- Take $\boxed{P_k}$ $\to$ $P_k = (\theta_k, \varphi_k)$

$\Updownarrow$

$$\left(\frac{\partial H}{\partial \theta}, \frac{\partial H}{\partial \varphi}\right)\Bigg|_{P_k} = \vec{0}$$

$(0, 0)$

Stage 1 check

$\Downarrow$

Critical point

$$\left[\Delta(P_k), \ \frac{\partial^2 H}{\partial \theta^2}\Bigg|_{P_k}\right] = \vec{0}$$

Stage 2 check

$\Downarrow$

$P_k \to$ minimum $\longleftrightarrow$ $\underbrace{I_k = (J_1^k, J_2^k, J_3^k)}_{\text{given set of MOI}}$

If $I_k \to$ has minima P_k points

$\Downarrow$

$\boxed{\text{valid}}$ MOIs I_k

$$\begin{aligned}\mathcal{H} \ =& \ \frac{I}{2}(A_1 + A_2) + A_3 I^2 + I(I - \frac{1}{2}) \sin^2\theta \left(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3\right) \\ &+ \frac{j}{2}(A_2 + A_3) + A_1 j^2 - 2A_1 I j \sin\theta - V\frac{2j-1}{j+1} \sin\left(\gamma + \frac{\pi}{6}\right).\end{aligned} \tag{1}$$

Free terms $\to$ T_0

$T_1 : f(\theta)$

$T_2 : g(\theta, \varphi)$

$$H = T_0 + T_1(\theta) + T_2(\theta, \varphi)$$

$$\frac{\partial^2 H}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(\frac{\partial H}{\partial \theta}\right) = \frac{\partial}{\partial \theta}\left(T_1' + \frac{\partial}{\partial \theta} T_2(\theta, \varphi)\right) \quad (A)$$

$$\frac{\partial^2 H}{\partial \theta^2} = H_{\theta\theta} = T_1'' + \frac{\partial^2 T_2(\theta, \varphi)}{\partial \theta^2}$$

$$\frac{\partial^2 H}{\partial \varphi^2} = \frac{\partial}{\partial \varphi}\left(\frac{\partial H}{\partial \varphi}\right) = \frac{\partial}{\partial \varphi} T_2' = T_2''$$

$$\Delta H = \frac{\partial^2 H}{\partial \theta^2}\frac{\partial^2 H}{\partial \varphi^2} - \left(\frac{\partial^2 H}{\partial \theta \partial \varphi}\right)^2 \quad (B)$$

$$\frac{\partial^2 H}{\partial \varphi^2} = H_{\varphi\varphi}$$

$$P_k \in [J_k]$$

$$\updownarrow$$

$$\underline{(A),(B)} \Rightarrow P_k = (\theta_0, \varphi_0) \begin{cases} \Delta H|_{P_k} > 0 \\ H_{\theta\theta} \end{cases} \Rightarrow P_k \rightarrow \text{minimal point}$$

- find the analytical expressions for $\frac{\partial^2 H}{\partial \theta^2}$, $\frac{\partial H}{\partial \varphi^2}$

$$\Downarrow$$

$$\mathcal{H} = \frac{I}{2}(A_1 + A_2) + A_3 I^2 + I(I - \frac{1}{2}) \sin^2\theta \left(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3\right)$$
$$+ \frac{j}{2}(A_2 + A_3) + A_1 j^2 - 2A_1 I j \sin\theta - V\frac{2j-1}{j+1} \sin\left(\gamma + \frac{\pi}{6}\right). \quad (1)$$

$$w_1 = f(I, A_1, A_2; \varphi) \qquad w_2$$

$$\frac{\partial^2 H}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(I(I-1)\sin 2\theta \left(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3\right) - 2A_1 I j \cos\theta\right)$$

$$= \frac{\partial}{\partial \theta}\left[w_1 \sin 2\theta - w_2 \cos\theta\right] = w_2 \sin\theta + 2 w_1 \cos 2\theta$$

$$\frac{\partial^2 H}{\partial \theta^2} = w_0 \cos 2\theta + w_2 \sin\theta$$

$$w_0 = 2 w_1$$

$$\mathcal{H} = \frac{I}{2}(A_1 + A_2) + A_3 I^2 + I(I - \frac{1}{2}) \sin^2\theta \left(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3\right) + \frac{j}{2}(A_2 + A_3) + A_1 j^2 - 2A_1 I j \sin\theta - V \frac{2j-1}{j+1} \sin\left(\gamma + \frac{\pi}{6}\right). \qquad (1)$$

$w_0 \; 2$

$$\frac{\partial^2 H}{\partial \varphi^2} = I(I-1)\sin^2\theta \, \frac{\partial}{\partial \varphi}\left[2A_2 \sin\varphi\cos\varphi - 2A_1 \sin\varphi\cos\varphi\right]$$

$$= w_e \frac{\partial}{\partial \varphi}\left[2 \sin\varphi\cos\varphi - 2A_1 \sin\varphi \cos\varphi\right]$$

$$= w_0 (A_2 - A_1) \frac{\partial}{\partial \varphi} \sin 2\varphi$$

$$\frac{\partial^2 H}{\partial \varphi^2} = w_0 (A_2 - A_1)\, 2\cos 2\varphi$$