CYRR 304 **Name:**

Homework 1, Spring 2024

"*The purpose of computing is insight, not numbers.*" RICHARD HAMMING

Homework 1 has questions 1 through 1 with a total of of 8 points. Your recorded score will be scaled to twenty points.

The point value for each question or part of a question is in the box following each question or part of a question. Neatly write your answers on your own paper, being careful to clearly label each part of each question. Digitize your work as a pdf, and turn it into Canvas. This work is due **Saturday 27 January** at 11:59 PM.

1. An IEEE number of the type binary16 has the form

$$(-1)^s (b_0.b_1 b_2 \cdots b_{10})_2 \times 2^e,$$

where the sign bit $s \in \{0,1\}$, the exponent $e \in \{-14,-13,\cdots,14,15\}$ and each bit $b_k \in \{0,1\}$. There are ten bits in the fractional part of the number, and just like for binary64 numbers, the leading bit b_0 is not stored and defaults to one.

The exponent range of -14 to 15 contains 30 integers. And just like for a binary64 number, there are two values of the exponent reserved for special conditions, including the number zero, overflow, and the like. That makes for a total of 32 distinct values for the exponent.

[1] (a) How many bits must be used to store the exponent in computer memory? Explain.

[1] (b) How many bits must be used to store the entire binary16 number in computer memory? Explain.

[1] (c) Find the *largest* number of the type binary16.

[1] (d) Find the *smallest* positive normalized number of the type binary16. Remember that the leading bit of a normalized number is one.

[1] (e) Find the *smallest* positive denormalized number of the type binary16. Remember that the leading bit of a denormalized number is zero. Also, the exponent of a denormalized binary16 is -14.

[1] (f) Find the *machine epsilon* ε_m for a number of the type binary16.

[1] (g) We have $\mathrm{Fl}(1/10) = (-1)^0(1.1001100110)_2 \times 2^{-4}$. Express $\mathrm{Fl}(1/10)$ as a rational number (that is, as a quotient of integers).

[1] (h) Show that $|\mathrm{Fl}(1/10) - 1/10| \le \varepsilon_m$.