Distance Estimators

(1) Euclidean distance.

$A = (x_1, y_1) \quad B = (x_2, y_2)$

$$d = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

If polar coordinates

$p = (r_1, \theta_1) \quad q = (r_2, \theta_2)$

$$d = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_1 - \theta_2)}$$

it is used in method of least squares, regression analysis, loss func^n etc.

(2) Chebyshev distance / Tchebychev dist.

$$d = \max(|x_2 - x_1|, |y_2 - y_1|)$$

(3) Mahalanobis distance

$\vec{x} = (x_1, x_2, x_3 \ldots x_N)^T$

$\vec{y} = (y_1, y_2, y_3 \ldots y_N)^T$

S = Covariance matrix

$$d(\vec{x}, \vec{y}) = \sqrt{(\vec{x} - \vec{y}) S^{-1} (\vec{x} - \vec{y})}$$

If the covariance matrix is identity, then Mahalanobis dist. reduces the Euclidean distance

$$d(\vec{x}, \vec{y}) = \sqrt{\sum_{i=1}^{N} \frac{(x_i - y_i)^2}{S_i^2}}$$

$$\sigma = \sqrt{\frac{\Sigma (x_i - \mu)^2}{N}}$$

S_i = std. deviation of x_i & y_i

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(4) Taxicab metric / Manhattan distance

$\vec{x} = (x_1, x_2 \ldots x_N)$

$\vec{y} = (y_1, y_2 \ldots y_N)$

$$d(\vec{x}, \vec{y}) = \sum_{i=1}^{N} |x_i - y_i|$$

(5) Minkowski distance

$$D(\vec{x}, \vec{y}) = \left(\sum_{i=1}^{N} |x_i - y_i|^p\right)^{1/p}$$

for $p \geq 1$ Minkowski dist. is a distance function.

if $p < 1$ it is not a distance function.

$p = 1$ or 2

if $p \to \infty$ then, we obtain chebyshev dist.

$$\lim_{p \to \infty} \left(\sum_{i=1}^{N} |x_i - y_i|^p\right)^{1/p} = \max_{i=1}^{N} |x_i - y_i|$$

if $p \to -\infty$.

$$\lim_{p \to -\infty} \left(\sum_{i=1}^{N} |x_i - y_i|^p\right)^{1/p} = \min_{i=1}^{N} |x_i - y_i|$$

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(6) Haversine formula:
Used to find dist. betn 2 points on a sphere given their latitudes & longitudes (GPS)

Central angle (θ) betn any 2 points on a sphere is:

$$\theta = \frac{d}{r}$$

d = dist. betn 2 points of a phere
r = radius of the sphere = 6371 km
or 3961 miles

haversine formula i.e hav (θ) using lat & long.

$$\boxed{hav(\theta) = hav(\varphi_2 - \varphi_1) + \cos(\varphi_1) \cdot \cos(\varphi_2) \cdot hav(\lambda_2 - \lambda_1)}$$

where $\varphi_1, \varphi_2 \rightarrow$ lat. of point 1 & point 2
$\lambda_1, \lambda_2 \rightarrow$ long. " "

$$hav(\theta) = \sin^2\left(\frac{\theta}{2}\right) = \frac{1 - \cos(\theta)}{2}$$

$$\therefore d = 2r \cdot \arcsin\left(\sqrt{hav(\theta)}\right)$$

$$\Rightarrow d = 2r \cdot \arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos(\varphi_1) \cdot \cos(\varphi_2) \cdot \sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}\right)$$

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