Database Management Systems (CSN-351)
Relational Database Design (contd. 3)

BTech 3rd Year (CS) + Minor + Audit

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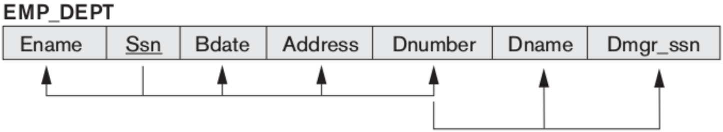
Closure

Definition (Closure)

The set of all dependencies that include F (the set of functional dependencies that are specified on relation schema R) as well as all dependencies that can be inferred from F is called the closure of F; it is denoted by F^+.

Inference of FDs



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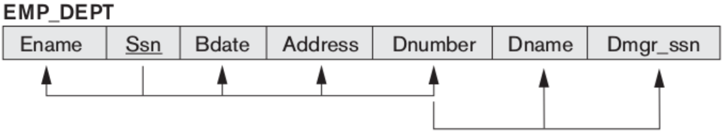

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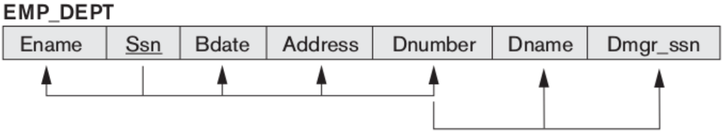

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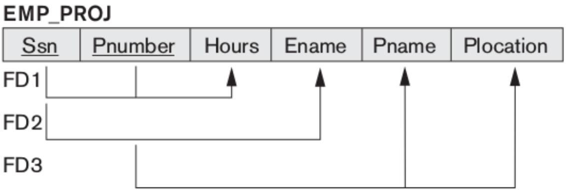
- IR5 (union or additive rule): $\{X \to Y, X \to Z\} \models X \to YZ$.
- IR6 (pseudotransitive rule): $\{X \to Y, WY \to Z\} \models WX \to Z$.

Algorithm: Determining X^+, the Closure of X under F

Input: A set F of FDs on a relation schema R, and a set of attributes X, which is a subset of R.

```
X+ := X;
repeat
    oldX+ := X+;
    for each functional dependency Y → Z in F do
        if X+ ⊇ Y then X+ := X+ ∪ Z;
until (X+ = oldX+);
```

Determining X^+



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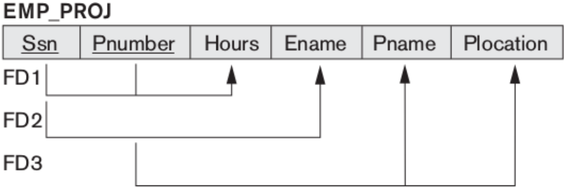
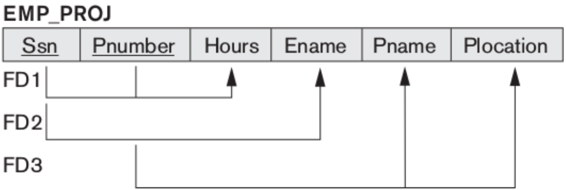


$$F = \{\text{Ssn} \rightarrow \text{Ename}, \\ \text{Pnumber} \rightarrow \{\text{Pname}, \text{Plocation}\}, \\ \{\text{Ssn}, \text{Pnumber}\} \rightarrow \text{Hours}\}$$

Determining X^+



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$\{\text{Ssn}\}^+ = \{\text{Ssn}, \text{Ename}\}$
$\{\text{Pnumber}\}^+ = \{\text{Pnumber}, \text{Pname}, \text{Plocation}\}$
$\{\text{Ssn}, \text{Pnumber}\}^+ = \{\text{Ssn}, \text{Pnumber}, \text{Ename}, \text{Pname}, \text{Plocation}, \text{Hours}\}$

Cover

Definition (Cover)

A set of functional dependencies F is said to cover another set of functional dependencies E if every FD in E is also in F^+ ; that is, if every dependency in E can be inferred from F; alternatively, we can say that E is covered by F.

Equivalence

Definition (Equivalence)

Two sets of functional dependencies E and F are equivalent if $E^+ = F^+$. Therefore, equivalence means that every FD in E can be inferred from F, and every FD in F can be inferred from E; that is, E is equivalent to F if both the conditions — E covers F and F covers E — hold.

Are these FD sets Equivalent?

$F = \{A \to C, AC \to D, E \to AD, E \to H\}$
and $G = \{A \to CD, E \to AH\}$.

Minimal FD

A set of functional dependencies F is minimal if it satisfies the following conditions:

1. Every dependency in F has a single attribute for its right-hand side.
2. We cannot replace any dependency $X \to A$ in F with a dependency $Y \to A$, where Y is a proper subset of X, and still have a set of dependencies that is equivalent to F.
3. We cannot remove any dependency from F and still have a set of dependencies that is equivalent to F.

Minimal Cover

Definition (Minimal Cover)

A minimal cover of a set of functional dependencies E is a minimal set of dependencies (in the standard canonical form and without redundancy) that is equivalent to E. We can always find at least one minimal cover F for any set of dependencies E.

Algorithm: Finding a Minimal Cover F for a Set of Functional Dependencies E

Input: A set of functional dependencies E.

1. Set $F := E$.
2. Replace each functional dependency $X \to \{A_1, A_2, \ldots, A_n\}$ in F by the n functional dependencies $X \to A_1, X \to A_2, \ldots, X \to A_n$.
3. For each functional dependency $X \to A$ in F
   for each attribute B that is an element of X
   if $\{ \{F - \{X \to A\} \} \cup \{ (X - \{B\}) \to A\} \}$ is equivalent to F
   then replace $X \to A$ with $(X - \{B\}) \to A$ in F.
4. For each remaining functional dependency $X \to A$ in F
   if $\{F - \{X \to A\} \}$ is equivalent to F,
   then remove $X \to A$ from F.

Find Minimal Cover of the Given Set of FDs

$$E : \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$$

Algorithm: Finding a Key K for R Given a set F of Functional Dependencies

Input: A relation R and a set of functional dependencies F on the attributes of R.

1. Set $K := R$.
2. For each attribute A in K

   {compute $(K - \text{A})^+$ with respect to F;

   if $(\text{K} - \text{A})^+$ contains all the attributes in R, then set K := K – {A} };