

Can I do X?

MATH 202

April 3, 2024

“The law is reason unaffected by desire.”

Aristotle

Everything which is not forbidden is allowed

Generally, in the legal system everything that is not explicitly forbidden is legal.

- 👉 For example, in Kearney, backyard chickens are prohibited by city code, so chickens in town are illegal,
- 👉 but painting your house purple is legal because purple houses aren't mentioned in city code.

Of course, I'm not a lawyer, so . . .

But math is different

- 👉 But in math, most things that are not explicitly allowed are forbidden.
- 👉 In the context of math, the answer to the question “Can I do X ?” is “if there is a rule for it, sure, if not, no way.”

Exhaustive rules

Generally, the aim of algebra is to enumerate everything that is allowed. That means if it's not listed as rule, it's not true.

- 👉 The enumeration of the rules of exponents in our QRS attempts to be exhaustive, so if something isn't listed likely it's not true.
- 👉 Also, generally it's the aim of algebra to condense rules to a minimal set of rules.
- 👉 That means that sometimes something in algebra might be provably true from a minimal set of rules, but not explicitly stated as a rule.

For every means for every

Rubbish or nonrubbish:

$$(\forall a \in \mathbb{R}_{\neq 0}) \left(\frac{a+b}{a} = 1 + b \right) \equiv \text{True}$$

This is abject rubbish. To show this, choose $a = 2$ and $b = 5$, then

$$\left[\frac{2+5}{2} = 1+5 \right] \equiv \left[\frac{7}{2} = 6 \right] \equiv [7 = 12] \equiv \text{False}.$$

👉 Checking a special case for a “for every” statement is a powerful way to possibly show that it is false.

Can I do ...

Theorem (multiplicative cancellation)

We have

$$(\forall a, c \in \mathbb{R}_{\neq 0}, b \in \mathbb{R}) \left(\frac{ab}{ac} = \frac{b}{c} \right) \equiv \text{True}.$$

- 👉 In words, this says that a common nonzero *multiplicative* factor in a numerator and denominator can be “canceled.”
- 👉 Functionally, we write $\frac{ab}{ac} = \frac{b}{c}$.
- 👉 Provided that a and c are nonzero, replacing $\frac{ab}{ac}$ by $\frac{b}{c}$ in any statement doesn't change the meaning (or truth value) of the statement.

Avoiding slang

- 👉 The verb “canceled” is mathematical slang—its use is convenient, but it’s subject to abuse.
- 👉 A problem with slang is that it is often poorly defined and misused. An example of misuse is the **bogus cancellation**

$$\frac{a+b}{a+c} = \frac{\cancel{a}+b}{\cancel{a}+c} = \frac{b}{c}.$$

- 👉 Our rule says that a common *multiplicative factor* in the numerator and denominator can be canceled, but in this example, the common term is additive, not multiplicative.