

Homework 4, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 4 has questions 1 through 4 with a total of 20 points. Edit this file and append your answers using LaTeX. Be sure to fill in your name. Upload the converted pdf of your work to Canvas. This assignment is due *Saturday 17 September at 11:59 PM*.

Link to your Overleaf work: XXX

- 5 1. Show that the union of sets (each a subset of the reals) that are bounded above is bounded above. Specifically, show that if A and B are both bounded above, $A \cup B$ is bounded above.
- 5 2. Using induction, we can show that for any positive integer n , if A_1, A_2, \dots, A_n are bounded above, then $\cup_{k=1}^n A_k$ is bounded above. Find an example of sets A_1, A_2, A_3, \dots such that $\cup_{k=1}^{\infty} A_k$ is not bounded above, but each set A_1, A_2, A_3, \dots is bounded above.
- 5 3. Let A and B be subsets of \mathbf{R} and let A and B be bounded above. Show that
$$\text{lub}(A \cup B) = \max(\text{lub}(A), \text{lub}(B)).$$
- 5 4. Show that $(\exists x \in [0, 1]) (\forall r \in \mathbf{R}_{<0}) ((x - r, x + r) \not\subset [0, 1])$.