TRANSITION RATES FOR A TRIAXIAL ROTOR PLUS A SINGLE QUASI-PARTICLE

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E2 and M1 matrix elements are considered for the system consisting of a triaxial rotor and a single quasiparticle. While B(M1) values are more closely related to the signature-splitting of the quasiparticle spectra, E2 matrix elements exhibit directly the information on the shape of the system.

The possible occurrence of triaxial nuclear shapes is an issue of considerable interest in current studies of nuclei with large angular momentum. In the present note we point out the richness of the information contained in the E2 moments between yrast states especially with respect to the experimental determination of triaxial shapes. For example, a considerable amount of signature dependence in the $B(\mathrm{E2}, \Delta I = 1)$ transitions was reported [1] already more than one year ago in the analysis of the negative-parity band of ^{157}Ho. Such a strong signature dependence seemed impossible to be obtained theoretically [2], if one assumed an axially-symmetric shape for the nucleus. However, we now believe that the reported signature dependence may be taken as an evidence for the deviation of the nuclear shape from axial symmetry.

First of all, it may be useful to examine the E2 operator in a triaxial rotor. By writing the quadrupole operator quantized along the rotation axis (the intrinsic three-axis) as $\hat{Q}_\mu$ (Q_ν), one has the relation

$$\hat{Q}_\mu = D^2_{\mu 0} Q_0 + (D^2_{\mu 2} + D^2_{\mu -2}) Q_2 \ , \tag{1}$$

where the deformation of the rotor is expressed by the parameters Q_0 and Q_2. One then gets the quadrupole operators

$$\begin{aligned} &\hat{Q}_0 = -\tfrac{1}{2} Q_0 + (3/2)^{1/2} Q_2 \ , \\ &\quad \text{for static quadrupole moment} \ , \\ &\hat{Q}_{\pm 1} = [\mp (3/2)^{1/2} Q_0 \pm Q_2] \cdot (I_3/I) \ , \\ &\quad \text{for } \Delta I = \pm 1 \text{ transitions} \ , \\ &\hat{Q}_{\pm 2} = (3/8)^{1/2} Q_0 + \tfrac{1}{2} Q_2 \ , \\ &\quad \text{for } \Delta I = \pm 2 \text{ transitions} \ . \end{aligned} \tag{2}$$

From the expression (2) we note the following points: (a) In the case of an axially-symmetric shape all the E2 moments (with $\Delta I = 0, \pm 1, \pm 2$) are determined by one parameter Q_0, while for a triaxial shape the ratio of static moments to collective E2 transition (with $\Delta I = \pm 2$) depends significantly on the ratio of Q_0 to Q_2. (b) The effect of $Q_2 \neq 0$ (i.e. $\gamma \neq 0$) is such that the quantity $\hat{Q}_{\pm 1}$ is reduced (enhanced) if the quantity $\hat{Q}_{\pm 2}$ is enhanced (reduced). Therefore the ratio of the $B(\mathrm{E2}; I \to I-1)$-value to the $B(\mathrm{E2}; I \to I-2)$-value can be used as a sensitive measure of the γ-value. (c) The expression (2) gives immediately the values of the E2 matrix elements averaged over the two signatures but a more detailed analysis of the tunnelling between ± values of I_3 is necessary in order to describe the signature dependent effects. (d) By using the definition [3] of Q_2 and Q_0 in terms of β and γ, the expression for $\hat{Q}_{\pm 2}$ in (2) can be seen to be proportional to $\cos(\gamma + 30^\circ)$.

Now, we take into account explicitly the intrinsic state describing a single quasiparticle. In the following we consider the "high-j" single-particle shell which has played a special role in the analysis of yrast-spectroscopy. The intrinsic hamiltonian [4] is written as

$$H_{\mathrm{intr}} = \sum_{\nu} a_\nu^+ a_\nu (\epsilon_\nu - \lambda) + \frac{\Delta}{2} \sum_{\mu,\nu} \delta(\bar{\mu}, \nu) \cdot (a_\mu^+ a_\nu^+ + a_\nu a_\mu) , \quad (3)$$

where ϵ_ν are the one-particle energies for a single-particle with angular momentum j moving in a general triaxially-deformed quadrupole potential

$$V(\gamma) = [\kappa/j(j+1)] \times \{[3j_3^2 - j(j+1)] \cos\gamma - \sqrt{3}(j_2^2 - j_1^2) \sin\gamma\} , \quad (4)$$

where κ is used as an energy unit. (The value of κ depends on the size of quadrupole (Y_{20}) deformation and may be something between 2 and 2.5 MeV.)

In the following we use a particle-rotor model and thus treat angular momentum as a good quantum-number. Our hamiltonian is written as

$$h = H_{\mathrm{intr}} + \sum_{k=1}^{3} \frac{\hbar^2}{2\mathcal{I}_k} (I_k - j_k)^2 , \quad (5)$$

where I_k is the k-component of the total angular momentum and $\mathcal{I}_k$ is the inertial parameter associated with rotation about the kth axis. Though it is, in general, possible that the principal axes of the density distribution do not coincide with those of the inertial parameters, we do not consider this possibility in the present note. The presence of the recoil term in the hamiltonian (5) is essential in the particle-rotor model, but it is also seen that the recoil term produces γ-dependent renormalization of the single-particle energies. We do not discuss here the details of the particle-rotor model and its connection with the cranking model, but would emphasize that the cranking model as usually formulated involves the neglect of correlations that appear to be crucial for the description of the E2 matrix elements considered in the present note. In particular we are thinking of the detailed correlation of the vectors $\boldsymbol{I}$ and $\boldsymbol{j}$ as seen in the intrinsic coordinate system.

The basic formulae for the particle-rotor model with triaxial shape can be found in refs. [3] and [5]. In the numerical calculations such as in ref. [5] the moments of inertia of hydrodynamical type

$$\mathcal{I}_k = \tfrac{4}{3} J_0 \sin^2(\gamma - \tfrac{2}{3}\pi k) , \quad (6)$$

are conventionally assumed. If one takes the expressions (4) and (6), it is known to be sufficient to investigate the region $0^\circ \leqslant \gamma \leqslant 60^\circ$. The expression (6) means that $\mathcal{I}_1(\gamma)$ is largest for $0^\circ \leqslant \gamma \leqslant 60^\circ$, while the rigid moments of inertia calculated from the ellipsoidal shape of the core, which gives rise to the one-body potential (4), has the largest component around the two-axis. That means, in available numerical calculations such as those in ref. [5] only the region of the so-called "rotation around an intermediate axis" (namely, the "$-60^\circ \leqslant \gamma \leqslant 0^\circ$ region" in the sense of the cranking model [6]) has been investigated. In order to investigate the "$0^\circ \leqslant \gamma \leqslant 60^\circ$ region" in the sense of the cranking model, we invert the sign of γ in the expression of $\mathcal{I}_k(\gamma)$ in (6) while keeping the expression $V(\gamma)$ in (4). Thus, in the following numerical evaluation we use the expression (6) for the moments of inertia and employ the sign of γ corresponding to the convention of the cranking model, namely

$$\begin{aligned} &\gamma > 0: \; V(\gamma) \leftrightarrow \mathcal{I}_k(-\gamma) , \\ &\gamma = 0: \; V(\gamma=0) \leftrightarrow \mathcal{I}_3 = 0 , \quad \mathcal{I}_1 = \mathcal{I}_2 , \\ &\gamma < 0: \; V(\gamma) \leftrightarrow \mathcal{I}_k(\gamma) . \end{aligned} \quad (7)$$

We stress here that the moments of inertia of the type (6) is an assumption and, in fact, numerical results have often a crucial dependence [‡1] on the form of $\mathcal{I}_k(\gamma)$ assumed. In any case, in the present note we do not go to very large values of $|\gamma|$.

We diagonalize the hamiltonian (5) by using the BCS one-quasi-particle states for the particle wave functions. Therefore, we can not go to very high spin states, but the calculation is sufficient for drawing conclusions about the problems treated in the present note.

In fig. 1 we show examples of calculated B(E2) values for the $j = h_{11/2}$ shell. For $\gamma = 0$ the energy of the single-particle level with $\Omega_3 = 7/2$ is $(0.03)\kappa$. Thus, very roughly speaking, the upper part of the graph [with $\lambda = (-0.10)\kappa$] may correspond to the negative-parity yrast states of odd-A Ho-isotopes, while

[‡1] This was mentioned also by Meyer-ter-Vehn in ref. [5].

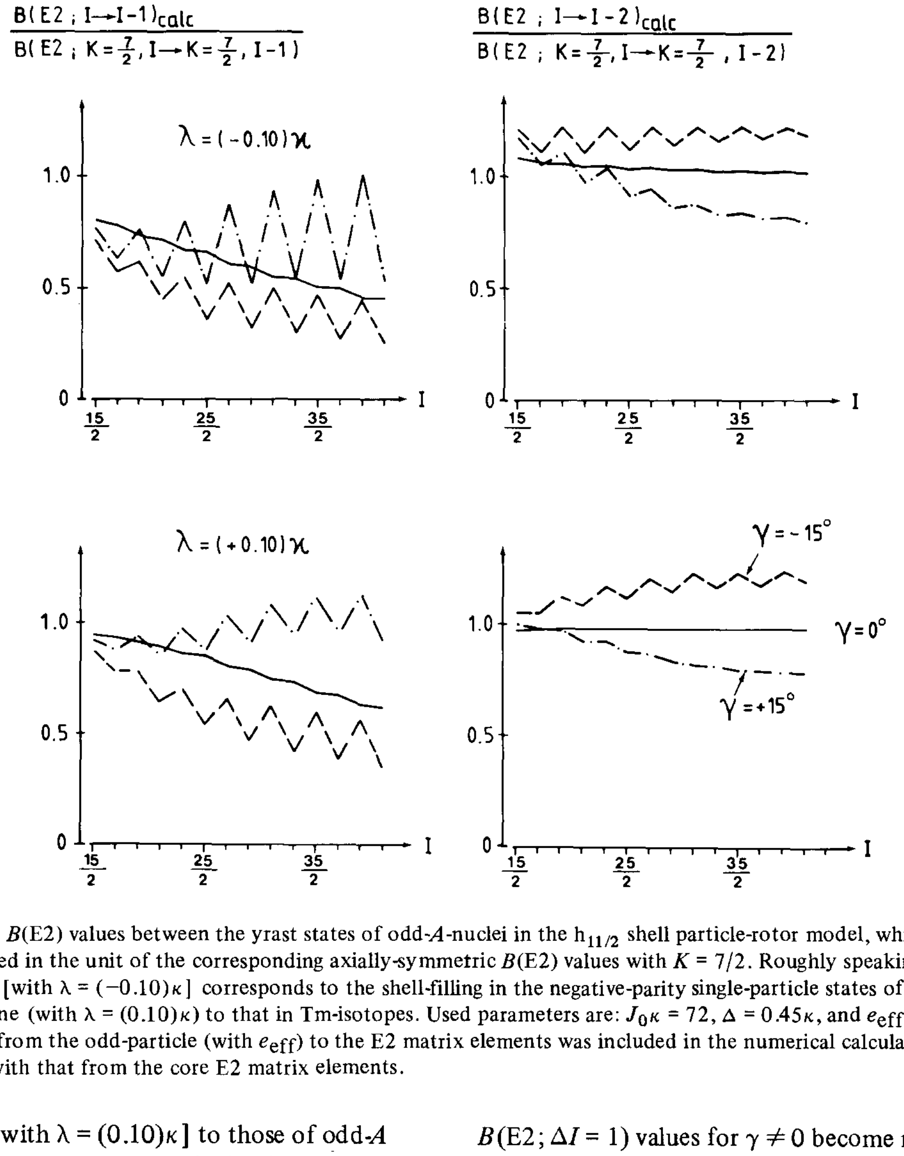

Fig. 1. Calculated B(E2) values between the yrast states of odd-A-nuclei in the $h_{11/2}$ shell particle-rotor model, which are, for convenience, expressed in the unit of the corresponding axially-symmetric B(E2) values with K = 7/2. Roughly speaking, the upper part of the graph [with $\lambda = (-0.10)\kappa$] corresponds to the shell-filling in the negative-parity single-particle states of Ho-isotopes, while the lower one (with $\lambda = (0.10)\kappa$) to that in Tm-isotopes. Used parameters are: $J_0\kappa = 72$, $\Delta = 0.45\kappa$, and $e_{\mathrm{eff}} = 2e$. Though the contribution from the odd-particle (with e_{eff}) to the E2 matrix elements was included in the numerical calculation, it is very small compared with that from the core E2 matrix elements.

the lower one [with $\lambda = (0.10)\kappa$] to those of odd-A Tm-isotopes. For convenience, B(E2) values are expressed in the unit of the corresponding axially-symmetric B(E2) values with K = 7/2, which are smoothly-varying functions of I.

Conclusions, which can be drawn from numerical examples such as those in fig. 1 and are not obtained from expression (2), are:

(1) For $\gamma = 0$, B(E2) values have a negligible amount of signature dependence both for $\Delta I = 1$ and $\Delta I = 2$ transitions, while for $\gamma \neq 0$ a considerable amount of signature dependence ("zigzag") appears especially in $B(\mathrm{E2}; \Delta I = 1)$ values. The "zigzag" which appears in $B(\mathrm{E2}; \Delta I = 1)$ values for $\gamma \neq 0$ become much more conspicuous for the presently investigated region of I, if the Fermi level is lower in the $h_{11/2}$ shell. In the example in fig. 1 the signature dependence of the $B(\mathrm{E2}; \Delta I = 1)$ values for $\gamma < 0$ is in phase with the one for $\gamma > 0$. However, it becomes out of phase when the rotational perturbation becomes larger. (Namely, the curve for $\gamma < 0$ changes the phase of "zigzag" with increasing I.) The conspicuous signature dependence of the calculated $B(\mathrm{E2}; \Delta I = 1)$ values comes from the constructive or destructive phase between the core E2 matrix elements with $\Delta K = 0$ and $\Delta K = 2$. – This structure of the wave functions does

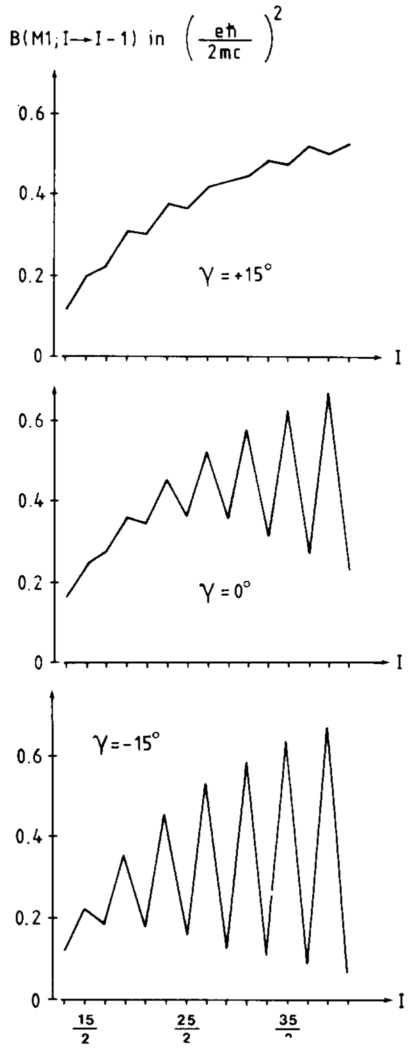

Fig. 2. Calculated B(M1) values between the proton yrast states in the $h_{11/2}$ shell particle-rotor model. Used parameters are: $\lambda = (0.10)\kappa$, $J_0\kappa = 72$, $\Delta = 0.45\,\kappa$, $g_s = 3.91$, $g_\ell = 1.0$, and $g_R = 0.42$.

not exist in the simplest version of cranking model.

(2) The B(E2; $\Delta I = 2$) values may approach the classical value of the γ-dependence, which is proportional to $[\cos(\gamma + 30°)]^2$, for larger values of I. By comparing the ratio $[\cos(15°)/\cos(45°)]^2 = 1.87$ with the ratio of $B(\mathrm{E2}, I \to I-2, \gamma = -15°)/B(\mathrm{E2}; I \to I-2, \gamma = +15°)$ estimated from fig. 1, we see that the B(E2; $\Delta I = 2$) values approach to the classical values very slowly with increasing I.

(3) Calculated values of the static quadrupole moments approach to the expression $\hat{Q}_0$ in (2), also very slowly with increasing I. The calculated values show a very small signature dependence.

In fig. 2 we show calculated B(M1) values for one of the cases in fig. 1, namely for $\lambda = (0.10)\kappa$. It can be seen that the information extracted from the B(M1) values is quite independent of that from the B(E2) values. While the B(E2) values exhibit directly information on the shape of the system, the B(M1) values are more closely related [2,7] to the signature splitting of the quasiparticle spectra. However, if the shape of the system deviates considerably from axial symmetry, the effective g-factor may have an appreciable dependence on the direction of the rotation axis as seen in the intrinsic space. Since we know very little about this dependence, the usefulness of B(M1) values for studying the nuclear structure of triaxial systems may be limited.

It would be extremely interesting to measure the ratio of the $B(\mathrm{E2}; I \to I-1)$ value to the $B(\mathrm{E2}; I \to I-2)$ value in the same nucleus both before and after the band-crossing of the core, in order to say something about the change of the γ-value at the band-crossing. It is our strong hope to get more data so that we could learn more about the triaxial shape in nuclei by using the ideas described in the present note. It is intended to analyze the available experimental data [1,8] in a subsequent publication.

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