

CYRR 304  
Homework 7, Spring 2024

Name:

*"When you're good to others, you're best to yourself."*

BENJAMIN FRANKLIN

Homework 7 has questions 1 through 4 with a total of 20 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 23 March** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas.

The degree four Bernstein polynomials  $P_0$  through  $P_4$  are defined by

$$P_0(x) = (1-x)^4, \quad P_1(x) = 4x(1-x)^3, \quad P_2(x) = 6x^2(1-x)^2, \quad P_3(x) = 4x^3(1-x), \quad P_4(x) = x^4. \quad (1)$$

The degree of each Bernstein polynomial is four, and it can be proven that every polynomial of degree four or less is a linear combination of these five polynomials. On the interval  $[0, 1]$ , the Bernstein polynomials are non negative. This property makes them well suited to approximating functions that are nonnegative on  $[0, 1]$ .

- 5 1. Use Gadfly to graph  $P_0$  through  $P_4$  on the interval  $[0, 1]$ .
- 5 2. Find the real numbers  $c_0, c_1, c_2, c_3$ , and  $c_4$  such that the function

$$F(x) = \sum_{k=0}^4 c_k P_k(x) \quad (2)$$

minimizes

$$\sum_{k=0}^4 \left( c_k P\left(\frac{k}{10}\right) - \sin\left(\frac{\pi k}{10}\right) \right)^2. \quad (3)$$

Be sure to find the condition number of the coefficient matrix of the normal equations.

- 5 3. Use Gadfly to graph both  $y = \sin(\pi x)$  and  $y = \sum_{k=0}^4 c_k P_k(x)$  on the interval  $[0, 1]$ , where the numbers  $c_0$  through  $c_4$  are the numbers that you found in Question 2.
- 5 4. Use Gadfly to graph  $y = \left| \sin(\pi x) - \sum_{k=0}^4 c_k P_k(x) \right|$ , where the numbers  $c_0$  through  $c_4$  are the numbers that you found in Question 2. Visually find  $\max_{x \in [0, 1]} \left( \left| \sin(\pi x) - \sum_{k=0}^4 c_k P_k(x) \right| \right)$ .