MATH 202 **Name:**____________________________

Practice Exam 1 **Row:**_____________________________

1. The force F required to lift sack of potatoes x feet from the ground level on planet Betazed is $F(x) = \frac{5}{1000+x}$. Find the work done by lifting the sack of potatoes from $x = 0$ to $x = 1000$.

Solution: For a position dependent force F, the work required to move something from a to b is $\int_a^b F(x)\,\mathrm{d}x$. So

$$\begin{aligned} \text{work} &= \int_0^{1000} \frac{5}{1000+x}\,\mathrm{d}x, \\ &= 5\ln(1000+x)\Big|_{x=0}^{x=1000}, \\ &= 5\left(\log(2000) - \log(1000)\right), \\ &= \ln(32). \end{aligned}$$

Generally, I would say that $\ln(32)$ is simpler than is $5\left(\log(2000) - \log(1000)\right)$.

The gravitational force near the surface of Planet Betazed is unusual–but this is a math problem.

$\boxed{5}$ 2. Find the work done moving a 107 kg mass from $x = -2$ to $x = 5$ if the position dependent force is $F(x) = \begin{cases} x & x < 1 \\ 1 & x \geq 1 \end{cases}$, where the units of force are Newtons and the units of distance are meters.

Solution:

$$\text{work} = \frac{5}{2}.$$

$\boxed{5}$ 3. Find the *numerical value* of $\displaystyle\int_3^8 \frac{1}{10-5x}\mathrm{d}x$.

Solution:

$$\int_3^8 \frac{1}{10-5x}\mathrm{d}x = \frac{\ln(5)}{5} - \frac{\ln(30)}{5}.$$

$\boxed{5}$ 4. Find a general solution to the DE $y\dfrac{\mathrm{d}y}{\mathrm{d}x} = x$.

Solution:

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + c,$$

where $c \in \mathbf{R}$.

5. Find a formula for each derivative:

$\boxed{5}$ (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(xe^{x^2}\right)$

Solution:

$$2x^2e^{x^2} + e^{x^2}$$

$\boxed{5}$ (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\exp(x)+\exp(-x)}{2}\right)$

Solution:

$$\frac{\exp(x)-\exp(-x)}{2}$$

$\boxed{5}$ (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\ln(x)\right)$

Solution:

$$1+\ln(x).$$

(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\left(\dfrac{1+x}{1-x}\right)$

Solution:

$$\frac{2}{1-x^2}.$$

$\boxed{5}$ 6. Find the numerical value of $\int_1^2 1+\ln(x)\,\mathrm{d}x$. **Hint:** Look at your answer to part 'a' of the previous question.

Solution:

$$2\log(2)$$

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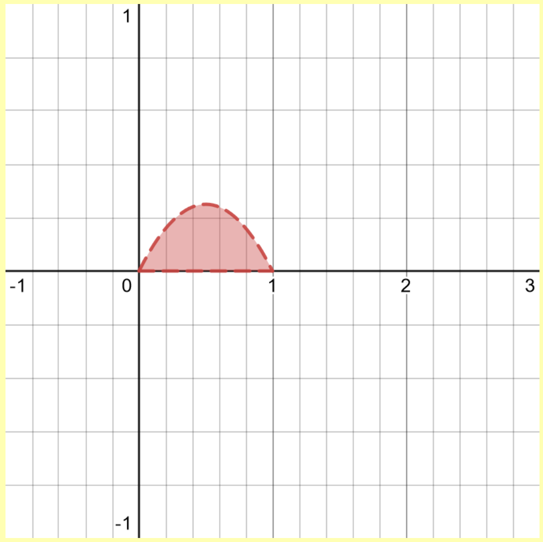
7. Let Q be the portion of the xy plane described by $0 \le x \le 1$ and $0 \le y \le x(1-x)$.

$\boxed{5}$ (a) Draw a nicely *labeled* picture of the set Q.

Solution:



$\boxed{5}$ (b) Using *disks* (that is strips *perpendicular* to the axis of rotation), find the *volume* of the solid generated by rotating Q about the x axis. Express the result as a *definite integral*, but **do not** find the numerical value of the definite integral.

Solution:
$$\pi \int_0^1 x^2(1-x)^2\,dx.$$

$\boxed{5}$ (c) Using *shells* (that is, strips *parallel* to the axis of rotation), find the *volume* of the solid generated by rotating Q about the x axis. Express the result as a *definite integral*, but **do not** find the numerical value of the definite integral.

Solution:
$$2\pi \int_0^{1/4} y\sqrt{1-4y}\,dy.$$

$\boxed{5}$ (d) Using a strip that is parallel to the y axis, find area of Q.

Solution:
$$\frac{1}{6}. \tag{1}$$

$\boxed{5}$ (e) Using a strip that is parallel to the y axis, find the y coordinate of the centroid of Q.

Solution:
$$\frac{1}{10}. \tag{2}$$

$\boxed{5}$ (f) Using a strip that is parallel to the y axis, find the x coordinate of the centroid of Q.

Solution:

$$\frac{1}{2}. \tag{3}$$

$\boxed{5}$ 8. Express the arclength of the portion of the hyperbola $x^2 - y^2 = 1$ with endpoints $(x = 2, y = \sqrt{3})$ and $(x = 3, y = \sqrt{8})$. Do not attempt to find the numerical value of the definite integral.

Solution:

$$\int_2^3 \sqrt{\frac{2x^2-1}{x^2-1}}\, \mathrm{d}x\,.$$

$\boxed{5}$ 9. Show that $y^5 = x + c$ is a general solution to the DE $y^4 \frac{\mathrm{d}y}{\mathrm{d}x} = 1/5$.

Solution: Differentiating $y^5 = x + c$ gives $5y^4 \frac{\mathrm{d}y}{\mathrm{d}x} = 1$. Dividing that by 5 yields the given DE.