## Greek Characters

| Name | Symbol | Typical use(s) |
|------|--------|----------------|
| alpha | $\alpha$ | angle, constant |
| beta | $\beta$ | angle, constant |
| gamma | $\gamma$ | angle, constant |
| epsilon | $\epsilon$ or $\varepsilon$ | angle, constant |
| theta | $\theta$ or $\vartheta$ | angle, constant |
| pi | $\pi$ or $\pi$ | circular constant |
| phi | $\phi$ or $\varphi$ | angle, constant |

## Named Sets

| empty set | $\varnothing$ |
|-----------|---------------|
| real numbers | $\mathbf{R}$ |
| ordered pairs of reals | $\mathbf{R}^2$ |
| integers | $\mathbf{Z}$ |
| positive integers | $\mathbf{Z}_{>0}$ |
| positive real numbers | $\mathbf{R}_{>0}$ |

## Set Symbols

| Meaning | Symbol |
|---------|--------|
| is a member | $\in$ |
| subset | $\subset$ |
| intersection | $\cap$ |
| union | $\cup$ |

## Intervals

For numbers $a$ and $b$, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a,b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$
$$(a,b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$
$$[a,b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

## Logic Symbols

| Meaning | Symbol |
|---------|--------|
| negation | $\neg$ |
| and | $\wedge$ |
| or | $\vee$ |
| implies | $\implies$ |
| equivalent | $\equiv$ |
| for all | $\forall$ |
| there exists | $\exists$ |

## Exponents

For $a, b > 0$ and $m, n$ real:

$$a^0 = 1, \qquad 0^a = 0$$
$$1^a = 1, \qquad a^n a^m = a^{n+m}$$
$$a^n/a^m = a^{n-m}, \qquad (a^n)^m = a^{n \cdot m}$$
$$a^{-m} = 1/a^m, \qquad (a/b)^m = a^m/b^m$$
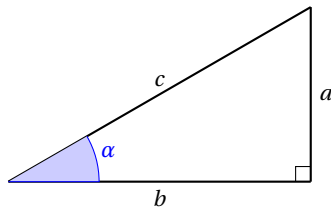
## Polar to Cartesian

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

For $r > 0$ and $0 \leq \theta < 2\pi$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \begin{cases} 2\pi - \arccos(x/r) & \text{if } y < 0 \\ \arccos(x/r) & \text{if } y \geq 0 \end{cases}$$

## Right Triangle Trigonometry



$$\sin(\alpha) = a/c \qquad \cos(\alpha) = b/c \qquad \tan(\alpha) = a/b$$
$$\csc(\alpha) = c/a \qquad \sec(\alpha) = c/b \qquad \cot(\alpha) = b/a$$

## Trigonometric Identities

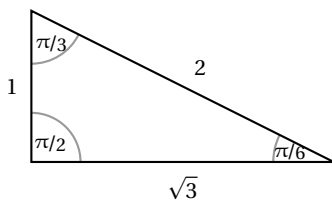$$\sin^2(x) + \cos^2(x) = 1$$
$$2\cos^2(x) = 1 + \cos(2x)$$
$$2\sin^2(x) = 1 - \cos(2x)$$
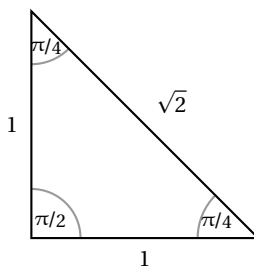$$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
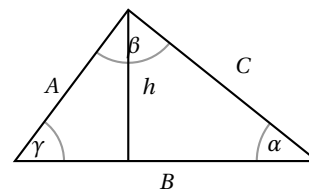$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

## Famous Triangles

**The 30-60-90 triangle**



**The 45-45-90 triangle**



## Laws of Cosine & Sine



**Law of cosines**

$$C^2 = A^2 + B^2 - 2AB\cos(\gamma)$$

**Law of sines**

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

**Area**

$$\text{Area} = hB/2 = AB\sin(\gamma)/2$$

## Solution of Equations

**Algebraic**

$$[ab = 0] \equiv [a = 0 \text{ or } b = 0]$$
$$[a^2 = b^2] \equiv [a = b \text{ or } a = -b]$$
$$\left[\frac{a}{b} = 0\right] \equiv [a = 0 \text{ and } b \neq 0]$$
$$\left[\frac{a}{b} = \frac{c}{d}\right] \equiv [ad = bc \text{ and } b \neq 0 \text{ and } d \neq 0]$$
$$[|a| = |b|] \equiv [a = b \text{ or } a = -b]$$
$$[\sqrt{a} = b] \equiv [a = b^2 \text{ and } b \geq 0]$$

For $a \neq 0$,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

**Trig**

$$[\cos(a) = 0] \equiv [a = (k - 1/2)\pi, k \in \mathbf{Z}]$$
$$[\sin(a) = 0] \equiv [a = k\pi, k \in \mathbf{Z}]$$
$$[\tan(a) = 0] \equiv [a = k\pi, k \in \mathbf{Z}]$$
$$[\cos(a) = b] \equiv [a = \pm\arccos(b) + 2k\pi, k \in \mathbf{Z}]$$
$$[\sin(a) = b] \equiv [a = \arcsin(b) + 2k\pi, k \in \mathbf{Z} \text{ or }$$
$$a = -\arcsin(b) + (2k + 1)\pi, k \in \mathbf{Z}]$$
$$[\tan(a) = b] \equiv [a = \arctan(b) + k\pi, k \in \mathbf{Z}]$$

## Vectors

**Dot product:** $\langle x_1, y_1 \rangle \cdot \langle x_2, y_2 \rangle = x_1 x_2 + y_1 y_2$

**Length:** $\|\langle x, y \rangle\| = \sqrt{x^2 + y^2}$

**Unit Vectors:** $\mathbf{i} = \langle 1, 0 \rangle, \quad \mathbf{j} = \langle 0, 1 \rangle$

**Angle:** $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos(\vartheta)$, where $(\vartheta)$ is the acute angle between $\mathbf{a}$ and $\mathbf{b}$.
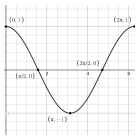
# Graphs

**Cosine, sine, and tangent**



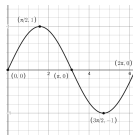Figure 1: Graph of $y = \cos(x)$ on $[0, 2\pi]$.



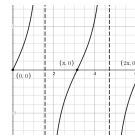Figure 2: Graph of $y = \sin(x)$ on $[0, 2\pi]$.



Figure 3: Graph of $y = \tan(x)$ on $[0, 2\pi]$.

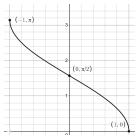**Arccosine, arcsine, and arctangent**



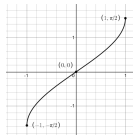Figure 4: Graph of $y = \arccos(x)$ on $[-1, 1]$.


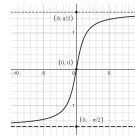
Figure 5: Graph of $y = \arcsin(x)$ on $[-1, 1]$.



Figure 6: Graph of $y = \arctan(x)$ on $[-10, 10]$.

# Unit Circle