MATH 202, Spring 2024 | **Name:** ____________________
In class work 23 | **Row and Seat**:____________________

"Life is full of surprises, but never when you need one." CALVIN (BILL WATTERSON)

In class work **23** has questions **1** through **6** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 19 April 13:20*.

$\boxed{1}$ 1. Use the fact that for all real x the equation $\mathrm{e}^x = \displaystyle\sum_{k=0}^{\infty} \frac{1}{k!} x^k$ is an identity to find a power series representation for e^{-x^2}.

$\boxed{1}$ 2. Define a function erf by the definite integral $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-t^2}\, \mathrm{d}t$. Find a power series representation for erf. Find the radius of convergence of this power series.

$\boxed{1}$ 3. With a bit of faith and luck, maybe a good approximation to erf is the sum of its first 101 terms. Use Desmos to graph this approximation for $-6 < x < 6$. Actually, the function erf is known to Desmos. Plot a graph of both the sum of the first 101 terms of the power series for erf along with the function erf. As best you can, reproduce the graphs here. Incidentally, the sum of the first 101 terms is $\sum_{k=0}^{100}$, not $\sum_{k=0}^{101}$.

$\boxed{1}$ 4. Based on the Desmos graph of erf, what is your conjecture for the value of $\lim_{x \to \infty} \mathrm{erf}(x)$?

$\boxed{1}$ 5. For "large" values of x, explain why $\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N} \frac{(-1)^k}{(2k+1)(k!)} x^{2k+1}$ is not a good approximation to erf no matter how large we make N.

Remember: No matter the degree of a polynomial, its limit toward infinity is determined by the term of the polynomial with the highest power. For example, provided $a_{101} \neq 0$, we have

$$\lim_{x\to\infty} \left(a_o + a_1 x + a_2 x^2 + \cdots + a_{101} x^{101}\right) = \lim_{x\to\infty} a_{101} x^{101} = \begin{cases} -\infty & a_{101} < 0 \\ \infty & a_{101} > 0 \end{cases}.$$

$\boxed{1}$ 6. Find a formula for the derivative of erf; that is find a formula for erf$'$. Make your formula as "simple" as you can. You might like to return the initial definition as a definite integral; that is $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$