

• wobbling motion \rightarrow collective mode that appears when moments of inertia I_λ of all three principal axes of the nuclear density distribution are unequal

- for a given α, m $I \rightarrow$ uniform rotation about the axis with largest MOI corresponds to minimal energy (A)



\rightarrow at a larger energy: the axis (A) precesses (wobbles) about the space-fixed angular-momentum axis \vec{I} .

\downarrow in quantal systems

$\rightarrow \vec{I}$ wobbles around the medium (m) axis in the body fixed frame.



• appearance of rotational bands that correspond to successive excitations of wobbling phonons (m_ω)

• alternating signature $\alpha = \alpha_0 + m_\omega$ which determines the spin sequence $I = \alpha \hbar$ even m_ω .

\rightarrow adjacent wobbling bands $m_\omega + 1 / m_\omega$ are connected by $\Delta I = 1 \hbar$ transitions \rightarrow large $E2$ component (collective)

generated by the wobbling motion of the entire charged body.

- all odd nuclei have an odd nucleon occupying a high- j orbital

Qualitative study of wobbling regime:

- dependence of E_{wob} on I



J_{\parallel} = axis of uniform rotation (MOI)

J_{\perp} = MOIs of the two perpendicular axes

Frozen alignment (FA) \rightarrow particle's a.m. \vec{j} is rigidly aligned with the axis

$$E_{\text{rot}} = A_{\parallel} R_{\parallel}^2 + A_{\perp} R_{\perp}^2$$

- exciting the first wobbling quantum \Rightarrow change $J_{\parallel} : I \rightarrow I-1$

The geometry of the precession cones in Fig. 1 implies a change of R_{\perp}^2 from 0 to $\approx 2I$ and of R_{\parallel}^2 from $(I-j)^2$ to $(I-j-1)^2$, which gives an increase in the rotor energy by

$$E_{\text{wobb}} = (A_{\perp} - A_{\parallel})2I + 2\bar{j}A_{\parallel}, \quad \bar{j} = j + 1/2. \quad (2)$$

The expression

$$E_{\text{wobb}} = \sqrt{[(A_{\perp 1} - A_{\parallel})2I + 2\bar{j}A_{\parallel}][(A_{\perp 2} - A_{\parallel})2I + 2\bar{j}A_{\parallel}]} \quad (3)$$

obtained in Ref. [17] for three different moments of inertia $\mathcal{J}_{\perp 1}, \mathcal{J}_{\perp 2}, \mathcal{J}_{\parallel}$, is the geometric mean value of the wobbling energies given in Eq. (2).

It should be understood that the frozen alignment scenario discussed here is an idealization to illustrate the longitudinal and transverse coupling schemes in a transparent way. The odd particle responds to the inertial forces, changing its orientation to a certain degree. Nevertheless, the qualitative classification remains valid. Wobbling is characterized by collectively enhanced $I \rightarrow I-1$, $E2$ transitions from the wobbling to the yrast band, where the wobbling energy increases (decreases) for LW (TW).

In case of the simple and the longitudinal wobblers [Figs. 1(a) and 1(c)] the precession cone revolves about the m axis with the largest moment of inertia. As $A_{\parallel} < A_{\perp}$, the wobbling energy E_{wobb} increases with I . For the case of transverse wobbling [Fig. 1(d)], the precession cone revolves about the s (or l) axis, which has a smaller moment of inertia than that for rotation about the m axis. In this case, $A_{\parallel} > A_{\perp}$ and the wobbling energy E_{wobb} decreases with I until zero, where the mode becomes unstable.

The expression

$$E_{\text{wobb}} = \sqrt{[(A_{\perp 1} - A_{\parallel})2I + 2\bar{j}A_{\parallel}][(A_{\perp 2} - A_{\parallel})2I + 2\bar{j}A_{\parallel}]} \quad (3)$$

The signature-partner bands represent another type of excitation involving a partial dealignment of the odd particle with respect to its preferred axis [Fig. 1(b)]; for those, the connecting $\Delta I = 1$ transitions are of predominant $M1$ character, with very little, if any, $E2$ admixture.

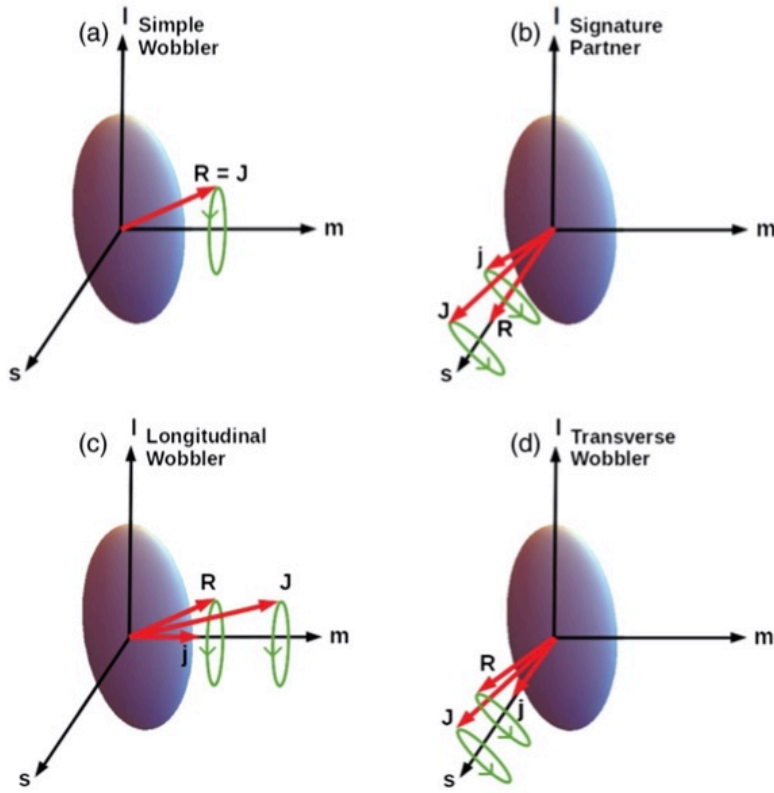


FIG. 1. Angular momentum geometry of (a) simple wobbler, (b) signature partner, (c) longitudinal, and (d) transverse wobbler in the body fixed frame, where l , m , and s correspond to the long, medium, and short axis, respectively. R , j , and J are the rotor, odd particle, and total angular momentum, respectively.

Behavior of the collective rotor in wobbling motion

E. Streck,¹ Q. B. Chen,^{1,*} N. Kaiser,^{1,†} and Ulf-G. Meißner^{2,3,4,‡}

¹Physik-Department, Technische Universität München, D-85747 Garching, Germany

²Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

³Institute for Advanced Simulation, Institut für Kernphysik, Jülich Center for Hadron Physics and JARA-HPC,

Forschungszentrum Jülich, D-52425 Jülich, Germany

⁴Ivane Javakhishvili Tbilisi State University, 0186 Tbilisi, Georgia



(Received 9 May 2018; revised manuscript received 4 September 2018; published 12 October 2018)

The behavior of the collective rotor in wobbling motion is investigated within the particle-rotor model for the nucleus ^{135}Pr by transforming the wave functions from the K representation to the R representation. After reproducing the experimental energy spectra and wobbling frequencies, the evolution of the wobbling mode in ^{135}Pr , from transverse at low spins to longitudinal at high spins, is illustrated by the distributions of the total angular momentum in the intrinsic reference frame (azimuthal plot). Finally, the coupling schemes of the angular momenta of the rotor and the high- j particle for transverse and longitudinal wobbling are obtained from the analysis of the probability distributions of the rotor angular momentum (R plots) and their projections onto the three principal axes (K_R plots).

DOI: 10.1103/PhysRevC.98.044314

E. Angular momentum coupling schemes

From the above analysis of energy expectation values of the intrinsic Hamiltonian \hat{H}_{intr} , azimuthal plots $\mathcal{P}(\theta, \varphi)$ of the total angular momentum, and the R plots and three K_R plots for the rotor angular momentum, one can deduce the following features in the transverse wobbling region:

- (i) the single-particle (angular momentum) is aligned with the s axis;
- (ii) the average rotor angular momentum is more than $1\hbar$ (and less than $2\hbar$) longer in the wobbling band with spin $I + 1$ than in the yrast band with spin I ;
- (iii) the projection of the rotor angular momentum onto the I axis is very small;
- (iv) the rotor angular momenta in yrast states (with I) and wobbling states (with $I + 1$) have similar components along the s axis. For neighboring states with $I - 2$ and I , the component R_s differs by about $2\hbar$;
- (v) the component R_i increases by about $2\hbar$ from an yrast state I to a wobbling state $I + 1$. In addition, R_i in the yrast state I is about $1\hbar$ smaller than its value in the wobbling state $I - 1$.

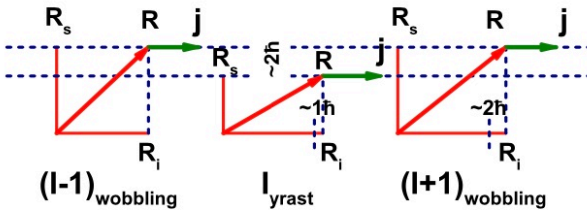


FIG. 10. Similar as Fig. 9, but for the longitudinal wobbling motion.

On the other hand, for longitudinal wobbling one finds the following features:

- (i) the proton particle (angular momentum) is aligned with the i axis;
- (ii) the average value of R_s is about $4\hbar$ in the yrast band and about $6\hbar$ in the wobbling band.
- (iii) the increment of R_i from an yrast state with $I - 1$ to a wobbling state with I is about $1\hbar$.

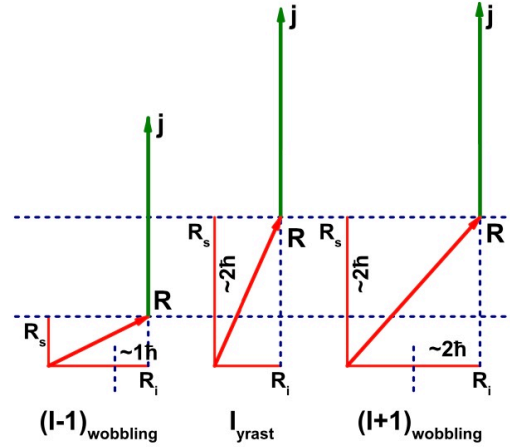


FIG. 9. Schematic illustration of the coupling scheme of the angular momenta j and R of the high- j particle and the rotor for the transverse wobbling in an yrast state with I and two wobbling states with $I \pm 1$. The total angular momentum is $I = R + j$.

