Estimating Causal Effects Using Proxy Interference Networks

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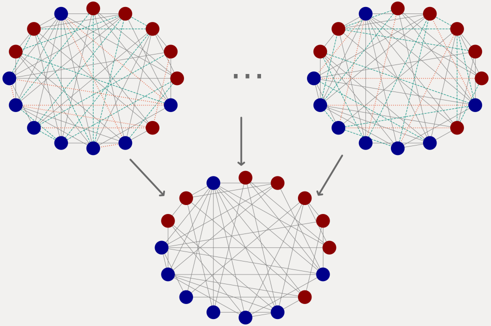

Background

- **Causal Inference.** Estimate the effect of treatment on an outcome.
- **Interference.** Treatment of one unit affect the outcomes of others.
- Treatments spreads through a network.
  - Nodes: units; Edges: magntiude of pairwise interfernce.

- **Causal Inference.** Estimate the effect of treatment on an outcome.
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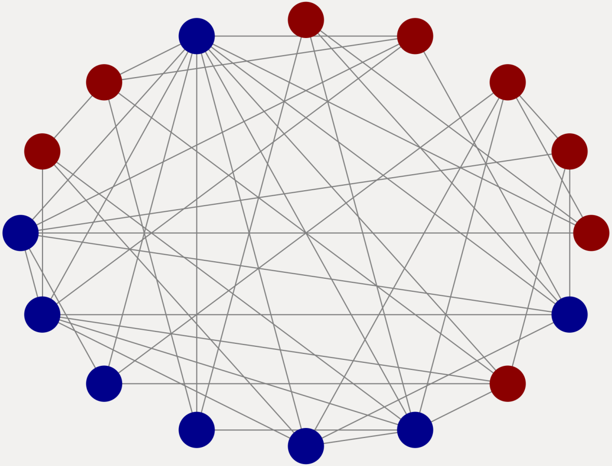
- Examples:
  - *Social networks.* Transmission of information, behavior, encouragements, etc.
  - *Epidemiology.* Mitigating spread of infectious diseases or addictive drugs.
  - A/B testing in marketplaces.



The Challenge

- Accurately measuring social networks is challenging.
- We observe only proxy measurements of the true network.
  - Measurements error.
  - Multiple sources of data.
  - Multilayer networks.
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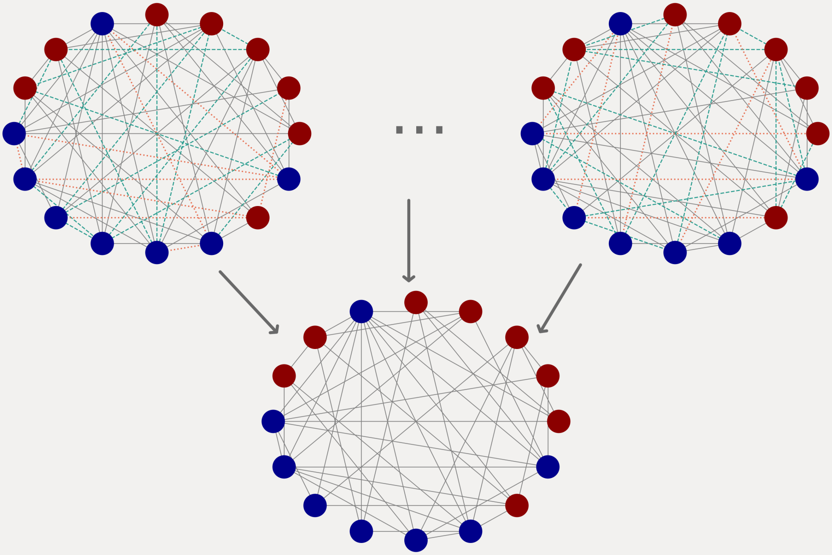
How can we estimate causal effects using proxy networks?



Illustrative Example – Paluck et al. (2016)

- Field experiement in 56 middle-schools.
- Study how anti-conflict education spread through social networks.
- Measured social networks using self-reported friendships.
  - Bi-layer networks: frequently interacted and best friends.
  - Measured at pre- and post-intervention period.

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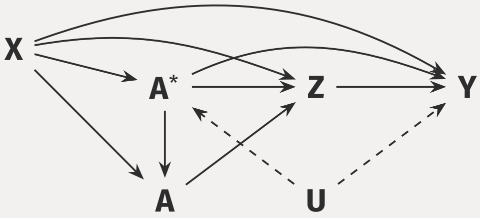
- Measured social networks using self-reported friendships.
  - Bi-layer networks: frequently interacted and best friends.
  - Measured at pre- and post-intervention period.
- Which of the networks, if any, is the true network?
- **Objective:** Estimate the intervention effects using the proxy networks.

Formal Setup

- Finite population $i \in \{1, \dots, N\}$.
- Treatments: $\mathbf{Z} \in \{0, 1\}^N$.
- Outcomes: $\mathbf{Y} \in \mathbb{R}^N$.
- Covariates/features: $\mathbf{X}$.
- True interference network: $\mathbf{A}^* \in \{0, 1\}^{N \times N}$.
- Proxy networks: $\mathbf{A} = \left(\mathbf{A}^1, \dots, \mathbf{A}^B\right)$.

Structural Causal Model

- Population-level directed acyclic graph (DAG):



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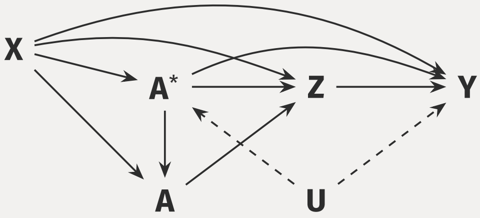

- Requires probabilistic models:
  1. *True network.* $p\left(\mathbf{A}^* | \mathbf{X}, \theta\right)$.
  2. *Proxy networks.* $p\left(\mathbf{A} | \mathbf{A}^*, \mathbf{X}, \gamma\right)$.
  3. *Outcomes.* $p\left(\mathbf{Y} | \mathbf{Z}, \mathbf{A}^*, \mathbf{X}, \eta\right)$.

Causal Estimands

Causal effects are the impact of hypothetical interventions on **Z**.

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1. **Static.** $\mathbb{E}\left[Y_i | do(\mathbf{Z} = \mathbf{z}), \mathbf{X}, \mathbf{A}^*\right]$.
   - Treating all ($\mathbf{z} = 1$) versus none ($\mathbf{z} = 0$).
2. **Dynamic.** $\mathbb{E}\left[Y_i | do(\mathbf{Z} = h(\mathbf{X}, \mathbf{A}^*)), \mathbf{X}, \mathbf{A}^*\right]$.
   - Treating units with specific features, e.g., above certain age.
3. **Stochastic.** $\mathbb{E}_{\pi_\alpha(\mathbf{Z})}\mathbb{E}\left[Y_i | do(\mathbf{Z}), \mathbf{X}, \mathbf{A}^*\right]$.
   - Expected impact of randomly treating α_1 vs α_0 percent of units.

Bayesian Inference

- Observed data $\mathbf{O} = (\mathbf{Y}, \mathbf{Z}, \mathbf{X}, \mathbf{A})$. Latent variables $(\mathbf{A}^*, \eta, \gamma, \theta)$.

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- Posterior distribution:

$$\begin{aligned} p(\mathbf{A}^*, \eta, \gamma, \theta | \mathbf{O}) \propto\ & p(\mathbf{Y}|\mathbf{Z}, \mathbf{A}^*, \mathbf{X}, \eta)p(\eta) \\ & \times p(\mathbf{A}|\mathbf{A}^*, \mathbf{X}, \gamma)p(\gamma) \\ & \times p(\mathbf{A}^*|\mathbf{X}, \theta)p(\theta). \end{aligned}$$

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- Propose two sampling schemes:
  1. *Modularization.* “break” the posterior into smaller, more manageable parts.
  2. *Gibbs sampling.* Sample discrete with *Local Informed Proposals*.

Simulations

Two figures: MAPE of estimated treatment effects and MAE of exposure mapping.

Data Analysis

Results of Paluck et al. (2016) analysis.

Figure 1: Figure caption.

	Heading 1	Heading 2
Row 1	v_{11}	v_{12}
Row 2	v_{21}	v_{22}
Row 3	v_{31}	v_{32}

Table 1: Table caption.

Thanks for using **Focus**!

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Appendix

More math details.