

## Homework 1, Fall 2022

Homework 1 has questions 1 through 2 with a total of 25 points.

**Very neatly** hand write your solutions, digitize them (pdf works the best—please no \*.HEIC files. Canvas cannot display them), and submit the digitized copy to Canvas. This assignment is due *Saturday 27 August at 11:59 PM*.

I have neither given nor received unauthorized assistance on this assignment.

1. A function  $F$  is increasing on its domain provided

$$(\forall x, y \in \text{dom}(F)) (x < y \implies F(x) \leq F(y)).$$

5 (a) Without using negation (the symbol  $\neg$ ), write the negation of

$$(\forall x, y \in \text{dom}(F)) (x < y \implies F(x) \leq F(y))$$

in symbolic form. For assistance with the logic, see the section “Tautologies” in our class Quick Reference.

5 (b) Show that the function  $x \in [-1, 1] \mapsto |x|$  is not increasing on its domain.

2. A function  $F$  is subadditive on its domain provided

$$(\forall x, y \in \text{dom}(F)) (F(x + y) \leq F(x) + F(y)).$$

5 (a) Without using negation (the symbol  $\neg$ ), write the negation of

$$(\forall x, y \in \text{dom}(F)) (F(x + y) \leq F(x) + F(y))$$

in symbolic form.

5 (b) Show that the function  $x \in \mathbf{R} \mapsto x^2$  is not subadditive on its domain.

5 (c) Show that the function  $x \in \mathbf{R} \mapsto |x|$  is subadditive on its domain. To do this, you may use the triangle inequality without proving it.