20.10.2020

Digital Image Processing (CSE/ECE 478)

Lecture-19: Image Restoration

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Many slides borrowed from Vineet Gandhi @CVIT!

degraded images

ideal image

Blurred image

- What caused the image to blur?
  - Camera: translation, shake, out-of-focus …
  - Environment: scattered and reflected light
  - Device noise: CCD/CMOS sensor and circuitry
  - Quantization noise

degraded images

ideal image

Blurred image

- What caused the image to blur?
  - Camera: translation, shake, out-of-focus ...
  - Environment: scattered and reflected light
  - Device noise: CCD/CMOS sensor and circuitry
  - Quantization noise
- Can we improve the image, or "undo" the effects?

Degradations

• original

• optical blur

• motion blur

• spatial quantization (discrete pixels)

• additive intensity noise

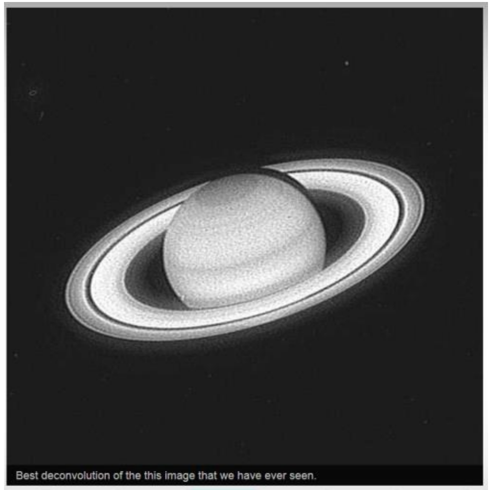
Examples (Optical Blur)

Photo by Rachel Been

Lens Blur selfie, background focus | Lens Blur selfie, foreground focus

Interesting read: Light Field Cameras

Examples



Courtesy: NASA

Examples (Restoration from camera shake)

Courtesy: Cho et al. ICCV 2007

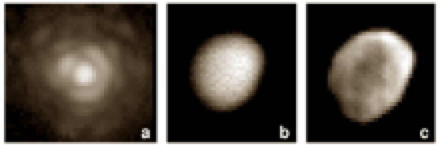
Examples (Atmospheric conditions)

Single Image Haze Removal [He et al. CVPR 2009]

image restoration

- started from the 1950s
- application domains
  - Scientific explorations
  - Legal investigations
  - Film making and archival
  - Image and video (de-)coding
  - …
  - Consumer photography

Example of image restoration
Asteroid Vesta



- related problem: image reconstruction in radio astronomy, radar imaging and tomography

[Banham and Katsaggelos 97]

Original image

Blurred image

- Image enhancement: “improve” an image subjectively.

Original image

Blurred image

- Image enhancement: "improve" an image subjectively.
- Image restoration: remove distortion from image in order to go back to the "original" → objective process.

a model for image distortion

- Image restoration
  - Use a priori knowledge of the degradation
  - Modeling the degradation and apply the inverse process
  - Formulate and evaluate objective criteria of goodness

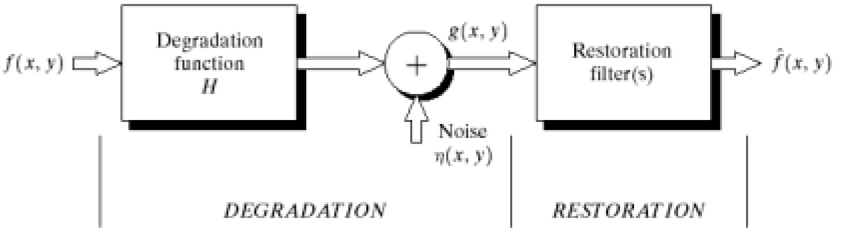
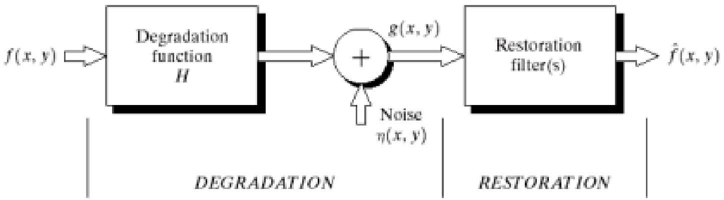


FIGURE 5.1 A model of the image degradation/restoration process.

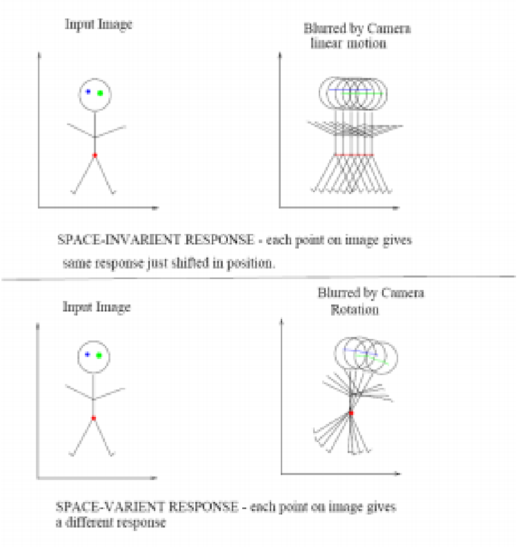
$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

$\longrightarrow$ design restoration filters such that $\hat{f}(x, y)$ is as close to $f(x, y)$ as possible.



usual assumptions for the distortion model

- Noise
  - Independent of spatial location
    - Exception: periodic noise …
  - Uncorrelated with image
- Degradation function H
  - Linear
  - Position-invariant



Mathematical Model of Image Degradation/Restoration

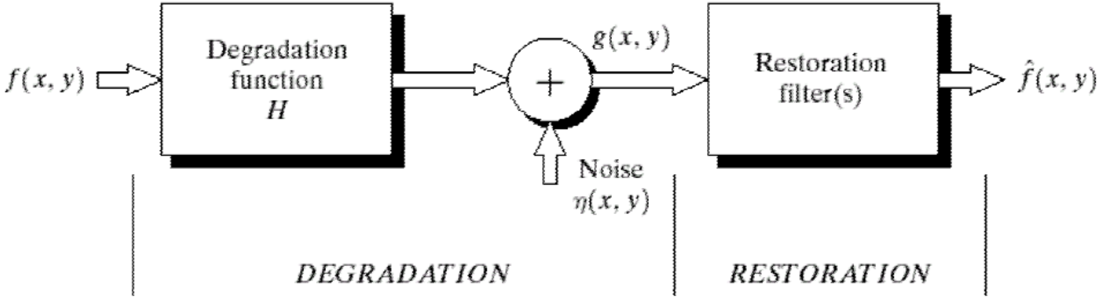


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

Mathematical Model of Image Degradation/Restoration

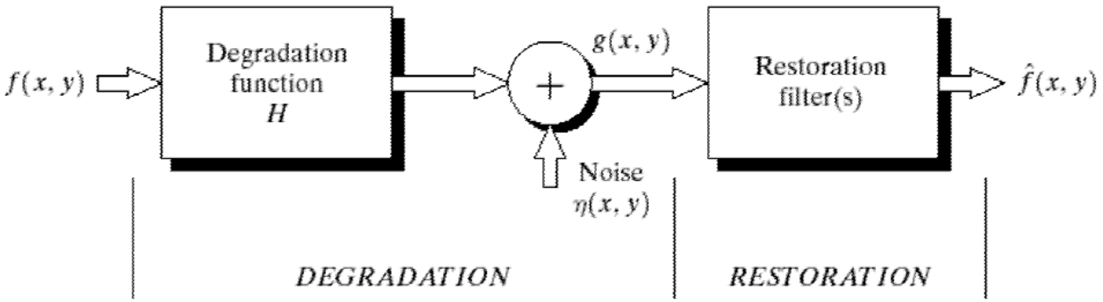


FIGURE 5.1 A model of the image degradation/restoration process.

If H is a linear, position-invariant process

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

Mathematical Model of Image Degradation/Restoration

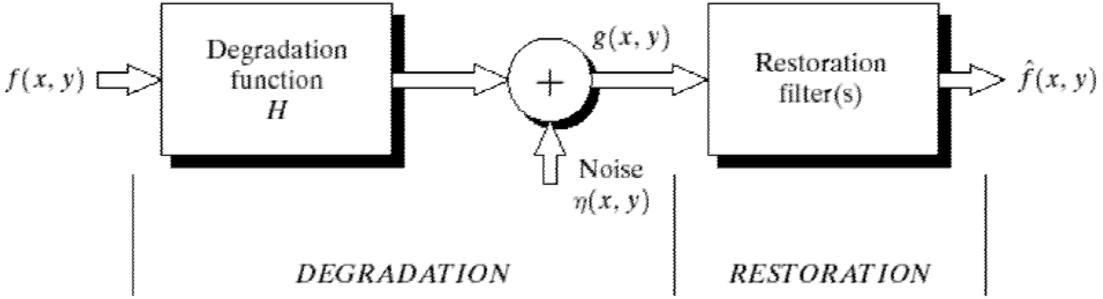


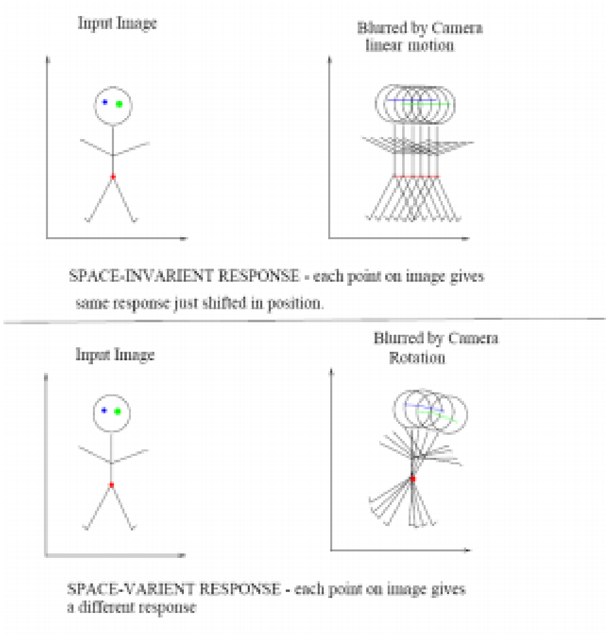
FIGURE 5.1 A model of the image degradation/ restoration process.

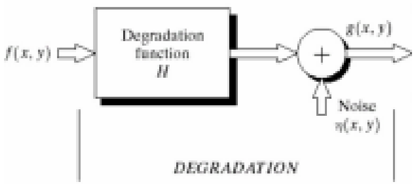
$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

usual assumptions for the distortion model

- Noise
  - Independent of spatial location
    - Exception: periodic noise ...
  - Uncorrelated with image
- Degradation function H
  - Linear
  - Position-invariant

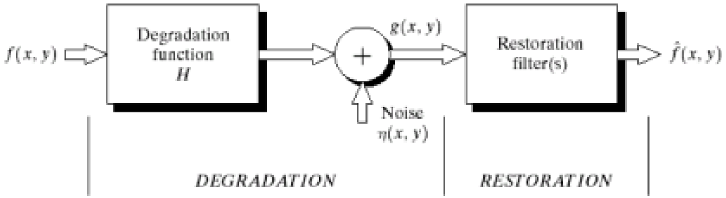




divide-and-conquer step #1: image degraded only by noise.

Noise based Degradation

- Assuming H is identity, model reduces to:



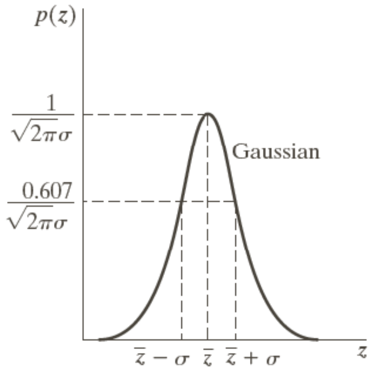
$$g(x,y) = f(x,y) + \eta(x,y)$$

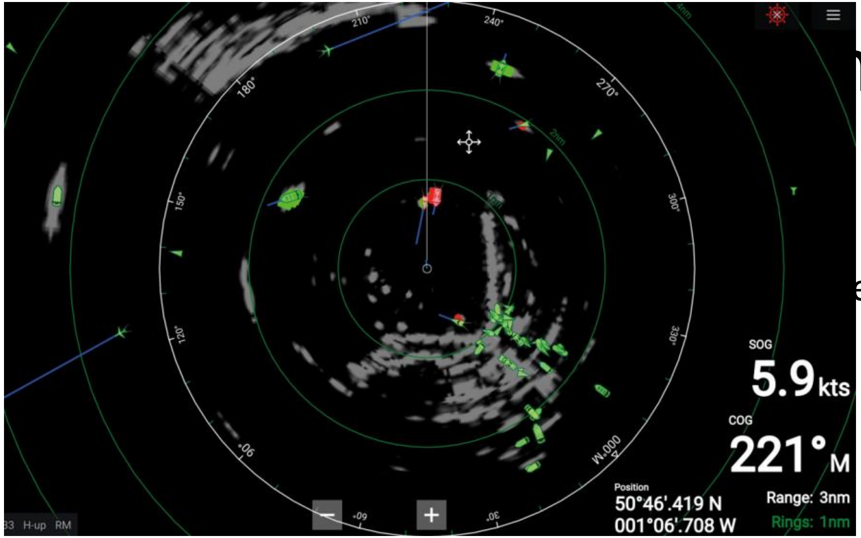
$$G(u,v) = F(u,v) + N(u,v)$$

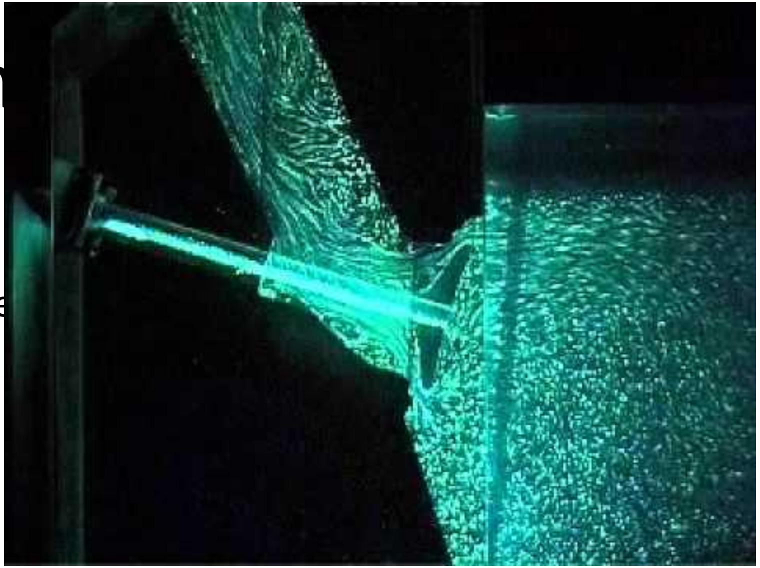
Noise Models

- Gaussian (normal) Noise
  - widely used due to mathematical convenience

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$



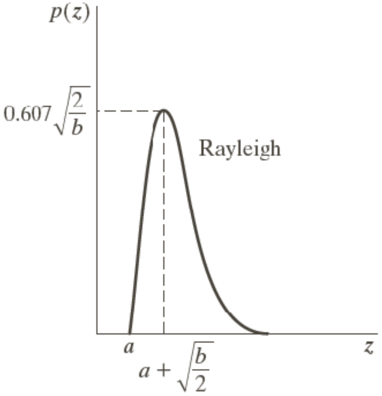




- Rayleigh Noise [Radar, Velocity images]

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b/4}$ Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$



Useful for modelling skewed histograms

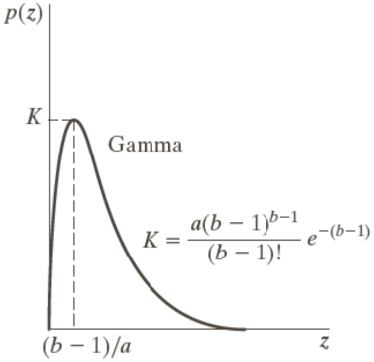
Noise Models

- Erlang (Gamma) Noise [Laser images]

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

b positive integer

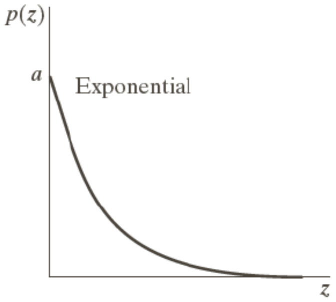


Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$

- Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

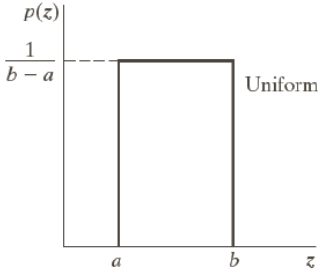


Noise Models

- Uniform Noise [quantization, most unbiased]

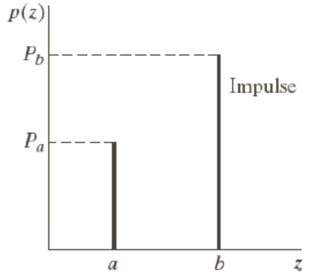
$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} = \dfrac{a+b}{2}$ Variance: $\sigma^2 = \dfrac{(b-a)^2}{12}$



- Impulse (salt-and-pepper) Noise [sync errors in digitization or transmission, sensor malfunction]

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



$P_a = P_b \Rightarrow$ *unipolar* noise

Noise Models

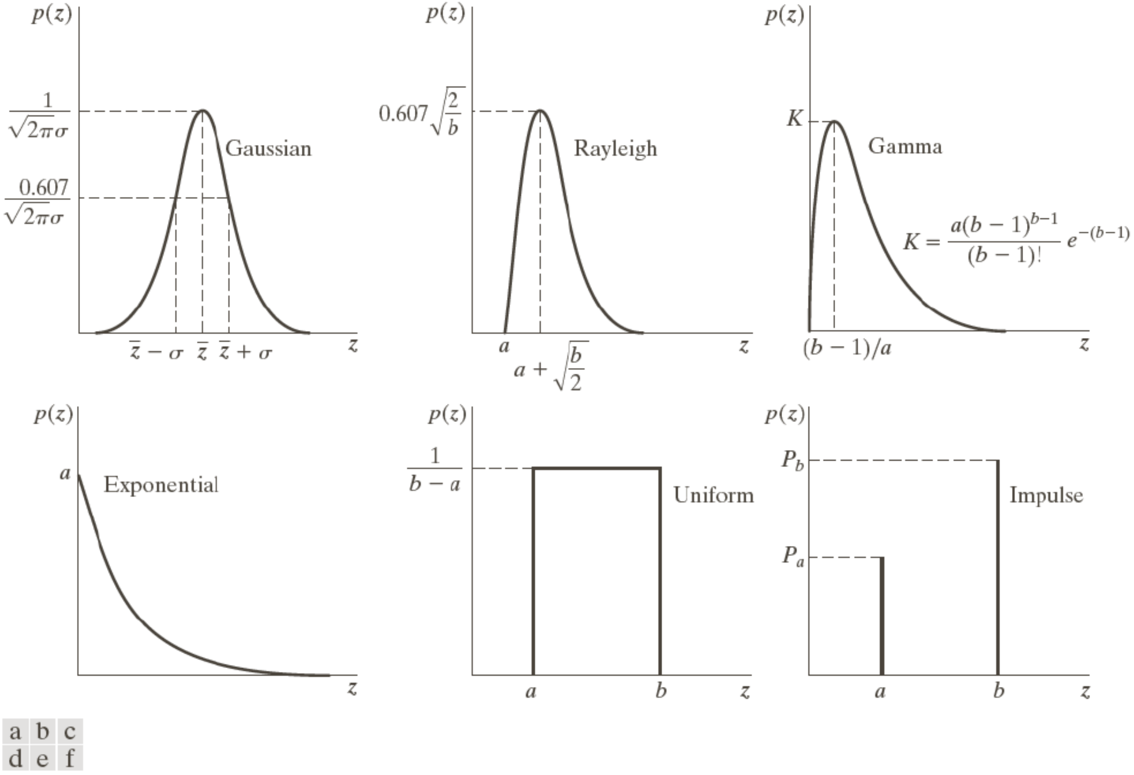


FIGURE 5.2 Some important probability density functions.

Illustration of Noise Models

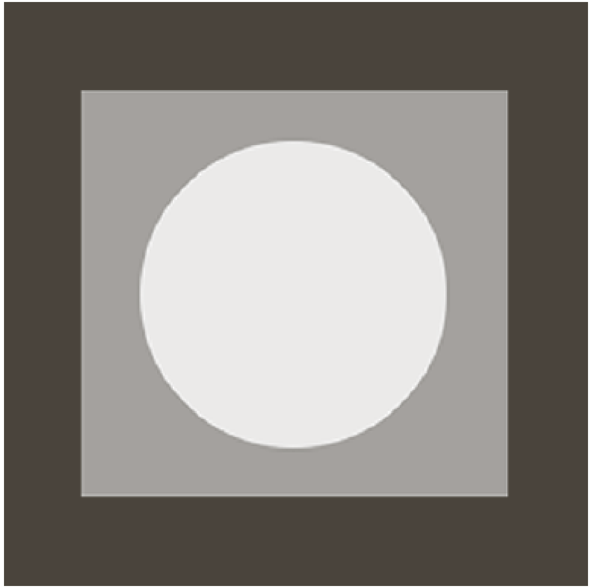
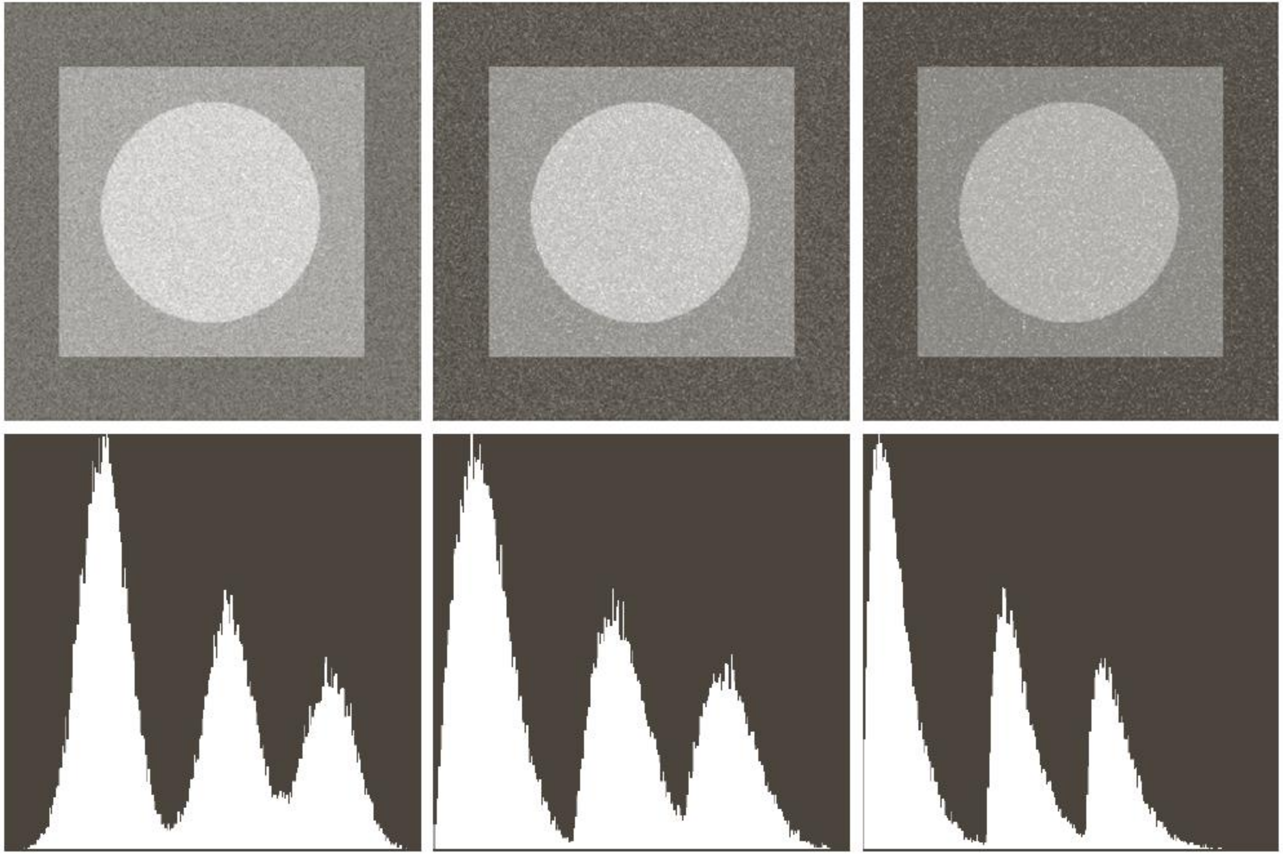


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Illustration of Noise Models



Gaussian Rayleigh Gamma

Illustration of Noise Models

Visually similar.

Not easy to determine noise model from appearance

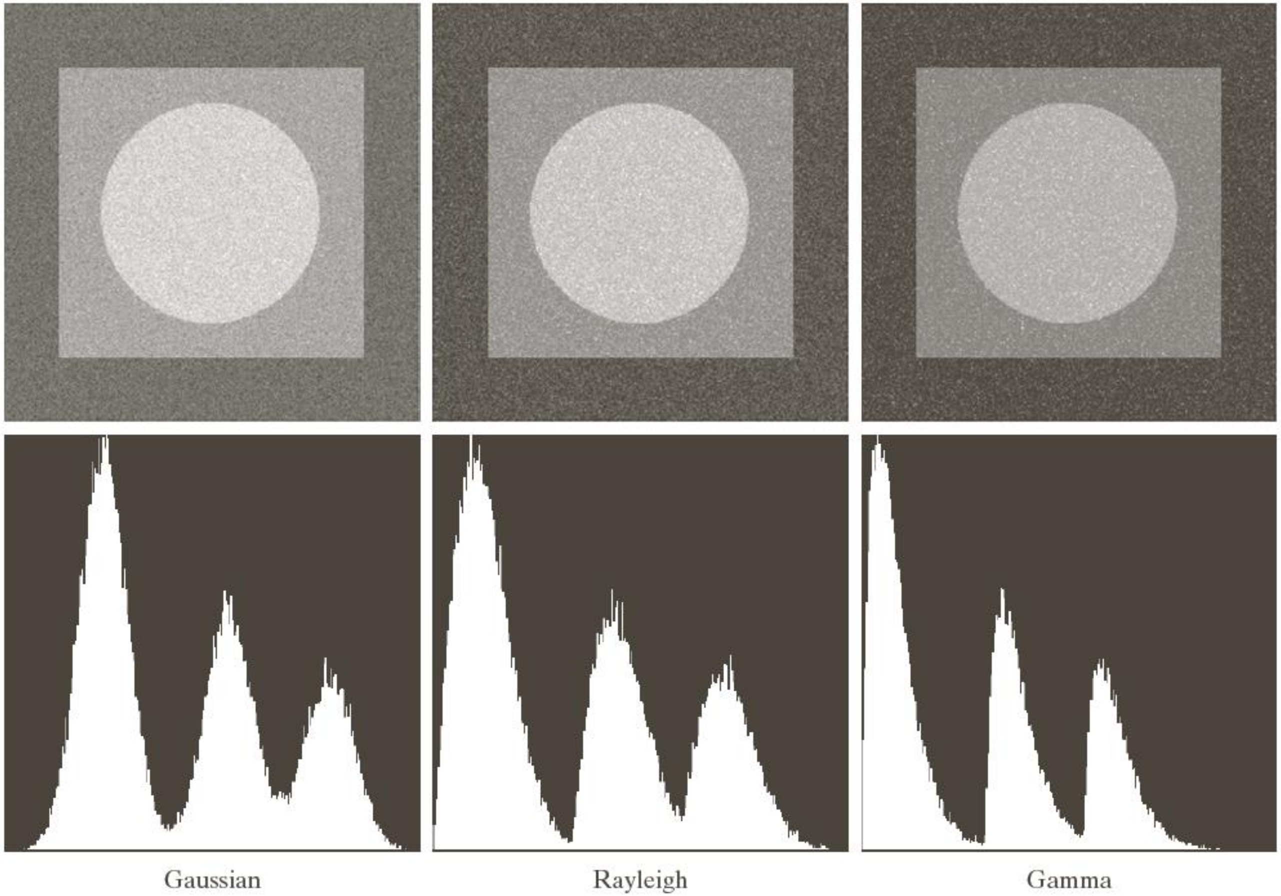
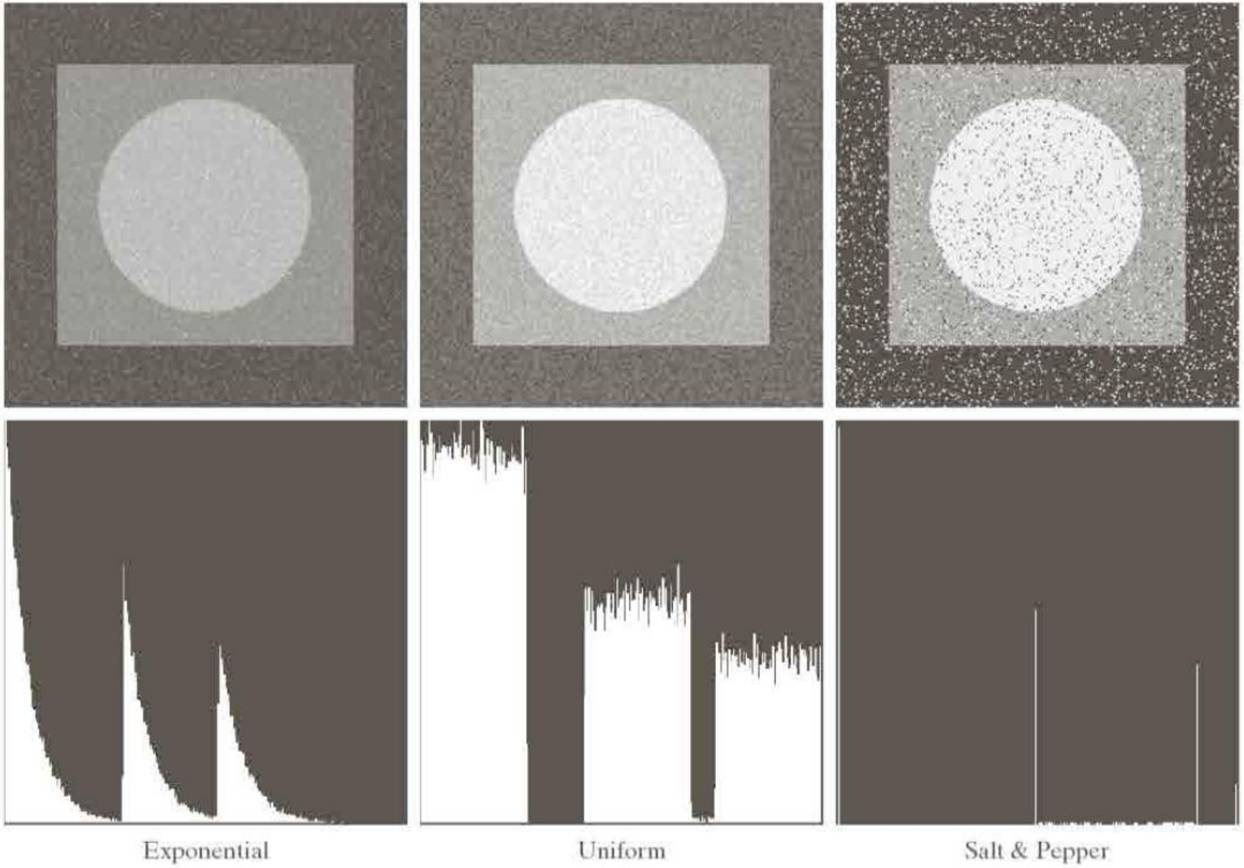


Illustration of Noise Models

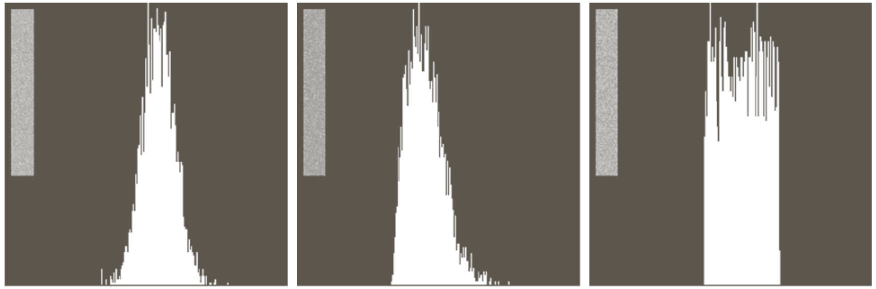


How to study system noise

- Imaging system available
  - Noise Calibration: Capture a set of ‘flat environments’ (e.g. solid gray board, object at fixed location)
  - Select the model with better statistical test scores (Akaike Information Criteria (AIC) or Likelihood Ratio Test (LRT))
-

How to study system noise

- Only images available
  - Estimate from patches of constant intensity
  - For impulse noise
    - Use a mid-gray patch/area

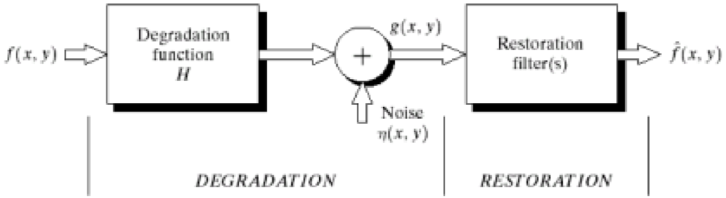


a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in presence of noise only

- Assuming H is identity, model reduces to:



$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

Restoration (in presence of noise only)

- mean filters

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

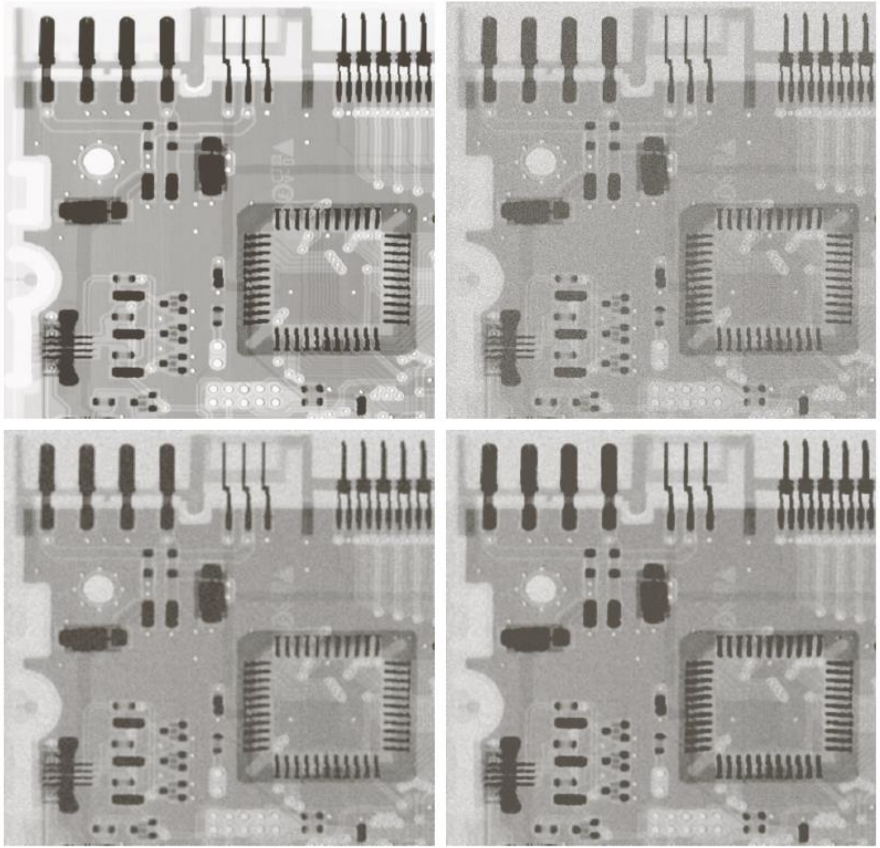
Geometric mean filter

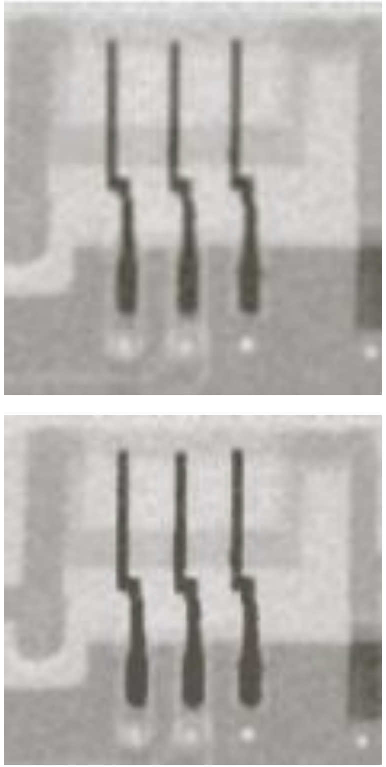
$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Restoration (in presence of noise only)

a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)





Restoration (in presence of noise only)

- mean filters

<u>Harmonic mean filter</u>

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

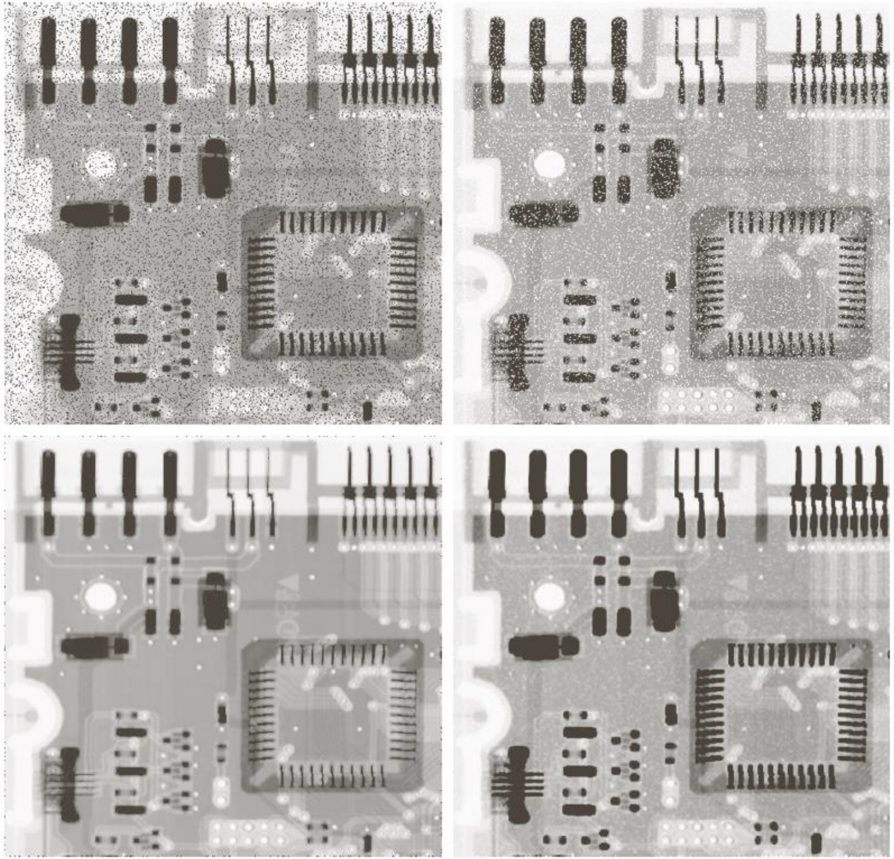
Works well for salt noise or Gaussian noise, but fails for pepper noise

<u>Contraharmonic mean filter</u>

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q = *order* of the filter
Good for salt-and-pepper noise.
Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$
NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

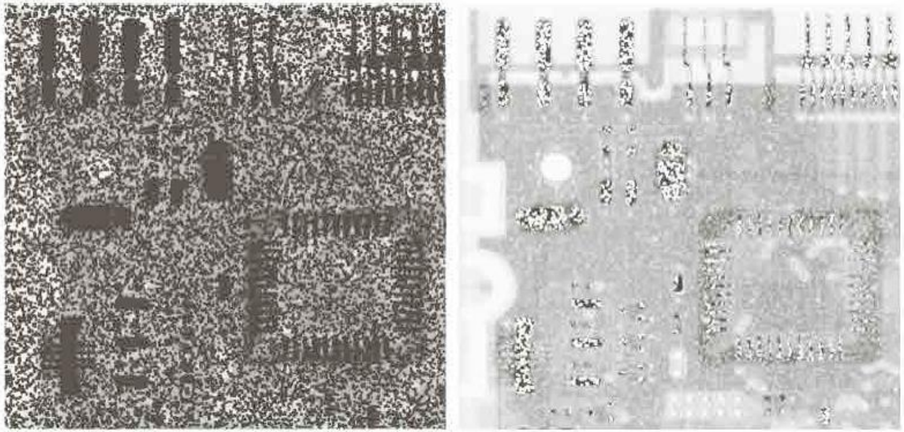
Restoration (in presence of noise only)



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Restoration (in presence of noise only)

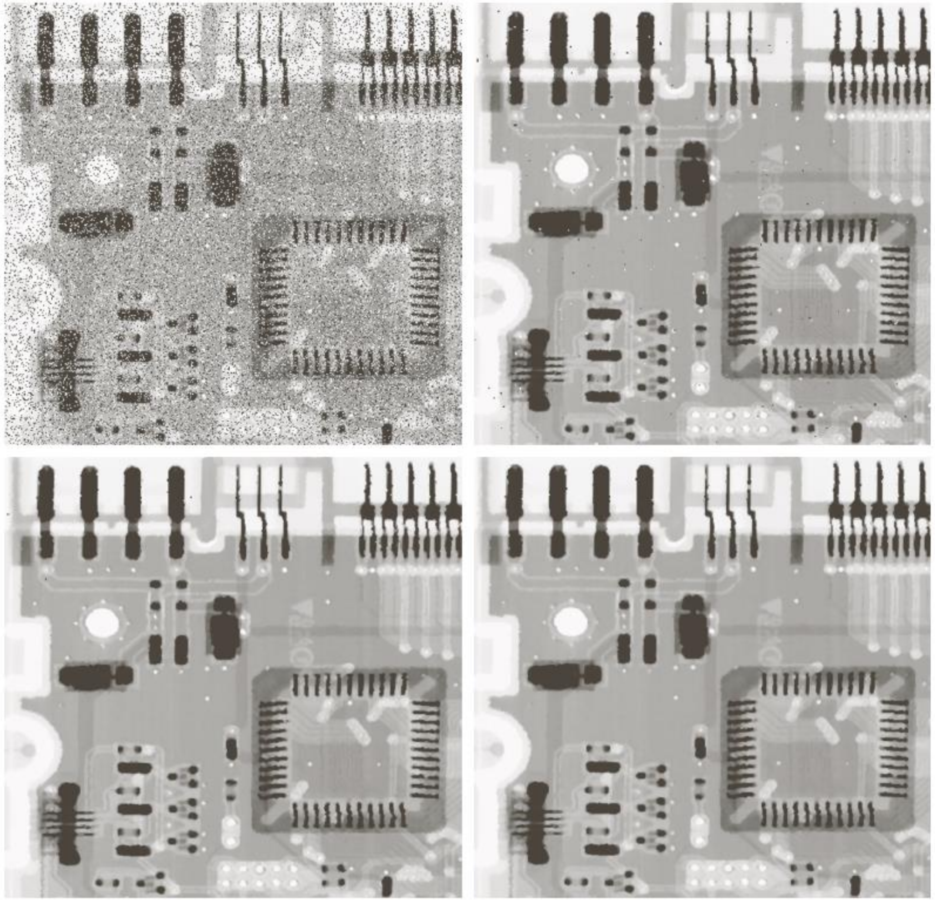


a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Restoration (in presence of noise only)

- Median filter



a	b
c	d

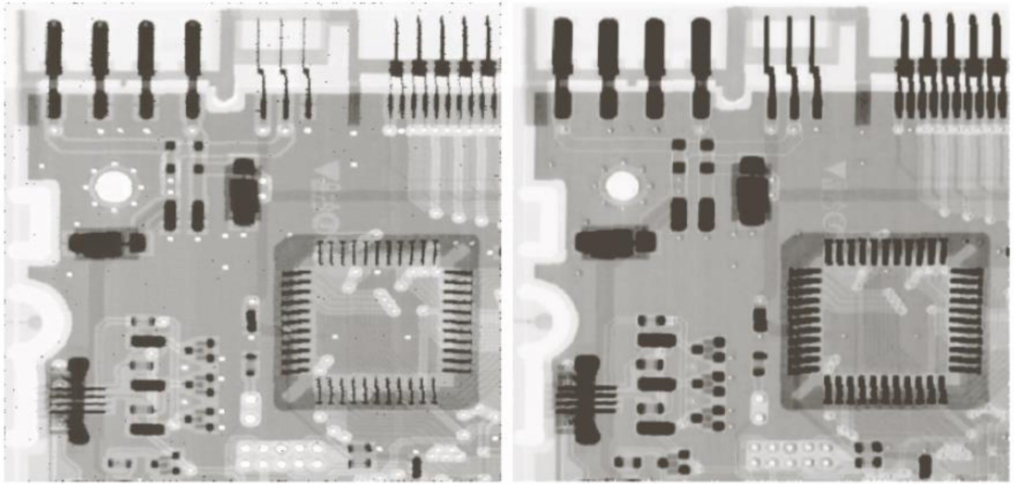
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Restoration (in presence of noise only)

- Max, Min filters

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter Min filter

Restoration (in presence of noise only)

- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max\{g(s,t)\}_{(s,t)\in S_{xy}} + \min\{g(s,t)\}_{(s,t)\in S_{xy}}\right]$$

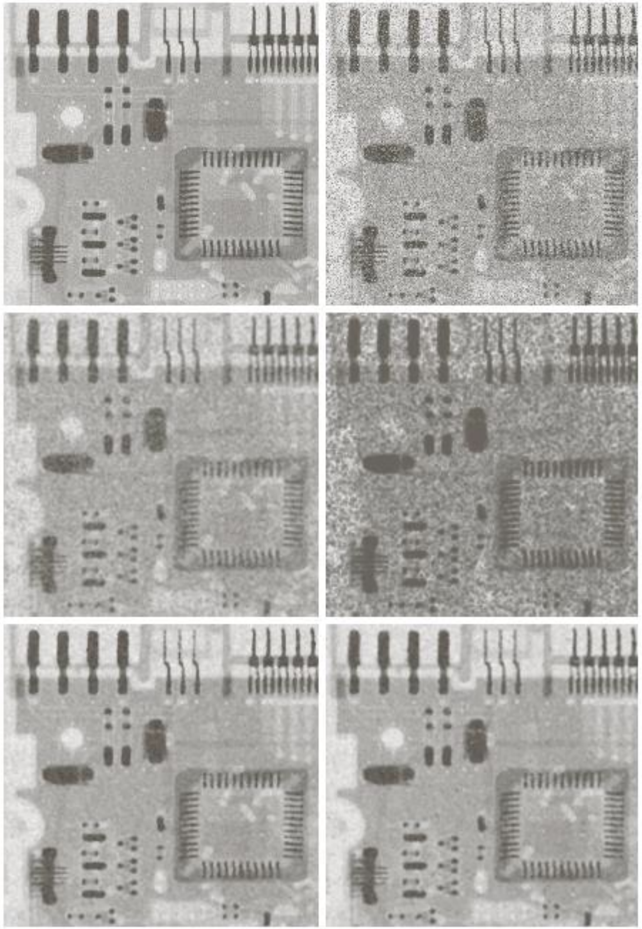
Best for Uniform or Gaussian noise

- Alpha trimmed filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Where g_r represents the image *g* in which the *d/2* lowest and *d/2* highest intensity values in the neighbourhood S_{xy} were deleted

Restoration (in presence of noise only)



original

Original + salt and pepper noise

Arithmetic mean filter

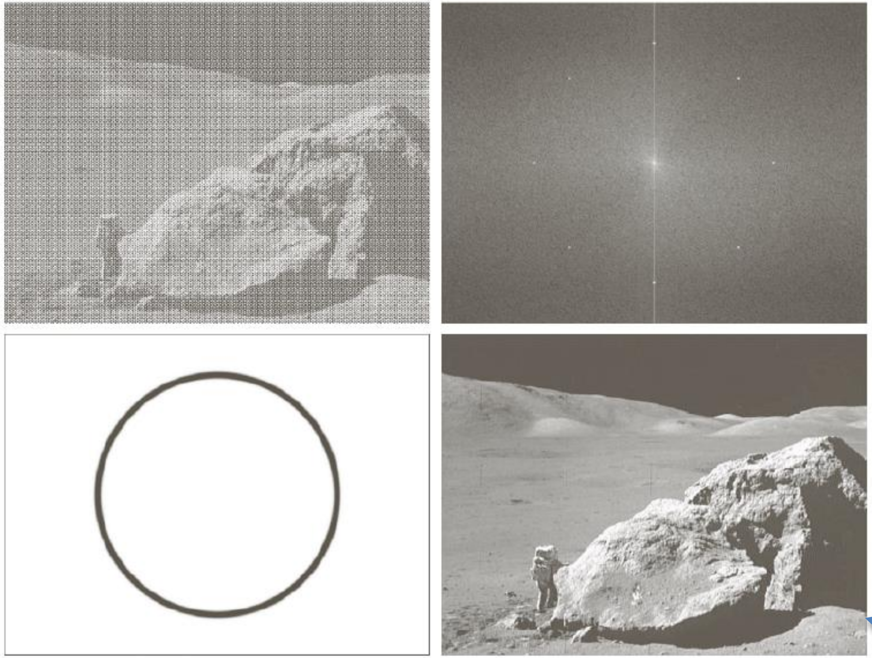
Geometric mean filter

Median filter

Alpha Trimmed filter

Restoration (in presence of noise only)

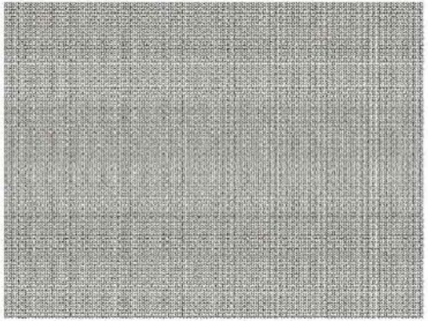
- Band pass/reject



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Very difficult to get result of this quality via spatial domain filtering using small convolutional masks

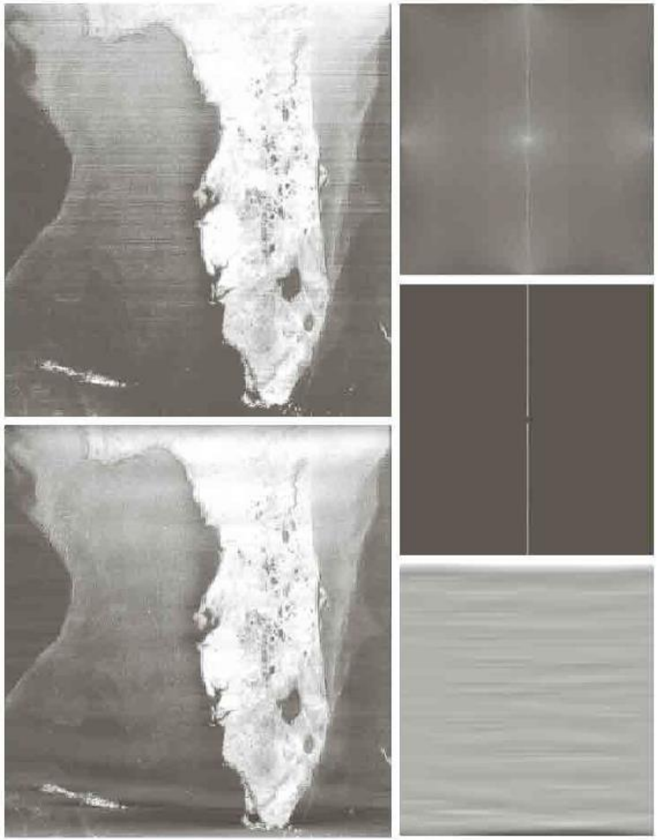
Restoration (in presence of noise only)

- Notch pass/reject

spectrum

Degraded image

Filtered image



Notch pass filter

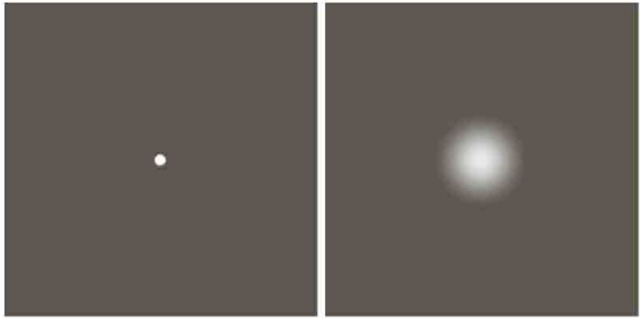
Spatial noise pattern

a b
c
e d

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Estimation of degradation function

- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration
  - Mathematical modelling



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Motion Blur

- Exposure
- If amount of light hitting the sensor changes significantly over exposure period → Motion Blur
- Causes (one or more of)
  - Camera motion
  - Subject motion
  - (Drastic) change in Lighting condition

Motion blur effect

```
#define filterWidth 9
#define filterHeight 9

double filter[filterHeight][filterWidth] =
{
  1, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 1, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 1, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 1, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 1, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 1, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 1, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1,
};

double factor = 1.0 / 9.0;
double bias = 0.0;
```

https://lodev.org/cgtutor/filtering.html

Estimation by Modeling (uniform motion blurring)

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

$$G(u,v) = F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

Estimation by Modeling (uniform motion blurring)

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

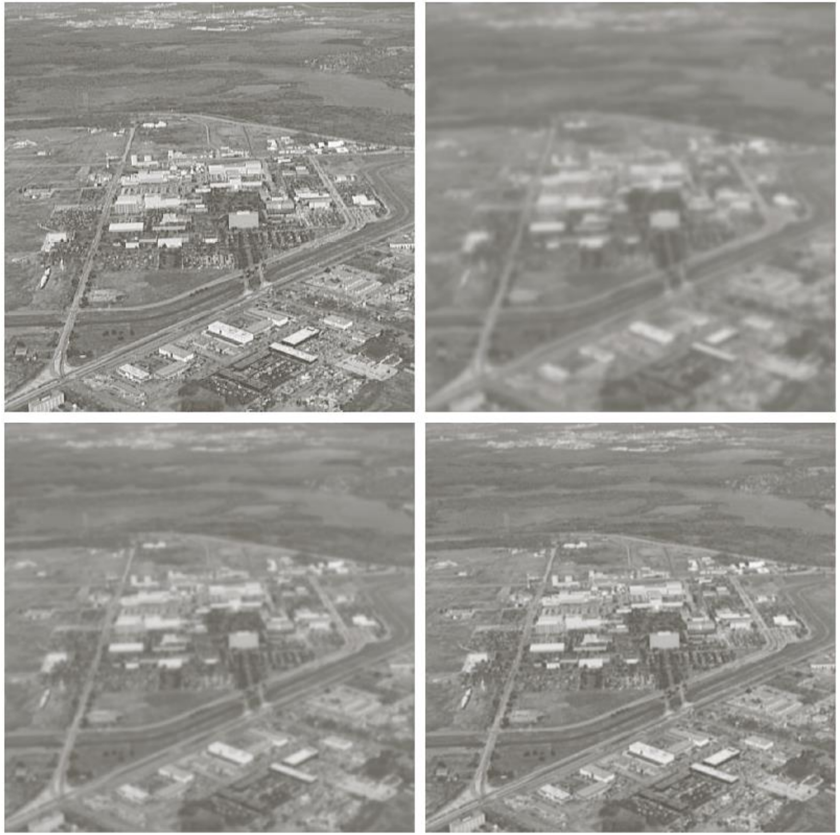
putting, $x_0(t) = at/T$ and $y_0(t) = bt/T$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\, e^{-j\pi(ua+vb)}$$

a b

FIGURE 5.26
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Estimation by Modeling (atmospheric turbulence)



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

Model of Image Degradation/Restoration

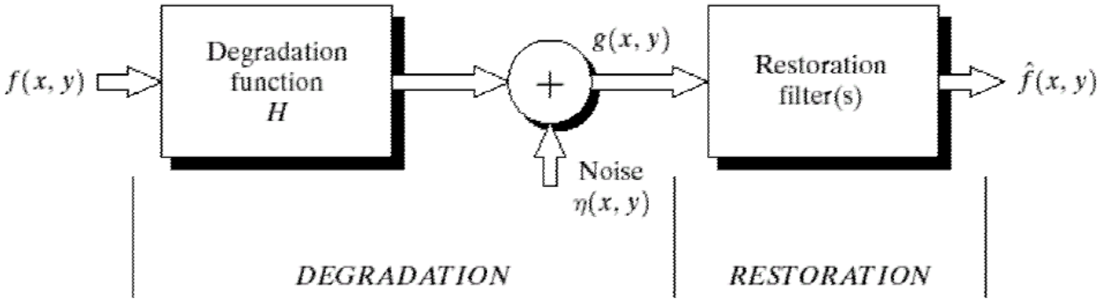


FIGURE 5.1 A model of the image degradation/ restoration process.

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Recovering image (in presence of both Noise and degradation)

- Even if we know the degradation function we cannot recover the un-degraded image!!

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \quad \Rightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Two problems:

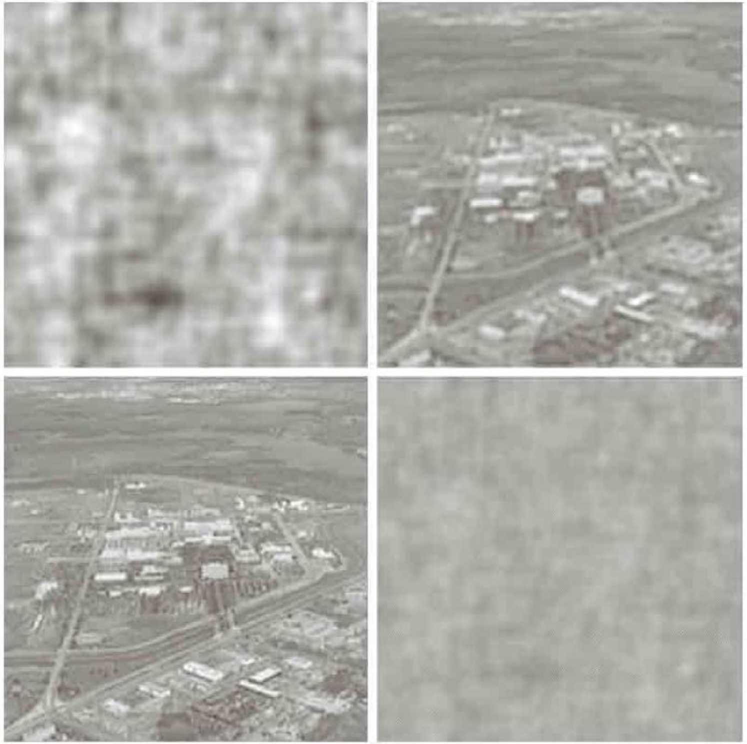
1. N(u,v) is a random function whose fourier transform is not known
2. If degradation has zero or small values → N(u,v)/H(u,v) will dominate

Recovering image (in presence of both Noise and degradation)

Degraded Image
(with known model)

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



No explicit provision for handling noise!

Weiner filter

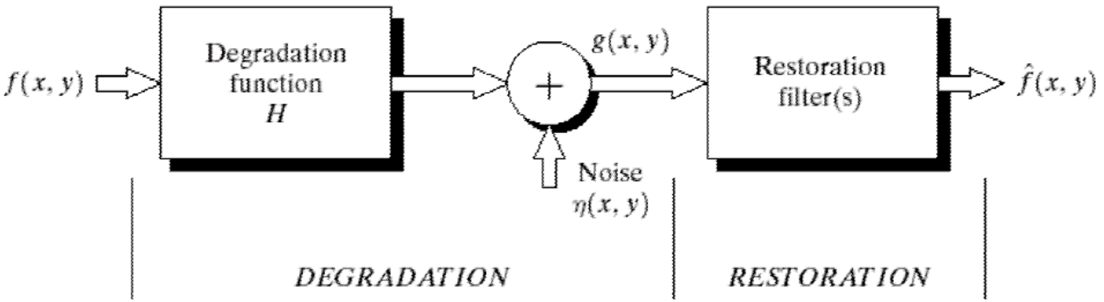


FIGURE 5.1 A model of the image degradation/ restoration process.

Consider image and noise as random variables

$$e^2 = E\{(f - \hat{f})^2\}$$

Assumption:

- Noise and image are uncorrelated

Weiner filter

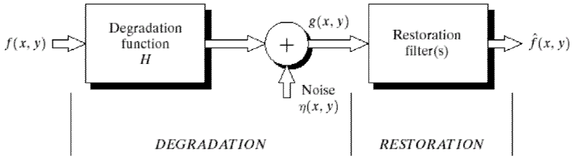


FIGURE 5.1 A model of the image degradation/restoration process.

$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function e is given by: $H^*(u,v)$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image

Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Weiner filter

Weiner filter

blur + add. noise | Inverse filtering | Wiener filtering

Reduced noise variance

Reduced noise variance

a b c
d e f
g h i

Scribe List

2019201021
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