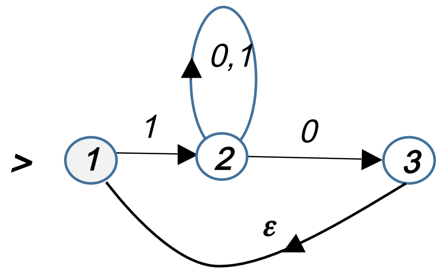
**SABANCI UNIVERSITY**

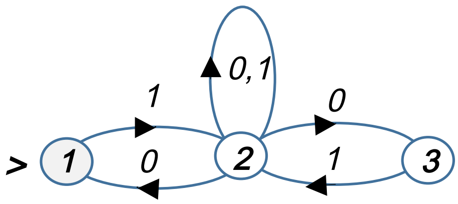
**Faculty of Engineering and Natural Sciences**
**CS 302 Automata Theory**
**Fall 2017**

# *Midterm Answers*

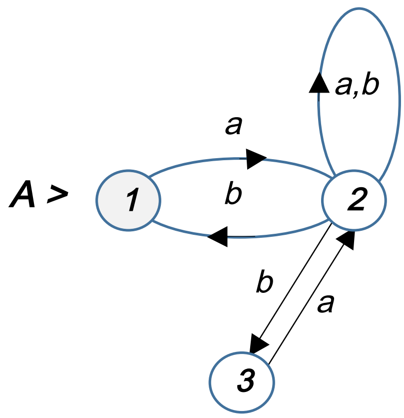
## Answer 1 (*35* points)

***(a)*** *(10 pts)* ***E = (1.(0+1)*.0)****

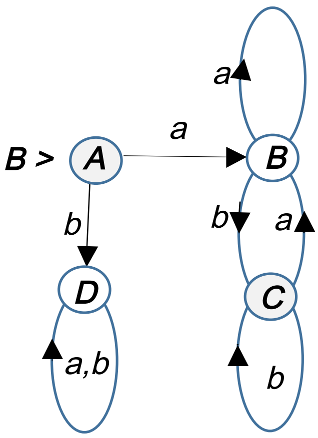



***(b)*** *(25 pts)*



| *q* | σ | *q′* |
|---|---|---|
| *1=A* | *a* | *2* |
| *1* | *b* | ∅ |
| *2=B* | *a* | *2* |
| *2* | *b* | *1,2,3* |
| *1,2,3=C* | *a* | *2* |
| *1,2,3* | *b* | *1,2,3* |
| ∅ = *D* | *a* | ∅ |
| ∅ | *b* | ∅ |




| | A | B | C | D |
|---|---|---|---|---|
| A | | **1** | **2** | **1** |
| B | | | **1** | **2** |
| C | | | | **1** |
| D | | | | |

Hence the *DFA* ***B*** above is already a minimal state machine.

## Question 2 (*30* points)

***(a)*** *(15 pts)* $L = \{w \in (0+1)^* \mid \#0s \text{ in } w > \#1s \text{ in } w\}$

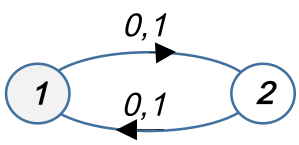
***L*** is a not *regular* language and we prove this using the pumping lemma (PL).

Let $n$ be given by the PL and choose $w = 0^{n+1}1^n$ . Then $w \in L$ and $|w| \geq n$ therefore by the PL :
$w = xyz$ , $|xy| \leq n$ , $|y| \geq 1$ and $xy^jz \in L$ for all $j = 0,1,\ldots$
But by the choice of $w$ : $x = 0^p$ , $y = 0^q$ where $p+q \leq n$ and $q \geq 1$ and $z = 0^{n+1-p-q}\ 1^n$ and therefore for $j=0$ , $xy^jz = xz = 0^p\ 0^{n+1-p-q}\ 1^n = 0^{n+1-q}\ 1^n \notin L$ , a contradiction to the PL since $n+1-q \leq n$

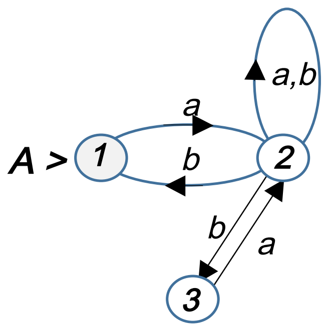
using $q \geq 1$.

***(b)*** *(15 pts)* $L = \{w \in (0+1)^* \mid \#0s \text{ in } w + \#1s \text{ in } w = \textbf{an even number}\}$

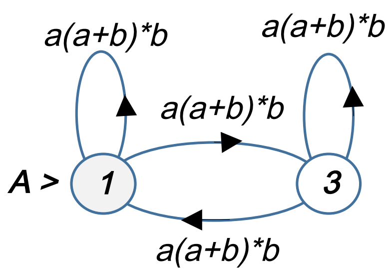
The following NFA accepts ***L***



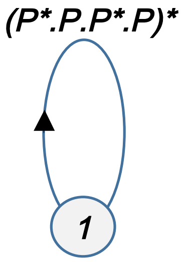

## Question 3 *(35 pts)*




Eliminate state 2 :



Eliminate state 3 :




where ***P := a(a+b)*b*** and since ***(P*.P.P*.P)* = P**** answer is :

***E = (a(a+b)*b)****