MATH 250 **Name:**________________________

Review for Exam III **Row:**________________________

Exam III has questions 1 through 7 with a total of 54 points. This exam is printed on both sides of the paper.

1. From a litter of eight border collies, how many subsets of puppies with cardinality four are there? Explain.

2. A five digit integer has the form $d_1 d_2 d_3 d_4 d5$, where $d_1, d_2 \ldots, d_5 \in \{0,1,2,3,4,5,6,7,8,9\}$ and $d_1 \neq 0$. In each of the following cases, determine the number of five digit integers that satisfy the given condition. **Clearly explain your work with a few sentences.**

$\boxed{2}$ (a) No additional restrictions on the digits.

$\boxed{2}$ (b) There are no repeated digits.

$\boxed{10}$ 3. Let A, B and C be finite sets. Given that $\operatorname{card}(A) = 46$, $\operatorname{card}(B) = 107$, and $\operatorname{card}(A \cap B) = 12$, find $\operatorname{card}(A \cup B)$.

$\boxed{10}$ 4. Find a bijection from $[0, 1]$ to $[-1, 1]$.

$\boxed{10}$ 5. Show that the set $\mathbf{Z}$ is countable. To do this, you must construct a one-to-one function from $\mathbf{Z}$ to $\mathbf{Z}_{\geq 0}$.

$\boxed{10}$ 6. A recursive definition of a sequence F is $F_n = \begin{cases} 1 & \text{if } n = 0 \\ \frac{1}{4}F_{n-1} + \frac{3n+1}{4} & \text{if } n \in \mathbf{Z}_{\geq 1} \end{cases}$. Use induction to show that $(\forall n \in \mathbf{Z}_{\geq 0})\left(F_n = n + \frac{1}{4^n}\right)$.

$\boxed{10}$ 7. Let A and B be countable sets. Show that $A \cup B$ is countable.