**MATH 460** **Name:**

**Homework 12, Fall 2022**

I have neither given nor received unauthorized assistance on this assignment.

Homework 12 has questions 1 through 3 with a total of 15 points. Please neatly handwrite your solutions, digitize them and submit the digitized copy to Canvas. This work is due *Wednesday 30 November at 11:59* P M.

5 1. Suppose $F \in \mathrm{C}^2_{\mathbf{R}}$ and let $a \in \mathbf{R}$. Use the L'Hôpital rule to show that

$$\lim_{x\to 0} \frac{F(x) - 2F(0) + F(-x)}{x^2} = F''(0).$$

Remember that $F \in \mathrm{C}^2_{\mathbf{R}}$ means that $F, F'$, and $F''$ are continuous on $\mathbf{R}$.

5 2. Define $G(x) = x|x|$. Show that $G'$ is not differentiable at zero. You may use the fact that $G' = x \in \mathbf{R} \mapsto |x|$.

5 3. Define $G(x) = x|x|$. Show that

$$\lim_{x\to 0} \frac{G(x) - 2G(0) + G(-x)}{x^2} = 0.$$