

# 432 Class 12 Slides

[github.com/THOMASELOVE/2020-432](https://github.com/THOMASELOVE/2020-432)

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# Setup

```
library(magrittr); library(janitor); library(here)
library(knitr)
library(naniar)
library(broom)

library(mice)
  # mice = multiple imputation through chained equations

library(tidyverse)

theme_set(theme_bw())
```

# Today's Goals

Use multiple imputation to deal with missing data in fitting:

- linear regression with `lm`
- logistic regression with `glm`

using the `mice` package. (MICE = Multiple Imputation through Chained Equations)

## Useful (if somewhat dated) Sources

- <https://thomasleeper.com/Rcourse/Tutorials/mi.html>.
- <https://stats.idre.ucla.edu/r/faq/how-do-i-perform-multiple-imputation-using-predictive-mean-matching-in-r/>

# Multiple Imputation: Potential and Pitfalls

### Multiple imputation for missing data in epidemiological and clinical research: potential and pitfalls

*In this article, we review the reasons why missing data may lead to bias and loss of information in epidemiological and clinical research. We discuss the circumstances in which multiple imputation may help by reducing bias or increasing precision, as well as describing potential pitfalls in its application. Finally, we describe the recent use and reporting of analyses using multiple imputation in general medical journals, and suggest guidelines for the conduct and reporting of such analyses.*

- <https://www.bmj.com/content/338/bmj.b2393>
- <https://doi.org/10.1136/bmj.b2393>

# Types of Missing Data (from Sterne et al.)

- **Missing completely at random** There are no systematic differences between the missing values and the observed values. For example, blood pressure measurements may be missing because of breakdown of an automatic sphygmomanometer.
- **Missing at random** Any systematic difference between the missing values and the observed values can be explained by differences in observed data. For example, missing blood pressure measurements may be lower than measured blood pressures but only because younger people may be more likely to have missing blood pressure measurements.
- **Missing not at random** Even after the observed data are taken into account, systematic differences remain between the missing values and the observed values. For example, people with high blood pressure may be more likely to miss clinic appointments because they have headaches

“Missing at random” is an assumption that justifies the analysis, not a property of the data.

# Pitfalls When Using Multiple Imputation (Sterne et al.)

## Data that are missing not at random

- Some data are inherently missing not at random because it is not possible to account for systematic differences between the missing values and the observed values using the observed data.
- In such cases multiple imputation may give misleading results. Those results can be either more or less misleading than a complete case analysis.
- For example, consider a study investigating predictors of depression. If individuals are more likely to miss appointments because they are depressed on the day of the appointment, then it may be impossible to make the missing at random assumption plausible, even if a large number of variables is included in the imputation model.
- Where complete cases and multiple imputation analyses give different results, the analyst should attempt to understand why, and this should be reported in publications.

# Ways to Deal with Missing Data (from Sterne et al.)

- There are circumstances in which analyses of **complete cases** will not lead to bias. Missing data in predictor variables do not cause bias in analyses of complete cases if the reasons for the missing data are unrelated to the outcome. Specialist methods to address missing data may lessen the loss of precision and power resulting from exclusion of individuals with incomplete predictor variables but are not required in order to avoid bias.
- **Single Imputation** of missing values usually causes standard errors to be too small, since it fails to account for the fact that we are uncertain about the missing values.
- If we assume data are missing at random, then unbiased and statistically more powerful analyses (compared with analyses based on complete cases) can generally be done by including individuals with incomplete data.



# Multiple Imputation (from Sterne et al.)

- Multiple imputation ... aims to allow for the uncertainty about the missing data by creating several different plausible imputed data sets and appropriately combining results obtained from each of them.
- The first stage is to create multiple copies of the dataset, with the missing values replaced by imputed values. The imputation procedure must fully account for all uncertainty in predicting the missing values by injecting appropriate variability into the multiple imputed values; we can never know the true values of the missing data.

# Multiple Imputation (from Sterne et al.)

- The second stage is to use standard statistical methods to fit the model of interest to each of the imputed datasets. Estimated associations in each of the imputed datasets will differ because of the variation introduced in the imputation of the missing values, and they are only useful when averaged together to give overall estimated associations. Standard errors are calculated using Rubin's rules, which take account of the variability in results between the imputed datasets, reflecting the uncertainty associated with the missing values.
  - Valid inferences are obtained because we are averaging over the distribution of the missing data given the observed data.

## A Small Example (Sterne et al.)

Consider, for example, a study investigating the association of systolic blood pressure with the risk of subsequent coronary heart disease, in which data on systolic blood pressure are missing for some people.

The probability that systolic blood pressure is missing is likely to:

- decrease with age (doctors are more likely to measure it in older people),
- decrease with increasing body mass index, and
- decrease with history of smoking (doctors are more likely to measure it in people with heart disease risk factors or comorbidities).

If we assume that data are missing at random and that we have systolic blood pressure data on a representative sample of individuals within strata of age, smoking, body mass index, and coronary heart disease, then we can use multiple imputation to estimate the overall association between systolic blood pressure and coronary heart disease.

# Today's Data

```
fram_raw <- read_csv(here("data/framingham.csv")) %>%  
  clean_names()
```

See <https://www.framinghamheartstudy.org/> for more details.

This particular data set has been used by lots of people, in many different settings, and variations of it are all over the internet. I don't know who the originators were.

# Data Cleanup

```
fram_10 <- fram_raw %>%  
  mutate(educ = fct_recode(factor(education),  
                            "Some HS" = "1",  
                            "HS grad" = "2",  
                            "Some Coll" = "3",  
                            "Coll grad" = "4")) %>%  
  mutate(obese = as.numeric(bmi >= 30)) %>%  
  rename(smoker = "current_smoker",  
         cigs = "cigs_per_day",  
         stroke = "prevalent_stroke",  
         highbp = "prevalent_hyp",  
         chol = "tot_chol",  
         sbp = "sys_bp", dbp = "dia_bp",  
         hrate = "heart_rate",  
         chd10 = "ten_year_chd") %>%  
  select(sbp, chd10, educ, smoker, cigs, bp_meds,  
         chol, bmi, obese, glucose)
```

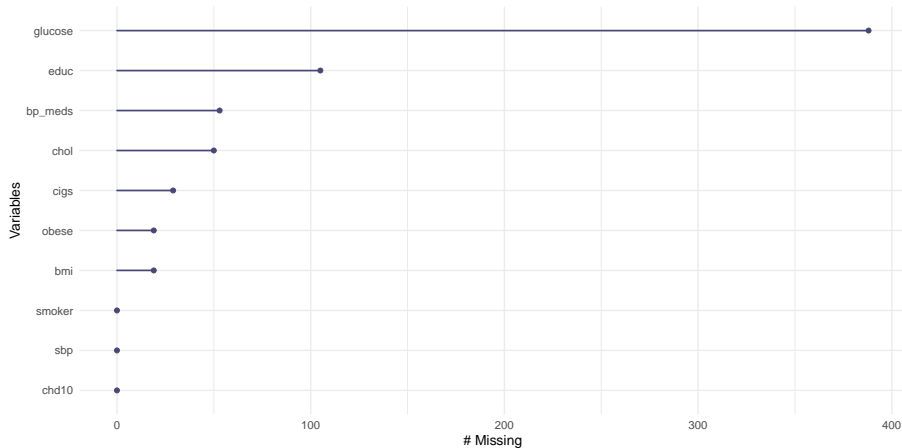
## Data Descriptions (variables we'll use today)

The variables describe  $n = 4238$  adult subjects who were examined at baseline and then followed for ten years to see if they developed incident coronary heart disease during that time.

Variable	Description
educ	four-level factor: educational attainment
smoker	1 = current smoker at time of examination, else 0
cigs	number of cigarettes smoked per day
bp_meds	1 = using anti-hypertensive medication at time of exam
chol	total cholesterol (mg/dl)
sbp	systolic blood pressure (mm Hg)
bmi	body mass index in $kg/m^2$
obese	1 if subject's bmi is 30 or higher, else 0
glucose	blood glucose level in mg/dl
chd10	1 = coronary heart disease in next 10 years

# Which variables are missing data?

```
gg_miss_var(fram_10)
```



# Counts of Missing Data, by Variable

```
miss_var_summary(fram_10) %>%  
  filter(n_miss > 0)
```

```
# A tibble: 7 x 3  
  variable n_miss pct_miss  
  <chr>     <int>    <dbl>  
1 glucose    388     9.16  
2 educ      105     2.48  
3 bp_meds     53     1.25  
4 chol       50     1.18  
5 cigs       29     0.684  
6 bmi        19     0.448  
7 obese      19     0.448
```

## Track missingness with shadow

```
fram_10_sh <- bind_shadow(fram_10)
```



# Two Key Settings for Multiple Imputation

- Use linear regression to predict sbp accounting for missingness via multiple imputation
  - Predictors include glucose, obese, educ, and smoker.
- Use logistic regression to predict chd10 accounting for missingness via multiple imputation
  - Predictors include glucose, bp\_meds, chol, bmi, cigs and educ

## Setting 1: Linear Model for sbp

## Model 2 (CC): Two-predictor model for sbp

Suppose we ignore the missingness and just run the model on the data with complete information on sbp, glucose and obese.

```
m2_cc <- fram_10_sh %$% lm(sbp ~ glucose + obese)
```

```
tidy(m2_cc, conf.int = TRUE) %>% select(-statistic) %>%  
  kable(digits = 3)
```

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	121.671	1.244	0	119.232	124.110
glucose	0.111	0.015	0	0.082	0.139
obese	13.532	1.045	0	11.484	15.580

## Edited Summary of Model 2 (CC)

Suppose we ignore the missingness and just run the model.

Residual standard error: 21.42 on 3833 degrees of freedom  
(402 observations deleted due to missingness)

Multiple R-squared: 0.05857, Adjusted R-squared: 0.05808

F-statistic: 119.2 on 2 and 3833 DF, p-value:  $< 2.2e-16$

```
glance(m2_cc) %>%  
  select(r.squared, adj.r.squared, AIC, BIC) %>%  
  kable(digits = c(4, 4, 0, 0))
```

r.squared	adj.r.squared	AIC	BIC
0.0586	0.0581	34401	34426

## Model 4 (CC): Four-predictor model for sbp

```
m4_cc <- fram_10_sh %$%  
  lm(sbp ~ glucose + obese + smoker + educ)  
  
tidy(m4_cc, conf.int = TRUE) %>% select(-statistic) %>%  
  kable(digits = 3)
```

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	127.107	1.388	0	124.385	129.829
glucose	0.106	0.015	0	0.078	0.135
obese	12.304	1.066	0	10.213	14.395
smoker	-4.704	0.699	0	-6.075	-3.332
educHS grad	-3.698	0.833	0	-5.332	-2.065
educSome Coll	-4.724	1.010	0	-6.704	-2.744
educColl grad	-5.954	1.158	0	-8.225	-3.683

## Edited Summary of Model 4 (CC)

Suppose we ignore the missingness and just run the model.

Residual standard error: 21.2 on 3733 degrees of freedom  
(498 observations deleted due to missingness)

Multiple R-squared: 0.08257, Adjusted R-squared: 0.0811

F-statistic: 56 on 6 and 3733 DF, p-value:  $< 2.2e-16$

```
glance(m4_cc) %>%  
  select(r.squared, adj.r.squared, AIC, BIC) %>%  
  kable(digits = c(4, 4, 0, 0))
```

r.squared	adj.r.squared	AIC	BIC
0.0826	0.0811	33466	33516

# Subset of Variables to be used in our models 2 and 4

```
fram_sub <- fram_10 %>%  
  select(sbp, glucose, obese, educ, smoker)  
  
miss_var_summary(fram_sub)
```

```
# A tibble: 5 x 3  
  variable n_miss pct_miss  
  <chr>      <int>    <dbl>  
1 glucose    388     9.16  
2 educ      105     2.48  
3 obese      19     0.448  
4 sbp         0      0  
5 smoker      0      0
```

## Create multiple imputations for this subset

```
set.seed(4322020)
```

```
fram_mice24 <- mice(fram_sub, m = 20)
```

```
iter imp variable
```

1	1	glucose	obese	educ
1	2	glucose	obese	educ
1	3	glucose	obese	educ
1	4	glucose	obese	educ
1	5	glucose	obese	educ
1	6	glucose	obese	educ
1	7	glucose	obese	educ
1	8	glucose	obese	educ
1	9	glucose	obese	educ
1	10	glucose	obese	educ
1	11	glucose	obese	educ
1	12	glucose	obese	educ
1	13	glucose	obese	educ



# Summary Information about Imputation Process

```
summary(fram_mice24)
```

Class: mids

Number of multiple imputations: 20

Imputation methods:

sbp	glucose	obese	educ	smoker
""	"pmm"	"pmm"	"polyreg"	""

PredictorMatrix:

	sbp	glucose	obese	educ	smoker
sbp	0	1	1	1	1
glucose	1	0	1	1	1
obese	1	1	0	1	1
educ	1	1	1	0	1
smoker	1	1	1	1	0

## Run Model 2 on each imputed data frame

```
m2_mods <- with(fram_mice24, lm(sbp ~ glucose + obese))  
summary(m2_mods)
```

```
# A tibble: 60 x 5
```

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	122.	1.17	104.	0.
2	glucose	0.111	0.0137	8.10	7.07e-16
3	obese	13.4	0.986	13.6	4.54e-41
4	(Intercept)	121.	1.16	104.	0.
5	glucose	0.120	0.0136	8.78	2.32e-18
6	obese	13.2	0.984	13.4	2.13e-40
7	(Intercept)	122.	1.17	104.	0.
8	glucose	0.108	0.0136	7.89	3.90e-15
9	obese	13.5	0.984	13.7	6.50e-42
10	(Intercept)	121.	1.18	103.	0.

```
# ... with 50 more rows
```

## Pool Results across the 20 imputations

```
m2_pool <- pool(m2_mods)
summary(m2_pool, conf.int = TRUE, conf.level = 0.95)
```

	estimate	std.error	statistic	df
(Intercept)	121.1956852	1.24145482	97.62392	1251.744
glucose	0.1154547	0.01466823	7.87107	1050.433
obese	13.2926318	0.99045467	13.42074	4115.498

	p.value	2.5 %	97.5 %
(Intercept)	0.00000e+00	118.7601235	123.631247
glucose	8.65974e-15	0.0866723	0.144237
obese	0.00000e+00	11.3508052	15.234458

## Model 2 (Complete Cases vs. Multiple Imputation)

```
tidy(m2_cc, conf.int = TRUE) %>% kable(digits = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	121.671	1.244	97.792	0	119.232	124.110
glucose	0.111	0.015	7.577	0	0.082	0.139
obese	13.532	1.045	12.954	0	11.484	15.580

```
summary(m2_pool, conf.int = TRUE, conf.level = 0.95) %>%  
  select(-df) %>% kable(digits = 3)
```

	estimate	std.error	statistic	p.value	2.5 %	97.5 %
(Intercept)	121.196	1.241	97.624	0	118.760	123.631
glucose	0.115	0.015	7.871	0	0.087	0.144
obese	13.293	0.990	13.421	0	11.351	15.234

# More Details on Multiple Imputation Modeling

m2\_pool

Class: mipo      m = 20

	estimate	ubars	b
(Intercept)	121.1956852	1.3856592382	1.481436e-01
glucose	0.1154547	0.0001906054	2.338243e-05
obese	13.2926318	0.9718971045	8.669863e-03

	t	dfcom	df	riv
(Intercept)	1.5412100706	4235	1251.744	0.112257637
glucose	0.0002151569	4235	1050.433	0.128808274
obese	0.9810004604	4235	4115.498	0.009366584

	lambda	fmi
(Intercept)	0.100927729	0.102360806
glucose	0.114109966	0.115791878
obese	0.009279665	0.009760773

- fmi = fraction of missing information due to nonresponse

## Model 4 run on each imputed data frame

```
m4_mods <- with(fram_mice24, lm(sbp ~ glucose + obese))  
summary(m4_mods)
```

```
# A tibble: 60 x 5
```

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	122.	1.17	104.	0.
2	glucose	0.111	0.0137	8.10	7.07e-16
3	obese	13.4	0.986	13.6	4.54e-41
4	(Intercept)	121.	1.16	104.	0.
5	glucose	0.120	0.0136	8.78	2.32e-18
6	obese	13.2	0.984	13.4	2.13e-40
7	(Intercept)	122.	1.17	104.	0.
8	glucose	0.108	0.0136	7.89	3.90e-15
9	obese	13.5	0.984	13.7	6.50e-42
10	(Intercept)	121.	1.18	103.	0.

```
# ... with 50 more rows
```

## Pool Results across the five imputations

```
m4_pool <- pool(m4_mods)
summary(m4_pool, conf.int = TRUE, conf.level = 0.95)
```

	estimate	std.error	statistic	df
(Intercept)	121.1956852	1.24145482	97.62392	1251.744
glucose	0.1154547	0.01466823	7.87107	1050.433
obese	13.2926318	0.99045467	13.42074	4115.498

	p.value	2.5 %	97.5 %
(Intercept)	0.00000e+00	118.7601235	123.631247
glucose	8.65974e-15	0.0866723	0.144237
obese	0.00000e+00	11.3508052	15.234458

# Complete Cases Results

```
tidy(m4_cc, conf.int = TRUE) %>% select(-statistic) %>%  
  kable(digits = 3)
```

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	127.107	1.388	0	124.385	129.829
glucose	0.106	0.015	0	0.078	0.135
obese	12.304	1.066	0	10.213	14.395
smoker	-4.704	0.699	0	-6.075	-3.332
educHS grad	-3.698	0.833	0	-5.332	-2.065
educSome Coll	-4.724	1.010	0	-6.704	-2.744
educColl grad	-5.954	1.158	0	-8.225	-3.683



# More Details on Multiple Imputation Modeling

m4\_pool

Class: mipo      m = 20

	estimate	ubars	b
(Intercept)	121.1956852	1.3856592382	1.481436e-01
glucose	0.1154547	0.0001906054	2.338243e-05
obese	13.2926318	0.9718971045	8.669863e-03

	t	dfcom	df	riv
(Intercept)	1.5412100706	4235	1251.744	0.112257637
glucose	0.0002151569	4235	1050.433	0.128808274
obese	0.9810004604	4235	4115.498	0.009366584

	lambda	fmi
(Intercept)	0.100927729	0.102360806
glucose	0.114109966	0.115791878
obese	0.009279665	0.009760773

- fmi = fraction of missing information due to nonresponse

## Estimate $R^2$ or adjusted $R^2$ ?

```
pool.r.squared(m2_mods)
```

```
      est      lo 95      hi 95 fmi  
R^2 0.05923026 0.04601681 0.07387826 NaN
```

```
pool.r.squared(m2_mods, adjusted = TRUE)
```

```
      est      lo 95      hi 95 fmi  
adj R^2 0.05878594 0.04561876 0.07339091 NaN
```

```
pool.r.squared(m4_mods)
```

```
      est      lo 95      hi 95 fmi  
R^2 0.05923026 0.04601681 0.07387826 NaN
```

```
pool.r.squared(m4_mods, adjusted = TRUE)
```

```
      est      lo 95      hi 95 fmi  
adj R^2 0.05878594 0.04561876 0.07339091 NaN
```

## Compare Model 4 to Model 2 after imputation

The models must be nested for this to be appropriate. We'll use the Wald test after a linear regression fit.

```
fit4 <- with(fram_mice24,  
             expr = lm(sbp ~ glucose + obese + smoker + educ))  
fit2 <- with(fram_mice24,  
             expr = lm(sbp ~ glucose + obese))  
  
pool.compare(fit4, fit2, method = "wald")$pvalue
```

```
      [,1]  
[1,]      0
```

## Setting 2: Logistic Model for chd10

## Model 3 (CC): Three-predictor model for chd10

Suppose we ignore the missingness and just run the model on the data with complete information on glucose, bp\_meds and cigs.

```
m3_cc <- fram_10_sh %$% glm(chd10 ~ glucose + bp_meds + cigs,  
                             family = binomial)
```

```
tidy(m3_cc, exponentiate = TRUE, conf.int = TRUE) %>%  
  select(-statistic) %>% kable(digits = 3)
```

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	0.063	0.151	0	0.046	0.084
glucose	1.011	0.002	0	1.007	1.014
bp_meds	2.736	0.211	0	1.791	4.105
cigs	1.015	0.004	0	1.007	1.022

## Model 6 (CC): Six-predictor model for chd10

```
m6_cc <- fram_10_sh %$% glm(chd10 ~ glucose + bp_meds + cigs +  
                             educ + chol + bmi,  
                             family = binomial)  
  
tidy(m6_cc, exponentiate = TRUE, conf.int = TRUE) %>%  
  select(-statistic) %>% kable(digits = 3)
```

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	0.012	0.410	0.000	0.005	0.026
glucose	1.009	0.002	0.000	1.006	1.013
bp_meds	2.418	0.217	0.000	1.564	3.676
cigs	1.016	0.004	0.000	1.009	1.023
educHS grad	0.595	0.117	0.000	0.472	0.747
educSome Coll	0.634	0.144	0.001	0.475	0.835
educColl grad	0.753	0.157	0.070	0.550	1.017
chol	1.005	0.001	0.000	1.003	1.007
bmi	1.034	0.011	0.003	1.012	1.057

## Subset of Variables to be used in our models 3 and 6

```
fram_sub36 <- fram_10_sh %>%  
  select(chd10, glucose, bp_meds, cigs, educ, chol, bmi)  
  
miss_var_summary(fram_sub36)
```

```
# A tibble: 7 x 3  
  variable n_miss pct_miss  
  <chr>      <int>    <dbl>  
1 glucose     388     9.16  
2 educ       105     2.48  
3 bp_meds      53     1.25  
4 chol        50     1.18  
5 cigs        29     0.684  
6 bmi         19     0.448  
7 chd10        0      0
```

# Create multiple imputations for this subset

```
set.seed(432202036)
```

```
fram_mice36 <- mice(fram_sub36, m = 10)
```

```
iter imp variable
```

1	1	glucose	bp_meds	cigs	educ	chol	bmi
1	2	glucose	bp_meds	cigs	educ	chol	bmi
1	3	glucose	bp_meds	cigs	educ	chol	bmi
1	4	glucose	bp_meds	cigs	educ	chol	bmi
1	5	glucose	bp_meds	cigs	educ	chol	bmi
1	6	glucose	bp_meds	cigs	educ	chol	bmi
1	7	glucose	bp_meds	cigs	educ	chol	bmi
1	8	glucose	bp_meds	cigs	educ	chol	bmi
1	9	glucose	bp_meds	cigs	educ	chol	bmi
1	10	glucose	bp_meds	cigs	educ	chol	bmi
2	1	glucose	bp_meds	cigs	educ	chol	bmi
2	2	glucose	bp_meds	cigs	educ	chol	bmi
2	3	glucose	bp_meds	cigs	educ	chol	bmi



# Summary information about Imputation Process

```
summary(fram_mice36)
```

Class: mids

Number of multiple imputations: 10

Imputation methods:

chd10	glucose	bp_meds	cigs	educ	chol
" "	"pmm"	"pmm"	"pmm"	"polyreg"	"pmm"
bmi					
"pmm"					

PredictorMatrix:

	chd10	glucose	bp_meds	cigs	educ	chol	bmi
chd10	0	1	1	1	1	1	1
glucose	1	0	1	1	1	1	1
bp_meds	1	1	0	1	1	1	1
cigs	1	1	1	0	1	1	1
educ	1	1	1	1	0	1	1
chol	1	1	1	1	1	0	1

## Run Model 3 on each imputed data frame

```
m3_mods <- with(fram_mice36,  
                glm(chd10 ~ glucose + bp_meds + cigs,  
                    family = binomial))  
summary(m3_mods)
```

# A tibble: 40 x 5

	term	estimate	std.error	statistic	p.value
	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	(Intercept)	-2.89	0.147	-19.7	1.87e-86
2	glucose	0.0117	0.00156	7.52	5.68e-14
3	bp_meds	1.05	0.201	5.23	1.71e- 7
4	cigs	0.0151	0.00343	4.38	1.16e- 5
5	(Intercept)	-2.72	0.138	-19.7	3.03e-86
6	glucose	0.00974	0.00146	6.68	2.39e-11
7	bp_meds	1.06	0.198	5.34	9.35e- 8
8	cigs	0.0148	0.00344	4.30	1.73e- 5
9	(Intercept)	-2.85	0.148	-19.3	5.08e-83
10	glucose	0.0112	0.00157	7.11	1.15e-12

## Pool Results across the 10 imputations

```
m3_pool <- pool(m3_mods)
summary(m3_pool, exponentiate = TRUE,
        conf.int = TRUE, conf.level = 0.95) %>%
  select(-df) %>% kable(digits = c(3, 3, 2, 2, 3, 3))
```

	estimate	std.error	statistic	p.value	2.5 %	97.5 %
(Intercept)	0.061	0.156	-17.99	0	0.045	0.082
glucose	1.011	0.002	6.35	0	1.007	1.014
bp_meds	2.865	0.202	5.21	0	1.927	4.259
cigs	1.015	0.003	4.41	0	1.008	1.022

## Comparing Model 3 Results

### Complete Cases

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	0.063	0.151	0	0.046	0.084
glucose	1.011	0.002	0	1.007	1.014
bp_meds	2.736	0.211	0	1.791	4.105
cigs	1.015	0.004	0	1.007	1.022

### After Multiple Imputation

	estimate	std.error	p.value	2.5 %	97.5 %
(Intercept)	0.061	0.156	0	0.045	0.082
glucose	1.011	0.002	0	1.007	1.014
bp_meds	2.865	0.202	0	1.927	4.259
cigs	1.015	0.003	0	1.008	1.022

## Run Model 6 on each imputed data frame

```
m6_mods <- with(fram_mice36, glm(chd10 ~ glucose +  
                                bp_meds + cigs + educ + chol + bmi,  
                                family = binomial))  
  
summary(m6_mods)
```

# A tibble: 90 x 5

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	-4.29	0.379	-11.3	1.23e-29
2	glucose	0.0109	0.00154	7.04	1.92e-12
3	bp_meds	0.917	0.205	4.47	7.91e- 6
4	cigs	0.0166	0.00344	4.82	1.47e- 6
5	educHS grad	-0.547	0.109	-5.02	5.18e- 7
6	educSome Coll	-0.443	0.133	-3.33	8.61e- 4
7	educColl grad	-0.245	0.146	-1.68	9.29e- 2
8	chol	0.00417	0.000952	4.38	1.16e- 5
9	bmi	0.0270	0.0106	2.56	1.05e- 2
10	(Intercept)	-4.19	0.376	-11.1	7.78e-29

## Pool Results across the 10 imputations

```
m6_pool <- pool(m6_mods)
summary(m6_pool, exponentiate = TRUE,
        conf.int = TRUE, conf.level = 0.95) %>%
  select(-df) %>% kable(digits = c(3, 3, 2, 2, 3, 3))
```

	estimate	std.error	statistic	p.value	2.5 %	97.5 %
(Intercept)	0.013	0.384	-11.23	0.00	0.006	0.028
glucose	1.010	0.002	6.03	0.00	1.007	1.013
bp_meds	2.510	0.207	4.45	0.00	1.674	3.764
cigs	1.017	0.003	4.87	0.00	1.010	1.024
educHS grad	0.591	0.110	-4.76	0.00	0.476	0.734
educSome Coll	0.675	0.134	-2.93	0.00	0.519	0.879
educColl grad	0.811	0.147	-1.42	0.15	0.608	1.082
chol	1.004	0.001	4.40	0.00	1.002	1.006
bmi	1.031	0.011	2.84	0.00	1.009	1.052

# Comparing Model 6 Results

## Complete Cases

term	estimate	std.error	p.value	conf.low	conf.high
(Intercept)	0.012	0.410	0.000	0.005	0.026
glucose	1.009	0.002	0.000	1.006	1.013
bp_meds	2.418	0.217	0.000	1.564	3.676
cigs	1.016	0.004	0.000	1.009	1.023
educHS grad	0.595	0.117	0.000	0.472	0.747
educSome Coll	0.634	0.144	0.001	0.475	0.835
educColl grad	0.753	0.157	0.070	0.550	1.017
chol	1.005	0.001	0.000	1.003	1.007
bmi	1.034	0.011	0.003	1.012	1.057

# Comparing Model 6 Results

## After Multiple Imputation

	estimate	std.error	p.value	2.5 %	97.5 %
(Intercept)	0.013	0.384	0.000	0.006	0.028
glucose	1.010	0.002	0.000	1.007	1.013
bp_meds	2.510	0.207	0.000	1.674	3.764
cigs	1.017	0.003	0.000	1.010	1.024
educHS grad	0.591	0.110	0.000	0.476	0.734
educSome Coll	0.675	0.134	0.003	0.519	0.879
educColl grad	0.811	0.147	0.155	0.608	1.082
chol	1.004	0.001	0.000	1.002	1.006
bmi	1.031	0.011	0.005	1.009	1.052



## Compare Model 6 to Model 3 after imputation

Again, these models need to be nested. We'll use the likelihood ratio test after a logistic regression fit.

```
fit6 <- with(fram_mice36,
             expr = glm(chd10 ~ glucose + bp_meds + cigs +
                        educ + chol + bmi, family = binomial))
fit3 <- with(fram_mice36,
             expr = glm(chd10 ~ glucose + bp_meds + cigs,
                        family = binomial))

pool.compare(fit6, fit3, method = "likelihood")$pvalue

[1] 8.610224e-12
```

# Pitfalls When Using Multiple Imputation (Sterne et al.)

## Omitting the outcome variable from the imputation procedure

Often an analysis explores the association between one or more predictors and an outcome but some predictors have missing values.

- Here, the outcome carries information about the missing values of the predictors and this information must be used.
- Consider a model relating systolic blood pressure to time to coronary heart disease, fitted to data that have some missing values of systolic blood pressure.
  - When missing systolic blood pressure values are imputed, individuals who develop coronary heart disease should have larger values, on average, than those who remain disease free.
  - Failure to include the coronary heart disease outcome and time to this outcome when imputing the missing systolic blood pressure values would falsely weaken the association between systolic blood pressure and coronary heart disease.

# Pitfalls When Using Multiple Imputation (Sterne et al.)

## Dealing with non-normally distributed variables

Many multiple imputation procedures assume that data are normally distributed, so including non-normally distributed variables may introduce bias.

- A pragmatic approach here is to transform such variables to approximate normality before imputation and then transform the imputed values back to the original scale.
- Different problems arise when data are missing in binary or categorical variables. Some procedures handle these types of missing data better than others.

# Options within `mice` for imputation approaches

Default methods include:

- `pmm` predictive mean matching (default choice for quantitative variables)
- `logreg` logistic regression (default for binary categorical variables)
- `polyreg` polytomous logistic regression (for nominal multi-categorical variables)
- `polr` proportional odds logistic regression (for ordinal categories)

but there are `cart` methods and many others available, too.

# Pitfalls When Using Multiple Imputation (Sterne et al.)

## Plausibility of missing at random assumption

- For example, the missing at random assumption may be reasonable if a variable that is predictive of missing data in a covariate of interest is included in the imputation model, but not if the variable is omitted from the model.
- Multiple imputation analyses will avoid bias only if enough variables predictive of missing values are included in the imputation model.
- It is sensible to include a wide range of variables in imputation models, including all variables in the substantive analysis, plus, as far as computationally feasible, all variables predictive of the missing values themselves and all variables influencing the process causing the missing data.

# Guidelines for reporting, I (Sterne et al.)

How should we report on analyses potentially affected by missing data?

- Report the number of missing values for each variable of interest, or the number of cases with complete data for each important component of the analysis. Give reasons for missing values if possible, and indicate how many individuals were excluded because of missing data when reporting the flow of participants through the study. If possible, describe reasons for missing data in terms of other variables (rather than just reporting a universal reason such as treatment failure.)
- Clarify whether there are important differences between individuals with complete and incomplete data—for example, by providing a table comparing the distributions of key exposure and outcome variables in these different groups
- Describe the type of analysis used to account for missing data (eg, multiple imputation), and the assumptions that were made (eg, missing at random)

# Guidelines for reporting, II (Sterne et al.)

How should we report on analyses that involve multiple imputation?

- Provide details of the imputation modeling (software used, key settings, number of imputed datasets, variables included in imputation procedure, etc.)
- If a large fraction of the data is imputed, compare observed and imputed values.
- Where possible, provide results from analyses restricted to complete cases, for comparison with results based on multiple imputation. If there are important differences between the results, suggest explanations.
- It is also desirable to investigate the robustness of key inferences to possible departures from the missing at random assumption, by assuming a range of missing not at random mechanisms in sensitivity analyses.

# Next Up

- Minute Paper after Class 12 due Wednesday at 2 PM
- For those of you who still need to do work on your proposal, the next revision deadline is 9 AM Wednesday
- You'll have access to Quiz 1 at 5 PM Wednesday
- No class Thursday. Next Tuesday's class will be about ridge regression and the lasso.