## Greek characters

| Name | Symbol | Typical use(s) |
|---|---|---|
| alpha | $\alpha$ | angle, constant |
| beta | $\beta$ | angle, constant |
| gamma | $\gamma$ | angle, constant |
| delta | $\delta$ | limit definition |
| epsilon | $\epsilon$ or $\varepsilon$ | limit definition |
| theta | $\theta$ or $\vartheta$ | angle |
| pi | $\pi$ or $\varpi$ | circular constant |
| phi | $\phi$ or $\varphi$ | angle, constant |

## Named sets

| | |
|---|---|
| empty set | $\varnothing$ |
| real numbers | $\mathbf{R}$ |
| ordered pairs | $\mathbf{R}^2$ |

| | |
|---|---|
| integers | $\mathbf{Z}$ |
| positive integers | $\mathbf{Z}_{>0}$ |
| positive reals | $\mathbf{R}_{>0}$ |

## Set symbols

| Meaning | Symbol |
|---|---|
| is a member | $\in$ |
| subset | $\subset$ |
| intersection | $\cap$ |

| Meaning | Symbol |
|---|---|
| union | $\cup$ |
| complement | superscript$^{\mathrm{C}}$ |
| set minus | $\setminus$ |

## Logic symbols

| Meaning | Symbol |
|---|---|
| negation | $\neg$ |
| and | $\wedge$ |
| or | $\vee$ |
| implies | $\implies$ |

| Meaning | Symbol |
|---|---|
| equivalent | $\equiv$ |
| iff | $\iff$ |
| for all | $\forall$ |
| there exists | $\exists$ |

## Truth Tables

| $P$ | $Q$ | $P \wedge Q$ | $P \vee Q$ | $P \implies Q$ | $P \equiv Q$ |
|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $T$ | $T$ | $F$ |
| $F$ | $F$ | $F$ | $F$ | $T$ | $T$ |

## Tautologies

$$\neg\neg P \equiv P$$
$$(P \vee P) \equiv P$$
$$(P \wedge P) \equiv P$$
$$(P \equiv Q) \equiv (Q \equiv P)$$
$$(P \implies Q) \equiv (P \vee \neg Q)$$
$$(P \not\Longrightarrow Q) \equiv (\neg P \wedge Q)$$
$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$
$$(P \implies Q) \equiv (\neg Q \implies \neg P)$$
$$(P \not\Longrightarrow Q) \equiv (P \wedge \neg Q)$$
$$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$$
$$\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$$
$$\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$$

## Vacuous Truth

$$(\forall x \in \varnothing)(P(x)) \equiv \text{True}$$
$$(\exists x \in \varnothing)(P(x)) \equiv \text{False}$$

## Arithmetic properties

$$(\forall a, b \in \mathbf{R})(a + b = b + a) \qquad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a + (b + c) = (a + b) + c) \qquad \text{commutivity}$$
$$(\forall a, b \in \mathbf{R})(ab = ba) \qquad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a(bc) = (ab)c) \qquad \text{commutivity}$$
$$(\forall a, b, c \in \mathbf{R})(a(b + c) = ab + ac) \qquad \text{distrutity}$$

## Function notation

| | |
|---|---|
| $\mathrm{dom}(F)$ | domain of function $F$ |
| $\mathrm{range}(F)$ | range of function $F$ |
| $\mathrm{C}_A$ | set of continuous functions on set $A$ |
| $\mathrm{C}^1_A$ | set of differentiable functions on set $A$ |
| $A \to B$ | set of functions from $A$ to $B$ |

## Set operators

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$
$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$
$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$
$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

## Generalized set operators

Each member of a set $\mathcal{C}$ is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists B \in \mathcal{C})(z \in B)\}$$
$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall B \in \mathcal{C})(z \in B)\}$$

Theorem: $\bigcup_{A \in \mathcal{C}} A^{\mathrm{C}} = \left(\bigcap_{A \in \mathcal{C}} A\right)^{\mathrm{C}}$

## Functions applied to sets

Let $A \subset \mathrm{dom}(F)$ and $B \subset \mathrm{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$
$$F^{-1}(B) = \{x \in \mathrm{dom}(F) \mid F(x) \in B\}$$

## Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \le |x| + |y|$$
$$\big||x| - |y|\big| \le |x - y|$$

## Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \le x\}$$
$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \ge x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$
$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$

## Elementary function properties

**Increasing** $(\forall\, x, y \in A)(x < y \implies F(x) \leq F(y))$. For strictly increasing, replace $F(x) \leq F(y)$ with $F(x) < F(y)$.

**Decreasing** $(\forall\, x, y \in A)(x < y \implies F(x) \geq F(y))$ For strictly decreasing, replace $F(x) \geq F(y)$ with $F(x) > F(y)$.

**One-to-one** $(\forall\, x, y \in \mathrm{dom}(F))(F(x) = F(y) \implies x = y)$

## Equivalence relations

Let $R \in A \times A \to \{\mathrm{true}, \mathrm{false}\}$. We say

**reflective** $(\forall x \in S)(x \, R \, x)$

**symmetric** $(\forall x, y \in S) x \, R \, y \implies y \, R \, x$

**transitive** $(\forall x, y, z \in S) x \, R \, y \land y \, R \, z \implies x \, R \, z$

**Equivalence class** $[x] = \{s \in S \mid s \, R \, x\}$

## Axioms

**Well-ordering** Every nonempty set of positive integers contains a least element.

**Induction** $(\forall n \in \mathbf{Z}_{\geq 0})(P(n))$ if and only if $P(0) \land (\forall n \in \mathbf{Z}_{\geq 0})(P(n) \implies P(n+1))$.