MATH 202, Fall 2023 **Name:** ______________________

In class work 12 **Row and Seat**:______________________

"Study hard what interests you the most in the most undisciplined, irreverent and original manner possible." RICHARD FEYNMAN

In class work **12** has questions **1** through **3** with a total of **4** points.Turn in your work at the end of class *on paper*. This assignment is due *Thursday 5 October 13:20*.

$\boxed{2}$ 1. Find the *numeric value* of the integral $\int_0^\infty \frac{x}{1+x^4}\,\mathrm{d}x$. **Hint:** To find an antiderivative of $\int \frac{x}{1+x^4}\,\mathrm{d}x$, use the substitution $z = x^2$.

Solution: Let's begin by finding an antiderivative; once we found it, we'll use the FTC along with a limit to find the value of the improper integral. We have

$$\begin{aligned}
\int \frac{x}{1+x^4}\,\mathrm{d}x &= \frac{1}{2}\int \frac{1}{1+(x^2)^2}\,\mathrm{d}x^2, && \left(x\mathrm{d}x = \frac{1}{2}\mathrm{d}x^2\right),\\
&= \frac{1}{2}\int \frac{1}{1+z^2}\,\mathrm{d}z, && \text{(replace } x^2 \text{ by } z\text{)}\\
&= \frac{1}{2}\arctan(z), && \text{(standard antiderivative)}\\
&= \frac{1}{2}\arctan(x^2) && \text{(replace} z \text{ by } x^2\text{).}
\end{aligned}$$

Second, we take on the improper integral:

$$\begin{aligned}
\int_0^\infty \frac{x}{1+x^4}\,\mathrm{d}x &= \lim_{a\to\infty}\int_0^a \frac{x}{1+x^4}\,\mathrm{d}x,\\
&= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(x^2)\Big|_0^a\right.,\\
&= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(a^2) - \frac{1}{2}\arctan(0)\right),\\
&= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(a^2)\right),\\
&= \frac{\pi}{4}
\end{aligned}$$

$\boxed{1}$ 2. Show that $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ converges. To do this, use a comparison test with $\frac{\alpha}{1+x^2}$, where α is a number that you cleverly choose.

Solution: For all real numbers x, we have $27 \le 28 + \cos(x) \le 29$. Let's (cleverly) choose α to be 29. Then for all real numbers x, we have

$$0 \le \frac{28+\cos(x)}{1+x^2} \le \frac{29}{1+x^2}. \tag{1}$$

But $\int_0^\infty \frac{29}{1+x^2}\,\mathrm{d}x$ converges, so $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ converges.

Be careful We only know that $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ is a real number, but the comparison test **doesn't** tell us its value. We'll it does tell us that

$$\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x \le \int_0^\infty \frac{29}{1+x^2}\,\mathrm{d}x = \frac{29\pi}{2} \approx 45.553093477052.$$

Numerical integration gives us the approximation $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x \approx 44.560$

$\boxed{1}$ 3. Show that $\int_1^\infty \frac{107+\mathrm{e}^{-x}}{1+x^2}\,\mathrm{d}x$ converges. To do this, use a limit comparison test.

Solution: We know that $\int_1^\infty \frac{1}{1+x^2}\,\mathrm{d}x$ converges. And for all real $x \ge 1$ we have $\frac{107+\mathrm{e}^{-x}}{1+x^2} > 0$ and $\frac{1}{1+x^2} > 0$. Finally, everything in sight is continuous; so look at

$$\begin{aligned}\lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{\frac{107+\mathrm{e}^{-x}}{1+x^2}} &= \lim_{x\to\infty} \frac{1}{107+\mathrm{e}^{-x}}, \\ &= \frac{1}{107}.\end{aligned}$$

So $\int_1^\infty \frac{107+\mathrm{e}^{-x}}{1+x^2}\,\mathrm{d}x$ converges.