Advanced Calculus I

MATH 460–01

Fall 2023

Instructor: Barton Willis, PhD, Professor of Mathematics
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Office Hours: Monday, Wednesday, and Friday 10:00 AM– 11:00 AM, Tuesday and Thursday 9:30 AM– 11:00 AM, and by appointment.

Important Dates

First Homework due	27 August
Exam 1	23 September
Exam 2	4 November
Exam 3	2 December
Final exam	14 December 8:00 AM – 10:00 AM

Grading

Your course grade will be based on weekly homework sets, three midterm exams, and a comprehensive final exam; specifically:

Weekly Homework: *12 fifteen point assignments*	180 (total)
Mid-term exams 1,2, and 3: *100 points each*	300 (total)
Comprehensive Final exam	150 (total)

If we end the term with less than 180 points for homework, your homework point total will be scaled to a total of 180.

The following table shows the *minimum* number of points (out of 630) that are required for each of the twelve letter grades D- through A+. For example, a point total of 546 points will earn you a grade of B+, and a point total of 567 points will earn you a grade of A-. If you earn a point total of 377 or less, you a failing course grade.

D-	378	B-	504
D	399	B	525
D+	420	B+	546
C-	441	A-	567
C	462	A	588
C+	483	A+	624

Class meeting time and place

This class meets Monday, Wednesday, and Friday from 9:05 AM– 9:55 AM in Discovery Hall, room 386.

Course Resources

Our textbook is *An Introduction to Analysis*, 2$^{\text{nd}}$ edition, Waveland Press, Prospect Heights, Illinois, 2002 (ISBN 13: 978-1-57766-232-7) by James Kirkwood. The book by the same author and title, but published by PWS Publishing Company (Boston, 1995, ISBN 13: 0-534-94422-1) is identical.

Some homework assignments for this course will need to be typeset. To do this, you will need to create a *no cost* account on Overleaf (`https://www.overleaf.com/`). For a tutorial for using Overleaf, see `https://www.overleaf.com/tutorial`.

Course Calendar

Generally, we'll adhere to the scheduled exam dates even if we are ahead or behind with coursework. When we are ahead or behind, the topics on the exams will be appropriately adjusted.

Notices:

(a) *Exams will be given on* ***Friday*** *of the week they are assigned.*

(b) Homework (labelled **HW**) will be due one minute before midnight on Saturday of the week they are assigned.

(c) The final exam will be given on 14 December 8:00 AM – 10:00 AM.

Week	Week Starting	Section(s)	Topic(s)	Assessments
1	21 August		Logic, Proof methods, and Overleaf	**HW 1**
2	28 August	§1.1 – 1.3	Sets, Functions, Real numbers, Completeness	**HW 2**
3	4 September	§2.1 – §2.2	Sequences & Subsequences	**HW 3**
4	11 September	§2.1 – §2.2	Sequences & Subsequences	**Exam 1**
5	18 September	§2.3	Bolzano-Weierstrass	**HW 4**
6	25 September	§3.1	Topology	**HW 5**
7	2 October	§3.1	Topology	**HW 6**
8	9 October	§4.1	Limits and Continuity	**Exam 2**
9	16 October	§4.1	Limits and Continuity	**HW 7**
10	23 October	§5.1	Derivatives	**HW 8**
11	30 October	§5.1	Derivatives	**HW 9**
12	6 November	§5.2	Some Mean Value Theorems	**Exam 3**
13	13 November	§6.1	Some Mean Value Theorems	**HW 10**
14	20 November	§6.2	The Riemann Integral	**HW 11**
15	27 November		The Riemann Integral	**HW 12**
16	4 December		Catch up or review	
17	11 December			**Final Exam**

University Policies

For the UNK's Policies and statements on: Attendance Policy, Academic Honesty Policy, Reporting Student Sexual Harassment, Sexual Violence or Sexual Assault, Students with Disabilities, Students Who are Pregnant, and UNK Statement of Diversity & Inclusion, you *must read* `https://www.unk.edu/academic_affairs/asa_forms/course-policies-and-resources.php`.

Policies

Unless an assessment is *explicitly* stated to be a group project, *all work you turn in for a grade must be your own.* If you need assistance in completing a homework assignment, you may ask me for help. Googling for answers, seeking help from the Learning Commons or other faculty members, or using solution keys from previous terms (either from UNK or other universities) is also prohibited. Violation of these rules will result in earning a grade of zero on the assessment. Each homework assignment you turn in for a grade must include the statement:

$\boxed{\text{I have neither given nor received unauthorized assistance on this assignment.}}$

If two assignments are so similar that only collaboration could explain their similarities, both assignments will receive a grade of zero. Using unauthorized materials or communication devices (cell phone, for example) while taking a test will earn you a grade of zero on that assessment.

1. Regular in person class attendance is required. If you are ill or need to miss class due to athletics, please let me know ahead of time and I will make an effort to put the class on Zoom.

2. For examinations and in class assignments, show your work. *No credit will be given for multi-step problems without the necessary work. Your solution must contain enough detail so that I am convinced that you could correctly work any similar problem.* Also erase or clearly mark any work you want me to ignore; otherwise, I'll grade it.

3. The work you turn in is expected to be *accurate, complete, concise, neat,* and *well-organized. You will not earn full credit on work that falls short of these expectations.*

4. Class cancellations due to weather, illness, or other unplanned circumstances may require that we make adjustments to the course calendar, exam dates, and due dates or specifics for course assessments.

5. Extra credit is not allowed.

6. For examinations, you may use a teacher provided quick reference sheet, but no other reference materials. You may also use a pencil, eraser, and a scientific calculator. For examinations, your phone and all such devices must be turned off and *out of sight.*

7. Generally, if you are ill or absent for any reason (including athletics), you must turn in your in class work on time. Permission to turn in work late must be made in advance, otherwise late in class work will count zero points.

8. During class time, please refrain from using electronic devices. If your device usage distracts your classmates, I will ask you to put it away. If it's my impression that you are often not paying attention in class, I reserve the right to decline to help you during office hours.

9. The final examination will be *comprehensive* and it will be given during the time scheduled by the University. Except for *extraordinary circumstances* you must take the exam at this time.

10. If you have questions about how your work has been graded, make an appointment with me immediately.

11. Please regularly check Canvas to verify that your scores have been recorded correctly. If I made a mistake in recording one of your grades, I'll correct it provided you saved your paper.

Barton Willis, PhD

Professor of Mathematics and Statistics

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Blocks marked "Appointments" (yellow highlight) means *usually* available to make appointments. Blocks with gray highlights mean not available.

Fall 2022

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00					
9:15	**Advanced Calculus I** 9:05-9:55, DSCH 386	**Not Available** 9:05-9:30	**Advanced Calculus I** 9:05-9:55, DSCH 386	**Not Available** 9:05-9:30	**Advanced Calculus I** 9:05-9:55, DSCH 386
9:30		**Office Hours** 9:30–11:00, DSCH 368		**Office Hours** 9:30–11:00, DSCH 368	
9:45					
10:00	**Office Hours** 10:00–11:00, DSCH 368		**Office Hours** 10:00–11:00, DSCH 368		**Office Hours** 10:00–11:00, DSCH 368
10:15					
10:30					
10:45					
11:00	**Lunch**	**Lunch**	**Lunch**	**Lunch**	**Lunch**
11:15					
11:30					
11:45					
12:00					
12:15	**Calculus I** 12:20-13:10, DSCH 383		**Calculus I** 12:20-13:10, DSCH 383		**Calculus I** 12:20-13:10, DSCH 383
12:30		**Calculus I** 12:30-13:20, DSCH 383		**Calculus I** 12:30-13:20, DSCH 383	
12:45					
13:00					
13:15	**Appointments** 13:15-15:00, DSCH 368		**Appointments** 13:15-15:00, DSCH 368		**Appointments** 13:15-15:00, DSCH 368
13:30					
13:45					
14:00		**Departental Meeting** 14:00-15:00		**Departental Meeting** 14:00-15:00	
14:15					
14:30					
14:45					
15:00					
15:15	**Not Available** 15:15-17:00		**Not Available** 15:15-17:00	**Not Available** 15:15-17:00	
15:30					
15:45					
16:00					
16:15					
16:30					
16:45					
17:00					

Greek characters

Name	Symbol	Typical use(s)
alpha	α	angle, constant
beta	β	angle, constant
gamma	γ	angle, constant
delta	δ	limit definition
epsilon	ϵ or ε	limit definition
theta	θ or ϑ	angle
pi	π or ϖ	circular constant
phi	ϕ or φ	angle, constant

Named sets

empty set	$\varnothing$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$

integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive reals	$\mathbf{R}_{>0}$

Set symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$

Meaning	Symbol
union	$\cup$
complement	superscript$^{\mathrm{C}}$
set minus	$\setminus$

Intervals

For numbers a and b, we define the intervals:

$$(a,b) = \{x \in \mathbf{R} \mid a < x < b\}$$
$$[a,b) = \{x \in \mathbf{R} \mid a \le x < b\}$$
$$(a,b] = \{x \in \mathbf{R} \mid a < x \le b\}$$
$$[a,b] = \{x \in \mathbf{R} \mid a \le x \le b\}$$

Logic symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\implies$

Meaning	Symbol
equivalent	$\equiv$
iff	$\iff$
for all	$\forall$
there exists	$\exists$

Tautologies

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$(P \implies Q) \equiv (\neg Q \implies \neg P)$$
$$P \not\Rightarrow Q \equiv P \wedge \neg Q$$
$$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$$
$$\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$$
$$\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$$

Function notation

$\mathrm{dom}(F)$	domain of function F
$\mathrm{range}(F)$	range of function F
C_A	set of continuous functions on set A
C^1_A	set of differentiable functions on set A
$A \to B$	set of functions from A to B

Generalized set operators

Each member of a set $\mathcal{C}$ is a set:

$$\bigcup_{A \in \mathcal{C}} A = \{z \mid (\exists B \in \mathcal{C})(z \in B)\}$$
$$\bigcap_{A \in \mathcal{C}} A = \{z \mid (\forall B \in \mathcal{C})(z \in B)\}$$

Theorem: $\displaystyle\bigcup_{A \in \mathcal{C}} A^{\mathrm{C}} = \left(\bigcap_{A \in \mathcal{C}} A\right)^{\mathrm{C}}$

Functions applied to sets

Let $A \subset \mathrm{dom}(F)$ and $B \subset \mathrm{range}(F)$:

$$F(A) = \{F(x) \mid x \in A\}$$
$$F^{-1}(B) = \{x \in \mathrm{dom}(F) \mid F(x) \in B\}$$

Triangle inequalities

For all $x, y \in \mathbf{R}$, we have

$$|x + y| \le |x| + |y|$$
$$\big||x| - |y|\big| \le |x - y|$$

Floor and ceiling

Definitions:

$$\lfloor x \rfloor = \max\{k \in \mathbf{Z} \mid k \le x\}$$
$$\lceil x \rceil = \min\{k \in \mathbf{Z} \mid k \ge x\}$$

Properties:

$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(x < n \iff \lfloor x \rfloor < n)$$
$$(\forall x \in \mathbf{R}, n \in \mathbf{Z})(n < x \iff n < \lceil x \rceil)$$

Bounded sets

Bounded below A set A is *bounded below* provided $(\exists M \in \mathbf{R})(\forall x \in A)(M \le x)$.

Bounded above The set A is *bounded above* provided $(\exists M \in \mathbf{R})(\forall x \in A)(x \le M)$.

Bounded A set is *bounded* if it is bounded below and bounded above.

Elementary function properties

Increasing $(\forall x, y \in A)(x < y \implies F(x) \le F(y))$. For strictly increasing, replace $F(x) \le F(y)$ with $F(x) < F(y)$.

Decreasing $(\forall x, y \in A)(x < y \implies F(x) \ge F(y))$ For strictly decreasing, replace $F(x) \ge F(y)$ with $F(x) > F(y)$.

One-to-one $(\forall x, y \in \mathrm{dom}(F))(F(x) = F(y) \implies x = y)$

Subadditive $(\forall x, y \in \mathrm{dom}(F))(F(x + y) \le F(x) + F(y))$

Bounded above $(\exists M \in \mathbf{R})(\forall x \in \mathrm{dom}(F))(F(x) \le M)$

Bounded below $(\exists M \in \mathbf{R})(\forall x \in \mathrm{dom}(F))(M \le F(x))$

Topology

Open ball $\mathrm{ball}(a,r) = \{x \in \mathbf{R} \mid -r + a < x < r + a\}$

Punctured ball $\mathrm{ball}'(a,r) = \mathrm{ball}(a,r) \setminus \{a\}$

Open set A subset A of $\mathbf{R}$ is *open* provided

$$(\forall x \in A)\,(\exists r \in \mathbf{R}_{>0})(\mathrm{ball}(x,r) \subset A)\,.$$

Closed set A subset A of $\mathbf{R}$ is *closed* provided $\mathbf{R} \setminus A$ is open.

Limit point A number a is a *limit point* of a set A provided $(\forall r \in \mathbf{R}_{>0})\big(\text{ball}'(a,r) \cap A \neq \varnothing\big)$.

Boundary point A number a is a *boundary point* of a set A provided

$$(\forall r \in \mathbf{R}_{>0})\Big(\text{ball}(a,r) \cap A \neq \varnothing \wedge \text{ball}(a,r) \cap A^{\mathrm{C}} \neq \varnothing\Big).$$

Set closure $\overline{A} = A \cup \text{LP}(A)$, were $\text{LP}(A)$ is the set of limit points of A.

Open cover A set $\mathcal{C}$ is an open cover of a set A provided

(a) every member of $\mathcal{C}$ is an open set
(b) $A \subset \bigcup_{B \in \mathcal{C}} B$

Compact A set A is compact provided for every open cover $\mathcal{C}$ of A, there is a finite subset $\mathcal{C}'$ of $\mathcal{C}$ such that $\mathcal{C}'$ is an open cover of A.

Least and greatest bounds

For any subset A of $\mathbf{R}$:

glb $z = \text{glb}(A)$ provided

(a) z is an lower bound for A
(b) if x is a lower bound for A then $x \leq z$

lub $z = \text{lub}(A)$ provided

(a) z is an upper bound for A
(b) if x is a upper bound for A then $z \leq x$

Sequences

Bounded A sequence F is bounded if $\text{range}(F)$ bounded.

Monotone A sequence is monotone if it either increases or decreases.

Cauchy A sequence F is Cauchy provided

(a) for every $\varepsilon \in \mathbf{R}_{>0}$
(b) there is $n \in \mathbf{Z}$
(c) such that for all $k, \ell \in \mathbf{Z}_{>n}$
(d) $|F_k - F_\ell| < \varepsilon$

Converges A sequence F converges provided

(a) there is $L \in \mathbf{R}$
(b) and $n \in \mathbf{Z}$
(c) such that for all $k \in \mathbf{Z}_{>n}$
(d) $|F_k - L| < \varepsilon$.

Functions

Continuous A function F is continuous at a provided

(a) $a \in \text{dom}(F)$ and
(b) for every $\varepsilon \in \mathbf{R}_{>0}$
(c) there is $\delta \in \mathbf{R}_{>0}$
(d) such that for all $x \in \text{ball}(a,\delta) \cap \text{dom}(F)$
(e) we have $F(x) \in \text{ball}(F(a), \epsilon)$.

Uniformly continuous A function F is uniformly continuous on a set A provided

(a) $A \subset \text{dom}(F)$; and
(b) for every $\varepsilon \in \mathbf{R}_{>0}$
(c) there is $\delta \in \mathbf{R}_{>0}$
(d) such that for all $x, y \in A$ and $|x - y| < \delta$
(e) we have $|F(x) - F(y)| < \epsilon$.

Limit A function F has a limit toward a provided

(a) a is a limit point of $\text{dom}(F)$; and
(b) there is $L \in \mathbf{R}$
(c) such that for every $\varepsilon \in \mathbf{R}_{>0}$
(d) there is $\delta \in \mathbf{R}_{>0}$
(e) such that for all $x \in \text{ball}'(a,\delta)$
(f) we have $F(x) \in \text{ball}(L, \epsilon)$.

Differentiable A function F is differentiable at a provided

(a) $a \in \text{dom}(F)$; and
(b) there is $\phi \in \text{dom}(F) \to \mathbf{R}$
(c) such that ϕ is continuous at a and
(d) $(\forall x \in \text{dom}(F))(F(x) = F(a) + (x - a)\phi(x))$.

Riemann sums

Partition A set $\mathcal{P}$ is a partition of an interval $[a,b]$ provided

(a) the set $\mathcal{P}$ is finite
(b) every member of $\mathcal{P}$ is an open interval
(c) the members of $\mathcal{P}$ are pairwise disjoint
(d) $\bigcup_{I \in \mathcal{P}} \overline{I} = [a,b]$

Let F be a bounded function on an interval $[a,b]$ and let $\mathcal{P}$ be a partition of $[a,b]$.

Lower sum $\underline{S}(\mathcal{P}) = \sum_{I \in \mathcal{P}} \text{glb}\big(F(\overline{I})\big) \times \text{length}(I)$

Upper sum $\overline{S}(\mathcal{P}) = \sum_{I \in \mathcal{P}} \text{lub}\big(F(\overline{I})\big) \times \text{length}(I)$

Riemann sum $\sum_{I \in \mathcal{P}, x^\star \in \overline{I}} F(x^\star) \times \text{length}(I)$

Axioms

Completeness Every nonempty subset A of $\mathbf{R}$ that is bounded above has a least upper bound.

Well-ordering Every nonempty set of positive integers contains a least element.

Induction $(\forall n \in \mathbf{Z}_{\geq 0})(P(n))$ if and only if $P(0) \wedge (\forall n \in \mathbf{Z}_{\geq 0})(P(n) \implies P(n+1))$.

Named theorems

Archimedean $(\forall x \in \mathbf{R})(\exists n \in \mathbf{Z})(n > x) \equiv \text{true}$.

Bolzano–Weirstrass Every bounded real valued sequence has a convergent subsequence.

Heine–Borel A subset of $\mathbf{R}$ is compact iff it is closed and bounded.

Cauchy completeness Every Cauchy sequence in $\mathbf{R}$ converges.

Monotone convergence Every bounded monotone sequence converges.

Intermediate value theorem If $F \in \mathrm{C}_{[a,b]}$, then for all $y \in [\min(F(a), F(b)), \max(F(a), F(b))]$ there is $x \in [a,b]$ such that $F(x) = y$.

Mean Value If $F \in \mathrm{C}_{[a,b]} \cap \mathrm{C}^1_{(a,b)}$, there is $\xi \in (a,b)$ such that $(b-a)F'(\xi) = F(b) - F(a)$.