MATH 250 **Name:**
Homework 8, Spring 2023

The more I read, the more I acquire, the more certain I am that I know nothing.
FRANÇOIS-MARIE AROUET

I have neither given nor received unauthorized assistance on this assignment.

Homework 8 has questions 1 through 3 with a total of 30 points. For this assignment, **use Overleaf to typeset your work and upload a pdf to Canvas.** The assignment is due Saturday 8 April at 11:59 pm.

10 1. Let A, B, and C be countable sets. Show that $A \times B \times C$ is countable. Remember that $A \times B \times C$ is the set of three-tuples; specifically

$$A \times B \times C = \{(a, b, c) \mid a \in A, b \in B, c \in C\}.$$

Hint You need to find $\Phi \in A \times B \times C \to \mathbf{Z}_{\geq 0}$ that is one-to-one. But you do *not* need to make Φ onto.

10 2. Let A and B be nonempty finite sets. Show that $A \setminus B$ is finite. (The sets needn't be nonempty, but assuming they are both nonempty eliminates two special cases.)

Solution:

10 3. Let A and B be nonempty finite sets. Show that $A \cup B$ is finite. (Again, the sets needn't be nonempty.)

Solution:

Hints There is $M \in \mathbf{Z}_{\geq 1}$ and a bijection $\Phi \in A \to 1 \ldots M$. Similarly, there is $N \in \mathbf{Z}_{\geq 1}$ and a bijection $\Psi \in B \to 1 \ldots N$. For some integer L, you need to define a bijection from $A \cup B$ to $1 \ldots L$. If A and B were disjoint, you could define

$$\Omega = x \in A \cup B \mapsto \begin{cases} \Phi(x) & x \in A \\ \Psi(x) + M & x \in B \end{cases}.$$

Then Ω is a bijection from $A \cup B$ to $1 \ldots M + N$. But this doesn't work if $A \cap B \neq \varnothing$. Actually when A and B aren't disjoint, Ω isn't a function–if $w \in A \cap B$, what's the value of $\Omega(w)$? But the definition of Ω can be tweaked to make Ω a bijection.

Solution: