Answers of homework Exercises

Module FREQ: Frequency Response FIR filter

Course: Signals Processing Basics (5ESE0)

Notes:

- During the contact hours complete workout of exercises can be explained by a tutor on request.
- The symbol [P] in the margin of an exercise denotes there is a pencast available.

Exercise 1

$$|H(\mathrm{e}^{\mathrm{j}\theta})| = 9 - 6\cos(\theta) + 2\cos(2\theta) \text{ and } \angle\{H(\mathrm{e}^{\mathrm{j}\theta})\} = -2\theta$$

Exercise 2

a.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = -1 + 2\mathrm{e}^{-\mathrm{j}\theta} - \mathrm{e}^{-\mathrm{j}2\theta}$$

b.

$$y[n] = 10\cos(\frac{\pi}{2}n - \frac{\pi}{6})$$

c.

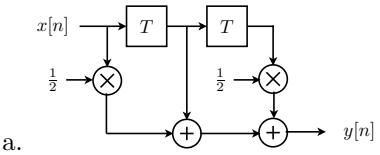
$$y[n] = h[n] = -\delta[n] + 2\delta[n-1] - \delta[n-2]$$

d.

$$y[n] = u[n] \star h[n] = -\delta[n] + \delta[n-1]$$

Exercise 3

a.

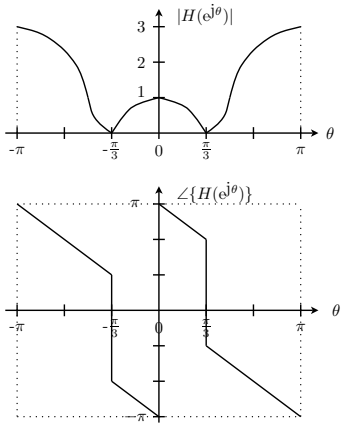


b.

$$y[n] = 12\cos(\frac{\pi}{3}n - \frac{\pi}{2})$$

Exercise 4

$$H(\mathrm{e}^{\mathrm{j}\theta}) = (1 - 2\cos(\theta)) \cdot \mathrm{e}^{-\mathrm{j}\theta}$$

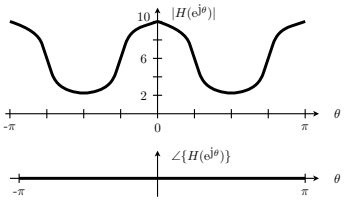


Exercise 5

a.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = 6 + 4\cos(2\theta)$$

b.



c.

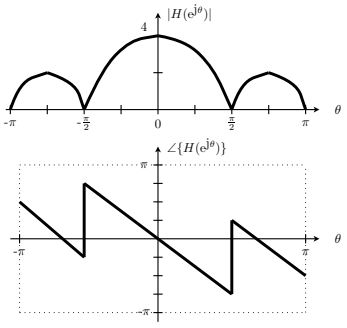
$$y[n] = 100 - 20\cos(\frac{\pi}{2}(n-1))$$

Exercise 6

a.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = \left(\frac{\sin(2\theta)}{\sin(\frac{1}{2}\theta)}\right) \cdot \mathrm{e}^{-\mathrm{j}(\frac{3}{2})\theta}$$

b.



c. $\theta_0 = \frac{\pi}{2}$ since

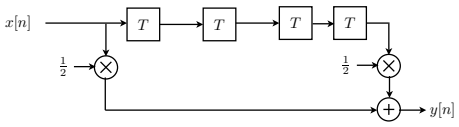
$$x[n] = 1 + 2\cos(n\frac{\pi}{2}) \quad \mapsto \quad y[n] = 4$$

Exercise 7

a. Band-stop filter.

b.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = \frac{1}{2} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}4\theta} \quad \circ\!\!-\!\!\circ \quad h[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-4]$$



Exercise 8

a.

$$y_3[n] = x_3[n-1] - x_3[n-2] \quad \Rightarrow \quad h_3[n] = \delta[n-1] - \delta[n-2]$$

b.

$$H_1(\mathrm{e}^{\mathrm{j}\theta}) = 1 + \mathrm{e}^{-\mathrm{j}2\theta} \qquad H_2(\mathrm{e}^{\mathrm{j}\theta}) = 7\mathrm{e}^{-\mathrm{j}5\theta} + 7\mathrm{e}^{-\mathrm{j}6\theta}$$

c.

$$H(\mathrm{e}^{\mathrm{j}\theta}) \quad = \quad 7\mathrm{e}^{-\mathrm{j}6\theta} - 7\mathrm{e}^{-\mathrm{j}10\theta}$$

d.

$$h[n] = 7\delta[n-6] - 7\delta[n-10]$$

Exercise 9

[P2]

a.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = 1 + \mathrm{e}^{-\mathrm{j}3\theta} \quad \circ\!\!-\!\!\circ \quad h[n] = \delta[n] + \delta[n-3]$$

b. The system filters out frequencies $\theta = \pm\pi$ and $\theta = \pm\frac{\pi}{3}$.

c.

$$y[n] = 6 + \delta[n-2] + \delta[n-5] + \sqrt{2}\cos(\frac{\pi}{2}n + \frac{\pi}{2})$$

Exercise 10

a.

$$H_1(\mathrm{e}^{\mathrm{j}\theta}) = 1 - \alpha\mathrm{e}^{-\mathrm{j}\theta}$$

b.

$$H_2(\mathrm{e}^{\mathrm{j}\theta}) = \frac{1 - \alpha^6\mathrm{e}^{-\mathrm{j}6\theta}}{1 - \alpha\mathrm{e}^{-\mathrm{j}\theta}}.$$

c.

$$H(\mathrm{e}^{\mathrm{j}\theta}) = 1 - \alpha^6\mathrm{e}^{-\mathrm{j}6\theta} \quad \circ\!\!-\!\!\circ \quad h[n] = \delta[n] - \alpha^6\delta[n-6]$$

d.

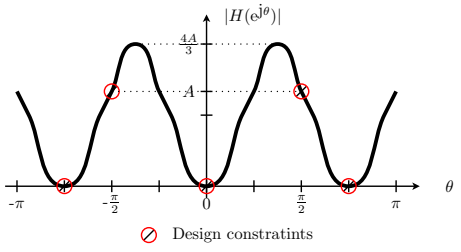
$$h[n] = h_1[n] \star h_2[n] = \delta[n] - \alpha^6\delta[n-6]$$

Exercise 11

$$y(t) = 20 + 6\sqrt{2}\cos(2000\pi t + \frac{\pi}{12}).$$

Exercise 12

a.



b. The filter must be the first filter $H_1(e^{j\theta})$ and we must choose $f_{s,a}$.