

## **Homework #6** due December 28 before recitation

### **Question 1**

Consider the CFG  $G = (V, \Sigma, R, S)$  where  $V = \{S, A, B, C, D, E\}$ ,  $\Sigma = \{a, b, c\}$  and  $R$  is as given below

$R$  :

$S \rightarrow AE \mid EB \mid C$

$A \rightarrow aA \mid a$

$B \rightarrow Bb \mid b$

$C \rightarrow Cc$

$D \rightarrow aCb \mid a \mid b \mid c$

$E \rightarrow aEb \mid e$

(a) Remove all the null productions of  $G$ , if any, and call the result  $G_1$ .

(b) Remove all the unitary productions of  $G_1$ , if any, call the result  $G_2$

(c) Remove all the non-generative and non-reachable symbols of this grammar, if any, and call the result  $G_3$ .

(d) Compute the Chomsky Normal Form of  $G_3$  using your results above.

(e) State in the simplest possible way the language generated by  $G$

### **Question 2**

Show that the languages  $L_1, L_2 \subseteq \{a, b, c, d\}^*$  given below are context-free languages (CFL) :

$L_1 = \{a^n b^n c^m d^m; n, m \geq 0\}$  ;  $L_2 = \{a^n b^m c^m d^n; n, m \geq 0\}$

Is the language  $L = (\omega \in \{a, b, c, d\}^* \mid \text{in } \omega : \#a's = \#b's \wedge \#c's = \#d's)$  a CFL ? If so find a CFG that generates  $L$  or a PDA that accepts  $L$ ; if not prove your claim using the pumping lemma for CFGs.

### **Question 3**

A CFG is called right linear if **all** productions are of the form  $A \rightarrow aB$  or  $A \rightarrow e$  and called left linear if **all** productions are of the form  $A \rightarrow Ba$  or  $A \rightarrow e$  where  $A, B \in V$  and  $a \in T$  and  $e$  is the empty string. Show that both right linear and left linear grammars generate regular languages. Specify finite state machines corresponding respectively to right and left linear grammars.

**Main Text:** Exercise 6.4.1 (a), (c); 6.4.2, 7.1.3, 7.1.4, 7.2.1 (b), (c), 7.4.3(b), (c)