MATH 202, Fall 2023 **Name:**______________________________

Practice Exam **Row and Seat:**__________________________

Warning: For the most part, I've only given answers, not solutions–this is BOB (back-of-book) fashion. Of course, for your exam you will need to **show your work**.

1. Define a region Q of the xy-plane by $Q = \{(x, y) | 0 \le y \le x\sin(x), 0 \le x \le \pi\}$
   (a) Find area(Q).
   (b) Find the x coordinate of the centroid of Q.
2. Find the value of each indefinite or definite integral.
   (a) $\displaystyle\int xe^{x^2}\,\mathrm{d}x =$

   (b) $\displaystyle\int_0^1 \frac{x}{\left(1+x^2\right)^{3/2}}\,\mathrm{d}x =$

   (c) $\displaystyle\int x\sqrt{1-x^2}\,\mathrm{d}x =$

(d) $\displaystyle\int \tan^{-1}(x)\,\mathrm{d}x =$

(e) $\displaystyle\int x\ln(x)\,\mathrm{d}x =$

(f) $\displaystyle\int_0^1 xe^{-x}\,\mathrm{d}x =$

(g) $\displaystyle\int \frac{1}{(x+5)(x+9)}\,\mathrm{d}x =$

(h) $\displaystyle\int \cos^2(x)\,\mathrm{d}x =$

(i) $\displaystyle\int \cos^3(x)\sin(x)\,\mathrm{d}x =$

3. Find the numerical value of each improper integral.

(a) $\int_0^\infty x\mathrm{e}^{-x^2}\,\mathrm{d}x$

(b) $\int_0^\infty x\mathrm{e}^{-x}\,\mathrm{d}x$

(c) $\int_{-\infty}^\infty \frac{1}{x^2+9}\,\mathrm{d}x$

(d) $\int_0^1 \frac{1}{x^{9/10}}\,\mathrm{d}x$

$\boxed{2}$ 4. When Morwenna graduates from UNK and starts her first job, she expects to earn a starting annual salary of \$42,000. She plans to work for 42 years and she expects to earn a 3% raise each year. Thus, in her n$^{\text{th}}$ year of work, her salary is $42,000 \times 1.03^{n-1}$. During Morwenna's 42 years of labor, how much will she earn?

5. Given a formula for a sequence b, find its limit. Show all of your work.

$\boxed{2}$ (a) $b_n = \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k.$

$\boxed{2}$ (b) $b_n = \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$.

6. Show that the sequence whose formula is $a_k = \sqrt{k^2 + 3k + 1} - k$ converges. Show all of your work.

$\boxed{2}$ 7. Determine if the sequence whose formula is $b_k = k \ln\left(1 + \frac{8}{k}\right)$ converges. If it does, find its limit. As always, show your work.

8. A sequence c is defined recursively by

$$c_n = \begin{cases} 2 & n = 0 \\ 5 & n = 1 \\ 5c_{n-1} - 6c_{n-2} & n = 2, 3, 4, \ldots \end{cases}$$

$\boxed{2}$ (a) Find the numeric values of c_2, c_3, and c_4.

$\boxed{2}$ 9. Find the *numeric value* of the integral $\int_0^\infty \frac{x}{1+x^4}\,\mathrm{d}x$. **Hint:** To find an antiderivative of $\int \frac{x}{1+x^4}\,\mathrm{d}x$, use the substitution $z = x^2$.

$\boxed{1}$ 10. Show that $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ converges. To do this, use a comparison test with $\frac{\alpha}{1+x^2}$, where α is a number that you cleverly choose.

$\boxed{1}$ 11. Show that $\int_1^\infty \frac{107+\mathrm{e}^{-x}}{1+x^2}\,\mathrm{d}x$ converges. To do this, use a limit comparison test.

12. Use the integral test to show that the series $\sum_{k=0}^{\infty} \frac{1}{1+k^2}$ converges.