MATH 202, Fall 2023 **Name:** _____
In class work 14 **Row and Seat**: _____

"You are never dedicated to something you have complete confidence in."

ROBERT M. PIRSIG

In class work **14** has questions **1** through **3** with a total of **8** points.Turn in your work at the end of class *on paper*. This assignment is due *Thursday 12 October 13:20*.

2 1. When Morwenna graduates from UNK and starts her first job, she expects to earn a starting annual salary of $42,000. She plans to work for 42 years and she expects to earn a 3% raise each year. Thus, in her n^{th} year of work, her salary is $42,000 \times 1.03^{n-1}$. During Morwenna's 42 years of labor, how much will she earn?

2. Given a formula for a sequence b, find its limit. Show all of your work.

2 (a) $b_n = \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k$.

2 (b) $b_n = \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$.

2 3. The Newton–Raphson method[1] is a way to find an approximate solution to an equation $F(x) = 0$. Specifically, the method starts with a guess for the solution, call it a_1, and then refines the guess with values $a_2, a_3, \ldots$. This sequence is called a *Newton sequence*. If all goes well, the sequence a converges to solution to $F(x) = 0$. Specifically for $F(x) = x^2 - 2$ and an initial guess of 1, the Newton sequence is defined recursively by

$$a_{n+1} = \begin{cases} 1 & n = 0 \\ a_n - \frac{a_n^2 - 2}{2a_n} & n > 0 \end{cases}.$$

Assuming that the sequence a converges to a positive number, find the numerical value of $\lim_{n\to\infty} a_n$. Use the fact that if $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{n+1} = L$.

Many of you will have the urge to "simplify" $a_n - \frac{a_n^2-2}{2a_n}$ to $\frac{a_n^2+2}{2a_n}$ or possibly to $\frac{a_n}{2} + \frac{1}{a_n}$. Doing so is an OK thing to do, but I suggest doing a bit more 'T' from GNAT[2] before you give into your urge to simplify.

[1] Raphson invented the method before Newton. If you didn't learn the Newton–Raphson method in Calculus I, I should tell the Office of Student Records to expunge your MATH 115 credit.

[2] GNAT = Graphical, Numerical, Algebraic, Think. I possibly invented the acronym, but Deborah Hughes Hallett invented, or at least popularized, the concept.