Social Networks & Recommendation Systems

XI. Agent-based models.

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Lecture

Epidemics – introduction

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- Classically we assume that every agent is in contact with each other, and then the variables describe the system in which $\beta, \gamma > 0$

$$\begin{cases} \frac{dS}{dt} &= -\beta SI, \\ \frac{dI}{dt} &= \beta SI - \gamma I, \\ \frac{dR}{dt} &= \gamma I. \end{cases}$$

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- β probability that in a single time step a healthy individual will become infected from a sick neighbor.
- γ the probability that the sick agent will recover in a single time step.
- Model can be described, in the mean-field approach, as follows

$$\frac{dI(t)}{dt} = \left[\beta\left(\langle k\rangle \frac{I(t)}{N}\right)\right] S(t) - \gamma I(t).$$

SIS model on the graphs: analysis

Let's change variables

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where $i(t) = I(t)/N$ and $s(t) = S(t)/N$

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Let us ask two questions:

- Under what conditions does an epidemic burst?
- What happens to the number of infected people in the large time limit?

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  should be compared with the model parameters

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Then the equation takes the form

$$\frac{di_k(t)}{dt} = \beta k Q_I s_k(t) - \gamma i_k(t).$$

SIS model on scale-free networks – asymptotic

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Which is very bad news...

Why?

Very simple bibliometric model

BA algorithm modification

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- At each time step, the agent publishes works that are cited according to the rich get richer.
- Is it consistent with empirical data? (visualization)

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Illustration

Thank you for your attention!

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