Advanced Calculus, Fall 2022

Review for Exam II

1. Show that the sequence $k \in \mathbf{Z}_{>0} \mapsto \frac{k+1}{k+5}$ converges.

Solution: We'll show that

$$(\exists\, L \in \mathbf{R})(\forall\, \varepsilon \in \mathbf{R}_{>0})(\exists\, n \in \mathbf{Z})\,(\forall\, k \in \mathbf{Z}_{>n})\left(\left|\frac{k+1}{k+5} - L\right| < \varepsilon\right).$$

Choose $L = 1$. Let $\varepsilon \in \mathbf{R}_{>0}$. Choose $n = \left\lceil \frac{4}{\varepsilon} \right\rceil$. Let $k \in \mathbf{Z}_{>n}$. We have

$$\begin{aligned}
\left|\frac{k+1}{k+5} - L\right| &= \left|\frac{k+1}{k+5} - 1\right|, && \text{(substitution)} \\
&= \frac{4}{k+5}, && \text{(algebra)} \\
&= \varepsilon. && \text{(algebra)}
\end{aligned}$$