MATH 202, Spring 2024 **Name:** _____

In Class work **Row and Seat:** _____

1. The force F required to lift a large bag of carrots depends on the distance x the bag is to the center of the earth. Given that $F(x) = \frac{10^8}{x^2}$, find

   (a) The work required to lift the carrots from $x = 10^8$ feet to $x = 3 \times 10^8$ feet.

   (b) The work required to lift the carrots from $x = 10^8$ feet to $x = \infty$; that is, compute $\int_{10^8}^{\infty} F(x)\,\mathrm{d}x$.

2. Define a region Q of the xy-plane by $Q = \{(x, y) | 0 \leq y \leq xe^x, 0 \leq x \leq 1\}$

   (a) Find area(Q).

   (b) Find the x coordinate of the centroid of Q.

3. Find the value of each indefinite or definite integral.

(a) $\displaystyle\int_0^1 \frac{x}{\left(1+x^2\right)^{1/3}}\,\mathrm{d}x =$

(b) $\displaystyle\int x\sqrt{1-x^2}\,\mathrm{d}x =$

(c) $\displaystyle\int \sin^{-1}(x)\,\mathrm{d}x =$

(d) $\displaystyle\int x\ln(x)\,\mathrm{d}x =$

(e) $\int_0^1 xe^{-x}\,\mathrm{d}x =$

(f) $\int \frac{1}{(x+5)(x+9)}\,\mathrm{d}x =$

(g) $\int \cos^2(x)\,\mathrm{d}x =$

(h) $\int \cos^3(x)\sin(x)\,\mathrm{d}x =$

4. Find the numerical value of each improper integral.

(a) $\int_0^\infty x\mathrm{e}^{-x}\,\mathrm{d}x$

(b) $\int_{-\infty}^\infty \frac{1}{x^2+9}\,\mathrm{d}x$

(c) $\int_0^1 \frac{1}{x^{9/10}}\,\mathrm{d}x$

5. Given a formula for a sequence b, find its limit. Show all of your work.

2 (a) $b_n = \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k$.

2 (b) $b_n = \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$.

6. Use the integral test to show that the series $\sum_{k=0}^{\infty} \frac{1}{1+k^2}$ converges.