

# SMAI-M20-Lec 15 Review questions

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## Review Question - I (one, none or more correct)

1. Let  $X = UDV^T$ . Then
  - 1.1 Columns of U are eigenvectors of  $X^T X$
  - 1.2 Columns of V are eigenvectors of  $X^T X$
  - 1.3 Rows of U are eigenvectors of  $X^T X$
  - 1.4 Rows of V are eigenvectors of  $X^T X$
  - 1.5 None of these

Ans: B  $X^T X = VD^2 V^T \implies X^T X V = VD^2 = V\Lambda$

## Review Question - II (one, none or more correct)

Let  $X = UDV^T$ . Then

1. Columns of  $U$  are eigenvectors of  $XX^T$
2. Columns of  $V$  are eigenvectors of  $XX^T$
3. Rows of  $U$  are eigenvectors of  $XX^T$
4. Rows of  $V$  are eigenvectors of  $XX^T$
5. None of these

Ans: A  $XX^T = UD^2U^T \implies XX^T U = UD^2 = U\Lambda$

## Review Question - III (one, none or more correct)

Consider  $X$  to be a square matrix of size  $n \times n$  and  $X = UDV^T$ .

1. Both  $X^T X$  and  $XX^T$  have the same eigenvalues
2. Both  $X^T X$  and  $XX^T$  have the same eigenvectors
3.  $X$ ,  $XX^T X$  and  $XX^T$  have the same eigenvalues
4.  $D^2$  contains the eigenvalues of  $X^T X$  on its diagonal
5.  $D$  contains the eigenvalues of  $X^T X$  on its diagonal
6. None of these

Ans: AD

## Review Question - IV (one, none or more correct)

Consider  $X$  to be a square matrix of size  $n \times n$  and  $X = UDV^T$ .  
Then:

1. If  $\text{rank}(X) = n$ ,  $D$  has all non-zero entries in diagonal.
2. If  $\text{rank}(X) = k$ ,  $D$  has  $k$  zeros in diagonal
3. If  $\text{rank}(X) = k$ ,  $D$  has  $n - k$  zeros in diagonal
4. if  $\text{rank}(X) = n$  but  $|A|$  is a very small number then,  $D$  takes the form  $D = \text{diag}(d_1, d_2, \dots, \epsilon)$  where  $\epsilon$  is a very small number
5. None of these

Ans: ACD

Suppose you want to apply PCA to your data  $X$  which is in 2D and you decompose  $X$  as  $UDV^T$ . Then,

1. PCA can be useful if all elements of  $D$  are equal
2. PCA can be useful if all elements of  $D$  are not equal
3.  $D$  is not full-rank if all points in  $X$  lie on a straight line
4.  $V$  is not full-rank if all points in  $X$  lie on a straight line
5.  $D$  is not full-rank if all points in  $X$  lie on a circle
6. None of these

Ans: BC