MATH 460 **Name:** (append your name)
Homework 1, Spring 2020

Homework 1 has questions 1 through 6 with a total of 18 points. Edit this file and append you answers using LaTeX.

Examples

Question Write the statement *There is a positive real number x such that* $x^2 = 2$ in symbolic form;
Answer $(\exists x \in \mathbf{R})(x^2 = 2)$.

Question In symbolic form, write the negation of your answer to the previous question. Use the negation rules $\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$ and $\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$ to transform the your symbolic form (replace the left side by the right side.)
Answer $(\forall x \in \mathbf{R})(x^2 \neq 2)$.

$\boxed{3}$ 1. Write the statement *For every positive real number x, there is a positive real number y such that* $y < x$ in symbolic form.

Solution:

$$(\forall x \in \mathbf{R}_{>0})(\exists y \in \mathbf{R}_{>0})(y < x)$$

$\boxed{3}$ 2. Write the negation statement *For every positive real number x, there is a positive real number y such that* $y < x$ in symbolic form. Use the negation rules $\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$ and $\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$ to transform the your symbolic form (replace the left side by the right side.)

Solution:

$$(\exists \in \mathbf{R}_{>0})(\forall y \in \mathbf{R}_{>0})(y \geq x)$$

$\boxed{3}$ 3. Write the statement $(\forall x, y \in \mathbf{R})(x^2 = y^2 \implies x = y)$ as an English sentence that doesn't use logical symbols.

Solution: For all real numbers x and y, if $x^2 = y^2$, we have $x = y$.

$\boxed{3}$ 4. Write the statement $\neg(\forall x, y \in \mathbf{R})(x^2 = y^2 \implies x = y$ as an English sentence that doesn't use logical symbols. Again, use the negation rules $\neg(\forall x \in A)(P(x)) \equiv (\exists x \in A)(\neg P(x))$ and $\neg(\exists x \in A)(P(x)) \equiv (\forall x \in A)(\neg P(x))$ to transform the your symbolic form (replace the left side by the right side.)

Solution: For there are real numbers x and y, such that if $x^2 = y^2$ and $x \neq y$.

$\boxed{3}$ 5. Let F be a real valued function. Write the *contrapositive* of the statement *If F is continuous at zero, then F is differentiable at zero.* as an English sentence.

Solution: If F is not differentible at zero, then F is not continuous at zero.

$\boxed{3}$ 6. Let F be a real valued function. Write the *converse* of the statement *If F is continuous at zero, then F is differentiable at zero.* as an English sentence.

Solution: If F is differentible at zero, then F is continuous at zero.