

TRIGONOMETRIC PRODUCT RULES

For all $x, y \in \mathbf{R}$, we have

$$\cos(x) \cos(y) = \frac{\cos(x+y)}{2} + \frac{\cos(x-y)}{2},$$

$$\cos(x) \sin(y) = \frac{\sin(x+y)}{2} - \frac{\sin(x-y)}{2},$$

$$\sin(x) \sin(y) = \frac{\cos(x-y)}{2} - \frac{\cos(x+y)}{2}.$$

INTEGRATION REDUCTION RULES

$$\int \tan(x)^n dx = \begin{cases} \ln(|\sec(x)|) & n = 1 \\ \frac{\tan(x)^{n-1}}{n-1} - \int \tan(x)^{n-2} dx & n \neq 1 \end{cases}$$

$$\int \sec(x)^n dx = \begin{cases} \ln(|\tan(x) + \sec(x)|) & n = 1 \\ \frac{\sec(x)^{n-2} \tan(x)}{n-1} + \frac{n-2}{n-1} \int \sec(x)^{n-2} dx & n \neq 1 \end{cases}$$

PRODUCTS OF TAN AND SEC

$$\int \tan(x)^{2m+1} \sec(x)^n dx = \int (u^2 - 1)^m u^{n-1} du, \text{ where } u = \sec(x)$$

$$\int \tan(x)^m \sec(x)^{2n} dx = \int u^m (u^2 + 1)^{2n-2} du, \text{ where } u = \tan(x)$$

$$\int \tan(x)^{2m} \sec(x)^n dx = \int (\sec(x)^2 - 1)^m \sec(x)^n du, \text{ expand}$$

