

Homework 4, Spring 2023

I have neither given nor received unauthorized assistance on this assignment.

Homework 4 has questions 1 through 4 with a total of 20 points. For this assignment, *neatly handwrite* your work on your own paper, digitize it, and upload it to Canvas.

- 5 1. Without explicitly using \neg , write the negation of

$$(\forall x \in \mathbf{R}) (\exists y \in \mathbf{R}) (|x| < |y|).$$

Solution:

- 5 2. Show that the statement

$$(\forall x \in \mathbf{R}) (\exists y \in \mathbf{R}) (|x| < |y|).$$

is false by showing that its negation is true.

Solution:

- 5 3. Show that the statement

$$(\exists L \in \mathbf{R}) (\exists m \in \mathbf{R}) (\forall x \in \mathbf{R}) \left(\frac{x+9}{x+11} < L \right)$$

is true.

Solution:

- 5 4. Given that the statement

$$(\forall x \in \mathbf{R}) (|\sin(x)| \leq 1)$$

is true, show that

$$(\exists m \in \mathbf{R}) (\forall x \in \mathbf{R}) (|\sin(x) + x| \leq m|x|)$$

Solution: