MATH 202, Fall 2023 **Name:** ______________________

In class work 18 **Row and Seat**:______________________

“ *“Life is full of surprises, but never when you need one.”* CALVIN (BILL WATTERSON)

In class work **18** has questions **1** through **6** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 2 November 13:20.*

$\boxed{1}$ 1. Use the fact that for all real x the equation $\mathrm{e}^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$ is an identity to find a power series representation for e^{-x^2}.

$\boxed{1}$ 2. Define a function erf by the definite integral $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-t^2} \, \mathrm{d}t$. Find a power series representation for erf. Find the radius of convergence of this power series.

$\boxed{1}$ 3. With a bit of faith and luck, maybe a good approximation to erf is the sum of its first 101 terms. Use Desmos to graph this approximation for $-6 < x < 6$. Actually, the function erf is known to Desmos. Plot a graph of both the sum of the first 101 terms of the power series for erf along with the function erf. As best you can, reproduce the graphs here.

$\boxed{1}$ 4. Based on the Desmos graph of erf, what is your conjecture for the value of $\lim_{x\to\infty} \operatorname{erf}(x)$?

$\boxed{1}$ 5. Find the numerical value of $\lim\limits_{x\to\infty} \dfrac{2}{\sqrt{\pi}} \sum\limits_{k=0}^{100} \dfrac{(-1)^k}{(2k+1)(k!)} x^{2k+1}$. For "large" values of x, explain why $\frac{2}{\sqrt{\pi}} \sum_{k=0}^{\mathrm{N}} \frac{(-1)^k}{(2k+1)(k!)} x^{2k+1}$ is not a good approximation to erf no matter how large we make N.

Remember: No matter the degree of a polynomial, its limit toward infinity is determined by the term of the polynomial with the highest power. For example, provided $a_{1000000} \neq 0$, we have

$$\lim_{x\to\infty} \left(a_o + a_1 x + a_2 x^2 + \cdots + a_{1000000} x^{1000000}\right) = \lim_{x\to\infty} a_{1000000} x^{1000000}.$$

$\boxed{1}$ 6. Find a formula for the derivative of erf; that is find a formula for erf$'$. Make your formula as "simple" as you can.