Feasibility of an Alternative Satellite Range-Rate Determination Method Using the Doppler Shifted Transmitted Signal

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Abstract

A satellite-carrier-frequency based method was set up to determine the change in magnitude of the position vector between the ground station and the Delfi-C3 (the range-rate). The results were compared with the Two-Line Element (TLE) set, provided by the United States Space Surveillance Network. The raw satellite radio signal data was first split into equal time blocks of 0.1 [s], to each of which the Fast Fourier Transform was applied to obtain the frequency energy spectrum. After a series of filtering was applied, the carrier frequency was extracted and used in the Doppler shift relation to find the range-rate. The results were then compared with TLE computations, which were derived from the propagated orbit and the ground stations position obtained from GPS measurements. During the range-rate evaluation, it was found that there was approximately 6 [s] difference between the Doppler calculated time of minimum distance (t_0) and the TLE one. After trying to remove the source of this error, the Doppler tracking was found to deviate up to 300 [m/s] from the TLE. This shows that the Doppler tracking method is not yet feasible. However, this deviation may be reduced by using better equipment.

Keywords: Satellite tracking, Range-rate, Orbit determiniation, TLE, Doppler shift, Fast Fourier Transform, Carrier frequency

I. Introduction

The United States Space Surveillance Network (USSSN) makes the position of space objects in the compact form of Two-Line Element (TLE) data freely available. TLE data generally relies on radar systems and optical observation, with an accuracy of 1 [km] [1] [2]. This is currently the method used by (among others) the Delft University of Technology's ground station to get information about the Delfi-C3 satellite's orbit. From this TLE data the range-rate can be determined. This research is motivated to find an alternative method to calculate the range-rate that is more accurate than the TLE method. Furthermore, this alternative method should allow a ground station to determine a satellite's orbit using its own resources, independent of the USSSN.

The Delfi-C3, launched in 2008, currently transmits a radio signal to Earth. The ground station, which comprises one S-band antenna and two Very High Frequency antennas, records the signal as a complex WAV file [3]. From this signal, the frequency spectrum can be extracted. The Delfi-C3 signal's relative motion affects the ground station's observed carrier frequency due to the Doppler effect [4]. These changes in the frequency spectrum can be used to determine the range-rate, which is the change in magnitude of the position vector between the ground station and the Delfi-C3. This method for range-rate determination, the Doppler method, can be done using only the ground stations available resources.

The purpose of this research is to investigate whether it is feasible to use the observed carrier frequency of the Delfi-C3 to determine its range-rate with current ground station equipment with a higher accuracy than the USSSN's TLE data. If this experiment poses a successful result, at least as accurate compared to TLE, further steps can be taken to determine the position from the range-rate.

II. Methodology

This research is dedicated to determining the range-rate. This range-rate will be obtained using two different methods, Doppler tracking and TLE. Fig. 1 gives a overview of both processes.

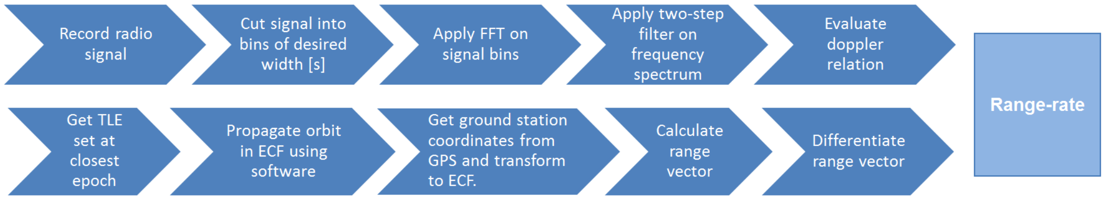

Fig. 1 – *Overview of research method*

Range-rate determination by Doppler tracking

The radio was tuned to a center frequency of 145.870 [MHz] with a sampling frequency F_s of 50 [kHz] (specified by the radio),the recording lasted for exactly 900 [s] starting at UTC 11:00:02 GPS time. The WAV file was read into MathWorks$'$ Matlab (version 2013b). Since the recording uses a complex signal representation (phasors), the signal was converted to absolute amplitude over time.

To extract information about the frequency over time, the read-in signal was split up into desired time intervals (bins). The amount of points per bin is the time resolution in [s] times F_s. Following this a Discrete Fourier Transform was done on each bin using the built-in Matlab function *fft* which uses the Discrete Fourier Transform as in 1, for details see [5] and [6]. The resulting frequency spectra were saved and visualized resulting in a frequency-time amplitude spectrum.

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi f_0 t} \tag{1}$$

In order to know the frequency shift over time, this raw data had to be filtered. For this the following algorithm was used: The range of clearly visible signal was specified, and the frequency with the maximum energy (hypothetically the carrier signal's frequency) was found within this range for each bin. To filter out the actual carrier frequency, the maximum energies were regressed with a 13th degree polynomial. This degree is chosen such that the regression will not catch too much noise (higher degree) but also maintains the rough signal shape (lower degree). Then, a searchwidth was defined around this polynomial. In this searchwidth a new maximum was selected for each bin. This procedure was then repeated with decreasing searchwidth according to equation 2. In this equation S [Hz] is the search range, S_0 [Hz] is the initial range, i [-] is the number of iterations, v [-] is the search speed and S_f [Hz] is the final range. The iterations were stopped when the averaged difference between two succeeding polynomials was less than 0.1 [Hz]. This resulted an initial filtered signal. The search speed, initial range and final range allowed for tuning of the search algorithm, which was needed when using different time resolutions.

$$S = S_0 e^{(i \cdot v)} + S_f \tag{2}$$

As a secondary filter, the amount of energy of the apparent center frequency (after the initial filter) was observed. Only the time bins with a clearly present maximum energy are passed through the secondary filter (the initial filtering could have converged to noise frequency close to the center frequency when there is no clear carrier frequency present in that time bin). The threshold for this filter was variable since it depended on the average energy of the carrier frequency. For a time resolution of 0.1 [s], the initial searchwidth was set to 7000 [Hz], final searchwidth to 10[Hz] and search speed to 0.6. The secondary threshold was set to 0.03 [J]. These values were acquired by tuning the parameters. This time resolution will also be used to evaluate the range-rate.

With the results of the the data filtering, the range-rate was calculated with equation 3 [7]. In this equation $\dot{\rho}$ is the range-rate [m/s], f_r [Hz] is the received frequency, f_t [Hz] is the transmitted frequency and c is the speed of light in vacuum which has a value of 299792458 [m/s]. The speed of light was assumed to be constant between the satellite and the ground station. The actual frequency data points were used instead of the polynomial regression to determine the range-rate.

$$\dot{\rho} = c(1 - \frac{f_r}{f_t}) \tag{3}$$

The last unknown was the transmitted frequency. In order to calculate this transmitted frequency the second derivative of the fitted 13th degree polynomial on the filtered data was set to zero. This was done because it was known that the received frequency equals the transmitted frequency when the range-rate was zero, and the range-rate is zero when the calculated second derivative of this fitted curve is also zero. The calculated second derivative had multiple roots due to noise at the end of the time interval. It was observed that the time of minimal distance should be between 400 [s] and 500 [s]. For this reason the root of the second derivative was searched in this interval. After obtaining the time of minimal distance the transmitted frequency was calculated using the fitted curve. This frequency was used to calculate the range-rate using relation (3).

Range-rate determination by United States Space Surveillance Network TLE data files

The range-rate of the satellite was also determined with another method, with the use of Two-Line Element (TLE) sets. The satellite orbit was propagated using third-party script *Satellite Orbit Computation* [8]. The TLE from epoch nearest to the desired time domain of the orbit was obtained from Space-Track [9]. This was then used to provide the software with the most up-to-date input variables. This TLE set is displayed in Fig. 2.

1 32789U 08021G 13043.23567977 .00002998 00000-0 33546-3 0 5568
2 32789 097.7832 108.6098 0013368 354.1491 037.6069 14.87584278259547

Fig. 2 – *This TLE, referenced to 2013-02-13 05:39:23 UTC, is used to propagate the orbit over future time. The TLE contains all the needed orbital parameters. Explanation of the different elements in the TLE can be found in literature [10].*

The orbit was propagated in the Moving Earth Centered Frame (ECF) on the time domain from 2013-02-13 11:00:02 UTC to 2013-02-13 11:15:02 UTC with a time step of 0.01 [s]. After that a check was performed by plotting the orbit on a textured 3D Earth image [11].

The GPS coordinates of the ground station were obtained and converted to Cartesian coordinates. GPS measurements were done with a GPS device next to the antennae of the ground station to obtain a latitude, longitude and height. The accuracy of the GPS device was $\pm$ 3 [m]. The built-in function *geocradius* gave the radius of the earth at the latitude of the ground station, based on the WGS-84 model [12]. The height of ground station is 95 [m] and was added to this radius. The station's position vector in the ECF in x,y and z coordinates, was calculated from the latitude, longitude, and the distance from the earth's center to the ground station using trigonometric relations for spherical coordinates.

A series of range vectors in x, y, and z direction was created by subtracting the ground station position vector from the series of satellite position vectors. The final range vector was obtained using relation 4, in which X, Y and Z are the range vectors in the Cartesian coordinate system. The range is denoted as [ρ] and range-rate as [$\dot{\rho}$]. The data from the resulting range vector is then differentiated, yielding the range-rate using relation 5.

$$\rho = \sqrt{X^2 + Y^2 + Z^2} \tag{4}$$

$$\dot{\rho} = \frac{\rho(i+1) - \rho(i)}{dt} \tag{5}$$

This produced the rate of change of the range between two points, at the time of the first point for each subsequent pair. This was done for all points, giving a series of range-rate data. This method produces range-rates in between two time points. This would cause a shift of the range-rate. To account for this shift the TLE will be propagated using a time step of 0.01 [s] and the range-rates are assigned to the correct time (i.e. the first range-rate value will be assigned to 0.005 [s]).

III. Results

Our first general observation was that the signal started (satellite came into view of the antenna) at 225 [s], and ended at 800 [s]. This could be obtained from the frequency spectrum over time. Therefore, only the data between 225 and 800 [s] was used to test the range-rate calculation method. Secondly, it was observed that after 325 [s] the signal was interrupted, as can be seen from Fig. 3.

Frequency spectrum over time

The sidelobes and center frequency could be distinguished from the rest of the data. These were recognised by the larger amounts of energy, as may be seen in Fig. 3. Furthermore, it was noticed that the gradient had its maximum between 400 and 500 [s]. According to the Doppler effect, the change in frequency is maximum when the relative velocity is minimum. Therefore, the point of zero distance should lie within this range of 400 to 500 [s]. Interesting to notice is the deviation of the dataset to the signal at approximately 250 [s]; the loss of the signal caused an error.

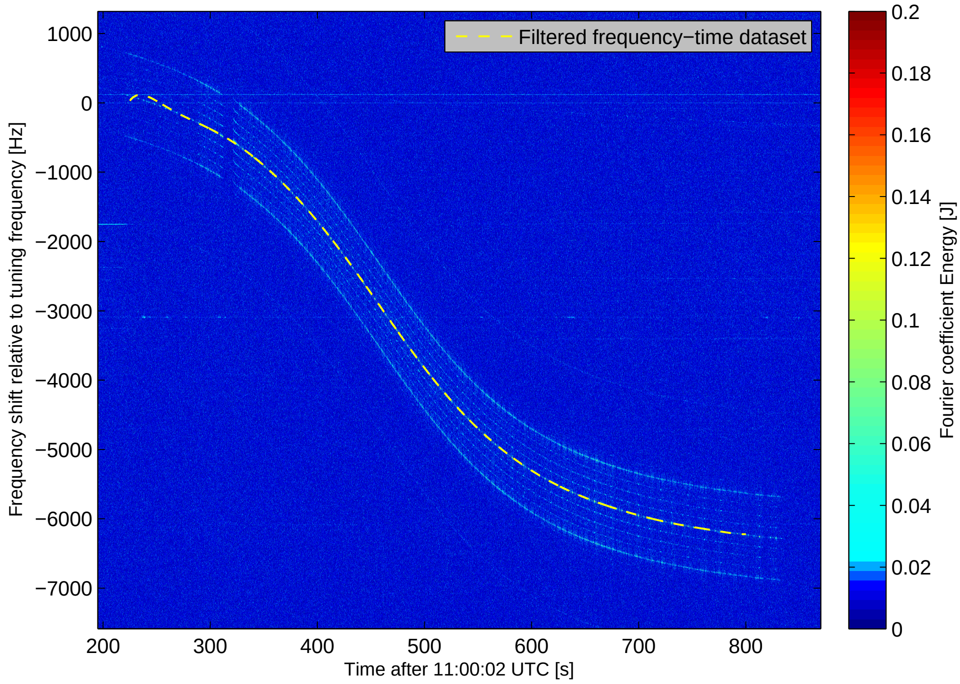

Fig. 3 – *The frequency spectrum plotted versus time. Different color codes are used for different magnitudes of the fourier coefficients. The dashed yellow line is the final fit of the frequency-time dataset. The frequencies are relative to the tuning frequency of 145.87 Mhz.*

The filtering process' first iteration polynomial fit may be seen as the red line in Fig. 4. From this figure, firstly, it may be noticed that the polynomial regression of the raw data still has a significant error. Secondly, it may be observed that as the amount of noise increases, the error also increases.

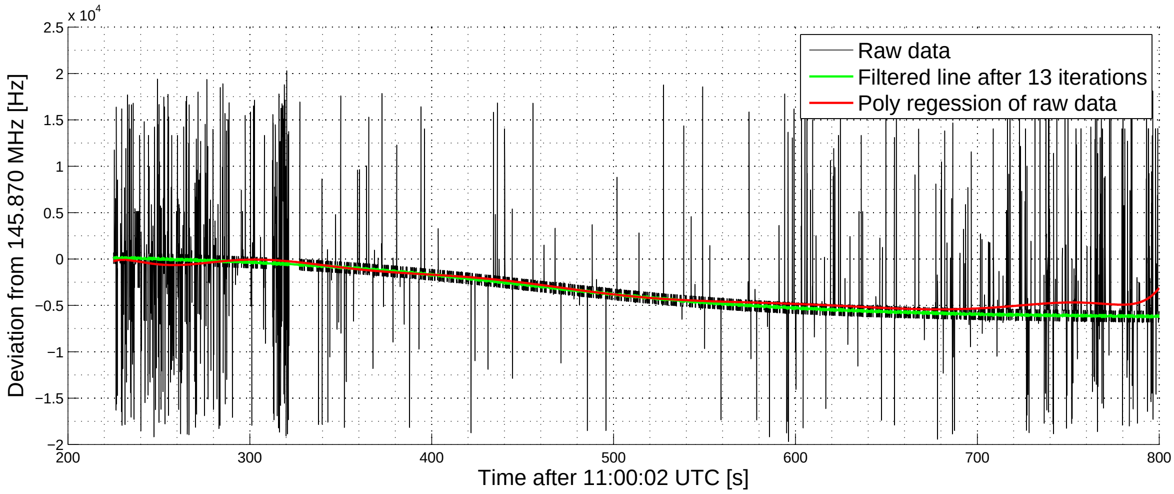

Fig. 4 – *The raw data, initial fit and final fit plotted versus time. The raw data consists of the frequencies with the maximum energy for each bin. The initial fit was done on the raw data with a 13 degree polynomial and the final fit was obtained after the data is filtered by using 13 iterations.*

After 13 iterations, the initial filtering process produced the green dataset in Fig. 5 (which is the same as the green line in Fig.4 . After the use of the secondary filtering process, the data points that are noise but lie on the line of the center frequency, were filtered out. This filtering process may be seen in Fig. 5. The data points marked in purple were found to be eligible data points and the yellow line is the resulting best fit of the frequency-time dataset. An indication for the quality of the fit can be extracted from the difference between the regression through this final dataset and the dataset itself. It has an average difference of $1.4E-5$ [Hz] and a standard deviation of 6.0 [Hz]

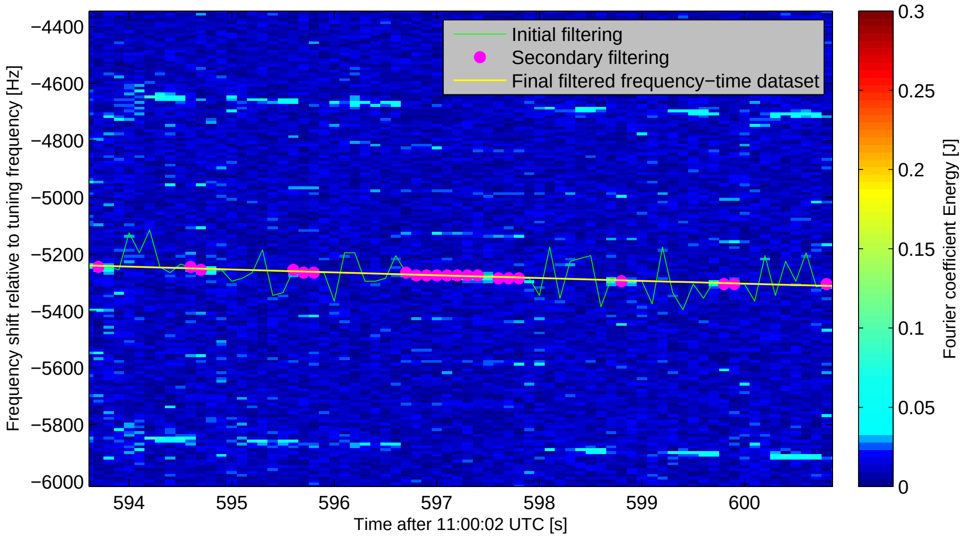

Fig. 5 – *The frequency-time spectrum, as obtained from Fig. 3, zoomed in at 594 up to 600 seconds after 11:00:02 UTC. The maxima that are obtained from noise in the initial filtering, were filtered out by the secondary filtering.*

Range-rate evaluation

The range-rate was obtained from the filtered frequency-time dataset. By plotting the range-rates obtained from the Doppler method and the TLE, Fig. 6 was obtained. The black line is the range-rate calculated from the TLE, and the blue line is the calculated range-rate by using the developed method.

It was observed that the time of minimum distance (t_0), was 458.2 [s] for the Doppler method, and 464.2 [s] for the TLE. Thus, the difference between the t_0 of the Doppler method and the one of the TLE, is 6 [s]. The time of minimum distance is at a range-rate of 0 [m/s]. If one would shift the t_0 of the Doppler method, such that it is equal to the one of the TLE, one would obtain the red line in Fig. 6.

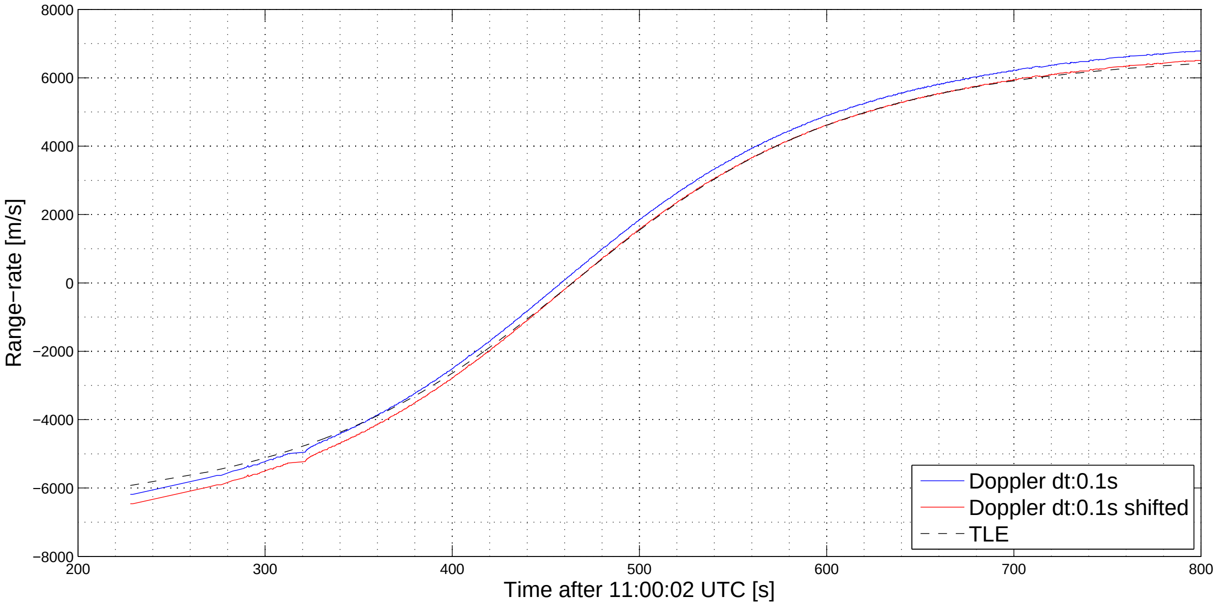

Fig. 6 – *The comparison between the TLE, the Doppler range-rate with and without t_0 shift. The shifted Doppler line was obtained by assuming a t_0 equal to the one of the TLE.*

Fig. 7 shows the difference in meters per second between the TLE, the shifted and unshifted Doppler range-rates. Firstly, it may be noticed that at 325 [s], the moment the signal is lost, the range-rate deviates significantly from the TLE's range-rate. Secondly, it may be observed that both the shifted and unshifted Doppler range-rate errors are more steady after t_0. For the original Doppler range rate, the deviation is about 300 [m/s] whereas the 'corrected' Doppler range rate shows an deviation of 50 [m/s].

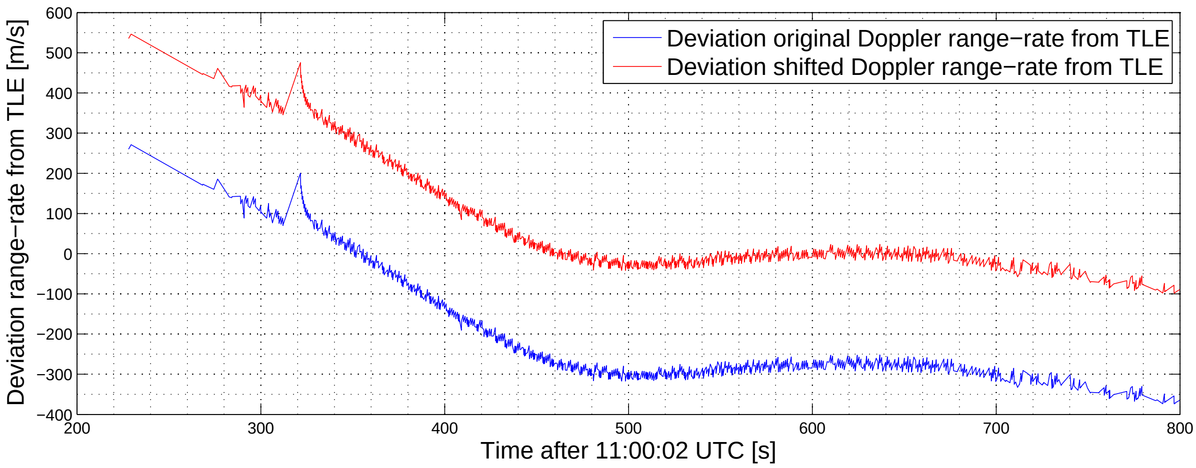

Fig. 7 – *The deviations of the Doppler range-rates from the one calculated by the TLE. The values of the range-rate of the TLE have been subtracted from the range-rates calculated using the Doppler method.*

Finally, if one would plot the gradient before and after 325 [s], one would see that these do not overlap. The gradient before the 325 [s] is shifted upwards by 40 [m/s]. The loss of the signal could be explained by the Delfi-C3 communication system shutting off. When this happens, the crystal in the Delfi-C3 (which ensures a constant clock tick) is slow in getting back to its specified frequency.

IV. Discussion

The first step in calculating the range-rate of the satellite using the Doppler effect on the carrier frequency was to filter the raw data. By using the two filter system, the actual frequency of the carrier signal over time can be extracted from the raw data with an accuracy of 2-3 times the frequency resolution. For a time resolution of 0.1 [s], the frequency resolution is 10 [Hz]. Increasing the time resolution beyond a certain point may eventually increase the error size. Also, extraction of the actual carrier frequency is limited by the frequency difference between the main and side lobes, for the Delfi-C3 this is 600 [Hz] (Fig. 5). This means that the theoretical limit of the time resolution would be $\frac{1}{600} = 0.002$ [s], because at this point the side lobe and main frequency can theoretically still be discerned. We were however unable to test this because at this time resolution the energy difference between the noise and the side lobes/carrier frequency was not large enough to be detected by the filtering setup. The best time resolution we could acquire was 0.05 [s], at smaller time intervals the filtering produced too much noise which resulted in large errors of approximately 1 [km/s] in the range-rate.

During the range-rate calculation, we found that this alternative method produces qualitatively sound results; the shapes of the two range-rates in Fig. 6 are very similar. However, we observed that our Doppler calculated range-rate showed a systematic deviation from the TLE range-rate of about 6 [s]. This implied that the Doppler calculation took the time of Delfi-C3's minimum distance to the ground station 6 [s] earlier than predicted by TLE. With an orbital velocity of about 7.5 [km/s], this gives a position error of 45 [km]. A method that results in a position error of 45 [km] is not accurate enough to be useful for most applications, making it an invalid alternative. This error source needs to be investigated further.

In the Doppler shift calculations, assumptions where made concerning the signal propagation through the atmosphere. The speed of light was assumed to be the speed of light in vacuum, 299,792,458 [m/s]. In reality the speed of light varies and is slower than light in a vacuum due to the atmosphere.The signal also bends as it enters the atmosphere because of tropospheric and ionospheric refraction. The total time delay due to change of medium was found to be in the order of tens of microseconds for the troposphere and hundreds of microseconds for the ionosphere. This leads to a maximum total error in the magnitude of $\pm$ 15 [m/s] for the Doppler range-rate data.

In the calculations that were done in this study, the data was fitted with a curve, which introduced approximation errors. The second derivative was taken of this curve, meaning that the transmitted frequency used in the calculations is not the actual transmitted frequency. This will alter the values of the calculated range-rate. The standard deviation of the fitted curve on the filtered frequency data was 6 [Hz]. This standard deviation is quite low, which means that the error will be relatively low.

In trying to rule out these errors, we determined the t_0 from the TLE calculations and manually entered this into the Doppler range-rate calculation. The results are shown in Fig. 7. At the t_0 of 464 [s], the difference in range-rate is logically 0 [m/s], however at the rest of the time the difference is in the order of 100 [m/s]. At the beginning of the time interval 225 [s] to about 464 [s] the error is notably bigger, about 300 [m/s]. This could be explained by polar atmosphere characteristics (radiation, magnetic field), exact numbers were not looked into. In the rest of the signal, 464 [s] to 800 [s], the approximation is better. The difference between the two range-rates is there about 50 [m/s].

Not only in the Doppler calculated range-rate errors could be made. The calculations using the TLE data also contain errors. The position of the ground station was measured using a GPS receiver that was accurate to 3 [m], leading to a maximum error of 3 [m] for the ground station's location. This error will propagate to the final results. Another possible error source comes from the TLE data itself. The data was obtained by taking orbital elements at a certain time instance and using it to predict the orbit, this leads to errors in the prediction.

The Doppler method is not yet accurate enough to be feasible for satellite positioning. The shift error of 6 [s] needs to be researched further, this error could be improved with better ground station equipment in order to minimize the error between observation and filtering/computations (e.g. more accurate F_s and better linking of time of recording) The other discussed errors should be (further) quantified to assess their contributions and to allow fitting solutions to be found. Using a model of the ionospheric and tropospheric delay would lower the time delay error. Furthermore, using a model for the polar atmosphere characteristics would improve the range-rate results. The filtering process has a low standard deviation, 6 [Hz], which means it is useful for future use at time resolution up to 0.05 [s].

V. Conclusion

The feasibility of using the Doppler tracking as a (alternative) method for range-rate determination was investigated, which might be a first step for a new orbit determination and tracking method. Therefore, the range-rate of the Delfi-C3 relative to the ground station was determined. The calculation produced an error of 6 [s] in estimating the time of minimum distance to the ground station and thereby a large 300 [m/s] error in the range-rate, compared to the values obtained from the TLE method. Thus it can be concluded that the method is not feasible. However, when this error was managed to be ruled out, the Doppler method calculated range-rate differed much less from the one obtained from the TLE method. From this it follows that the Doppler method might be a feasible alternative method if the time error would be reduced. The final filter of the proposed method extracted the carrier frequency of the Delfi-C3 with a standard deviation of 6 [Hz]. This result shows that the filter of the proposed method processed the raw data adequately, and has a high potential for future use. To make a final statement about the feasibility, the source of the time error needs to be investigated further by quantifying all possible errors. Besides that, the theoretical limit time resolution was not researched since the designed filters were not capable enough. Therefore, better filters might be used as a recommendation to get closer to the theoretical limit of the time resolution.

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