Grammar for mathematicians and other humans

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MATH 460

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Write proofs as (English) sentences

Write proofs as (English) sentences; specifically:

- Every sentence must start with a word, not a mathematical expression.
- Every sentence of a proof must end with a *period or a semicolon.*
- Except for enumeration, generally mathematical expressions should be separated by a word or phrase.

Examples

Wrong: m,n integers
Correct: Let m and n be integers.

Wrong: $x > 0$, we have $x + 1 > 0$.
Correct: Since $x > 0$, we have $x + 1 > 0$.

Wrong: If $x \in A$, $x \in B$.
Correct: If $x \in A$, then $x \in B$.

No poetry

Ode To Tomatoes by Pablo Neruda

The street
filled with tomatoes,
midday,
summer,
light is
halved
like
a
tomato

☞ Write proofs as regular text, not as poetry with wide margins.

Example

$$\textbf{Wrong:} \quad \text{Let } \varepsilon > 0.$$
$$\text{Choose } \delta = \varepsilon/3.$$
$$\text{For } |x - 1| < \delta \text{ we have } x < 1 + \delta.$$

Correct: Let $\varepsilon > 0$. Choose $\delta = \varepsilon/3$. For $|x - 1| < \delta$ we have $x < 1 + \delta$.

Say what you mean

Try reading your text out loud. Make sure it makes sense.

Examples

Wrong: Let $x \in A \subset B$.
Correct Let $x \in A$. Since $A \subset B$, we have $x \in B$.

Wrong: Let $k > 0$ be an integer.
Correct: Let k be a positive integer.

The sentence

Let x be a member of A is a subset of B.

is nonsense. So is

Let k is greater than zero be an integer.

First waffle rule

The first waffle is never perfect; neither is the first attempt at a proof. Revise your work until it is as close to perfect as you can make it.

- But first be sure your work is logical–correcting the form of illogical work is a waste of time.
- By all means, if it helps you construct a proof, draw pictures and diagrams filled with lines and arrows.
- But do not include your scratch work in the final copy.
- In a quest for perfection, mathematicians have been known to write math on resturant menus, unpaid bills, and on birth certificates.

Pick-and-show idiom

Anytime you need to show one set is a subset of another, you should use the "pick-and-show" idiom; it looks like this:

Proposition *Let A and B be sets and suppose H_1, H_2, $\ldots$, and H_n. Then $A \subset B$.*

Proof *If $x \in A$, we have (deductions made using the facts H_1 through H_n); therefore $x \in B$.*

Here, the statements H_1 through H_n are the hypothesis of the proposition. To demonstrate set equality, use the pick-and-show idiom twice.

Pick-and-show shown

Pick-and-show example

Proposition Let A and B be sets with $A \subset B$. Then $B^{\mathrm{C}} \subset A^{\mathrm{C}}$.

Proof If $x \in B^{\mathrm{C}}$, then $x \notin B$. Since $x \notin B$ and $A \subset B$, we have $x \notin A$; therefore $x \in A^{\mathrm{C}}$.

- The *conclusion* of the proposition is $B^{\mathrm{C}} \subset A^{\mathrm{C}}$. Thus pick-and-show starts with "If $x \in B^{\mathrm{C}}$."
- The hypothesis is $A \subset B$. Starting pick-and-show starting with 'If $x \in A^{\mathrm{C}}$" is the exit ramp to nowhere.

Jeep[1] idiom

To show that there is only one thing of some object, assume Thing_1 and Thing_2 are these objects and show that $\text{Thing}_1 = \text{Thing}_2$

There's only one

Proposition There is at most one empty set.

Proof Suppose E and E' are empty sets. Since E is empty, we have $E \subset E'$. Similarly since E' is empty, we have $E' \subset E$; therefore $E = E'$.

[1] there's only one