CYRR 304 **Name:**

Homework 7, Spring 2024

"When you're good to others, you're best to yourself." BENJAMIN FRANKLIN

Homework 7 has questions 1 through 4 with a total of of 20 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 23 March** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas.

The degree four Bernstein polynomials P_0 through P_4 are defined by

$$P_0(x) = (1-x)^4, \quad P_1(x) = 4x(1-x)^3, \quad P_2(x) = 6x^2(1-x)^2, \quad P_3(x) = 4x^3(1-x), \quad P_4(x) = x^4. \tag{1}$$

The degree of each Bernstein polynomial is four, and it can be proven that every polynomial of degree four or less is a linear combination of these five polynomials. On the interval $[0,1]$, the Bernstein polynomials are non negative. This property makes them well suited to approximating functions that are nonnegative on $[0,1]$.

$\boxed{5}$ 1. Use Gadfly to graph P_0 through P_4 on the interval $[0,1]$.

$\boxed{5}$ 2. Find the real numbers c_0, c_1, c_2, c_3, and c_4 such that the function

$$F(x) = \sum_{k=0}^{4} c_k P_k(x) \tag{2}$$

minimizes

$$\sum_{k=0}^{4} \left(c_k P\left(\frac{k}{10}\right) - \sin\left(\frac{\pi k}{10}\right)\right)^2. \tag{3}$$

Be sure to find the condition number of the coefficient matrix of the normal equations.

$\boxed{5}$ 3. Use Gadfly to graph both $y = \sin(\pi x)$ and $y = \sum_{k=0}^{4} c_k P_k(x)$ on the interval $[0,1]$, where the numbers c_0 through c_4 are the numbers that you found in Question 2.

$\boxed{5}$ 4. Use Gadfly to graph $y = \left|\sin(\pi x) - \sum_{k=0}^{4} c_k P_k(x)\right|$, where the numbers c_0 through c_4 are the numbers that you found in Question 2. Visually find $\max_{x \in [0,1]} \left(\left|\sin(\pi x) - \sum_{k=0}^{4} c_k P_k(x)\right|\right)$.