

I have neither given nor received unauthorized assistance on this assignment.

Homework 11 has questions 1 through 1 with a total of 15 points. Please neatly hand-write your solutions, digitize them and submit the digitized copy to Canvas. This work is due *Saturday 19 November at 11:59 P.M.*

1. Define

$$F = x \in \mathbf{R} \mapsto \begin{cases} \frac{x}{2} & x \in \mathbf{Q} \\ x & x \in \mathbf{R} \setminus \mathbf{Q} \end{cases}. \quad (1)$$

You may freely use the fact that in between any two distinct real numbers there is a rational number; and the fact that in between any two distinct real numbers there is an irrational number.

- 5 (a) Show that F is Lipschitz continuous at zero; thus show that

$$(\exists M \in \mathbf{R}) (\exists r \in \mathbf{R}_{>0}) (\forall x \in \text{ball}(0, r)) (|F(x) - F(0)| \leq M|x|).$$

- 5 (b) Show that F is continuous at zero.

- 5 (c) Show that F is not differentiable at zero.