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Dynamical moments of inertia associated with wobbling motion in the triaxial superdeformed nucleus

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Abstract. The three moments of inertia associated with the wobbling mode built on the triaxial superdeformed states in Lu–Hf region are investigated by means of the cranked shell model plus random-phase approximation to the configurations with aligned quasiparticle(s). The result indicates that it is crucial to take into account the direct contribution to the moments of inertia from the aligned quasiparticle(s) so as to realize $\mathcal{J}_x > \mathcal{J}_y$ in positive-γ shapes.

PACS. 21.10.Re Collective levels – 21.60.Jz Hartree-Fock and random-phase approximations

The wobbling motion is a decisive evidence of stable triaxial deformations in rapidly rotating nuclei as discussed by Bohr and Mottelson about thirty years ago. Naming the main rotational axis the x axis, its excitation energy is given by

$$\hbar\omega_{\rm wob} = \hbar\omega_{\rm rot}\sqrt{\frac{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}},$$

where $\omega_{\rm rot}$ is the rotational frequency of the main rotation about the x axis and $\mathcal{J}$s are the three moments of inertia. Although there had been no definite experimental information about it so far, the first firm evidence in the triaxial superdeformed (TSD) states in ^{163}Lu was reported last year[1]. Whereas the expression above requires $\mathcal{J}_x > \mathcal{J}_y$ and $\mathcal{J}_z$, the observed TSD band is thought of as $\gamma > 0$, which leads to $\mathcal{J}_x < \mathcal{J}_y$ in the irrotational moments of inertia,

$$\mathcal{J}_k^{\rm irr} = \frac{4}{3}\mathcal{J}_0 \sin^2(\gamma + \frac{2}{3}\pi k),$$

with $k = 1 – 3$ denoting the x, y and z principal axes, that are believed to describe realistic nuclei well.

Aiming at solving this puzzle, we performed a random-phase approximation (RPA) calculation in the rotating frame[2, 3, 4, 5] to study the dynamical moments of inertia, $\mathcal{J}_y^{(\rm eff)}(\omega_{\rm wob})$ and $\mathcal{J}_z^{(\rm eff)}(\omega_{\rm wob})$, determined simultaneously with the excitation energy $\hbar\omega_{\rm wob}$ by the dispersion equation,

$$(\hbar\omega_{\rm wob})^2 = (\hbar\omega_{\rm rot})^2 \frac{(\mathcal{J}_x - \mathcal{J}_y^{(\rm eff)}(\omega_{\rm wob}))(\mathcal{J}_x - \mathcal{J}_z^{(\rm eff)}(\omega_{\rm wob}))}{\mathcal{J}_y^{(\rm eff)}(\omega_{\rm wob})\mathcal{J}_z^{(\rm eff)}(\omega_{\rm wob})},$$

which is derived from the coupled RPA equation of motion. We have developed a computer code for the RPA to excitation

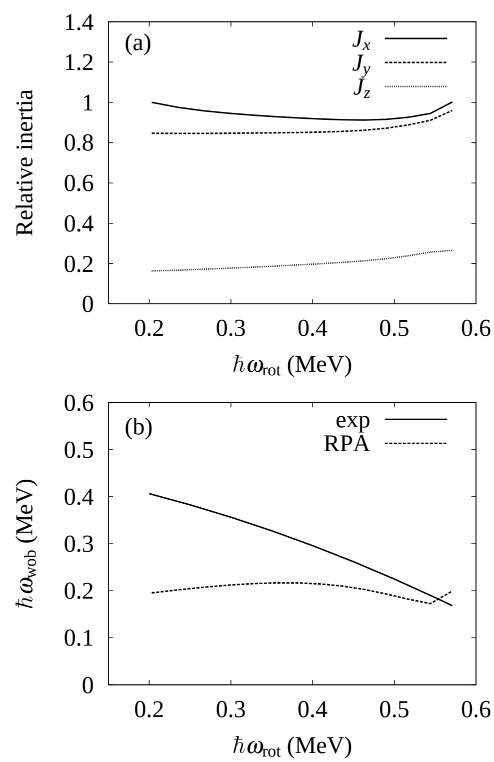

Fig. 1. (a) Calculated moments of inertia, and (b) experimental and calculated excitation energies of the wobbling mode in ^{163}Lu. Note that the proton BC crossing occurs at around $\hbar\omega_{\rm rot} \geq 0.55$ MeV in the calculation. (Taken from Ref.[6].)

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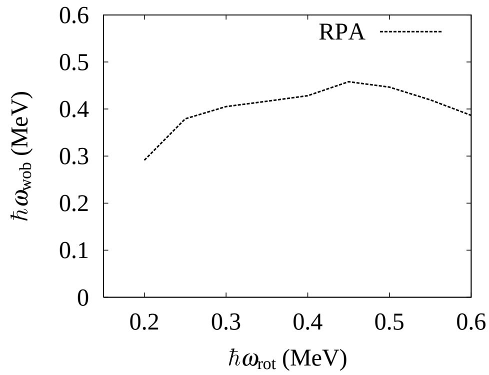

Fig. 2. Calculated excitation energy of the wobbling mode in ^{168}Hf.

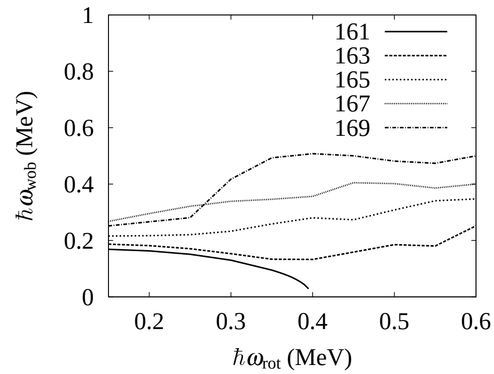

Fig. 3. Calculated excitation energies of the wobbling modes in $^{161-169}$Lu.

modes built on configurations with arbitrary number of aligned quasiparticles (QPs). In this talk, we first study the dynamical moments of inertia, associated with the wobbling motion, of the whole system including the odd $\pi i_{13/2}$ quasiparticle. We did not perform full selfconsistent calculations for given force strengths, but fixed them in such a way to guarantee the decoupling of the Nambu-Goldstone modes for a given mean field. Thus there is no ambiguity about the choice of force strengths for the RPA calculations. Our calculation[6] adopting $\epsilon_2 =$ 0.43 and $\gamma = 20^\circ$ [1] proved that the contribution from the alignment of the intruder $\pi i_{13/2}$ quasiparticle, $\varDelta\mathcal{J}_x = i_{\rm QP}/\omega_{\rm rot}$, superimposed on an irrotational-like inertia of the 0QP part, made the total $\mathcal{J}_x$ larger than $\mathcal{J}_y$ as shown in Fig.1(a).

Another virtue of the present framework is that the $\omega_{\rm rot}$ dependence of $\mathcal{J}$s is introduced automatically even when the mean field parameters are fixed. In particular, the decrease of $\mathcal{J}_x - \mathcal{J}_y$ due to a near constancy of the aligned angular momentum of the intruder $\pi i_{13/2}$ quasiparticle, $i_{\rm QP}$, leads to the decrease of $\omega_{\rm wob}$ as in Fig.1(b), whereas $\omega_{\rm rot}$-independent $\mathcal{J}$s lead to $\omega_{\rm wob} \propto \omega_{\rm rot}$.

Next, in order to demonstrate that this mechanism applies also to 2QP states in even-even nuclei, we present the result for ^{168}Hf although the character of the excited TSD bands has not been clarified experimentally[8]. Figure 2 shows the result. Thanks to one more aligned QP in comparison with the odd-A Lu, the calculated $\omega_{\rm wob}$ is larger. In addition, the $(\nu j_{15/2})^2$ alignment occurs at around $\hbar\omega_{\rm rot}$ = 0.45 MeV, this makes $\omega_{\rm wob}$ even larger.

Finally, we briefly mention the N-dependence since some new experimental information was presented in this conference. A preliminary calculation indicates that the $(\nu j_{15/2})^2$ aligns at lower $\omega_{\rm rot}$ in heavier isotopes, and consequently $\omega_{\rm wob}$ becomes larger in heavier isotopes as shown in Fig.3. Here, Nilsson model parameters slightly different from those in Fig.1 are used. Details will be discussed in Ref.[9].

To summarize, we have performed the RPA calculation in the rotating frame to the triaxial superdeformed odd-A nucleus ^{163}Lu, in which the wobbling motion was observed for the first time, and discussed the physical conditions for its appearance. We have confirmed that the proton $i_{13/2}$ alignment that makes $\mathcal{J}_x > \mathcal{J}_y^{\rm (eff)}$ is indispensable for the appearance of the wobbling mode in this nucleus with a positive-γ shape. This mechanism has been shown to apply also to even-even nuclei. A qualitative prediction for the N-dependence in odd-A Lu isotopes has been briefly mentioned.

[1] This shape lead to transition quadrupole moments $Q_{\rm t} = 10.9$ – 11.3 eb for $\hbar\omega_{\rm rot} = 0.20$ – 0.57 MeV in accordance with the data, $Q_{\rm t} = 10.7 \pm 0.7$ eb [7]. But it was reported by P. Fallon in this conference that a new measurement indicates that $Q_{\rm t}$ may be somewhat smaller.

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