

## Results - trajectories

The intersection of the energy ellipsoid  $\mathcal{H}$  (given the three moments of inertia and a defined spin state) with the angular momentum sphere (of radius  $I$ ) gives the trajectory of a rotational state within  $^{163}\text{Lu}$ .

Because the trajectories can provide information with regards to the wobbling regime inside the nucleus, we studied the evolution of the trajectories for all the energy states in the wobbling spectrum of  $^{163}\text{Lu}$  (that is, the four triaxial strongly deformed bands).

### Working with projections

However, instead of representing the *ellipsoid + sphere*, and highlighting the intersection between these surfaces in the 3-dimensional space generated by the components of the angular momentum (i.e.  $x_1$ ,  $x_2$  and  $x_3$ ), we will focus on projecting the intersection curves onto a plane of reference. Depending on the wobbling regime, a different plane will be chosen. More precisely, if the ellipsoid is intersecting the sphere in such a way that the obtained trajectory is around axis  $x_1$ , then the  $x_3Ox_2$  plane will be chosen to project the obtained contours. On the other hand, if the intersection curve is a trajectory around the axis  $x_3$ , the  $x_1Ox_2$  will be considered the projecting plane. Examples for both cases will be shown in the sections below.

## 1 Trajectories for a given spin-state

The first step of the present study is to obtain all the possible trajectories of a single spin state, when the energy of that state increases arbitrarily. In the present calculations, a spin state with angular momentum  $I = 25/2$  was chosen.



**Info:** Within the calculation process of the trajectories, we used the moments of inertia obtained by the fitting procedure of the  $^{163}\text{Lu}$  wobbling spectrum.

It is worth mentioning that for a given spin-state, the magnitude of the energy can produce one of two outcomes:

- If the energy  $E$  is smaller than  $E_{trans}$ , then the system rotates around the axis with the largest MOI, that is the 1-axis.
- If the energy  $E$  is larger than  $E_{trans}$ , then the system changes its rotation around the axis with the smallest MOI, that is the 3-axes.

The  $E_{trans}$  represents the energy at which system changes the wobbling regime: it depends on the the spin and the three moments of inertia. The trajectories that were obtained within our calculations, for a fixed spin-state, involve both cases of wobbling regimes. As a result, we have contour lines that need to be projected on both planes which were discussed above.

In Figures 1, 2, the spin-state  $I = 25/2$  is studied with an increase in energy. For small energies, the system rotates around  $x_1$ , so the ellipsoid+sphere intersection is projected onto the  $x_3Ox_2$  plane. When the energy of the nucleus is larger than the *transition phase*, then the nucleus changes its axis of rotation to  $x_3$ . As a result, the trajectories now must be projected onto the  $x_1Ox_2$  plane. Note that the thickness of the curves is directly proportional to the magnitude of the energy. This was configured just for illustrative purposes.

## 2 The real trajectories of $^{163}\text{Lu}$

Now that the evolution of the trajectories with energy while keeping the same spin-state was shown, it is worth studying the evolution of all the trajectories from the wobbling spectrum of  $^{163}\text{Lu}$ . Namely, each of the four TSD bands (TSD1, TSD2, TSD3, TSD4) need to be represented in terms of *projected trajectories* either on the plane  $x_3Ox_2$  (if the energy  $E_{TSD}$  that corresponds to the spin  $I$  is smaller than the one at which wobbling regime changes), or the plane  $x_1Ox_2$  (if the energy is larger than the value at which wobbling regime changes).

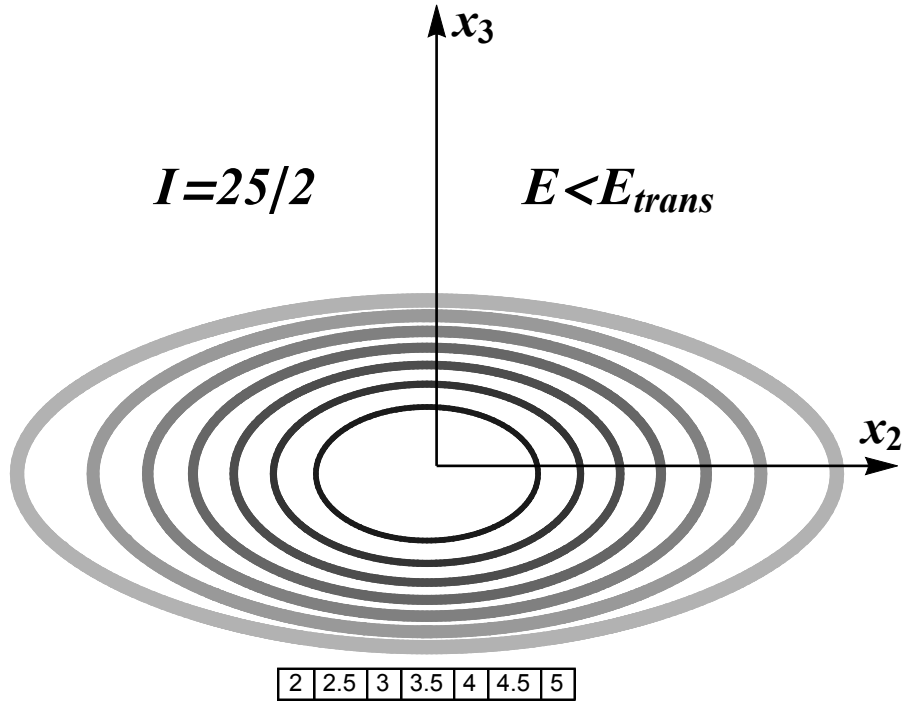


Figure 1: The trajectories for a given spin-state, evaluated at arbitrary energies, when the system rotates around  $x_1$ , when the energy of the state is small. Numerical values can be seen in the bottom portion.

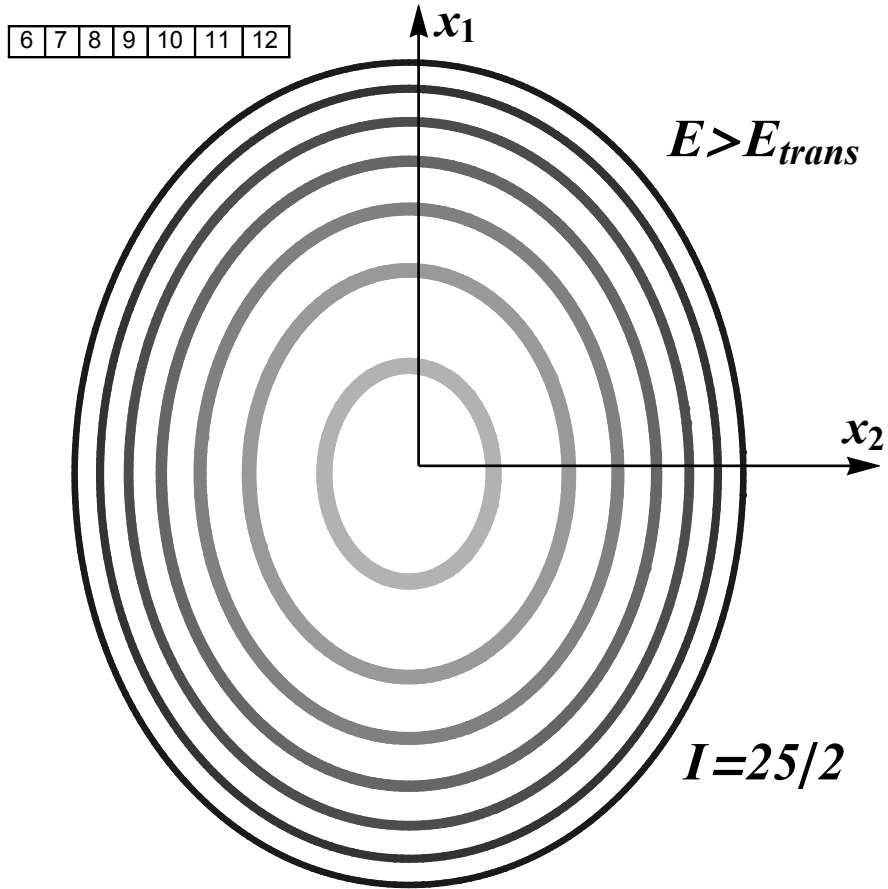


Figure 2: The trajectories for a given spin-state, evaluated at arbitrary energies, when the system rotates around  $x_3$ , when the energy of the state is large. Numerical values can be seen in top left corner.

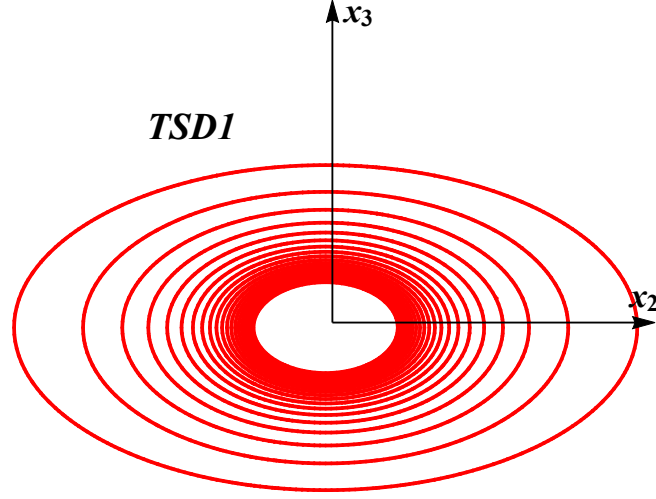


Figure 3: The states from TSD1 band are represented as projected trajectories onto the corresponding plane.

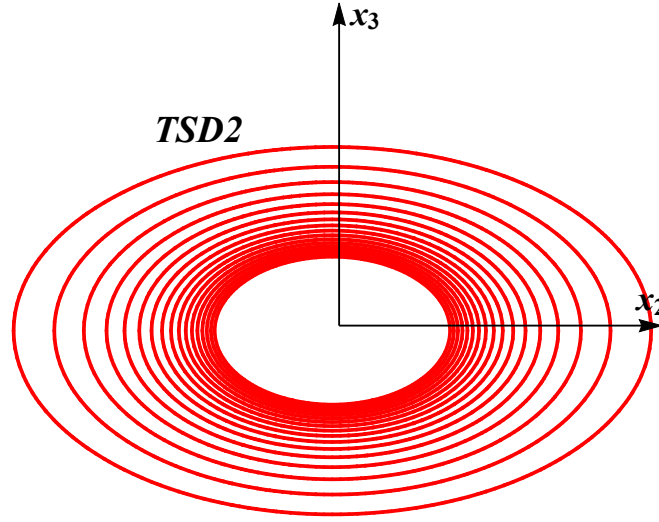


Figure 4: The states from TSD2 band are represented as projected trajectories onto the corresponding plane.



**Info:** For each band in particular, we used the numerical value of the wobbling energies that were previously calculated with a fitting procedure.

It is worth mentioning that for  $^{163}\text{Lu}$ , using the obtained theoretical values for the wobbling energies, **all the states produce trajectories that rotate around  $x_1$  axis**. As a result, all the TSD bands in the nucleus will only have projections onto  $x_3Ox_2$  plane. Results are shown in the Figures 3, 4, 5, 6, where for each band, the states are represented by the red contours (that represent the trajectories of the nucleus in the angular momentum space).

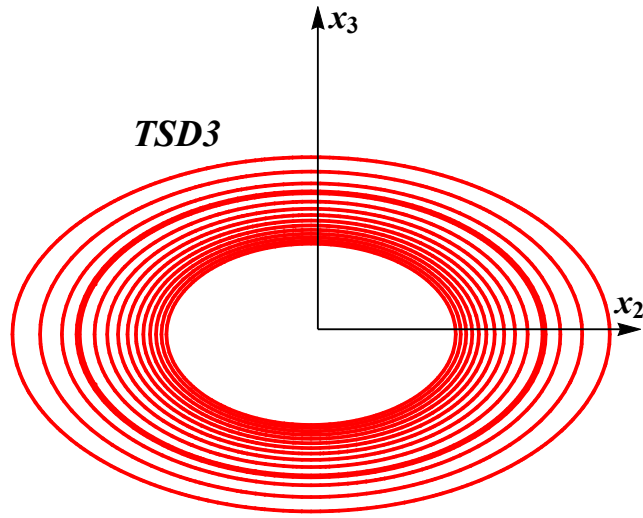


Figure 5: The states from TSD3 band are represented as projected trajectories onto the corresponding plane.

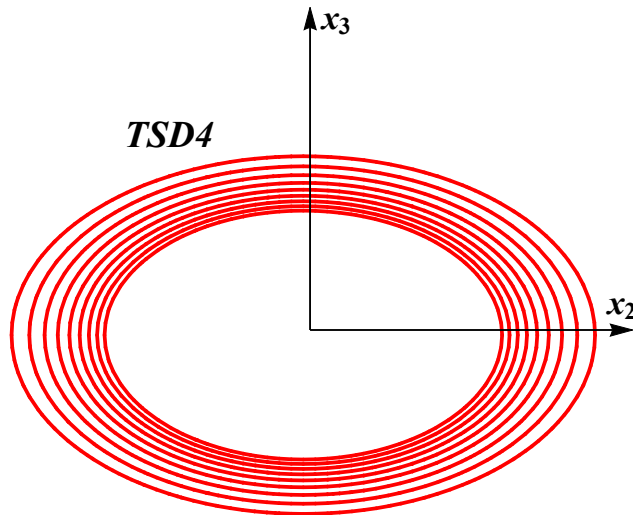


Figure 6: The states from TSD4 band are represented as projected trajectories onto the corresponding plane.