New Results Concerning Collective Motion in Triaxial Nuclei

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Nuclear Deformation

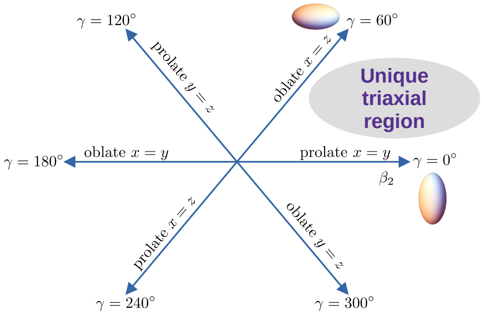
Most of the nuclei are either *spherical* or *axially symmetric* in their ground-state.
Deformation parameter β (*Bohr, 1969*): preserves axial symmetry

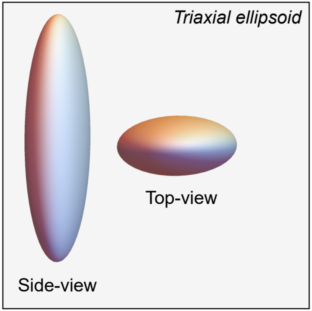
Figure 1: **spherical:** $\beta = 0$ **prolate:** $\beta > 0$ **oblate:** $\beta < 0$

Nuclear Triaxiality

Non-axial shape

Deviations from symmetric shapes can occur across the chart of nuclides $\rightarrow$ **triaxial nuclei**.
The triaxiality parameter γ (*Bohr, 1969*): departure from axial symmetry





Fingerprints for Triaxiality

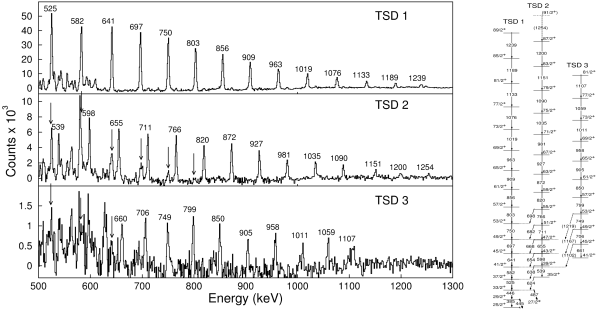
- Experimentally, stable triaxial nuclei represent a real challenge
- Clear signatures for confirming stable triaxiality in nuclei
  1. Chiral symmetry breaking (*Frauendorf, 1997*)
  2. **Wobbling motion** (*Bohr & Mottelson, 1975*)

Wobbling Motion (WM)

- Unique to non-axial nuclei
- Predicted 50 years ago for even-A nuclei
- First experimental evidence for ^{163}Lu (*Ødegård*, 2001)
- Currently: confirmed wobblers within the mass regions $A \approx [100, 130, 160, 180]$.

Triaxial Rotor Energy

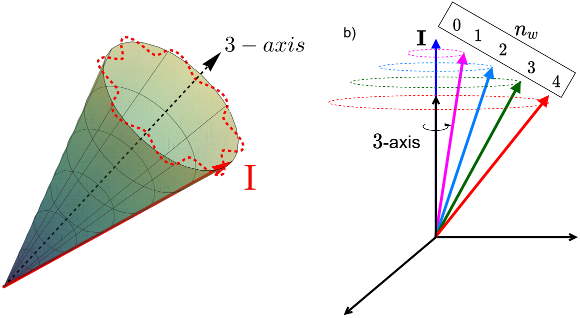
- A triaxial nucleus can rotate about any of the three axes
- Rotation about the axis with **the largest moment of inertia** (MOI) is energetically the most favorable: $E_{\mathrm{rot}} \propto \frac{\hbar^2}{2\mathcal{J}_{\mathrm{max}}} I(I+1)$
- MOI anisotropy $\rightarrow$ the *main rotation* around $\mathcal{J}_{\mathrm{max}}$ is disturbed by the other two axes $\rightarrow$ *total motion of the rotating nucleus has an oscillating behavior*



Figures from Schönwaßer et al., 2001

Wobbling Motion

- Total angular momentum **I** disaligned w.r.t. body-fixed axes
- The a.m. **precesses** and **wobbles** around the axis with $\mathcal{J}_{\text{max}}$
- The precession of **I** can increase by **tilting**
- Tilting by an energy quanta $\sim$ *vibrational character* $\rightarrow$ **wobbling phonon** $n_w = 0, 1, 2 ...$

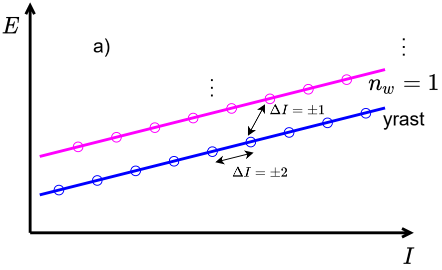


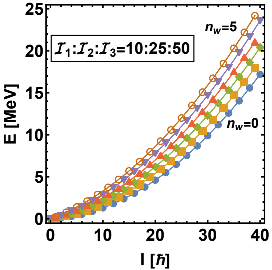
Wobbling Spectrum

Even-A Nuclei

- Employing the Harmonic Approximation *(Bohr, 1969)*
- $\hat{H}$ composed of a *rotational* part and *harmonic oscillation* (i.e., wobbling) part:

$$\hat{H} = \frac{\hbar^2}{2\mathcal{J}_{\text{max}}} I(I+1) + \hbar\omega_{\text{wob}}\left(n_w + \frac{1}{2}\right), n_w = 0, 1, 2, \ldots \quad (1)$$





New Results for A=130

Recent findings for even-even nuclei

- Two wobbling bands have been identified experimentally in ^{130}**Ba** (*Petrache et al., 2019*)
- DFT+PRM description of the wobbling motion described the excited spectra (*Chen et al., 2019*)
- Stable triaxiality for $\beta = 0.24$ and $\gamma = 21.5°$

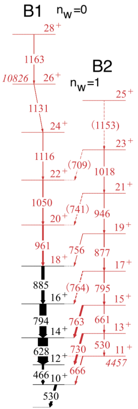


Figure from Petrache et al., 2019

New Results for A=130 II

Harmonic Approximation

- Employed an energy spectrum of harmonic type according to Eq. 1:

$$E_{I,n_w} = \frac{\hbar^2}{2\mathcal{J}_3} I(I+1) + \hbar\omega_{\text{wob}}\left(n_w + \frac{1}{2}\right) \tag{2}$$

- wobbling frequency - linear dependence on I (fixed MOI ordering $\mathcal{J}_3 > \mathcal{J}_{1,2}$)

$$\hbar\omega_{\text{wob}}(I) = 2f(\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3) \cdot I \tag{3}$$

New Results for A=130 III

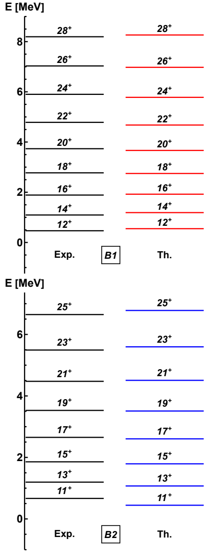
Harmonic Approximation

- Reproduced the excited spectra for $\{B1, B2\}$
- Fix a *free parameter set*: $\mathcal{P} = [\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3]$
- Adopt a fitting procedure:

$$\chi^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)^2}{E_{\text{exp}}^{(i)}} \tag{4}$$

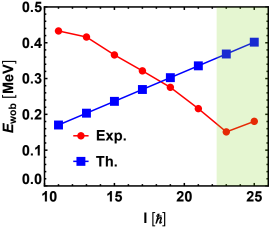
Results for ^{130}Ba **PRELIMINARY!**

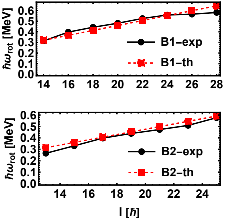
$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$	$[\hbar^2 \text{MeV}^{-1}]$
27	22	**43**	

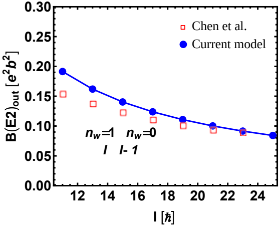


Poenaru, 2022, unpublished

- **Left:** Wobbling energy* $E_{\mathrm{wob}}(I) = E_{\mathrm{B2}}(I) - E_{\mathrm{B1}}(I)$
- **Center:** Rotational frequency $\hbar\omega_{\mathrm{rot}}$ - collective
- **Right:** Interband transition probabilities $I_{\mathrm{B2}} \rightarrow I_{\mathrm{B1}}$







Poenaru, 2022, unpublished

Electromagnetic transitions

Even-A nuclei conclusions

- $\beta = 0.24$ and $\gamma = 21.5^\circ$ were adopted for *quadrupole moments* and *transition probabilities*
- PRM values are calculated by Chen et al through a similar approach of minimizing the χ^2-function
- Main rotation occurs along 3-axis
- Approximation only reproduces E_{wob} in the high-spin limit

I	$B(E2)_{\mathrm{out}}/B(E2)_{\mathrm{in}}$		
	Th.	PRM*	Exp.
11	0.37	-	-
13	0.32	0.51	0.32
15	0.27	0.42	0.36
17	0.24	0.35	0.22
19	0.21	0.29	0.22
21	0.19	0.25	0.41
23	0.18	-	-
25	0.16	-	-

Poenaru, 2022, unpublished

Wobbling Motion in Odd-Mass Nuclei

Particle-Rotor-Model

- The precessional motion of $\mathbf{I} = \mathbf{R} + \mathbf{j}$ is caused by the coupling of an *even-even triaxial core* + *odd-particle*
- System: [even-even core] + [odd quasi-particle moving in a quadrupole deformed mean-field generated by the core]

$$\hat{H} = H_{\text{core}} + H_{\text{sp}} = \sum_{k=1,2,3} \frac{1}{2\mathcal{J}_k}\left(\hat{I}_k - \hat{j}_k\right)^2 + \\ + \epsilon_j + \frac{V}{j(j+1)}\left[\cos\gamma\left(3\hat{j}_3^2 - j^2\right) - \sqrt{3}\sin\gamma\left(\hat{j}_1^2 - \hat{j}_2^2\right)\right] \tag{5}$$

for ^{163}Lu: *R Poenaru, AA Raduta, IJMPE, 2021.*

for ^{167}Pr: *A. A. Raduta and R. Poenaru, Phys Rev C, 2020.*

for ^{135}Pr: *A. A. Raduta and R. Poenaru, Journal of Physics G, 2020.*

Classical energy

Energy spectrum for odd mass nuclei

- Total energy is given in terms of **two wobbling frequencies**: $\Omega_1 \rightarrow$ core and $\Omega_2 \rightarrow$ odd-particle
- The frequencies represent the harmonic-like behavior of the wobbling motion in odd-A nuclei

Energy spectrum

$$E_I = \epsilon_j + \mathcal{H}_{\text{min}}(I) + \hbar\Omega_1^I\left(n_{w_1} + \frac{1}{2}\right) + \hbar\Omega_2^I\left(n_{w_2} + \frac{1}{2}\right) \tag{6}$$

Results for A=163

Wobbling Motion in ^{163}Lu

- Four wobbling bands TSD1,2,3,4
- Stable minimum at $\beta = 0.38$ and $\gamma = 20^\circ$

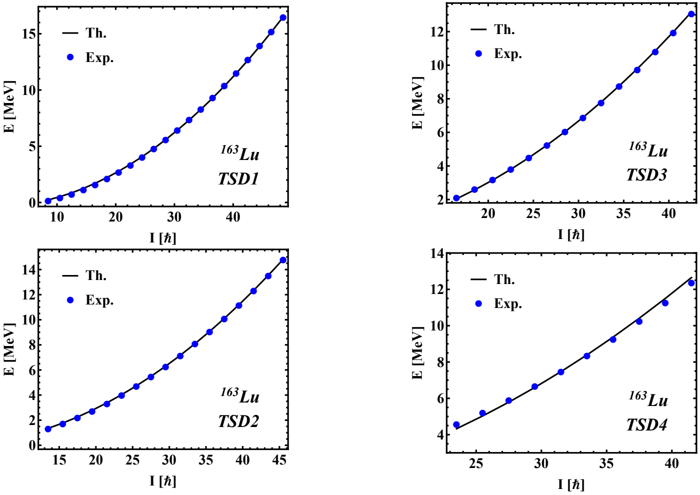
Model parametrization

- 5 free parameters: $\mathcal{P}_{\text{fit}} = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, V, \gamma\}$
- $\gamma^{\text{fit}} = 22^\circ \rightarrow \gamma^{\text{exp}} = 20^\circ$
- $V^{\text{fit}} = 2.1$MeV (literature: $0 < V < 8$ MeV)
- the odd proton $i_{13/2}$ of positive parity couples to the triaxial core and generates TSD1,2,3,4

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$	$[\hbar^2\text{MeV}^{-1}]$
72	15	7	

R Poenaru, AA Raduta, IJMPE, 2021

Results for A=163 II

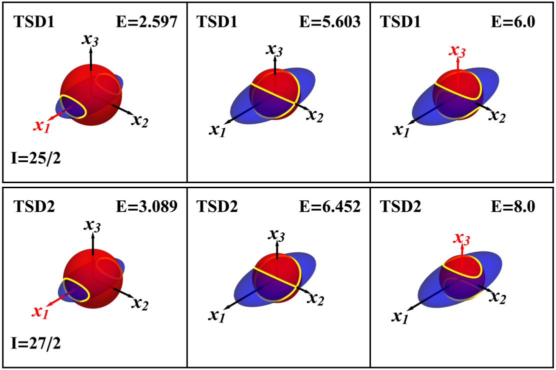


R Poenaru, AA Raduta - Rom. J. of Phys,66 (7-8), 2021

- Energy and angular momentum: constants of motion

$$E = \mathcal{S}_1 x_1^2 + \mathcal{S}_2 x_2^2 + \mathcal{S}_3 x_3^2 + \mathcal{S}_0^{\text{core+sp}} , \quad (7)$$

$$I^2 = x_1^2 + x_2^2 + x_3^2 . \quad (8)$$

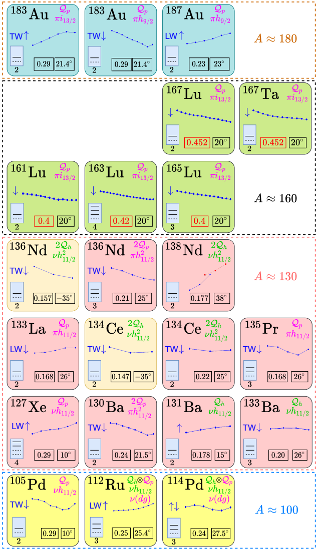


R Poenaru, AA Raduta - Rom. J. of Phys,66 (9-10), 2021

Conclusions

- Energy spectrum for even-even nuclei was obtained using a *harmonic approximation*
- Energy spectrum for odd-A nuclei was obtained using *PRM*
- 1 oscillatory motion for even-A $\longrightarrow$ $\hbar\omega_{\text{wob}}$
- 2 oscillatory motions for odd-A $\longrightarrow$ $\hbar\Omega_{1,2}$
- odd-A: γ obtained self-consistently (agreement with exp.)
- geometrical interpretations: consistent with quantal studies (*Lawrie et al. 2020*)

Chart of wobblers



Wobbling nuclei (up to date)

Poenaru, 2022, in progress