Track-to-Track fusion.

2. Derivation of optimal Track fusion.

$\hat{x}_1(K|K), \hat{x}_2(K|K), \ldots \hat{x}_n(K|K) \rightarrow$ remote tracks

for target $x(K)$ observed by 'n' sensors.

The optimal fusion is a conditional mean given the track estimate & the correspond'g covariance,

$$\hat{x}_F(K|K) = E\{x(K) | \hat{x}_i, P_i; i = 1, 2, \ldots n\}.$$

$\hat{x}_F \rightarrow$ fused estimate.
$P_F \rightarrow$ " estimates covariance
$\hat{x}_i \rightarrow$ individual track estimate
$P_i \rightarrow$ correspond'g covariance of individual track estimate.

for simplicity of derivation, assume that the posterior probab. density function is of the standard normal "N",

$$\text{ie } f(x|\hat{x}_i) = \phi[(x - \hat{x}_F) / \sigma_F]$$

$$P_F = \sigma_F^2$$

The posterior density $f(x|\hat{x}_i)$ is proportional to joint density function.

ie $f(x-\hat{x}_1, x-\hat{x}_2, \ldots, x-\hat{x}_n)$ which is a likelihood function expressed by

$$f \propto \exp\left\{-\frac{1}{2}(x-\hat{x}_1, x-\hat{x}_2, \ldots x-\hat{x}_n)^T \Sigma^{-1} (x-\hat{x}_1, x-\hat{x}_2, \ldots x-\hat{x}_n)\right\}$$

Scanned by CamScanner

Denote, $e = [I, I, \ldots I]^T$,
$\Sigma = [E\{(x - \hat{x}_i)(x - \hat{x}_j)^T\}]^n$ and
$\mu = (\hat{x}_1, \hat{x}_2, \ldots \hat{x}_n)^T$

joint density function can be expressed as:
$$f(\mu - ex) \propto \exp\{-\tfrac{1}{2}(\mu - ex)^T \Sigma^{-1} (\mu - ex)\}.$$

matrix dimensions:

$x = (2d \times 1)$, $\hat{x}_F = (2d \times 1)$.
$\mu = (2dn \times 1)$, $P_i = (2d \times 2d)$.
$e = (2dn \times 2d)$
$\Sigma = (2dn \times 2dn)$

where,
d = no. of state vector elements.

Eg: $d = 2$ if (P, V) & $d = 3$ if (P, V, a)
position, velocity, acceleration

$2 \rightarrow$ indicates the dimension of co-ord. s/m i.e (x, y).

$\hat{x}_F \rightarrow$ can be obtained by differentiating the likelihood funcⁿ w.r.t x from

$l = (\mu - ex)^T \Sigma^{-1} (\mu - ex)$ and
$$\frac{\partial l}{\partial x} = -2 \cdot e^T \Sigma^{-1} (\mu - ex) = 0$$

$$\therefore \hat{x}_F = (e^T \Sigma^{-1} e)^{-1} (e^T \Sigma^{-1} x) \quad \Big\} \text{--- (1)}$$
$$P_F = (e^T \cdot \Sigma^{-1} \cdot e)^{-1}$$

(1) — is the General expression for n-sensors S/M, as compared the Bar-Shalom's 2 Sensor case.

$$\hat{x}_F = (P_1 + P_2 - P^{12} - P^{21})^{-1} \left[(P^2 - P^{12}) \hat{x}_1 + (P^1 - P^{21}) \hat{x}_2 \right]$$

$$\hat{P}_F = e^T \Sigma^{-1} e = tr(\Sigma^{-1})$$
$$= (P^1 P^2 - P^{12} P^{21})^{-1} (P^1 + P^2 - P^{12} - P^{21})$$

where,

P^i = auto-covariances of remote track i.

P^{ij} = cross-correlation betⁿ track i & j.

for uncorrelated tracks, $P^{ij} = 0$.

$\Sigma \rightarrow$ the co-variance matrix is a block diagonal matrix.

In this case the S/M can be completely decoupled with state vector size of 2d:

$$\hat{x}_F = P_F \left[\sum_{j=1}^{n} (P^j)^{-1} \hat{x}_j \right] \quad \Big\} \text{--- (2)}$$
$$P_F = \left[\sum_{i=1}^{n} (P^i)^{-1} \right]^{-1}$$

3. Estimation of Covariances.

The optimal track fusion requires knowledge about s/m Covariance Σ.
which consists of remote covariance (P^i) &
cross covariance (P^{ij}).

$$\Sigma = (P^{ij})$$

where,

$$P^{ij} = E\{(x-\hat{x}_i)(x-\hat{x}_j)^T\}.$$

3.1 Estimation of AutoCovariance.

If the remote trackers do not provide the confidence of track quality in the form of estimation error covariance.

a) Austere track fusion

If track estimation errors are independent, then the joint density func^n is expressed as a product of density func^n of each tracker.

if variances of estimators are identical ~~then~~.
i.e $P^i = \sigma^2$, then.

$$\left.\begin{aligned} \hat{x}_F &= \frac{1}{n}\sum_{i=1}^{n}\hat{x}_i \\ P_F &= \sigma^2/n. \end{aligned}\right\} \quad (3).$$

It provides an optimal fusion for independent tracks when the sensors have ~~independent~~ identical measurement characteristics.

b) Estimation by time averaging

When the target dynamic is exactly modeled and filter residual is statistically equivalent to white noise.

Thus, the filter covariances can be estimated from KF smoothing eqn's.

$$\hat{x}_i(K|K-1),\ \hat{x}_i(K|K),\ K_i(K);\ H_i(K)$$

$$\hat{x}_i(K|K) = \hat{x}_i(K|K-1) + K_i(K)\left[Z_i(K) - H_i(K)\,\hat{x}_i(K|K-1)\right]$$

$$\boxed{\begin{array}{l} x' = x + K.y \\ \text{where, } y = Z - H.x \end{array}}$$

the filter residual → difference bet^n smoother & predicted states as follows.

$$\hat{x}_i(K|K) - \hat{x}_i(K|K-1) = K_i(K)\underbrace{\left[Z_i(K) - H_i(K)\,\hat{x}_i(K|K-1)\right]}_{V_i(K)}$$

$$= K_i(K).V_i(K)$$

Smoothing error covariance :-

$$\Delta_i(K) = E\{[\hat{x}_i(K|K) - \hat{x}_i(K|K-1)][\hat{x}_i(K|K) - \hat{x}_i(K|K-1)]^T\}$$

$$\Rightarrow K_i(K)\,S_i(K).\,K_i(K)^T$$

where,

$$S_i(K) = E\{V_i(K).V_i(K)^T\}$$

$$K_i(K) = P_i(K|K-1)\,H_i(K)^T.\,S_i(K)^{-1}$$

K-1)]

we get,

$$S_i(K)\cdot K_i(K) = P^i(K|K-1)\cdot H_i(K)^T$$

$$\therefore \Delta_i(K) = P^i(K|K-1)\cdot H_i(K)^T\cdot K_i(K)^T$$

↳ Smoothing error Covariance.

Predicted Covariance →

$$P^i(K|K-1) = (K_i H_i)^{-1}\,\Delta_i(K);$$

where, $\Delta_i(K)$ is computed by time averaging.

$$\Delta_i(K) = \frac{1}{K}\sum_{j=K}^{K}\left[\hat{x}_i(j|j) - \hat{x}_i(j|j-1)\right]\left[\hat{x}_i(j|j) - \hat{x}_i(j|j-1)\right]^T$$

c) Recursive Estimation

from KF eqn's, Covariance update & prediction are computed by.

$$P^i(K|K) = (I-K_iH_i)\,P^i(K|K-1)\,(I-K_iH_i)^T + K_iR_iK_i^T \quad (4)$$

$$P^i(K+1|K) = F_i(K)\,P^i(K|K)\cdot F_i(K)^T + L_i(K)\,Q_i(K)\,L_i(K)^T \quad (5)$$

K → Kalman Gain.

H → Measurement matrix.

$$\left[\begin{array}{l} P' = (I-KH)P \\ P' = F\cdot P\cdot F^T + Q \end{array}\right] \leftarrow \text{from KF Eqn's}$$

classmate

Date

Page

d) Estimation by Steady State Conditions

Impose steady state conditions to (4) & (5)

$P^i(K+1|K+1) = P^i(K|K)$ and

$P^i(K+1|K) = P^i(K|K-1)$

the steady state covariance converges to constant matrix

$$P^i(K+1|K) = Q_i \begin{bmatrix} \alpha T^4/\beta^2 & T^3/\beta \\ T^3/\beta & T^2(2\alpha+\beta)/2\beta \end{bmatrix}$$

$Q_i \longrightarrow$ process noise matrix covariance.

$T \rightarrow$ Sampling time

α & $\beta \rightarrow$ Smoothing Gain.

3-2 Estimation of cross-~~correlation~~ covariance (P^{ij})

Dynamic s/m:

$$X_i(K+1) = F_i X_i(K) + \underbrace{L_i \xi_i(K)}_{\text{process noise.}}$$

$$Z_i(K) = H_i X_i(K) + \eta_i(K)$$

$\hookrightarrow y = Z - HX$

Cross Covariance b/w Sensors:

$$P^{ij}(K+1|K) = F_i(K) P^{ij}(K|K) F_j(K)^T + \underbrace{L_i Q_{ij} L_i^T}_{\text{process noise Covariance}}$$

$$P^{ij}(K|K) = (I - K_i H_i) P^{ij}(K|K-1)(I - K_j H_j)^T$$

3.3 Track fusion without cross Covariance (P^{ij})

a) Bar-Shalom method-

Fused s/m track & Covariance are expressed by :-

$$P(K|K)^{-1} \hat{X}(K|K) - P(K|K-1)^{-1} \cdot \hat{X}(K|K-1)$$

$$= \sum_{i=1}^{n} \left[P^i(K|K)^{-1} \hat{X}_i(K|K) - P^i(K|K-1)^{-1} \hat{X}_i(K|K-1) \right]$$

and.

$$P(K|K)^{-1} - P(K|K-1)^{-1} = \sum_{i=1}^{n} \left[P^i(K|K)^{-1} - P^i(K|K-1)^{-1} \right]$$

b) Pseudo. measurement method.

This method is an inverse operation that converts a target estimate back to sensor measurement by :-

$$K(K)^{+} [\hat{X}(K|K) - \hat{X}(K|K-1)] + H\hat{X}(K|K-1),$$

where, $K(K)^{+} \rightarrow$ pseudo inverse.