Wobbling Phenomenon in Odd-Mass Nuclei

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Triaxiality

- Nuclear shapes: most of the nuclei are spherical or axially symmetric in the ground state.

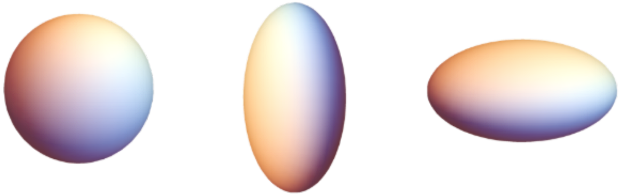


Figure 1: **Spherical:** $\beta_2 = 0$; **Prolate:** $\beta_2 > 0$; **Oblate:** $\beta_2 < 0$

Deformation parameters

There are also deviations from *axial symmetric shapes* $\rightarrow$ **triaxial shapes** (e.g. no symmetry axis).

Nuclear surface - axially-asymmetric shape

- For nuclei with the three principal axes of different lengths (*axial asymmetry*), the γ deformation parameter emerges.
- γ is a measure of asymmetry between the three *moments of inertia* of the nucleus.

Shape parameters

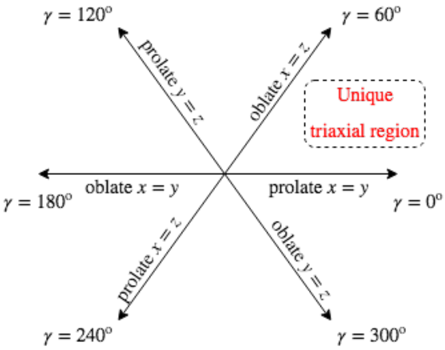


Figure 2: The (β, γ) plane divided into six equivalent parts.

Wobbling motion - clear signature for triaxiality

- A triaxial nucleus can rotate about any of the three axes.
- Rotation about the axis with the largest moment of inertia (MOI) is energetically the most favorable.
- The other two axes contribute to the total nuclear motion (due to the anisotropy between the MOIs) $\rightarrow$ This motion has an oscillating behavior.
- Spectrum: $E = E_{\text{rot}} + E_{\text{wob}}\left(n_w + \frac{1}{2}\right)$.

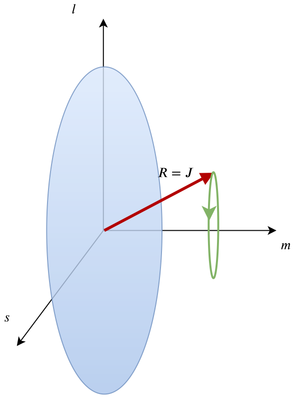


Figure 3: A simple wobbler.

Wobbling motion

Wobbling motion (WM)

- Uniquely associated to triaxial structures.
- It was theoretically predicted by Bohr and Mottelson more than 50 years ago (for the even-A case).
- Experimentally confirmed for ^{163}Lu in 2001.

Experimental evidence

In present, few wobblers are experimentally confirmed in the mass regions: $A \approx 130, 160, 180$.

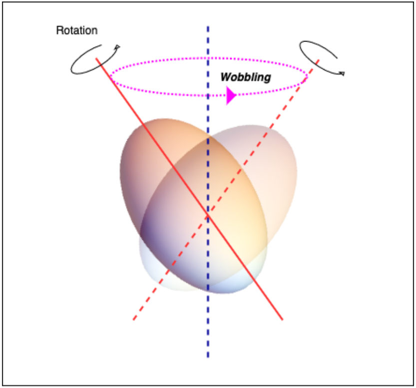


Figure 4: Schematic representation for the nuclear wobbling motion.

Wobbling motion

Triaxial nuclei

The rotational angular momentum is NOT aligned along any of the body-fixed axes: it **precesses** and **wobbles** around the axes with the largest MOI.

Wobbling bands

Sequences of $\Delta I = 2\hbar$ rotational bands that are built on different *wobbling phonon excitations*.

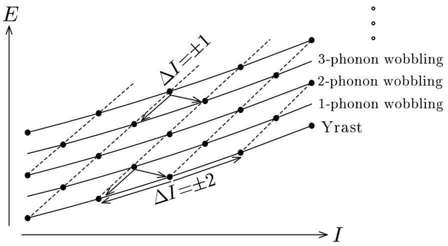


Figure 5: Rotational-band structures of the wobbling motion.

Particle-rotor coupling

Wobbling motion in odd-A nuclei

- Coupling of a nucleon from a high j-shell with the triaxial rotor core is crucial for the wobbling phenomenon.
- The odd particle's angular momentum couples to the rotor, driving the nucleus to large deformations and it also stabilizes the shape.

Odd particle

- For nuclei with $A \approx 160$, the odd $\pi i_{13/2}$ is the *intruder* which drive the isotope to very high deformations (up to $n_w = 3$ wobbling phonon number).
- For nuclei with $A \approx 180$, the odd $\pi h_{9/2}$ and $\pi i_{13/2}$ are the *intruders* which drive the system to very high deformations.

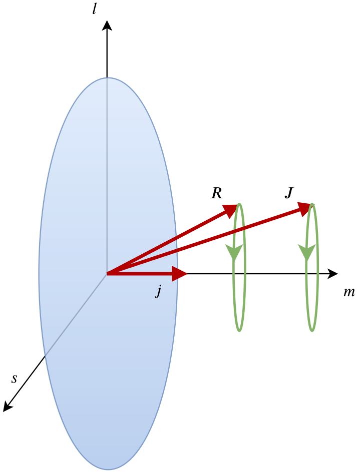
Wobbling regimes

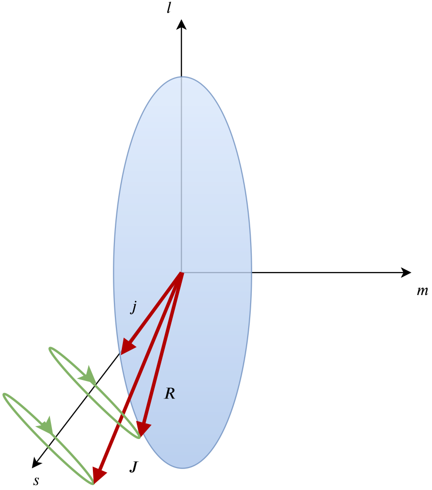
Frauendorf et al (2014) formulated two possible wobbling modes in the case of $odd - A$ nuclei.

Longitudinal wobbling (WL) the quasiparticle's angular momentum j is *aligned* with the rotational axis of the system (m-axis for triaxial nuclei)

Transverse wobbling (TW) the quasiparticle's a.m. j is *perpendicular* to the rotational axis (m); so it is parallel with either the long (l) axes or the short (s axes).

LW vs TW - graphical representation





Theoretical framework

The odd-mass system consists of an even-even core (described by a triaxial rotor Hamiltonian H_{rot}) and a single j-shell nucleon described by its single-particle Hamiltonian H_{sp}.

Total system:

$$H = H_{\text{rot}} + H_{\text{sp}} = \tag{1}$$

$$\sum_{k=1,2,3} A_k (I_k - j_k)^2 + \epsilon_j + \frac{V}{j(j+1)} \left[\cos\gamma(3j_3^2 - \mathrm{j}^2) - \sqrt{3}\sin\gamma(j_1^2 - j_2^2)\right] \tag{2}$$

Solving the Hamiltonian in semi-classical approach for ^{163}Lu: *R. Poenaru and A. A. Raduta, International Journal of Modern Physics E, 2150033, 2021*.

For ^{135}Pr: *A. A. Raduta and R. Poenaru, Journal of Physics G, 015106, 2020*.

Energy function - Semiclassical approach

The eigenvalues for $\hat{H} = H_{rot} + H_{sp}$ are obtained by solving the *variational principle*:

$$\delta \int_0^t \langle \Psi_{IjM} | \hat{H} - i \frac{\partial}{\partial t'} | \Psi_{IjM} \rangle \, dt' = 0. \tag{3}$$

The trial function $|\Psi_{IjM}\rangle$ is a direct product $|\psi\rangle_{\text{core}} \otimes |\phi\rangle_{\text{s.p.}}$. From Eq. 3, one obtains the *classical equations of motion* in the canonical form:

$$\frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi} \; ; \; \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r} \tag{4}$$

$$\frac{\partial \mathcal{H}}{\partial t} = \dot{\psi} \; ; \; \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{t} \tag{5}$$

Classical energy function

Considering the *energy function* as the average of the Hamiltonian on the trial function:

$$\mathcal{H} \equiv \langle \Psi_{IjM} | H | \Psi_{IjM} \rangle \tag{6}$$

Harmonic frequencies

- The energy function is *minimal* in the point $\mathcal{P}_{min}$ when $A_1 < (A_2, A_3)$.
- Linearizing the equations of motion around the minimum point of $\mathcal{H}$, one obtains a harmonic motion for the system, with the *frequencies* given by:

$$\Omega^4 + B\Omega^2 + C = 0 \tag{7}$$

- B and C functions that depend on: **the inertia factors A_k, single particle potential strength V and the triaxiality parameter γ.**

Semiclassical wobbling energies

Harmonic Frequencies II

- The two real and positive solutions for Eq. 7: Ω_1 and Ω_2 are called *the wobbling frequencies*.
- They represent the harmonic-like behavior of the wobbling motion in odd-A nuclei.
- In our model, the pair of frequencies (Ω_1^I, Ω_2^I) is associated with the rotational motion of the core and the odd particle, respectively.

Energy spectrum

$$E_I = \epsilon_j + \mathcal{H}_{\min}(I) + \left[\Omega_1^I\left(n_{w_1} + \frac{1}{2}\right) + \Omega_2^I\left(n_{w_2} + \frac{1}{2}\right)\right] \tag{8}$$

Terms: single particle energy, energy in the minimum point and [harmonic-like motion].

preliminary Results for ^{183}Au

S. Nandi et. al. (PRC, 132501, 2020) - First Observation of Multiple Transverse Wobbling Bands in ^{183}Au.

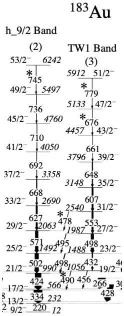


Figure 6: The first wobbling sequence (negative parity states).

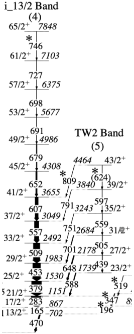


Figure 7: The second wobbling sequence (positive parity states).

Fitting the model

We tested our formalism by fitting the expression of the energy formula (Eq. 8) to the experimental data for the wobbling spectrum of both isotopes.

Parametrization: negative parity bands

- The odd-particle: $h_{j=9/2}$ proton.
- ground-state band: $(n_{w_1}, n_{w_2}) = (0, 0)$.
- second wobbling band: $(n_{w_1}, n_{w_2}) = (1, 0)$.

Parametrization: positive parity bands

- The odd-particle: $i_{j=13/2}$ proton.
- ground-state band: $(n_{w_1}, n_{w_2}) = (0, 0)$.
- second wobbling band: $(n_{w_1}, n_{w_2}) = (1, 0)$.

Results for ^{183}Au II

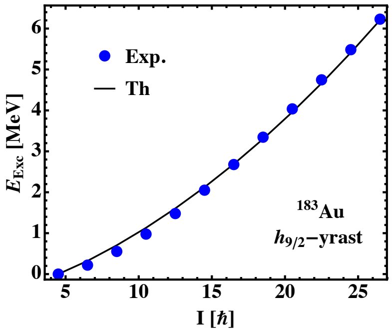


Figure 8: First band (negative parity).

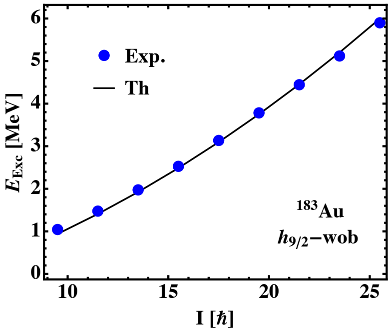


Figure 9: Second band (negative parity).

$rms \approx 79$ keV.

$\mathcal{I}_1$=86.71, $\mathcal{I}_2$=10.30, $\mathcal{I}_3$=4.36 $\hbar^2/\text{MeV}^{-1}$

$V = 0.1747387$ MeV, $\gamma = 21.23^o$ ($\gamma_{\text{exp}} \approx 20$).

preliminary Results for ^{183}Au III

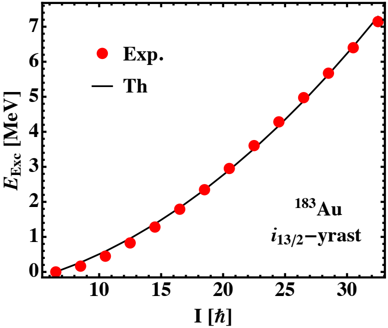


Figure 10: First band (positive parity).

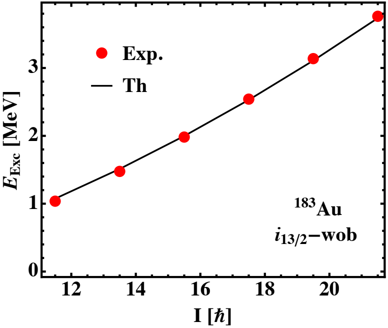


Figure 11: Second band (positive parity).

rms $\approx$ 73 keV.
$\mathcal{I}_1$=83.4, $\mathcal{I}_2$=3.63, $\mathcal{I}_3$=26 $\hbar^2/\text{MeV}^{-1}$
$V = 1.95$ MeV, $\gamma = 19.7^o$ ($\gamma_{\text{exp}} \approx 21$).

Parametrization

- *N. Senhsarma et. al. (PRL, 052501, 2020) showed evidence for Longitudinal Wobbling in 187Au.*
- Two such wobbling bands have been confirmed (Longitudinal character).
- The odd proton $h_{11/2}$ is coupling to the triaxial core.
- ground-state band: $(n_{w_1}, n_{w_2}) = (0, 0)$.
- second wobbling band: $(n_{w_1}, n_{w_2}) = (1, 0)$.

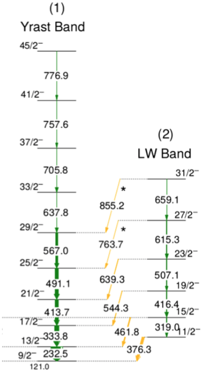


Figure 12: Experimental evidence of wobbling excitations in ^{187}Au.

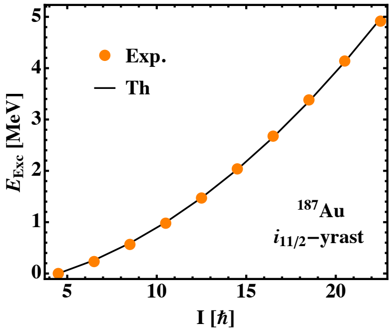


Figure 13: First band (positive parity).

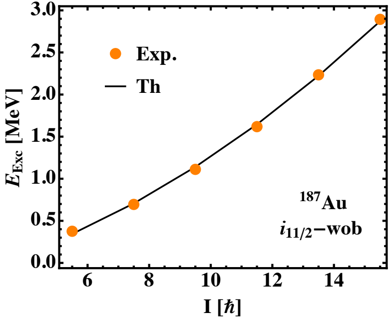
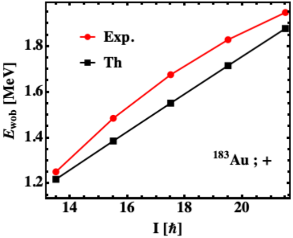


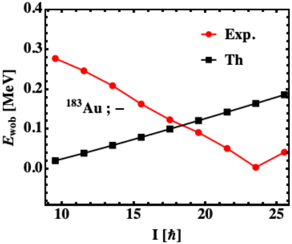
Figure 14: Second band (positive parity).

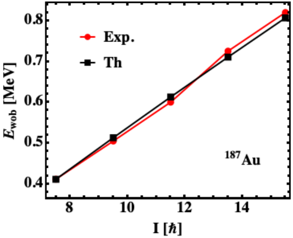
$rms \approx 23$ keV.
$\mathcal{I}_1$=55.1, $\mathcal{I}_2$=3.81, $\mathcal{I}_3$=19.56 $\hbar^2/\mathrm{MeV}^{-1}$
$V = 2.02$ MeV, $\gamma = 20^o$ ($\gamma_{exp} \in [20, 23]$)

Wobbling behavior for au isotopes

The behavior (i.e., Longitudinal or Transverse) is reflected (?) in the behavior of the *wobbling energy*: $E_{\mathrm{w}}(I) = E_I^1 - \frac{E_{I-1}^0 + E_{I+1}^0}{2}$







Wobbling character

- ^{183}Au (positive p.b.): longitudinal
- ^{183}Au (negative p.b.): mixed (tranvserse+longitudinal)
- ^{187}Au (positive p.b.): longitudinal

Conclusions

1. Analytical expression for the wobbling spectrum has been obtained for odd-A nuclei.
2. Rotor+odd-particle coupling is crucial for the wobbling mode to emerge.
3. Wobbling spectrum for three band-sequences (two in ^{183}Au and one in ^{187}Au) was numerically calculated $\rightarrow$ good agreement with experimental data).
4. Triaxiality parameter for each isotope γ was obtained via the fitting procedure $\rightarrow$ very close to the *accepted* value.
5. **Current model**: does not predict a change in the wobbling regime within a nuclear system. (i.e., the negative states from ^{183}Au).
6. $\mathcal{I}_{2,3}$ ordering: for ^{183}Au (negative p.b.) **reversed** when compared with the other two sequences $\rightarrow$ might indicate the *missing feature* in identifying the transverse part.
7. Ongoing debate on the test for TW/LW wobbling regimes:

Thank you for your attention!