

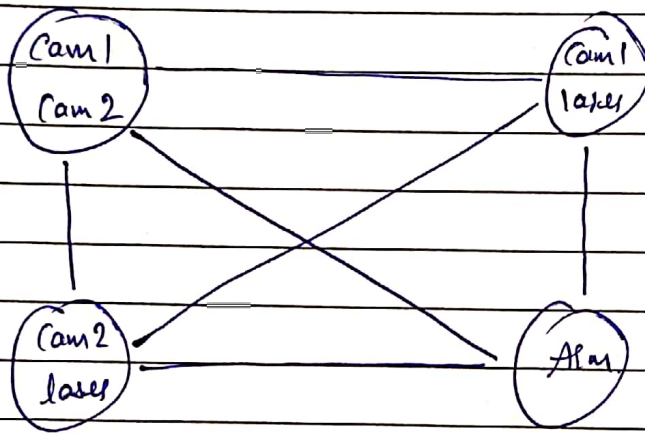
# Joint Calibration of Multiple Sensors

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Joint Calibration: The groups are formed using a unifying principle.

Each group is created the output 3D data. Made use of a fact that a sensor can appear in different groups to have redundancy. Then calibrate the entire system in one optimization.



- ✓ Although intrinsic and extrinsic calibration done separately.
- ✓ In this approach intrinsic & extrinsic calibration combined & performed simultaneously by posing a single calibration optimization objective, that is minimized through numerical optimization.

## Extrinsic Parameters

The extrinsic parameters of two co-ord. s/m. consists of a rotational <sup>matrix</sup> and a translation vector between the frames.

Assume the co-ord. of a point  $P$  is  $[x, y, z]^T$  in frame  $A$ , its co-ord. in frame  $B$  is:

$$R[x, y, z]^T + t \quad (1)$$

rotation matrix also satisfies 2 orthogonal constraint:

$$R^T \cdot R = I \quad (2)$$

## Intrinsic Parameters

We define a 3D system as a sensor or a group of sensors that gives us 3D data with known depth scales.

A single camera is not a 3D s/m. because it does not produce depth data. A 3D range finder or a stereo camera are 3D s/m.

$$\mathcal{I}(d, u) \in \mathbb{R} \quad (3)$$

$u$  = Calibration datum

$d$  = intrinsic model parameters eg: focal length, distortion, rotation & translation between diff. members of a group



## Extrinsic Calibration of 3D systems

$p^{(A)} = I_A(d_A, u^{(A)}) \rightarrow$  a point in frame A

$p^{(B)} = I_B(d_B, u^{(B)}) \rightarrow$  same " in frame B.

Now, the coord. of  $p^{(A)}$  in frame B under coord. transformation is

$$p^{(B)} = R p^{(A)} + t$$

As data may be noisy, we can model dist. as a normal random variable.

$$d = \| R p^{(A)} + t - p^{(B)} \|$$

Objective function for calibrating multiple sensors.

$$L_{ext}(R, t) = \sum_i \| R I_A(d_A, u_i^A) + t - I_B(d_B, u_i^B) \|^2 \quad (4)$$

## Intrinsic Calibration of 3D systems

The set of constraints we can use for intrinsic calibration of 3D system are distance preservation, collinearity, coplanarity.

In our framework, these constraints are enforced by likelihood estimations.



Each pixel has  $(u, v)$  coord.  
In our case

$$\begin{cases} u = u_i^0 \\ v = u_j^0 \end{cases}$$

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a. Distance preservation constraints.

$u_i^0, u_j^0 \rightarrow$  collected calibration data.

$\bar{d}_{ij} \rightarrow$  known dist. in 3D slm (ground truth).

$P_i, P_j \rightarrow$  2 points in 3D, given by intrinsic models of a 3D slm.

$d_{ij} \rightarrow$  dist. bet<sup>n</sup>  $P_i$  &  $P_j$  i.e. points

& is represented by random normal variable.

$$\text{i.e. } p(d_{ij}) = \frac{1}{Z} \exp\left(-\frac{\|d_{ij} - \bar{d}_{ij}\|^2}{\sigma^2}\right) \quad (5)$$

$u_i^0 = \{u_i^{(1)}, u_i^{(2)}\} \rightarrow$  corresponding pixels of 2 cameras images.

using  $u_i^0$ , slm can construct point in 3D i.e.  $P_i$ .

Similarly  $u_j^0$  gives  $P_j$ .

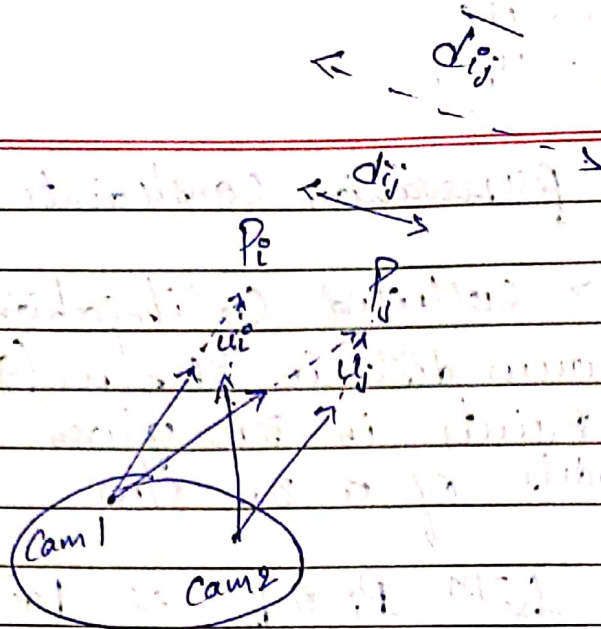
b. Collinearity and coplanarity constraints.

Distance  $d$  from the points to the line/plane represented by random normal variable.

$$\text{i.e. } p(d) = \frac{1}{Z} \exp\left(-\frac{\|d\|^2}{\sigma^2}\right) \quad (6)$$

Distance can be obtained by fitting a line or plane to a set of points via SVD - Singular Value decomposition.





$\bar{d}_{ij} \rightarrow$  Ground truth dist. (size of chessboard square)  
bet<sup>n</sup>  $P_i$  &  $P_j$

$d_{ij} \rightarrow$  hypothetical dist. given by calibration parameters.

By forcing  $d_{ij}$  to be close to  $\bar{d}_{ij}$ , we get good estimation of calibration parameters.

d. Combining all constraints.

" Likelihood of the constraints is (6)

$L(6)$

$$L(\alpha) = \sum_i \left( \|I(\alpha, u_i) - I(\alpha, u_j)\| - \bar{d}_{ij} \right)^2$$

$$+ \sigma_1^2 \sum_k \sum_{k \in L} d(I(\alpha, u_k), L)^2$$

$$+ \sigma_2^2 \sum_p \sum_{p \in P} d(I(\alpha, u_i), p)^2 \quad (7)$$

where,  $d(x, L) \rightarrow$  dist. from point  $x$  to line  $L$   
 $d(x, p) \rightarrow$  " " " " " " plane  $p$

## Joint Intrinsic and Extrinsic Calibration of 3D Systems

We first calibrate the intrinsics of each s/m, & then calibrate the extrinsics between them.

$$\text{Extr, Intr} (R, t, d_A, d_B) = \text{Intr} (d_A) + \text{Intr} (d_B) + T \text{Extr} (R, t, d_A, d_B) \quad (8)$$

## 3D - Systems

### a. Stereo Systems

It contains 2 or more cameras, with depth is found by triangulation.

Stere function for stereo systems with two cameras.

$$\text{Stereo} (\{R, t, d_{cam}^{(1)}, d_{cam}^{(2)}\}, \{u^{(1)}, u^{(2)}\}) = [x, y, z]^T \quad (9)$$

$d_{cam}^{(1)}, d_{cam}^{(2)} \rightarrow$  parameters of 2 cameras

$d_s = \{R, t, d_{cam}^{(1)}, d_{cam}^{(2)}\}$   
 $\rightarrow$  intrinsic parameters of stereo sm.

$u^{(1)}, u^{(2)} \rightarrow$  pixel co-ords in 2 images of 2 cameras.



b. Active triangulation s/m's

→ System contains laser scanner & Camera

→ A motor on the laser scanner records offset angles ( $\beta$ )

→ This system finds 3D location of image pixel

Active Triangulation ( $\{R, t, d_{cam}\}, \{\beta, u\} =$

$$[x, y, z]^T \quad (10)$$

$\beta, u \rightarrow$  Calibration data

$u \rightarrow$  Pixel coord.

$d_{cam} \rightarrow$  Camera Focal length

$R, t \rightarrow$  rotation & translation parameters

$d_{Active Triangulation} = \{R, t, d_{cam}\}$

$\rightarrow$  Extrinsic parameters

c. Range finder

→ Calibrated internally by the manufacturer & hence they do not have extrinsic parameters & do not need further calibration

$$Range\ Finder(u) = [x, y, z]^T$$

$u \rightarrow$  Calibration data, can be 'pixel' location in the depth map image