MATH 460 **Name:**

This past week, I reread a book of my youth: *Introduction to Mathematical Logic*, by Flora Dinkines (original copyright 1961). (My father, a non-mathematician scientist, regularly patrolled the used book bins for dime books.) Here are some fun questions that are adapted from the textbook.

1. Define the predicate

   $$Q = (x, y) \in \mathbf{R}^2 \mapsto [x + 2y = 4].$$

   For example, we have $Q(1,1) = [3 = 4] = \text{False}$ and $Q(2,1) = [4 = 4] = \text{True}$. Decide on the truth value of each the following statements; if the statement is true, prove it; if the statement is false, prove that its negation is true.

   (a) $(\forall x \in \mathbf{R}) \left(\exists y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

   (b) $(\exists x \in \mathbf{R}) \left(\forall y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

   (c) $(\forall x \in \mathbf{R}) \left(\exists y \in \mathbf{R}\right) \left(\neg Q(x, y)\right)$.

   (d) $(\forall x \in \mathbf{R}) \neg \left(\forall y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

2. Find examples of predicates P and Q such that the statement

   $$\forall x \, (P(x) \wedge Q(x)) \equiv (\forall x (P(x))) \wedge (\forall x (Q(x)))$$

   false. This shows that the existential qualifier does not distribute over the disjunction.

3. For any predicate P, show that

   $$(\exists x) \left(\exists y\right) \left(P(x, y)\right) \equiv \left(\exists y\right) (\exists x) \left(P(x, y)\right).$$

4. For any predicate P, show that

   $$(\forall x) \left(\forall y\right) \left(P(x, y)\right) \equiv \left(\forall y\right) (\forall x) \left(P(x, y)\right).$$

5. For any predicate P, show that

   $$(\exists x) \left(\forall y\right) \left(P(x, y)\right) \implies \left(\forall y\right) (\exists x) \left(P(x, y)\right).$$

6. Show there is a predicate P such that

   $$\left(\forall y\right) (\exists x) \left(P(x, y)\right) \implies (\exists x) \left(\forall y\right) \left(P(x, y)\right).$$

   is false.