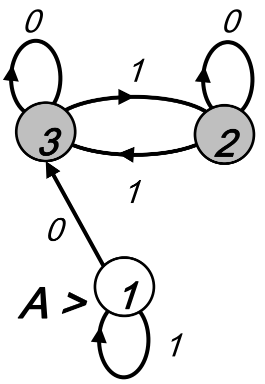
Fall 2016

SABANCI UNIVERSITY
Faculty of Engineering and Natural Sciences
CS 302 Automata Theory

ANSWERS TO THE FINAL EXAM

Question 1 (*20* points)



$A = (Q_A , \Sigma, \delta_A , q_{0A} , F_A)$ is given by the figure above where $q_{0A} = 1$ and $F_A = \{2,3\}$.

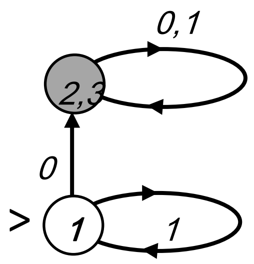
(a)

q	σ	q'
1	*0*	**3**
1	*1*	**1**
2	*0*	**2**
2	*1*	**3**
3	*0*	**3**
3	*1*	**2**

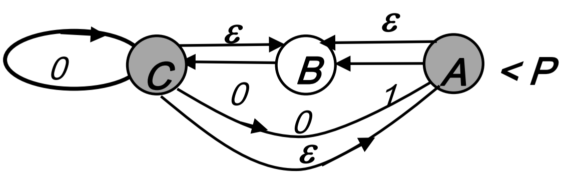
(b) By simple inspection : $L_A = 1^*.0.(0+1)^*$; $L_A^c = 1^*$

(c) Using Table Filling Algorithm below we observe that states ***2*** and ***3*** are equivalent hence minimal state DFA is as shown below.

	1	2	3
1		x	x
2			-----
3			



Question 2 (*20* points)



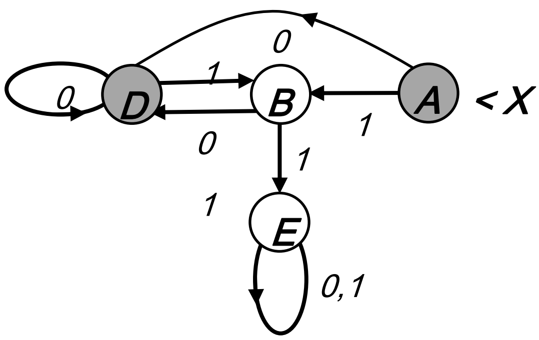
(a)

q	***σ***	***q'***
A	0	***ABC***
A	1	***B***
B	0	***ABC***
B	1	$\varnothing$
C	0	***A,B,C***
C	1	***B***

(b) *DFA* table below

q	***σ***	***q'***
A*	***0***	***ABC***
A	***1***	***B***
B	***0***	***ABC***
B	***1***	$\varnothing$
ABC(D*)	***0***	***ABC***
ABC	***1***	***B***
$\varnothing$***(E)***	***0***	$\varnothing$
$\varnothing$	***1***	$\varnothing$

DFA ***X*** is as below



Question 3 (*20* points)

CFG is $\boldsymbol{G = (\{S, A,B\},\{0,1\},R, S)}$ where the production set $\boldsymbol{R}$ is given below :

$\boldsymbol{R : S \to AB \; ; \; A \to 0A1 \,/ e \; ; \; B \to 1B0 \,/ e}$

(a)

null production elimination **:**

$\boldsymbol{S \to AB \,/ A \,/ B \; ; \; A \to ZERO\ A\ ONE \ / ZERO\ ONE \; ; \; B \to ONE\ B\ ZERO \,/ ONE\ ZERO}$

unit production elimination :

$\boldsymbol{S \to AB \,/ ZERO\ A\ ONE \ / ZERO\ ONE \ / ONE\ B\ ZERO \,/ ONE\ ZERO \ ;}$

$\boldsymbol{A \to ZERO\ A\ ONE \ / ZERO\ ONE \; ; \; B \to ONE\ B\ ZERO \,/ ONE\ ZERO}$

and the CNF is as below :

$\boldsymbol{S \to AB \,/ X\ ONE \ / ZERO\ ONE \,/ Y\ ZERO \,/ ONE\ ZERO \; ; \; A \to X\ ONE \,/ ZERO\ ONE \; ;}$

$\boldsymbol{B \to Y\ ZERO \,/ ONE\ ZERO \; ; \; X \to ZERO\ A \; ; \; Y \to ONE\ B \; ; \; ZERO \to 0 \; ; \; ONE \to 1}$

(b) *Note that* $\boldsymbol{L_G}$ *is given by* $\boldsymbol{L_G = \{0^n\ 1^{n+k}\ 0^k \; ; \; n,k \geq 0\}}$ *and the PDA* $\boldsymbol{P}$ *:*

$\boldsymbol{P = (\{q,f,p,r,s\},\{0,1\}, \ \{0,1,Z_0 , \ Z_1\}, \ \delta, \ q, \ Z_0 , \ \{f\})}$

accepts $\boldsymbol{L_G}$ *where* δ *is given below*

$\boldsymbol{(q, e , Z_0) \ \to (f, Z_1)}$

$\boldsymbol{(f, 0 , Z_1) \ \to (q, 0Z_1)}$

$\boldsymbol{(q, 0 , 0) \to (q, 00)}$

$\boldsymbol{(q, 1 , 0) \to (p, e)}$

$\boldsymbol{(p, 1 , 0) \to (p, e)}$

$\boldsymbol{(p, 1 , Z_1) \to (r, 1Z_1)}$

$\boldsymbol{(r, 1 , 1) \to (r, 11)}$

$\boldsymbol{(r, 0 , 1) \to (s, e)}$

$\boldsymbol{(s, 0 , 1) \to (s, e)}$

$\boldsymbol{(s, e, Z_1) \to (f, Z_1)}$

OR using the standard procedure of converting a *CGF* to a *PDA* ***P*** by *empty stack* :

$\boldsymbol{P = (\{q_0\},\{0,1\}, \ \{0,1,Z_0\}, \ \delta, \ q_0 , \ Z_0)}$

where δ *is given below*

$\boldsymbol{(q_0 , e, Z_0) \to (q_0 , SZ_0)}$

$\boldsymbol{(q_0 , e, S) \to (q_0, AB)}$

$\boldsymbol{(q_0 , e, A) \to (q_0, 0A1)}$

$\boldsymbol{(q_0 , e, A) \to (q_0, e)}$

$\boldsymbol{(q_0 , e, B) \to (q_0, 1B0)}$

$(q_0, e, B) \rightarrow (q_0, e)$

$(q_0, 0, 0) \rightarrow (q_0, e)$

$(q_0, 1, 1) \rightarrow (q_0, e)$

$(q_0, e, Z_0) \rightarrow (q_0, e)$

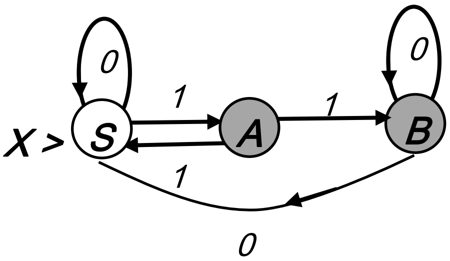
(c) *For the first solution* ***P*** *is already a* ***DPDA ;*** for the second it clearly is ***not*** a ***DPDA***.

Question 4 (*20* points)

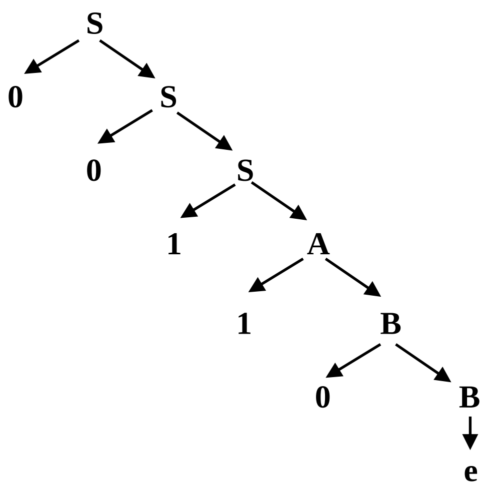
CFG is given as ***G = ({S, A, B},{0,1},R, S)*** where ***R*** is given below

$R : S \rightarrow 0S \,/\, 1A \;;\; A \rightarrow 1B \,/\, 1S \,/\, e \;;\; B \rightarrow 0B \,/\, 0S \,/\, e$

(a) *CFG* is a *right linear grammar* and is identified with an *NFA* as below where variables in ***G*** are identified with states in ***X*** and thus ***L_G*** is regular.



(b) There is a unique parse tree for the string 00110; however, there are other strings for which it is possible to construct two different parse trees (e.g. 11001). Hence, ***G*** is an *ambiguous grammar.*



Note: The answer *"This is a non-ambiguous grammar, because there is a unique parse tree for the string 00110"* will be accepted as well.

Question 5 (*20* points)

(a) $(s, \underline{\#}\, 0^n 1^n) \vdash_T (h, \underline{\#}\, 1^n 0^n)$

TM	*Condition*	*Next TMs*
$> A = R$	$\sigma = 0$	$1 . A$
	$\sigma = 1$	$0 . A$
	$\sigma = \#$	$L_{\#} . h$

(b) *We use a 2 tape TM as below and initial condition is :* $(s, \underline{\#}\, w , \underline{\#}\ )$

TM	*Condition*	*Next TMs*
$> A = R^1 R^2$	$\sigma^1 = \#$	C
	$\sigma^1 = 0$	$0^2 . A$
	$\sigma^1 = 1$	$L^1 . B$
$B = R^1 . L^2$	$\sigma^1 = 1 \wedge \sigma^2 = 0$	B
	$\sigma^1 = \# \wedge \sigma^2 = \#$	h
	else	C
$C = \#^1$	-	C