MATH 202, Fall 2023 **Name:** ______________________

In class work 20 **Row and Seat**: ______________________

In class work **20** has questions **1** through **3** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 7 November 13:20*.

"There's no problem so awful that you can't add some guilt to it and make it even worse."
CALVIN (BILL WATTERSON)

$\boxed{2}$ 1. Find the Taylor polynomial of order four centered at zero for the cosine function; that is find the polynomial $P_4(x) = \sum_{k=0}^{4} \frac{\cos^{(k)}(0)}{k!} x^k$. You'll need to find the numerical values of $\cos^{(0)}(0), \cos^{(1)}(0), \cos^{(2)}(0), \cos^{(3)}(0)$, and $\cos^{(4)}(0)$.

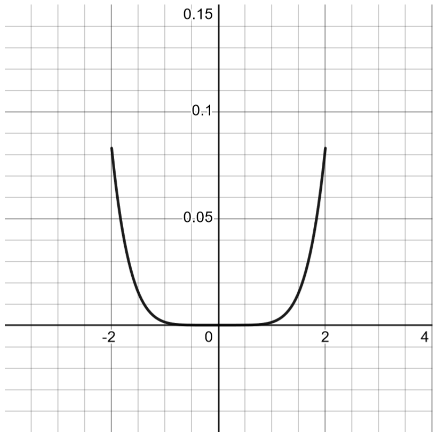
Solution: Let's make an nicely organized table of derivatives and their values at the center; here it is

n	$\cos^{(n)}(x)$	$\frac{\cos^{(n)}(0)}{n!}$
0	$\cos(x)$	1
1	$-\sin(x)$	0
2	$-\cos(x)$	$-\frac{1}{2}$
3	$\sin(x)$	0
4	$\cos(x)$	$\frac{1}{24}$

So $P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$.

$\boxed{2}$ 2. Use Desmos to graph $y = \cos(x)$ and $y = P_4(x)$, the Taylor polynomial of order four centered at zero for the cosine function. Reproduce the graph here. Use the graph to estimate maximum of $\max_{-2 \le x \le 2} |\cos(x) - P_4(x)|$.

Solution:



It appears the the maximum difference is about 0.08 (and it occurs at both endpoints).

$\boxed{2}$ 3. Find the Taylor polynomial of order four centered at zero for the *square* of the cosine function. You could *suffer* through the calculation by finding the first four derivatives of $\cos(x)^2$ and evaluate them at zero. Or you could use the fact we learned in class on Monday. Specifically, for $k \in \mathbf{Z}_{\geq 0}$ and infinitely differentiable functions F and G, define

$$a_k = \frac{F^{(k)}(0)}{k!}, \quad b_k = \frac{G^{(k)}(0)}{k!}.$$

Then for all $n \in \mathbf{Z}_{\geq 0}$, we have

$$\frac{(FG)^{(n)}(0)}{n!} = \sum_{k=0}^{n} a_k b_{n-k}.$$

You will want to use these formulae with $F = \cos$ and $G = \cos$. You can find the numbers a_o, a_1, a_2, a_3, a_4 and b_o, b_1, b_2, b_3, b_4 from the first question.

Solution: The easy way to expand $(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4)^2$ and to ignore all powers of x that are degree five or higher. Thus the 4th order TP for $\cos^2$ centered at zero is

$$\begin{aligned}(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4)^2 &= 1 + (-\frac{1}{2} - \frac{1}{2})x^2 + (\frac{1}{24} + \frac{1}{4} + \frac{1}{24})x^4 + \text{hot}, \\ &= 1 - x^2 + \frac{1}{3}x^4 + \text{hot}.\end{aligned}$$

In the physics and engineering communities, hot is a context dependent phrase meaning *higher order terms.* So our TP is $1 - x^2 + \frac{1}{3}x^4$