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TRIAXIALITY AND WOBBLING*

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The wobbling mode of the collective angular momentum expressed as a phonon excitation with a phonon quantum number, n_w, and wobbling frequency, $\hbar\omega_w$, is unique to the rotational motion of a triaxial nucleus. The presence of quasiparticle alignment introduces characteristic trends in the electromagnetic decay properties for transitions between bands with different wobbling quantum number. Evidence for the wobbling mode, and thereby triaxiality, has been obtained in several even-N Lu isotopes, ^{163}Lu being the best studied case with the strongest population of the wobbling excitations. As an important support for the wobbling interpretation, recent lifetime measurements in ^{163}Lu have shown that the quadrupole moments of the bands with $n_w = 0$ and $n_w = 1$ are very similar. Triaxial strongly deformed shapes are expected also for neighbouring Hf nuclei, but efforts to identify wobbling in the Hf isotopes where many bands are established, resembling those found in the Lu isotopes, has so far failed. To date the even-N Lu isotopes, ^{161,163,165,167}Lu, are the only nuclei in which wobbling excitations are identified.

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1. Introduction

Although most nuclei are spherical in their ground state divergence from both spherical and axial symmetry may be a rather common phenomenon for nuclei throughout the entire available mass region when excited states at higher angular momentum are considered. A direct determination of the shape parameters for the nuclear mass- and charge distribution is not in general attainable for excited states. Other less direct means must be invoked. The loss of spherical and axial symmetry will influence many spectroscopic

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observables, and an interplay of such measured quantities has often been used as evidence for triaxiality in nuclei. For well deformed nuclei triaxiality will have an effect on important quantities like *(i)* excitation energy of quasiparticles, *(ii)* the relation between signature partner bands and *(iii)* electromagnetic decay strength between bands. Yet such effects may arise from other structural changes than nuclear triaxiality, and stable static triaxiality is difficult to prove.

In recent years two different features both uniquely related to triaxiality have been at focus.

One such feature is breaking of chiral symmetry, *i.e.* a pair of chiral twin bands with identical structure and very close excitation energies may be expected under certain conditions: Coupling of the angular momenta of the individual valence particles and the collective rotation may bring the total spin out from any of the principal planes. This can give rise to both left- and right-handed orientation of the angular momentum vectors [1] and is discussed in particular in odd-odd nuclei in the $A \sim 130$ region [2]. A beautiful selection rule for electromagnetic transitions is recently obtained theoretically for idealistic chiral pair bands [3].

A triaxial nucleus may rotate collectively about any axis, and another unique effect of triaxiality has to do with the extra degree of freedom from transferring some collective angular momentum from the axis with the largest moment of inertia to the two other axes. This transferred angular momentum is quantized, and a family of rotational bands with increasing number of wobbling phonon excitations is expected. Wobbling has been identified in a number of even-N Lu-isotopes, including even a second phonon wobbling excitation in a couple of cases [4–8].

2. Wobbling

The quantized wobbling motion of a triaxial nucleus, of which the classical analog is the motion of an asymmetric top, has been treated in details in Ref. [9]. In the high-spin limit, and neglecting the intrinsic structure, the excitation energy can be separated into the rotation about the principal axis with the largest moment of inertia and the wobbling motion:

$$E(I, n_w) = I(I+1)\frac{\hbar^2}{2\Im_x} + \hbar\omega_w \left(n_w + \tfrac{1}{2}\right) ,$$

where n_w is the wobbling phonon number and ω_w the wobbling frequency which depends on the three moments of inertia with respect to the principal axes,

$$\hbar\omega_w = \hbar\omega_{\rm rot}\sqrt{(\Im_x - \Im_y)(\Im_x - \Im_z)/(\Im_y \Im_z)}\,.$$

The rotational frequency, $\hbar\omega_{\mathrm{rot}}$, is expressed as $I/\Im_x$. The formulation of the rotational degree of freedom of a triaxial nucleus as wobbling excitations corresponds to a 1-dimensional harmonic oscillator, as can be seen from the expressions given above.

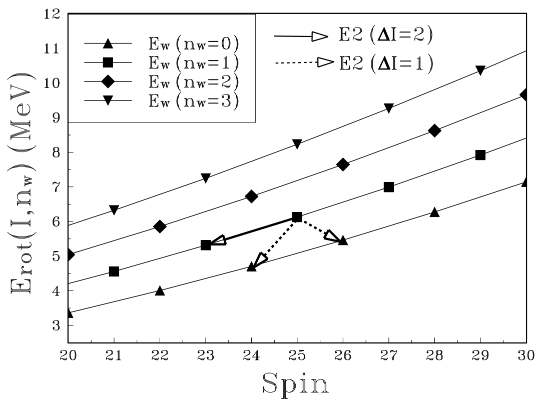

Fig. 1. Schematic illustration of a family of wobbling bands.

The presence of aligned particles will favour a specific (triaxial) shape depending on the degree of shell filling in the high-j sub-shell, and states with high-j aligned particles may appear close to the yrast line since the rotational energy required to build up a certain total angular momentum is relatively small.

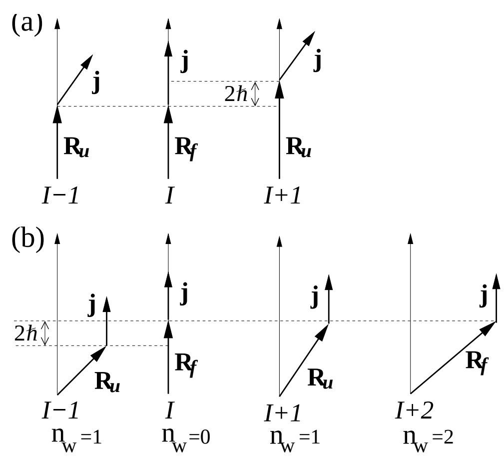

Fig. 2. Coupling scheme for an aligned particle ($\vec{j}$) and a rotor ($\vec{R}$) to the total angular momentum $\vec{I}$, where $\vec{I} = \vec{R} + \vec{j}$. In (a) the cranking-like and (b) the wobbling scenario, respectively, are schematically illustrated [10, 11].

In this case, following the work of Ref. [11], in which a high-j particle is coupled to a triaxial rotor, the energy can be written as

$$E(I, n_w, j) = I(I+1)\frac{\hbar^2}{2\Im_x} + \hbar\omega_w \left(n_w + \tfrac{1}{2}\right) + \frac{\hbar^2}{2\Im_x}(-2I_x j_x + j_x^2)\,.$$

The energy difference between bands with $\Delta n_w = 1$,

$$\Delta E(I, j) = \hbar\omega_w + 2j_x\frac{\hbar^2}{2\Im_x}$$

is larger than in the case of no aligned angular momentum. As shown in Ref. [10] the wobbling excitation may compete with the familiar cranking-like excitation which will have $\vec{j}$ tilted away from the x-axis. These two alternative coupling schemes are illustrated in Fig. 2.

In the particle-rotor calculations the moments of inertia have been released from hydrodynamical constraint, and values close to rigid have been determined such that the experimental energies of the bands with $n_w = 0$ and $n_w = 1$ (in ^{163}Lu) are reproduced on the average. This is justified since with an appreciable divergence from axial symmetry rigid moments of inertia are not in conflict with the required symmetry.

In Refs. [10, 11] matrix elements are calculated for in-band and out-of-band transitions for the wobbling excitations in ^{163}Lu which has an aligned proton in the $i_{13/2}$ orbital. A triaxial shape $(\epsilon, \gamma) \sim (0.4, 20°)$ is assumed in agreement with expectations from potential energy surface calculations with the cranking code, Ultimate Cranker (UC) [12], and the measured in-band transition quadrupole moments. The calculated M1 transition strength is quite small, in agreement with the wobbling picture, whereas the $\Delta n_w = 1$ E2 transition matrix elements are sizable and the phonon rules almost born out, in accordance with the 1-dim harmonic vibrational description.

$$B(\mathrm{E2}, n_w = 2, I \to n_w = 1, I-1) \sim 2 \cdot B(\mathrm{E2}, n_w = 1, I-1 \to n_w = 0, I-2)$$

$$B(E2, n_w = 2, I \to n_w = 0, I-2) \sim 0$$

In addition, the out-of-band to in-band ratios are for large triaxiality with $\gamma > 15°$ found to be independent of ϵ but strongly dependent on γ, while keeping the n_w/I dependence inherent in the wobbling phonon description. These ratios are more easily compared to the experimental data

$$\frac{B(\mathrm{E2}, \Delta n_w = 1)}{B(\mathrm{E2},\ \Delta n_w = 0)} \propto \frac{n_w}{I} \cdot \frac{\sin^2(\gamma + 30°)}{\cos^2(\gamma + 30°)}\,.$$

The reduced $I \to I-2$ transition probabilities within a band with fully aligned spin and the rotation axis as quantization axis ($K = I$) can be written [9, 13] as

$$B(\mathrm{E2}, n_w = 0) = \frac{5}{16\pi}(e\hat{Q}_2)^2 \langle II2-2|I-2I-2\rangle^2 = \frac{5}{16\pi}(e\hat{Q}_2)^2 \frac{2I-3}{2I+1}\,,$$

where $\hat{Q}_2$ is the quadrupole transition operator quantized along the rotation axis. For the wobbling bands with the collective angular momentum titled away from the rotation axis one has $K = I - n_w$. For the one-phonon band this results for $I \to I-2$ in

$$\begin{aligned} B(\mathrm{E2}, n_w = 1) &= \frac{5}{16\pi}(e\hat{Q}_2)^2 \langle II-12-2|I-2I-3\rangle^2 \\ &= \frac{5}{16\pi}(e\hat{Q}_2)^2 \frac{(2I-3)(2I-4)}{(2I+1)2I}\,. \end{aligned}$$

This effect causes the expected in-band $B(\mathrm{E2})$ values for the one-phonon wobbling band to be ~ 7 % lower than the values for the fully aligned band at $I \sim 30\hbar$, with identical intrinsic structures for the two bands [13].

3. Experimental evidence for wobbling

The signatures in the experimental data which may identify wobbling phonon excitations are a 'family of bands' with similar properties. In particular, these bands should have similar dynamical moments of inertia and alignments. More important are the electromagnetic properties of the decay between bands. For the $\Delta I = 1 (\Delta n_w = 1)$ transitions the decomposition in E2 and M1 amplitudes must be determined. This requires angular correlation, angular distribution ratio and linear polarization measurements, for decay branches out of weakly populated bands, a challenge that demands the large state-of-the-art detector arrays to be utilized.

Wobbling has been identified in the even-N Lu-isotopes ^{161,163,165,167}Lu (*cf.* Fig. 3) with the data for ^{163}Lu providing the best studied case so far. ^{163}Lu has the strongest populated $\pi i_{13/2}$ band with a two-phonon wobbling excitation identified. The data for the neighbouring isotopes are less complete, and for ^{161}Lu the connection to the normal deformed structures is not completely firm at present.

3.1. The best wobbler, ^{163}*Lu*

For ^{163}Lu a partial level scheme is shown in Fig. 4, including a comparison of band properties (Fig. 4b) documenting that TSD1,TSD2 and TSD3 do indeed form a family. When compared to calculations we note that the model of Ref. [11] is not able to reproduce the obvious anharmonicity revealed in the excitation energy of TSD3 relative to TSD2, being considerably

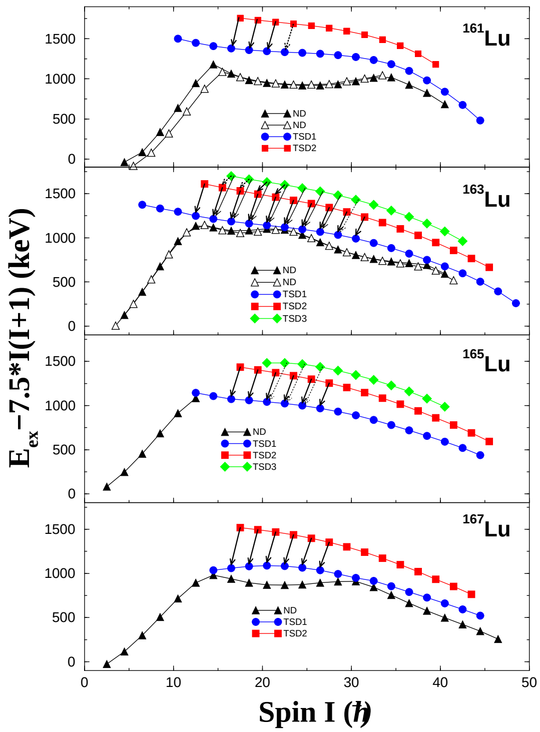

Fig. 3. Wobbling excitations in ^{161,163,165,167}Lu [4–8]. A rigid rotor reference is subtracted from the excitation energies. Identified interband transitions are marked by arrows.

smaller than the excitation of TSD2 above TSD1 *cf.* Fig. 3. Furthermore, it is not possible to reproduce the measured spin dependence of the wobbling frequency with spin-independent moments of inertia.

The most important comparison though deals with the electromagnetic transition rates (Fig. 4c), from which the agreement between experiment and calculations appear very satisfactory, bearing out the phonon picture. Note that the B(E2) ratios for the $n_w = 2 \rightarrow n_w = 1$ transition from TSD3 to TSD2 is even slightly higher than expected although the error is large. Possible anharmonicity in the second phonon excitation revealed in the smaller separation in energy between TSD3 and TSD2 relative to the separation between TSD2 and TSD1, seems not to affect the transition rate. Most likely, one may find excitations in the neighbourhood of TSD3, which influence the excitation energies, whereas transition rates corresponding to possible admixed amplitudes are likely to be small and therefore do not significantly perturb the transition rates expected for the wobbling excitations. The measured B(M1) values from TSD2 to TSD1 are quite small, consistent with the wobbling excitation.

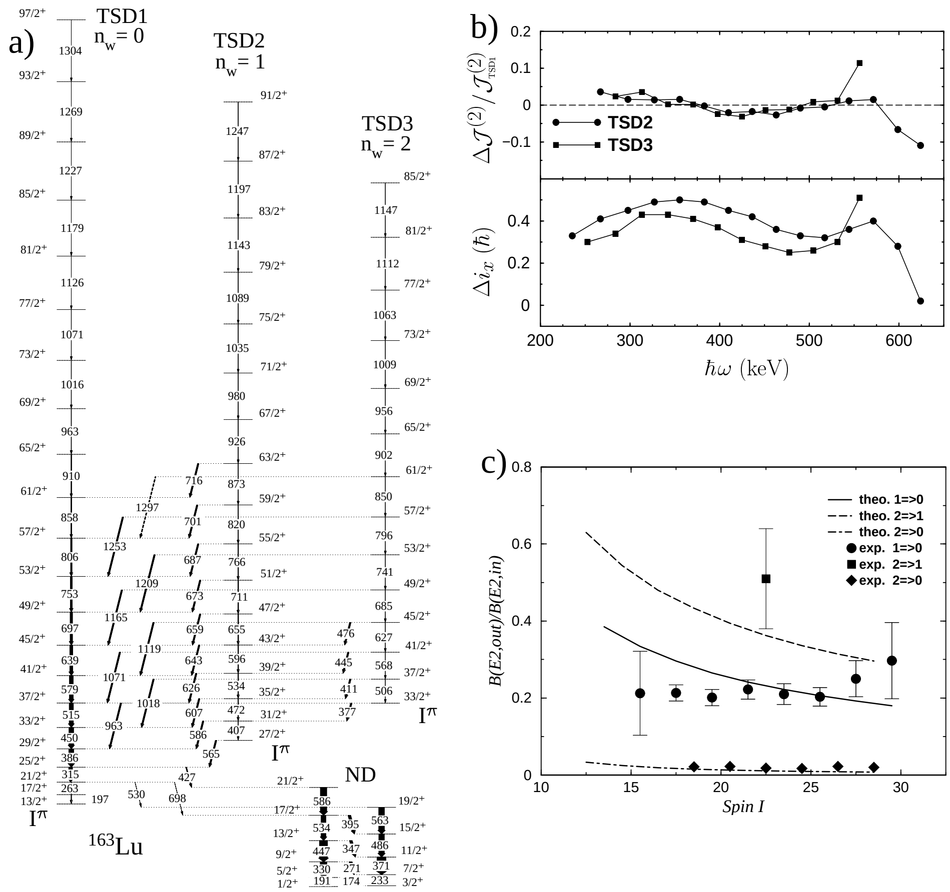

Fig. 4. (a) Partial level scheme for ^{163}Lu [4,5] showing the two excited TSD bands and their decay mutually and to TSD1. (b) Relative differences in dynamic moments of inertia (top) and alignment (bottom) between TSD1 and the excited TSD bands. (c) Experimental and calculated [11] ratios of B(E2) values for out-of band transitions, $n_w = 1 \rightarrow n_w = 0$, $n_w = 2 \rightarrow n_w = 1$ and $n_w = 2 \rightarrow n_w = 0$ to the corresponding in-band B(E2) values with $\Delta n_w = 0$. For the lowest and 2 highest experimental points with $n_w = 1 \rightarrow n_w = 0$ the mixing ratios are not measured but assumed to be identical to the average of values measured for $17.5 \leq I \leq 25.5$.

3.2. The triaxial shape of ^{163}*Lu*

New measurements of lifetimes in ^{163}Lu for states in TSD1 and TSD2 covering a region in spin of $20.5 \leq I \leq 34.5$ have been performed, allowing an additional and important test of the identicality of the intrinsic structure of the one-phonon wobbler band and its basis ($n_w = 0$) [13,14]. With transition quadrupole moments extracted as described in the preceeding section the

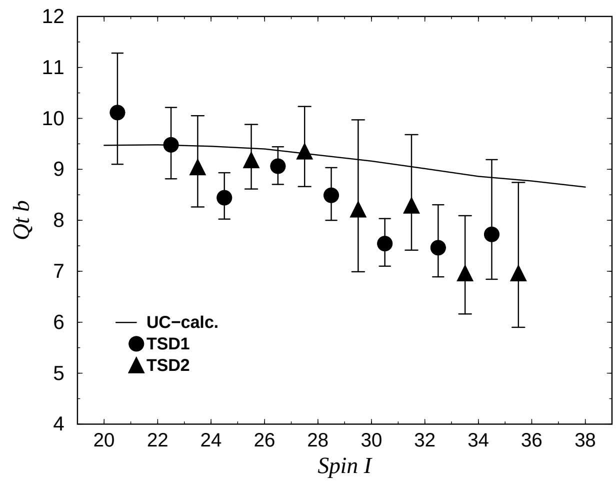

Fig. 5. Comparison of experimental values of transition quadrupole moments $Q_t(=\sqrt{3/8}\hat{Q}_2)$ for TSD1 and TSD2 in ^{163}Lu [13,14] The values for TSD1 may be compared to cranking calculations shown by the solid line.

in-band $\hat{Q}_2$-values are identical within errors. The values for TSD1 may be compared to predictions from cranking calculations. The decrease in the experimental values of $Q_t(=\sqrt{8/3}\cdot\hat{Q}_2)$ with spin is qualitatively reproduced by the calculation, in which the decrease stems mainly from an increase in γ from $\sim 19.5°$ to $\sim 21.5°$ over the spin-range shown in Fig. 5. A slightly larger increase in γ would comply with the experimental Q_t-values.

Since the $B(\mathrm{E2,out})/B(\mathrm{E2,in})$ ratios are approximately independent of ϵ but strongly influenced by γ we show these for the $n_w = 1 \rightarrow n_w = 0$ transitions compared to the particle-rotor calculations of Ref. [11] for $\gamma = 15°, 20°$ as in Fig. 5, and 30°. It appears that the flat behaviour rather than a $1/I$-dependence in the data can be explained if γ increases from $\sim 16°$ to $\sim 22°$ over the exposed spin-range. Therefore, a slightly stronger spin-dependence in the triaxiality γ than expected from cranking calculations explains simultaneously the decrease in transition quadrupole moments. The $1/I$-dependence inherent in the $B(\mathrm{E2,out})/B(\mathrm{E2,in})$ values for decay between wobbling phonon excitations is thereby restored.

4. The Lu-Hf region

In addition to the wobbling excitations shown in Fig. 3 for the even-N Lu isotopes quasiparticle excitations of negative parity with suggested configuration assignments are found in the triaxial strongly deformed minima of ^{163}Lu [15] and ^{167}Lu [16, 17]. Furthermore, two connected TSD bands with

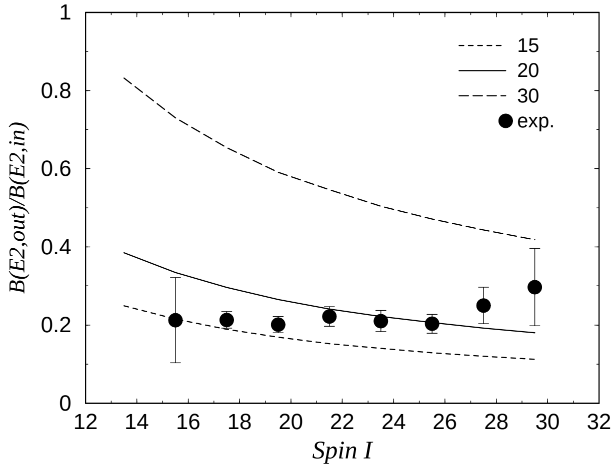

Fig. 6. Experimental and calculated [11] ratios of B(E2) values for out-of band transitions, $n_w = 1 \to n_w = 0$, to the corresponding in-band B(E2) values with $\Delta n_w = 0$. Calculated values are shown for $\gamma = 15°, 20°$ and $30°$.

assigned quantun numbers and configurations are established in ^{164}Lu [18]. In a ll these cases the odd proton is occupying the $i_{13/2}$ orbital, and quasineutron excitations include the $i_{13/2}$ and $h_{9/2}$ orbitals. The role of the $j_{15/2}$ orbital at high spin is also discussed.

Triaxial strongly deformed minima are expected also in the neighbouring Hf nuclei, and extensive searches in the presumably most favourable region, $^{164-166}$Hf, have not given any evidence for such structures. In the heavier Hf isotopes $^{168-174}$Hf $\sim$ 15 bands which may be of TSD-type have been identified. None of these bands have been connected to known structures, and they have therefore no firm determination of excitation energy, spin or parity. As an important exception, one band in ^{175}Hf, which is found to be nearly isospectral with the strongest populated TSD band in ^{174}Hf, has been connected to known normal deformed bands [19].

The large deformation has been verified for the strongest populated band in ^{168}Hf [20]. Very recently even larger transition quadrupole moments $Q_t \sim$ 13 b have been determined for several of the 8 bands in ^{174}Hf [21, 22], and the same value is found for the connected band, Band 2, shown in the level scheme of ^{175}Hf [19] (*cf.* Fig. 7). This band has unambigous spin, parity and excitation energy determined, and the surprising fact is that Band 2 covers a spin region, $I = 79/2 - 127/2$, which is 12–15 units higher than the spins observed in the TSD bands of the Lu isotopes (*cf.* Figs. 3 and 4) whereas the latter are similar to those observed for Band 1 in ^{175}Hf. The relative alignment is 10–12 units larger for Band 2 than for Band 1 [19]. Except for

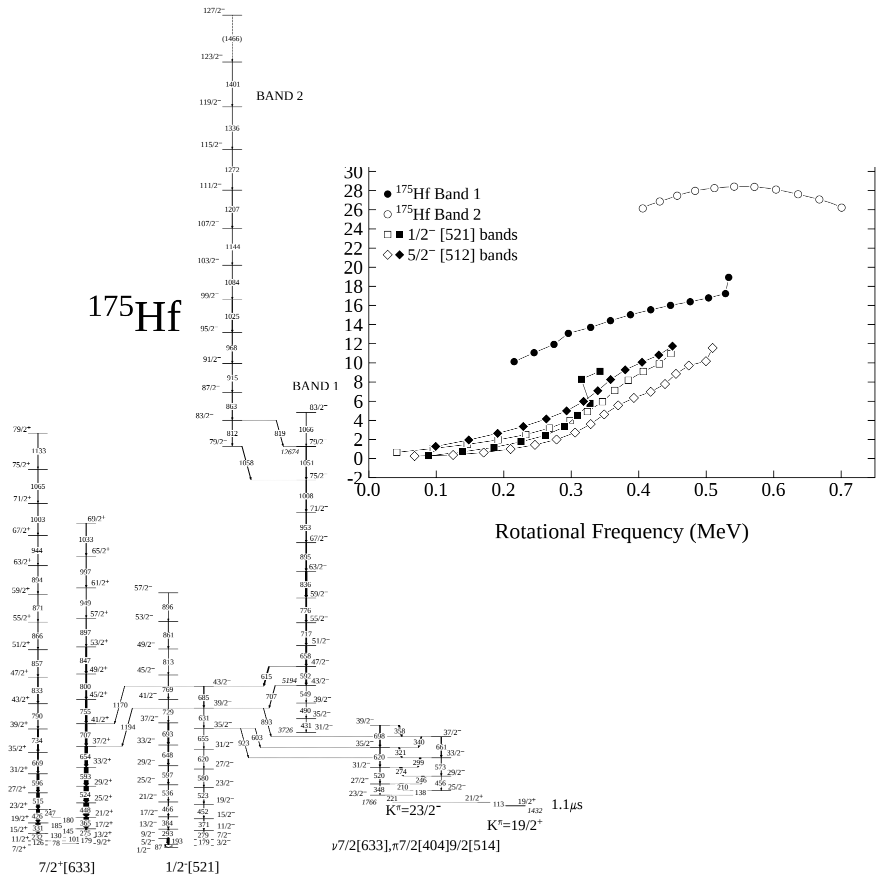

Fig. 7. Partiel level scheme of ^{175}Hf [19]. Band 2 is connected to known structures through Band 1. The inset shows the measured relative alignment *vs* rotational frequency for selected bands.

Band 1 in ^{175}Hf, the dynamical moments of inertia, $J^{(2)}$ for the — possibly triaxial — strongly deformed bands in the Hf nuclei, show a decrease with spin, whereas the values of $J^{(2)}$ for all the TSD bands in the Lu isotopes are increasing or constant as a function of spin.

The Hf-isotopes with suggested TSD structures are more neutron rich than the Lu-isotopes for which the triaxiality has been verified by the wobbling excitations. Therefore one expects the strongly shape-driving $j_{15/2}$ neutron orbital to come into play in the triaxial local minimum, and according to cranking calculations the expected triaxiality for the heaviest ^{174,175}Hf is as large as $\sim 30°$. The measured values of Q_t for the four strongest populated bands in ^{174}Hf and the one band in ^{175}Hf are considerably larger than calculated [19, 22].

With these new experimental results concerning (triaxial) strongly deformed bands in Hf-isotopes becoming available there seems to be an important qualitative difference to the neighbouring Lu nuclei. Of particular importance is the spin-range covered, but also much larger transition quadrupole moments have been found. Therefore, the intrinsic structure in these bands in the Hf-isotopes is most likely of more complex multi-quasiparticle structure. Whether they correspond to triaxial mimima is an open question. So far any proof for triaxiality by the identification of wobbling excitations has failed.

5. Summary and outlook

Triaxiality is documented in even-N Lu isotopes through the identification of of wobbling phonon excitations. The degree of freedom exploited in the wobbling excitation mode offers the possibility for determining three different moments of inertia for the rotational motion of a triaxial nucleus, a new challenge to theory. The region of stable triaxiality around Lu provides a testing ground for models to describe the interplay of shells driving the nuclear shape towards triaxiality.

Experiments have shown that large deformation exists in the heavier Hf isotopes, but the excitations found here have much higher spin than the TSD bands in Lu. The structures are most likely more complex, and no wobbling excitations have been identified.

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