CS 302 Recitation 7

November 26, 2020

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Contradicts with rule 1.

Problem 2

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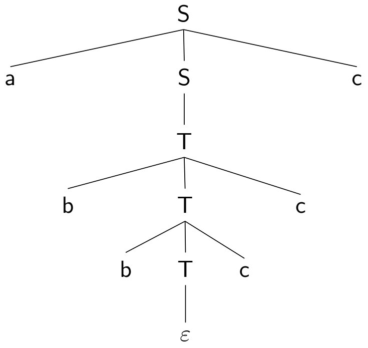

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$T_1 \to bT_1c \mid T_1c \mid \varepsilon$

$S_2 \to aS_2c \mid aT_2 \mid bT_2 \mid aS_2$

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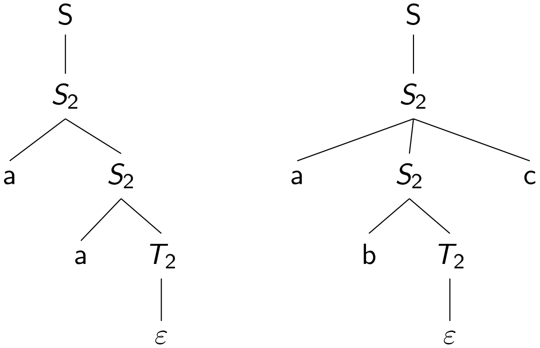

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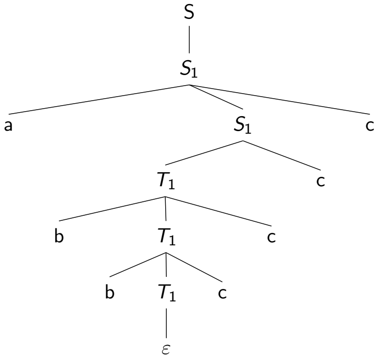

Problem 4

$$S \to AA$$
$$A \to aSa \mid a$$

Show that given grammar is ambiguous.

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Two different left-most derivation for the string a^5

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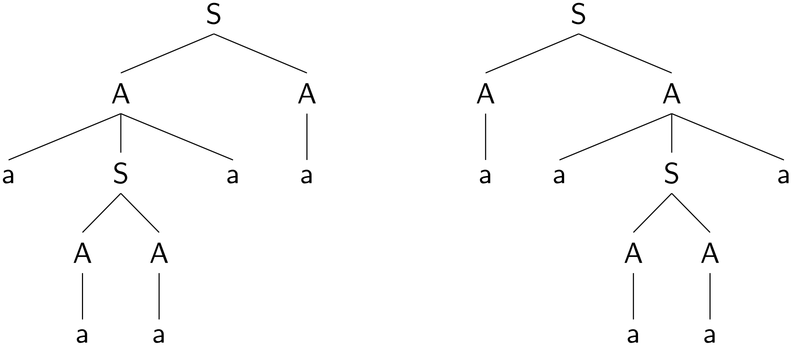

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CFG to PDA

- For eah variable A in G: $\delta(q, \varepsilon, A) = \{(q, \beta) \mid A \to \beta \textit{ is a production from } G\}$
- For each terminal a in G: $\delta(q, \varepsilon, A) = \{(q, \beta) \mid A \to \beta \textit{ is a production from } G\}$

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$$T \rightarrow bTc \mid \varepsilon$$

$$\delta(q, \varepsilon, S) = \{(q, aSc), (q, T)\}$$
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Input Letters:$\{a, b, c\}$, Stack Symbols: $\{a, b, c, S, T\}$
Initial and Final State:q, Start Symbol:S, Set of States:$\{q\}$