Theorem 1. *In a population of gossipers with only one media, the evolution of the gossipers' opinion distribution follows the following equation,*

$$\frac{\partial p(x,t)}{\partial t} = \xi \Psi_g(x,t) + (1-\xi)\Psi_m(x,t) \tag{1}$$

where

$$\Psi_g(x,t) = \frac{{d_g}^3}{3}\alpha_g(\alpha_g - 1)\frac{\partial^2\left(p^2(x,t)\right)}{\partial x^2}, \tag{2}$$

$$\Psi_m(x,t) = \begin{cases} \frac{1}{1-\alpha_m}p(\frac{1}{1-\alpha_m}x - \frac{\alpha_m}{1-\alpha_m}y,t) - p(x,t) & x \in I_1 \\ -p(x,t) & x \in I_2 \end{cases} \tag{3}$$

with $I_1 = \{x||x-y| < (1-\alpha_m)d_m\}$ *and* $I_2 = \{x|d_m \geq |x-y| \geq (1-\alpha_m)d_m\}$.

Proof. Using the mean field (MF) approximations[Lasry and Lions, 2007] theories, partial derivative of opinion distribution $p(x,t)$ can be described by the master equation

$$\frac{\partial \mathrm{p}}{\partial t} = \int_{-d}^{d} dy\left[W_{x+y\to x}p(x+y) - W_{x\to x+y}p(x)\right] \tag{4}$$

here $W_{x+y\to x}$ is the transition rate from $x+y$ to x and $W_{x+y\to x}dxdy$ is the probability of jumping to the interval $(x+y, x+y+dx)$ during the time interval $(t, t+dt)$ given that it is at the point $x+y$ at time t. $W_{x\to x+y}$ is defined similarly.

At each step, a gossiper get information from other gossiper chosen randomly with probability ξ or a media that the gossiper followed with probability $1-\xi$. Let $w^{[g]}$ and $w^{[m]}$ be the probability that the particle jumps caused by gossiper and media respectively. Then we have

$$\begin{aligned} W_{x\to x+y} &= \xi w^{[g]}_{x\to x+y} + (1-\xi)w^{[m]}_{x\to x+y} \\ W_{x+y\to x} &= \xi w^{[g]}_{x+y\to x} + (1-\xi)w^{[m]}_{x+y\to x} \end{aligned} \tag{5}$$

Simplify the equation 4, we can get

$$\begin{aligned} \frac{\partial \mathrm{p}}{\partial t} &= \xi\Psi_g(x,t) + (1-\xi)\Psi_m(x,t) \\ &= \xi \int dy\left[w^g_{x+y\to x}p(x+y) - w^g_{x\to x+y}p(x)\right] + \\ &(1-\xi)\int dy\left[w^m_{x+y\to x}p(x+y) - w^m_{x\to x+y}p(x)\right] \end{aligned} \tag{6}$$

The first integral $\Psi_g(x,t)$ in the right side of equation 6 stands for the evolution of the gossiper opinion distribution contributed by other gossipers, which obey the following dynamics [Weisbuch *et al.*, 2001]:

$$\Psi_g(x,t) = \frac{{d_g}^3}{3}\alpha_g(\alpha_{gg} - 1)(p^2)'' \tag{7}$$

here $(p^2)'' = \frac{\partial^2 \mathrm{p}^2}{\partial x^2}$ is the second partial derivatives of p with respect to x. α_g is the convergence parameter and its ranges from 0 to 0.5. d_g is the threshold.

Then we focus on the second integral $\Psi_m(x,t)$ in the right side of equation 6, which stands for the evolution of the gossiper opinion distribution contributed by medias.

Assume that the media is j and is located in $u_j(u_j = x + d_j)$, then the probability density function of media j can be defined using Dirac delta function $q(x) = \delta(x - u_j)$. The Dirac delta function $\delta(x)$[Hassani, 2000] is used to model a tall narrow spike function (an impulse) and other similar abstractions such as a point charge, point mass or electron point, and meets the following conditions

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases} \tag{8}$$

$$\int_{-\infty}^{+\infty} \delta(x)dx = 1 \tag{9}$$

then the transition rate $w^{[m]}_{x+y\to x}$ from $x+y$ to x will be,

$$\begin{aligned} &w^{[m]}_{x+y\to x} \\ &= \int q(x+z)\delta\left(x - [(x+y) + \alpha_m((x+z) - (x+y))]\right)dz \\ &= \int q(x+z)\frac{1}{\alpha_m}\delta\left(z - \frac{\alpha_m - 1}{\alpha_m}y\right)dz \end{aligned} \tag{10}$$

$\delta(x - [(x+y) + \alpha_m((x+z) - (x+y))])$ in equation 10 indicates the situation that an agent with opinion $x+z$ transact to x influenced by media in $x+y$. $q(x+z)$ is the probability density function of media in $x+z$. Similarity to the definition of $w_{x\to x+y}$, we have

$$\begin{aligned} &w^{[m]}_{x\to x+y} \\ &= \int q(x+z)\delta\left((x+y) - (x + \alpha_m((x+z) - x))\right)dz \\ &= \int q(x+z)\frac{1}{\alpha_m}\delta\left(z - \frac{1}{\alpha_m}y\right)dz \end{aligned} \tag{11}$$

Combine equations 8-11, we obtain

$$\begin{aligned} &\Psi_m(x,t) = \int dy\left[w^m_{x+y\to x}p(x+y) - w^m_{x\to x+y}p(x)\right] \\ &= \frac{1}{\alpha_m}\int_{-\alpha_m d_m}^{\alpha_m d_m} p(x+y)\delta\left(\frac{\alpha_m - 1}{\alpha_m}y - (u_j - x)\right)dy \\ &- \frac{1}{\alpha_m}\int_{-\alpha_m d_m}^{\alpha_m d_m} p(x)\delta\left(\frac{1}{\alpha_m}y - (u_j - x)\right)dy \\ &= \begin{cases} \frac{1}{1-\alpha_m}p(\frac{1}{1-\alpha_m}x - \frac{\alpha_m}{1-\alpha_m}u_j) - p(x) & x \in I_1 \\ -p(x) & x \in I_2 \end{cases} \end{aligned} \tag{12}$$

here $I_1 = \{x||x - u_j| < (1-\alpha_m)d_m\}$, $I_2 = \{x|d_m \geq |x - u_j| \geq (1-\alpha_m)d_m\}$.

Combining with equation 7, we finally complete the proof. □

References

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