Paper presentation: "Combinatorial algorithm for counting small induced graphs and orbits" (Demšar & Hočevar, 2017)

Graphlets and Orbits

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Paper

- Combinatorial algorithm for counting small induced graphs and orbits (Demšar, and Hočevar) 2017.

Background

- Network motifs
- What is a small induced graph?
- What is an orbit?

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- Triangles in $O(n^3)$ (Itai and Rodeh, 1978)

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- What is an orbit?
- Triangles in $O(n^3)$ (Itai and Rodeh, 1978)
- $\mathrm{O}(n^{k/3})$ (Nesetril and Poljak, 1985)

Motivation

- Node similarity

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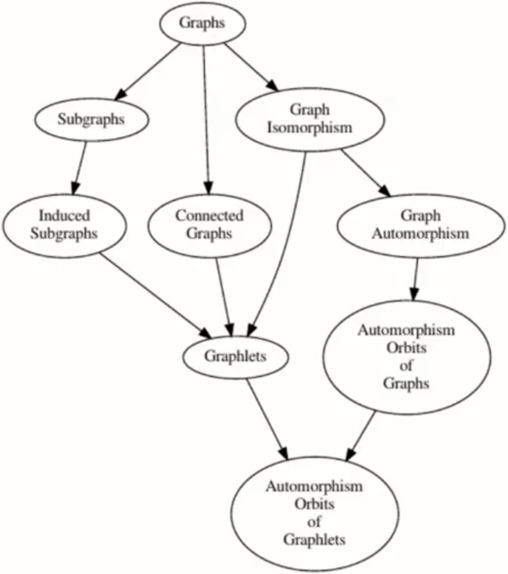
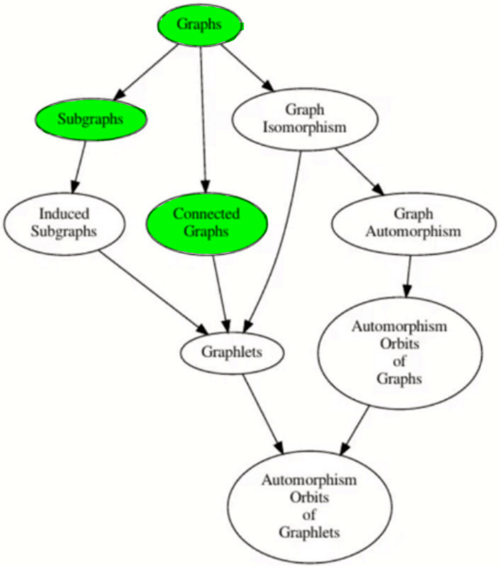
- Role of the node

Overview


Graphs
Subgraphs
Graph Isomorphism
Induced Subgraphs
Connected Graphs
Graph Automorphism
Graphlets
Automorphism Orbits of Graphs
Automorphism Orbits of Graphlets

Overview

- 
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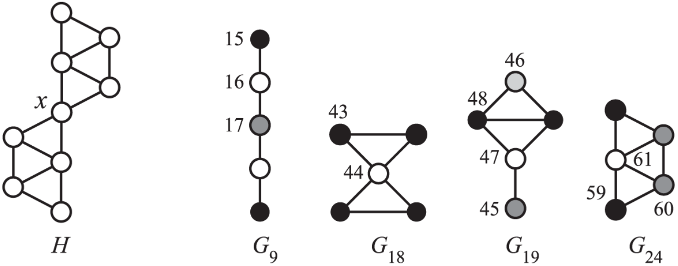
Induced Subgraphs

- Let $H = (V, E)$ be the host graph

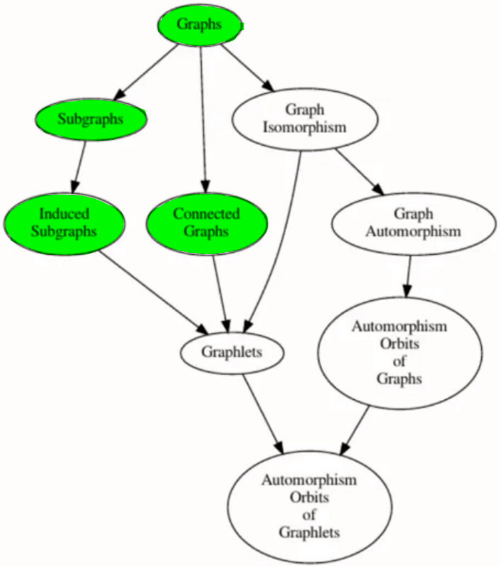
Induced Subgraphs

- Let H = (V, E) be the host graph
- Remove vertices and maintain edges



(Figure 2 - Demšar & Hočevar)

So far we've covered,

- 

Isomorphisms

- **Definition:** Graphs G and H are isomorphic if there is a structure that preserves a one-to-one correspondence between the vertices and edges

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- Same # of vertices
- Same # of edges
- Same vertex degree

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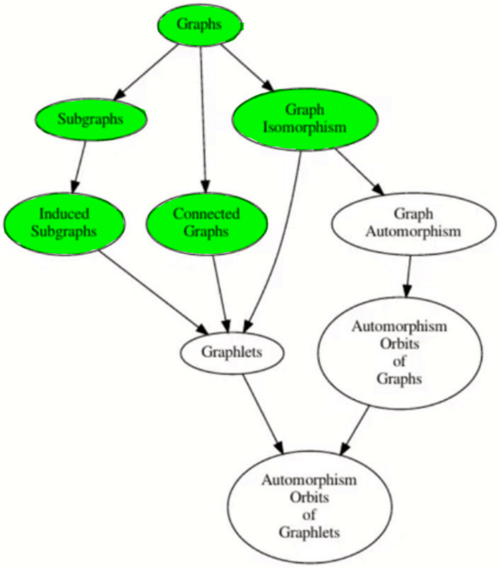
- 

Graphlets

- **Definition:** The set of all simple graphs (graphlets) on k nodes is G_k.

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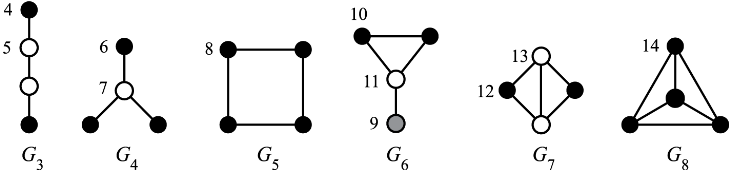
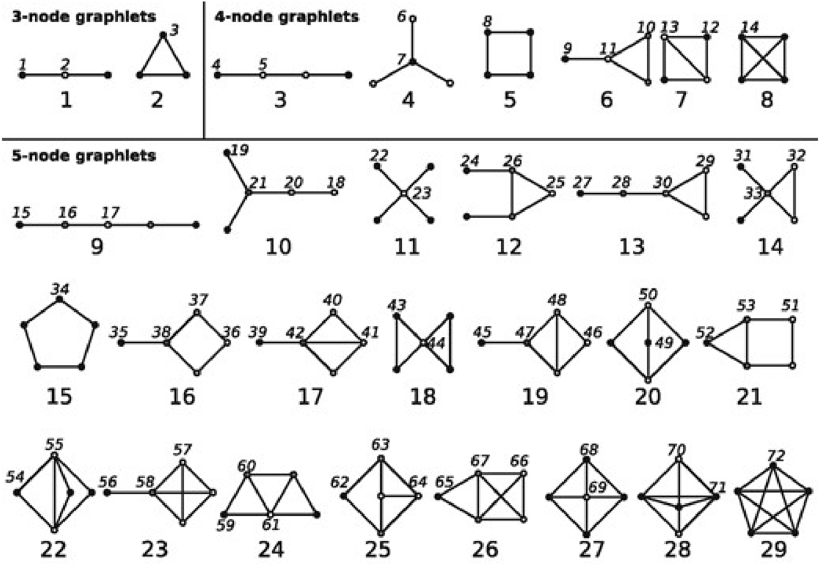
- **Additional contraints**
- "Small" graphlets
- k $\in$ [1, 5]
- Graphlets on k = 4 nodes,



(Figure 1 - Demšar & Hočevar)

Graphlets


3-node graphlets
4-node graphlets
5-node graphlets
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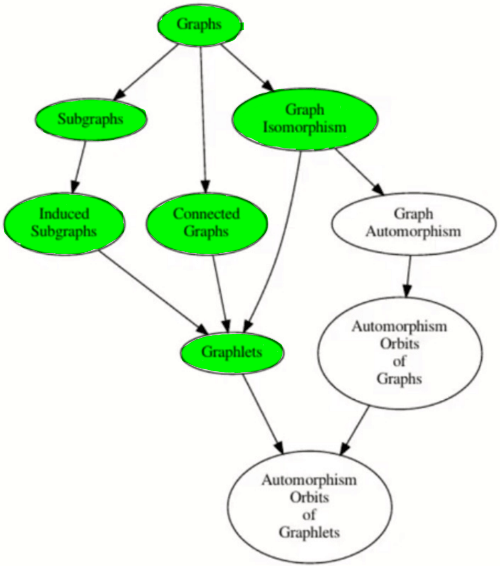
- 

Graph Automorphisms

- **Recall:** graph isomorphisms

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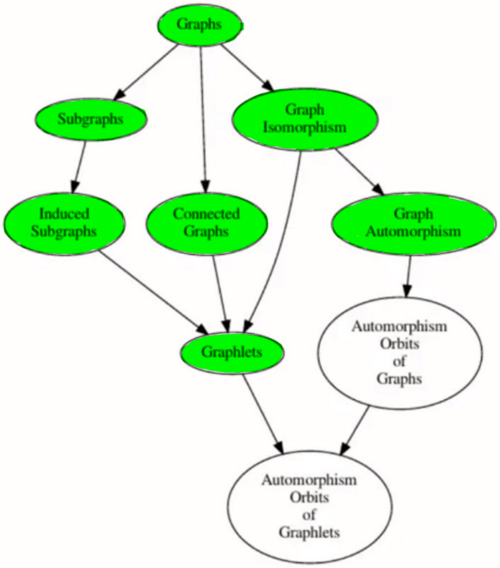
- $\mathrm{Aut}(G) = \mathrm{Iso}(G, G)$

So far we've covered,

- 

Automorphism Orbit (aka Orbit)

- What do graph automorphisms tell us about node orbit?

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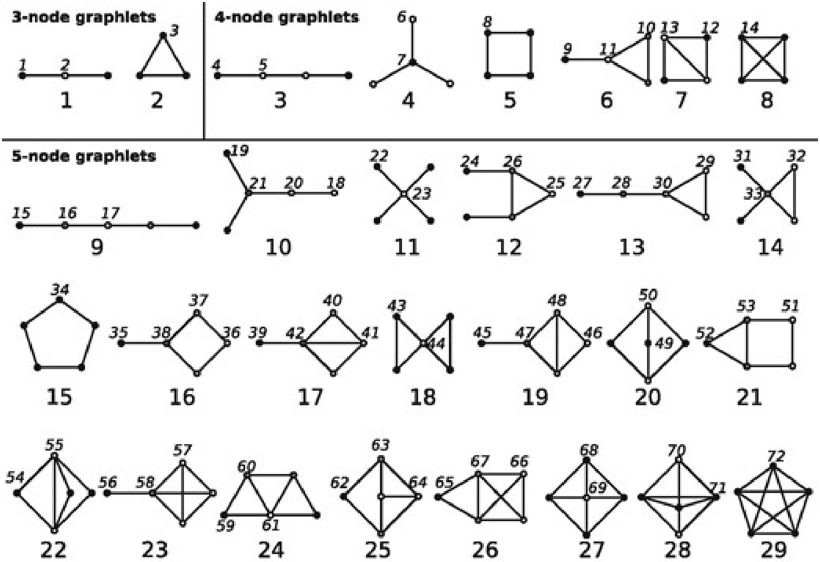
- $$Orb(G, v) = \{\omega \; \epsilon \; V \mid \exists \sigma \; \epsilon \; Aut(G) : \sigma(v) = \omega\} \qquad (1)$$
- "The orbit of a node v in a graphlet G is the set of nodes that can be mapped onto v by some automorphism of the graphlet."

Automorphism Orbit (aka Orbit)



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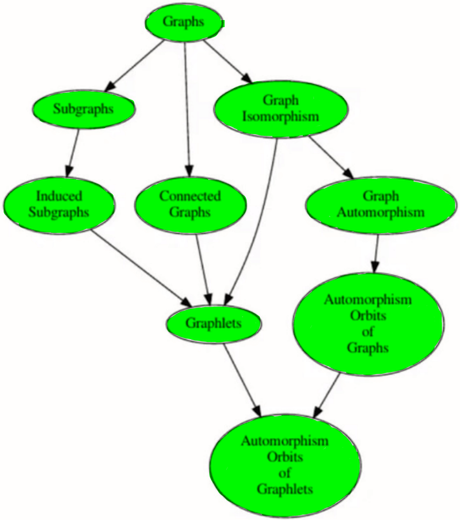
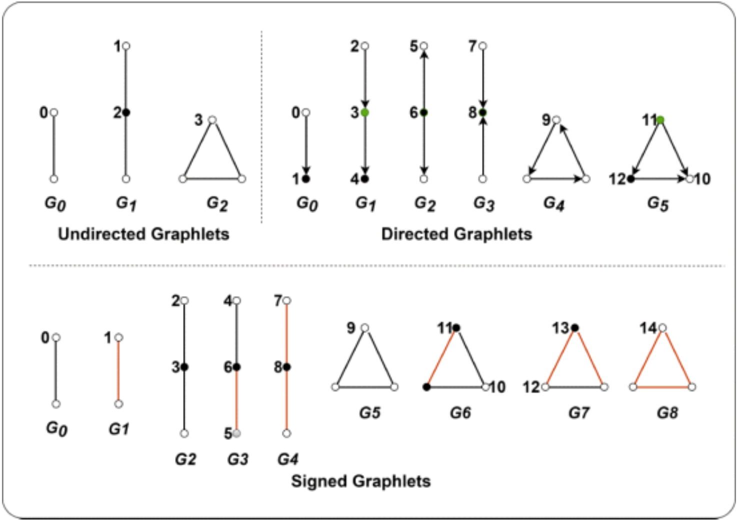
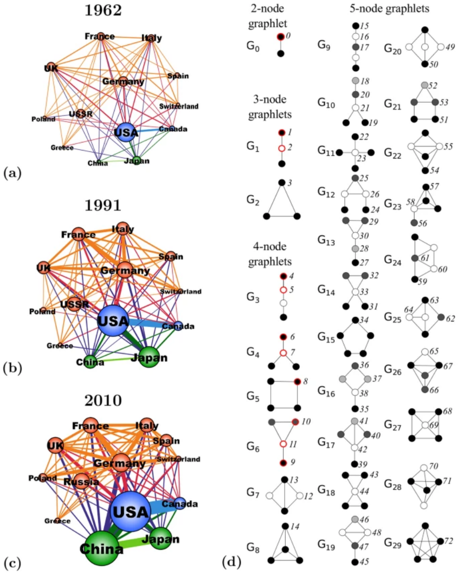
- 

Applications of Automorphism Orbit



Applications of Automorphism Orbit

Dynamic Trade Networks (Yaverogolu et al. 2014)



References

Tomaž Hočevar and Janez Demšar.
Combinatorial algorithm for counting small induced graphs and orbits.
PLOS ONE, 12(2):1–17, 02 2017.

Ine Melckenbeeck, Pieter Audenaert, Didier Colle, and Mario Pickavet.
Efficiently counting all orbits of graphlets of any order in a graph using autogenerated equations.
Bioinformatics, 34(8):1372–1380, 11 2017.

Ömer Nebil Yaveroäa Lu, Noël Malod-Dognin, Darren Davis, Zoran Levnajic, Vuk Janjic, Rasa Karapandza, Aleksandar Stojmirovic, and Nataša Pržulj.
Revealing the hidden Language of complex networks.
Scientific Reports, 4:1–9, 2014.