MATH 202, Fall 2023 **Name:** ______________________________

In class work 8 **Row and Seat**: ______________________________

In class work **8** has questions **1** through **4** with a total of **15** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 21 September 13:20.*

Notice: $\cos(x)^2$ means $(\cos(x))^2$. It *doesn't* mean $\cos(x^2)$. Our textbook writes this expression as $\cos^2(x)$. Either way is OK.

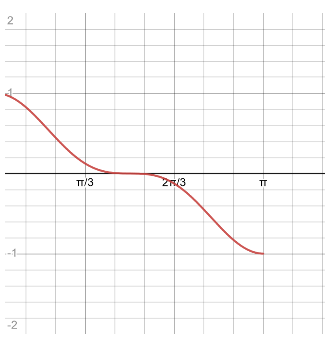
$\boxed{2}$ 1. Find the area of the region $\{(x, y) | 0 \leq y \leq \sin(x)^2 \text{ and } 0 \leq x \leq \pi\}$.

Solution:

$$\begin{aligned}
\text{Area} &= \int_0^\pi \sin(x)^2 \, \mathrm{d}x, && \text{(area formula)} \\
&= \int_0^\pi \frac{1}{2} - \frac{1}{2}\cos(2x) \, \mathrm{d}x, && \text{(double angle)} \\
&= \frac{1}{2}x - \frac{1}{4}\sin(2x) \Big|_0^\pi, && \text{(known antiderivatives)} \\
&= \frac{\pi}{2}. && \text{(algebra)}
\end{aligned}$$

$\boxed{2}$ 2. Use Desmos to graph $y = \cos(x)^3$ on the interval $[0, \pi]$. Based on the graph, make a pretty good guess for the numerical value of $\int_0^\pi \cos(x)^3 \, \mathrm{d}x$. Duplicate the graph here and justify your guess.

Solution:



The area below the x-axis appears to equal the area above the x-axis. My guess is that $\int_0^{\pi} \cos(x)^3 \, \mathrm{d}x = 0$.

$\boxed{2}$ 3. Find the numerical value of $\int_0^{\pi} \cos(x)^3 \, \mathrm{d}x$.

Solution: Let's first find an antiderivative

$$\begin{aligned}
\int \cos(x)^3 \, \mathrm{d}x &= \int \cos(x)(1 - \sin(x)^2) \, \mathrm{d}x, \\
&= \int 1 - z^2 \, \mathrm{d}z, && (z = \sin(x) \text{ and } \mathrm{d}z = \cos(x)\mathrm{d}x) \\
&= z - \frac{1}{3} z^3, \\
&= \sin(x) - \frac{1}{3} \sin(x)^3
\end{aligned}$$

So $\int_0^{\pi} \cos(x)^3 \, \mathrm{d}x = 0$.

3 4. Use the identities

$$\sin(x)\cos(y) = \frac{\sin\left(y+x\right) - \sin\left(y-x\right)}{2},$$
$$\sin(x)\sin(y) = -\frac{\cos\left(y+x\right) - \cos\left(y-x\right)}{2},$$
$$\cos(x)\cos(y) = \frac{\cos\left(y+x\right) + \cos\left(y-x\right)}{2}.$$

to find the values of each of the following definite integrals

2 (a) $\int_0^{2\pi} \sin(5x)\cos(x)\,\mathrm{d}x$.

Solution:

$$\int_0^{2\pi} \sin(5x)\cos(x)\,\mathrm{d}x = \int_0^{2\pi} \frac{\sin(6x) + \sin(4x)}{2}\,\mathrm{d}x = -\frac{\cos(6x)}{12} - \frac{\cos(4x)}{8}\bigg|_0^{2\pi} = 0.$$

Actually for any integer n, we have $\int_0^{2\pi} \sin(nx)\,\mathrm{d}x = 0$.

2 (b) $\int_0^{2\pi} \cos(5x)\cos(x)\,\mathrm{d}x$.

Solution: Let's use the nice fact that for any nonzero integer n, we have $\int_0^{2\pi} \cos(nx)\,\mathrm{d}x = 0$

$$\int_0^{2\pi} \cos(5x)\cos(x)\,\mathrm{d}x = \int_0^{2\pi} \frac{\cos(6x) + \cos(4x)}{2}\,\mathrm{d}x = 0.$$

2 (c) $\int_0^{2\pi} \cos(5x)^2\,\mathrm{d}x$.

Solution: Again, use the nice fact that for any nonzero integer n, we have $\int_0^{2\pi} \cos(nx)\,\mathrm{d}x = 0$.

$$\int_0^{2\pi} \cos(5x)^2\,\mathrm{d}x = \int_0^{2\pi} \frac{1}{2} + \frac{1}{2}\cos(10x)\,\mathrm{d}x = \pi.$$