Forecasting at Scale

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1 Prophet's goals and alternative

2 Prophet model

3 Stan model and fitting

4 Future Talk's

5 Further reading

Prophet's goals and alternative

Prophet library

The Prophet library[1] is a model and a framework, that targets

- non-experts with background business knowledge

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- less knowledge about time series is required
- easy to configure (analyst in the loop)
- easily interpretable parameters
- flexible for a wide range of business problems

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Alternative

The `forecast` R-package contains some alternative for automated times series models.

- `auto.arima`[2], fits mutliple ARIMA models and take the best fit

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- `snaive`[4], random walk model with seasonality

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[3] Hyndman, Koehler., Snyder & Grose, 2002, 'A state space framework for automatic forecasting using exponential smoothing methods'

[4] De Livera, Hyndman. & Snyder, 2011, 'A state space framework for automatic forecasting using exponential smoothing methods'

Prophet model

Generalized additive models

Prophet uses a generalized additive model (GAM)[5]

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t$$

- $y(t)$ target

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- $y(t)$ target
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- $h(t)$ represents holiday effects, i.e. irregular schedules over one or more days
- ϵ_t error term (not accomodated by the model)

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The trend model

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- piecewise linear model

Nonlinear Saturating growth (basic)

A basic logistic growth model is given by

$$g(t) = \frac{C}{1 + exp(-k(t - m))}$$

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where

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- m is an offset parameter

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- incoporate trend changes by explicite defined change points
- change point can be set by analyst (e.g. product launches) or automatically selected

Nonlinear Saturating growth (time dependent carrying capacity)

TBA - this is not explained in the non reviewed paper

Nonlinear Saturating growth (non constant growth rate - idea)

let $s_1, \ldots, s_n$ be change points (time stamps).

define $\delta \in \mathbb{R}^n$ (vector of growth rate adjustments)

define growth rate at time t by

$$k(t) := k + \sum_{j: s_j < t} \delta_j$$

where k is the base growth rate

Nonlinear Saturating growth (non constant growth rate - mathematically correct)

define $a(t) \in \{0,1\}^n$ by

$$a_j(t) := \begin{cases} 1, & \text{for } t \leq s_j \\ 0, & otherwise \end{cases}$$

then rate at time t is given by

$$k(t) := k + a(t)^T \delta$$

Nonlinear Saturating growth (non constant growth rate - mathematically correct 2)

The offset parameter m is than adjusted according to the change points by:

$$\gamma_i := \left(s_j - m - \sum_{l<j} \gamma_j\right) \left(1 - \frac{k + \sum_{l<j} \delta_l}{k + \sum_{l \leq j} \delta_l}\right)$$

Nonlinear Saturating growth (non constant growth rate - mathematically correct 3)

Putting all together we recieve :

$$
\begin{aligned}
g(t) &= \frac{C(t)}{1 + exp(-k(t)(t - m(t))} \\
&= \frac{C(t)}{1 + \exp(-(k + a(t)^T\delta)(t - (m + a(t)^T\gamma)))}
\end{aligned}
$$

Linear trend with Changepoints

using the change points from above a piecewise linear model is given by

$$
\begin{aligned}
g(t) &= k(t) \cdot t + m(t) \\
&:= (k + a(t)^T \delta) * t + (m + a(t)^T \gamma)
\end{aligned}
$$

To do

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where

- $P(\delta|Data)$ is the conditional probability to observe δ given $Data$
- $P(Data|\delta)$ is the likelihood, which can be interpreted as the probability to observe $Data$ given δ

Automatic changepoint selection

Change points are automatically detected by putting a sparse prior on δ, i.e.

$$\delta_j \sim \mathit{Laplace}(0, \tau)$$

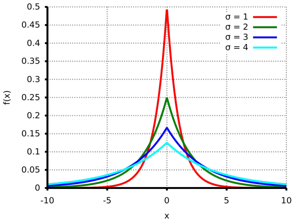

Figure 1: LaPlace density function from wikipedia, here $\sigma = \tau$

The seasonal model

The seasonality is approximated by a (truncated) Fourier serie:

$$s(t) = \sum_{i=1}^{N} (a_n \cos(\frac{2\pi it}{P}) + b_n \sin(\frac{2\pi it}{P}))$$

where

- P is the period in days, for example P =7 for weekly seasonality

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- P is the period in days, for example P =7 for weekly seasonality
- N truncates the series and is a model parameter for the user to adjust the fitting

The seasonal model (fitting)

Fitting requires to estimate $2N$ parameters $\beta = (a_1, b_1, \ldots, a_N, b_N)$.

This is done by constructing a matrix of seasonality vectors for each t in our historical and future data, e.g. for weekly seasonality and $N = 3$

$$X(t) := (\cos(\frac{2\pi(1)t}{7}), \ldots, \sin(\frac{2\pi(3)t}{7}))$$

Hence the seasonal component is

$$s(t) = X(t) \cdot \beta$$

where the prior on β is normaly distributed: $\beta \sim \text{Normal}(0, \sigma^2)$.

Fourier series

TBF

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- the functions $e_k(x) := \exp(\frac{2\pi ikx}{P})$ is a basis of a (dense) subspace in the vector space of P periodic functions
- the basis functions e_k can be expressed as a function in sin and cos

Fourier series

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- recall how is an element in an vector space is represented in a basis
- the functions $e_k(x) := \exp(\frac{2\pi ikx}{P})$ is a basis of a (dense) subspace in the vector space of P periodic functions
- the basis functions e_k can be expressed as a function in sin and cos
- convergence of Fourier series

"Holidays and Events"

TBF

Stan model and fitting

Stan model and fitting

Stan is a platform for Bayesian inference using MCMC sampling and more.

Stan model with logistic growth trend model

```
model {
// Priors
k ~ normal(0, 5);
m ~ normal(0, 5);
epsilon ~ normal(0, 0.5);
delta ~ double_exponential(0, tau);
beta ~ normal(0, sigma);
// Logistic likelihood
y ~ normal(C ./ (1 + exp(-(k + A * delta)
                        .* (t - (m + A * gamma)))) +
            X * beta, epsilon);
}
```

Stan fitting for prophet

- Prophet uses the L-BFGS algorithm from Stan to fit the GAM

Stan fitting for prophet

- Prophet uses the L-BFGS algorithm from Stan to fit the GAM
- BFGS = Broyden–Fletcher–Goldfarb–Shanno algorithm, quasi Newton method

Future Talk's

regarding Prophet

- how the time dependet carrying capacity is determined ?

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- how additional regressors are included ?

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- how outliers are handeled ?

theoretical background

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- Stan models
- GAMS
- Baysian models
- L-BFGS
- Marcov chain Monte Carlo

Further reading

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- The textbook for forecast R-package, but with a lot of theory and practice