MATH 202 **Name:** _____

Calculus Practice IV, Fall 2023

Here is an opportunity for you to maintain your calculus skills over the summer. If you complete these problems, digitize your work, and submit your work to Canvas, I will send you my solutions. If you need some help with these questions, email me with your questions (willisb@unk.edu)

Completing this work is optional, and it does not enter into your class grade in any way–this work is not a bonus, extra credit, or anything like that.

1. The graph in Figure 1 shows the graph of a wild and crazy function (the red curve) whose domain is $[-4,4]$ that we'll unimaginatively call F. Define a function G by

$$G(x) = \int_{-4}^{x} F(s)\,\mathrm{d}s.$$

As best you can, draw a graph of G.

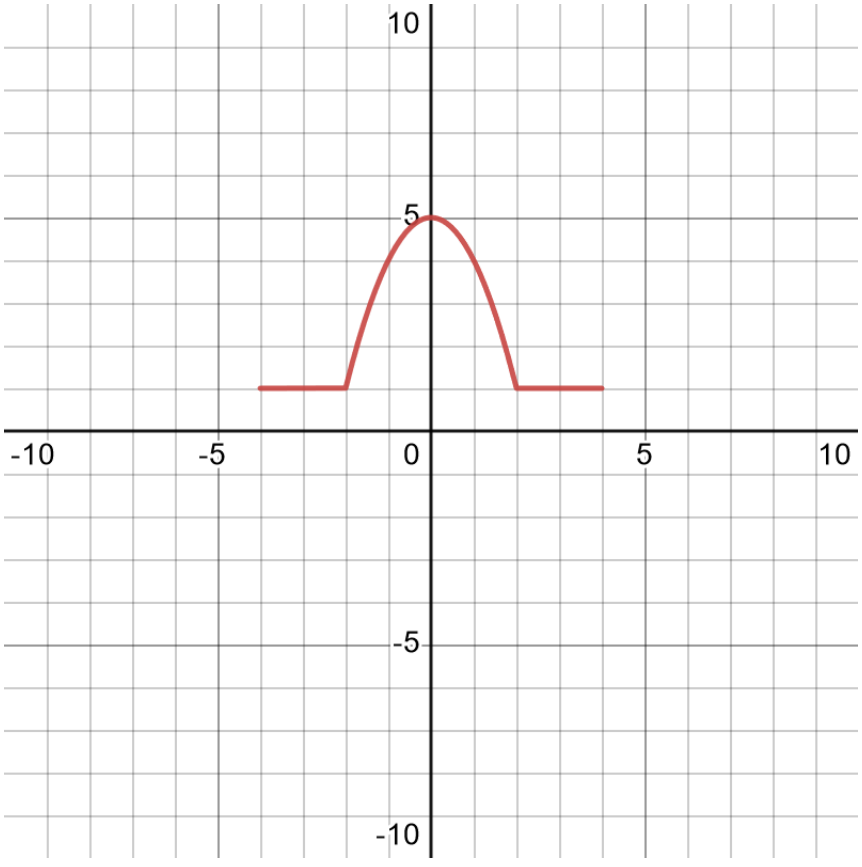


Figure 1: Graph of some wild and crazy function.

Solution:

To start, let's review the properties of a function G that is defined by

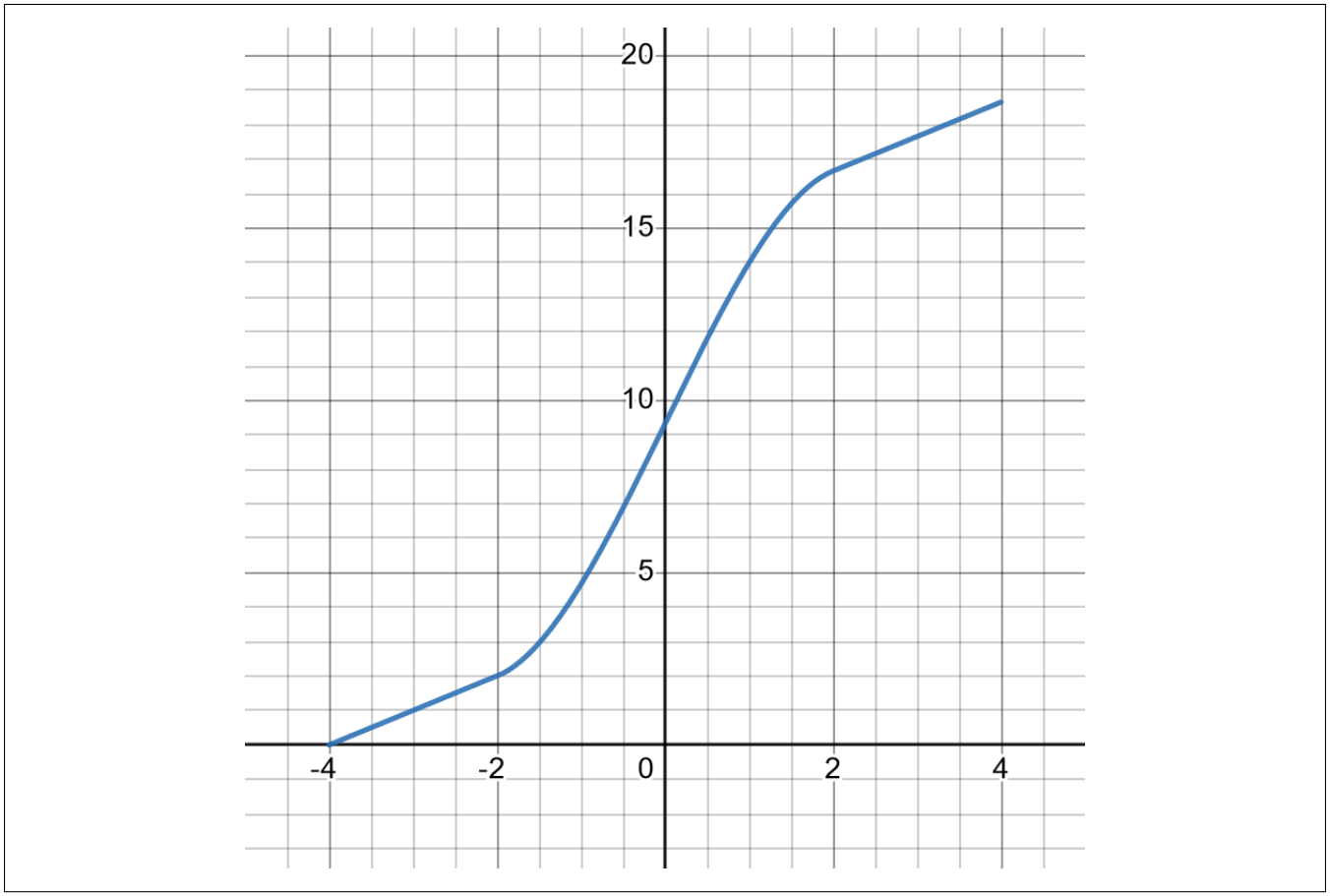
$$G(x) = \int_a^x F(s)\,\mathrm{d}s,$$

where F is integrable on an interval $[a,b]$. We have:

- $G(a) = 0$
- G is continuous on $[a,b]$.
- G is differentiable everywhere that F is continuous.
- Wherever F is continuous, we have $G'(x) = F(x)$.

Specifically for our function F given graphically in Figure 1, we conclude that (b) $G(-4) = 0$, (b) G is continuous on $[-4,4]$ (c)$G'(x) = F(x)$ on $[-4,4]$. Further, since $F(x) > 0$, we see that $G'(x) > 0$. This means that G is an increasing function. The steepest part of the graph of G is at 0, where $G'(0) = 5$. Finally, the derivative of G is constant on the interval $[-4,-1]$, so G is a linear function on this interval. The same is true on the interval $[1,4]$.

Putting all this together, a graph of G looks something like


20
15
10
5
-4
-2
0
2
4