

## Greek Characters

Name	Symbol	Typical use(s)
alpha	$\alpha$	angle, constant
beta	$\beta$	angle, constant
gamma	$\gamma$	angle, constant
delta	$\delta$	limit definition
epsilon	$\epsilon$ or $\varepsilon$	limit definition
theta	$\theta$ or $\vartheta$	angle
pi	$\pi$ or $\pi$	circular constant
phi	$\phi$ or $\varphi$	angle, constant

## Named Sets

empty set	$\emptyset$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

## Set Symbols

Meaning	Symbol
is a member	$\in$
subset	$\subset$
intersection	$\cap$
union	$\cup$
set minus	$\setminus$

## Intervals

For numbers  $a$  and  $b$ , we define the intervals:

$$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$$

## Logic Symbols

Meaning	Symbol
negation	$\neg$
and	$\wedge$
or	$\vee$
implies	$\Rightarrow$
equivalent	$\equiv$
iff	$\Leftrightarrow$
for all	$\forall$
there exists	$\exists$

## Exponents

For  $a, b > 0$ ,  $x \in \mathbf{R}$ , and  $m, n$  real:

$$a^0 = 1, \quad 0^a = 0$$

$$1^a = 1, \quad a^n a^m = a^{n+m}$$

$$a^n / a^m = a^{n-m}, \quad (a^n)^m = a^{n \cdot m}$$

$$a^{-m} = 1/a^m, \quad (a/b)^m = a^m / b^m$$

$$\sqrt{x^2} = |x|$$

## Trigonometric Identities

$$\cos(x)^2 + \sin(x)^2 = 1$$

$$2 \cos(x)^2 = 1 + \cos(2x)$$

$$2 \sin(x)^2 = 1 - \cos(2x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\operatorname{arccot}(x) = \pi/2 - \arctan(x)$$

$$\operatorname{arccsc}(x) = \pi/2 - \arcsin(1/x)$$

$$\operatorname{arcsec}(x) = \arccos(1/x)$$

## Limits

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

$$\lim_{x \rightarrow \infty} e^x = \infty \quad \lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} \ln(x) = \infty \quad \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

## Derivatives

### Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x) \tan(x)$
$\csc(x)$	$-\cot(x) \csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\operatorname{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\operatorname{arsinh}(x)$	$1/\sqrt{1+x^2}$
$\operatorname{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

## General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1}(x)$	$1/f'(f^{-1}(x))$

## Antiderivatives

$$\int a \, dx = ax$$

$$\int x^a \, dx = \frac{1}{1+a} x^{a+1}, \quad \text{if } a \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x|$$

$$\int \cos(x) \, dx = \sin(x)$$

$$\int \sin(x) \, dx = -\cos(x)$$

$$\int \tan(x) \, dx = \ln|\sec(x)|$$

$$\int \sec(x) \, dx = \ln|\tan(x) + \sec(x)|$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)|$$

$$\int \cot(x) \, dx = \ln|\sin(x)|$$

$$\int |x| \, dx = x|x|/2$$

## Sums

For  $k, m, n \in \mathbf{Z}_{>0}$

$$\sum_{k=0}^{n-1} 1 = n$$

$$\sum_{k=0}^{n-1} k = \frac{(n-1)n}{2}$$

$$\sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

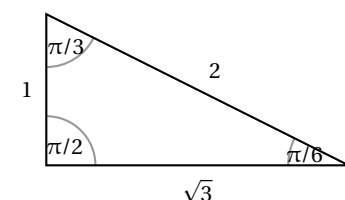
$$\sum_{k=0}^{n-1} x^k = \frac{1-x^n}{1-x}, \quad x \neq 1$$

## Logarithms

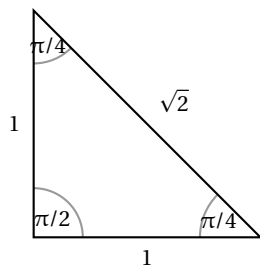
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

## Famous Triangles

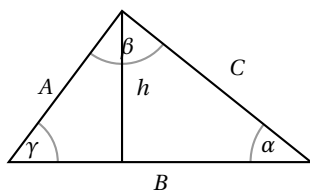
### The 30-60-90 triangle



## The 45-45-90 triangle



## Laws of Cosine & Sine



**Law of cosine:**  $C^2 = A^2 + B^2 - 2AB \cos(\gamma)$

**Law of sines:**  $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

**Area:**  $\text{Area} = \frac{1}{2}hB = \frac{1}{2}AB \sin(\gamma)$

## Hyperbolic Functions

$$2 \cosh(x) = \exp(x) + \exp(-x)$$

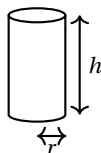
$$2 \sinh(x) = \exp(x) - \exp(-x)$$

$$\tanh(x) = \cosh(x)/\sinh(x)$$

$$\cosh(x)^2 - \sinh(x)^2 = 1$$

## Volumes

### Right Circular Cylinder

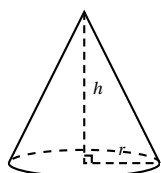


**Volume:**  $V = \pi r^2 h$

**Area:** (not including circular ends)

$$A = 2\pi r h$$

### Cone



**Volume:**  $V = \frac{1}{3}\pi r^2 h$

**Area** (not including circular base)

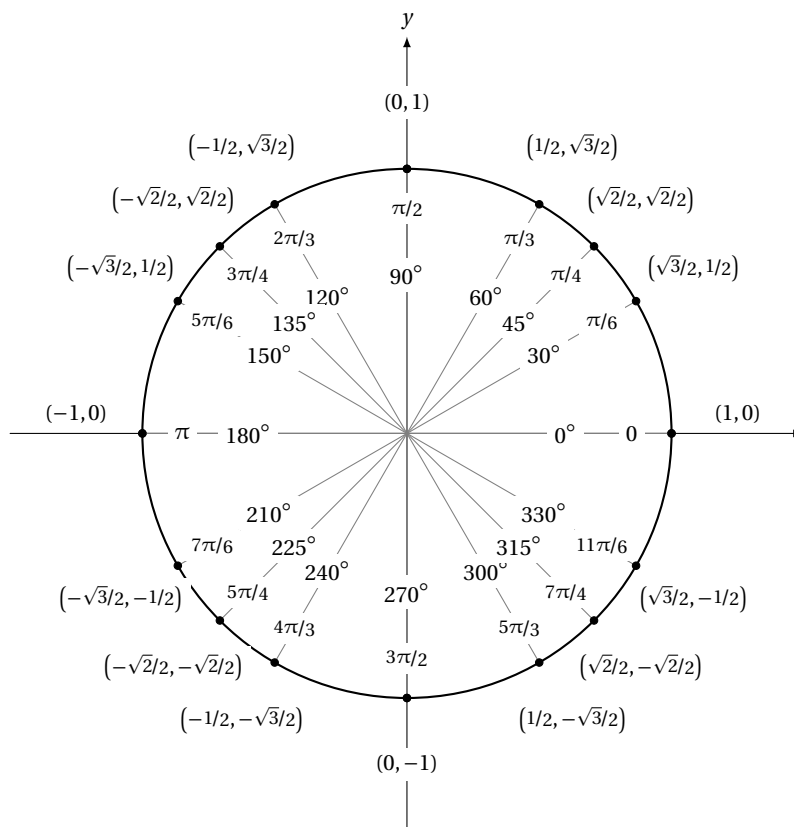
$$A = \pi r \sqrt{r^2 + h^2}$$

### Sphere

**Area:**  $A = 4\pi r^2$

**Volume:**  $V = \frac{4\pi}{3}r^3$

## Unit Circle



## Applications

Arclength of curve  $y = f(x)$  with  $a \leq x \leq b$

$$= \int_a^b \sqrt{1 + f'(x)^2} dx$$

For the region  $Q$  of the  $xy$  plane given by

$$Q = \{(x, y) \mid f(x) \leq y \leq g(x) \wedge a \leq x \leq b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x) dx$$

Assuming  $0 \leq f(x)$  and rotating about the  $x$ -axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2 dx$$

Assuming  $0 \leq a < b$  and rotating about the  $y$ -axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x)) dx$$

Centroid

$$\text{Area}(Q) \times \bar{x} = \int_a^b x(g(x) - f(x)) dx$$

$$\text{Area}(Q) \times \bar{y} = \frac{1}{2} \int_a^b (g(x)^2 - f(x)^2) dx$$

For the region  $Q$  of the  $xy$  plane given by

$$Q = \{(x, y) \mid f(y) \leq x \leq g(y) \wedge a \leq y \leq b\},$$

interchange  $x$  and  $y$  in *all* the previous formulas.

## Geometric series

$$\sum_{k=0}^{\infty} z^k = \begin{cases} \frac{1}{1-z} & z \in (-1, 1) \\ \infty & z \in [1, \infty) \end{cases}.$$

When  $z \in (-\infty, -1]$ , the series  $\sum_{k=0}^{\infty} z^k$  diverges.

## Divergence Test

If  $\lim_{k \rightarrow \infty} a_k \neq 0$ , then the series  $\sum a_k$  diverges.

## P-series

The series  $\sum_{k=1}^{\infty} \frac{1}{k^p}$  converges when  $p \in (1, \infty)$ ; otherwise it diverges.

## Ratio Test

Let  $a$  be a sequence with  $0 \notin \text{range}(a)$ . Define  $L = \lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right|$ .

- $L \in [0, 1) \Rightarrow \sum |a_k|$  converges.
- $L \in (1, \infty) \Rightarrow \sum a_k$  diverges.

## Limit Comparison Test

Let  $a$  and  $b$  be positive sequences. Define  $L = \lim_{k \rightarrow \infty} \frac{a_k}{b_k}$ .

- If  $L \in \mathbf{R}_{>0}$  and  $\sum a_k$  converges then  $\sum b_k$  converges.
- If  $L \in \mathbf{R}_{>0}$  and  $\sum a_k$  diverges then  $\sum b_k$  diverges.
- If  $L = 0$  and  $\sum b_k$  converges, then  $\sum a_k$  converges
- If  $L = \infty$  and  $\sum b_k$  diverges, then  $\sum a_k$  diverges.

## Alternating Series Test

Let  $a$  be a positive and eventually decreasing sequence. Then  $\sum (-1)^k a_k$  converges iff  $\lim_{k \rightarrow \infty} a_k = 0$ .

## Taylor and MacLaurin Series

If a function  $F$  is infinitely differentiable at  $a$ , its Taylor series centered at  $a$  is

$$\sum_{k=0}^{\infty} \frac{F^{(k)}(a)}{k!} (x-a)^k.$$

When  $a$  is zero, the Taylor series is also known as the MacLaurin Series.

## Polar to Cartesian

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

For  $r > 0$  and  $0 \leq \theta < 2\pi$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} 2\pi - \arccos(x/r) & \text{if } y < 0 \\ \arccos(x/r) & \text{if } y \geq 0 \end{cases}$$

## Integrate Powers of Trig

Let  $m, n \in \mathbf{Z}_{\geq 0}$ . Then

- $\int \cos(x)^{2m} \sin(x)^{2n} dx = \int \left( \frac{1 + \cos(2x)}{2} \right)^m \left( \frac{1 - \cos(2x)}{2} \right)^n dx$
- $\int \cos(x)^{2m+1} \sin(x)^n dx = \int (1 - z^2)^m z^n dz$ , where  $z = \cos(x)$
- $\int \cos(x)^m \sin(x)^{2n+1} dx = \int z^m (1 - z^2)^n dz$ , where  $z = \sin(x)$
- $\int \sec(x)^n dx = \frac{1}{n-1} \sec(x)^{n-2} \tan(x) + \frac{n-2}{n-1} \int \sec(x)^{n-2} dx$
- $\int \tan(x)^{2m+1} \sec(x)^n dx = \int (z^2 - 1)^n z^{n-1} dz$ , where  $z = \tan(x)$
- $\int \tan(x)^{2m} \sec(x)^n dx = \int (\sec(x)^2 - 1)^m \sec(x)^n dx$

## Trig Substitutions

- $\int F\left(x, (1-x^2)^{n/2}\right) dx$ , use  $x = \sin(\theta)$ , where  $\theta \in [-\pi/2, \pi/2]$ , then integrate  $\int F(\sin(\theta), \cos(\theta)^n) \cos(\theta) d\theta$
- $\int F\left(x, (1+x^2)^{n/2}\right) dx$ , use  $x = \sinh(\theta)$ , where  $\theta \in \mathbf{R}$ , then integrate  $\int F(\sinh(\theta), \cosh(\theta)^n) \cosh(\theta) d\theta$
- $\int F\left(x, (x^2-1)^{n/2}\right) dx$ , use  $x = \sec(\theta)$ , then integrate  $\int F(\sec(\theta), \tan(\theta)^n) \sec(\theta), \tan(\theta) d\theta$