Probability

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✓ $p(x|y) = \cancel{p(x)/p(y)}\ \frac{p(x,y)}{p(y)}$

if x & y are independent.

✓ $p(x|y) = \frac{p(x).p(y)}{p(y)} = p(y)$

✓ Theorem of total probability.

$p(x) = \sum_y p(x|y).\ p(y)$ {discrete case

$p(x) = \int p(x|y).p(y)\,dy$ continuous case

✓ Bayes rule.

$p(x|y) = \frac{p(y|x).p(x)}{p(y)} = \frac{p(y|x).p(x)}{\sum_{x'} p(y|x').p(x')}$

(Discrete)

$p(x|y) = \frac{p(y|x).\ p(x)}{p(y)} = \frac{p(y|x).\ p(x)}{\int p(y|x').p(x')\,dx'}$

(continuous)

~~Functions: 1) Initialization~~
~~2) Predict, Sigma points~~

$p(y)$ does not depend on x. Thus, the factor $p(y)^{-1}$ is often written as a 'normalizer' ~~equation~~ variable denoted by (η)

$$\Rightarrow p(x|y) = \eta\,(p(y|x) \cdot p(x))$$

* We simply use normalizer 'η' to indicate that the final result has to be normalized to 1.

✓ Expectation of random variable X is given by:

$$E(X) = \sum_x x \cdot p(x).$$

$$E(X) = \int x \cdot p(x) \cdot dx.$$

$$E(aX+b) = a \cdot E[X] + b.$$

$$Cov[X] = E[X - E[X]]^2 = E[X^2] - E[X]^2$$

✓ for a discrete random variable, entropy is given by:

$$H(P) = E[-\log_2 p(x)] = -\sum_x p(x) \log_2 p(x)$$

✓ Conditioning Bayes rule:

$$p(x|y,z) = \frac{p(y|x,z)\; p(x|z)}{p(y|z)}$$

$$p(x, y|z) = p(x|z) \cdot p(y|z)$$

$$p(x|z) = p(x|z, y)$$

$$p(y|z) = p(y|z, x)$$

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$$p(x, y|z) = p(x|z) \cdot p(y|z) \not\Rightarrow p(x,y) = p(x) \cdot p(y)$$

✓ belief over state variable x_t.

$\Rightarrow bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$

W.K.T

$$\underbrace{p(a|b)}_{\text{posterior}} = \frac{\overbrace{p(b|a)}^{\text{likelihood}} \cdot \overbrace{p(a)}^{\text{prior}}}{\underbrace{p(b)}_{\text{Normalizing constant}}}$$

calculate a posterior before incorporating z_t, just after executing the control u_t. hence posterior is denoted as:

$\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$

↳ predicts the state at time t based on the previous state posterior, before incorporating the measurement at time t.

Calculating $bel(x_t)$ from $\overline{bel}(x_t)$ is called measurement update.

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Algorithm for Bayes filter

Algorithm Bayes-filter ($bel(x_{t-1}), u_t, z_t$):

for all x_t do:

$\overline{bel}(x_t) = \int p(x_t | u_t, x_{t-1}) \, bel(x_{t-1}) \, dx$

$bel(x_t) = \eta \, p(z_t | x_t) \cdot \overline{bel}(x_t)$

endfor

return $bel(x_t)$.

Derivation of Bayes filter

$$p(x_t | z_{1:t}, u_{1:t}) = p(x_t | z_t, z_{1:t-1}, u_{1:t})$$

$$= \frac{p(z_t | x_t, z_{1:t-1}, u_{1:t}) \, p(x_t | z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})}$$

$$= \eta \, p(z_t | x_t, z_{1:t-1}, u_{1:t}) \cdot p(x_t | z_{1:t-1}, u_{1:t})$$

$$p(z_t | x_t, z_{1:t-1}, u_{1:t}) = p(z_t, u_t).$$

↳ If we (hypothetically) knew the state x_t and were interested in predicting the measurement z_t, no partner control i/p's provide us information. measurement

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$$p(x_t | z_{1:t}, u_{1:t}) = \eta \cdot p(z_t | x_t) \cdot p(x_t | z_{1:t-1}, u_{1:t})$$

hence,

$$bel(x_t) = \eta \, p(z_t | x_t) \cdot \overline{bel}(x_t)$$

Next, expand term $\overline{bel}(x_t)$.

$$\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$$
$$= \int p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) \cdot p(x_{t-1} | z_{1:t-1}, u_{1:t}) \cdot dx_{t-1}$$

W.K.T $\left\{ p(A) = \int p(A|B') \cdot p(B') \, dB' \right.$.

$$p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t).$$

↳ This implies if we know x_{t-1}, past measurements & controls convey no information regarding x_t. This gives above equation

Here we retain the control i/p u_t, since it does not kill the state x_{t-1}.

$$\overline{bel}(x_t) = \int p(x_t | x_{t-1}, u_t) \cdot p(x_{t-1} | z_{1:t-1}, u_{1:t-1}) \, dx_{t-1}$$