Advanced Calculus, Fall 2022 **Name:**____________________________
Practice Exam II **Row and Seat:**____________________________

1. Show that the sequence $k \in \mathbf{Z}_{>0} \mapsto \frac{k+1}{k+5}$ converges.
2. Give an example of a convergent subsequence of $k \in \mathbf{Z}_{>0} \mapsto (-1)^k$.
3. Show that sequence $k \in \mathbf{Z}_{>0} \mapsto \begin{cases} k! & k < 1000 \\ \frac{k+1}{k+5} & k \geq 1000 \end{cases}$ converges.
4. Use the QRS definition of an open set to show that interval $(0,1)$ is open.
5. Use the QRS definition of a closed set to show that interval $[0,1]$ is closed.
6. Use the QRS definitions of open and closed to show that the set $\mathbf{R}$ is open and closed.
7. Use the QRS definition of a boundary point to show that $\partial(0,1] = \{0,1\}$. Use this result to explain why $(0,1]$ is not closed.
8. Use the QRS definition to show that $0 \notin \mathrm{LP}(\mathbf{Z})$.
9. Show that the function $F(x) = \begin{cases} -1, & \text{if } x < 5, \\ 1, & \text{if } x \geq 5 \end{cases}$ does not have a limit toward 5.
10. Show that the function $F(x) = x^2$ has a limit toward 2.
11. Show that the set $(0,\infty)$ is not compact by showing that there is an open cover of $(0,\infty)$ that has no finite subcover.
12. Show that if a subset of $\mathbf{R}$ is not bounded, it is not compact. Do this using the definition of compact that involves open covers.
13. Show that the union of two compact sets is compact. Do this using the definition of compact that involves open covers.
14. Show that if sets A and B are closed, so it $A \cup B$.
15. Give an example of open sets $\mathcal{G}_1, \mathcal{G}_3, \mathcal{G}_3, \ldots$ such that the intersection $\bigcap_{k \in \mathbf{Z}_{>0}} \mathcal{G}_k$ is not open.

16. Let $F : \mathbf{Z} \to \mathbf{R}$ and let $F(x) = \sqrt[3]{x^{14} + 1066} + \sqrt[43]{x^2 + 1776}$. Either prove or disprove: The function F has a limit toward 1.

17. Define $F = x \in \mathbf{Z} \mapsto \sqrt[3]{x^{14} + 1066} + \sqrt[43]{x^2 + 1776}$. Show that F is not continuous at 1.

18. Let F be a convergent sequence and let $\alpha \in \mathbf{R}$. Show that αF is a convergent sequence.

19. Let $|F|$ be a convergent sequence. Show that $|F|$ is a convergent sequence.

20. Use the inequality $|\sqrt{a} - \sqrt{b}| \leq \sqrt{|a - b|}$, for $a, b > 0$ to show that the function

    $$F = x \in \mathbf{R} \mapsto \sqrt{1 + x},$$

    is continuous at 1.