Homework #5

(1) Let *M* be the *PDA* defined by $Q = \{q, q_0, q_1, q_2\}, \Sigma = \{a,b\}, \Gamma = \{a\}, F := \{q, q_1\}.$

$$\delta(q_0, a, Z_0) = \{(q, Z_0)\}$$

$$\delta(q, a, Z_0) = \{(q, aZ_0)\}$$

$$\delta(q, a, a) = \{(q, aa)\}$$

$$\delta(q, b, a) = \{(q_1, e)\}$$

$$\delta(q_1, b, a) = \{(q_1, e)\}$$

$$\delta(q_1, b, Z_0) = \{(q_2, e)\}$$

- *a*) Describe the language accepted by *M*.
- *b*) Trace all computations of the strings *aab*, *abb*, *aba* in *M*.
- c) Show that *aaabb*, $aaab \in L(M)$.
- (2) Construct *PDA*s that accept each of the following languages.

a)
$$\{a^{\mathbf{i}}b^{\mathbf{j}}\mid 0\leq i\leq j\}$$

$$b) \; \{a^{\boldsymbol{i}}c^{\boldsymbol{j}}b^{\boldsymbol{i}} \mid i,j \geq 0\}$$

c)
$$\{a^{i}b^{j}c^{k} \mid i+k=j\}$$

$$d) \quad \{a^{\mathbf{i}}b^{\mathbf{j}} \mid 0 \le i \le j \le 2i\}$$

(3) $L = \{w \in \{a, b\}^* \mid at \ least \ one \ prefix \ of \ w \ contains \ strictly \ more \ b's \ than \ a's.\}.$

For example, baa, abb, abbbaa are in L, but aab, aabbab are not in L.

- **a)** Construct a PDA that accepts \boldsymbol{L} by final state.
- **b)** Construct a PDA that accepts \boldsymbol{L} by empty stack.
- **(4)** From the main text *Exercises* **6.3.4**, **7.1.3**, **7.2.1**(*b*,*c*)