CP1: Structure of the Rational Numbers

Notation

1. Draw a rectangle to represent a garden. Shade in $\frac{3}{5}$ of the garden. Without changing the shading, show why $\frac{3}{5}$ of the garden is the same as $\frac{12}{15}$ of the garden.

Comparison

1. Argue, in terms of the meaning of "whole", "numerator", and "denominator" why cross-multiplication "works" to decide which of two different positive fractions is the greater.
2. Give examples of pairs of fractions for which cross-multiplication is unnecessary to determine which is greater.

Operations

The following stories involve the integers 20 and 4. Keeping the stories the same, replace the number 20 by the number $\frac{9}{11}$ and replace the 4 with the number $\frac{2}{7}$ and comment on whether it makes logical, if not necessarily practical, sense. Now switch the replacements, i.e. replace 20 with $\frac{2}{7}$, etc.., and repeat the exercise.

1. Fred has 4 gallons of gas in his tank, which has a capacity of 20 gallons. If he fills it up today, how many gallons will he buy?
2. Fred is currently $20 in debt and he receives a check for $4. What is Fred's financial worth now?
3. A cake recipe calls for 20 cups of flour. If Susie is going to make 4 recipes, how much flour will she need?
4. Each container of sludge holds 4 gallons. If we must store 20 gallons of sludge, how many containers will we need?
5. We have 20 gallons of sludge to store. If 4 equal-sized sludge containers exactly store all the sludge, how many gallons of sludge will fit into one container?

Structure

What algebraic properties do the rational numbers enjoy that the integers do not?

Order and density

1. Try to find the largest rational number smaller than $\frac{3}{7}$. Explain why you can or can't do so.
2. How many rational numbers are there between $\frac{3}{7}$ and $\frac{4}{7}$? Find 3 of them.