

# New Results Concerning Collective Motion in Triaxial Nuclei

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# Nuclear Deformation

Most of the nuclei are either *spherical* or *axially symmetric* in their ground-state.

Deformation parameter  $\beta$  (Bohr, 1969): preserves axial symmetry

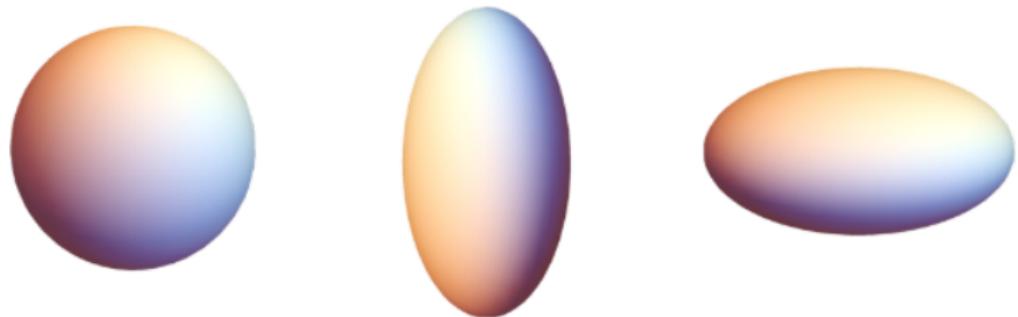


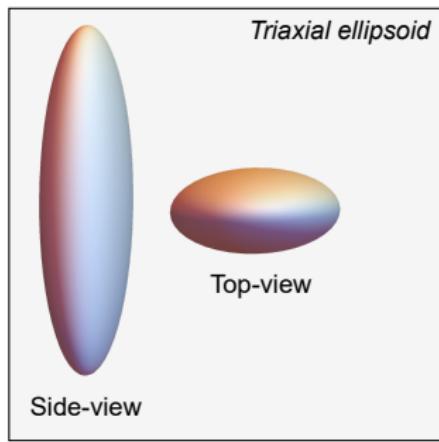
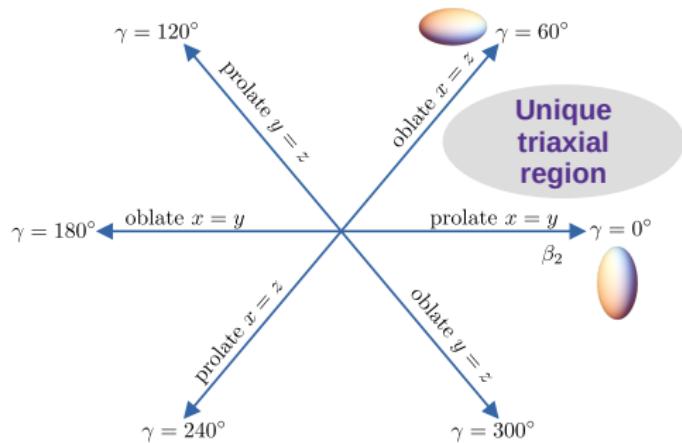
Figure 1: **spherical:**  $\beta = 0$  **prolate:**  $\beta > 0$  **oblate:**  $\beta < 0$

# Nuclear Triaxiality

## Non-axial shape

Deviations from symmetric shapes can occur across the chart of nuclides → **triaxial nuclei**.

The triaxiality parameter  $\gamma$  (*Bohr, 1969*): departure from axial symmetry



# Fingerprints for Triaxiality

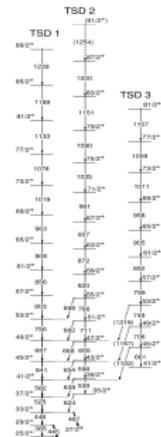
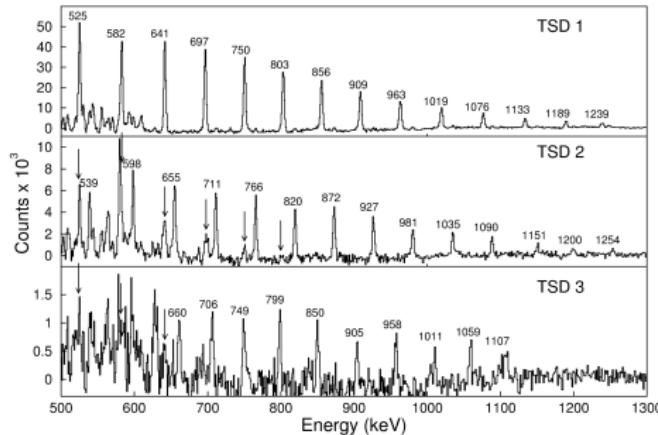
- Stable triaxial nuclei represent a real challenge
- Clear signatures for confirming stable triaxiality in nuclei
  - ① Chiral symmetry breaking (*Frauendorf, 1997*)
  - ② **Wobbling motion** (*Bohr & Mottelson, 1975*)

## Wobbling Motion (WM)

- Unique to non-axial nuclei
- Predicted 50 years ago for even- $A$  nuclei
- First experimental evidence for  $^{163}\text{Lu}$  (*Ødegård, 2001*)
- Currently: confirmed wobblers within the mass regions  
 $A \approx [100, 130, 160, 180]$ .

# Triaxial Rotor Energy

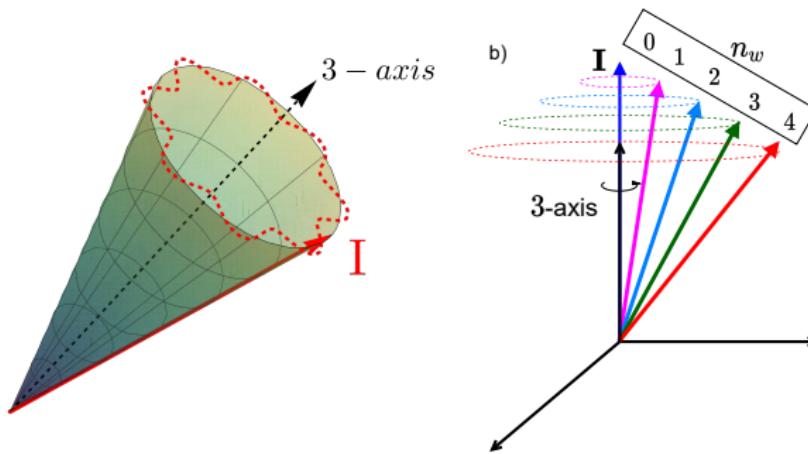
- A triaxial nucleus can rotate about any of the three axes
- Rotation about the axis with **the largest moment of inertia** (MOI) is energetically the most favorable:  $E_{\text{rot}} \propto \frac{\hbar^2}{2J_{\max}} I(I+1)$
- MOI anisotropy → the *main rotation* around  $J_{\max}$  is disturbed by the other two axes → *total motion of the rotating nucleus has an oscillating behavior*



Figures from Schönwaßer et al., 2001

# Wobbling Motion

- Total angular momentum  $\mathbf{I}$  disaligned w.r.t. body-fixed axes
- The a.m. **precesses** and **wobbles** around the axis with  $\mathcal{J}_{\max}$
- The precession of  $\mathbf{I}$  can increase by **tilting**
- Tilting by an energy quanta  $\sim$  *vibrational character*  $\rightarrow$  **wobbling phonon**  $n_w = 0, 1, 2\dots$

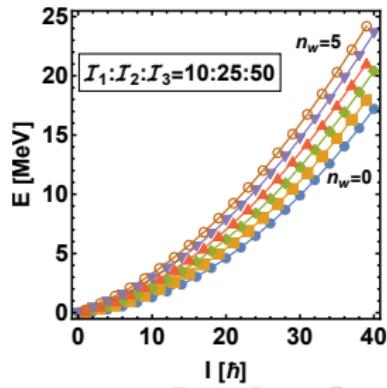
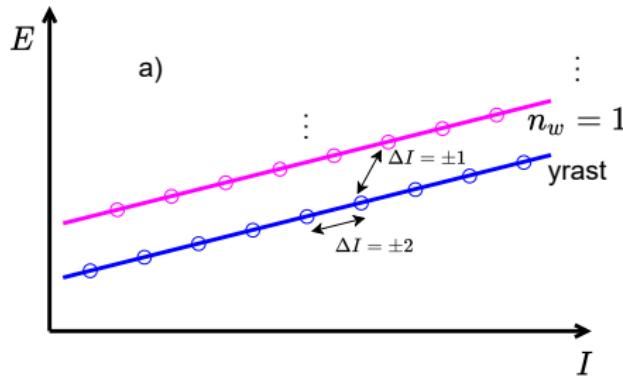


# Wobbling Spectrum

## Even-A Nuclei

- Employing the Harmonic Approximation (*Bohr, 1969*)
- $\hat{H}$  composed of a *rotational* part and *harmonic oscillation* (i.e., wobbling) part:

$$\hat{H} = \frac{\hbar^2}{2J_{\max}} I(I+1) + \hbar\omega_{\text{wob}} \left( n_w + \frac{1}{2} \right), \quad n_w = 0, 1, 2, \dots \quad (1)$$



# New Results for A=130

## Recent findings for even-even nuclei

- Two wobbling bands have been identified experimentally in  $^{130}\text{Ba}$  (*Petrache et al., 2019*)
- DFT+PRM description of the wobbling motion described the excited spectra (*Chen et al., 2019*)
- Stable triaxiality for  $\beta = 0.24$  and  $\gamma = 21.5^\circ$

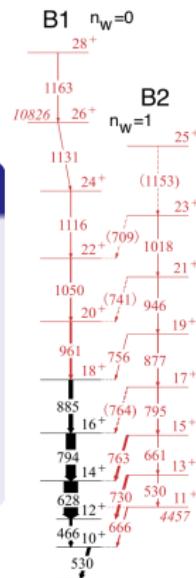
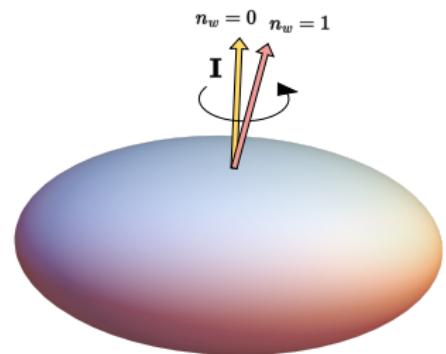


Figure from Petrache  
et al., 2019

## New Results for A=130 II

- Employed an energy spectrum of harmonic type according to Eq. 1:

$$E_I = \frac{\hbar^2}{2\mathcal{J}_3} I(I+1) + \hbar\omega_{\text{wob}} \left( n_w + \frac{1}{2} \right)$$



- wobbling frequency - linear dependence on  $I$  (fixed MOI ordering  $\mathcal{J}_3 > \mathcal{J}_{1,2}$ )

$$\hbar\omega_{\text{wob}}(I) = 2f(\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3) \cdot I$$

# New Results for A=130 III

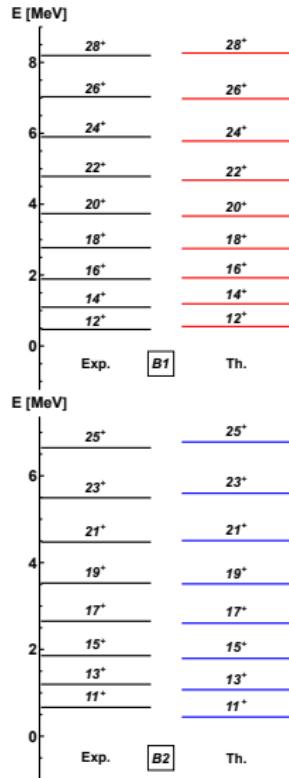
## Harmonic Approximation

- Reproduced the excited spectra for  $\{B1, B2\}$
- Fix a *free parameter set*:  $\mathcal{P} = [\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3]$
- Adopt a fitting procedure:

$$\chi^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left( E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)} \right)^2}{E_{\text{exp}}^{(i)}} \quad (2)$$

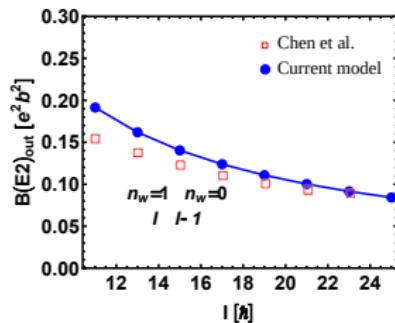
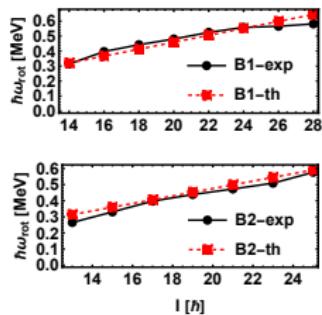
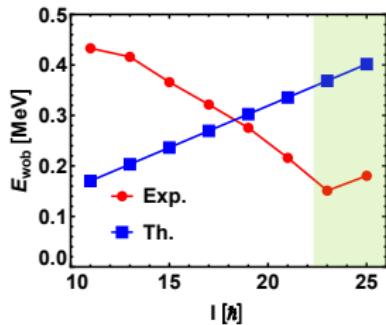
## Results for $^{130}\text{Ba}$ PRELIMINARY!

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$	$[\hbar^2 \text{MeV}^{-1}]$
27	22	43	



# New Results for A=130 IV

- **Left:** Wobbling energy\*  $E_{\text{wob}}(I) = E_{\text{B}2}(I) - E_{\text{B}1}(I)$
- **Center:** Rotational frequency  $\hbar\omega_{\text{rot}}$  - collective
- **Right:** Interband transition probabilities  $I_{\text{B}2} \rightarrow I_{\text{B}1}$



Poenaru, 2022, unpublished

# Electromagnetic transitions

## Even-A nuclei conclusions

- $\beta = 0.24$  and  $\gamma = 21.5^\circ$  were adopted for *quadrupole moments and transition probabilities*
- PRM values are calculated by Chen et al through a similar approach of minimizing the  $\chi^2$ -function
- Main rotation occurs along 3-axis
- Approximation only reproduces  $E_{\text{wob}}$  in the high-spin limit

$I$	$B(E2)_{\text{out}}/B(E2)_{\text{in}}$		
	Th.	PRM*	Exp.
11	0.37	-	-
13	0.32	0.51	0.32
15	0.27	0.42	0.36
17	0.24	0.35	0.22
19	0.21	0.29	0.22
21	0.19	0.25	0.41
23	0.18	-	-
25	0.16	-	-

Poenaru, 2022, unpublished

# Wobbling Motion in Odd-Mass Nuclei

## Particle-Rotor-Model

- The precessional motion of  $\mathbf{I} = \mathbf{R} + \mathbf{j}$  is caused by the coupling of an even- $A$  triaxial core + odd-particle
- System: [even- $A$  core] + [odd quasi-particle moving in a quadrupole deformed mean-field generated by the core]

$$\hat{H} = H_{\text{core}} + H_{\text{sp}} = \sum_{k=1,2,3} \frac{1}{2J_k} \left( \hat{I}_k - \hat{j}_k \right)^2 + \\ + \epsilon_j + \frac{V}{j(j+1)} \left[ \cos \gamma \left( 3\hat{j}_3^2 - j^2 \right) - \sqrt{3} \sin \gamma \left( \hat{j}_1^2 - \hat{j}_2^2 \right) \right] \quad (3)$$

for  $^{163}\text{Lu}$ : R Poenaru, AA Raduta, IJMPE, 2021.

for  $^{167}\text{Pr}$ : A. A. Raduta and R. Poenaru, Phys Rev C, 2020.

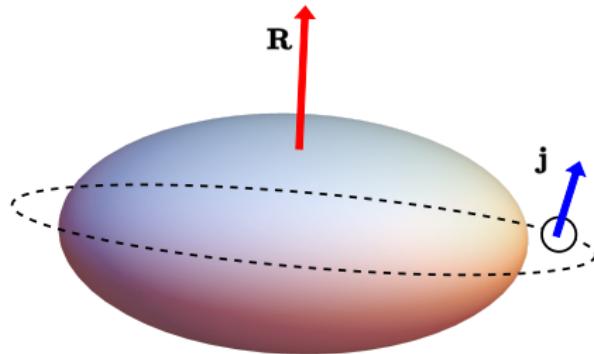
for  $^{135}\text{Pr}$ : A. A. Raduta and R. Poenaru, Journal of Physics G, 2020.

# Classical energy

## Energy spectrum for odd mass nuclei

- Total energy is given in terms of **two wobbling frequencies**:  
 $\Omega_1 \rightarrow$  core and  $\Omega_2 \rightarrow$  odd-particle

$$E_I = \epsilon_j + \mathcal{H}_{\min}(I) + \hbar\Omega_1^I \left( n_{w1} + \frac{1}{2} \right) + \hbar\Omega_2^I \left( n_{w2} + \frac{1}{2} \right)$$



# Results for A=163

## Wobbling Motion in $^{163}\text{Lu}$

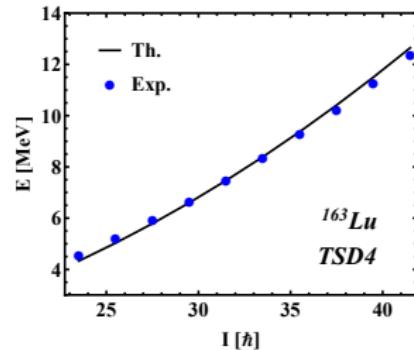
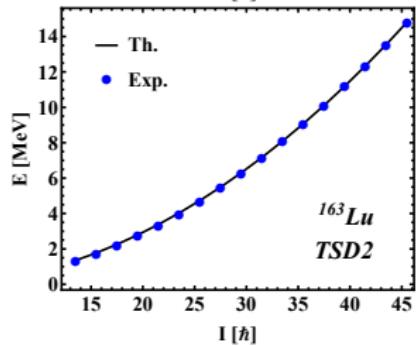
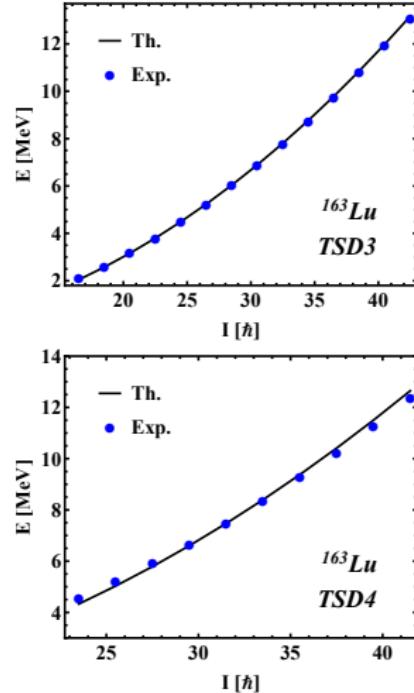
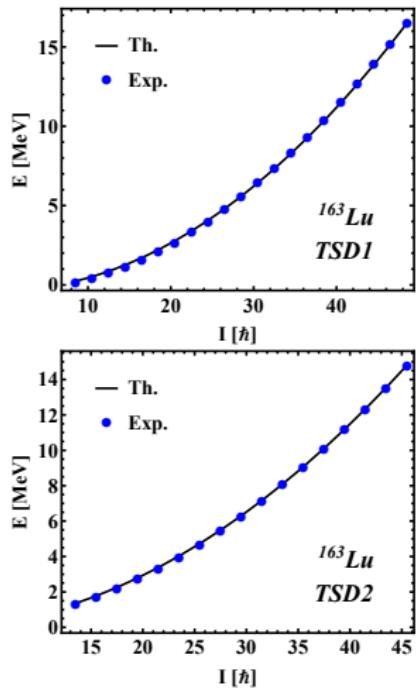
- Four wobbling bands TSD1,2,3,4
- Stable minimum at  $\beta = 0.38$  and  $\gamma = 20^\circ$

## Model parametrization

- 5 free parameters:  $\mathcal{P}_{\text{fit}} = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, V, \gamma\}$
- $\gamma^{\text{fit}} = 22^\circ \rightarrow \gamma^{\text{exp}} = 20^\circ$
- $V^{\text{fit}} = 2.1 \text{ MeV}$  (literature:  $0 < V < 8 \text{ MeV}$ )
- the odd proton  $i_{13/2}$  of positive parity couples to the triaxial core and generates TSD1,2,3,4

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$	$[\hbar^2 \text{MeV}^{-1}]$
72	15	7	

# Results for A=163 II

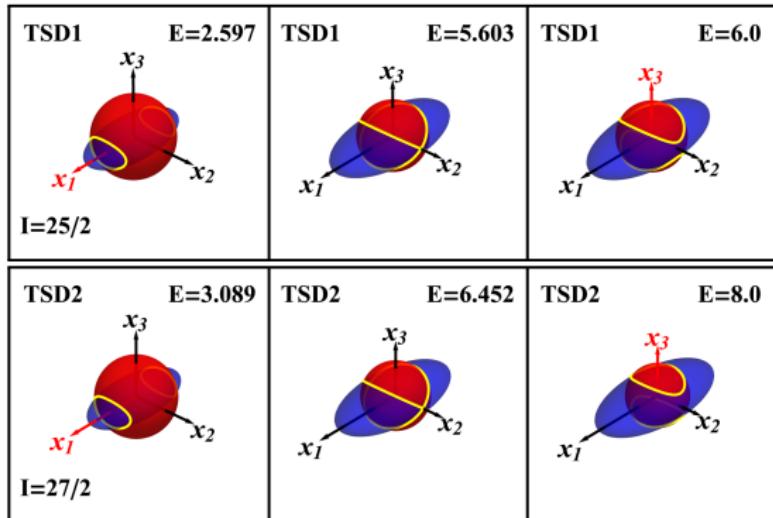


# Results for A=163 III

- Energy and angular momentum: constants of motion

$$E = \mathcal{S}_1 x_1^2 + \mathcal{S}_2 x_2^2 + \mathcal{S}_3 x_3^2 + \mathcal{S}_0^{\text{core+sp}} , \quad (4)$$

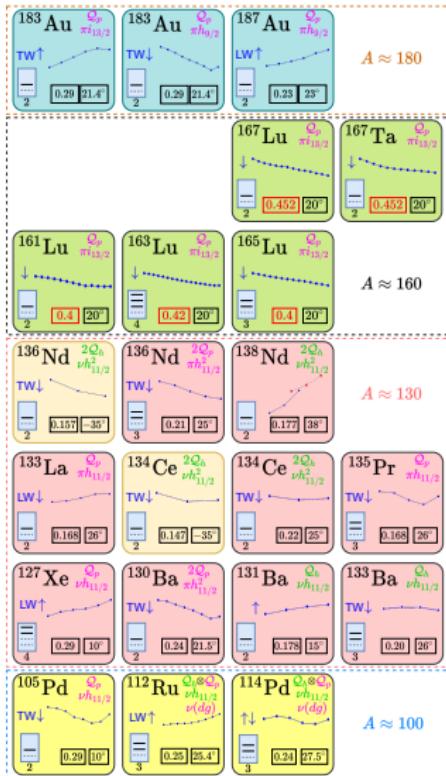
$$I^2 = x_1^2 + x_2^2 + x_3^2 . \quad (5)$$



# Conclusions

- Energy spectrum for even-even nuclei was obtained using a *harmonic approximation*
- Energy spectrum for odd- $A$  nuclei was obtained using *PRM*
- 1 oscillatory motion for even- $A$   $\longrightarrow \hbar\omega_{\text{wob}}$
- 2 oscillatory motions for odd- $A$   $\longrightarrow \hbar\Omega_{1,2}$
- odd- $A$ :  $\gamma$  obtained self-consistently (agreement with exp.)
- geometrical interpretations: consistent with quantal studies (*Lawrie et al. 2020*)

# Chart of wobblers



Thank you for your attention!

Wobbling nuclei (up to date)

Poenaru, 2022, in progress