BLG 202E Numerical Methods in CE
2019/2020 Spring
Homework - 4

Due 31.05.2020 23:59

Policy:

- In Case of Cheating and Plagiarism Strong **disciplinary action will be taken**.
- Upload your solutions through Ninova. Homeworks sent via e-mail and late submissions will not be accepted.
- Prepare a report including all your solutions, codes and their results.
- You are asked to upload a .ipynb file (Jupyter Notebook) and a .pdf file (report) to Ninova.
- You should write all your codes in Python language using Jupyter notebook. You can install Jupyter Notebook by following these steps on this documentation. If you are not familiar with Jupyter Notebook, you can check this tutorial.
- You do not have to use Latex for the report but if you use Latex, you will get 10% more points. You can use this Latex template for the report.
- If you do not use Latex, the handwritten parts of the solutions must be presented on a paper legibly and scanned clearly. 10% penalty will be applied for illegible reports.

1. **[30 points]**

Points	x	y
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	-0.60	1.04
6	-1.04	0.60
7	-1.20	0.00

The data points above are the results obtained from a chemical experiment. Considering the quadratic line that passes through points 4, 5, 6 when drawn on a standard coordinate system, interpolate the value of y where x = 0.90 with Newton' s divided difference method and a second order polynomial. Calculate the absolute relative approximate error for the second order polynomial interpolation.

For your questions about the question 1: Mehmet Koca (koca19@itu.edu.tr)

2. **[35 points]**

   (a) Derive second, fourth and sixth order centered difference formulas for the first derivative of f at x_0 using Taylor's expansion. Calculate the truncation error for each formula.

   (b) Implement a method that received a function of one variable, a number, approximation order (second, fourth and sixth) and step size (h). Using this method, calculate the derivative of $e^x cos(2x)$ at 1 for values $h = 10^{-k}, k = 1, ..., 10$ and different approximation order. Then calculate absolute error for each approximation order and compare the results with theoretical findings. Also generate a graph of different order approximations' accuracy as in Figure 14.1 from our textbook. Explain your observations briefly.

For your questions about the question 2: Abdullah Ekrem Okur (okurabd@itu.edu.tr)

3. **[35 points]**
Consider the numerical differentiation of the function $f(x) = c(x)e^{x/\pi}$ defined on $[0, \pi]$, where

$$c(x) = j \text{ , } .25(j-1)\pi \leq x < .25j\pi$$

for $j = 1, 2, 3, 4$.

(a) Contemplating a difference approximation with step size $h = n/\pi$, explain why it is a very good idea to ensure that n is an integer multiple of 4, $n = 4l$.

(b) Consider approximating $q(x_i)$ in terms of values of $c(x)$ and $g(x)$, where

$$q(x) = -[c(x)g'(x)]'$$

is known to be square integrable (but not necessarily differentiable) on $[0, \pi]$. The function g is assumed given, and it has some jumps of its own to offset those of $c(x)$ so as to create a smoother function $\phi(x) = c(x)g'(x)$. The latter is often termed the *flux function* in applications. Evaluate the merits (or lack thereof) of the difference approximation

$$h^{-1}\left[\frac{c_{i+1/2}(g_{i+1}-g_i)}{h} - \frac{c_{i-1/2}(g_i-g_{i-1})}{h}\right]$$

with g_i, $g_{i\pm1}$, and $c_{i\pm1/2}$ appropriately defined for $i = 1, ..., n-1$.

For your questions about the question 3: Ruşen Halepmollası (halepmollasi@itu.edu.tr,)