**MATH 460** **Name:**
**Homework 3, Fall 2022**

I have neither given nor received unauthorized assistance on this assignment.

Homework 3 has questions 1 through 4 with a total of 20 points. Edit this file and append you answers using LaTeX. Be sure to fill in your name. Upload the converted pdf of your work to Canvas. This assignment is due *Saturday 10 September at 11:59* PM.

**Link to your Overleaf work:** XXX

5 1. Show that $(\forall x \in (-1,1))\,(\exists r \in \mathbf{R}_{>0})\,((x-r, x+r) \subset (-1,1))$.

**Solution:**

5 2. Define $S = \{(-k,k) | k \in \mathbf{Z}_{>0}\}$. Show that $\bigcup_{q \in S} q = \mathbf{R}$.

**Solution:**

5 3. On $\mathbf{Z}^2$ define the binary operators $+$ and $\times$ by

$$(a,b) + (c,d) = (a+c, b+d),$$
$$(a,b) \times (c,d) = (ac + 2bd, ad + bc).$$

These operators are commutative and associative. Additionally, the additive identity is $(0,0)$, the multiplicative identity is $(1,0)$, every member of $\mathbf{Z}^2$ has an additive identity, and multiplication distributes over addition. Given these facts, show that $(\mathbf{Z}^2, +, \times)$ is a field. The only thing left to show is that every member $\mathbf{Z}^2$ except for the additive identity has a multiplicative inverse. To prove this, you might like to use the fact that $\sqrt{2}$ is not rational.

**Solution:**

5 4. Show that the complex field is not ordered. Hint: Suppose it is. Let $P$ be its positive set. Since $\mathrm{i} \neq 0$, either $\mathrm{i} \in P$ or $-\mathrm{i} \in P$. Show that both $\mathrm{i} \in P$ or $-\mathrm{i} \in P$ are contradictions.

**Solution:**