MATH 250 **Name:**
Homework 7, Spring 2023

"To learn, one must be humble. But life is the great teacher." JAMES JOYCE

$\boxed{\text{I have neither given nor received unauthorized assistance on this assignment.}}$

Homework 7 has questions 1 through 4 with a total of 70 points. For this assignment, *neatly hand write* your work on your own paper, digitize it, and upload it to Canvas. This assignment is due Saturday 1 April at 11:59 PM.

1. Let A and B be subsets of $\mathbf{R}$. Any function from A to B is called a *real valued function of one real variable.* For any two such functions F and G, define

$$F + G = x \in \operatorname{dom}(F) \cap \operatorname{dom}(G) \mapsto F(x) + G(x),$$
$$FG = x \in \operatorname{dom}(F) \cap \operatorname{dom}(G) \mapsto F(x)G(x),$$
$$\frac{F}{G} = x \in \operatorname{dom}(F) \cap \operatorname{dom}(G) \setminus \{x \in \mathbf{R} | G(x) = 0\} \mapsto \frac{F(x)}{G(x)}.$$

$\boxed{10}$ (a) Find an example of two *real valued functions of one real variable* F and G such that both F and G are one-to-one, but $F + G$ is not one-to-one. Justify your example.

$\boxed{10}$ (b) Find an example of two *real valued functions of one real variable* F and G such that both F and G are one-to-one, but FG is not one-to-one. Justify your example.

$\boxed{10}$ (c) Define $F = x \in \mathbf{R}_{\neq 0} \mapsto x + \frac{1}{x}$. Find $\operatorname{dom}(1/F)$.

2. On the set $\mathbf{R}$ define the equivalence relation $(x \sim y) \equiv \sin(x) = \sin(y)$. Give an explicit representation for the equivalence class $[1/2]$. (Okay–the set $[1/2]$ has infinitely many members, so no, you can't list all the members, but you can express the set $[1/2]$ in set builder notation.)

3. Define $\Phi = k \in \mathbf{Z} \mapsto \begin{cases} 2k & k \leq 0 \\ 2k+1 & k > 0 \end{cases}$.

$\boxed{10}$ (a) Is Φ one-to-one? If so, prove it; if not, prove that too.

$\boxed{10}$ (b) Find $\operatorname{range}(\Phi)$.

4. The prime factorization of an integer is unique. On consequence of this is that if $2^k \times 3^\ell = 2^{k'} \times 3^{\ell'}$, where $k, \ell, k,$ and k' are non negative integers, then $k = k'$ and $\ell = \ell'$. Define a function $\Psi = (k, n) \in \mathbf{Z}_{\geq 0} \times \mathbf{Z}_{\geq 0} \mapsto 2^k 3^n$.

$\boxed{10}$ (a) Show that the function Ψ is one-to-one.

$\boxed{10}$ (b) Show that range$(\Psi) \neq \mathbf{Z}_{\geq 0}$