Determining the accuracy of range-rate solutions from Doppler tracking compared to the NASA TLE solution

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Abstract

Satellites can be tracked in their along track direction using Doppler tracking by determining the range-rate of a satellite, which is the variation of the range with respect to time. Measurements were performed to determine how accurate the range-rate from Doppler tracking is compared to NASA Two-Line Element (TLE) solutions, using the TU Delft satellite Delfi-n3Xt. This is done by using a Fourier transform on the waveform audio file that was obtained from the emitted signal of the Delfi-n3Xt during a single pass. From this, the actual shifted carrier frequency from the satellite was subtracted. Then a range-rate is calculated using the Doppler formula.
Along with this range-rate, another range-rate is obtained from the NASA TLE. Taking the difference between these two datasets results in an error of the Doppler tracking range-rate with respect to the NASA TLE range-rate. It shows that the accuracy of Doppler tracking is not yet high enough to substitute the NASA TLE range-rate solutions, which could be accounted to atmospheric delays.

Keywords: Delfi-n3Xt - Satellite tracking - Doppler tracking - NASA TLE - Range-rate

Abbreviations & Symbols

DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite
ERA	Earth Rotation Angle
GEO	Geostationary Earth orbit
GPS	Global Positioning System
LEO	Low Earth orbit
MEO	Medium Earth orbit
NASA	National Aeronautics and Space Administration
NORAD	North American Aerospace Defense Command
SGP4	Simplified General Perturbation model 4th generation
TCA	Time of Closest Approach
TLE	Two-Line Element

a	equatorial radius
b	polar radius
c	speed of light
f	received frequency
f_0	transmitted frequency
T_u	time in Julian days from the year 2000
Δv	velocity of the source relative to the receiver
φ	geodetic latitude

I. Introduction

The most common-used method to determine the location of a satellite is either the Global Positioning System (GPS) or the NASA Two-Line Element (TLE) solutions. TLE is a standard originally specified by the North American Aerospace Defence Command (NORAD) and specifies the orbital elements that determine how the satellite will move through time. However, there are some disadvantages to these systems prediciting the satellites' orbits. One disadvantage of the GPS-system is its limited possible use. Since GPS-satellites are in (high-) Medium Earth Orbit (MEO) the GPS-network can only be used to track Low Earth Orbit (LEO) and lower MEO-satellites. Satellite orbits that have a larger semi major axis cannot be tracked by the GPS-satellites, simply because their communication equipment cannot face the GPS satellites at the other side of the Earth. Furthermore a GPS system adds complexity to a satellite and consumes power.

The NASA TLE system is used in combination with a 4th generation Simplified General Perturbation model (SGP4). The SGP4 model is a model that can predict the position of a satellite through time using a single data point containing a position, velocity and orbital decay. Using radar systems, NASA determines the position of the satellite in space at multiple times. With these measurements NASA can calculate the orbital decay more accurately and improve the accuracy of position and velocity predictions [1]. The orbital elements that fully determine the orbit of a satellite (inclination, eccentricity, semi-major axis, longitude of the ascending node, argument of periapsis and mean anomaly), are combined with the orbital decay information and current position of the satellite in the standard TLE format. This TLE dataset can then be used to propagate the state at that time through time to find the position and velocity of the satellite at any given time [2]. One problem of the TLE data is that for it to be accurate, it requires position measurements of several times, which means the TLE data is not accurate the first couple of days after launch. Also, TLE data that is propagated back in time results in more accurate position predictions than that propagated forward in time [3]. Therefore, the orbital data found for past dates from the TLE data is a good point of reference to compare other methods to. The fact that TLE solutions are made available only once a day makes real time tracking inaccurate.

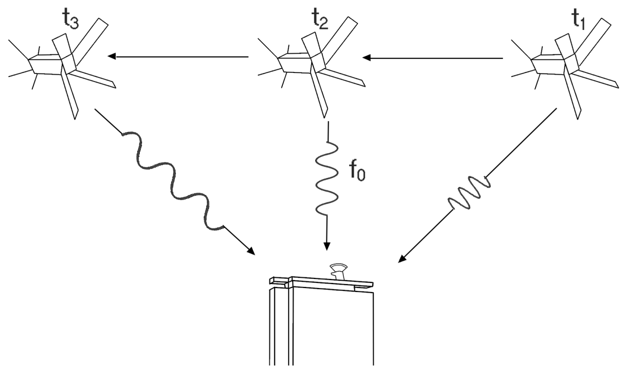

Figure 1: *Doppler tracking is a way of tracking a signals frequency, which is higher when the source is approaching (t_1) and lower when the source is distantiating (t_3). At t_2 the signal is the unshifted transmitted frequency (f_0).*

Doppler tracking is an alternative way of determining the orbital track by determining the satellite velocity relative to the signal receiver. A signal can be sent from a ground station to a satellite or vice versa. Comparing the transmitted frequency (f_0) of the signal to the frequency as it is being received the radial component of the satellite's velocity relative to the ground station can be determined: a received frequency higher than the transmission frequency means the satellite is moving towards the ground station while a lower frequency implies that the satellite is travelling away from the station. A received frequency that is equal to the transmitted frequency means the satellite has no radial velocity with respect to the ground station, this happens at the Time of Closest Approach (TCA). This can be seen in Figure 1. When the range-rate of the satellite, calculated using Doppler tracking, is found with respect to multiple ground stations, the orbit and location of the satellite can be determined. One typical system employing this type of satellite tracking is the DORIS network [4].

Doppler tracking can also be used to calculate the range-rate and therefore the along track position of a satellite, using the data from a single pass. This article will verify how accurate this method of determining the satellite's range-rate is, in comparison with the aforementioned NASA TLE solutions.

II. Description of the data

The signal of one pass of the Delfi-n3Xt over the Delft groundstation was recorded. The pass occurred on 21 November 2013 from 10:16:46 till 10:38:19 UTC. The data was filtered of any frequencies that are not of interest. After that, the resulting frequencies that were centered on the theoretical carrier frequency of the satellite, were now centered on 0 Hz. The obtained spectrum are the frequencies relative to the theoretical carrier frequency. Negative frequencies, when converted to an amplitude-time signal, result in a complex signal. This complex signal is stored in a .wav waveform audio file, where the left channel represents the real part, and the right channel represents the imaginary part of the signal.

To be able to get insight into the accuracy of the computed range-rate, reference data of the NASA TLE is used. The TLE data is stored in a database which can be found on www.space-track.org [5]. Since there was no TLE data available for 21 November the data of 23 November was used. Data from 21 November is missing because the Delfi-n3Xt was launched on this day and the lack of data points hinders NASA from publishing the TLE solutions. The data of 24 and 25 November was used to calculate the uncertainty in the prediction. Using the combination of these three TLE data sets, the position given by the NASA TLE was propagated back in time using the SGP4 model.

III. Method

In order to compare the Doppler tracking method with the NASA TLE-solution range-rates have to be calculated over time for both methods. For the Doppler tracking this process is described in subsections A, B, and C. The method for calculating the range-rate using TLE data can be found in subsections D and E.

A. Converting intensity-time signal to intensity-frequency

To calculate the Doppler shift an intensity-frequency format was needed, which was done using a Discrete Fourier Transform (DFT). This is a mathematical tool that allows to find the intensities of different frequencies in an interval of a intensity-time signal [6]. The signal was divided in pieces of 1 second long. For every second, the DFT gave a spectrum where the signal of the satellite was clearly visible. Plotting the spectrum over time, a waterfall plot was constructed which can be seen in Figure 3. In this plot the signal can be seen as a constant band of high intensity, roughly 2.7 kHz wide. The center of this band contains the real received signal. The frequency shift due to the Doppler effect is the difference between the 0 Hz frequency of the plot and the signal.

B. Signal subtraction

From the spectrum representation of the signal it is possible to subtract the received satellite signal as a frequency per time-interval. A band of 2.7 kHz with the highest sum of intensities

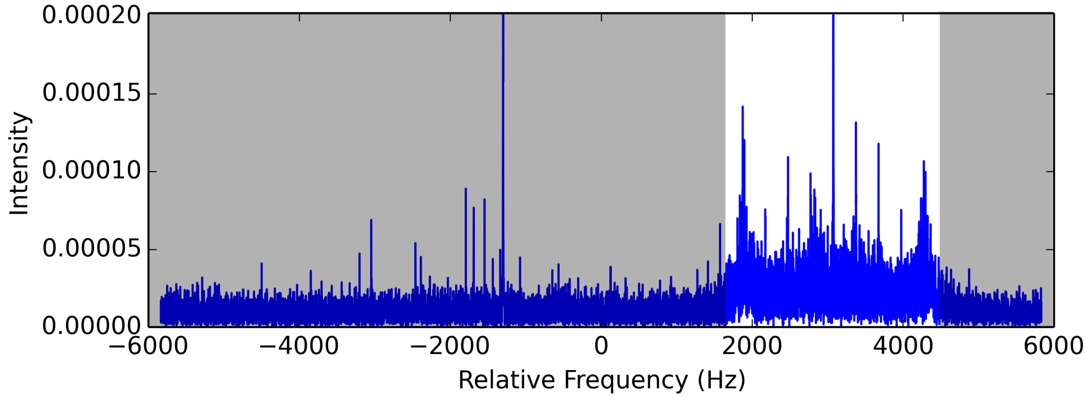

Figure 2: *Time interval at 720 seconds, showing the Fourier transformed signal and showing the bandwidth of 2700 Hz.*

is found in each time-interval. The band of 2.7 kHz is chosen because the signal is 2.7 kHz wide (Fig 2). This interval is searched for the maximum intensity peak, which represents the received satellite signal in that time interval. Doing this for all time intervals the received satellite frequency per time interval can be computed.

C. Generating range-rates

From the filtered received frequency-interval data the range-rate with reference to time can be computed. On a given time the received satellite frequency is linked to the emitted frequency of the satellite by Doppler's Eq. 1 [7]. In which f is the received frequency, f_0 is the carrier frequency and ΔV is the range-rate, which is the unknown factor. This formula assumes the speed of light in vacuum and no relativistic influences.

$$f = (1 + \frac{\Delta v}{c}) f_0 \tag{1}$$

For the carrier frequency three different values are used. The first value is the theoretical carrier frequency, which is the frequency theoretically emited by the satellite, 145.87 MHz. The second one is the frequency at which the range-rate following from the Doppler tracking is zero. A third carrierfrequency value is found at the TCA of the TLE solution.

D. Uncertainties in the TLE data

For the range-rate as given by the SGP4 model, TLE data sets are used from 23, 24 and 25 November. These data sets are propagated to the time of the satellite pass. The difference between these three solutions allows to calculate the uncertainty of a TLE solution.

The difference between analogous position vectors of two consecutive days can be expressed as the total uncertainty. This uncertainty can be expressed in three different directions: radial direction, crosstrack direction, and alongtrack direction. The along-track uncertainty was obtained by multiplying the difference in true anomaly with the orbital radius [8]. The radial uncertainty was obtained by computing the difference in the radii of two consecutive TLE data sets, when comparing analogous points in time. The crosstrack uncertainty was obtained by Eq. 2, here ε

represents the uncertainty.

$$\varepsilon_{total}^2 = \varepsilon_{crosstrack}^2 + \varepsilon_{alongtrack}^2 + \varepsilon_{radial}^2 \tag{2}$$

E. TLE range-rate

The TLE data file contains the position and velocity of the Delfi-n3Xt obtained from the NASA TLE in the J2000 reference system. This was first converted from the J2000 to an Earth fixed reference system using Eq. 3 [9].

$$ERA(T_u) = 2\pi(0.7790572732640 + 1.00273781191135448 T_u) \tag{3}$$

In this same Earth fixed reference system the position of the ground station was calculated using elementary algebra. This was done by calculating the radius of the Earth and adding the height of the ground station (95 m). The radius of the Earth at the position of the ground station was calculated using Eq. 4.

$$R = R(\varphi) = \sqrt{\frac{(a^2 \cos\varphi)^2 + (b^2 \cos\varphi)^2}{(a \cos\varphi)^2 + (b \cos\varphi)^2}} \tag{4}$$

Using these two positions and the velocity of the satellite, the range-rate was determined. In Eq. 4, R is the radius of the Earth as a function of geodetic latitude, φ. The a resembles the equatorial radius and b the polar radius.

IV. Results

As a result of the simulation and analysis four range-rates are computed. The first one being the range-rate of the satellite from the NASA TLE data (TLE) and the second the range-rate of the satellite calculated from the Doppler shift in the signal of the .wav file using the theoretical carrier frequency of 145.87 MHz (Model 1). The third is the range-rate of the satellite, determined by now using a different carrier frequency, obtained by using the frequency at the calculated TCA of the satellite as the carrier frequency (Model 2). The fourth being the range-rate of the satellite calculated by overlaying the TCA of the TLE and the Doppler curves (Model 3). These four range-rates are plotted in Figure 4. With this, the error is found by calculating the absolute difference between the NASA TLE range-rate and the other three satellite range-rates separately, this is plotted in Figure 5.

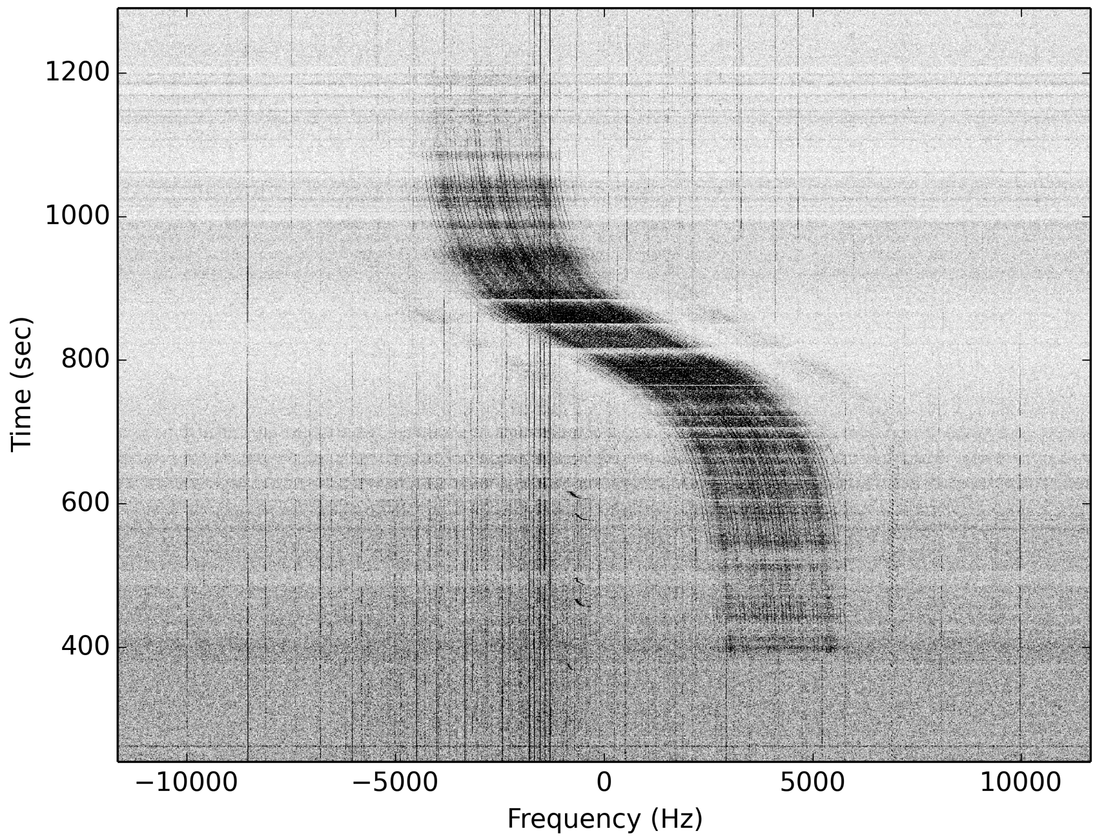

Figure 3: *2D waterfall plot of the signal versus time of a single pass of the Delfi-n3Xt, Time in seconds on the y-axis and frequency in Hz on the x-axis. The plot is centered on 145.87 MHz.*

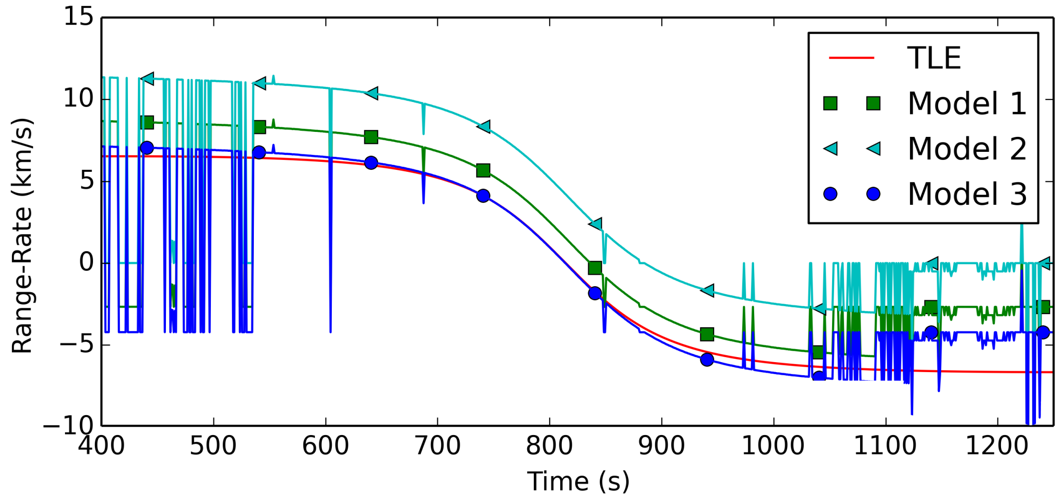

Figure 4: *Range-rate versus time of the Delfi-n3Xt by NASA TLE (unmarked line), Doppler tracking (model 1), Doppler tracking using a different carrier frequency (model 2) and Doppler tracking shifted using the TCA of the TLE (model 3).*

By using the TLE data of 23 November an uncertainty exists, this uncertainty will vary over time. The uncertainty in the along-track direction has an average value of 0.63 km, with a standard deviation of 0.16 km. In the radial direction the average uncertainty is 0.13 km, with a standard deviation of 0.07 km. The value of the uncertainty in the cross-track direction is negligible.

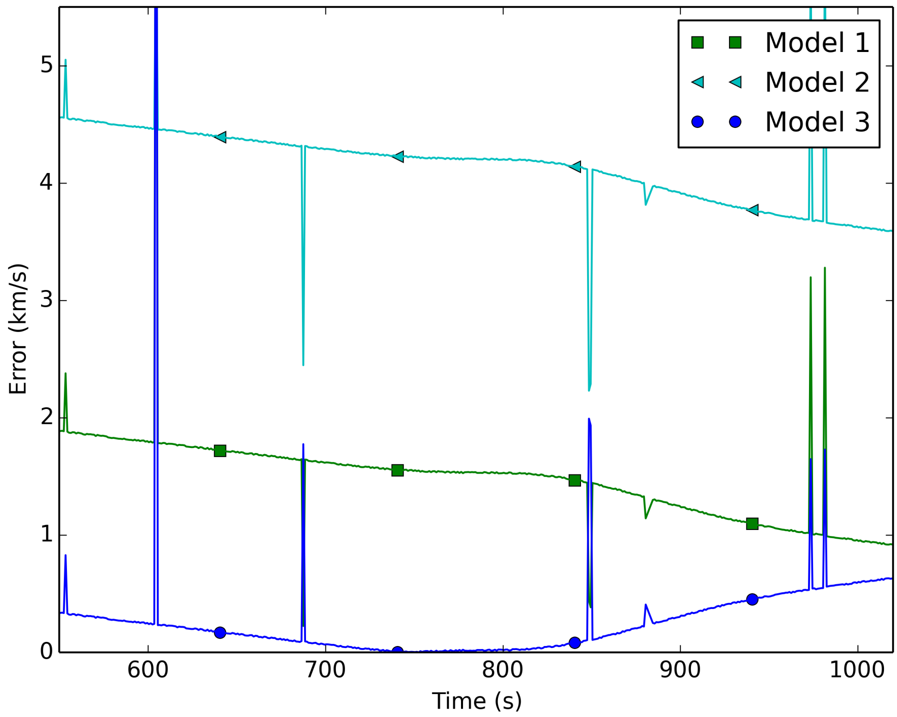

Figure 5: *Difference in range-rate of the Delfi-n3Xt by NASA TLE and different calculated Doppler tracking models.*

V. Discussion of the results

In Figure 3 activity can be seen on a constant band all along the time domain. This could be caused by an interferential broadcast of an unknown third party. This is possible because the signal sent by the Delfi-n3Xt is located in the band of 144-146 MHz, a frequency band allocated to amateur radio broadcasts [10]. The Doppler shift due to range-rate is clearly visible in Figure 3 as denoted by the S-shaped curves of the measured intensities. Multiple intensity bands occur simultaneously although only one signal is sent by the Delfi-n3Xt. This is due to frequency intermodulation effects [11]. In Figure 4 it can be seen that between 400 and 600 seconds the dominating frequency alternates between the satellite frequency and a constant interference frequency. This interference frequency is also shown in Figure 3. Using the method outlined in Ref. [12] it was found that the frequency shift due to relativity effects at a range-rate of 8 km/s is only 0.5 Hertz. Compared to the frequency shifts seen in Figure 3 this is negligible. Between 600 and 1100 seconds the graph follows the satellite signal, except for two spots around 850 seconds. From Figure 3 it can be seen that around this time the signal intensity fades out periodically. This might be due to sender and receiver pointing offset losses. During the time when the antenna does not receive the full signal intensity the frequency with the highest intensity is the same interference frequency as found between 400 and 600 seconds.

As can also be seen from Figure 4 there is a discrepancy between the experimental data and the range-rate deduced from the TLE measurements. Several causes might be present: relativity, atmospheric refraction, Earth's magnetic field activity, solar activity and noise. Solar activity and magnetic storm activity were low on November 21 of 2013 [13]. From this it can be deduced that

the ionospheric activity on that day was not abnormally high. Nonetheless the ionosphere induces an additional frequency shift on the signal, because of the refraction of the radio waves [14].
In Figure 5, the error between the TLE and the improved experimental Doppler values decreases over time. A cause of this might be atmospheric and ionospheric interferences. The atmospheric interference is a minimum at the TCA, whereas the ionospheric interference keeps reducing over time as the satellite moves towards the equator. The ionospheric error is larger when the satellite flies over the magnetic poles of the Earth since the electron density at the poles is larger than at other parts of the Earth [15].

The error of model 1 is 1-2 km/s, the error for model 2 is 3-5 km/s and the error for model 3 is at most 1 km/s (fig 4). Model 1 is the original model, obtained from the data. However, this model still has a significant error, it has the same curvature as model 3 but is on each point in time shifted by 1-2 km/s upwards. Therefore model 2 was produced to come up with a better estimation, using the frequency of model 1 at the TCA as the carrier frequency. However, this model seems to have a larger error, it is shifted by 2-3 km/s upwards. Model 3 was produced overlaying the TCA of the TLE and the experimental data. This model shows that the data has an increasing error of atmost 1 km/s the further the satellite is distant from the groundstation, this could be accounted for by including the effects of atmospheric interference. So the biggest error between model 1 and model 3 is the shift, this error originates from a different received signal frequency. This is the only variable which can alter the range-rate (equation 1). The received frequency is shifted by atmospheric refraction.

VI. Conclusion

As a conclusion on the determination of the accuracy of the range-rate solutions using Doppler tracking compared to NASA TLE, it can be said that Doppler tracking is not as accurate as the NASA TLE method. The data from model 1 shows a difference of 1-2 km/s on the range-rate, this is too much for satellite applications like GPS. However because this difference is a fixed amount on each point in time, it is possible to substract this difference (obtaining model 3) and use this data for low accuracy demanding applications like getting an estimation on the position of a satellite in the along-track direction.

It is recommended to analyse the data of more passes of the Delfi-n3Xt, so systematic errors can be detected with greater confidence. Like DORIS, Doppler tracking can be used in accuracy demanding satellite application, but more passes of a satelite are needed [4]. Additionally, detailed models can be implemented to account for errors caused by atmospheric refraction and other disturbances. Models like the Klobuchar model for the ionospheric refraction [16] combined with the methods presented in Ref. [14] are suggested to be used in further research.

VII. Acknowledgments

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