MATH 460 **Name:**

Homework 1, Fall 2022

Homework 1 has questions 1 through 2 with a total of 25 points.

Very neatly hand write your solutions, digitize them (pdf works the best–please no *.HEIC files. Canvas cannot display them), and submit the digitized copy to Canvas. This assignment is due *Saturday 27 August at 11:59* PM.

I have neither given nor received unauthorized assistance on this assignment.

1. A function F is increasing on its domain provided

$$\big(\forall x, y \in \operatorname{dom}(F)\big)\big(x < y \implies F(x) \le F(y)\big).$$

$\boxed{5}$ (a) Without using negation (the symbol $\neg$), write the negation of

$$\big(\forall x, y \in \operatorname{dom}(F)\big)\big(x < y \implies F(x) \le F(y)\big).$$

in symbolic form. For assistance with the logic, see the section "Tautologies" in our class Quick Reference.

Solution:

$$\big(\exists x, y \in \operatorname{dom}(F)\big)\big((x < y) \wedge (F(x) > F(y))\big).$$

$\boxed{5}$ (b) Show that the function $x \in [-1,1] \mapsto |x|$ is not increasing on its domain.

Solution: We'll show that

$$\big(\exists x, y \in \operatorname{dom}(F)\big)\big((x < y) \wedge (|x| > |y|)\big).$$

Choose $x = -1$ and $y = 0$. We have

$$\big[(x < y) \wedge (|x| > |y|)\big] \equiv [(-1 < 0) \wedge (|-1| > |0|)] \equiv \text{True}.$$

2. A function F is subadditive on its domain provided

$$\big(\forall x, y \in \operatorname{dom}(F)\big)\big(F(x+y) \le F(x) + F(y)\big).$$

$\boxed{5}$ (a) Without using negation (the symbol $\neg$), write the negation of

$$\big(\forall x, y \in \operatorname{dom}(F)\big)\big(F(x+y) \le F(x) + F(y)\big)$$

in symbolic form.

Solution:

$$\left(\exists x, y \in \operatorname{dom}(F)\right)\left(F(x+y) > F(x) + F(y)\right).$$

$\boxed{5}$ (b) Show that the function $x \in \mathbf{R} \mapsto x^2$ is not subadditive on its domain.

Solution: We'll show that

$$\left(\exists x, y \in \operatorname{dom}(F)\right)\left((x+y)^2 > x^2 + y^2\right).$$

Choose $x = 1$ and $y = 1$. We have

$$\left[(x+y)^2 > x^2 + y^2\right] \equiv [4 > 1 + 1] \equiv \text{True}.$$

$\boxed{5}$ (c) Show that the function $x \in \mathbf{R} \mapsto |x|$ is subadditive on its domain. To do this, you may use the triangle inequality without proving it.

Solution: We'll show that

$$\left(\forall x, y \in \mathbf{R}\right)\left(|x+y| \leq |x| + |y|\right).$$

This is the triangle inequality.