MATH 460 **Name:**
Homework 9, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 9 has questions 1 through 2 with a total of 10 points. You must typeset your work using Overleaf. This work is due *Saturday 29 October at 11:59* PM.

Link to your Overleaf work: XXX

5 1. A function F is said to be *Lipschitz continuous* on a set A provided that $A \subset \mathrm{dom}(F)$ and

$$(\exists m \in \mathbf{R}_{>0})\left(\forall x, y \in A\right)\left(|F(x) - F(y)| < m|x - y|\right).$$

Show that if F is Lipschitz continuous on $[0,1]$, it is uniformly continuous on $[0,1]$.

5 2. Show that the square root function is not Lipschitz continuous on $[0,1]$. You will need to show that

$$(\forall m \in \mathbf{R}_{>0})\left(\exists x, y \in A\right)\left(|\sqrt{x} - \sqrt{y}| \geq m|x - y|\right).$$

Hint: Choose $y = 0$. Given $m \in \mathbf{R}_{>0}$, you will need to choose $x \in [0,1]$ such that $\sqrt{x} \geq mx$.