EVIDENTIAL REASONING
Chapter for the Handbook of Legal Reasoning

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Contents

When a suspect appears in front of a criminal court, there is a very high probability that he will be found guilty. In the Netherlands, for instance, the conviction rate of suspects that appear in criminal courts is reported to be around 95% year after year.[1] In the United States, the conviction rate in federal courts has been roughly 75% and in Japan has reached as high a rate as 99%.[2] This does not mean that the fact-finders deciding about the facts of a case have an easy job. Whether laypeople, such as jury members selected from the general public, or professionals, often experienced judges having completed postgraduate education, all face the difficulties associated with handling the evidence that is presented in court. What to do with conflicting testimonies? Does an established DNA match outweigh the testimony that the suspect was not on the crime scene? How to coherently interpret a large body of evidence? When is there enough evidence to convict 'beyond a reasonable doubt'?

The primary aim of this chapter is to explain the nature of evidential reasoning, the characteristic difficulties encountered, and the tools to address these difficulties. Our focus is on evidential reasoning in criminal cases. There is an extensive scholarly literature on these topics, and it is a secondary aim of the chapter to provide readers the means to find their way in historical and ongoing debates.

1 SETTING THE STAGE

We set the stage by using two important and often encountered kinds of evidence as an illustration: eyewitness testimony and DNA profiling. These two kinds of evidence will be used to establish a list of central questions that structure the exposition that follows.

1.1 Eyewitness testimony

Eyewitness testimony has always been a central source of information in criminal proceedings. It typically takes the form of oral statements by the witness in court, in response to questions by the prosecution, the defense, the court, and sometimes, albeit rarely, the jury. Eyewitness testimony can also come in the form of reports of oral examinations written in the pre-court stages of the criminal investigation.

Eyewitness testimony can provide information about what has happened on the scene of the crime. Here is an example.

> Q: Can you describe what happened that day?
>
> A: I was in the park and suddenly heard a lot of noise, very close by. I saw two men quarreling, shouting. Suddenly one of them pulled a gun, and I heard a shot. The other man fell to the ground. The shooter looked around, looked me in the eye, and then started to run.

> Q: Can you describe the shooter?
>
> A: He was a young men, in his twenties, I think. Tall, blonde, with a very white skin, and unusually blue eyes. He looked unhealthy, with bad teeth, like a drug addict. He was wearing a perfectly ironed Armani shirt, which surprised me.

The information contained in the testimony can be more or less detailed, and on its basis the fact finders can form a hypothesis about what happened. Still, it remains a hypothesis. There are many reasons why the hypothetical events reconstructed on the basis of the testimony might not be true. Typical reasons against the truth of the events reported by an eyewitness include that a witness has wrongly interpreted what she saw, that time has distorted her memories, or that the witness is intentionally lying.

1.2 DNA profiling

DNA profiling has become an important tool in courts. The evidential relevance of a DNA profile stems from the fact that, although most of the structure of DNA is shared among all human beings (more than 99%), the variations that do exist are very specific for each individual.

A *DNA profile* is determined by analyzing a number of specific locations, the so-called *loci*, of a DNA molecule. Different countries use different sets of *core loci* for their DNA profiles. For instance, the CODIS system in the United States uses 13 core loci, to be expanded to 20 core loci in 2017.[3] At each specific locus, a different *allele* might occur. For instance, one locus used in the profiles stored in forensic DNA databases in the United States is CSF1PO. This locus has up to 16 allele types, depending on how often the molecular sequence AGAT is repeated at that location.[4] A DNA profile, then, consists in a list of allele types for a certain number of select core loci.

How is the statistical frequency of a DNA profile estimated? Many countries have created extensive reference databases that contain million of DNA profiles. Each specific DNA profile is rare, and reference databases of profiles are used to estimate how rare a profile is. This is a two step process. First, the number of occurrences of each allele at each core locus in the reference database are counted. This gives an estimate of the proportional frequency of each allele at each core locus in the population. Second, the measured proportional frequencies for the alleles at the core loci are multiplied. This gives an estimate of the frequency of the DNA profile, or to use a more common terminology, the *Random Match Probability* of the DNA profile. These Random Match Probabilities are the numbers reported by forensic experts in courts, and the smaller they are, the higher the evidential value of the profile is taken to be. The sets of core loci have been chosen such that Random Match Probabilities are typically very small, for instance, in the order of 1 in 50 billion, amply exceeding the number of people on our planet.

A key assumption underlying the model—used when multiplying the estimated probabilities of specific alleles—is that there are no dependencies among the alleles at different loci in the population considered. This assumption does not always hold, for instance, in a population with family relations. Scientists have also established certain dependencies among the profiles within ethnic groups. Moreover, testing the independence assumption can be hard, and require the assessment of more profiles than reasonably possible.

With this background in place, suppose now that a trace of blood is found on the crime scene, and that the DNA profile created from the trace matches the DNA profile of the suspect. Using the match, the fact finders can form the hypothesis that the suspect is the source of the blood trace, and the Random Match Probability associated with the profile provides a measure of the evidential strength of the match. Importantly, the hypothesis that can be formed on the basis of a DNA match is rather circumscribed. It is limited to the suspect being the *source* of the trace and should not be confused with the hypothesis that the suspect is *guilty*, at least absent other information about how the trace got there. Further, the hypothesis itself that the suspect is the source need not be true. The suspect and the perpetrator, though different people, might share the same DNA profile, either because they are identical twins or because, though unrelated, they happen to share the same profile.[5] We should also be wary of laboratory errors and false positive matches.

1.3 Central questions

Using the two kinds of evidence as an illustration, we can now provide a list of central questions about evidential reasoning in the law.

Question 1: How should we handle conflicting evidence? It often occurs that the evidence provides conflicting perspectives on the crime. For instance, a witness claims that the criminal has blond hair, but the suspect whose DNA matched that of the trace at the crime scene, has dark hair. What to do in case of such conflicts?

Question 2: How should we handle the strength of the evidence? Some evidence is stronger than other evidence. This is most obvious in the case of DNA evidence, where DNA profiles come with different Random Match Probabilities. But also some eyewitness testimonies are stronger than others. For instance, the description of a criminal by a witness who could only view the crime scene in bad lighting conditions, is of lesser value. How to address the strength of evidence?

Question 3: How should we coherently interpret the available evidence? A DNA profile match can support that the suspect is the source, and a witness can add information about how the crime was committed. In general, there is a lot of evidence that needs to be coherently combined in order to make sense of what has happened. How do we combine all information in a coherent whole?

Question 4: How should we decide about the facts given the evidence? When are we done?

After a careful and exhaustive investigation in the pretrial and trial phases of the criminal proceedings, the question arises when a decision can be made and what that decision is. When is the burden of proof met? What is the meaning of "beyond a reasonable doubt"? When have we collected enough evidence to make a decision?

The plan for the paper is as follows. In the next section (Section 2), we discuss three normative frameworks that can help us understand how to correctly handle the evidence. In the remaining sections, the four questions we set out above will be discussed, consecutively, while emphasizing the role of the three normative frameworks (Sections 3, 4, 5 and 6).

2 THREE NORMATIVE FRAMEWORKS

In this section, we discuss three normative frameworks for the correct handling of the evidence: arguments; probabilities; and scenarios. We shall only emphasize their distinctive theoretical strengths. Arguments can naturally capture the dialogical dimension, by modeling relations of support and attack. Probabilities are better suited to quantify the value of the evidence. Finally, scenarios are best in offering a coherent and holistic interpretation of large bodies of evidence.

2.1 Arguments

The first normative framework that we discuss uses arguments as primary tool. Arguments are best analyzed in a dialogical setting, for they contain reasons that *support* or *attack* a certain conclusion of interest. For instance, when a witness reports that she saw the suspect at the crime scene, this evidence constitutes a reason for the conclusion that the suspect was, in fact, at the crime scene. But if the DNA profile found at the crime scene does not match the suspect's DNA profile, this constitutes a reason attacking the conclusion. An argument with a supporting and an attacking reason is represented in Figure 1.

The analysis of the structure of arguments goes back to the early twentieth century when John Henry Wigmore developed his famous evidence charts (Wigmore, 1913, 1931). The work by the New Evidence Scholarship (Anderson et al., 2005) continued from Wigmore's insights. Independently, and not focusing on evidence in criminal cases, the structure of arguments for and against conclusions was formalized and computationally studied by the philosopher John Pollock (1987, 1995). The work by Pollock stimulated an extensive literature on the formal and computational study of arguments for and against conclusions (van Eemeren et al., 2014b).

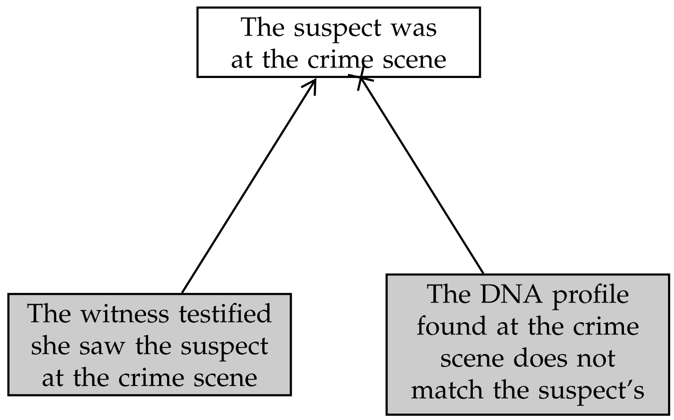

Figure 1: Arguments with supporting and attacking reasons

2.2 Probabilities

The second normative framework uses probabilities as the main tool. In handling evidence in court, a crucial question from the probabilistic perspective is, how probable is a certain hypothesis H given a body of evidence E? This is the *conditional probability* of H given E, or in symbols, $P(H|E)$. Another crucial question is, how does the probability of H change in light of evidence E? This *probability change* is expressed by the difference between the conditional probability $P(H|E)$ and the so-called *prior probability* $P(H)$. Both questions can be addressed with the famous Bayes' theorem:

$$\Pr(H|E) = \frac{\Pr(E|H)}{\Pr(E)} \cdot \Pr(H)$$

This formula—which can be easily proven from the probability axioms[6]—shows how the conditional probability $\Pr(H|E)$ of hypothesis H given evidence E can be computed by the prior probability $\Pr(H)$ and the factor $\Pr(E|H)/\Pr(E)$.

The interest in probabilistic calculations as a tool for the good handling of the evidence has recently been stimulated by the statistics related to DNA profiling, and by some infamous miscarriages of justice that involved statistics, in particular the Lucia de Berk and Sally Clark cases (Dawid et al., 2011; Fenton, 2011; Schneps and Colmez, 2013). The interest is not new (Tillers, 2011), and can in fact be traced back to early forensic science in the late nineteenth century (Taroni et al., 1998). To what extent probabilistic calculations have a place in courts has always been, and remains, the subject of debate.[7]

2.3 Scenarios

Finally, the third normative framework centers around scenario analysis. In a scenario, a coherent account of what may have happened in a case is made explicit. Scenario analysis proves helpful when considering a complex case and its evidence. For instance, the following brief scenario can help to make sense of a murder case:

The robber killed the victim when caught during a robbery.

Such a coherent explanation of the evidence in the form of a scenario can be regarded as a sense-making tool for handling cases with a large dossier.

In particular, legal psychology has contributed to our knowledge about the role of scenarios in handling the evidence (Bennett and Feldman, 1981; Pennington and Hastie, 1993a). Scenarios were shown to be misleading, as experiments showed that a false scenario told in a sensible chronological order was more easily believed than a true scenario that was told in a random order. Still, the legal psychologists Wagenaar et al. (1993) emphasised the usefulness of scenario analysis for the rational handling of the evidence, using the technique in their work on debunking dubious case decisions. Scenario analysis is connected with inference to the best explanation (Pardo and Allen, 2008).

Further readings The three normative frameworks (Anderson et al., 2005; Dawid et al., 2011; Kaptein et al., 2009).

Arguments: Wigmore charts (Wigmore, 1913, 1931). The New Evidence Scholarship (Anderson et al., 2005). Formal and computational study of arguments (Pollock, 1987, 1995). Informal and formal argumentation theory (van Eemeren et al., 2014a).

Probabilities: Evidence and probabilities Mortera and Dawid (2007); Schum (1994) Statistics in the law (Fenton, 2011). Miscarriages of justice involving statistics (Dawid et al., 2011; Schneps and Colmez, 2013). Debates on whether probabilistic calculations have a place in courts Finkelstein and Fairley (1970); Tribe (1971), back in 1970's, and more recently, the 2012 special issue of *Law, Probability and Risk*; Vol. 4, No. 2.

Scenarios: Scenarios in evidential reasoning (Bennett and Feldman, 1981; Pennington and Hastie, 1993a,b). Scenarios and miscarriages of justice (Wagenaar et al., 1993). Inference to the best explanation (Pardo and Allen, 2008). Hypothetical explanations of the evidence (Thagard, 1989).

Combined approaches: Combining arguments and scenarios (Bex, 2011; Bex et al., 2010). Bayesian networks for evidential reasoning (Fenton et al., 2013; Hepler et al., 2007). Combining arguments, scenarios and probabilities (Timmer et al., 2015; Verheij, 2014; Verheij et al., 2016; Vlek et al., 2014).

3 CONFLICTING EVIDENCE

In many situations, it is clear what the facts are. In a simple case of tax evasion, for example, it will be easy to establish whether you filed for taxes on time and whether your employer paid you 100,000 dollars in 2015. Only in special circumstances, such as administrative errors, there will be something to dispute here. But cases that are litigated in court are typically more complicated. Disputes emerge because the two parties—who then become the defense and the prosecution in a criminal trial—introduce evidence that support conflicting reconstructions of the facts. In this section, we illustrate how each of the three frameworks can represent and model conflicts between different pieces of evidence.

3.1 Arguments

In the argument-based framework, the handling of conflicting evidence is analyzed in terms of arguments for and against a certain conclusion.

Three kinds of attack can be distinguished: rebutting, undercutting and undermining. Consider a crime case, where a witness testified she saw the suspect at the crime scene. The witness testimony constitutes a reason supporting the conclusion that the suspect indeed was at the crime scene. There are three parts to this argument: the conclusion; the reason (also called the premise); and the connection between the reason and the conclusion. Each of these three parts can be attacked.

First, the conclusion can be attacked. For example, suppose DNA testing shows that the suspect does not genetically match with the traces found at the crime scene. Such an attacking reason is called a *rebutting attack*. It supports the opposite conclusion, namely that the suspect was *not* at the crime scene. Second, the reason itself can be attacked, although this is a bit more difficult to imagine. For instance, if the witness never actually testified that she saw the suspect at the crime scene, this attacks the very existence of the supporting reason itself. This kind of attack is referred to as *undermining attack*. Third, the connection between the reason and the conclusion can be attacked. The fact that the lighting conditions were bad when the witness saw the crime, is an example of such an attack, referred to as an *undercutting attack*. In contrast with a rebutting attack, an undercutting attack provides no support for the opposite conclusion. In the example, if the lighting conditions were bad, there is no reason explicitly supporting that the suspect was not at the crime scene. The attacking reason, here the bad lighting conditions, is also referred to as an exclusionary reason. The three examples of the different kinds of attack are shown in Figure 2.

Three kinds of support can be distinguished: multiple, subordinated and coordinated. Just as attacking reasons can target the conclusion of an argument, its supporting reason or the connection between the two, additional reasons can provide further support for each of these parts. Additional reasons can be seen as responses to attacking reasons or as reasons strengthening an existing argument.

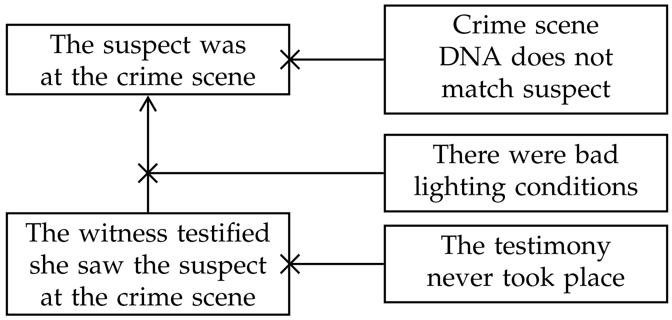

Figure 2: Three kinds of attack

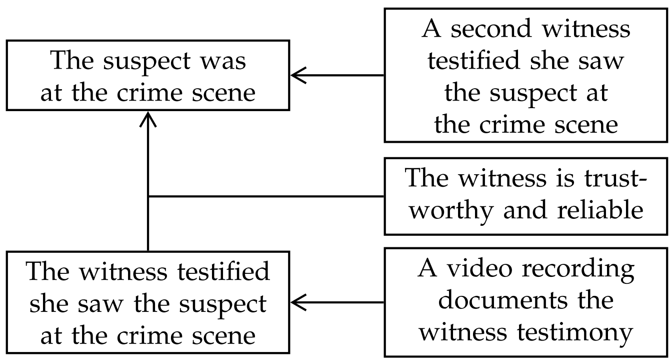

Figure 3: Three kinds of (further) support

Consider, once again, the argument that the suspect was at the crime scene because the witness reports that she saw the suspect at the crime scene. First, the conclusion can be further supported, for example, by a second witness testimony. If a conclusion is supported by more than one reason, this is referred to as *multiple support*. Second, the reason itself can be supported, for example, by a video recording of the witness testimony itself. Support of the reason itself is called *subordinating support*. Finally, the connection between the reason and the conclusion can be further supported, for example, by another testimony that the witness has always been trustworthy and reliable. Support for the connection between the reason and the conclusion does not have a standard name, but is closely related to a third named kind of support: *coordinated support*. In coordinated support, the support for the conclusion consists of at least two supporting reasons which, in their conjunctive combination, provide support for the conclusion. Coordinated support is distinguished from multiple support because in the latter each supporting reason provides support for the conclusion by itself. Figure 3 shows the three kinds of (further) support. (Multiple and subordinated support are graphically visualized with an arrow, whereas coordinated support is shown with a line. An arrow indicates the unnamed kind of support of the connection between reason and conclusion.)

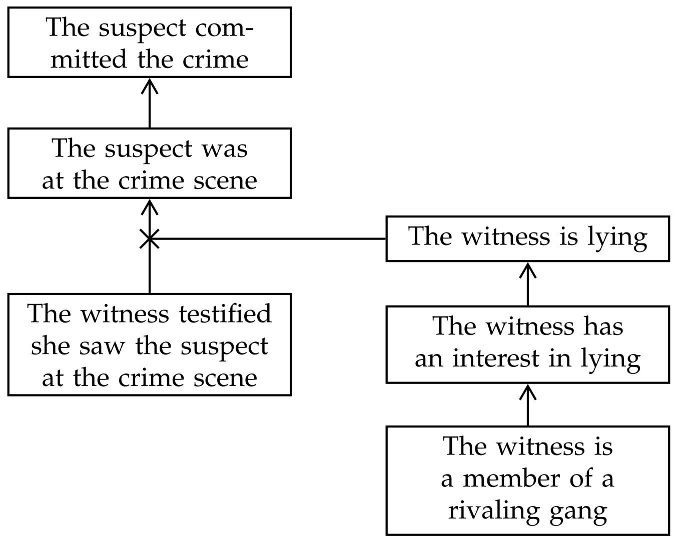

Figure 4: Supporting and attacking reasons can be chained.

Arguments have structures, involving complexes of supporting and attacking reasons. So far we have looked at a simple argument, consisting of a reason and a conclusion, along with three types of attacking reasons and three types of symmetric supporting reasons. But an argument can also be more complex, for example, it can contain *chains of reasons*.

Consider, once again, the example of a witness who reports that she saw the suspect at the crime scene. The witness testimony constitutes a reason supporting the conclusion that the suspect was at the crime scene, and this conclusion—in turn—functions as a reason that supports the conclusion that the suspect committed the crime. This chain of supporting reasons is graphically depicted in Figure 4, on the left. Attacking reasons can also be chained. For example, when it is discovered that the witness is a member of a rivaling gang, this constitutes a reason for concluding that the witness has an interest in lying, and further, for concluding that the witness is in fact lying (Figure 4, on the right). This conclusion attacks—undercuts, to be precise—the connection between the witness testimony and the conclusion the suspect was at the crime scene.

Further readings Argument structure and diagrams (Freeman, 1991; Toulmin, 1958; Wigmore, 1913, 1931). Defeasible reasoning and nonmonotonic logic (Gabbay et al., 1994; Pollock, 1987). Rebutting and undercutting attack (Pollock, 1987, 1995). Undermining attack (Bondarenko et al., 1997). Formal evaluation of defeasible arguments (Dung, 1995; Pollock, 1987, 1995; Prakken, 2010). Argumentative dialogue (Hage, 2000; Prakken, 1997; Toulmin, 1958; Walton and Krabbe, 1995). Accrual of reasons and weighing (Hage, 1997; Pollock, 1995; Prakken, 2005; Verheij, 1996). Argument diagramming and evaluation software (Gordon et al., 2007; Kirschner et al., 2003; Pollock, 1995; Reed and Rowe, 2004; van Gelder, 2003; Verheij, 2005).

3.2 Scenarios

In the scenarios normative framework, the handling of conflicting evidence is analyzed by considering different scenarios about what may have happened.

Typically there are different, conflicting scenarios about what has happened. This happens when the prosecution and the defence present different scenarios about a murder case, as follows:

S_1: The suspect killed the victim when caught during a robbery.

S_2: The suspect was at home with his wife.

The former is a guilty suspect scenario, and the latter an alibi scenario. They are conflicting since only one of them can be true. Here the conflict is connected to the different sides, prosecution and defence. But there can also be conflicting scenarios on the same side, for instance when there are multiple suspects:

S_1: Suspect 1 killed the victim when caught during a robbery.

S_2: Suspect 2 killed the victim when caught during a robbery.

Scenarios can be contradicted by evidence. Let's consider a murder case with two alternative scenarios: a former partner murder scenario S_1 where the victim's former partner murders the victim, and a robbery murder scenario S_2 where a robber murders the victim when caught during a robbery. If the robber's partner presents an alibi (evidence E_1), this evidence contradicts scenario S_2. This evidence is independent from scenario S_1.

Scenarios can explain evidence. Both scenarios explain the starting point of the murder investigation: the victim's body found at the crime scene (evidence E_2), in the sense that finding the body is expected assuming either scenario to be true. Assume now that a match is found between the DNA profile of a tissue trace found under the victim's finger nails and that of his former partner (evidence E_3). This evidence is explained by scenario S_1, but not by S_2, of which evidence E_3 is independent.

Further readings Scenarios in evidential reasoning (Bennett and Feldman, 1981; Pennington and Hastie, 1993a,b). Scenarios and miscarriages of justice (Wagenaar et al., 1993). Inference to the best explanation (Pardo and Allen, 2008). Hypothetical explanations of the evidence (Thagard, 1989).

3.3 Probabilities

In the probabilistic normative framework, the handling of conflicting evidence is analyzed in terms of the conditional probabilities that connect the evidence and the hypotheses.

Evidential support and attack can be characterized as "probability change". A piece of evidence E supports an hypothesis H whenever E raises the probability of H, or in symbols, $P(H|E) > P(H)$. For example, a witness testifies that she saw the defendant around the crime scene at the time of the crime. The testimony supports the hypothesis that the defendant is guilty. This can be described probabilistically, as follows:

$$P(\textit{guilt}|\textit{testimony}) > P(\textit{guilt}).$$

By contrast, a piece of evidence E attacks a hypothesis H whenever E lowers the probability of H, or in symbols, $P(H|E) < P(H)$. For example, if a DNA test shows no match between the traces found at the crime scene and the defendant, this evidence attacks the hypothesis that the defendant is guilty. Probabilistically,

$$P(\textit{guilt}|\textit{no DNA match}) < P(\textit{guilt}).$$

Evidential support and attack can be characterized as "likelihood ratio". There is another characterization of evidential support and attack. Instead of comparing the initial probability $P(H)$ and the probability $P(H|E)$ of the hypothesis given the evidence, a so-called likelihood ratio of the form $P(E|H)/P(E|\neg H)$ can also be used. On this account, E supports H whenever the likelihood ratio $\frac{P(E|H)}{P(E|\neg H)}$ is greater than one. Intuitively, this means that the presence of the evidence is more probable if the hypothesis is true than if the hypothesis is false. By contrast, a piece of evidence E attacks a hypothesis H whenever the likelihood ratio is lower than one. This means that the presence of the evidence is less likely if the hypothesis is true than if the hypothesis is false. For the two examples considered earlier, we have:

$$\frac{P(\textit{testimony}|\textit{guilt})}{P(\textit{testimony}|\neg\textit{guilt})} > 1, \text{ and}$$

$$\frac{P(\textit{no DNA match}|\textit{guilt})}{P(\textit{no DNA match}|\neg\textit{guilt})} < 1.$$

These two characterizations of evidential support/attack—in terms of probability increase/decrease, and in terms of a likelihood ratio greater/lower than one—are in fact equivalent. For the following statements hold:

$$P(H|E) > P(H) \text{ iff } \frac{P(E|H)}{P(E|\neg H)} > 1.$$ [8]

$$P(H|E) < P(H) \text{ iff} \frac{P(E|H)}{P(E|\neg H)} < 1.$$

The conflict between two pieces of evidence can be described probabilistically. Two pieces of evidence come into conflict with one another insofar as one supports a hypothesis and the other attacks the same hypothesis. The conflict can be described probabilistically, in that one piece of evidence increases the probability of the hypothesis, while the other decreases it, or equivalently, the likelihood ratio is positive (for one piece of evidence) and negative (for the other). For example, the testimony that the defendant was around the crime scene conflicts with the lack of a DNA match. Probabilistically, the testimony increases the probability of the defendant's guilt (or equivalently, the likelihood ratio is greater than one), while the lack of a DNA match decreases the probability of the same hypothesis (or equivalently, the likelihood ratio is lower than one).

Further readings On confirmation theory and accounts of evidential support (Carnap, 1950; Crupi, 2015; Fitelson, 1999).

4 EVIDENTIAL VALUE

The evidence found in a criminal investigation has different levels of evidential value: some evidence is very strong, other not so much. How is evidential value handled in each of the three normative frameworks? That is the topic of this section.

4.1 Probability

In the probabilistic framework, evidential value is measured using the numeric calculus of probability theory.

The incremental evidential value is measured by the probabilistic difference. The incremental value of evidence for, or against, a hypothesis can be quantified probabilistically in various ways. One approach considers the difference between the probability of the hypothesis with and without the evidence, that is, $P(H|E) - P(H)$. The larger the positive difference, the higher the value of the evidence for the hypothesis. An alternative approach is given by the likelihood ratio $P(E|H)/P(E|\neg H)$. For any value greater than one, the higher the likelihood ratio, the higher the value of the evidence for the hypothesis.

By contrast, a negative difference $P(H|E) - P(H)$ and a likelihood ratio lower than one quantify the value of the evidence *against* a hypothesis. The larger the negative difference and the lower the likelihood ratio (for any value below one), the higher the value of the evidence against the hypothesis.

The overall evidential value is measured by the overall conditional probability. In contrast with the incremental evidential value of evidence that is measured by a probabilistic difference or likelihood ratio, the overall evidential value of the full body of evidence is measured by the conditional probability of the hypothesis given the evidence. The higher, or lower, the probability $P(H|E)$, the higher the overall value of the evidence for, or against, the hypothesis. If there is a series of pieces of evidence $E_1, \ldots, E_n$, the overall evidential value of the evidence is measured as $P(H|E_1, \ldots, E_n)$.

Overall and incremental evidential value should not be confused. To illustrate, suppose we have strong evidence E_1 for the hypothesis H that a suspect was at the crime scene, for instance, security camera footage in which the suspect is easily recognizable. In this case, the overall evidential value $P(H|E_1)$ of the evidence is high. If this is the only evidence, then also the incremental evidential value is high: before the evidence is considered, the hypothesis is not strongly supported, i.e. $P(H)$ is low, whereas after the evidence is considred, the hypothesis is strongly supported, i.e. $P(H|E_1)$ is high. In this case, the overall and incremental evidential value of E_1 are both high. But suppose a witness testifies that the defendant was not at the crime scene (evidence E_2), but as it turns out, the witness is unreliable as a known accomplice of the suspect. Consider now the overall evidential value $P(H|E_1, E_2)$ of the two pieces of evidence together. This will not have changed much when compared to $P(H|E_1)$. As a result, the incremental evidential value of E_2 is low, while still the overall evidential value $P(H|E_1, E_2)$ is high, even though E_2 did not contribute much.

The difference between overall and incremental evidential value can be especially confusing when there is a single piece of evidence and there is a misalignment between the two values. Consider the hypothesis $\neg H$ that the suspect was not at the crime scene and the evidence E_2, the testimony of the unreliable witness. Then $P(\neg H)$ is high and also $P(\neg H|E_2)$ is high. Uncritically interpreted, the high value of $P(\neg H|E_2)$ suggests that the testimony of the unreliable witness has a high evidential value. But incrementally E_2 did not change much. The hypothesis $\neg H$ is, in totality, still strongly supported after the incrementally weak evidence E_2, since the hypothesis was already strongly supported before that evidence.

DNA evidence has high incremental evidential value, but there are complications. As an illustration, we discuss the likelihood ratio of a DNA match. When introduced in court, a DNA match comes with an estimated Random Match Probability (RMP). One way to interpret this probability is as the probability that a random person, who had nothing to do with the crime, would match. Now, with some simplifications (on these later), the evidential value of the DNA match M in favor of the suspect being the source of the sample S, in terms of a likelihood ratio, is as follows

$$\frac{P(M|S)}{P(M|\neg S)} = \frac{1}{RMP}.$$

The numerator $P(M|S)$ equals 1 because we assume that if the defendant is the source of the sample, the lab test will report a match. As for the denominator, putting $P(M|\neg S) = RMP$ is plausible because the probability that a match would be reported assuming that the defendant was *not* the source is roughly the same as the chance that a random person—someone who had no contact with the victim—would match anyway. For example, if the RMP is 1 in 200 million, the likelihood ratio would be

$$\frac{P(M|S)}{P(M|\neg S)} = \frac{1}{\frac{1}{\text{200 million}}} = \text{200 million.}$$

Since the likelihood ratio in question is a high number, the DNA match in favor of the suspect being the source has a high evidential value. More generally, a low RMP corresponds to a match with a rare profile, hence has a high evidential value.

Still, even with a low RMP one should beware of the complications when using a DNA match to establish the guilt of a suspect. Consider the following, non-equivalent hypotheses:

1. The *lab reports* that the defendant's genetic profile matches with the crime traces;
2. The defendant's genetic profile *truly matches* with the crime traces;
3. The defendant is the *source* of the traces; and
4. The defendant *visited* the crime scene;
5. The defendant is *guilty*.

The inferential path from 'reported match' to 'guilt', passing through the intermediate steps 'true match', 'source' and 'visit', is a long one, and each step comes with sources of error that undermine the inference along the way.

First, the inference from 'reported match' to 'true match' depends on the question whether the laboratory has made a mistake. A key source of lab mistakes originates in human error, much less rare than a good DNA profile. Second, the inference from 'true match' to 'source' can go wrong in several ways. Of course, even a rare match can be accidental, as measured by the RMP. But another cause of a match without the suspect being the source occurs in cases of close family relations. For instance, a suspect's genetically identical twin has identical DNA, and twins are much less rare than good DNA profiles. Third, the inference from 'source' to 'visiting the crime scene' is not infallible. In particular, the traces can have been accidentally transferred to the crime scene or have been planted there. Fourth, the inference from 'visiting the crime scene' to 'guilt' can go wrong in many ways, because having visited a crime scene is not nearly the same as having committed the crime investigated.

Further readings Introductions to using probability for weighing evidence (Dawid, 2002; Finkelstein and Fairley, 1970; Mortera and Dawid, 2007). Critique of the probabilistic approach (Allen and Pardo, 2007; Cohen, 1977; Tribe, 1971). Prosecutor's fallacy (Thompson and Shumann, 1987). Introduction to DNA evidence (Kaye and Sensabaugh, 2000; Wasserman, 2008). Different hypotheses for evaluating DNA evidence (Cook et al., 1998; Evett et al., 2000; Koehler, 1993). Probabilistic analyses of DNA evidence (Balding, 2005; Buckleton, 2005; Robertson and Vignaux, 1995). Lab errors for DNA evidence (Thompson et al., 2003). Match is not all-or-nothing judgment (Kaye, 1993). Uniqueness of DNA profiles (Kaye, 2013; Weir, 2007). How DNA evidence can be synthesized and implanted (Frumkin et al., 2009). Cold hit controversy in DNA evidence cases (Balding and Donnely, 1996; NRC, 1996). Comparison between DNA evidence and fingerprints (Zabell, 2005). Probabilistic analyses of eyewitness testimony (Friedman, 1987; Schum, 1994; Schum and Starace, 2001).

4.2 Arguments

The evidential value of arguments can be analyzed in terms of the strength of the reasons they are built from, but also by asking critical questions about the reasons of the argument, its conclusion and the connection between reasons and conclusion.

The reasons used can be conclusive or defeasible. A reason is conclusive when, given the reason, its conclusion is guaranteed. The main type of conclusive reasons correspond to deductive, logically valid reasoning. An example of a conclusive reason occurs in the logically valid argument from the reasons 'John is shot' and 'If someone is shot, he dies', to the conclusion 'John dies'. Its logical validity is connected to the underlying logical structure of the argument: from 'A' and 'A implies B', conclude 'B'.

Many reasons are not conclusive, but defeasible. There are circumstances in which the conclusion does not follow, although the reason obtains. The reason 'The witness reports to have seen the suspect at the crime scene' supports the conclusion 'The suspect was at the crime scene', but does not guarantee that conclusion, because—for instance—the witness could have made a mistake. A defeasible reason can provide *prima facie* justification for a conclusion, which might later be withdrawn in light of countervailing reasons. Reasons that occur in so-called *adductive arguments* are also defeasible, where abductive arguments can be thought of as providing an explanation. For example, from ‘John’s DNA matches the crime trace’ conclude ‘John left the trace’. The fact that John left the trace is put forward as an explanation for the fact that John’s DNA matches the trace. Abductive arguments are typically defeasible because there often are alternative explanations. Someone with the same genetic profile as John might have left the trace.

Arguments can be evaluated by asking critical questions Consider again the one-step argument from the reason ‘The witness reports that she saw the suspect at the crime’ to the conclusion ‘The suspect was at the crime scene’. Critical questions that can be asked about the argument include ‘Are there reasons to doubt the suspect was not at the crime scene, such as an alibi?’, ‘Are there reasons to doubt that the witness testimony supports the conclusion that the suspect was at the crime scene, for instance is the witness lying?’ and ‘Are there reasons to doubt the witness testimony, such as a fraudulent report?’. The first of these questions is directed at the argument’s conclusion, the second at the argument step from reason to conclusion, and the third at the argument’s reason. These different kinds of critical questions are connected to the three kinds of argument attack discussed in Section 3.1 (see in particular Figure 2, page 10). Suppose that initially it is believed that the suspect was at the crime scene because of the witness testimony. A positive answer to any of the questions will weaken the support for the conclusion that the suspect was at the crime scene, perhaps up to the point of it no being believable.

It can be subject to debate whether a reason supports or attacks a conclusion. Whether a reason supports a conclusion is associated with an underlying general rule. For instance, the argument from a witness testimony (the reason) to the suspect’s being at the crime scene (the conclusion) is connected to the general rule that what witnesses say can generally be believed. Following Toulmin (1958)’s terminology, such general rules making explicit how to get from the reason to the conclusion are referred to as warrants. Support for a warrant is called the backing of the warrant.

More generally, the supporting or attacking relation between a reason and its conclusion can be supported or attacked. This gives rise to four different combinations: support of a supporting relation, support of an attacking relation, attack of a supporting relation, and attack of an attacking relation. In Figure 5, these situations are illustrated by two opposite witness testimonies.

Further readings Nonmonotonic reasoning (Gabbay et al., 1994). Prima facie reasons, undercutting and rebutting defeaters (Pollock, 1987, 1995). Warrants and backings (Toulmin, 1958). Argument schemes and critical questions (Walton et al., 2008).

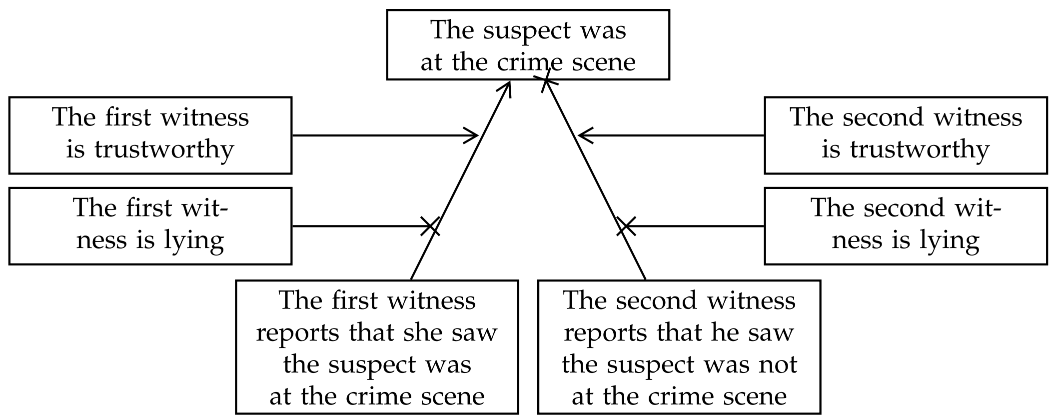

Figure 5: Arguments about whether a reason is supporting or attacking

4.3 Scenarios

The evidential value of a scenario depends on how well it matches up with the evidence. This matching up can be understood in three ways: a scenario's consistency with the evidence; its power to explain the evidence; and its evidential support. Let us consider each in turn.

Scenarios must be consistent with the evidence. We can evaluate a scenario by checking whether it is consistent with the evidence presented in a case. If a witness testifies that the defendant was at home with his girlfriend at 6 PM, while according to the scenario proposed by the prosecutor, the defendant was at the crime scene at 6 PM, the two are inconsistent. More precisely, insofar as the evidence is taken at face value—that is, the witness is taken to be truthful—the scenario is inconsistent with the evidence. A *prima facie* inconsistency between the evidence and a proposed scenario need not be damning for the scenario's credibility, but at least, it calls for further investigating. It might, of course, turn out that the witness was untruthful or simply confused about the timing. If so, the evidence will be discarded, not the scenario. But, there are cases in which the evidence cannot be easily discarded. Suppose a video recording shows the defendant went into a bank and committed a robbery, and instead, the scenario proposed by the defense denies precisely that. The defense will be hard pressed to justify the proposed scenario given the inconsistency with the evidence.

Scenarios can be more or less plausible, logically consistent, cohesive, normal While consistency with the evidence, explanatory power and evidential support measure how well a scenario matches up with the evidence, *plausibility* measures how well a scenario matches up with our background assumptions and knowledge of the world. The events that constitute a scenario must be linked by relations of temporal order and causality. These relations, at the very least, should not violate the laws of nature or common sense. If a scenario asserts that the same individual was in two different locations at the same time, or moved from one location to another in too short amount of time, the scenario would lack plausibility. Lack of plausibility can become so pronounced that it amounts to a lack of *logical consistency*, for example, claiming that the defendant had and did *not* have a motive for killing the victim

The plausibility of a scenario is weakened when the components of the scenario, its sub-scenarios, cannot be easily combined together. This could also be understood as a lack of *cohesiveness* among the components of a scenario. Suppose the defendant is said to have a lovely relation with the victim the day before, while the next day the defendant is depicted as killing the victim out of anger. The scenario lacks cohesiveness because it cannot explain why anger followed a peaceful interaction the day before. So, not only the scenario should explain the evidence, but certain components of a scenario should explain others. Typically we expect that events prior in time explain events later in time.

Plausibility should not be reduced to *normality*. Plausibility has certainly something to do with what happens most of the time, but criminal cases are often about odd coincidences, unexpected and improbable occurrences. Indeed, a scenario according to which the defendant covered 500 mile of distance by car is more normal than a scenario in which the defendant covered the same distance by foot. But the former need not be more plausible than the latter. Consistency with the laws of nature and common sense, whether the parts of the scenario "hang together" nicely (cohesiveness and internal consistency) are also indicators of plausibility.

The more evidence a scenario can explain, the better. When a case comprises several items of evidence, a scenarios's credibility depends on how it can account for the whole mass of evidence. The more items of evidence a scenario can account for, preferably from both the prosecutor and the defense, the better the scenario. This depends on the scenario's consistency with the evidence, explanatory power and evidential support. These criteria were already discussed earlier. The difference now is that consistency, explanatory power and evidential support should be measured relative to the totality of the evidence.

Suppose two items of evidence must be explained: first, the presence of fingerprints at the crime scene; and second the fact that those fingerprints match with the defendant. Suppose scenario S_1 comprises the event that the defendant visited the crime scene, while S_2 compromises the event that some other individual visited the crime scene. Both scenarios explain the presence of fingerprint traces, but only S_1 can explain the fact that the traces match with the defendant. Now, a third scenario S_3 in which another individual visited the crime and implanted fingerprints that match the defendant can explain both items of evidence. As far as explanatory power goes, scenarios S_1 and S_3 are equal, although further evidence may distinguish them. For example, if a witness testified that she saw the defendant walk towards the location of the crime when the crime was committed, scenario S_3 could not easily explain the testimony, while S_1 could. So, absent other evidence, scenario S_1 explains more evidence than the other competing scenarios.

Further readings Explanation in the deductive nomological model (Hempel and Oppenheim, 1948). Explanation and causality (Salmon, 1984). Abduction and inference to the best explanation (Lipton, 1991). More the philosophical literature on scientific explanation (Woodward, 2014). Two directions of fit (Wells, 1992).

5 COHERENTLY INTERPRETING THE EVIDENCE

The dossiers of criminal cases can be large, and the coherent interpretation of the evidence in such a dossier can be daunting, whichever normative framework is used. For each framework, we discuss how the coherent interpretation of the evidence can be addressed.

5.1 Scenarios

Scenarios can provide coherent interpretations that make sense of the evidence.

Scenarios are clusters of events, ordered in time and connected by causal relations. Consider again a murder example with two suspects: the victim's former partner, who killed the victim after a fight (S_1), and a robber who killed the victim when caught during a robbery (S_2). Scenario S_1 can be made explicit as a sequence of four hypothetical consecutive events (Figure 6). First, the victim and his former partner have a relation (H_1); then they break up (H_2); subsequently, there is a fight (H_3); and finally, the victim is killed by his former partner (H_4). Scenario S_2 can be analyzed as consisting of three events. First, the robber enters the victim's house (H_5); then the victim accidentally encounters the robber (H_6); and finally, the robber kills the victim (H_7). Some of the events in these chronologically ordered scenarios are causally connected. For instance, in the first scenario, the killing is caused by the break up and the fight; and in the second scenario by the accidental encounter.

Scenarios can be more or less complete. Another criterion to evaluate scenarios is their *completeness*. Since scenarios are discursive arrangement of events, ordered according to temporal and causal relations, they may contain gaps in time, space and causality. A scenario may not describe the defendant's whereabouts between 4 and 6 PM, while it describes, rather precisely, what the defendant did at 7 PM, immediately before the killing

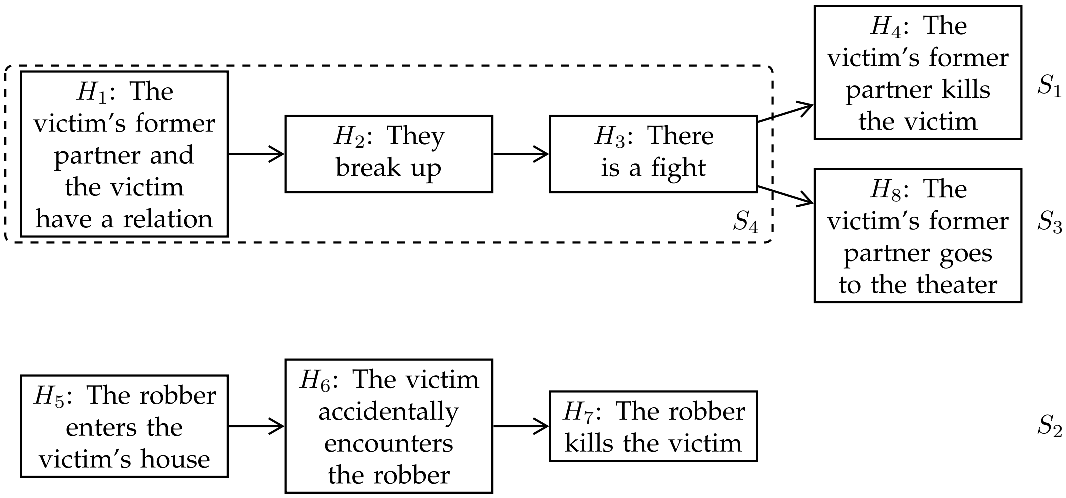

Figure 6: Alternative scenarios and their structure

took place. The temporal gap between 4 and 6 PM makes it less complete than a scenario which describes the defendant's whereabouts between 4 and 7 PM without gaps. Yet, this might not be the notion of completeness that is important here.

In a murder case, the identity of the perpetrator, the motive, the modus operandi of the crime and the weapon are relevant items which a scenario should specify. The lack of any of these from a proposed scenario would make it incomplete. The law itself sometimes requires that specific items be proven, for example, in criminal cases both the *mens rea* and *actus reus* must be proven. Still, the law does not say whether the defendant's whereabouts the day before or a month before the murder are relevant. They could be relevant to establish motive or they might not. This depends on the circumstances of the case. So, how is completeness a criterion to evaluate a scenario? Some suggest that scenarios must follow certain patterns, schematic structure or scripts. For example, in most violent crimes, we can identify an initial moment of conflict, which triggers a specific psychological reaction that gives rise to the formation of an intention, which, in turn, later results in the violent act. On this account, a scenario is complete whenever it has all of its parts, at least given an appropriate scenario script. The notion of completeness, then, overlaps with that of plausibility and cohesiveness of a scenario.

All in all, evaluating a scenario required different levels of analysis: consistency with the evidence; explanatory power (predictive power and causal fit); evidential support; plausibility (and also, cohesiveness, internal consistency and normality); completeness.

Scenarios considered may or may not solve a case, and show which evidence is legally relevant. A criminal case is only solved when the legally relevant circumstances can be proven. For instance, in murder cases, it should be proven who killed the victim and why. Of the scenarios considered, the former partner murder scenario S_1 and the robbery murder scenario S_2 explain how and why the murder happened, but the alibi scenario S_3 does not. In a crime investigation, it can be hard to find evidence that proves a sufficiently detailed scenario about what has happened. For instance, in the example, it is initially clear that a murder had happened, but not by who. Because of the break up, a scenario that answers the why-question is considered: the former partner murder scenario S_1. That scenario is initially corroborated by the DNA match E_1, but then breaks down by the use of the bank card E_2 that proves the alibi scenario S_3. Only when the robber is caught and confesses having killed the victim (evidence E_3)—possibly much later, it becomes clear what has happened. The body found E_0 and the confession E_3 are relevant for answering the legally relevant questions who killed the victim and why. The other evidence considered, the DNA match E_1 and the use of the bank card E_2, have played a relevant role in the investigation, but do not support or contradict robbery murder scenario S_2.

Further readings Explanation and unification in philosophy of science (Friedman, 1974). Coherence in epistemology (BonJour, 1985). The crossword puzzle analogy for coherently evaluating a mass of evidence (Haack, 2008). Explanatory coherence (Thagard, 2001). The story model (Pennington and Hastie, 1993b). Scenarios as scripts (Wagenaar et al., 1993). Scenarios in legal cases (Griffin, 2013). Worries about scenarios in law (Velleman, 2003).

5.2 Arguments

An analysis of a case in terms of arguments can become very complex. This was already noted by Wigmore, when he developed his charting method for analyzing the evidence in a criminal case (Wigmore, 1913, 1931). Figure 7 provides an example of a Wigmore diagram (1931). Here Wigmore has analyzed a murder case (Commonwealth v. Umilian, 1901). Jedrusik, the victim, was the author of a letter in which he falsely advised a priest that Umilian had a wife and children (back in the country from which they emigrated), conflicting with Umilians intention to marry. In the chart, Z stands for the charge that Umilian killed Jedrusik. The node at 8 represents a revengeful murderous emotion toward Jedrusik. At 18, it is represented that the marriage is in the end performed, reducing his feelings of revenge. This claim is supported by the (unspecified) evidence at 18.1. The diagram contains some two dozen nodes. Diagrams for more complex cases can contain many more nodes.

The evaluation of an argument can depend on its subarguments. Given an argumentative analysis of the case, one would like to know which conclusions follow, and which don't. The structure of a complex of arguments influences the evaluation of the arguments,

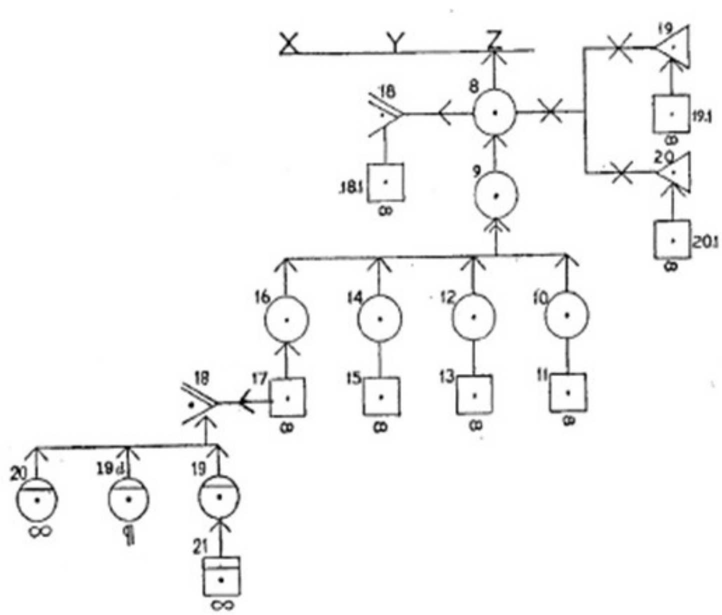

Figure 7: A Wigmore chart

and in particular, the subarguments of a larger argument determine the evaluation of the whole. For example, consider the argument in Figure 4 (page 11). There the witness testimony supported the intermediate conclusion that the suspect was at the crime scene, which in turn supported the conclusion that the suspect committed the crime. So there is an argument consisting of two steps. In the example, the first of these steps is attacked by a counterargument involving the lying of the witness. As a result of this attack, the argument to the intermediate conclusion that the suspect was at the crime scene breaks down and its conclusion does not follow. As a result, also the larger two-step argument no longer supports its conclusion, which hence does not follow. Since the one-step subargument does not successfully support the intermediate conclusion, also the whole two-step argument for the final conclusion does not successfully support its conclusion.

The evaluation of an argument can depend on chains of attacks. When an argument is successfully attacked, it no longer successfully supports its conclusion. But attack can be chained, since the attack itself can be countered by a further attack. When an attack is successfully attacked, the original argument can be reinstated, in the sense that it again successfully supports its conclusion. Figure 9 shows an example. A first witness, Witness A, reports that the suspect was at the crime scene. Given only this information, there is good reason to assume that the suspect was at the crime scene. However, there is a second witness, Witness B, who reports that Witness A is lying. Given these two reasons, based on the witness reports by A and B, it is no longer successfully supported that the suspect was at the crime scene. If now there is a third witness, Witness C, who reports that Witness B is lying, the attack is countered. Witness B is no longer believed, so there is no reason to conclude that Witness A is lying. As a result, A's report can again support its conclusion that the suspect was at the crime scene. The original argument based on A's report is reinstated.

Conflicts between reasons can be addressed by exceptions, preferences and weighing [NEEDS FIXING]. The counterarguments to a reason that result from asking critical questions give rise to conflicts between reasons. Sometimes conflicts of reasons can be resolved in the sense that it can be determined which conclusions follow from the conflicting reasons. We distinguish three kinds of conflicts.

In the first kind of conflicts between reasons, there is a reason attacked by an exclusionary reason, i.e., an attack of the undercutting kind that goes against the connection between the reason and its conclusion (cf. Section 3.1). This situation is shown at the top of Figure 8: There is a witness reporting that the suspect was at the crime scene, but there is evidence that the witness is lying. In that case, the conclusion that the suspect was at the crime scene does not follow, since there is no supporting reason for it. The reason (the witness report itself) and the exception (the lying of the witness) both hold as they are assumptions. In this situation of a reason with an undercutting attack by an exclusionary reason, the conflict of reasons is resolved by the exception expressed in the exclusionary reason.

In the second kind of conflicts between reasons, there is one reason for a conclusion and another against the conclusion. For instance, there are two witnesses with opposite reports about whether the suspect was at the crime scene or not (see the middle of Figure 8). This is an example of a rebutting attack (Section 3.1). The conflict of reasons is unresolved given only the reasons. Further information about the reasons is required to resolve the conflict. The resolution of such a conflict of reasons can be thought of as the preference of one over the other. A reason can be preferred over another, for instance when it is stronger. In the example, the preference (in the figure indicated by the $<$-sign) is argued for by the claim that one of the witnesses is more reliable than the other. In this case, the conclusion follows that the suspect was not at the crime scene, given this conflict of reasons and its resolution.

The third kind of conflicts between reasons discussed here involves more than two reasons. For instance, there are more than two witnesses, with conflicting reports (Figure 8, bottom). In the figure, both sides are supported by two witness reports. Resolving such a conflict can be thought of as a weighing of the reasons, generalizing the preference between two conflicting reasons. Again, it is concluded that the suspect was not at the crime scene, given this resolution of this conflict of reasons.

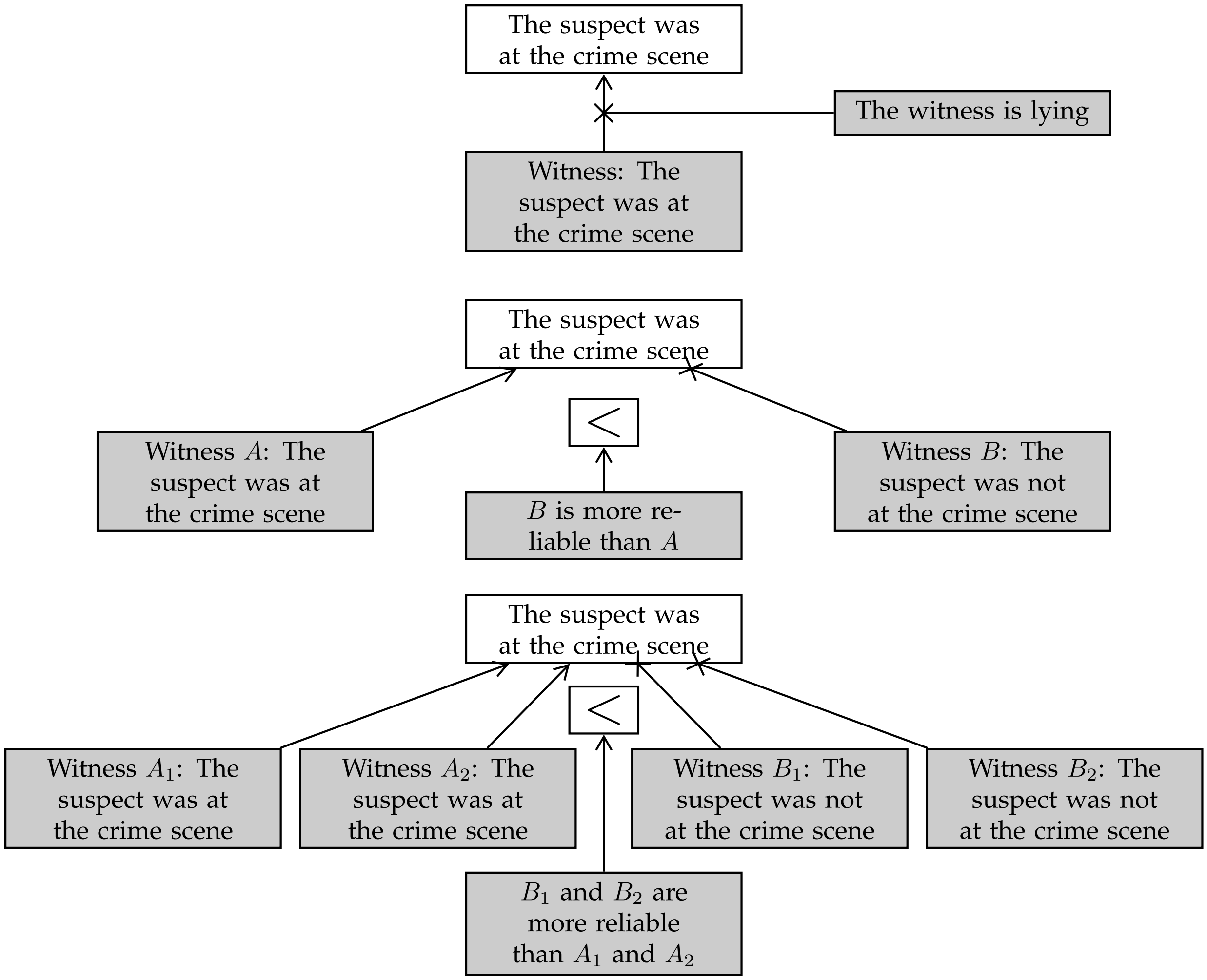

Figure 8: Three kinds of conflicts and their resolution

Further readings Argument structure and their evaluation (Pollock, 1995). Formalizing argumentation (Prakken and Vreeswijk, 2002). Evaluating argument attack (Dung, 1995). Formal argumentation models (Gordon et al., 2007; Prakken, 2010; Simari and Loui, 1992; Verheij, 2003; Vreeswijk, 1997).

5.3 Probability

The probability calculus provides formal rules for the coherent interpretation of the evidence.

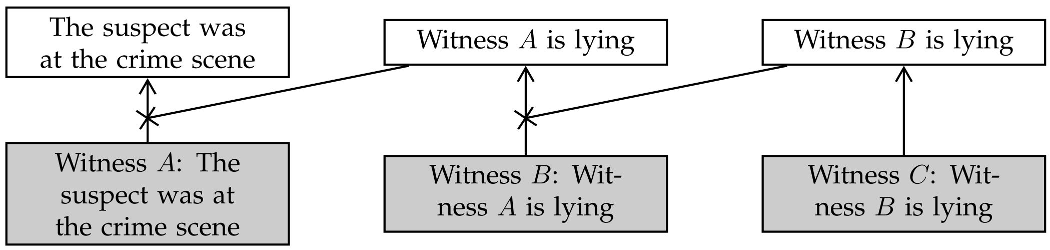

Figure 9: Reinstatement

The likelihood ratio formula shows how to find the posterior odds given the evidence. The odds of a hypothesis H are defined as the ratio $P(H)/P(\neg H)$ of the probability of the hypothesis and the probability of its negation. Given evidence E, the odds $P(H)/P(\neg H)$ of the hypothesis unconditioned on the evidence are called the *prior odds* of the hypothesis, and the odds $P(H|E)/P(\neg H|E)$ of the hypothesis conditioned on the evidence the *posterior odds*.[9] It follows from Bayes' theorem (Section 2.2) that the posterior odds can be found by multiplying the prior odds with the likelihood ratio:

$$\frac{P(H|E)}{P(\neg H|E)} = \frac{P(E|H)}{P(E|\neg H)} \cdot \frac{P(H)}{P(\neg H)}.$$ [10]

This formula shows that an estimate of the (incremental) evidential value of the evidence for a hypothesis, expressed by the likelihood ratio $\frac{P(E|H)}{P(E|\neg H)}$, does not by itself give an estimate of the posterior odds of a hypothesis. One needs an estimate of the prior odds too.

To arrive at the *posterior probability* $P(H|E)$ of a hypothesis conditional on the evidence, from the posterior odds $\frac{P(E|H)}{P(E|\neg H)}$ of the hypothesis given the evidence, the following formula applies:

$$P(H|E) = \frac{\frac{P(E|H)}{P(E|\neg H)}}{1 + \frac{P(E|H)}{P(E|\neg H)}}.$$

Consider an example. The incremental evidential value of a DNA match, relative to the hypothesis that the defendant is guilty, is given by the the likelihood ratio $\frac{P(M|G)}{P(M|\neg G)}$. Suppose this ratio is assigned a numerical value, as follows:

$$\frac{P(M|G)}{P(M|\neg G)} = \frac{1}{\frac{1}{2{,}000{,}000}} = 2,000,000.$$

Suppose, also, that the prior odds are as follows:

$$\frac{P(G)}{P(\neg G)} = \frac{\frac{1}{200{,}000}}{\frac{199{,}999}{200{,}000}} \approx \frac{1}{200,000}.$$

The posterior odds of the hypothesis given the match $\frac{P(G|M)}{P(\neg G|M)}$ are therefore as follows:

$$\frac{P(G|M)}{P(\neg G|M)} = \frac{P(M|G)}{P(M|\neg G)} \cdot \frac{P(G)}{P(\neg G)} \approx 2,000,000 \cdot \frac{1}{200,000} = \frac{20}{1}.$$

So the poster probability of the hypothesis is as follows:

$$P(G|M) = \frac{\frac{P(M|G)}{P(M|\neg G)}}{1 + \frac{P(M|S)}{P(M|\neg G)}} \approx \frac{20}{1+20} \approx 0.95.$$

A generalization of the formula shows how to handle more pieces of evidence. So far we have considered only one piece of evidence. For two pieces of evidence E_1 and E_2, the formula can be generalized by multiplying the likelihood ratios of the individual pieces of evidence, as follows:

$$\frac{P(H|E_1 \wedge E_2)}{P(\neg H|E_1 \wedge E_2)} = \frac{P(E_2|H)}{P(E_2|\neg H)} \cdot \frac{P(E_1|H)}{P(E_1|\neg H)} \cdot \frac{P(H)}{P(\neg H)}.$$

This simple generalization holds provided that the two pieces of evidence are independent conditional on the hypothesis, that is, $P(E_2|H) = P(E_2|H \wedge E_1)$. (More on this below.)

To illustrate, consider now two pieces of evidence: a DNA match and a witness testimony. The DNA match, call it M, holds between the crime traces and the defendant, and the witness, call it W, in her testimony asserts that the defendant was seen at the crime scene. Both pieces of evidence, intuitively, support the hypothesis G that the defendant is guilty. To assign an explicit numerical value, assume the DNA match has a likelihood ratio $\frac{P(M|G)}{P(M|\neg S)}$ of 2 million, and the witness testimony a likelihood ratio $\frac{P(W|G)}{P(W|\neg G)}$ of 1,000. These numbers are purely illustrative, but are needed to perform the probabilistic calculations.[11] Finally, assume the two pieces are independent conditional on the hypothesis G, that is, $P(W|G) = P(W|G \wedge M)$.

The combined (incremental) evidential value of the two pieces of evidence is given by multiplying the two likelihood ratios, that is, $2,000,000 \times 1,000 = 2,000,000,000$, which is a higher value than the two pieces considered independently. If the prior odds $\frac{P(G)}{P(\neg G)}$ are roughly $\frac{1}{200,000}$ as before, the posterior odds are therefore as follows:

$$\frac{P(G|M| \wedge W)}{P(\neg S|M \wedge W)} = \frac{P(M|G)}{P(M|\neg S)} \cdot \frac{P(W|G)}{P(W|\neg G)} \cdot \frac{P(G)}{P(\neg G)} \approx 2,000,000 \cdot 1,000 \cdot \frac{1}{200,000} = \frac{20,000}{1}.$$

So the poster probability of the hypothesis given the two pieces of evidence is as follows:

$$P(G|M \wedge W) \approx \frac{20,000}{1+20,000} \approx 0.99.$$

Compare this probability with $P(G|M)$, which was 0.95, a lower value. The probability calculus can offer a numerical representation of the intuitive fact that two pieces of evidence, taken together, have a higher (overall) evidential value than one piece of evidence alone.

If the two pieces of evidence are no independent, the likelihood ratio formula for two pieces of evidence takes the following, more general form:

$$\frac{P(H|E_1 \wedge E_2)}{P(\neg H|E_1 \wedge E_2)} = \frac{P(E_2|H \wedge E_1)}{P(E_2|\neg H \wedge E_1)} \cdot \frac{P(E_1|H)}{P(E_1|\neg H)} \cdot \frac{P(H)}{P(\neg H)}.$$

It is easy to see the first, simple generalization follows from the second, assuming independence between the two pieces of evidence conditional on the hypothesis of interest. The first generalization does not always hold because the evidential value of a piece of evidence, as measured by the likelihood ratio, can change in the face of other evidence. The problem with the second formula, however, is that it is much more difficult to apply.

The formula can be generalized to calculate the probability of a composite hypothesis, although one should we wary of the independence assumptions used. Consider now two hypotheses, H_1 and H_2, each supported by an item of evidence, E_1 and E_2, respectively. Can the likelihood ratio formula be generalized to cases with two items of evidence and two hypotheses combined? Assuming that the two pieces of evidence are independent, conditional on the hypotheses, and the two hypotheses are also independent, it is enough to simply multiply the two likelihood ratios, as follows:

$$\frac{P(H_1 \wedge H_2|E_1 \wedge E_2)}{P(\neg H|E_1 \wedge E_2)} = \frac{P(E_2|H_2)}{P(E_2|\neg H_2)} \cdot \frac{P(E_1|H_1)}{P(E_1|\neg H_1)} \cdot \frac{P(H_1)}{P(\neg H_1)} \cdot \frac{P(H_2)}{P(\neg H_2)}.$$ [12]

To illustrate, suppose a DNA match links the defendant to the crime scene, while an email written by the defendant indicates he had a plan to kill the victim. Each of these item of evidence support a different hypothesis, abbreviated *contact* and *plan*. Assuming that the two items of evidence are independent, conditional on the hypotheses of interest, and assuming that the two hypotheses are also independent, we have:

$$\frac{P(\mathit{match} \wedge \mathit{email}|\mathit{contact} \wedge \mathit{plan})}{P(\mathit{match} \wedge \mathit{email}|\neg(\mathit{contact} \wedge \mathit{plan}))} = \frac{P(\mathit{match}|\mathit{contact})}{P(\mathit{match}|\neg \mathit{contact})} \times \frac{P(\mathit{email}|\mathit{plan})}{P(\mathit{email}|\neg \mathit{plan})}$$

Suppose, for the sake of argument, the (incremental) evidential value of the DNA match in favor of the hypothesis *contact*, and the (incremental) evidential value of the defendant's email in favor of the hypothesis *plan*, are as follows:

$$\frac{P(\mathit{match}|\mathit{contact})}{P(\mathit{match}|\neg \mathit{contact})} = 36 \text{ and } \frac{P(\mathit{email}|\mathit{plan})}{P(\mathit{email}|\neg \mathit{plan})} = 36$$

where the numbers are, as usual, illustrative. The combined (incremental) evidential value of *match* and *email* in favor of the hypothesis consisting of *contact* and *plan*, is $36 \times 36 = 1296$. For the value of the prior odds $\frac{P(contact \wedge plan)}{P(\neg(contact \wedge plan))}$, assume that

$$\frac{P(contact)}{P(\neg contact)} = 0.1/0.9 \text{ and } \frac{P(plan)}{P(\neg plan)} = 0.1/0.9.$$

Now, putting everything together, we get

$$\begin{aligned} \frac{P(contact \wedge plan | match \wedge email)}{P(\neg(contact \wedge plan) | match \wedge email)} &= \frac{P(match \wedge email | contact \wedge plan)}{P(match \wedge email | \neg(contact \wedge plan))} \times \frac{P(contact \wedge plan)}{P(\neg(contact \wedge plan))} \\ 0.93/0.07 &= 1296 \times \frac{0.1 \times 0.1}{1-(0.1 \times 0.1).} \end{aligned}$$

So, not only does the probability calculus offers us an elegant way to combine different pieces of evidence, but also to combine different hypotheses in larger scenarios. These combinations, however, rests on assumptions about the independence of the hypotheses and of the pities of evidence. One should be wary that these assumptions do not always hold. Consider a scenario that consists of several elements, for instance an accidental encounter with a robber (H_1) followed by the robber killing the victim (H_2). When the two events are independent, the probability of the composite hypothesis $H_1 \wedge H_2$ is, by definition of probabilistic independence, the product of the probabilities of the individual elements. But the elements of a scenario need not be independent. In the example, it seems reasonable to assume that an accidental encounter with a robber increases the probability of being killed, albeit slightly. Using formal notation, $P(H_2) < P(H_2|H_1)$. Hence also $P(H_1) \cdot P(H_2) < P(H_1 \wedge H_2)$ and H_1 and H_2 are not independent. This point also holds when considering the probability of a composite hypothesis conditioned on evidence E. It does not in general hold that $P(H_1|E) \cdot P(H_2|E) = P(H_1 \wedge H_2|E)$, only when the elements of the composite hypothesis are independent given the evidence.

Further readings The conjunction paradox (Cohen, 1977) and a response (Dawid, 1987). Coherence and probability (Bovens and Hartmann, 2003). Bayesian networks (Taroni et al., 2006). Probabilistic analysis of an entire legal case (Kadane and Schum, 1996). On the use of probability in law (Fenton, 2011).

6 REASONING AND DECISION MAKING

So far we have focused on how the evidence can be evaluated and combined, and how inferences can be drawn. But once the evidence has been introduced at trial, examined and cross examined, it comes a time when the fact finders, either a trained judge or a group of lay jurors, must reason from the evidence, reach a conclusion and decide whether to convict or acquit the defendant. The decision criterion is defined by law and consists of a standard of proof, sometimes also called burden of persuasion. The criterion for criminal cases in common law countries is *proof beyond a reasonable doubt*, and a similar criterion exists outside the common law. If the decision makers are persuaded of the defendant's guilt beyond a reasonable doubt, they should convict, or else they should acquit.

Paraphrases of the formulation 'proof beyond a reasonable doubt' abound in the case law. And yet, it is unclear whether they improve our understudying. The US Supreme Court might have been right when, in Holland v. United States (1954), 348 U.S. 121, it wrote that that 'attempts to explain the term "reasonable doubt" do not result in making it any clearer' (140). The three frameworks we considered—probability, arguments and narratives—can be used to characterize more precisely the standard of proof, although they are not immune from shortcomings, as we shall soon see.

Further readings Evidence law manuals (Fisher, 2008; Méndez, 2008). Criminal Procedure manuals (Allen et al., 2005). Character evidence and its exclusion (Redmayne, 2015).

6.1 Probability

The guilt probability is estimated by weighing the evidence with the probability calculus. On the probabilistic framework, the goal is to estimate the probability of the defendant's guilt based on all the available evidence. The estimation begins with the lowest possible value for the guilt probability, prior to considering any evidence. As more evidence is presented, the guilt probability moves upwards or downwards depending on whether the evidence is incriminating or exculpatory. When all the evidence is considered, a final guilt probability value is reached, all things considered. This forms the basis for the decision to convict or acquit.

The value of the guilt probability is arrived at by applying Bayes' theorem a repeated number of times and by plugging the values of the probabilities that are needed. Sometimes these probabilities are known because they are based on estimated frequencies, but sometimes they are not. For example, $P(G)$ is required to calculate $P(G|E)$. This is the probability of the defendant's guilt regardless of the evidence presented at trial. What should $P(G)$ be? For technical reasons, it cannot be zero, but it also cannot be 50% because of the presumption of innocent. Arguably, $P(G)$ should be relatively low, but how low? This remains hotly debated.

The decision criterion is a guilt probability threshold. In probabilistic terms, proof of guilt beyond a reasonable doubt means that the defendant's *probability of guilt*, given the evidence presented at trial, meets a threshold, say, $>$99% or $>$99.9%. A numerical value for the threshold can be identified using expected utility theory. Let $c(CI)$ be the cost of convicting an innocent and $c(AG)$ the cost of acquitting a guilty defendant. For a conviction to be justified, the expected cost of convicting an innocent must be lower than the expected cost of acquitting an innocent, that is,

$$P(G|E) \cdot c(AG) > [1 - P(G|E)] \cdot c(CI).$$

The inequality holds just in case

$$\frac{P(G|E)}{1 - P(G|E)} > \frac{c(CI)}{c(AG)}.$$

Suppose $\frac{c(CI)}{c(AG)} = \frac{99}{1}$, as might be more appropriate in a criminal case in which the conviction of an innocent defendant is regarded as far worse than the acquittal of a guilty defendant. Then, the inequality hold only if $P(G)$ meets the threshold 99%. More complicated models are also possible, but the basic idea is that the probability required for a conviction is a function of weighing the costs that would result from an erroneous decision.

It is not clear how to estimate all the required probabilities. The characterization is simple, crisp and elegant, but a too literal interpretation of it is problematic. If a probabilistic threshold is understood as a criterion which the decision makers should mechanically apply whenever they confront the decision to convict or acquit, two difficulties arise. The first difficulty is that assigning a probability value to guilt itself might not be feasible. As seen earlier, the starting probability $P(G)$ cannot be easily determined, and even if this value could be known, other probability values might remain unknown. One solution here is that instead of aiming for a unique guilt probability, we can simply aim for an interval of admissible probabilities given the evidence. More generally, the estimation of the probability of guilt can be viewed as an idealized process, a regulative ideal which can improve the precision of legal reasoning.

Another problem with the probabilistic characterization is that it does not take into account the so-called weight of the evidence, that is, whether the evidential basis contains all the evidence in the case or just a partial subset of the evidence. The guilt probability will vary dramatically depending on the evidence that is used to estimate it. It is tempting to suggest that the guilt probability must be based on a body of evidence that is complete, or at least as complete as reasonably possible. And yet, it is unclear how to characterize this notion.

Further readings Probabilistic accounts of the burden of proof (Cheng, 2013; Hamer, 2004; Kaplan, 1968; Kaye, 1986, 1999). Critique of probabilistic accounts (Cohen, 1977; Haack, 2014; Ho, 2008; Nesson, 1979; Pardo and Allen, 2008; Stein, 2005; Thomson, 1986). On the question whether the threshold should be variable (Kaplow, 2012; Picinali, 2013).

The problem of priors (Finkelstein and Fairley, 1970; Friedman, 2000). A critique of the proof beyond a reasonable doubt as understood in the law (Larry Laudan, 2006). History of beyond a reasonable doubt standard (Shapiro, 1991; Whitman, 2008). Other measures, weight, resiliency and completeness of the evidence (Kaye, 1999; Stein, 2005).

6.2 Arguments

Suporting and attacking reasons are collected and weighed. In a court of law, the prosecutor puts forward a conclusion and offers supporting reasons. The opposing parts responds by offering attacking reasons. The dialectical process can be complex. As seen earlier, there are different attacking reasons: undermining, undercutting and rebutting. The process is complex also because it can be iterated. A conclusion can be attacked by an attacking reasons, and the latter in turn can be attacked. And so on. When the dialectical process reaches an equilibrium point and the opposing parties have nothing more to contribute, the status of a claim and its supporting reasons can be assessed.

On the argument based framework, the goal is to consider all the available reasons, by representing them in a comprehensive argumentation graph that keeps track of the relations of support and attack. The two competing theories of the cases, the prosecutor's and the defense's theory, will each be supported by a set of reasons. The argument framework, through the aid of argument graphs, allow us to compare the relative strength of the reasons in favor of one side of the case or the other. This comparison of the two sides forms the basis for the trial decision.

Defeating all attacks is the criterion for meeting the standard of proof. In order to establish the defendant's guilt beyond a reasonable doubt, all the attacks against the conclusion that the defendant is guilty must be defeated. Now, whether an attack is defeated is not always an all or nothing affair. It is often a matter of degrees. If the reasons for guilt are slightly stronger than all their attacks, this would not be enough yet. To meet the demands of the standard of proof beyond a reasonable doubt, the supporting reasons must be significantly stronger than all their attacks. On the other hand, defeating all the attacks with absolute certainty would be too much to expect. So, more realistically, all attacks must be defeated in an almost definitive way. Here, it seems, we need to reintroduce some threshold, even though not in an explicitly probabilistic or numerical way.

It is not clear when to stop collecting supporting and attacking reasons. The argumentation framework is rather realistic. The idea that meeting the standard of proof requires to answer all attacks against the conclusion that the defendant is guilty is natural enough. A problem is that if the opposing party puts forward no attacks, meeting the standard of proof would be effortless. A possible response here is that the attacks must be all the attacks which a reasonable objector could in principle put forward, not just the attacks that in fact are put forward. But who is this 'reasonable objector'?

Another problem consists in identifying the threshold. While the probability based account can identify a specific probability threshold, at least in theory, by applying the principle of expected utility theory, the argumentation based framework cannot. How could the principle of expected utility theory be applied to the argument framework as well?

Further readings Carneades model (Gordon and Walton, 2009; Gordon et al., 2007). Justified argument model (Prakken and Sartor, 2007, 2009).

6.3 Scenarios

Competing scenarios are collected and compared. On the narrative framework, the two parties will put forward competing scenarios, at least two or possibly more than two. This is partly problematic because in a criminal case, the defense does not have the burden of proof. So it might well be that the defense puts forward a scenario that weakens the prosecutor's scenario, but that is not a scenario that proves innocence. Be that as it may, the various competing scenarios will be evaluated along the different criteria we identified, such as, consistency with the evidence, explanatory power, plausibility, cohesiveness, etc. The question arises, which scenario should be selected among the competitors?

The best explanatory scenario is the rule of decision. We can picture the process of evaluation of the competing scenarios as a process of elimination. At the beginning, several scenarios are viable, but as more evidence is considered and the scrutiny of each scenario continues, fewer scenarios will survive. The goal would be to select one scenario, or at least a limited set of scenarios, so that the answer to the question 'guilty or not?' would be univocal. On this picture, a scenario meets the demands of the standard of proof whenever it is the *only* scenario left.

But, once again, we confront a recurrent problem. The selection of one scenario is not always an all or nothing affair. The term 'abduction' or the expression 'inference to the best explanation' is sometimes used in this context. The basic idea is that, when confronted with two or more competing scenarios, the best explanation must be chosen. The notion of 'best explanation' here is wide ranging. It includes criteria such as consistency with the evidence, explanatory power (predictive power and causal fit), evidential support, plausibility, completeness, etc. Other criteria might also play a role, such the simplicity of the scenario. So, the best explanation is the scenario that fares better on some combination of these criteria. This is a matter of degrees. The scenarios get higher or lower scores relative to the applicable evaluation criteria. The scenario that gets the best score, and that meets a suitable threshold level, should be selected.

It is not clear how to find the scenarios. The process of scenario selection resembles how jurors reason in trial proceedings, whereas despite its clear mathematical underpinnings, it is hard to relate probability to judicial proceedings: jurors do not naturally quantify guilt, and it is difficult to quantify it even if we wanted to. Still, a problem with the scenario approach is that the method by which a scenario is selected is not entirely transparent. The different criteria, such as consistency, explanatory power, coherence etc. can pull the decision makers in opposite directions. For example, a scenario might be better in terms of explanatory power, while another might be more plausible. What to do, then? One might wonder whether the criterion for the best scenario should simply be this: the most probable scenario which meets a sufficiently high probability threshold. If so, this would not be very different from the probability based account of the standard of proof.

Further readings Inference to the best explanation (Lipton, 1991). Application of inference to the best explanation to legal reasoning (Pardo and Allen, 2008). Narrative based account of proof beyond a reasonable doubt (Allen, 2010; Allen and Stein, 2013).

7 SUMMARY AND CONCLUSION

We have discussed evidential reasoning in the law. For this, we have distinguished three normative frameworks: one focusing on the arguments for and against the positions taken, the second using probabilities to assess the evidential value of the evidence, and the third considering the scenarios that best explain the evidence.

We discussed four main themes: conflicting evidence, evidential value, the coherent interpretation of the evidence, and reasoning and decision making. For each theme, we discussed how they can be addressed in each of the three frameworks. We summarize our discussions for each theme, using the highlighted phrases in the preceding sections.

Conflicting evidence

Arguments The arguments for and against different positions have structure, involving complexes of reasons supporting and attacking positions. Three kinds of support can be distinguished: multiple, subordinated and coordinated support. Three kinds of attack can be distinguished: rebutting, undercutting, and undermining attack.

Scenarios Scenarios are clusters of events, ordered in time and connected by causal relations. Scenarios can explain a piece of evidence or be contradicted by it. Scenarios considered may or may not solve a case, and show which evidence is legally relevant.

Probabilities Evidential support and attack can be characterized as "probability changes". Evidential support and attack can be characterized as "likelihood ratio". The conflict between two pieces of evidence can be described probabilistically.

Evidential value

Probabilities The evidential value of a piece of evidence is measured by probabilistic difference or likelihood ratio ('incremental evidential value'). The evidential value of all the evidence, in its totality, is measured by the overall conditional probability ('overall evidential value'). DNA evidence can have a high incremental evidential value, but one should beware of complications.

Arguments The reasons used in arguments have different evidential value. Some are conclusive, others defeasible. An argument can be tested by asking critical questions and the evaluation of the argument depends on the answers given. It can be subject to debate whether a reason supports or attacks a conclusion.

Scenarios Scenarios must be consistent with the evidence, taken at face value. Scenarios must explain the evidence. Scenarios can be evaluated against two directions of fit.

Coherently interpreting the evidence

Scenarios The more evidence a scenario can explain, the better. Scenarios can be more or less plausible, logically consistent, cohesive, normal. Scenarios can be more or less complete.

Arguments The evaluation of an argument can depend on its subarguments. The evaluation of an argument can depend on chains of attacks. Conflicts between reasons can be addressed by exceptions, preferences and weighing.

Probabilities Independent items of evidence can be combined by multiplying the likelihood ratios. Items of evidence can be combined even if they are not independent. Different items of evidence for different hypotheses can also be combined.

Reasoning and decision making

Probabilities The guilt probability is estimated by weighing the evidence within the probability calculus. The decision criterion is a guilt probability threshold. It is not clear how to estimate all the required probabilities.

Arguments Arguments and counterarguments are collected and weighed. Defeating all counterarguments is the criterion for meeting the standard of proof. It is not clear when to stop collecting arguments and counterarguments.

Scenarios Competing scenarios are collected and compared. The best explanatory scenario is the rule of decision. It is not clear how to find the scenarios.

With the thematic discussion of the three normative frameworks, we have aimed to show how each framework contributes to the understanding of conflicting evidence, evidential value, the coherent interpretation of the evidence, and reasoning and decision making. In this way, we hope to contribute to the further development of the three frameworks. In our perspective, there is no need to choose between the frameworks, since each adds to the normative analysis of evidential reasoning. At the same time, there is room for further studies of how the three normative frameworks relate to each other, and how they can be integrated into an overall normative perspective on evidential reasoning.

NOTES

[1]Source: CBS, the Dutch central bureau of statistics, publishing its data at www.cbs.nl.

[2]**SOURCE TO BE ADDED**

[3]See `www.fbi.gov/services/laboratory/biometric-analysis/codis`.

[4]See `www.cstl.nist.gov/strbase/str_CSF1PO.htm`.

[5]At a rate of a dozen or more twin births per 1000 live births, identical twins are not that rare. Source `https://en.wikipedia.org/wiki/Twin#Statistics`.

[6]Bayes' theorem follows quickly using the definition of conditional probability. We have $\Pr(E|H) = \Pr(H \wedge E)/\Pr(H)$. Here we use logical conjunction $\wedge$ to write the combined event H and E. Hence, $\Pr(H \wedge E) = \Pr(E|H) \cdot \Pr(H)$. It follows that $\Pr(H|E) = \Pr(H \wedge E)/\Pr(E) = \Pr(E|H) \cdot \Pr(H)/\Pr(E)$, proving Bayes' theorem.

[7]A recent instance of the debate concerns the R v T case, where the UK Court of Appeal restricted the use of Bayes' theorem in courts to cases with a solid statistical foundation such as DNA; see the 2012 special issue of Law, Probability and Risk; Vol. 4, No. 2. For a 1970s instance of the debate, see Finkelstein and Fairley (1970); Tribe (1971).

[8]To see why, recall that

$$\frac{P(H|E)}{P(\neg H|E)} = \frac{P(E|H)}{P(E|\neg H)} \cdot \frac{P(H)}{P(\neg H)},$$

which implies

$$\frac{P(E|H)}{P(E|\neg H)} > 1 \text{ iff } \frac{P(H|E)}{P(\neg H|E)} > \frac{P(H)}{P(\neg H)}.$$

For one direction, if $P(H|E) > P(H)$, then $1-P(H|E) < 1-P(H)$. This means that $\frac{P(H|E)}{1-P(H|E)} > \frac{P(H)}{1-P(H)}$, and thus $\frac{P(H|E)}{P(\neg H|E)} > \frac{P(H)}{P(\neg H)}$. So, by the equivalence above, $\frac{P(E|H)}{P(E|\neg H)} > 1$. For the other direction, if $\frac{P(E|H)}{P(E|\neg H)} > 1$, then $\frac{P(H|E)}{P(\neg H|E)} > \frac{P(H)}{P(\neg H)}$, again by the equivalence above. The latter is the same as $\frac{P(H|E)}{1-P(H|E)} > \frac{P(H)}{1-P(H)}$. To establish $P(H|E) > P(H)$, suppose for contradiction that $P(H|E) \leq P(H)$, which implies $1 - P(H|E) \geq 1 - P(H)$. This means that $\frac{P(H|E)}{1-P(H|E)} \leq \frac{P(H)}{1-P(H)}$. This contradicts $\frac{P(H|E)}{1-P(H|E)} > \frac{P(H)}{1-P(H)}$, and thus $P(H|E) > P(H)$.

[9]The language of 'priors' and 'posteriors' is standard and suggests an temporal ordering. However, these probabilities are all assigned at the same time; they are not temporally ordered. Contrast this with *Bayesian updating* which is the controversial epistemological thesis that the probability of H *after* considering evidence E at time t_2 must be equivalent to the condition probability $P(H|E)$ earlier at time t_1.

[10]To derive the likelihood ratio, one first applies Bayes' theorem to both H and $\neg H$. We get $\Pr(H|E) = \Pr(E|H) \cdot \Pr(H)/\Pr(E)$ and $\Pr(\neg H|E) = \Pr(E|\neg H) \cdot \Pr(\neg H)/\Pr(E)$. Using these, we find:

$$\frac{\Pr(H|E)}{\Pr(\neg H|E)} = \frac{\Pr(E|H) \cdot \Pr(H)/\Pr(E)}{\Pr(E|\neg H) \cdot \Pr(\neg H)/\Pr(E)} = \frac{\Pr(E|H) \cdot \Pr(H)}{\Pr(E|\neg H) \cdot \Pr(\neg H)},$$

proving the likelihood ratio formula.

[11]Of course, there remains the question of how the numbers can obtained and whether the numbers needed to carry out the calculations are always available in the first place. This is a topic of debate.

[12]

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