CYRR 304 **Name:**
Homework 3, Spring 2024

> *"Compound interest is the eighth wonder of the world. He who understands it, earns it; he who doesn't, pays it".* attributed to ALBERT EINSTEIN

Homework 3 has questions 1 through 1 with a total of of 15 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 10 Feb** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas. To convert you Jupyter notebook to HTML, do File –> Download as –> HTML. (For the File menu, look toward the upper right corner.)

1. This week's assignment is motivated by a problem in financial math. But we'll skip the details of the origin of the problem and skip directly to the numerical analysis. For various value of the number q, we need to solve the equation

$$q = z + z^2 + z^3 + z^4 \tag{1}$$

for z. Let's use fixed point iteration to do do this. There are lots of ways to convert this equation to fixed point form; here is one way

$$z = q - \left(z^2 + z^3 + z^4\right). \tag{2}$$

$\boxed{5}$ (a) Suppose $q = 4.35$. Use Gadfly to graph both $z \mapsto z$ and $z \mapsto q - \left(z^2 + z^3 + z^4\right)$ on the same graph. Do you think the fixed point sequence will converge? Explain.

$\boxed{5}$ (b) For best convergence of a fixed point sequence for a function F, we would like the derivative of F be zero at the fixed point. And that gives and idea. Instead of using the fixed point method on the function $F = z \mapsto q - \left(z^4 + z^3 + z^2 + z\right)$, let's find the fixed point sequence of $G = z \mapsto F(z) + s(F(z) - z)$, where s is some real number. The functions F and G have the same fixed points–let's choose the number s so that $G'(1) = 0$. Thus $0 = F'(1) + s(F'(1) - 1)$. But $F'(1) = -9$, so we want $s = -\frac{9}{10}$. This choice gives $G = z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$.
Use Gadfly to graph both $z \mapsto z$ and $z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$ on the same graph. Do you think this fixed point sequence will converge? Explain.

$\boxed{5}$ (c) Used fixed point iteration to find the fixed point of $z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$.