MATH 202, Spring 2024 **Name:** ______________________________

In class work 13 **Row and Seat**:____________________________

In class work **13** has questions **1** through **3** with a total of **6** points.

"A linguistic construction is called referentially transparent when for any expression built from it, replacing a subexpression with another one that denotes the same value does not change the value of the expression." (Wikipedia)

Theorem 1 (CT) *Let a and b be seqences. And suppose that* $(\forall k \in \mathbf{Z}_{\geq 0})\,(0 \leq a_k \leq b_k)$. *Then* $\sum b$ *converges* $\implies \sum a$ *converges.*

Theorem 2 (LCT) *Let a and b be seqences. And suppose that there is and integer N such that for all* $k \in \mathbf{Z}_{\geq N}$, *we have* $0 < a_k$ *and* $0 < b_k$. *Then*

- $\lim_{k\to\infty}\left(\frac{a_k}{b_k}\right) \in \mathbf{R}_{>0} \implies \left(\sum a \text{ converges} \equiv \sum b \text{ converges}\right)$
- $\lim_{k\to\infty}\left(\frac{a_k}{b_k}\right) = 0 \implies \left(\sum b \text{ converges} \implies \sum a \text{ converges}\right)$
- $\lim_{k\to\infty}\left(\frac{a_k}{b_k}\right) = \infty \implies \left(\sum a \text{ converges} \implies \sum b \text{ converges}\right)$

$\boxed{2}$ 1. Use the CT to show that $\sum_{k=0}^{\infty} \frac{1}{k^2+k+28}$ converges.

$\boxed{2}$ 2. Use the LCT to show that $\sum_{k=0}^{\infty} \begin{cases} (-1)^k k! & k < 10^9 \\ \frac{1}{k^8+1} & k \geq 10^9 \end{cases}$ converges.

$\boxed{2}$ 3. After an arduous calcuation spanning tweleve pages of engineering paper, my friend Lilly Poole has (correctly) shown that

$$\int_0^\infty \frac{1}{x^8+1}\,\mathrm{d}x = \frac{\pi}{8\sin\left(\frac{\pi}{8}\right)}.$$

After that, Ms. Poole (spouse of Mr. Wade Poole) cocludes that $\sum_{k=0}^{\infty} \frac{1}{1+k^8} = \frac{\pi}{8\sin\left(\frac{\pi}{8}\right)}$. Is Ms. Poole's conclusion correct? Explain.