(4) Consider the *Deterministic Finite Automata* ,

$A = (Q_A, \Sigma_A, \delta_A, q_{0A}, F_A)$ and $B = (Q_B, \Sigma_B, \delta_B, q_{0B}, F_B)$ with

$Q_A \cap Q_B = \phi$, $\Sigma_A \cap \Sigma_B = \phi$ where ϕ stands for the null set.

Let $L_A \subseteq \Sigma_A^*$ and $L_B \subseteq \Sigma_B^*$ be the languages accepted by A and B respectively and define the interleaved language:

$L_A \parallel L_B := \{s \; \varepsilon \; (\Sigma_A \cup \Sigma_B)^* \mid s \uparrow_A \varepsilon \; L_A \text{ and } s \uparrow_B \varepsilon \; L_B \}$

where $s \uparrow_A$ and $s \uparrow_B$ stand for the projection of s on Σ_A and Σ_B obtained by erasing all the symbols of s in Σ_B and Σ_A respectively.

$$s \in (\overline{\Sigma_A \cup \Sigma_B})^*$$

$$S: 100ab10$$

$$S\uparrow_A \qquad 10010$$

$$S\uparrow_B \qquad ab$$

s   erase   erase $s' \in \Sigma_B$

$$\Sigma_B = \{a,b\}$$

$$\Sigma_A = \{1,0\}$$

$A \parallel B$ that accepts $L_A \parallel L_B$

$$w$$

$$w\uparrow_A \; \in L_A$$
$$w\uparrow_B \; \in L_B$$

$A: w\uparrow_A$
$B: w\uparrow_B$

$$A\parallel B = \left(Q_{A\parallel B}, \Sigma_{A\parallel B}, q_{0A\parallel B}, \delta_{A\parallel B}, F_{A\parallel B} \right)$$

$$Q_{A\parallel B} = Q_A \times Q_B$$

$$\Sigma_{A\parallel B} = \Sigma_A \cup \Sigma_B$$

$$\delta_{A\parallel B}\left((q_A, q_B), \sigma \right) = \left(q_A, \delta_B(q_B) \right)$$

$$q_{0A\parallel B} = (q_{0A}, q_{0B})$$

$$F_{A\parallel B} = F_A \times F_B$$

$$\left(\delta_A(q_A), q_B \right) \quad \sigma \in \Sigma_A$$

$$\left(q_A, \delta_B(q_B) \right) \quad \sigma \in \Sigma_B$$

DFA accepts $L = (0.1)^* \| (a.b)^*$

X accepts $L((0.1)^*)$

Y accepts $L((a.b)^*)$

$q_x, q_y = q_{xy}$
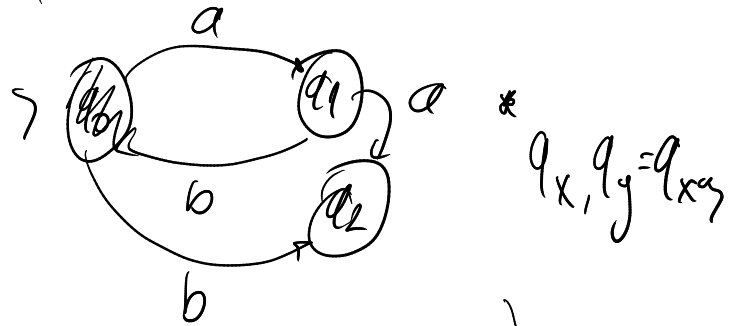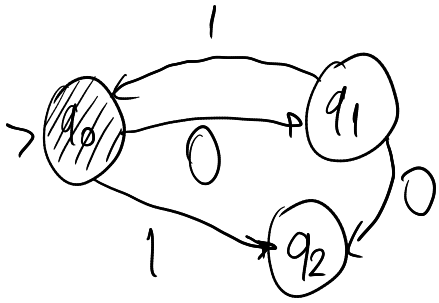
$Q_{X\|Y} = \{ q_{00}, q_{11}, q_{12}, q_{10}, q_{01}, q_{22}, q_{21}, q_{02}, q_{20} \}$

$\Sigma_{X\|Y} = \{ 0, 1, a, b \}$

$q_{Q_{X\|Y}} = q_{00}$ $F_{X\|Y} = \{ q_{00} \}$

q	σ	q'
q_{00}	0. 1 a b	q_{10} q_{20} q_{01} q_{02}
⋮	⋮	⋮
q_{11}	⋮	⋮
⋮		⋮
q_{22}		

Q11 $L = \{ a^n b^m : n \neq m \}$

→ RE
→ RL
→ CFG generate L

$n \neq m$ $\boxed{n > M}$ $\boxed{m > n}$

$n > M$ $S_a \to a S_a b \mid A$ $A \to a A \mid a$

$n = M$
$a^n b^n$ $S \to a S b \mid \varepsilon$ | $S_b \to a S_b b \mid B$ $m > n$
$n \geq 0$
 $\underline{}$ $B \to Bb \mid b$
 $a S b$
 $a a S b b$

$S_{ab} \to S_a \mid S_b$ $n > M$
$M \neq n$ $m > n$
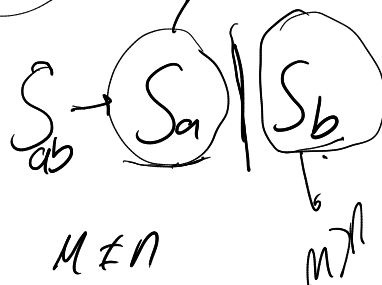
- DFA $\equiv$ ϵNFA $\equiv$ NFA   accepts regular language

RLs are generated RE

- PDA   accepts CFL

CFLs are generated CFG

$$L = \{a^i b^j c^k \mid i,j,k \geq 0 \quad i=j \text{ or } i=k\}$$

$$L = L' \cup L'' \qquad L' = \{a^i b^i c^k \mid i,k \geq 0\} \checkmark$$

$$L'' = \{a^i b^j c^i \mid i,j \geq 0\}$$

PDA P' accepts L'

$$(q'_0, a, Z_0) \rightarrow (q_1, aZ_0) \qquad (q_2, c, Z_0) \rightarrow (f, Z_0)$$

$$(q_1, a, a) \rightarrow (q_1, aa) \qquad (f, c, Z_0) \rightarrow (f, Z_0)$$

$$(q_1, b, a) \rightarrow (q_2, e) \qquad (q_2, e, Z_0) \rightarrow (f, Z_0)$$

$$(q_2, b, a) \rightarrow (q_2, e)$$

P^* PDA accepts $L^a = \{a^i b^j c^i \mid i, j \geq 0\}$

$(q_0^*, a, z_0) \rightarrow (q_1, a z_0)$

$(q_1, a, a) \rightarrow (q_1, aa)$

$(q_1, b, a) \rightarrow (q_2, a)$

$(q_1, e, a) \rightarrow (q_2, a)$

$(q_2, b, a) \rightarrow (q_2, a)$

$(q_2, c, a) \rightarrow (q_3, e)$

$(q_3, c, a) \rightarrow (q_3, e)$

$(q_3, e, z_0) \rightarrow (f, z_0)$

$L' \cup L^a = L$

$(q_{00}, e, z_0) \rightarrow (q_0^a, z_0)$

$(q_{00}, e, z_0) \rightarrow (q_0', z_0)$

$L = \{a^M b^{2n} c^{3n} d^p \mid p > M \quad M, n \geq 1\}$

$L' = \{a^M d^p \mid p > M\}$

$S' \rightarrow aS'd \mid D \qquad D \rightarrow Dd \mid d \qquad S'$ gene. L'

$L^a \rightarrow \{b^{2n} c^{3n} \mid n \geq 1\}$

$S^a \rightarrow bb\, S\, ccc \mid bb\, ccc$

$\rightarrow S'' \rightarrow aS''d \mid S''$