MATH 202 **Name:** _____

Calculus Practice II, Fall 2023

Here is an opportunity for you to maintain your calculus skills over the summer. If you complete these problems, digitize your work, and submit your work to Canvas, I will send you my solutions. If you need some help with these questions, email me with your questions (willisb@unk.edu)

Completing this work is optional, and it does not enter into your class grade in any way–this work is not a bonus, extra credit, or anything like that.

1. Use a substitution to find each antiderivative.

   (a) $\int x\cos(x^2)\,\mathrm{d}x$

   Solution: Choose $z = x^2$. Then $\mathrm{d}z = 2x\mathrm{d}x$. So

   $$\int x\cos(x^2)\,\mathrm{d}x = \int \frac{1}{2}\cos(z)\,\mathrm{d}z = \frac{1}{2}\sin(z) = \frac{1}{2}\sin(x^2).$$

   We should develop the habit of checking antiderivatives by differentiation; we have

   $$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\sin(x^2)\right) = \frac{1}{2}\times 2x\cos(x^2) = x\cos(x^2).$$

   (b) $\int \frac{x}{1+x^2}\,\mathrm{d}x$

   Solution: Choose $z = 1 + x^2$. Then $\mathrm{d}z = 2x\mathrm{d}x$. So

   $$\int \frac{x}{1+x^2}\,\mathrm{d}x = \int \frac{1}{2z}\,\mathrm{d}z = \frac{1}{2}\ln(z) = \frac{1}{2}\ln(x^2+1).$$

   (c) $\int x|x^2-1|\,\mathrm{d}x$

   Solution: Choose $z = x^2 - 1$. So $\mathrm{d}z = 2x\mathrm{d}x$. So

   $$\int x|x^2-1|\,\mathrm{d}x = \int \frac{1}{2}|z|\,\mathrm{d}z = \frac{1}{4}z|z| = \frac{1}{4}\left(x^2-1\right)|x^2-1|$$

   Here we used the not so well known fact that

   $$\int |x|\,\mathrm{d}x = \frac{1}{2}x|x|.$$

   (d) $\int x^2\sqrt{x^3+1}\,\mathrm{d}x$

Solution: Choose $z = x^3 + 1$. Then $\mathrm{d}z = 2x^2\mathrm{d}x$. So

$$\int x^2\sqrt{x^3+1}\,\mathrm{d}x = \int \frac{1}{3}\sqrt{z}\,\mathrm{d}z = \frac{2z^{\frac{3}{2}}}{9} = \frac{2\left(x^3+1\right)^{\frac{3}{2}}}{9}.$$