Homework Exercises

Module CNAP: Complex Numbers And Phasors

Course: Signals Processing Basics (5ESE0)

Notes:

- Only the answers are available for students. No pdf document with complete workout available for students.
- During the contact hours complete workout of exercises can be explained on request.
- The symbol [P] in the margin of an exercise denotes there is a pencast available.

Exercise 1

Fig.1 shows a plot of a sinusoidal signal. From the plot, determine the values for the amplitude A, phase φ, and frequency ω needed in the representation:

$$x(t) = A\cos(\omega t + \varphi).$$

Give the answer as numerical values, *including the units* where applicable.

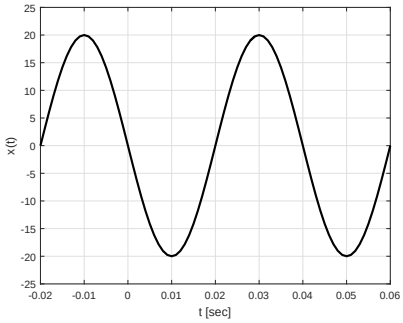

Figure 1: Sinusoidal signal

Exercise 2

You can derive the real $\Re e\{\cdot\}$ and imaginary part $\Im m\{\cdot\}$ of complex exponentials for all angles that are multiples of $\pi/4$ and $\pi/6$ without a calculator if you learn only a few specific values by heart.

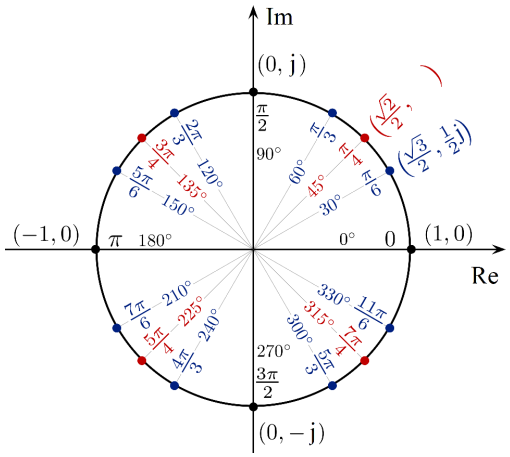

Figure 2: Unit circle in complex plane

[P1]

a. Try to invent a simple trick to remember the values for $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ when $\theta = \frac{\pi}{3}, \frac{\pi}{4}$ or $\frac{\pi}{6}$?

b. From Fig.2, it follows that $e^{j\frac{\pi}{6}} = \frac{1}{2}\sqrt{3} + j\frac{1}{2}$. With this knowledge, can you determine $\Re e\{e^{j\frac{2\pi}{3}}\}$ and $\Im m\{e^{j\frac{2\pi}{3}}\}$

c. Determine $\Im m\{e^{j\frac{\pi}{4}}\}$ and with this and previous answers try to fill all missing numbers of Fig.2.

Exercise 3
Convert the following complex numbers to polar form $z = re^{j\theta}$:

a. $z = 0 + 2j$

b. $z = -\frac{3}{2} - \frac{\sqrt{3}}{2}j$

Exercise 4
Convert the following complex numbers to Cartesian form ($z = x + jy$).

a. $z = \sqrt{2}e^{j\frac{3\pi}{4}}$

b. $z = 3e^{-j\frac{\pi}{2}}$

Exercise 5
Given the complex numbers z_1 and z_2, derive z_3 for each of the following cases:

a. $z_1 = 3 + 4j, \quad z_2 = e^{j\frac{\pi}{6}}, \quad z_3 = z_1 + z_2.$

b. $z_1 = \sqrt{8}e^{j\frac{3\pi}{4}}, \quad z_2 = -2 - 2j \quad z_3 = z_1 - z_2.$

[P2] c. $z_1 = -3j, \quad z_2 = 2e^{-j\frac{3\pi}{2}}, \quad z_3 = \frac{z_1}{z_2}.$

Exercise 6
Evaluate the Cartesian expression of the following complex numbers.

a. $z = 2j\left(j + \frac{1}{j}\right)$

b. $z = \left(\frac{\sqrt{2}}{2}(1 + j)\right)^{1003}$

[P3] c. $z = \frac{j^{-1} - j^{-2}}{j^{-3} + j^{-4}}$

Exercise 7
Simplify the following complex-valued expressions.

a. For $z = -5 + 13j$, evaluate $z \cdot z^* = |z|^2$.

b. For $z = -2 + 5j$, evaluate $\Re e\{z \cdot e^{-j\frac{\pi}{2}}\}$.

c. For $z = je^{-j\frac{\pi}{3}}$, evaluate $\Im m\{z\}$.

Exercise 8
Determine the complex roots of the following equations:

a. $z^2 + 36 = 0$

[P4] b. $z^2 + 8z + 20 = 0$

Exercise 9
Define $x(t)$ as $x(t) = 2\cos(300\pi t + \frac{3\pi}{4}) + 2\sqrt{2}\cos(300\pi(t+0.005))$. Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega t + \varphi)$ by finding the numerical values of A and φ, as well as ω.

Exercise 10
Solve the following equations for r and θ

$$-3 + 2\mathrm{j} - re^{\mathrm{j}\pi} + 4e^{-\mathrm{j}\theta} = 0$$

Make sure to provide *all* possible solutions.

Exercise 11
[P5] Obtain all possible real-valued solutions for M and ψ that satisfy the following equation.

$$5\cos(\omega t) = M\cos\left(\omega t - \frac{\pi}{6}\right) + 5\cos\left(\omega t + \psi\right).$$