Forty Seven Questions for Dr. Willis

1. **Question** Did I miss anything in class last week?

   Answer Yes. Please read the poem "Did I Miss Anything?" by Tom Wayman.

2. **Question** Will that be on the test?

   Answer Maybe. If you are concerned about a topic that is briefly mentioned in class that is not in the homework or textbook, please ask me about its importance. But it is, I think, important for me to show you how the topics we are learning relate to other disciplines as well as to understand the historical and human side of the creative effort involved in creating (or discovering) mathematics. So yes, not every mathematical morsel we learn in class will be "on the test."

3. **Question** On the test, do you want interval notation, set builder notation, or a pictorial representation?

4. **Answer** Unless the question explicitly says which form, you are free to express yourself in whatever way you wish.

5. **Question** Is this OK, or do I need to simplify it?

6. **Answer** Mabye. Giving guidelines on what it means to simplified is tough. But if you (a) do all rational arithmetic (b) combine all like terms, (c) simplify all "famous" values of functions, for example $\cos(0) = 1$ and $\sqrt{81} = 9$, and (d) make an effort to simplify all vanishing expressions to zero (for example $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2}$ simplifies to zero), you'll be OK.

7. **Question** Will we review the entire dead week?

   Answer Maybe. Once we finish all sections in the course calendar, we'll review for the remainder of the term. But it's likely that we'll need to use at least part of dead week to cover new material.

8. **Question** Will you give us a review before every exam?

   Answer Yes. The review will likely not have a complete solution key, but yes there will be a review. The review for the final exam will be the union of all the other reviews. We might spend some class time before an exam for review, but it's unlikely we'll spend the entire class period reviewing.

9. **Question** Can I get extra credit?

   Answer No. Our syllabus gives our grading scale and assessments. Course grades for all students will be assigned according to the syllabus.

10. **Question** Will you sign my grade check form?

    Answer Yes. Please help me out and look up your course grade on Canvas and show it to me.

11. **Question** My major is X and I have to take Calculus II. I've heard that Calculus II is much harder than Calculus I. Should I switch majors?

    Answer No, not for that reason. Calculus II is no more intellectually challenging than Calculus I. All science majors have required classes (for example, Physical Chemistry, Quantum Mechanics, etc) that are *far* more difficult than Calculus II.

12. **Question** Isn't $1/0$ *really* equal to infinity?

    Answer No, the rule $1/0 = \infty$ is rubbish. If we adopted $1/0 = \infty$ as a rule and kept the other usual rules of arithmetic, we'd be able to derive statements that are manifestly false (things like $1 = 2$).

13. **Question** Isn't $1/\infty$ *really* equal to zero?

    Answer No, $1/\infty = 0$ is rubbish. The reason is much the same as for why $1/0 = \infty$ is rubbish. Possibly the confusion stems from the limit fact that if $\lim_{x\to a} F(x) = 1$ and $\lim_{x\to a} G(x) = \infty$, we have $\lim_{x\to a} \frac{F(x)}{G(x)} = 0$. Arguably, the calculation $\lim_{x\to a} \frac{F(x)}{G(x)} = \frac{1}{\infty} = 0$ is a forgivable faux pas, but this limit fact doesn't imply that $1/\infty = 0$.

    Question Is 0^0 undefined?

    Answer Sometimes, but not always. In the context of a summation, such as $\sum_{k=0}^{n} x^k$, the first term is x^0. If 0^0 is undefined, the first term of the sum is undefined when $x = 0$. But in the context of a sum of powers of a variable, we almost always want $x^0 = 1$ for all x. In other contexts, 0^0 is undefined. Mathematical notation, like all natural languages, is context dependent.

14. **Question** What is $\infty - \infty$?

    Answer It is rubbish.

15. **Question** Is infinity a number?

    Answer Yes, infinity is a number. It's not a *real* number, but it is a number.

16. **Question** Can a function be infinity?

    Answer No. Functions and numbers are distinct objects, so a function can *never* be a number. It is possible for infinity to be in the range of a function, for example, $x \in \mathbf{R} \mapsto \begin{cases} x & \text{if } x \neq 2 \\ \infty & \text{if } x = 2 \end{cases}$. Although infinity is in the range of this function, the function certainly isn't infinity. Failing to distinguish a function from a number is an example of *conflating*.

17. **Question** What do you mean by an identity?

    Answer An equation is an identity provided its solution set is "everything." For equations involving one real variable, "everything" generally means the set of all real numbers. For example, the solution set to $0x = 0$ is $\mathbf{R}$, so $0x = 0$ is an identity. But the solution set to $\frac{x}{x} = 1$ is the set of all real numbers *except* for zero, so $\frac{x}{x} = 1$ isn't an identity.

    A relaxed notion of an identity is an equation whose solution set is the same as its natural domain. For that meaning, $\frac{x}{x} = 1$ is an identity because both its solution set and its natural domain are $\mathbf{R}_{\neq 0}$.

18. **Question** To prove that an equation is an identity, I was told to "only work with one side at a time." Why is that?

    Answer First of all, to prove that an equation is an identity, we need to (i) solve the equation and (ii) show that its solution set is the same as its natural domain. During the process of solving, we can't do anything that enlarges the solution set, but doing anything that maintains the solution set is allowed.

    The requirement to "only work with one side at a time" is an effort to disallow doing things that enlarge the solution set of the given equation. One such example is multiplying both sides of an equation by zero. Another example is squaring both sides of an equation. Indeed multiplying both sides of $x = 1$ by zero enlarges the solution set from $\{1\}$ to $\mathbf{R}$. And that's a huge enlargement. Another example is squaring both sides of $x = 1$ enlarges the solution set from $\{1\}$ to $\{-1, 1\}$.

    Instead of the adage to "only work with one side at a time," students would be much better served if we gave them guidance that was firmly rooted in logic than to "only work with one side at a time."

19. **Question** Why is $0! = 1$?

    Answer Because it's useful and it extends the validity of a useful identity. For positive integers n greater than two, we have the identity $n! = n(n-1)!$. If we replace n by one in this identity, we get $0! = 1$.

20. **Question** I saw a proof that $0 = 1$. Is it correct?

    Answer No, it is rubbish. Most likely the so-called proof involved a hidden divide by zero.

21. **Question** My cousin told me about a strategy for playing Keno that is guaranteed to win. Does it work?

    Answer No. Many such methods involve doubling your bet repeatedly. This doubling scheme is an effective way to convert a large fortune into a small fortune. Actually, there is a sure-fire way to earn a small fortune playing Keno–the trick is to start with a large fortune.

22. **Question** If a sentence ends with a factorial function, should a period follow the explanation point?

    Answer Yes. But the sentence should be re-written to not end that way. Writing "$x = 0!$." is nerd humor for the equivalent statement "$x = 1$."

23. **Question** Does $\ln x + y$ mean $(\ln x) + y$ or $\ln(x + y)$?

    Answer The standard is that is $\ln x + y = (\ln x) + y$. But $\ln xy = \ln(xy)$. All this is too confusing, so I suggest using parentheses.

24. **Question** Is it true that $a \cdot b + c$ means $a(b + c)$, but $ab + c$ means $(ab) + c$?

    Answer No. Both $a \cdot b + c$ and $ab + c$ mean $(ab) + c$. Possibly before about 1940 or so, using a centered dot for multiplication had a lower precedence than for addition. But at least in the United States, this is no longer true.

25. **Question** Is time the fourth dimension?

    Answer No, not really. For many physical theories, it's convenient to lump the three spatial dimensions together with the time, forming a vector with four components. But in mathematics, vectors can have any number of components and we don't impose any particular meaning to the components. So I wouldn't say that time is *the* fourth dimension.

26. **Question** Can you see the fourth dimension?

    Answer No, absolutely not.

27. **Question** Is mathematics *invented* or *discovered*?

    Answer I don't know. Meta-mathematical questions don't interest me all that much. It isn't, I think, a question that is worth pondering.

28. **Question** Do numbers exist?

    Answer I don't know. Again, it isn't, I think, a question that is worth pondering.

29. **Question** Do the Fibonacci numbers frequently appear in nature?

    Answer No, not really. For an explanation, read the article "Fibonacci Flim-Flam," by Donald E. Simanek.

30. **Question** Are there deep connections between mathematics and music?

    Answer No, not really. The connections that are known aren't, I would say, particularly deep.

31. **Question** Billy had 53 socks. Eleven of them were brown. The rest were navy blue. He never sorts them, and keeps them all in his drawer. In the morning, what's the greatest number of socks that he has to pull out before he gets a matching pair?

    Answer It depends. Is Billy color blind? Are the lights on? What does matching mean? Does a wool sock match with a cotton sock? It matters.

32. **Question** What's the next number in the sequence $2, 5, 8, 11, \ldots$?

    Answer It's a silly problem that deserves a silly answer–any number is just as logical as any other. The formula for the sequence might be $k \in \mathbf{Z}_{\geq 0} \mapsto \begin{cases} 3k+2 & \text{if } k \leq 3 \\ \sqrt{2} & \text{if } k > 3 \end{cases}$. That would make the next term $\sqrt{2}$. Arguably, the formula $k \in \mathbf{Z}_{\geq 0} \mapsto 3k+2$ is in some sense the most simple, so by the law of parsimony (the simplest explanation is most likely the right one), the next term is 14. But this problem doesn't have a unique solution, so the law of parsimony doesn't apply.

33. **Question** Is it true that $1+2+3+\cdots = -1/12$?

    Answer Maybe. For one meaning of the concept of convergence, the answer is yes; for other meanings, including the meaning that you will learn (or learned) in calculus, the answer is no. Some physical theories use the notion of convergence for which it is true that $1+2+3+\cdots = -1/12$.

34. **Question** Is $\sqrt{9} = \pm 3$?

    Answer Maybe. But for this class, it's best to stick with the so-called principal square root–that means $\sqrt{9} = 3$, not $\sqrt{9} = \pm 3$.

35. **Question** Why does my graphing calculator not show the left side of the graph of $y = x^{2/3}$.

    Answer It's because your calculator is using what is usually known as the principal branch for the 2/3 power and not the so-called real-branch rule. Using the real branch rule, we have $(-1)^{2/3} = ((-1)^2)^3 = 1^3 = 1$. But using principal branch, we have $(-1)^{2/3} = \frac{\sqrt{3}}{2}\mathrm{i} - \frac{1}{2}$ If you would like to understand this, please enroll in MATH 365.

36. **Question** Do I need a graphing calculator for this class?

    Answer No. When we need a computer drawn graph, we'll use an online tool.

37. **Question** What other classes to you teach?

    Answer At UNK I have taught College Algebra, Plane Trigonometry, Calculus I, II, and III, Applied Calculus I, Foundations of Math, Differential Equations, Abstract Algebra, Complex Analysis, Numerical Analysis, Linear Algebra, and Advanced Calculus I. Additionally, I've taught the final third of Math for Elementary Teachers I as well as Probability and Statistics. At another university, I taught classes in partial differential equations and a year long applied mathematics course for graduate students in engineering.

38. **Question** Is it true that four current UNK faculty took a math class from you?

    Answer This was true at one time, but currently I only know of two who took a class from me. I've been told that there is one more, but I don't know who this is.

39. **Question** Why did you choose to be a math teacher?

    Answer It was a combination of having something close to love for the discipline, growing up in a scientifically based family, and learning from some strong science teachers in high school and college.

40. **Question** What is your favorite comfort food?

    Answer For cool weather, it's cornbread (with butter and honey), chili, collard greens, chow-chow, and peach cobbler; for warm weather, it's cold slaw, baked beans, sliced tomatoes, buttered corn on the cob, and peach cobbler.

41. **Question** What is your favorite bird?

    Answer Goldfinches–I grow sunflowers for them and in late summer they visit my garden and gobble up the crop.

42. **Question** Do bees visit your gardens?

    Answer Yes–my pollinator garden attracts lots of bees. I love watching them and I wish I could learn all the different species that visit my flowers. I don't have hives for honey bees, but I'd like to try that someday.

43. **Question** What is your favorite programming language?

    Answer Common Lisp, definitely. And this is fortunate because I'm a developer for a computer algebra system that is written in Common Lisp. My second favorite language is Julia.

44. **Question** Is it true that you hit your head on a TV in class and passed out?

    Answer True, I bled but I didn't pass out.

45. **Question** Will you tell the story about the time you chased a turkey?

    Answer No, the turkey story has been retired.

46. **Question** Is there anything that totally creeps you out?

    Answer Bats (the flying mammal kind) come pretty close.

47. **Question** Has somebody really asked all these questions?

    Answer Except for this question, yes.