# Extrinsic Calibration of a Camera and a LiDAR from natural Scenes



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## Lidar - Camera projection model

### A. Camera model

Assuming the Camera is calibrated, given a pixel point (u,v) on the camera image plane, we can recover the orientation of the vector X emanating from the effective viewpoint to the corresponding 3D point.

Given a 3D point $\lambda X$, we project it onto the Camera image plane (u,v).

$$\lambda X = \lambda [x, y, z]^T = F(u,v) \quad (1)$$

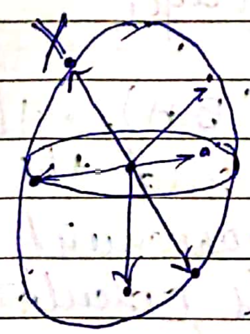
$$[u, v]^T = F^{-1}(X) \quad (2)$$

where, $\lambda \rightarrow$ depth factor

$\|X\| \rightarrow 1$

$F \rightarrow$ This function depends on the camera used.

We assume that for every sensed pixel we know the orientation of the correspondent vector X on the unit sphere centred in the mirror frame.



## B. Laser model

3-D range sensor used is a custom-built. It is composed of laser scan mounted on a rotating support.

### Sensor model:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} C_i C_j & -C_i S_j & S_i & C_i d_x + S_i d_z \\ S_j & C_j & 0 & 0 \\ -S_i C_j & S_i S_j & C_i & -S_i d_x + C_i d_z \end{bmatrix}}_{(3 \times 4)} \underbrace{\begin{bmatrix} \rho_{ij} \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{(4 \times 1)} \quad (3)$$

$C_i = \cos(\varphi_i)$, $C_j = \cos(\theta_j)$, $S_i = \sin(\varphi_i)$, $S_j = \sin(\theta_j)$

where,

$\rho_{ij} \rightarrow$ $j^{th}$ measured distance with corresponding orientation $\theta_j$ in the $i$th scan plane, which makes the angle $\varphi_j$ with the horizontal plane.

The offset of the external rotation axis from the laser frame has components $d_x$ and $d_z$

$[x, y, z]^T \rightarrow$ coord's of each measured point.

### Bearing angle Image

Bearing angle (BA) $\rightarrow$ the angle between the laser beam and the segment joining 2 consecutive measurement points.

This angle is calculated for each point in the depth matrix

$$BA_i = \arccos \frac{f_i - f_{i-1} \cos d\varphi}{\sqrt{f_i^2 - f_{i-1}^2 - f_i f_{i-1} \cos d\varphi}} \quad (4)$$

where,

$f_i \rightarrow i$-th depth value in the selected trace of the depth ~~image~~ matrix & $d\varphi$ is the corresponding angle increment.

Performing this calculation for all point in the depth matrix will lead to an image which is refered to as BA image.

Hence, these will be used next for extracting corresponding features

## Extrinsic Lidar - Camera Calibration

### A - Data collection

calibration technique needs a single acquisition of both the laser scanner & the omnidirectional camera. The acquisition target can be any natural scene with a sufficient number of distinguishable key points i.e edges, depth discontinuities ...

At the end of the visual correspondance pairing, we have n laser points in the laser frame and their correspondent points on the camera image plane.

We write these points as:

$$O_c = [O_{c,1}; O_{c,2}, \ldots O_{c,n}]$$

$$O_L = [O_{L,1}, O_{L,2}, \ldots O_{L,n}]$$

$$d_L = [d_{L,1}, d_{L,2}, \ldots d_{L,n}] \quad (5)$$

where, $O_c$ & $O_L$ → orientation vectors of camera & laser points in their respective reference frames.

$d_L$ → point distances in the laser frame.

B. Extrinsic Calibration:

Extrinsic Calibration of Camera & lidar consists in finding the Rotation R and translation T between the lidar and camera frame that minimizes a certain reprojection error function.

$$\min_{R,T} \frac{1}{2} \sum_{i=1}^{n} \| m_i - \hat{m}(R, T, P_i) \|^2 \quad (6)$$

where, $\hat{m}(R, T, P_i)$ → reprojection onto the image plane of the laser point $P_i$.

However, the reprojection error is not theoretically optimal in our application because the resolution of camera is not uniform.

A better error function uses the Riemmann metric.

This metric minimizes the difference of the bearing angles of the camera points & the " " " " lidar " after reprojection into image, i.e :

$$\min_{R,T} \frac{1}{2} \sum_{i=1}^{n} \left\| \arccos\left( O_{C,i}^{T} \cdot O_{CL,i} \right) \right\|^2 \qquad (7)$$

Where, $O_{CL} \rightarrow$ unit form orientation vector of $\hat{m}(R,T,P_i)$