Exam III has questions 1 through 6 with a total of 60 points.

10   1. Using a limit of *Newton quotient,* show that the function $x \in \mathbf{R} \mapsto x^2 + 5x$ is differentiable at 2. *Justify* each step of your calculation with a *word or a phrase.*

10   2. Using a limit of *Newton quotient,* show that the function $x \in \mathbf{R} \mapsto |5 - x| + x$ is differentiable at 2. *Justify* each step of your calculation with a *word or a phrase.*

$\boxed{10}$ 3.   Given the fact that the function $x \in \mathbf{R} \mapsto \max(1, x)$ is *continuous* on $\mathbf{R}$, show that the function $F = x \in \mathbf{R} \mapsto x \max(1, x)$ is *differentiable* at zero.

10   4. Show that the function

$$x \in \mathbf{R} \mapsto x \begin{cases} x\cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is *differentiable* at zero.

$\boxed{10}$ 5. Let F be a function and let $a \in \text{dom}(F)$. Show that if F is continuous at a and $F(a) > 0$, then

$$(\exists r \in \mathbf{R}_{>0})\,(\forall x \in \text{ball}(a, r) \cap \text{dom}(F))\,(F(x) > 0)$$

(We used this fact to prove the reciprocal rule for derivatives.)

10   6. Give an example of a function F that is not differentiable at zero, but the function $x \in \mathrm{dom}(F) \mapsto F(x)^2$ is differentiable at zero.