# CS 302 Recitation 3

November 1, 2020

# Problem 1

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- $$\big(P(Pb+b)+\varepsilon+Pb+b\big)\big(P(Pb+b)\big)^*$$
- Distributive Law

$$\big(P(Pb+b)+\varepsilon\big)\big(P(Pb+b)\big)^*+\big(Pb+b\big)\big(P(Pb+b)\big)^*$$

# Problem 1 cont.

- $$\left(P(Pb+b)+\varepsilon\right)\left(P(Pb+b)\right)^{*}+\left(Pb+b\right)\left(P(Pb+b)\right)^{*}$$

# Problem 1 cont.

- $$\color{red}{\left(P(Pb+b)+\varepsilon\right)}\left(P(Pb+b)\right)^* + \left(Pb+b\right)\left(P(Pb+b)\right)^*$$
- By $(\varepsilon + L)L^* = L^*$,

$$\left(P(Pb+b)\right)^* + \left(Pb+b\right)\left(P(Pb+b)\right)^*$$

# Problem 1 cont.

- $$\left(P(Pb+b)+\varepsilon\right)\left(P(Pb+b)\right)^* + \left(Pb+b\right)\left(P(Pb+b)\right)^*$$
- By $(\varepsilon + L)L^* = L^*$,
  $$\left(P(Pb+b)\right)^* + \left(Pb+b\right)\left(P(Pb+b)\right)^*$$
- Since $Pb + b = aa^*b + b = (aa^* + \varepsilon)b = a^*b$,
  $$\left(aa^*a^*b\right)^* + a^*b\left(aa^*a^*b\right)^*$$

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- Since $Pb + b = aa^*b + b = (aa^* + \varepsilon)b = a^*b$,

$$\left(aa^*a^*b\right)^* + a^*b\left(aa^*a^*b\right)^*$$

- $a^*a^* = a^*$

$$(aa^*b)^* + a^*b(aa^*b)^*$$

# Problem 1 cont. (2)

- $$(aa^*b)^* + a^*b(aa^*b)^*$$

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- $$(aa^* b)^* + a^* b(aa^* b)^*$$

- Distributive Law

$$(a^* b + \varepsilon)(aa^* b)^*$$

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$$(aa^*b + b + \varepsilon)(aa^*b)^*$$
- By $(\varepsilon + L)L^* = L^*$,
$$(b + \varepsilon)(aa^*b)^*$$

# Problem 2

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1) x.y^i.z \in A,\ i = 0, 1, 2, \ldots$
2. $2) |y| > 0,$
3. $3) |x.y| \leq p$

- Prove that language $A_1 = \{0^n 1^m 0^n \mid m, n \geq 0\}$ is regular or not.

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- $$s = x.y.z = 0^p10^p$$

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- Prove that language $A_1 = \{0^n 1^m 0^n \mid m, n \geq 0\}$ is regular or not.
- $$s = x.y.z = 0^p 1 0^p$$
- By condition 3, x consists of all 0's, and y consists of all 0's.

$$x = 0^a, y = 0^b, z = 0^c 1 0^p$$

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- By condition 3, x consists of all 0's, and y consists of all 0's.
$$x = 0^a, y = 0^b, z = 0^c 1 0^p$$
- For $x.y.z \in A_1$, $x.y.y.z = 0^{a+2b+c} 1 0^p$

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- $$s = x.y.z = 0^p 1 0^p$$
- By condition 3, x consists of all 0's, and y consists of all 0's.

$$x = 0^a, y = 0^b, z = 0^c 1 0^p$$

- For $x.y.z \in A_1$, $x.y.y.z = 0^{a+2b+c} 1 0^p$
- Contradicts with Condition 1!

# Problem 3

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For pumping length $p$ and $s = x.y.z \in A$,

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- Prove that language Prove that language $A_2 = \{a^{2^n} \mid n \geq 0\}$ is regular or not.

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For pumping length $p$ and $s = x.y.z \in A$,

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$$s = x.y.z = a^{2^p}$$
- By definition of the language, the string from language $A_2$ with minimum length greater than $s$, is $a^{2p+1}$

$$|a^{2^{p+1}}| = |a^{2 \cdot 2^p}| = 2 \cdot |s|$$

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$$|a^{2^{p+1}}| = |a^{2 \cdot 2^p}| = 2 \cdot |s|$$
- Since $x.y.z = a^{2^{p+1}} \in A_2$, the statement $x.y.y.z \in A_2$ must also be true, by condition 1. Contradiction! Since $|x.y| \leq p, |y| > 0,$

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- $$s = x.y.z = a^{2^p}$$
- By definition of the language, the string from language $A_2$ with minimum length greater than $s$, is $a^{2^{p+1}}$
$$|a^{2^{p+1}}| = |a^{2 \cdot 2^p}| = 2 \cdot |s|$$
- Since $x.y.z = a^{2^{p+1}} \in A_2$, the statement $x.y.y.z \in A_2$ must also be true, by condition 1. Contradiction! Since $|x.y| \leq p, |y| > 0,$
- $x.y.y.z$ can be neither $a^{2^p}$ nor $a^{2^{p+1}}$

# Problem 4

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1) x.y^i.z \in A,\ i = 0, 1, 2, \ldots$
2. $2) |y| > 0,$
3. $3) |x.y| \leq p$

- Prove that language $A_3 = \{0^m 1^n \mid m \neq n \ and \ m, n \geq 0\}$ is regular or not.

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- The basic idea:

$$s = x.y.z = 0^p 1^d, d > p$$

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- The basic idea:

$$s = x.y.z = 0^p1^d, d > p$$

- By condition 3, x consists of all 0's, and y consists of all 0's.

$$x = 0^a, y = 0^b, z = 0^c1^d, a + b + c = p$$

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$$s = x.y.z = 0^p 1^d, d > p$$

- By condition 3, x consists of all 0's, and y consists of all 0's.

$$x = 0^a, y = 0^b, z = 0^c 1^d, a + b + c = p$$

- Find a $x.y^i.z = 0^{a+bi+c} 1^d$ s.t. $a + bi + c = d$ and prove by contradiction.

# Problem 4 cont.

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

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- Find a $x.y^i.z = 0^{a+bi+c}1^d$ s.t. $a + bi + c = d$
- Problem:

$$bi = d - a - c$$

# Problem 4 cont.

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- Find a $x.y^i.z = 0^{a+bi+c}1^d$ s.t. $a + bi + c = d$
- Problem:

$$bi = d - a - c$$

- We must choose such a $d$ that it satisfies this equation for each $a, b, c$.

# Problem 4 cont.

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1) x.y^i.z \in A,\ i = 0, 1, 2, \ldots$
2. $2) |y| > 0,$
3. $3) |x.y| \leq p$

- Find a $x.y^i.z = 0^{a+bi+c}1^d$ s.t. $a + bi + c = d$
- Problem:

$$bi = d - a - c$$

- We must choose such a $d$ that it satisfies this equation for each $a, b, c$.
-

$$d = p + p!$$

$$x = 0^a, y = 0^b, z = 0^c 1^{p+p!}, a + b + c = p$$

# Problem 4 cont.

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1)x.y^i.z \in A$, $i = 0, 1, 2, \ldots$
2. $2)|y| > 0$,
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- Problem:

$$bi = d - a - c$$

- We must choose such a $d$ that it satisfies this equation for each $a, b, c$.
-

$$d = p + p!$$

$$x = 0^a, y = 0^b, z = 0^c1^{p+p!}, a + b + c = p$$

- For $i = \frac{p!}{b} + 1$:

$$x.y^{\frac{p!}{b}+1}.z = 0^a0^{p!+b}0^c1^{p!} = 0^{a+b+c+p!}1^{p+p!} = 0^{p+p!}1^{p+p!}$$

# Problem 4 altn.

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1) x.y^i.z \in A$, $i = 0, 1, 2, \ldots$
2. $2) |y| > 0$,
3. $3) |x.y| \leq p$

- Let $C$ be the complement of language $A_3$. By closure property, if $A_3$ is regular, $C$ is also regular.

# Problem 4 altn.

## Pumping Lemma

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- Let $C$ be the complement of language $A_3$. By closure property, if $A_3$ is regular, $C$ is also regular.
- $$B = \{0^n 1^n \mid n \geq 0\} = C \cap 0^* 1^*$$

  By closure property, if $C$ and $0^*1^*$ are regular, $B$ is also regular.

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## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

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$$B = \{0^n 1^n \mid n \geq 0\} = C \cap 0^* 1^*$$
By closure property, if $C$ and $0^*1^*$ are regular, $B$ is also regular.
- $B$ is not regular $\wedge$ $0^*1^*$ is regular $\Rightarrow$ $C$ is not regular.

# Problem 4 altn.

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

1. $1) x.y^i.z \in A,\ i = 0, 1, 2, ...$
2. $2) |y| > 0,$
3. $3) |x.y| \leq p$

- Let $C$ be the complement of language $A_3$. By closure property, if $A_3$ is regular, $C$ is also regular.
-
$$B = \{0^n 1^n \mid n \geq 0\} = C \cap 0^* 1^*$$
By closure property, if $C$ and $0^*1^*$ are regular, $B$ is also regular.
- $B$ is not regular $\wedge$ $0^*1^*$ is regular $\Rightarrow$ $C$ is not regular.
- We proved $A_3$ is not regular.

# Problem 5

## Pumping Lemma

For pumping length $p$ and $s = x.y.z \in A$,

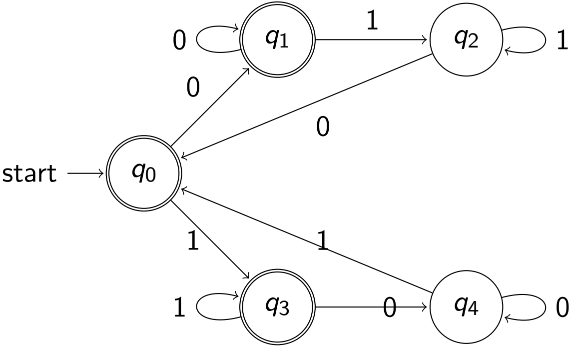
1. $1)x.y^i.z \in A,\ i = 0, 1, 2, ...$
2. $2)|y| > 0,$
3. $3)|x.y| \leq p$

- Prove that language $A_4 = \{w \mid$ $w$ *has an equal number of occurrences of* $01's$ *and* $10's$ *as substrings*$\}$ is regular or not.

# Problem 5 cont.

- It is regular!

- It is regular!

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