MATH 460   **Name:**

Riemann integral practice for final

"I'm killing time while I wait for life to shower me with meaning and happiness."

<div align="right">CALVIN (BILL WATTERSON)</div>

1. Define $f = x \in [0,2] \longmapsto \begin{cases} 1 & x \neq 1 \\ 99 & x = 1 \end{cases}$. Given a positive number ε, explicitly find

   a partition P of $[0,2]$ such that $\overline{S}(f,P) - \underline{S}(f,P) < \varepsilon$

2. Define $f = x \in [0,2] \longmapsto \begin{cases} x & x \in Q \\ 0 & x \notin Q \end{cases}$. Show that f is not Riemann integrable on

   $[0,2]$.

3. Define $f = x \in [0,3] \longmapsto \begin{cases} 3 & 0 \leq x < 1 \\ 4 & 1 \leq x < 2 \\ x+2 & 2 \leq x \leq 3 \end{cases}$. Define $g = x \in [0,3] \longmapsto \int_0^x f(t)\,dt$.

   (a) Find a formula for g.

   (b) Sketch a graph of g.

   (c) Show that g is not differentiable at 1.

   (d) Show that g is differentiable at 2.

4. Define $Q = [0,3] \longmapsto \int_0^x \lfloor t \rfloor\,dt$. Sketch a pretty good graph of Q.