MATH 460 **Name:**
Homework 5, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 5 has questions 1 through 3 with a total of 30 points. Neatly **handwrite your solutions**, digitize your work, and turn it into Canvas. You do **not** need to use LaTeX for this assignment. This work is due *Saturday 1 October at 11:59* PM.

$\boxed{5}$ 1. Show that $(\forall x \in \mathbf{R})\left(\max(0, x) = \frac{x+|x|}{2}\right)$. Likely you will want to consider the cases $x < 0$ and $x \geq 0$ separately.

Solution:

$\boxed{5}$ 2. Let F be a convergent sequence. Show that the sequence $k \in \mathbf{Z}_{\geq 0} \mapsto \max(F_k, 0)$ converges. **Hint:** Use the result of Question 1.

Solution:

3. Define $H = n \in \mathbf{Z}_{>0} \mapsto \sum_{k=1}^{n} \frac{1}{k}$.

$\boxed{5}$ (a) Use the fact that $(\forall x \in \mathbf{R}_{\geq 1})\left(\ln(x+1) - \ln(x) \leq \frac{1}{x}\right)$ to show that the sequence H is not bounded above (and consequently does not converge). To do this, you will use some standard calculus facts about telescoping sums and about the natural logarithm. One fact that you might use is the fact that the natural logarithm is not bounded above.

Solution:

$\boxed{5}$ (b) Show that $(\forall \varepsilon \in \mathbf{R}_{>0})\,(\exists n \in \mathbf{Z})\,(\forall k \in \mathbf{Z}_{>n})\,(|H_{k+1} - H_k| < \varepsilon)$.

Solution:

$\boxed{5}$ (c) Show that the sequence H is not Cauchy.

Solution:

$\boxed{5}$ (d) Draw a graph that shows that the fact $(\forall x \in \mathbf{R}_{\geq 1})\left(\ln(x+1) - \ln(x) \leq \frac{1}{x}\right)$ is due to the fact that the natural logarithm function is concave down. **Hint:** For the graph $y = \ln(x)$ and a positive number k, show the tangent line at the point $(x = k, y = \ln(k))$ and the secant line through the points $(x = k, y = \ln(k))$ and $(x = k+1, y = \ln(k+1))$.