Answers of homework Exercises

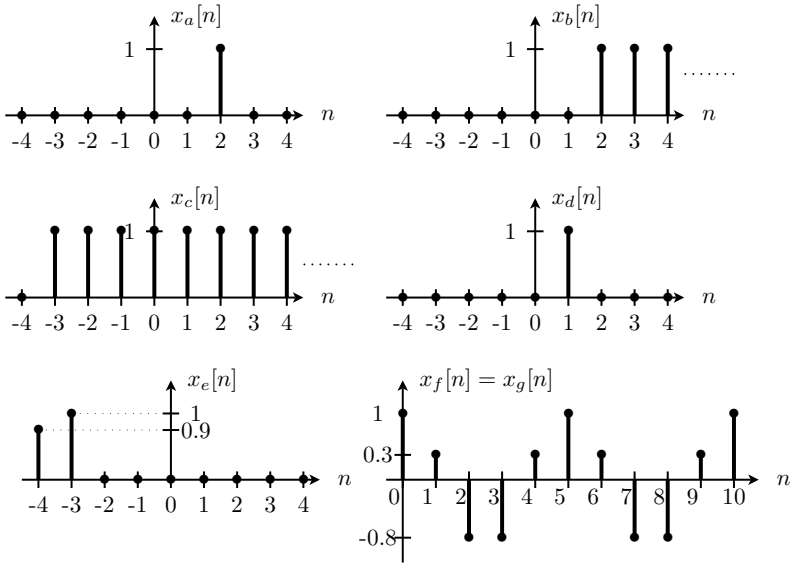
Module FIR: Finite Impulse Response

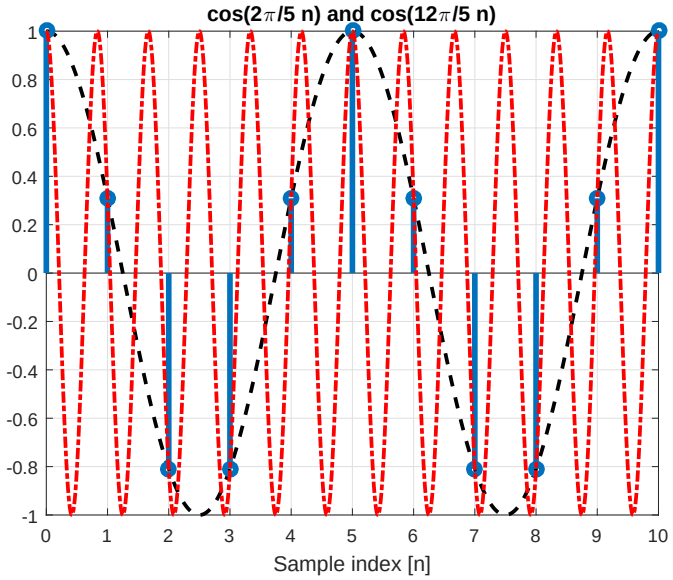
Course: Signals Processing Basics (5ESE0)

Notes:

- During the contact hours complete workout of exercises can be explained by a tutor on request.
- The symbol [P] in the margin of an exercise denotes there is a pencast available.

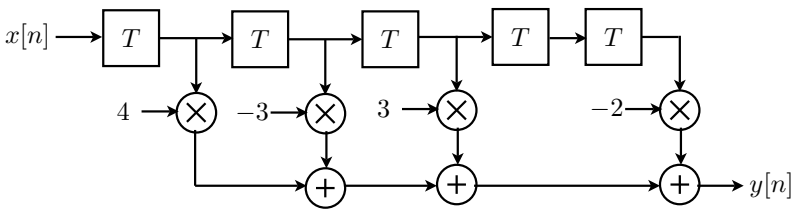
Exercise 1





Exercise 2

a)

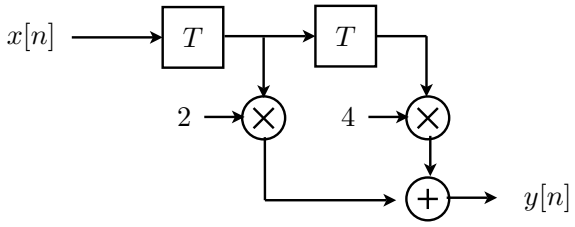


b)

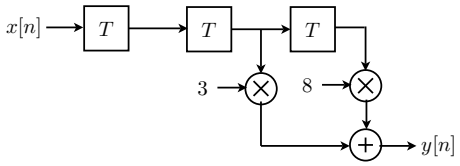
$$y[n] = 7x[n] - 2x[n-1] + 3x[n-2] - 3x[n-3]$$

[P1] c)

$$y[n] = 2x[n-1] + 4x[n-2]$$

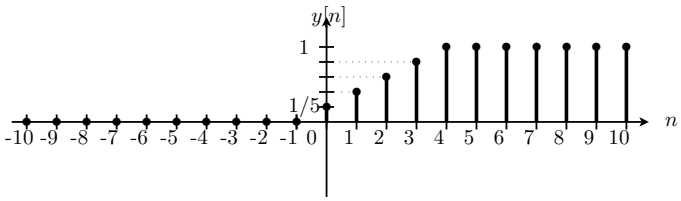


Exercise 3



Exercise 4

a)



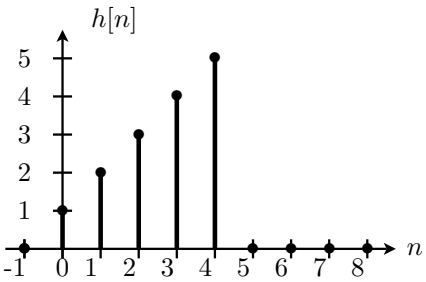
b)

$$y[n] = \begin{cases} 0 & n < 0 \\ \frac{1}{L}(n+1) & 0 \le n \le L-1 \\ 1 & n > L-1 \end{cases}$$

Exercise 5

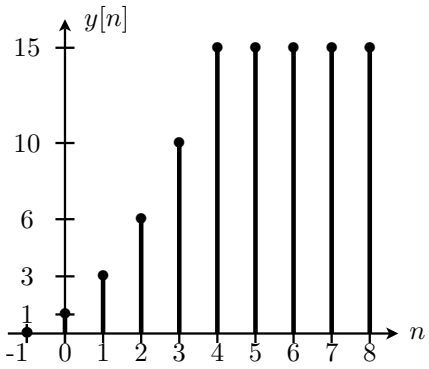
a. $b_0 = 1, b_1 = 2, b_2 = 3, b_3 = 4, b_4 = 5$ and $b_n = 0$ for $n < 0$ and $n > 4$.

b. $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$



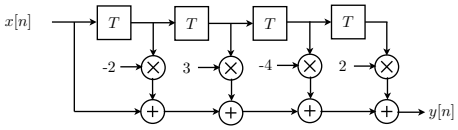
c.

$$y[n] = \delta[n] + 3\delta[n-1] + 6\delta[n-2] + 10\delta[n-3] + 15u[n-4]$$



Exercise 6

[P2] a)

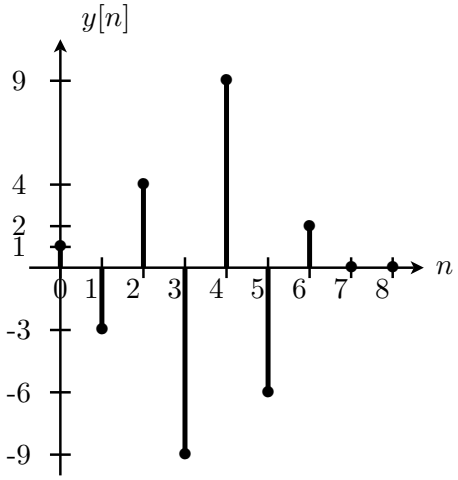


b)

$$h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - 4\delta[n-3] + 2\delta[n-4]$$

c)

$$y[n] = \delta[n] - 3\delta[n-1] + 4\delta[n-2] - 9\delta[n-3] + 9\delta[n-4] - 6\delta[n-5] + 2\delta[n-6]$$



Exercise 7

a.

$$A_1 = A_2 = 4 \; ; \; \theta_1 = -\theta_2 = \theta_x = \frac{\pi}{6} \; ; \; \phi_1 = -\phi_2 = -\frac{\pi}{6}$$

b. Thus $A = 8$, $\theta_y = \theta_x = \frac{\pi}{6}$ and $\phi = -\frac{\pi}{6}$. The output frequency θ_y is equal to the input frequency θ_x.

Exercise 8

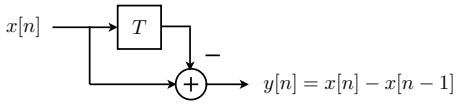
	Linear	Time-Invariant	Causal
a	Yes	Yes	No
b	No	Yes	No
c	No	Yes	Yes
d	Yes	No	Yes
e	Yes	Yes	Yes

[8b=P3]

Exercise 9

$L = 3$ and $\{b_0, b_1, b_2, b_3\} = \{0, 1, 2, 3\}$,, while all other coefficients are equal to zero.

Exercise 10



Exercise 11

We can set up a linear set of 3 equations with two unknown b_0 and b_1:

$$\begin{aligned} b_0 &= 1 \\ b_0 + b_1 &= 0 \\ b_1 &= 0 \end{aligned}$$

which leads to a contradiction. Thus we can not make such an FIR.

Exercise 12

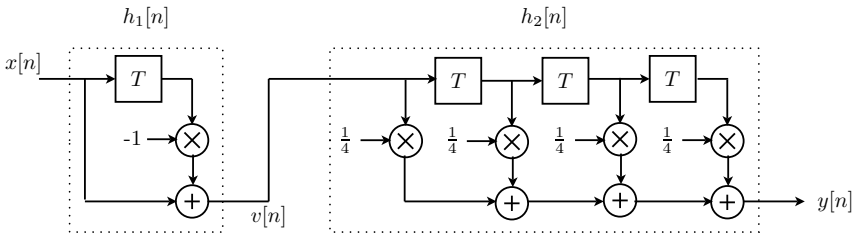
$y[n] = \delta[n-1] + \delta[n-3] - \delta[n-4] - \delta[n-6]$

Exercise 13

a. $v[n] = x[n] - x[n-1]$

b. $h_2[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

c.

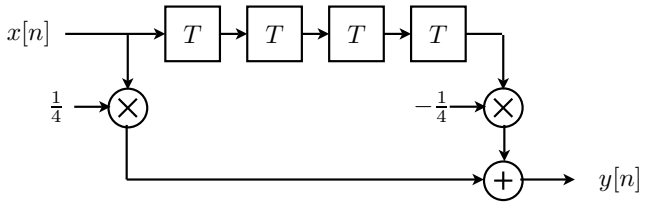


The cascaded system has 4 delays and 5 multiplications.

d.

$$h[n] = \frac{1}{4}\delta[n] - \frac{1}{4}\delta[n-4]$$

e. A signal flow graph of the combined system is given in the figure.



This realization contains 4 delays and two multipliers.

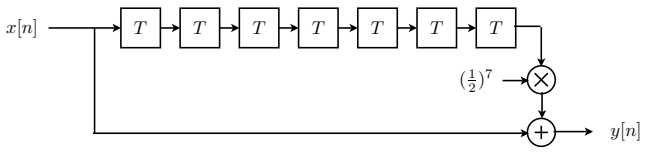
Exercise 14

[P4]

a.

$$h[n] = \delta[n] - \delta[n-10]$$

b. $y[n] = x[n] - x[n-10]$.



c.

Exercise 15

Filter $h_1[n]$ has a 'low pass' character, since it passes low frequencies and it attenuates high frequencies.
Filter $h_2[n]$ has a 'high pass' character, since it passes high frequencies and it attenuates low frequencies.