Fall 2017

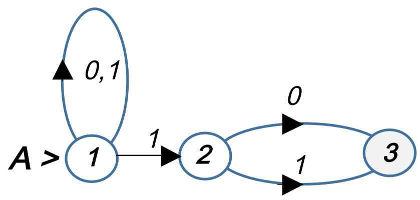
SABANCI UNIVERSITY
Faculty of Engineering and Natural Sciences
CS 302 Automata Theory

Answers to Final Examination

Answer 1 (*25* points)

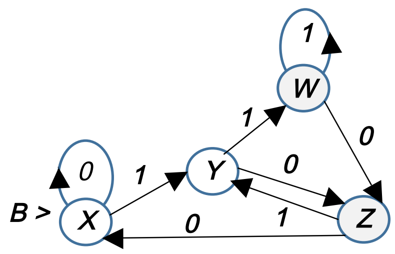
(a) *(8 pts)* State the full definition of a ***nondeterministic finite automaton*** *(NFA)*,
A = (Q, Σ, δ , s, F) including the *extended transition function* **δE** and the condition for a string ***u∈Σ**** to be accepted by ***A*** *(in your definition the domain and range of the transition functions* **δ** *and* **δE** *should be explicitly and clearly specified)*.
SEE THE RELEVANT SLIDES.

(b) *(5pts)* Construct an *NFA **A*** with at most ***3*** states that accepts the *regular expression*
E = ***(1+0)*.1.(1+0)*** interpreted as a language.



(c) *(6 pts)* Construct a *DFA **B*** that accepts the language ***E*** in part ***(b)***.

q	σ	*q'*
X=1	*0*	*1*
1	*1*	*1,2*
Y=1,2	*0*	*1,3*
2	*1*	*1,2,3*
Z=1,3	*0*	*1*
1,3	*1*	*1,2*
W=1,2,3	*0*	*1,3*
1,2,3	*1*	*1,2,3*



(d) *(6 pts)* Construct a *minimal state DFA* ***C*** that accepts the language ***E*** in part ***(b)***.

The table below shows that ***B*** is a *minimal state machine*, hence ***C=B*** is a solution.

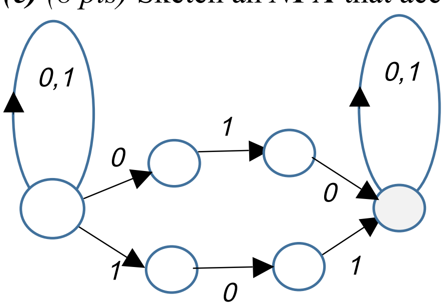
	X	Y	Z	W
X		*2*	*1*	*1*
Y			*1*	*1*
Z				*2*
W				

Answer 2 (*25* points)

(a) *(9 pts)* Specify, clearly and with all the details, a *context-free grammar (CFG)*,
G = (V, T, R, S) that generates the set of *all* ***regular expressions*** over some finite set ***Σ***.
SEE THE RELEVANT SLIDE
(b) *(8 pts)* Write down a regular expression ***E*** over the set ***{0,1}*** corresponding to the language ***L*** where for each string ***u*** in ***L*** , ***010*** OR ***101*** is *substring* of ***u***.

E = (1+0)*.((1.0.1)+(0.1.0)).(1+0)*

(c) *(8 pts)* Sketch an ***NFA*** that accepts the language ***L*** in part ***(b)*** above .



Answer 3 (*25 points)*

(a) *(7 pts)* State clearly the *definition* of a *PDA* ***P*** accepting a language ***L*** by *final state*; and by *empty stack* .

SEE THE RELEVANT SLIDES

(b) *(10 pts) (*Compute a *PDA* ***P*** that accepts the language $L = (\omega \in (0+1)^* \mid \#1\text{s} = \#0\text{s in } \omega)$ by *final state*.

P transitions (f is the only final state) :

$(q_0, 0, Z_0) \rightarrow (q_0, 0Z_0)$
$(q_0, 1, Z_0) \rightarrow (q_0, 1Z_0)$

$(q_0, 1, 1) \rightarrow (q_0, 11)$
$(q_0, 0, 0) \rightarrow (q_0, 00)$
$(q_0, 1, 0) \rightarrow (q_0, e)$
$(q_0, 0, 1) \rightarrow (q_0, e)$
$(q_0, 0, 1) \rightarrow (q_0, e)$

$(q_0, e, Z_0) \rightarrow (f, Z_0)$

(c) *(8 pts)* Is your ***PDA*** a deterministic ***PDA (DPDA)*** ? If not try to modify it so that it is a ***DPDA***.

Because of transitions $(q_0, 0, Z_0) \rightarrow (q_0, 0Z_0)$ ***(or*** $(q_0, 1, Z_0) \rightarrow (q_0, 1Z_0)$***) and*** $(q_0, e, Z_0) \rightarrow (f, e)$ the above is not a ***DPDA*** .

To make it so, we modify by replacing the first **2** transitions by the following **2** transitions :

$(f, 0, Z_0) \rightarrow (q_0, 0Z_0)$
$(f, 1, Z_0) \rightarrow (q_0, 1Z_0)$

So that computation necessarily starts with the last transition $(q_0, e, Z_0) \rightarrow (f, Z_0)$ and proceeds after that if input list is not empty.

Answer 4 (*25* points)

Consider the language $L = (\omega \in \{a,b,c\}^* /\ \omega = a^{k+1} b^k c^{k-1}\ ;\ k > 0\ )$

(a) *(15 pts)* Choosing a appropriate $\omega \in L$ *(if you do not show explicitly your choice of* ω *you get NO credit !)* and using the *pumping lemma* show that L is NOT a *context-free language*.

By *PL* for *CFL*s given the integer n choose $\omega = a^{n+1}\ b^n\ c^{n-1} \in L$ then $|\omega| > n$ ***hence by PL :***

(1) $\omega = xuvwz = a^{n+1}\ b^n\ c^{n-1}$

(2) $|uvw| \leq n\ ;\ |uw| > 0$

(3) $xu^j v w^j z \in L\ ,\ \ \forall j \geq 0$

Hence the following **2** possibilities exist for ***uvw :***

(i) $uvw = a^r b^t$, $r+t > 0$ and so $xvz = a^{n+1-q}\ b^{n-q'} c^{\ n-1}$ where $q+q' > 0$

(ii) $uvw = b^r c^t$, $r+t > 0$ and so $xvz = a^{n+1} b^{n-q}\ c^{n-1-q'}$ where $q+q' > 0$

For all these possibilities $xu^j v w^j z \notin L$ for $j=0$ a ***contradiction*** to ***PL*** ! Hence ***L*** is not a *CFL !*

(b) *(10 pts)* Compute a *2*-tape *NDTM* TM ***M*** that decides $L = (\omega \in \Sigma /\ \omega = uvu\ ,\ u,v \in \Sigma^*,\ u \neq e)$

Initially ***M*** has the configuration $(s,\ \underline{\#}\ \omega\ ,\ \underline{\#})$ and $d \notin \Sigma$

$M > A = R^1$	$\sigma^1 = \#$	h_{NO}
	$\sigma^1 \neq \#$	B
B	$\sigma^1 \neq \#$	$R^1 B$
	$\sigma^1 \neq \#$	C
	$\sigma^1 = \#$	h_{NO}
$C = R^1$	$\sigma^1 \neq \#$	$d^1\ C$
	$\sigma^1 \neq \#$	D
	$\sigma^1 = \#$	h_{NO}
D	$\sigma^1 := X \neq \#$	$d^1\ R^2\ X^2\ D$
	$\sigma^1 = \ \#$	$L_\#^1\ L_\#^2\ E$
$E = R^1\ R^2$	$\sigma^1 = \sigma^2$	E
	$\sigma^1 = d \wedge\ \sigma^2 = \#$	h_{YES}
	else	h_{NO}