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Unscented Kalman filter (UKF)

CTRV model

State vector (x)

$$x = (P_x, P_y, v, \psi, \dot{\psi})$$

position (P_x, P_y); velocity (v), yaw angle (ψ), yaw rate or turn rate ($\dot{\psi}$)

prediction : [Generate sigma points
predict sigma points
predict mean & covariance

update : [Predict measurement
update state

(1) Generate sigma points

$$\underset{(5\times 11)}{X_{k|k}} = x_{k|k} \;\; x_{k|k} + \sqrt{(\lambda+n_x)\cdot P_{k|k}} \;\; x_{k|k} - \sqrt{(\lambda+n_x)P_{k|k}}$$

$$x_{k|k} = \begin{bmatrix} P_x \\ P_y \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(object state mean)

for initialization

$\lambda = 3 - n_x.$

$n_x = 5$

no. of sigma points $= 2(n_x)+1$
$= 11$

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$P_{k|k}$ $=$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
(5×5)
~~predicted~~
Object state
covariance
(Initialisation)

Augment State

$x_{a,k} = \begin{bmatrix} p_x \\ p_y \\ v \\ \varphi \\ \dot{\varphi} \\ \nu_a \\ \nu_{\ddot{\varphi}} \end{bmatrix}$
(7×1)

~~process~~ noise vector

$\nu_a \rightarrow$ longitudinal acceleration noise

$\nu_{\ddot{\varphi}} \rightarrow$ Yaw acceleration noise

$n_a = 7.$

n_σ (no. of sigma points) $= 2(n_a)+1$
$= \underline{15}$

Augment covariance matrix, $P_{a,k|k} = \begin{bmatrix} P_{k|k} & 0 \\ 0 & Q \end{bmatrix}$
(7×7)

$Q = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_{\ddot{\varphi}}^2 \end{bmatrix}$
(2×2)

(process noise covariance matrix)

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Generate Augment sigma points

$$\mathcal{X}_{a,k|k} = \left[x_{a,k|k} \quad x_{a,k|k} + \sqrt{(\lambda+n_a) P_{a,k|k}} \quad x_{a,k|k} - \sqrt{(\lambda+n_a) P_{a,k|k}} \right]$$

(7×15)

$$\lambda = 3 - n_a$$

matrix of sigma points in state space.

② Predict sigma points

$$\mathcal{X}_{k+1|k} = f(\mathcal{X}_k, \nu_k)$$

(process model)

$$\mathcal{X}_{k+1|k} = \mathcal{X}_{a,k|k} + \int_{t_k}^{t_{k+1}} g(x)\, dt + \nu_k$$

$$\dot{x} = g(x) = \begin{bmatrix} \dot{P_x} \\ \dot{P_y} \\ \dot{V} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi \cdot V \\ \sin\psi \cdot V \\ 0 \\ \dot{\psi} \\ 0 \end{bmatrix}$$

$$\nu_k = \begin{bmatrix} \frac{1}{2} \Delta t^2 \cdot \cos\psi_k \cdot \nu_a \\ \frac{1}{2} \Delta t^2 \cdot \sin\psi_k \cdot \nu_a \\ \Delta t \cdot \nu_a \\ \Delta t^2 \cdot \nu_{\ddot{\psi}} \\ \Delta t \cdot \nu_{\ddot{\psi}} \end{bmatrix}$$

process noise vector

③ Predict Mean & Covariance

a) Predicted state mean.

$$x_{k+1|k} = \sum_{i=0}^{2 \cdot n_a} w_i \cdot X_{k+1|k,i}$$

$$\underset{\text{weights}}{w_i} = \frac{\lambda}{\lambda + n_a} = \frac{1}{2(\lambda + n_a)} \quad i = 0 \ldots n_a$$

b) Predicted state Covariance

$$P_{k+1|k} = \sum_{i=0}^{2 \cdot n_a} w_i \left(X_{k+1|k,i} - x_{k+1|k}\right) \cdot \left(X_{k+1|k,i} - x_{k+1|k}\right)^T$$

④ Predict Measurement

a) $\underset{(3 \times 15)}{Z_{k+1|k}} = h(x_{k+1})$

(measurement model)

for LiDAR

$$\underset{(2 \times 1)}{h(x_{k+1})} = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

for RADAR.

$$\underset{(3 \times 1)}{h(x_{k+1})} = \begin{bmatrix} \sqrt{P_x^2 + P_y^2} \\ \arctan\left(P_y / P_x\right) \\ \dfrac{P_x V_x + P_y V_y}{\sqrt{P_x^2 + P_y^2}} \end{bmatrix}$$

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$n_\sigma = 2(n_a)+1 = 15$

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i.e we pass earlier Generated sigma points through measurement model the get matrix of sigma points in measurement space

b) Predict Measurement mean

$$z_{k+1|k} = \sum_{i=0}^{2 \cdot n_\sigma} w_i \cdot Z_{k+1|k,i}$$

b) Predict Measurement Covariance

$$S_{k+1|k} = \sum_{i=0}^{2 \cdot n_\sigma} w_i \left(Z_{k+1|k,i} - z_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T + R$$

$$R_{LIDAR} = \begin{bmatrix} \sigma_{px}^2 & 0 \\ 0 & \sigma_{py}^2 \end{bmatrix}$$

$$R_{RADAR} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_\varphi^2 & 0 \\ 0 & 0 & \sigma_{\dot\rho}^2 \end{bmatrix}$$

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(5) Update state

a) $T_{k+1|k} = \sum_{i=0}^{2n_a} w_i \left(X_{k+1|k,i} - x_{k+1|k}\right) \cdot \left(Z_{k+1|k,i} - z_{k+1|k}\right)^T$

cross correlation between sigma points in state space & measurement space

b) $K_{k+1|k} = T_{k+1|k} \cdot S_{k+1|k}^{-1}$

Kalman Gain.

c) state update

$\underset{(5\times1)}{x_{k+1|k+1}} = x_{k+1|k} + K_{k+1|k} \cdot \left(Z_{k+1(sensor)} - z_{k+1|k}\right)$

$Z_{LIDAR} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$

$Z_{RADAR} = \begin{bmatrix} \rho \\ \varphi \\ \dot{\varphi} \end{bmatrix}$

d) Covariance matrix update

$\underset{(5\times5)}{P_{k+1|k+1}} = P_{k+1|k} - K_{k+1|k} \cdot S_{k+1|k} \cdot K_{k+1|k}^T$

Process_measurement ( )
{ (1). Initialization ( )
(2). prediction ( )

{

a) Augment_sigma_points ( )
b) predict_sigma_points ( )
c) predict mean and Covariance ( )

}

(3). update_LiDAR ( )
{

a) PredictMeasurement ( )

b) update_state ( )

}

(4) update_Radar ( )
{ a) PredictMeasurement ( )

b) update_state ( )

}
}

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Initialization()
{
we initialize ① state vector
② state covariance matrix

Yaw change.

① Initialization ()
② predict sigmapoints()

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