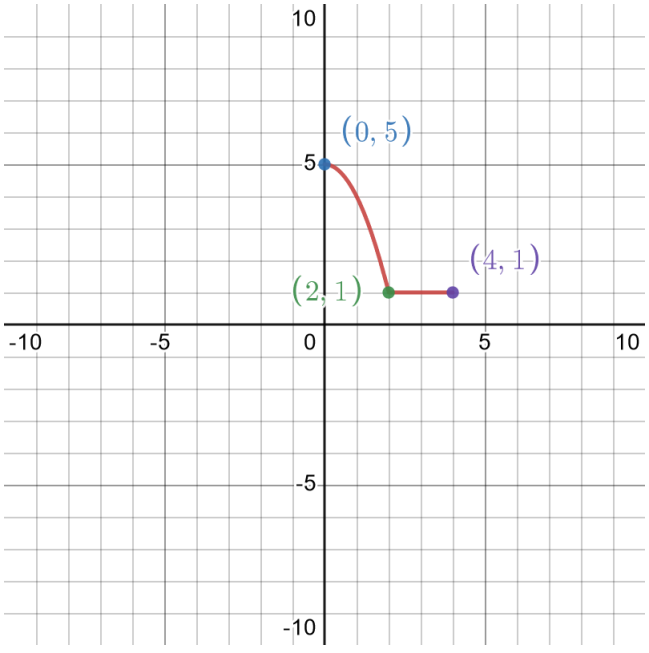
MATH 202, Fall 2023 **Name:** _____
In class work week 1 (b) **Row and Seat:** _____

In class work **1 (b)** has questions **1** through **2** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 24 August 13:20*.

1. Define a function F by $F(x) = \begin{cases} 5 - x^2 & 0 \le x \le 2 \\ 1 & 2 < x \le 4 \end{cases}$.

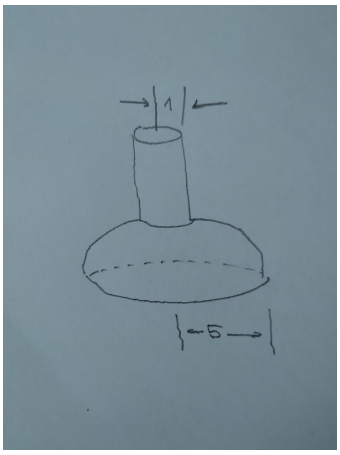
[1] (a) Sketch a graph of F. Notice $\text{dom}(F) = [0,4]$, so don't extend the graph to the left of zero or to the right of four.

Solution: Here is a pretty good graph with several labeled points. Although the graph is defined by a split rule, it appears to be continuous. You can check that an alternative formula for F is $F(x) = \max(5 - x^2, 1)$. It's a fact that isn't in our textbook, but since each argument to max is the formula to a continuous function, the function F is also continuous.



[1] (b) The graph of F is revolved about the x-axis, forming a solid of revolution. As best you can, draw a picture of this solid.

Solution: Keeping with the chemistry glassware theme, it looks like a boiling flask


1
5

1 (c) Find the numerical value of the volume of the solid generated by revolving the graph of F about the x-axis. You may use strips that perpendicular or parallel to the axis of rotation–the choice is yours.

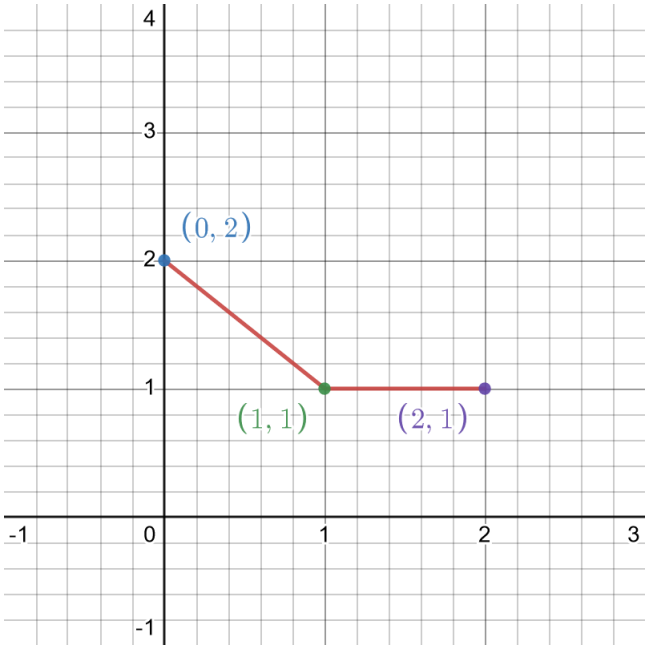
Solution: Let's use strips that are perpendicular to the axis of rotation. We have

$$\begin{aligned} V &= \pi \int_0^1 (5-x^2)^2 \, \mathrm{dx} + \pi \int_1^4 \mathrm{dx}, \\ &= \pi \frac{x^5}{5} - \pi \frac{10x^3}{3} + 25x|_0^1 + x|_1^4, \\ &= \pi \frac{328}{15} + 3\pi, \\ &= \frac{476\pi}{15}. \end{aligned}$$

2. Define a function G by $G(x) = \begin{cases} 2-x & 0 \le x \le 1 \\ 1 & 1 < x \le 2 \end{cases}$.

1 (a) Sketch a graph of G. Notice $\text{dom}(G) = [0,2]$, so don't extend the graph to the left of zero or to the right of two.

Solution:



1 (b) The graph of G is revolved about the y-axis, forming a solid of revolution. As best you can, draw a picture of this solid.

It looks like a cylindrical cake with a Hershey kiss on the top.

1 (c) Find the numerical value of the volume of the solid generated by revolving the graph of G about the y-axis. You may use strips that perpendicular or parallel to the axis of rotation–the choice is yours.

Solution: Let's use strips that are parallel to the axis of rotation. We have

$$\begin{aligned} V &= 2\pi \int_0^1 x(2-x)\,\mathrm{d}x + 2\pi \int_1^2 x\,\mathrm{d}x \\ &= \frac{13\pi}{3}. \end{aligned}$$