Particle Swarm Optimization

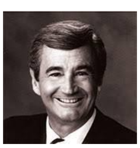
Inspiration

The idea behind PSO is to mimic the behaviour of animals like birds in navigating and foraging through social learning, by observing the behaviour of nearby birds who appear to be near the destination/food source.

PARTICLE SWARM OPTIMIZATION

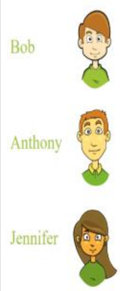
Developed in 1995 by

Dr. James Kennedy

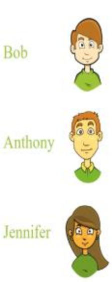


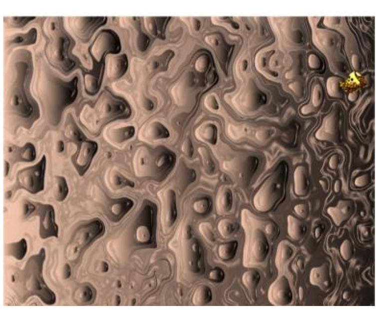
Dr. Eberhart

PSO Search Startegy

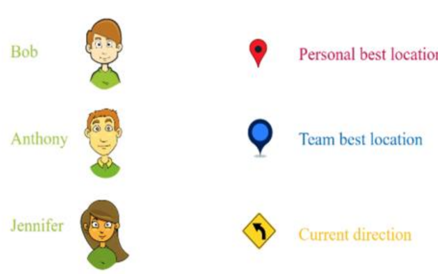


PSO Search Strategy





PSO Search Strategy



PSO Search Strategy

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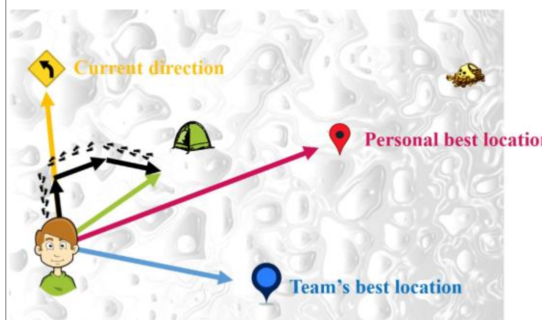

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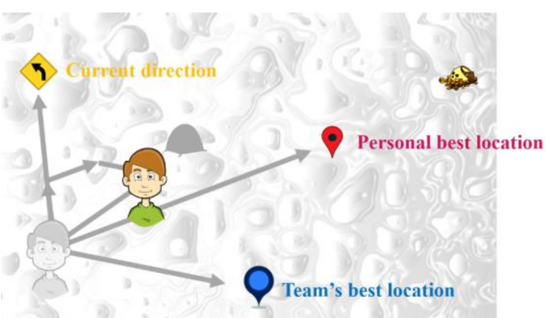

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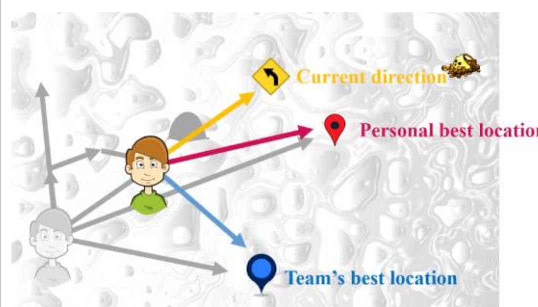

PSO Search Strategy

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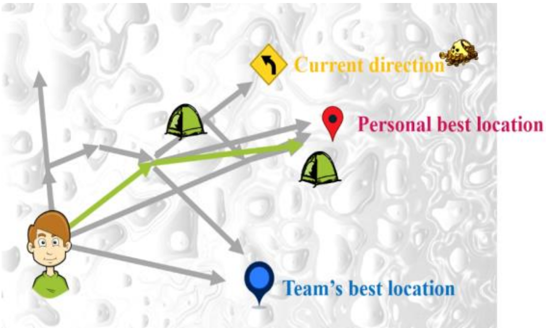

PSO Search Strategy

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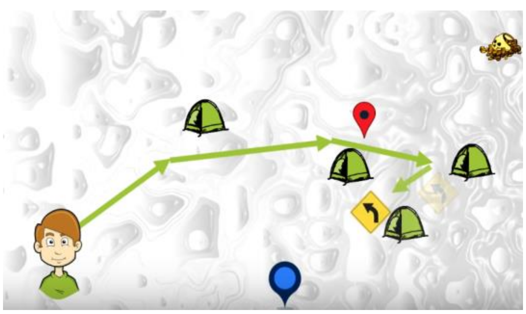

PSO Search Strategy

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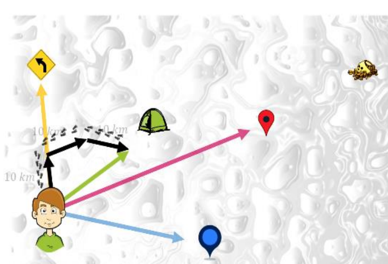
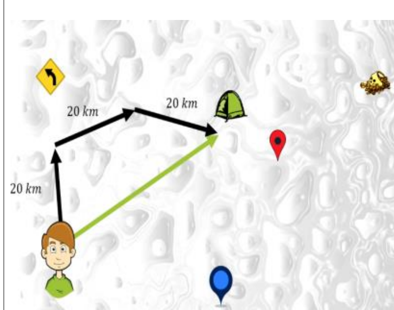


$2 \times r \times 10\ km$
$r\ in\ [0,1]$

PSO Search Strategy



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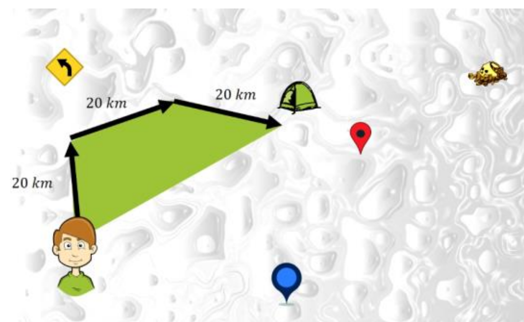
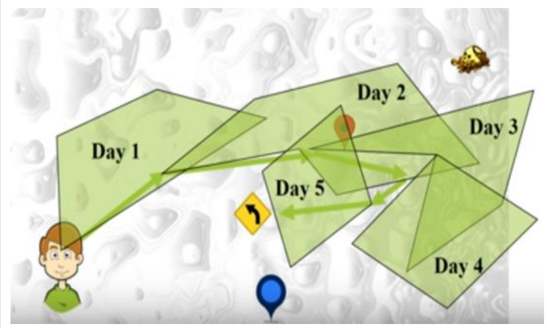
PSO Search Strategy



PSO Search Strategy



PSO Search Strategy

PSO Search Strategy

Bob $\overrightarrow{X_1^d} = [x_1^d, y_1^d]$

Anthony $\overrightarrow{X_2^d} = [x_2^d, y_2^d]$

Jennifer $\overrightarrow{X_3^d} = [x_3^d, y_3^d]$

$$\overrightarrow{X_i^d} = [x_i^d, y_i^d, z_i^d, \ldots]$$

PSO Search Strategy

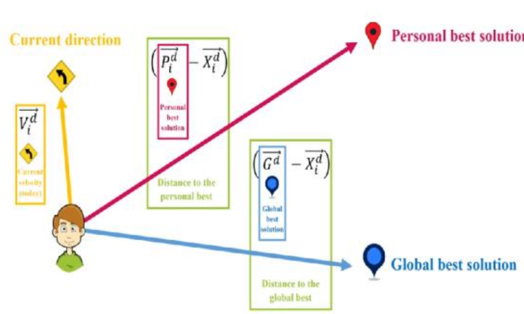
Velocity Update Equation

$$\underbrace{\overrightarrow{V_i^{d+1}}}_{\text{Next velocity (tomorrow)}} = r_1 \underbrace{\overrightarrow{V_i^d}}_{\text{Current velocity (today)}} + 2r_2 \underbrace{\left(\overrightarrow{P_i^d} - \overrightarrow{X_i^d}\right)}_{\text{Distance to the personal best}} + 2r_3 \underbrace{\left(\overrightarrow{G^d} - \overrightarrow{X_i^d}\right)}_{\text{Distance to the global best}}$$

Personal best solution

Global best solution

PSO Search Strategy



PSO Search Strategy

Position Update Equation

$$\overrightarrow{X_i^{d+1}} = \overrightarrow{X_i^{d}} + \overrightarrow{V_i^{d+1}}$$

Position in day *d+1* | Position in day *d* | Velocity in day *d+1*

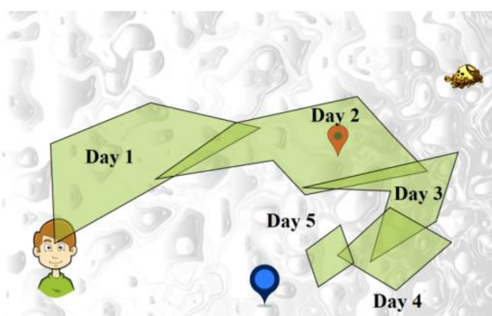
PSO Search Strategy

$$\overrightarrow{X_i^{t+1}} = \overrightarrow{X_i^{t}} + \overrightarrow{V_i^{t+1}}$$

$$\overrightarrow{V_i^{t+1}} = w\overrightarrow{V_i^{t}} + c_1 r_1 \left(\overrightarrow{P_i^{t}} - \overrightarrow{X_i^{t}}\right) + c_2 r_2 \left(\overrightarrow{G^{t}} - \overrightarrow{X_i^{t}}\right)$$

Cognitive component **Social component**

PSO Search Strategy



❑PSO uses a population of individuals, to search feasible region of the function space.

❑Population is called ***swarm***
❑Individuals are called ***particles.***

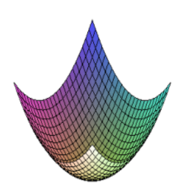
❑ Though the PSO algorithm has been shown to perform well, there is no mathematical proof of its convergence.

Basic Flow of PSO

1. Initialize the ***swarm*** from the solution space.
2. Evaluate ***fitness*** of individual ***particles.***
3.Find ***gbest***, ***pbest*** and update the ***velocity.***
4. Move each ***particle*** to a new ***position***.
5. **Go to step 2**, and repeat until convergence or a stopping condition is satisfied.

Understanding PSO: Step by Step

$$Minimize\ f(x) = x_1^2 + x_2^2$$
$$-5 \le x_1, x_2 \le 5$$
$$f\ \min = 0\ at\,(0,0)$$



1. Initialization of The Population

Let the population size be 5.
Generate 5 particles randomly in the feasible search space.

Let $S = \{X_1, X_2, X_3, X_4, X_5\}$

$X_1 = (2.7045, 4.8030)$
$X_2 = (4.5974, 2.8793)$
$X_3 = (1.8710, 4.0528)$
$X_4 = (1.6400, 1.3202)$
$X_5 = (3.3392, 0.9963)$

2. Evaluation of Fitness

Evaluate fitness of each particle :

$f(X_1) = 2.7045^2 + 4.8030^2 = 30.3831$

Similarly

$f(X_2) = 29.4265$
$f(X_3) = 19.9258$
$f(X_4) = 4.4325$ Call X_4 as *gbest*.
$f(X_5) = 12.1429$

Best value is at X_4 with $f_{min} = 4.4325$

Pbest is same as X

Generating The Velocities

Generate the Velocity vector uniformly in the range [0,1]

$V = \{V_1, V_2, V_3, V_4, V_5\}$
$V_1 = (v_{11}, v_{12}) = (0.4752, 0.6987)$
$V_2 = (v_{21}, v_{22}) = (0.4141, 0.4020)$
$V_3 = (v_{31}, v_{32}) = (0.7797, 0.9433)$
$V_4 = (v_{41}, v_{42}) = (0.6183, 0.4749)$
$V_5 = (v_{51}, v_{52}) = (0.2530, 0.9398)$

Evaluating the Velocity and Position Equations

First Particle:
Velocity Update for 1st Component:

$$v_{11} = v_{11} + c_1r_1(pbest_{11} - x_{11}) + c_2r_2(gbest_{11} - x_{11})$$
$$v_{11} = 0.4752 + 2*0.34*(2.7045 - 2.7045) + 2*0.86*(1.6400 - 2.7045) = -1.35574$$

Position Update for 1st component:

$$x_{11} = x_{11} + v_{11}$$
$$x_{11} = 2.7045 + (-1.35574) = 1.34876$$

First Particle:
Velocity Update for 2nd Component:

$$v_{12} = v_{12} + c_1r_1(pbest_{12} - x_{12}) + c_2r_2(gbest_{12} - x_{12})$$
$$v_{12} = 0.6987 + 2*0.47*(4.8030 - 4.8030) + 2*0.91*(1.3202 - 4.8030) = -4.368362$$

Position Update for 2nd component:

$$x_{12} = x_{12} + v_{12}$$
$$x_{12} = 4.8030 + (-4.368362) = 0.434638$$

So, $X_1 = (1.34876, 0.434638)$

Second Particle:

$$v_{21} = 0.4141 + 2*0.34*(4.5974 - 4.5974) + 2*0.86*(1.6400 - 4.5974) = -4.672628$$
$$x_{21} = 4.5974 + (-4.672628) = -0.075228$$

$$v_{22} = 0.4020 + 2*0.12*(2.8793 - 2.8793) + 2*0.06*(1.3202 - 2.8793) = 0.214908$$
$$x_{22} = 2.8793 + (-0.1593) = 3.094208$$

So, $X_2 = (-0.075228, 3.094208)$

Third Particle:

v_{31} = 0.7797 + 2*0.98*(1.8710 – 1.8710) + 2*0.86*(1.6400 – 1.8710)
= 0.38238
x_{31} = 1.8710 + 0.38238
= 2.25338

v_{32} = 0.9433 + 2*0.69*(4.0528 – 4.0528) + 2*0.34*(1.3202 – 4.0528)
= – 0.914868
x_{32} = 4.0528 + (– 0.914868)
= 3.137932

So, X_3= (2.25338, 3.137932)

Fourth Particle:

v_{41} = 0.6183 + 2*0.18*(1.6400 – 1.6400) + 2*0.23*(1.6400 – 1.6400)
= 0.6183
x_{41} = 1.6400 + 0.6183
= 2.2583

v_{42} = 0.4749 + 2*0.61*(1.3202 – 1.3202) + 2*0.04*(1.3202 – 1.3202)
= 0.4749
X_{42} = 1.3202 + 0.4749
= 1.7951

So, X_4= (2.2583, 1.7951)

Fifth Particle:

v_{51} = 0.2530 + 2*0.09*(3.3392 – 3.3392) + 2*0.39*(1.6400 – 3.3392)
= – 1.072376
x_{51} = 3.3392 + (– 1.072376)
= 2.266824

v_{52} = 0.9398 + 2*0.65*(0.9963 – 0.9963) + 2*0.10*(1.3202 – 0.9963)
= 1.00458
x_{52} = 0.9963 + 1.00458
= 2.00088

So, X_5= (2.266824, 2.00088)

The positions and the corresponding fitness values obtained after the evaluation :

Position	Fitness
X_1 = (1.34876,0.43638)	**$f(X_1)$ = 2.00806**
X_2 = (-0.075228, 3.094208)	$f(X_2)$ = 9.57978
X_3 = (2.25338, 3.137932)	$f(X_3)$ = 14.9243
X_4 = (2.2583, 1.7951)	$f(X_4)$ = 8.32266
X_5 = (2.266824, 2.00088)	$f(X_5)$ = 9.12401

X_1 is the new *gbest*.

Initial Swarm	Updated swarm
X_1 = (2.7045, 4.8030)	X_1 = (1.34876,0.43638)
X_2 = (4.5974, 2.8793)	X_2 = (-0.075228, 3.094208)
X_3 = (1.8710, 4.0528)	X_3 = (2.25338, 3.137932)
X_4 = (1.6400, 1.3202)	X_4 = (2.2583, 1.7951)
X_5 = (3.3392, 0.9963)	X_5 = (2.266824, 2.00088)
Corresponding fitness is	Corresponding fitness is
$f(X_1)$ = 30.3831	**$f(X_1)$ = 2.00806**
$f(X_2)$ = 29.4265	$f(X_2)$ = 9.57978
$f(X_3)$ = 19.9258	$f(X_3)$ = 14.92430
$f(X_4)$ = 4.4325	$f(X_4)$ = 8.32266
$f(X_5)$ = 12.1429	$f(X_5)$ = 9.12401

The values of *pbest* for each particle is as following:

$pbest_1$ = (1.34876, 0.434638)
$pbest_2$ = (–0.075228, 3.094208)
$pbest_3$ = (2.25338, 3.137932)
$pbest_4$ = (1.6400, 1.3202)
$pbest_5$ = (2.266824, 2.00088)