MATH 202, Fall 2023 **Name:** ____________________

In class work 16 **Row and Seat:** ____________________

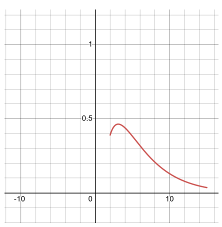
"The place to improve the world is first in one's own heart and head and hands, and then work outward from there." ROBERT M. PIRSIG

In class work **16** has questions **1** through **3** with a total of **7** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 24 October 13:20.*

1. Define a function F by $F(x) = \frac{\ln(x)}{\left(\frac{4}{3}\right)^x}$.

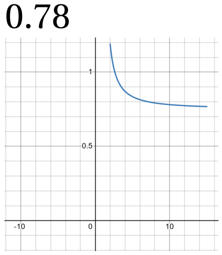
1 (a) Use Desmos to graph $y = F(x)$ for $2 \le x \le 15$. Reproduce the graph here. Based on the graph, what is your guess for the numeric value of $\lim_{x\to\infty} \frac{\ln(x)}{\left(\frac{4}{3}\right)^x}$?

Solution: The graph indicates that $\lim_{x\to\infty} \frac{\ln(x)}{\left(\frac{4}{3}\right)^x} = 0$.



1 (b) Use Desmos to graph $y = \frac{F(x+1)}{F(x)}$ for $2 \le x \le 15$. Reproduce the graph here. Based on the graph, what is your guess for the numeric value of $\lim_{x\to\infty} \frac{F(x+1)}{F(x)}$?

Solution: We have $\frac{F(x+1)}{F(x)} = \frac{3}{4}\frac{\ln(x+1)}{\ln(x)}$. The graph indicates that $\lim_{x\to\infty} \frac{F(x+1)}{F(x)} \approx 0.78$



1 (c) Use the l'Hôpital rule to find the numeric value of $\lim_{x\to\infty} \frac{F(x+1)}{F(x)}$.

Solution:

$$\begin{aligned}
\lim_{x\to\infty} \frac{3}{4}\frac{\ln(x+1)}{\ln(x)} &= \lim_{x\to\infty} \frac{3}{4}\frac{\frac{1}{x+1}}{\frac{1}{x}}, \\
&= \lim_{x\to\infty} \frac{3}{4}\frac{x}{x+1}, \\
&= \frac{3}{4}.
\end{aligned}$$

1 (d) Use the *ratio test* to determine if the series $\sum_{k=2}^{\infty} F(k)$ converges or diverges.

Solution: Since $\lim_{x\to\infty} \frac{3}{4}\frac{\ln(x+1)}{\ln(x)} \in [0,1)$, the series $\sum_{k=2}^{\infty} F(k)$ converges.

2. Use the *ratio test* to determine if each series converges or diverges.

1 (a) $\sum_{k=0}^{\infty} \frac{2^k}{3^k+8}$

Solution:

$$\begin{aligned}\lim_{k\to\infty} \frac{2\left(3^k+8\right)}{3^{k+1}+8} &= \lim_{k\to\infty} \frac{2\ln(3)3^k}{\ln(3)3^{k+1}},\\ &= \lim_{k\to\infty} \frac{2}{3},\\ &= \frac{2}{3}.\end{aligned}$$

$\boxed{1}$ (b) $\displaystyle\sum_{k=0}^{\infty} \frac{((k)!)^3}{(3k)!} 14^k$

Solution:

$$\lim_{k\to\infty} \frac{14(k+1)^2}{3\,(3k+1)\,(3k+2)} = \frac{14}{27}.$$

So $\sum_{k=0}^{\infty} \frac{((k)!)^3}{(3k)!} 14^k$ converges.

$\boxed{1}$ 3. Find the numeric value of the $\displaystyle\lim_{k\to\infty} \frac{((k)!)^3}{(3k)!} 14^k$. Justify your answer.

Since $\sum_{k=0}^{\infty} \frac{((k)!)^3}{(3k)!} 14^k$ converges, we have $\displaystyle\lim_{k\to\infty} \frac{((k)!)^3}{(3k)!} 14^k = 0$.