Database Management Systems (CSN-351)
Indexing and Hashing

BTech 3rd Year (CS) + Minor + Audit

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Hash indices: Based on a uniform distribution of values across a range of buckets. The bucket to which a value is assigned is determined by a function, called a *hash function*.

Indexing Techniques Evaluation Factors

- Access types
- Access time
- Insertion time
- Deletion time
- Space overhead

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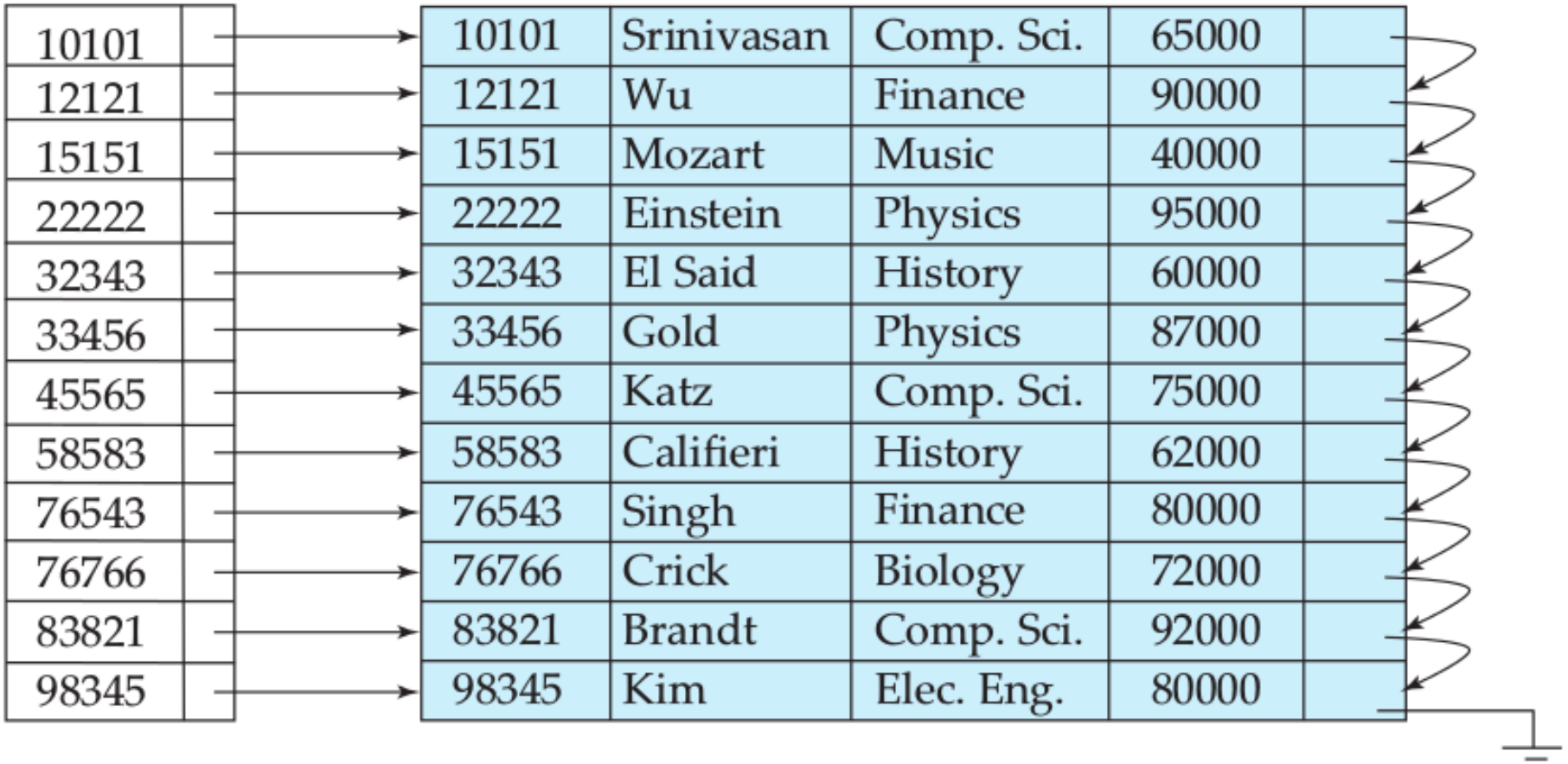
Index-sequential file: ordered sequential file with a primary index.

Dense Index Files

Dense index: Index record appears for every search-key value in the file.



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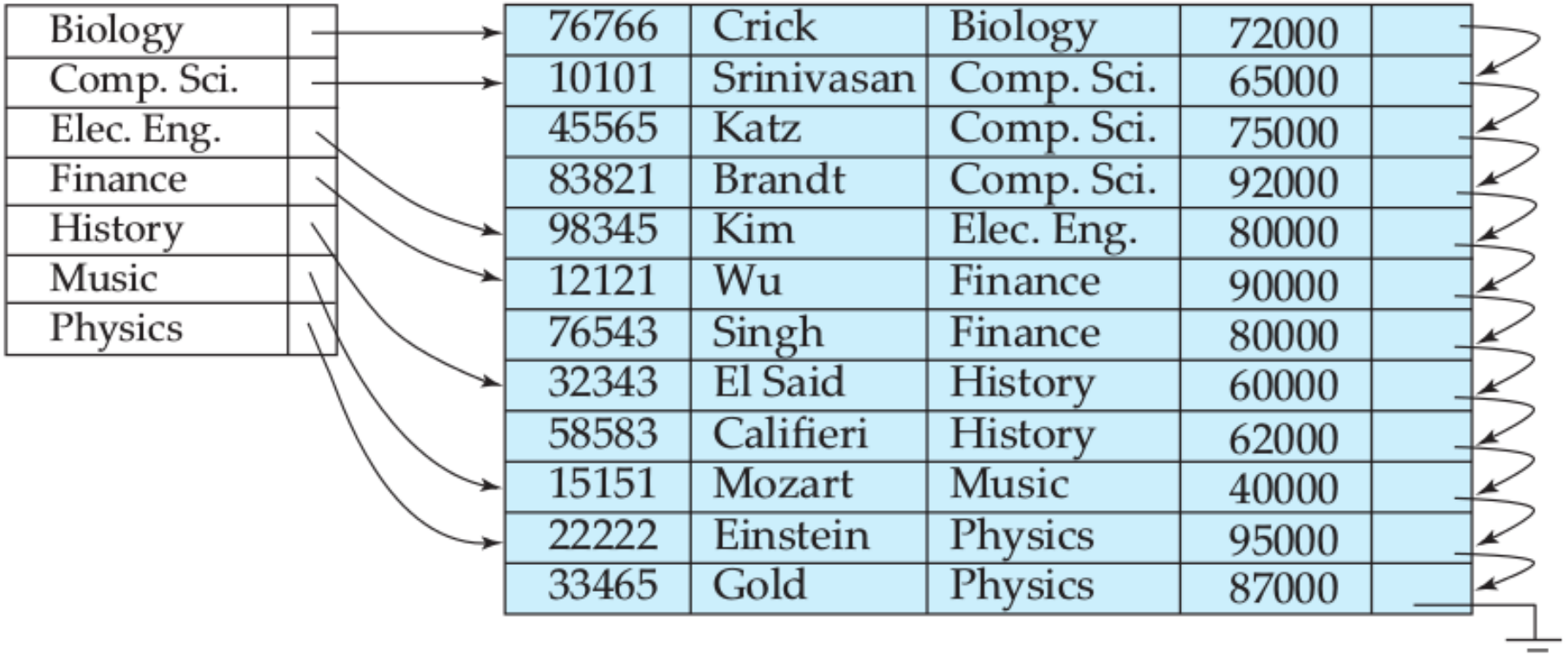
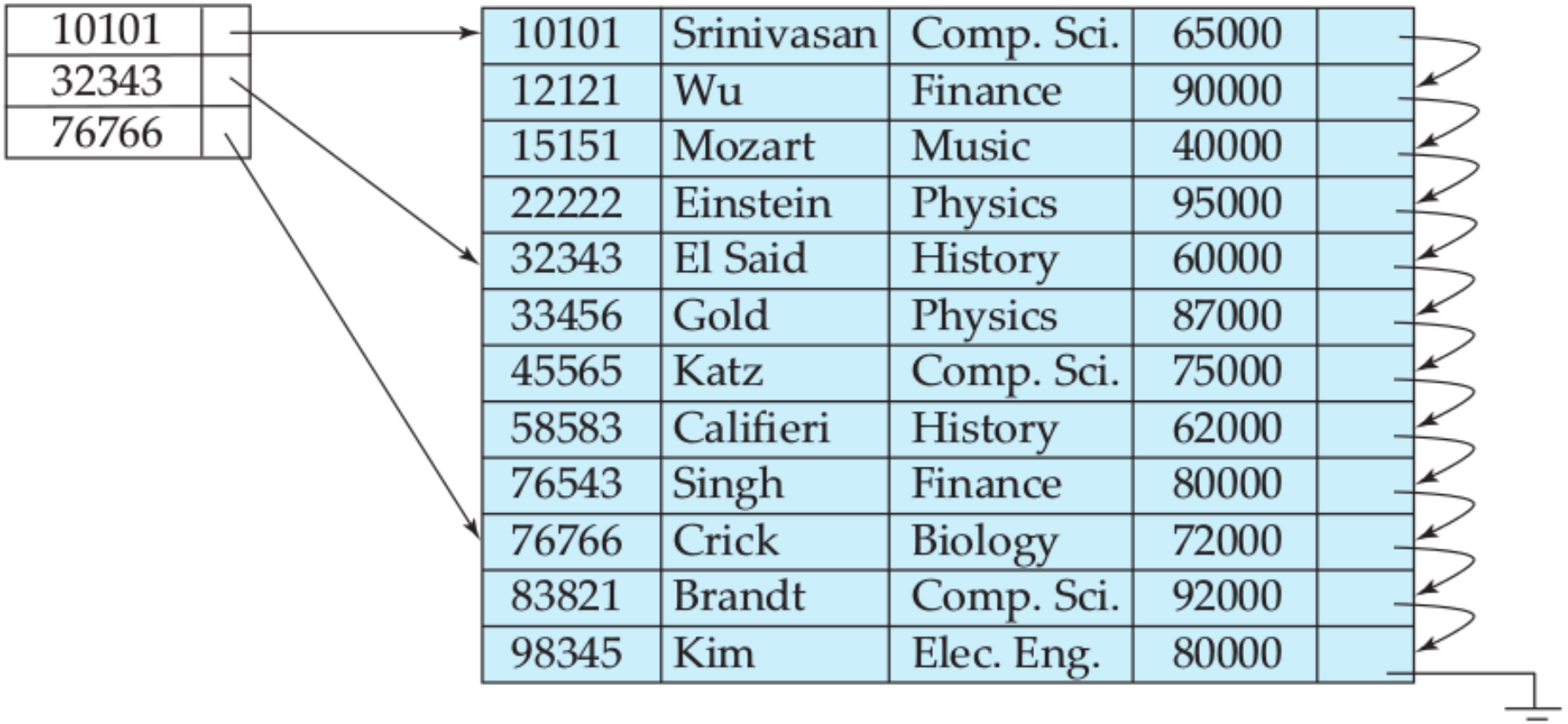


Sparse Index Files

Sparse Index: contains index records for only some search-key values.

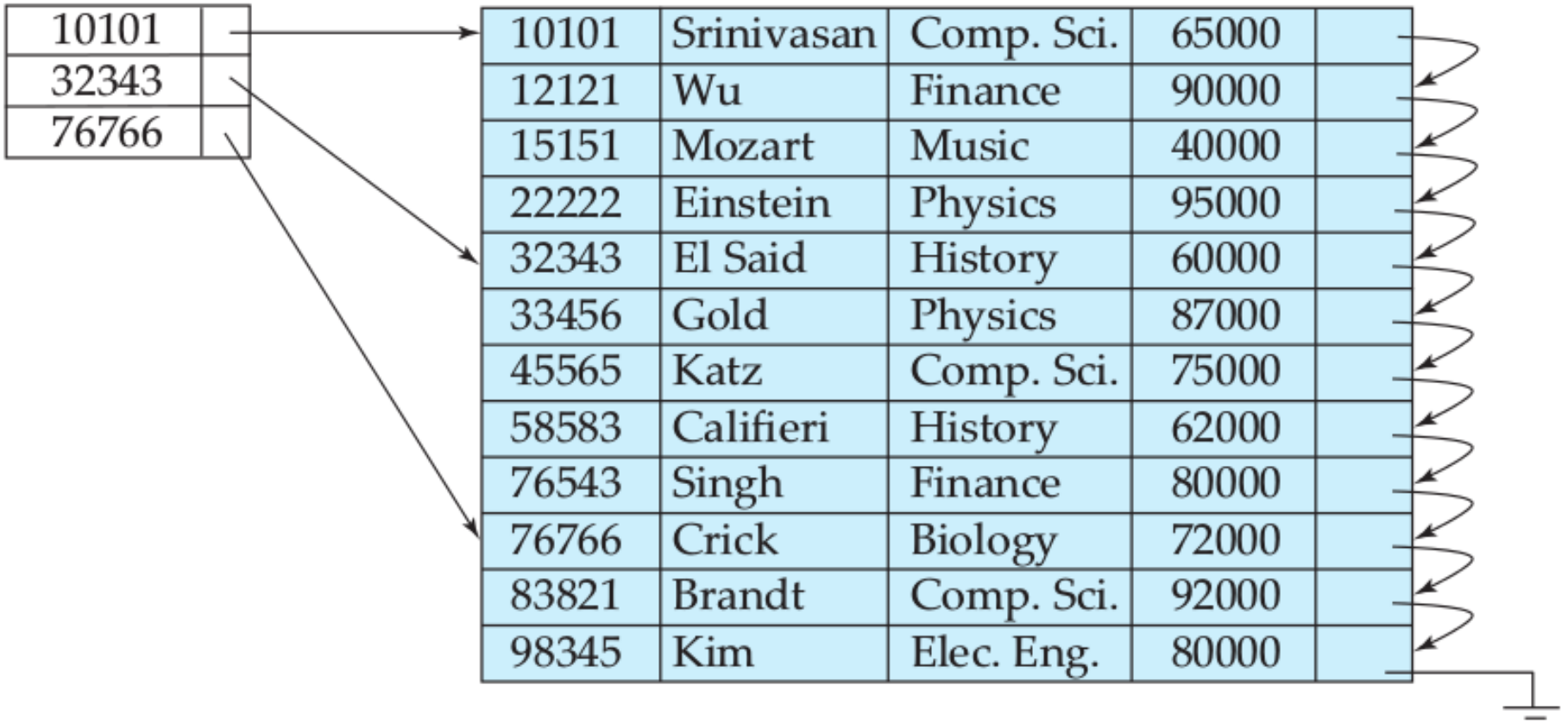
Applicable when records are sequentially ordered on search-key.



Sparse Index Files

To locate a record with search-key value `K` we:

- Find index record with largest search-key value $< K$.
- Search file sequentially starting at the record to which the index record points.



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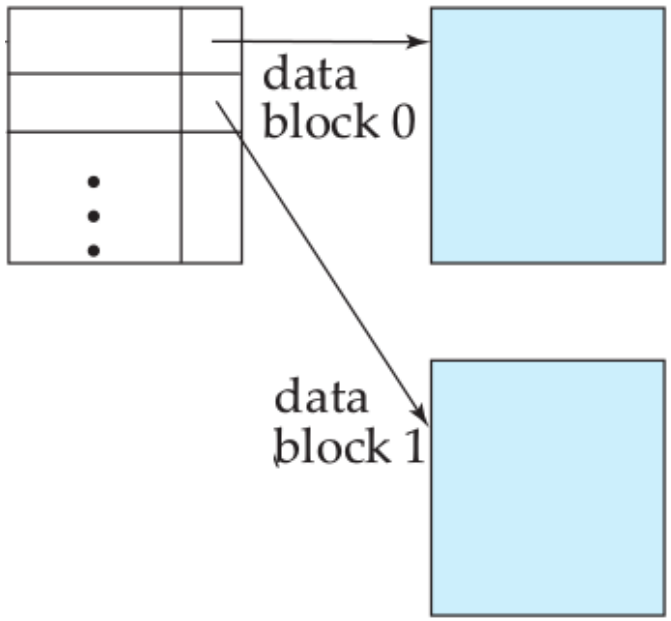
Good tradeoff: sparse index with an index entry for every block in file, corresponding to the smallest search-key value in the block.



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Indices at all levels must be updated on insertion or deletion from the file.

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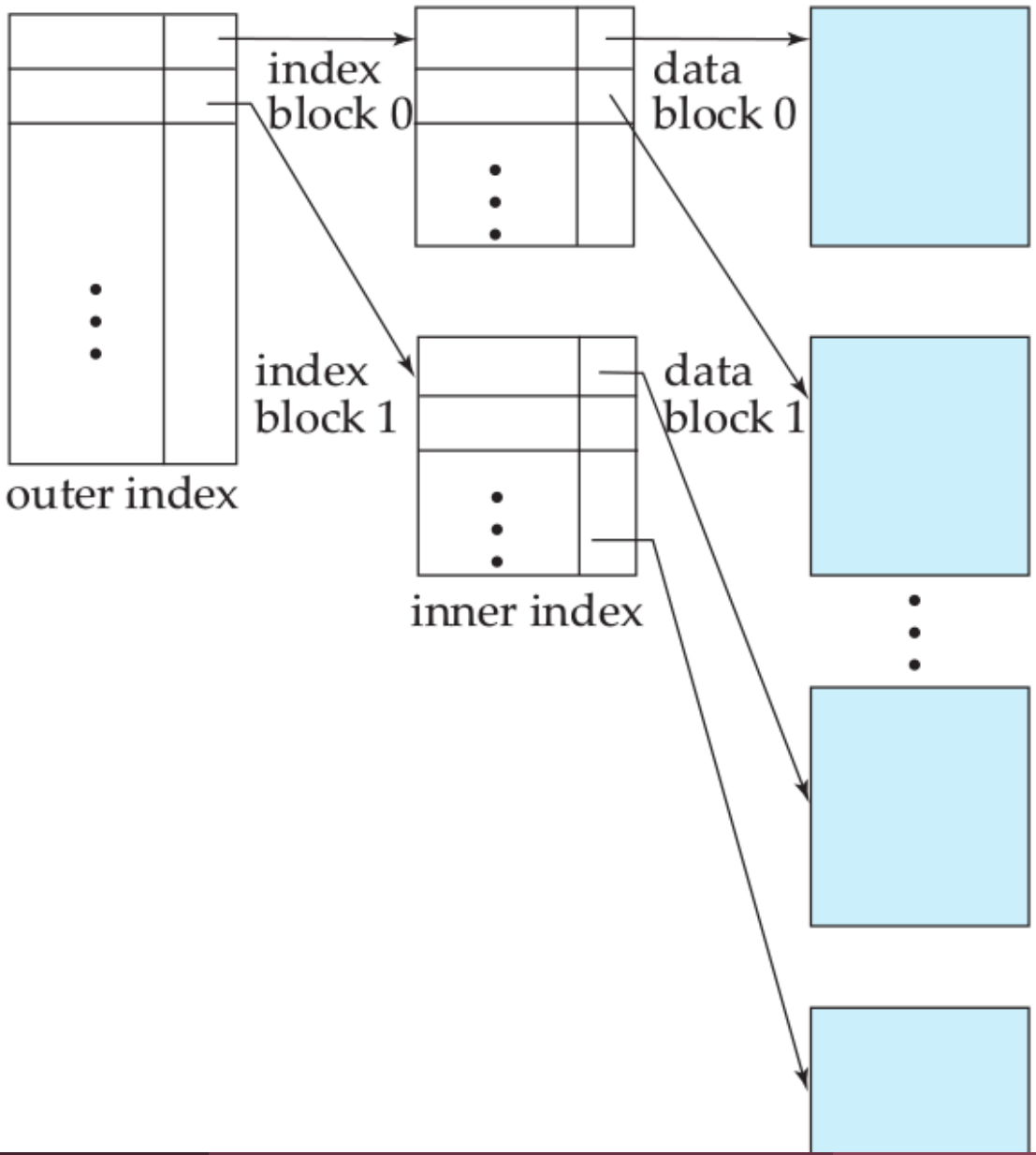

Index Update: Deletion

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If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

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Multilevel insertion and deletion: algorithms are simple extensions of the single-level algorithms.

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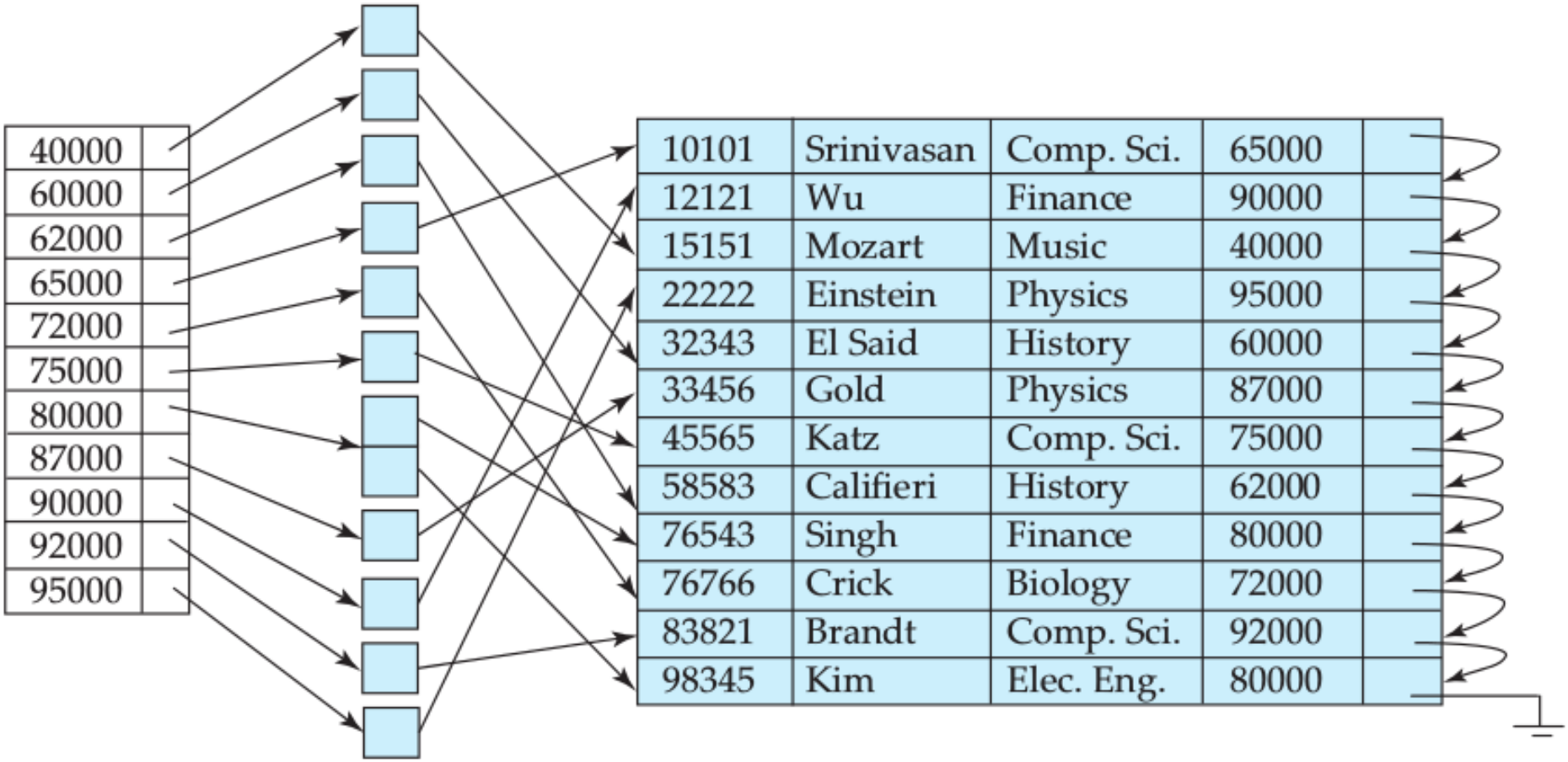
We can have a **secondary index** with an index record for each search-key value.

Secondary Indices

Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.

Secondary indices have to be dense



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Sequential scan using primary index is efficient, but a sequential scan using a secondary index is expensive.

- Each record access may fetch a new block from disk.
- Block fetch requires about 5 to 10 milliseconds, versus about 100 nanoseconds for memory access.

Resource

Data Structures Visualization Gallery,
Department of Computer Science, University of San Francisco

`https://www.cs.usfca.edu/~galles/visualization/Algorithms.html`

Question 1

A hash table of length 10 uses open addressing with hash function $h(k) = k \mod 10$, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

- 46, 42, 34, 52, 23, 33
- 34, 42, 23, 52, 33, 46
- 46, 34, 42, 23, 52, 33
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Answer: **30**

Question 3

The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k \mod 10$ and linear probing. What is the resultant hash table?

Question 4

Consider a hash table of size seven, with starting index zero, and a hash function $(3x + 4)$ mod 7. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing?

Question 5

Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function $x \mod 10$, which of the following statements are true?

i. 9679, 1989, 4199 hash to the same value

ii. 1471, 6171 hash to the same value

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Answer: **11**