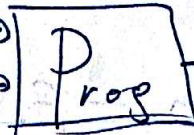


CCFG

cost $w(\cdot)$



$$T_i \xrightarrow{f_i} U_i \quad f_i=5$$

$$A \rightarrow (acb + a)$$

$$\begin{aligned} S &\rightarrow (aABb) \\ A &\rightarrow (aCb) \\ B &\rightarrow (bAc) \\ C &\rightarrow (A|B|c) \end{aligned}$$

Constrained CFG



$$\begin{aligned} A &\rightarrow cb \\ A &\rightarrow a \end{aligned}$$

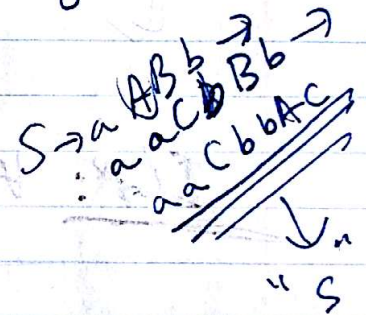
$$w(a)=1 \quad w(b)=2 \quad w(c)=3$$

Goal: find a string s that can be produced by the CCFG (obeying production capacity constraints) with maximal weight

$$w(s) = \sum_{s_i \in s} w(s_i)$$

$$\operatorname{argmax}_{s \in \Sigma^*} w(s)$$

$$\Sigma = \{a, b, c\}$$



- String Gen
- 1) Find w^* subject to capacity constraints
 - 2) Find s s.t. $w(s) = w^*$

$$w^* = 12$$

$$w^* = \max_{in(s)} w(a) \cdot (f_1 + f_2 + f_3) + w(b) \cdot (f_1 + f_2 + f_4 + f_5) + w(c) \cdot (f_4 + f_8)$$

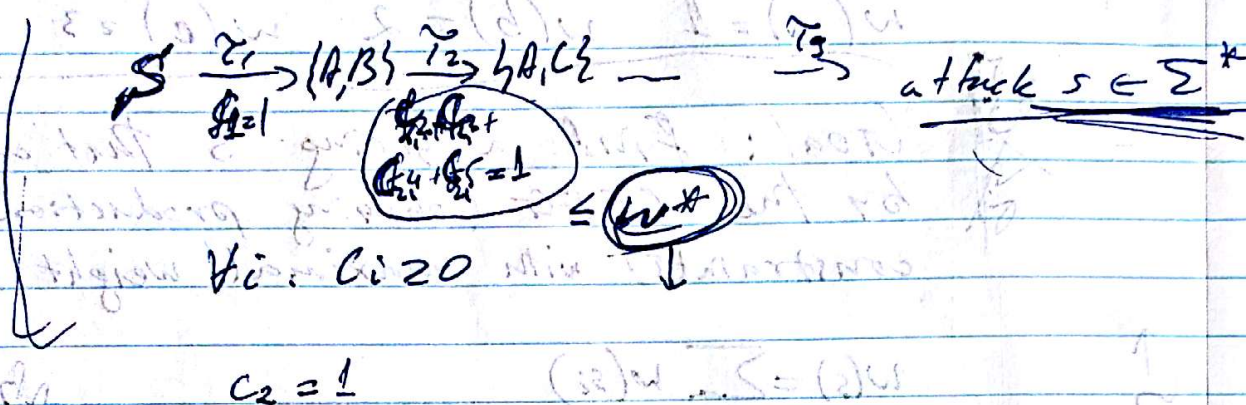
$$\begin{aligned} in(A): & f_1 + f_4 + f_6 = f_2 + f_3 & out(A) \\ in(B): & f_1 + f_2 = f_4 + f_5 & out(B) \\ in(C): & f_2 = f_6 + f_7 + f_8 & out(C) \end{aligned}$$

$\forall i, f_i \geq 0$, then plus capacity constraints

$$\begin{aligned} f_1 &= 1 \\ f_2 &= 1 \quad f_3 = 0 \\ f_4 &= 2 \quad f_5 = 0 \\ f_6 &= 0 \quad f_7 = 2 \quad f_8 = 3 \end{aligned} \quad \left\{ \begin{aligned} 0 &\leq f_1 \leq 1 \\ 0 &\leq f_2 \leq 2 \\ 0 &\leq f_3 \leq 1 \\ 0 &\leq f_4 \leq 2 \end{aligned} \right. \quad \left\{ \begin{aligned} 0 &\leq f_5 \leq 1 \\ 0 &\leq f_6 \leq 2 \\ 0 &\leq f_7 \leq 2 \\ 0 &\leq f_8 \end{aligned} \right.$$

2) String Generation - Find string s
(or ^{the} sequence of prod. rules) s.t. $w(s) = w^*$

→ Encode the production sequence (seq. of rules) as a transition system



ILP: $\forall e_{ij} \in \{0, 1\}$

\Rightarrow Attack String s

09/04/2025

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$$\{a\} + \{b\} = \{a, b\} \quad w(b) = 2$$

$$w(c) = 3$$

$$C \xrightarrow{b^*} A_2 \mid B_2 \mid C \quad b^8$$

$$b_0, b_1, \dots, w^* = \max [w(a)]$$

$$f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 + f_8 = 1$$

$$f_1 + f_2 + f_3 + f_4 + f_5 + f_6 = f_2 + f_3$$

$$f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 = f_4 + f_5$$

$$f_2 = f_6 + f_7 + f_8$$

Ex: $4 \text{ W } f_1 \rightarrow ($

10

2

[illegible]

100

0-27

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Objective :

$$w(a)\{f_1 + f_2 + f_3\} + w(b)\{f_1 + f_2 + f_4 + f_5\}$$

$$+ w(c)\{f_4 + f_8\}$$

$$= f_1[w(a) + w(b)] + f_2[w(a) + w(b)] + f_3[w(a)] \\ + f_4[w(b) + w(c)] + f_5[w(b)] + f_6[0] \\ + f_7[0] + f_8[w(c)]$$

$$= 3f_1 + 3f_2 + f_3 + 5f_4 + 2f_5 + 0f_6 \\ + 0f_7 + 3f_8$$

~~W*~~

Solution : $W^* : 23$

$$\begin{aligned} f_1 &= 1 \\ f_2 &= 2 \\ f_3 &= 1 \\ f_4 &= 2 \\ f_5 &= 0 \\ f_6 &= 0 \\ f_7 &= 1 \\ f_8 &= 1 \end{aligned}$$

$$\begin{aligned} f_1 &\geq 0 \\ f_2 &\geq 0 \\ f_3 &\geq 0 \\ f_4 &\geq 0 \\ f_5 &\geq 0 \\ f_6 &\geq 0 \\ f_7 &\geq 0 \\ f_8 &\geq 0 \end{aligned}$$

2

$$S \rightarrow AS \mid AB$$

$$A \rightarrow aB$$

$$S \xrightarrow{R_2} AS \mid \xrightarrow{R_1} AB$$

$$A \rightarrow aB$$

$$B \rightarrow b$$

Production rules \rightarrow converted to vectors

$S(1, 0, 0) \rightarrow$ Start with an S having 1S and 0A's & 0B's

$$(-1, 1, 1)$$

 R_1
 R_2

$$(0, 1, 0)$$

$$AB(0, 1, 1)$$

$$AS(1, 1, 0)$$

 R_2

$$(0, 1, 0)$$

$$AAS(1, 2, 0)$$

Example

$$S \rightarrow \overset{R_3}{AB} \mid \overset{R_2}{BA} \mid \overset{R_1}{SS}$$

$$A \rightarrow a \text{ (2) } R_4 \rightarrow (-1, 1, 1)$$

$$B \rightarrow b \text{ (2) } R_5$$

$$S \ (1, 0, 0)$$

$$\downarrow R_1$$

$$(2, 0, 0)$$

$$\downarrow R_2$$

$$(1, 1, 1)$$

$$\downarrow R_3$$

$$(0, 2, 2)$$

$$\downarrow R_4$$

$$R_4 - R_4 \rightarrow R_5 - R_5 \rightarrow (0, 0, 0)$$

$$(0, \dots, 0, 0, 0) \equiv (0, \dots, 0, 0, 0)$$

$$(0, \dots, 0, 0, 0) \equiv (0, \dots, 0, 0, 0)$$

$$(1, 0, 0)$$

$\downarrow R_3$

$$(0, 1, 1)$$

$\downarrow R_4, R_5$

$$(0, 0, 0)$$

Example

22	AB	BA	← 2
①	②	③	
AB	④	⑤	← 1
AB	⑥	⑦	← 8

ILP & LP framing

$$S(1, 0, 0, 0, \dots, 0)$$

$$\downarrow g_{11}, g_{12}, \dots, g_{1R}$$

$$(x_{21}, x_{22}, \dots, x_{2N})$$

$$\downarrow g_{11}, g_{12}, \dots, g_{1R}$$

$$(x_{31}, x_{32}, \dots, x_{3N})$$

$$\downarrow g_{11}, g_{12}, \dots, g_{1R}$$

$$(x_{41}, x_{42}, \dots, x_{4N})$$

$$\downarrow$$

$$(\dots)$$

$$(x_{F1}, x_{F2}, \dots, x_{FN}) \equiv (0, 0, 0, \dots, 0)$$

where $F = \sum f_i$ (sum over all the frequencies)

At each step:

$$(1, 0, 0, \dots, 0)$$

$$+ \quad \xrightarrow{\quad AB \quad} \quad \xrightarrow{\quad BA \quad}$$

$$\left[g_{11}(\text{actual rule}) + g_{12}(\text{actual rule}) + \dots + g_{1N}(\text{actual rule}) \right]$$

$$\equiv (x_{21}, x_{22}, x_{23}, \dots, x_{2N})$$

$$(x_{21}, x_{22}, x_{23}, \dots, x_{2N})$$

$$+ \left[g_{11}(\text{actual rule}) + g_{12}(\dots) + \dots \right]$$

$$\equiv (x_{31}, x_{32}, x_{33}, \dots, x_{3N})$$

↓
ILP(0,1) decides
at each step which
g to be 1 (all other
g's would be non-
zero)