MATH 460 **Name:**
Homework 10, Fall 2022

I have neither given nor received unauthorized assistance on this assignment.

Homework 10 has questions 1 through 5 with a total of 20 points. Please neatly handwrite your solutions, digitize them and submit the digitized copy to Canvas. This work is due *Saturday 12 November at 11:59* PM.

[5] 1. Prove the product rule for derivatives. Specifically show that if f and g are differentiable at a, then fg is differentiable at a. To do this use the Carathéodory definition of the derivative. You may freely use the fact that the sum and product of continuous functions is continuous.

[5] 2. Show that the function $x \in \mathbf{R} \mapsto x|x|$ is differentiable at zero.

[5] 3. Let $f, g \in \mathbf{R} \to \mathbf{R}$. Either prove or find a counterexample: *If fg is differentiable at zero, either f is differentiable at zero or g is differentiable at zero.*

[5] 4. Suppose $f, g \in \mathbf{R} \to \mathbf{R}$ are differentiable at zero and suppose $f(0) = 0, g(0) = 0$, and $g'(0) \neq 0$. Use the Carathéodory definition of the derivative to prove the L'Hôpital little theorem:

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{f'(0)}{g'(0)}.$$

5. (Bonus) There is at least one factual claim in the text

`http://mathonline.wikidot.com/carath-eodorys-differentiation-criterion`

that is rubbish and at least one grammatical error (I'd say far more than one). What are these errors?