Fall 2019

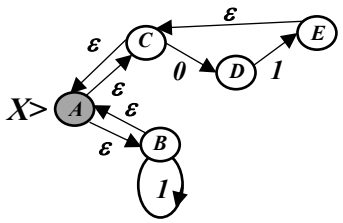
SABANCI UNIVERSITY
Faculty of Engineering and Natural Sciences
CS 302 Automata Theory

Final Answers

Answer1 *(25 points)*

(a)*(6 pts)* ***X*** is given below.

initial state = final state =A



(b)*(6 pts)*

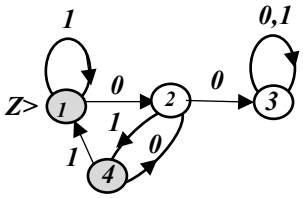
NFA ***Y*** *has the table below*

*Note the new initial state(s) = ε-closure(**A**) =* ***ABC***

state	*input*	*next state*
*>A**	*0*	*D*
	1	*ABC*
>B	*0*	*D*
	1	*ABC*
>C	*0*	*D*
	1	*ABC*
D	*0*	*null*
	1	*ABCE*
E	*0*	*D*
	1	*ABC*

(c)*(7 pts) DFA* ***Z*** equivalent to ***Y*** is given below.

state	*input*	*next*
*>ABC=1**	*0*	*D=2*
	1	*ABC=1*
D=2	*0*	*Null =3*
	1	*ABCE=4*
*ABCE=4**	*0*	*D=2*
	1	*1*
Null=3	*0*	*Null=3*
	1	*Null=3*

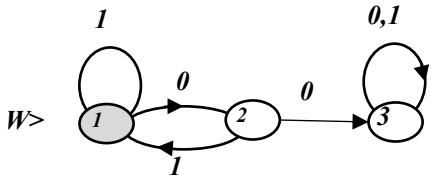


(d)*(6 pts)* Minimal state DFA ***W*** that is equivalent to ***Z*** is computed using table filling algorithm.

1 2 3 4

	1	*1*	*eq*	*1*
		2	*1*	*2*
			1	*3*
				4

The above filled table shows that state ***1*** is equivalent to state ***4*** . Hence minimal state *DFA* ***W*** is given below.



The language symbolized by ***E*** (and accepted by ***W***) is : *set of all strings in which a* ***0*** *is always immediately followed by a* ***1****.*

Answer 2 (*25* points)

(a)*(13 Xpts)* Consider the *CFG,* $G = (V,T,R,S)$ where

$V = \{Zero, One, S\}$, $T = \{0,1\}$ and R is given by the following productions:

$S \rightarrow 0\ Zero\ S \mid 1\ One\ S \mid e$; $Zero \rightarrow 0\ Zero\ Zero \mid 1$; $One \rightarrow 1\ One\ One \mid 0$

Leftmost derivation of 011010 is given below.

$S \Rightarrow 0\ Zero\ S \Rightarrow 0\ 1\ S \Rightarrow 0\ 1\ 1\ One\ S \Rightarrow 0\ 1\ 1\ 0\ S \Rightarrow 0\ 1\ 1\ 0\ 1\ One\ S \Rightarrow 0\ 1\ 1\ 0\ 1\ 0\ S$

$\Rightarrow 0\ 1\ 1\ 0\ 1\ 0$

The language $L_G = (\omega \in \{0,1\}^* \mid$ #0's = #1's in $\omega)$

(b)*(12 pts)* Construct a *PDA* $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ that accepts the language :

$L = (\omega \in \{0,1\}^* \mid \omega = 0^n 1^{n+1}, n \geq 0)$, by final state.

$P = (\{q_0, f\}, \{0,1\}, \{Z_0, 0,1\}, \delta, q_0, Z_0, \{f\})$ where the transitions of δ are given below:

$(q_0, 0, X) \rightarrow (q_0, 0X)\ ; X=0,Z_0$

$(q_0, 1, 0) \rightarrow (q_0, e)$

$(q_0, 1, Z_0) \rightarrow (f, Z_0)$

P is definitely a ***DPDA***

Answer 3 (*25 points)*

(a) *(7 pts)* See the slides.

(b) *(8pts)* Consider the *CFG*, $G=(\{X,S\}, \{a,b,c\}, R, S)$ where R is composed of the following productions :

$S \rightarrow aSc \mid X \mid e\ ;\ X \rightarrow aXb \mid e$

Leftmost derivation of a^3bc^2

$S \Rightarrow aSc \Rightarrow aaScc \Rightarrow aaXcc \Rightarrow aaaXbcc \Rightarrow aaaebcc = a^3bc^2$

$L_G = \{ a^r b^n c^m\ ;\ r = n+m\ ,\ n,m \geq 0$ *integers* $\}$

(c) *(10 pts)* Compute the *Chomsky Normal Form* for G.

Get rid of null productions

S → aSc | X | ac ; X→ aXb | ab

Get rid of unit production

S → aSc | aXb | ab | ac ; X→ aXb | ab

And more :

S → ASC | AXB |AB | AC ; X→ AXB | AB ; A→ a ; B → b ; C → c

S → AD | AE |AB | AC ; D → SC ; E→ XB ; X→ AE | AB ; A→ a ; B → b ; C → c

Done.

Question 4 (*25* points)

(a) *(15 pts) For input* ***w = ab*** *the step-by-step computation is given below :*

 # ab |-- # a b |-- # $ b |-- # $ b # |-- # $ b # # |-- # $ b # a |-- # $ b # a |-- # a b # a

|-- # a b # a |-- # a $ # a |-- # a $ # a |-- # a $ # a # |-- # a $ # a b |-- # a $ # a b |-- # a b # a b

|-- # a b # a b |-- # a b # a b # |--$_{Leftshift}$ # a b a b # |-- # a b a b

(b) *(10 pts) The general solution is:*

(s, # w) |--$_{M}$* (h, # w w)