Joint Homeworks #7,#8 *due January 17, 23.59*

Question 1

Describe a single tape *TM* ***M*** in both graphical and compositional tabular forms that *decides* the language $L = \{u \in \Sigma_0^* \mid u = u^R\}$. Take the initial *ID* as: $(s, \underline{\#}u)$ **).** Verify your result by exhibiting the step-by-step tape contents for $u = abcde$

(*Hint: you may use an extra symbol such as* $\$ \notin \Sigma_0$ *to facilitate your computation)*

Question 2

Describe a TM ***M*** in both graphical and compositional tabular forms that performs the following computation:

$(s, \$w\underline{\#}) \vdash^*_M (h, \$u\underline{\#})$

where ***u*** is obtained from ***w*** by compressing all blank (#) symbols in ***w*** and ***$*** is a special symbol not used in ***w***. Verify your result by exhibiting the step-by-step tape contents for the input

$w = \#ab\#c\#a\#a$

Question 3

(a) Construct a 2-tape *DTM* ***M*** that decides the language below (*specify the TM in tabular compositional form)* $L_2 = \{\omega \in (0+1)^* \mid w = u.u\ ,\ u \in (0+1)^*\}$

(b) Repeat part (a) for $L_N = \{\omega \in (0+1)^* \mid w = u^N,\ u \in (0+1)^*\}$ where ***N*** is a given integer constant.

Question 4

Construct a *TM* ***M*** (***multitape*** *and/or* ***nondeterministic*** *if necessary!)* that decides the language below (*specify the TM in tabular compositional form)*.

$L_n = (\omega \in \{a,b,c,d\}^* \mid w = a^n b^m c^n d^m\ ,\ n, m > 0\ )$

Question 5

Construct *TMs* in compositional tabular forms (***multitape*** *and/or* ***nondeterministic*** *if necessary!)* that perform the following computations:

(i) $(s, \underline{\#}w) \vdash^*_M (h, \underline{\#}w^R)$

(ii) $(s, \underline{\#}w) \vdash^*_M (h, \underline{\#}ww)$

(iii) $(s, \underline{\#}w) \vdash^*_M (h, \underline{\#}w\#w^R)$

(iv) $(s, \underline{\#}w) \vdash^*_M (h, \underline{\#}a^n b^n)$ *where* $\omega \in (a+b)^*$ *the number of* ***a****s and* ***b****s in* ***w*** *are both equal to the integer* $n > 0$

(v) Same question as in part (iv) except that the initial $\omega \in (a+b+c+d)^*$.

Question 6

Construct in *tabular compositional form* the details of a ***3-tape*** universal TM that uses the tape alphabet $\boldsymbol{\Sigma_U = \{\#,0,1,\$,',', (,)\}}$ with *7* characters **,** where ***tape 1*** simulates the actual tape contents of the simulated DTM in encoded form ; ***tape 2*** is read-only and has the contents of the *transition function* of the simulated *TM* in encoded form ; and ***tape 3*** has the contents of the encoded *current state* of the simulated TM ; all as described in class. To make life simple choose the encoding of the *states* and the *input alphabet* of the simulated machine in fixed number of binary digits by adding zeros to the left of the codes if necessary (else you will have expansion and compression problems in simulated tape contents).