MATH 202, Fall 2023 **Name:** ____________________

In class work 25 **Row and Seat**:____________________

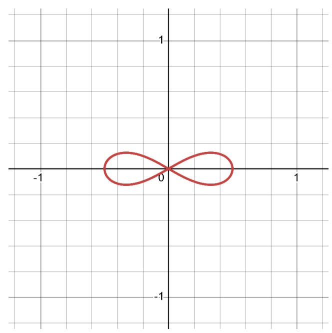
In class work **25** has questions **1** through **1** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 30 November at 13:20*.

"As we express our gratitude, we must never forget that the highest appreciation is not to utter words, but to live by them." JOHN F. KENNEDY

1. In polar coordinates, an equation of a curve $\mathscr{C}$ is $r = \sqrt{\frac{1}{4} - \sin(\theta)^2}$.

$\boxed{2}$ (a) Use Desmos to draw a graph of this polar equation. As best you can, reproduce the graph here.

Solution: The curve is a Lemniscate of Booth. It looks like the infinity symbol. Its graph is shown below.



Notice that the natural domain of $r = \sqrt{\frac{1}{4} - \sin(\theta)^2}$ is $\theta \in [-\frac{\pi}{6}, -\frac{\pi}{6}] \cup [\frac{5\pi}{6}, \frac{7\pi}{6}]$; outside this set, the value of r is not real. Desmos recognizes this and skips over the territory where r isn't real.

$\boxed{2}$ (b) Find all solutions to $0 = \sqrt{\frac{1}{4} - \sin(\theta)^2}$ with $\theta \in [0, 2\pi]$. These solutions give all the points on the curve that intersect the origin. To find *all* solutions to this equation, use the *source of all knowledge* (**SOAK**), that is, the unit circle.

Solution:

$$
\begin{aligned}
\left[0 = \sqrt{\frac{1}{4} - \sin(\theta)^2}\right] &= \left[0 = \frac{1}{4} - \sin(\theta)^2\right], && \text{(square root fact)}\\
&= \left[0 = \left(\frac{1}{2} - \sin(\theta)\right)\left(\frac{1}{2} + \sin(\theta)\right)\right], && \text{(factor)}\\
&= \left[-\frac{1}{2} = \sin(\theta) \vee -\frac{1}{2} = \sin(\theta)\right], && \text{(factor and solve)}\\
&= \left[\theta = \frac{11\pi}{6}, \theta = \frac{\pi}{6}, \theta = \frac{5\pi}{6}, \theta = \frac{7\pi}{6}\right]. && \text{(SOAK)}
\end{aligned}
$$

$\boxed{2}$ (c) For each intersection of $\mathscr{C}$ with the origin, find the slope of the tangent line. Using Desmos, verify that you have found the correct tangent lines. **Note:** Desmos refuses[1] to graph a polar curve of the form $\theta = f(r)$. And it particular, it will not graph the polar curve $\theta = \frac{\pi}{4}$, for example. To workaround this, you'll need to find the cartesian equation of the tangent lines.

Solution: For the polar curve $r = f(\theta)$ and assuming $f(\theta_o) =$, we have

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{\theta=\theta_o} = \tan(\theta_o). \tag{1}$$

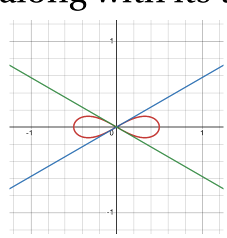
So the tangents to the curve at the origin are

$$y = \tan\left(\frac{\pi}{6}\right)x, \quad y = \tan\left(\frac{5\pi}{6}\right)x.$$

Or simplifying these, we have

$$y = \frac{1}{\sqrt{3}}x, \quad y = -\frac{1}{\sqrt{3}}x.$$

Here is a picture of the curve along with its tangent lines at the origin.



[1] I think Desmos should hire some UNK CS graduates to fix this.

Optional For extra fun, find a cartesian equation of the curve $\mathscr{C}$. Show that for $x \in [-\frac{1}{2}, \frac{1}{2}]$, a cartesian equation of the curve is $y = \pm\frac{\sqrt{\sqrt{64x^2+9}-8x^2-3}}{2^{\frac{3}{2}}}$. And show that the other two solutions are not real. Finally, are there any values of x that allow the nested radical $\sqrt{\sqrt{64x^2+9}-8x^2-3}$ to denest?