Ground State Rotational Bands of Deformed e-e Nuclei

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Abstract

In the frame work of the hydrodynamical model, a new model of the ground state rotational bands of deformed e-e nuclei is developed by introducing the variable moment of inertia, and the effect of β - and γ - vibrational bands. The model is applied to calculate the energies of the ground state band of ^{158}Dy . The results of our calculations are in close agreement with data compared with other existing models.

Introduction

Recently Abzouzy and Antony^[1] have shown that the semi-empirical expression of Sood^[2] works well for Th and U nuclei, which is obtained from the power series of angular momentum factor I(I+1) as

$$\frac{C}{B} = \frac{D}{C} = \dots = N \frac{B}{A} \tag{1}$$

where A, B, C, D are the coefficients of successive order correction terms^[3]. They have then assigned N to depend on two parameters

$$N = a - bI \tag{2}$$

and deduced their values^[1]: a = 2.55 and b = 0.05 for the best fit.

Theory and formalism

A more systematic calculations can be carried out by considering only the Rotational-Vibrational model RVM of Bohr and Mottelson^[4]. This was done by adding the effect of nuclear rotation^[2] on the moment of inertia, thus one writes

$$E(J) = A(J)J(J+1) - B(J)J^{2}(J+1)^{2}$$
(3)

where

$$A(J) = \frac{\hbar^2}{2 \, \jmath(J)} \tag{4}$$

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$$B(J) = \frac{4A(J)^{3}}{(\hbar\omega_{\gamma})^{2}} + \frac{12A(J)^{3}}{(\hbar\omega_{\beta})^{2}}$$
 (5)

here $\hbar\omega_{\beta}$ and $\hbar\omega_{\gamma}$ are the head energies of the β - and γ -vibrations^[5], respectively. For convenience, we rewrite Eq. (3) as

$$E(J) = A(J)J(J+1)\left(1 - \frac{B(J)}{A(J)}J(J+1)\right)$$
 (6)

where the ratio $\frac{B(J)}{A(J)}$ takes the form

$$\frac{B(J)}{A(J)} = \frac{\hbar^4}{\mathcal{I}(J)^2} \left[\frac{1}{(\hbar \omega_{\gamma})^2} + \frac{3}{(\hbar \omega_{\beta})^2} \right]. \tag{7}$$

In order to write $\mathcal{J}(J)^{-1}$ and $\mathcal{J}(J)^{-2}$ in terms of the ground state value $\mathcal{J}_0^{[6]}$, which corresponds to J = 0, we make Taylor series expansion, giving

$$\frac{1}{J(J)} = \frac{1}{J_0} \frac{1}{1 + \sigma_1 J + \sigma_2 J^2 + \cdots}$$
 (8)

$$\frac{1}{\mathcal{J}(J)^2} = \frac{1}{\mathcal{J}_0^2} \left(1 - 2\sigma_1 J - 2\sigma_2 J^2 + 3\sigma_1^2 J^2 + 6\sigma_1 \sigma_2 J^3 - \dots \right) \tag{9}$$

where, we have introduced the Softness parameter^[7], defined by

$$\sigma_n = \frac{1}{n!} \frac{1}{\mathcal{I}_0} \frac{\partial^n \mathcal{I}(J)}{\partial J^n} \bigg|_{J=0} . \tag{10}$$

Keeping only the first order terms in σ_n , Eqs. (8) and (9) become

$$\frac{1}{\mathcal{I}(J)} = \frac{1}{\mathcal{I}_0} \frac{1}{1 + \sigma_1 J} \tag{11}$$

$$\frac{1}{J(J)^2} = \frac{1}{J_0^2} (1 - 2\sigma_1 J). \tag{12}$$

Inserting Eqs. (11) and (12) into (4) and (7), respectively, and rearranging Eq. (6), one easily obtains

$$E(J) = \frac{A_0}{1 + \sigma_1 J} J(J+1) \left[1 - \frac{B_0}{A_0} (1 - 2\sigma_1 J) J(J+1) \right]$$
 (13)

which can be simplified to give

$$E(J) = \frac{A_0}{1 + \sigma_1 J} J(J+1) - cA_0^3 \frac{(1 - 2\sigma_1 J)}{1 + \sigma_1 J} J^2 (J+1)^2$$
 (14)

where,
$$A_0 = \frac{\hbar^2}{2J_0}$$
 and $B_0 = cA_0^3 = \left[\frac{4}{(\hbar\omega_{\gamma})^2} + \frac{12}{(\hbar\omega_{\beta})^2}\right]A_0^3$.

This third order nonlinear expression, Eq. (14), can be used to calculate the ground state rotational bands of deformed *e-e* nuclei.

Results

We have applied Eq. (14) to calculate the energies of the ground state band of ^{158}Dy . In these calculations, we have treated A_0 , σ_1 and c as three free parameters, which can be determined by fitting the first excited states 2^+ , 4^+ , and 6^+ with the available data. In Fig. 1 we schematically present the results of our calculations along with the experimentally observed energies and the prediction of other existing models^[8]. The results of our model are in close agreement with data than the other models.

$A_0 = 17.04$	2 KeV	$\sigma_1 = 13$.	806 c=	= 3.440 ×10	⁻⁶ KeV ⁻²
<i>J</i> ^π 18 ⁺	_	VMI 3943.0			Present work 3866.3
16+	3190.7	3263.8	3248.5	3329.3	3221.0
14+	2612.6	2628.6	2623.7	2674.9	2613.3
12+	2049.2	2041.7	2043.5	2071.9	2043.1
10+	1519.9	1508.8	1513.5	1526.2	1516.2
8+	1044.1	1036.9	1041.4	1045.0	1043.1
6+	637.87	637.87	637.87	637.87	637.87
4 ⁺ ————————————————————————————————————	98 94	98 94	98.94	317.26 98.94 0.0	317.26 98.94 0.0

Fig.1. Experimental and calculated energy levels of ¹⁵⁸Dy in [KeV].

Conclusion

The present model Eq. (14) is practically fit to predict the ground state rotational bands of almost all deformed e-e nuclei, and can also be applied to nuclei where the energies of β - and γ - vibrations are experimentally available. With the known values of $\hbar\omega_{\beta}$ and $\hbar\omega_{\gamma}$, contained in parameter c of Eq. (14), the model reduces to only two parametric expression, hence, the higher spin states can be evaluated by fitting only the first excited states 2^+ and 4^+ with experimental data.

References

- 1) A. Abzouzi and M. S. Antony, Phys. Rev. C37 (1988) 401.
- 2) P. C. Sood, Phys. Rev. 161 (1967) 1063.
- O. Nathan and S. G. Nilson, Alpha, Beta, and Gamma Ray Spectroscopy, Ed. K. Siegbahn, North Holland (1965) 601.
- 4) A. Bohr and B. R. Mottelson, Nuclear Structure Vol-11, Nuclear Deformation, Benjamin, New York (1984).
- 5) R. K. Sheline, Rev. Mod. Phys. 32 (1960) 1.
- 6) J. S. Batra and R. K. Gupta, Phys. Rev. C43 (1991) 43.
- 7) H. Morinaga, Nucl. Phys. 75 (1966) 385.
- 8) Dennis Banatsos and Abraham Klein, Atomic Data and Nuclear Data Tables 30 (1984) 271.