MATH 202, Fall 2023 **Name:** ______________________

In class work 10 **Row and Seat:** ______________________

In class work **10** has questions **1** through **4** with a total of **8** points. Turn in your work at the end of class *on paper*.

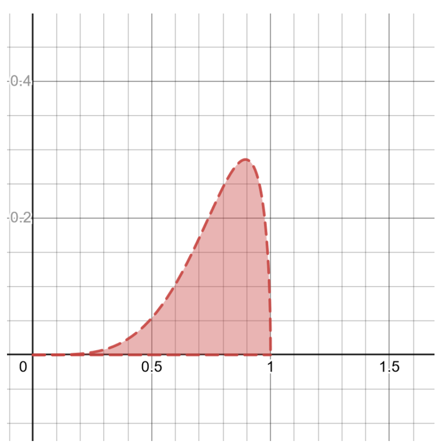
This assignment is due *Tuesday 26 September 13:20*.

Here are some results that you might like to use

$$
\begin{aligned}
\cos(x)^2 &= \frac{\cos(2x)}{2} + \frac{1}{2},\\
\cos(x)^4 &= \frac{\cos(4x)}{8} + \frac{\cos(2x)}{2} + \frac{3}{8},\\
\sin(x)^2 &= \frac{1}{2} - \frac{\cos(2x)}{2},\\
\cos(x)^2\sin(x)^2 &= \frac{1}{8} - \frac{\cos(4x)}{8},\\
\cos(x)^4\sin(x)^2 &= -\frac{\cos(6x)}{32} - \frac{\cos(4x)}{16} + \frac{\cos(2x)}{32} + \frac{1}{16},\\
\sin(x)^4 &= \frac{\cos(4x)}{8} - \frac{\cos(2x)}{2} + \frac{3}{8},\\
\cos(x)^2\sin(x)^4 &= \frac{\cos(6x)}{32} - \frac{\cos(4x)}{16} - \frac{\cos(2x)}{32} + \frac{1}{16},\\
\cos(x)^4\sin(x)^4 &= \frac{\cos(8x)}{128} - \frac{\cos(4x)}{32} + \frac{3}{128}.
\end{aligned}
$$

$\boxed{2}$ 1. Use Desmos to sketch the region Q defined as $Q = \{(x, y) \mid 0 \le y \le x^4\sqrt{1-x^2} \text{ and } 0 \le x \le 1\}$. Duplicate the graph here.

Solution:



$\boxed{2}$ 2. Find area(Q).

Solution: We need to find Area$(Q) = \int_0^1 x^4\sqrt{1-x^2}\,dx$. Let $x = \sin(\theta)$. Then when $x = 0$, we have $\theta = 0$; and

$x = 1$, implies $\theta = \pi/2$. So We have

$$\begin{aligned}
\int_0^1 x^4 \sqrt{1-x^2}\,\mathrm{d}x &= \int_0^{\pi/2} \sin(x)^4 \cos(x)^2 \,\mathrm{d}x, \\
&= \int_0^{\pi/2} \frac{\cos(6\theta)}{32} - \frac{\cos(4\theta)}{16} - \frac{\cos(2\theta)}{32} + \frac{1}{16}\,\mathrm{d}\theta \\
&= \frac{\sin(6\theta)}{192} - \frac{\sin(4\theta)}{64} - \frac{\sin(2\theta)}{64} + \frac{\theta}{16}\Big|_0^{\pi/2}, \\
&= \frac{\pi}{32}.
\end{aligned}$$

$\boxed{2}$ 3. Using your graph, make a pretty good guess for the x-coordinate to the centroid of Q.

Solution: I think the x coordinate of the center of the centroid is close to the relative maximum of the function; so I'm going to guess that $\overline{x} \approx \frac{9}{10}$

$\boxed{2}$ 4. Find the x-coordinate to the centroid of Q.

Solution: We need to evaluate $\int_0^1 x^5 \sqrt{1-x^2}\,\mathrm{d}x$. The MATH 115 way to do this is to substitute $z = 1 - x^2$. That works OK. The MATH 202 way is to substitute $x = \sin(\theta)$. Let's try the MATH 202 way:

$$\begin{aligned}
\int x^5 \sqrt{1-x^2}\,\mathrm{d}x &= \int \sin(\theta)^5 \cos(\theta)^2 \,\mathrm{d}\theta \\
&= \int \sin(\theta)(1-\cos(\theta)^2)^2 \cos(\theta)^2 \,\mathrm{d}\theta \\
&= -\int (1-z^2)^2 z^2 \,\mathrm{d}z, \\
&= -\int z^2 - 2z^4 + z^6 \,\mathrm{d}z, \\
&= -\frac{1}{3}z^3 + \frac{2}{5}z^5 - \frac{1}{7}z^7, \\
&= -\frac{1}{3}\cos(\theta)^3 + \frac{2}{5}\cos(\theta)^5 - \frac{1}{7}\cos(\theta)^7, \\
&= -\frac{1}{3}\cos(\arcsin(x))^3 + \frac{2}{5}\cos(\arcsin(x))^5 - \frac{1}{7}\cos(\arcsin(x))^7,
\end{aligned}$$

So for the definite integral, we have

$$\int_0^1 x^5 \sqrt{1-x^2}\,\mathrm{d}x = (0+0+0) - \left(-\frac{1}{3} + \frac{2}{5} - \frac{1}{7}\right) = \frac{8}{105} \tag{1}$$

So $\text{Area}(Q)\overline{x} = \frac{8}{105}$, so $\overline{x} = \frac{8}{105} \times \frac{32}{\pi} = \frac{256}{105\pi} \approx 0.7760698177433374$.