Fall 2020

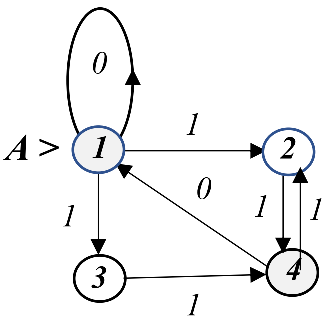
18.1.2021

SABANCI UNIVERSITY
Faculty of Engineering and Natural Sciences
CS 302 Automata Theory

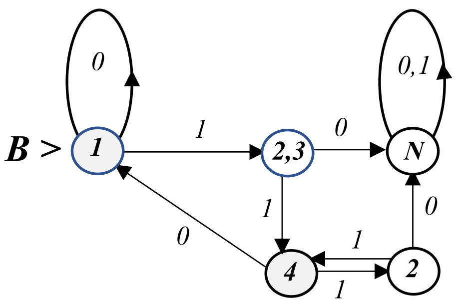
Answer 1 (25 points)

(a) *(5 pts) See the relevant slides*

(b) *(3 pts) It is not a DFA since, for example, there are no transitions at state 3.*

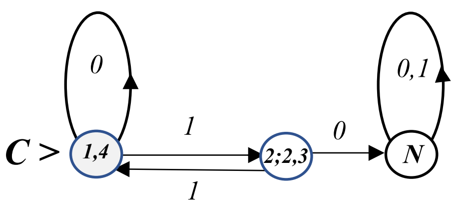


(c) *(12 pts) Equivalent DFA **B** is given below.*



	2	2,3	N	1	4
2			1	0	0
2,3			1	0	0
N				0	0
1					
4					

*Minimal state machine **C** is given below using **2≡2,3** and **1≡4***



(d) *(5 pts) Take any sequence in **{0,1}*** and replace each '**1**' in it by **'1.1'***

Answer 2 (25 points)

Consider the CFG, $\mathbf{G = (V,T,R,S)}$ *where*

$\mathbf{V = \{A, B, S\}}$, $\mathbf{T = \{a,b\}}$ *and* $\mathbf{R}$ *is given by the following productions:*

$\mathbf{S \rightarrow AB}$

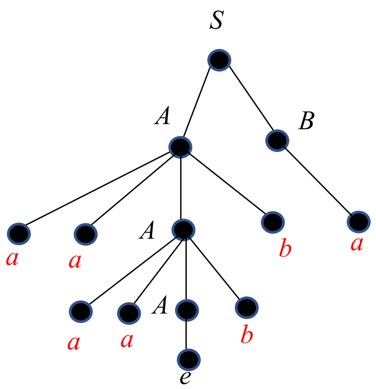
$\mathbf{A \rightarrow aaAb \mid e}$

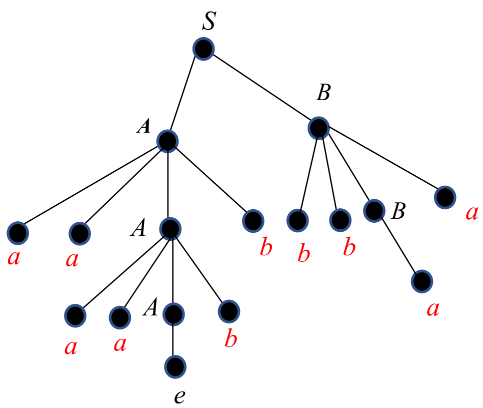
$\mathbf{B \rightarrow bbBa \mid a}$

(a) *(7 pts)* $\mathbf{L_G = \{a^{2n}\, b^{n+2k}\, a^{k+1} \; ; \; n,k \geq 0 \}}$

(b) *(8 pts)* ***(i)*** $\mathbf{n=2}$ *and* $\mathbf{k=0}$ *implies* $\mathbf{u_1 = a^4b^2a \in L_G}$; ***(ii)*** $\mathbf{n}$ *must be* **2** *and* $\mathbf{k}$ *must be* **1** *which yields the string* $\mathbf{a^4b^4a^2}$.

The parse trees for $\mathbf{u_1}$ *and* $\mathbf{u_2}$ *are given below*





(c) *(10 pts)* $\mathbf{P = (\{q_0, q, f\}, \{a,b\}, \{a,b,Z_0\}, \delta, q_0, Z_0, \{f\})}$ *and* δ *is given by :*

$\mathbf{(q_0 , e , Z_0) \rightarrow (q_0 , SZ_0)}$

$\mathbf{(q_0 , a , a) \rightarrow (q_0 , e)}$

$\mathbf{(q_0 , b , b) \rightarrow (q_0 , e)}$

$\mathbf{(q_0 , e, S) \rightarrow (q_0 , AB)}$

$\mathbf{(q_0 , e, A) \rightarrow \{(q_0 , aaAb), (q_0 ,e)\}}$

$\mathbf{(q_0 , e, B) \rightarrow \{(q_0 , bbBa), (q_0 ,a)\}}$

$\mathbf{(q_0 , e, Z_0) \rightarrow (f, Z_0)}$

Answer 3 (*25 points)*

(a) *(15 pts)* L_1 is not a *CFL* which we prove using the pumping lemma. Hence assume $N > 0$ as the pumping lemma integer and choose $z = a^N b^N c^N \in L_1$ and $|z| = 3N > N$ as required by the ***PL*** . By the ***PL,*** $z = uvwxy$, where $|vwx| \leq N$ and $|vx| > 0$. Hence since $z = uvwxy = a^N b^N c^N$ it follows that :

Case 1 : $vwx = a^m$ *or* $vwx = b^m$ *or* $vwx = c^m$ where $m \leq N$ ***; OR***

Case 2 : $vwx = a^i b^j$ *or* $vwx = b^i c^j$ where $i+j \leq N$

According to ***PL*** we must have $uwy \in L_1$ and if ***Case 1*** holds :

$uwy = a^{N-p} b^N c^N$ or $= a^N b^{N-p} c^N$ or $a^N b^N c^{N-p}$ where $p = |vx| > 0$, so that

$uwy \neq a^m b^{m+k} c^m$ contradicting the ***PL*** . If however ***Case 2*** holds then

$uwy = a^{N-i} b^{N-j} c^N$ or $a^N b^{N-i} c^{N-j}$ where $0 < i+j \leq N$ so that again

$uwy \neq a^m b^{m+k} c^m$ for any value of $m,k \geq 0$ which again contradicts the ***PL*** .

On the other hand $L_2 = \{a^n b^k c^n \ ; \ n,k \geq 0\}$ is a *CFL* generated by the *CFG*

$G=(\{S,X\},\{a,b,c\},R,S)$ where R is given by

$S \rightarrow aSc \mid X \ ; \ X \rightarrow bX \mid e$

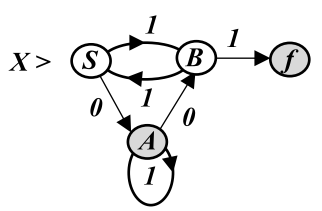
This can be converted to a *CNF* grammar $G' = (\{S,X,Y,A,B,C\},\{a,b,c\}, R',S\ )$ with R' as below :

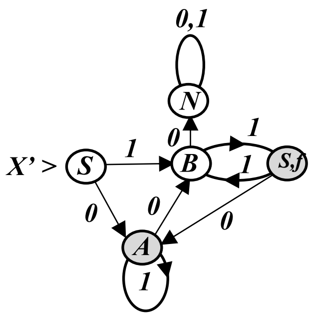
$S \rightarrow YC \mid BX \mid b \ ; \ Y \rightarrow AS \ ; \ X \rightarrow BX \mid b \ ; \ A \rightarrow a \ ; \ B \rightarrow b \ ; \ C \rightarrow c$

(b) *(10 pts)* For the right linear grammar with the productions

$S \rightarrow 0A \mid 1B \ ; \ A \rightarrow 0B \mid 1A \mid e \ ; \ B \rightarrow 1S \mid 1$

the NFA X that accepts L_G and the *DFA* X' are given below. The table below shows $Y=X'$ is already a minimal state *DFA*.





	S	*B*	*N*	*A*	*S,f*
S		*1*	*1*	*0*	*0*
B			*1*	*0*	*0*
N				*0*	*0*
A					*0*
S,f					

Answer 4 (*25* points)

(a) *(10 pts)* See the relevant slide.

(b) *(7 pts)*

Label TM	*Condition*	*Next TM*
$M > A{=}R$	$\sigma=0$	$\#.R_{\#}.L.B$
	$\sigma=\#$	h_{YES}
	$\sigma=1$	h_{NO}
B	$\sigma=1$	$\#.\ L_{\#}.\ A$
	$\sigma\neq 1$	h_{NO}

(c) *(8 pts)*

Label TM	*Condition*	*Next TM*
$M > A{=}R$	$\sigma=0$	$\#.R_{\#}.L.B$
	$\sigma=\#$	h_{YES}
	$\sigma=1$	h_{NO}
B	$\sigma=1$	$\#.L.C$
	$\sigma\neq 1$	h_{NO}
C	$\sigma=1$	$\#.\ L_{\#}.\ A$
	$\sigma\neq 1$	h_{NO}