

CYRR 304
Homework 7, Spring 2024

Name:

"When you're good to others, you're best to yourself."

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Homework 7 has questions 1 through 4 with a total of 20 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 23 March** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas.

The degree four Bernstein polynomials P_0 through P_4 are defined by

$$P_0(x) = (1-x)^4, \quad P_1(x) = 4x(1-x)^3, \quad P_2(x) = 6x^2(1-x)^2, \quad P_3(x) = 4x^3(1-x), \quad P_4(x) = x^4. \quad (1)$$

The degree of each Bernstein polynomial is four, and it can be proven that every polynomial of degree four or less is a linear combination of these five polynomials. On the interval $[0, 1]$, the Bernstein polynomials are non negative. This property makes them well suited to approximating functions that are nonnegative on $[0, 1]$.

- 5 1. Use Gadfly to graph P_0 through P_1 on the interval $[0, 1]$.
- 5 2. Find the real numbers c_0, c_1, c_2, c_3, c_4 , and c_5 such that the function

$$F(x) = \sum_{k=0}^5 c_k P_k(x) \quad (2)$$

minimizes

$$\sum_{k=0}^5 \left(c_k P\left(\frac{k}{10}\right) - \sin\left(\frac{\pi k}{10}\right) \right)^2. \quad (3)$$

Be sure to find the condition number of the coefficient matrix of the normal equations.

- 5 3. Use Gadfly to graph both $y = \sin(\pi x)$ and $y = \sum_{k=0}^5 c_k P_k(x)$ on the interval $[0, 1]$, where the numbers c_0 through c_5 are the numbers that you found in Question 2.
- 5 4. Use Gadfly to graph $y = \left| \sin(\pi x) - \sum_{k=0}^5 c_k P_k(x) \right|$, where the numbers c_0 through c_5 are the numbers that you found in Question 2. Visually find $\max_{x \in [0, 1]} \left(\left| \sin(\pi x) - \sum_{k=0}^5 c_k P_k(x) \right| \right)$.