

In class work 2 has questions 1 through 2 with a total of 10 points. Turn in your work at the end of class *on paper*. This assignment is due *Tuesday 30 January 13:20*.

- 5 1. Let b be a positive number. Find the length of the curve $y = b\left(\frac{x}{b}\right)^{3/2}$ where $0 \leq x \leq b$. Possibly you will be more successful if you begin by expressing y in an equivalent form: $y = b^{-1/2}x^{3/2}$ **Hint:** The quantity b is a length. To make the dimensions correct, the answer *must* be of the form $b \times \text{number}$; if not, check your work!

Solution: A better alternative to $y = a\left(\frac{x}{a}\right)^{3/2}$ might be $y = \frac{x^{3/2}}{\sqrt{a}}$. We have

$$\frac{dy}{dx} = \frac{3}{2\sqrt{a}}x^{1/2}. \quad (1)$$

That makes

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{9x}{4a} \quad (2)$$

The arclength s is

$$s = \int_0^a \sqrt{1 + \frac{9x}{4a}} dx,$$

Let's substitute $z = 1 + \frac{9x}{4a}$. Then $dz = \frac{9}{4a}dx$. Solving for dx gives $dx = \frac{4a}{9}dz$. And one more detail: we know the limits of integration for x , but we need them for z . When $x = 0$, we have $z = 1$. And when $x = a$, we have $z = 1 + \frac{9}{4} = \frac{13}{4}$. We're ready:

$$\begin{aligned} &= \frac{4a}{9} \int_1^{13/4} \sqrt{z} dz, \\ &= \frac{3}{2} z^{3/2} \Big|_{z=1}^{z=13/4}, \\ &= \frac{4a}{9} \frac{3}{2} \left(\left(\frac{13}{4} \right)^{3/2} - 1 \right), \end{aligned}$$

- 5 2. Find the surface area of the solid generated by rotating the curve $y = \sqrt{x}$ where $0 \leq x \leq 1$ about the x-axis.

Solution: A better alternative to $y = a\sqrt{\frac{x}{a}}$ might be $y = \sqrt{a}\sqrt{x}$. We have

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{a}x^{-1/2}. \quad (3)$$

So

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{a}{4x}}. \quad (4)$$

The surface area is

$$\begin{aligned} \text{Area} &= 2\pi \int_0^a \sqrt{a}\sqrt{x} \sqrt{1 + \frac{1}{4x}} dx \\ &= 2\pi\sqrt{a} \int_0^a \sqrt{x + \frac{a}{4}} dx \\ &= \frac{\pi}{6} (5^{3/2} - 1) a^2. \end{aligned}$$