CYRR 304 **Name:**
Homework 3, Spring 2024

"Fast is fine, but accuracy is everything." WYATT ERP

Homework 3 has questions 1 through 1 with a total of of 20 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 10 Feb** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas. To convert you Jupyter notebook to HTML, do File –> Download as –> HTML. (For the File menu, look toward the upper right corner.)

1. On the first day of each month, Patsy invests \$100 in Larry's Pretty Good mutual fund (LPG). The share price varies each day, so let's say that on the first of January the share price is p_0 dollars per share, the first of February, the price is p_1, and ... and p_{11} on the first of December. So on 31 December, Patsy owns

$$100 \sum_{k=0}^{11} \frac{1}{p_k} \tag{1}$$

shares, and the value of these shares is $100 p_{12} \sum_{k=0}^{11} \frac{1}{p_k}$, where p_{12} is the share price on the last day of the year. Patsy would like to compare her investment to an investment that has a fixed rate of return of r. She calculates that had she similarly invested \$100 on the first day of each month to an investment with a fixed rate of return, at the end of the year, her investment would be worth

$$100r \frac{(1+r)^{1/12}}{(1+r)^{1/12}-1}. \tag{2}$$

So to find the APR of her mutual fund return, Patsy wants to solve

$$100 p_{12} \sum_{k=0}^{11} \frac{1}{p_k} = 100r \frac{(1+r)^{1/12}}{(1+r)^{1/12}-1}. \tag{3}$$

for r. Of course, we can cancel the multiplicative factor of 100, so Patsy needs to solve the equation

$$p_{12} \sum_{k=0}^{11} \frac{1}{p_k} = r \frac{(1+r)^{1/12}}{(1+r)^{1/12}-1}. \tag{4}$$

for r. The stuff on the left is known, so let's name it q. So Patsy needs to solve

$$q = r \frac{(1+r)^{1/12}}{(1+r)^{1/12}-1}. \tag{5}$$

No amount of algebra tricks allow Patsy to solve this equation, so she decides to use a numerical method. Her favorite method is fixed point iteration, but her first step is to convert the equation to a fixed point form. There are lots of ways to do this–we need to pick one and try it. A fairly obvious way to convert to fixed point form is to add $r - q$ to both sides of this equation; thus

$$r = r \frac{(1+r)^{1/12}}{(1+r)^{1/12}-1} - q + r. \tag{6}$$

5 (a) Suppose $q = 13.5$. (This means after investing \$1,200 for the year, her account value at the end of the year is \$1,350, for a total gain of \$150.). Use Gadfly to graph both $r \mapsto r$ and $r \mapsto r\frac{(1+r)^{1/12}}{(1+r)^{1/12}-1} - 13.5 + r$. Based on this graph, do you expect that a fixed point sequence will converge?

5 (b) Use fixed point iteration to solve $r = r\frac{(1+r)^{1/12}}{(1+r)^{1/12}-1} - 13.5 + r$ for r.

5 (c) Write Julia code that takes the twelve monthly share prices $p_0, p_1, \ldots p_{12}$ and returns $p_{12}\sum_{k=1}^{11}\frac{1}{p_k}$.

5 (d) Write a Julia function **Patsy(q)** that uses fixed point iteration to solve $r = r\frac{(1+r)^{1/12}}{(1+r)^{1/12}-1} - q + r$.