MATH 202, Spring 2024 **Name:** ____________________
In class work 3 **Row and Seat**: ____________________

In class work **3** has questions **1** through **2** with a total of **6** points. Turn in your work at the end of class *on paper*. This assignment is due *Thursday 1 February 13:20.*

$\boxed{2}$ 1. Evaluate the definite integral $\int_0^2 x\sqrt{1+x^2}\,\mathrm{d}x$ by using the substitution $z = 1 + x^2$.

Solution: Method I

Let $z = 1 + x^2$. Then $\mathrm{d}z = 2x\mathrm{d}x$. The integrand has a factor of $x\mathrm{d}x$, so let's solve $\mathrm{d}z = 2x\mathrm{d}x$ for $x\mathrm{d}x$. Thus $x\mathrm{d}x = \frac{1}{2}\mathrm{d}z$. When $x = 0$, we have $z = 1$; when $x = 2$, we have $z = 5$. So

$$\int_0^2 x\sqrt{1+x^2}\,\mathrm{d}x = \int_1^5 \frac{1}{2}\sqrt{z}\,\mathrm{d}z, \qquad \text{(substitution)} \tag{1}$$

$$= \frac{1}{2} \times \frac{2}{3} z^{3/2}\Big|_{z=1}^{z=5}, \qquad \text{(FTC)} \tag{2}$$

$$= \frac{1}{3}\left(5^{3/2} - 1\right). \qquad \text{(algebra)} \tag{3}$$

Method II For Method II, we first find an anti derivative by using a substitution, second we return the original variable, and finally we use the FTC. Let $z = 1 + x^2$. Then $\mathrm{d}z = 2x\mathrm{d}x$. The integrand has a factor of $x\mathrm{d}x$, so let's solve $\mathrm{d}z = 2x\mathrm{d}x$ for $x\mathrm{d}x$. Thus $x\mathrm{d}x = \frac{1}{2}\mathrm{d}z$. So

$$\int x\sqrt{1+x^2}\,\mathrm{d}x = \int \frac{1}{2}\sqrt{z}\,\mathrm{d}z, \qquad \text{(substitution)} \tag{4}$$

$$= \frac{1}{2} \times \frac{1}{3} z^{3/2}, \qquad \text{(power rule for antiderivatives)} \tag{5}$$

$$= \frac{1}{3}(1+x^2)^{3/2}. \qquad \text{(return to original variable)} \tag{6}$$

Now we can use the FTC

$$\int_0^2 x\sqrt{1+x^2}\,\mathrm{d}x = \frac{1}{3}(1+x^2)^{3/2}\Big|_{x=0}^{x=2}, \tag{7}$$

$$= \frac{1}{3}\left(5^{3/2} - 1\right). \tag{8}$$

2. The force required to extend a spring is proportional to the amount of extension.

2 (a) If a force of 10 Newtons extends the spring 0.03 meters, find the formula for the force F required to extend the spring x meters.

Solution: The force F as a function of displacement x has the form $F = kx$, where k is a number. From the given data, we have $10 = k \times 0.03$. So $k = \frac{1000}{3}$ N/m.

2 (b) Find the *work* required to extend the spring 0.05 meters. If you don't know, the MKS unit of work is the Joule (which is Newton × meter). (To maybe make this more tangible: To lift a two pound sack of sugar three feet up, the work required is about one Joule.)

Solution:

$$W = \int_0^{0.05} \frac{1000}{3} x \, dx = \frac{1000}{6} x^2 \Big|_{x=0}^{x=0.05} = \frac{5}{12} \text{ J.} \tag{9}$$