5.1

proposition - a sentence that has a defined logical (boolean) value

statement - see proposition

propositional connective - logical operators such as negation, conjunction, disjunction

conjunction - logical and. True iff both statements are true

disjunction - logical or. True when at least one of the statements is true.

exclusive or - True iff both statements are different

implication - $s_1 \implies s_2$. False when $s1 = True$, $s_2 = False$ and true otherwise.

antecedent - the first statement in a implication.

consequent - the second statement in a implication.

vacuously true - a statement that is true because the antecedent was false

logical equivalence - if and only if, true when both statements have the same

logical value - true or false

tautology an expression that's always true

5.2

x	y	$x \implies y$	$\neg x$	$\neg x \vee y$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

Hence $x \implies y \iff \neg x \vee y$

5.3

x	y	$x \vee y$	$x \wedge (x \vee y)$
F	F	F	F
F	T	T	F
T	F	T	T
T	T	T	T

Hence $x \wedge (x \vee y) \iff x$

5.4

a) $(\neg(true \wedge false)) \vee true \to true$

b) $(true \wedge false) \vee true \to true$

c) $true \wedge (false \vee true) \to true \wedge true \to true$

d) $(\neg true) \vee \neg(\neg(false) \wedge true) \to false \vee \neg(true \wedge true) \to \neg(true) \to false$

e) $false \implies (true \vee false) \to true$

f) $(true \vee false) \implies true \to true \implies true \to true$

g) $true \implies (\neg(true) \wedge \neg(false)) \to true \implies (false \wedge true) \to true \implies false \to false$

h) $false \vee (\neg(true) \implies true) \to false \implies true \to true$

5.5

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A := (if B then C) and (if C then B);
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5.6

a)

$$\begin{array}{rl} p \wedge \neg(p \wedge q) & \\ p \wedge ((\neg p) \vee (\neg q)) & \text{De Morgan} \\ p \wedge (\neg p) \vee p \wedge (\neg q) & \text{Distributive} \\ False \vee p \wedge (\neg q) & \text{Complement} \\ p \wedge (\neg q) & \text{Identity} \end{array}$$

b)

$$\begin{array}{rl} (p \implies q) \wedge p & \\ (\neg p \vee q) \wedge p & \text{Conditional as disjunction} \\ (\neg p \wedge p) \vee (q \wedge p) & \text{Distributive} \\ false \vee (q \wedge P) & \text{Complement} \\ q \wedge p & \text{Identity} \end{array}$$

c)

$$\begin{aligned}
&(p \vee q) \wedge (p \vee \neg q) \\
((p \vee q) \wedge p) \vee ((p \vee q) \wedge \neg q) \quad & \text{Distributive} \\
(p \vee q) \vee ((p \vee q) \wedge \neg q) \quad & \text{Absorption} \\
(p \vee q) \vee ((\neg q \wedge p) \vee (\neg q \wedge q)) \quad & \text{Distributive} \\
(p \vee q) \vee ((\neg q \wedge p) \vee False) \quad & \text{Complement} \\
(p \vee q) \vee (\neg q \wedge p) \quad & \text{Identity} \\
(p \vee q) \vee (p \wedge \neg q) \quad & \text{Commutative} \\
p \vee (q \vee (p \wedge \neg q)) \quad & \text{Associative} \\
p \vee ((q \vee p) \wedge (q \wedge \neg q)) \quad & \text{Distributive} \\
p \vee ((q \vee p) \wedge False) \quad & \text{Complement} \\
p \vee False \quad & \text{Universal bound} \\
p \quad & \text{Identity}
\end{aligned}$$

d)

$$\begin{aligned}
p \vee (q \wedge \neg p) \quad & \\
(p \vee q) \wedge (p \vee \neg p) \quad & \text{Distributive} \\
(p \vee q) \wedge True \quad & \text{Complement} \\
(p \vee q) \quad & \text{Identity}
\end{aligned}$$

e)

$$\begin{aligned}
p \wedge \neg(p \wedge \neg q) \quad & \\
p \wedge (\neg p \vee q) \quad & \text{De Morgan} \\
(p \wedge \neg p) \vee (p \wedge q) \quad & \text{Distributive} \\
False \vee (p \wedge q) \quad & \text{Complement} \\
p \wedge q \quad & \text{Identity}
\end{aligned}$$

f)

$$
\begin{aligned}
&\neg(p \wedge q) \wedge (p \vee \neg q) & \\
&(\neg p \vee \neg q) \wedge (p \vee \neg q) & \text{De Morgan} \\
&((\neg p \vee \neg q) \wedge p) \vee ((\neg p \vee \neg q) \wedge \neg q) & \text{Distributive} \\
&((\neg p \vee \neg q) \wedge p) \vee \neg q & \text{Absorption} \\
&((\neg p \wedge p) \vee (\neg q \wedge p)) \vee \neg q & \text{Distributive} \\
&(False \vee (\neg q \wedge p) \vee \neg q & \text{Complement} \\
&(\neg q \wedge p) \vee \neg q & \text{Identity} \\
&\neg q & \text{Absorption}
\end{aligned}
$$

5.7

a) $P \wedge Q$

b) $P \wedge \neg R$

c) $\neg(P \wedge Q)$

d) $P \wedge R$

e) $P \implies Q$

f) $P \implies Q$

g) $Q \implies P$

h) $Q \implies P$

i) $(P \wedge Q) \implies R$

j) $P \wedge Q \wedge R$

k) $P \wedge (Q \implies R)$

5.8

argument - a set of statements (premises) that support the truth of a conclusion $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \implies c$

premise - see argument

conclusion - see argument

valid argument - an argument where the implication is a tautology

sound argument - an argument that is valid and all of it's premises are true

inference rule - basic arguments such as modus ponens

5.9

a)

$$(A \implies B) \land (C \lor \neg B) \land A \implies C \quad \text{- The argument to validate}$$

$$\begin{aligned}
A \implies B &\quad \text{Premise} && (1)\\
C \lor \neg B &\quad \text{Premise} && (2)\\
A &\quad \text{Premise} && (3)\\
\neg A \lor B &\quad \text{Conditional as disjunction applied to 1} && (4)\\
\neg A \lor C &\quad \text{Resolution applied to 4 and 2} && (5)\\
\therefore C &\quad \text{Disjunctive syllogism applied to 5 and 3} && (6)
\end{aligned}$$

b)

$$A \land (\neg B \implies \neg A) \implies B \quad \text{- The argument to validate}$$

$$\begin{aligned}
A &\quad \text{Premise} && (1)\\
\neg B \implies \neg A &\quad \text{Premise} && (2)\\
A \implies B &\quad \text{Contrapositive applied to 2} && (3)\\
\therefore B &\quad \text{Modus ponens applied to 3 and 1} && (4)
\end{aligned}$$

c)

$$\neg(A \lor \neg B) \land (B \implies C) \implies (\neg A \land C) \quad \text{- The argument to validate}$$

$$\begin{aligned}
\neg(A \lor \neg B) &\quad \text{Premise} && (1)\\
B \implies C &\quad \text{Premise} && (2)\\
\neg A \land B &\quad \text{De Morgan applied to 1} && (3)\\
\therefore \neg A \land C &\quad \text{Modus ponens applied to 2 and 3} && (4)
\end{aligned}$$

d)

$$\neg A \land (A \lor B) \implies B \quad \text{- The argument to validate}$$

$$\begin{aligned}
\neg A &\quad \text{Premise} && (1)\\
(A \lor B) &\quad \text{Premise} && (2)\\
\therefore B &\quad \text{Disjunctive syllogism applied to 1 and 2} && (3)
\end{aligned}$$

e)

$$(\neg A \implies \neg B) \land B \land (A \implies C) \implies C \quad \text{- The argument to validate}$$

$$\begin{aligned}
\neg A \implies \neg B & \quad \text{Premise} && (1)\\
B & \quad \text{Premise} && (2)\\
A \implies C & \quad \text{Premise} && (3)\\
B \implies A & \quad \text{Contrapositive 1} && (4)\\
B \implies C & \quad \text{Hypothetical syllogism 4 and 3} && (5)\\
\therefore C & \quad \text{Modus ponens 5 and 2} && (6)
\end{aligned}$$

5.10

$c(x)$ x is a car

$m(x)$ x is a motorcycle

$f(x)$ x is fast

a) All cars are fast

$$\forall x(c(x) \implies f(x))$$

b) Some motorcycles are fast

$$\exists x(m(x) \land f(x))$$

c) All cars are fast but no motorcycle is fast

$$\forall x(c(x) \implies f(x) \land m(x) \implies \neg f(x))$$

d) Only motorcycles are fast

$$\forall x(f(x) \implies m(x))$$

e) No car is fast

$$\forall x(c(x) \implies \neg f(x))$$

f) If every car is fast, then every motorcycle is fast

$$\forall x(c(x) \implies f(x)) \implies \forall y(m(y) \implies f(y)))$$

g) Some motorcycles are not fast

$$\exists x(m(x) \land \neg f(x))$$

h) If no car is fast, then some motorcycles are not fast

$$\forall x(c(x) \implies \neg f(x)) \implies \exists y(m(y) \land \neg f(y))$$

5.11

$b(x)$ x is a bee

$f(x)$ x is a flower

$s(x)$ x stings

$p(x, y)$ x pollinates y

a) All bees sting.
$$\forall x(b(x) \implies s(x))$$

b) Some bees sting.
$$\exists x(f(x) \wedge s(x))$$

c) Only bees sting.
$$\forall x(s(x) \implies b(x))$$

d) Some bees pollinate all flowers
$$\exists x \forall y(p(x, y))$$

e) All bees pollinate some flowers.
$$\forall x \exists y(p(x, y))$$

5.12

$e(x)$ x is an even integer

a) Some integers are even
$$(\exists i \in \mathbb{Z})(e(i))$$

b) All integers are even.
$$(\forall i \in \mathbb{Z})(e(x)))$$

c) Some integers are odd.
$$(\exists i \in \mathbb{Z})(\neg e(i))$$

d) Some integers are both even and odd.
$$(\exists i \in \mathbb{Z})(e(i) \wedge \neg e(x))$$

Is this even expressible using only the $e(\bullet)$ predicate? It only makes sense when odd is expressible by $\neg e(\bullet)$ but for 0 (an integer both odd and even) $e(0) \wedge \neg e(0)$ cannot be true (because $e(\bullet)$ cannot be true while $\neg e(\bullet)$ is also true.

e) The sum of an even integer and 12 is an even integer

$$(\forall i \in \mathbb{Z})(e(x) \implies e(x+12) \ni \mathbb{Z})$$

f) The sum of any two even integers is an even integer.

$$(\forall x, y \in \mathbb{Z})(e(x) \wedge e(y) \implies e(x+y) \ni \mathbb{Z})$$