MATH 202, Fall 2023 **Name:** _____
Exam 4 Practice **Row and Seat**: _____

"If people sat outside and looked at the stars each night, I'll bet they'd live a lot differently."
CALVIN (BILL WATERSON)

1. For the parametricaly defined curve $\begin{cases} x = 3t \\ y = 9t^2 + t \end{cases}$ $-\infty < t < \infty$, eliminate the parameter t. Sketch the resulting curve in the xy cartesian coordinate system.

   Solution: Solving $x = 3t$ for t yields $t = x/3$. Substituting this into $y = 9t^2 + t$ gives $y = x^2 + x/3$. For a graph, try Desmos.

2. For the parametricaly defined curve $\begin{cases} x = 3\cos(t) \\ y = 9\sin(t) \end{cases}$ $0 \le t \le 2\pi$, eliminate the parameter t. Sketch the resulting curve in the xy cartesian coordinate system.

   Solution: $(x/3)^2 + (y/9)^2 = 1$. For a graph, try Desmos.

3. For the curve $x^3 + y^3 - 3xy = 48$, find the numerical value of $\left.\dfrac{dy}{dx}\right|_{x=2,y=4}$

Solution: $\left.\frac{dy}{dx}\right|_{x=2,y=4} = 0.$

4. For the parametrically defined curve $\begin{cases} x = -\sqrt{1+t} \\ y = \sqrt{3t} \end{cases}$ $0 \le t < \infty$, find the numerical values of $\left.\frac{dy}{dx}\right|_{t=3}$ and $\left.\frac{d^2y}{dx^2}\right|_{t=3}$.

Solution: See your class notes.

5. Represent the arc length of the polar curve $r = a(1 - \sin(\theta))$ as a definite integral. Here a is a positive real number.

Solution:

$$\sqrt{2}\,|a| \int_0^{2\pi} \sqrt{1 - \sin(theta)}\,\mathrm{d}\theta.$$

6. Find all points on the polar curve $r = 3 - 2\sin(\theta)$ that have a horizontal tangent line.

Solution: $\theta = \frac{\pi}{2}, \theta = \frac{\pi}{2}, \theta = \arcsin\left(\frac{3}{4}\right), \theta = \pi - \arcsin\left(\frac{3}{4}\right)$

7. Find area of the region bounded by the polar curve $r = 3 - 2\sin(\theta)$

Solution: $\frac{11}{2}\pi$.

8. Find the area bounded by the polar curve $r = \cos(4\theta)$

Solution: $\frac{\pi}{2}$