MATH 202, Fall 2023 **Name:** ______________________

In class work week 2(a) **Row and Seat**: ______________________

In class work **2(a)** has questions **1** through **2** with a total of **10** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 29 August 13:20.*

$\boxed{5}$ 1. Let a be a positive number. Find the length of the curve $y = a\left(\frac{x}{a}\right)^{3/2}$ where $0 \le x \le a$.

Solution: A better alternative to $y = a\left(\frac{x}{a}\right)^{3/2}$ might be $y = \frac{x^{3/2}}{\sqrt{a}}$. We have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2\sqrt{a}}x^{1/2}. \tag{1}$$

That makes

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1 + \frac{9x}{4a} \tag{2}$$

The arclength s is

$$s = \int_0^a \sqrt{1 + \frac{9x}{4a}}\,\mathrm{d}x,$$

Let's substitute $z = 1 + \frac{9x}{4a}$. Then $\mathrm{d}z = \frac{9}{4a}\mathrm{d}x$. Solving for $\mathrm{d}x$ gives $\mathrm{d}x = \frac{4a}{9}\mathrm{d}z$. And one more detail: we know the limits of integration for x, but we need them for z. When $x = 0$, we have $z = 1$. And when $x = a$, we have $z = 1 + \frac{9}{4} = \frac{13}{4}$. We're ready:

$$= \frac{4a}{9}\int_1^{13/4} \sqrt{z}\,\mathrm{d}z,$$

$$= \frac{3}{2}z^{3/2}\Big|_{z=1}^{z=13/4},$$

$$= \frac{4a}{9}\frac{3}{2}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right),$$

$\boxed{5}$ 2. Let a be a positive number. Find the surface area of the solid generated by rotating the curve $y = a\sqrt{\frac{x}{a}}$ where $0 \le x \le a$ about the x-axis.

A better alternative to $y = a\sqrt{\frac{x}{a}}$ might be $y = \sqrt{a}\sqrt{x}$. We have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\sqrt{a}x^{-1/2}. \tag{3}$$

So

$$\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} = \sqrt{1+\frac{a}{4x}}. \tag{4}$$

The surface area is

$$\begin{aligned}
\text{Area} &= 2\pi\int_0^a \sqrt{a}\sqrt{x}\sqrt{1+\frac{1}{4x}}\,\mathrm{d}x \\
&= 2\pi\sqrt{a}\int_0^a \sqrt{x+\frac{a}{4}}\,\mathrm{d}x \\
&= \frac{\pi}{6}\left(5^{3/2}-1\right)a^2.
\end{aligned}$$