Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or π	circular constant
phi	ϕ or φ	angle, constant

Named Sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

Intervals

For numbers a and b, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a,b) = \{x \in \mathbf{R} \mid a \le x < b\}$$

$$(a,b] = \{x \in \mathbf{R} \mid a < x \le b\}$$

$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$
equivalent	$\equiv$
for all	$\forall$
there exists	$\exists$

Exponents

For $a, b > 0$ and m, n real:

$$a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}$$

$$(a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m$$

$$(a/b)^m = a^m / b^m$$

Trigonometric Identities

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$2\cos(x)^2 = 1 + \cos(2x)$$

$$2\sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\arccos(x) = \pi/2 - \arcsin(x)$$

$$\operatorname{arccot}(x) = \pi/2 - \arctan(x)$$

$$\operatorname{arccsc}(x) = \pi/2 - \operatorname{arcsec}(x)$$

Limits

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$$

$$\lim_{x\to\infty} \mathrm{e}^x = \infty \qquad \lim_{x\to-\infty} \mathrm{e}^x = 0$$

$$\lim_{x\to\infty} \ln(x) = \infty \qquad \lim_{x\to 0^+} \ln(x) = -\infty$$

Derivatives

Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1\prime}(x)$	$1/f'(f^{-1}(x))$

Antiderivatives

$$\int a\,\mathrm{d}x = ax$$

$$\int x^a\,\mathrm{d}x = \frac{1}{1+a}x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x}\,\mathrm{d}x = \ln|x|$$

$$\int \cos(x)\,\mathrm{d}x = \sin(x)$$

$$\int \sin(x)\,\mathrm{d}x = -\cos(x)$$

$$\int \tan(x)\,\mathrm{d}x = \ln|\sec(x)|$$

$$\int \sec(x)\,\mathrm{d}x = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x)\,\mathrm{d}x = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x)\,\mathrm{d}x = \ln|\sin(x)|$$

$$\int 2|x|\,\mathrm{d}x = x|x|$$

$$\int 2\lfloor x \rfloor\,\mathrm{d}x = (2x-1)\lfloor x \rfloor - \lfloor x \rfloor^2$$

$$\int 2\lceil x \rceil\,\mathrm{d}x = (2x+1)\lceil x \rceil - \lceil x \rceil^2$$

Sums

For $k, m, n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)\,n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)\,n\,(2n-1)}{6}$$

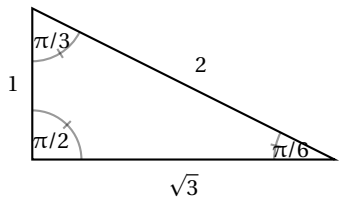
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, x \neq 1$$

Logarithms

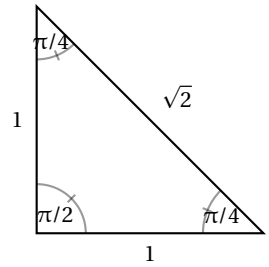
$$\log_a(x) = \frac{1}{\ln(a)}\ln(x)$$

Famous Triangles

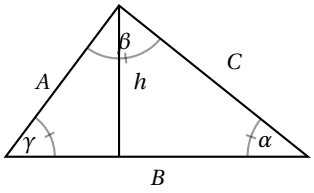
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine

$$C^2 = A^2 + B^2 - 2AB\cos(\gamma)$$

Law of sines

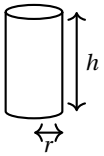
$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Areas and volumes

$$\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB\sin(\gamma)$$

Volumes

Right Circular Cylinder



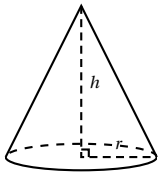
Volume

$$\mathrm{V} = \pi r^2 h$$

Area, not including areas of circular ends

$$\mathrm{A} = 2\pi r h$$

Cone



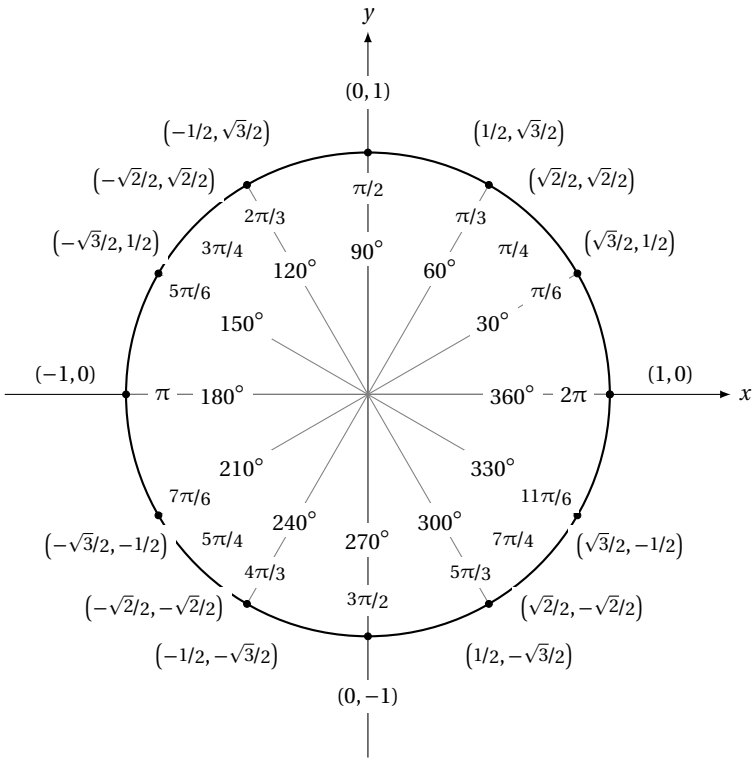
Volume

$$\mathrm{V} = \frac{1}{3}\pi r^2 h$$

Area, not including area of circular base:

$$\mathrm{A} = \pi r\left(r + \sqrt{r^2 + h^2}\right)$$

Unit Circle



Applications

Arclength of curve $y = F(x)$ with $a \le x \le b$

$$= \int_a^b \sqrt{1 + F'(x)^2}\,\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x, y) \mid f(x) \le y \le g(x) \wedge a \le x \le b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x)\,\mathrm{d}x$$

Assuming $0 \le f(x)$ and rotating about the x-axis

$$\text{Vol}(Q) = \pi\int_a^b g(x)^2 - f(x)^2\,\mathrm{d}x$$

Assuming $a \ge 0$ and rotating about the y-axis

$$\text{Vol}(Q) = 2\pi\int_a^b x(g(x) - f(x))\,\mathrm{d}x$$

Centroid

$$\text{Area}(Q) \times \overline{x} = \int_a^b x\left(g(x) - f(x)\right)\mathrm{d}x,$$

$$\text{Area}(Q) \times \overline{y} = \frac{1}{2}\int_a^b \left(g(x)^2 - f(x)^2\right)\mathrm{d}x.$$