

21.08.2020

# Digital Image Processing (CSE/ECE 478)

## Lecture-3: Recap/Discussion

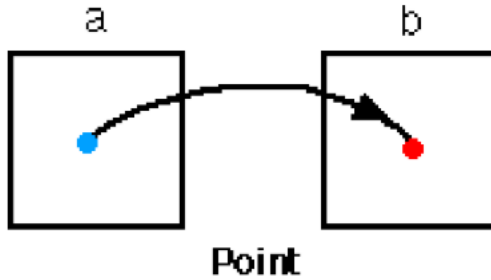
Ravi Kiran

Center for Visual Information Technology (CVIT), IIIT Hyderabad



# Spatial Domain Processing

- ▶ Manipulating Pixels Directly in Spatial Domain
- ▶ 3 approaches
- ▶ 1. Point to Point



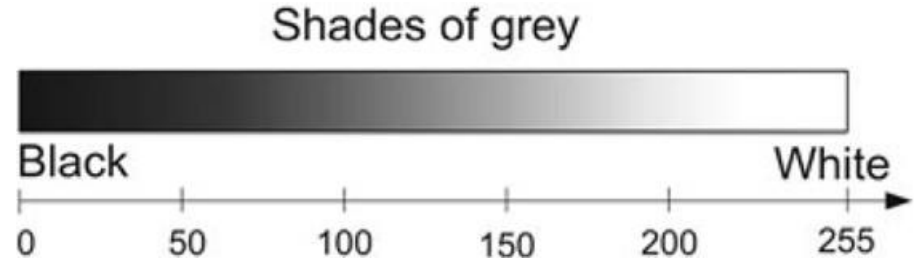
# Linear Intensity Transforms

►  $T(z) = z + K$

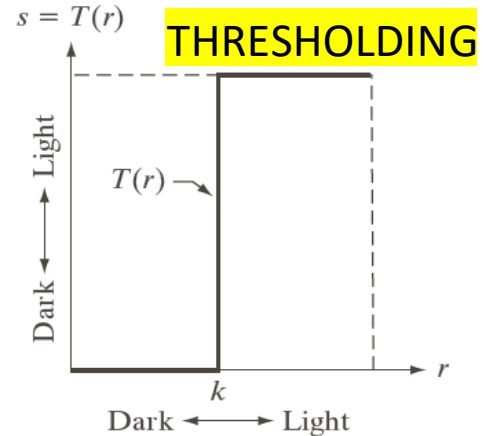
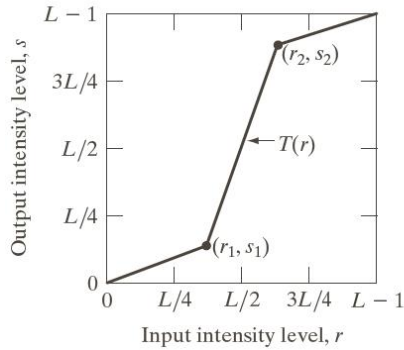
►  $T(z) = z - K$

►  $T(z) = Kz$

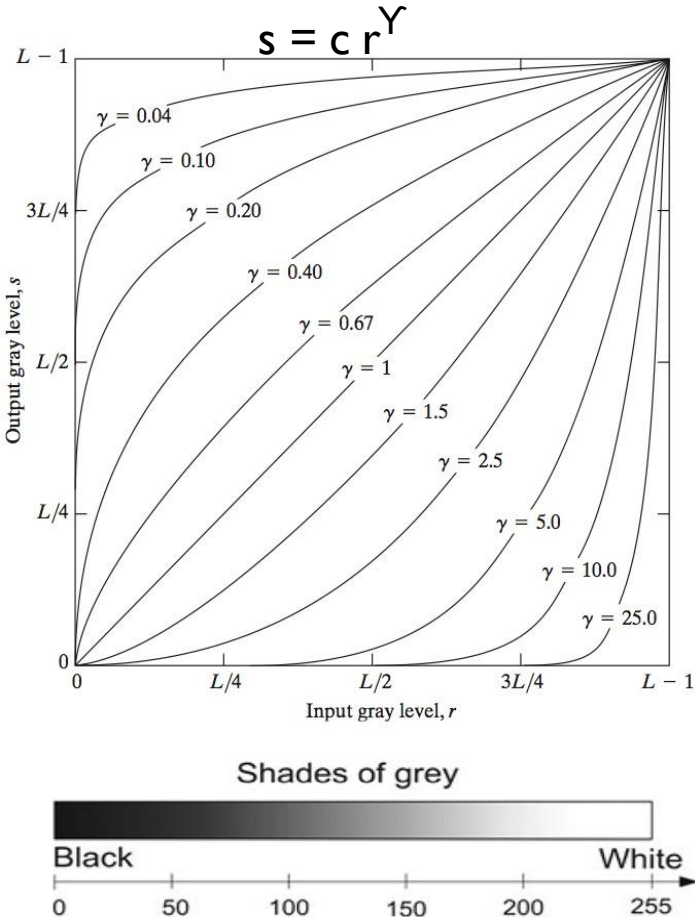
►  $T(z) = K_1z + K_2$



# Piecewise-Linear Transformations



# Power-Law Transformations

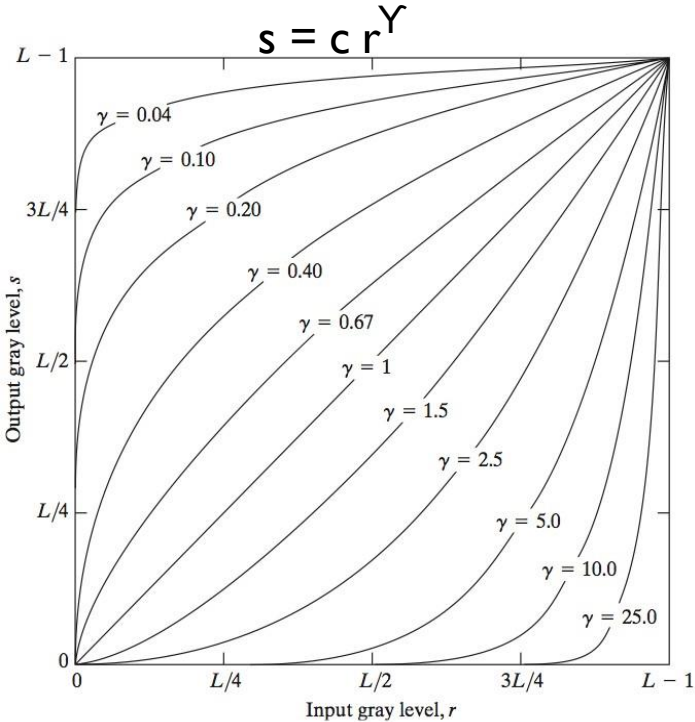


a b  
c d

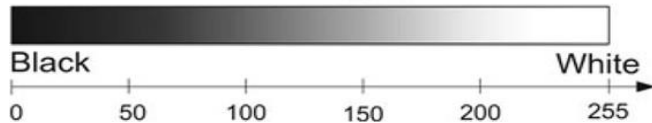
**FIGURE 3.9**  
(a) Aerial image.  
(b)–(d) Results of  
applying the  
transformation in  
Eq. (3.2-3) with  
 $c = 1$  and  
 $\gamma = 3.0, 4.0$ , and  
 $5.0$ , respectively.  
(Original image  
for this example  
courtesy of  
NASA.)



# Power-Law Transformations



Shades of grey



Demo:

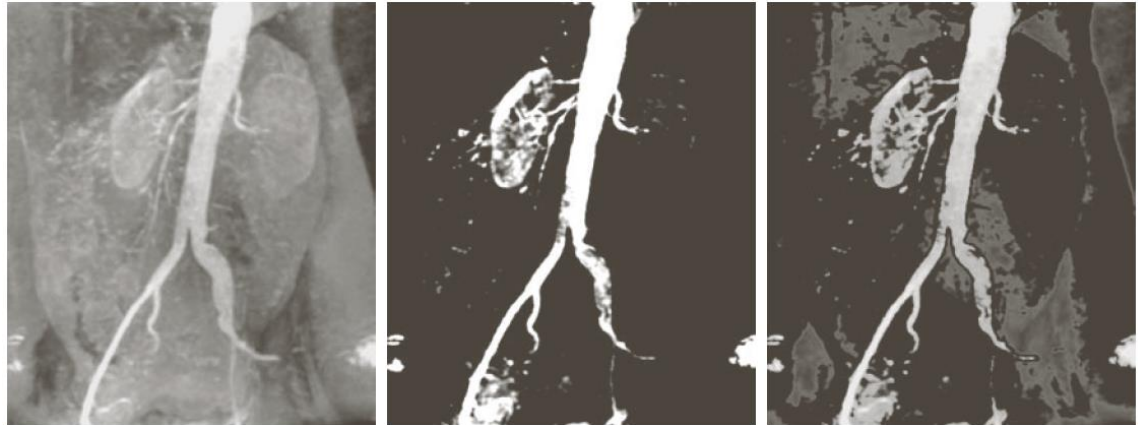
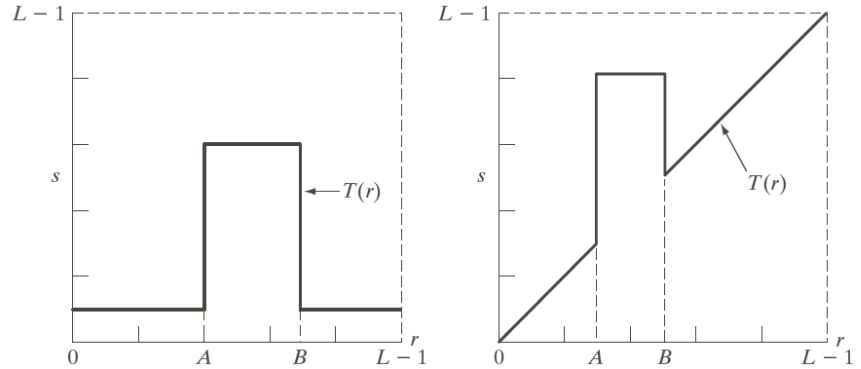
<https://colab.research.google.com/drive/11qL0VKleZnONtPuxAryAf9WkUC7kEMI#scrollTo=aU5WQaqOpSCr&line=12&uniqifier=1>



# Intensity Slicing

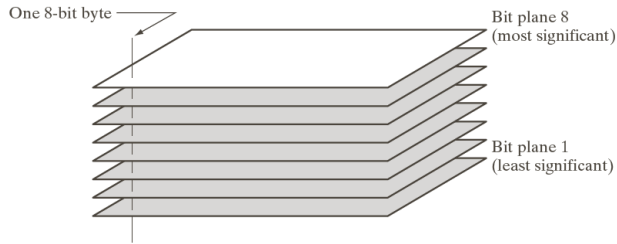
a b

**FIGURE 3.11** (a) This transformation highlights intensity range  $[A, B]$  and reduces all other intensities to a lower level. (b) This transformation highlights range  $[A, B]$  and preserves all other intensity levels.



a b c

# Bit plane slicing



a	b	c
d	e	f
g	h	i

**FIGURE 3.14** (a) An 8-bit gray-scale image of size  $500 \times 1192$  pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.



21.08.2020

# Digital Image Processing (CSE/ECE 478)

## Lecture-4: Histogram Processing

Ravi Kiran

Center for Visual Information Technology (CVIT), IIIT Hyderabad



# Piecewise-Linear Transformations

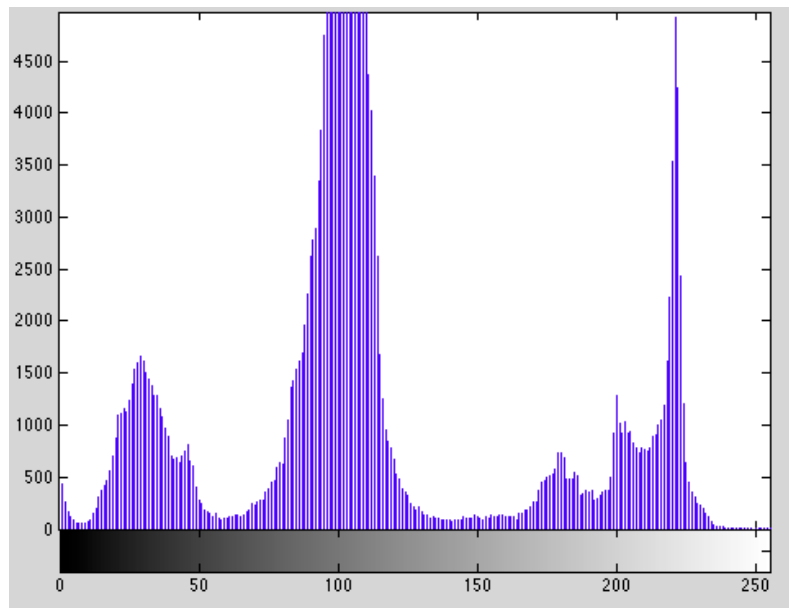
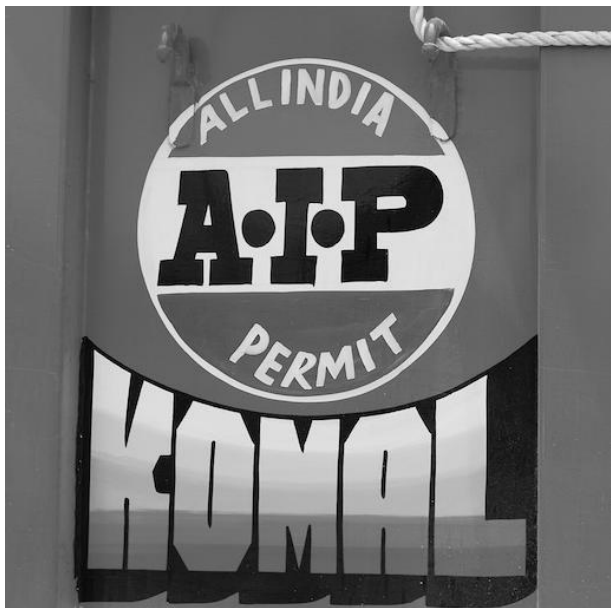


# Histogram: An image representation + visualization

$$h_r(i) = n_i$$

$i \rightarrow$  intensity value, range  $[0, L-1]$

$n_i \rightarrow$  number of pixels with intensity  $i$



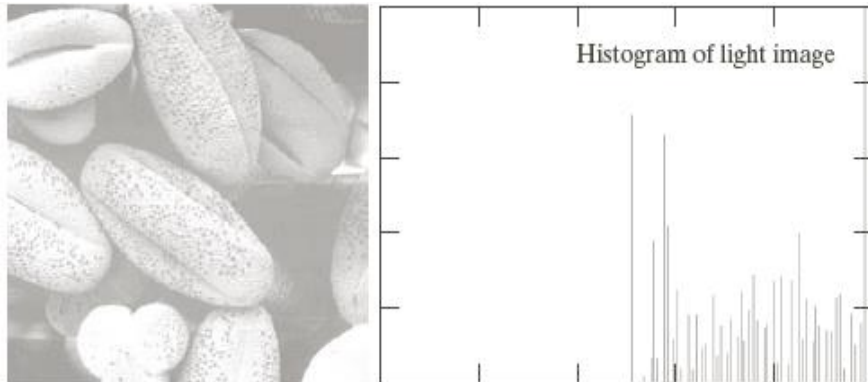
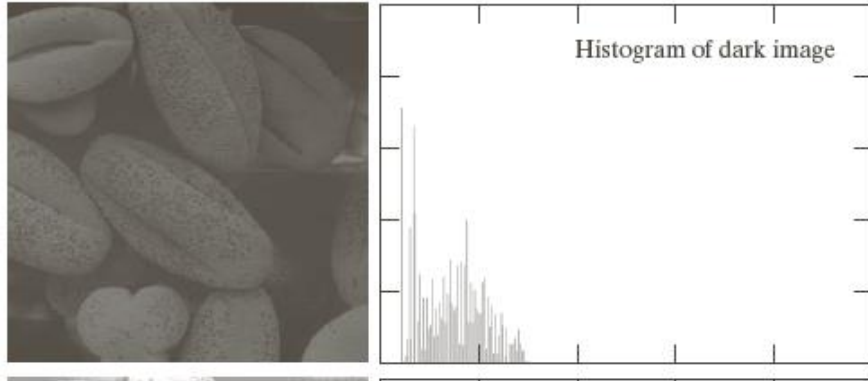
# Histograms

- ▶ What can we infer from histograms?



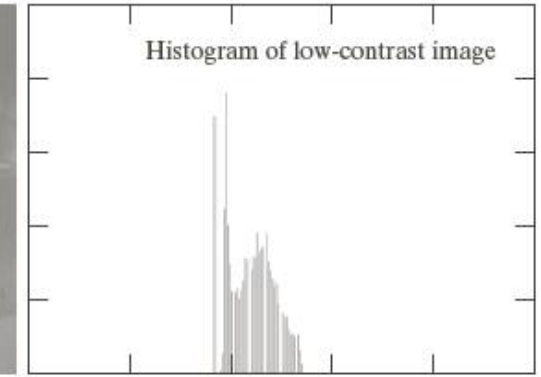
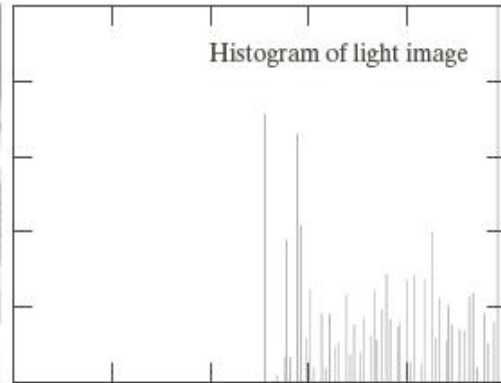
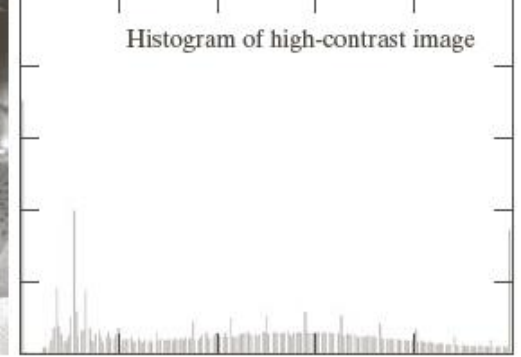
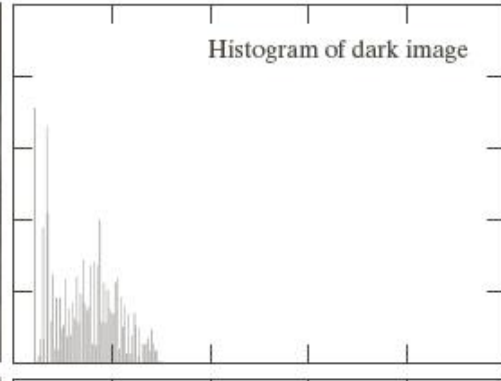
Histogram viewing standard in most DSLR cameras

# Histograms and Contrast



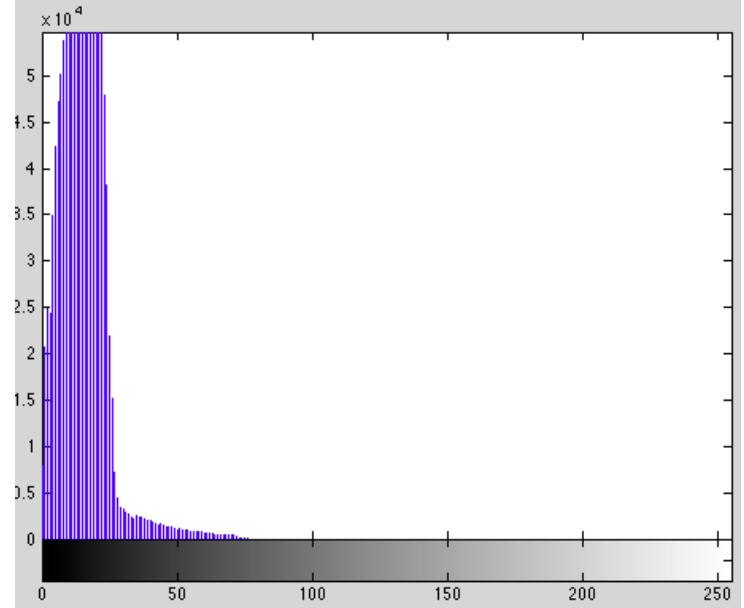
$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

# Histograms and Contrast



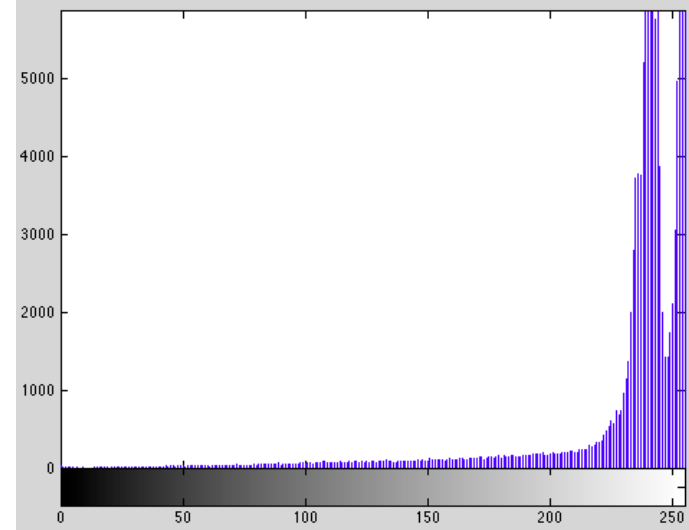


# Histograms



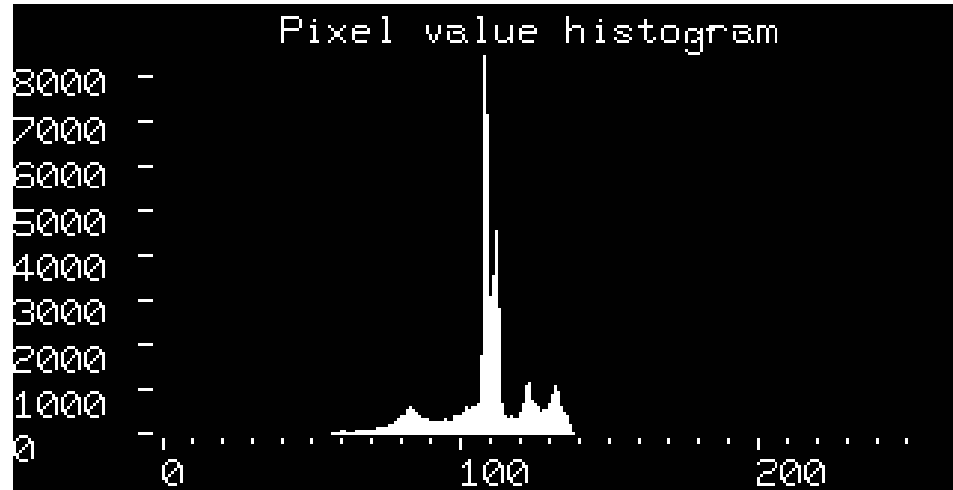
Under exposure

# Histograms



Over exposure

# A low-contrast image and its histogram



# Contrast Stretching



$$f_{\text{ac}}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

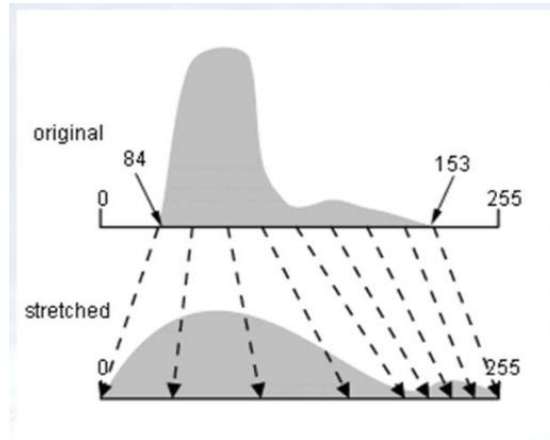
# Contrast Stretching

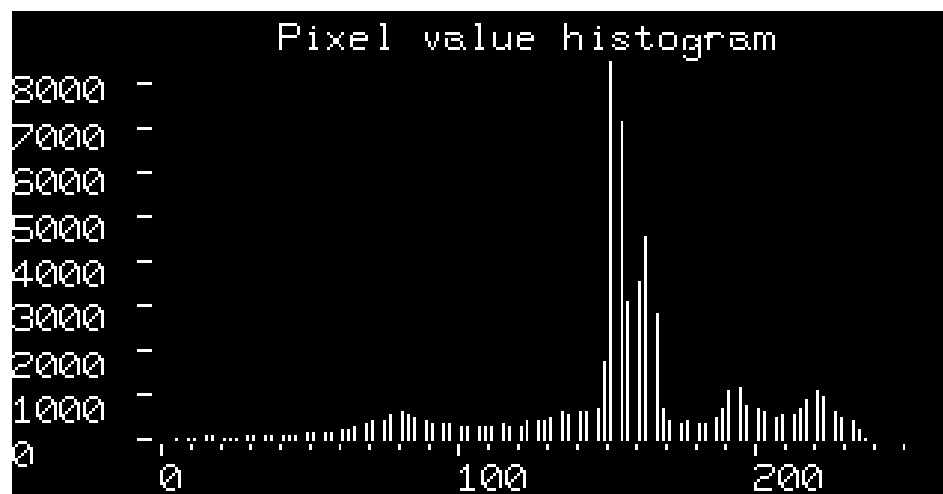
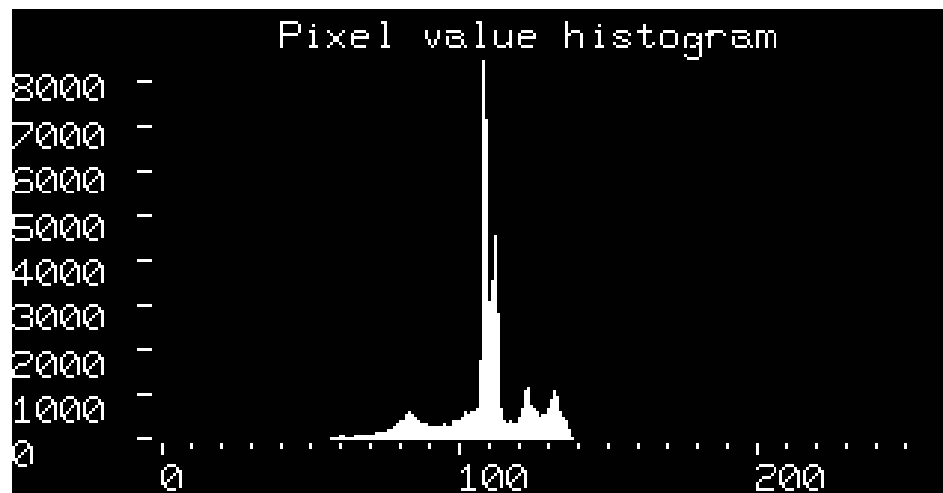


$$f_{\text{ac}}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

If  $a_{\text{min}} = 0$  and  $a_{\text{max}} = 255$

$$f_{\text{ac}}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$







# Contrast Stretching

Suppose we have a single pixel with intensity 255 in the original intensity range.

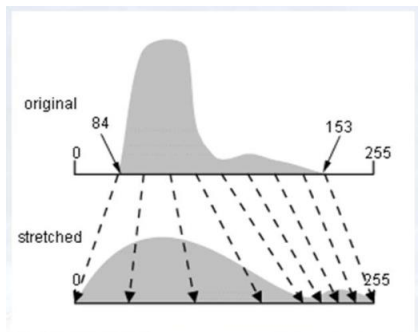
What happens ?



$$f_{ac}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

If  $a_{\text{min}} = 0$  and  $a_{\text{max}} = 255$

$$f_{ac}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$



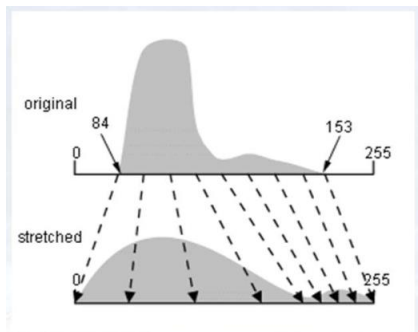
# Contrast Stretching



$$f_{\text{ac}}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

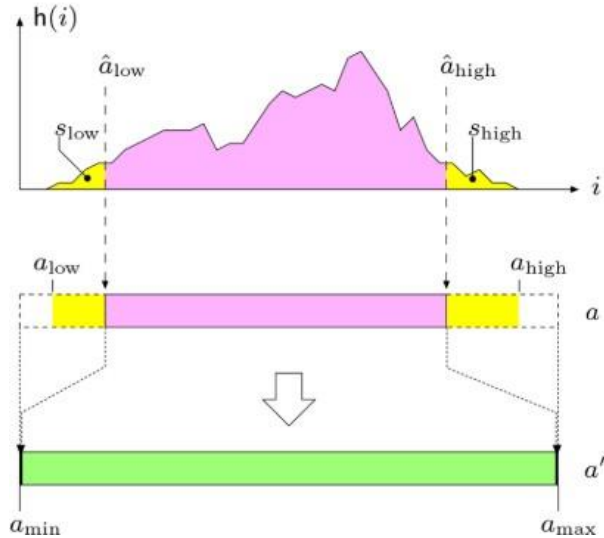
If  $a_{\text{min}} = 0$  and  $a_{\text{max}} = 255$

$$f_{\text{ac}}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$



Suppose we have a single pixel with intensity 0 in the original intensity range.  
What happens ?

# Contrast Stretching ver. 2

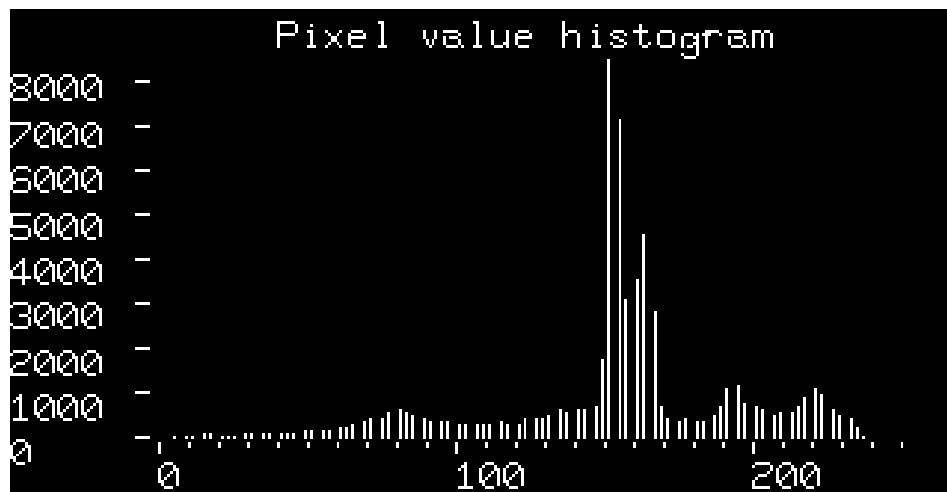
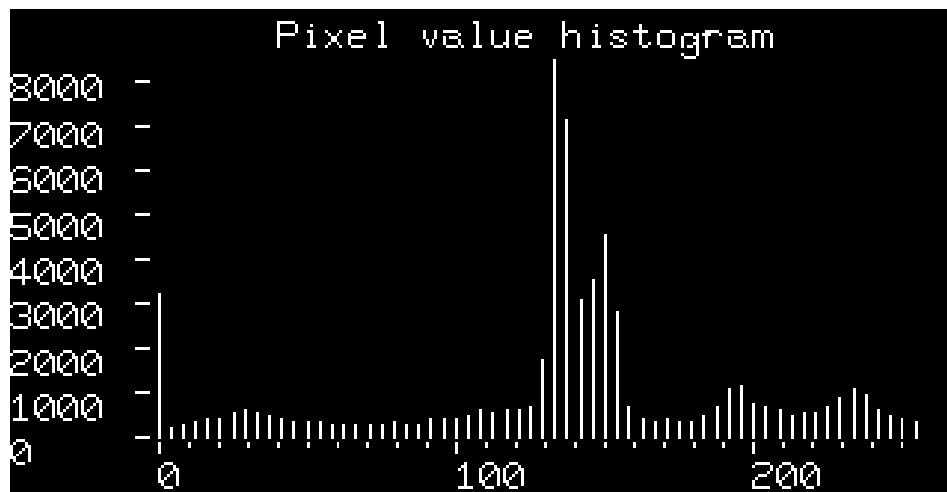


$$\hat{a}_{low} = \min\{i \mid H(i) \geq M \cdot N \cdot s_{low}\}$$

$$\hat{a}_{high} = \max\{i \mid H(i) \leq M \cdot N \cdot (1 - s_{high})\}$$

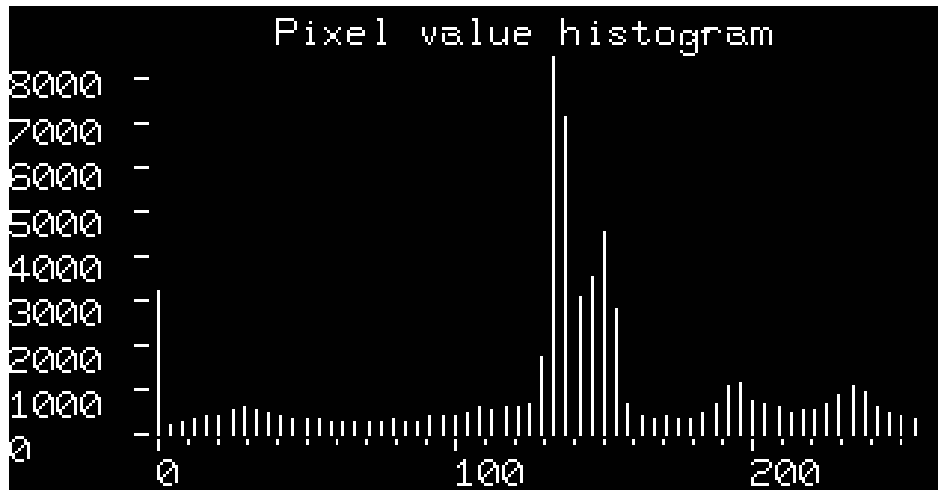
$$f_{mac}(a) = \begin{cases} a_{min} & \text{for } a \leq \hat{a}_{low} \\ a_{min} + (a - \hat{a}_{low}) \cdot \frac{a_{max} - a_{min}}{\hat{a}_{high} - \hat{a}_{low}} & \text{for } \hat{a}_{low} < a < \hat{a}_{high} \\ a_{max} & \text{for } a \geq \hat{a}_{high} \end{cases}$$

Ver. 2



# Are all intensities well represented ?

Ver. 2



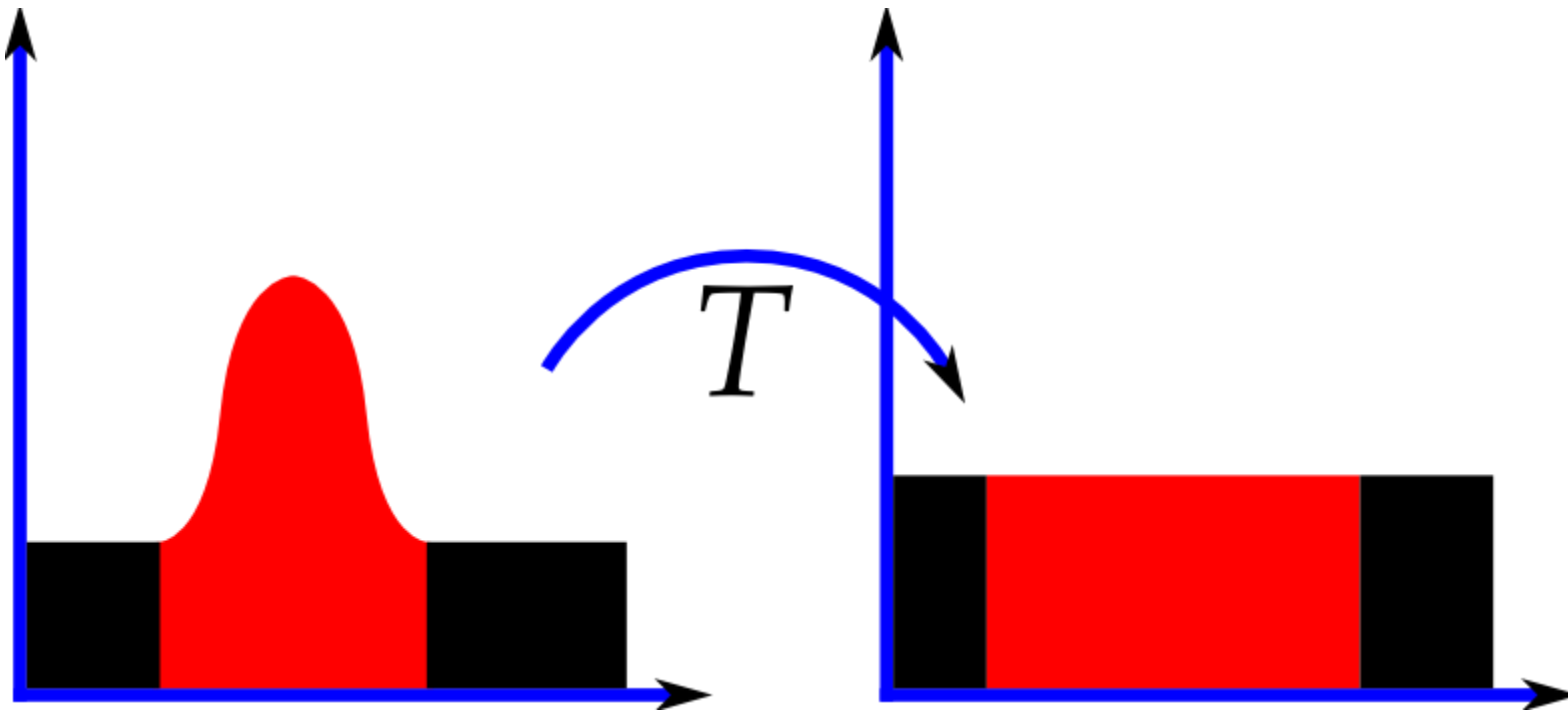


ALL INTENSITIES

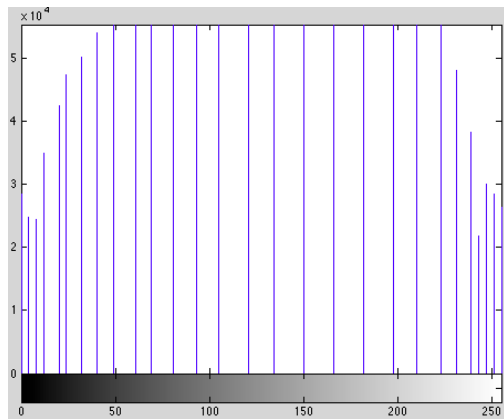
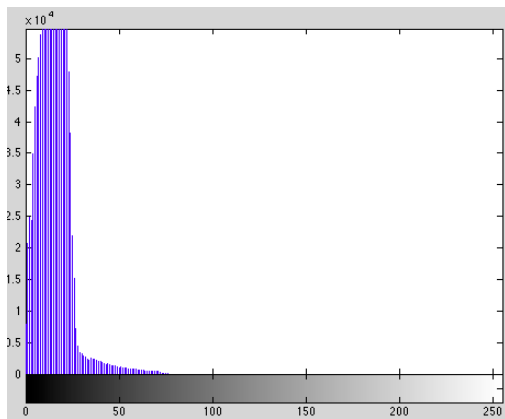
**MATTER**



# Histogram Equalization



# Histogram Equalization



# The issue with contrast stretching



$$f_{ac}(a) = a_{\min} + (a - a_{\text{low}}) \cdot \frac{a_{\max} - a_{\min}}{a_{\text{high}} - a_{\text{low}}}$$

If  $a_{\min} = 0$  and  $a_{\max} = 255$

$$f_{ac}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$

# Histogram Equalization

# Histogram Equalization

# Histogram Equalization



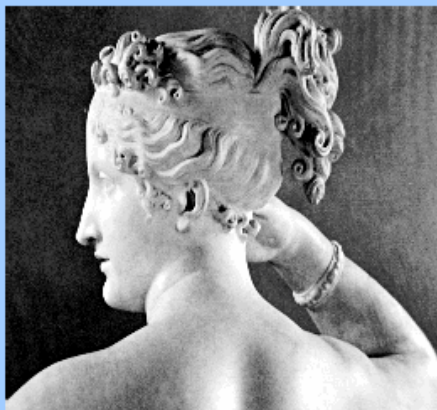
# Histogram Equalization



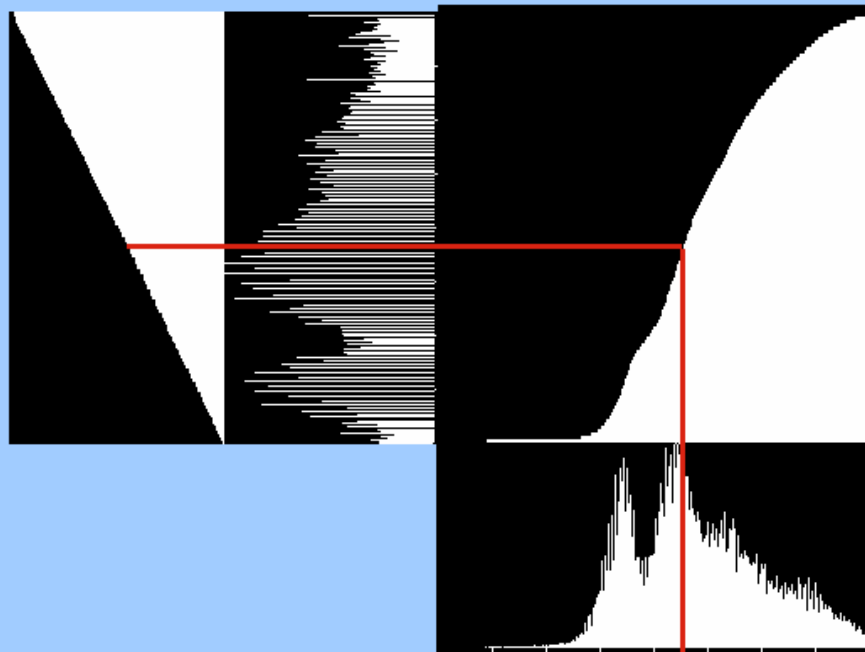
Contrast  
Stretching



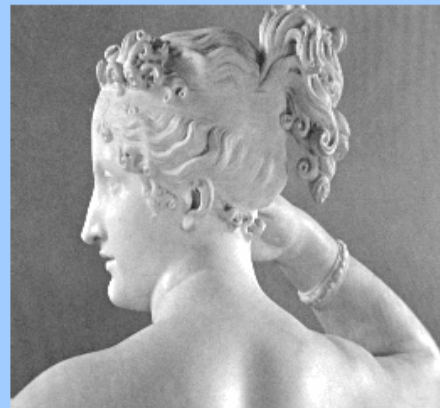
Histogram  
Equalization



Equalized histogram



Histogram of original image

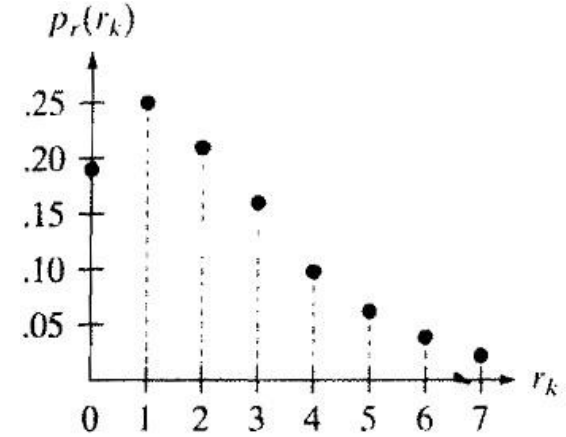


# Histogram Equalization - Example

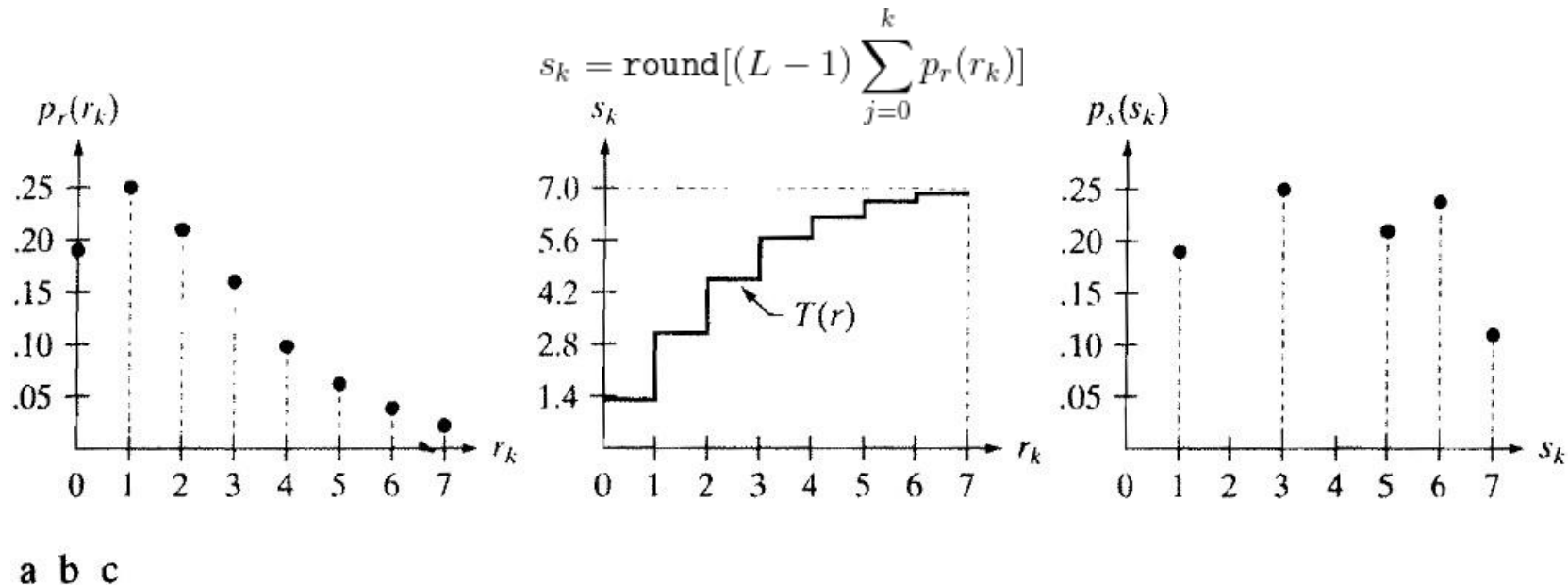
64 x 64 image

3-bits / pixel

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



# Histogram Equalization - Example



**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

# Histogram Equalization

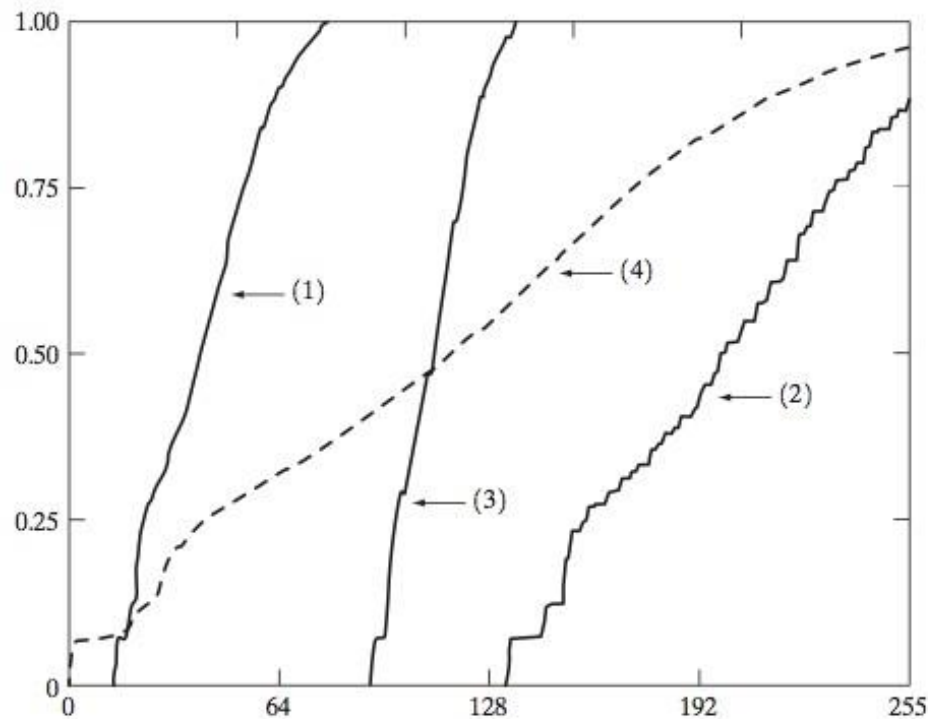
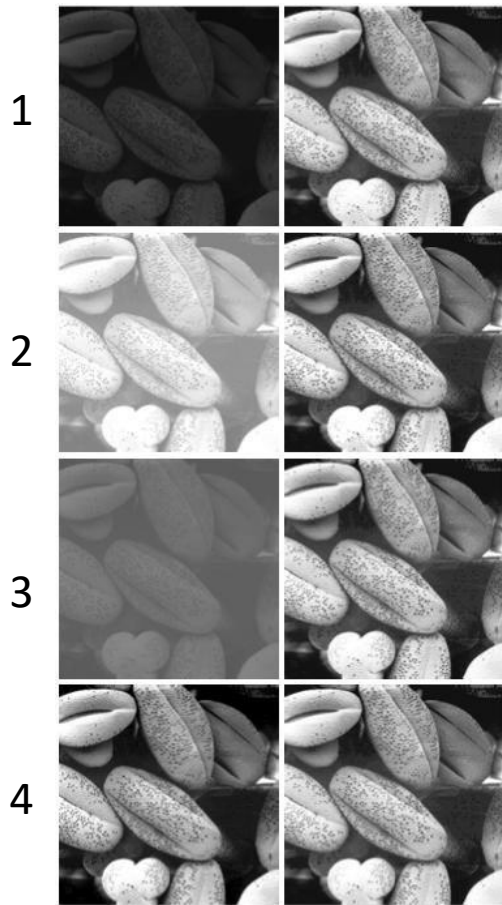


Image Courtesy: Gonzalez and Woods

# Histogram Equalization v/s Contrast Enhancement

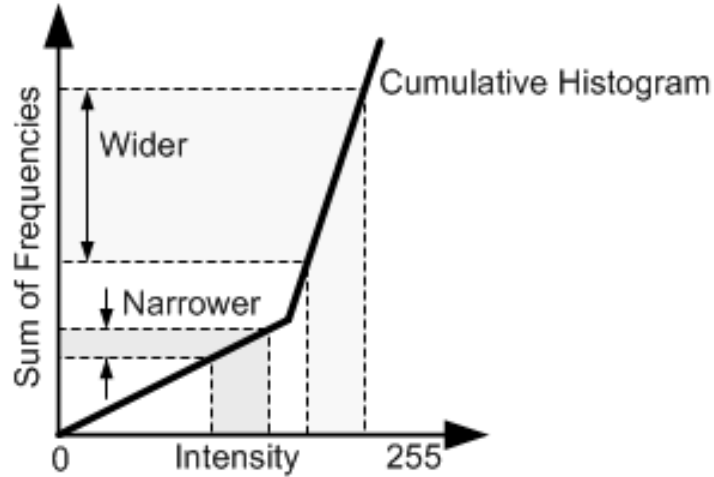


Contrast Enhancement



Histogram equalization

# Histogram Equalization : A Visual Explanation

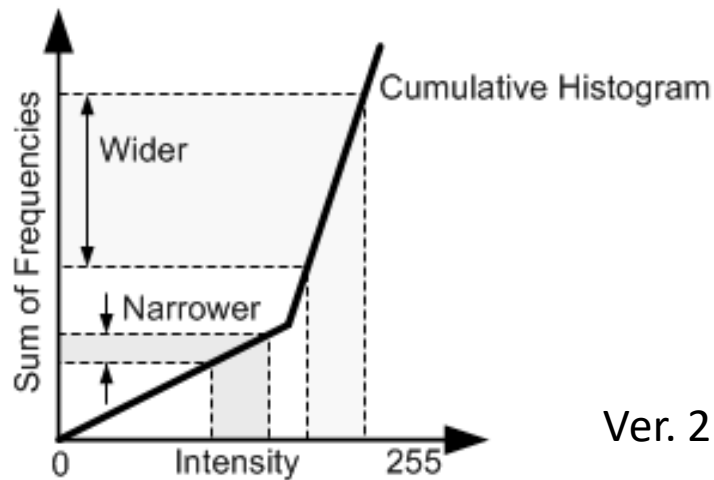


$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$s_k = T(r_k) = \text{round} \left( (L - 1) \sum_{j=0}^{j=k} p_r(r_j) \right)$$

# Histogram Equalization (ver. 2)

$$h[i] = \text{constant}, \quad 0 \leq i \leq L - 1$$



$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$s_k = T(r_k) = \text{round} \left( (L - 1) \underbrace{\sum_{j=0}^{j=k} p_r(r_j)} \right)$$

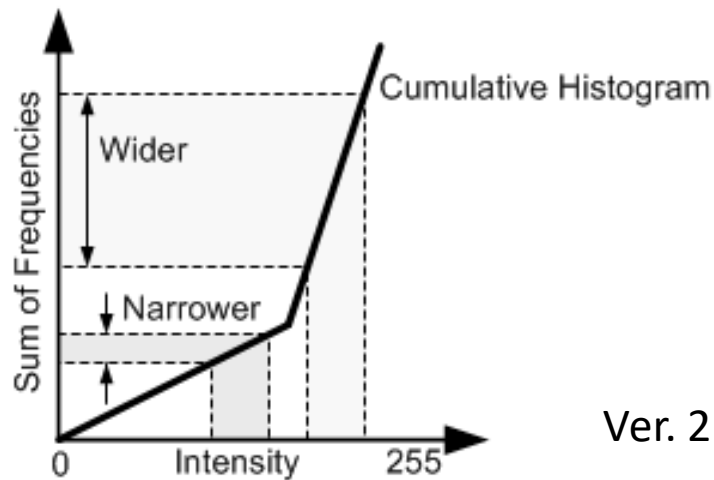
Ver. 2

$$s_k = T(r_k) = \text{round} \left( (L - 1) * \frac{\text{cdf}(r_k) - \text{cdf}_{\min}}{1 - \text{cdf}_{\min}} \right)$$



# Histogram Equalization (ver. 2)

$$h[i] = \text{constant}, \quad 0 \leq i \leq L - 1$$



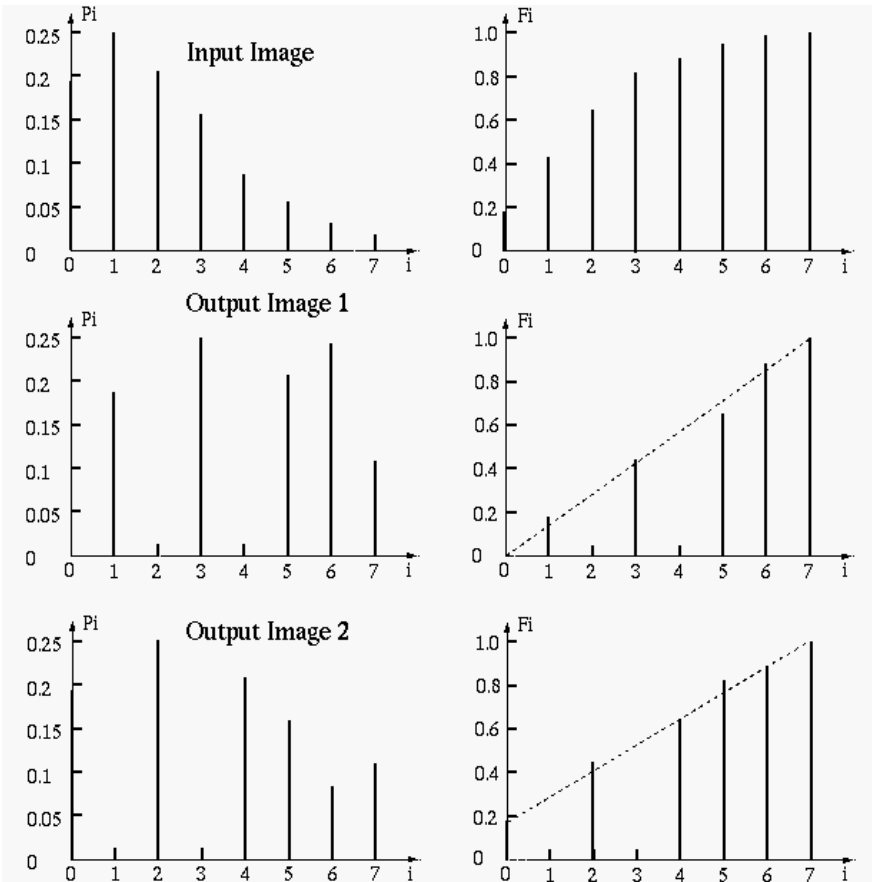
$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$s_k = T(r_k) = \text{round} \left( (L - 1) \underbrace{\sum_{j=0}^{j=k} p_r(r_j)} \right)$$

Ver. 2 
$$s_k = T(r_k) = \text{round} \left( (L - 1) * \frac{cdf(r_k) - cdf_{min}}{1 - cdf_{min}} \right)$$

$$cdf_{min} = p_r(r_a) \text{ where } r_a = \min\{r_t | p_r(r_t) > 0\}; 0 \leq r_t \leq (L - 1)$$

# Histogram Equalization (default v/s ver. 2)



$$s_k = T(r_k) = \text{round} \left( (L-1) \underbrace{\sum_{j=0}^{j=k} p_r(r_j)} \right)$$

**Ver. 2**  $s_k = T(r_k) = \text{round} \left( (L-1) * \frac{\text{cdf}(r_k) - \text{cdf}_{\min}}{1 - \text{cdf}_{\min}} \right)$

$\text{cdf}_{\min} = p_r(r_a)$  where  $r_a = \min\{r_t | p_r(r_t) > 0\}; 0 \leq r_t \leq (L-1)$

# Histogram Equalization

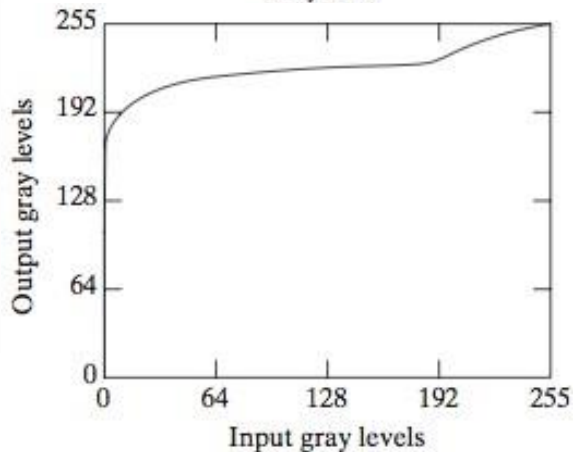
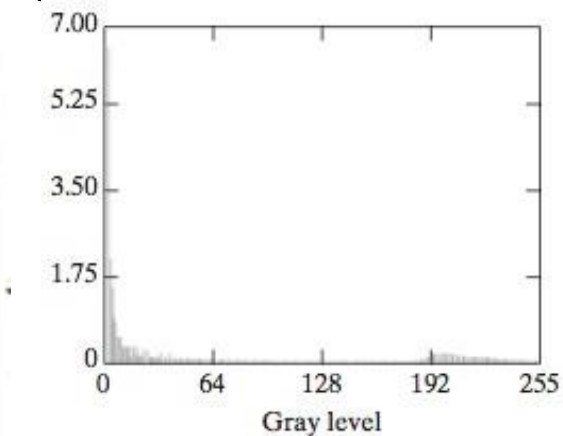
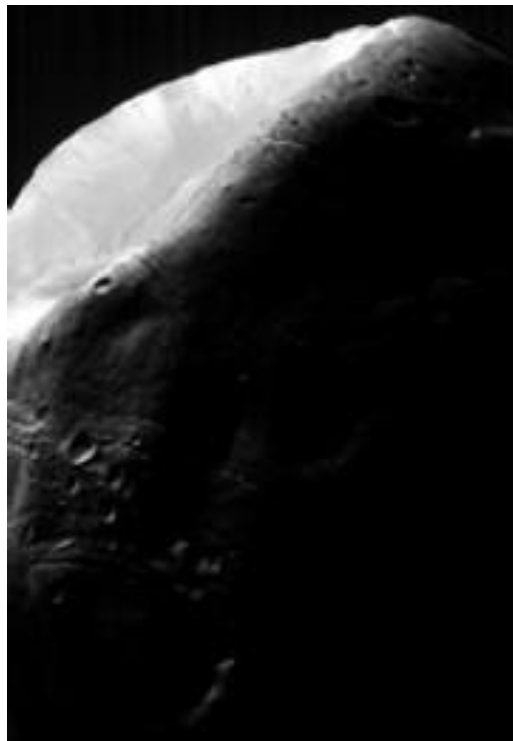
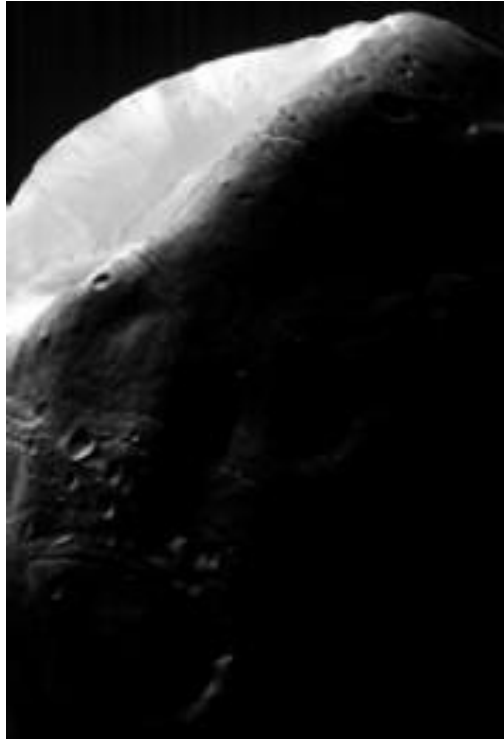
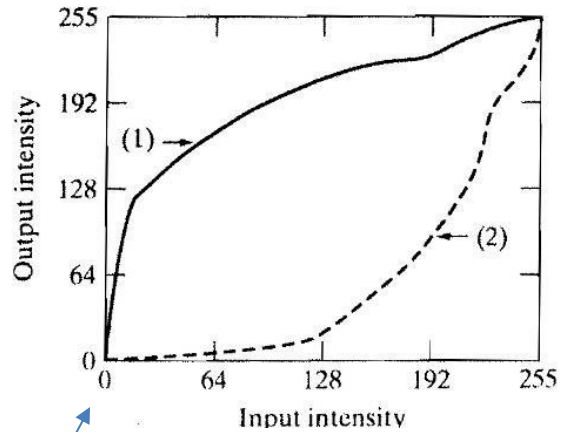


Image Courtesy: Gonzalez and Woods

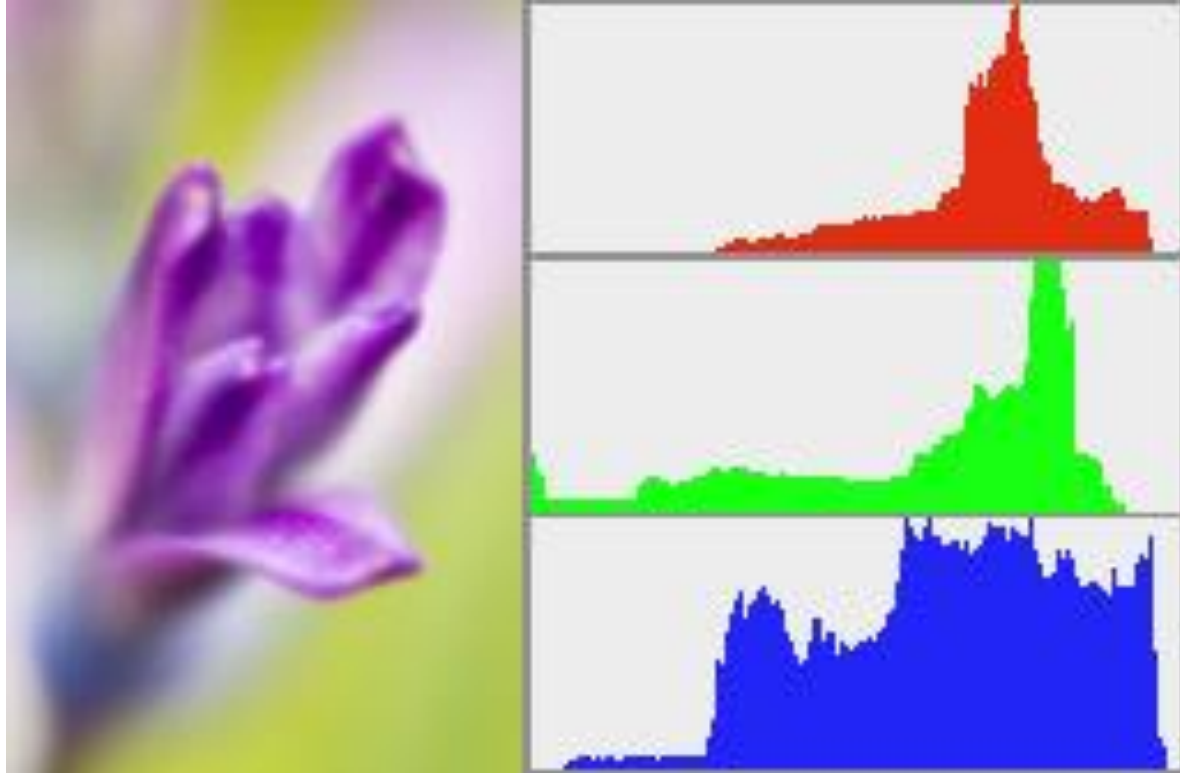
# Histogram specification

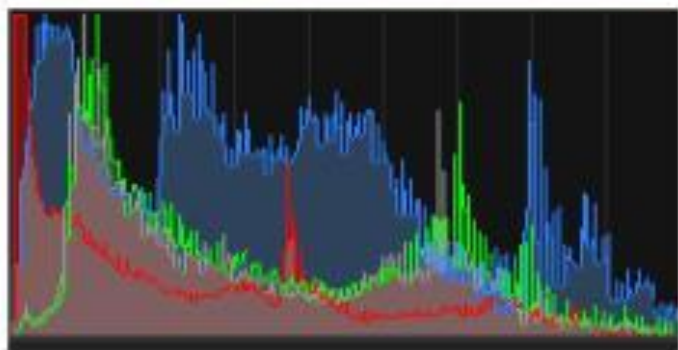
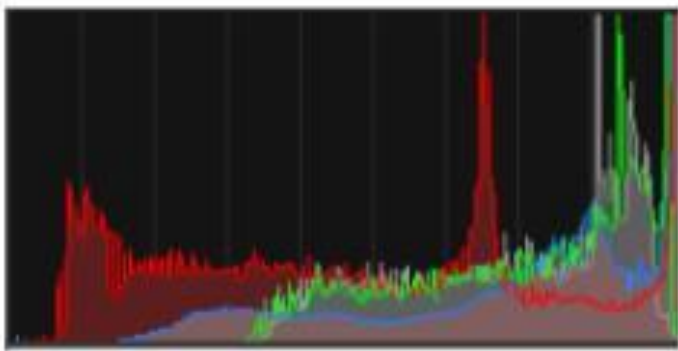
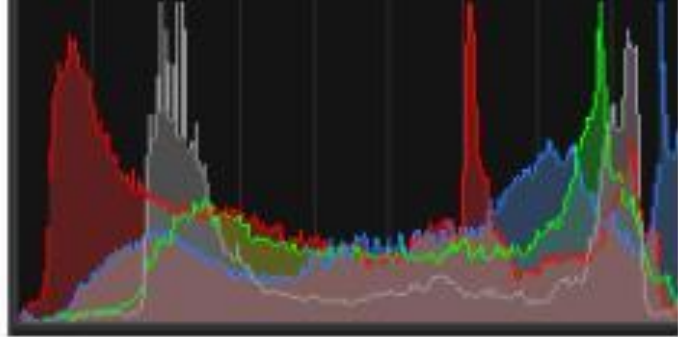
## Histogram Specification / Matching [Section 3.3.2]



Compare with the curves we saw for contrast enhancement. What's the difference ?

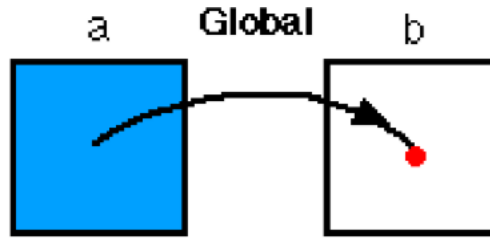
# Histograms for RGB images





# Histogram Processing

- ▶ Global to Point





# Histogram : Discussion

- A visualization
- A useful statistical representation of image intensities
  - Not dependent on image size
- Drawbacks
  - No spatial information
  - Intensity-centric
  - Raw (unnormalized form): Image-size dependent
- Equalization:
  - An image 'normalization' approach
  - Improves global contrast, but can also boost noise

# References

- ▶ GW Chapter – 3.3.1 to 3.3.3
- Transformations of Random Variables
  - <http://www.randomservices.org/random/dist/Transformations.html>
  - Section 1 of <http://www.cs.cmu.edu/~minx/transform.pdf>
  - Leibnitz Integration Rule :  
[https://en.wikipedia.org/wiki/Leibniz\\_integral\\_rule#Alternative\\_derivation](https://en.wikipedia.org/wiki/Leibniz_integral_rule#Alternative_derivation)
  - [Univariate transformation of a random variable](#)

# Scribe Group

20171172
20171205
20171208
2018101002
2018101003
2018101005

# Mini Quiz 1 Link

<https://forms.office.com/Pages/ResponsePage.aspx?id=vDsaA3zPK06W7IZ1VVQKHNFN1LYrWjxAktM68Sb-hiFUOEdKVEIEOU8xTjNZTjNCUDFRTjhHQ09BNC4u>