MATH 202, Fall 2023 **Name:** ____________________

In class work 15 **Row and Seat**: ____________________

"The pencil is mightier than the pen."

ROBERT M. PIRSIG

In class work **15** has questions **1** through **9** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 19 October 13:20*.

Warning: For the most part, I've only given answers, not solutions. This allows you to check your answers. Of course, for the exam, you must show all of your work.

1. Use *integration by parts* to find an antiderivative of each of the following:

   (a) $\int x\mathrm{e}^{-x}\,\mathrm{d}x$

   Solution:

   $$\int x\mathrm{e}^{-x}\,\mathrm{d}x = -(x+1)\,\mathrm{e}^{-x}$$

   (b) $\int x^2\mathrm{e}^{-x}\,\mathrm{d}x$

   Solution:

   $$\int x^2\mathrm{e}^{-x}\,\mathrm{d}x = -\left(x^2+2x+2\right)\mathrm{e}^{-x}$$

2. Define a region of the xy plane Q by $Q = \{(x, y) | 0 \leq y \leq e^{-x} \text{ and } 0 \leq x \leq 5\}$. **Hint:** For both parts of this question, use an answer from Question 1.

   (a) Find Area(Q)

   Solution:

   $$\text{Area}(Q) = \int_0^5 e^{-x}\, dx = 1 - \frac{1}{e^5}$$

   (b) Find the x coordinate of the centroid of Q.

   Solution:

   $$\text{Area}(Q)\,\overline{x} = \int_0^5 x e^{-x}\, dx = 1 - \frac{6}{e^5}.$$

   So

   $$\overline{x} = \frac{1 - \frac{6}{e^5}}{1 - \frac{1}{e^5}}.$$

3. Find a formula for each antiderivative.

(a) $\int \frac{x+9}{(x+4)\,(x+5)}\,\mathrm{d}x$ (Use partial fractions).

Solution:

$$\int \frac{x+9}{(x+4)\,(x+5)}\,\mathrm{d}x = 5\ln(|x+4|) - 4\ln(|x+5|).$$

(b) $\int \frac{x^3}{\sqrt{1\text{-}x^2}}\,\mathrm{d}x$. (Use the substitution $x = \sin(\theta)$, where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$.)

Solution: It's OK to leave the expression in terms of a composition of a trigonometric function with an inverse trigonometric function. But here is it is explicitly as a algebraic function:

$$\int \frac{x^3}{\sqrt{1\text{-}x^2}}\,\mathrm{d}x = -\frac{x^2\sqrt{1-x^2}}{3} - \frac{2\sqrt{1-x^2}}{3}.$$

4. Find the limit of each sequence a whose formula is

(a) $a_n = \frac{(2n-1)(7n+1)}{n^2+1}$

Solution:

$$\lim_{n\to\infty} a_n = 14.$$

(b) $a_n = n \ln\left(1 + \frac{\sqrt{2}}{n}\right)$

Solution:

$$\lim_{n\to\infty} a_n = \sqrt{2}.$$

(c) $a_n = \sqrt{n^2 + 46n + 1} - n$

Solution:

$$\lim_{n\to\infty} a_n = 23.$$

5. Give an example of a sequence a such that $\lim\limits_{k\to\infty} a_k = 0$ and $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} a_k = \infty$.

Solution: An example is $a_k = \frac{1}{k}$. We have $\lim\limits_{k\to\infty} \dfrac{1}{k} = 0$ and

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k} = \infty.$$

6. Give an example of a sequence a such that $\lim\limits_{k\to\infty} a_k = 0$ and $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} a_k$ is a real number.

Solution: An example is $a_k = \frac{1}{k^2}$. Then

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k^2} \text{ converges .}$$

7. Show that the series $\sum_{k=1}^{\infty} \sqrt{k^2+46k+1}-k$ diverges. Justify your answer.

Solution: Since $\lim_{k\to\infty}\left(\sqrt{k^2+46k+1}-k\right) \neq 0$, the series $\sum_{k=1}^{\infty} \sqrt{k^2+46k+1}-k$ diverges.

8. Find the numerical value of the sum $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$.

Solution: $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k = 15$

9. Find the numerical value for each improper integral.

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{81+x^2}\,\mathrm{d}x.$

Solution: $\displaystyle\int_{-\infty}^{\infty} \frac{1}{81+x^2}\,\mathrm{d}x = \frac{\pi}{9}$

(b) $\displaystyle\int_{0}^{\infty} \sin(x)\mathrm{e}^{-x}\,\mathrm{d}x.$

Solution: $\displaystyle\int_{0}^{\infty} \sin(x)\mathrm{e}^{-x}\,\mathrm{d}x = \frac{1}{2}$