## **Greek Characters**

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
epsilon	$\epsilon$ or $\epsilon$	angle, constant
theta	$\theta$ or $\vartheta$	angle, constant
pi	$\pi$ or $\pi$	circular constant
phi	$\phi$ or $\varphi$	angle, constant

## Named Sets

empty set	Ø
real numbers	R
ordered pairs of reals	$\mathbf{R}^2$
integers	Z
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

## Set Symbols

Meaning	Symbol
is a member	€
subset	_
intersection	n
union	U

## **Intervals**

For numbers a and b, we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \le x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \le b\}$$

$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

## Logic Symbols

Meaning	Symbol
negation	7
and	٨
or	V
implies	$\Rightarrow$
equivalent	=
for all	A
there exists	] Э

### Exponents

For a, b > 0 and m, n real:

$$a^{0} = 1,$$
  $0^{a} = 0$   
 $1^{a} = 1,$   $a^{n}a^{m} = a^{n+m}$   
 $a^{n}/a^{m} = a^{n-m},$   $(a^{n})^{m} = a^{n\cdot m}$   
 $a^{-m} = 1/a^{m},$   $(a/b)^{m} = a^{m}/b^{m}$ 

## Polar to Cartesian

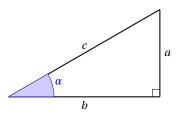
$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

For r > 0 and  $0 \le \theta < 2\pi$ 

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} 2\pi - \arccos(x/r) & \text{if } y < 0\\ \arccos(x/r) & \text{if } y \ge 0 \end{cases}$$

# Right Triangle Trigonometry



$$\sin(\alpha) = a/c$$
  $\cos(\alpha) = b/c$   $\tan(\alpha) = a/b$   
 $\csc(\alpha) = c/a$   $\sec(\alpha) = c/b$   $\cot(\alpha) = b/a$ 

## Trigonometric Identities

$$\sin(x)^{2} + \cos(x)^{2} = 1$$

$$2\cos(x)^{2} = 1 + \cos(2x)$$

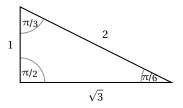
$$2\sin(x)^{2} = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

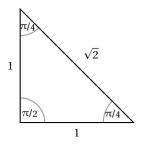
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

## Famous Triangles

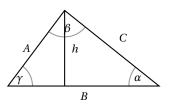
## The 30-60-90 triangle



## **The 45-45-90 triangle**



## Laws of Cosine & Sine



#### Law of cosines

$$C^2 = A^2 + B^2 - 2AB\cos(\gamma)$$

#### Law of sines

$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

#### Area

Area = 
$$hB/2 = AB\sin(\gamma)/2$$

## Solution of Equations

### Algebraic

$$[ab = 0] \equiv [a = 0 \text{ or } b = 0]$$

$$[a^2 = b^2] \equiv [a = b \text{ or } a = -b]$$

$$\left[\frac{a}{b} = 0\right] \equiv [a = 0 \text{ and } b \neq 0]$$

$$\left[\frac{a}{b} = \frac{c}{d}\right] \equiv [ad = bc \text{ and } b \neq 0 \text{ and } d \neq 0]$$

$$[|a| = |b|] \equiv [a = b \text{ or } a = -b]$$

$$\left[\sqrt{a} = b\right] \equiv [a = b^2 \text{ and } b \geq 0]$$

For  $a \neq 0$ ,

$$\left[ax^2 + bx + c = 0\right] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

### Trig

$$\begin{aligned} & \left[\cos(a) = 0\right] \equiv \left[a = (k - 1/2)\pi, k \in \mathbf{Z}\right] \\ & \left[\sin(a) = 0\right] \equiv \left[a = k\pi, k \in \mathbf{Z}\right] \\ & \left[\tan(a) = 0\right] \equiv \left[a = k\pi, k \in \mathbf{Z}\right] \\ & \left[\cos(a) = b\right] \equiv \left[a = \pm \arccos(b) + 2k\pi, k \in \mathbf{Z}\right] \\ & \left[\sin(a) = b\right] \equiv \left[a = \arcsin(b) + 2k\pi, k \in \mathbf{Z}\right] \\ & a = -\arcsin(b) + (2k + 1)\pi, k \in \mathbf{Z}\right] \\ & \left[\tan(a) = b\right] \equiv \left[a = \arctan(b) + k\pi, k \in \mathbf{Z}\right] \end{aligned}$$

## Vectors

**Dot product:** 
$$\langle x_1, y_1 \rangle \cdot \langle x_2, y_2 \rangle = x_1 x_2 + y_1 y_2$$
  
**Length:**  $\|\langle x, y \rangle\| = \sqrt{x^2 + y^2}$   
**Unit Vectors:**  $\mathbf{i} = \langle 1, 0 \rangle$ ,  $\mathbf{j} = \langle 0, 1 \rangle$ 

**Angle:**  $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos(\theta)$ , where  $(\theta)$  is the acute angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

## Graphs

### Cosine, sine, and tangent



Figure 1: Graph of  $y = \cos(x)$  on  $[0, 2\pi]$ .



Figure 2: Graph of  $y = \sin(x)$  on  $[0, 2\pi]$ .



Figure 3: Graph of  $y = \tan(x)$  on  $[0, 2\pi]$ .

### Arccosine, arcsine, and arctangent



Figure 4: Graph of  $y = \arccos(x)$  on [-1, 1].



Figure 5: Graph of  $y = \arcsin(x)$  on [-1, 1].



Figure 6: Graph of  $y = \arctan(x)$  on [-10, 10].

# Unit Circle

