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Digital Image Processing (CSE/ECE 478)

Lecture-19b: Image Restoration

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Many slides borrowed from Vineet Gandhi @CVIT!

Mathematical Model of Image Degradation/Restoration

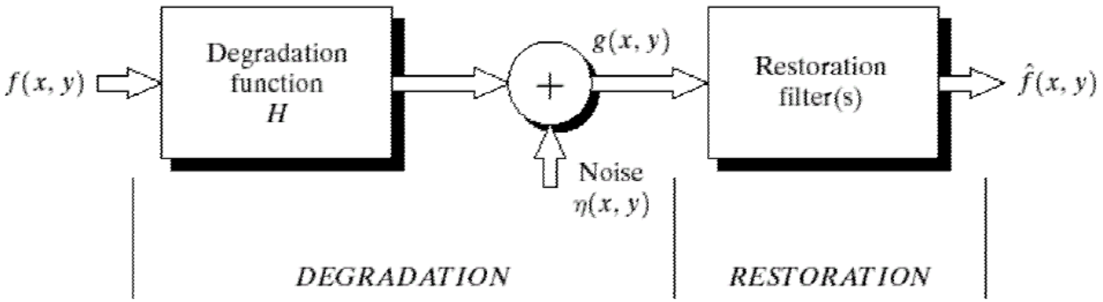
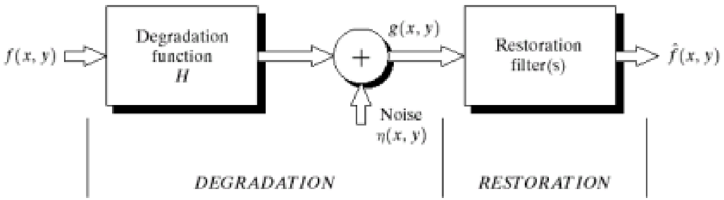


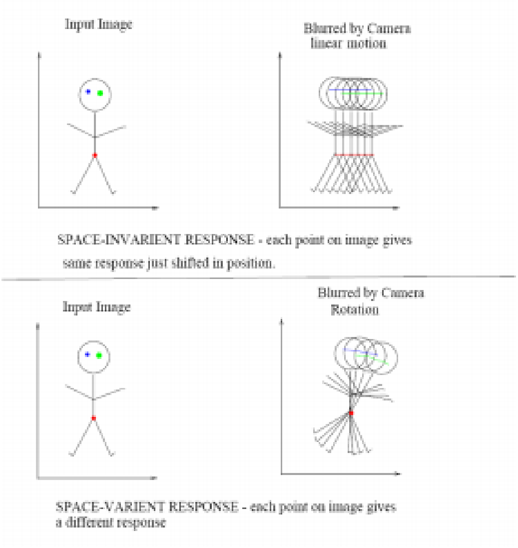
FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$



usual assumptions for the distortion model

- Noise
  - Independent of spatial location
    - Exception: periodic noise …
  - Uncorrelated with image
- Degradation function H
  - Linear
  - Position-invariant



Mathematical Model of Image Degradation/Restoration

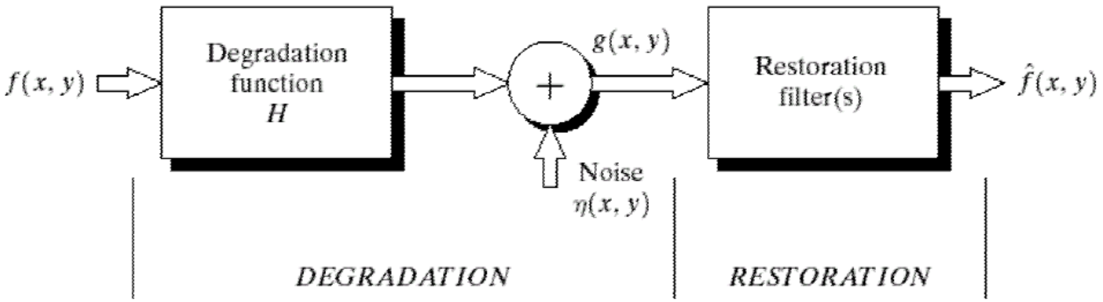


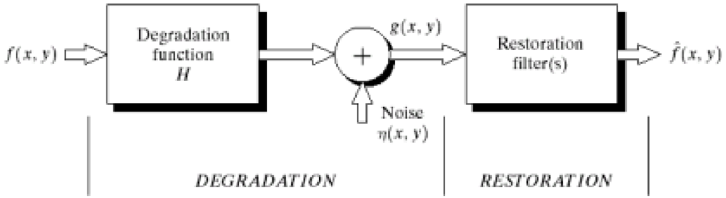
FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = \underbrace{h(x, y) \star f(x, y)} + \underbrace{\eta(x, y)}$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Noise based Degradation

- Assuming H is identity, model reduces to:



$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

Noise Models

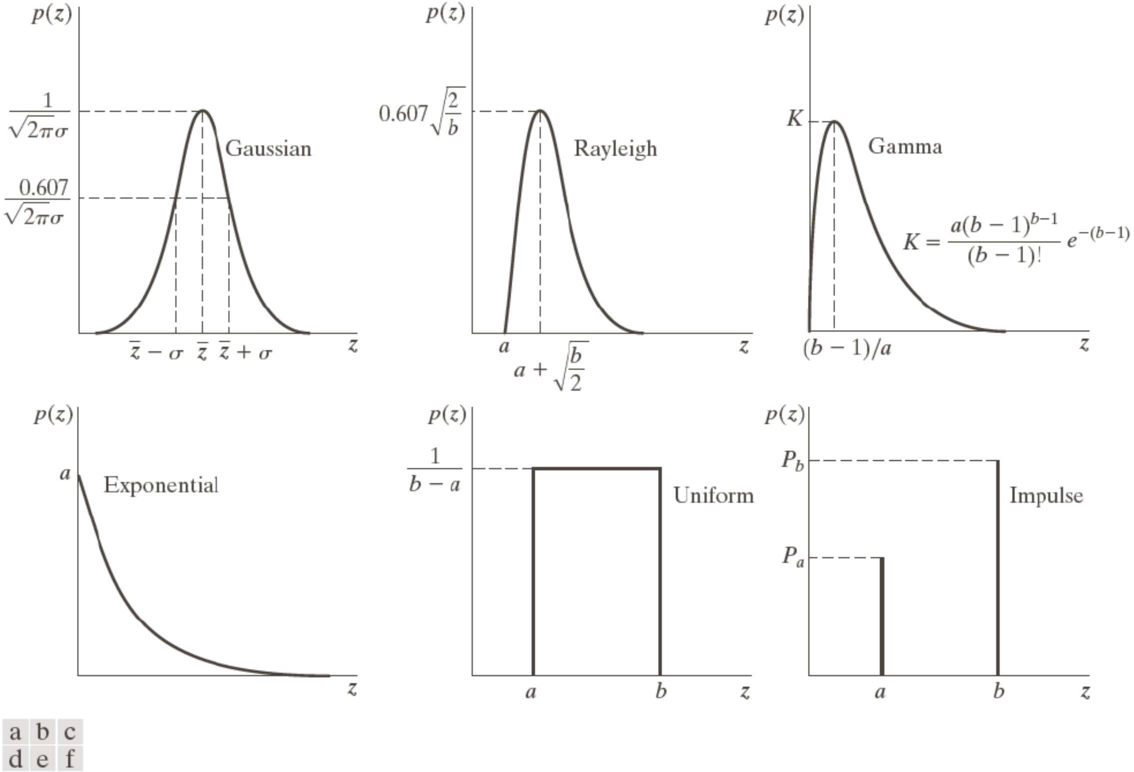


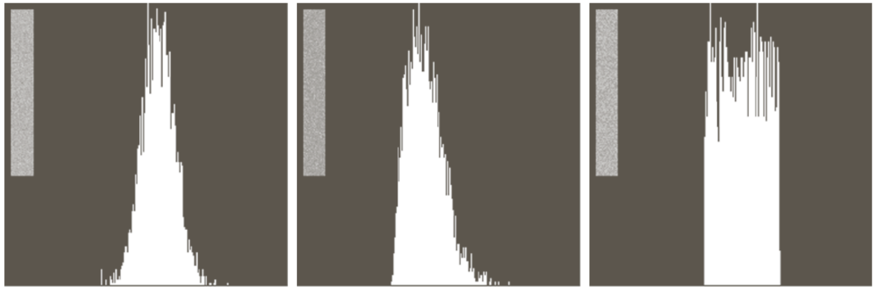
FIGURE 5.2 Some important probability density functions.

How to study system noise

- Imaging system available
  - Noise Calibration: Capture a set of ‘flat environments’ (e.g. solid gray board, object at fixed location)
  - Select the model with better statistical test scores (Akaike Information Criteria (AIC) or Likelihood Ratio Test (LRT))
-

How to study system noise

- Only images available
  - Estimate from patches of constant intensity
  - For impulse noise
    - Use a mid-gray patch/area

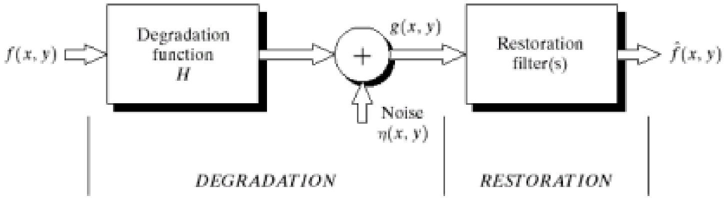


a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in presence of noise only

- Assuming H is identity, model reduces to:



$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

Restoration (in presence of noise only)

- mean filters

- Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Restoration (in presence of noise only)

- mean filters
  - Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise or Gaussian noise, but fails for pepper noise

  - Contraharmonic mean filter

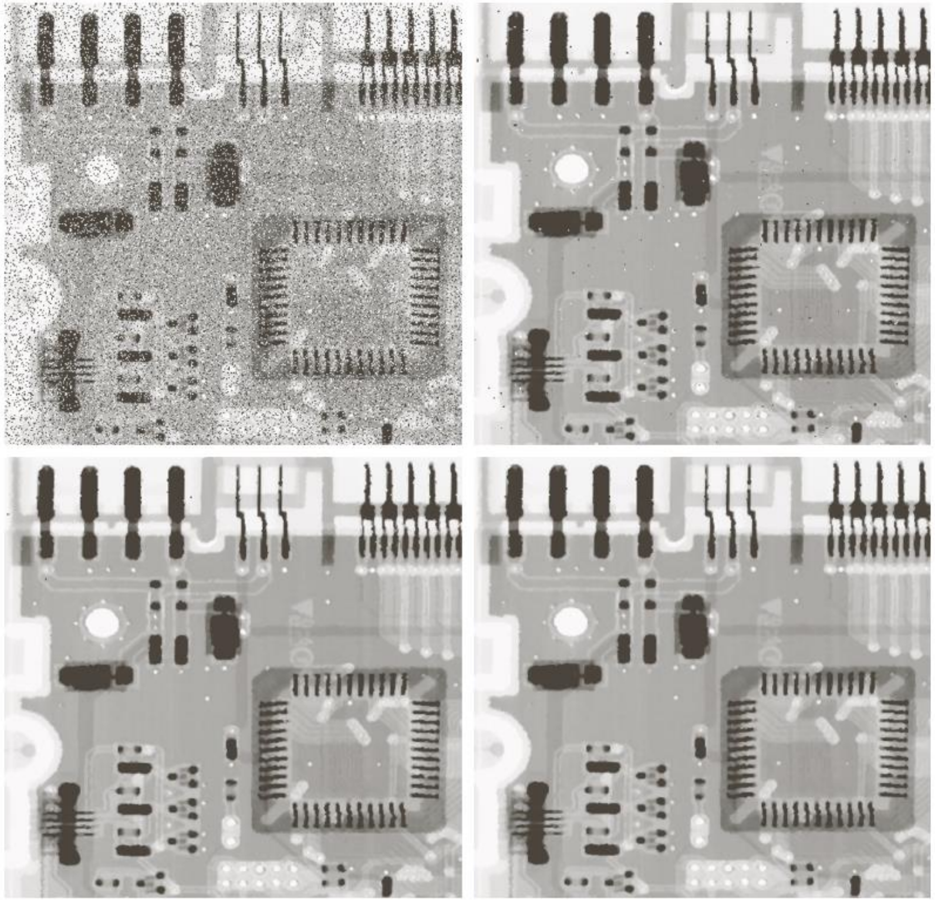
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q

Q = *order* of the filter
Good for salt-and-pepper noise.
Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$
NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

Restoration (in presence of noise only)

- Median filter



a b
c d

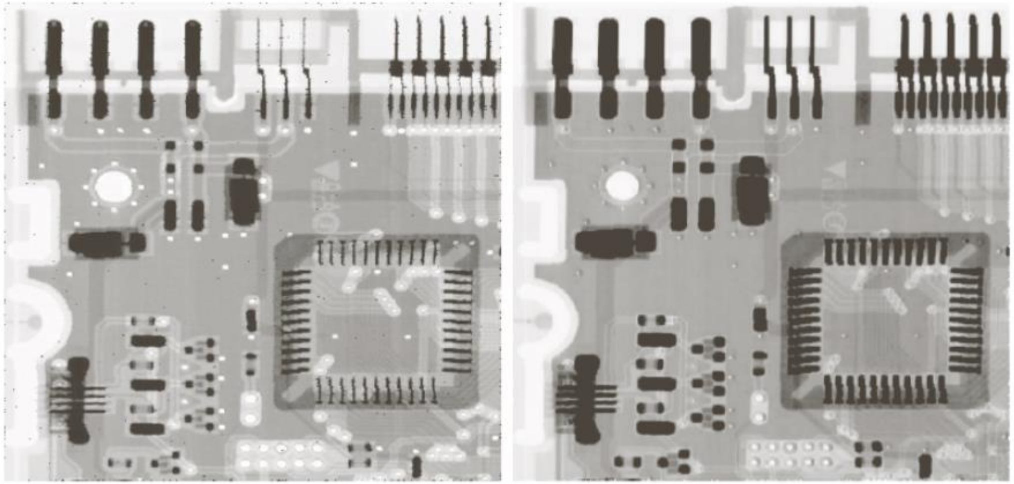
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Restoration (in presence of noise only)

- Max, Min filters

a b

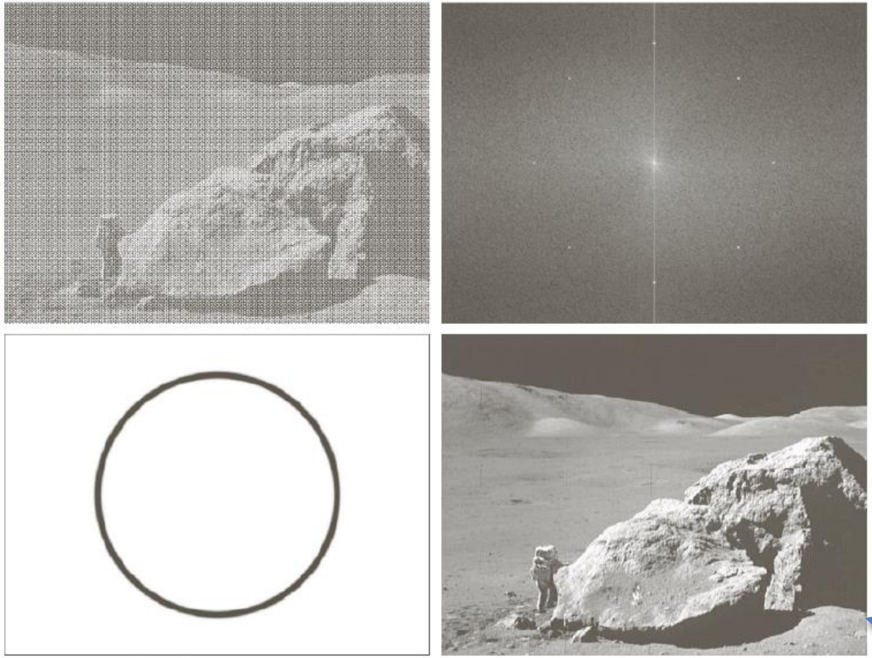
FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter Min filter

Restoration (in presence of noise only) – periodic noise

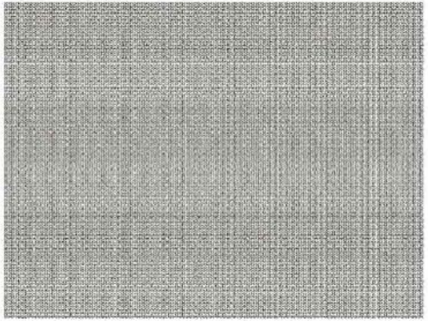
- Band pass/reject



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



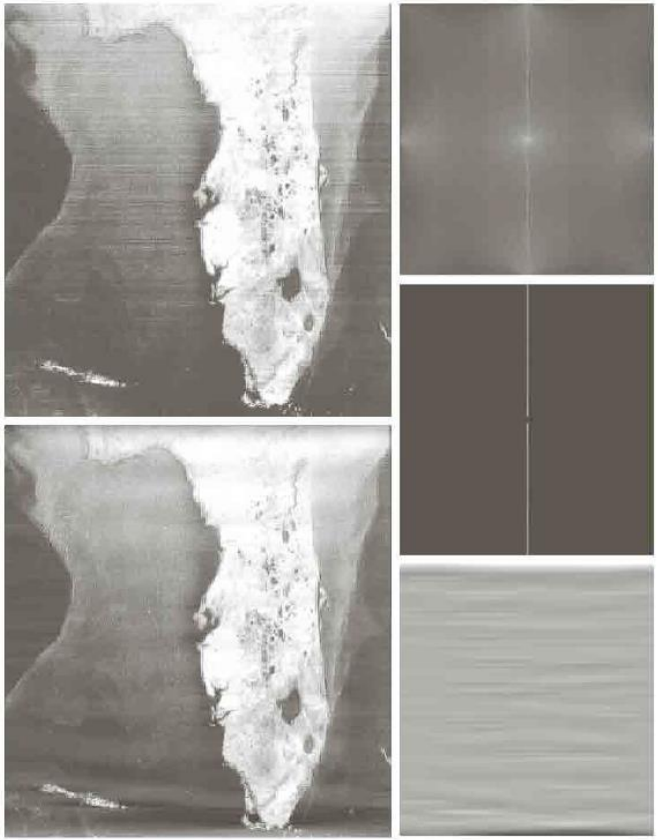
Very difficult to get result of this quality via spatial domain filtering using small convolutional masks

Restoration (in presence of noise only) - periodic noise

- Notch pass/reject

spectrum

Degraded image



Filtered image

Notch pass filter

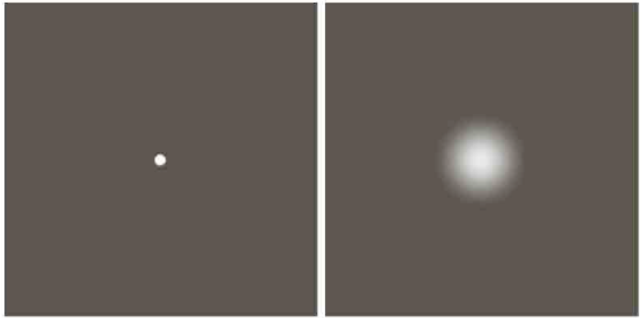
Spatial noise pattern

a b c e d

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Estimation of degradation function

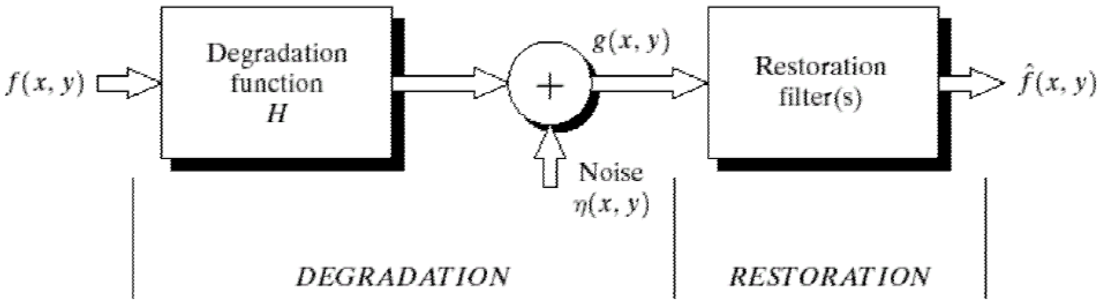
- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration ($H(u,v) = G(u,v)/A$; A = strength of impulse input)
  - Mathematical modelling



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Mathematical Model of Image Degradation/Restoration



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$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Estimation by Modeling (uniform motion blurring)

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

$$G(u,v) = F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

Estimation by Modeling (uniform motion blurring)

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

putting, $x_0(t) = at/T$ and $y_0(t) = bt/T$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\, e^{-j\pi(ua+vb)}$$

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Estimation by Modeling (atmospheric turbulence)

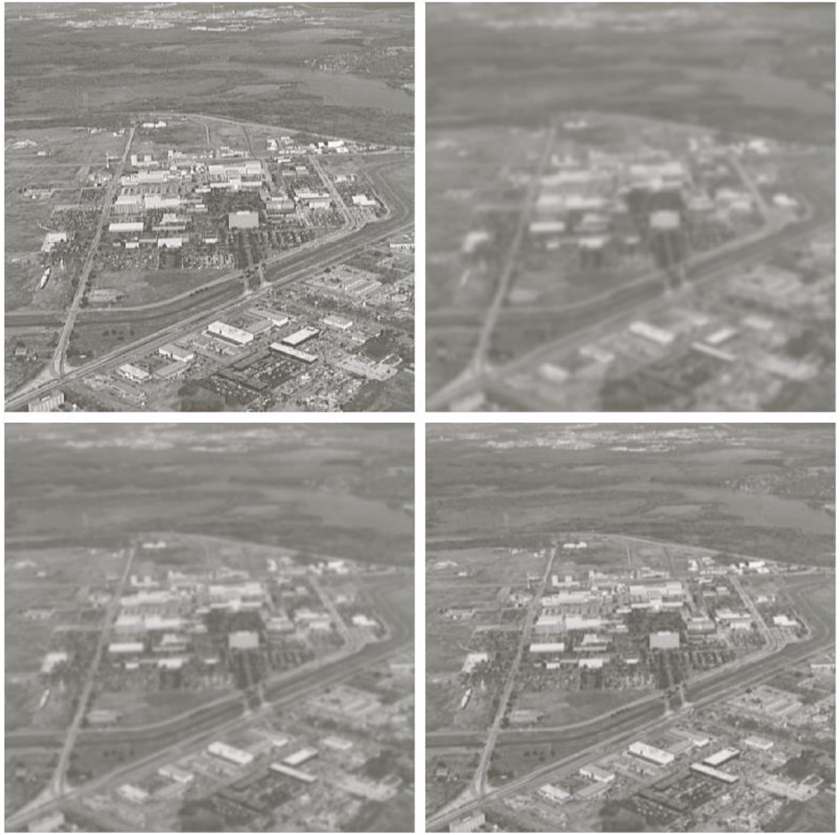


FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$\underline{H(u,v)} = e^{-k(u^2+v^2)^{5/6}}$$

Recovering image : Inverse Filtering

$G = HF + N$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Recovering image (in presence of both Noise and degradation)

- Even if we know the degradation function we cannot recover the un-degraded image!!

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \quad \Rightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Two problems:

1. N(u,v) is a random function whose fourier transform is not known
2. If degradation has zero or small values → N(u,v)/H(u,v) will dominate

Weiner filter

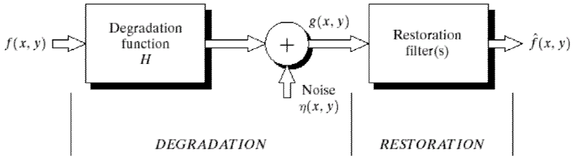


FIGURE 5.1 A model of the image degradation/restoration process.

$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function e is given by: $H^*(u,v)$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image

Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Inverse v/s Weiner filter

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

$\sigma^2 = 500$

blur + add. noise | Inverse filtering | Wiener filtering

Reduced noise variance

Reduced noise variance

a b c
d e f
g h i

Image Restoration

$$f \longrightarrow \boxed{H} \longrightarrow g$$

$H(u,v)$
$G(u,v)$

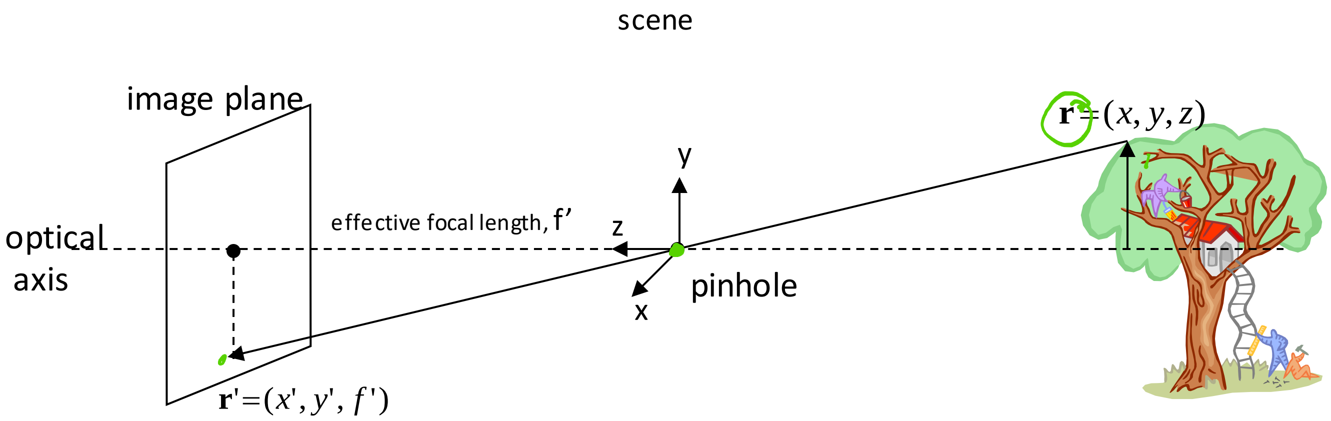
$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Inverse problems

Known / Given	Problem type
H,g	Recovery
g	Blind recovery
g, H partially	Semi blind recovery
f,g	System identification

Geometric Distortion

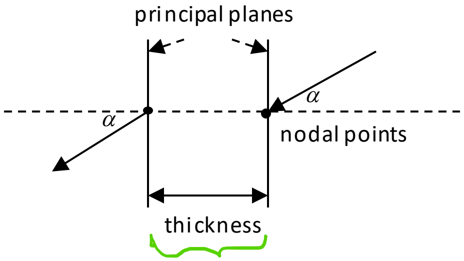
Pinhole and the Perspective Projection



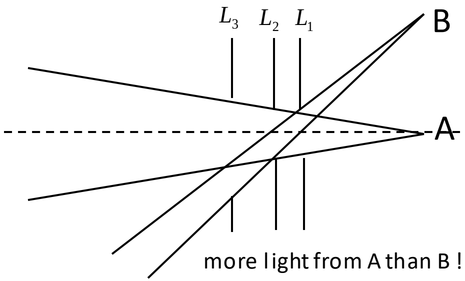
$$\frac{\mathbf{r}'}{f'}=\frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'}=\frac{x}{z} \qquad \frac{y'}{f'}=\frac{y}{z}$$

Common Lens Related Issues - Summary

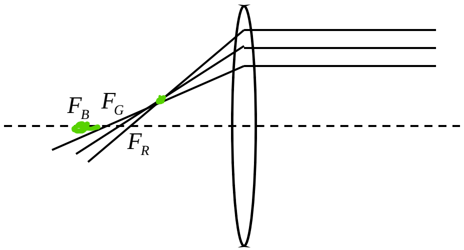
Compound (Thick) Lens



Vignetting



Chromatic Abberation



Lens has different refractive indices for different wavelengths.

Radial and Tangential Distortion

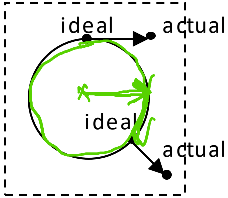


image plane

Lens Glare

- Stray interreflections of light within the optical lens system.
- Happens when very bright sources are present in the scene.

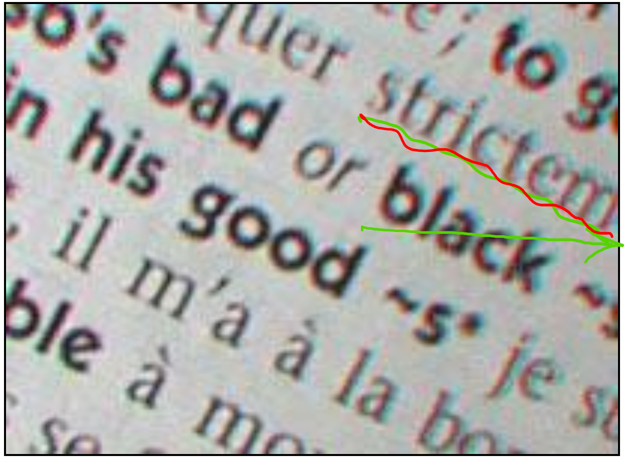
Reading: http://www.dpreview.com

Vignetting

photo by Robert Johnes

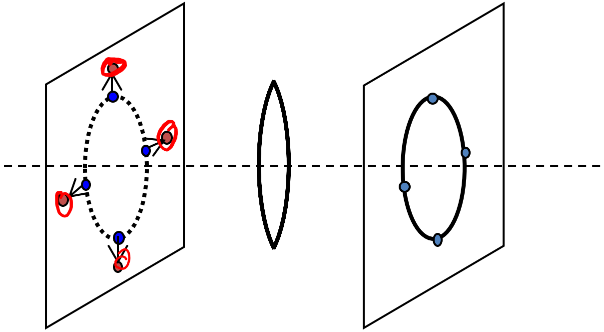
Chromatic Aberrations

longitudinal chromatic aberration
(axial)

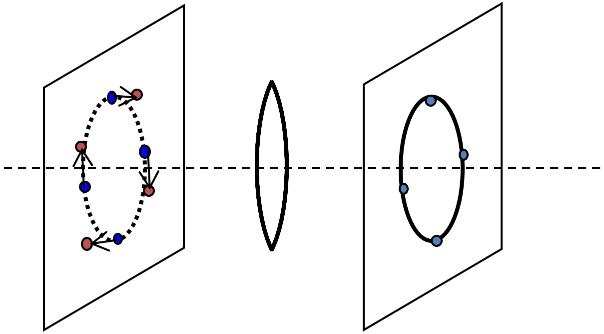


transverse chromatic aberration
(lateral)

Geometric Lens Distortions



Radial distortion



Tangential distortion

Photo by Helmut Dersch

Both due to lens imperfection

Rectify with geometric camera calibration

who said distortion is a bad thing?

blur ...

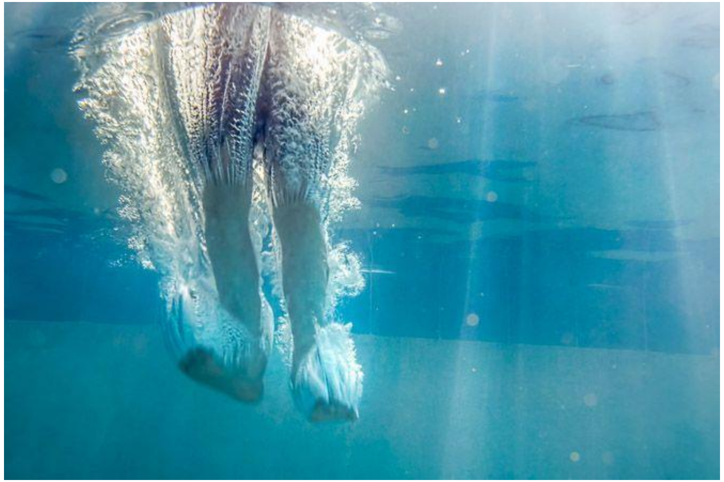
noise ...

geometric ...









References

- http://www.robots.ox.ac.uk/~az/lectures/ia/lect3.pdf
- https://www.ece.iastate.edu/~namrata/EE528_Spring07/ImageRestoration1.pdf
- http://www.ee.columbia.edu/~xlx/ee4830/notes/lec7.pdf
- DIP Ch. 5 (G&W)