1. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of $F(\{-2,-1,0,1,2\})$.

Solution:

$$F(\{-2,-1,0,1,2\} = \{F(-2),F(-1),F(0),F(1),F(2)\} = \{4,1,0,1,4\} = \{0,1,4\}.$$

2. Define $F = x \in \mathbf{R} \mapsto x^2$. Enumerate the members of $F^{(-1)}(\{0,1,4\})$.

Solution: The solution set to $F(x) = 4$ is $\{-2,2\}$; the solution set to $F(x) = 1$ is $\{-1,1\}$; and the solution set to $F(x) = 0$ is $\{0\}$. So

$$F^{(-1)}(\{0,1,4\}) = \{-2,-1,0,1,2\}.$$

3. Show that

$$(\forall\, a \in \mathbf{R}_{>0})\,(\exists\; m \in \mathbf{R})\,(\forall\; x \in \mathbf{R}_{\geq 0})\left(\sqrt{x} \leq \sqrt{a} + m(x-a)\right).$$

Hints: You might like to use the facts:

$$\begin{aligned}
\left[\sqrt{x} \leq \sqrt{a} + m(x-a)\right] &\equiv \left[\sqrt{x} - \sqrt{a} - m(x-a) \leq 0\right], \\
&\equiv \left[\sqrt{x} - \sqrt{a} - m(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a}) \leq 0\right], \\
&\equiv \left[(\sqrt{x}-\sqrt{a})(1 - m(\sqrt{x}+\sqrt{a}) \leq 0\right].
\end{aligned}$$

Solution: Let $a \in \mathbf{R}_{>0}$. Choose $m = \frac{1}{2\sqrt{a}}$. We have

$$\begin{aligned}
\left[\sqrt{x} \leq \sqrt{a} + m(x-a)\right] &\equiv \left[\sqrt{x} - \sqrt{a} - m(x-a) \leq 0\right], && \text{(algebra)} \\
&\equiv \left[\sqrt{x} - \sqrt{a} - m(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a}) \leq 0\right], && \text{(algebra)} \\
&\equiv \left[(\sqrt{x}-\sqrt{a})(1 - m(\sqrt{x}+\sqrt{a}) \leq 0\right], && \text{(factor)} \\
&\equiv \left[(\sqrt{x}-\sqrt{a})\frac{\sqrt{a}-\sqrt{x}}{2\sqrt{a}} \leq 0\right], && \text{(substitution)} \\
&\equiv \left[-\frac{(\sqrt{x}-\sqrt{a})^2}{2\sqrt{a}} \leq 0\right], && \text{(factor)} \\
&= \text{True}.
\end{aligned}$$

4. Show that for all sets A and B that $(B \setminus A = B) \implies (A \cap B = \emptyset)$. **Hint:** Try proving the contrapositive.

Solution: We'll show that $A \cap B \neq \varnothing \implies B \setminus A \neq B$. Since $A \cap B \neq \varnothing$, there is x such that $x \in A$ add $x \in B$; thus $x \notin B \setminus A$ and $x \in B$. Since there is a member of B that isn't a member of $B \setminus A$, we've shown that $B \setminus A \neq B$.