If $A = UDV^T$, then A^TA is:

(A) UD^2U^T

(B) VD^2V^T

(C) A square matrix

(D) is always full rank

(E) none of the above

Consider a set of general vectors $\mathbf{a}_i \in R^d$. (assume all elements are some random numbers in the range of $[0, 1]$) $\mathbf{b}$ is another such vector. Consider the matrix:

$$\mathbf{A} = \sum_{i=1}^{k} \mathbf{a}_i \mathbf{a}_i^T + \sum_{i=k+1}^{d} \mathbf{b}\mathbf{b}^T$$

What is the effective rank of $\mathbf{A}$

(A) k

(B) 1

(C) none of the above

(D) $k + 1$

(E) d

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$$\mathbf{A} = \sum_{i=1}^{k} 10^{-k} \mathbf{a}_i \mathbf{a}_i^T + \sum_{i=k+1}^{d} \mathbf{b} \mathbf{b}^T$$

What is the effective rank of $\mathbf{A}$

(A) $k+1$

(B) k

(C) d

(D) 1

(E) none of the above

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(A) 1

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What is the effective rank of $\mathbf{A}$

(A) 1

(B) none of the above

(C) k

(D) d

(E) $k+1$