

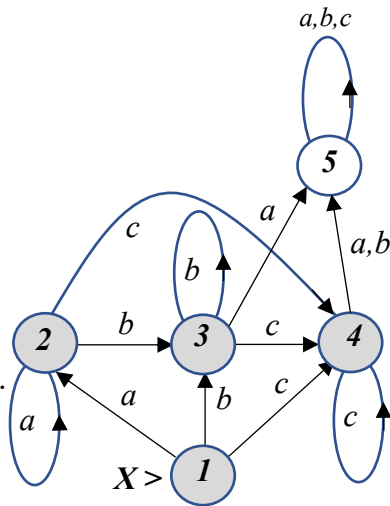
# SABANCI UNIVERSITY

## Faculty of Engineering and Natural Sciences CS 302 Automata Theory Fall 2020

### Remote Midterm Answers

#### Answer 1 (50 points)

(a) (15 pts) A DFA  $X$  that accepts  $L_1$  is given below ; hence  $L_1$  is regular.



(b) (15 pts)  $L_2$  is not regular. Choose  $w = a^{2n} c^n$  which is in  $L_2$  and has length  $3n > n$ . By the pumping lemma  $w = xyz$  where  $|xy| \leq n$  and  $|y| > 0$ ; hence  $xyz = a^{2n} c^n$  implies that  $xy = a^p$  and  $y = a^q$  with  $p \leq n$  and  $q > 0$ . Hence  $x = a^{p-q}$ ,  $y = a^q$  and  $z = a^{2n-p} c^n$  therefore  $xy^j z$  for  $j=0$  is :  $xz = a^{p-q} a^{2n-p} c^n = a^{2n-q} c^n$  and since  $q > 0$  occurrence of  $a$ 's are less than twice that of  $c$ 's and thus  $xz$  is not in  $L_2$  as demanded by the pumping lemma and so  $L_2$  is NOT regular.

(c) (20 pts)  $G = (\{S, B\}, \{a, b, c\}, R, S)$  where the production set  $R$  is given by

$R : S \rightarrow aaSc \mid B \mid e ; B \rightarrow bB \mid e$

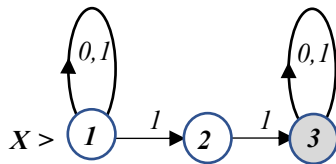
## Answer 2 (50 points)

(a) (25 pts)  $E = (0+1)^*1.1.(0+1)^*$ .

$E$

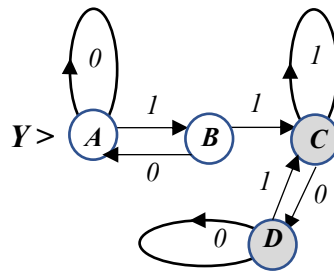
$\rightarrow E * E \rightarrow (E) * E \rightarrow (E+E) * E \rightarrow (0+E) * E \rightarrow (0+1) * E \rightarrow (0+1) * E.E \rightarrow (0+1) * 1.E$   
 $\rightarrow (0+1) * 1.E.E \rightarrow (0+1) * 1.1.E \rightarrow (0+1) * 1.1.E * \rightarrow (0+1) * 1.1.(E) * \rightarrow (0+1) * 1.1.(E+E) * \rightarrow ($   
 $0+1) * 1.1.(0+E) * \rightarrow (0+1) * 1.1.(0+1) *$

(b) (25 pts) NFA for  $E$  is given below

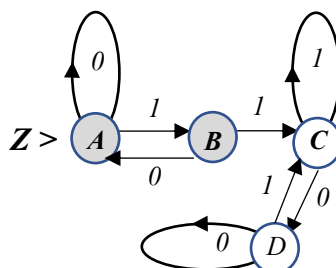


This is converted to a DFA  $Y$  using the table below

state	input	next
$>1=A$	0	1
1	1	1,2
$1,2=B$	0	1
1,2	1	1,2,3
$1,2,3=C^*$	0	1,3
1,2,3	1	1,2,3
$1,3=D^*$	0	1,3
1,3	1	1,2,3



The complement language corresponds to a DFA where final and non-final states of  $Y$  are interchanged yielding the DFA  $Z$  below



To find the minimal state DFA we use the table filling algorithm as below

C          D          A          B

	Eq	0	0
		0	0
			1

Hence **D** and **C** are equivalent states and a minimal state *DFA W* is :

