Homework exercises

Module FIR: Finite Impulse Response

Course: Signals Processing Basics (5ESE0)

Notes:

- Only the answers are available for students. No pdf document with complete workout available for students.
- During the contact hours complete workout of exercises can be explained on request.
- The symbol [P] in the margin of an exercise denotes there is a pencast available.

Exercise 1

Remember the following most important basic digital signals:

$$\text{Unit impulse} \quad : \quad \delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Unit step} \quad : \quad u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Sinusoidal} \quad : \quad x[n] = A\cos(\theta n + \phi) \ \text{ with } A = \text{Amplitude} \ ; \ \theta = \text{Relative frequency} \ ; \ \phi = \text{Phase}$$

Make a plot (rough sketch) of the following signals:

a. $x[n] = \delta[n-2]$ in the range $n = -4, -3, \cdots, 3, 4$

b. $x[n] = u[n-2]$ in the range $n = -4, -3, \cdots, 3, 4$

c. $x[n] = u[n+3]$ in the range $n = -4, -3, \cdots, 3, 4$

d. $x[n] = u[n-1] - u[n-2]$ in the range $n = -4, -3, \cdots, 3, 4$

e. $x[n] = (0.9)^{(-n-3)}u[-n-3]$ in the range $n = -4, -3, \cdots, 3, 4$

f. $x[n] = \cos(\frac{2\pi}{5}n)$ in the range $n = 0, 1, \cdots, 9, 10$

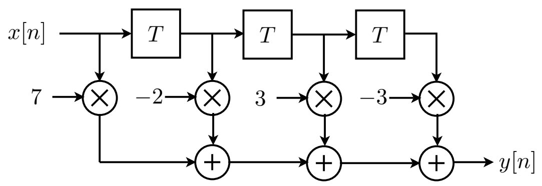
g. $x[n] = \cos(\frac{12\pi}{5}n)$ in the range $n = 0, 1, \cdots, 9, 10$

Exercise 2

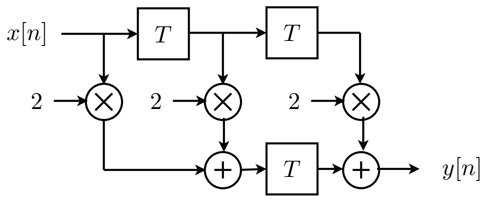
a. For the following Difference Equation (DE), draw a representation of this system in a signal flow graph (block diagram):

$$y[n] = 4x[n-1] - 3x[n-2] + 3x[n-3] - 2x[n-5].$$

b. Write a simple formula for the DE defined by the signal flow graph as depicted in the figure.



c. Write a simple formula for the DE defined by the signal flow graph as depicted in the figure and draw a signal flow graph (block diagram) of this system which contains only 2 delay blocks T.



[P1]

Exercise 3
Draw the signal flow graph (block diagram) of the system which can be described by the following DE:

$$y[n] = \sum_{k=1}^{3}(k^2 - 1)x[n-k]$$

Exercise 4
The DE of a "running average" filter is given by:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] \quad \text{with integer } L > 0$$

The input signal of this system is the unite step function, thus $x[n] = u[n]$.

a. Compute and make a sketch of the numerical values of $y[n]$ over the range $-5 \leq n \leq 10$, assuming that $L = 5$.

b. Derive, for $n \geq 0$, a general equation for $y[n]$ that applies for any filter length L.

Exercise 5
The DE which describes the input- output relation of an FIR filter is given by the following equation:

$$y[n] = \sum_{k=0}^{4}(k+1)x[n-k]$$

The input signal of this system is the unite step function, thus $x[n] = u[n]$.

a. Determine the filter coefficients $\{b_k\}$ of this FIR filter.

b. Determine the impulse response, $h[n]$, for this FIR filter and make a (stem) plot of $h[n]$ versus n over the range $-1, 0, \cdots, 8$.

c. Use convolution to compute $y[n]$ and make a (stem) plot of $y[n]$ versus n over the range $-1, 0, \cdots, 8$.

Exercise 6
[P2] The DE which describes the input- output relation of an FIR filter is given by the following equation:

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4].$$

a. Draw the signal flow graph (block diagram) of this FIR filter.

b. Determine the impulse response $h[n]$ for this FIR filter.

c. Use convolution to determine the output due to the input

$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

Plot the output sequence $y[n]$ for $-3 \leq n \leq 10$.

Exercise 7
An FIR filter is described by the DE:

$$y[n] = x[n-1] + x[n-3] + x[n-5]$$

Evaluate $y[n]$ when the input is a sinusoidal signal with frequency $\theta_x = \frac{\pi}{6}$: $x[n] = 4\cos(\theta_x n + \frac{\pi}{3})$. In this example it may be possible to evaluate $y[n]$ by inserting the expression for $x[n]$ into the DE and using goniometric relations. We can also try to evaluate the output samples by explicitly evaluating all the samples of $x[n]$ and finding a general expression for $y[n]$ via the convolution sum. In general however both of these approaches may become very difficult.
An alternative approach, which in general is much easier, will be discussed in the next Chapter. In the next Chapter you will learn about the *frequency response* of LTI systems, which is a powerful tool to characterize filters. In this exercise we will catch a glimpse of this approach and for this reason you have to evaluate $y[n]$ in this exercise by following steps:

a. Split the sinusoidal expression of $x[n]$ with Euler into two phasor components:

$$x[n] = x_1[n] + x_2[n] = 2\mathrm{e}^{\mathrm{j}(\frac{\pi}{6}n+\frac{\pi}{3})} + 2\mathrm{e}^{-\mathrm{j}(\frac{\pi}{6}n+\frac{\pi}{3})}$$

Use phasor calculations, to evaluate the resulting individual output signals $y_1[n]$, which is the result of $x_1[n]$, and $y_2[n]$, which is the result of $x_2[n]$. Write these output signals in the following general form:

$$\begin{aligned} y_1[n] &= A_1\mathrm{e}^{\mathrm{j}(\theta_1 n+\phi_1)} \\ y_2[n] &= A_2\mathrm{e}^{\mathrm{j}(\theta_2 n+\phi_2)} \end{aligned}$$

Evaluate the parameters $A_1, \theta_1, \phi_1, A_2, \theta_2$ and ϕ_2.

b. The general expressions of $y_1[n]$ and $y_2[n]$ can be used to evaluate $y[n]$, which can be expressed as follows:

$$y[n] = A\cos(\theta_y n + \phi)$$

Evaluate the parameters A, θ_y and ϕ.
How are the relative frequencies of the input θ_x and output θ_y related?

Exercise 8
For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

a. $y[n] = x[n-2] + 2x[n] + x[n+2]$

[P3] b. $y[n] = (x[-n])^2$

c. $y[n] = \mathrm{e}^{x[n]}$

d. $y[n] = x[n]\cos(0.2\pi n)$

e. $y[n] = -x[n+1] + x[n] - x[n-1]$

Exercise 9
You are given the following expression which describes the DE of an FIR filter:

$$y[n] = \sum_{k=0}^{L} b_k x[n-k].$$

When testing this FIR filter with an input signal $x[n] = \delta[n+1] + \delta[n-1]$, the observed output equals $y[n] = \delta[n] + 2\delta[n-1] + 4\delta[n-2] + 2\delta[n-3] + 3\delta[n-4]$. Determine the coefficients b_k of the FIR filter. What is the value of L?

Exercise 10
If the input signal to an FIR filter ie equal to the unit step function, thus $x[n] = u[n]$, give the signal flow graph (block scheme) of the FIR filter which produces the output $y[n] = \delta[n]$.

Exercise 11
Prove that there is no FIR possible that can process the input $x[n] = \delta[n] + \delta[n-1]$ to give the output $y[n] = \delta[n]$.

Exercise 12
You are given the following expression which describes the DE of an FIR filter:

$$y[n] = \sum_{k=0}^{L} b_k x[n-k].$$

When testing this FIR filter with an input signal $x_1[n] = \delta[n] - \delta[n-1]$, the observed output equals $y_1[n] = \delta[n-1] - \delta[n-4]$.
When the input signal equals:

$$x[n] = \begin{cases} (-1)^n & \text{for } n = 0,1,2,3 \\ 0 & \text{elsewhere} \end{cases}$$

use LTI properties to determine the output $y[n]$, thus without explicit evaluating the impulse response $h[n]$ of the filter.
TIP: First express $x[n]$ in terms of $x_1[n]$.

Exercise 13
The diagram in Fig.1 depicts a *cascade connection* of two LTI systems; i.e., the output of the

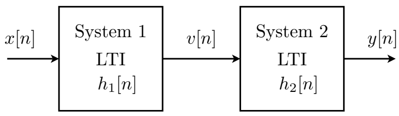


Figure 1: Cascade of two LTI systems

first system is the input to the second system, and the overall output is the output of the second system. Suppose that System 1 has impulse response,

$$h_1[n] = \begin{cases} 1 & \text{for } n = 0 \\ -1 & \text{for } n = 1 \\ 0 & \text{elsewhere} \end{cases}$$

and System 2 is described by the DE:

$$y[n] = \frac{1}{4}v[n] + \frac{1}{4}v[n-1] + \frac{1}{4}v[n-2] + \frac{1}{4}v[n-3]. \tag{1}$$

a. Determine the DE of System 1; i.e., the equation that relates $v[n]$ to $x[n]$.

b. From the DE in equation(1), determine the impulse response $h_2[n]$ of System 2.

c. Give a signal flow graph of System 1 cascaded with a signal flow graph of System 2. How many multipliers and delays do you count?

d. Determine the impulse response $h[n]$ of the overall cascade system.

e. Give a signal flow graph of the overall system. How many multipliers and delays do you count?

Exercise 14

[p4] Consider again the cascade connection of two LTI systems, as shown in Fig.1. For this exercise System 1 is a "first-order difference" filter described by the DE

$$v[n] = x[n] - x[n-1]$$

and System 2 is described by the impulse response

$$h_2[n] = u[n] - u[n-10].$$

a. Determine the impulse response $h[n]$ of the overall cascaded system.

b. Obtain a single DE that relates $y[n]$ to $x[n]$.

Exercise 15

In this exercise, we will get a glimpse of discrete-time LTI systems can be used to process

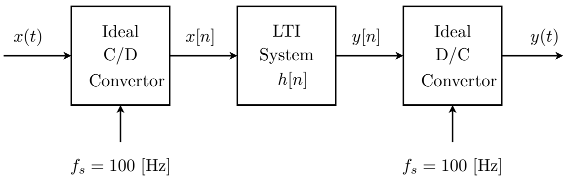


Figure 2: Digital signal processing of a continuous-time signal

continuous-time signals (using C/D and D/C converters), as is shown in Fig.2. This exercise will catch a glimpse of the next chapter, in which you will learn about the *frequency response* of LTI systems, which is a powerful tool to characterize filters. The sampling frequency of both converters is 100[Hz]. Compute the output signal $y(t)$ for the following situations.

- $x_1(t) = 1$ and $h_1[n] = \delta[n] + \delta[n-1]$.
- $x_1(t) = 1$ and $h_2[n] = \delta[n] - \delta[n-1]$.
- $x_2(t) = \cos(100\pi t)$ and $h_1[n] = \delta[n] + \delta[n-1]$.
- $x_2(t) = \cos(100\pi t)$ and $h_2[n] = \delta[n] - \delta[n-1]$.

Based on these results, explain the behavior of both filters $h_1[n]$ and $h_2[n]$ for the two given frequencies $x_1(t)$ (DC) and $x_2(t)$.