

The recurrence relationship is given as:

$$x(n) = 1 + \frac{1}{x(n-1)}$$

For  $x(0) = x_0 \in \mathbb{R}$ .

As  $n \rightarrow \infty$   $x(n) \rightarrow x(n-1)$ .

$$x = 1 + \frac{1}{x}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$$\begin{aligned}\Delta &= (-1)^2 - 4 \cdot 1 \cdot (-1) \\ &= 5\end{aligned}$$

$$\begin{aligned}x_1 &= \frac{-(-1) - \sqrt{5}}{2 \cdot 1} \\ &= \frac{1 - \sqrt{5}}{2}\end{aligned}$$

$$x_2 = \frac{1 + \sqrt{5}}{2}$$

$\forall x_0 \in \mathbb{R}$

$$x(1) = 1 + \frac{1}{x_0} > 0$$

That leaves us with  $x_2$  which is the definition of  $\varphi$ .