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Unscented Kalman filter

Constant Velocity model

$$x = \begin{bmatrix} P_x \\ P_y \\ V_x \\ V_y \end{bmatrix}$$

1. Generate Sigma points

$$X_{k|k} = x_{k|k}. \quad x_{k|k} + \sqrt{(\lambda + n_x) P_{k|k}}.$$

$$x_{k|k} - \sqrt{(\lambda + n_x) P_{k|k}}$$

$$x_{k|k} = \begin{bmatrix} P_x \\ P_y \\ 0 \\ 0 \end{bmatrix}$$

Initial state vector.

$\lambda = 3 - n_x.$

$n_x = 4$

$\therefore$ no. of sigma points

$= 2 \times n_x + 1$

$= 9$

$$P_{k|k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Initial state covariance matrix

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Augment State

$$x_{a,k|k} = \begin{bmatrix} P_x \\ P_y \\ V_x \\ V_y \\ V_{a_x} \\ V_{a_y} \end{bmatrix}$$
(6×1)

V_{a_x} → longitudinal acceleration noise

V_{a_y} → Yaw-acceleration noise

$\left.\begin{matrix} V_{a_x} \\ V_{a_y} \end{matrix}\right\}$ noise vector.

- $n_a = 6$.
- $\lambda = 3 - n_a$.

no. of sigma points
$= 2 \cdot n_a + 1$
$= 13$.

$$P_{a,k|k} = \begin{bmatrix} P_{k|k} & 0 \\ 0 & Q \end{bmatrix}$$
(6×6)

$$Q = \begin{bmatrix} \sigma_{a_x}^2 & 0 \\ 0 & \sigma_{a_y}^2 \end{bmatrix}$$
(2×2)

Generate Augment Sigma points

$X_{a,k|k} = x_{a,k|k}$. $x_{a,k|k} + \sqrt{(\lambda + n_a) P_{a,k|k}}$
(6×13)

. $x_{a,k|k} - \sqrt{(\lambda + n_a) P_{a,k|k}}$

(2) Predict Sigma points

$$\dot{x} = g(x) = \begin{bmatrix} \dot{P_x} \\ \dot{P_y} \\ \dot{V_x} \\ \dot{V_y} \end{bmatrix} = \begin{bmatrix} V_x \\ V_y. \\ 0 \\ 0 \end{bmatrix}$$

$$\nu_k = \begin{bmatrix} 1/2 \quad \Delta t^2 \cdot \nu_{a_x} \\ 1/2 \quad \Delta t^2 \cdot \nu_{a_y} \\ \Delta t \cdot \nu_{a_x}. \\ \Delta t \cdot \nu_{a_y} \end{bmatrix}$$

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$$X_{k+1|k} = X_{a,k|k} + \int_{t_k}^{t_{k+1}} g(x)\cdot dt + V_k$$

$$= X_{a,k|k} + \begin{bmatrix} \int_{t_k}^{t_{k+1}} V_x(t)\cdot dt \\ \int_{t_k}^{t_{k+1}} V_y(t)\cdot dt \\ 0 \\ 0 \end{bmatrix} + V_k$$

$$= X_{a,k|k} + \begin{bmatrix} V_x \\ V_y \\ 0 \\ 0 \end{bmatrix} + V_k$$

③ Predict mean & Covariance

a). Predicted state mean

$$x_{k+1|k} = \sum_{i=0}^{2 \cdot n_a} w_i \cdot X_{k+1|k,i}$$

b) Predicted State Covariance

$$P_{k+1|k} = \sum_{i=0}^{2 \cdot n_a} w_i (X_{k+1|k,i} - x_{k+1|k}) (X_{k+1|k,i} - x_{k+1|k})^T$$

④ Predict measurement

a). $Z_{k+1|k} = h(x_{k+1})$

Radar → (3×13)

LiDAR → (2×13)

b) Predict measurement mean :

$$z_{k+1|k} = \sum_{i=0}^{2 \cdot n_a} w_i \cdot Z_{k+1|k,i}$$

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c) Predict measurement Covariance

$$S_{k+1|k} = \sum_{i=0}^{2\cdot n_a} W_i \left(Z_{k+1|k,i} - z_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T + R_{sensor}$$

(5) Update State.

a) $$T_{k+1|k} = \sum_{i=0}^{2\cdot n_a} W_i \left(X_{k+1|k,i} - x_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T$$

b) $$K_{k+1|k} = T_{k+1|k} \cdot S_{k+1|k}^{-1}$$ (Kalman Gain).

c) State update

$$\underset{(4\times 1)}{x_{k+1|k+1}} = x_{k+1|k} + K_{k+1|k}\left(z_{k+1(sensor)} - z_{k+1|k}\right)$$

$$Z_{LIDAR} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \qquad Z_{RADAR} = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

d) Covariance matrix update

$$\underset{(4\times 1)}{P_{k+1|k+1}} = P_{k+1|k} - K_{k+1|k} \cdot S_{k+1|k} \cdot K_{k+1|k}^T$$

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$$R_{LIDAR} = \begin{bmatrix} \sigma_{px}^2 & 0 \\ 0 & \sigma_{py}^2 \end{bmatrix}$$
(2×2)

$$R_{RADAR} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_\psi^2 & 0 \\ 0 & 0 & \sigma_{\dot\psi}^2 \end{bmatrix}$$
(3×3)

$$h(x_{k+1})_{LIDAR} = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$
(2×1)

$$h(x_{k+1})_{RADAR} = \begin{bmatrix} \sqrt{P_x^2 + P_y^2} \\ \arctan\left(P_y / P_x \right) \\ \dfrac{P_x V_x + P_y . V_y}{\sqrt{P_x^2 + P_y^2}} \end{bmatrix}$$
(3×1)

$$= X_{a,k|k} + \begin{bmatrix} V_x \\ V_y \\ 0 \\ 0 \end{bmatrix} + V_k$$

→ Total 500 Samples.

Case 1: 'P_x' constant

→ for 1 → 100 Samples,
Kept P_x, ρ, $\cos\psi$ constant.

→ Value of 'P' matrix corresponding to P_x, P_y, V_x, V_y is decreasing at each timestep irrespective of road scenario.

→ 'Q' matrix is constant.

→ Value of 'P' matrix is much closer to 'zero'

Case 2: 'P_y' constant (Kept, P_y, $\rho * \sin\psi$ constant)

→ for 101 → 200 Samples

Scenario is same as constant 'P_x'
Value of 'P' matrix corresponding the state vector variables, much closer to '0'.

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Case 3 : 'V_x' Constant (Kept $V_x \cos\varphi$ Constant).
→ for 201 → 300 samples
→ Scenario is same as 'P_x' & 'P_y' constant

Case 4 : 'V_y' Constant (Kept $V_y * \sin\varphi$ Const).
→ for 301 → 400 samples.
→ Scenario is same.

Case 5 : (P_x, V_x) Constant.
→ Kept (P_x, $V \cdot \cos\varphi$, $f \cdot \cos\varphi$ constant)
→ for 400 → 420 samples.

→ Value of 'P' matrix corresponding the 'P_x' & 'P_y' → we can observe they was increase & decrease slightly.

where as 'V_x' & 'V_y' decreasing continuously

Case 6 : (P_y, V_y) Constant
→ Kept (P_y, $V \cdot \sin\varphi$, $f \cdot \sin\varphi$ constant)
→ for 421 → 440 samples

→ Similar scenario as we observed in case 5.

→ Again 'P' matrix is constant, even in case 5.

Case 7 : P_x, P_y, V_x, V_y changing.
→ samples 440 → 500.
All variables of 'P' matrix corresponding the state vector are varying (increase,

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decrease) depending on road scenario
ie when the road is straight values
are decreases at each timestep,
When the road is curvy, values increase
& decreases ie we can observe continuous
variations.