CS 302
QUIZ 7

26 November, 2019

ANSWERS

(a) (5 points) See slide set #7

(b) (5 points) $\mathbf{L = (\omega \in \{0,1\}^* \mid \omega = a^{k+2}\, b^{\,k+1}\, c^{\,k}, k \geq 0\,)}$ is NOT a CFL.

Assume it is ; then there is some $\mathbf{n > 0}$ and we choose $\mathbf{w = a^{n+2}b^{n+1}c^{n}}$, hence

$\mathbf{|w| = 3n+3 > n}$ and by PL $\mathbf{w = uvwxy}$ and

(i) $\mathbf{|vwx| \leq n}$;(ii) $\mathbf{|vx| > 0}$ and (iii) $\mathbf{u\, v^{\,j}\, w\, x^{\,j}\, y \in L}$ for all $\mathbf{j \geq 0}$ and in particular for $\mathbf{j = 0}$.

We show that $\mathbf{uwy \notin L}$, contradicting (iii) which implies that $\mathbf{L}$ is not a CFL.

Note that because of (i) either : (1) $\mathbf{vwx = a^m}$ or $\mathbf{= b^m}$ or $\mathbf{= c^m}$ where $\mathbf{m \leq n}$ **; OR**

(2) $\mathbf{vwx = a^{\,i}\, b^{\,j}}$ **;** or $\mathbf{= b^{\,i}\, c^{\,j}}$ where $\mathbf{i+j \leq n}$. But then because of (ii) with $\mathbf{p = |vx| > 0}$

if (1) holds then $\mathbf{uwy = a^{n+2-p}\; b^{n+1}\, c^{n}}$; or $\mathbf{uwy = a^{n+2}\; b^{n+1-p}\, c^{n}}$; or $\mathbf{uwy = a^{n+2}\; b^{n+1}\, c^{n-p}}$ **;**

OR if (2) holds then $\mathbf{uwy = a^{n+2-r}\, b^{n+1-t}\, c^{n}}$; or $\mathbf{uwy = a^{n+2}\, b^{n+1-r}\, c^{n-t}}$ where $\mathbf{r+t = p > 0}$.

Hence for all the cases above $\mathbf{uwy \notin L}$ and the result follows !