

# **Topics in Applied Optimization**

Applications to Machine Learning, Vision, and Data Analytics

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Pawan Kumar

CSTAR, IIIT-H

## Course Information

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  - 4.5 Basic Linux Utilities (assumed)

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1. Online Randomized Quizzes: 30 % (Most of Boyd's Book)

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4. Assignments: 25 % (Take Home Assignments)

## References

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- Convex Optimization by Boyd and Vanderberghe (main)
- Convex Optimization by Bertsekas (some part)
- Deep Learning, Bengio, Hinton, Goodfellow (some part)
- Numerical Optimization, Jorge Nocedal, Stephen J. Wright (some part)
- Non-convex Optimization, Jain
- Non-Smooth Optimization, Boyd (Online notes)

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  - Recommender systems (Matrix completion problem)
  - Non-linear optimization in computer vision
  - Plenty more applications during the semester...

## Have you seen optimization problem before?

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$$\begin{aligned} & \text{maximize } f(x) = -x^2 + 4x - 3 \\ & \text{subject to } x \in [-1, 1]. \end{aligned}$$

What is the solution?

# What is an Optimization Problem?

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- $C$  is called constrained set

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$$\begin{aligned} & \text{minimize } f(x) \\ & \text{subject to } x \in C \end{aligned}$$

Here

- $f(x)$  is called objective function
- $C$  is called constrained set
- We want to find  $x$  such that  $f(x)$  is minimized

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- Learn to model a problem as an optimization problem ([Modelling](#))
  - Find suitable objective function
  - Find suitable set of constraints
- Learn algorithms to solve the optimization problem ([Solver](#))
  - Identify the problem class: linear/nonlinear, smooth/nonsmooth, convex/nonconvex
  - Propose a suitable solver for the given class

# Optimization = Problem Solving

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**Problem:** Of all rectangles of area 100, which has the smallest perimeter?

**Modelling:**

- Objective function:

- Constraint set:

- Final Model:

# Optimization = Problem Solving

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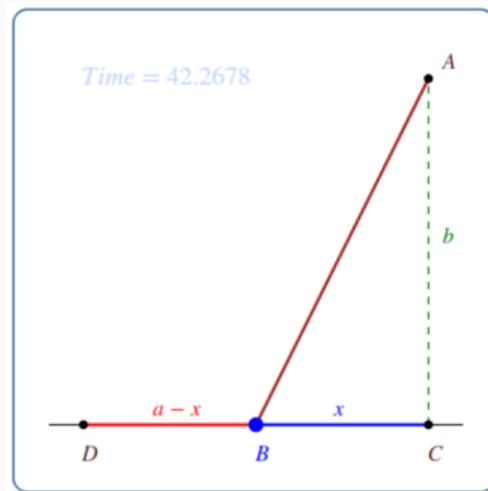
**Solution:**

# **Optimization = Problem Solving**

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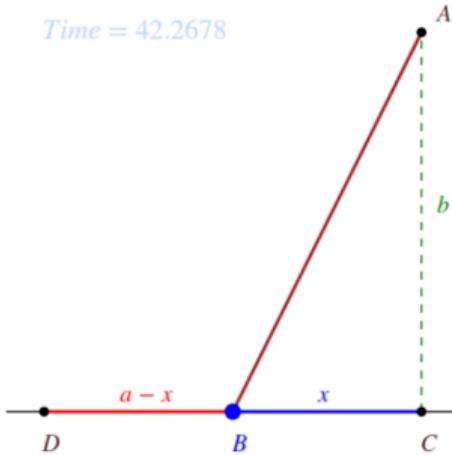
# Optimization = Problem Solving

**Problem:** Suppose you want to reach a point  $A$  that is located across the sand from a nearby road. Suppose that the road is straight, and  $b$  is the distance from  $A$  to the closest point  $C$  on the road. Let  $v$  be your speed on the road, and let  $w$ , which is less than  $v$ , be your speed on the sand. Right now you are at the point  $D$ , which is a distance  $a$  from  $C$ . At what point  $B$  should you turn off the road and head across the sand in order to minimize your travel time to  $A$ ?

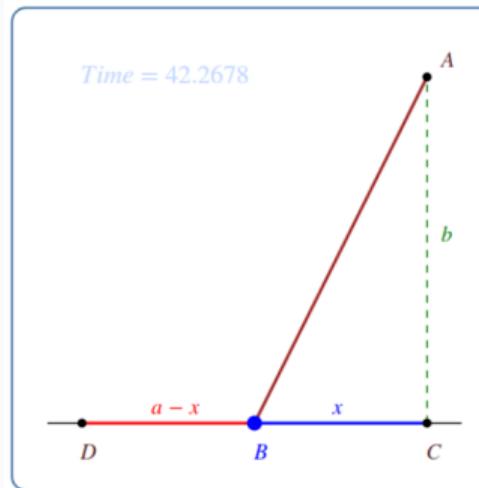


- How do we model this problem as optimization problem?
- How do we solve the formulated optimization problem?

Time = 42.2678



# Optimization = Problem Solving



- Objective function:
- Constraints:
- Model:

# Optimization = Problem Solving

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Consider the following temperature data of a city in a day:

Time	Rescaled Time	Temperature
9:00	0:00	15
11:00	2:00	30
14:00	4:00	34
16:00	6:00	33
18:00	8:00	21
22:00	12:00	18

Predict temperature at 14:30.

- Objective function:
- Constraints:
- Model: