

## Homework 1, Fall 2023

Homework 1 has questions 1 through 4 with a total of 40 points. When I record your grade, I will scale it to twenty points. For details of the grading scheme for this assignment, please see the section 'Grading rubric' of our syllabus.

Revise, proofread, revise again (and again), *neatly* hand write your solutions, digitize your work, and up load the converted pdf of your work to Canvas. This work is due **Saturday 26 August** at 11:59 P.M.

- 10 1. For the statement  $(\exists M \in \mathbf{R}) (\forall x \in \mathbf{R}_{\geq 0}) \left( \frac{5x}{x+1} \leq M \right)$ , explain why

*Proof.* Choose  $M = \frac{5x}{x+1}$ . Let  $x \in \mathbf{R}_{\geq 0}$ . We have

$$\left[ \frac{5x}{x+1} \leq M \right] \equiv \left[ \frac{5x}{x+1} \leq \frac{5x}{x+1} \right], \quad \text{(substitution for } M\text{)}$$

$$\equiv \text{True.} \quad \text{(syntactic equality).}$$

□

is *abject rubbish*.

- 10 2. Write a correct proof of  $(\exists M \in \mathbf{R}) (\forall x \in \mathbf{R}_{\geq 0}) \left( \frac{5x}{x+1} \leq M \right)$ .

- 10 3. Without explicitly using negation (either  $\neg$  or anything equivalent to negation), write the negation of the statement

$$(\exists M \in \mathbf{R}_{<5}) (\forall x \in \mathbf{R}_{\geq 0}) \left( \frac{5x}{x+1} < M \right).$$

Unlike the previous questions, the number  $M$  in this question must be *less* than five. Also, the final inequality is now a strict inequality (equality is not allowed). These differences are *not* typos.

- 10 4. Show that the statement

$$(\exists M \in \mathbf{R}_{<5}) (\forall x \in \mathbf{R}_{\geq 0}) \left( \frac{5x}{x+1} < M \right).$$

is *false* by showing that its negation is true.