MATH 202 **Name:**____________________________

In Class work 7 **Row:**____________________________

1. The force F required to lift sack of potatoes x feet from the ground level on planet Yavin is $F(x) = \frac{5}{(1000+2x)^2}$. Find the work done by lifting the sack of potatoes from $x = 0$ to $x = 1000$.

Solution: For a position dependent force F, the work required to move something from a to b is $\int_a^b F(x)\,\mathrm{d}x$. So

$$\begin{aligned} \text{work} &= \int_0^{1000} \frac{5}{(1000+2x)^2}\,\mathrm{d}x, \\ &= -\left.\frac{5}{2(1000+2x)}\right|_{x=0}^{x=1000}, \\ &= \frac{1}{600}. \end{aligned}$$

$\boxed{5}$ 2. Find the work done moving a 107 kg mass from $x = -2$ to $x = 5$ if the position dependent force is $F(x) = \begin{cases} 5 & x < 1 \\ 8 & x \geq 1 \end{cases}$, where the units of force are Newtons and the units of distance are meters.

Solution:

$$\text{work} = \int_{-2}^{1} 5\mathrm{d}x + \int_1^5 8\mathrm{d}x = 47.$$

$\boxed{5}$ 3. Find the *numerical value* of $\int_4^7 \frac{1}{3x-10}\mathrm{d}x$.

Solution:

$$\int_4^7 \frac{1}{3x-10}\mathrm{d}x = \frac{1}{3}\ln\left(\frac{11}{2}\right)$$

$\boxed{5}$ 4. Find a general solution to the DE $y\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2$.

Solution:

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 + c,$$

where $c \in \mathbf{R}$.

5. Find a formula for each derivative:

$\boxed{5}$ (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{-x^2}\right)$

Solution:

$$-\left(\left(2\,x^2 - 1\right)\,e^{-x^2}\right)$$

$\boxed{5}$ (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\exp(x) - \exp(-x)}{2}\right)$

Solution:

$$\frac{\exp(x) + \exp(-x)}{2}$$

$\boxed{5}$ (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}(x\ln(x))$

Solution:

$$1 + \ln(x).$$

(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\left(\dfrac{1+x}{1-x}\right)$

Solution:

$$-\frac{2}{1-x^2}.$$

$\boxed{5}$ 6. Find the numerical value of $\int_1^2 1 + \ln(x)\,\mathrm{d}x$. **Hint:** Look at your answer to part ‘c’ of the previous question.

Solution:

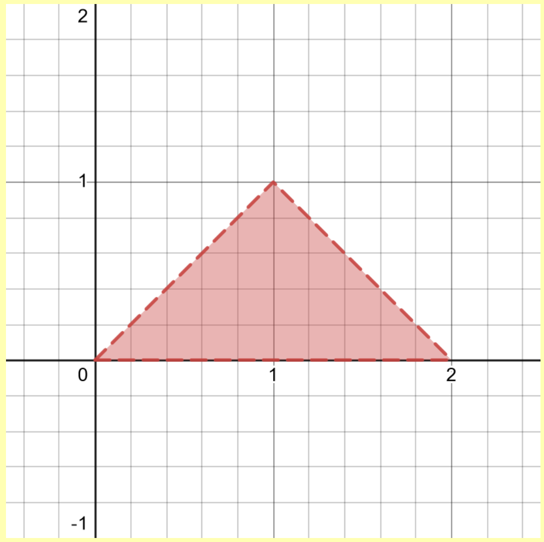
$$2\log(2)$$

.

7. Let Q be the portion of the xy plane described by $0 \le x \le 1$ and $0 \le y \le 1 - |x-1|$.

$\boxed{5}$ (a) Draw a nicely *labeled* picture of the set Q.

Solution:



$\boxed{5}$ (b) Using *disks* (that is strips *perpendicular* to the axis of rotation), find the *volume* of the solid generated by rotating Q about the x axis. Express the result as a *definite integral*, but **do not** find the numerical value of the definite integral.

Solution:

$$\pi \int_0^1 (1-|x-1|)^2 \, \mathrm{d}x.$$

$\boxed{5}$ (c) Using *shells* (that is, strips *parallel* to the axis of rotation), find the *volume* of the solid generated by rotating Q about the x axis. Express the result as a *definite integral*, but **do not** find the numerical value of the definite integral.

Solution: To find the length of the strip, we need to solve $y = 1-|x-1|$ for x. The solutions are $x = y$ and $x = 2-y$. The length of the strip is $2-y-y$ or $2-2y$.

$$2\pi \int_0^1 y(2-2y) \, \mathrm{d}y.$$

$\boxed{5}$ (d) Using a strip that is parallel to the y axis, find area of Q.

Solution:

$$\int_0^1 (2-2y) \, \mathrm{d}y = 1. \tag{1}$$

$\boxed{5}$ (e) Using a strip that is parallel to the y axis, find the y coordinate of the centroid of Q.

Solution:

$$\text{Area}\,\overline{y} = \int_0^1 y(2-2y)\,\mathrm{d}y = \frac{1}{3}. \tag{2}$$

Since the area is one, $\overline{y} = \frac{1}{3}$

$\boxed{5}$ (f) Using a strip that is parallel to the y axis, find the x coordinate of the centroid of Q.

Solution:

$$\text{Area}\,\overline{x} = \frac{1}{2}\int_0^1 \left((2-y)^2 - y^2\right)\mathrm{d}y = 1. \tag{3}$$

$\boxed{5}$ 8. Express the arclength of the portion of the curve $y = x^2$ with $-1 \le x \le 1$. Do not attempt to find the numerical value of the definite integral.

Solution:

$$\int_{-1}^1 \sqrt{1+4x^2}\,\mathrm{d}x$$

$\boxed{5}$ 9. Show that $y^5 = x + c$ is a general solution to the DE $y^4 \frac{\mathrm{d}y}{\mathrm{d}x} = 1/5$.

Solution: Differentiating $y^5 = x + c$ gives $5y^4 \frac{\mathrm{d}y}{\mathrm{d}x} = 1$. Dividing that by 5 yields the given DE.