MATH 250 **Name:**
Homework 5, Spring 2023

"To learn, one must be humble. But life is the great teacher." JAMES JOYCE

I have neither given nor received unauthorized assistance on this assignment.

Homework 5 has questions 1 through 5 with a total of 7 points. For this assignment, *neatly handwrite* your work on your own paper, digitize it, and upload it to Canvas. This assignment is due Saturday 11 March at 11:59 PM.

[1] 1. Show that $\{(2,3)\}$ is a transitive relation on $\mathbf{Z}$.

[1] 2. Show that $\{(1,3),(3,4),(1,4)\}$ is a transitive relation on $\mathbf{Z}$.

[1] 3. Give an example of a set A and transitive relations R and R' on A such that $R \cup R'$ is not a transitive relation on A. (Hint: An example is hiding in plain sight.)

4. On the set $\mathbf{R}$, define a relation $\{(a,b) \in \mathbf{R} \times \mathbf{R} \mid a \neq b\}$.

[1] (a) Is this relation *reflexive*? If so, prove it; if not, give an example that shows that it is not reflexive.

[1] (b) Is this relation *symmetric*? If so, prove it; if not, give an example that shows that it is not symmetric.

[1] (c) Is this relation *transitive*? If so, prove it; if not, give an example that shows that it is not transitive.

[1] 5. Let F be the set of all real valued functions from $\mathbf{R}$ to $\mathbf{R}$. Define a relation E on F as

$$E = \{(f,g) \mid f - g \text{ is a constant function}\}.$$

If E is an equivalence relation on F, prove it; if not, give an example that shows that it is not an equivalence relation.

To say that a function f is constant means $(\exists\, C \in \mathbf{R})\,(\forall\, x \in \mathbf{R})\,\big(f(x) = C\big)$.