Trigonometric product rules

For all $x, y \in \mathbf{R}$, we have

$$\cos(x)\cos(y) = \frac{\cos(x+y)}{2} + \frac{\cos(x-y)}{2},$$
$$\cos(x)\sin(y) = \frac{\sin(x+y)}{2} - \frac{\sin(x-y)}{2},$$
$$\sin(x)\sin(y) = \frac{\cos(x-y)}{2} - \frac{\cos(x+y)}{2}.$$

Integration reduction rules

$$\int \tan(x)^n \, dx = \begin{cases} \ln(|\sec(x)|) & n = 1 \\ \dfrac{\tan(x)^{n-1}}{n-1} - \displaystyle\int \tan(x)^{n-2} \, dx & n \neq 1 \end{cases}$$

$$\int \sec(x)^n \, dx = \begin{cases} \ln(|\tan(x) + \sec(x)|) & n = 1 \\ \dfrac{\sec(x)^{n-2}\tan(x)}{n-1} + \dfrac{n-2}{n-1}\displaystyle\int \sec(x)^{n-2} \, dx & n \neq 1 \end{cases}$$

Products of tan and sec

$$\int \tan(x)^{2m+1} \sec(x)^n \, dx = \int (u^2-1)^m u^{n-1} \, du, \text{ where } u = \sec(x)$$
$$\int \tan(x)^m \sec(x)^{2n} \, dx = \int u^m (u^2+1)^{2n-2} \, du, \text{ where } u = \tan(x)$$
$$\int \tan(x)^{2m} \sec(x)^n \, dx = \int (\sec(x)^2-1)^m \sec(x)^n \, du, \text{ expand}$$