Greek Characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
lambda	λ	Lagrange multiplier
pi	π or π	circular constant
phi	ϕ or φ	angle

Named Sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
ordered triples	$\mathbf{R}^3$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

Intervals

For numbers a and b, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a,b) = \{x \in \mathbf{R} \mid a \le x < b\}$$

$$(a,b] = \{x \in \mathbf{R} \mid a < x \le b\}$$

$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\Longrightarrow$
equivalent	$\equiv$
for all	$\forall$
there exists	$\exists$

Exponents

For $a, b > 0$ and m, n real:

$$a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}$$

$$(a^n)^m = a^{n\cdot m}$$

$$a^{-m} = 1/a^m$$

$$(a/b)^m = a^m / b^m$$

Trigonometric Functions

Special values of trig functions

x	$\cos(x)$	$\sin(x)$	$\tan(x)$
0	1	0	0
$\pi/6$	$\sqrt{3}/2$	$1/2$	$1/\sqrt{3}$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
$\pi/3$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$
$\pi/2$	0	1	dne
$2\pi/3$	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}$
$3\pi/4$	$-1/\sqrt{2}$	$1/\sqrt{2}$	-1
$5\pi/6$	$-\sqrt{3}/2$	$1/2$	$-1/\sqrt{3}$
π	-1	0	0
$7\pi/6$	$-\sqrt{3}/2$	$-1/2$	$1/\sqrt{3}$
$5\pi/4$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1
$4\pi/3$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}$
$3\pi/2$	0	-1	dne
$5\pi/3$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}$
$7\pi/4$	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1
$11\pi/6$	$\sqrt{3}/2$	$-1/2$	$-1/\sqrt{3}$

Special values of inverse trig

x	$\arccos(x)$	$\arcsin(x)$
-1	π	$-\pi/2$
$-\sqrt{3}/2$	$5\pi/6$	$-\pi/3$
$-1/2$	$2\pi/3$	$-\pi/6$
0	$\pi/2$	0
$1/2$	$\pi/3$	$\pi/6$
$\sqrt{3}/2$	$\pi/6$	$\pi/3$
1	0	$\pi/2$

Special values arctan and arccotan

x	$\arctan(x)$	$\mathrm{arccot}(x)$
$-\sqrt{3}$	$-\pi/3$	$-\pi/6$
-1	$-\pi/4$	$-\pi/4$
$-1/\sqrt{3}$	$-\pi/6$	$-\pi/3$
0	0	$\pi/2$
$1/\sqrt{3}$	$\pi/6$	$\pi/3$
1	$\pi/4$	$-\pi/4$
$\sqrt{3}$	$\pi/3$	$\pi/6$

Trigonometric identities

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$\cos(x)^2 = \frac{1}{2}(1+\cos(2x))$$

$$\sin(x)^2 = \frac{1}{2}(1-\cos(2x))$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

Limits

$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x} = 0$$

$$\lim_{x\to\infty} e^x = \infty$$

$$\lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to\infty} \ln(x) = \infty$$

$$\lim_{x\to 0^+} \ln(x) = -\infty$$

$$\lim_{x\to 0}(1+x)^{1/x} = e$$

Derivatives

General Cases

$F(x)$	$F'(x)$
$af(x)+bg(x)$	$af'(x)+bg'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x)-f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1\prime}(x)$	$1/f'(f^{-1}(x))$

Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\mathrm{arcsec}(x)$	$1/\left(\sqrt{x^2-1}\,\lvert x\rvert\right)$
$\mathrm{arccsc}(x)$	$-1/\left(\sqrt{x^2-1}\,\lvert x\rvert\right)$
$\mathrm{arccot}(x)$	$-1/(x^2+1)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$
$\log_a(x)$	$1/x\ln(a),\ a\in\mathbf{R}_{>0}$
x^a	ax^{a-1}

Antiderivatives[1]

$$\int a\,\mathrm{d}x = ax$$
$$\int x^a\,\mathrm{d}x = \frac{1}{1+a}x^a, \quad \text{if } a \neq -1$$
$$\int \frac{1}{x}\,\mathrm{d}x = \ln|x|$$
$$\int \cos(x)\,\mathrm{d}x = \sin(x)$$
$$\int \sin(x)\,\mathrm{d}x = -\cos(x)$$
$$\int \tan(x)\,\mathrm{d}x = \ln|\sec(x)|$$
$$\int \sec(x)\,\mathrm{d}x = \ln|\tan(x)+\sec(x)|$$
$$\int \csc(x)\,\mathrm{d}x = -\ln|\csc(x)+\cot(x)|$$
$$\int \cot(x)\,\mathrm{d}x = \ln|\sin(x)|$$
$$\int 2|x|\,\mathrm{d}x = x|x|$$
$$\int 2\lfloor x\rfloor\,\mathrm{d}x = (2x-1)\lfloor x\rfloor - \lfloor x\rfloor^2$$
$$\int 2\lceil x\rceil\,\mathrm{d}x = (2x+1)\lceil x\rceil - \lceil x\rceil^2$$

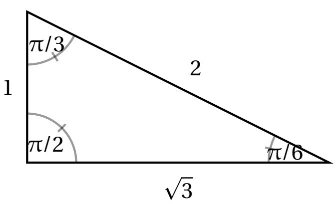
Sums

For $k, m, n \in \mathbf{Z}_{>0}$ and $\alpha, \beta \in \mathbf{R}$

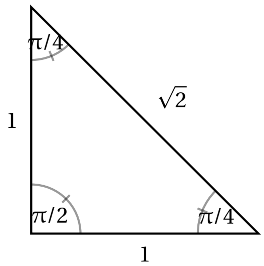
$$\sum_{k=0}^{n-1} 1 = n$$
$$\sum_{k=0}^{n-1} k = \frac{(n-1)\,n}{2}$$
$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)\,n\,(2n-1)}{6}$$
$$\sum_{k=m}^{n} a_k = \sum_{k=0}^{n-m} a_{k+m}$$
$$\sum_{k=m}^{n} \alpha a_k + \beta b_k = \alpha \sum_{k=m}^{n} a_k + \beta \sum_{k=m}^{n} b_k$$

Famous Triangles

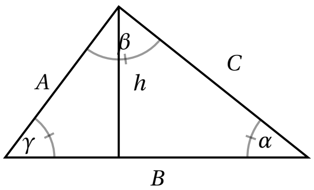
The 30-60-90 triangle



The 45-45-90 triangle



Laws of Cosine & Sine



Law of cosine

$$C^2 = A^2 + B^2 - 2AB\cos(\gamma)$$

Law of sines

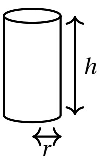
$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Areas and volumes

$$\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB\sin(\gamma)$$

Volumes

Right Circular Cylinder



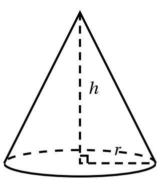
Volume

$$V = \pi r^2 h$$

Area, not including areas of circular ends

$$A = 2\pi r h$$

Cone



Volume

$$V = \frac{1}{3}\pi r^2 h$$

Area, not including area of circular base:

$$A = \pi r\left(r + \sqrt{r^2+h^2}\right)$$

Applications

Arclength of curve $y = F(x)$ with $a \le x \le b$

$$= \int_a^b \sqrt{1+F'(x)^2}\,\mathrm{d}x$$

For the region Q of the xy plane given by

$$Q = \{(x,y) \mid f(x) \le y \le g(x) \land a \le x \le b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x)\,\mathrm{d}x$$

Assuming $0 \le f(x)$ and rotating about the x-axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2\,\mathrm{d}x$$

Assuming $a \ge 0$ and rotating about the y-axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x))\,\mathrm{d}x$$

Centroid

$$\text{Area}(Q) \times \overline{x} = \int_a^b x\left(g(x) - f(x)\right)\mathrm{d}x,$$
$$\text{Area}(Q) \times \overline{y} = \frac{1}{2}\int_a^b \left(g(x)^2 - f(x)^2\right)\mathrm{d}x.$$

For the region described by

$$Q = \{(x,y) \mid f(y) \le x \le g(y) \land a \le y \le b\},$$

interchange x and y in all the previous formulas. Specifically we have

$$\text{Area}(Q) = \int_a^b g(y) - f(y)\,\mathrm{d}y$$

Assuming $0 \le f(y)$ and rotating about the y-axis

$$\text{Vol}(Q) = \pi \int_a^b g(y)^2 - f(y)^2\,\mathrm{d}y$$

Assuming $a \ge 0$ and rotating about the x-axis

$$\text{Vol}(Q) = 2\pi \int_a^b y(g(y) - f(y))\,\mathrm{d}y$$

Centroid

$$\text{Area}(Q) \times \overline{y} = \int_a^b y\left(g(y) - f(y)\right)\mathrm{d}y$$
$$\text{Area}(Q) \times \overline{x} = \frac{1}{2}\int_a^b \left(g(y)^2 - f(y)^2\right)\mathrm{d}y$$

[1] Valid on any interval on which the antiderivative is continuous.