Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Truth Tables

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \equiv Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Tautologies

$$\neg\neg P \equiv P$$
$$(P \vee P) \equiv P$$
$$(P \wedge P) \equiv P$$
$$(P \equiv Q) \equiv (Q \equiv P)$$
$$(P \implies Q) \equiv (P \vee \neg Q)$$
$$(P \not\implies Q) \equiv (\neg P \wedge Q)$$
$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$
$$(P \implies Q) \equiv (\neg Q \implies \neg P)$$
$$(P \not\implies Q) \equiv (P \wedge \neg Q)$$
$$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$$
$$\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$$
$$\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$$

Vacuous Truth

$$(\forall x \in \varnothing)(P(x)) \equiv \text{True}$$
$$(\exists x \in \varnothing)(P(x)) \equiv \text{False}$$

Arithmetic properties

$(\forall a, b \in \mathbf{R})(a + b = b + a)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c)$ commutivity

$(\forall a, b \in \mathbf{R})(ab = ba)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c)$ commutivity

$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac)$ distrutity

Function notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F
C_A	set of continuous functions on set A
C^1_A	set of differentiable functions on set A
$A \to B$	set of functions from A to B

Set operators

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$
$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$
$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$
$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

Generalized set operators

Each member of a set $\mathcal{C}$ is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists B \in \mathcal{C})(z \in B)\}$$
$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall B \in \mathcal{C})(z \in B)\}$$

Theorem: $\displaystyle\bigcup_{A \in \mathcal{C}} A^{\mathrm{C}} = \left(\bigcap_{A \in \mathcal{C}} A\right)^{\mathrm{C}}$

Functions applied to sets

Let $A \subset \mathrm{dom}(F)$ and $B \subset \mathrm{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$
$$F^{-1}(B) = \{x \in \mathrm{dom}(F) \mid F(x) \in B\}$$

Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \leq |x| + |y|$$
$$\big||x| - |y|\big| \leq |x - y|$$

Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \leq x\}$$
$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \geq x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$
$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$

Elementary function properties

Increasing $(\forall\, x, y \in A)(x < y \implies F(x) \leq F(y))$. For strictly increasing, replace $F(x) \leq F(y)$ with $F(x) < F(y)$.

Decreasing $(\forall\, x, y \in A)(x < y \implies F(x) \geq F(y))$ For strictly decreasing, replace $F(x) \geq F(y)$ with $F(x) > F(y)$.

One-to-one $(\forall\, x, y \in \mathrm{dom}(F))(F(x) = F(y) \implies x = y)$

Equivalence relations

Let $R \in A \times A \to \{\mathrm{true}, \mathrm{false}\}$. We say

reflective $(\forall x \in S)(x\, R\, x)$

symmetric $(\forall x, y \in S) x\, R\, y \implies y\, R\, x$

transitive $(\forall x, y, z \in S) x\, R\, y \wedge y\, R\, z \implies x\, R\, z$

Equivalence class $[x] = \{s \in S \mid s\, R\, x\}$

Axioms

Well-ordering Every nonempty set of positive integers contains a least element.

Induction $(\forall n \in \mathbf{Z}_{\geq 0})(P(n))$ if and only if $P(0) \wedge (\forall n \in \mathbf{Z}_{\geq 0})(P(n) \implies P(n+1))$.