Eur. Phys. J. Plus (2015) **130**: 117
DOI 10.1140/epjp/i2015-15117-2

The European Physical Journal Plus

Regular Article

ΔI = 1 staggering in signature partner pairs of super-deformed rotational bands in the A = 190 mass region

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Received: 19 February 2015 / Revised: 20 April 2015
Published online: 23 June 2015 –

Abstract. Several signature partner pairs in super-deformed rotational bands in the $A = 190$ mass region have exhibited $\Delta I = 1$ staggering effects in its transition energies. A total of twenty signature partner pairs of super-deformed (SD) rotational bands in the $A = 190$ mass region were investigated in this study. The intrinsic structure and the band head moment of inertia J_0 of these signature partner pairs were found to be identical. The band head spin I_0 and the band head moment of inertia J_0 of these pairs were assigned by using the VMI (variable moment of inertia) equation. The $\Delta I = 1$ staggering was also examined through the staggering index $S(I)$ formula, where interlinking transition energies between signature partner pairs were experimentally known. A large amplitude staggering was observed in these signature partner pairs. The paper indicates the possibility of a high signature splitting property and will be useful for further studies.

1 Introduction

Studies on super-deformed (SD) rotational bands are an interesting area of theoretical and experimental physics. So far, over 85 super-deformed rotational bands (SDRBs) have been reported from Au, Hg, Tl, Pb, Bi and Po nuclei in the $A = 190$ mass region alone [1,2]. Many hanging transitions with regularly spaced high-spin gamma-ray sequences were also observed in this area. However there are many unresolved problems like assigning spin and parity, identifying excitation energies relative to their ground state. This could be due to non-observable discrete linking transitions between SD state and the low-lying normal deformed (ND) state. Currently gamma-ray energies are the only experimentally available information, while calculating spin values using theoretical methods is one of the alternative approaches. Researchers have made several efforts to assigning spins of SD bands in the $A = 190$ mass region. Some of these methods were successful, like energy expansion $I\,(I + 1)$ formula [3], energy power series in terms of angular momenta [4,5], ω^2 expansion [6,7], ab formula [8–11], super-symmetric algebraic model including many-body interactions (SAM) [12–18], etc. The variable moment of inertia (VMI) equation was used in our previous studies to assign the band head spin I_0 value calculated directly from experimental spectra [19,20]. This method could successfully predict the band head spin I_0 value of 66 SD rotational bands for 22 isotopes of Au, Hg, Tl, Pb, Bi and Po in the $A = 190$ mass region. Our results were in good agreement with experimental spectra and other band head spin values available in the literature. Surprisingly, while plotting the root mean square deviation (rms) for these bands, similar intrinsic structures were observed suggesting towards signature partner pairs. The band head moment of inertia J_0 of these pairs was found to be identical. A total of 20 signature partner pairs were investigated in Hg, Tl and Pb isotopes in the $A = 190$ mass region.

Signature partner pairs are two identical bands with fixed signatures α leading to de-excitation of certain pairs of bands influenced by neighbouring nuclei. This is often observed in discrete state nuclei with extreme gamma-ray energy values. Among the signature partners, transition energies in one band may fall very close to midpoint energies of an adjacent band. Supplement to our finding are several spectroscopic studies reporting unexpected odd-even staggering effects among SD bands in this region.

The staggering effect in its transition energies consists of the sequences of states differing by two units of the angular momentum which are displaced relative to each other. This effect is often seen in long rotation sequences

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where the expected regular behaviour of energy levels with respect to spin or rotational frequencies is disturbed. This leads to splitting of the rotational sequence into two parts with states separated by $\Delta I = 4$ bifurcations resulting in a zigzag behaviour of transition energies ($E\gamma$). It was reported that some SD bands show slight $\Delta I = 2$ staggering (also called $\Delta I = 4$ bifurcation), that is the band energy of spin sequences in $I = I_0 + 4n$ ($n = 0, 1, 2, 3, \dots$) is displaced relative to the spin sequence $I = I_0 + 4n + 2$. Therefore it is considered as a function of rotational frequency or spin.

The amplitude of staggering depends on rotational bands in this region, which is related to the signature quantum numbers $r = e^{i\alpha\Pi}$, defining the admissible spin sequences in $I = \alpha \pm 2n$ ($n = 0, 1, 2\dots$). Usually two signature partner bands, with $r = +1(\alpha = 0)$ and $r = -1(\alpha = 1)$ in even mass nuclei and $r = \pm 1$ ($\alpha = \pm 1/2$) in odd mass nuclei, were observed, which are often separated by signature splitting energies. Staggering occurs at high rotational frequencies and is associated with $\Delta I = 1$ signature splitting. $\Delta I = 1$ staggering is relative to the displacement of energy levels with odd spin ($I = 1, 3, 5$) and even spin ($I = 2, 4, 6$) rotational bands. Many other studies have reported $\Delta I = 2$ staggering in SD bands in the $A = 190$ mass region [21–26]. Their spin (I) levels were relatively dependent on neighbouring $I \pm 2$. That is as discussed above: $\Delta I = 2$ staggering spin levels follow the $I = I_0 + 2$, $I_0 + 6$, $I_0 + 10$ pattern, which is relatively displaced with its neighbouring band $I = I_0$, $I_0 + 4$, $I_0 + 8$.

In this study we are verifying the possibility of a $\Delta I = 1$ signature splitting feature in signature partner pairs among SD bands using the staggering index formula. Our study is limited to only eight pairs (from Hg, Tl and Pb isotopes) among the twenty pairs which were showing $\Delta I = 1$ staggering. This is because experimentally known interlinking transition energies were available only for these bands. The formalism used to determine I_0, C and J_0 values are reported in sect. 2. The $\Delta I = 1$ staggering in the signature partner pairs in SD bands is mentioned in sect. 2.1. In sect. 3, I_0, C and J_0 values and the plots highlighting $\Delta I = 1$ staggering among signature partner pairs are reported. Our conclusion is given in sect. 4.

2 Theoretical framework

In the VMI model equation the energy levels of rotational bands is given as [27–29]

$$E_I = E_0 + \frac{I(I+1) - I_0(I_0+1)}{2J_I} + \frac{C(J_I - J_0)^2}{2}\,, \tag{1}$$

where E_0 is the band head energy of the rotational band, J_0 is the ground-state moment of inertia parameter, the variable J_I is the moment of inertia of the nucleus for each spin value and C is the restoring force constant, while, for SD bands, the transition energy is expressed as

$$E\gamma(I \to I-2) = E(I) - E(I-2). \tag{2}$$

Therefore, we get [19]

$$E\gamma(I \to I-2) = \frac{[I(I+1) - (I-2)(I-1)]}{2J_0} - \frac{[(I-2)(I-1)]^2 - [I(I+1)]^2}{8CJ_0^4}\,. \tag{3}$$

We have considered only two parameters: J_0 and the softness parameter C. These two parameters were determined by fitting least square procedures of all the known energies in the observed spectra. The band head spin I_0 is determined in terms of observed transition energies ratio. The transition energy ratio (R) is expressed as

$$R = \frac{E\gamma(I+2 \to I)}{E\gamma(I \to I-2)}\,. \tag{4}$$

The root mean square (rms) deviations of transition energies for different spin values were also calculated. The calculated energies were in good agreement with the observed values when I_0 was assigned accurately. The rms value is lowest for band head spin value. If I_0 shifts away from the accurate value by ± 1, a rapid shift in the rms deviation can be observed. The root mean square deviation is expressed as

$$\chi = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_\gamma^{\rm cal}(I_i) - E_\gamma^{\rm exp}(I_i)}{E_\gamma^{\rm exp}(I_i)}\right|^2}\,, \tag{5}$$

where n is the total number of transitions involved in the fitting. It was observed that the rms deviation plots are nearly the same for the intrinsic structure in the signature partners.

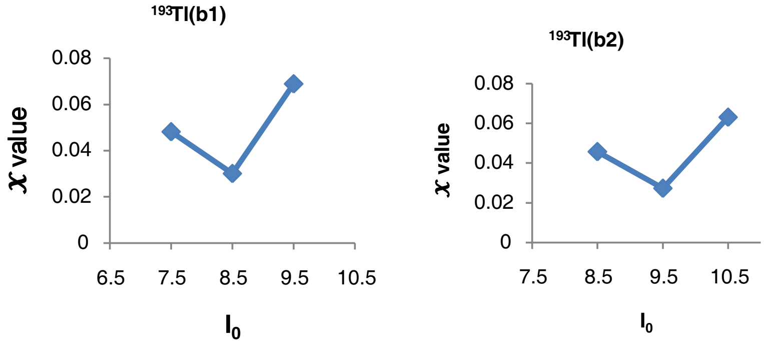

Fig. 1. Rms deviation plots for various spin assignments showing the signature partner pairs in ^{193}Tl (band 1) and ^{193}Tl (band 2) nuclei.

2.1 ΔI = 1 staggering in signature partner pairs of SD bands

Several studies have used $\Delta^2 E\gamma(I)$, which is the difference between the average $I+2 \rightarrow I \rightarrow I-2$ transition energies in one band and the $I+1 \rightarrow I-1$ transition energies in its signature partner pair [9,30,31]. In this paper we have calculated energies in staggering index $S(I)$ which could relate the experimentally interlinking transition energies between two signature partner pairs. $\Delta I = 1$ staggering in signature partner pairs in SD bands are represented in terms of energies in staggering index $S(I)$ [32] defined as

$$S(I) = [E(I) - E(I-1)] - \frac{[E(I+1) - E(I)] + [E(I-1) - E(I-2)]}{2}\,. \tag{6}$$

3 Results

Using the VMI equation, the band head spin value I_0 and transition energies were calculated for 66 SD rotational bands of 22 isotopes of Au, Hg, Tl, Pb, Bi and Po in the $A = 190$ mass region, as reported in our previous works [19, 20]. Around twenty signature partner pairs of Hg, Tl and Pb isotopes were investigated from these SD bands. The two parameters, band head moment of inertia J_0 and restoring constant (C), were calculated from experimentally known transition energies using the best fit method (BFM). The rms deviation was obtained from the calculated transition energies with experimental spectra, and it was used to verify the band head spin value I_0 assignment for signature partner pairs in these bands.

For each pair of SD bands, the rms deviation was plotted against the I_0 value which showed similar intrinsic structure suggesting that they are signature partner pairs. In these signature partner pairs, transition energies in one band were falling very close to the midpoint energies of adjacent transitions in the other band. For example, in fig. 1, ^{193}Tl of band 1 is comparable with ^{193}Tl band 2.

The band head moment of inertia J_0, the restoring constant (C) and the I_0 value of signature partner pairs were reported in table 1. J_0 of signature partner pairs were found to be identical suggesting they do have the same intrinsic structure, which is always sensitive to the spin assignment. The restoring constant (C) is almost the same in signature partner pairs of ^{193}Hg (band 3 and band 4), ^{194}Hg (band 2 and band 3), ^{195}Hg (band 1 and band 2), ^{195}Hg (band 3 and band 4), ^{194}Tl (band 1 and band 2), ^{194}Tl (band 3 and band 4), ^{193}Pb (band 5 and band 6) and ^{194}Pb (band 2 and band 3). We also observed that the transition energies for signature partner pairs in SD bands vary in a regular pattern with increasing spin values. Therefore large amplitude staggering patterns at $\Delta I = 1$ are possible, which suggests signature splitting in most of these pairs.

The staggering index $S(I)$ formula is the interlinking between the transition energies. For $\Delta I = 1$ staggering in signature partner pairs, the staggering index $S(I)$ was plotted against spin values. The results are shown in fig. 2. As mentioned earlier, staggering property was studied for 8 cases of 20 signature partner pairs of Hg, Tl and Pb isotopes in the $A = 190$ mass regions as experimental information was available only for them. They are ^{193}Hg (band 1) and ^{193}Hg (band 2), ^{191}Tl (band 1) and ^{191}Tl (band 2), ^{193}Tl (band 1) and ^{193}Tl (band 2), ^{195}Tl (band 1) and ^{195}Tl (band 2), ^{196}Pb (band 2) and ^{196}Pb (band 3), ^{197}Pb (band 1) and ^{197}Pb (band 2), ^{193}Hg (band 3) and ^{193}Hg (band 4), ^{193}Pb (band 5) and ^{193}Pb (band 6). Most of the signature partner pairs show large amplitude staggering. The staggering pattern observed in SD bands are similar to normal deformed bands where the Coriolis force plays an important role [33]. Further studies in this area are required to confirm these findings.

Table 1. The band head moment of inertia (J_0) and band head spin (I_0) of 20 signature partners pairs in the $A = 190$ mass region, where b1, b2, b3, ..., are given for band 1, band 2, band 3, ..., respectively.

Signature partners	Assigned I_0(VMI)	(J_0) $\hbar^2$ KeV^{-1}	C (10^6) KeV^3
^{191}Hg (b2)	10.5	0.0951	4.57
^{191}Hg (b3)	11.5	0.0943	7.95
^{193}Hg (b1)	9.5	0.0934	5.47
^{193}Hg (b2)	10.5	0.0935	8.17
^{193}Hg (b3)	9.5	0.0932	8.28
^{193}Hg (b4)	10.5	0.0936	8.10
^{194}Hg (b2)	8	0.0940	8.02
^{194}Hg (b3)	9	0.0940	8.02
^{195}Hg (b1)	11.5	0.0936	6.80
^{195}Hg (b2)	10.5	0.0934	6.52
^{195}Hg (b3)	10.5	0.0979	13.70
^{195}Hg (b4)	11.5	0.0980	13.40
^{191}Tl (b1)	11.5	0.0927	9.02
^{191}Tl (b2)	10.5	0.0921	8.31
^{193}Tl (b1)	8.5	0.0960	8.43
^{193}Tl (b2)	9.5	0.0961	9.71
^{194}Tl (b1)	12	0.0988	11.78
^{194}Tl (b2)	9	0.0999	11.14
^{194}Tl (b3)	10	0.0948	7.24
^{194}Tl (b4)	9	0.0948	7.24
^{194}Tl (b5)	8	1.007	7.41
^{194}Tl (b6)	9	1.010	9.70
^{195}Tl (b1)	5.5	0.0955	11.03
^{195}Tl (b2)	6.5	0.0950	7.70
^{193}Pb (b3)	10.5	0.0946	9.68
^{193}Pb (b4)	11.5	0.0940	7.70
^{193}Pb (b5)	8.5	0.0930	8.35
^{193}Pb (b6)	9.5	0.0930	8.42
^{194}Pb (b2)	10	0.0944	6.60
^{194}Pb (b3)	11	0.0944	6.91
^{195}Pb (b1)	7.5	0.0987	13.17
^{195}Pb (b2)	6.5	0.0980	2.77
^{195}Pb (b3)	7.5	0.0905	4.49
^{195}Pb (b4)	8.5	0.0924	7.20
^{196}Pb (b2)	8	0.0918	7.20
^{196}Pb (b3)	9	0.0918	3.40
^{197}Pb (b1)	5.5	0.0987	13.17
^{197}Pb (b2)	4.5	0.0975	10.35
^{193}Hg (b1)	9.5	0.0930	5.47
^{193}Pb (b5)	8.5	0.0930	8.35

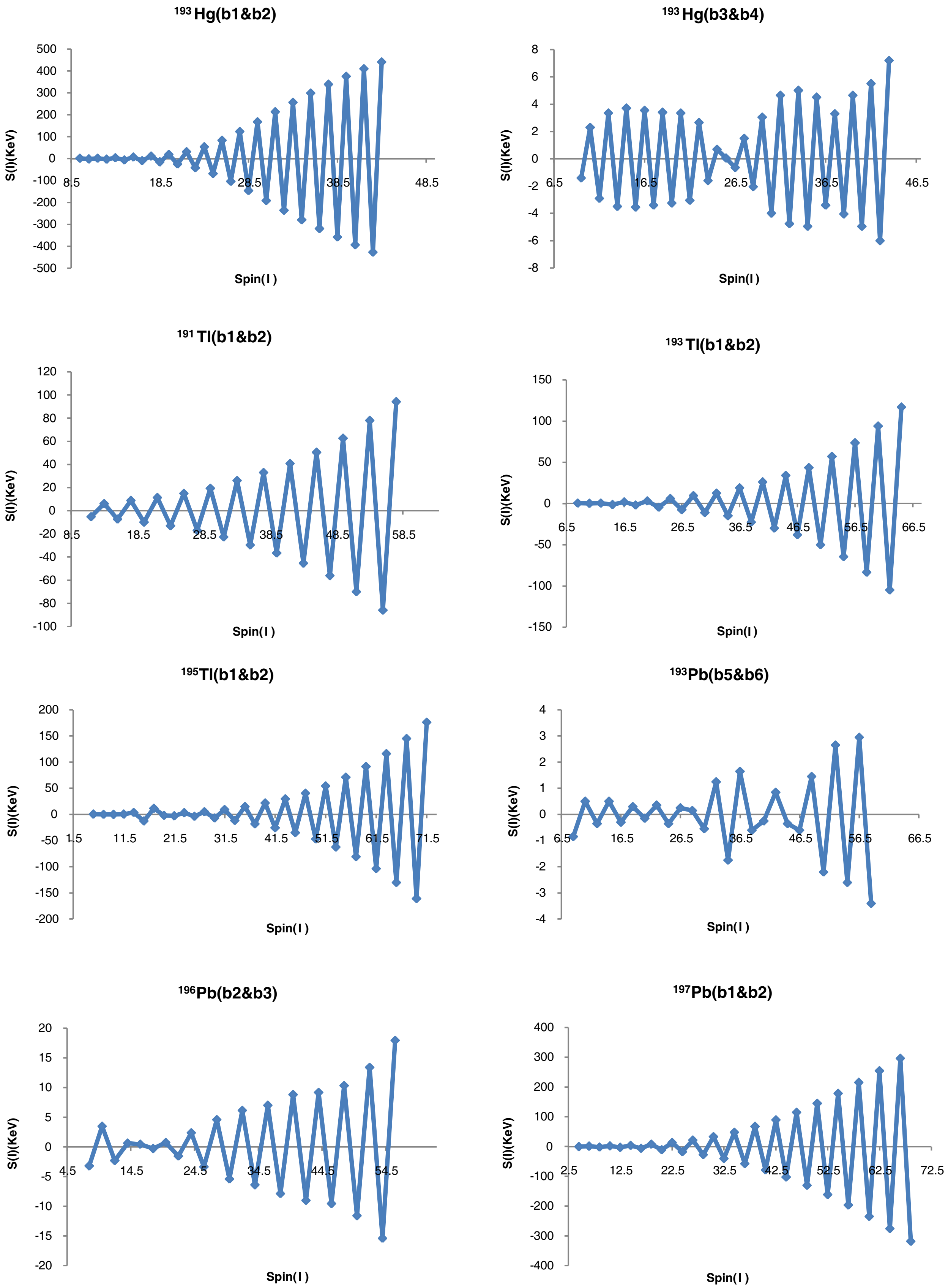

Fig. 2. The $\Delta I = 1$ staggering index $S(I)$ as a function of spin (I) for the signature partners in the $A = 190$ mass region.

4 Conclusion

This paper reports twenty signature partner pairs in SD bands for Hg, Tl and Pb isotopes in the $A = 190$ mass region. For each pair of SD bands the rms deviation was plotted against spin I_0 value, showing similar pattern in the rms plots, confirming signature partner property. Band head moments of inertia J_0 and intrinsic structure of these twenty signature partner pairs are found to be identical. $\Delta I = 1$ staggering in signature partner pairs for isotopes of Hg, Tl and Pb were verified through staggering index formula in 8 cases as experimentally known linking transition energies were available only for these cases. Most of these signature partner pairs show a large amplitude staggering pattern. For most cases, $\Delta I = 1$ staggering starts from low values at low spin values, it gradually increase as the spin increases with the exception of ^{193}Hg (band 3 and band 4) and ^{193}Pb (band 5 and band 6) .We will discuss the staggering phenomena in a following paper.

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