MATH 202, Fall 2023 **Name:** ____________________

In class work week 5(a) **Row and Seat:** ____________________

In class work **5(a)** has questions **1** through **4** with a total of **8** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 19 September 13:20*.

$\boxed{2}$ 1. Use IBP to find an antiderivative of the inverse cosine function; that is find $\int \cos^{-1}(x)\,\mathrm{d}x$. To do the IBP, integrate one and differentiate $\cos^{-1}(x)$.

Solution: In tabular form, IBP gives

	D	**I**
$+$	$\cos^{-1}(x)$	1
$-$	$-\frac{1}{\sqrt{1-x^2}}$	x

So

$$\int \cos^{-1}(x)\,\mathrm{d}x = x\cos^{-1}(x) + \int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x,$$
$$= x\cos^{-1}(x) - \sqrt{1-x^2}.$$

For $\int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$, let $z = 1-x^2$. That gives $\int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x = -\sqrt{1-x^2}$.

$\boxed{2}$ 2. Find the area of the region $\{(x,y) | 0 \le y \le \cos^{-1}(x) \text{ and } -1 \le x \le 1\}$. A pretty good graph of this region is

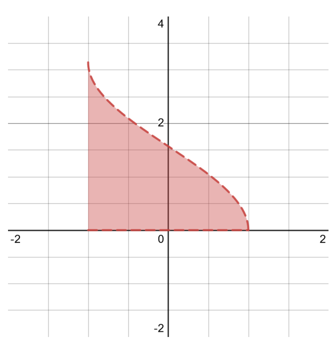

Figure 1: A pretty good graph of $\{(x,y) | 0 \le y \le \cos^{-1}(x) \text{ and } -1 \le x \le 1\}$.

Solution:

We have

$$\int_{-1}^{1} \cos^{-1}(x)\,\mathrm{d}x = x\cos^{-1}(x) - \sqrt{1-x^2}\Big|_{x=-1}^{x=1} = \cos^{-1}(1) + \cos^{-1}(-1) = \pi.$$

$\boxed{2}$ 3. Find the area of the region $\{(x,y) | \cos^{-1}(x) \le y \le \pi \text{ and } -1 \le x \le 1\}$. Try doing this by being clever. A pretty good graph of this region is

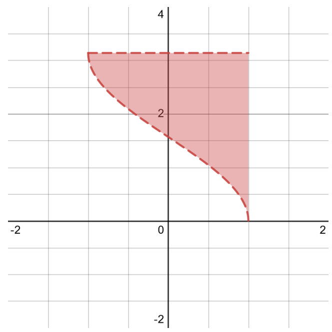

Figure 2: A pretty good graph of $\{(x,y) | \cos^{-1}(x) \le y \le \pi \text{ and } -1 \le x \le 1\}$

Solution: To find the area of the given region, from the $2\times\pi$ rectangle with vertices $(-1,0),(1,0),(1,\pi)$, and $(-1,\pi)$, we subtract the area we found in the previous question. So this area is also π.

$\boxed{2}$ 4. Use IBP to find $\int x(1-x)\sin(\pi x)\,\mathrm{d}x$. To do this, differentiate $x(1-x)$ and integrate $\sin(\pi x)$. If you enjoy doing more work than needed, expand the integrand as $\int x\sin(\pi x)\,\mathrm{d}x$ - $\int x^2\sin(\pi x)\,\mathrm{d}x$.

	D	**I**
$+$	$x-x^2$	$\sin(\pi x)$
$-$	$1-2x$	$-\frac{1}{\pi}\cos(\pi x)$
$+$	-2	$\frac{1}{\pi^2}\sin(\pi x)$
$+$	0	$-\frac{1}{\pi^3}\cos(\pi x)$

So

$$\int x\sin(\pi x)\,\mathrm{d}x = \frac{1}{\pi^3}\left(\pi^2x^2 - \pi^2 x - 2\right)\cos(\pi x) - \frac{1}{\pi^2}(2x-1)\sin(\pi x).$$