Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

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1. Use power series to find the numerical value of each limit. You might like to use the facts

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots,$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots,$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \frac{7x^5}{256} - \frac{21x^6}{1024} + \cdots$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + \cdots$$

$\boxed{1}$ (a) $\displaystyle \lim_{x \to 0} \frac{\sin(x) - x + x^3/6}{x^5}$

$\boxed{1}$ (b) $\displaystyle \lim_{x \to 0} \frac{\sin(3x^2)}{1 - \cos(2x)}$

2. Use the *ratio* test to determine the radius of convergence of each power series.

1   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!} x^k$

1   (b) $\displaystyle\sum_{k=0}^{\infty} k! x^k$

1   3. Find the numerical value of $\displaystyle\sum_{k=3}^{\infty} \left(\frac{1}{10}\right)^k$.

4. The de Jonquiére function Li_4 can be defined by $Li_4(x) = \sum\limits_{k=1}^{\infty} \dfrac{x^k}{k^4}$ and $dom(Li_4) = (-1, 1)$

$\boxed{1}$ (a) Find the *numerical value* of $Li_4(0)$.

$\boxed{1}$ (b) Find the *numerical value* of $\text{Li}_4'(0)$.

$\boxed{1}$ (c) Find the *numerical value* of $\text{Li}_4''(0)$.

5. Determine convergence or divergence of each series. Fully justify your work quoting theorems from our class.

$\boxed{1}$ (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{8k+2}$

$\boxed{1}$ (b) $\displaystyle\sum_{k=0}^{\infty}(2k+1)(-1)^k$

$\boxed{1}$ (c) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k + 3^k}$

6. For all real numbers x, we have $\cos(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$. Define a wild and crazy function Q as $Q(x) = \int_0^x \cos\left(t^4\right) dt$.

$\boxed{1}$ (a) Find a power series centered at zero for Q.

1   (b) Find the radius of convergence for the power series you found in the previous question.

1   7. For all $x \in (-1, 1)$, we have $\sqrt{1+x} = \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} x^k$. Find the numerical value of

$$\sum_{k=0}^{42} \binom{\frac{1}{2}}{k}\binom{\frac{1}{2}}{42-k}.$$