## Wobbling motion in triaxial superdeformed nuclei

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**Abstract.** We discuss some characteristic features of the wobbling motion excited on the triaxial superdeformed Lu nucleus. We show how these features are connected to the moments of inertia microscopically calculated by means of the quasiparticle RPA in the rotating frame.

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The concept of independent single particle motions in mean field potentials serves as a starting point of nuclear structure theories. Single particle energy levels vary as functions of various parameters of the mean field potential; bunch, degenerate, and form shell gaps. This is called shell structure. Shell structure is the clue to various nuclear phenomena. One of the most famous examples is the occurrence of the superdeformation with the axis ratio 2:1 observed in the  $A\sim150$  mass region. In this case we consider single particle energy levels in an axially symmetrically deformed potential, for example the anisotropic harmonic oscillator one. These levels strongly degenerate at some deformations that correspond to simple integer axis ratios. Consequently shell gaps are formed. This occurs also in more realistic potentials. Since it is difficult for nucleons to excite across the gap, shell-closed configurations become stable.

A similar situation can be thought of in triaxially  $(Y_{22})$  deformed potential when varying the triaxial parameter  $\gamma$  from  $0^{\circ}$  to  $60^{\circ}$ . Actually, a realistic energy surface calculation [1] predicts the existence of stable configurations with  $\varepsilon_2 \sim 0.4$  and  $\gamma \sim 20^{\circ}$ , where  $\varepsilon_2$  stands for a parametrization of the  $Y_{20}$  deformation. When rotation sets in, the triaxial parameter requires three times larger range  $(-120^{\circ} \leq \gamma \leq +60^{\circ})$  in the so-called Lund convention) according to the relation between the axis of rotation and that of deformation. Therefore  $\gamma = +20^{\circ}$  and  $-20^{\circ}$  in rotating systems represent different physical situations. Actually, according to Ref. [1], the energy minimum at  $\gamma \simeq +20^{\circ}$  is stabler than that at  $\gamma \simeq -20^{\circ}$ .

The signal of triaxial deformation has long been sought for but the result has been ambiguous. From a theoretical viewpoint, however, Bohr and Mottelson predicted the existence of the nuclear wobbling motion in rapidly rotating triaxially deformed systems [2]. This is a quantum analog of the one that has been known in classical mechanics [3]. A candidate of the configuration with  $\varepsilon_2 \sim 0.4$  and  $|\gamma| \sim 20^\circ$ , which is called the triaxial superdeformation (TSD), has been known in a Lu isotope for years [4]. In 2001 an excited TSD band was reported for the first time in  $^{163}$ Lu [5]. In this work, ex-

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tremely strong interband electric quadrupole transitions with about a hundred Weisskopf units were measured and therefore this was thought of as a clear evidence of a collective wobbling motion and consequently of triaxial deformation.

A characteristic feature of the wobbling motion is its excitation energy given by

$$\hbar\omega_{\text{wob}} = \hbar\omega_{\text{rot}}\sqrt{\frac{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}},$$
(1)

when we name the axis of the main rotation the x axis. Here  $\omega_{\rm rot}$  is the rotational frequency of the main rotation and  $\mathcal{J}$ s are moments of inertia. In order for  $\omega_{\rm wob}$  to be real, the order of  $\mathcal{J}$ s is constrained; for example  $\mathcal{J}_x > \mathcal{J}_y$ ,  $\mathcal{J}_z$  or  $\mathcal{J}_x < \mathcal{J}_y$ ,  $\mathcal{J}_z$ . In the case of the rotor model,  $\mathcal{J}$ s are given by hand. As the input, a few models of nuclear moments of inertia are known. Among them, aside from its magnitude, the  $\gamma$  dependence in the irrotational model,

$$\mathcal{J}_k^{\text{irr}} = 4B\beta^2 \sin^2(\gamma + \frac{2}{3}\pi k),\tag{2}$$

with k=1-3 denoting the x-z principal axes, B the irrotational mass parameter, and  $\beta$  a deformation parameter similar to  $\varepsilon_2$ , is believed to be appropriate for the collective motion. Note that the overall magnitude is not relevant to Eq. (1). Its  $\gamma$  dependence is shown in Fig. 1. This figure suggests choosing  $\gamma \simeq -20^\circ$  out of  $|\gamma| \simeq 20^\circ$  because of  $\mathcal{J}_x > \mathcal{J}_y > \mathcal{J}_z$ , whereas the potential energy surface calculation mentioned above suggests  $\gamma \simeq +20^\circ$ . In order to solve this puzzle, we need a framework that determines  $\mathcal{J}$  s microscopically.

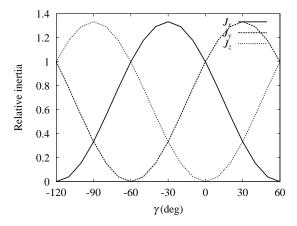


FIGURE 1. Irrotational model moments of inertia.

We adopt the cranking model plus random phase approximation (RPA) in the manner of Marshalek [6]. The one-body Hamiltonian is given by

$$h' = h - \hbar \omega_{\rm rot} J_x \,, \tag{3}$$

here h denotes the Nilsson plus BCS Hamiltonian. We perform the RPA to the residual pairing plus quadrupole interaction. According to signature ( $\pi$  rotation about the x axis)

symmetry, only the  $\alpha = 1$   $(r = \exp[-i\pi\alpha] = -1)$  sector of the interaction,

$$H_{\text{int}}^{(-)} = -\frac{1}{2} \sum_{K=1,2} \kappa_K^{(-)} Q_K^{(-)\dagger} Q_K^{(-)}, \qquad (4)$$

is relevant for the description of the wobbling motion. The equation of motion,

$$\left[h' + H_{\text{int}}^{(-)}, X_n^{\dagger}\right]_{\text{RPA}} = \hbar \omega_n X_n^{\dagger} , \qquad (5)$$

0.4

0.5

0.6

for the n-th eigenmode  $X_n^{\dagger}$  leads to a pair of coupled equations for the transition amplitudes. Then, by assuming  $\gamma \neq 0$ , this can be cast into the form [6],

$$\hbar\omega_{\text{wob}} = \hbar\omega_{\text{rot}}\sqrt{\frac{\left(\mathcal{J}_{x} - \mathcal{J}_{y}^{(\text{eff})}(\omega_{\text{wob}})\right)\left(\mathcal{J}_{x} - \mathcal{J}_{z}^{(\text{eff})}(\omega_{\text{wob}})\right)}{\mathcal{J}_{y}^{(\text{eff})}(\omega_{\text{wob}})\mathcal{J}_{z}^{(\text{eff})}(\omega_{\text{wob}})}},$$
(6)

for n = wob. See Ref. [7] for details. The form of Eq. (6) is evidently parallel to Eq. (1) but here  $\mathcal{J}_{y,z}^{(\mathrm{eff})}(\omega_{\mathrm{wob}})$  are dynamical ones that are determined simultaneously with  $\omega_{\mathrm{wob}}$ . In this sense Eq. (6) is a highly nonlinear equation and it is not trivial whether a collective wobbling solution is obtained from it or not.

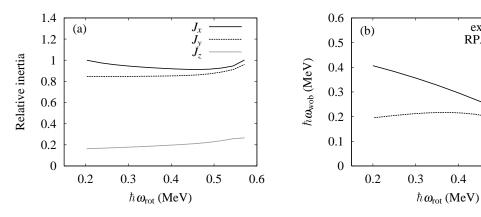
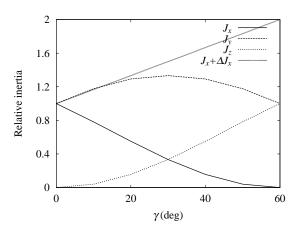


FIGURE 2. (a) Calculated moments of inertia, and (b) experimental and calculated excitation energies of the wobbling mode in  $^{163}$ Lu. Note that the proton BC crossing occurs at around  $\hbar\omega_{\rm rot} \geq 0.55$  MeV in the calculation. Data are taken from Refs. [5, 8]. (Taken from Ref. [9].)

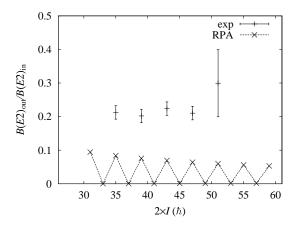
We obtained an extremely collective solution for  $^{163}Lu$  by adopting  $\varepsilon_2=0.43,\ \gamma=0.43$  $+20^{\circ}$ , and  $\Delta_n = \Delta_p = 0.3$  MeV. The result is shown in Fig. 2. Figure 2(a) graphs the moments of inertia. This figure indicates  $\mathcal{J}_x > \mathcal{J}_y > \mathcal{J}_z$ . Then what is the relation to the irrotational  $\gamma$  dependence that is believed to be realistic? The key is that  $^{163}$ Lu can be regarded as the system consisting of the collective rotor and one quasiparticle. Contrastively to the case of the particle rotor model in which only the rotor is responsible for the moment of inertia, in the present model the last odd quasiparticle also bears inertia. Thus, the calculated moments of inertia in Fig. 2(a) can be interpreted as a superposition of an irrotational-like one  $(\mathcal{J}_x < \mathcal{J}_y)$  of the rotor and an additional



**FIGURE 3.** Schematic drawing of the alignment contribution from the last odd quasiparticle to the moment of inertia.

alignment contribution (mainly to  $\mathcal{J}_x$ ) from the last odd quasiparticle as schematically depicted in Fig. 3.

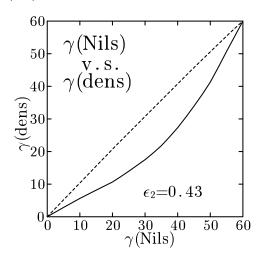
Next we discuss the  $\omega_{\rm rot}$  dependence of  $\omega_{\rm wob}$  presented in Fig. 2(b). When the moments of inertia are independent of  $\omega_{\rm rot}$ ,  $\omega_{\rm wob}$  is proportional to  $\omega_{\rm rot}$ . This in turn indicates that the actual moments of inertia depend on  $\omega_{\rm rot}$ . Figure 2(a) shows that the calculated  $\mathcal{J}$ s do depend on  $\omega_{\rm rot}$ . Seemingly their dependence is weak, the decrease of  $\mathcal{J}_x - \mathcal{J}_y^{(\rm eff)}$  makes  $\omega_{\rm wob}$  a flat or decreasing function.



**FIGURE 4.** Interband E2 transition rates for I (wobbling)  $\to I \pm 1$  (yrast) transitions as functions of  $2 \times \text{spin } I$ . They are divided by the in-band  $E2(I \to I - 2)$  transition rates. Experimental values were taken from Ref. [8]. Noting that the states I+1 (yrast) are slightly higher in energy than I (wobbling) at I > 51/2 and  $B(T_{\lambda}; I \to I + 1) \simeq B(T_{\lambda}; I + 1 \to I)$  at high spins, we plotted those for  $I \to I + 1$  at the places with the abscissae I+1 in order to show clearly their characteristic staggering behavior. (Taken from Ref.[9].)

Now we come to the  $B(E2)_{\rm out}/B(E2)_{\rm in}$  ratio. Here  $B(E2)_{\rm out}$  means the reduced transition rate of the interband electric quadrupole transition between the wobbling and the yrast TSD bands, while  $B(E2)_{\rm in}$  means that of the in-band one. Therefore this ratio measures the collectivity of the wobbling excitation. Figure 4 compares the experimental and the theoretical ratios. Although in this calculation the calculated ones are factor 2 –

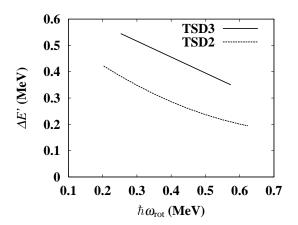
3 smaller, recently we found its reason. The origin of the discrepancy is the difference of the physical meaning of the triaxial parameter  $\gamma$ . We adopted in the above calculation  $\gamma = +20^{\circ}$  of the Nilsson potential but we found that the discrepancy is resolved if  $\gamma = +20^{\circ}$  of the density distribution is adopted because their relation is not a diagonal straight line (Fig. 5). That is,  $\gamma(\text{dens}) = +20^{\circ}$  corresponds to  $\gamma(\text{Nils}) \simeq +30^{\circ}$ , and larger  $\gamma(\text{Nils})$  leads to larger  $\gamma(\text{Nils})$ 



**FIGURE 5.** Relation between  $\gamma$  of the Nilsson potential and that of the density distribution of the nucleus (solid curve).

Finally we mention the anharmonicity in the observed wobbling spectrum. In Ref. [11] the two phonon wobbling excitation was reported. The data exhibits strong anharmonicity as shown in Fig. 6. This might indicate softness of the potential energy surface. As a numerical experiment we examined a calculation for  $^{162}$ Yb, which is the system with the last odd quasiparticle in  $^{163}$ Lu removed. In this nucleus we did not obtain a wobbling solution. This result is quite natural because this nucleus does not have the last odd quasiparticle that produces the additional contribution to  $\mathcal{J}_x$  and that consequently leads to the existence of the wobbling motion by making  $\mathcal{J}_x - \mathcal{J}_y^{\text{(eff)}} > 0$ . Actually we confirmed that the angular momentum vector is tilted in this nucleus following the instability of the wobbling motion [12].

To summarize, we have discussed some characteristics of the one phonon and the two phonon wobbling excitations in the triaxial superdeformed nucleus,  $^{163}$ Lu, which are determined by the behavior of the moments of inertia. First we have shown that the wobbling motion in positive  $\gamma$  nuclei emerges thanks to the alignment contribution to the moment of inertia superimposed on the collective contribution. Second we have discussed that the decreasing behavior of the observed excitation energy of the one phonon wobbling is brought about by the rotational frequency dependence of the dynamical moments of inertia. Possible  $\omega_{\rm rot}$  dependence of  $\gamma$  would also affect  $\omega_{\rm wob}$ . Thirdly we have pointed out the importance of self-consistency between density and potential in determining an appropriate value of  $\gamma$  that determines the transition strength. Finally we have discussed a possible "phase transition" to the tilted axis rotation regime, associated with the instability with respect to the wobbling degree of freedom.



**FIGURE 6.** Experimental excitation energies of the two phonon (TSD3) and the one phonon (TSD2) wobbling states relative to the yrast triaxial superdeformed (TSD1) states in <sup>163</sup>Lu. Data are taken from Ref. [11]. (Taken from Ref. [12].)

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