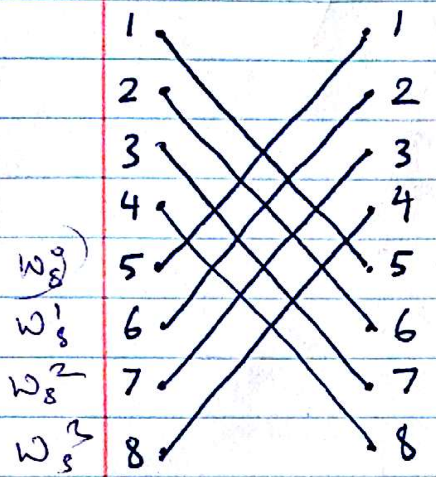
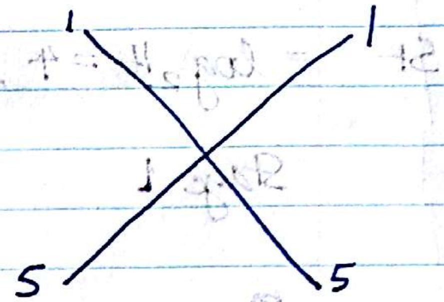
Butterfly mechanism for computing FFT:



For each X

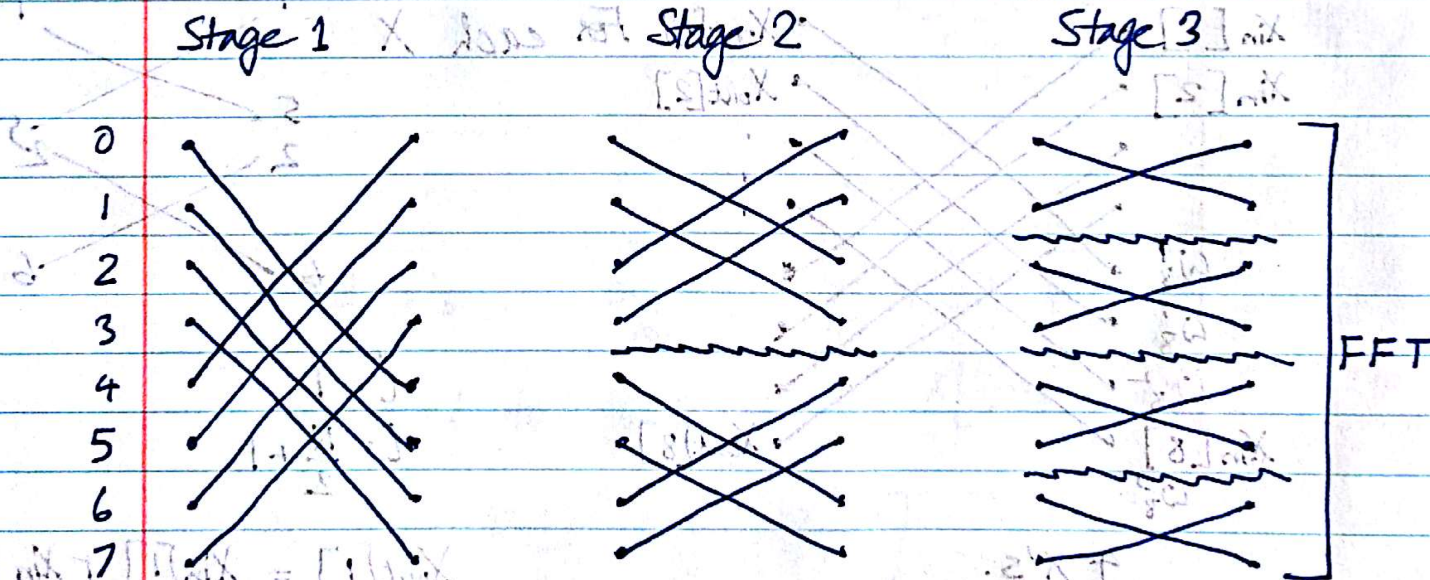


$X_{out}(1) = X_{in}(1) + X_{in}(5)$

$X_{out}(5) = [X_{in}(1) + X_{in}(5)] * W^k_n$

where $W^k_n = e^{-j \cdot 2\pi k/N}$

where $k =$



$X_{in} = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]$

$St = \log_2 16 = 4$ stages

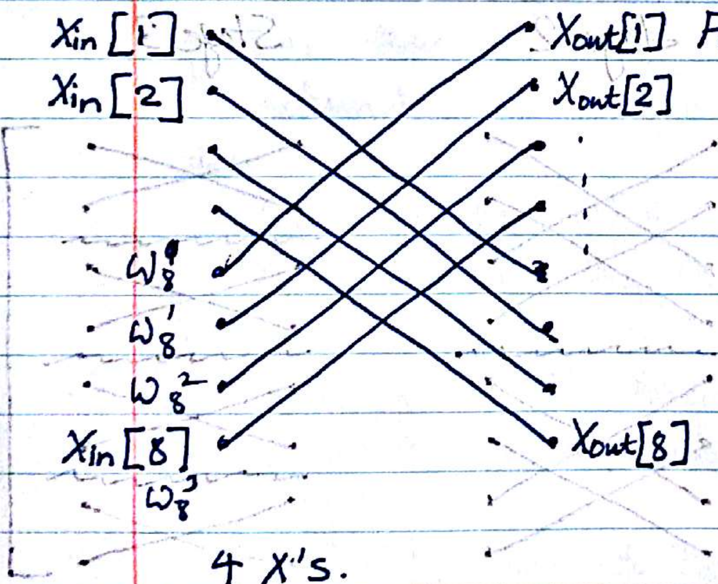
Stage 1

or

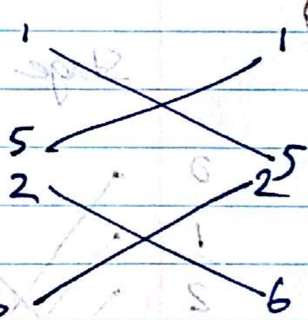
$X_{in} = [1, 2, 3, 4, 5, 6, 7, 8]$

$St = \log_2 8 = 3$ stages

Stage 1



For each X



$i = 1$

$i = \frac{N}{2} + 1$

$X_{out}[1] = X_{in}[1] + X_{in}[5]$

$X_{out}[5] = \{X_{in}[1] + X_{in}[5]\} e^{-\frac{2\pi i k}{N}}$

$K = 0$

$X_{out}[2] = X_{in}[2] + X_{in}[6]$

$= \{ \qquad \} e^{-\frac{2\pi i k}{N}}$

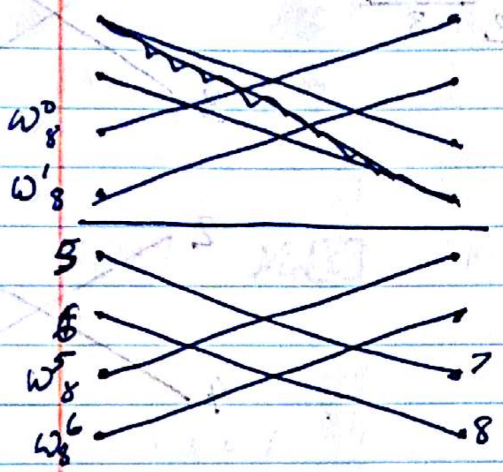
$K = 1$

$\frac{N}{2}$ X's. for $i = 1$ to $\frac{N}{2}$,

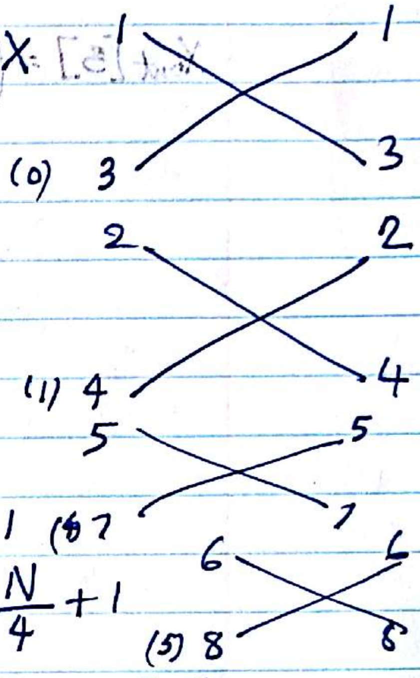
$\{ X_{out}[i] = X_{in}[i] + X_{in}[\frac{N}{2} + i]$

$X_{out}[\frac{N}{2} + i] = [\qquad] e^{-\frac{2\pi j (i-1)}{N}} \}$

Stage 2



For each X:



$i = 1$

$i = \frac{N}{4} + 1$

$$X_{out}[1] = X_{in}[1] + X_{in}[3]$$

$$X_{out}[3] = \{X_{in}[1] + X_{in}[3]\} e^{-\frac{2\pi j k}{N}}$$

$K = 0$

$\frac{N}{2^2}$

$\frac{N}{2}$ X's

for $i = 1$ to $\frac{N}{4}$

$i = 1$, 2

$$X\left[\frac{N}{2}+i\right] = X[5] + X\left[\frac{N}{2}+\frac{N}{4}+i\right]$$

$$X\left[\frac{N}{2}+\frac{N}{4}+i\right] = \left\{X[5] + X\left[\frac{N}{2}+\frac{N}{4}+i\right]\right\} \times \left[e^{-\frac{2\pi j\left(\frac{N}{2}+i-1\right)}{N}}\right]$$

$$X_{out}[i] = X_{in}[i] + X_{in}\left[\frac{N}{4}+i\right]$$

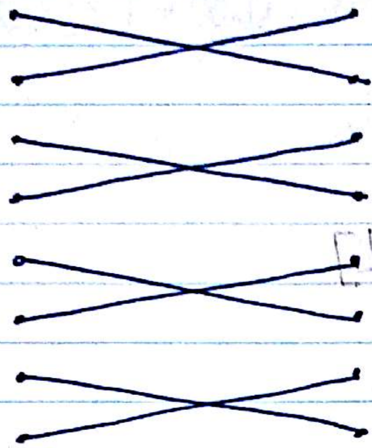
$i = 1$	1	1	3
$i = 2$	2	2	4
	3	3	

$\frac{N}{4} = 2$

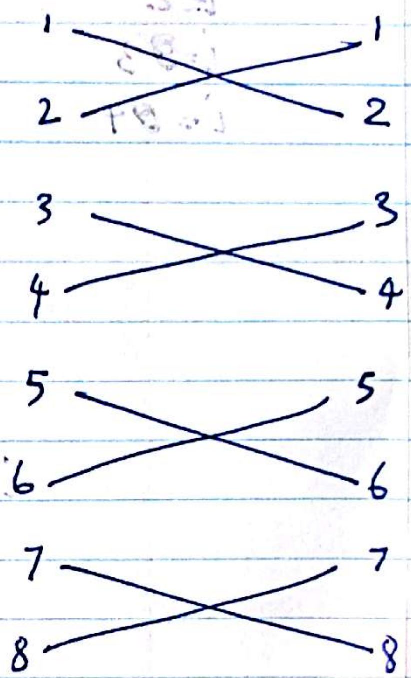
$$X_{out}\left[\frac{N}{4}+i\right] = \left\{X_{in}[i] + X_{in}\left[\frac{N}{4}+i\right]\right\} e^{-\frac{2\pi j(i-1)}{N}}$$

$i = 1$	3	1	3
$i = 2$	4	2	4

Stage 3



For each X



$i = 1$ to $\frac{N}{2^{stage}}$

$j = 1$ to 2^{stage}.

$i = 1$ to 1

$j = 2^3$

$$\overset{1}{X_{out}[i]} = \overset{1}{X_{\cancel{out}\,in}[i]} + \overset{2}{X_{in}\left[\tfrac{N}{8}+i\right]}$$

$$\overset{2}{X_{out}\left[\tfrac{N}{8}+i\right]} = \left\{\overset{1}{X_{in}[i]} + \overset{2}{X_{in}\left[\tfrac{N}{8}+i\right]}\right\}e$$

$$\overset{3}{X_{out}\left[\tfrac{N}{4}+i\right]} = X_{in}\left[\overset{\frac{N}{4}+i}{\cancel{\quad}}\right] + X_{in}\left[\tfrac{N}{4}\cancel{+6}+\tfrac{N}{8}+i\right]$$

$$\overset{4}{X_{out}\left[\tfrac{N}{8}+\tfrac{N}{4}+i\right]} = \left\{X_{in}\left[\tfrac{N}{4}+i\right] + X_{in}\left[\tfrac{N}{8}+\tfrac{N}{4}+i\right]\right\}e^{[}$$

$$\overset{5}{X_{out}\left[\tfrac{N}{2}+i\right]} = \left\{X_{\overset{in}{\cancel{out}}}\left[\tfrac{N}{2}+i\right] + X_{\overset{in}{\cancel{out}}}\left[\tfrac{N}{2}+\tfrac{N}{8}+i\right]\right.$$

$$\overset{6}{X_{out}\left[\tfrac{N}{2}+\tfrac{N}{8}+i\right]}$$

$$\overset{7}{X_{out}}\left[\overset{4}{\tfrac{N}{2}}+\overset{2}{\tfrac{N}{4}}+\overset{1}{\cancel{\tfrac{N}{8}}}+i\right] = X_{in}\left[\tfrac{N}{2}+\tfrac{N}{4}+i\right] + X_{in}\left[\tfrac{N}{2}+\tfrac{N}{4}+\tfrac{N}{8}+i\right]$$

$$i \rightarrow i \quad , \quad i+\frac{N}{8}$$

$$i+\frac{N}{4} \rightarrow i+\frac{N}{4} \quad , \quad i+\frac{N}{8}+\frac{N}{4}$$

$$i+\frac{N}{2} \rightarrow i+\frac{N}{2} \quad , \quad i+\frac{N}{8}+\frac{N}{2}$$

$$i+\frac{N}{2}+\frac{N}{4} \rightarrow i+\frac{N}{4}+\frac{N}{2} \quad , \quad i+\frac{N}{8}+\frac{N}{4}+\frac{N}{2}$$

i

$i+\frac{N}{8}$

$i+\frac{N}{4}$

$i+\frac{N}{4}+\frac{N}{8}$

$i+\frac{N}{2}$

$i+\frac{N}{2}+\frac{N}{8}$

$i+\frac{N}{2}+\frac{N}{4}$

$i+\frac{N}{2}+\frac{N}{4}+\frac{N}{8}$