MATH 202, Fall 2023
In class work 8

Name: ______________________________
Row and Seat: ______________________

In class work **8** has questions **1** through **4** with a total of **15** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 21 September 13:20*.

Notice: $\cos(x)^2$ means $(\cos(x))^2$. It *doesn't* mean $\cos(x^2)$. Our textbook writes this expression as $\cos^2(x)$. Both notations are OK. Also, for arguments that are products, it's traditional to drop the parenthesis for the trig functions and to write $\cos x$ for $\cos(x)$, for example. I think this is the exit ramp to perdition (that is utter ruin).

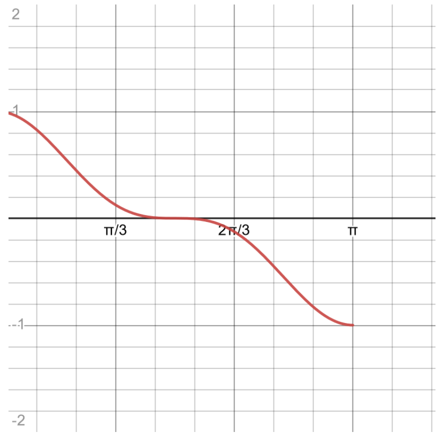
$\boxed{2}$ 1. Find the area of the region $\{(x, y) \mid 0 \le y \le \sin(x)^2 \text{ and } 0 \le x \le \pi\}$.

Solution:

$$\begin{aligned}
\text{Area} &= \int_0^\pi \sin(x)^2 \, \mathrm{d}x, && \text{(area formula)} \\
&= \int_0^\pi \frac{1}{2} - \frac{1}{2}\cos(2x) \, \mathrm{d}x, && \text{(double angle)} \\
&= \frac{1}{2}x - \frac{1}{4}\sin(2x) \Big|_0^\pi, && \text{(known antiderivatives)} \\
&= \frac{\pi}{2}. && \text{(algebra)}
\end{aligned}$$

$\boxed{2}$ 2. Use Desmos to graph $y = \cos(x)^3$ on the interval $[0, \pi]$. Based on the graph, make a pretty good guess for the numerical value of $\int_0^\pi \cos(x)^3 \, \mathrm{d}x$. Duplicate the graph here and justify your guess.

Solution:



The area that the graph bounds that is below the x-axis appears to equal the area the graph bounds that is above the x-axis. My guess is that $\int_0^\pi \cos(x)^3 \, dx = 0$.

We could prove this by making a change of variable $x = z + \pi/2$ and showing that the new integrand is odd and the new interval is symmetric with respect to the origin.

$\boxed{2}$ 3. Find the numerical value of $\int_0^{\pi} \cos(x)^3 \, \mathrm{d}x$.

Solution: Let's first find an antiderivative and second find the value of the definite integral.

$$\begin{aligned}
\int \cos(x)^3 \, \mathrm{d}x &= \int \cos(x)(1-\sin(x)^2) \, \mathrm{d}x, \\
&= \int 1 - z^2 \, \mathrm{d}z && (z = \sin(x) \text{ and } \mathrm{d}z = \cos(x)\mathrm{d}x) \\
&= z - \frac{1}{3}z^3, \\
&= \cos(x) - \frac{1}{3}\cos(x)^3
\end{aligned}$$

So $\int_0^{\pi} \cos(x)^3 \, \mathrm{d}x = 0$.

$\boxed{3}$ 4. Use the identities

$$\sin(x)\cos(y) = \frac{\sin\left(y+x\right)-\sin\left(y-x\right)}{2},$$
$$\sin(x)\sin(y) = -\frac{\cos\left(y+x\right)-\cos\left(y-x\right)}{2},$$
$$\cos(x)\cos(y) = \frac{\cos\left(y+x\right)+\cos\left(y-x\right)}{2}$$

to find the values of each of the following definite integrals

$\boxed{2}$ (a) $\int_0^{2\pi} \sin(5x)\cos(x)\,\mathrm{d}x$.

> **Solution:**
>
> $$\int_0^{2\pi} \sin(5x)\cos(x)\,\mathrm{d}x = \int_0^{2\pi} \frac{\sin(6x)+\sin(4x)}{2}\,\mathrm{d}x = -\frac{\cos(6x)}{12} - \frac{\cos(4x)}{8}\Big|_0^{2\pi} = 0.$$
>
> Actually for any integer n, we have $\int_0^{2\pi} \sin(nx)\,\mathrm{d}x = 0$. Using that fact, we don't even need the antiderivative to determine that the value is zero.

$\boxed{2}$ (b) $\int_0^{2\pi} \cos(5x)\cos(x)\,\mathrm{d}x$.

> **Solution:** Let's use the nice fact that for any nonzero integer n, we have $\int_0^{2\pi} \cos(nx)\,\mathrm{d}x = 0$
>
> $$\int_0^{2\pi} \cos(5x)\cos(x)\,\mathrm{d}x = \int_0^{2\pi} \frac{\cos(6x)+\cos(4x)}{2}\,\mathrm{d}x = 0.$$

$\boxed{2}$ (c) $\int_0^{2\pi} \cos(5x)^2\,\mathrm{d}x$.

> **Solution:** Again, use the nice fact that for any nonzero integer n, we have $\int_0^{2\pi} \cos(nx)\,\mathrm{d}x = 0$.
>
> $$\int_0^{2\pi} \cos(5x)^2\,\mathrm{d}x = \int_0^{2\pi} \frac{1}{2} + \frac{1}{2}\cos(10x)\,\mathrm{d}x = \pi.$$