Dear Dr. Holub:
I apologize that this Homework is a few decades late. I don't have a copy of the class syllabus (I'd guess there wasn't one), so I don't know what your late policy is, but I hope that you will accept it at least for partial credit.

Sincerely,
Barton Willis, PhD
Professor of Mathematics

I have neither given nor received unauthorized assistance on this assignment.

1. Show that

$$\left(\forall\ x, y \in \mathbf{R}\right)\left(\frac{|x+y|}{1+|x+y|} \le \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}\right).$$

Solution: Let $x, y \in \mathbf{R}$. We have

$$\begin{aligned}
\left[\frac{|x+y|}{1+|x+y|} \le \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}\right] &\equiv \left[\frac{|x+y|}{1+|x+y|} - \frac{|x|}{1+|x|} - \frac{|y|}{1+|y|} \le 0\right] && \text{(algebra)} \\
&\equiv \left[\frac{|x+y| - |x| - |y| - |x||y|(2+|x+y|)}{(1+|x+y|)(1+|x|)(1+|y|)} \le 0\right] && \text{(lots of algebra)} \\
&\equiv \text{True} && \text{(triangle inequality)}
\end{aligned}$$

To justify the last line, we need the triangle inequality (that makes $|x+y| - |x| - |y|$ negative) along with the fact that the term $-|x||y|(2+|x+y|)$ is also negative. Finally, the denominator is positive because it's a product of positive terms.