MATH 460 **Name:**
Homework 11, Fall 2023

Homework 11 has questions 1 through 1 with a total of 20 points. This work is due **Saturday 25 November** at 11:59 PM. **For this assignment, neatly handwrite your solutions and submit a digitized version to Canvas.**

1. Let $F = \in [0,1] \mapsto x^2$. For any $n \in \mathbf{Z}_{>0}$, define a partition P_n of $[0,1]$, by $\{{}^{k}/_{n} \mid k \in \mathbf{Z}_{\geq 0, \leq n}\}$. For this partition, the length of each subinterval is $1/n$. So a general Riemann sum has the form

$$\frac{1}{n}\sum_{k=0}^{n-1} F(c_k),$$

where $c_k \in [{}^{k}/_{n}, {}^{(k+1)}/_{n}]$. Choosing $c_k = {}^{k}/_{n}$ gives the left point Riemann sum L_n. Specifically

$$L_n = \frac{1}{n}\sum_{k=0}^{n-1} F(k/n)$$

And choosing $c_k = {}^{(k+1)}/_{n}$ gives the right point Riemann sum L_n. Specifically

$$R_n = \frac{1}{n}\sum_{k=0}^{n-1} F(k+1/n)$$

Changing the sum index k to $k-1$ gives an alternative formula for R_n. It is

$$R_n = \frac{1}{n}\sum_{k=1}^{n} F(k/n)$$

10 (a) Find a simple, and I mean simple representation for $R_n - L_n$. **Hint:** When I'm overwhelmed, I go to my happy place. For sums, my happy place is to list a few terms:

$$L_n = \frac{1}{n}\left(F(0/n) + F(1/n) + F(2/n) + \cdots + F((n-1)/n)\right),$$
$$R_n = \frac{1}{n}\left(F(1/n) + F(2/n) + F(3/n) + \cdots + F(1)\right).$$

10 (b) Let $\varepsilon \in \mathbf{R}_{>0}$. Show that there is $n \in \mathbf{Z}_{>0}$ such that $|R_n - L_n| < \varepsilon$.