

## Greek Characters

Name	Symbol	Typical use(s)
alpha	$\alpha$	angle, constant
beta	$\beta$	angle, constant
gamma	$\gamma$	angle, constant
epsilon	$\epsilon$ or $\varepsilon$	angle, constant
theta	$\theta$ or $\vartheta$	angle, constant
pi	$\pi$ or $\pi$	circular constant
phi	$\phi$ or $\varphi$	angle, constant

## Named Sets

empty set	$\emptyset$
real numbers	$\mathbf{R}$
ordered pairs of reals	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

## Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$

## Intervals

For numbers  $a$  and  $b$ , we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

## Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$
equivalent	$\equiv$
for all	$\forall$
there exists	$\exists$

## Exponents

For  $a, b > 0$  and  $m, n$  real:

$$a^0 = 1, \quad 0^a = 0$$

$$1^a = 1, \quad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \quad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \quad (a/b)^m = a^m / b^m$$

## Polar to Cartesian

$$x = r \cos(\theta)$$

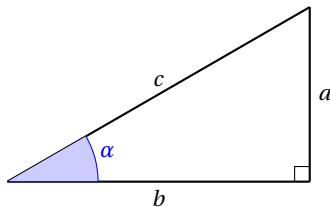
$$y = r \sin(\theta)$$

For  $r > 0$  and  $0 \leq \theta < 2\pi$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} 2\pi - \arccos(x/r) & \text{if } y < 0 \\ \arccos(x/r) & \text{if } y \geq 0 \end{cases}$$

## Right Triangle Trigonometry



$$\sin(\alpha) = a/c \quad \cos(\alpha) = b/c \quad \tan(\alpha) = a/b$$

$$\csc(\alpha) = c/a \quad \sec(\alpha) = c/b \quad \cot(\alpha) = b/a$$

## Trigonometric Identities

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$2 \cos(x)^2 = 1 + \cos(2x)$$

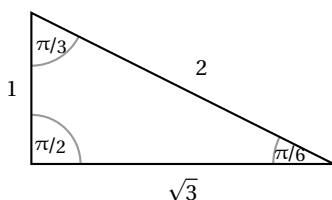
$$2 \sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

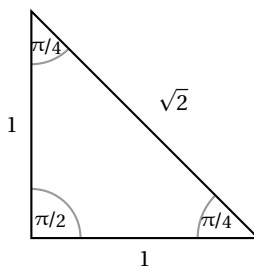
$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

## Famous Triangles

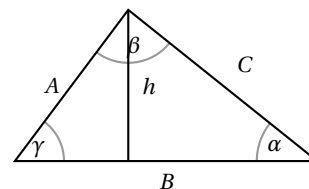
### The 30-60-90 triangle



### The 45-45-90 triangle



## Laws of Cosine & Sine



### Law of cosines

$$C^2 = A^2 + B^2 - 2AB \cos(\gamma)$$

### Law of sines

$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

### Area

$$\text{Area} = hB/2 = AB \sin(\gamma)/2$$

## Solution of Equations

### Algebraic

$$[ab = 0] \equiv [a = 0 \text{ or } b = 0]$$

$$[a^2 = b^2] \equiv [a = b \text{ or } a = -b]$$

$$\left[\frac{a}{b} = 0\right] \equiv [a = 0 \text{ and } b \neq 0]$$

$$\left[\frac{a}{b} = \frac{c}{d}\right] \equiv [ad = bc \text{ and } b \neq 0 \text{ and } d \neq 0]$$

$$[|a| = |b|] \equiv [a = b \text{ or } a = -b]$$

$$[\sqrt{a} = b] \equiv [a = b^2 \text{ and } b \geq 0]$$

For  $a \neq 0$ ,

$$[ax^2 + bx + c = 0] \equiv \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

### Trig

$$[\cos(a) = 0] \equiv [a = (k - 1/2)\pi, k \in \mathbf{Z}]$$

$$[\sin(a) = 0] \equiv [a = k\pi, k \in \mathbf{Z}]$$

$$[\tan(a) = 0] \equiv [a = k\pi, k \in \mathbf{Z}]$$

$$[\cos(a) = b] \equiv [a = \pm \arccos(b) + 2k\pi, k \in \mathbf{Z}]$$

$$[\sin(a) = b] \equiv [a = \arcsin(b) + 2k\pi, k \in \mathbf{Z} \text{ or } a = -\arcsin(b) + (2k + 1)\pi, k \in \mathbf{Z}]$$

$$[\tan(a) = b] \equiv [a = \arctan(b) + k\pi, k \in \mathbf{Z}]$$

## Vectors

**Dot product:**  $\langle x_1, y_1 \rangle \cdot \langle x_2, y_2 \rangle = x_1 x_2 + y_1 y_2$

**Length:**  $\|\langle x, y \rangle\| = \sqrt{x^2 + y^2}$

**Unit Vectors:**  $\mathbf{i} = \langle 1, 0 \rangle, \quad \mathbf{j} = \langle 0, 1 \rangle$

**Angle:**  $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos(\theta)$ , where  $(\theta)$  is the acute angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

# Graphs

## Cosine, sine, and tangent

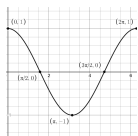


Figure 1: Graph of  $y = \cos(x)$  on  $[0, 2\pi]$ .

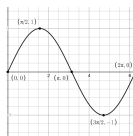


Figure 2: Graph of  $y = \sin(x)$  on  $[0, 2\pi]$ .

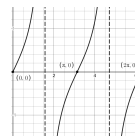


Figure 3: Graph of  $y = \tan(x)$  on  $[0, 2\pi]$ .

## Arccosine, arcsine, and arctangent

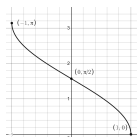


Figure 4: Graph of  $y = \arccos(x)$  on  $[-1, 1]$ .

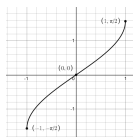


Figure 5: Graph of  $y = \arcsin(x)$  on  $[-1, 1]$ .

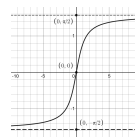


Figure 6: Graph of  $y = \arctan(x)$  on  $[-10, 10]$ .

# Unit Circle

