CYRR 304 **Name:**
Homework 4, Spring 2024

"Compound interest is the eighth wonder of the world. He who understands it, earns it; he who doesn't, pays it". attributed to ALBERT EINSTEIN

Homework 4 has questions 1 through 1 with a total of of 15 points. Your recorded score will be scaled to twenty points. The point value for each question or part of a question is in the box following each question or part of a question. This work is due **Saturday 17 Feb** at 11:59 PM.

For this assignment, convert your Jupyter notebook (a IPYNB file) to HTML and submit the HTML file to Canvas. To convert you Jupyter notebook to HTML, do File –> Download as –> HTML. (For the File menu, look toward the upper right corner.)

1. This week's assignment is motivated by a problem in financial math. But we'll skip the details of the origin of the problem and skip directly to the numerical analysis. For various value of the number q that are close to four, we need to solve the equation

$$q = z + z^2 + z^3 + z^4 \tag{1}$$

for z. When $q = 4$, the solution to the equation $q = z + z^2 + z^3 + z^4$ is $z = 1$. From now on, we'll choose $q = 4.35$.

Let's use fixed point iteration to do this. There are lots of ways to convert this equation to fixed point form; here is one way

$$z = 4.35 - \left(z^2 + z^3 + z^4\right). \tag{2}$$

$\boxed{5}$ (a) Use Gadfly to graph both $z \mapsto z$ and $z \mapsto 4.35 - \left(z^2 + z^3 + z^4\right)$ on the same graph. Recalling that the solution is near $z = 1$, try looking at this graph on the interval $[0, 1.5]$. Do you think the fixed point sequence will converge? Explain. **Remember** for a fixed point $x^\star$ for a function F, the fixed point sequence will converge to $x^\star$ when $|F'(x^\star)| < 1$ and the initial value of the sequence is sufficiently close to the fixed point.

$\boxed{5}$ (b) For best convergence of a fixed point sequence for a function F, we would like the derivative of F be zero at the fixed point. And that gives an idea. Instead of using the fixed point method on the function

$$F(z) = 4.35 - \left(z^4 + z^3 + z^2\right), \tag{3}$$

let's find the fixed point sequence of

$$G(z) = F(z) + s(F(z) - z), \tag{4}$$

where s is some real number. Provided that $s \neq -1$, the functions F and G have identical fixed points. For best convergence of a fixed point sequence for G, we want the derivative of G to vanish at the fixed point. We know that the fixed point is near 1, so let's choose the number s so that $G'(1) = 0$. Thus, $0 = F'(1) + s(F'(1) - 1)$. But $F'(1) = -9$, so we want $s = -\frac{9}{10}$. This choice gives $G = z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$.

Use Gadfly to graph both $z \mapsto z$ and $z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$ on the same graph. Do you think this fixed point sequence will converge? Explain.

$\boxed{5}$ (c) Used fixed point iteration to find the fixed point of $z \mapsto \frac{1}{10}F(z) + \frac{9}{10}z$.