

22.09.2020

Digital Image Processing (CSE/ECE 478)

Lecture-12: Morphological Operations

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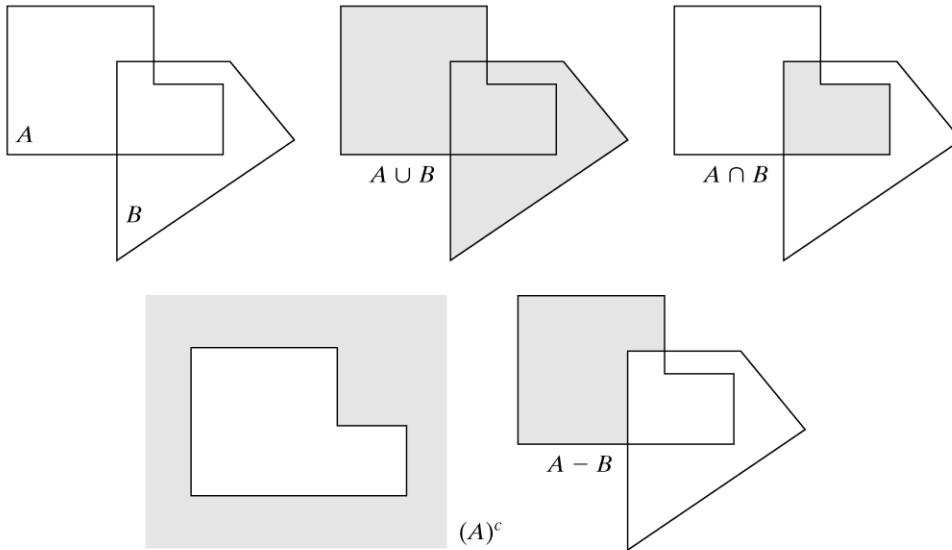
Introduction to Morphological Operators

Image – Set of Pixels

- Basic idea:
 - Object/Region = set of pixels (or coordinates of pixels)
- 0 = background
- 1 = foreground



Object = set of pixels (or coordinates of pixels)



a b c
d e

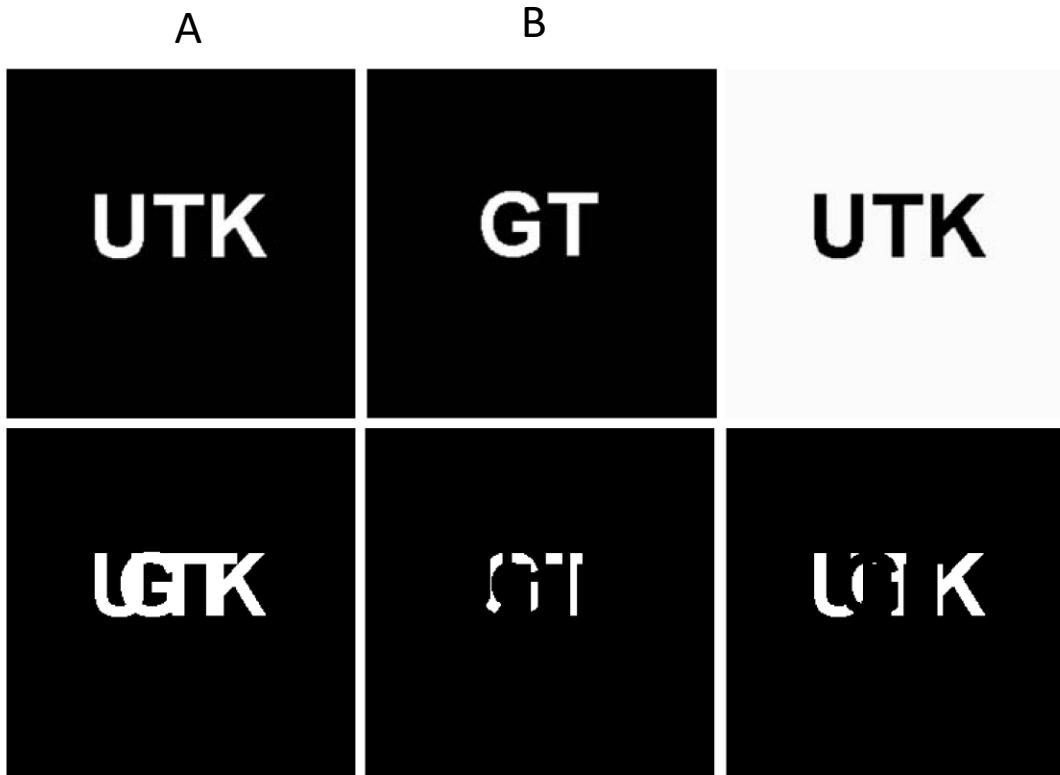
FIGURE 9.1

- (a) Two sets A and B .
- (b) The union of A and B .
- (c) The intersection of A and B .
- (d) The complement of A .
- (e) The difference between A and B .

Basic operations on shapes

From: Digital Image Processing, Gonzalez, Woods And Eddins

Set Operations on Binary Images



Structuring Element

3x3

1	1	1
1	1	1
1	1	1

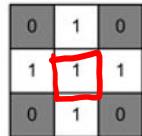
5x5

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

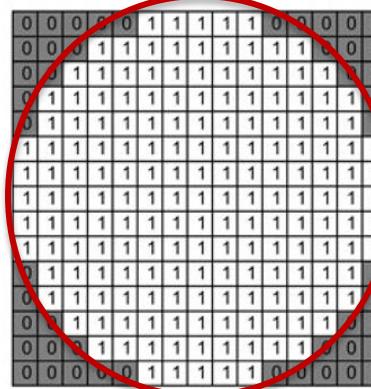
15x15

Box

Disc

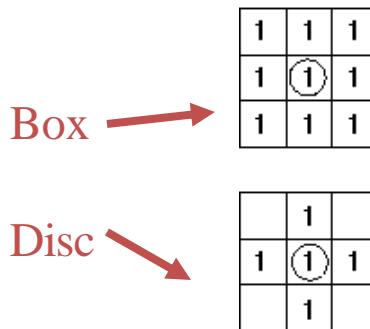
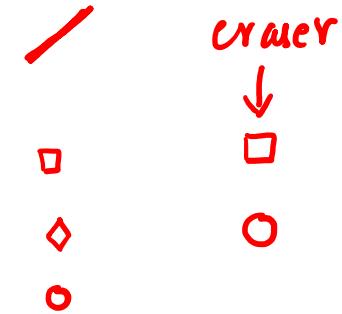


0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0



Structuring Element (Kernel)

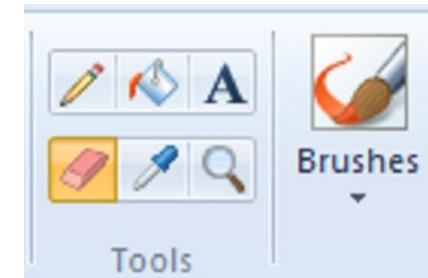
- Can have varying sizes
- Have an origin
- Usually, element values are 0,1 and none(!)
 - For thinning, other values are possible
- Empty spots in the Structuring Elements are *don't care's!*



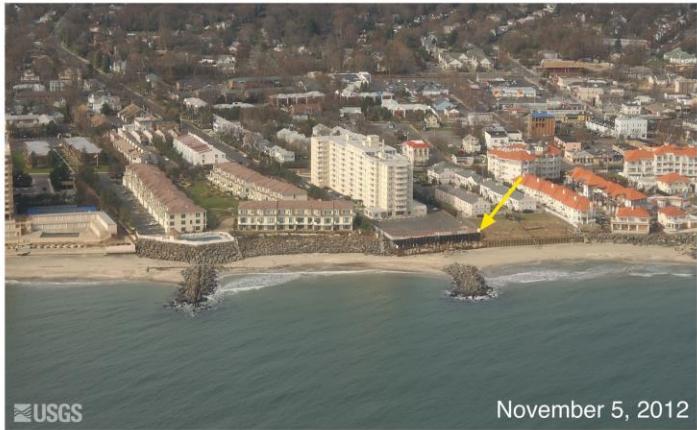
	1	1	1	1		
1	1	1	1	1	1	
1	1	1	1	1	1	1
1	1	1	1	(1)	1	1
1	1	1	1	1	1	1
	1	1	1	1	1	

1	1	
1	(0)	
1		0

1	1	1
1	(0)	1
1	1	1



Erosion



Erosion



SE

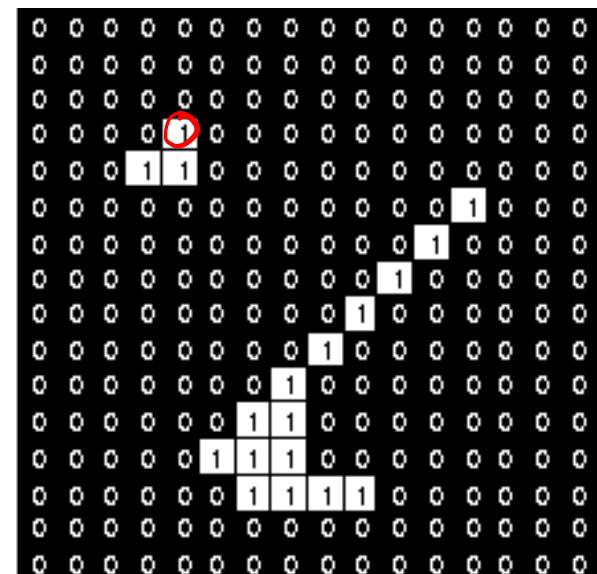
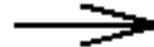
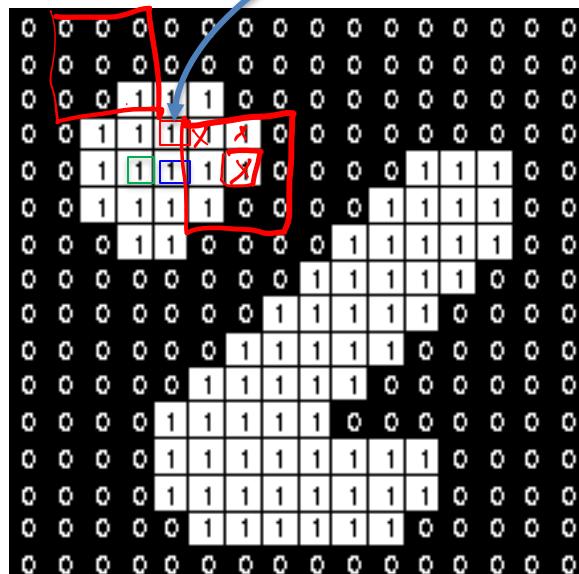
Erosion : Effect

1	1	1
1	1	1
1	1	1

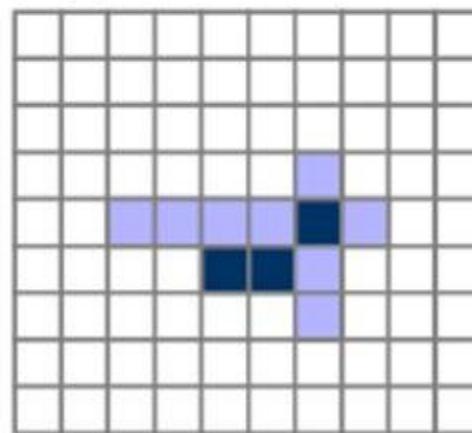
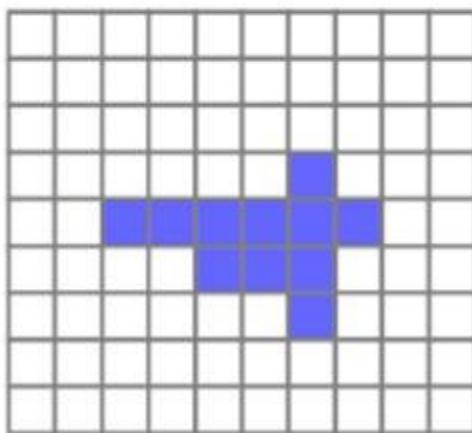
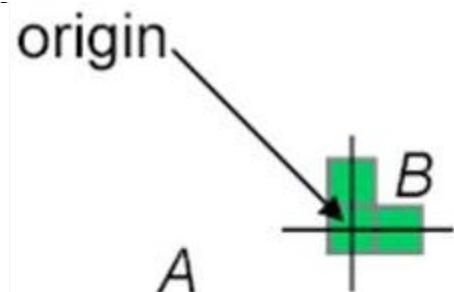
Set of coordinate points =

{ (-1, -1), (0, -1), (1, -1),
(-1, 0), (0, 0), (1, 0),
(-1, 1), (0, 1), (1, 1) }

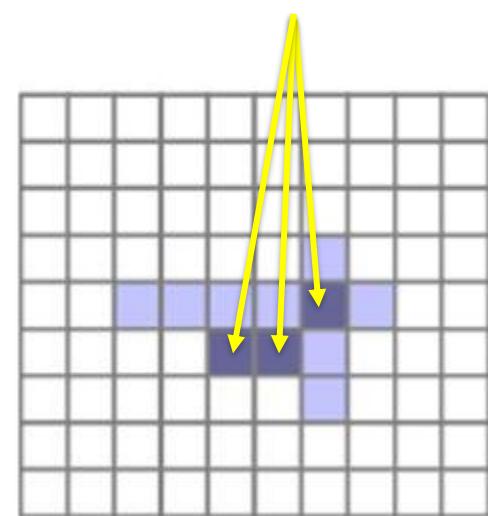
If, for a particular location of Structuring Element (SE) origin, SE lies fully within the region, retain the location, else set to 0



SEs operate wrt an origin

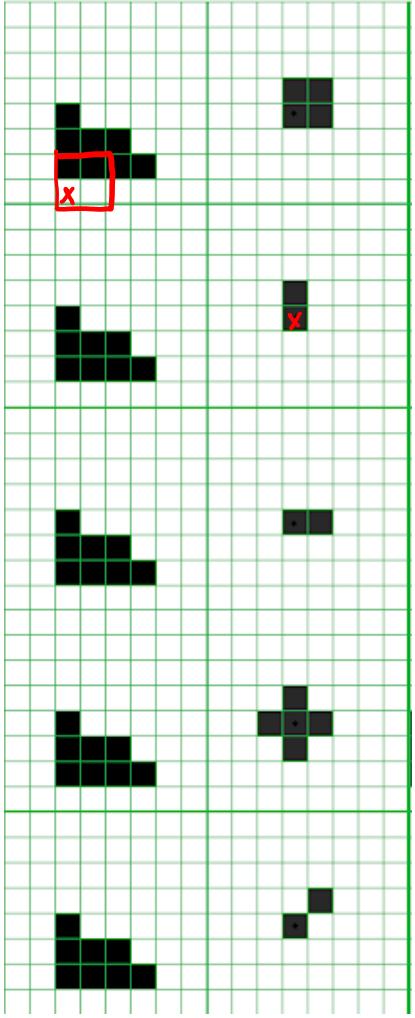


Pixels active after erosion

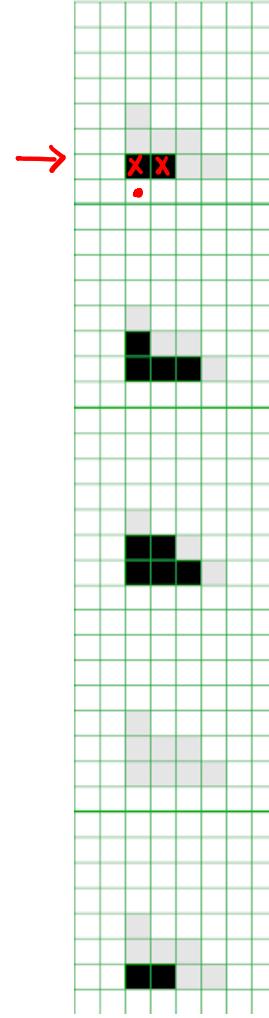


ORIGINAL IMAGE

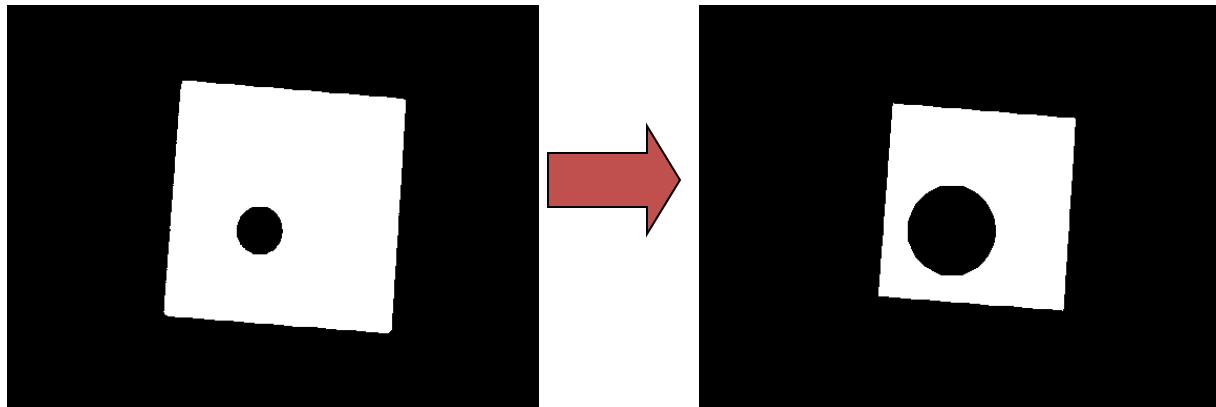
STRUCTURING ELEMENT



EROSION



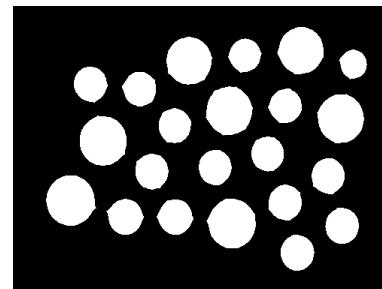
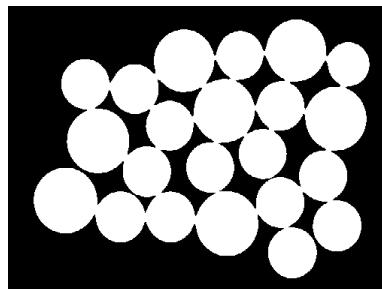
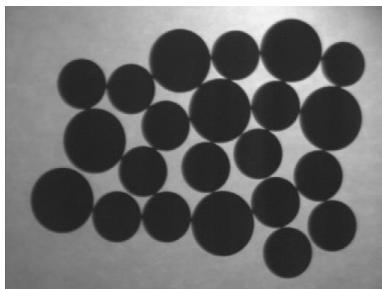
Another example of erosion



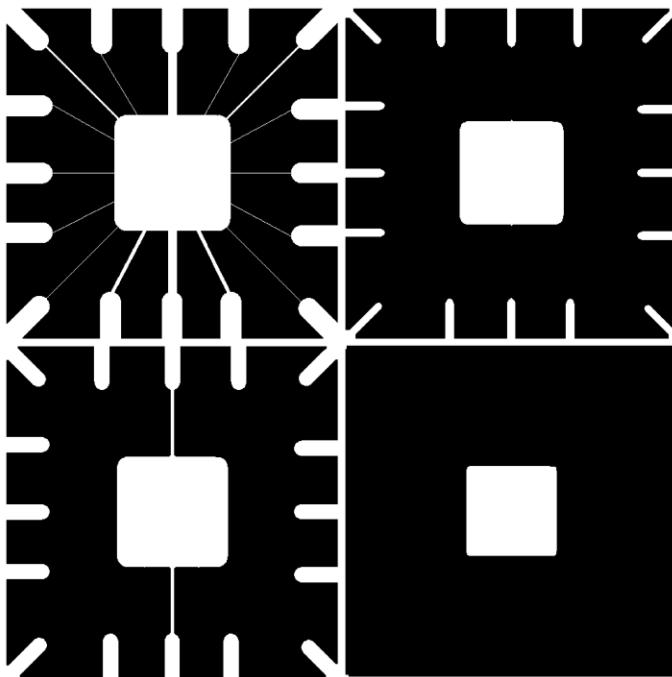
Erosion → Image gets darker

Example: Counting coins

- Difficult because they touch each other!
- Solution: Binarization and Erosion separates them!



Erosion - example



a	b
c	d

FIGURE 9.8 An illustration of erosion.

- (a) Original image.
- (b) Erosion with a disk of radius 10.
- (c) Erosion with a disk of radius 5.
- (d) Erosion with a disk of radius 20.



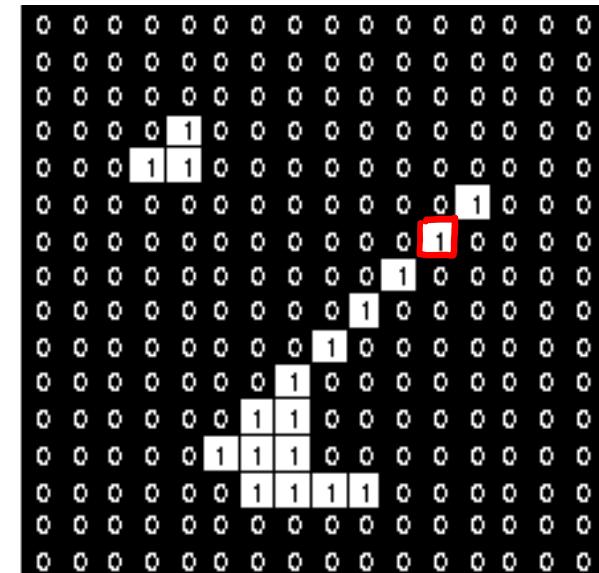
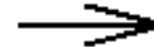
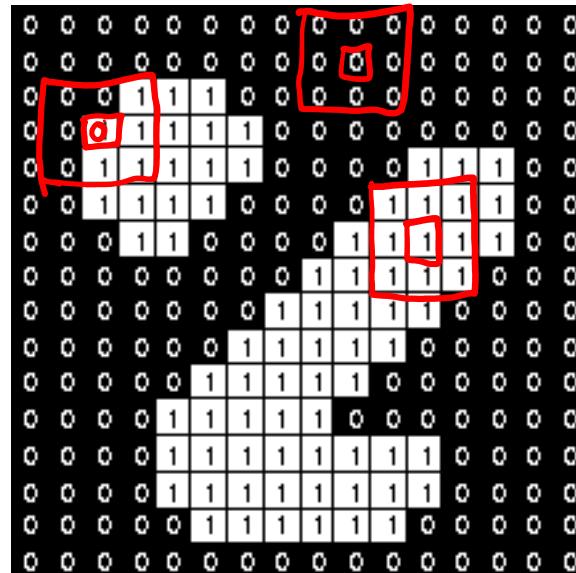
From: Digital Image Processing, Gonzalez, Woods
And Eddins

Erosion : Operation (min filter)

1	1	1
1	1	1
1	1	1

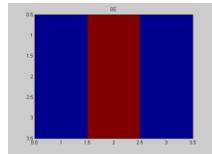
Set of coordinate points =

```
{ (-1, -1), (0, -1), (1, -1),
  (-1, 0), (0, 0), (1, 0),
  (-1, 1), (0, 1), (1, 1) }
```

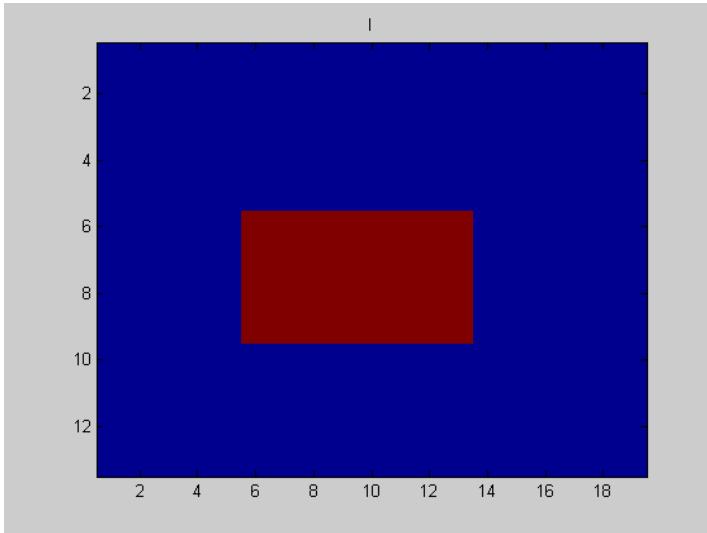


MATLAB code

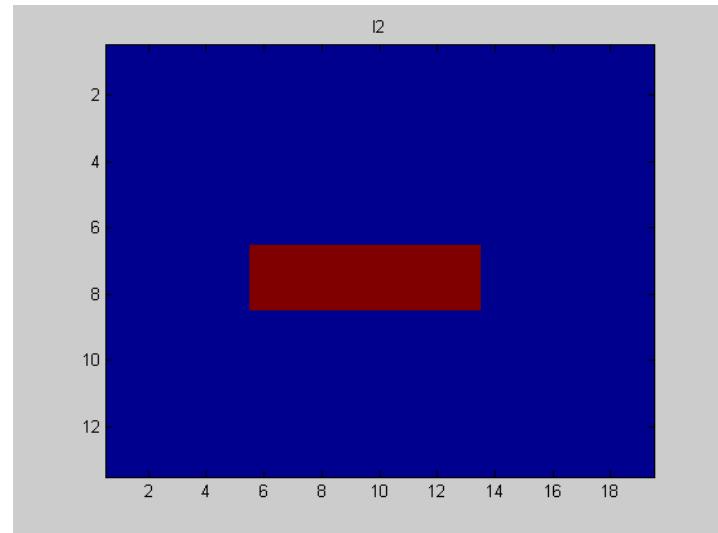
0 1 1 0



SE = 3x3



I2



I3=imerode(I2,SE);

Erosion

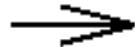
- Shrinks foreground objects
- Foreground holes are enlarged
- Small (relative to structuring element size) foreground objects are removed.

Dilation

1	1	1
1	1	1
1	1	1

A 15x15 binary matrix where the central 3x3 area contains all 1's. The rest of the matrix is filled with 0's.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1



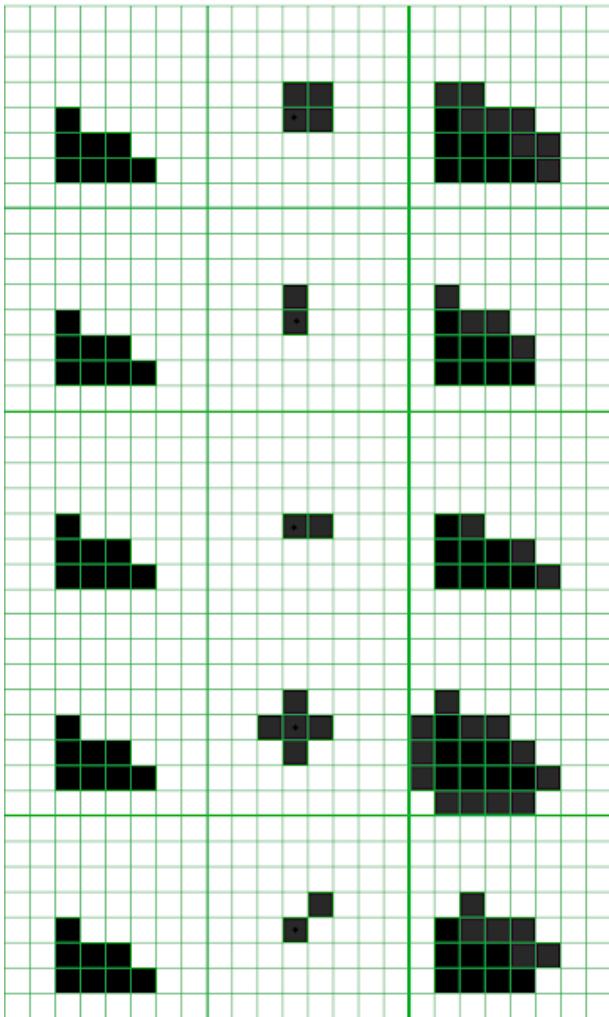
A 15x15 binary matrix resulting from the dilation operation. The central 3x3 area contains all 1's, and the 5x5 kernel has been applied to the entire matrix. The original 1's from the input matrix remain at their original positions, while the 0's are replaced by 1's from the kernel. The 1's in the kernel overlap with the 1's in the input matrix.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

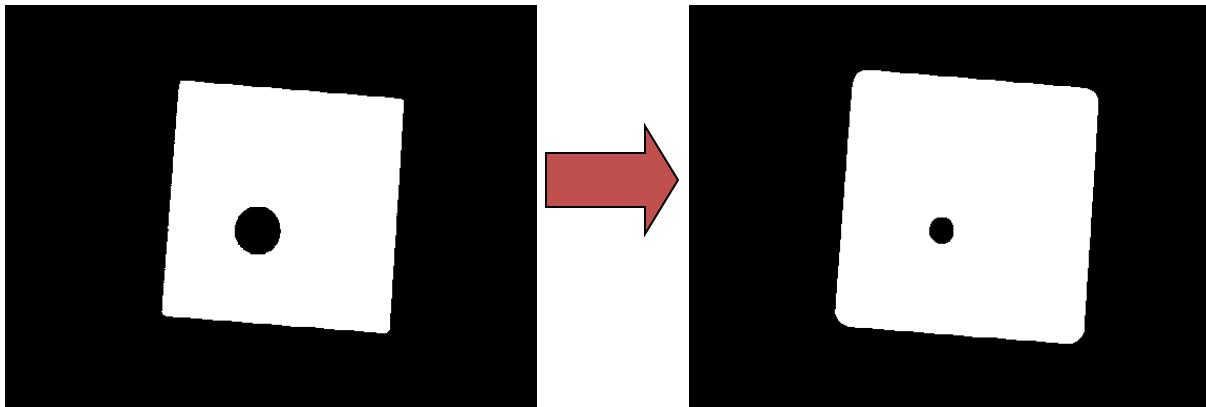
ORIGINAL IMAGE

STRUCTURING
ELEMENT

DILATION



Dilation Example



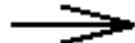
- Image gets lighter, more uniform intensity
- NOTE-1: SE = disk
- NOTE-2: Multiple iterations of dilation

Dilation (max filter)

1	1	1
1	1	1
1	1	1

A 16x16 binary matrix representing an image. A 3x3 kernel of ones is applied across the entire matrix. The result is a 16x16 matrix where every element is 1. The original input values are 0.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1



A 16x16 binary matrix representing the result of the dilation operation. The result is a 16x16 matrix where every element is 1. The original input values are 0.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



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0	1	0
1	1	1
0	1	0

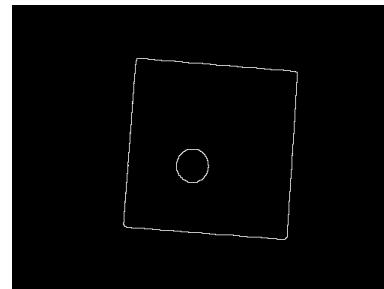
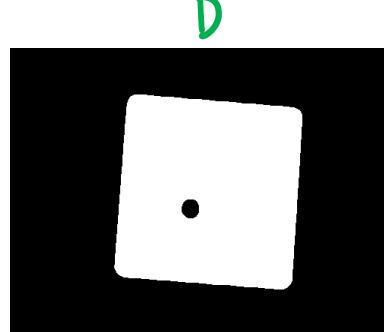
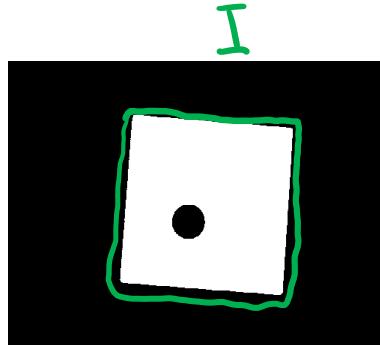
Figure 1: Image before (left) and after (right) dilation with the structuring element shown at the bottom

Dilation

- Expands foreground objects
- Foreground holes are shrunk

Boundary Detection

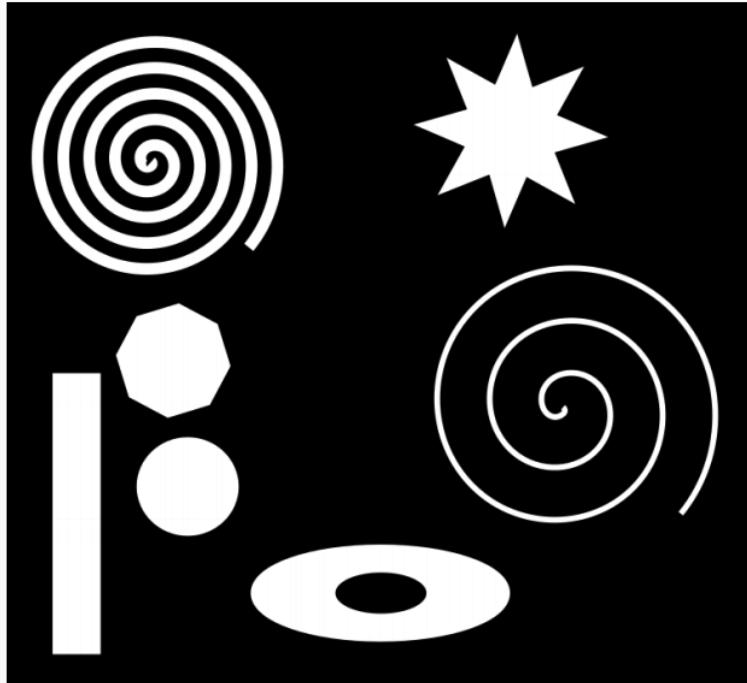
1. Dilate input image
2. Subtract input image from dilated image $D - I$
3. Boundaries remain!



Can use erosion also ..



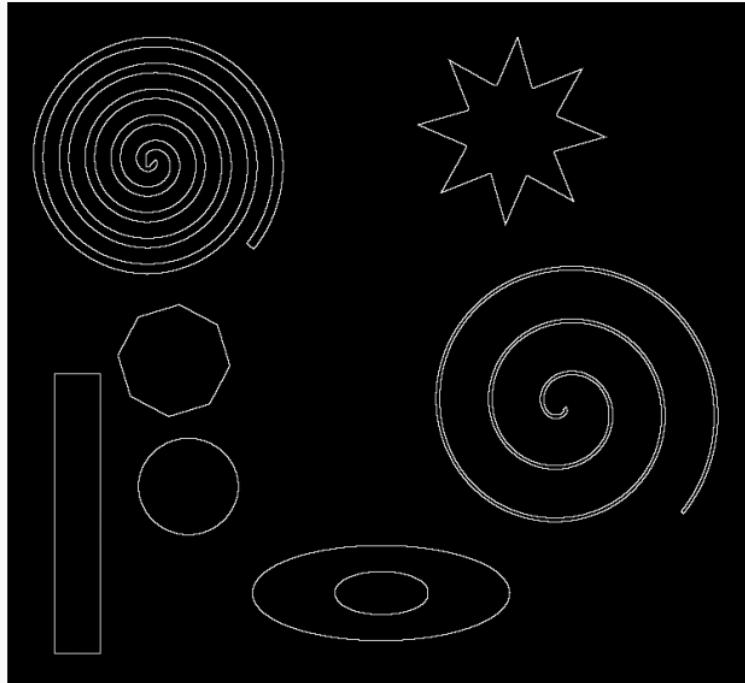
Fig 3: (a) Original Image (linkon.tif) (B) After erosion operation (C) Boundary Extraction with the help of Erosion.



(a) f

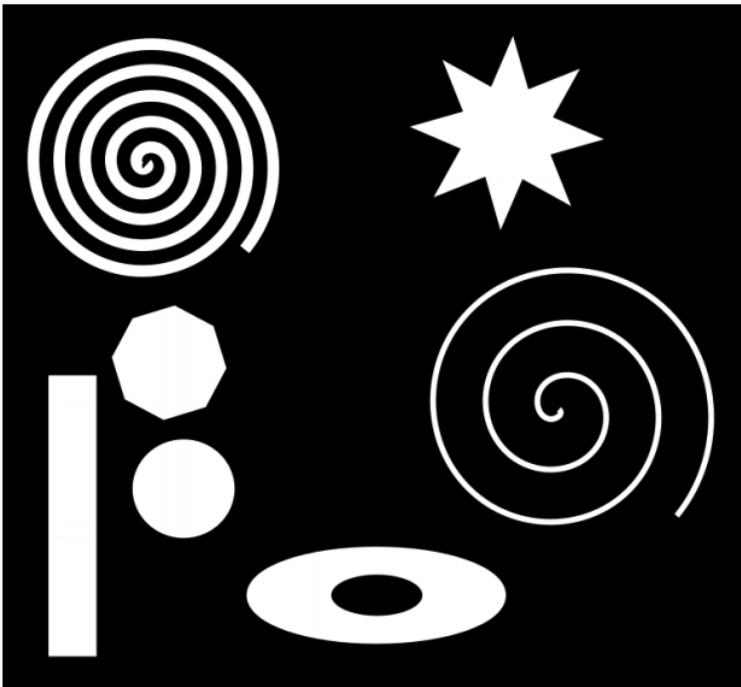
1	1	1
1	1	1
1	1	1

(b) s



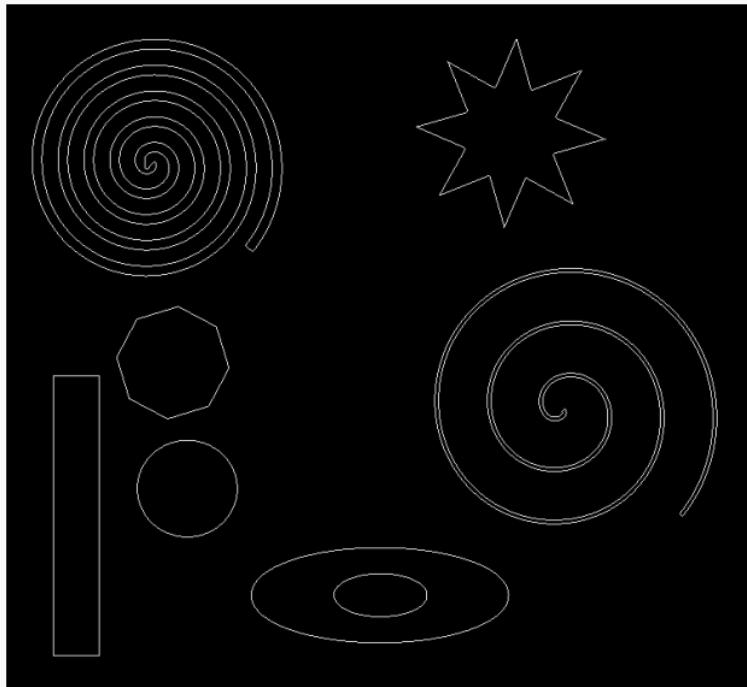
(c) $f - (f \ominus s)$

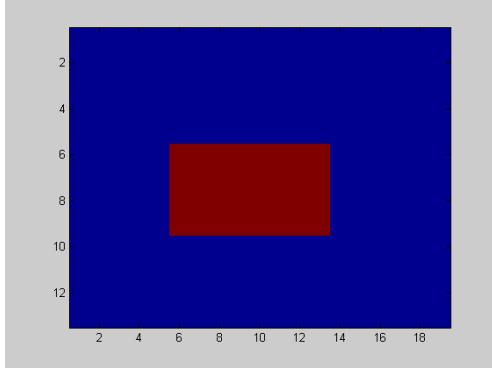
8-connectivity



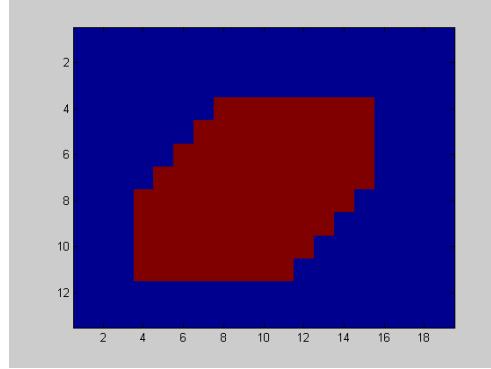
0	1	0
1	1	1
0	1	0

(b) s

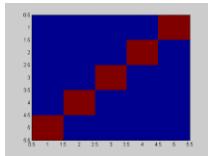




I



I2



SE

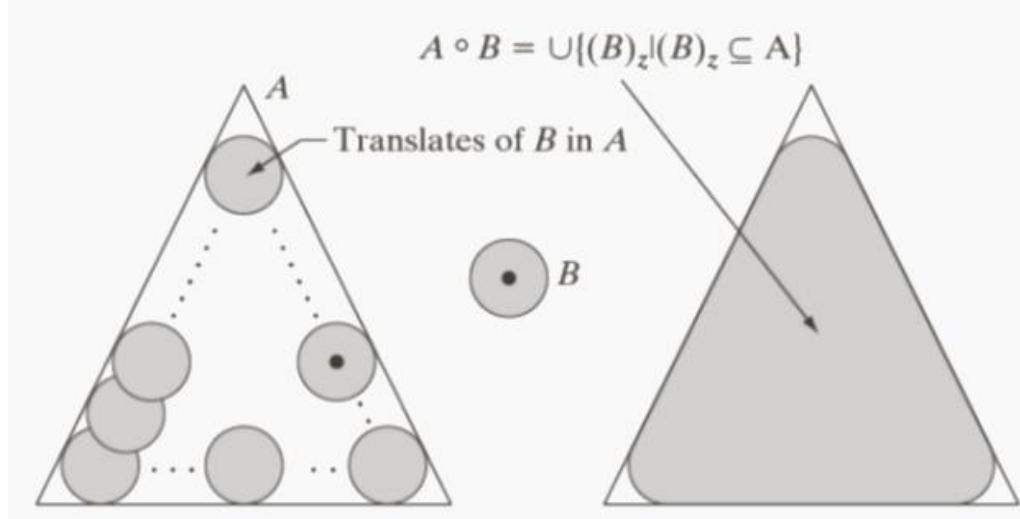
```
>> I(6:9,6:13)=1;  
>> figure, imagesc(I)  
>> I2=imdilate(I,SE);  
>> figure, imagesc(I2)
```

Opening and Closing

- Important operations
- Derived from the fundamental operations
 - Dilation ✓
 - Erosion ✓

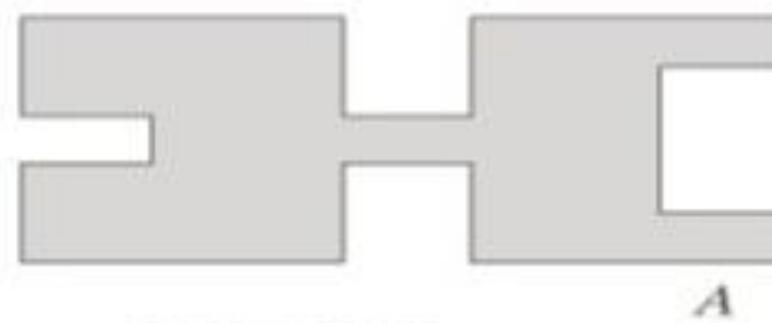
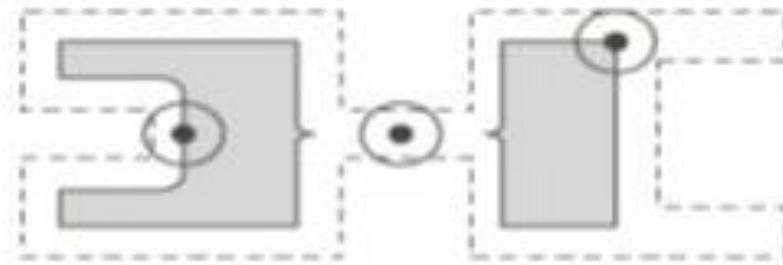
Opening

- Take the structuring element (SE) and slide it around *inside* each foreground region.
 - All pixels which can be covered by the SE with the SE being entirely within the foreground region will be preserved.
 - All foreground pixels which can *not* be reached by the structuring element without lapping over the edge of the foreground object will be eroded away!



Opening

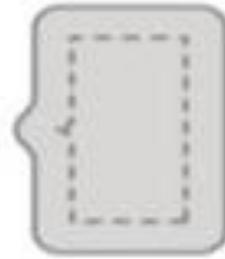
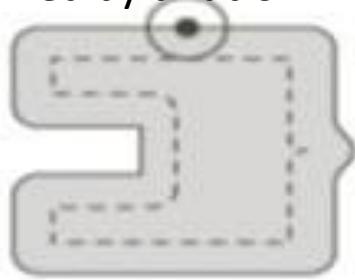
Erosion



A



Followed by dilation ...



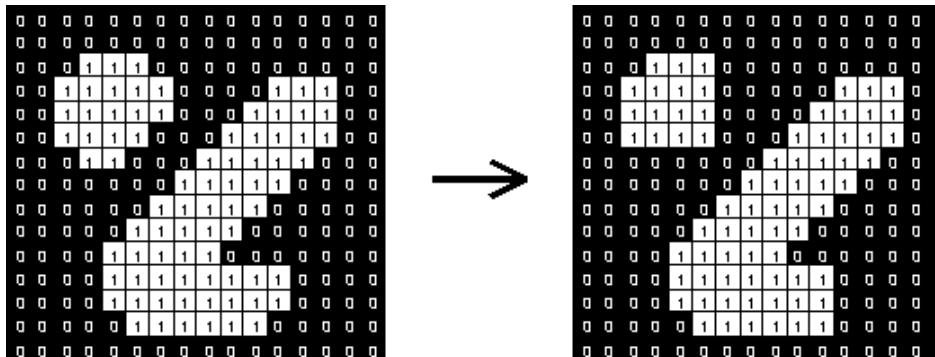
$A \ominus B$



$$A \circ B = (A \ominus B) \oplus B$$

Opening

- Structuring element: 3x3 square



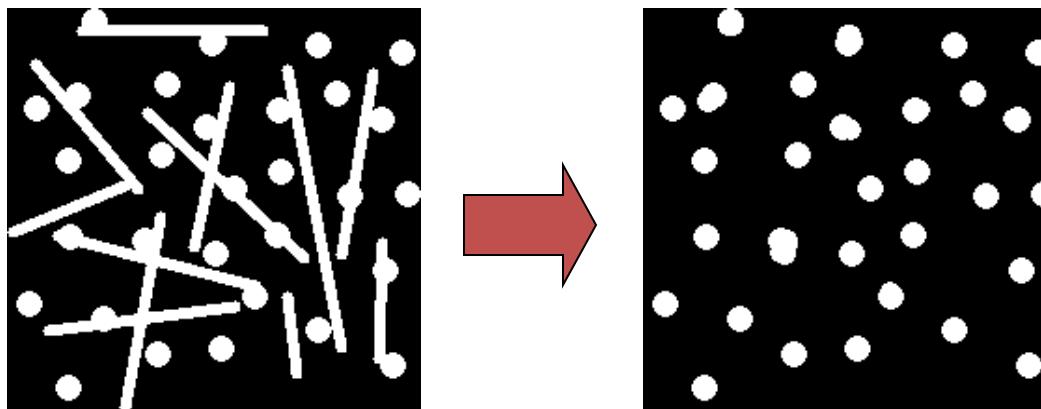
Take the structuring element (SE) and slide it around *inside* (each) foreground region

All foreground pixels which can *not* be reached by the structuring element without lapping over the edge of the foreground object will be eroded away!

All pixels which can be covered by the SE with the SE being entirely within the foreground region will be preserved.

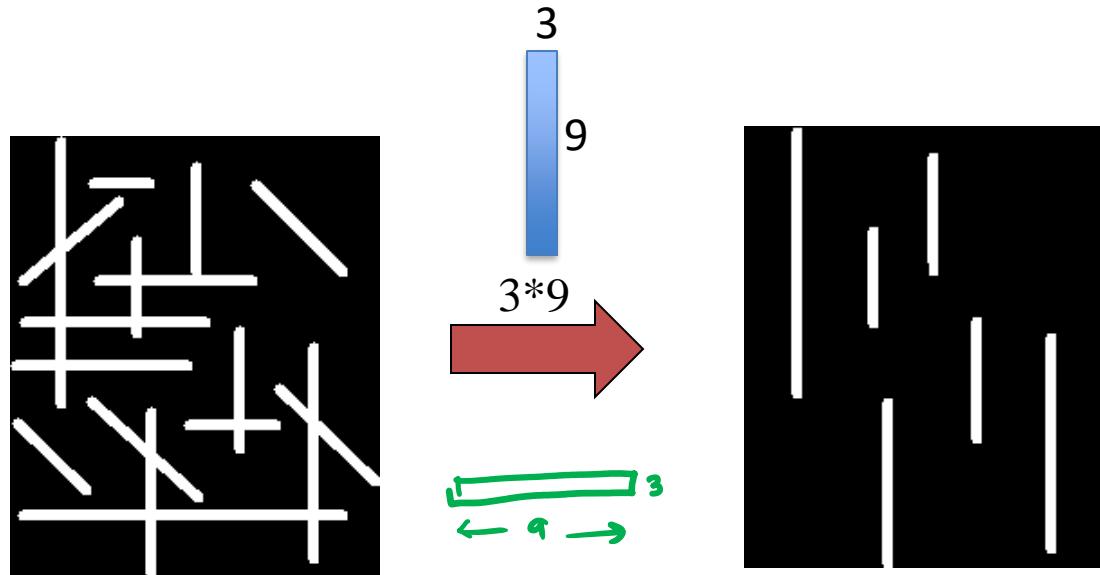
Opening: Example

- Opening with a 11 pixel diameter disc



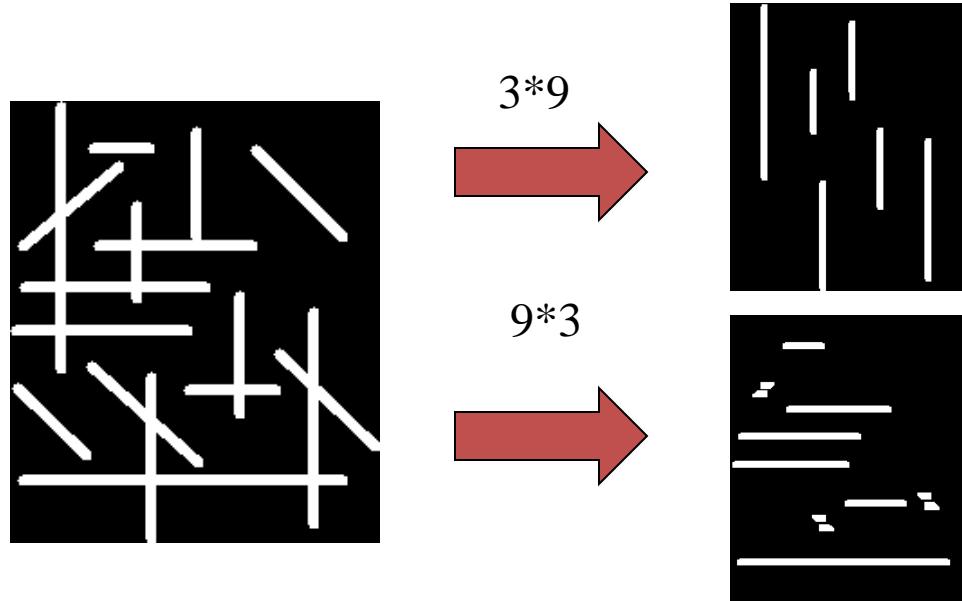
Opening: Another Example

- 3x9 Structuring Element

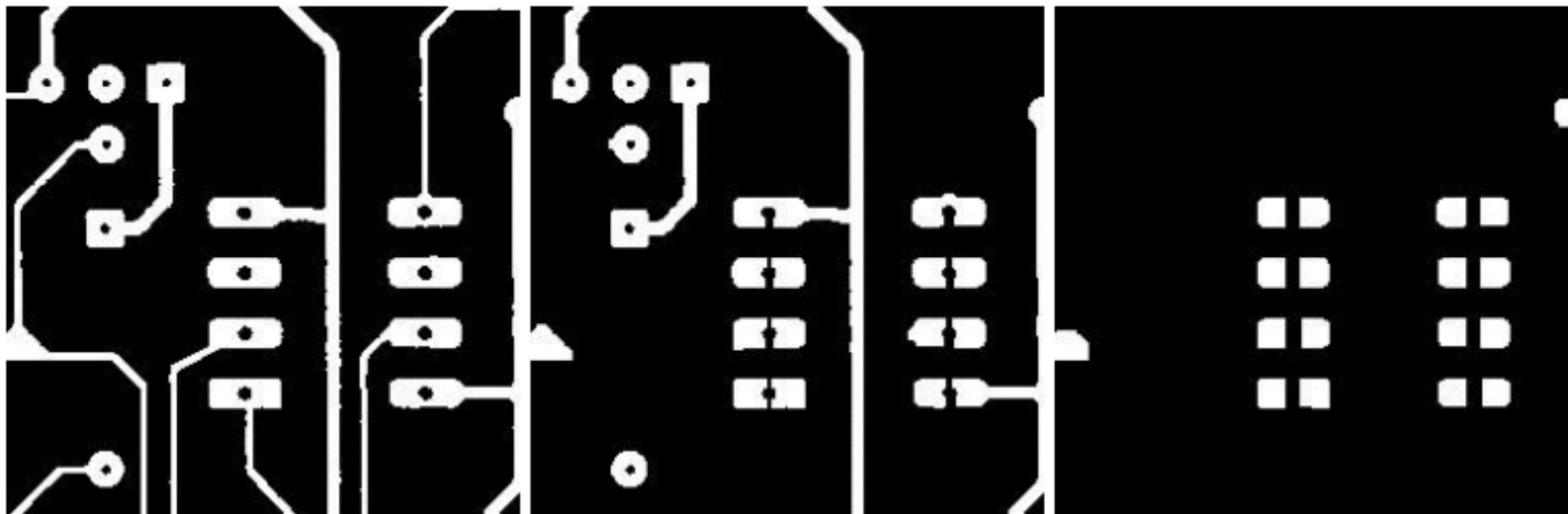


Opening: Another Example

- 3x9 and 9x3 Structuring Element



Opening



Binary image f

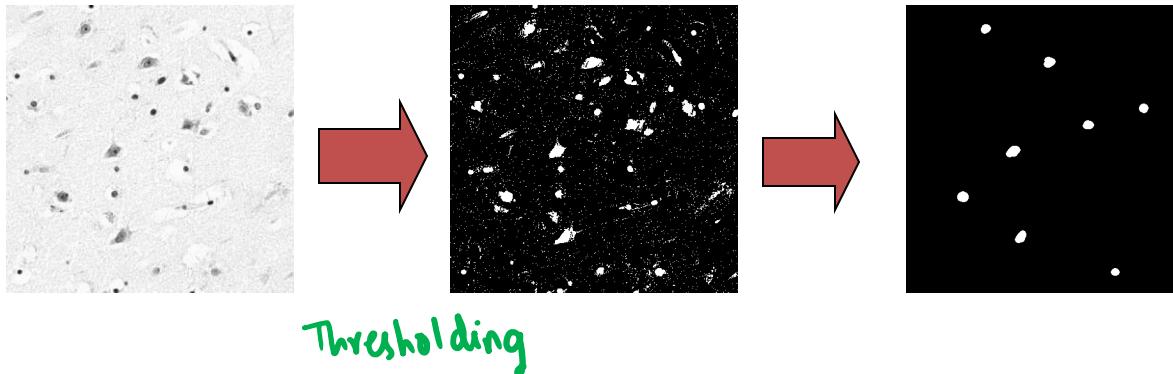
$f \circ s$ (5x5 square)

$f \circ s$ (9x9 square)

Results of opening with a square structuring element (www.mmorph.com/html/morph/mmopen.html).

Use Opening for Separating Blobs

- Use large structuring element that fits into the big blobs
- Structuring Element: 11 pixel disc



Opening

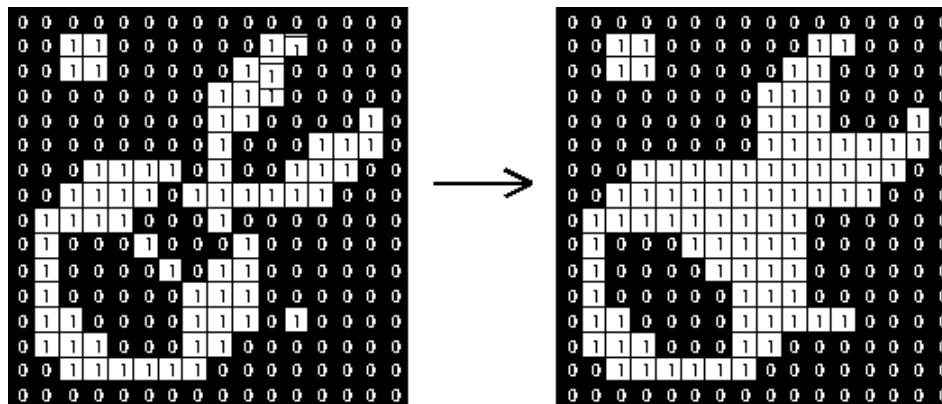
- Similar to Erosion
 - Spot and noise removal
 - Less destructive
- Erosion followed by Dilation *using the same SE*
 - the *same structuring element for both operations.*
- Opening is **idempotent**: Repeated application has no further effects!

$$\underline{B} = \text{Open}(I)$$

$$B = \text{Open}(B)$$

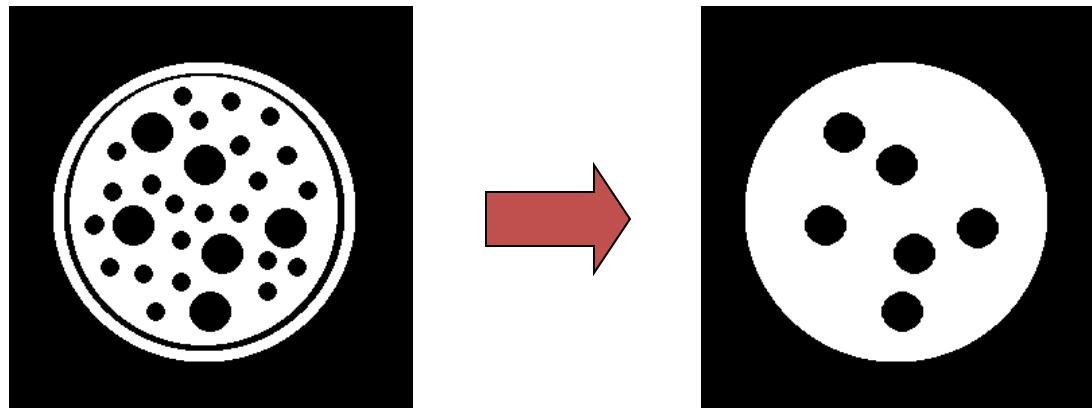
Closing (Dilation then Erosion)

- Structuring element: 3x3 square



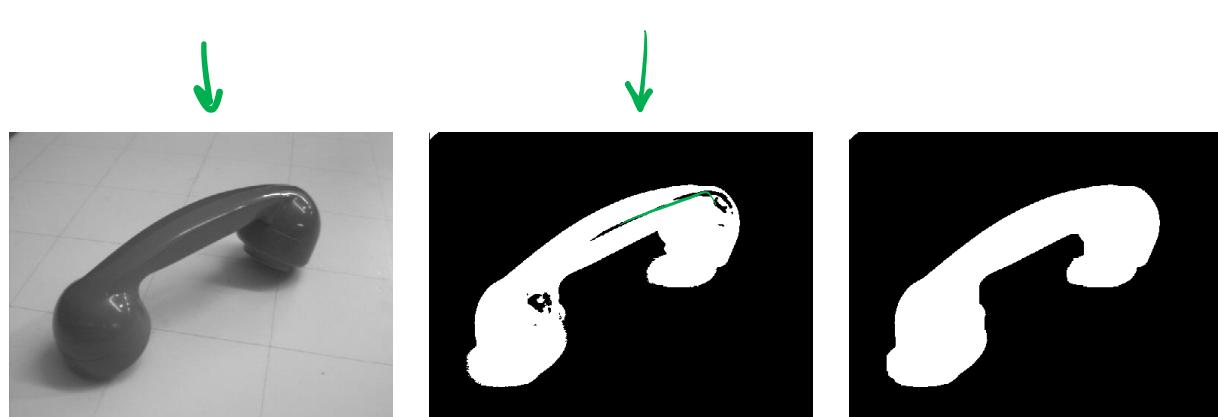
Closing: Example

- Closing operation with a 22 pixel disc
- Closes small holes in the foreground



Closing Example 1

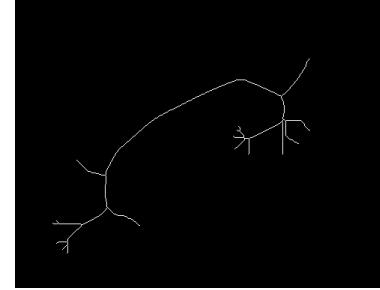
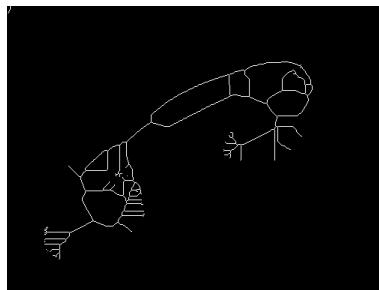
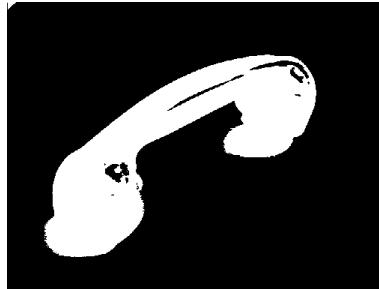
1. Threshold
2. Closing with disc of size 20



Thresholded closed

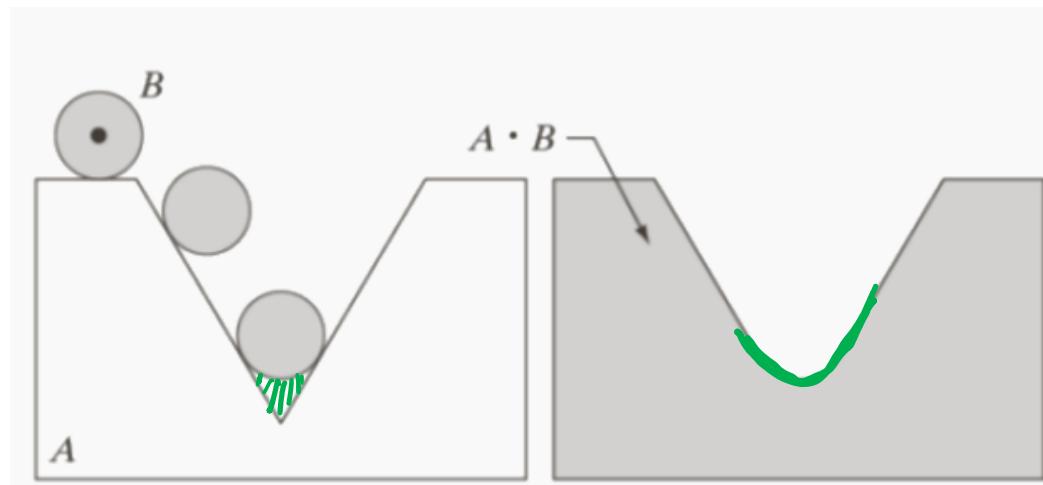
Application of Closing

- Good for further processing: E.g. Skeleton operation looks better for closed image!



Closing

- Take the structuring element (SE) and slide it around outside each foreground region.
- Closing = Regions not covered by the structuring element when doing the above operation.



Opening & Closing

- Opening is the *dual* of closing
- *i.e.* opening the foreground pixels with a particular structuring element
- is equivalent to closing the background pixels with the same element.

0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	0	0	0
0	1	1	1	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	1	1	1	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

(a) f

1	1	1
1	1	1
1	1	1

(b) s

0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

(c) $f \circ s$

1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	0	0	1	1
1	0	0	0	0	1	1	1	0	1	1
1	0	0	0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1
1	1	0	2	1	1	1	1	0	0	1
1	1	0	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	0	0	1	1
1	0	0	0	0	1	1	1	1	1	1

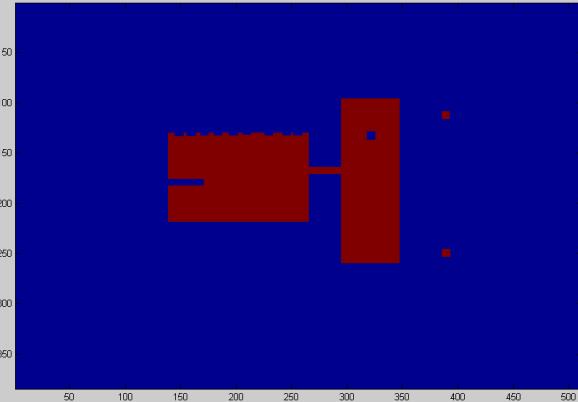
(a) f^c

1	1	1
1	1	1
1	1	1

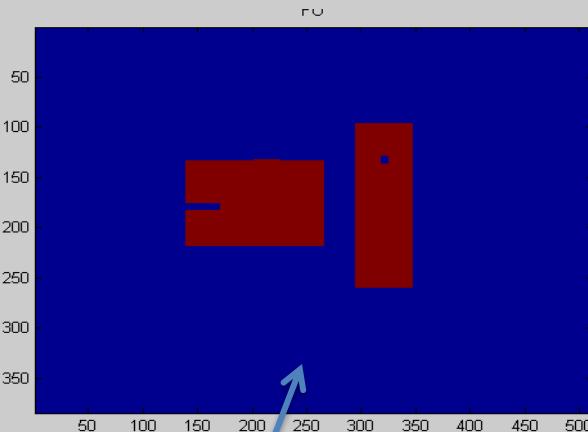
(b) s

1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1

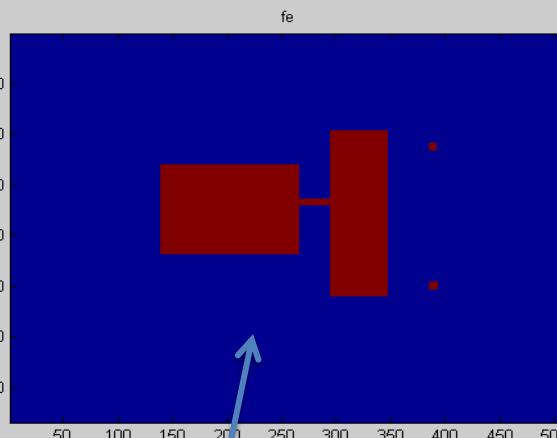
(c) $f^c \bullet s$



ORIGINAL



OPENING



CLOSING

Fingerprint problem



FIGURE 9.11 (a) Noisy fingerprint image. (b) Opening of image. (c) Opening followed by closing. (Original image courtesy of the National Institute of Standards and Technology.)

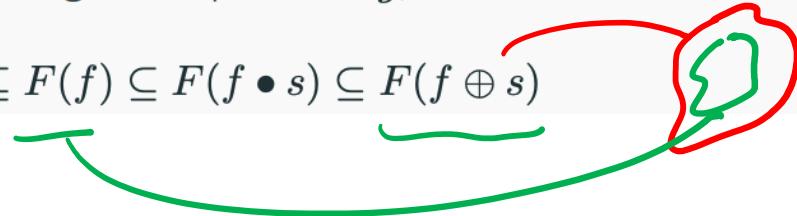
MAGNITUDE RELATIONS

- Dilation and closing are *extending operations*, meaning that foreground pixels are added to the image.
- Erosion and opening are *narrowing operations*, meaning that foreground pixels are removed.
- For a binary image f and a binary structuring element s , we have that

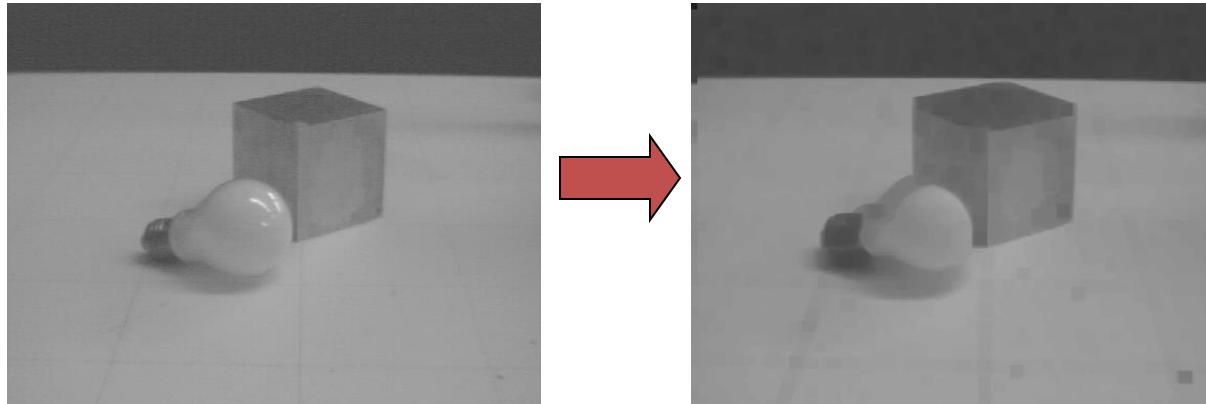
$$\rightarrow \underbrace{(f \ominus s)(x)}_{\text{EROSION}} \leq \underbrace{(f \circ s)(x)}_{\text{OPENING}} \leq \underbrace{f(x)}_{\text{CLOSING}} \leq \underbrace{(f \bullet s)(x)}_{\text{DILATION}} \leq (f \oplus s)(x) \leftarrow$$

- On a similar note, if $F(g)$ is the set of foreground pixels in g ,

$$F(f \ominus s) \subseteq F(f \circ s) \subseteq F(f) \subseteq F(f \bullet s) \subseteq F(f \oplus s)$$

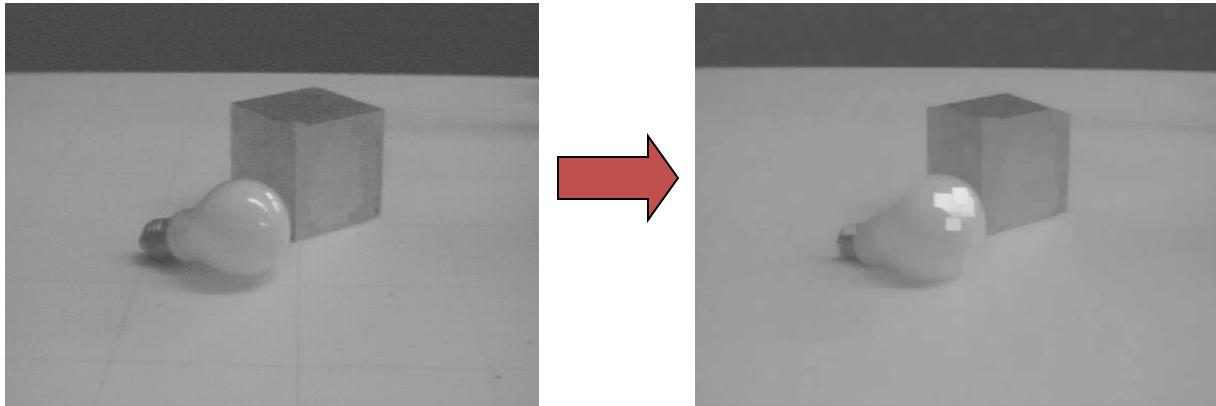


Erosion on Gray Value Images



- min filter
- Images get darker!

Dilation on Gray Value Images



- max filter
- More uniform intensity

References

- G&W, 3rd Ed., 9.1-9.3, 9.6
- <https://in.mathworks.com/help/images/morphological-dilation-and-erosion.html>
- https://scikit-image.org/docs/dev/auto_examples/applications/plot_morphology.html#sphx-glr-auto-examples-applications-plot-morphology-py

Scribe List

2018102018
2018102019
2018102022
2018102027
2018102028
2018102031