

**Advanced Calculus I**  
**Homework Assignment fake**  
**Due Never 2022**

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*This assignment has questions 1 through 5 with a total of 10 points. The box to the left of each question gives the point value for the question.*

**Link to Overleaf:** <https://www.overleaf.com/9215671327dffffbvstwbvp>

- 2 1. Let  $m, n$  be odd integers. Show that  $m^2 + n^2$  is even.

**Solution:** Since  $m$  and  $n$  are odd integers, there are integers  $k$  and  $\ell$  such that  $m = 2k + 1$  and  $n = 2\ell + 1$ . We have

$$\begin{aligned} m^2 + n^2 &= (2k + 1)^2 + (2\ell + 1)^2, && \text{(substitution)} \\ &= 4k^2 + 4k + 4\ell^2 + 4\ell + 2, && \text{(expand)} \\ &= 2(2k^2 + 2k + 2\ell^2 + 2\ell + 1). && \text{(factor)} \end{aligned}$$

We've shown that  $m^2 + n^2$  is twice the integer  $2k^2 + 2k + 2\ell^2 + 2\ell + 1$ ; thus  $m^2 + n^2$  is even.

- 2 2. Let  $n$  be an odd integer. Then  $n^2 - 1$  is divisible by 8.
- 2 3. Give two examples of sentences that are *not* statements.

**Solution:**

1. Border collies are the *best* dogs.
2. Mathematics is discovered, not invented.

- 2 4. Give two examples of sentences that are statements.

**Solution:**

1. The absolute value function is continuous at zero.
2. Every perfect integer is even.

- 2 5. Let  $P$  be a statement. Show that  $P \equiv \neg\neg P$  is a tautology.

**Solution:** A truth table is

$P$	$\neg P$	$\neg\neg P$	$P \equiv \neg\neg P$
$T$	$F$	$T$	$T$
$F$	$T$	$F$	$T$

Since the final column is all true, we've shown that  $P \equiv \neg\neg P$  is a tautology.