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* Covariance matrices of Extended Kalman Filter for state vector = ([Px, Py, Vx, Vy]^{T}).

(1) State covariance matrix (P).

Initial state covariance matrix (P_0):

$$P_0 = \begin{array}{c|cccc} & P_x & P_y & V_x & V_y \\ \hline P_x & 1 & 0 & 0 & 0 \\ P_y & 0 & 1 & 0 & 0 \\ V_x & 0 & 0 & 1 & 0 \\ V_y & 0 & 0 & 0 & 1 \end{array}$$

✓ Since our state vector dimension is 4x1, hence our 'P' matrix dimension is 4x4.

✓ We initialized diagonal elements of 'P_0' to unity, because variables of state vector are independent to each other.

✓ We used Constant Velocity (CV) motion model, hence considered Position & Velocity i.e Px, Py, Vx, Vy in state vector.

Now, P_{k_p} i.e predicted state covariance matrix is:

$$P_{k_p} = F \cdot P_{k-1} \cdot F^{T} + Q_{k-1}$$

where, F is state transition matrix it is designed based on state transition law's.

Q_k = process noise covariance matrix.

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State Transition Equations

$$\begin{cases} P_x' = P_x + V_x \Delta t + \frac{1}{2} a_x \Delta t^2 \\ P_y' = P_y + V_y \Delta t + \frac{1}{2} a_y \Delta t^2 \\ V_x' = V_x + a_x \Delta t \\ V_y' = V_y + a_y \Delta t \end{cases}$$

where, $\Delta t = t_2 - t_1$

$a_x = \frac{V_{x_2} - V_{x_1}}{t_2 - t_1}$

$a_y = \frac{V_{y_2} - V_{y_1}}{t_2 - t_1}$

we know that,

$s = ut + \frac{1}{2} at^2$

$V = u + at$.

$$\therefore F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore U_K = \underset{(4\times1)}{\begin{bmatrix} \frac{1}{2} a_x \Delta t^2 \\ \frac{1}{2} a_y \Delta t^2 \\ a_x \Delta t \\ a_y \Delta t \end{bmatrix}} = \underbrace{\underset{(4\times2)}{\begin{bmatrix} \frac{1}{2} \Delta t^2 & 0 \\ 0 & \frac{1}{2} \Delta t^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}}}_{G} \underbrace{\underset{(2\times1)}{\begin{bmatrix} a_x \\ a_y \end{bmatrix}}}_{a}$$

$$Q_v = \begin{bmatrix} \sigma_{ax}^2 & 0 \\ 0 & \sigma_{ay}^2 \end{bmatrix}$$

σ_{ax}^2 = Error in acceleration (a_x)

σ_{ay}^2 = Error in acceleration (a_y).

$$\therefore \underset{(4\times4)}{Q_K} = \underset{(4\times2)}{G} \cdot \underset{(2\times2)}{Q_V} \cdot \underset{(2\times4)}{G^T}$$

* Variations in the values of 'P' matrix.

✓ Ideally, the values of 'P' matrix goes on reducing at each iteration, when the road is straight & the vehicle is moving at constant velocity.

✓ But in real-time roads won't be straight always, it may be curvy, more curvy. Also vehicle not always moves with constant velocity depending on traffic & road conditions may accelerate or decelerate, in such conditions the values of 'P' matrix may increase gradually & then decreases gradually.

✓ This happens because filter sometimes at specific conditions of roads (mostly non-linear) breaks or we can say ~~fails~~ fails to handle (couldn't better linearize the non-linearity).

* Variations in the Values of 'Q' matrix

- ✓ Once the Variable (σ_{ax}^2, σ_{ay}^2, Δt) in Q matrix are evaluated, they remain same and does not vary at any conditions.
- ✓ Different road scenarios & vehicle speed doesn't make any difference for 'Q' matrix.
- ✓ 'Q' matrix represents the errors that are expect in designing the state transition Equations
- ✓ σ_{ax}^2, σ_{ay} → are Co-variance (error) in the acceleration, these values we are going to assign initially i.e while designing the filter.

  Δt → It is the time difference at which the filter receives sensor measurement, this value also fixed while designing the filter.

- ✓ Hence values of 'Q' matrix never changes, It's objective is, at each iteration values of 'Q' matrix summed up with values of 'P_{k-1}' matrix (previous state covariance matrix) to give P_{kp} (predicted state covariance matrix).

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* Ideal mmWave RADAR gives $(\rho, \psi, \dot{\rho})$ (ie radial distance, radial velocity and yaw angle)

$$\boxed{\begin{aligned} P_x &= \rho \cdot \cos(\psi) \\ P_y &= \rho \cdot \sin(\psi) \\ V_x &= \dot{\rho} \cdot \cos(\psi) \\ V_y &= \dot{\rho} \cdot \sin(\psi) \end{aligned}}$$

* Case of 3-D radar, which gives (ρ, ψ, ϕ) (ie radial distance, yaw angle & Elevation angle).

$$r = \sqrt{x^2+y^2+z^2} \quad \text{--- (1)}$$

$$\psi = \arctan\left(\frac{y}{x}\right) \quad \text{--- (2)}$$

$$\phi = \arctan\left(\frac{z}{(x^2+y^2)^{1/2}}\right) \quad \text{--- (3)}$$

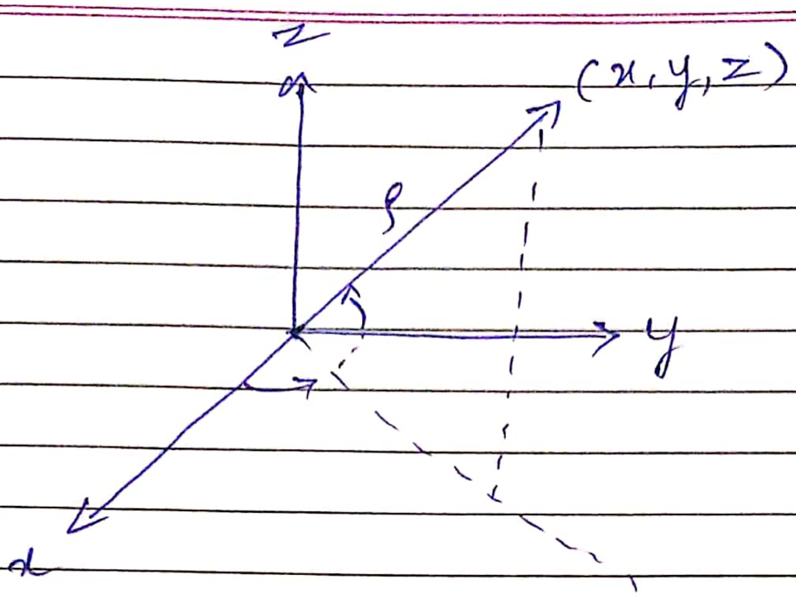
Eqⁿ (1) — (3) in Cartesian co-ordinates.

~~xxxx~~, $x = r\cos\psi.\cos\phi$

$y = r\sin\psi\cos\phi$

$z = r.\sin\psi$

$r = \rho$ (radial distance)


z
(x, y, z)
ρ
y
x