An Efficient Particle filter for the OOSM (Out of Sequence measurement) problem.

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II System models and problem formulation

A- System models

$$X_k = [x_{k,1}, \ldots, x_{k,M}]^T$$

M = Diamention of target state.

for time k and k−1, the evolutⁿ of target state is.

$$X_k = g(X_{k-1}) + W_{k,k-1} \quad \text{——— (1)}$$

g → state transition function

$W_{k,k-1}$ → process noise from time k−1 to k

At time k, the state is sensed by N sensors according to measurement model.

$$Z_k^i = h_i(X_k^i) + V_k^i \quad \text{——— (2)}$$

i = sensors, {i = 1, 2, -- N}.
hi = non linear measurement funⁿ of iᵗʰ sensor.

Z_k^i is the measurement @ time k. & @ sensor, i, {i=1,2, -- N}.

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B - Problem formulation

Posterior probability density func^n $p(x_K | Z_{1:K})$:

$$p(X_K|Z_{1:K}) = \int p(X_K|X_{K-1}) \cdot p(X_{K-1}|Z_{1:K-1})\, dX_{K-1} \quad (3)$$

(from markov assumption)

(Posterior) (likelihood) (prior)

$$p(X_K|Z_{1:K}) = \frac{p(Z_K|x_K) \cdot p(X_K|Z_{1:K-1})}{p(Z_K|Z_{1:K-1})} \quad (4)$$

$\{p(X_K|Z_{1:K-1})\}$ (normalizing constants)

(from Bayes rule)

an earlier measurement $z(\tau)$ arrives with time stamps τ, such that

$$K - l < \tau < K - l + 1 \quad (n)$$

where,

l = some positive integer

$z(\tau)$ = l-step-lag OOSM

Our goal is to update the pdf $p(X_K|Z_{1:K})$

Of the current state X_K with OOSM $z(\tau)$ to obtain $p(X_K|Z_{1:K}, z(\tau))$.

(To find solution for this --)

To add new term 'd'

$$p(a|b,c) = \int p(a|d) \cdot p(d|b,c)$$ ← (8)

{Thm. of Total prob.} ← (9)

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III Exact Bayesian Solution for OOSM problem using Bayes rule

$$p(X_k | Z_{1:k}, Z(\tau)) = \frac{p(Z(\tau) | X_k, Z_{1:k})\, p(X_k | Z_{1:k})}{p(Z(\tau) / Z_{1:k})} \quad (6)$$

where,

$$p(Z(\tau) | Z_{1:k}) = \int p(Z(\tau) | X_k, Z_{1:k}) \cdot p(X_k | Z_{1:k})\, dX_k \quad (7)$$

(Total prob. thm)

(7) is a normalizing constant doesn't depend on X_k, $p(Z(\tau) | X_k, Z_{1:k})$ is called asynchronous likelihood func^n (ALF)

$p(X_k | Z_{1:k})$ calculated in (3) & (4),

Now, our task is to obtain ALF $p(Z(\tau) | X_k, Z_{1:k})$

Introducing the target state $X(\tau)$, thus ALK is expressed as.

$$p(Z(\tau) | X_k, Z_{1:k}) = \int p(Z(\tau) | X(\tau)) \cdot p(X(\tau) | X_k, Z_{1:k}) \cdot dX(\tau) \quad (8)$$

Bayes rule → conditioning Bayes rule + Theorem of total probability

likelihood func^n $p(Z(T)|X(T))$ is known from (2), term $p(X(T)|X_K, Z_{1:K})$ is regarded as smoothing probability density func^n (pdf).

By employing the Chapman Kolmogorov eq^n & Bayes formula, the smoothing pdf denoted as

$$p(X(T)|X_K, Z_{1:K}) = \int \overset{①}{p(X(T)|X_{K-l+1}, Z_{1:K-1})} \cdot \overset{②}{p(X_{K-l+1}|X_K, Z_{1:K})}\, dX_{K-l+1} \quad (9)$$

Where,

① $p(X(T)|X_{K-l+1}, Z_{1:K-1}) =$

$$\frac{p(X_{K-l+1}|X(T))\, p(X(T)|Z_{1:K-1})}{\int p(X_{K-l+1}|X(T))\, p(X(T)|Z_{1:K-1})\, dX(T)} \quad (10)$$

Bayes rule.

State transition kernel, which is obtained from the target motion model.

② $p(X_{K-l+1}|X_K, Z_{1:K})$, it can be calculated recursively. i-e

$$p(X_n | X_k, Z_{1:k}) =$$

$$\int p(X_n | X_{n+1}, Z_{1:n})\, p(X_{n+1} | X_k, Z_{1:k})\, dX_n$$

$$= p(X_n | Z_{1:n}) \int \frac{p(X_{n+1} | X_k, Z_{1:k}) \cdot p(X_{n+1} | X_n)}{\int p(X_n | Z_{1:n}) \cdot p(X_{n+1} | X_n)\, dX_n}\, dX_{n+1} \qquad (11)$$

for, $n = k-1, k-2, \ldots, k-l+1$

here, with $n = k-1$, the term

$$p(X_n | X_k, Z_{1:k}) = p(X_{k-1} | X_k, Z_{1:k-1})$$

$$= \frac{p(X_k | X_{k-1}) \cdot p(X_{k-1} | Z_{1:k-1})}{\int p(X_k | X_{k-1})\, p(X_{k-1} | Z_{1:k-1})\, dX_k}$$

(Bayes rule) $\qquad (12)$

$\therefore$ with initial $n = k-1$ & substitute it into $p(X_n | X_k, Z_{1:k})$ in (12), then recursively implementing (11), thus the term $p(X_{k-l+1} | X_k, Z_{1:k})$ can be found.

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IV The Efficient particle filtering algorithm

Particle filter is a useful tool for implementing a recursive Bayesian filter. Considering it has an excellent performance in non-linear systems, an implementation of the exact Bayesian filter using particle filter is done here.

dX_{n+1}

dX_n

Due to smoothing prob. density func^n (pdf) $p(X_n | X_K, Z_{1:K})$ in (11) is required to be calculated when implementing the exact Bayesian solut^n, a General Gaussian smoother is desired.

{ i.e for smoothing pdf, General Gaussian smoother is needed.

also smoothing pdf is required to implement Bayesian solution for OOSM problem }.

(refer (3))

Sample-based approximation of the posterior pdf $p(X_K | Z_{1:K})$ is expressed as

$$p(X_K | Z_{1:K}) = \sum_{q=1}^{Q} w_{K|K}^{(q)} \delta(X_K - X_K^{(q)}) \quad (18)$$

where, $q = 1, 2, \ldots Q$ is index of particle. $X_K^{(q)}$ @ time K,

Q = particle size

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$\omega_{k|k}^{(q)}$ = particle weight of q^{th} particle.

particles are drawn according to:

$X_k^{(q)} \sim p(X_k \mid X_{k-1}^{(q)})$.

Then substituting (13) into (6) yields.
{Now, refer (6)}

$$p(X_k \mid Z_{1:k}, Z(\tau)) = \frac{\sum_{q=1}^{Q} \omega_{k|k}^{(q)}\, p(Z(\tau) \mid X_k^{(q)}, Z_{1:k})\, \delta(X_k - X_k^{(q)})}{\sum_{q=1}^{Q} \omega_{k|k}^{(q)}\, p(Z(\tau) \mid X_k^{(q)}, Z_{1:k})} \quad (14)$$

i = index of i^{th} particle.

Thus, expected posterior pdf can be ~~expressed~~ approximated as:

$$p(X_k \mid Z_{1:k}, Z(\tau)) = \sum_{q=1}^{Q} \omega_{k|k,\tau}^{(q)}\, \delta(X_k - X_k^{(q)}) \quad (15)$$

where, $\omega_{k|k,\tau}^{(q)} \propto \omega_{k|k}^{(q)}\, p(Z(\tau) \mid X_k^{(q)}, Z_{1:k})$ (16)

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Now, the Goal is to obtain $p(Z(T) | X_K^{(q)}, Z_{1:K})$ i.e the weight of the Asynchronous likelihood func^n with particle sample $X_K = X_K^{(q)}$

A. Calculate the particle weight of ALF.

To calculate ALF, the integral in (8) has to be computed.

Based on this we express $p(Z(T)|X_K, Z_{1:K})$ as {refer (8)}.

$$p(Z(T)|X_K, Z_{1:K}) =$$

$$\int \frac{p(Z(T)|X(T)) \cdot p(X(T)|X_K^{(q)}, Z_{1:K})}{\pi(X(T) | X_K^{(q)}, Z_{1:K})} \times$$

$$\pi(X(T)|X_K^{(q)}, Z_{1:K})$$

(17).