

## NAMED SETS

empty set	$\emptyset$
real numbers	$\mathbf{R}$
ordered pairs	$\mathbf{R}^2$
integers	$\mathbf{Z}$
positive integers	$\mathbf{Z}_{>0}$
positive real numbers	$\mathbf{R}_{>0}$

## EXPONENTS

For  $a, b \in \mathbf{R}_{>0}$ ,  $x \in \mathbf{R}$ , and  $m, n \in \mathbf{R}$ ,

$$\begin{aligned} a^0 &= 1, & 0^a &= 0 \\ 1^x &= 1, & a^n a^m &= a^{n+m} \\ a^n / a^m &= a^{n-m}, & (a^n)^m &= a^{n \cdot m} \\ a^{-m} &= 1/a^m, & (a/b)^m &= a^m / b^m \\ \sqrt{x^2} &= |x| \end{aligned}$$

## TRIGONOMETRIC IDENTITIES

We define  $\text{dom}(\text{arccot}) = (0, \pi)$ .

$$\begin{aligned} (\cos(x))^2 + (\sin(x))^2 &= 1 \\ 2(\cos(x))^2 &= 1 + \cos(2x) \\ 2(\sin(x))^2 &= 1 - \cos(2x) \\ (\cos(x))^2 - (\sin(x))^2 &= \cos(2x) \\ \sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ \text{arccot}(x) &= \pi/2 - \arctan(x) \\ \text{arccsc}(x) &= \arcsin(1/x) \\ \text{arcsec}(x) &= \arccos(1/x) \\ \arcsin(x) + \arccos(x) &= \pi/2 \\ \text{arcsec}(x) + \text{arccsc}(x) &= \pi/2 \end{aligned}$$

## HYPERBOLIC FUNCTIONS

$$\begin{aligned} 2 \cosh(x) &= \exp(x) + \exp(-x) \\ 2 \sinh(x) &= \exp(x) - \exp(-x) \\ \tanh(x) &= \sinh(x)/\cosh(x) \\ \cosh(x)^2 - \sinh(x)^2 &= 1 \end{aligned}$$

## LOGARITHMS

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

## DERIVATIVES

### Specific cases

$F(x)$	$F'(x)$
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec(x)^2$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc(x)^2$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(x^2+1)$
$\cosh(x)$	$\sinh(x)$
$\sinh(x)$	$\cosh(x)$
$\tanh(x)$	$1/\cosh(x)^2$
$\text{arccosh}(x)$	$1/\sqrt{x^2-1}$
$\text{arcsinh}(x)$	$1/\sqrt{1+x^2}$
$\text{arctanh}(x)$	$1/(1-x^2)$
$\exp(x)$	$\exp(x)$
$\ln(x)$	$1/x$

### General Cases

$F(x)$	$F'(x)$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$1/g(x)$	$-g'(x)/g(x)^2$
$f(x)/g(x)$	$(g(x)f'(x) - f(x)g'(x))/g(x)^2$
$f(g(x))$	$g'(x)f'(g(x))$
$f^{-1}(x)$	$1/f'(f^{-1}(x))$

## ANTIDERIVATIVES

$$\begin{aligned} \int a \, dx &= ax \\ \int x^a \, dx &= \frac{1}{a+1} x^{a+1}, \quad \text{if } a \neq -1 \\ \int \frac{1}{x} \, dx &= \ln|x| \\ \int \cos(x) \, dx &= \sin(x) \\ \int \sin(x) \, dx &= -\cos(x) \\ \int \tan(x) \, dx &= \ln|\sec(x)| \\ \int \sec(x) \, dx &= \ln|\tan(x) + \sec(x)| \\ \int \csc(x) \, dx &= -\ln|\csc(x) + \cot(x)| \\ \int \cot(x) \, dx &= \ln|\sin(x)| \\ \int |x| \, dx &= x|x|/2 \end{aligned}$$

## SUMS

For  $n \in \mathbf{Z}_{>0}$

$$\begin{aligned} \sum_{k=0}^{n-1} 1 &= n, & \sum_{k=0}^{n-1} k &= (n-1)n/2 \\ \sum_{k=0}^{n-1} k^2 &= (n-1)n(2n-1)/6, \\ \sum_{k=0}^{n-1} x^k &= \frac{1-x^n}{1-x}, \quad x \neq 1 \\ \sum_{k=0}^{\infty} z^k &= \begin{cases} \frac{1}{1-z} & z \in (-1, 1) \\ \infty & z \in [1, \infty] \end{cases} \end{aligned}$$

When  $z \in (-\infty, -1]$ , the series  $\sum_{k=0}^{\infty} z^k$  diverges.

## APPLICATIONS

Arc length of curve  $y = f(x)$  with  $a \leq x \leq b$

$$= \int_a^b \sqrt{1 + f'(x)^2} \, dx$$

For the region  $Q$  of the  $xy$  plane given by

$$Q = \{(x, y) \mid f(x) \leq y \leq g(x) \wedge a \leq x \leq b\},$$

we have

$$\text{Area}(Q) = \int_a^b g(x) - f(x) \, dx$$

Assuming  $0 \leq f(x)$  and rotating about the  $x$ -axis

$$\text{Vol}(Q) = \pi \int_a^b g(x)^2 - f(x)^2 \, dx$$

Assuming  $0 \leq a < b$  and rotating about the  $y$ -axis

$$\text{Vol}(Q) = 2\pi \int_a^b x(g(x) - f(x)) \, dx$$

Centroid

$$\text{Area}(Q) \times \bar{x} = \int_a^b x(g(x) - f(x)) \, dx$$

$$\text{Area}(Q) \times \bar{y} = \frac{1}{2} \int_a^b (g(x)^2 - f(x)^2) \, dx$$

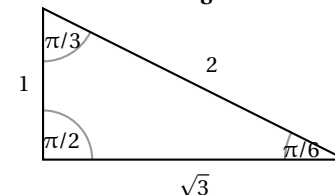
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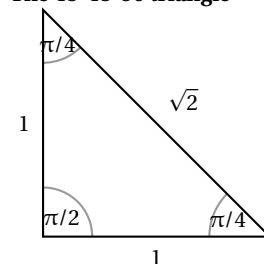
interchange  $x$  and  $y$  in *all* the previous formulas.

## FAMOUS TRIANGLES

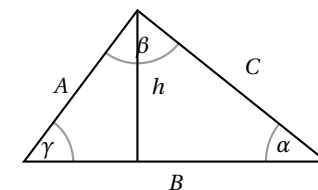
### The 30-60-90 triangle



### The 45-45-90 triangle



## LAWS OF COSINE & SINE



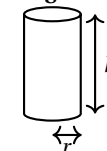
**Law of cosine:**  $C^2 = A^2 + B^2 - 2AB \cos(\gamma)$

**Law of sines:**  $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B} = \frac{\sin(\gamma)}{C}$

**Area:**  $\text{Area} = 1/2 h B = 1/2 AB \sin(\gamma)$

## VOLUMES

### Right Circular Cylinder



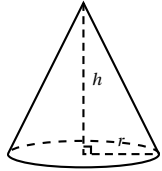
**Volume:**  $V = \pi r^2 h$

**Area:** (not including circular ends)  $A = 2\pi r h$

### Sphere

**Area:**  $A = 4\pi r^2$  **Volume:**  $V = \frac{4\pi}{3} r^3$

## Cone



**Volume:**  $V = \pi r^2 h/3$

### Area

$A = \pi r \sqrt{r^2 + h^2}$  (not including circular base).

## P-SERIES, DIVERGENCE TEST, RATIO TEST, COMPARISON, & AST

The series  $\sum_{k=1}^{\infty} \frac{1}{k^p}$  converges when  $p \in (1, \infty)$ ; otherwise it diverges.

If  $\lim_{k \rightarrow \infty} a_k \neq 0$ , the series  $\sum a_k$  diverges.

Let  $a$  be a sequence with  $0 \notin \text{range}(a)$ . Define  $L = \lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right|$ .

- $L \in [0, 1) \Rightarrow \sum |a_k|$  converges.
- $L \in (1, \infty] \Rightarrow \sum a_k$  diverges.

Let  $a$  and  $b$  be positive sequences. Define  $L = \lim_{k \rightarrow \infty} \frac{a_k}{b_k}$ .

- If  $L \in \mathbf{R}_{>0}$  and  $\sum a_k$  converges then  $\sum b_k$  converges.
- If  $L \in \mathbf{R}_{>0}$  and  $\sum a_k$  diverges then  $\sum b_k$  diverges.
- If  $L = 0$  and  $\sum b_k$  converges, then  $\sum a_k$  converges.
- If  $L = \infty$  and  $\sum b_k$  diverges, then  $\sum a_k$  diverges.

Let  $a$  be a positive and eventually decreasing sequence. Then  $\sum (-1)^k a_k$  converges if and only if  $\lim_{k \rightarrow \infty} a_k = 0$ .

## TAYLOR AND MACLAURIN SERIES

If a function  $F$  is infinitely differentiable at  $a$ , its Taylor series centered at  $a$  is

$$\sum_{k=0}^{\infty} \frac{F^{(k)}(a)}{k!} (x-a)^k.$$

When  $a$  is zero, the Taylor series is also known as the MacLaurin Series.

## POLAR TO CARTESIAN

$$x = r \cos(\theta) \quad y = r \sin(\theta)$$

For  $r > 0$  and  $0 \leq \theta < 2\pi$

$$r = \sqrt{x^2 + y^2}, \quad \theta = \begin{cases} 2\pi - \arccos(x/r) & \text{if } y < 0 \\ \arccos(x/r) & \text{if } y \geq 0 \end{cases}$$

## INTEGRATE POWERS OF TRIG

Let  $m, n \in \mathbf{Z}_{\geq 0}$ . Then

- $\int \cos(x)^{2m} \sin(x)^{2n} dx = \int \left( \frac{1 + \cos(2x)}{2} \right)^m \left( \frac{1 - \cos(2x)}{2} \right)^n dx$
- $\int \cos(x)^{2m+1} \sin(x)^n dx = \int (1 - z^2)^m z^n dz$ , where  $z = \sin(x)$
- $\int \cos(x)^m \sin(x)^{2n+1} dx = - \int z^m (1 - z^2)^n dz$ , where  $z = \cos(x)$

- $\int \sec(x)^n dx = \frac{1}{n-1} \sec(x)^{n-2} \tan(x) + \frac{n-2}{n-1} \int \sec(x)^{n-2} dx$ , provided  $n \neq 1$ .
- $\int \tan(x)^{2m+1} \sec(x)^n dx = \int (z^2 - 1)^m z^{n-1} dz$ , where  $z = \sec(x)$
- $\int \tan(x)^{2m} \sec(x)^n dx = \int (\sec(x)^2 - 1)^m \sec(x)^n dx$ .

## TRIG SUBSTITUTIONS

- $\int F\left(x, (1-x^2)^{n/2}\right) dx$ , use  $x = \sin(\vartheta)$ , where  $\vartheta \in [-\pi/2, \pi/2]$ , then integrate  $\int F(\sin(\vartheta), \cos(\vartheta)^n) \cos(\vartheta) d\vartheta$
- $\int F\left(x, (1+x^2)^{n/2}\right) dx$ , use  $x = \sinh(\vartheta)$ , where  $\vartheta \in \mathbf{R}$ , then integrate  $\int F(\sinh(\vartheta), \cosh(\vartheta)^n) \cosh(\vartheta) d\vartheta$
- $\int F\left(x, (x^2-1)^{n/2}\right) dx$ , use  $x = \sec(\vartheta)$ , then integrate  $\int F(\sec(\vartheta), \tan(\vartheta)^n) \sec(\vartheta) \tan(\vartheta) d\vartheta$

## UNIT CIRCLE

