MATH 202, Fall 2023 **Name:** _____

In class work 10 **Row and Seat**: _____

In class work **10** has questions **1** through **1** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 28 September 13:20*.

1. Find each antiderivative:

2 (a) $\int \frac{x+1}{(x-3)(x-9)}\,\mathrm{d}x$

Solution:

$$\int \frac{x+1}{(x+3)(x-9)}\,\mathrm{d}x = \int \frac{1}{6(x+3)} + \frac{5}{6(x-9)}\,\mathrm{d}x = \frac{\log(|x+3|)}{6} + \frac{5\log(|x-9|)}{6}$$

2 (b) $\int \frac{1}{(x-1)^2 x}\,\mathrm{d}x$

Solution:

$$\int \frac{1}{(x-1)^2 x}\,\mathrm{d}x = \int \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\,\mathrm{d}x = \ln(|x|) - \frac{1}{x-1} - \ln(|x-1|)$$

2 (c) $\int \frac{1}{x^2+18x+1}\,\mathrm{d}x$

Hint: The factorization of $x^2+18x+1$ is the gnarly $\left(x-4\sqrt{5}+9\right)\left(x+4\sqrt{5}+9\right)$. So the pfd has the form

$$\frac{1}{x^2+18x+1} = \frac{A}{x-4\sqrt{5}+9} + \frac{B}{x+4\sqrt{5}+9}$$

And sure, you can solve the problem starting with this. An alternative is to make a substitution $x = z+\alpha$, where α is a "magic" number you choose to transform the problem into comfort math. And comfort math might be

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\alpha}\tanh^{-1}\left(\frac{x}{\alpha}\right)\right) = \frac{1}{\alpha^2-x^2}.$$

Solution:

$$\begin{aligned}\int \frac{1}{x^2+18x+1}\,\mathrm{d}x &= \int \frac{1}{(z+\alpha)^2+18(z+\alpha)+1}\,\mathrm{d}z \\ &= \int \frac{1}{z^2+(18+2\alpha)+18\alpha+1}\,\mathrm{d}z,\end{aligned}$$

Choose $\alpha = -9$. Then

$$\begin{aligned}&= \int \frac{1}{z^2-80}\,\mathrm{d}z, \\ &= -\frac{1}{\sqrt{80}}\tanh^{-1}\left(z/\sqrt{80}\right), \\ &= -\frac{1}{\sqrt{80}}\tanh^{-1}\left((x+9)/\sqrt{80}\right)\end{aligned}$$