MATH 460 **Name:**
Homework 4, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 4 has questions 1 through 4 with a total of 20 points. Edit this file and append you answers using LaTeX. Be sure to fill in your name. Upload the converted pdf of your work to Canvas. This assignment is due *Saturday 17 September at 11:59* PM.

Link to your Overleaf work: XXX

$\boxed{5}$ 1. Show that the union of sets that are bounded above is bounded above. Specifically, show that if A and B are subsets of $\mathbf{R}$ and both A and B are bounded above, $A \cup B$ is bounded above.

$\boxed{5}$ 2. Using induction, we can show that for any a finite union of sets that are bounded above is bounded above. Specifically, if for every positive integer n the sets $A_1, A_2, \ldots A_n$ are bounded above, $\cup_{k=1}^{n} A_k$ is bounded above.

Your task is to find an example of sets $A_1, A_2, A_3, \ldots$ such that $\cup_{k=1}^{\infty} A_k$ is *not* bounded above, but each set $A_1, A_2, A_3, \ldots$ is bounded above.

$\boxed{5}$ 3. Let A and B be subsets of $\mathbf{R}$ and let A and B be bounded above. Show that

$$\operatorname{lub}(A \cup B) = \max(\operatorname{lub}(A), \operatorname{lub}(B)).$$

$\boxed{5}$ 4. Show that $(\exists x \in [0,1])\,(\forall r \in \mathbf{R}_{<0})\,((x - r, x + r) \not\subset [0,1])$. We'll learn in a bit that this is a proof that the set $[0,1]$ is not open.