28.08.2020

Digital Image Processing (CSE/ECE 478)

Lecture-6: Spatial Filtering

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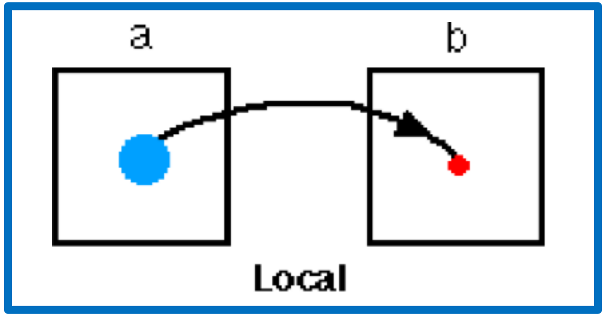
Announements

- TAs
  - Meher Shashwat Nigam
  - Soumyasis Gun
  - Adithya Arun
  - Surendra Gopireddy

Announcements

- Mini Quiz – 2 today
- Tutorial Slot : 5pm, Saturday

- **Neighborhood to Point**



Spatial Domain Filtering

Mean/Average Filter (Smoothing)

```
M = 3
For each valid location [x,y] in S
        a ← Average of intensities in a M x M neighborhood centered on [x,y]
        D[x,y] = round(a)
```

120	190	140	150	200
17	21	30	8	27
89	123	150	73	56
10	178	140	150	18
190	14	76	69	87

I

x

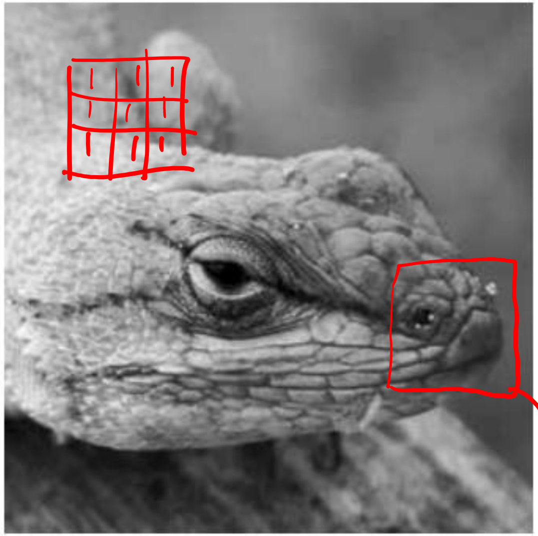
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

	98			

dst

http://199.116.233.101/index.php?title=File:Animated-kernels.gif



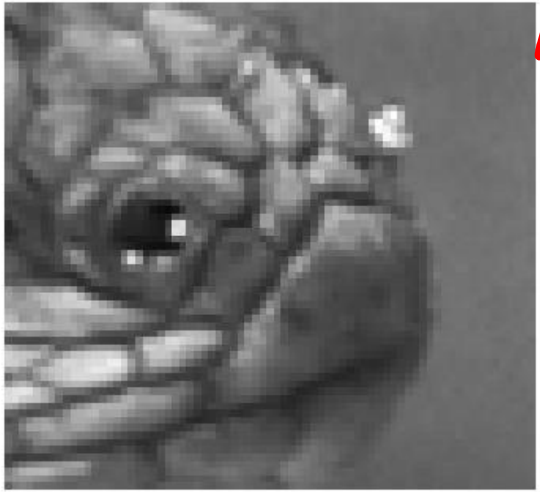
I

$$\frac{1}{9} * \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

3 x 3

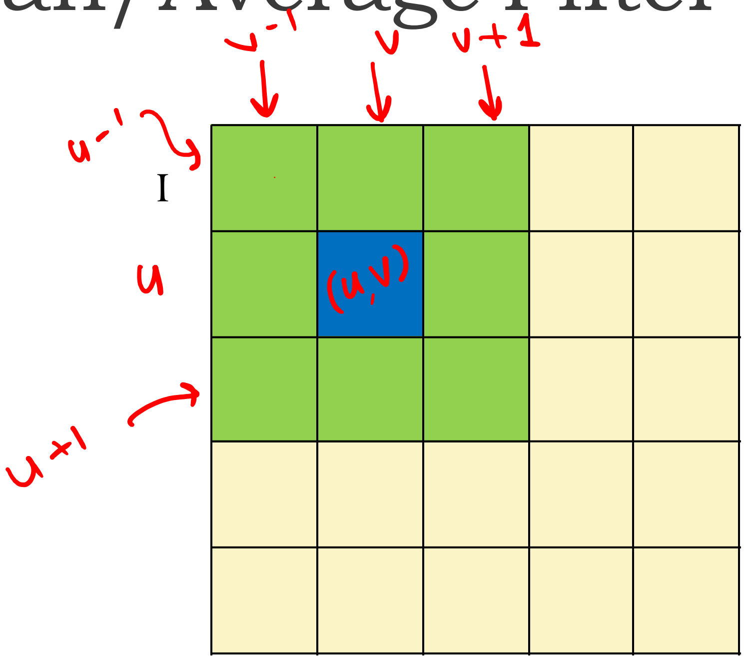
5 x 5

dst



$$\frac{1}{25} \quad \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ \vdots \end{matrix}$$

Mean / Average Filter



Note: Coefficients sum to 1

H(F)

H → 3 x 3

Weight Mask / Kernel / Filter

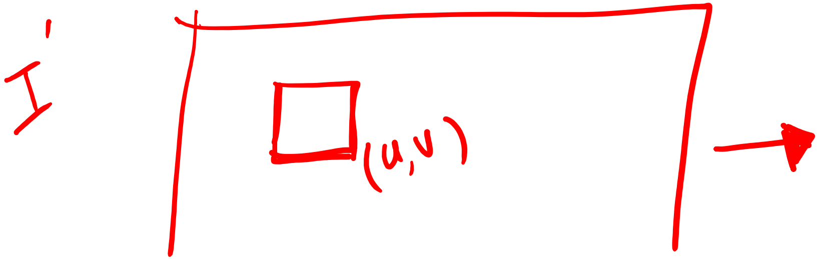
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

F

F

mask coefficients

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$



$$I'(u,v) \leftarrow \sum_{j=-1}^{1} \sum_{i=-1}^{1} \underbrace{I(u+i, v+j) \boxed{\cdot\ H(i,j)}}$$

Effect of Mask Size

Original Image

[3x3]

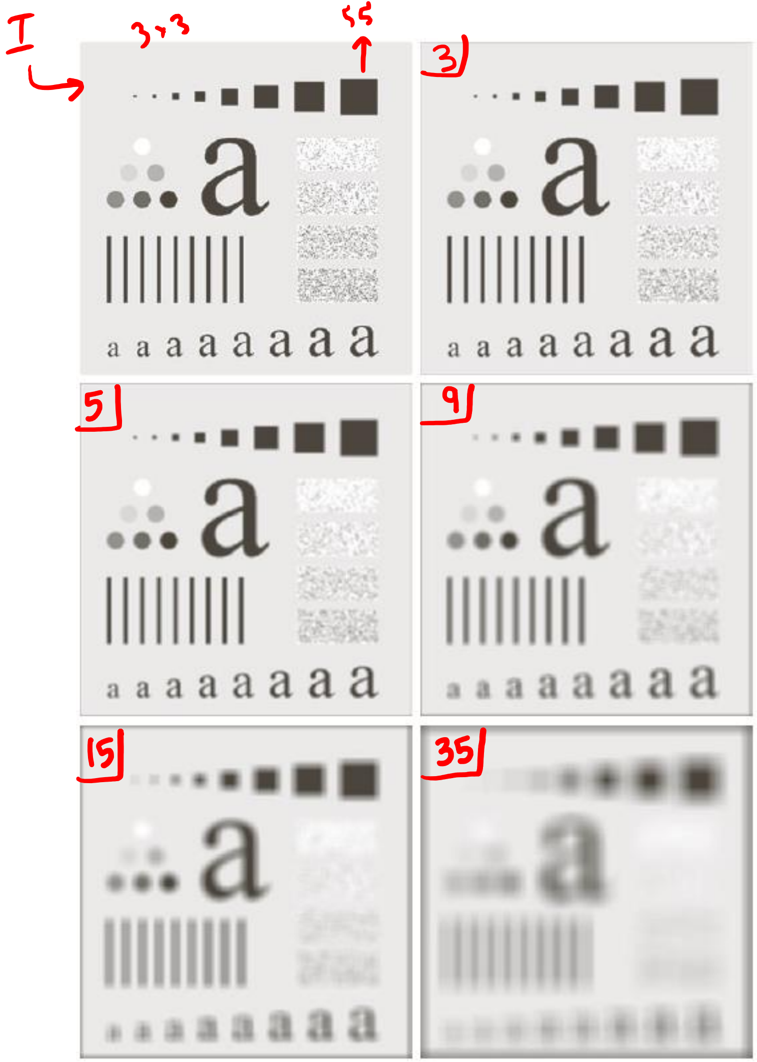
[5x5]

[7x7]

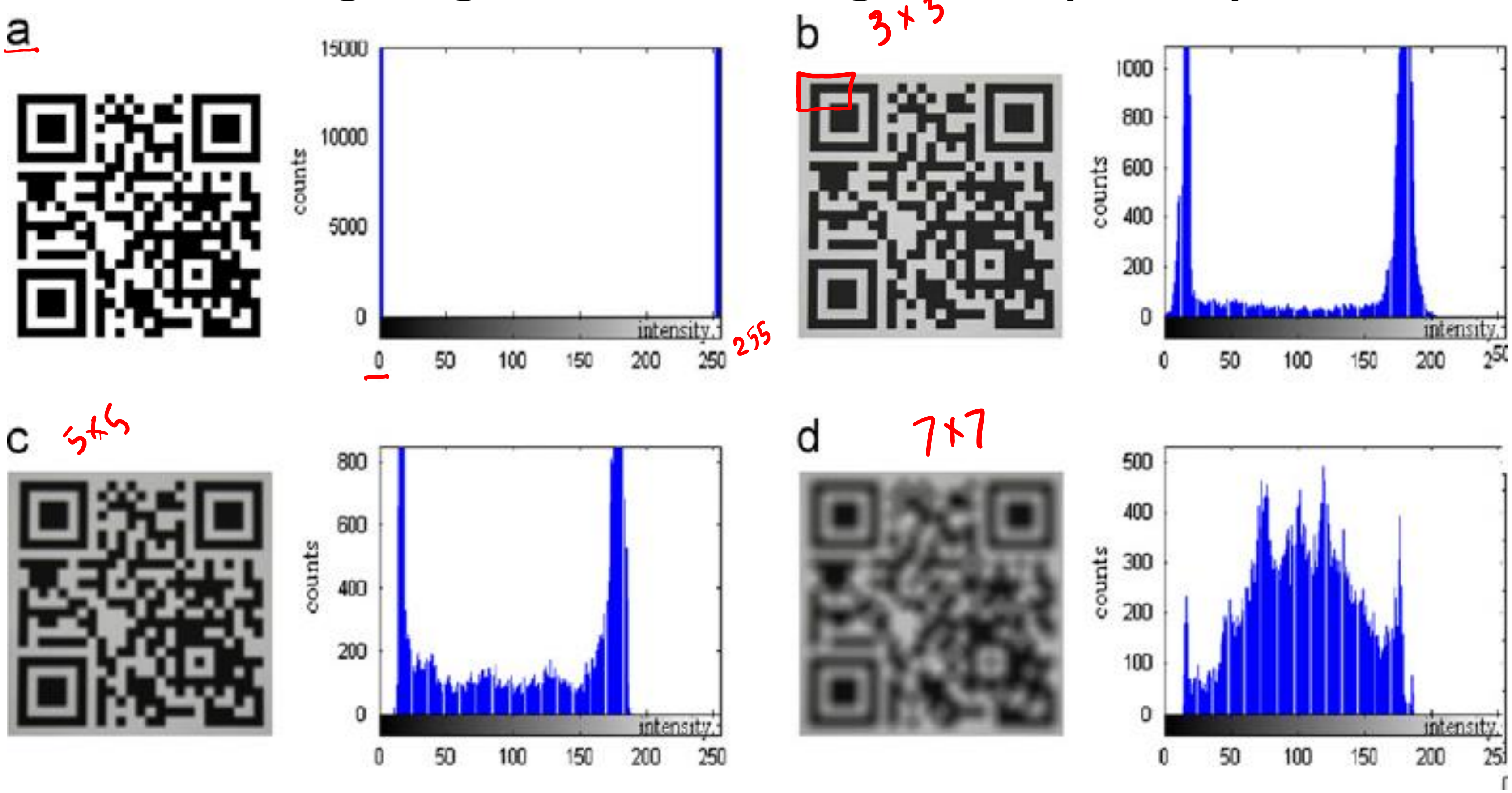
Square averaging filter

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a	b
c	d
e	f



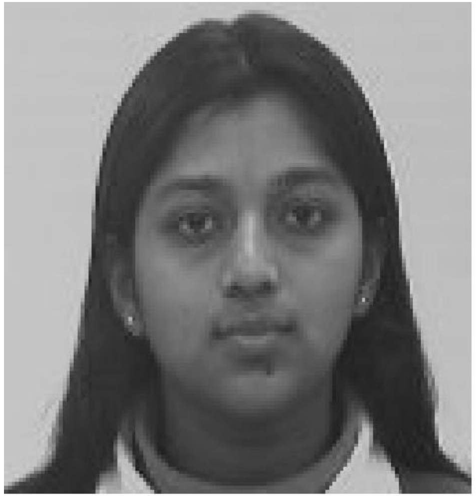
Averaging – a histogram perspective



https://www.researchgate.net/figure/Histograms-of-barcode-images-with-different-levels-of-blur-a-a-synthetic-QR-code-image_fig3_224201499

Repeated Averaging Using Same Filter

5×5



I

Before

3×3

After

3×3

After repeated averaging

NOTE: Can get the effect of larger filters by smoothing repeatedly with smaller filters

Weighted Averaging

$$I'(u,v) = \frac{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} I(u+i, v+j) \cdot H(i,j)}{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} H(i,j)}$$

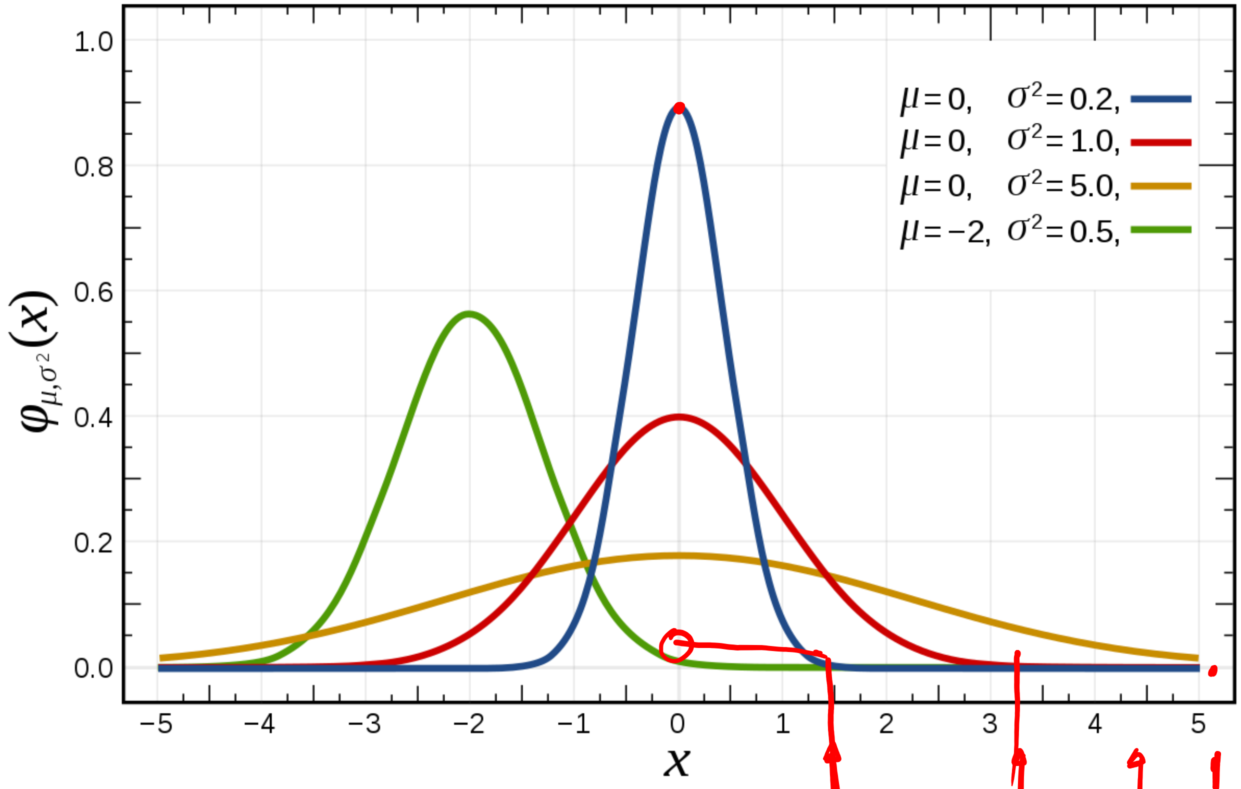
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Standard average

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Weighted average

Gaussian Function (1-D)



7x7

3 3

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ, σ

$\rightarrow 3\sigma$

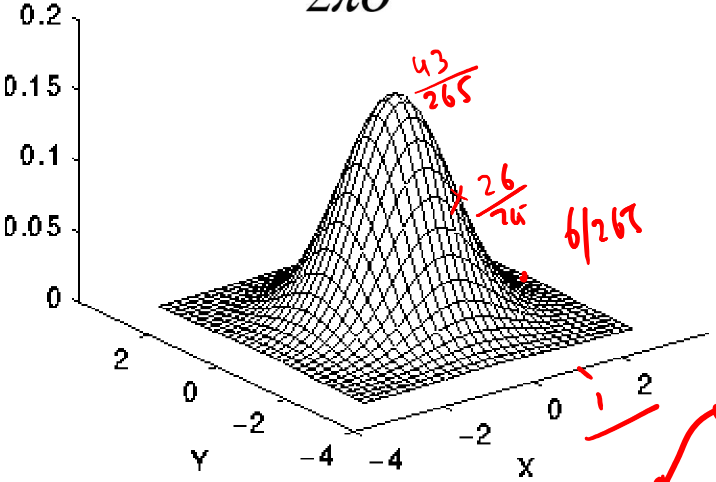
$\rightarrow 4\sigma$

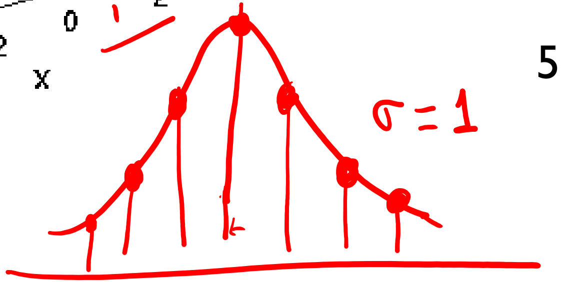
5.6

Gaussian Smoothing

- Mask weights are samples of a zero-mean 2-D Gaussian

$$G(x,y) = \frac{1}{2\pi\sigma^2}\exp\{-(x^2+y^2)/2\sigma^2\}$$





$\frac{1}{265}$

1	4	6	4	1
4	16	26	16	4
6	26	43	26	6
4	16	26	16	4
1	4	6	4	1

5

5×5 Gaussian filter, σ=1

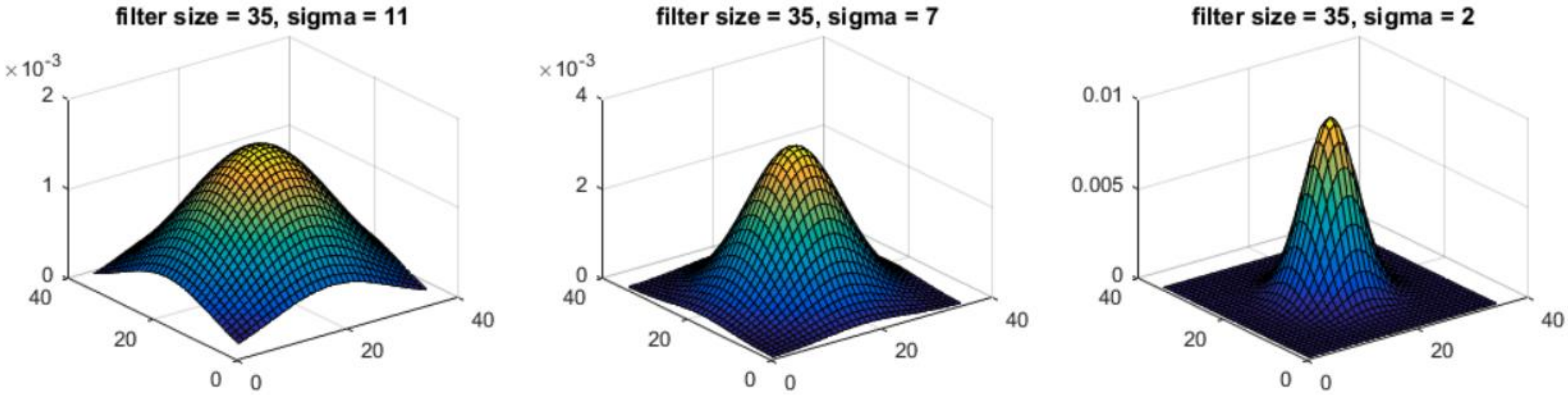
Gaussian Smoothing – Effect of sigma

$$G(x,y) = \frac{1}{2\pi\sigma^2}\exp\{-(x^2+y^2)/2\underline{\sigma}^2\}$$

gauss(σ, n)

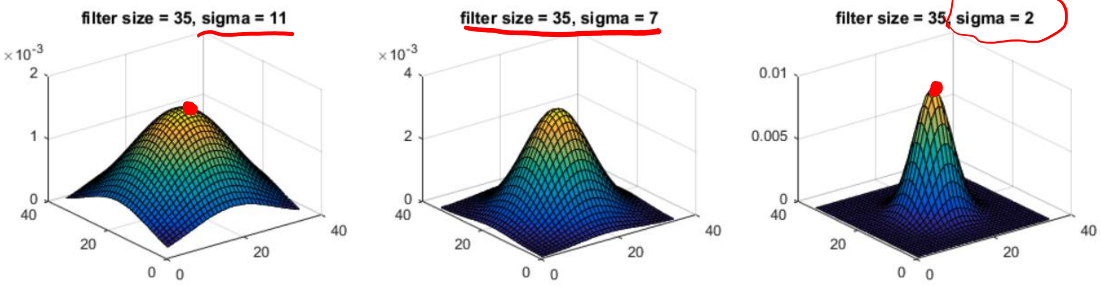
0.2

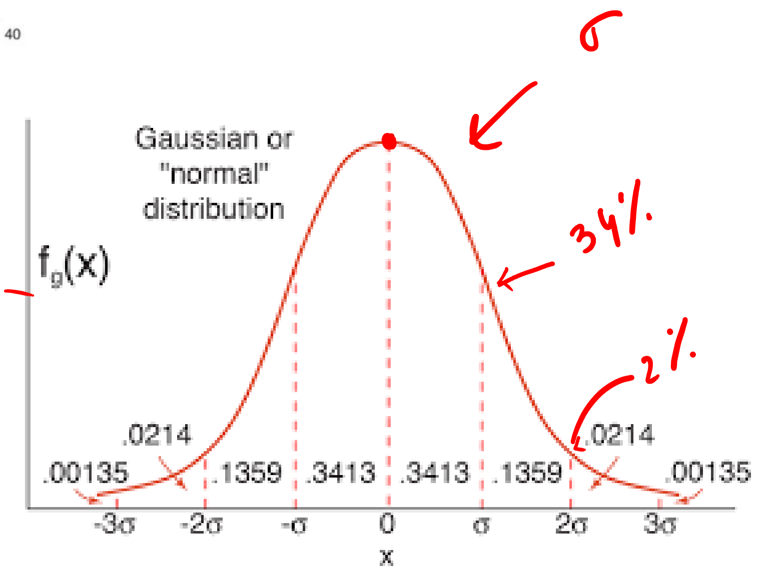
5



Gaussian Smoothing – Effect of sigma

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$





Gaussian Smoothing – Effect of sigma

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2+y^2)/2\sigma^2\}$$

I

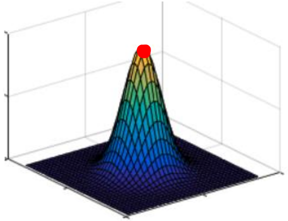
J =

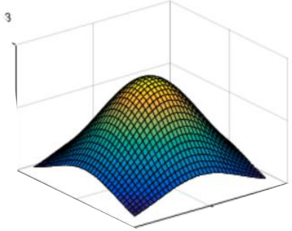
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Original Image
(Sigma 0)

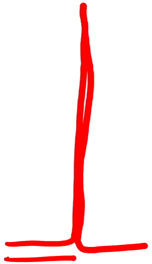
Gaussian Blur
(Sigma 0.7)

Gaussian Blur
(Sigma 2.8)

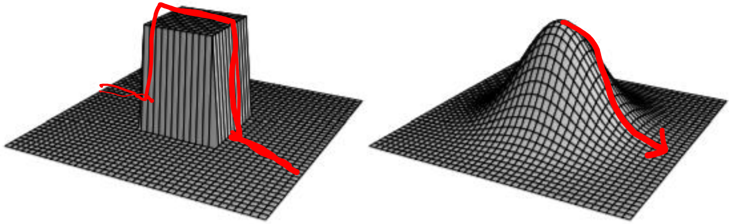


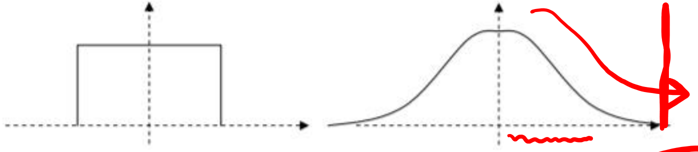


1



Averaging vs Gaussian filters





5 x 5

g

$\frac{1}{9}$

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

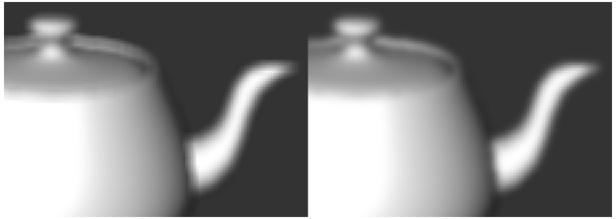
255

0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0

σ

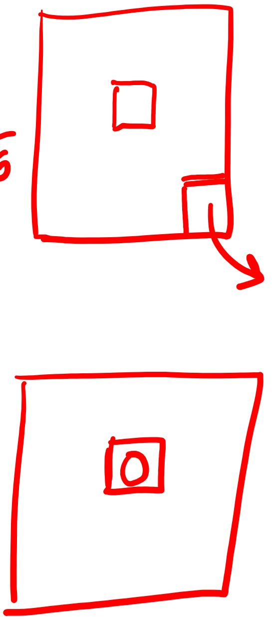
gaussian

3x3



Smoother intensity transitions

$\frac{1}{5}$

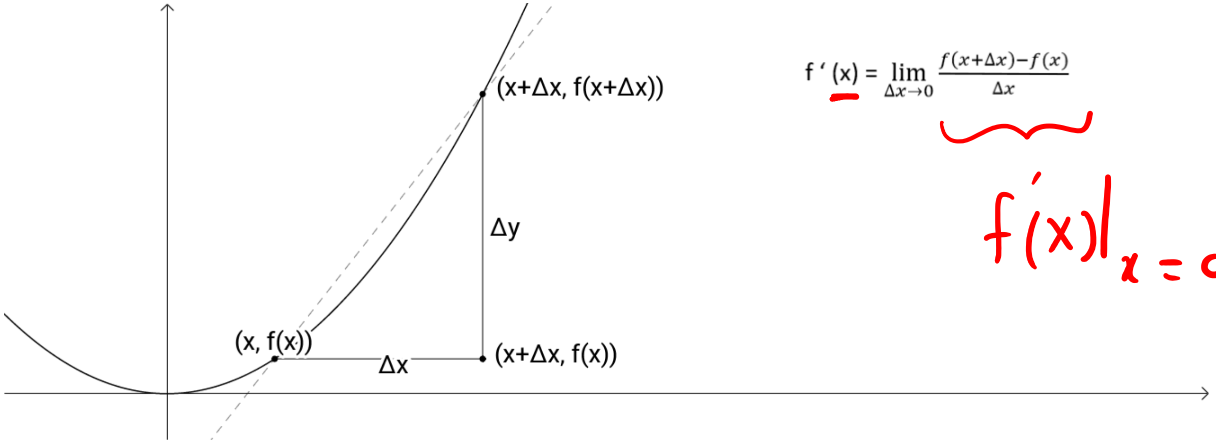


255

$\frac{255}{5}$

$\frac{255}{40}$

Recap: Derivatives

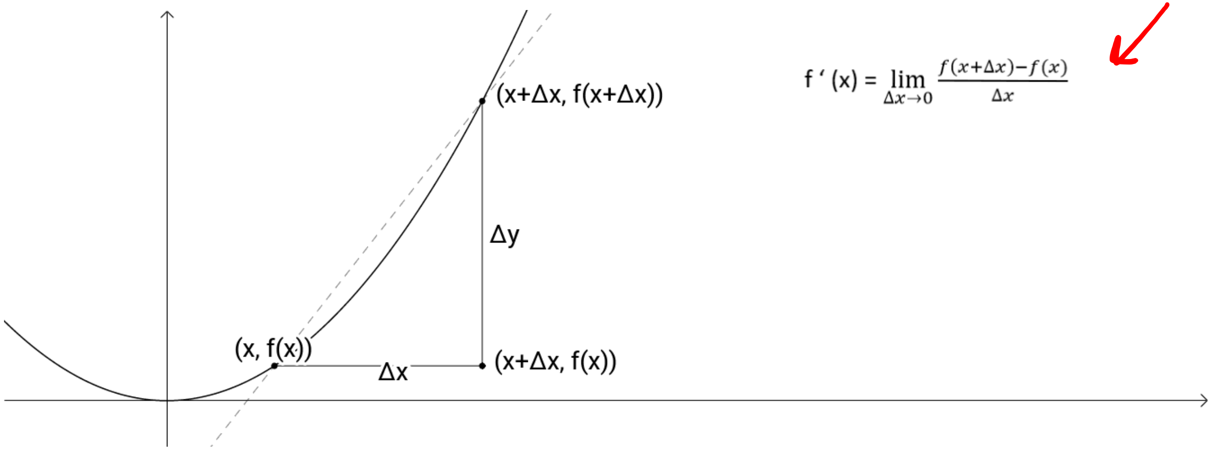


$$f'(x) = \underbrace{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$f(x)$

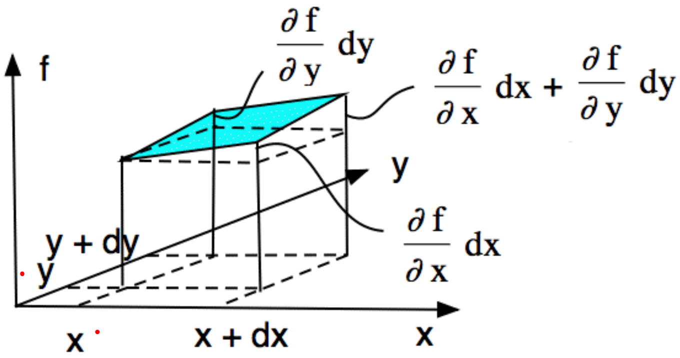
$f'(x)\Big|_{x=a}$

Recap: Derivatives



$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$f(x,y)$



$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

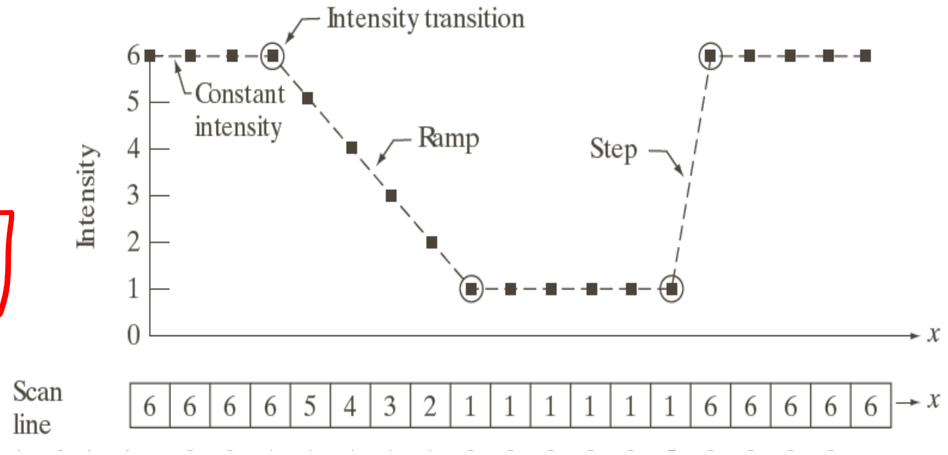
First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\underline{\Delta x}, y) - f(x,y)}{\Delta x}$$

x

y

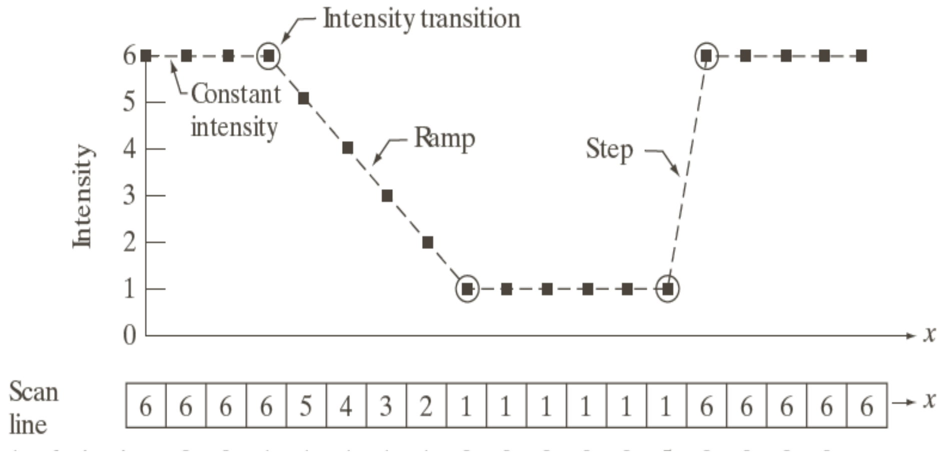
$$\frac{\partial f(x,y)}{\partial x} \sim \underline{f[x+\underline{1}, y] - f[x,y]}$$



First Derivative (Digital approximation)

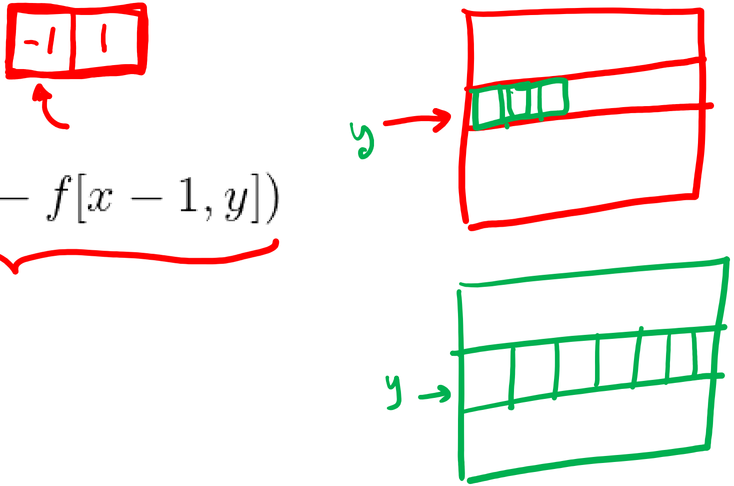
$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$



Second Derivative (Digital Approximation)

$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$



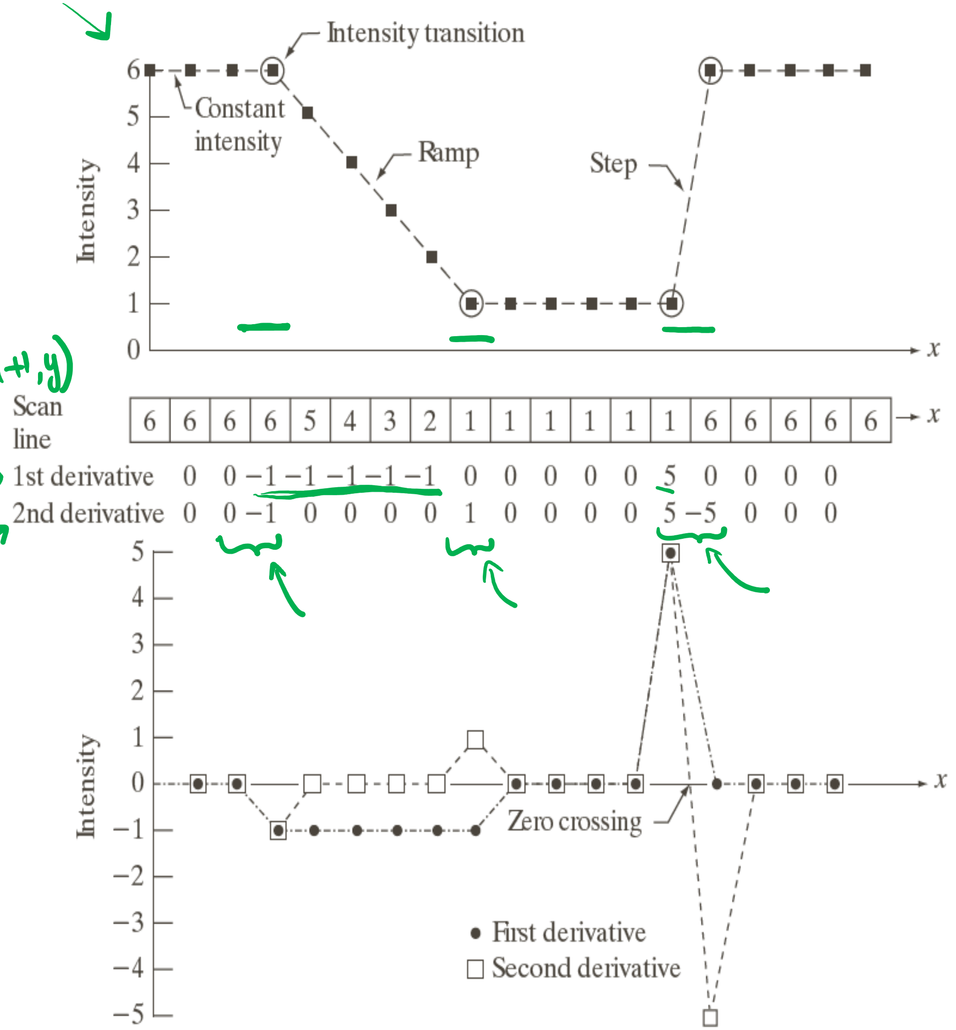
First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim \underbrace{f[x+1,y] - f[x,y]}$$

$\boxed{1 \mid -1}$

f(x,y) f(x+1,y)



Second Derivative (Digital Approximation)

$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$

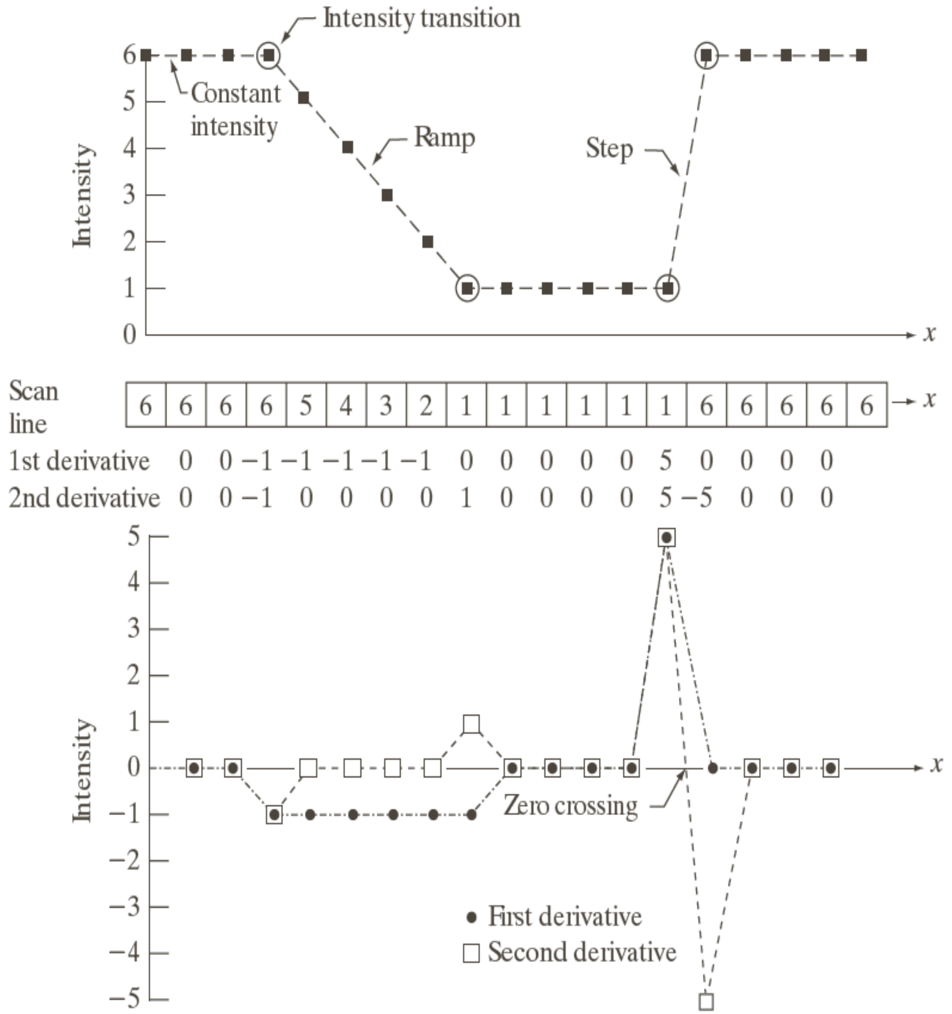
First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$

Second Derivative (Digital Approximation)

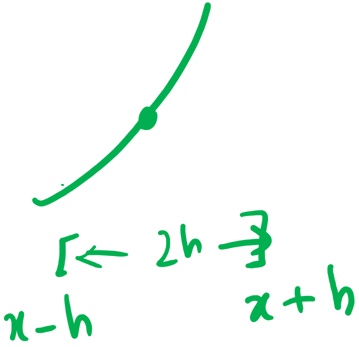
$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$



Alt: Derivative as symmetric Difference

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$



$$\lim_{h \to 0} \frac{\boxed{1} f(x+h) + \boxed{0} \cdot f(x) \boxed{-} f(x-h)}{2h}$$

$$\begin{array}{ccc} -1 & 0 & 1 \\ \cdot & \cdot & \cdot \\ x-h & x & x+h \end{array}$$

Image Gradient and Edges

$$\frac{f(x+h,y)-f(x-h,y)}{2h} \Rightarrow \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline\end{array}$$

x-derivative

$$\frac{f(x,y+h)-f(x,y-h)}{2h} \Rightarrow \begin{array}{|c|}\hline -1 \\ \hline 0 \\ \hline 1 \\ \hline\end{array}$$

y-derivative

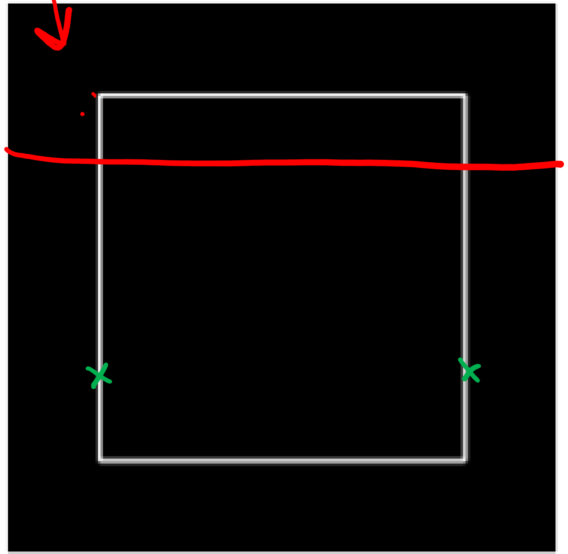
$\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline\end{array}$ $\boxed{0}$ edge

2 55 255 251

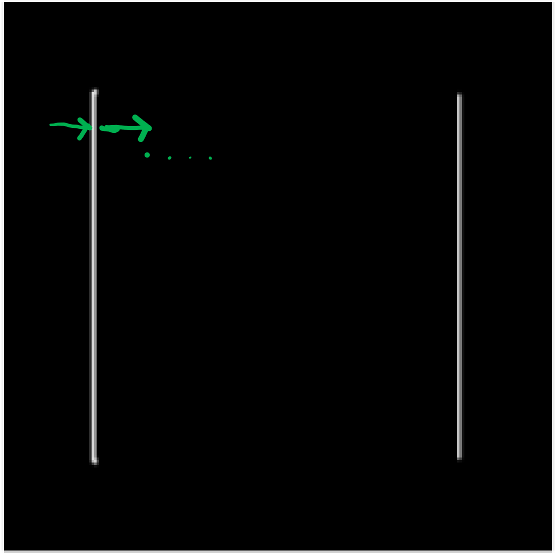
0 0 255 ● 0 0

$\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline\end{array}$

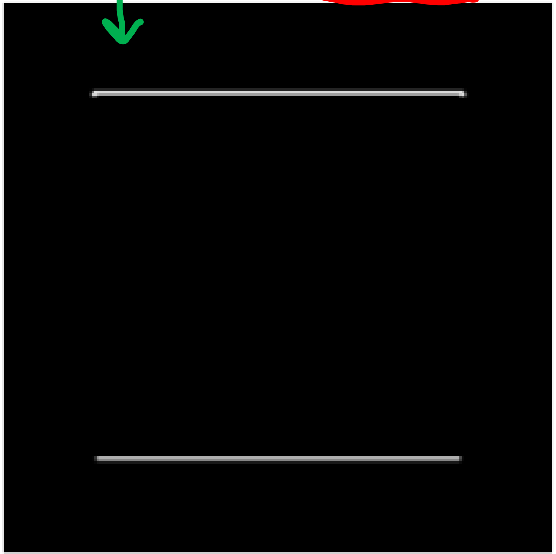
→ 255 0 -255 0



Image



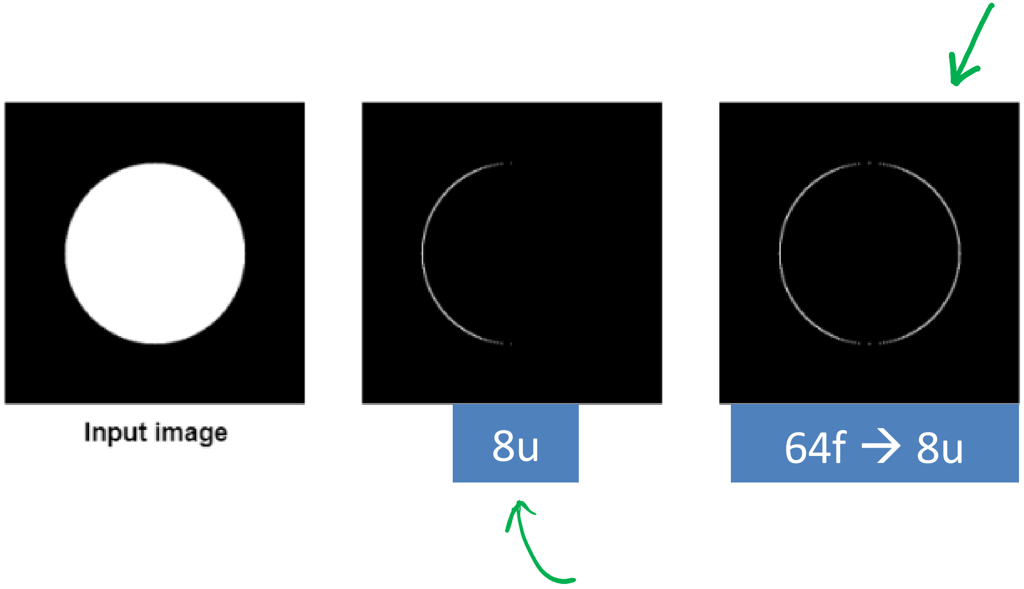
Gradient in x



Gradient in y

https://i1.wp.com/theailearner.com/wp-content/uploads/2019/05/gradxy.png?resize=768%2C280&ssl=1

Edge 'Image'



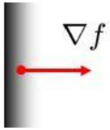
https://i0.wp.com/theailearner.com/wp-content/uploads/2019/05/Sobel64.png?w=731&ssl=1

Image gradient

The gradient of an image:

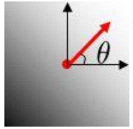
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity

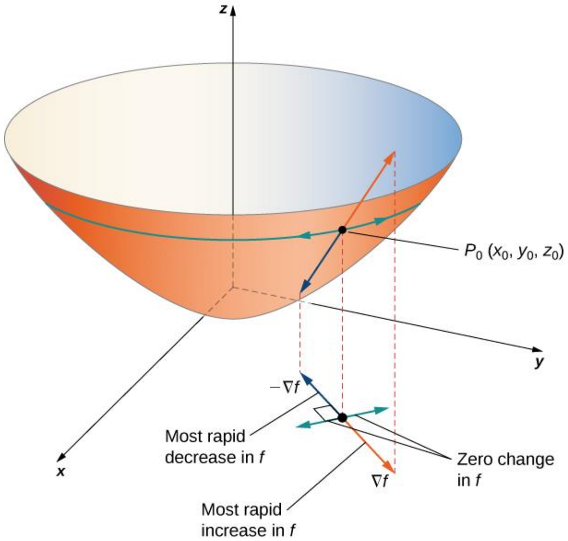


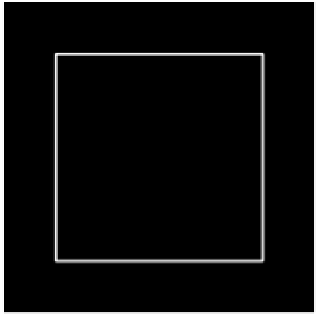
$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

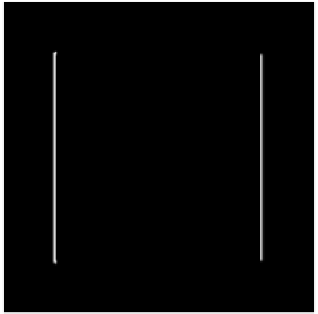


$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$





Image



Gradient in x

Gradient in y

Prewitt Edge Filter

Dr. Prewitt

https://nihrecord.nih.gov/sites/recordNIH/files/pdf/1984/NIH-Record-1984-03-13.pdf

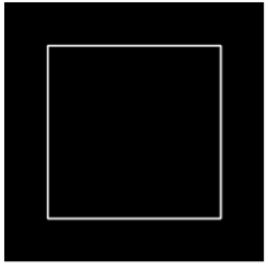
-1	0	+1
-1	0	+1
-1	0	+1

G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

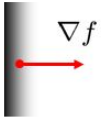
Edge is perpendicular to gradient



Image

Gradient in x

Gradient in y



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

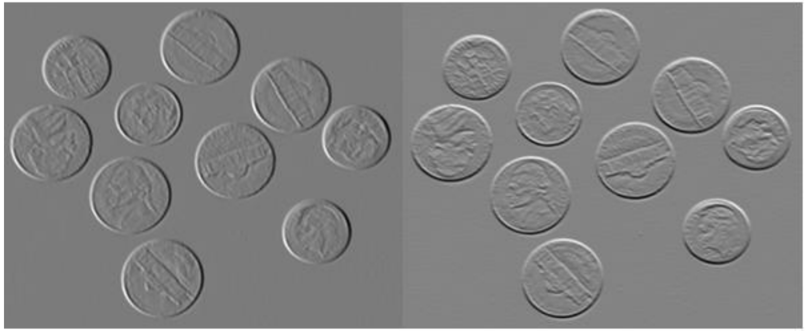
-1	0	+1
-1	0	+1
-1	0	+1

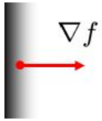
G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

Edge is perpendicular to gradient





$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

-1	0	+1
-1	0	+1
-1	0	+1

G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

Scribe List

2018101029
2018101033
2018101034
2018101035
2018101037
2018101039