

# Topics in Applied Optimization

Applications to Machine Learning, Vision, and Data Analytics

---

Pawan Kumar

CSTAR, IIIT-H

## Course Information

---

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**
  - 4.1 Basic Linear Algebra (review)

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**
  - 4.1 Basic Linear Algebra (review)
  - 4.2 Multivariable Calculus, Vector and Matrix Calculus (review)



# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**
  - 4.1 Basic Linear Algebra (review)
  - 4.2 Multivariable Calculus, Vector and Matrix Calculus (review)
  - 4.3 Basic Probability and Statistics (review)

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**
  - 4.1 Basic Linear Algebra (review)
  - 4.2 Multivariable Calculus, Vector and Matrix Calculus (review)
  - 4.3 Basic Probability and Statistics (review)
  - 4.4 Familiarity with C/C++, Python, and Matlab (assumed!)

# Course Information

---

1. **Name:** Topics in Applied Optimization (a.k.a TAO)
2. **Code:** CSE484
3. **Credit:** 4
4. **Prerequisites:**
  - 4.1 Basic Linear Algebra (review)
  - 4.2 Multivariable Calculus, Vector and Matrix Calculus (review)
  - 4.3 Basic Probability and Statistics (review)
  - 4.4 Familiarity with C/C++, Python, and Matlab (assumed!)
  - 4.5 Basic Linux Utilities (assumed)

# Weightages

---

1. Online Randomized Quizzes: 30 % (Most of Boyd's Book)

# Weightages

---

1. Online Randomized Quizzes: 30 % (Most of Boyd's Book)
2. Online Written End-Sem: 20 %

# Weightages

---

1. Online Randomized Quizzes: 30 % (Most of Boyd's Book)
2. Online Written End-Sem: 20 %
3. Projects: 25 % (Paper reading, Coding, and Presentation)

# Weightages

---

1. **Online Randomized Quizzes:** 30 % (Most of Boyd's Book)
2. **Online Written End-Sem:** 20 %
3. **Projects:** 25 % (Paper reading, Coding, and Presentation)
4. **Assignments:** 25 % (Take Home Assignments)

# References

---

- Convex Optimization by Boyd and Vanderberghe (main)
- Convex Optimization by Bertsekas (some part)
- Deep Learning, Bengio, Hinton, Goodfellow (some part)
- Numerical Optimization, Jorge Nocedal, Stephen J. Wright (some part)
- Non-convex Optimization, Jain
- Non-Smooth Optimization, Boyd (Online notes)



## Some Remarks:

- Attendance is **not** compulsory,

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained
  - Generative models (Variational Autoencoders, GANs)



## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained
  - Generative models (Variational Autoencoders, GANs)
    - GAN: Two player min-max optimization problem; saddle point problems

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained
  - Generative models (Variational Autoencoders, GANs)
    - GAN: Two player min-max optimization problem; saddle point problems
  - Recommender systems (Matrix completion problem)

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained
  - Generative models (Variational Autoencoders, GANs)
    - GAN: Two player min-max optimization problem; saddle point problems
  - Recommender systems (Matrix completion problem)
  - Non-linear optimization in computer vision

## Some Remarks:

- Attendance is **not** compulsory, but there will be **surprise quizzes!**
- Highly recommended to attend classes!
- Linear algebra, Advanced calculus, and Prob. and stat will be reviewed in first two classes
  - Take extra care to brush up!
- Previous knowledge of Machine learning is **not** necessary
- Sample applications to be covered:
  - Classification and regression problems (LR, ANN, SVMs)
    - ANN: Non-convex and non-linear
    - SVM: Usually convex, but constrained
  - Generative models (Variational Autoencoders, GANs)
    - GAN: Two player min-max optimization problem; saddle point problems
  - Recommender systems (Matrix completion problem)
  - Non-linear optimization in computer vision
  - Plenty more applications during the semester...

## Have you seen optimization problem before?

---

$$\begin{aligned} &\text{maximize } f(x) = -x^2 + 4x - 3 \\ &\text{subject to } x \in [-1, 1]. \end{aligned}$$

What is the solution?

# What is an Optimization Problem?

---

# What is an Optimization Problem?

---

A problem of the form

$$\begin{array}{ll}\text{minimize} & f(x) \\ \text{subject to} & x \in C\end{array}$$

# What is an Optimization Problem?

---

A problem of the form

$$\begin{array}{ll}\text{minimize} & f(x) \\ \text{subject to} & x \in C\end{array}$$

Here

- $f(x)$  is called objective function



# What is an Optimization Problem?

---

A problem of the form

$$\begin{array}{ll}\text{minimize} & f(x) \\ \text{subject to} & x \in C\end{array}$$

Here

- $f(x)$  is called objective function
- $C$  is called constrained set

# What is an Optimization Problem?

---

A problem of the form

$$\begin{array}{ll}\text{minimize} & f(x) \\ \text{subject to} & x \in C\end{array}$$

Here

- $f(x)$  is called objective function
- $C$  is called constrained set
- We want to find  $x$  such that  $f(x)$  is minimized

# Objectives of this course

---

# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))

# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function

# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function
  - Find suitable set of constraints

# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function
  - Find suitable set of constraints
- Learn algorithms to solve the optimization problem ([Solver](#))

# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function
  - Find suitable set of constraints
- Learn algorithms to solve the optimization problem ([Solver](#))
  - Identify the problem class: linear/nonlinear, smooth/nonsmooth, convex/nonconvex



# Objectives of this course

---

- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function
  - Find suitable set of constraints
- Learn algorithms to solve the optimization problem ([Solver](#))
  - Identify the problem class: linear/nonlinear, smooth/nonsmooth, convex/nonconvex
  - Propose a suitable solver for the given class

# Optimization = Problem Solving

---

**Problem:** Of all rectangles of area 100, which has the smallest perimeter?

**Modelling:**

- Objective function:
- Constraint set:
- Final Model:

# Optimization = Problem Solving

---

**Problem:** Of all rectangles of area 100, which has the smallest perimeter?

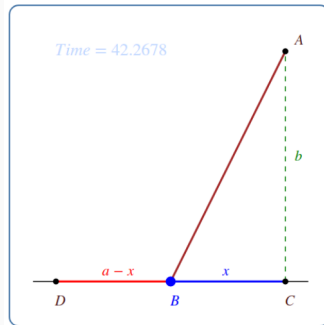
**Solution:**

# Optimization = Problem Solving

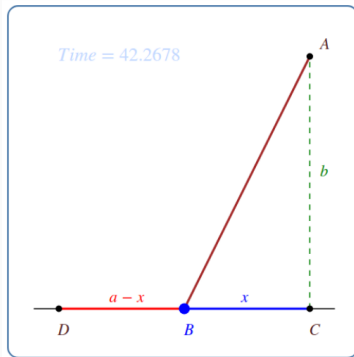
---

# Optimization = Problem Solving

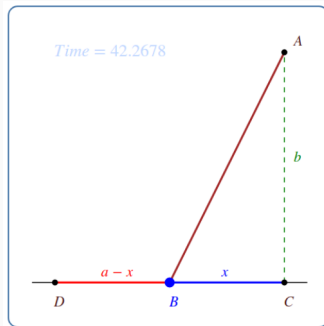
**Problem:** Suppose you want to reach a point  $A$  that is located across the sand from a nearby road. Suppose that the road is straight, and  $b$  is the distance from  $A$  to the closest point  $C$  on the road. Let  $v$  be your speed on the road, and let  $w$ , which is less than  $v$ , be your speed on the sand. Right now you are at the point  $D$ , which is a distance  $a$  from  $C$ . At what point  $B$  should you turn off the road and head across the sand in order to minimize your travel time to  $A$ ?



- How do we model this problem as optimization problem?
- How do we solve the formulated optimization problem?



# Optimization = Problem Solving



- Objective function:
- Constraints:
- Model:

# Optimization = Problem Solving

---

Consider the following temperature data of a city in a day:

Time	Rescaled Time	Temperature
9:00	0:00	15
11:00	2:00	30
14:00	4:00	34
16:00	6:00	33
18:00	8:00	21
22:00	12:00	18

Predict temperature at 14:30.

- Objective function:
- Constraints:
- Model: