MATH 202, Fall 2023 **Name:** ______________________________

In class work 18 **Row and Seat**:______________________________

"The universe is a big place, perhaps the biggest." KURT VONNEGUT

In class work **18** has questions **1** through **2** with a total of **5** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 31 October 13:20*.

1. The zero order Bessel function J_0 is defined by its power series.[1] The power series is

$$\mathrm{J}_0(x) = \sum_{k=0}^{\infty} \frac{1}{(k!)^2}\left(-\frac{x^2}{4}\right)^k = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147456} - \frac{x^{10}}{14745600} + \cdots$$

The radius of convergence of this power series is infinity.

$\boxed{1}$ (a) Find the numerical values of $\mathrm{J}_0(0), \mathrm{J}_0'(0)$, and $\mathrm{J}_0''(0)$.

$\boxed{1}$ (b) When x has a modest magnitude, say $-10 < x < 10$, the summand $\frac{1}{(k!)^2}\left(-\frac{x^2}{4}\right)^k$ converges to zero very quickly. For example, when $x = 10$ and $k = 100$, the numeric value of the summand is about 7.1×10^{-177}. So it's reasonable to conjecture that $\mathrm{J}_0(x) \approx \sum_{k=0}^{100} \frac{1}{(k!)^2}\left(-\frac{x^2}{4}\right)^k$. Use Desmos to graph $y = \sum_{k=0}^{100} \frac{1}{(k!)^2}\left(-\frac{x^2}{4}\right)^k$ and reproduce the graph here.

[1] This function is named in honor of Friedrich Bessel, a German mathematician and scientist who lived from 1784–1846. Bessel functions arise in mechanical vibrations of a drum and electromagnetic waves in a coaxial cable.

2. A function $\mathscr{E}$ is defined by a power series; specifically $\mathscr{E}(x) = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$. The radius of convergence of this power series is infinity.

$\boxed{1}$ (a) Find the numerical value of $\mathscr{E}(0)$.

$\boxed{1}$ (b) Find a power series for $\mathscr{E}'$ and show that for all real numbers x, we have $\mathscr{E}'(x) = \mathscr{E}(x)$.

$\boxed{1}$ (c) The only solution to the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = y$ and $y|_{x=0} = 1$ is $y = \exp(x)$. What is a simple formula for the function $\mathscr{E}$?