I: `https://www.unk.edu/about/equity-access-diversity.php`

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Blocks marked "Appointments" (light yellow highlight) means usually available to make appointments.

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00–9:45	**MATH 102** 9:05-9:55 DSCH 116	**Unavailable** 8:00-12:00	**MATH 102** 9:05-9:55 DSCH 116	**Unavailable** 8:00-12:00	**MATH 102** 9:05-9:55 DSCH 116
10:00–10:45	**Office Hours** 10:00-11:00 DSCH 368		**Office Hours** 10:00-11:00 DSCH 368		**Office Hours** 10:00-11:00 DSCH 368
11:00–11:45	**Lunch** 11:00-12:00 Off campus	**Lunch** 11:00-12:00 Off campus	**Lunch** 11:00-12:00 Off campus	**Lunch** 11:00-12:00 Off campus	**Lunch** 11:00-12:00 Off campus
12:00–12:45	**Appointments** 12:00-13:20 DSCH 368	**Office Hours** 12:00-14:00 DSCH 368	**Appointments** 12:00-13:20 DSCH 368	**Office Hours** 12:00-14:00 DSCH 368	**Appointments** 12:00-13:20 DSCH 368
13:00–13:45	**MATH 365-01** 13:25-14:15 DSCH 386		**MATH 365-01** 13:25-14:15 DSCH 386		**MATH 365-01** 13:25-14:15 DSCH 386
14:00–14:45		**Meeting** 14:00-15:00		**Meetings** 14:00-15:00	
15:00–15:45	**MATH 250-01** 14:30-15:20 DSCH 386	**Appointments** 15:00-16:30 DSCH 368	**MATH 250-01** 14:30-15:20 DSCH 386	**Appointments** 15:00-16:30 DSCH 368	**MATH 250-01** 14:30-15:20 DSCH 386
16:00	**Unavailable** 15:30-16:30		**Unavailable** 15:30-16:30		**Unavailable** 15:30-16:30

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or Π	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Truth Tables

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \equiv Q$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

Tautologies

$$\neg\neg P \equiv P$$
$$(P \vee P) \equiv P$$
$$(P \wedge P) \equiv P$$
$$(P \equiv Q) \equiv (Q \equiv P)$$
$$(P \implies Q) \equiv (P \vee \neg Q)$$
$$(P \not\iff Q) \equiv (\neg P$$

landQ

$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$
$$(P \implies Q) \equiv (\neg Q \implies \neg P)$$
$$(P \not\implies Q) \equiv (P \wedge \neg Q)$$
$$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$$
$$\neg(\forall\, x \in A)(P(x)) \equiv (\exists\, x \in A)(\neg P(x))$$
$$\neg(\exists\, x \in A)(P(x)) \equiv (\forall\, x \in A)(\neg P(x))$$

Arithmetic properties

$(\forall a, b \in \mathbf{R})(a + b = b + a)$	commutivity
$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$	commutivity
$(\forall a, b \in \mathbf{R})(ab = ba)$	commutivity
$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$	commutivity
$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$	distrutity

Function notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F
C_A	set of continuous functions on set A
C^1_A	set of differentiable functions on set A
$A \to B$	set of functions from A to B

Set operators

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$
$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$
$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$
$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

Generalized set operators

Each member of a set $\mathcal{C}$ is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists\, B \in \mathcal{C})(z \in B)\}$$
$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall\, B \in \mathcal{C})(z \in B)\}$$

Theorem: $\displaystyle\bigcup_{A \in \mathcal{C}} A^{\mathrm{C}} = \left(\bigcap_{A \in \mathcal{C}} A\right)^{\mathrm{C}}$

Functions applied to sets

Let $A \subset \mathrm{dom}(F)$ and $B \subset \mathrm{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$
$$F^{-1}(B) = \{x \in \mathrm{dom}(F) \mid F(x) \in B\}$$

Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \le |x| + |y|$$
$$\big||x| - |y|\big| \le |x - y|$$

Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \le x\}$$
$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \ge x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$
$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$

Elementary function properties

Increasing $(\forall\, x, y \in A)(x < y \implies F(x) \le F(y))$. For strictly increasing, replace $F(x) \le F(y)$ with $F(x) < F(y)$.

Decreasing $(\forall\, x, y \in A)(x < y \implies F(x) \ge F(y))$ For strictly decreasing, replace $F(x) \ge F(y)$ with $F(x) > F(y)$.

One-to-one
$(\forall\, x, y \in \mathrm{dom}(F))(F(x) = F(y) \implies x = y)$

Equivalence relations

Let $R \in A \times A \to \{\text{true}, \text{false}\}$. We say

reflective $(\forall x \in S)(x\,R\,x)$

symmetric $(\forall x, y \in S) x\,R\,y \implies y\,R\,x$

transitive $(\forall x, y, z \in S) x\,R\,y \wedge y\,R\,z \implies x\,R\,z$

Equivalence class $[x] = \{s \in S \mid s\,R\,x\}$

Axioms

Well-ordering Every nonempty set of positive integers contains a least element.

Induction $(\forall n \in \mathbf{Z}_{\geq 0})(P(n))$ if and only if $P(0) \wedge (\forall n \in \mathbf{Z}_{\geq 0})(P(n) \implies P(n+1))$.