Basant Vishwakarma

Amicable Numbers

OVERVIEW

Amicable numbers are two different natural numbers related in such a way that the sum of the proper divisors of each is equal to the other number. That is, σ(a)=b and σ(b)=a, where σ(n) is equal to the sum of positive divisors of n.

The smallest pair of amicable numbers is (220, 284). They are amicable because the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71 and 142, of which the sum is 220. (A proper divisor of a number is a positive factor of that number other than the number itself. For example, the proper divisors of 6 are 1, 2, and 3.)

1. σ(220) = 1+2+4+5+10+11+20+22+44+55+110 = 284
2. σ(220) = 1+2+4+71+142 = 220

RULES FOR GENERATION

Euler's rule

While these rules do generate some pairs of amicable numbers, many other pairs are known, so these rules are by no means comprehensive.

In particular, the two rules below produce only even amicable pairs, so they are of no interest for the open problem of finding amicable pairs coprime to 210 = 2*3*5*7, while over 1000 pairs coprime to 30 = 2*3*5 are known

$$p = (2^{(n-m)} + 1)\times 2^{(m)} - 1$$
$$q = (2^{(n-m)} + 1)\times 2^{n} - 1$$
$$r = (2^{(n-m)} + 1)^2\times 2^{(m+n)} - 1$$

Where $n > m > 0$ are integers and p, q, and r are prime numbers, then $2^n\times p\times q$ and $2^n\times r$ are a pair of amicable numbers. Thābit ibn Qurra's theorem corresponds to the case $m = n - 1$. Euler's rule creates additional amicable pairs for (m,n) = (1,8), (29,40) with no others being known. Euler overall found 58 new pairs to make all the by then existing pairs into 61

REGULAR PAIRS

Let (m, n) be a pair of amicable numbers with $m < n$, and write $m = gM$ and $n = gN$ where g is the greatest common divisor of m and n. If M and N are both coprime to g and square free then the pair (m, n) is said to be regular. Otherwise, it is called irregular or exotic. If (m, n) is regular and M and N have i and j prime factors respectively, then (m, n) is said to be of type (i, j).

For example, with $(m, n) = (220, 284)$, the greatest common divisor is 4 and so $M = 55$ and $N = 71$. Therefore, $(220, 284)$ is regular of type $(2, 1)$.