SABANCI UNIVERSITY
Faculty of Engineering and Natural Sciences
CS 302 Automata Theory

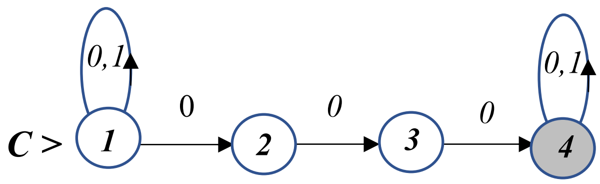
SAMPLE FINAL ANSWERS

Answer 1 (25 points)

(a) *We approach the problem by designing an NFA that accepts* $\mathbf{L}^c$ ***(****the complement of* **L)** *which is given by the following regular expression* **E**

E = *(1+0)*.0.0.0.(1+0)**

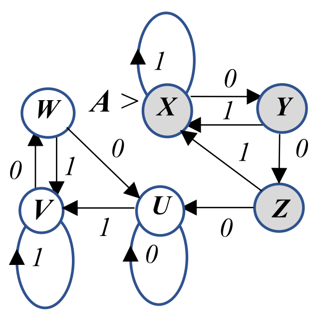
This is accepted by the NFA **C** *below*



The DFA corresponding to **C** *above is as below*

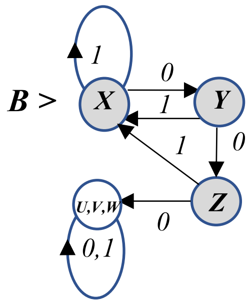
State	*input = 0*	*input = 1*
> 1=X	*1,2 =Y*	*1*
1,2	*1,2,3 =Z*	*1*
1,2,3	*1,2,3,4 =U**	*1*
1,2,3,4	*1,2,3,4*	*1, 4 =V**
1, 4	*1,2,4=W**	*1,4*
1,2,4	*1,2,3,4*	*1,4*

A *is obtained from* **C** *by interchanging the final and non-final states to account for the complement language as shown below.*



(b) *The minimal state machine* ***B*** *is obtained using the table filling algorithm as below where* $\boldsymbol{U \equiv V \equiv W}$.

X	Y	Z	U	V	W
	3	2	1	1	1
		2	1	1	1
			1	1	1
				-	-
					-



Answer 2 (*25* points)

(a) *(10 pts)* The language corresponding to income exceeding ***0*** TL is :

$\boldsymbol{L = (w \in \{0,1\}^* \mid \#1\text{'s in } w > \#0\text{'s in } w)}$

Since this is not a regular language an *NFA* cannot be used for detection. Proof of this is straightforward using the *pumping lemma* for regular languages.

(b) *(15 pts) A PDA* $\boldsymbol{P = (\{q_0, f\}, \{0,1\}, \{0,1,Z_0\}, \delta, Z_0, q_0, \{f\})}$ *where* δ *is given by :*

$\boldsymbol{(q_0, 0, Z_0) \rightarrow (q_0, 0Z_0)}$

$\boldsymbol{(q_0, 0, 0) \rightarrow (q_0, 00)}$

$\boldsymbol{(q_0, 1, 0) \rightarrow (q_0, e)}$

$\boldsymbol{(q_0, 1, Z_0) \rightarrow (f, Z_0)}$

$\boldsymbol{(f, 0, Z_0) \rightarrow (q_0, Z_0)}$

$\boldsymbol{(f, 1, Z_0) \rightarrow (f, 1Z_0)}$

$\boldsymbol{(f, 1, 1) \rightarrow (f, 11)}$

$\boldsymbol{(f, 0, 1) \rightarrow (f, e)}$

Then ***P*** *accepts (or detects) the winning sequence by* ***final state****. Moreover* ***P*** *is a DPDA.*
(Note that according to the phrasing of the question the 'bell' rings after a transition ***iff P*** *is in the final state* ***f****.)*

Answer 3 (*25 points)*

*A CFG **G = ({E,T,F,I} , {+,*, (,) , x,y,z} , R, E)** where the productions **R** are as follows :*

E → E + T | T ; T → T*F | F ; F → I |(E) | I(E) ; I → x|y|z

***(a)** There are no nullable variables hence we start with unit pairs:*

***(E,T),(E,F),(E,I),(T,F) ,(T,I) ,(F,I)** hence we have the following productions :*

E → E + T | T*F | (E)| I(E) | x|y|z

T → T*F | (E) | I(E) |x|y|z

F → (E) | I(E) | x|y|z

Next step is to replace non-single RHS terminals by non-terminals

Plus → +
*Mult → **
[→ (
] →)

Hence

E → E Plus T | T Mult F | [E] | I[E] | x|y|z

T → T Mult F | [E] | I [E] |x|y|z

F → [E] | I [E] | x|y|z

After ensuring two items on RHS the final CNF is given below

E → E A |T B | [C | I D | x|y|z ; A → Plus T ; B → Mult F ; C → E] ; D → [C

T → TB | [C | I D | x|y|z

F → [C | I D | x|y|z;

I → x|y|z

Plus → +
*Mult → **
[→ (
] →)

***(b)** (13 pts)*

Given :

E → E + T | T ; T → T*F | F ; F → I |(E) | I(E) ; I → x|y|z

First remove left recursions by right recursions

E → E + T | T** by : **E → T B ; B → +T B | e

T → T*F | F *by* ***: T → F C ; C → * F C | e***

Then substitute by using appropriate productions eventually to replace the first nonterminals in a production by terminals.

E → FCB *or* ***E → ICB |(E)CB | I(E)CB*** *or:*

E → x-y-z-CB | (E)CB | x-y-z (E) CB *. . . (1)*

T → FC *or*

T → x-y-z-C | (E)C | x-y-z (E) C *. . . (2)*

F → x-y-z | (E) | x-y-z (E) *. . . (3)*

Finally get rid of the nullable productions for ***B*** *and* ***C***

B → +T B | +T *. . . (4)*

C → * F C | *F *. . . (5)*

and in (1) and (2) replace ***CB*** *(and* ***C****) by 4 possibilities (2 possibilities) where none; one or both of* ***B*** *and* ***C*** *are replaced by* ***e****.*

(1),(2), (3), (4) and (5) constitute the GNF

Answer 4 (*25* points)

(a) *(10 pts)*

Label TM	Condition	TM
$A > R$	$\sigma = 0$	$1.A$
	$\sigma = 1$	$0.A$
	$\sigma = \#$	$L_{\#}.h$

(b) *(15 pts)*

Label TM	Condition	TM
$A = R^1_{\#}$	-	B
$B = L^1.R^2$	$\sigma^1 = x \neq \#$	$x^2.B$
	$\sigma = \#$	$L^2_{\#}.C$
$C = R^1.R^2$	$\sigma^1 = \sigma^2 \neq \#$	C
	$\sigma^1 = \sigma^2 = \#$	h_{YES}
	$else$	h_{NO}