Estimating Causal Effects Using Proxy Interference Networks

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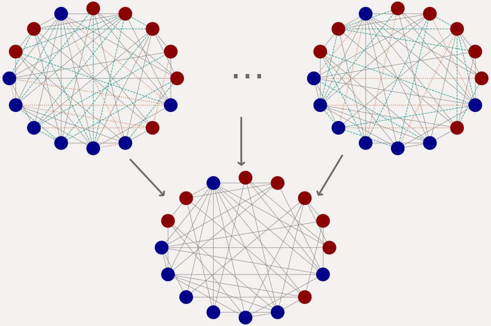

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Illustrative Example – Paluck et al. (2016)

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- **Objective:** Estimate the intervention effects using the proxy networks.

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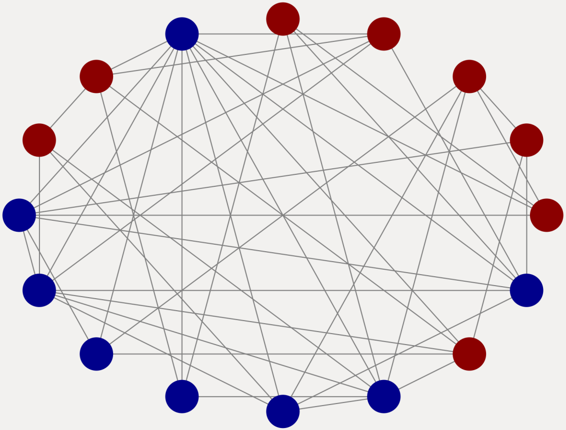
- Treatments spreads through a **network**.
    - Nodes: units. Edges: pairwise interactions.
- Examples:
    - *Social networks.* Information & behavior spread.
    - *Public health.* transmisstion of infectious diseases or addictive drugs.
    - A/B testing in marketplaces.



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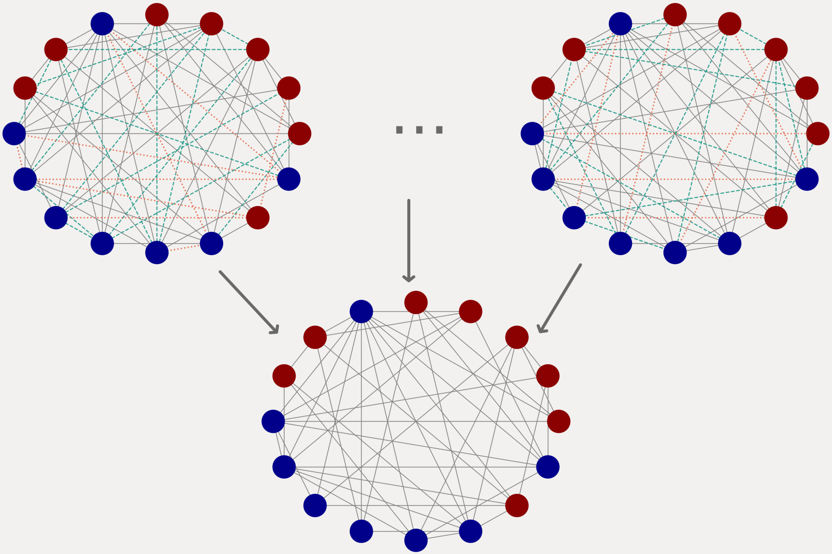
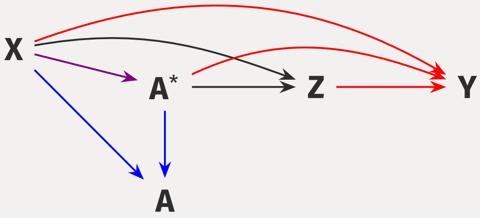
How can we estimate causal effects using proxy networks?



Formal Setup

- Finite population $i \in \{1, \dots, N\}$.
- Treatments: $\mathbf{Z} \in \{0, 1\}^N$.
- Outcomes: $\mathbf{Y} \in \mathbb{R}^N$.
- Covariates/features: $\mathbf{X}$.
- True interference network: $\mathbf{A}^* \in \{0, 1\}^{N \times N}$.
- Proxy networks: $\mathbf{A} = \left(\mathbf{A}^1, \dots, \mathbf{A}^B\right)$.

- Population-level directed acyclic graph (DAG):



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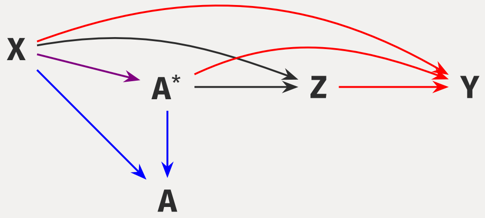

- Requires probabilistic models:
  1. *True network.* $p(\mathbf{A}^*|\mathbf{X}, \theta)$.
  2. *Proxy networks.* $p(\mathbf{A}|\mathbf{A}^*, \mathbf{X}, \gamma)$.
  3. *Outcomes.* $p(\mathbf{Y}|\mathbf{Z}, \mathbf{A}^*, \mathbf{X}, \eta)$.

Causal Estimands

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2. **Dynamic.** $\mathbb{E}\left[Y_i | do(\mathbf{Z} = h(\mathbf{X}, \mathbf{A}^*)), \mathbf{X}, \mathbf{A}^*\right]$.
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3. **Stochastic.** $\mathbb{E}_{\pi_\alpha(\mathbf{Z})}\mathbb{E}\left[Y_i | do(\mathbf{Z}), \mathbf{X}, \mathbf{A}^*\right]$.
   - Randomly treating α_1 vs α_0 percent of units.

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Bayesian Inference

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- Propose two sampling schemes:
  1. *Modularization.* "break" the posterior into smaller, more manageable parts.
  2. *Gibbs sampling.* Sample discrete with *Local Informed Proposals*.

Simulations

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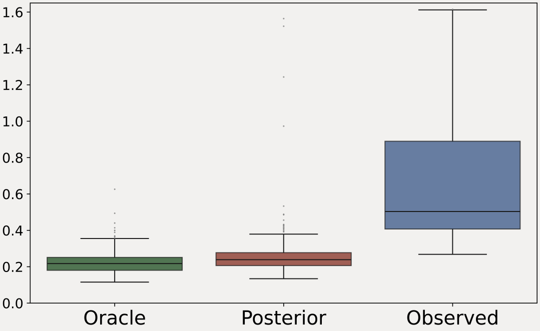

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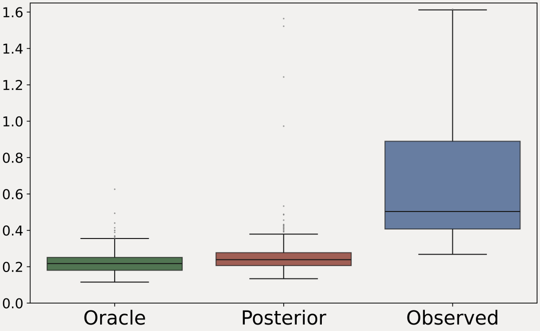

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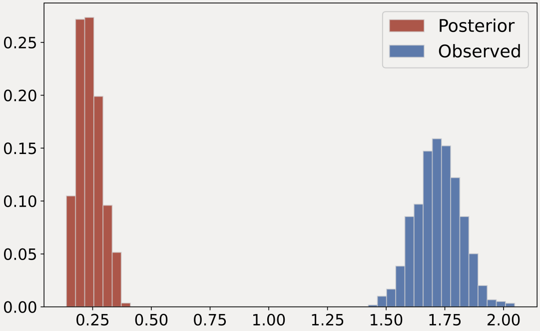

Figure 2: MAE (←) of network statistics.

Data Analysis - Paluck et al. (2016)

- Outcome is indicator of anti-conflict behavior.
- Stochastic estimand.
- Four available networks.
  - Use one as ”Observed” true network.
  - Analysis using different combination of the four proxy networks.

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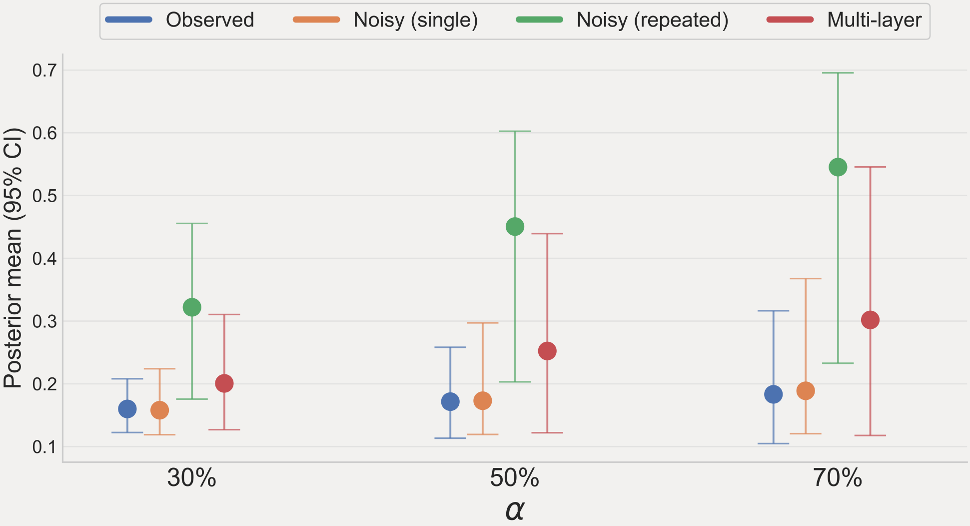

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- Correctly measuring social relations is often impossible.
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- Bayesian framework for inference. Computation is challenging.

Thank You!

Appendix

More math details.