

Show that $\int_1^2 \sqrt{1 + \frac{1}{4x}} \, dx = \int_1^{\sqrt{2}} \sqrt{1 + 4x^2} \, dx$. Try to do this in three different ways:

- (a) using lots of grubby calculus
- (b) using a modest amount of calculus
- (c) not much calculus

Of course, “lots of grubby calculus” has no particular meaning, but see how much and how little calculus you can use to find a solution.