Review for Exam I

1. Show that

$$(\forall k \in \mathbf{Z}_{>0}) \left(\frac{1}{k} \le \frac{1}{k} - \frac{1}{k+1} \right).$$

2. Show that

$$(\forall x \in (-\infty, 1)) \, (\exists r \in \mathbf{R}) \, ((x - r, x + r) \subset (-\infty, 1)) \, .$$

3. Let A, B be subsets of $\mathbf{R}$ and let A be bounded above. Show that $A \setminus B$ is bounded above.

4. Give an example of subsets A, B be subsets of $\mathbf{R}$ such that $A \setminus B$ is bounded above, but A is not bounded above.

5. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of the set

$$F(\{-4, -1, 0, 1, 4\}).$$

6. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of the set

$$F^{-1}(\{-4, -1, 0, 1, 4\}).$$

7. Using the definition from the QRS, show that the sequence $k \in \mathbf{Z}_{\ge 0} \mapsto \frac{k-6}{k+28}$ converges.

8. Show that the sequence $n \in \mathbf{Z}_{>0} \mapsto \sum_{k=1}^{n} \frac{1}{k^2}$ is bounded above. To do this, use the fact that for all positive integers k, we have $\frac{1}{k} \le \frac{1}{k} - \frac{1}{k+1}$.

9. Either show that the sequence

$$k \in \mathbf{Z} \mapsto \sin(\pi k)$$

converges or that it diverges. For either case, you're proof will must use the definition from the QRS.

10. Show that the sequence

$$k \in \mathbf{Z} \mapsto \begin{cases} k! & k < 100 \\ \frac{1}{k} & k \ge 100 \end{cases}$$

converges. You must use the definition in the QRS.

11. Show that

$$(\forall a \in \mathbf{R}) \, (\exists m \in \mathbf{R}) \, (\forall x \in \mathbf{R}) \left(x^2 - a^2 \le m(x - a) \right).$$