

## Homework 10, Fall 2023

Homework 10 has questions 1 through 3 with a total of 30 points. This work is due **Saturday 11 November** at 11:59 P.M. **For this assignment, neatly handwrite your solutions and submit a digitized version to Canvas.**

- 10 1. Define a function  $F = x \in \mathbf{R} \mapsto \begin{cases} 0 & x \leq 2 \\ 2 & x > 2 \end{cases}$ . Find  $F^{-1}((-1/2, 1/2))$ . Explain why this shows that  $F$  is not continuous on  $\mathbf{R}$ . Instead of a proof, you may use a convincing picture to find  $F^{-1}((-1/2, 1/2))$ . See Theorem 4–5 in our textbook.
- 10 2. Define  $F = x \in [0, 1] \mapsto x$ . Find an open set  $\mathcal{G}$  such that  $F^{-1}((-1/2, 1/2)) = \mathcal{G} \cap [0, 1]$ . Instead of a proof, you may use a convincing picture to find  $F^{-1}((-1/2, 1/2))$ . This shows that  $F^{-1}((-1/2, 1/2))$  is open relative to  $\text{dom}(F)$ .
- 10 3. Define  $F = (2x-1) \begin{cases} \frac{1}{8}(2x+1) & x < 1/2 \\ -\frac{1}{8}(2x-3) & x \geq 1/2 \end{cases}$ . Show that  $F$  is differentiable at  $1/2$ . To do this you can either use a limit of a Newton quotient or you can show that the function  $x \in \mathbf{R} \mapsto \begin{cases} \frac{1}{8}(2x+1) & x < 1/2 \\ -\frac{1}{8}(2x-3) & x \geq 1/2 \end{cases}$  is continuous at  $1/2$ .