INSTITUTE OF PHYSICS PUBLISHING PHYSICA SCRIPTA
Phys. Scr. **T125** (2006) 139–141 doi:10.1088/0031-8949/2006/T125/032

Vibrational excitations and tilted rotation in ^{163}Lu

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Received 15 July 2005
Accepted for publication 1 September 2005
Published 28 June 2006
Online at stacks.iop.org/PhysScr/T125/139

Abstract
Using a microscopic self-consistent model, we analyse the excitation built upon the rotational band in ^{163}Lu, which has been identified as a wobbling excitation on top of the rotation of a triaxial, strongly deformed shape. We find that the presence of pairing correlations substantially affects the energy of the excitation. Our calculations predict the onset of tilted rotation at a critical rotational frequency where the energy of the excitation approaches zero.

PACS numbers: 21.60.−n, 21.60.Jz

The analysis of the interplay of a nuclear shape and the orientation of angular momentum led to various important discoveries [1, 2]. Non-axial deformation can give rise to a new type of dynamics involving the orientation degree of freedom. This includes wobbling excitations [1, 3, 4] which are created by a fluctuation of the angular momentum direction around one of the principal axes of a deformed nucleus. Their characteristic feature is strong E2 transitions with $\Delta I = \pm 1\hbar$ connecting the excited (wobbling) states with the vacuum states. One may suggest for non-axial shapes a possible transition from a wobbling mode with a rotational axis fluctuating around one principal axis towards a tilted rotation [5]. The recent interest in wobbling excitations was sparked by the discovery of a particular excited rotational band above a rotational band in ^{163}Lu [4], which is associated with a triaxial, strongly deformed (TSD) nuclear shape. The lowest band is called the TSD1 band, while the excited one is denoted as the TSD2 band. The theoretical description of those band structures was done in terms of the phenomenological, particle plus rotor model [6] and a non self-consistent, microscopic analysis (within a mean field plus random phase approximation approach) [7, 8]. Based on the solution of the microscopic equation in [8], it was concluded that the pairing correlations do not affect the wobbling phonon, and this should be considered as a specific feature of such excitations. On the other hand, it was also found that the wobbling excitations are very sensitive to a single particle alignment. It is however well known that the alignment decreases the pairing correlations. One of our goals is to clarify how pairing correlation affects the excitation spectrum in a calculation based on the self-consistent cranking + random phase approximation (CRPA). We will also study a possible transition to a tilted solution.

We start with a pairing + QQ Hamiltonian [9]:

$$\hat{H} = \sum_k \epsilon_k \hat{c}_k^\dagger \hat{c}_k - \frac{\kappa}{2}\hat{Q}\cdot\hat{Q} - \sum_{\tau=n,p} G_\tau \hat{P}_\tau^\dagger \hat{P}_\tau. \qquad (1)$$

Here ϵ_k is the single-particle energy of the spherical modified oscillator Hamiltonian $\hat{h}_{\text{sph}}$ [10]. The operators $\hat{c}_k^\dagger(\hat{c}_k)$ are fermion creation (annihilation) operators, with the suffix $k(l)$ labelling a complete set of quantum numbers. The quadrupole residual interaction, $\hat{Q}\cdot\hat{Q}$ is a sum over the five components of the quadrupole operators. The pairing operator $\hat{P}$ has the form $\hat{P}_\tau^\dagger = \sum_{k>0} \hat{c}_{k\tau}^\dagger \hat{c}_{\bar{k}\tau}^\dagger$. The index $\bar{k}$ refers to the time conjugated state. To study the rotational properties of the system we performed the transformation into the rotating frame $\hat{H}' = \hat{H} - \vec{\omega}\cdot\vec{\hat{J}}$, where $\vec{\hat{J}}$ represents the angular momentum operators and $\vec{\omega} = \omega(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ is the rotational angular frequency vector. ω is the magnitude of the rotational frequency and θ and φ are tilt angles. The mean field part of H' can be written as

$$\hat{h}'_{\text{MF}} = \hat{h}_0 - \vec{\omega}\cdot\vec{\hat{J}} - \tfrac{2}{3}\hbar\omega_0\epsilon_2(\hat{Q}_0\cos\gamma - \hat{Q}_2\sin\gamma) - \sum_\tau \Delta_\tau(\hat{P}_\tau^\dagger + \hat{P}_\tau) - \lambda_\tau \hat{N}_\tau, \qquad (2)$$

where Δ_τ is the pair field strength and λ_τ is the constraint for the particle numbers. Further, ϵ_2 and γ denote the deformation parameters in the intrinsic frame of reference defined by the self-consistency conditions (see details in [2]). Using the Bogoliubov transformation we obtain Hartree–Bogoliubov equations that are solved for the TSD1 band in ^{163}Lu, with and without the pairing field. The major shells $N = 4$–6 are considered for both protons and neutrons. We used $\Delta N = 0$ quadrupole interaction only. This type of interaction provides quite reasonable results and is sufficient for studying the pairing effects on low-lying excitations. The pairing strength is adjusted to reproduce the global fit of the odd–even mass differences for the ground state of even–even nuclei [11]. The proton pair field is reduced by 20% due to the odd proton. The quadrupole interaction strength is fitted to reproduce the ground state deformation obtained with Nilsson–Strutinsky calculations.

For $\hbar\omega < 0.5$ MeV, we find a principal axis rotation as the lowest solution. At $\hbar\omega \approx 0.25(0.4)$ MeV the proton (neutron) pair field disappears (see figure 1). We also find increasing γ deformation and a slowly changing ϵ_2 deformation along the band, see figure 2. At small rotational frequencies strong proton and neutron pair fields affect the deformation. In contrast to the unpaired case, the calculations with the pairing forces predict a small γ-deformation at low rotational frequencies. Since the triaxial minimum is quite shallow in the unpaired calculations, the presence of the pair field is enough to almost restore the axial symmetry. With the increase of the rotational frequency the equilibrium deformations manifest a triaxially, strongly deformed shape of ^{163}Lu. Once the self-consistent mean cranking solutions are found, we apply the quasi-boson approximation in standard way [9] in order to construct the vibrational excitations by the RPA approach. The CRPA Hamiltonian is diagonalized by solving the equations of motion. The RPA equations have several spurious solutions related to the symmetries, i.e. the rotational invariance and the particle number conservation, broken in the mean field calculations. If the mean field problem is solved with a high accuracy the spurious solutions will be completely decoupled from the physical solutions.

Our results, as shown in figure 3, without the pairing correlations are similar to the ones obtained in [7]. The lowest solution is of negative signature and is the only collective negative signature solution below 1 MeV excitation energy. The calculations predict an almost constant phonon excitation energy up to $\hbar\omega \approx 0.4$ MeV. Above this value we obtain a rapid decrease of the excitation energy, which leads to transition into a stable tilted solution, see figure 3. The transition to a tilted solution is a clear signal that the lowest RPA solution before the onset of tilted rotation is a fluctuation of the angular momentum relative to the intrinsic frame in the φ-direction. This is consistent with an interpretation of this solution as a wobbling excitation. In contrast to the conclusion of [8], the pairing interaction dramatically changes the results at small rotational frequencies. The excitation energy is substantially larger at small rotational frequencies, while it decreases with the increase of the rotational frequency. The rate of the reduction as a function of the rotational frequency is slightly faster than is seen in the experiment. After the collapse of both pair fields at $\hbar\omega \approx 0.4$ MeV, the

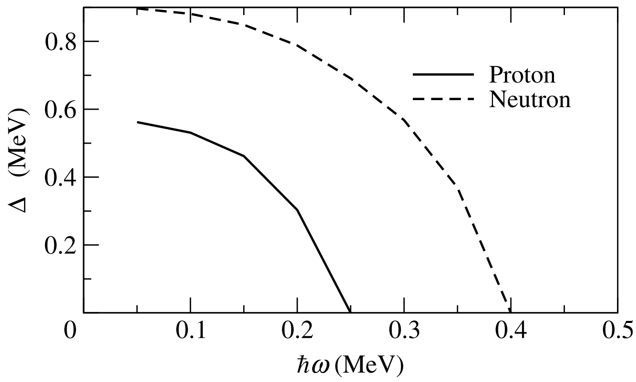

Figure 1. The neutron and proton pair field in the TSD1 band in ^{163}Lu.

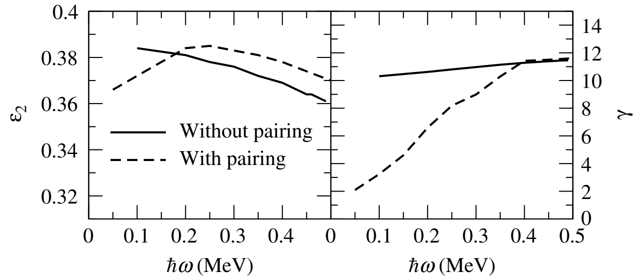

Figure 2. The deformation parameters ϵ_2 and γ, associated with the TSD1 band in ^{163}Lu.

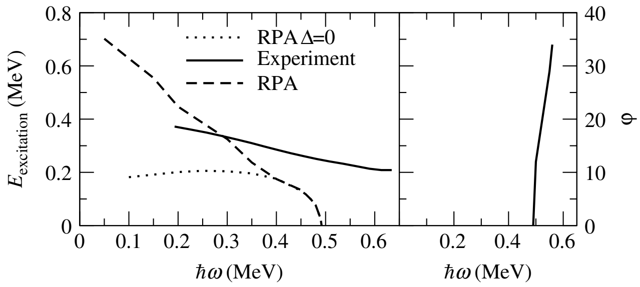

Figure 3. The excitation energy of the TSD2 band relative to the TSD1 band in ^{163}Lu. Results are compared to the experimental [4]. In the right panel we show the tilt angle ϕ.

paired and unpaired calculations predict similar results. Large values of the excitation energy at small rotational frequencies are brought about by strong pairing fields that reduce γ deformation. The ratio of the in-band M1 and E2 transitions to out of band E2 transitions for the lower part of the TSD2 band is shown in figure 4. We use a charge of 0.5 (1.5) for neutrons (protons). Our calculated M1 transition rate is ten times larger than that seen in the experiment, and we get a (four times) too small E2 transition rate.

Summarizing, the self-consistent treatment of the excitations in the CRPA provides a satisfactory description of the experimental excitation energy of the TSD2 band in ^{163}Lu which is usually interpreted as a wobbling excitation. We conclude that the pairing interactions change the energy of the phonon, at least, indirectly by changing the γ deformation. The E2 strength is too weak in our calculation and the M1 transition rate is much too strong. In addition, our calculations indicate the onset of a tilted rotation in ^{163}Lu above a critical

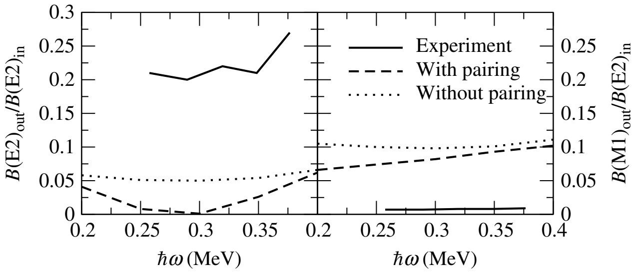

Figure 4. The ratio of the transition probability from the TSD2 band to the lower band and the in-band transitions. The results are compared with the experimental data [4].

rotational frequency, preceded by the RPA excitation energy going to zero. Further investigations on the structure of these excitations is in progress.

Acknowledgments

DA acknowledges support from EPSRC (UK). RGN acknowledges support from the Ramón y Cajal programme and from DGI (Spain).

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