# Recit-10

December 14, 2020

### Definition

A context-free grammar is in Chomsky normal form if all productions are of the form

$$A \to BC$$

or

$$A \to a$$

where $A, B, C \in V$ and $a \in T$.

*Q1)* Convert the following CFG to Chomsky Normal Form;

$$S \to ASB$$

$$A \to aAS|a|\epsilon$$

$$B \to SbS|A|bb$$

# Q1

## Chomsky Normal Form

Eliminate $\epsilon$ productions, $A \rightarrow \epsilon$;

$$S \rightarrow ASB|SB$$

$$A \rightarrow aAS|a|aS$$

$$B \rightarrow SbS|A|bb|\epsilon$$

Next eliminate $B \rightarrow \epsilon$;

$$S \rightarrow ASB|SB|AS|S$$

$$A \rightarrow aAS|a|aS$$

$$B \rightarrow SbS|A|bb$$

### Recall

Algorithm for computing the CNF of a grammar $G$;

1. eliminate (a) epsilon productions, (b) unit productions, (c) useless symbols (first non generating then non reachable).
2. For every production including a terminal, call $t$, replace $t$ symbol with a variable $G_t$ that generates corresponding the terminal. And introduce the production $G_t \rightarrow t$.
3. Replace every production of the type $A \rightarrow B_1 B_2 \ldots Bn$ for $n \geq 3$ with the productions: $A \rightarrow B_1 C_1$, $C_1 \rightarrow B_2 C_2$, ... , $C_{n-2} \rightarrow B_{n-1} B_n$ where $C_i$, $i = 1, \ldots, n-2$ are new variables.

Next, remove all unit rules! Begin by removing $B \rightarrow A$;

$$S \rightarrow ASB|SB|AS|S$$

$$A \rightarrow aAS|a|aS$$

$$B \rightarrow SbS|aAS|a|aS|bb$$

We can directly eliminate $S \rightarrow S$;

$$S \rightarrow ASB|SB|AS$$

$$A \rightarrow aAS|a|aS$$

$$B \rightarrow SbS|aAS|a|aS|bb$$

## Recall

Algorithm for computing the CNF of a grammar $G$;

1. eliminate (a) epsilon productions, (b) unit productions, (c) useless symbols (first non generating then non reachable).
2. For every production including a terminal, call $t$, replace $t$ symbol with a variable $G_t$ that generates corresponding the terminal. And introduce the production $G_t \rightarrow t$.
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Replace each non terminal $t$ with variables $G_t$ and introduce the rule $G_t \to t$;

$$S \to ASB|SB|AS$$

$$A \to G_aAS|G_a|G_aS$$

$$B \to SG_bS|G_aAS|G_a|G_aS|G_bG_b$$

$$G_a \to a$$

$$G_b \to b$$

## Recall

Algorithm for computing the CNF of a grammar $G$;

1. eliminate (a) epsilon productions, (b) unit productions, (c) useless symbols (first non generating then non reachable).
2. For every production including a terminal, call $t$, replace $t$ symbol with a variable $G_t$ that generates corresponding the terminal. And introduce the production $G_t \to t$.
3. Replace every production of the type $A \to B_1B_2 \dots Bn$ for $n \geq 3$ with the productions: $A \to B_1C_1$, $C_1 \to B_2C_2$, ... , $C_{n-2} \to B_{n-1}B_n$ where $C_i$, $i = 1, \dots, n-2$ are new variables.

Step 3)

$$S \rightarrow AC_1|SB|AS$$

$$A \rightarrow G_aC_2|G_a|G_aS$$

$$B \rightarrow SC_3|G_aC_2|G_a|G_aS|G_bG_b$$

$$G_a \rightarrow a$$

$$G_b \rightarrow b$$

$$C_1 \rightarrow SB$$

$$C_2 \rightarrow AS$$

$$C_3 \rightarrow G_bS$$

Replace each non terminal $t$ with variables $G_t$ and introduce the rule $G_t \rightarrow t$;

$$S \rightarrow ASB|SB|AS$$

$$A \rightarrow G_aAS|G_a|G_aS$$

$$B \rightarrow SG_bS|G_aAS|G_a|G_aS|G_bG_b$$

$$G_a \rightarrow a$$

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$$S \to AC_1|SB|AS$$
$$A \to G_aC_2|G_a|G_aS$$
$$B \to SC_3|G_aC_2|G_a|G_aS|G_bG_b$$
$$G_a \to a$$
$$G_b \to b$$
$$C_1 \to SB$$
$$C_2 \to AS$$
$$C_3 \to G_bS$$

**Definition**

A context-free grammar is in Chomsky normal form if all productions are of the form

$$A \to BC$$

or

$$A \to a$$

where $A, B, C \in V$ and $a \in T$.

CYK algorithm checks the membership of a given string $w$ for a given grammar $G$ as follows;

**Recall**

1. Algorithm works only if the grammar is in CNF.
2. Given $w = a_1 a_2 a_3 \ldots a_n$, define $w_{ij} = a_i \ldots a_j$.
3. Define subsets of $V$, $V_{ij} = \{A \in V : A \Rightarrow^* w_{ij}\}$
4. $w \in L(G) \iff S \in V_{1n}$ where $n = |w|$.

*Q2)* Given the string $w = aabbb$ and the grammar $G$;

$$S \to AB$$

$$A \to BB|a$$

$$B \to AB|b$$

check if $w \in L(G)$.

$$V_{ij} = \cup\{X : X \to YZ \; s.t. \; Y \in V_{ik}, Z \in Vk+1,j\} \text{ for } k = i, \ldots, j-1$$

$$S \to AB$$
$$A \to BB|a$$
$$B \to AB|b$$

**Recall**

1. Algorithm works only if the grammar is in CNF.
2. Given $w = a_1 a_2 a_3 ... a_n$, define $w_{ij} = a_i ... a_j$.
3. Define subsets of $V$, $V_{ij} = \{A \in V : A \Rightarrow^* w_{ij}\}$
4. $w \in L(G) \iff S \in V_{1n}$ where $n = |w|$.

Similar to the first step analysis method; first define $V_{ii}$ sets.

$w_{11} = a,\ w_{22} = a,$
$w_{33} = b,\ w_{44} = b,\ w_{55} = b$

$V_{11} = A,\ V_{22} = A,$
$V_{33} = B,\ V_{44} = B,\ V_{55} = B$

$w_{12} = aa$, $w_{23} = ab$, $w_{34} = bb$, $w_{45} = bb$.

$V_{12} = \{X : X \to YZ, Y \in \{A\}, Z \in \{A\}\}$, there is no $AA$ in the rules.
$V_{12} = \emptyset$

$V_{23} = \{X : X \to YZ, Y \in \{A\}, Z \in \{B\}\}$,
since we have $S \to AB$ and $B \to AB$;
$V_{23} = \{S, B\}$

Similarly;
$V_{34} = \{A\}$, $V_{45} = \{A\}$

$w_{13} = aab$, $w_{24} = abb$, $w_{35} = bbb$

$V_{24} := V_{22}.V_{34} \cup V_{23}.V_{44}$

$\{X : X \to YZ, Y \in V_{22}, Z \in V_{34}\} =$
$= \{X : X \to YZ, Y \in \{A\}, Z \in \{A\}\} = \emptyset$, no production for $AA$.

$\{X : X \to YZ, Y \in V_{23}, Z \in V_{44}\} =$
$= \{X : X \to YZ, Y \in \{S, B\}, Z \in V_B\} = \{A\}$ since $A \to BB$, no production for $SB$.

$V_{24} = \emptyset \cup \{A\} = \{A\}$

Similarly,
$V_{13} = \{S, B\}$
$V_{35} = \{S, B\}$

$w = aabbb$

Similarly, you can find $V_{25} = \{S, B\}$, $V_{14} = \{A\}$

$V_{15} := V_{14}.V_{55} \cup V_{13}.V_{45} \cup V_{12}.V_{35} \cup V_{11}.V_{25}$

$\{X : X \to YZ, Y \in V_{14}, Z \in V_{55}\} =$
$= \{X : X \to YZ, Y \in \{A\}, Z \in \{B\}\} = \{S, B\}$, since $S$ *and* $B \to AB$.

$\{X : X \to YZ, Y \in V_{11}, Z \in V_{25}\} =$
$= \{X : X \to YZ, Y \in \{A\}, Z \in V_{S,B}\} = \{S, B\}$ since $S$ *and* $B \to AB$, and no production for $AS$.

...

$s \in V_{15}$

DONE! $S \in V_{15}$

# Q3

Construct a PDA that accepts the languages of the following grammar. It is already in GNF.

$$S \rightarrow aABA|aBB$$

$$A \rightarrow bA|b$$

$$B \rightarrow cB|c$$

**Definition**

A context-free grammar is said to be in Greibach normal form if all productions have the form

$$A \rightarrow \alpha x$$

where $a \in T$ and $x \in V^*$

# Q3

$$S \to aABA | aBB$$

$$A \to bA | b$$

$$B \to cB | c$$

$Q = \{q_0, q_1\},\ \Sigma = \{a, b, c\},\ \Gamma = \{A, B\},\ F = \{q_1\}$

$\delta(q_0, a, e) = (q_1, ABA),\ \delta(q_0, a, e) = (q_1, BB)$
$\delta(q_1, b, A) = (q_1, A),\ \delta(q_1, b, A) = (q_1, e)$
$\delta(q_1, c, B) = (q_1, B),\ \delta(q_1, c, B) = (q_1, e)$