MATH 202, Fall 2023 **Name:** ______________________

In class work 12 **Row and Seat**: ______________________

> *"Study hard what interests you the most in the most undisciplined, irreverent and original manner possible."* RICHARD FEYNMAN

In class work **12** has questions **1** through **3** with a total of **4** points.Turn in your work at the end of class *on paper*. This assignment is due *Thursday 5 October 13:20*.

[2] 1. Find the *numeric value* of the integral $\int_0^\infty \frac{x}{1+x^4}\,\mathrm{d}x$. **Hint:** To find an antiderivative of $\int \frac{x}{1+x^4}\,\mathrm{d}x$, use the substitution $z = x^2$.

Solution: Let's begin by finding an antiderivative; once we found it, we'll use the FTC along with a limit to find the value of the improper integral. We have

$$\begin{aligned}\int \frac{x}{1+x^4}\,\mathrm{d}x &= \frac{1}{2}\int \frac{1}{1+(x^2)^2}\,\mathrm{d}x^2, && \left(\frac{1}{2}\mathrm{d}x^2 = 2x\mathrm{d}x\right)\\ &= \frac{1}{2}\int \frac{1}{1+z^2}\,\mathrm{d}z,\\ &= \frac{1}{2}\arctan(z),\\ &= \frac{1}{2}\arctan(x^2).\end{aligned}$$

Second, we take on the improper integral:

$$\begin{aligned}\int_0^\infty \frac{x}{1+x^4}\,\mathrm{d}x &= \lim_{a\to\infty}\int_0^a \frac{x}{1+x^4}\,\mathrm{d}x,\\ &= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(x^2)\Big|_0^a\right.,\\ &= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(a^2) - \frac{1}{2}\arctan(0)\right),\\ &= \lim_{a\to\infty}\left(\frac{1}{2}\arctan(a^2)\right),\\ &= \frac{\pi}{4}\end{aligned}$$

[1] 2. Show that $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ converges. To do this, use a comparison test with $\frac{\alpha}{1+x^2}$, where α is a number that you cleverly choose.

Solution: For all real numbers x, we have $27 \le 28 + \cos(x) \le 29$. Let's (cleverly) choose α to be 29. Then for all real numbers x, we have

$$0 \le \frac{28 + \cos(x)}{1 + x^2} \le \frac{29}{1 + x^2}. \tag{1}$$

But $\int_0^\infty \frac{29}{1+x^2}\,\mathrm{d}x$ converges, so $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ converges.

Be careful We only know that $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x$ is a real number, but the comparison test **doesn't** tell us its value. We'll it does tell us that

$$\int_0^\infty \frac{28 + \cos(x)}{1 + x^2}\,\mathrm{d}x \le \int_0^\infty \frac{29}{1 + x^2}\,\mathrm{d}x = \frac{29\pi}{2} \approx 45.553093477052.$$

Numerical integration gives us the approximation $\int_0^\infty \frac{28+\cos(x)}{1+x^2}\,\mathrm{d}x \approx 44.560$

$\boxed{1}$ 3. Show that $\int_1^\infty \frac{107+e^{-x}}{1+x^2}\,dx$ converges. To do this, use a limit comparison test.

Solution: We know that $\int_1^\infty \frac{1}{1+x^2}\,dx$ converges. And for all real $x \geq 1$ we have $\frac{107+e^{-x}}{1+x^2} > 0$ and $\frac{1}{1+x^2} \geq 0$. Finally, everything in sight is continuous; so look at

$$\lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{\frac{107+e^{-x}}{1+x^2}} = \lim_{x\to\infty} \frac{1}{107+e^{-x}},$$
$$= \frac{1}{107}.$$

So $\int_1^\infty \frac{107+e^{-x}}{1+x^2}\,dx$ converges.