432 Class 19 Slides

github.com/THOMASELOVE/2020-432

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Today: Ordinal Logistic Regression and Using `rms`

Setup

```r
library(knitr); library(janitor); library(magrittr)
library(caret)
library(rms)
library(nnet)
library(MASS)
library(broom)
library(tidyverse)
```

```r
asbestos <- read_csv("data/asbestos.csv") %>% type.convert()
```

Asbestos Exposure in the U.S. Navy

These data describe 83 Navy workers, engaged in jobs involving potential asbestos exposure.

- The workers were either removing asbestos tile or asbestos insulation, and we might reasonably expect that those exposures would be different (with more exposure associated with insulation removal).
- The workers either worked with general ventilation (like a fan or naturally occurring wind) or negative pressure (where a pump with a High Efficiency Particulate Air filter is used to draw air (and fibers) from the work area.)
- The duration of a sampling period (in minutes) was recorded, and their asbestos exposure was measured and classified in three categories:
  - low exposure ($<$ 0.05 fibers per cubic centimeter),
  - action level (between 0.05 and 0.1) and
  - above the legal limit (more than 0.1 fibers per cc).

Source Simonoff JS (2003) *Analyzing Categorical Data*. New York: Springer, Chapter 10.

Our Outcome and Modeling Task

We'll predict the ordinal Exposure variable, in an ordinal logistic regression model with a proportional odds assumption, using the three predictors

- Task (Insulation or Tile),
- Ventilation (General or Negative pressure) and
- Duration (in minutes).

Exposure is determined by taking air samples in a circle of diameter 2.5 feet around the worker's mouth and nose.

Summarizing the Asbestos Data

We'll make sure the Exposure factor is ordinal...

```
asbestos$Exposure <- factor(asbestos$Exposure, ordered=T)
summary(asbestos[,2:5])
```

```
        Task                     Ventilation      Duration    
 Insulation:46   General             :34    Min.   : 30.0  
 Tile      :37   Negative pressure:49    1st Qu.: 85.0  
                                          Median :138.0  
                                          Mean   :147.1  
                                          3rd Qu.:212.5  
                                          Max.   :300.0  
                 Exposure  
 (1) Low exposure     :45  
 (2) Action level     : 6  
 (3) Above legal limit:32  
```

The Proportional-Odds Cumulative Logit Model

We'll use the `polr` function in the `MASS` library to fit our ordinal logistic regression.

- Clearly, Exposure group (3) Above legal limit, is worst, followed by group (2) Action level, and then group (1) Low exposure.
- We'll have two indicator variables (one for Task and one for Ventilation) and then one continuous variable (for Duration).
- The model will have two logit equations: one comparing group (1) to group (2) and one comparing group (2) to group (3), and three slopes, for a total of five free parameters.

Equations to be Fit

The equations to be fit are:

$$log(\frac{Pr(Exposure \leq 1)}{Pr(Exposure > 1)}) = \beta_{0[1]} + \beta_1 Task + \beta_2 Ventilation + \beta_3 Duration$$

and

$$log(\frac{Pr(Exposure \leq 2)}{Pr(Exposure > 2)}) = \beta_{0[2]} + \beta_1 Task + \beta_2 Ventilation + \beta_3 Duration$$

where the intercept term is the only piece that varies across the two equations.

- A positive coefficient β means that increasing the value of that predictor tends to *lower* the Exposure category, and thus the asbestos exposure.

Fitting the Model with the `polr` function in `MASS`

```
model.A <- polr(Exposure ~ Task + Ventilation + Duration,
                data=asbestos)
```

Model Summary

```
> summary(model.A)

Re-fitting to get Hessian

Call:
polr(formula = Exposure ~ Task + Ventilation + Duration, data = asbestos)

Coefficients:
                                 Value Std. Error t value
TaskTile                     -2.251333   0.644792 -3.4916
VentilationNegative pressure -2.156979   0.567540 -3.8006
Duration                     -0.000708   0.003799 -0.1864

Intercepts:
                                         Value   Std. Error t value
(1) Low exposure|(2) Action level        -2.0575  0.6611    -3.1123
(2) Action level|(3) Above legal limit -1.5111  0.6344    -2.3820

Residual Deviance: 99.87952
AIC: 109.8795
```

Explaining the Model Summary

The first part of the output provides coefficient estimates for the three predictors.

```
                                Value Std. Error t value
TaskTile                    -2.251333   0.644792 -3.4916
VentilationNegative pressure -2.156979   0.567540 -3.8006
Duration                    -0.000708   0.003799 -0.1864
```

- The estimated slope for Task = Tile is -2.25. This means that Task = Tile provides less exposure than does the other Task (Insulation) so long as the other predictors are held constant.
- Typically, we would express this in terms of an odds ratio.

Odds Ratios and CI for Model A

```r
exp(coef(model.A))
```

```
                 TaskTile VentilationNegative pressure 
                0.1052589                    0.1156740 
                 Duration 
                0.9992922 
```

```r
exp(confint(model.A))
```

```
Waiting for profiling to be done...


Re-fitting to get Hessian

                                  2.5 %    97.5 %
TaskTile                     0.02718379 0.3538549
VentilationNegative pressure 0.03641039 0.3427734
Duration                     0.99187230 1.0069533
```

tidy for polr models exponentiates by default...

```r
tidy(model.A, conf.int = TRUE)
```

term	estimate	std.error	statistic
(1) Low exposure|(2) Action level	-2.057	0.661	-3.112
(2) Action level|(3) Above legal limit	-1.511	0.634	-2.382
Duration	-0.001	0.004	-0.186
TaskTile	-2.251	0.645	-3.492
VentilationNegative pressure	-2.157	0.568	-3.801

term	conf.low	conf.high	coefficient_typ
(1) Low exposure|(2) Action level	NA	NA	zeta
(2) Action level|(3) Above legal limit	NA	NA	zeta
Duration	-0.008	0.007	coefficient
TaskTile	-3.605	-1.039	coefficient
VentilationNegative pressure	-3.313	-1.071	coefficient

Assessing the Ventilation Coefficient

```
                                Value Std. Error t value
TaskTile                    -2.251333   0.644792 -3.4916
VentilationNegative pressure -2.156979   0.567540 -3.8006
Duration                    -0.000708   0.003799 -0.1864
```

Similarly, the estimated slope for Ventilation = Negative pressure (-2.16) means that Negative pressure provides less exposure than does General Ventilation. We see a relatively modest effect (near zero) associated with Duration.

Summary of Model A: Estimated Intercepts

```
Intercepts:
                                             Value   Std. Error t va
(1) Low exposure|(2) Action level           -2.0575  0.6611     -3.1
(2) Action level|(3) Above legal limit -1.5111  0.6344     -2.3
```

The first parameter (-2.06) is the estimated log odds of falling into category (1) low exposure versus all other categories, when all of the predictor variables (Task, Ventilation and Duration) are zero. So the first estimated logit equation is:

$$log(\frac{Pr(Exposure \leq 1)}{Pr(Exposure > 1)}) =$$

$$-2.06 - 2.25[Task = Tile] - 2.16[Vent = NP] - 0.0007 Duration$$

Summary of Model A: Estimated Intercepts

```
Intercepts:
                                            Value   Std. Error t va
(1) Low exposure|(2) Action level          -2.0575  0.6611     -3.1
(2) Action level|(3) Above legal limit -1.5111  0.6344     -2.3
```

The second parameter (-1.51) is the estimated log odds of category (1) or (2) vs. (3). The estimated logit equation is:

$$log(\frac{Pr(Exposure \leq 2)}{Pr(Exposure > 2)}) =$$

$$-1.51 - 2.25[Task = Tile] - 2.16[Vent = NP] - 0.0007 Duration$$

Comparing Model A to an "Intercept only" Model

```r
model.1 <- polr(Exposure ~ 1, data=asbestos)
anova(model.1, model.A)
```

```
Likelihood ratio tests of ordinal regression models

Response: Exposure
                             Model Resid. df Resid. Dev   Test
1                                1        81  147.61971
2 Task + Ventilation + Duration        78   99.87952 1 vs 2
     Df LR stat.     Pr(Chi)
1
2     3 47.74019 2.41857e-10
```

Comparing Model A to Model without Duration

```r
model.TV <- polr(Exposure ~ Task + Ventilation, data=asbestos)
anova(model.A, model.TV)
```

```
Likelihood ratio tests of ordinal regression models

Response: Exposure
                             Model Resid. df Resid. Dev   Test
1             Task + Ventilation        79   99.91421
2 Task + Ventilation + Duration        78   99.87952 1 vs 2
     Df    LR stat.    Pr(Chi)
1
2     1 0.03469471 0.8522368
```

Is a Task*Ventilation Interaction helpful?

```
model.TxV <- polr(Exposure ~ Task * Ventilation, data=asbestos
anova(model.TV, model.TxV)
```

```
Likelihood ratio tests of ordinal regression models

Response: Exposure
                 Model Resid. df Resid. Dev   Test    Df
1 Task + Ventilation        79   99.91421
2 Task * Ventilation        78   99.64326 1 vs 2     1
   LR stat.   Pr(Chi)
1
2 0.2709469 0.6026973
```

`asbestos` Likelihood Ratio Tests

Model	Elements	DF	Deviance	Test	p
1	Intercept	81	147.62	–	–
2	D	80	142.29	vs 1	0.021
3	T	80	115.36	vs 1	< 0.0001
4	V	80	115.45	vs 1	< 0.0001
5	T+V	79	99.91	vs 4	< 0.0001
6	T*V	78	99.64	vs 5	0.60
7	T+V+D	78	99.88	vs 5	0.85

- T = Task
- V = Ventilation
- D = Duration

In-Sample Predictions with our T+V model

```
model.TV <- polr(Exposure ~ Task + Ventilation,
                 data=asbestos)

asbestos <- asbestos %>%
  mutate(TV_preds = predict(model.TV))

asbestos %>% tabyl(TV_preds, Exposure) %>%
  adorn_title() %>% kable()
```

	Exposure		
TV_preds	(1) Low exposure	(2) Action level	(3) Above legal
(1) Low exposure	42	3	10
(2) Action level	0	0	0
(3) Above legal limit	3	3	22

Accuracy of These Classifications?

```
asbestos %>% tabyl(TV_preds, Exposure) %>%
  adorn_title() %>% kable()
```

	Exposure		
TV_preds	(1) Low exposure	(2) Action level	(3) Above legal
(1) Low exposure	42	3	10
(2) Action level	0	0	0
(3) Above legal limit	3	3	22

- Predicting Low exposure led to 42 right and 13 wrong.
- We never predicted Action Level
- Predicting Above Legal Limit led to 22 right and 6 wrong.

Total: 64 right, 19 wrong. Accuracy $= 64/83 = 77.1\%$

5-fold cross-validation for polr model?

```
set.seed(2020432)
train.control <- trainControl(method = "cv", number = 5)
modTV_cv <- train(Exposure ~ Task + Ventilation,
                      data = asbestos, method = "polr",
                  trControl = train.control)
```

Results of 5-fold cross-validation `modTV_cv`

```
Ordered Logistic or Probit Regression

83 samples
 2 predictor
 3 classes: '(1) Low exposure', '(2) Action level', '(3) Above

No pre-processing
Resampling: Cross-Validated (5 fold)
Summary of sample sizes: 67, 66, 66, 67, 66
Resampling results across tuning parameters:

  method     Accuracy   Kappa
  cauchit    0.7477941  0.5069165
  cloglog    0.7125000  0.4463752
  logistic   0.7477941  0.5069165
  loglog     0.7713235  0.5470383
  probit     0.7477941  0.5069165
```

Which `kappa` is that?

Fleiss' `kappa`, or κ describes the extent to which the observed agreement between the predicted classifications and the actual classifications exceeds what would be expected if the predictions were made at random.

- Larger values of κ indicate better model performance ($\kappa = 0$ indicates very poor agreement between model and reality, κ near 1 indicates almost perfect agreement.)

```
Resampling results across tuning parameters:

  method     Accuracy   Kappa
  cauchit    0.7477941  0.5069165
  cloglog    0.7125000  0.4463752
  logistic   0.7477941  0.5069165
  loglog     0.7713235  0.5470383
  probit     0.7477941  0.5069165
```

Is the proportional odds assumption reasonable?

Alternative: fit a multinomial model?

```
mult_TV <- multinom(Exposure ~ Task + Ventilation,
                    data = asbestos, trace = FALSE)
```

View the Multinomial Model?

```
mult_TV
```

```
Call:
multinom(formula = Exposure ~ Task + Ventilation, data = asbes
    trace = FALSE)

Coefficients:
                      (Intercept)  TaskTile
(2) Action level       0.05268936 -1.160153
(3) Above legal limit  2.07821627 -2.699743
                      VentilationNegative pressure
(2) Action level                         -2.316099
(3) Above legal limit                    -2.496044

Residual Deviance: 98.08263
AIC: 110.0826
```

In-Sample Predictions with the multinomial T+V model

```
asbestos <- asbestos %>%
  mutate(TVmult_preds = predict(mult_TV))

asbestos %>% tabyl(TVmult_preds, Exposure) %>%
  adorn_title() %>% kable()
```

	Exposure		
TVmult_preds	(1) Low exposure	(2) Action level	(3) Above legal
(1) Low exposure	42	3	10
(2) Action level	0	0	0
(3) Above legal limit	3	3	22

Compare Models with Likelihood Ratio Test?

```
(LL_multTV <- logLik(mult_TV)) # multinomial model: 6 df
```

```
'log Lik.' -49.04131 (df=6)
```

```
(LL_polrTV <- logLik(model.TV)) # polr model: 4 df
```

```
'log Lik.' -49.9571 (df=4)
```

```
(G = -2 * (LL_polrTV[1] - LL_multTV[1]))
```

```
[1] 1.831584
```

```
pchisq(G, 2, lower.tail = FALSE)
```

```
[1] 0.4001996
```

No statistically detectable difference in quality of fit ($p = 0.4$) between the proportional odds model and the more complex multinomial logistic regression model.

Using `rms` to fit ordinal logistic regression models

Proportional Odds Ordinal Logistic Regression with `lrm`

```
d <- datadist(asbestos)
options(datadist = "d")

model_TV_LRM <- lrm(Exposure ~ Task + Ventilation,
                    data = asbestos, x = TRUE, y = TRUE)

# note that Exposure must be an ordered factor
```

POLR results via `lrm` (slide 1)

```
model_TV_LRM
```

```
Logistic Regression Model

 lrm(formula = Exposure ~ Task + Ventilation,
      data = asbestos, x = TRUE, y = TRUE)

                                    Model Likelihood
                                       Ratio Test
 Obs                       83     LR chi2      47.71
  (1) Low exposure         45     d.f.             2
  (2) Action level          6     Pr(> chi2) <0.0001
  (3) Above legal limit    32
 max |deriv|            3e-10
```

POLR results via `lrm` (slide 2)

```
lrm(formula = Exposure ~ Task + Ventilation + Duration,
     data = asbestos, x = TRUE, y = TRUE)
```

```
 Discrimination    Rank Discrim.
    Indexes          Indexes
 R2        0.526    C       0.854
 g         2.064    Dxy     0.708
 gr        7.877    gamma   0.839
 gp        0.371    tau-a   0.396
 Brier     0.127
```

POLR results via `lrm` (slide 3)

```
 lrm(formula = Exposure ~ Task + Ventilation + Duration,
      data = asbestos, x = TRUE, y = TRUE)

                                Coef   S.E.   Wald Z Pr(>|Z|)
y>=(2) Action level              1.9713 0.4695  4.20  <0.0001
y>=(3) Above legal limit         1.4256 0.4348  3.28   0.0010
Task=Tile                       -2.2868 0.6173 -3.70   0.0002
Ventilation=Negative pressure -2.1596 0.5675 -3.81   0.0001
```

Ordinal Logistic Regression for T+V with orm

```
d <- datadist(asbestos)
options(datadist = "d")

model_TV_ORM <- orm(Exposure ~ Task + Ventilation,
                    data = asbestos, x = TRUE, y = TRUE)

# note that Exposure must be an ordered factor
```

Results for `model_TV_ORM` fit with `orm`

(I'll neaten these up on the next two slides.)

```
model_TV_ORM
```

```
Logistic (Proportional Odds) Ordinal Regression Model

 orm(formula = Exposure ~ Task + Ventilation, data = asbestos,
     x = TRUE, y = TRUE)

                              Model Likelihood          Discri
                                 Ratio Test                 In
 Obs               83        LR chi2      47.71     R2
  (1) Low exposure45         d.f.             2     g
  (2) Action level6          Pr(> chi2) <0.0001     gr
  (3) Above legal limit32    Score chi2   42.42     |Pr(Y>=medi
 Distinct Y         3        Pr(> chi2) <0.0001
 Median Y           1
 max |deriv| 6e-05
```

orm fit for T+V model (slide 1 of 2)

```
model_TV_ORM
```

```
Logistic (Proportional Odds) Ordinal Regression Model

orm(formula = Exposure ~ Task + Ventilation,
      data = asbestos, x = TRUE, y = TRUE)

                              Model Likelihood
                                 Ratio Test
 Obs                   83  LR chi2      47.71
  (1) Low exposure     45  d.f.             2
  (2) Action level      6  Pr(> chi2) <0.0001
  (3) Above legal limit 32  Score chi2   42.42
 Distinct Y      3         Pr(> chi2) <0.0001
 Median Y        1
 max |deriv| 6e-05
```

`orm` fit for `T+V` model (slide 2 of 2)

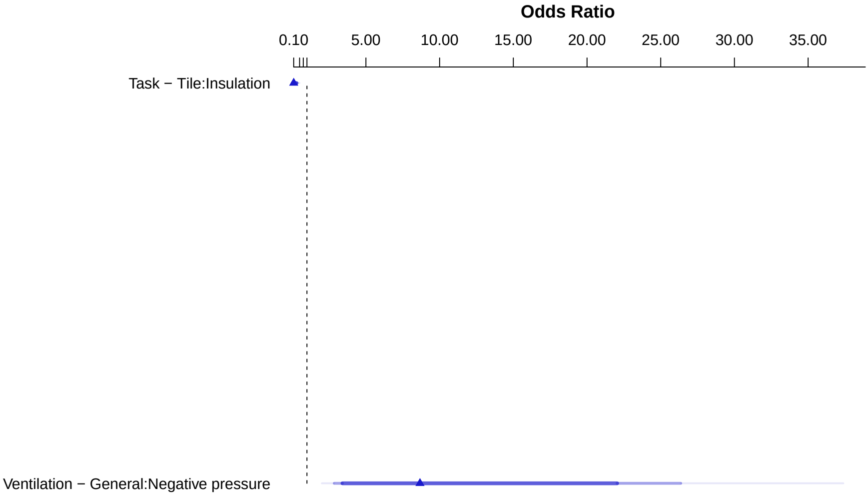
```
Logistic (Proportional Odds) Ordinal Regression Model

Discrimination Indexes
R2   0.526    rho   0.697
g    2.064    gr    7.877    |Pr(Y>=median)-0.5| 0.301

                               Coef    S.E.   Wald Z Pr(>|Z|)
 y>=(2) Action level            1.9713 0.4695  4.20  <0.0001
 y>=(3) Above legal limit       1.4256 0.4348  3.28   0.0010
 Task=Tile                     -2.2868 0.6173 -3.70   0.0002
 Ventilation=Negative pressure -2.1596 0.5675 -3.81   0.0001
```

Plot effects of the coefficients

```
plot(summary(model_TV_LRM))
```



rms::validate results from lrm

```r
set.seed(432)
validate(model_TV_LRM)
```

```
              index.                                   index.
                orig training    test optimism corrected  n
Dxy           0.7077   0.7175  0.7082   0.0093    0.6984 40
R2            0.5260   0.5426  0.5183   0.0243    0.5017 40
Intercept     0.0000   0.0000 -0.0279   0.0279   -0.0279 40
Slope         1.0000   1.0000  0.9464   0.0536    0.9464 40
Emax          0.0000   0.0000  0.0169   0.0169    0.0169 40
D             0.5627   0.5944  0.5515   0.0429    0.5199 40
U            -0.0241  -0.0241 -0.4004   0.3763   -0.4004 40
Q             0.5868   0.6185  0.9519  -0.3335    0.9203 40
B             0.1270   0.1234  0.1319  -0.0086    0.1356 40
g             2.0639   2.1722  2.0250   0.1472    1.9167 40
gp            0.3709   0.3746  0.3691   0.0055    0.3654 40
```

rms::validate results from orm

```r
set.seed(432)
validate(model_TV_ORM)
```

```
      index.orig training   test optimism index.corrected
rho       0.6970   0.7113 0.6976   0.0138          0.6833
R2        0.5260   0.5470 0.5183   0.0287          0.4973
Slope     1.0000   1.0000 0.9354   0.0646          0.9354
g         2.0639   2.1963 2.0260   0.1702          1.8937
pdm       0.3010   0.3181 0.3042   0.0139          0.2871
       n
rho   40
R2    40
Slope 40
g     40
pdm   40
```

Predictions (greater than or equal to)

```
head(predict(model_TV_LRM, type = "fitted"),3)
```

```
  y>=(2) Action level y>=(3) Above legal limit
1          0.07762357                 0.0464946
2          0.45306969                 0.3243171
3          0.45306969                 0.3243171
```

Predictions (individual)

```
head(predict(model_TV_LRM, type = "fitted.ind"),3)
```

```
  Exposure=(1) Low exposure Exposure=(2) Action level
1                0.9223764                 0.03112897
2                0.5469303                 0.12875255
3                0.5469303                 0.12875255
  Exposure=(3) Above legal limit
1                     0.0464946
2                     0.3243171
3                     0.3243171
```

Nomogram?

First, we'll create the functions to estimate the probabilities of falling into groups 1, 2, and 3.

```
model_TV_LRM$coef
```

```
          y>=(2) Action level      y>=(3) Above legal limit
                     1.971284                      1.425557
                    Task=Tile Ventilation=Negative pressure
                    -2.286807                     -2.159559
```

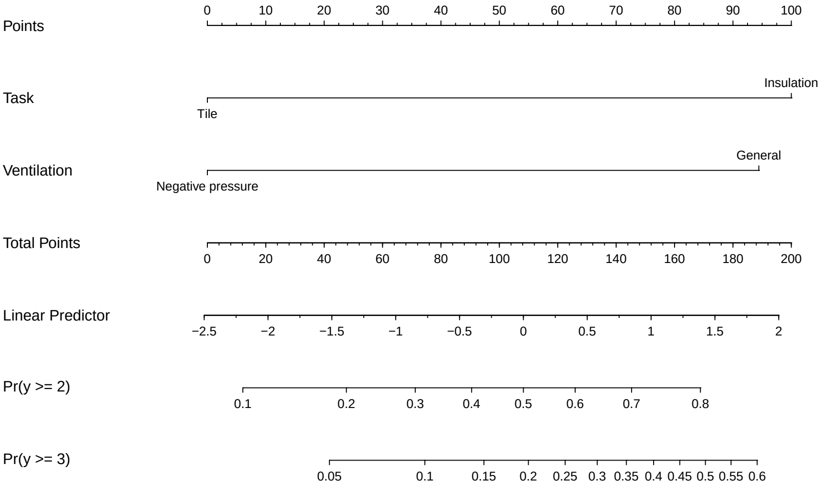
So `plogis` by default uses the first intercept shown, and to get the machine to instead use the second one, we need:

```
fun3 <- function(x) plogis(x - model_TV_LRM$coef[2])
```

Plot the Nomogram

```
plot(nomogram(model_TV_LRM,
         fun = list( 'Pr(y >= 2)' = plogis,
                     'Pr(y >= 3)' = fun3)))
```

Shown on next slide.


Points
0 10 20 30 40 50 60 70 80 90 100
Task
Tile
Insulation
Ventilation
Negative pressure
General
Total Points
0 20 40 60 80 100 120 140 160 180 200
Linear Predictor
−2.5 −2 −1.5 −1 −0.5 0 0.5 1 1.5 2
Pr(y >= 2)
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
Pr(y >= 3)
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6

Some Sources for Ordinal Logistic Regression

- A good source of information on fitting these models is https://stats.idre.ucla.edu/r/dae/ordinal-logistic-regression/
    - Another good source, that I leaned on heavily here, using a simple example, is https://onlinecourses.science.psu.edu/stat504/node/177.
    - Also helpful is https://onlinecourses.science.psu.edu/stat504/node/178 which shows a more complex example nicely.