

Quick reference on solving equations

The following table gives some hints on how to solve an equation for a variable x . The quantities X, Y, Z , and W match with any expression that involves x and the quantities a and b match with any number (or constant that doesn't depend on x). Sometimes you'll need to do some algebra (divide both sides by a nonzero number, factor, or other such things) to make the match.

Rule	Replace	With	Condition(s)
1	$0 = 1$	\emptyset	(none)
2	$0 = 0$	R	(none)
3	$ax = b$	$\left\{ \frac{b}{a} \right\}$	$a \neq 0$
4	$ax^2 + bx + c = 0$	$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$	$(a \neq 0)$ and $b^2 - 4ac \geq 0$
5	$XY = 0$	$X = 0$ or $Y = 0$	(none)
6	$\frac{X}{Y} = 0$	$X = 0$	$Y \neq 0$
7	$\frac{W}{X} = \frac{Y}{Z}$	$WZ = XY$	$X \neq 0$ and $Z \neq 0$
8	$X^2 = Y^2$	$X = -Y$ or $X = Y$	(none)
9	$X^2 = Y$	$X = -\sqrt{Y}$ or $X = \sqrt{Y}$	$Y \geq 0$
10	$ X = Y$	$X = Y$ or $X = -Y$	$Y \geq 0$
11	$\sqrt{X} = Y$	$X = Y^2$	$Y \geq 0$
12	$a^X = b$	$X = \frac{\ln(b)}{\ln(a)}$	$(0 < a < 1)$ or $(1 < a)$ and $(b > 0)$
13	$\ln(X) = a$	$X = \exp(a)$	
14	$\ln(X) + \ln(Y) = a$	$XY = \exp(a)$	$(X > 0)$ and $(Y > 0)$
15	$\cos(X) = a$	$X = 2\pi k + \cos^{-1}(a)$ or $X = 2\pi k - \cos^{-1}(a)$	$-1 \leq a \leq 1$ and $k \in \mathbf{Z}$
16	$\sin(X) = a$	$X = 2\pi k + \sin^{-1}(a)$ or $X = 2\pi k + \pi - \sin^{-1}(a)$	$-1 \leq a \leq 1$ and $k \in \mathbf{Z}$
17	$\tan(X) = a$	$X = 2\pi k + \tan^{-1}(a)$	$k \in \mathbf{Z}$

(a)

$$\begin{aligned} [(x-5)(x-7) = 0] &= [(x-5 = 0) \textbf{ or } (x-7 = 0)], && \text{(Rule 3)} \\ &= [(x = 5) \textbf{ or } (x = 7)], && \text{(Rule 1)} \end{aligned}$$

(b)

$$\begin{aligned} \left[\frac{6}{1+x^2} = 0 \right] &= [(6 = 0) \textbf{ and } 1+x^2 \neq 0], && \text{(Rule 4)} \\ &= \emptyset, && \text{(Rule 1)} \end{aligned}$$