"Instructions for living a life. Pay attention. Be astonished. Tell about it."

MARY OLIVER

Homework 8 has questions 1 through 2 with a total of of 25 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 30 March** at 11:59 PM.

1. You need a pretty good value of $\int_0^{10} \sin(x^2)\, dx$. To do this, you will use the n panel trapezoidal rule.

   5   (a) Use Gadfly to plot a graph of the *second derivative* of $x \mapsto \sin(x^2)$. Visually determine the value of $\max\limits_{x \in [0,10]} \left| 2\cos\left(x^2\right) - 4x^2 \sin\left(x^2\right) \right|$.

   5   (b) Find the least value for n needed so that the n-panel trapezoidal rule gives an error of no more than 10^{-8}.

   5   (c) Using that value of n, estimate the value of $\int_0^{10} \sin(x^2)\, dx$.

2. Let T_n be the n-panel trapezoidal rule value for $\int_0^1 \sqrt{x}\, dx$.

   5   (a) Sketch a graph of T on the interval $1 \ldots 500$.

   5   (b) The true value of $\int_0^1 \sqrt{x}\, dx$ is $2/3$. Sketch a graph of $n \mapsto n^2(T_n - 2/3)$ on the interval $1 \ldots 500$. Does it appear that the graph has a horizontal asymptote? Doesn't the theory say it should? Explain. (Hint is $x \mapsto \sqrt{x}$ sufficiently smooth on $[0,1]$?)