

$$a_1 = b_0q + a_0q + a_0p$$

$$b_1 = b_0p + a_0q$$

$$a_2 = b_1q + a_1q + a_1p$$

$$b_2 = b_1p + a_1q$$

$$a_2 = (b_0p + a_0q)q + (b_0q + a_0q + a_0p)q + (b_0q + a_0q + a_0p)p$$

$$b_2 = (b_0p + a_0q)p + (b_0q + a_0q + a_0p)q$$

Let's do a_2 first.

$$a_2 = (b_0p + a_0q)q + (b_0q + a_0q + a_0p)q + (b_0q + a_0q + a_0p)p$$

$$a_2 = b_0pq + a_0q^2 + b_0q^2 + a_0q^2 + a_0pq + b_0pq + a_0pq + a_0p^2$$

$$a_2 = b_0pq + b_0q^2 + b_0pq + a_0q^2 + a_0q^2 + a_0pq + a_0pq + a_0p^2$$

$$a_2 = b_0(q^2 + 2pq) + a_0(q^2 + 2pq) + a_0(p^2 + q^2)$$

Now we should have $q' = q^2 + 2pq$ and $p' = p^2 + q^2$, and this should check out on the b transformation.

$$b_2 = (b_0p + a_0q)p + (b_0q + a_0q + a_0p)q$$

$$b_2 = b_0p^2 + a_0pq + b_0q^2 + a_0q^2 + a_0pq$$

$$b_2 = b_0p^2 + b_0q^2 + 2a_0pq + a_0q^2$$

$$b_2 = b_0(p^2 + q^2) + a_0(q^2 + 2pq)$$

$$b_2 = b_0p' + a_0q'$$

This doesn't work out... but there's a little ambiguity in the problem description - maybe the transformations are sequential (first a then b) let's see if this checks out:

$$b_1 = b_0p + a_1q$$

$$b_2 = b_1p + a_2q$$

$$b_2 = (b_0p + a_1q)p + (b_1q + a_1q + a_1p)q$$

$$b_2 = b_0p^2 + a_1pq + b_1q^2 + a_1q^2 + a_1pq$$