MATH 202, Fall 2023 **Name:** ______________________________

In class work 20 **Row and Seat**: ______________________________

In class work **20** has questions **1** through **3** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 7 November 13:20*.

"There's no problem so awful that you can't add some guilt to it and make it even worse."
CALVIN (BILL WATTERSON)

$\boxed{2}$ 1. Find the Taylor polynomial of order four centered at zero for the cosine function; that is find the polynomial $P_4(x) = \sum_{k=0}^{4} \frac{\cos^{(k)}(0)}{k!} x^k$. You'll need to find the numerical values of $\cos^{(0)}(0), \cos^{(1)}(0), \cos^{(2)}(0), \cos^{(3)}(0)$, and $\cos^{(4)}(0)$.

$\boxed{2}$ 2. Use Desmos to graph $y = \cos(x)$ and $y = P_4(x)$, the Taylor polynomial of order four centered at zero for the cosine function. Reproduce the graph here. Use the graph to estimate maximum of $\max_{-2 \le x \le 2} |\cos(x) - P_4(x)|$.

$\boxed{2}$ 3. Find the Taylor polynomial of order four centered at zero for the *square* of the cosine function. You could *suffer* through the calculation by finding the first four derivatives of $\cos(x)^2$ and evaluate them at zero. Or you could use the fact we learned in class on Monday. Specifically, for $k \in \mathbf{Z}_{\geq 0}$ and infinitely differentiable functions F and G, define

$$a_k = \frac{F^{(k)}(0)}{k!}, \quad b_k = \frac{G^{(k)}(0)}{k!}.$$

Then for all $n \in \mathbf{Z}_{\geq 0}$, we have

$$\frac{(FG)^{(n)}(0)}{n!} = \sum_{k=0}^{n} a_k b_{n-k}.$$

You will want to use these formulae with $F = \cos$ and $G = \cos$. You can find the numbers a_o, a_1, a_2, a_3, a_4 and b_o, b_1, b_2, b_3, b_4 from the first question.