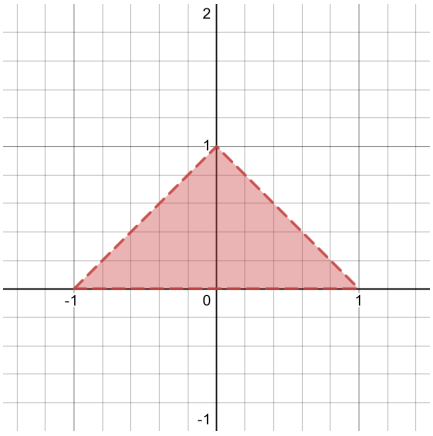
MATH 202, Fall 2023 **Name:** _____
In class work 4 **Row and Seat:** _____

In class work **4** has questions **1** through **1** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 5 September 13:20*.

1. Define a region Q of the xy plane by $Q = \{(x, y) \mid 0 \le y \le 1 - |x|, -1 \le x \le 1\}$.

[1] (a) Sketch the region Q in the xy plane.

Solution:



[2] (b) Make a conjecture about the location of the centroid of Q. Of course $\overline{x} \le 107$ and $\overline{y} \le 107$ is a conjecture, but try for something more specific.

Solution: The region q is symmetric with respect to the y axis. Surely this means that $\overline{x} = 0$.

The triangle is bottom heavy, so the y coordinate of the center of should be less than $1/2$.

[1] (c) Use junior high math (no calculus) to find area(Q).

Solution: area(Q) $= \frac{1}{2}$base $\times$ height $= 1$.

1 (d) Solve $M\overline{x} = \int_{-1}^{1} x(1-|x|)\,\mathrm{d}x$, where M is the area of Q, for $\overline{x}$. To evaluate the definite integral $\int_{-1}^{1} x(1-|x|)\,\mathrm{d}x$, use a fact about the integral of an odd function over a symmetric interval.

Solution: The integrand is odd and the interval is symmetric. So $\overline{x} = 0$.
To show that the integrand is odd, we need to show that $y = x(1-|x|)$ and $-y = (-x)(1-|-x|)$ are semantically the same. We have

$$\begin{aligned} \left[-y = (-x)(1-|-x|)\right] &\equiv \left[-y = (-x)(1-|x|)\right] && \text{(simplify absolute value)} \\ &\equiv \left[y = x(1-|x|)\right] && \text{(multiply by -1)} \end{aligned}$$

1 (e) Solve $M\overline{y} = \frac{1}{2}\int_{-1}^{1}(1-|x|)^2\,\mathrm{d}x$, where M is the area of Q, for $\overline{y}$. To do this, use the fun fact that $\displaystyle\int |x|\,\mathrm{d}x = \frac{1}{2}x|x|$.

Solution:

$$\begin{aligned} M\overline{y} &= \frac{1}{2}\int_{-1}^{1}(1-|x|)^2\,\mathrm{d}x, && \text{(given)} \\ &= \frac{1}{2}\int_{-1}^{1} 1-2|x|+|x|^2\,\mathrm{d}x, && \text{(expand)} \\ &= \frac{1}{2}\int_{-1}^{1} 1-2|x|+x^2\,\mathrm{d}x, && \text{(simplify } |x|^2\text{)} \\ &= \frac{1}{2}\left(x - x|x| + \frac{1}{3}x^3\right|_{x=-1}^{x=1}, && \text{(FTC)} \\ &= \frac{1}{2}\left(1-1+\frac{1}{3}\right) - \frac{1}{2}\left(-1+1-\frac{1}{3}\right), && \text{(paste)} \\ &= \frac{1}{3}. && \text{(arithmetic)} \end{aligned}$$

Since $M = 1$, we have $\overline{y} = \frac{1}{3}$.