**MATH 460** **Name:**

This past week, I reread a book of my youth (the actual copy): *Introduction to Mathematical Logic*, by Flora Dinkines. Here are some fun questions that are adapted from this textbook. This is *not* an assignment, bonus or otherwise. But if you would like a mid-July logic exercise that is more educational than solving another Sudoku puzzle, give these problems a try. And if you have questions about them, please let me know.

1. Define the predicate

$$Q = (x, y) \in \mathbf{R}^2 \mapsto [x + 2y = 4].$$

For example, we have $Q(1,1) = [3 = 4] = \text{False}$ and $Q(2,1) = [4 = 4] = \text{True}$. Decide on the truth value of each the following statements; if the statement is true, prove it; if the statement is false, prove that its negation is true.

   (a) $(\forall x \in \mathbf{R}) \left(\exists y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

   (b) $(\exists x \in \mathbf{R}) \left(\forall y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

   (c) $(\forall x \in \mathbf{R}) \left(\exists y \in \mathbf{R}\right) \left(\neg Q(x, y)\right)$.

   (d) $(\forall x \in \mathbf{R}) \neg \left(\forall y \in \mathbf{R}\right) \left(Q(x, y)\right)$.

2. Find examples of predicates $P$ and $Q$ such that the statement

$$(\forall x)\,(P(x) \vee Q(x)) \equiv (\forall x)\,(P(x)) \vee (\forall x)\,(Q(x))$$

is false. This shows that the existential qualifier does not distribute over the disjunction.

3. Does the existential qualifier distribute over the conjunction?

4. For any predicate $P$, show that

$$(\exists x) \left(\exists y\right) \left(P(x, y)\right) \equiv \left(\exists y\right) (\exists x) \left(P(x, y)\right).$$

5. For any predicate $P$, show that

$$(\forall x) \left(\forall y\right) \left(P(x, y)\right) \equiv \left(\forall y\right) (\forall x) \left(P(x, y)\right).$$

6. For any predicate $P$, show that

$$(\exists x)\left(\forall y\right)\left(P(x,y)\right) \implies \left(\forall y\right)(\exists x)\left(P(x,y)\right).$$

7. Show there is a predicate $P$ such that

$$\left(\forall y\right)(\exists x)\left(P(x,y)\right) \implies (\exists x)\left(\forall y\right)\left(P(x,y)\right).$$

   is false.