

Question 1

1-)

it says in the homework that input n will always be in the type of 3^k it is not possible for n to be an even number so it can't enter the else if ($n \% 2 == 0$) statement because the remainder is never 0 when divided by 2, so that else if is unnecessary.

let's say the function that calculate the number of stars when it's given a number " n " (which is in 3^k format) is $T(n)$, when we say that we see $T(3^0) = T(1) = 1$ this is the base case $T(1) = 1$

when the unnecessaryStars(n) function enters the ($n \% 2 == 1$) else if statement we can see it enters the recursion which prints 1 star for each recursion in the amount of times the n is divisible by 3 until it reaches to the 1; it prints 3 stars when each division step has ended. so we can express the $T(n)$ as $T(n) = 3 \cdot T(\frac{n}{3}) + 3$ except $n \neq 1$.

2-)

to find the closed form expression for the recurrence we will write the successive expansions.

1. $T(n) = 3 \cdot T(n/3) + 3$, let's put the $T(n)$ inside the n in $T(n/3)$

2. $T(n) = 3 \cdot (3 \cdot T(n/9) + 3) + 3$, let's do it again

3. $T(n) = 3^2 \cdot (3 \cdot T(n/27) + 3) + 3^2 + 3$, now we can see the pattern, after k amount of steps the closed form expression would be:

$$T(n) = 3^k \cdot T(n/3^k) + 3^k + 3^{k-1} + \dots + 3^1 \rightarrow \sum_{n=1}^k 3^k = \frac{1-3^{k+1}}{1-3}$$

since it is said that $n = 3^k$ we can solve the k and find $k = \log_3 n$. when we input $n = 3^k$ and $k = \log_3 n$, we can get rid of the geometric series because it converges to a constant value and for $3^k \cdot T(n/3^k)$ it also converges so the whole $T(n)$ converges to a constant so the time complexity can be expressed as $O(n)$

$T(n) = O(n)$, so we expressed the result as $O(n)$ by solving the recurrence by successive expansions.

Question 2

1-) Since this question is almost the same as question 1 i will do less explanations.

let's define $T(n)$ which gives the number of zeros that will be printed when given a 2^k type of n .
our base case is $T(0) = 1$.

`printZerosHelper` prints $(\log_2 n) + 1$ amount of zeros.
and there is two recurrence of $T(n-2)$ so
$$T(n) = (\log_2 n + 1) + 2 \cdot T(n-2).$$

2-)

$$1. T(n) = (\log_2(n) + 1) + 2 \cdot T(n-2)$$

$$2. T(n) = (\log_2(n) + 1) + 2 \cdot ((\log_2(n-2) + 1) + 2 \cdot T(n-4))$$

$$3. T(n) = (\log_2 n + 1) + 2 \cdot ((\log_2(n-2) + 1) + 4 \cdot ((\log_2(n-4) + 1) + 2 \cdot T(n-6)))$$

so if we do this for k steps:

after some amount of calculations we see that the common ratio of this geometric series is 2 which can be simplified as $2^{n/2} \cdot \log_2(n)$ so the $T(n) = O(2^{n/2} \cdot \log_2 n)$ time complexity.

Index of comments

2.1 You should have mentioned how you got $n/2$. What was the termination condition.