

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	\emptyset
real numbers	\mathbf{R}
ordered pairs	\mathbf{R}^2

integers	\mathbf{Z}
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	\in
subset	\subset
intersection	\cap

Meaning	Symbol
union	\cup
complement	superscript ^C
set minus	\setminus

Logic symbols

Meaning	Symbol
negation	\neg
and	\wedge
or	\vee
implies	\implies

Meaning	Symbol
equivalent	\equiv
iff	\iff
for all	\forall
there exists	\exists

Truth Tables

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \equiv Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Tautologies

$$\begin{aligned}
 \neg\neg P &\equiv P \\
 (P \vee P) &\equiv P \\
 (P \wedge P) &\equiv P \\
 (P \equiv Q) &\equiv (Q \equiv P) \\
 (P \implies Q) &\equiv (P \vee \neg Q) \\
 (P \not\implies Q) &\equiv (\neg P \\
 \text{land } Q & \\
 \neg(P \wedge Q) &\equiv (\neg P \vee \neg Q) \\
 (P \implies Q) &\equiv (\neg Q \implies \neg P) \\
 (P \not\implies Q) &\equiv (P \wedge \neg Q) \\
 (P \iff Q) &\equiv ((P \implies Q) \wedge (Q \implies P)) \\
 \neg(\forall x \in A)(P(x)) &\equiv (\exists x \in A)(\neg P(x)) \\
 \neg(\exists x \in A)(P(x)) &\equiv (\forall x \in A)(\neg P(x))
 \end{aligned}$$

Arithmetic properties

$$\begin{aligned}
 (\forall a, b \in \mathbf{R})(a + b &= b + a) && \text{commutivity} \\
 (\forall a, b, c \in \mathbf{R})(a + (b + c) &= (a + b) + c) && \text{commutivity} \\
 (\forall a, b \in \mathbf{R})(ab &= ba) && \text{commutivity} \\
 (\forall a, b, c \in \mathbf{R})(a(bc) &= (ab)c) && \text{commutivity} \\
 (\forall a, b, c \in \mathbf{R})(a(b + c) &= ab + ac) && \text{distrutivity}
 \end{aligned}$$

Function notation

$\text{dom}(F)$	domain of function F
$\text{range}(F)$	range of function F
C_A	set of continuous functions on set A
C_A^1	set of differentiable functions on set A
$A \rightarrow B$	set of functions from A to B

Set operators

$$\begin{aligned}
 A \cup B &= \{x \mid x \in A \vee x \in B\} \\
 A \cap B &= \{x \mid x \in A \wedge x \in B\} \\
 A \setminus B &= \{x \mid x \in A \wedge x \notin B\} \\
 A \times B &= \{(a, b) \mid a \in A \wedge b \in B\}
 \end{aligned}$$

Generalized set operators

Each member of a set \mathcal{C} is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists B \in \mathcal{C})(z \in B)\}$$

$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall B \in \mathcal{C})(z \in B)\}$$

Theorem: $\bigcup_{A \in \mathcal{C}} A^C = \left(\bigcap_{A \in \mathcal{C}} A\right)^C$

Functions applied to sets

Let $A \subset \text{dom}(F)$ and $B \subset \text{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$

$$F^{-1}(B) = \{x \in \text{dom}(F) \mid F(x) \in B\}$$

Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \leq |x| + |y|$$

$$||x| - |y|| \leq |x - y|$$

Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \leq x\}$$

$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \geq x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$

Elementary function properties

Increasing $(\forall x, y \in A)(x < y \implies F(x) \leq F(y))$. For strictly increasing, replace $F(x) \leq F(y)$ with $F(x) < F(y)$.

Decreasing $(\forall x, y \in A)(x < y \implies F(x) \geq F(y))$. For strictly decreasing, replace $F(x) \geq F(y)$ with $F(x) > F(y)$.

One-to-one

$$(\forall x, y \in \text{dom}(F))(F(x) = F(y) \implies x = y)$$

Equivalence relations

Let $R \in A \times A \rightarrow \{\text{true}, \text{false}\}$. We say

reflective $(\forall x \in S)(x R x)$

symmetric $(\forall x, y \in S)x R y \implies y R x$

transitive $(\forall x, y, z \in S)x R y \wedge y R z \implies x R z$

Equivalence class $[x] = \{s \in S \mid s R x\}$

Axioms

Well-ordering Every nonempty set of positive integers contains a least element.

Induction $(\forall n \in \mathbf{Z}_{\geq 0})(P(n))$ if and only if $P(0) \wedge (\forall n \in \mathbf{Z}_{\geq 0})(P(n) \implies P(n+1))$.