MATH 250 **Name:**______________________________

Exam I **Row:**______________________________

Exam I has questions 1 through 8 with a total of 100 points. This exam is printed on both sides of the paper.

1. **Show all of your work.** Do not expect to earn full credit for a correct answer without the needed work.
2. Divine intervention is *not* a substitute for showing your work.
3. If your answer is wrong, but your work shows me that you know the major steps in solving a problem, you will likely earn some partial credit.
4. Your work should convince me that not only could you correctly solve the given problem, but you could also solve any related problem.
5. If a question asks for a sentence, write your answer as an English sentence.
6. No talking, no sharing calculators, and no scratch paper.
7. Turn your phone off and put it out of sight.
8. Clear your desk of everything, except a pencil, eraser, and a calculator.
9. If you never make a mistake, you may use ink; otherwise use a pencil.
10. Do not unstaple the pages of your exam.
11. We'll all start at the same time–it's the polite thing to do.
12. Write your answers in the space provided.
13. If you do not want something graded, erase it or clearly cross it out.
14. You may stare at your feet, your paper, or the ceiling, but nowhere else.
15. If you wear a baseball cap, wear it backwards so I can see your eyes.
16. Work each problem correctly.
17. When you are finished, collect your things, place your exam paper in the folder on the front desk, and quietly leave the room.
18. After you turn in your paper, I will not answer questions about the test until after it is graded.
19. Read all directions and problems carefully.

1. True or False:

5 (a) ________ $\varnothing = \{\varnothing\}$.

5 (b) ________ $\varnothing \subset \{\varnothing\}$.

10 2. Write the *contrapositive* of the statement *If an integer n is even, then $2n+2$ is even.*

10 3. Write the *converse* of the statement *If an integer n is even, then $2n+2$ is even.*

10 4. Give an example of a *conditional statement that is true*, but whose converse is false.

5. Enumerate the members of each set:

$\boxed{10}$ (a) $\{1,2,\sqrt{5}\} \cap \{1,2,\sqrt{2023}\}$

$\boxed{10}$ (b) $\{1,2,\sqrt{5}\} \cup \{1,2,\sqrt{2023}\}$

$\boxed{10}$ (c) $\{1,2,\sqrt{5}\} \setminus \{1,2,\sqrt{2023}\}$

$\boxed{10}$ 6. Using a truth table, show that $P \implies Q$ is logically equivalent to $\neg Q \implies \neg P$.

$\boxed{10}$ 7. Let A and B be sets. Write the *contrapositive* of the statement $A \setminus B = A \implies A \cap B = \varnothing$.

$\boxed{10}$ 8. Let A and B be sets. Show that $(A \subset B) \wedge (B \subset C) \implies A \cap C$. I've started the proof for you

Proof Suppose $x \in A$; we'll show that $x \in C$.

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Truth Tables

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \equiv Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Tautologies

$$\neg\neg P \equiv P$$
$$(P \vee P) \equiv P$$
$$(P \wedge P) \equiv P$$
$$(P \equiv Q) \equiv (Q \equiv P)$$
$$(P \implies Q) \equiv (P \vee \neg Q)$$
$$(P \not\Longrightarrow Q) \equiv (\neg P \wedge Q)$$
$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$
$$(P \implies Q) \equiv (\neg Q \implies \neg P)$$
$$(P \not\Longrightarrow Q) \equiv (P \wedge \neg Q)$$
$$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$$
$$\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$$
$$\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$$

Vacuous Truth

$$(\forall x \in \varnothing)(P(x)) \equiv \text{True}$$
$$(\exists x \in \varnothing)(P(x)) \equiv \text{False}$$

Arithmetic properties

$$(\forall a, b \in \mathbf{R})(a + b = b + a) \quad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c) \text{commutivity}$$
$$(\forall a, b \in \mathbf{R})(ab = ba) \quad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c) \quad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac) \quad \text{distrutity}$$

Function notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F
C_A	set of continuous functions on set A
C^1_A	set of differentiable functions on set A
$A \to B$	set of functions from A to B

Set operators

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$
$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$
$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$
$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

Generalized set operators

Each member of a set $\mathcal{C}$ is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists B \in \mathcal{C})(z \in B)\}$$
$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall B \in \mathcal{C})(z \in B)\}$$

Theorem: $\bigcup_{A \in \mathcal{C}} A^{\mathrm{C}} = \left(\bigcap_{A \in \mathcal{C}} A\right)^{\mathrm{C}}$

Functions applied to sets

Let $A \subset \mathrm{dom}(F)$ and $B \subset \mathrm{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$
$$F^{-1}(B) = \{x \in \mathrm{dom}(F) \mid F(x) \in B\}$$

Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \leq |x| + |y|$$
$$\big||x| - |y|\big| \leq |x - y|$$

Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \leq x\}$$
$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \geq x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$
$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$