MATH 460 **Name:** Larry Kuúruks
Homework 2, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 2 has questions 1 through 4 with a total of 20 points. Edit this file and append you answers using LaTeX. Be sure to fill in your name. Upload the converted pdf of your work to Canvas. This assignment is due *Saturday 3 September at 11:59* PM.

Link to your Overleaf work: XXX

5 1. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of $F(\{-2,-1,0,1,2\})$.

Solution:

5 2. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of $F^{(-1)}(\{0,1,4\})$.

Solution:

5 3. Show that

$$(\forall\, a \in \mathbf{R}_{>0})\,(\exists\, m \in \mathbf{R})\,(\forall\, x \in \mathbf{R}_{\geq 0})\left(\sqrt{x} \leq \sqrt{a} + m(x-a)\right).$$

Hints: You might like to use the facts:

$$\begin{aligned}\left[\sqrt{x} \leq \sqrt{a} + m(x-a)\right] &\equiv \left[\sqrt{x} - \sqrt{a} - m(x-a) \leq 0\right], \\ &\equiv \left[\sqrt{x} - \sqrt{a} - m(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a}) \leq 0\right], \\ &\equiv \left[(\sqrt{x}-\sqrt{a})(1 - m(\sqrt{x}+\sqrt{a}) \leq 0\right]\end{aligned}$$

Solution:

5 4. Show that for all sets A and B that $(B \setminus A = B) \implies (A \cap B = \varnothing)$. **Hint:** Try proving the contrapositive.

Solution: