MATH 250 **Name:**
Homework 3, Spring 2023

"Mathematics is common sense." ERRETT BISHOP (1928–1983)

Errett Bishop was born in Newton, Kansas and was the son of a Wichita State mathematics professor. He is best known for his book *Foundations of Constructive Analysis*. Constructive mathematics rejects the Law of excluded middle.

I have neither given nor received unauthorized assistance on this assignment.

Homework 3 has questions 1 through 2 with a total of 10 points. For this assignment, use Overleaf to complete the assignment and upload the pdf to Canvas.

5 1. Show that for all positive integers p and q that $3q^2 \neq p^2$.

To do this, use the fact that every positive integer can be uniquely expressed in the form $3^e n$, where e is a nonnegative integer and n is not divisible by 3. For example, $24 = 3^1 \times 8$ (and 8 is not divisible by 3); and $963 = 3^2 \times 107$ (and 107 is not divisible by 3). Uniqueness of this representation means that

$$\left[3^e n = 3^{e'} n'\right] \equiv \left[(e = e') \wedge (n = n')\right].$$

Equivalently

$$\left[3^e n \neq 3^{e'} n'\right] \equiv \left[(e \neq e') \vee (n \neq n')\right].$$

Solution:

5 2. For all positive real numbers x and y with $x \neq y$, show that

$$\frac{x}{y} + \frac{y}{x} > 2.$$

To do this, assume that $\frac{x}{y} + \frac{y}{x} \leq 2$. Use some algebra facts to derive a contradiction.