BLG202E HOMEWORK № 3

Başar Demir, 150180080 13/05/2020

Problem 1

$$A = \begin{bmatrix} 3 & 1 & 6 \\ 6 & 2 & 4 \\ 9 & 3 & 2 \end{bmatrix}$$

0.1 The largest eigenvalue and the corresponding eigenvector

To find the the eigenvalues of the matrix A, we should substract λ from the diagonal of the matrix, then we should solve for its determinant equal to 0.

$$det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 1 & 6 \\ 6 & 2-\lambda & 4 \\ 9 & 3 & 2-\lambda \end{vmatrix} = 0$$

$$det(A - \lambda I) = (3-\lambda) \begin{vmatrix} 2-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} - \begin{vmatrix} 6 & 4 \\ 9 & 2-\lambda \end{vmatrix} + (6) \begin{vmatrix} 6 & 2-\lambda \\ 9 & 3 \end{vmatrix} = 0$$

$$det(A - \lambda I) = (3-\lambda)[(2-\lambda)^2 - 12] - [12 - 6\lambda - 36] + (6)[18 - 18 + 9\lambda] = 0$$

Characteristic Equation

$$det(A - \lambda I) = -\lambda^3 + 7\lambda^2 + 56\lambda = 0$$

If we solve this cubic equation, we get eigenvalues of A matrix. These are,

- $\lambda_1 = 0$
- $\lambda_2 = \frac{-\sqrt{273}+7}{2}$
- $\lambda_3 = \frac{\sqrt{273}+7}{2}$

The eigenvalue λ_3 is the biggest one.

Eigenvectors of λ_3

To find corresponding eigenvectors, $(A - \lambda_3 I)v = 0$ have to be solved.
$\lambda_3 = \frac{\sqrt{273}+7}{2} \approx 11.76$

$$(A - 11.76I)v = \begin{bmatrix} 3-11.76 & 1 & 6 \\ 6 & 2-11.76 & 4 \\ 9 & 3 & 2-11.76 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A - 11.76I)v = \begin{bmatrix} -8,76 & 1 & 6 \\ 6 & -9.76 & 4 \\ 9 & 3 & -9.76 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R2 + (\frac{50}{73})R1 \rightarrow R2 \begin{bmatrix} -8,76 & 1 & 6 \\ 0 & -9.07 & 8.10 \\ 9 & 3 & -9.76 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3 + (\frac{75}{73})R1 \rightarrow R3 \begin{bmatrix} -8,76 & 1 & 6 \\ 0 & -9.07 & 8.10 \\ 0 & 4.02 & -3.59 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3 + (\frac{75}{169})R2 \rightarrow R3 \begin{bmatrix} -8,76 & 1 & 6 \\ 0 & -9.07 & 8.10 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Finally we got reduced row echelon form, if we solve the linear equation by taking v_3 as t,

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0,78t \\ 0.89t \\ t \end{bmatrix} = \begin{bmatrix} 0,78 \\ 0.89 \\ 1 \end{bmatrix} t$$

The corresponding eigenvector for $\lambda 3$ is approximately equal to

$$\begin{bmatrix} 0,78 \\ 0.89 \\ 1 \end{bmatrix}$$

0.2 Singular Values of A

Singular values of the matrix A is equal to square roots of eigenvalues of A^TA or AA^T. Therefore, I will calculate A^TA and its eigenvalues.

$$AA^T = \begin{bmatrix} 3 & 1 & 6 \\ 6 & 2 & 4 \\ 9 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 6 & 9 \\ 1 & 2 & 3 \\ 6 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 46 & 44 & 42 \\ 44 & 56 & 68 \\ 42 & 68 & 94 \end{bmatrix}$$

Eigenvalues of AA^T

$$det(AA^T - \lambda I) = \begin{vmatrix} 46-\lambda & 44 & 42 \\ 44 & 56-\lambda & 68 \\ 42 & 68 & 94-\lambda \end{vmatrix} = 0$$

$$det(AA^T - \lambda I) = (46-\lambda) \begin{vmatrix} 56-\lambda & 68 \\ 68 & 94-\lambda \end{vmatrix} - 44 \begin{vmatrix} 44 & 68 \\ 42 & 94-\lambda \end{vmatrix} + 42 \begin{vmatrix} 44 & 56-\lambda \\ 42 & 68 \end{vmatrix} = 0$$

$$det(AA^T - \lambda I) = (46-\lambda)[(56-\lambda)(94-\lambda)-4624]-(44)[44(94-\lambda)-2856]+(42)[2992-(56-\lambda)(42)] = 0$$

Characteristic Equation

$$det(AA^T - \lambda I) = -\lambda^3 + 196\lambda^2 - 3840\lambda = 0$$

If we solve this cubic equation, we get eigenvalues of AA^T matrix. These are,

- $\lambda_1 = 0$
- $\lambda_2 = -2\sqrt{1441} + 98 \approx 22.079$
- $\lambda_3 = 2\sqrt{1441} + 98 \approx 173.921$

Therefore, the singular values that are square root of eigenvalues:

- $\sigma 1 = 0$
- $\sigma 2 = \sqrt{-2\sqrt{1441} + 98} \approx 4.6988$
- $\sigma 3 = \sqrt{2\sqrt{1441} + 98} \approx 13.1879$

0.3 Singular Value Decomposition of A

$$A = U\Sigma V^T$$

Singular Value Decomposition is based on two fundamental equations, which are:

- $A^T A = V\Sigma^T \Sigma V^T$
- $AV = U\Sigma$

First equation is basically an eigenvalue problem.

Finding V and Σ

In the b part of this question, we have already found eigenvalues of $A^T A$. So, we just have to calculate corresponding eigenvectors of these eigenvalues.

- For $\lambda_1 = 0$

$$det(A^T A - 0I) = \begin{bmatrix} 126 & 42 & 60 \\ 42 & 14 & 20 \\ 60 & 20 & 56 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R1 * \frac{1}{126} \to R1 \begin{bmatrix} 1 & 0.3333 & 0.4762 \\ 42 & 14 & 20 \\ 60 & 20 & 56 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R2-(42*R1)\rightarrow R2 \begin{bmatrix} 1 & 0.3333 & 0.4762 \\ 0 & 0 & 0 \\ 60 & 20 & 56 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3-(60*R1)\rightarrow R3 \begin{bmatrix} 1 & 0.3333 & 0.4762 \\ 0 & 0 & 0 \\ 0 & 0 & 27.4286 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3/27.4286\rightarrow R3 \begin{bmatrix} 1 & 0.3333 & 0.4762 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Finally we got reduced row echelon form, if we solve the linear equation by taking v_2 as t,

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.3333t \\ t \\ 0 \end{bmatrix} = \begin{bmatrix} -0.3333 \\ 1 \\ 0 \end{bmatrix} t$$

The corresponding eigenvector for $\lambda 1$ is approximately equal to

$$v_1 \approx \begin{bmatrix} \frac{-0.3333}{\sqrt{(-0.33)^2+1}} \\ \frac{1}{\sqrt{(-0.33)^2+1}} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.3162 \\ 0.9487 \\ 0 \end{bmatrix}$$

- For $\lambda_2 = 22.079$

$$det(A^TA-(22.079)I) = \begin{bmatrix} 103.921 & 42 & 60 \\ 42 & -8.079 & 20 \\ 60 & 20 & 33.921 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R1*\frac{1}{103.921}\rightarrow R1 \begin{bmatrix} 1 & 0.4042 & 0.5774 \\ 42 & -8.079 & 20 \\ 60 & 20 & 33.921 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R2-(42*R1)\rightarrow R2 \begin{bmatrix} 1 & 0.4042 & 0.5774 \\ 0 & -25.0534 & -4.2492 \\ 60 & 20 & 33.921 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3-(60*R1)\rightarrow R3 \begin{bmatrix} 1 & 0.4042 & 0.5774 \\ 0 & -25.0534 & -4.2492 \\ 0 & -4.2492 & -0.7207 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R2*\frac{1}{-25.0534}\rightarrow R2 \begin{bmatrix} 1 & 0.4042 & 0.5774 \\ 0 & 1 & 0.1696 \\ 0 & -4.2492 & -0.7207 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R3-(-4.2492*R2)\rightarrow R3\begin{bmatrix}1&0.4042&0.5774\\0&1&0.1696\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Finally we got reduced row echelon form, if we solve the linear equation by taking v_3 as t,

$$\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}\approx\begin{bmatrix}-0.5088t\\-0.1696t\\t\end{bmatrix}=\begin{bmatrix}-0.5088\\-0.1696\\1\end{bmatrix}t$$

The corresponding normalized eigenvector for $\lambda2$ is approximately equal to

$$v_2\approx\begin{bmatrix}\frac{-0.5088}{1.1348}\\\frac{-0.1696}{1.1348}\\\frac{1}{1.1348}\end{bmatrix}\approx\begin{bmatrix}-0.4484\\-0.1495\\0.8813\end{bmatrix}$$

- For $\lambda_3 = 173.921$

$$det(A^TA-(173.921)I)=\begin{bmatrix}-47.921&42&60\\42&-159.921&20\\60&20&-117.921\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R1*\frac{1}{-47.921}\rightarrow R1\begin{bmatrix}1&-0.8764&-1.2520\\42&-159.921&20\\60&20&-117.921\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R2-(42*R1)\rightarrow R2\begin{bmatrix}1&-0.8764&-1.2520\\0&-123.1104&72.5865\\60&20&-117.921\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R3-(60*R1)\rightarrow R3\begin{bmatrix}1&-0.8764&-1.2520\\0&-123.1104&72.5865\\0&72.5865&-42.7973\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R2*\frac{1}{-123.1104}\rightarrow R2\begin{bmatrix}1&-0.8764&-1.2520\\0&1&-0.5896\\0&72.5865&-42.7973\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R3-(72.5865*R2)\rightarrow R3\begin{bmatrix}1&-0.8764&-1.2520\\0&1&-0.5896\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Finally we got reduced row echelon form, if we solve the linear equation by taking v_3 as t,

$$\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}\approx\begin{bmatrix}1.7688t\\0.5896t\\t\end{bmatrix}=\begin{bmatrix}1.7688\\0.5896\\1\end{bmatrix}t$$

The corresponding eigenvector for $\lambda 3$ is approximately equal to

$$v_3 \approx \begin{bmatrix} \frac{1.7688}{2.1157} \\ \frac{0.5896}{2.1157} \\ \frac{1}{2.1157} \end{bmatrix} \approx \begin{bmatrix} 0.836 \\ 0.2787 \\ 0.4726 \end{bmatrix}$$

As a result, V and Σ values of the A have found. 0 singular values are not written to singular value matrix.

$$\Sigma \approx \begin{bmatrix} 13.1879 & 0 & 0 \\ 0 & 4.6988 & 0 \end{bmatrix} V \approx \begin{bmatrix} 0.836 & -0.4485 & -0.3162 \\ 0.2787 & -0.1495 & 0.9487 \\ 0.4726 & 0.8813 & 0 \end{bmatrix}$$

To find the last unknown of the SVD that is U, I will use second equation $AV = U\Sigma$. By the formula $U_i = \frac{1}{\sigma_i} AV_i$

$$U_1 \approx 0.0758 \begin{bmatrix} 3 & 1 & 6 \\ 6 & 2 & 4 \\ 9 & 3 & 2 \end{bmatrix} \begin{bmatrix} 0.836 \\ 0.2787 \\ 0.4726 \end{bmatrix} \approx \begin{bmatrix} 0.4262 \\ 0.5658 \\ 0.7053 \end{bmatrix}$$

$$U_2 \approx 0.2128 \begin{bmatrix} 3 & 1 & 6 \\ 6 & 2 & 4 \\ 9 & 3 & 2 \end{bmatrix} \begin{bmatrix} -0.4485 \\ -0.1495 \\ 0.8812 \end{bmatrix} \approx \begin{bmatrix} 0.8070 \\ 0.1138 \\ -0.5794 \end{bmatrix}$$

$$U \approx \begin{bmatrix} 0.4262 & 0.8070 \\ 0.5658 & 0.1138 \\ 0.7053 & -0.5794 \end{bmatrix}$$

The final form of the Singular Value Decomposition of matrix A:

$$A = U\Sigma V^T$$

$$\begin{bmatrix} 3 & 1 & 6 \\ 6 & 2 & 4 \\ 9 & 3 & 2 \end{bmatrix} \approx \begin{bmatrix} 0.4262 & 0.8070 \\ 0.5658 & 0.1138 \\ 0.7053 & -0.5794 \end{bmatrix} \begin{bmatrix} 13.1879 & 0 & 0 \\ 0 & 4.6988 & 0 \end{bmatrix} \begin{bmatrix} 0.836 & 0.2787 & 0.4726 \\ -0.4484 & -0.1495 & 0.8813 \\ -0.3162 & 0.9487 & 0 \end{bmatrix}$$

1 Problem 2

1.1 Linear Interpolation

$$p(x) = c_0 + c_1 x$$

I have chosen $t = 2.5$ and $t = 3.7$ data points. If I write, this data into equation:

$$p(2.5) = c_0 + c_1 2.5 = 100$$

$$p(3.7) = c_0 + c_1 3.7 = 130$$

If we solve this equations, we get $c_0 = 37.5$ and $c_1 = 25$. The interpolation function:

$$p(x) = 37.5 + 25x$$

Therefore,our speed prediction at $t = 3$ is

$$p(3) = 37.5 + 25 * 3 = \textbf{112.5}$$

1.2 Quadratic Interpolation

$$p(x) = c_0 + c_1 x + c_2 x^2$$

I have chosen $t = 2.2$, $t = 2.5$, $t = 3.7$ data points. If I write, this data into equation:

$$p(2.2) = c_0 + c_1 2.2 + c_2 (2.2)^2 = 80$$

$$p(2.5) = c_0 + c_1 2.5 + c_2 (2.5)^2 = 100$$

$$p(3.7) = c_0 + c_1 3.7 + c_2 (3.7)^2 = 130$$

If we transform this equations to matrix form:

$$\begin{bmatrix} 1 & 2.2 & 4.84 \\ 1 & 2.5 & 6.25 \\ 1 & 3.7 & 13.69 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 100 \\ 130 \end{bmatrix}$$

$$R2 - (R1) \rightarrow R2 \begin{bmatrix} 1 & 2.2 & 4.84 \\ 0 & 0.3 & 1.41 \\ 1 & 3.7 & 13.69 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 20 \\ 130 \end{bmatrix}$$

$$R3 - (R1) \rightarrow R3 \begin{bmatrix} 1 & 2.2 & 4.84 \\ 0 & 0.3 & 1.41 \\ 0 & 1.5 & 8.85 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 20 \\ 50 \end{bmatrix}$$

$$R3 - (5 * R2) \rightarrow R3 \begin{bmatrix} 1 & 2.2 & 4.84 \\ 0 & 0.3 & 1.41 \\ 0 & 0 & 1.8 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 20 \\ -50 \end{bmatrix}$$

If we solve the linear equation, we get:

$$c_0 = -219.4 c_1 = 197.2 c_2 = -27.8$$

$$p(x) = -219.4 + (197.2)x + (-27.8)(x)^2$$

Therefore,our speed prediction at $t = 3$ is

$$p(3) = -27.8 + (197.2)3 + (-219.4)(3)^2 \approx \textbf{122.22}$$

1.3 Cubic Interpolation

$$p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

I have chosen $t = 0$ $t = 2.2$, $t = 2.5$, $t = 3.7$ data points. If I write, this data into equation:

$$p(0) = c_0 + c_1(0) + c_2(0)^2 + c_3(0)^3 = 0$$

$$p(2.2) = c_0 + c_1(2.2) + c_2(2.2)^2 + c_3(2.2)^3 = 80$$

$$p(2.5) = c_0 + c_1(2.5) + c_2(2.5)^2 + c_3(2.5)^3 = 100$$

$$p(3.7) = c_0 + c_1(3.7) + c_2(3.7)^2 + c_3(3.7)^3 = 130$$

If we transform this equations to matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2.2 & 4.84 & 10.648 \\ 1 & 2.5 & 6.25 & 15.625 \\ 1 & 3.7 & 13.69 & 50.653 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 100 \\ 130 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.2 & 4.84 & 10.648 \\ 0 & 2.5 & 6.25 & 15.625 \\ 0 & 3.7 & 13.69 & 50.653 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 100 \\ 130 \end{bmatrix}$$

$$R2/2.2 \rightarrow R2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 2.5 & 6.25 & 15.625 \\ 0 & 3.7 & 13.69 & 50.653 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 100 \\ 130 \end{bmatrix}$$

$$R3 - (R2 * 2.5) \rightarrow R3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 0 & 0.75 & 0.525 \\ 0 & 3.7 & 13.69 & 50.653 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 9.090 \\ 130 \end{bmatrix}$$

$$R4-(R2*3.7) \to R3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 0 & 0.75 & 0.525 \\ 0 & 0 & 5.55 & 32.745 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 9.090 \\ -4.545 \end{bmatrix}$$

$$R3/0.75 \to R3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 0 & 1 & 4.7 \\ 0 & 0 & 5.55 & 32.745 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 12.121 \\ -4.545 \end{bmatrix}$$

$$R4-(R3*5.55) \to R4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 0 & 1 & 4.7 \\ 0 & 0 & 0 & 6.66 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 12.121 \\ -71.818 \end{bmatrix}$$

$$R4/6.66 \to R4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2.2 & 4.84 \\ 0 & 0 & 1 & 4.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 36.363 \\ 12.121 \\ -10.783 \end{bmatrix}$$

If we solve the linear equation, we get:

$$c_0 = 0 c_1 = -49.612 c_2 = 62.803 c_3 = -10.783$$

$$p(x) = (-49.612)x + (62.803)(x)^2 + (-10.783)(x)^3$$

Therefore,our speed prediction at $t = 3$ is

$$p(3) = (-49.612)3 + (62.803)(3)^2 + (-10.783)(3)^3 \approx \textbf{125.25}$$

1.4 Distance using Cubic Interpolation

By taking the integral over the time of velocity, it gives total distance of any moving object.

$$distance = \int_0^t v(t)dt$$

Before calculate the distance, we should make a conversion from km/h to m/s. We can take conversion factor as 0.277. If we take our equations integral from 0 to 3:

$$distance = 0.277 \int_0^3 (-49.612)t + (62.803)(t)^2 + (-10.783)(t)^3 dt$$

$$= 0.277\{(-49.612/2)t^2 + (62.803/3)(t)^3 + (-10.783/4)(t)^4 \Big|_0^3\}$$

$$= 0.277\{(-49.612/2)(3)^2 + (62.803/3)(3)^3 + (-10.783/4)(3)^4\} = \textbf{34.241 m}$$

2 Problem 3

In this problem, questions is solved based on Lagrange interpolation formula.

$$p(x) = \sum_{j=0}^{n} y_j L_j(x)$$

2.1 First Order Lagrange Polynomial

$$p(x) = y_0 L_0(x) + y_1 L_1(x)$$

I have chosen $t = 2.5$ and $t = 3.7$ data points. To determine Lagrange polynomials:

$$L_0 = a(x_0 - 3.7) = 1 \rightarrow L_0 = \frac{-1}{1.2}(x - 3.7)$$
$$L_1 = a(x_1 - 2.5) = 1 \rightarrow L_1 = \frac{1}{1.2}(x - 2.5)$$

Then $p(x)$ at $t = 3$

$$p(x) = 100\{\tfrac{-1}{1.2}(x - 3.7)\} + 130\{\tfrac{1}{1.2}(x - 2.5)\}$$
$$p(3) = 100\{\tfrac{-1}{1.2}(3 - 3.7)\} + 130\{\tfrac{1}{1.2}(3 - 2.5)\}$$
$$p(3) = 100\{\tfrac{-1}{1.2}(0.7)\} + 130\{\tfrac{1}{1.2}(0.5)\} = \mathbf{112.5}$$

2.2 Second Order Lagrange Polynomial

$$p(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

I have chosen $t = 2.2$, $t = 2.5$, $t = 3.7$ data points. If I write, this data into equation:

$$L_0 = a(x_0 - 2.5)(x_0 - 3.7) = 1 \rightarrow L_0 = \frac{1}{(0.3)(1.5)}(x - 2.5)(x - 3.7)$$
$$L_1 = a(x_1 - 2.2)(x_1 - 3.7) = 1 \rightarrow L_1 = \frac{-1}{(0.3)(1.2)}(x - 2.2)(x - 3.7)$$
$$L_2 = a(x_2 - 2.2)(x_2 - 2.5) = 1 \rightarrow L_2 = \frac{1}{(1.5)(1.2)}(x - 2.2)(x - 2.5)$$

Then $p(x)$ at $t = 3$

$$p(x) = \tfrac{80}{(0.3)(1.5)}(x - 2.5)(x - 3.7) + \tfrac{-100}{(0.3)(1.2)}(x - 2.2)(x - 3.7) + \tfrac{130}{(1.5)(1.2)}(x - 2.2)(x - 2.5)$$
$$p(3) = \tfrac{80}{(0.3)(1.5)}(3 - 2.5)(3 - 3.7) + \tfrac{-100}{(0.3)(1.2)}(3 - 2.2)(3 - 3.7) + \tfrac{130}{(1.5)(1.2)}(3 - 2.2)(3 - 2.5)$$
$$p(3) = \tfrac{80}{(0.3)(1.5)}(0.5)(-0.7) + \tfrac{-100}{(0.3)(1.2)}(0.8)(-0.7) + \tfrac{130}{(1.5)(1.2)}(0.8)(0.5) \approx \mathbf{122.222}$$

2.3 Third Order Lagrange Polynomial

$$p(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$$

I have chosen $t = 0$ $t = 2.2$, $t = 2.5$, $t = 3.7$ data points. If I write, this data into equation:

$$L_0 = a(x_0 - 2.2)(x_0 - 2.5)(x_0 - 3.7) = 1 \rightarrow L_0 = \frac{-1}{(2.2)(2.5)(3.7)}(x - 2.2)(x - 2.5)(x - 3.7)$$

$$L_1 = a(x_1 - 0)(x_1 - 2.5)(x_1 - 3.7) = 1 \rightarrow L_1 = \frac{1}{(2.2)(0.3)(1.5)}(x - 0)(x - 2.5)(x - 3.7)$$

$$L_2 = a(x_2 - 0)(x_2 - 2.2)(x_2 - 3.7) = 1 \rightarrow L_2 = \frac{-1}{(2.5)(0.3)(1.2)}(x - 0)(x - 2.2)(x - 3.7)$$

$$L_3 = a(x_3 - 0)(x_3 - 2.2)(x_3 - 2.5) = 1 \rightarrow L_3 = \frac{1}{(3.7)(1.5)(1.2)}(x - 0)(x - 2.2)(x - 2.5)$$

Then $p(x)$ at $t = 3$

$$p(x) =$$
$$\tfrac{80}{(2.2)(0.3)(1.5)}(x)(x-2.5)(x-3.7) + \tfrac{-100}{(2.5)(0.3)(1.2)}(x)(x-2.2)(x-3.7) + \tfrac{130}{(3.7)(1.5)(1.2)}(x)(x-2.2)(x-2.5)$$

$$p(x) = (80.808)(x)(x-2.5)(x-3.7) - (111.111)(x)(x-2.2)(x-3.7) + (19.519)(x)(x-2.2)(x-2.5)$$

$$p(x) = -10.784x^3 + 62.806x^2 - 49.61504x$$

$$p(3) = -10.784 * 3^3 + 62.806 * 3^2 - 49.61504 * 3 = \textbf{125.240}$$

2.4 Distance using Third Order Lagrange Polynomial

$$distance = \int_0^t v(t)dt$$

Before calculate the distance, we should make a conversion from km/h to m/s. We can take conversion factor as 0.277. If we take our equations integral from 0 to 3:

$$distance = 0.277 \int_0^3 -10.784t^3 + 62.806t^2 - 49.61504tdt$$

$$= 0.277\{(-10.784/4)t^4 + (62.806/3)t^3 - (49.61504/2)t^2 \Big|_0^3\}$$

$$= 0.277\{(-10.784/4)3^4 + (62.806/3)3^3 - (49.61504/2)3^2\} = \textbf{34.2400 m}$$