*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Present address: Physics Department, University of Tokyo, Tokyo, Japan.

‡Present address: Physics Department, University of Munich, Munich, West Germany.

[1]Decoupled bands are ones in which the angular momentum of the odd particle has been decoupled from the nuclear symmetry axis by the Coriolis interaction, and is aligned with the rotation axis or, more correctly, with the total spin axis.

[2]F. S. Stephens, in *Proceedings of the International Conference on Nuclear Physics, Munich, West Germany, 1973*, edited by J. de Boer and H. J. Mang (North Holland, Amsterdam, 1973), Vol. 2, p. 367.

[3]F. S. Stephens, R. M. Diamond, and S. G. Nilsson, Phys. Lett. 44B, 429 (1973).

[4]E. Grosse, F. S. Stephens, and R. M. Diamond, Phys. Rev. Lett. 31, 840 (1973); J. R. Leigh, R. M. Diamond, K. H. Maier, R. Nordhagen, and F. S. Stephens, Lawrence Berkeley Laboratory Report No. UCRL-20426, 1971 (unpublished).

[5]R. M. Diamond, F. S. Stephens, W. H. Kelly, and D. Ward, Phys. Rev. Lett. 22, 546 (1969).

[6]R. Nordhagen, G. Goldring, R. M. Diamond, K. Nakai, and F. S. Stephens, Nucl. Phys. A142, 577 (1970).

[7]A. Abragam and R. V. Pound, Phys. Rev. 92, 943 (1953).

Evidence for Asymmetric Shapes from High-Spin Odd-A Spectra*

J. Meyer ter Vehn,† F. S. Stephens, and R. M. Diamond

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 15 April 1974)

The model of a single-j nucleon coupled to an *asymmetric* rotor is applied to new experimental results on yrast bands in ^{187}Ir, ^{195}Au, and ^{197}Tl. The core asymmetry is shown to account for systematic trends in the high-spin spectra. Information on the shape dependence of moments of inertia is obtained which supports the idea of nuclear flow of irrotational type.

Rotational bands built on high-j states of unique parity in odd-A nuclei have a simple theoretical interpretation and can give rather detailed information about the nuclear shape and moments of inertia. In particular, this holds for nuclei with small deformations in the vicinity of closed shells in which the odd nucleon represents either a pure hole or a pure particle in the high-j orbital. It has been shown that a particle (hole) on a prolate (oblate) core tends to decouple from collective rotation by aligning its angular momentum with the rotation axis of the core.[1] This leads to decoupled bands with spin sequence j, $j+2$, $j+4$, ..., and energy spacings equal to those of the core. On the other hand, a hole (particle) in the prolate (oblate) core is strongly coupled and displays a normal rotational spectrum with spin sequence j, $j+1$, $j+2$, ...

The present calculation based on a single-j nucleon coupled to an asymmetric rotor shows that there is a continuous transition from decoupled to strongly coupled bands obtained by changing the shape of the nucleus from prolate to oblate through a series of asymmetric shapes. In this transition many levels change energy rather sharply relative to others; for example, the "un-favored" states, $I=j+1$, $j+3$, ... all drop considerably relative to the "favored" ones, $I=j$, $j+2$, ... This complex pattern of levels provides a severe test of the asymmetric-rotor model, and one of the objectives of this Letter is to apply this test to several nuclei in the $Z=80$ region. It is also important to realize that some new types of information can be extracted from these odd-A spectra, provided the model is applicable. This is basically because the γ dependence enters not only through the rotational Hamiltonian as in the even-even nuclei, but also through the Hamiltonian of the single particle. One can, therefore, easily differentiate between oblate and prolate shapes, and this determines γ in a range from 0–60° rather than only 0–30°. Furthermore, the level shifts, like the favored-unfavored one mentioned, are sensitive to the way the moments of inertia depend on shape, and thus can be used to test the assumption of irrotational flow.

The model Hamiltonian

$$H = \sum_{n=1}^{3} \frac{(J_n - j_n)^2}{2\mathcal{I}_n} + H_p \tag{1}$$

consists of the core rotational energy and the

particle's potential energy

$$H_p = k[(\cos\gamma)Y_0^2 + (\sin\gamma/\sqrt{2})(Y_2^2 + Y_{-2}^2)], \qquad (2)$$

where J_n, j_n are total and particle angular mo-
menta, and the Y_μ^2 are spherical harmonics. Ir-
rotational moments of inertia, $\vartheta_n = \frac{4}{3}\vartheta_0 \sin^2(\gamma$
$-n2\pi/3)$, for the axes $n = 1, 2, 3$ are chosen, and
the potential energy strength is taken as $k = (16\pi/$
$5)^{1/2}206A^{-1/3}\beta$ MeV consistent with empirical sin-
gle-particle spacings at $\gamma = 0$ for a nucleus with
mass A. The deformation parameter β, the
asymmetry parameter γ, and ϑ_0 can be connected
by the first 2^+ energy of the core which is given
for an asymmetric rotor[2] by

$$E_{2^+} = \frac{6\hbar^2}{2\vartheta_0} \frac{9 - [81 - 72\sin^2(3\gamma)]^{1/2}}{4\sin^2(3\gamma)}, \qquad (3)$$

where the inertial parameter is $6\hbar^2/2\vartheta_0 \cong 1225$
$\times A^{-7/3}\beta^{-2}$ MeV according to a general empirical
rule.[3] The eigenfunctions of Eq. (1) can be writ-
ten in the form

$$\psi_{IM} = \sum_{K,\Omega} C_{K,\Omega}^{(I,j)}$$
$$\times [D_{MK}^{(I)}\chi_\Omega^{(j)} + (-1)^{I-j}D_{M-K}^{(I)}\chi_{-\Omega}^{(j)}], \qquad (4)$$

where $\chi_\Omega^{(j)}$ denotes the single-particle wave func-
tion and $D_{MK}^{(I)}$ the rotational D functions. The
underlying D_2 symmetry group[4] restricts the
summation in Eq. (4) to $|K - \Omega| = 2m$, $m = 0, 1, 2,$
$\dots$, and $\Omega > 0$. A numerical solution of this mod-
el has been published earlier by Pashkevich and
Sardaryan[5] for $I \leq \frac{9}{2}$ and $j = \frac{5}{2}$. We have calculat-
ed all the yrast levels up to $I = \frac{23}{2}$ built on a $j = \frac{11}{2}$
single-particle state and show them in Fig. 1 as
functions of the asymmetry parameter γ for a
typical deformation, $\beta = 5A^{-2/3}$. The decoupled
limit at $\gamma = 0$ with core energies proportional to
6, 20, 42, $\dots$ for the favored $\frac{15}{2}$, $\frac{19}{2}$, $\frac{23}{2}$ states
relative to the $\frac{11}{2}$ band head is clearly seen, as
well as the strongly coupled limit at $\gamma = 60°$. The
unfavored $\frac{13}{2}$, $\frac{17}{2}$, $\frac{21}{2}$ states lie relatively high at γ
$= 0$, but come down sharply around $\gamma \approx 25°$. The
strongly coupled level order persists over the
whole range $30 \leq \gamma \leq 60°$. In this region, the parti-
cle angular momentum mainly points along the 2
axis which becomes the oblate symmetry axis at
$\gamma = 60°$, whereas the core angular momentum is
perpendicular to it, since the irrotational moment
of inertia about the 2 axis is small and vanishes
for $\gamma = 60°$. The low-spin levels corresponds to
opposite directions of core and particle angular
momentum and may also be grouped into favored
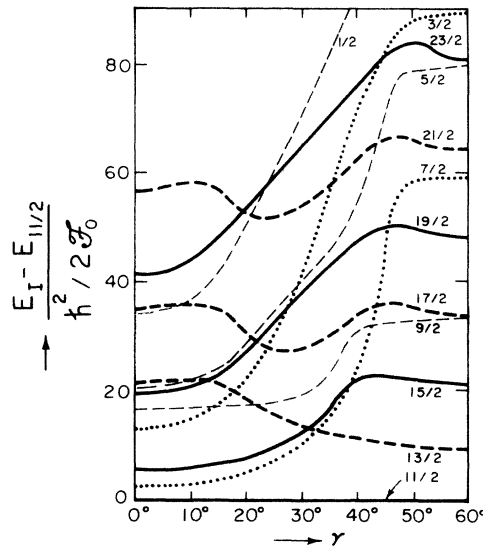$\frac{7}{2}$, $\frac{3}{2}$ and unfavored $\frac{9}{2}$, $\frac{5}{2}$, $\frac{1}{2}$ states.

FIG. 1. Spectrum of a $j = \frac{11}{2}$ particle coupled to an
asymmetric rotor with all yrast levels $I \leq \frac{23}{2}$ as func-
tions of γ.

Because of the characteristic behavior of these
different level groups as functions of γ, they
represent a sensitive test for the shape as well
as the moments of inertia. Unfortunately, up to
now there are only a few cases known where both
favored and unfavored states have been seen in
experiment. In Figs. 2 and 3, a comparison with
recent odd-A spectra is shown. The parameters
for the calculation are determined from the adja-
cent even nuclei: ϑ_0 and β from the first 2^+ en-
ergy, and γ from the energy ratio of the second
2^+ level and the nearest member of the ground-
state band. Thus, the calculated odd-A spectra
have no adjusted parameter. For comparison,
calculations with $\gamma = 0$ are also shown. They dif-
fer slightly from earlier results,[6] since the pres-
ent calculations do not include any pairing ef-
fects; the Fermi energy is placed at $-\infty$ $(+\infty)$ for
particles (holes) rather than at a finite value. All
experimental spectra—both odd and even mass
—are compressed as compared to the calculated
ones; this reflects considerable nonadiabatic ef-
fects not taken into account by the present rigid-
rotor model.

^{187}Ir (Fig. 2).—A decoupled band on a $\frac{9}{2}^-$ state
has been observed including three unfavored
states.[7] The band is consistent with an $h_{9/2}$ pro-
ton coupled to a prolate ^{186}Os core. The (β, γ)
fit, which reproduces the relative order of ground-
and γ-band levels up to 1.5 MeV in ^{186}Os, gives
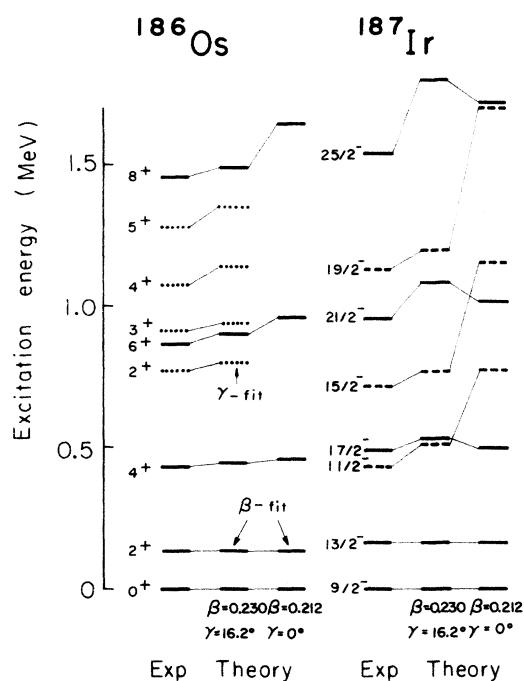comparably good agreement for ^{187}Ir. In particu-

FIG. 2. Comparison of calculated and experimental spectra in ^{187}Ir with parameters β and γ fitted to ^{186}Os.

lar, the unfavored $\frac{11}{2}$, $\frac{15}{2}$, $\frac{19}{2}$ states appear at lower energies in the correct order with the favored states, in contrast to the $\gamma = 0$ fit.

^{195}Au (Fig. 3).—The observed spectrum of negative-parity states represents a favorable case where both high-spin states from (HI, $xn\gamma$) experiments[8] and low-spin states from β decay[9] are known. The spectra of ^{191}Au and ^{193}Au are quite similar,[8] so that these states are characteristic for the Au nuclei in this mass region. The decoupled band is consistent with an $h_{11/2}$ proton hole in an oblate ^{196}Hg core. Applying the ^{196}Hg fit to the ^{195}Au calculation, the experimental level order is almost completely reproduced for high- as well as for low-spin states. Even levels at 962 and 1186 keV, tentatively assigned as second $\frac{11}{2}^-$ and $\frac{15}{2}^-$, occur in the expected places. Note that all the model states calculated to be in the energy region shown have been seen experimentally except for the $(\frac{5}{2}^-)_1$, $(\frac{7}{2}^-)_2$, $(\frac{9}{2}^-)_2$, and $(\frac{13}{2}^-)_2$ states with calculated energies 1507, 1672, 1440, and 1667 keV, respectively, relative to the $\frac{11}{2}^-$ energy. These could easily have been missed in both types of experimental population. The sharp decrease in energy of unfavored high-spin
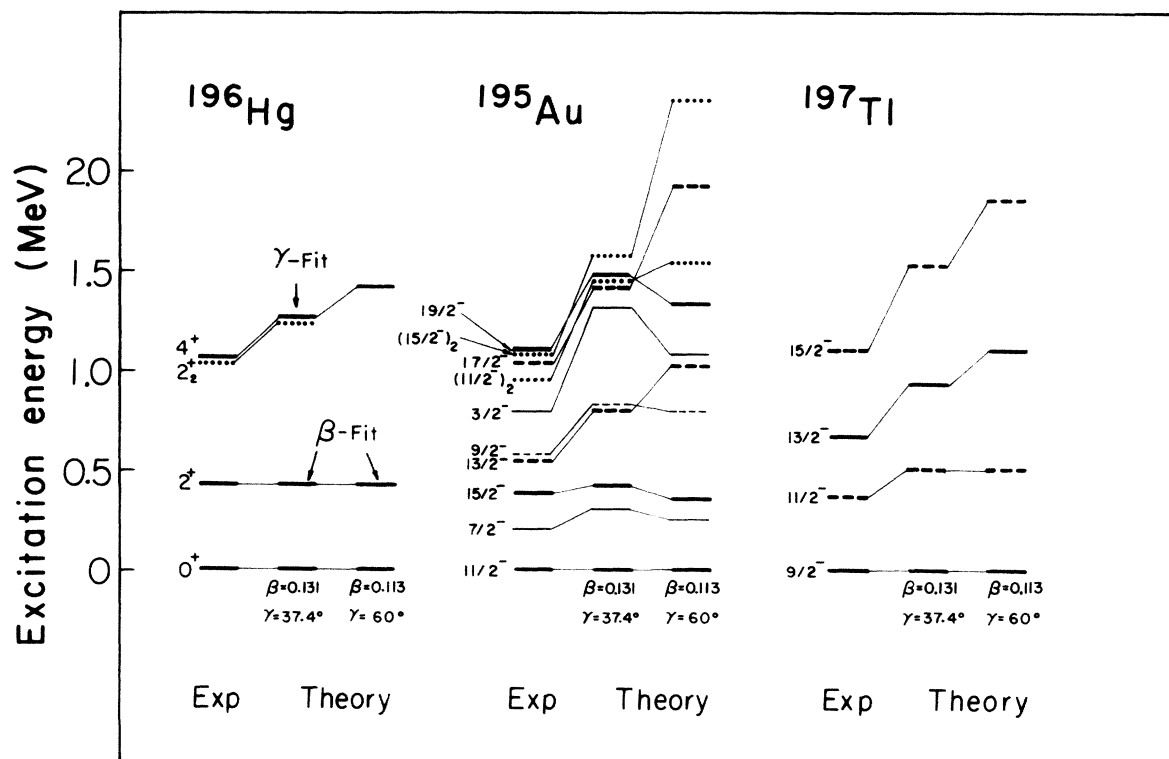

FIG. 3. Comparison of calculated and experimental spectra in ^{195}Au and ^{197}Tl with β and γ fitted to ^{196}Hg.

states, as compared to the $\gamma = 60°$ fit, occurs in agreement with experiment, and strongly supports an asymmetric shape. Again, the overall compression of the experimental spectrum as compared to the calculated one appears similar for ^{196}Hg and ^{195}Au.

^{197}Tl (Fig. 3).—The negative-parity band[10] in ^{197}Tl stems from the same ^{196}Hg core as for ^{195}Au, but now it is built on an $h_{9/2}$ particle instead of an $h_{11/2}$ hole. It represents the unique case of a strongly coupled rotational band on an oblate core, as discussed below. Although the level order would be reproduced by any γ within $30 < \gamma < 60°$, the characteristic approach of the $j+1$ and $j+2$ levels near $\gamma = 30°$ as seen in Fig. 1 is indeed present in the experimental spectrum.

The present results not only indicate triaxial shapes, but also show that the moments of inertia change like those of irrotational flow as the nucleus changes shape (γ). This is most apparent in the oblate region where, at $\gamma = 60°$, rigid flow would have its maximum moment of inertia about the oblate symmetry axis, whereas irrotational flow has a vanishing moment of inertia about the same axis. As a consequence, rigid flow would lead to decoupled bands at $\gamma = 60°$ and, in fact, for all γ, whereas irrotational flow yields strongly coupled bands in the oblate region as is seen, in fact, in the odd-mass Tl isotopes.

An open question remains about the nature of the observed shape asymmetry. Calculations[11] of potential energy surfaces in the $A = 190$ region do not show sharp minima at any γ. Therefore, one would expect soft, γ-fluctuating shapes rather than triaxial shapes with sharp γ. But it is then difficult to understand why the γ values for adjacent even and odd nuclei and within the odd-A spectra are so stable. Dynamic stabilization

of triaxial shapes is expected[12] for core spins $I \geq 6$; but this would affect only the highest-spin states of the examples studied. An answer might be that the actual potential energy minima at $\gamma \neq 0$ are more pronounced than predicted by existing calculations.

[1]F. S. Stephens, R. M. Diamond, J. R. Leigh, T. Kammuri, and K. Nakai, Phys. Rev. Lett. 29, 438 (1972).

[2]A . S. Davydov and G. F. Filippov, Nucl. Phys. 8, 237 (1958).

[3]L. Grodzins, Phys. Lett. 2, 88 (1962).

[4]J. P. Davidson, Collective Models of the Nucleus (Academic, New York, 1968).

[5]V. V. Pashkevich and R. A. Sardaryan, Nucl. Phys. 65, 401 (1965).

[6]F. S. Stephens, R. M. Diamond, D. Benson, Jr., and M. R. Maier, Phys. Rev. C 7, 2163 (1973).

[7]S. André, J. Boutet, J. Jastrzebski, J. Lukasiak, J. Rivier, C. Sebille-Schuck, Z. Sujkowski, and J. Treherne, in Proceedings of the International Conference on Nuclear Physics, Munich, Germany, 1973, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973), p. 196.

[8]P. O. Tjøm, M. Maier, D. Benson, Jr., F. S. Stephens, and R. M. Diamond, to be published.

[9]J. Frána, A. Špalek, M. Fišher, and A. Kokeš, Nucl. Phys. A165, 625 (1971).

[10]J. O. Newton, S. D. Cirilov, F. S. Stephens, and R. M. Diamond, Nucl. Phys. A148, 593 (1970), and to be published.

[11]K. Kumar and M. Baranger, Nucl. Phys. 110, 529 (1968).

[12]J. Meyer ter Vehn, to be published.