MATH 202 **Name:** ____________________

Calculus Practice I, Fall 2023

Here is an opportunity for you to maintain your calculus skills over the summer. If you complete these problems, digitize your work, and submit your work to Canvas, I will send you my solutions. If you need some help with these questions, email me with your questions (willisb@unk.edu)

Completing this work is optional, and it does not enter into your class grade in any way–this work is not a bonus, extra credit, or anything like that.

1. Find an equation of the tangent line to the curve $y = \sqrt{x^2+1}$ at the point $(x = 1, y = \sqrt{2})$.

Solution:

To find an equation of a line we need to know (a) its slope and (b) a point on the line. We're given a point on the line, so our main task is to find the slope of the tangent line. To do that we need to first find a formula for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and second we need to evaluate $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$. The chain rule tell us that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times 2x(x^2+1)^{-1/2} \tag{1}$$

And pasting in $x \to 1$, we have

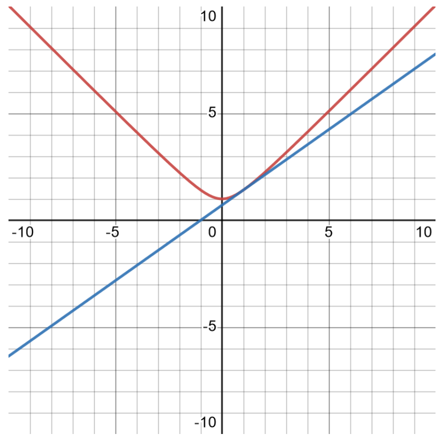
$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = \left.\frac{1}{2} \times 2x(x^2+1)^{-1/2}\right|_{x=1} = \frac{1}{\sqrt{2}}. \tag{2}$$

An equation of the given tangent line is

$$y - \sqrt{2} = \frac{1}{\sqrt{2}}(x-1) \tag{3}$$

Traditionally, we would simplify $\frac{1}{\sqrt{2}}$ to $\frac{\sqrt{2}}{2}$. But in this context, I don't see an advantage to doing this. So let us LIB (let it be).

As a check to our work, let's ask Desmos to draw graphs of both $y = \sqrt{1+x^2}$ and $y - \sqrt{2} = \frac{1}{\sqrt{2}}(x-1)$. Does the line and the curve appear to be tangent? Sure.


10
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2. Find each antiderivative.

   (a) $\int x^2 - x - 2\,\mathrm{d}x$

Solution:

$$\int x^2 - x - 2\,\mathrm{d}x = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x. \tag{4}$$

Some teachers will insist on the $+c$. I say let's just remember that all antiderivatives are undetermined up to an additive constant and for forget the silly $+c$ rule.

   (b) $\int (x-1)(x+2)\,\mathrm{d}x$

Solution:

   (c) $\int \frac{1+x^2}{x^2}\,\mathrm{d}x$

Solution: Expanding the integrand shows that this problem is identical to the previous.

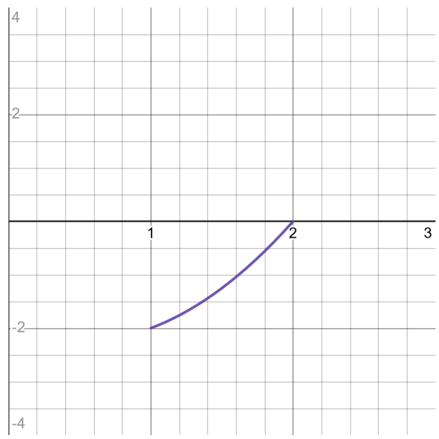
3. Find each definite integral.

(a) $\int_1^2 x^2 - x - 2\,\mathrm{d}x$

Solution: In the previous question, we found the antiderivative; so all we need to do is

$$\int_1^2 x^2 - x - 2\,\mathrm{d}x = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\Big|_{x=1}^{x=2} = -\frac{7}{6}. \tag{5}$$

Graphically, we can tell that this definite integral is negative. Why? The graph of the integrand is entirely below the x-axis, that's why. Here is a graph of the integrand



Actually, the graph of the integrand on the interval $[1,2]$ is pretty well approximated by a line segment joining $(x = 1, y = -2)$ and $(x = 2, y = 0)$. Doing so, we see that $\int_1^2 x^2 - x - 2\,\mathrm{d}x$ is pretty close to the negative of the area of a triangle with base one and height two.

(b) $\int_1^2 (x-1)(x+2)\,\mathrm{d}x$

Solution: Expanding the integrand, we see that we've done this problem before.

(c) $\int_1^4 \frac{1+x^2}{x^2}\,\mathrm{d}x$

4. For each function F, find the solution set of $F'(x) = 0$.

(a) $F(x) = x^2 + x + 3$

Solution:

(b) $F(x) = (x-3)(x^2+3)$

Solution:

(c) $F(x) = 2x + \frac{x}{x-2}$

Solution:

(d) $F(x) = \cos(x)\sin(x)$

Solution:

5. Find the value of each limit.

(a) $\lim_{x\to 4} \frac{\cos(x)+1}{x-3}$

Solution: The function is continuous at 4, so we can use DS (direct substitution); we have

$$\lim_{x\to 4} \frac{\cos(x)+1}{x-3} = \cos(4)+1. \tag{6}$$

If somebody wants a decimal approximation to $\cos(4)+1$ to any number of digits, they can do that.

(b) $\lim_{x\to 4} \frac{x^2-2x-3}{x-3}$

Solution: Again, the function is continuous at 4, so let's use DS; we have

$$\lim_{x\to 4} \frac{x^2-2x-3}{x-3} = 4^2 - 2\times 4 - 3 = 5. \tag{7}$$

(c) $\lim_{x\to \infty} \frac{5x^2+x+1}{7x^2+107}$