

“Singularity is almost invariably a clue.”

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In class work 11 has questions 1 through 2 with a total of 6 points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 3 October 13:20*.

1. These questions involve the region Q defined by

$$Q = \{(x, y) \mid 0 \leq y \leq \exp(-x), 0 \leq x < \infty\}.$$

For each of the following, you will need to evaluate an improper integral of the form

$\int_0^\infty F(x) \, dx$. You will need to evaluate such integrals using $\int_0^\infty F(x) \, dx = \lim_{a \rightarrow \infty} \int_0^a F(x) \, dx$.

- 1 (a) Sketch the region Q . Make a pretty good guess at the location of the *centroid* of Q .
- 1 (b) Find the *area* of the region Q .

1

(c) Find the x coordinate of the *centroid* of the region Q .

1

(d) Find the y coordinate of the *centroid* of the region Q .

2. The 1987 Edition of *Larry's Obscure Table of Obscure but Useful Integrals*, lists the antiderivative (reprinted with permission) $\int 2\lfloor x \rfloor dx = (2x - 1)\lfloor x \rfloor - \lfloor x \rfloor^2$.

- 1 (a) Use Desmos to graph $y = (2x - 1)\lfloor x \rfloor - \lfloor x \rfloor^2$ for $0 \leq x \leq 4$. Reproduce your graph here. Does the graph appear to be continuous? (When an antiderivative exists, it must be continuous.)

- 1 (b) Evaluate the definite integral $\int_0^{\sqrt{42}} 2x\lfloor x^2 + 1 \rfloor dx$.