MATH 202, Spring 2024 **Name:** ____________________

Exam 4 Practice **Row and Seat**:____________________

"If people sat outside and looked at the stars each night, I'll bet they'd live a lot differently."

CALVIN (BILL WATTERSON)

1. For the parametricaly defined curve $\begin{cases} x = 3t \\ y = 9t^2 + t \end{cases}$ $-\infty < t < \infty$, eliminate the parameter t. Sketch the resulting curve in the xy cartesian coordinate system.

> **Solution:** Solving $x = 3t$ for t yields $t = x/3$. Substituting this into $y = 9t^2 + t$ gives $y = x^2 + x/3$. The graph is an upward facing parabola with vertex $(x = -3/2, y = -9/4)$ and x-intercepts $x = -3$ and $x = 0$.

2. For the parametricaly defined curve $\begin{cases} x = 3\cos(t) \\ y = 9\sin(t) \end{cases}$ $0 \le t \le 2\pi$, eliminate the parameter t. Sketch the resulting curve in the xy cartesian coordinate system.

> **Solution:** $(x/3)^2 + (y/9)^2 = 1$. The graph is an ellipse with vertices $(x = -3, y = 0)$,$(x = 3, y = 0)$, $(x = 0, y = -9)$, and $(x = 0, y = 9)$.

3. For the parametrically defined curve $\begin{cases} x = -\sqrt{1+t} \\ y = \sqrt{3t} \end{cases}$ $0 \le t < \infty$, find the numerical values of $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=3}$ and $\left.\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right|_{t=3}$.

Solution: Before we start, let's gather our ingredients; we have

4. Represent the arc length of the polar curve $r = a(1 - \sin(\theta))$ as a definite integral. Here a is a positive real number.

Solution:

$$\sqrt{2}\,|a| \int_0^{2\pi} \sqrt{1 - \sin(\theta)}\, \mathrm{d}\theta.$$

5. Find all points on the polar curve $r = 3 - 2\sin(\theta)$ that have a horizontal tangent line.

Solution: $\theta = \frac{\pi}{2}, \theta = \arcsin\left(\frac{3}{4}\right), \theta = \pi - \arcsin\left(\frac{3}{4}\right)$

6. Find area of the region bounded by the polar curve $r = 3 - 2\sin(\theta)$

Solution: $\frac{11}{2}\pi$.

7. Find the area bounded by the polar curve $r = \cos(4\theta)$

Solution: $\frac{\pi}{2}$