

"The more I read, the more I acquire, the more certain I am that I know nothing."

VOLTAIRE

In class work 17 has questions 1 through 3 with a total of 6 points. Turn in your work at the end of class on paper. This assignment is due *Tuesday 26 October 13:20*.

1. Find the numerical value of $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$. Careful: This is an indeterminate form of the type 1^∞ . To start, I suggest that you use the technique $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} e^{x \ln(1 + \frac{1}{x})}$.

2. Use the *ratio* test to determine if the series $\sum_{k=0}^{\infty} \frac{\left(\frac{k}{3}\right)^k}{k!}$ converges or diverges.

- 1 3. Define a sequence s by $s_n = \sum_{k=1}^n \frac{(-1)^{k+1}}{\sqrt{k}}$. This is a convergent alternating series. Also define $s_\infty = \lim_{n \rightarrow \infty} s_n$
- 1 (a) Use Desmos to graph s on the interval $[1, 2, \dots, 150]$. Using Desmos, find the numeric values of s_{149} and s_{150} . As best you can, reproduce a cartoon of the graph of s .
- 1 (b) From the theory of convergent alternating series, we know that $s_{150} < s_\infty < s_{149}$. Looking at the graph of s , I would guess that s_∞ is pretty close to the arithmetic average of s_{150} and s_{149} ; that is $s_\infty \approx \frac{s_{150} + s_{149}}{2}$. Find the numeric value of $\frac{s_{150} + s_{149}}{2}$.
- 1 (c) Define a sequence w by $w_n = \frac{s_{n+1} + s_n}{2}$. With a bit of effort, we could prove that the sequence w is a convergent alternating sequence that converges to s_∞ . Use Desmos to graph the sequences s and w on the interval $[1, 2, \dots, 150]$. Which sequence would you say converges “faster”? As best you can, reproduce a cartoon graphs of s and w .