

**CYRR 304**  
**Homework 8, Spring 2024**

**Name:**

*"Instructions for living a life. Pay attention. Be astonished. Tell about it."*

MARY OLIVER

Homework 8 has questions 1 through 2 with a total of 25 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 30 March** at 11:59 PM.

1. You need a pretty good value of  $\int_0^{10} \sin(x^2) dx$ . To do this, you will use the  $n$  panel trapezoidal rule.

5 (a) Use Gadfly to plot a graph of the *second derivative* of  $x \mapsto \sin(x^2)$ . Visually determine the value of  $\max_{x \in [0,10]} |2 \cos(x^2) - 4x^2 \sin(x^2)|$ .

5 (b) Find the least value for  $n$  needed so that the  $n$ -panel trapezoidal rule gives an error of no more than  $10^{-8}$ .

5 (c) Using that value of  $n$ , estimate the value of  $\int_0^{10} \sin(x^2) dx$ .

2. Let  $T_n$  be the  $n$ -panel trapezoidal rule value for  $\int_0^1 \sqrt{x} dx$ .

5 (a) Sketch a graph of  $T$  on the interval  $1 \dots 500$ .

5 (b) The true value of  $\int_0^1 \sqrt{x} dx$  is  $2/3$ . Sketch a graph of  $n \mapsto n^2(T_n - 2/3)$  on the interval  $1 \dots 500$ . Does it appear that the graph has a horizontal asymptote? Doesn't the theory say it should? Explain. (Hint is  $x \mapsto \sqrt{x}$  sufficiently smooth on  $[0, 1]$ ?)