Fake Proofs

"If you can't prove what you want to prove, demonstrate something else and pretend that they are the same."

Darrell Huff

How to Lie with Statistics

Barton Willis, Attribution 4.0 International (CC BY 4.0), 2020

Prove a special case

Theorem

Every function that is differentiable at zero is continuous at zero.

Fake Proof

Let $F = x \in \mathbf{R} \mapsto x^2$. This function is differentiable at zero. And it's continuous at zero. We've shown that a function that is differentiable at zero is continuous at zero.

1. We can't prove an "every" statement by checking just one out of many cases.
2. Occasionally, students **mistakenly** believe that if the instructions are to *show* instead of "prove" that something is true, it's OK to verify the statement is true for one particular case. But no, "showing" that something is true and "proving" that is true are identical instructions.

Assume the conclusion

Theorem

Let A, B, and C be sets. If $A \subset B$ and $B \subset C$, we have $A \subset C$.

Fake Proof

Suppose $A \subset C$. [Rubbish Deleted]; therefore $A \subset C$.

1. Terrific–we just proved the tautology $(A \subset C) \implies (A \subset C)$.
2. A proof needs to flow from the hypothesis (in this case $A \subset B$ and $B \subset C$) to the conclusion $(A \subset C)$.

Assuming facts not in evidence

Theorem

Every continuous function has an antiderivative.

Fake Proof

Define $F(x) = a_0 + a_1 x + \cdots + a_n x^n$. An antiderivative of F is

$$a_0 x + \frac{1}{2} a_1 x^2 + \cdots + \frac{1}{1+n} a_n x^{n+1}.$$

Therefore, F has an antiderivative.

1. Assuming that F is a polynomial is assuming facts not in evidence.
2. Arguably, this fake proof falls under the "Prove a special case" category.

Proof by obfuscation

Theorem

For every $a, b \in \mathbf{R}$ with $a \neq b$ there is $x \in \mathbf{R}$ such that $\min(a, b) < x < \max(a, b)$.

Bad Form Proof

We the People of the United States, in Order to form a more perfect Union, establish Justice, insure domestic Tranquility, provide for the common defense, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish this Constitution for the United States of America. Choose $x = \frac{a+b}{2}$.

Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal. We have $x = \frac{a+b}{2} < \frac{\max(a,b)+\max(a,b)}{2} = \max(a, b)$.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. Similarly $\min(a, b) < x$. Therefore $\min(a, b) < x < \max(a, b)$.

Twisted Qualifiers

Theorem

We have

$$(\exists m \in \mathbf{R}_{>0}) \left(\forall x \in \mathbf{R}_{>0})(\left| \frac{x}{x+1} \right| \leq m|x| \right).$$

Fake Proof

Let $x > 0$. Choose $m = \frac{1}{1+x}$. Then $m > 0$, as required. Further,

$$\left[|\frac{x}{x+1}| \leq m|x| \right] \equiv \left[\left| \frac{x}{x+1} \right| \leq \left| \frac{x}{x+1} \right| \right] \equiv \text{True}.$$

1. We proved that $\left(\forall x \in \mathbf{R}_{>0} \right) \left(\exists m \in \mathbf{R}_{>0} \right) \left(\left| \frac{x}{x+1} \right| \leq m|x| \right)$.
2. In the actual proposition, the first introduced variable is m and the second is x. Since m comes first, it **cannot depend** on x.

Working backwards

Theorem

We have $3^{1/3} > 5^{1/5}$.

Fake Proof

We have

$$3^{1/3} > 5^{1/5} \implies (3^{1/3})^{15} > (5^{1/5})^{15} \implies 3^5 > 5^3 \implies 243 > 125 = \text{true!}$$

1. Congratulations–we just proved that if $3^{1/3} > 5^{1/5}$ then $243 > 125$.
2. This fake proof can be salvaged by starting with $243 > 125$ and reversing each implication.
3. But not all such "backward" proofs can be salvaged this way.
4. The suggestion to "start with what you want and work backwards" is sometimes OK, but often terrible advice.

Backwards redux

Proof

We have

$$\begin{aligned}
\left[3^{1/3} > 5^{1/5}\right] &\equiv \left[(3^{1/3})^{15} > (5^{1/5})^{15}\right] && (x \in \mathbf{R} \mapsto x^{15} \text{ is increasing })\\
&\equiv \left[3^5 > 5^3\right] && \text{(algebra)}\\
&\equiv [243 > 125] && \text{(algebra)}\\
&\equiv \text{true}
\end{aligned}$$

1. Isn't this proof also backwards? It starts with the conclusion!
2. Yes it starts with the conclusion, but it shows that $\left[3^{1/3} > 5^{1/5}\right] \equiv \text{true}$.
3. And that's fine.

Proof by Haiyu

Theorem

Let A, B, and C be sets. Then $A \setminus B \subset A$

Bad Form Proof

$$A \setminus B$$

$$A \setminus B \text{ takes away from } A$$

$$\text{So}$$

$$A \setminus B \subset A$$

1. Proofs aren't poems, so don't formate them as poetry.
2. Using a phrase such as " takes away from" is poetic license. Let's stick to using mathematical terminology.