CCFG $\rightarrow$ Prog $\rightarrow$

cost $w(\cdot)$ $\rightarrow$

$T_i \xrightarrow{f_i} U_i$ $f_i = 5$

$A \rightarrow (acb + a)$

Constrained CFG

$S \xrightarrow{f_1} (aAB\cancel{b})_1$

$A \xrightarrow{f_2} (aC\cancel{b})_2 \mid (a)^{f_3}$

$B \xrightarrow{f_4} (bA_2C)_2 \mid (b)^{f_5}$

$C \xrightarrow{f_6} (A)_2 \mid (B)_2 \mid c^{f_8}$

$A \rightarrow aCb$

$A \rightarrow a$

$w(a) = 1$ $w(b) = 2$ $w(c) = 3$

Goal: find a string s that can be produced by the CCFG (obeying production capacity constraints) with maximal weight

$w(s) = \sum_{s_i \in s} w(s_i)$

$\operatorname{argmax}_{s \in \Sigma^*} w(s)$

$\Sigma = \{a, b, c\}$

$S \rightarrow aABb$

$aaCbBb$

$aaCbbAC$

1) Find w^* ~~w(s)~~ subject to capacity constraints

String Gen. 2) Find s s.t. $w(s) = w^*$

$w^* = 12$

$w^* = \max \; w(a)\cdot(f_1 + f_2 + f_3) + w(b)\cdot(f_1 + f_2 + f_4 + f_5) + w(c)\cdot(f_4 + f_8)$

in(S): $1 = f_1$ out(S)

in(A): $f_1 + f_4 + f_6 = f_2 + f_3$ out(A)

in(B): $f_1 + f_7 = f_4 + f_5$ out(B)

in(C): $f_2 = f_6 + f_7 + f_8$ out(C)

$\forall i. f_i \geq 0$, then plus capacity constraints

$f_1 = 1$

$f_2 = 1$ $f_3 = 0$

$f_4 = 2$ $f_5 = 0$

$f_6 = 0$ $f_7 = 2$

$f_8 = 3$

$0 \leq f_1 \leq 1$ $0 \leq f_5$

$0 \leq f_2 \leq 2$ $0 \leq f_6 \leq 2$

$0 \leq f_3$ $0 \leq f_7 \leq 2$

$0 \leq f_4 \leq 2$ $0 \leq f_8$

2) String Generation — Find string s (or the sequence of prod. rules) s.t. $w(s) = w^*$

- Encode the production sequence (seq. of rules) as a transition system

$$S \xrightarrow[c_1 = 1]{\tau_1} \{A, B\} \xrightarrow{\tau_2} \{A, C\} \; \ldots \; \xrightarrow{\tau_3} \text{attack } s \in \Sigma^*$$

$c_{2,1} + c_{3,1} +$
$c_{4,1} + c_{5,1} = 1$

$\leq w^*$

$\forall i : c_i \geq 0$

$c_2 = 1$

ILP: $\forall c_{i,j} \in [0, 1]$ . . .

$\Longrightarrow$ Attack String s.

Constrained CFG problem

$S \xrightarrow{f_1} aABb|_1$ $\qquad w(a) = 1$

$A \xrightarrow{f_2} aCb_2 | a^{f_3}$ $\qquad w(b) = 2$

$B \xrightarrow{f_4} bAc_2 | b^{f_5}$ $\qquad w(c) = 3$

f_2, f_1, f_6 $\quad C \xrightarrow{f_6} A_2 | \overset{f_7}{B_2} | c^{f_8}$

$f_6, f_7, 1,$ $\quad W^* = \max [\, w(a)$

$f_1 \qquad\qquad = 1$

$f_1 + \cdot \cdot f_4 + \cdot + f_6 = f_2 + f_3$

$f_1 + \qquad f_7 = f_4 + f_5$

$f_2 = f_6 + f_7 + f_8$

$0 \leq f_1 \leq 1$ $\qquad f_1 \rightarrow ($

$0 \leq f_2 \leq 2$

$0 \leq f_3$

$0 \leq f_4 \leq 2$

$0 \leq f_5$

$0 \leq f_6$

$0 \leq f_7 \leq 2$

$0 \leq f_8$

Objective :

$$w(a)\{f_1 + f_2 + f_3\} + w(b)\{f_1 + f_2 + f_4 + f_5\}$$
$$+ w(c)\{f_4 + f_8\}$$

$$= f_1[w(a) + w(b)] + f_2[w(a) + w(b)] + f_3[w(a)]$$
$$+ f_4[w(b) + w(c)] + f_5[w(b)] + f_6[0]$$
$$+ f_7[0] + f_8[w(c)]$$

$$= 3f_1 + 3f_2 + f_3 + 5f_4 + 2f_5 + 0f_6$$
$$+ 0f_7 + 3f_8$$

$\cancel{W^*}$

Solution : W^* : 23

$f_1 = 1$
$f_2 = 2$
$f_3 = 1$
$f_4 = 2$
$f_5 = 0$
$f_6 = 0$
$f_7 = 1$
$f_8 = 1$

$$\cancel{S \xrightarrow{\quad} \overset{2}{AS} \mid AB}$$

$$\cancel{A \xrightarrow{\quad} aB}$$

$$S \longrightarrow \underset{R_2}{\overset{2}{AS}} \mid \underset{R_1}{\overset{2}{AB}}$$

$$A \longrightarrow aB \quad 4$$

$$B \rightarrow b$$

- Production rules $\longrightarrow$ converted to vectors

$S(1,0,0) \rightarrow$ Start with an S having 1 S and 0 A's & 0 B's

$$\begin{tikzcd}
S(1,0,0) \arrow[d, "{(-1,1,1)}"', "R_1"] \arrow[dr, "R_2", "{(0,1,0)}"'] & \\
AB(0,1,1) & AS(1,1,0) \arrow[d, "R_2"', "{(0,1,0)}"] \\
& AAS(1,2,0)
\end{tikzcd}$$

Example

$$S \rightarrow \overset{R_3}{\underset{3}{AB}} \mid \overset{R_2}{\underset{2}{BA}} \mid \overset{R_1}{\underset{1}{SS}}$$

$(-1, 1, 1)$

$$A \rightarrow a \quad 2 \quad R_4$$

$$B \rightarrow b \quad 2 \quad R_5$$

$$S\ (1, 0, 0) \xrightarrow{R_1} (2, 0, 0) \xrightarrow{R_2} (1, 1, 1) \xrightarrow{R_3} (0, 2, 2) \xrightarrow{R_4} \cdots R_4 \ldots R_4 \cdots \longrightarrow \cdots R_5 \ldots R_5 \cdots (0, 0, 0)$$

$$(1,0,0) \xrightarrow{R_3} (0,1,1) \xrightarrow{R_4,\ R_5} (0,0,0)$$

ILP & LP framing

$$S(1,0,0,0,\ldots\ldots,0)$$

$$\Big\downarrow\ r_{1i}, r_{12}, \ldots\ldots, r_{1R}$$

$$(x_{21}, x_{22}, \ldots\ldots, x_{2N})$$

$$\Big\downarrow\ r_{1i}, r_{12}, \ldots\ldots, r_{1R}$$

$$(x_{31}, x_{32}, \ldots\ldots, x_{3N})$$

$$\Big\downarrow\ r_{11}, r_{12}, \ldots\ldots, r_{1R}$$

$$(x_{41}, x_{42}, \ldots\ldots, x_{4N})$$

$$\Big\downarrow$$

$$(\ldots\ldots\ \ldots\ldots\ldots)$$

$$\Big\downarrow$$

$$(x_{F1}, x_{F2}, \ldots\ldots, x_{FN}) \equiv (0,0,0,\ldots\ldots,0)$$

where $F = \sum f_i$ (sum over all the frequencies)

At each step:

$$(1, 0, 0, \ldots, 0)$$

$$+$$

AB   BA

$$\left[r_{11}(\text{actual rule}) + r_{12}(\text{actual rule}) + \ldots\ldots\ r_{1k}(\text{actual rule}) \right]$$

$$\equiv (x_{21}, x_{22}, x_{23}, \ldots\ldots, x_{2N})$$

ILP (0, 1) decides at each step which r to be 1 (all other r's would be non-zero)

$$(x_{21}, x_{22}, x_{23}, \ldots\ldots, x_{2N})$$

$$+ \left[r_{11}(\text{actual rule}) + r_{12}(\ldots) + \ldots \right]$$

$$\equiv (x_{31}, x_{32}, x_{33}, \ldots\ldots, x_{3N})$$