BLG 202E Numerical Methods in CE
2019/2020 Spring
Homework - 5

Due: 15.06.2020 23:59 — Effect to the overall grade: 40%

Policy:

- In Case of Cheating and Plagiarism Strong **disciplinary action will be taken**.
- Upload your solutions through Ninova. Homeworks sent via e-mail and late submissions will not be accepted.
- Prepare a report including all your solutions, codes and their results.
- You are asked to upload a .ipynb file (Jupyter Notebook) and a .pdf file (report) to Ninova.
- You should write all your codes in Python language using Jupyter notebook. You can install Jupyter Notebook by following these steps on this documentation. If you are not familiar with Jupyter Notebook, you can check this tutorial.
- You do not have to use Latex for the report but if you use Latex, you will get 10% more points. You can use this Latex template for the report.
- If you do not use Latex, the handwritten parts of the solutions must be presented on a paper legibly and scanned clearly. 10% penalty will be applied for illegible reports.

1. **[25 points]**
Consider the approximation to the second derivative

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}$$

(a) There are in general two basic types of measured error: relative error and absolute error. Give their formulas, compare them and explain briefly which measure is more meaningful.

(b) Show that the discretization error for this formula is $\mathcal{O}(h)$.

(c) Calculate the second derivative of $tan(\frac{\pi}{4})$ using the formula above for $h = 1e-1, 1e-2, ..., 1e-9, 1e-10$. Then, draw a graph showing the difference between the expected error ($\mathcal{O}(h)$) and the actual error for each h values. You can use absolute error to calculate actual error. (Use Python for this part of the question.)

(d) Compare the measured error with theoretical findings. Have another error type occurred as a results of the procedure above. If so, give your reasoning.

(e) Determine whether this problem at near $\frac{\pi}{4}$ is well-conditioned or ill-conditioned. Explain why. Please justify your answer by coding or theoretically.

NOTE: Please add your codes and their results to the report. Otherwise, Python parts of your solution will not be graded.
NOTE 2: Please read Chapter 1 from our textbook.

For your questions about the question 1: Abdullah Ekrem Okur (okurabd@itu.edu.tr)

2. **[25 points]**
A machine stores floating-point numbers in a hypothetical 10-bit binary word. It employs the first bit for the sign of the number, the second one for the sign of the exponent, the next four for the exponent, and the last four for the magnitude of the mantissa.

(a) Find how 0.03754 will be represented in the floating-point 10-bit word.

(b) What is the decimal equivalent of the 10-bit word representation of part (a)?

(c) Confirm that the magnitude of the relative true error that results from approximate representation of 0.03754 in the 10-bit format is less than the machine epsilon.

For your questions about the question 2: Ruşen Halepmollası (halepmollasi@itu.edu.tr)

3. **[25 points]**
You are given a function $h(x) = x^2 + \frac{4}{25}$.

   (a) Find the fixed points of $h(x)$.

   (b) Which points you found in (a) will be converged by the fixed point iteration? Briefly explain your **solution** and your **assumptions**.

   (c) In order to reduce the convergence error by a factor of 10, how many iterations should you perform with the point/points you found in (b) approximately?

   (d) How can we see Newton's method as fixed point iteration? Explain the relationship between the methods and why this relationship is useful.

For your questions about the Question 3: Gülçin Baykal (baykalg@itu.edu.tr)

4. **[25 points]**
With the aim of expanding it's market share, a company goes for a research and decides to add 3 different kind of products to it's product range. The company wants to buy 200, 600 and 250 units of each product respectively to test the sales for only one month and makes a deal with a supplier which gives the lowest prices for each kind of product. However, company manager decides to test the sales for two more months and wants to buy 250, 500 and 200 units of each product respectively for the second month and 300, 350 and 400 units of each product respectively for the third month.

   Having increased the purchased amounts, company manager makes a bargain with the supplier and agrees to pay $ 55500, $ 51000 and $ 51000 for the first, second and third months respectively.

   As a candidate purchasing engineer, you are required to find the unit price for each product. Use Gaussian Elimination for your calculations, show each step explicitly and justify your answer.

For your questions about the question 4: Mehmet Koça (koca19@itu.edu.tr)