MATH 202, I	Fall 2024
In class worl	k 17

Name:	
Row and Seat	

"The more I read, the more I acquire, the more certain I am that I know nothing."

VOLTAIRE

In class work **17** has questions **1** through **3** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 26 October 13:20*.

1. Find the numerical value of $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$. Careful: This is an indeterminate form of the type 1^∞ . To start, I suggest that you use the technique $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} \mathrm{e}^{x\ln(1+\frac{1}{x})}$.

Solution:

$$\lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})} = \lim_{x \to \infty} \frac{\ln(1 + \frac{1}{x})}{\frac{1}{x}},$$

This is an indeterminate form of the type $\frac{0}{0}$, so let's try the l'Hôpital rule.

$$= \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{1}{x}} \left(-\frac{1}{x^2} \right)}{-\frac{1}{x^2}},$$

$$= 1$$

So
$$\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x = e$$
.

1 2. Use the *ratio* test to determine of the series $\sum_{k=0}^{\infty} \frac{\left(\frac{k}{3}\right)^k}{k!}$ converges or diverges.

Solution: Algebra is supposed to be easier than calculus, so let's start by simplifying

$$\frac{(\frac{k+1}{3})^{k+1}}{(k+1)!} \times \frac{k!}{(\frac{k}{3})^k} = \frac{1}{3}(\frac{k+1}{k})^k = \frac{1}{3}(1+\frac{1}{k})^k.$$

So
$$\lim_{k \to \infty} \frac{\left(\frac{k+1}{3}\right)^{k+1}}{(k+1)!} \times \frac{k!}{\left(\frac{k}{3}\right)^k} = \frac{1}{3}$$
. And so $\sum_{k=0}^{\infty} \frac{\left(\frac{k}{3}\right)^k}{k!}$ converges.

- 1 3. Define a sequence s by $s_n = \sum_{k=1}^n \frac{(-1)^{k+1}}{\sqrt{k}}$. This is a convergent alternating series. Also define $s_\infty = \lim_{n \to \infty} s_n$.
- (a) Use Desmos to graph s on the interval [1, 2, ..., 150]. Also use Desmos to find the numeric values of s_{149} and s_{150} . As best you can, reproduce a cartoon of the graph of s.

Solution:

(b) From the theory of convergent alternating series, we know that $s_{150} < s_{\infty} < s_{149}$. Looking at the graph of s, I would guess that s_{∞} is pretty close to the arithmetic average of s_{150} and s_{149} ; that is $s_{\infty} \approx \frac{s_{150} + s_{149}}{2}$. Find the numeric value of $\frac{s_{150} + s_{149}}{2}$.

Solution:

(c) Define a sequence w by $w_n = \frac{s_{n+1} + s_n}{2}$. With a bit of effort, we could prove that the sequence w is a convergent alternating sequence that converges to s_{∞} . Use Desmos to graph the sequences s and w on the interval [1, 2, ..., 150]. Which sequence would you say converges "faster"? As best you can, reproduce a cartoon graphs of s and w.