NUMBERS AND ALGEBRA (SUPPLEMENTS)

MATH 1165: AUTUMN 2014
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Contents

A Supplemental Activities

A.52 There Are Many Factors to Consider

Suppose we want to know how many factors 43, 560 has, but we don't need to list them all. In this activity we develop a method for computing the number of factors of a number.

A.52.1) Vic listed the following factors of 80: 1, 2, 4, 5, 8, 10, 20, 40, 80. What is missing? Describe your method of checking the list.

A.52.2) Factors of 60.

(a) List the factors of 60.

(b) Describe your method for ensuring that you have found all of the factors.

(c) Consider the prime factorization of 60 and the prime factorization of each of the factors. Describe what you notice.

A.52.3) Factors of 3^7.

(a) Is 2 a factor of 3^7? Say how you know.

(b) Is 7 a factor of 3^7? Say how you know.

(c) List all the factors of 3^7.

(d) Generalize: If p is a prime number, how many factors does p^n have? Explain your reasoning.

A.52.4) List all the factors of $3^2 \cdot 7^3$. Use your example to reason about the number of factors of $p^n q^m$, where p and q are both prime numbers.

A.52.5) List all the factors of $3^2 \cdot 7^3 \cdot 11$. Use your example to reason about the number of factors of $p^n q^m r^s$, where p, q, and r are all prime numbers.

A.52.6) How many factors does $43,560$ have? Use the example of $43,560$ to describe and explain a method for computing the number of factors that a number has.

A.53 Second Differences

In a previous activity, we developed strategies for finding the sum of arithmetic series. In this activity, we use arithmetic series to develop a formula for a sequence that has constant second differences. Then we demonstrate that all quadratic sequences have constant second differences.

A.53.1) Consider the sequence $f(n)$ given in the table below. In the rightmost column, Δ ("delta") means difference, computed by subtracting the current value of $f(n)$ from the next.

n	$f(n)$	Δ
0	4	3
1	7	3
2	10	3
3	13	3
4	16	3
5	19	

(a) Explain how $f(5)$ can be computed from the shaded cells in the table.

(b) Generalize your method to develop and explain a formula for $f(n)$.

(c) What was it about the differences that made this problem easy?

A.53.2) Consider the sequence $g(n)$ given in the table below.

n	$g(n)$	Δ	$\Delta\Delta$
0	1		
1	−2		
2	1		
3	10		
4	25		
5	46		
6	73		

(a) Compute Δ by subtracting the current value of $g(n)$ from the next.

(b) Explain the formula $\Delta(n) = g(n+1) - g(n)$.

(c) Check that the shaded cells sum to $g(5)$, and explain how that makes sense based upon how the Δ values were calculated.

(d) Because the Δ values ("first differences") are not constant, use the $\Delta\Delta$ column to compute the "differences of the differences" (also called "second differences").

(e) From the fact that the second differences are constant, develop an explicit formula for Δ in terms of n.

A.53.3) The same sequence $g(n)$ is given below, this time with a formula for Δ in terms of n.

n	$g(n)$	$\Delta(n) = 6n - 3$
0	1	−3
1	−2	3
2	1	9
3	10	15
4	25	21
5	46	27
6	73	

(a) Explain each of the following steps:

$$\begin{aligned} g(5) &= 1 + \Delta(0) + \Delta(1) + \Delta(2) + \Delta(3) + \Delta(4) \\ &= 1 + (6 \cdot 0 - 3) + (6 \cdot 1 - 3) + (6 \cdot 2 - 3) + (6 \cdot 3 - 3) + (6 \cdot 4 - 3) \\ &= 1 + 6 \cdot (0 + 1 + 2 + 3 + 4) + (-3 + -3 + -3 + -3 + -3) \\ &= 1 + 6 \cdot \frac{5 \cdot 4}{2} + 5 \cdot (-3) \end{aligned}$$

(b) Where do you see arithmetic series in the calculations you just explained?

(c) Generalize the above approach to yield an expression for $g(n)$.

(d) What kind of sequence is $g(n)$?

A.53.4) A general quadratic sequence $h(n)$ is given below.

n	$h(n) = an^2 + bn + c$	Δ	$\Delta\Delta$
0			
1			
2			
3			

(a) Compute the values of $h(n)$.

(b) Compute Δ by subtracting the next value of $h(n)$ from the current.

(c) Use the $\Delta\Delta$ column to compute the second differences.

(d) Generalize the result for first differences by computing $\Delta(n) = h(n+1) - h(n)$.

(e) Generalize the result for second differences by computing $\Delta\Delta(n) = \Delta(n+1) - \Delta(n)$.

(f) Explain how your work demonstrates that, for any quadratic sequence, the second differences must be constant.