MATH 202, Spring2024 **Name:** ____________________

In class work week 6 **Row and Seat**: ____________________

In class work **6** has questions **1** through **2** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 13 Feb 13:20.*

1. For the DE $3y^3\frac{\mathrm{d}y}{\mathrm{d}x} + 2x = 0$, do the following:

2 (a) Find a GS to the DE. Remember that a GS has one arbitrary constant.

Solution: We need to match the given DE to $A'(y)\frac{\mathrm{d}y}{\mathrm{d}x} = B'(x)$. To make the match, we need to subtract $2x$ from the DE. That gives $3y^3\frac{\mathrm{d}y}{\mathrm{d}x} = -2x$. So $A'(y) = 3y^3$ and $B'(x) = -2x$. Integrating, we find that $A(y) = \frac{3}{4}y^4$ and $B(x) = -x^2 + c$, where $c \in \mathbf{R}$. So a GS to the DE is

$$\frac{3}{4}y^4 = -x^2 + c. \tag{1}$$

Another GS is

$$3y^4 = -4x^2 + \pi c. \tag{2}$$

And another is

$$y^4 + 14 = -\frac{4}{3}x^2 + \pi c. \tag{3}$$

2 (b) Find a solution to the DE that satisfies $y = 2$ when $x = 1$.

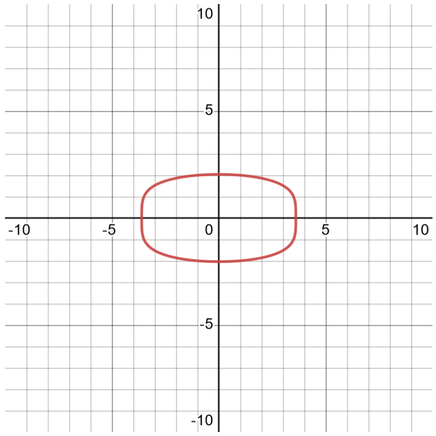
Solution: We need to paste $y = 2$ and $x = 1$ into a GS. Let's use the GS $\frac{3}{4}y^4 = -x^2 + c$.

$$\frac{3}{4}2^4 = -1^2 + c. \tag{4}$$

so $c = 13$. That makes this solution $\frac{3}{4}y^4 = -x^2 + 13$.

2 (c) Use Desmos to graph the solution you found in part 'b.'

Solution:



2. Find the numerical value of the definite integral $\int_4^8 \frac{1}{5x+1}\,\mathrm{d}x$.

Solution: Let $z = 5x + 1$. Then $\mathrm{d}z = 5\mathrm{d}x$. When $x = 4$, we have $z = 21$; and when $x = 8$, we have $z = 41$. So

$$\int_4^8 \frac{1}{5x+1}\,\mathrm{d}x = \int_{21}^{41} \frac{1}{5}\frac{1}{z}\,\mathrm{d}z = \frac{1}{5}\left(\ln(41) - \ln(21)\right). \tag{5}$$

Most folks would say that using the logarithm of a sum rule, a simpler answer is

$$\int_4^8 \frac{1}{5x+1}\,\mathrm{d}x = \frac{1}{5}\ln\left(\frac{41}{21}\right). \tag{6}$$

We could use the identity $y\ln(x) = \ln(x^y)$. That gives an answer

$$\ln\left(\sqrt[5]{\frac{41}{21}}\right), \tag{7}$$

but trading a division by 5 by a fifth root doesn't seem like a simplification.