CS 302 Recitation 9

December 7, 2020

Problem 0

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b) Construct a PDA accepted by final state for the same language.

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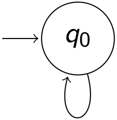
Accepted by empty stack:



$\varepsilon/P, 0P0$
$\varepsilon/P, 1P1$
$\varepsilon/P, \varepsilon$
$\varepsilon/P, 0$
$\varepsilon/P, 1$
$0/0, \varepsilon$
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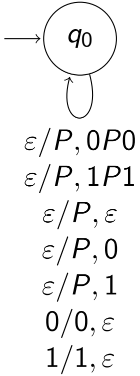

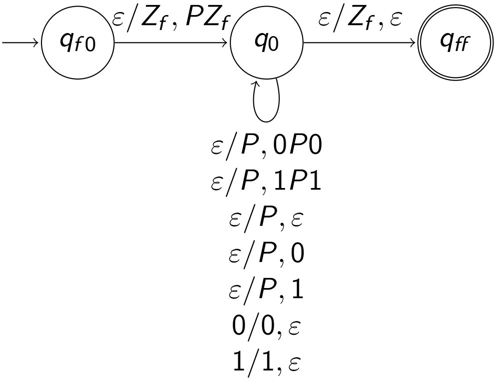

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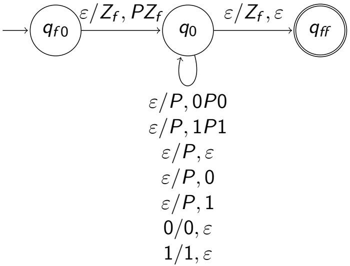

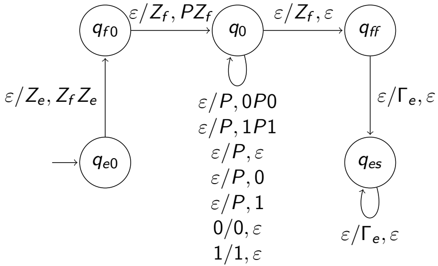

Problem 1

Determine whether the language $L = \{a^n b^n c^j : n \leq j\}$ is context-free or not.

Pumping Lemma Reminder

For $s = u.v.w.x.y \in L$ and pumping length p,

1. 1)$u.v^i.w.x^i.y \in L$, $i = 0, 1, 2, \ldots$
2. 2)$|v.x| > 0$,
3. 3)$|v.w.x| \leq p$

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  For $i = 0$, $u.w.y \notin L$.

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- Case 1: vx is a^q, b^q, c^q, or has only a and b with different numbers, or has only b and c:
  For $i = 0$, $u.w.y \notin L$.
- Case 2: vx only has a and b with equal numbers:
  For $i = 2$, $u.v^2.w.x^2.y \notin L$.

Problem 2

Determine whether the language $L = \{a^n b^j c^k : k = jn\}$ is context-free or not.

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For $s = u.v.w.x.y \in L$ and pumping length p,

1. $1)u.v^i.w.x^i.y \in L,\ i = 0, 1, 2, ...$
2. $2)|v.x| > 0,$
3. $3)|v.w.x| \leq p$

Problem 2

With pumping length p, $a^p b^p c^{p^2}$ is in the language.

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Case 1: vx is a^q, b^q, c^q, or only has a and b:
For $i = 0$, $u.w.y \notin L$.
Changing the length of only one type of letter is enough.
Changing the length of only a and b parts are enough.

Case 2: vx has only b and c:
For $i = 2$, $u.v^2.w.x^2.y \notin L$.
Let $\#b$'s and $\#c$'s in vx are q and r.
$u.v^2.w.x^2.y = a^p b^{p+q} c^{p^2+r}$
$p(p+q) = p^2 + r$
$p^2 + pq = p^2 + r$
$pq = r$, $0 < q < p$ and $0 < r < p$
Contradiction!

Problem 3

Determine whether the language $L = \{a^n : n \ is \ a \ prime\}$ is context-free or not.

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For $s = u.v.w.x.y \in L$ and pumping length p,

1. $1)u.v^i.w.x^i.y \in L, \ i = 0, 1, 2, ...$
2. $2)|v.x| > 0,$
3. $3)|v.w.x| \leq p$

Problem 3

With pumping length p, choose m s.t. $m \geq p$ and m is prime. a^m is in the language.

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$$|vx| = k > 0$$

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$$|vx| = k > 0$$
$$|u.v^i.w.x^i.y| = m + (i-1)k$$

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$$i = m + 1$$

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$$i = m + 1$$
$$|u.v^{m+1}.w.x^{m+1}.y| = m + (m+1-1)k = m + mk = m(k+1)$$

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Since $k > 0$, $m(k+1)$ is not prime.

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Contradiction!