MATH 460 **Name:**
Homework 8, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 8 has questions 1 through 4 with a total of 20 points. You must typeset your work using Overleaf. This work is due *Saturday 29 October at 11:59* PM.

Link to your Overleaf work: XXX

[5] 1. Show that the function $x \in \mathbf{R} \mapsto x^2$ is continuous on $\mathbf{R}$. That is show that

$$(\forall\, a \in \mathbf{R})\,(\forall \varepsilon \in \mathbf{R}_{>0})\,(\exists \delta \in \mathbf{R}_{>0})\,(\forall\, x \in \text{ball}(a,\delta))\,\left(|x^2 - a^2| < \varepsilon\right).$$

Solution:

[5] 2. Show that

$$(\forall\ \varepsilon \in \mathbf{R}_{>0})\,(\exists\ \delta \in \mathbf{R}_{>0})\,(\forall\ a, x \in \mathbf{R})\,\left(|x-a| < \delta \implies |x^2 - a^2| < \varepsilon\right)$$

is false. You might like to use the fact that solving the equations

$$x = \alpha + \frac{\delta}{4}, \quad a = \alpha - \frac{\delta}{4}, \quad x^2 - a^2 = 1$$

for x, a, and α yields

$$x = \frac{1}{\delta} + \frac{\delta}{4}, \quad a = \frac{1}{\delta} - \frac{\delta}{4}, \quad \alpha = \frac{1}{\delta}.$$

Solution:

[5] 3. Show that the function $x \in \mathbf{R} \mapsto \begin{cases} x & x \le 1 \\ 5 & x > 1 \end{cases}$ is not continuous at 1.

Solution:

[5] 4. Show that the function $x \in \mathbf{R} \mapsto \begin{cases} x & x \le 1 \\ 5 & x > 1 \end{cases}$ is continuous at 3.

Solution: