

CYRR 304
Homework 8, Spring 2024

Name:

“Instructions for living a life. Pay attention. Be astonished. Tell about it.”

MARY OLIVER

Homework 8 has questions 1 through 2 with a total of 25 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 30 March** at 11:59 PM.

1. You need a pretty good value of $\int_0^{10} \sin(x^2) dx$. To do this, you will use the n panel trapezoidal rule.

5 (a) Use Gadfly to plot a graph of the *second derivative* of $x \mapsto \sin(x^2)$. Visually determine the value of $\max_{x \in [0,10]} |2 \cos(x^2) - 4x^2 \sin(x^2)|$.

5 (b) Find the least value for n needed so that the n -panel trapezoidal rule gives an error of no more than 10^{-8} .

5 (c) Using that value of n , estimate the value of $\int_0^{10} \sin(x^2) dx$.

2. Let T_n be the n -panel trapezoidal rule value for $\int_0^1 \sqrt{x} dx$.

5 (a) Sketch a graph of T on the interval $1 \dots 500$.

5 (b) The true value of $\int_0^1 \sqrt{x} dx$ is $2/3$. Sketch a graph of $n \mapsto n^2(T_n - 2/3)$ on the interval $1 \dots 500$. Does it appear that the graph has a horizontal asymptote? Doesn't the theory say it should? Explain. (Hint is $x \mapsto \sqrt{x}$ sufficiently smooth on $[0, 1]$?)