

CYRR 304
Homework 6, Spring 2024

Name:

"To pay attention, this is our endless and proper work."

MARY OLIVER

Homework 6 has questions 1 through 6 with a total of 60 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 9 March** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas.

The sigmoid function is frequently used in machine learning, but oddly, there is not a single agreed definition of this function. For this assignment, we'll define the sigmoid function by

$$\text{sigmoid}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (1)$$

But the standard name of this function is erf, not sigmoid. The function erf is important in statistics. Oftentimes in machine learning, it's not important that the sigmoid function be especially accurate, but it is important for it to be super fast.

- 10 1. Use Gadfly to graph the function erf on the interval $[-1, 1]$. To evaluate erf using Julia, you will need to use the package manager to install the package `SpecialFunctions`. Based on the graph, does the function erf appear to be an even or odd function?
- 10 2. Find the third degree polynomial that interpolates the erf function. For the interpolation points, use $0, 1/3, 2/3, 1$.
- 10 3. Write a Julia function `sigmoid` that evaluates the third degree polynomial that you found in the previous question. Use the odd property of sigmoid to extend your function to accurately evaluate `sigmoid(x)` for $x \in [-1, 0]$.
- 10 4. Use Gadfly to graph $y = |\text{erf}(x) - \text{sigmoid}(x)|$. Use the graph to visually determine

$$\max_{x \in [-1, 1]} |\text{erf}(x) - \text{sigmoid}(x)|. \quad (2)$$

- 10 5. Use `@btime` to compare the time to evaluate `erf(0.6)` and `sigmoid(0.6)`. Is your function faster? If not, try again. Likely, if you defined your function with the `stab` operator `->`, it will be slow. Try defining it without the `stab` operator. To use `@btime`, you will need to use the package manager to install `BenchmarkTools`.
- 10 6. Find the third degree polynomial that interpolates the erf function, but this time for the interpolation points, use $\frac{1}{2} + \frac{1}{2} \cos\left(\frac{2k-1}{8}\pi\right)$ for $k \in \{1, 2, 3, 4\}$. Write a Julia function `sigmoid2` that evaluates the third degree polynomial that you found in this problem. Again, use the odd property of sigmoid to extend your function to accurately evaluate `sigmoid(x)` for $x \in [-1, 0]$. Is this function more accurate than the version that uses equally spaced interpolation points? Explain.