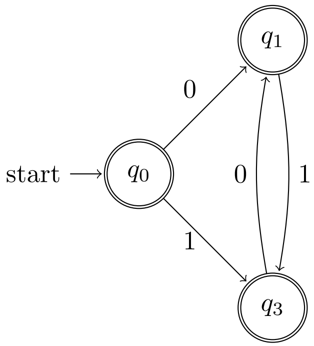
CS 302 Recitation 1

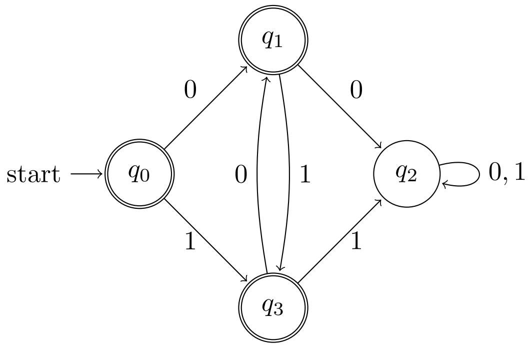
October 12, 2020

0.0.1 Some Notation

- State with double circle corresponds to a final state.
- ε^*: $ECLOSE$ function from lecture slides. For simplicity, we use a set as input, instead of calling function for each state and take union of results.For example, $\varepsilon^*(\{q_0, q_1\})$ means $ECLOSE(q_0) \cup ECLOSE(q_1)$

Problem 1.1. For alphabet $\Sigma = \{0, 1\}$, design a DFA/NFA accepts all strings that consecutive letters are never the same.

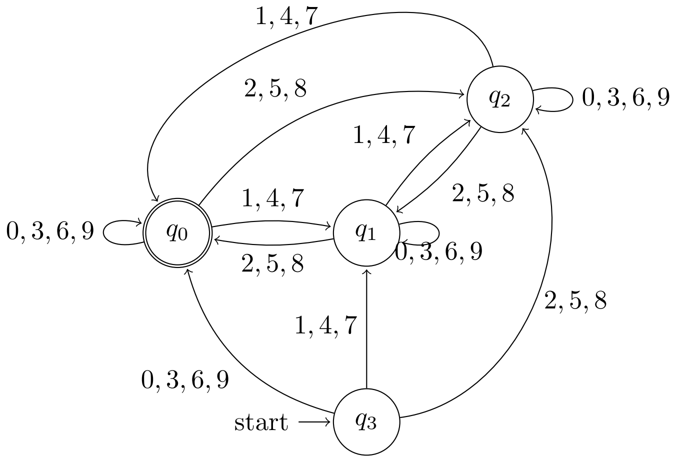




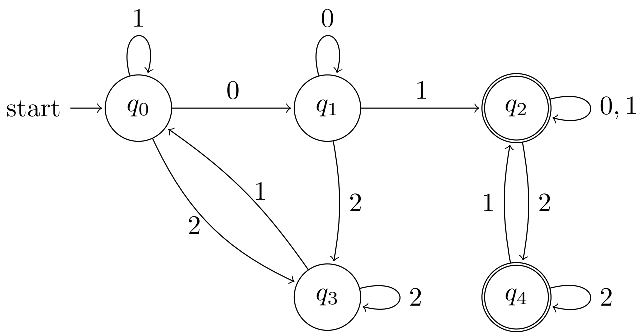
Problem 1.2. Give the transition table for DFA.

	0	1
$\rightarrow *q_0$	q_1	q_3
$*q_1$	q_2	q_3
q_2	q_2	q_2
$*q_3$	q_1	q_2

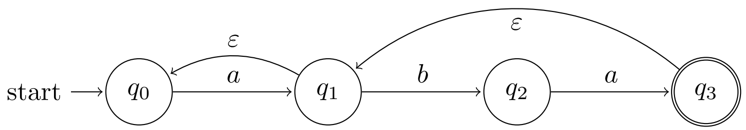
Problem 2. For the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, draw a DFA accepts the language $A = \{x \mid x \in \Sigma^* \ and\ x \mid 3\ \}$



Problem 3. For alphabet $\Sigma = \{0, 1, 2\}$, design a DFA/NFA accepts all strings that includes "01", but does not include "20".



Problem 4.1. Eliminate the ε-transitions from the given ε-NFA.

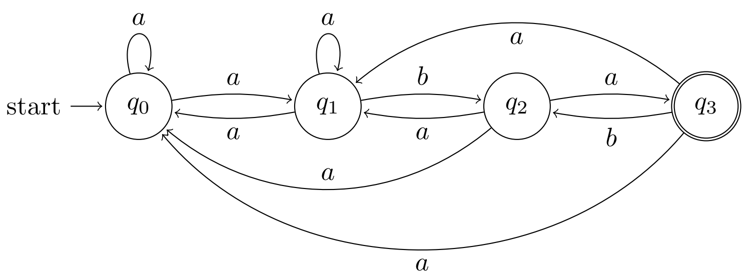


1. - $\varepsilon^*(\{q_0\}) = \{q_0\} = S$
     $\delta(S, a) = \{q_1\} = S_a$
     $\varepsilon^*(S_a) = \{q_0, q_1\}$
     $\delta(S, b) = \emptyset$

2. - $\varepsilon^*(\{q_1\}) = \{q_0, q_1\} = S$
     $\delta(S, a) = \{q_1\} = S_a$
     $\varepsilon^*(S_a) = \{q_0, q_1\}$
     $\delta(S, b) = \{q_2\} = S_b$
     $\varepsilon^*(S_b) = \{q_2\}$

3. - $\varepsilon^*(\{q_2\}) = \{q_2\} = S$
     $\delta(S, a) = \{q_3\} = S_a$
     $\varepsilon^*(S_a) = \{q_0, q_1, q_3\}$
     $\delta(S, b) = \emptyset$

4. - $\varepsilon^*(\{q_3\}) = \{q_0, q_1, q_3\} = S$
     $\delta(S, a) = \{q_1\} = S_a$
     $\varepsilon^*(S_a) = \{q_0, q_1\}$
     $\delta(S, b) = \{q_2\} = S_b$
     $\varepsilon(S_b) = \{q_2\}$

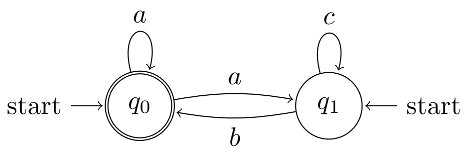


	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\emptyset$
q_1	$\{q_0, q_1\}$	$\{q_2\}$
q_2	$\{q_0, q_1, q_3\}$	$\emptyset$
$*q_3$	$\{q_0, q_1\}$	$\{q_2\}$

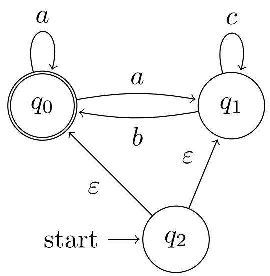
Problem 4.2. Create an equivalent DFA for the NFA in the previous step.

1. - $\delta(\{q_0\}, a) = \{q_0, q_1\} = S_1$
   - $\delta(\{q_0\}, b) = \emptyset$
2. - $\delta(S_1, a) = \{q_0, q_1\} = S_1$
   - $\delta(S_1, b) = \{q_2\} = S_2$
3. - $\delta(S_2, a) = \{q_0, q_1, q_3\} = S_3$
   - $\delta(S_2, b) = \emptyset$
4. - $\delta(S_3, a) = \{q_0, q_1\} = S_1$
   - $\delta(S_3, b) = \{q_2\} = S_2$

	a	b
$\rightarrow S_0$	S_1	S_4
S_1	S_1	S_2
S_2	S_3	S_4
$*S_3$	S_1	S_2
S_4	S_4	S_4



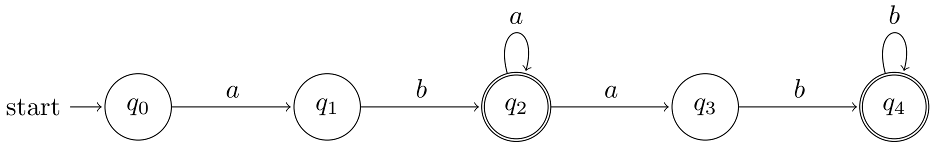
Problem 5. Create a DFA equivalent for the given NFA.



1. • $\varepsilon^*(\{q_2\}) = \{q_0, q_1, q_2\} = S_0$
   $\delta(S_0, a) = \{q_0, q_1\} = S_t$
   $\varepsilon^*(S_t) = \{q_0, q_1\} = S_1$
   $\delta(S_0, b) = \{q_0\} = S_t$
   $\varepsilon^*(S_t) = \{q_0\} = S_2$
   $\delta(S_0, c) = \{q_1\} = S_t$
   $\varepsilon^*(S_t) = \{q_1\} = S_3$
2. • $\varepsilon^*(S_1) = \{q_0, q_1\} = S_1$
   $\delta(S_1, a) = \{q_0, q_1\} = S_1$
   $\delta(S_1, b) = \{q_0\} = S_2$
   $\delta(S_1, c) = \{q_1\} = S_3$
3. • $\delta(S_2, a) = \{q_0, q_1\} = S_1$
   $\delta(S_2, b) = \emptyset$
   $\delta(S_2, c) = \emptyset$
4. • $\delta(S_3, a) = \emptyset$
   $\delta(S_3, b) = \{q_0\} = S_2$
   $\delta(S_3, c) = \{q_1\} = S_3$

	a	b	c
$\rightarrow S_0$	S_1	S_2	S_3
S_1	S_1	S_2	S_3
S_2	S_1	S_4	S_4
S_3	S_4	S_2	S_3
S_4	S_4	S_4	S_4

Problem 6. Create an NFA no more than 5 states that accepts the language $L = \{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$



	0	1
p	$\{q,s\}$	$\{q\}$
q	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
p	$\emptyset$	$\{p\}$

Problem 6. Convert the given NFA to DFA

	0	1
p	$\{q,s\}$	$\{q\}$
q	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
s	$\emptyset$	$\{p\}$
$\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$
$\{r,s\}$	$\{s\}$	$\{p\}$
$\emptyset$	$\emptyset$	$\emptyset$