MATH 202, Fall 2023 **Name:** ______________________

In class work 15 **Row and Seat**: ______________________

"The pencil is mightier than the pen."

ROBERT M. PIRSIG

In class work **15** has questions **1** through **9** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 19 October 13:20*.

1. Use *integration by parts* to find an antiderivative of each of the following:

   (a) $\int x\mathrm{e}^{-x}\,\mathrm{d}x$

   Solution:

   $$\int x\mathrm{e}^{-x}\,\mathrm{d}x = (\text{-}\,x\,\text{-}\,1)\,\mathrm{e}^{-x}$$

   (b) $\int x^2\mathrm{e}^{-x}\,\mathrm{d}x$

   Solution:

   $$\int x^2\mathrm{e}^{-x}\,\mathrm{d}x = \left(\text{-}\,x^2\,\text{-}\,2x\,\text{-}\,2\right)\%e^{-x}.$$

2. Define a region of the xy plane Q by $Q = \{(x, y)|0 \leq y \leq xe^{-x}$ and $0 \leq x \leq 5\}$. **Hint:** For both parts of this question, use an answer from Question 1.

   (a) Find Area(Q)

   Solution:

   $$\text{Area}(Q) = \int_0^5 xe^{-x}\,dx = 7e^{-5} - 2.$$

   (b) Find the x coordinate of the centroid of Q.

   Solution:

3. Find a formula for each antiderivative.

   (a) $\int \frac{x+9}{(x+4)(x+5)}\,\mathrm{d}x$ (Use partial fractions).

   Solution:

   (b) $\int \frac{x^3}{\sqrt{1-x^2}}\,\mathrm{d}x$. (Use the substitution $x = \sin(\theta)$, where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$.)

   Solution:

4. Find the limit of each sequence a whose formula is

(a) $a_n = \frac{(2n-1)(7n+1)}{n^2+1}$

Solution:

(b) $a_n = n \ln\left(1 + \frac{\sqrt{2}}{n}\right)$

Solution:

(c) $a_n = \sqrt{n^2 + 46n + 1} - n$

Solution:

5. Give an example of a sequence a such that $\lim_{k\to\infty} a_k = 0$ and $\lim_{n\to\infty} \sum_{k=1}^{n} a_k = \infty$.

Solution:

6. Give an example of a sequence a such that $\lim_{k\to\infty} a_k = 0$ and $\lim_{n\to\infty} \sum_{k=1}^{n} a_k$ is a real number.

Solution:

7. Show that the series $\sum_{k=1}^{\infty} \sqrt{k^2+46k+1} - k$ diverges. Justify your answer.

Solution:

8. Find the numerical value of the sum $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$.

Solution:

9. Find the numerical value for each improper integral.

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{81 + x^2}\,\mathrm{d}x.$

Solution:

(b) $\displaystyle\int_{0}^{\infty} \sin(x)\mathrm{e}^{-x}\,\mathrm{d}x.$

Solution: