Notation

We'll use some notation that isn't in our book. This notation is standard and it allows us to express facts with fewer words.

Notation	Meaning
$\mathrm{dom}(F)$	The *domain* of a function F; thus $\mathrm{dom}(F)$ is the set of all *inputs* to the function F.
$\mathrm{range}(F)$	The *range* of a function F; thus $\mathrm{range}(F)$ is the set of all *outputs* to the function F.
$F : A \to B$	This means: (i) F is a function, (ii) $\mathrm{dom}(F) = A$, and (iii) $\mathrm{range}(F) \subset B$.
$x \in A \mapsto F(x)$	Defines a function whose domain is the set A and whose formula is $F(x)$.
C_A	The set of functions that are continuous on the set A.
C_A^n	The set of functions that are continuous on A and whose first through n^{th} derivatives are also continuous on A.

Notes

(a) The notation $F : A \to B$ tells us that every output of F is a member of the set B, but it doesn't necessarily tell us its range.

(b) The notation $x \in [-1,1] \mapsto x^2$ defines a function without giving it a name. If you need to give it a name, use $F = x \in [-1,1] \mapsto x^2$.

Examples

(a) Every real number is a valid input to the natural exponential function exp; thus $\mathrm{dom}(\exp) = \mathbf{R}$.

(b) The set of outputs to the natural exponential function ln is $(0,\infty)$; thus $\mathrm{range}(\exp) = (0,\infty)$.

(c) The domain of the sine function is $\mathbf{R}$ and every output of sine is in $\mathbf{R}$; thus $\sin : \mathbf{R} \to \mathbf{R}$.

(d) The domain of the sine function is $\mathbf{R}$ and every output of sine is in $[-1,1]$; thus $\sin : \mathbf{R} \to [-1,1]$. Notice that $\mathrm{range}(\sin) = [-1,1]$.

(e) The domain of the natural logarithm ln is the interval $(0,\infty)$ and every output of ln is a real number; thus $\ln : (0,\infty) \to \mathbf{R}$.

(f) $x \in [-1,1] \mapsto x^2$ defines a function whose domain is the set $[-1,1]$ and whose output is the square of the input.

(g) Since the sine function is continuous on $\mathbf{R}$, we have $\sin \in C_{\mathbf{R}}$.

(h) Since the sine function is continuous on $[0,2\pi]$, we have $\sin \in C_{[0,2\pi]}$.

(i) $F \in C^2_{[-1,1]}$ is equivalent to $F \in C_{[-1,1]}$ and $F' \in C_{[-1,1]}$.

(j) The square root function $\sqrt{}$ is continuous on $[0,\infty)$, but its derivative is not; thus we have $\sqrt{} \in C_{[0,\infty)}$ and $\sqrt{} \notin C^1_{[0,\infty)}$.

Usage examples

(a) $F, G \in C_{[a,b]} \implies F + G \in C_{[a,b]}$.

In words, this says that the sum of functions that are continuous on an interval $[a,b]$ is continuous on the interval $[a,b]$.

(b) $F, G \in C_{[a,b]} \implies FG \in C_{[a,b]}$.

In words, this says that the product of functions that are continuous on an interval $[a,b]$ is continuous on the interval $[a,b]$.