MATH 460 **Name:**

Homework 5, Fall 2023

Homework 5 has questions 1 through 7 with a total of 70 points. When I record your grade, I will scale it to twenty points. For details of the grading scheme for this assignment, please see the section 'Grading rubric' of our syllabus.

Revise, proofread, revise again (and again), typeset your work using Overleaf, and upload the converted pdf of your compiled file work to Canvas. This work is due **Saturday 30 September** at 11:59 PM.

$\boxed{10}$ 1. Let F and G be sequences each with domain $\mathbf{Z}_{\geq 0}$ and suppose that

$$(\forall\, k \in \mathbf{Z}_{\geq 0})\,(F_k < G_k)$$

Show that if G is bounded above, then F is bounded above.

$\boxed{10}$ 2. Let F and G be sequences each with domain $\mathbf{Z}_{\geq 0}$ and suppose that

$$(\forall\, k \in \mathbf{Z}_{\geq 0})\,(F_k < G_k)$$

Show that if F is not bounded above, then G is not bounded above.

$\boxed{10}$ 3. Show that the sequence $n \in \mathbf{Z}_{\geq 1} \mapsto \sum_{k=1}^{n} (\ln(k+1) - \ln(k))$ is not bounded above. You may freely use the fact that the sequence $k \in \mathbf{Z}_{\geq 1} \mapsto \ln(k+1)$ is not bounded above.

$\boxed{10}$ 4. Use Desmos to draw graphs of both $y = 1/x$ and $y = \ln(x+1) - \ln(x)$ for $1 \leq x \leq 5$. Use your graph to conjecture whether $1/x > \ln(x+1) - \ln(x)$ or $1/x < \ln(x+1) - \ln(x)$ for all $x \in [1, \infty]$. Include your graph along with your conjecture.

Notice: One way to prove this is to use a MVT (mean value theorem). Since we're not there yet, for now, we'll use graphical evidence as a fairly convincing argument.

$\boxed{10}$ 5. Show that the sequence $n \in \mathbf{Z}_{\geq 1} \mapsto \sum_{k=1}^{n} \frac{1}{k}$ is not bounded above. Use the result from the previous question.

You may freely use the fact: Let F and G be sequences and suppose that for all $k \in \mathbf{Z}_{\geq 0}$ we have $F_k < G_k$, then for all $n \in \mathbf{Z}_{\geq 0}$ we have $\sum_{k=1}^{n} F_k < \sum_{k=1}^{n} G_k$.

$\boxed{10}$ 6. Show that the sequence $n \in \mathbf{Z}_{\geq 1} \mapsto \sum_{k=1}^{n} \frac{1}{k}$ does not converge.

$\boxed{10}$ 7. Let F be a sequence that converges to zero; further suppose that range$(F) \subset [0, \infty]$. Show that $\sqrt{F}$ converges to zero. We define $\sqrt{F} = k \in \mathbf{Z}_{\geq 0} \mapsto \sqrt{F_k}$.