

ROTATIONAL STATES IN EVEN ATOMIC NUCLEI

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Abstract: A theory of the energy states and the electromagnetic transitions between them is developed for nuclei which do not possess axial symmetry. It is shown that violation of axial symmetry does not significantly change the rotational states of axial nuclei and leads to the appearance of new energy states. The reduced probabilities for E2 and M1 transitions between various rotational states are computed.

1. Introduction

In some previous papers of the authors ¹⁻⁴) energy levels of non-spherical nuclei corresponding to collective excitations not involving violation of axial symmetry of the nuclei were investigated on basis of the generalized nuclear model proposed by A. Bohr and B. Mottelson ^{5,6}). It was shown that the rotational-vibrational energy of collective nuclear excited states is a function of only two parameters, viz., of the frequency of nuclear surface vibrations and of the ratio of the equilibrium deformation to the zero vibration amplitude. It seems natural to inquire to what extent these results are applicable if one takes into account possible violation of axial symmetry of the nucleus.

The problem of violation of axial symmetry of nuclei has been qualitatively treated in a number of papers ^{5,7,8}). Recently it has even become usual to ascribe nuclear excited states to the so-called γ -vibrations ^{9,10}). This type of assignment is usually based on the spin values of these levels and on the large probability of electromagnetic transitions which confirms the collective nature of the levels. No quantitative theory of γ -vibrations has been proposed.

In the present paper we investigate energy levels corresponding to rotation of the nucleus which does not entail changes of its internal state. It will be shown that violation of axial symmetry of even nuclei only slightly affects the rotational spectrum of the axial nucleus although some new rotational states with total angular momenta of 2, 3, 4... do appear. If the deviations from axial symmetry are small these levels lie very high and are not excited. As the deviations from axial symmetry are increased some of the additional levels become much lower. Thus, for example, the

ratio of the second excited state with spin 2 to the first state (which also exists in axial nuclei), varies from infinity to two. The probabilities for electric transitions between rotational levels in non-axial nuclei are evaluated in sec. 3. The probabilities for magnetic transitions are computed in sec. 4. From comparison of the proposed theory with the experimental data (sec. 5) it can be concluded that the properties of the experimentally observed energy states of even nuclei can be satisfactorily explained by assuming that these nuclei do not possess axial symmetry.

2. Energy States in Non-Axial Nuclei

Proceeding from the generalized nuclear model, consider the energy states of an even nucleus corresponding to rotation of the latter as a whole with no change of its internal state. The operator corresponding to the rotation energy of the nucleus has the form

$$H = \sum_{\lambda=1}^3 \frac{A J_{\lambda}^2}{2 \sin^2 \left(\gamma - \frac{2\pi}{3} \lambda \right)}, \quad (1.1)$$

where $A = \hbar^2 / 4B\beta^2$ is a quantity which has the dimension of energy; γ varies between 0 and $\pi/3$ and determines the deviation of the shape of the nucleus from axial symmetry; the J_{λ} are operators of the projections of the nuclear angular momenta on the axes of a coordinate system connected with the nucleus. The commutation rules for these projections differ from the rules for the projections in a space-fixed coordinate system by the signs in the right hand side.

According to (1.1), for $\gamma \neq 0$ or $\pi/3$ the nucleus should be regarded as an asymmetric top. In stationary states of the asymmetric top not one of the projections of the total angular momentum on axes 1, 2, 3 of the body-fixed coordinate system has a definite value and hence the energy levels cannot be specified by the values of $K = J_3$. Each value of the total angular momentum in the asymmetric top corresponds to $2J+1$ different energy levels. These levels can be classified with respect to the irreducible representations of group D_2 with symmetry elements C_2^1, C_2^2, C_2^3 corresponding to rotation through π about the coordinate axes 1, 2, 3, because operator (1.1) and the commutation between the J_{λ} are invariant with respect to this transformation group. Thus the energy levels of an asymmetric top split up into four types of levels which correspond to the four irreducible representations of group D_2 (see ref.¹¹), § 101 and ref.¹²).

In virtue of the symmetry conditions imposed on the wave function⁵), in even nuclei of the $2J+1$ different levels only those energy levels with a given J can exist which correspond to a completely symmetric

representation of group D_2 . Rotation states of the required symmetry will not exist if $J = 1$. Two such states will exist for $J = 2$, one for $J = 3$, three for $J = 4$, two for $J = 5$, four for $J = 6$, etc.

If the energy is expressed in units of $A = \hbar^2/4B\beta^2$ the energy of two levels of the required symmetry are, for $J = 2$, defined by the expression

$$\varepsilon_1(2) = \frac{9(1 - \sqrt{1 - \frac{8}{9} \sin^2(3\gamma)})}{\sin^2(3\gamma)}, \quad \varepsilon_2(2) = \frac{9(1 + \sqrt{1 - \frac{8}{9} \sin^2(3\gamma)})}{\sin^2(3\gamma)}. \quad (1.2)$$

The energy of a level with angular momentum $J = 3$ is given by

$$\varepsilon(3) = \frac{18}{\sin^2(3\gamma)}. \quad (1.3)$$

The three spin 4 energy levels are the roots of the third degree equation

$$\varepsilon^3 - \frac{90}{\sin^2(3\gamma)} \varepsilon^2 + \frac{48}{\sin^4(3\gamma)} [27 + 26 \sin^2(3\gamma)] \varepsilon - \frac{640}{\sin^4(3\gamma)} [27 + 7 \sin^2(3\gamma)] = 0.$$

The two spin 5 energy levels are given by the formula

$$\varepsilon_\tau(5) = \frac{45 \pm 9\sqrt{9 - 8 \sin^2(3\gamma)}}{\sin^2(3\gamma)}. \quad (1.4)$$

In (1.4) $\tau = 1$, if a minus sign is before the root and $\tau = 2$, if a plus is before it. The energy levels of states possessing an angular momentum equal to 6 are defined by a fourth degree equation which we shall not present here.

The following simple relation between the spin 2 and spin 3 energy levels follows from (1.2) and (1.3)

$$\varepsilon_1(2) + \varepsilon_2(2) = \varepsilon(3). \quad (1.5)$$

The energy levels of even nuclei computed on basis of the preceding formulas are plotted in fig. 1 as a function of γ . For $\gamma = 0$ the energy spectrum is identical to that of an axially-symmetric nucleus. For a fixed value of β violation of axial symmetry of the nucleus leads to an increase of the energy of the levels belonging to the axial nucleus. This increase of the level energy corresponds to a decrease of the effective moment of inertia of the nucleus or of the effective deformation parameter β_{eff} . For the first excited state of spin 2 the effective deformation parameter can be determined from the formula

$$\beta_{\text{eff}} = \beta \left[\frac{4 \sin^2(3\gamma)}{9 - \sqrt{81 - 72 \sin^2(3\gamma)}} \right]^{\frac{1}{2}}.$$

Besides the comparatively small change of the level energies of an axially-symmetric nucleus, violation of axial symmetry of the nucleus leads to the appearance of some new energy levels $\varepsilon_2(2)$, $\varepsilon(3)$, $\varepsilon_2(4)$ etc. By using the

dependence of $\epsilon_2(2)/\epsilon_1(2)$ on γ , one can determine the corresponding value of γ from the experimental value of the ratio. From fig. 1 the energy sequence of the spins and the energy values of the other nuclear levels can then be determined. Comparison of the theory with the experimental results will be performed in sec. 5.

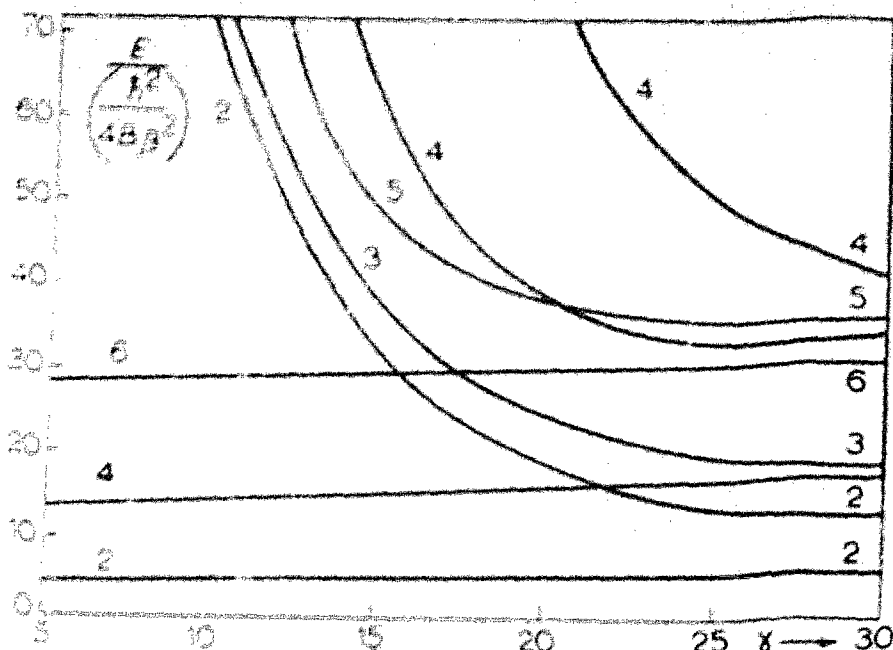


Fig. 1

3. Electric Transitions between Rotational Nuclear States

As is well known, measurements of the transition probabilities between electric states in a nucleus yield important information on the nature of the excited states. In particular, for elucidation of the nature of the second excited state of spin 2 in an even nucleus one may study the relative probability for transition from this level directly to the ground state or to the first excited state with a spin of 2. It has been assumed in a number of investigations that the first two spin 2 levels observed experimentally refer to one-phonon and two-phonon vibrations of the nuclear surface. In this case transition from the second state to the ground state can take place only as a result of violation of the oscillator approximation.

Assuming that both spin 2 levels can be assigned to rotational levels, we compute the ratio of the reduced transition probabilities for $\epsilon_2(2) \rightarrow \epsilon_1(2)$ and $\epsilon_2(2) \rightarrow \epsilon(0)$ as a function of the parameter γ and hence as a function of the ratio $\epsilon_2(2)/\epsilon_1(2)$ since the latter depends on γ .

In order to calculate the probability for E2 type transitions between the rotational levels the operator of the nuclear quadrupole moment

$$Q_{2\mu} = 2e \sqrt{\frac{4\pi}{5}} \sum_{i=1}^Z r_i^2 Y_{2\mu}(\vartheta_i, \varphi_i)$$

should be expressed in terms of the Eulerian angles defining the orientation of the nucleus and in terms of the collective coordinates with respect to axes connected to the nucleus. Employing the transformation of the spherical functions

$$Y_{2\mu}(\vartheta, \varphi) = \sum_{\nu} D_{\mu\nu}^2 Y_{2\nu}(\vartheta', \varphi')$$

corresponding to transition to a coordinate system connected with the nucleus and expressing the proton coordinates $r_i, \vartheta_i, \varphi_i$ in this system (assuming the protons to be uniformly distributed within the nucleus) through the collective coordinates a_ν where $a_0 = \beta \cos \gamma$, $a_1 = a_{-1} = 0$, $a_2 = a_{-2} = \beta \sin \gamma / \sqrt{2}$ by aid of the formula

$$a_\nu = \frac{4\pi}{3Z} \sum_{i=1}^Z \left(\frac{r_i}{R} \right)^2 Y_{2\nu}(\vartheta'_i, \varphi'_i)$$

we obtain the following expression for the μ^{th} component of the operator of the electric quadrupole moment

$$Q_{2\mu} = eQ_0 \left(D_{\mu 0}^2 \cos \gamma + \frac{D_{\mu 2}^2 + D_{\mu, -2}^2}{\sqrt{2}} \sin \gamma \right), \quad (2.1)$$

where

$$Q_0 = \frac{3ZR^2\beta}{\sqrt{5\pi}} \quad (2.2)$$

is the intrinsic electric quadrupole moment of an axial nucleus with a deformation parameter β ; $D_{\mu\nu}^2$ are generalized spherical functions (which depend on the Eulerian angles) defining the unitary transformation from a coordinate system fixed in space to a coordinate system fixed to the nucleus.

The wave functions of the rotational states of a non-axial nucleus can be written in the form

$$\begin{aligned} \psi_0 &= \frac{1}{\sqrt{8\pi^2}} \varphi(\beta, \gamma), \\ \psi_{21m} &= \left(\frac{5}{8\pi^2} \right)^{\frac{1}{2}} \varphi(\beta, \gamma) \left[a_1 D_{m0}^2 + b_1 \frac{D_{m2}^2 + D_{m, -2}^2}{\sqrt{2}} \right], \\ \psi_{22M} &= \left(\frac{5}{8\pi^2} \right)^{\frac{1}{2}} \varphi(\beta, \gamma) \left[a_2 D_{M0}^2 + b_2 \frac{D_{M2}^2 + D_{M, -2}^2}{\sqrt{2}} \right], \end{aligned} \quad (2.3)$$

where $\varphi(\beta, \gamma)$ is a function corresponding to the internal state of a nucleus which is assumed to be the same in all three rotational states.

$$\begin{aligned}
a_1 N_1 &= -[\sin \gamma \sin(3\gamma) + 3 \cos \gamma \cos(3\gamma) + \sqrt{9-8 \sin^2(3\gamma)}], \\
b_1 N_1 &= 3 \sin \gamma \cos(3\gamma) - \cos \gamma \sin(3\gamma), \\
N_1^2 &= 2\sqrt{9-8 \sin^2(3\gamma)}[\sqrt{9-8 \sin^2(3\gamma)} + \sin \gamma \sin(3\gamma) + 3 \cos \gamma \cos(3\gamma)], \\
a_2 N_2 &= \sqrt{9-8 \sin^2(3\gamma)} - \sin \gamma \sin(3\gamma) - 3 \cos \gamma \cos(3\gamma), \\
b_2 N_2 &= 3 \sin \gamma \cos(3\gamma) - \cos \gamma \sin(3\gamma), \\
N_2^2 &= 2\sqrt{9-8 \sin^2(3\gamma)}[\sqrt{9-8 \sin^2(3\gamma)} - \sin \gamma \sin(3\gamma) - 3 \cos \gamma \cos(3\gamma)].
\end{aligned} \tag{2.4}$$

As was previously noted, γ varies between 0 and $\pi/3$ and determines the deviation of the shape of the nucleus from axial symmetry. The axes of the ellipsoid employed to approximate the shape of the nucleus can be expressed through γ and β by the formula

$$R_\lambda = R \left[1 + \beta \sqrt{\frac{5}{4\pi}} \cos \left(\gamma - \frac{2\pi}{3} \lambda \right) \right], \quad \lambda = 1, 2, 3.$$

If $\gamma = 0$ the nucleus is an elongated ellipsoid of revolution with a symmetry 3 axis. If $\gamma = \pi/3$ the nucleus is an oblate ellipsoid of revolution with a symmetry 2 axis. The rotational states of a nucleus defined by operator (1.1) and the probabilities for electromagnetic transitions between them are the same for γ_1 and $\pi/3 - \gamma_1$. We therefore present the values of the various quantities in the interval between 0 and $\pi/6$.

In connection with the preceding it should be mentioned that one cannot decide on basis of measurements of the rotational energy of the nuclei or of the electromagnetic transitions between them whether the nucleus is an elongated or oblate ellipsoid. The only way to answer this question is to measure the mean values of the quadrupole electric moments in stationary states ($J, M = J$). In even nuclei the mean values of the quadrupole moments in the ground state are equal to zero. In the first excited state with spin 2 the mean value of the quadrupole moment is

$$Q_1 = -Q_0 \frac{6 \cos(3\gamma)}{7\sqrt{9-8 \sin^2(3\gamma)}},$$

where Q_0 is defined in (2.2). In the second excited state of spin 2 the mean value of the quadrupole moment has a different sign

$$Q_2 = -Q_1.$$

The reduced probability for the electric quadrupole transition $J\tau \rightarrow J'\tau'$ averaged over the initial polarization states of the nucleus is

$$B(E2; J\tau \rightarrow J'\tau') = \frac{5}{16\pi(2J+1)} \sum_{Mm\mu} |(J'\tau' m | Q_{2\mu} | J\tau M)|^2. \tag{2.5}$$

Since we assume that in this transition the internal state of the nucleus does not change, the reduced transition probability can be expressed through the

mean value of β and γ in the $\varphi(\beta, \gamma)$ state with help of (2.5). Inserting (2.3) into (2.5) and using (2.4) we find the following values for the reduced electric quadrupole transition probabilities expressed in $e^2 Q_0^2 / 16\pi$ units. This unit corresponds to the reduced electric quadrupole transition probability in an axially symmetric nucleus between rotational levels of spin 2 and 0.

$$b(E2; 21 \rightarrow 0) = \frac{B(E2; 21 \rightarrow 0)}{\left(\frac{e^2 Q_0^2}{16\pi}\right)} = \frac{1}{2} \left[1 + \frac{3 - 2 \sin^2(3\gamma)}{\sqrt{9 - 8 \sin^2(3\gamma)}} \right], \quad (2.6)$$

$$b(E2; 22 \rightarrow 0) = \frac{1}{2} \left[1 - \frac{3 - 2 \sin^2(3\gamma)}{\sqrt{9 - 8 \sin^2(3\gamma)}} \right], \quad (2.7)$$

$$b(E2; 22 \rightarrow 21) = \frac{1}{7} \frac{\sin^2(3\gamma)}{9 - 8 \sin^2(3\gamma)}. \quad (2.8)$$

It is interesting to note that

$$b(E2; 21 \rightarrow 0) + b(E2; 22 \rightarrow 0) = 1.$$

The ratio $\varepsilon_2(2)/\varepsilon_1(1)$, the relative reduced transition probabilities (2.6) and (2.8) and the ratio of the reduced electric quadrupole transition probabilities $b(E2; 22 \rightarrow 21)/b(E2; 22 \rightarrow 0)$ are listed in table 1 for several

TABLE 1

γ	$\frac{\varepsilon_2(2)}{\varepsilon_1(2)}$	$b(E2; 21 \rightarrow 0)$	$b(E2; 22 \rightarrow 0)$	$b(E2; 22 \rightarrow 21)$	$\frac{b(E2; 22 \rightarrow 21)}{b(E2; 22 \rightarrow 0)}$
0	∞	1.000	0	0	1.43
5	64.2	0.993	0.0074	0.011	1.49
10	15.9	0.972	0.028	0.051	1.79
15	6.85	0.947	0.053	0.143	2.70
20	3.73	0.933	0.067	0.357	5.35
22.5	2.93	0.937	0.0625	0.563	19.02
24	2.59	0.948	0.052	0.782	15.1
25	2.41	0.955	0.0425	0.865	20.6
26	2.26	0.968	0.0324	1.01	31.2
28	2.07	0.99	0.010	1.28	126
29	2.01	0.996	0.004	1.41	363
30	2.00	1.000	0	1.43	∞

values of γ . From the data in table 1 it follows that the reduced probability (2.6) for transition from the first excited state to the ground state only slightly changes when axial symmetry of the nucleus is violated. The reduced probability (2.7) for transition from the second excited state of spin 2 directly to the ground state vanishes for $\gamma = 0^\circ$ or 30° and comprises about 5—7 % of the corresponding reduced transition probability between the first excited and ground energy states when $\gamma = 15—24^\circ$. The reduced E2 transition probability from the second excited spin 2 level to the first

excited level is very small for $\gamma \approx 0$ and subsequently rapidly increases and for $\gamma \approx 20^\circ$ equals approximately 40 % of the reduced ground state transition probability in the axial nucleus and about 140 % for $\gamma \approx 30^\circ$. Of special interest is the ratio of the reduced probabilities, $b(E2; 22 \rightarrow 21)/b(E2; 22 \rightarrow 0)$ as this quantity does not depend on the degree of population of level $\epsilon_2(2)$ and can be directly measured.

With help of the explicit form of the wave function for energy level $\epsilon(3)$

$$\psi_{3m} = \sqrt{\frac{7}{16\pi^2}} \varphi(\beta\gamma) [D_{22}^2 - D_{2-2}^2],$$

the reduced electric quadrupole transition probabilities can be computed (in our units)

$$b(E2; 3 \rightarrow 22) = \frac{25}{18} \left(1 + \frac{3 - 2 \sin^2(3\gamma)}{\sqrt{9 - 8 \sin^2(3\gamma)}} \right);$$

$$b(E2; 3 \rightarrow 21) = \frac{25}{18} \left(1 - \frac{3 - 2 \sin^2(3\gamma)}{\sqrt{9 - 8 \sin^2(3\gamma)}} \right).$$

The values of these probabilities and their ratio are presented in table 2. These reduced probabilities satisfy the relation

$$b(E2; 3 \rightarrow 22) + b(E2; 3 \rightarrow 21) = \frac{25}{14}.$$

TABLE 2

γ	$b(E2; 3 \rightarrow 21)$	$b(E2; 3 \rightarrow 22)$	$\frac{b(E2; 3 \rightarrow 21)}{b(E2; 3 \rightarrow 22)}$
10	0.0332	1.77	0.0075
15	0.025	1.69	0.056
20	0.12	1.67	0.072
25	0.078	1.70	0.044
29	0.0096	1.72	0.039
30	0	1.78	0

4. Magnetic Dipole Transitions between Rotational States

According to Bohr and Mottelson ⁵⁾ the operator of the magnetic dipole moment corresponding to collective motions in an even nucleus is defined by the formula

$$\mathfrak{M}(1\mu) = \mu_0 g_R \int \mathbf{R}(\mathbf{r}) \nabla(r Y_{1\mu}) d\mathbf{r} \quad (3.1)$$

where $\mu_0 = eh/2Mc$ is the nuclear magneton; g_R is the gyromagnetic ratio corresponding to collective motion of the nucleons in the nucleus;

$$\mathbf{R}(\mathbf{r}) = BR_0^{-5} \sum_{\lambda} \hat{\mathbf{z}}_{\lambda} [\mathbf{r} \nabla(r^2 Y_{2\lambda})]$$

is the angular momentum density.

Introducing the operator $L = -i[r\nabla]$ we can write

$$[r\nabla(r^2 Y_{2\lambda})] = ir^2 L Y_{2\lambda} \text{ and } \nabla(r Y_{1\mu}) = (-1)^\mu \sqrt{\frac{3}{4\pi}} e_\mu.$$

Inserting these values into (3.1) and employing the relation

$$(e_\mu L) Y_{2\lambda} = (-1)^\mu \sqrt{6} (21, \lambda + \mu, -\mu | 2\lambda) Y_{2, \lambda + \mu},$$

we reduce (3.1) to the form

$$\mathfrak{M}(1\mu) = \mathfrak{M}_0(1\mu) + \mathfrak{M}_1(1\mu) + \dots \quad (3.2)$$

where

$$\mathfrak{M}_0(1\mu) = \frac{\mu_0 g_R}{2} \sqrt{\frac{3}{\pi}} J_\mu, \quad (3.3)$$

$$\mathfrak{M}_1(1\mu) = \mu_0 g_R \frac{5\sqrt{6}}{7\pi} \sum_\nu (21\mu - \nu, \nu | 1\mu) \alpha_{\mu-\nu}^* J_\nu. \quad (3.4)$$

Here the J_ν are projections of the total angular momentum which in the classical theory are expressed through the coordinates α_μ (in a space-fixed coordinate system these coordinates characterize the deviation of the shape of the nucleus from a spherical one) and through the corresponding velocities $\dot{\alpha}_\mu$ by the formula

$$J_\nu = i\sqrt{6}(-1)^\nu B \sum_\mu \dot{\alpha}_\mu \alpha_{\mu+\nu} (21, \mu + \nu, -\nu | 2\mu),$$

where B is the mass parameter of the theory; $(21, \mu + \nu, -\nu | 2, \mu)$ is the vector addition coefficient. In quantum theory the J_ν are operators. The action of these operators on wave functions D_{MK}^J , in terms of which functions (2.3) of an asymmetric top are expressed, is defined by the equality

$$J_\nu D_{MK}^J = (-1)^\nu \sqrt{J(J+1)} (J1M + \nu, -\nu | J, M) D_{M+\nu, K}^J.$$

The reduced magnetic dipole radiation probability from the state 22 to state 21 defined by functions (2.3) is

$$B(M1; 22 \rightarrow 21) = \frac{1}{5} \sum_{m' \mu m} |(21m' | \mathfrak{M}(1\mu) | 22m)|^2. \quad (3.5)$$

It is easy to verify that operator (3.3) does not contribute to this probability. Inserting (3.4) and (2.3) into (3.5) we get

$$B(M1; 22 \rightarrow 21) = \frac{90}{49\pi^2} \mu_0^2 g_R^2 \beta^2 \frac{\sin^2(3\gamma)}{9 - 8\sin^2(3\gamma)}. \quad (3.6)$$

From (3.6) it follows that in an axial nucleus ($\gamma = 0$) the reduced magnetic dipole radiation probability is zero and increases with γ reaching a maximal value at $\gamma = 30^\circ$.

If the value of γ is determined from the ratio E_{22}/E_{21} of the energies of two spin levels, one can evaluate the value of the gyromagnetic ratio g_R for

collective nucleon motion by using (3.6) and the results of measurement of the reduced magnetic dipole transition probabilities.

If one, furthermore, takes into account that according to (2.8) the reduced electric quadrupole radiation probability for transitions between the same levels is

$$B(E2; 22 \rightarrow 21) = \frac{10e^2 Q_0^2}{7\pi} \frac{\sin^2(3\gamma)}{[9 - 8\sin^2(3\gamma)]},$$

one may determine the ratio of the reduced probabilities

$$\frac{B(M1; 22 \rightarrow 21)}{B(E2; 22 \rightarrow 21)} = \frac{80}{7} \left(\frac{\mu_0 g_R}{eZ R_0^2} \right)^2, \quad (3.7)$$

where R_0 is the nuclear radius. It is interesting to note that (3.7) is not dependent on γ and β .

From (3.7) the ratio of the intensities of γ quanta emitted in both types of radiation is

$$\frac{T(M1)}{T(E2)} = \frac{80}{0.21k^2} \left(\frac{\mu_0 g_R}{eZ R_0^2} \right)^2, \quad (3.8)$$

where

$$k = \frac{E_{22} - E_{21}}{\hbar c}.$$

5. Comparison with Experiment

The results obtained in the preceding sections were based on the assumption that during rotation of a nucleus its internal state does not change. This assumption can be only approximately true and is the more accurate the farther the rotational energy levels are located from levels corresponding to excitation of the internal states of the nucleus and possessing the same J values, parities, etc.

Experimental values of the ratio of the energy of the second spin 2 level to that of the first level are listed in table 3. The value of parameter γ can be computed from this ratio with help of formulas (1.2). Employing (2.7) and (2.8) and making use of this value of γ one can evaluate $\{b(E2; 22 \rightarrow 21)/b(E2; 22 \rightarrow 0)\}_{\text{theor}}$. Comparison of these ratios with the experimental ones (column 7) indicates that the theory satisfactorily describes the experimentally observed rapid variation of the ratio of the reduced probabilities for transitions from a nucleus to one with a different γ value.

As can be seen by comparing columns 4 and 5 of table 3 for many nuclei the sum of the energies of the two spin 2^+ levels is equal to the energy of the spin 3 level, as required by equation (1.5), with an accuracy to 1 %. The deviation of 5 % observed in the Cd^{114} nucleus can be explained by the effect of three other levels located near level 2 and possessing spins of 4, 0 and 2.

From fig. 1 it can be seen that for $\gamma < 21.5^\circ$ the second excited 2^+ level should lie above level 4^+ , whereas for $\gamma > 21.5^\circ$ it should lie below this level. The level 3^+ should always be higher than level 4^+ . These rules are valid for

TABLE 3

Nucleus	$\frac{\epsilon_2(2)}{\epsilon_1(2)}$	γ°	Experimental keV		$\frac{b(E2; 22 \rightarrow 21)}{b(E2; 22 \rightarrow 0)}$		ref.
			$\epsilon_1(2)$ + $\epsilon_2(2)$	$\epsilon(3)$			
					theor.	exper.	
Pt ¹⁹⁶	1.97	30	1040	—	∞	> 2500	9, 24
Pt ¹⁹⁴	1.94	30	929	920.9	∞	163	13
Xe ¹³⁶	2.0	30	1445	—	∞	197	14
Te ¹³²	2.14	27.5	2040	—	104	200	23
Cd ¹¹⁴	2.17	26.75	1771	1860	78	77.9	15
Se ⁷⁶	2.2	26.6	1760	—	68	23.5	16
Te ¹³²	2.25	26.5	1760	—	66	78.2	17
Hg ¹⁸⁰	2.66	23.2	1500	—	13	25	23
Gd ¹⁶⁴	3.01	21.4	1133	1140	7.2	—	18
W ¹⁸⁶	8.08	13.9	1000	1000	2.0	—	19
Sm ¹⁵²	8.9	13.48	1206	1226	1.9	1.7	18, 22
Dy ¹⁶⁶	11	12.2	1051	1047	1.9	2.38	10
W ¹⁸²	12.1	11.6	1322	1331	1.6	1.59	20
Pu ²³⁸	23.4	8.13	1074	1076	1.2	1.3—1.5	25
Pu ²⁴⁰	23.7	8.0	1063	1060	1.2	—	19

all the nuclei in table 3 for which the positions of levels with spins of 2, 4, 3 are known.

It is interesting to note that for $\gamma = 30^\circ$ the theory predicts an equidistant spacing of energy levels $\epsilon_1(2)$, $\epsilon_2(2)$, $\epsilon(3)$. A level spacing of this type can also be derived in the oscillator approximation for the energy of surface vibrations. However in this case the $\epsilon_2(2)$ and $\epsilon(3)$ levels should be degenerate with spin values of respectively 0, 2, 4 and 0, 2, 3, 4, 6.

Unfortunately, the experimental data which can be used to determine the ratio $b(E2; 3 \rightarrow 21)/b(E2; 3 \rightarrow 22)$ are quite sparse. We know this ratio only for Kr^{81 21}) for which it equals 0.016. This corresponds to a value of γ which slightly exceeds 29. A value $\gamma > 29$ agrees with the observed value $\epsilon_2(2)/\epsilon_1(2) = 1.9$ if it be taken into account that in this nucleus adiabatic conditions apparently are not rigorously fulfilled.

From formulas (2.6) and (2.7) one can compute the ratio of the reduced probabilities, $b(E2; 22 \rightarrow 0)/b(E2; 21 \rightarrow 0)$; it equals the ratio $b(E2; 0 \rightarrow 22)/b(E2; 0 \rightarrow 21)$ of the reduced probabilities for the inverse transitions which appear during Coulomb excitation of the nuclei. This ratio is maximal for γ values lying between 15° and 25° (the variation of γ in this interval corresponds to $6.28 > \epsilon_2(2)/\epsilon_1(2) > 2.4$) and is very small for $\gamma < 5^\circ$ and $\gamma > 29^\circ$. The values of the ratio $b(E2; 0 \rightarrow 22)/b(E2; 0 \rightarrow 21)$ presented in ref.²³) and derived from Coulomb excitation data and also the theoretical

values of γ computed from (2.6) and (2.7) for the corresponding energy ratio, $\epsilon_2(2)/\epsilon_1(2)$, are presented in table 4. It is apparent from the table that the theoretical values satisfactorily agree with the experimental ones.

TABLE 4

Nucleus	$\frac{\epsilon_2(2)}{\epsilon_1(2)}$	γ	$\frac{b(E2; 0 \rightarrow 22)}{b(E2; 0 \rightarrow 21)}$	
			exper.	theor.
Os ¹⁹⁰	2.9	20	0.04	0.072
Ru ¹⁹⁰	2.55	23.7	0.024	0.057
Ru ¹⁹⁴	2.56	24	0.013	0.055
Cd ¹¹⁶	2.35	25.05	0.021	0.044
Ru ¹⁹²	2.24	25.07	0.015	0.043
Pd ¹⁹⁶	2.2	26.5	0.017	0.027
Pd ¹⁰⁸	2.18	27	0.01	0.02
Cd ¹¹⁴	2.17	27	0.02	0.02

In order to compare the experimental ratio of the intensities of magnetic dipole and electric quadrupole radiations in the $2^+ \rightarrow 2^+$ transition with the values which formula (3.8) yields, we put $\mu_0 = 5.05 \times 10^{-24}$ erg · gauss⁻¹ $g_R = 0.4$; $R_0 = 1.2A^{1/3} \times 10^{-13}$ cm. The $T(M1)/T(E2)$ ratios computed in this way are given in table 5 (third column). The intensity of the E2 transition

TABLE 5

Nucleus	$E_{22} - E_{21}$ keV	$\frac{T(M1)}{T(E2)}$	exper. % of E2 transition
Se ⁷⁶	643	9.8×10^{-2}	98 ± 1
Te ¹²²	693	1.9×10^{-2}	92 ± 4
Os ¹⁸⁶	627	6.5×10^{-3}	99 ± 1
Os ¹⁸⁸	480	1.04×10^{-2}	99.6

relative to the total intensity of the $22 \rightarrow 21$ transition derived from the data of Lindqvist and Marklund²⁵) is given in the last column of the table.

Comparison of the predictions of the theory with the experimental data presently known to us thus confirms the assumption that some even nuclei do not possess axial symmetry.

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