MATH 202, Fall 2023 **Name:** ____________________

In class work 11 **Row and Seat**: ____________________

"Singularity is almost invariably a clue." SHERLOCK HOLMES

In class work **11** has questions **1** through **2** with a total of **6** points.Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 3 October 13:20*.

1. These questions involve the region Q defined by

$$Q = \{(x, y) \mid 0 \le y \le \exp(-x), 0 \le x < \infty\}.$$

For each of the following, you will need to evaluate an improper integral of the form $\int_0^\infty F(x)\,\mathrm{d}x$. You will need to evaluate such integrals using $\lim_{a\to\infty} \int_0^a F(x)\,\mathrm{d}x$.

$\boxed{1}$ (a) Sketch the region Q. Make a pretty good guess at the location of the *centroid* of Q.

$\boxed{1}$ (b) Find the *area* of the region Q.

$\boxed{1}$ (c) Find the x coordinate of the *centroid* of the region Q.

$\boxed{1}$ (d) Find the y coordinate of the *centroid* of the region Q.

2. The floor function rounds a real number x down to the next integer that is less than or equal to x. For example, $\lfloor \pi \rfloor = 3$ and $\lfloor 3 \rfloor = 3$. The 1987 (Edition B) of *Larry's Obscure Table of Obscure but Useful Integrals*, lists the antiderivative (reprinted here with permission) $\int \lfloor x \rfloor \, dx = \frac{1}{2}(2x-1)\lfloor x \rfloor - \frac{1}{2}\lfloor x \rfloor^2$.

$\boxed{1}$ (a) Use Desmos to graph $\frac{1}{2}(2x-1)\lfloor x \rfloor - \frac{1}{2}\lfloor x \rfloor^2$. for $0 \leq x \leq 4$. Reproduce your graph here. Does the graph appear to be continuous? (When an antiderivative exists, it must be continuous.)

$\boxed{1}$ (b) Evaluate the definite integral $\int_0^{\sqrt{42}} 2x \lfloor x^2 + 1 \rfloor \, dx$.