MATH 202, Fall 2023 **Name:** ______________________

In class work 17 **Row and Seat:**______________________

"The more I read, the more I acquire, the more certain I am that I know nothing." VOLTAIRE

In class work **17** has questions **1** through **3** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 26 October 13:20.*

$\boxed{1}$ 1. Find the numerical value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$. Careful: This is an indeterminate form of the type 1^∞. To start, I suggest that you use the technique $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} \mathrm{e}^{x\ln(1+\frac{1}{x})}$.

> **Solution:**
>
> $$\lim_{x\to\infty} \mathrm{e}^{x\ln(1+\frac{1}{x})} = \lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}},$$
>
> This is an indeterminate form of the type $\frac{0}{0}$, so let's try the l'Hôpital rule.
>
> $$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}},$$
> $$= 1.$$
>
> So $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \mathrm{e}$.

$\boxed{1}$ 2. Use the *ratio* test to determine of the series $\sum_{k=0}^{\infty} \frac{\left(\frac{k}{3}\right)^k}{k!}$ converges or diverges.

> **Solution:** Algebra is supposed to be easier than calculus, so let's start by simplifying
>
> $$\frac{(\frac{k+1}{3})^{k+1}}{(k+1)!} \times \frac{k!}{(\frac{k}{3})^k} = \frac{1}{3}(\frac{k+1}{k})^k = \frac{1}{3}(1+\frac{1}{k})^k.$$
>
> So $\lim_{x\to\infty} \frac{(\frac{k+1}{3})^{k+1}}{(k+1)!} \times \frac{k!}{(\frac{k}{3})^k} = \frac{1}{3}$. And so $\sum_{k=0}^{\infty} \frac{\left(\frac{k}{3}\right)^k}{k!}$ converges.

$\boxed{1}$ 3. Define a sequence s by $s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{\sqrt{k}}$. This is a convergent alternating series. Also define $s_\infty = \lim_{n\to\infty} s_n$.

$\boxed{1}$ (a) Use Desmos to graph s on the interval $[1, 2, \ldots, 150]$. Also use Desmos to find the numeric values of s_{149} and s_{150}. As best you can, reproduce a cartoon of the graph of s.

Solution:

$\boxed{1}$ (b) From the theory of convergent alternating series, we know that $s_{150} < s_\infty < s_{149}$. Looking at the graph of s, I would guess that s_∞ is pretty close to the arithmetic average of s_{150} and s_{149}; that is $s_\infty \approx \frac{s_{150}+s_{149}}{2}$. Find the numeric value of $\frac{s_{150}+s_{149}}{2}$.

Solution:

$\boxed{1}$ (c) Define a sequence w by $w_n = \frac{s_{n+1}+s_n}{2}$. With a bit of effort, we could prove that the sequence w is a convergent alternating sequence that converges to s_∞. Use Desmos to graph the sequences s and w on the interval $[1, 2, \ldots, 150]$. Which sequence would you say converges "faster"? As best you can, reproduce a cartoon graphs of s and w.