MATH 202, Fall 2023 **Name:** ______________________

In class work 5 **Row and Seat**: ______________________

In class work **5** has questions **1** through **2** with a total of **10** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 7 September 13:20*.

1. Find a formula for F' given

$\boxed{2}$ (a) $F(x) = \ln(x^2+1)$

> **Solution:** The chain rule when the outer function is the natural logarithm is
>
> $$\big(\ln(f(x))\big)' = \frac{f'(x)}{f(x)}.$$
>
> Using this, we have
>
> $$\begin{aligned} F'(x) &= \frac{(x^2+1)'}{x^2+1}, && \text{(chain rule)} \\ &= \frac{2x}{x^2+1}. && \text{(polynomial derivative)} \end{aligned}$$

$\boxed{2}$ (b) $F(x) = x\ln(x^2+1)$

> **Solution:** For our first step, we need to use the product rule:
>
> $$\begin{aligned} F'(x) &= (x)'\ln(x^2+1) + x\big(\ln(x^2+1)\big)', && \text{(product rule)} \\ &= \ln(x^2+1) + x\frac{2x}{x^2+1}, && \text{(polynomial derivative and part `a')} \\ &= \ln(x^2+1) + \frac{2x^2}{x^2+1}. && \text{(generally viewed as simplification)} \end{aligned}$$

$\boxed{2}$ (c) $F(x) = \frac{\ln(1+x)-\ln(1-x)}{2}$

> **Solution:** For our first step, let's not use the quotient rule; instead let's use

outativity

$$\begin{aligned}
F'(x) &= \frac{1}{2}\left(\ln(1+x) - \ln(1-x)\right)', && \text{(outative rule)}\\
&= \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x}\right), && \text{(chain rule)}\\
&= \frac{1}{1-x^2}. && \text{(generally viewed as simplification)}
\end{aligned}$$

2. Find the numerical values of the definite integrals

$\boxed{2}$ (a) $\int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x$

Solution: The integrand doesn't match a standard integrands, so let's try a substitution. Since we are substituting into a definite integral, there are **four** ingredients:

1. $z = 1 + x^2$
2. $\mathrm{d}z = 2x\mathrm{d}x$; alternatively $x\mathrm{d}x = \frac{1}{2}\mathrm{d}z$
3. $x = 0 \implies z = 1 + 0^2 = 1$
4. $x = 1 \implies z = 1 + 1^2 = 2$

With that, we have

$$\begin{aligned}
\int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x &= \int_1^2 \frac{1}{2}\frac{1}{z}\,\mathrm{d}z,\\
&= \frac{1}{2}\ln(z)\Big|_{z=1}^{z=2},\\
&= \frac{1}{2}\left(\ln(2) - \ln(1)\right),\\
&= \frac{1}{2}\ln(2).
\end{aligned}$$

Another answer is $\ln(\sqrt{2})$, but one logarithm and one square root is less simple than is one logarithm and one divide. It's OK, to use $\int \frac{1}{z}\,\mathrm{d}z = \ln(|z|)$, but I recognized that the definite integral is over a positive interval, so forget about the absolute value.

$\boxed{2}$ (b) $\int_{-4}^{-2} \frac{1}{x+10}\,\mathrm{d}x$

Solution: The integrand doesn't match a standard integrands, so let's try a substitution. Since we are substituting into a definite integral, there are **four** ingredients:

1. $z = x + 10$
2. $\mathrm{d}z = \mathrm{d}x$
3. $x = -4 \implies z = -4 + 10 = 6$
4. $x = -2 \implies z = -2 + 10 = 8$

So

$$\int_{-4}^{-2} \frac{1}{x+10}\,\mathrm{d}x = \int_{6}^{8} \frac{1}{z}\,\mathrm{d}z = \ln(8) - \ln(6) = \ln\left(\frac{4}{3}\right).$$

Since $\ln(8) - \ln(6)$ involves two logarithms and $\ln\left(\frac{4}{3}\right)$ involves only one, generally, $\ln\left(\frac{4}{3}\right)$ is a simplification of $\ln(8) - \ln(6)$