CS 302
REMOTE QUIZ 8

21 December, 2020

ANSWERS

(a) (5 pts) L_1 is a CFL generated by CFG $G = (\{S,A\},\{a,b,c,d\}, R,S)$ where R is given by :

$S \rightarrow aSd \mid B \; ; B \rightarrow bBc \mid e$

(b) (5 pts) L_2 is not a CFL which we prove using the pumping lemma. Assume L_2 is CFL and let n be given and choose $z = a^{n+2} b^{n+1} c^n$ so that $|z| = 3n+3 > n$ as required by the PL

By the PL $z = uvwxy = a^{n+2} b^{n+1} c^n$. Since by the PL $|vwx| \leq n$ we have **2** cases :

Case 1 $vwx = a^t$; or $vwx = b^t$; or $vwx = c^t$ where $t \leq n$

Case 2 $vwx = a^j b^i$; or $vwx = b^j c^i$;

Again by PL $|vx| = r > 0$ so :

Case 1 $|uwy| = a^{n+2-r} b^{n+1} c^n$; or $= a^{n+2-} b^{n+1-r} c^n$; or $= a^{n+2-} b^{n+1-} c^{n-r}$

Case 2 $|uwy| = a^{n+2-p} b^{n+1-q} c^n$ or $= a^{n+2-} b^{n+1-p} c^{n-q}$; $p+q = r > 0$

For all cases $uwy \notin L_2$ contradicting the assumption that L_2 is CFL as demanded by PL.