

## Homework 8, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 8 has questions 1 through 4 with a total of 20 points. You must typeset your work using Overleaf. This work is due *Saturday 29 October at 11:59 PM*.

**Link to your Overleaf work:** XXX

- 5 1. Show that the function  $x \in \mathbf{R} \mapsto x^2$  is continuous on  $\mathbf{R}$ . That is show that

$$(\forall a \in \mathbf{R}) (\forall \varepsilon \in \mathbf{R}_{>0}) (\exists \delta \in \mathbf{R}_{>0}) (\forall x \in \text{ball}(a, \delta)) (|x^2 - a^2| < \varepsilon).$$

- 5 2. Show that

$$(\forall \varepsilon \in \mathbf{R}_{>0}) (\exists \delta \in \mathbf{R}_{>0}) (\forall a, x \in \mathbf{R}) (|x - a| < \delta \implies |x^2 - a^2| < \varepsilon)$$

is false. You might like to use the fact that the solving the equations

$$x = \alpha + \frac{\delta}{4}, \quad a = \alpha - \frac{\delta}{4}, \quad x^2 - a^2 = 1$$

for  $x, a$ , and  $\alpha$  yields

$$x = \frac{1}{\delta} + \frac{\delta}{4}, \quad a = \frac{1}{\delta} - \frac{\delta}{4}, \quad \alpha = \frac{1}{\delta}.$$

- 5 3. Show that the function  $x \in \mathbf{R} \mapsto \begin{cases} x & x \leq 1 \\ 5 & x > 1 \end{cases}$  is not continuous at 1.

- 5 4. Show that the function  $x \in \mathbf{R} \mapsto \begin{cases} x & x \leq 1 \\ 5 & x > 1 \end{cases}$  is continuous at 3.