Answers of homework Exercises

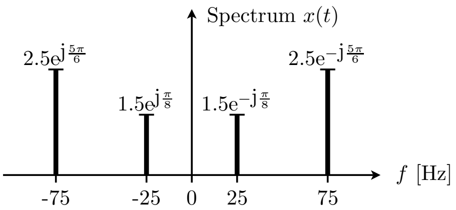
Module SPAFS: Spectrum And Fourier Series

Course: Signals Processing Basics (5ESE0)

Notes:

- During the contact hours complete workout of exercises can be explained by a tutor on request.
- The symbol [P] in the margin of an exercise denotes there is a pencast available.

Exercise 1



Exercise 2

Spectrum	(a)	(b)	(c)	(d)	(e)
Fundamental Frequency [Hz]	0.5	1	0.5	1.2	0.6
Signal	(3)	(5)	(1)	(2)	(4)

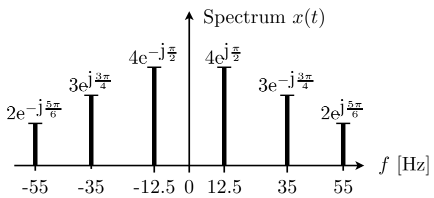
Exercise 3

$$x(t) \quad = \quad 8\sqrt{2}\cos(1000\pi t + \pi/4) + 4\cos(400\pi t - \pi/2)$$

Exercise 4

[P1]

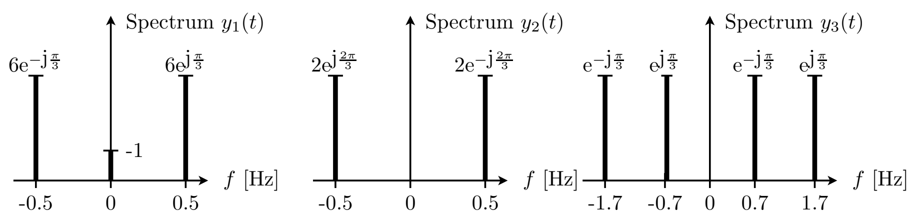
a. .



b.

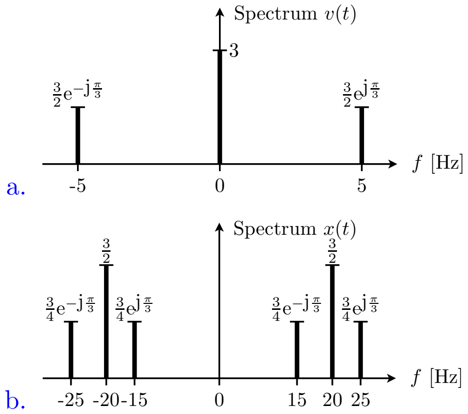
$$x(t) \quad = \quad 8\cos(2\pi 12.5t + \frac{\pi}{2}) + 6\cos(2\pi 35t - \frac{3\pi}{4}) + 4\cos(2\pi 55t + \frac{5\pi}{6})$$

Exercise 5



Exercise 6

[P2]



Exercise 7

a.

$$x(t) = 6 + 6\cos(400\pi t + \frac{\pi}{2}) + 8\cos(100\pi t)$$

b. $T_0 = 1/50$ [sec]

c.

$$\alpha_0 = 6 \ ; \ \alpha_1 = \alpha^*_{-1} = 4 \ ; \ \alpha_4 = \alpha^*_{-4} = 3e^{j\frac{\pi}{2}}$$

All other Fourier weight α_k are equal to zero.

Exercise 8

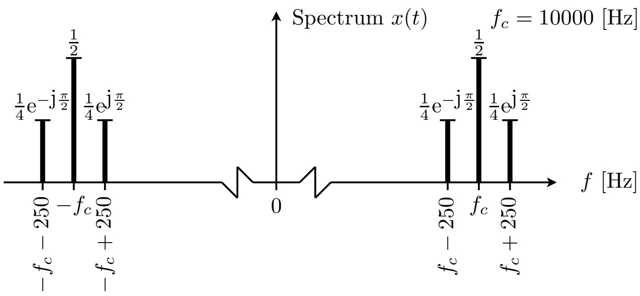
a. $T_0 = 1/50$[sec]

b. $\alpha_k = 0$ except for $\alpha_0 = 1$, $\alpha_3 = \alpha_{-3} = 3/2$ and $\alpha_5 = \alpha^*_{-5} = e^{-j\frac{3\pi}{4}}$

Exercise 9

$\omega_0 = 200\pi$ [rad/sec]. Furthermore $\alpha_k = 0$ except for $\alpha_2 = \alpha^*_{-2} = 2e^{-j\frac{\pi}{2}}$ and $\alpha_5 = \alpha^*_{-5} = 4\sqrt{2}e^{j\frac{\pi}{4}}$.

Exercise 10



a.

b. $F_0 = 250$ [Hz]. Furthermore $\alpha_k = 0$ except for $\alpha_{39} = \alpha^*_{-39} = \frac{1}{4}e^{-j\frac{\pi}{2}}$; $\alpha_{40} = \alpha_{-40} = \frac{1}{2}$; $\alpha_{41} = \alpha^*_{-41} = \frac{1}{4}e^{j\frac{\pi}{2}}$

Exercise 11

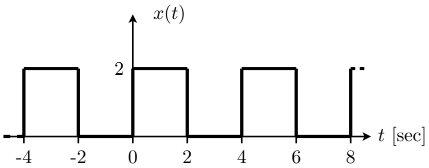
a. Yes, $x(t)$ is periodic with $T_{0,x} = 40$ [msec].

b. The frequency of the new sinusoid is different from all the frequencies in the spectrum of $x(t)$. The signal $y(t)$ is periodic with $T_{0,y} = 200$ [msec].

c. $w(t)$ is not periodic .

Exercise 12

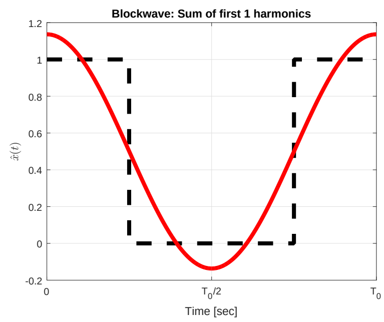
[P3] a.



b. $\alpha_0 = 1$

c. $\alpha_k = \frac{1-(-1)^k}{k\pi} e^{-j\frac{\pi}{2}}$

d. The result for $N = 1$ is depicted in the figure:



e. $\beta_0 = 2\alpha_0 - 1$ and for $k \neq 0$ $\beta_k = 2(-1)^k \alpha_k$.

Exercise 13

a.

$$\beta_0 = 2\alpha_0 + 3$$
$$\beta_k = 2\alpha_k \qquad \text{for all } k \text{ except } k = 0.$$

b.

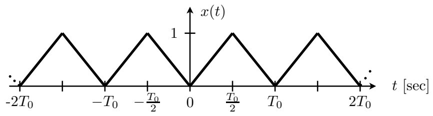
$$\gamma_k = \alpha_k e^{-jk\frac{\pi}{2}} \qquad k = 0, \pm 1, \pm 2, \cdots$$

Exercise 14
All values of α_k are equal to zero except for $\alpha_1 = \alpha_{-1} = \frac{3}{8}$ and $\alpha_3 = \alpha_{-3} = \frac{1}{8}$.

Exercise 15
R1 - R4, R2 - R7, R3 - R5, R6 - R8.

Exercise 16

a. The signal is a triangular wave form as depicted in the figure.



b. $\alpha_0 \frac{1}{2}$

c. $\alpha_k = \frac{(-1)^k - 1}{\pi^2 k^2}$

d. The approximation $\hat{x}(t)$ of the original square wave with the first harmonic is depicted in the figure: