

Roles of triaxiality and residual interaction in signature inversions of $A \sim 130$ odd-odd nuclei

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Abstract

Rotational bands with $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations are studied using a particle-rotor model in which a proton and a neutron quasiparticles interacting through a zero-range force are coupled with a triaxial rotor. It is shown for ^{124}Cs that one can reproduce the signature dependence of energy and $B(M1)/B(E2)$ ratio best when one takes into account γ -deformations with irrotational-flow moment of inertia in addition to the proton-neutron interaction proposed by Semmes and Ragnarsson. Including both effects, a systematic calculation of signature splittings is performed for Cs, La isotopes and $N=75$ isotones to be compared with experiments. Discussions are also done on the deficiencies of the cranking model concerning its applicability to signature inversion phenomena in odd-odd nuclei.

1 Introduction

In the cranking model, the order of single-particle energy levels can be reversed between a favored-signature state and its unfavored-signature partner when the axial symmetry is violated and the shortest axis is chosen as the rotation axis[1, 2, 3]. This type of rotation is called a *positive- γ rotation* in so-called Lund convention[4]. The *signature-inversion phenomenon* is expected to manifest itself in experiment as the reversion of the staggering of rotational spectrum because, as discussed in sect. 2.1, the signature quantum number can usually be associated with the parity of the total angular momentum I (or $I - \frac{1}{2}$ for odd- A nuclei).

Signature inversions have been found systematically in regions of mass number $A \sim 130$ and $A \sim 160$. They occur only in multi-quasiparticle (qp) bands. For 2qp bands of odd-odd nuclei, inversions take place in low-spin regions including the bandhead states. For odd- A nuclei, they happen at higher spins than the first backbendings, *i.e.*, in 3qp bands consisting

of two rotation-aligned qp's of proton or neutron and a deformation-aligned qp of the other type of nucleon. A brief review of the experimental facts is given in ref. [5].

When one tries to explain these signature inversions in terms of triaxiality, one encounters a serious problem that, assuming as usual that the moment of inertia has the same shape dependence as that of irrotational flow, the nucleus prefers to rotate around its intermediate-length principal axis, not the shortest one. This type of rotation, which is called a negative- γ rotation, increases the signature splitting in the normal direction rather than reverse it.

An *ad hoc* prescription to realize a positive- γ rotation is to exchange the components of the moment of inertia between the shortest axis and the intermediate-length axis by hand[6]. In this paper, a moment of inertia having such shape dependence is called a γ -reversed one.

The irrotational-flow moment of inertia is much more reasonable than the γ -reversed one. The former is naturally derived by separating the quadrupole oscillations around a spherical shape between rotation and vibration[7]. It is also supported by a microscopic calculation based on the cranking formula[8].

Our previous calculation using a particle-triaxial-rotor model, too, supports it[9]: We have treated all the nucleons in intruder orbitals ($\pi h_{11/2}$ and $\nu i_{13/2}$) in a microscopic way. These nucleons give rise to as much as half of the moment of inertia of the entire nucleus. This particle-rotor system does not show any signature inversions, even if the rotor is given a γ -reversed moment of inertia, implying that a complete shell-model configuration-mixing calculation, if possible, will probably be approximated better with an irrotational-flow moment of inertia rather than a γ -reversed one.

Therefore ingredients other than triaxiality have to be taken into account. Considering that inversions are always found in multi-qp bands, the residual interaction between qp's seems important. Hamamoto has analyzed a 3qp band in ^{157}Ho employing a $Q_p \cdot Q_n$ force as the residual interaction[10]. She concluded that an interaction strength larger than the standard one by an order of magnitude was necessary to reproduce the signature inversion. Ikeda and Shimano have succeeded in reproducing signature inversions in 3qp bands using the standard $Q_p \cdot Q_n$ interaction, but they have needed to change the sign of the rotation- γ -vibration coupling[11], which corresponds to the usage of a γ -reversed moment of inertia.

Semmes and Ragnarsson have employed a zero-range residual interaction in a model in which a proton and a neutron qp's are coupled with a triaxial rotor and applied the model to the $(\pi h_{11/2})^1(\nu i_{13/2})^1$ band in ^{152}Eu [12, 13] and the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{120}Cs [14]. These applications are successful in reproducing the angular momentum (I_{inv}) at which the signature splitting changes the sign.

In this paper, we adopt the framework of Semmes and Ragnarsson and intend to improve their results. Although they concluded that the best way is to employ the proton-neutron (pn) interaction without γ -deformations, we expect that triaxiality must be considerably affecting signature-dependent quantities so long as low-energy γ -bands exist in the neighboring

even-even nuclei. We choose an odd-odd nucleus ^{124}Cs , whose experimental spin-assignments are very reliable[5], and perform an intensive study to see what kind of combination of γ -deformations and pn interactions is most suitable to reproduce signature dependence in energy and $B(M1)/B(E2)$ ratio. Then, using the best combination, we make a systematic calculation of signature splittings for Cs, La isotopes and $N=75$ isotones and compare the results with experiment.

Signature inversion phenomena in odd-odd nuclei have also been studied in other theoretical frameworks, e.g., the cranking model[15] and the interacting-boson-fermion model[16].

In sect. 2, in order to stress the necessity to use the particle-rotor model, we point out two deficiencies of the cranking model concerning its applicability to signature-inversion phenomena. In sect. 3, we check the reasonableness of the strength of the pn interaction proposed by Semmes and Ragnarsson. The model is explained in sect. 4. The results of calculations are given in sects. 5. The contents of this paper are summarized in the last section.

2 Inadequacy of the cranking model

In this section, we point out two shortcomings of the cranking model, which make us question the applicability of the model to signature inversion phenomena in odd-odd nuclei. The aim is to illustrate the necessity to invoke a particle-rotor model rather than the cranking model, although the former is more phenomenological and yet more laborious to solve than the latter.

2.1 Non-unique correspondence between signature and the parity of I

First, we discuss the relation between the parity of the total angular momentum I (or $I - \frac{1}{2}$ for odd- A systems) and the signature quantum number. The former is what is observed in experiments, while the latter is peculiar to the cranking model, in which the nucleus is assumed to perform a uniform rotation around a space-fixed axis. The signature quantum number is defined as the eigenvalue of an operator,

$$\hat{\mathcal{R}}_x \equiv \exp(-i\pi\hat{I}_x), \quad (1)$$

which rotates the wavefunction of the system by 180° around the x -axis. The x -axis is taken to be parallel to the cranking axis, which coincides with one of the principal axes of the quadrupole deformation. \mathcal{R}_x is a good quantum number in the cranking model because both of the Coriolis field ($-\omega_{\text{rot}}\hat{I}_x$) and the Nilsson potential commute with $\hat{\mathcal{R}}_x$. Now, if $I_x = I$ for the states (of the real nucleus, not of the cranking model) under consideration, the signature and the parity of I obviously have one-to-one correspondence. There seems

to be no justifications for identifying the two quantities which does not depend on the assumption of $I_x = I$.

The condition $I_x = I$ is not fulfilled, however, when the fluctuation $\langle \hat{I}_y^2 + \hat{I}_z^2 \rangle$ is not negligible, say larger than $\sim 2I$. ($I_y^2 + I_z^2$ takes on I for $I_x = I$ and $3I - 1$ for $I_x = I - 1$. At the middle point, $I_y^2 + I_z^2 = 2I - \frac{1}{2}$.) For example, it is not negligible when quasiparticles are excited in orbitals having large Ω quantum number and/or when a triaxially deformed nucleus does three-dimensional rotations¹. One should be very careful in applying the cranking model to these cases.

The deviation of I_x from I is larger at smaller spins, while signature inversions in odd-odd nuclei take place at low spins. This is one of the reasons why we employ the particle-rotor model, to which $I_x \neq I$ does not matter.

When we explain the results of calculations with particle-rotor model, we use the word *signature* to signify the parity of I . For example, concerning $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations, states with odd (even) I are called favored (unfavored) signature states.

2.2 Mixing of the single-particle signature

Second, we examine the ability of the cranking model to treat 2qp configurations. In the cranking model, it is assumed that each particle has a definite signature. We show in this subsection, however, that the signature of each particle can be mixed completely, even when the total signature remains a rather good quantum number.

We consider only pure- j orbitals. For our purpose, it is necessary to distinguish between rotation-aligned (RA) orbitals and deformation-aligned (DA) ones because they play different roles.

RA orbitals are labeled by the x -component m_x of the angular momentum and are signature eigenstates:

$$\hat{\mathcal{R}}_x |j, m_x\rangle = e^{-i\pi m_x} |j, m_x\rangle. \quad (2)$$

Orbitals $\{|j, j\rangle, |j, j-2\rangle, \dots, |j, -j+1\rangle\}$ have a signature $e^{-i\pi j}$, while orbitals $\{|j, j-1\rangle, |j, j-3\rangle, \dots, |j, -j\rangle\}$ have the opposite signature $e^{i\pi j}$. The former set of orbitals is called the *favored* signature states because it includes the orbital $|j, j\rangle$, which has the smallest expectation value of the Coriolis field $(-\omega_{\text{rot}} \hat{j}_x)$. The latter set is called the *unfavored* signature states.

For DA orbitals, the signature eigenstates are obtained by taking linear combinations of a pair of degenerated Nilsson orbitals, whose z -component of the angular momentum Ω is a good quantum number,

$$|j, |\Omega|, \mathcal{R}_x = e^{\pm i\pi j}\rangle = 2^{-1/2}(|j, \Omega\rangle \mp |j, -\Omega\rangle). \quad (3)$$

¹ A familiar example is the γ -band of the triaxial rotor[17]. Although all of the three signatures r_1 , r_2 , and r_3 are taken to be +1[7], the band includes odd-spin members.

It can be shown in the lowest order perturbation theory that orbitals with $\mathcal{R}_x = e^{-i\pi j}$ ($e^{i\pi j}$) are favored (unfavored) by the Coriolis field, in agreement with the favored (unfavored) signature for RA orbitals.

We consider 2qp configurations of $(j_p)^1(j_n)^1$ type consisting of a proton and a neutron. For each qp of kind τ (=p or n), we take into account a favored orbital f_τ and an unfavored one u_τ . If they are RA orbitals, we take $|f_\tau\rangle = |j_\tau, m_x = j_\tau\rangle$ and $|u_\tau\rangle = |j_\tau, m_x = j_\tau - 1\rangle$. In case of DA orbitals, those of eq.(3) are used. There are four intrinsic states $\{f_p f_n, u_p u_n, f_p u_n, u_p f_n\}$ (the signature quartette).

In fig. 1, we show how the quartette is grouped into rotational bands. It is supposed that $j_p - j_n$ is even. The first portion shows a case in which two qp's are in RA orbitals. Because collective transitions of the rotor do not occur between states with different RA orbitals, there exist four bands based on each member of the quartette.

The second portion treats a case where the proton (neutron) qp is in RA (DA) orbitals. The third portion is obtained by exchanging the types (RA or DA) of orbitals between proton and neutron. In both cases, the difference of the signature of the RA orbital makes different bands, while that of the DA orbital gives rise to the odd-even staggering within each band.

The independent-qp picture has been applicable so far. It no longer holds when both qp's are in DA orbitals, as shown in the last portion. The signature quartette should be mixed up to construct states labeled with $K = |\Omega_p + \Omega_n|$, because the rotational energy depends on K as $E_I \propto I(I+1) - K^2$. The band with $K = K_> \equiv ||\Omega_p| + |\Omega_n||$ has a wavefunction $f_p f_n + u_p u_n$ for odd I and $f_p u_n + u_p f_n$ for even I , while the band with $K = K_< \equiv ||\Omega_p| - |\Omega_n||$ has $f_p f_n - u_p u_n$ for odd I and $u_p f_n - f_p u_n$ for even I (not normalized). Therefore the cranking model cannot treat K -bands with multi-qp intrinsic states ².

In practice, ideal (*i.e.*, pure- m_x) RA orbitals hardly exist and the wavefunctions of 2qp bands inevitably contain the components of K -bands. In this respect, too, it is worth while using the particle-rotor model for odd-odd nuclei.

Let us discuss briefly about the signature quartette. In all the portions of the figure except the first one, the phase of the odd-even staggering of the higher-lying band is opposite to that of the lower-lying one. This feature may be useful to tell which pair of bands makes the signature quartette. The $(\pi h_{11/2})^1(\nu h_{11/2})^1$ bands in Cs isotopes are roughly classified to the second case from the fermi levels. For ¹²⁴Cs, our calculation predicts that the first excited band of this configuration is at about 0.6 MeV from the yrast band. The odd-even staggerings of the lowest two bands are of opposite phase for spins both below and above I_{inv} . They seem to make a signature quartette.

² Although the tilted-axis cranking model[18] can produce multi-qp K -band states, it does not conserve "the signature quantum number" for the π -rotation around the cranking axis, not to mention of the parity of I .

3 The proton-neutron interaction

In refs. [12, 13, 14, 19], a zero-range interaction of the following form ³ has been used as the residual force between the unpaired proton and neutron in odd-odd nuclei.

$$V_{\text{pn}} = 4\pi\sqrt{\frac{\pi}{2}}b^3\delta(\mathbf{r}_{\text{p}} - \mathbf{r}_{\text{n}})(u_0 + u_1\boldsymbol{\sigma}_{\text{p}} \cdot \boldsymbol{\sigma}_{\text{n}}), \quad (4)$$

where $b \equiv (\hbar/m\omega)^{1/2}$. Boisson and Piepenbring have obtained $u_1 = -0.8$ MeV[19] through least-squares fits to the GM shifts[21], leaving u_0 undetermined. Semmes and Ragnarsson have proposed a strength $u_0 = -7.2$ MeV[12] by determining the ratio u_0/u_1 from $V_T/V_S \sim 0.6$, the experimental tendency of the ratio of the spin-triplet energy to the singlet one[22]. They have found that this u_0 leads to accurate reproductions of I_{inv} for ^{120}Cs [14] and ^{152}Eu [12, 13].

3.1 How strong is the V_{pn} of Semmes and Ragnarsson ?

In ref.[10], it has been concluded that a $Q_{\text{p}}Q_{\text{n}}$ interaction more than ten times as strong as the standard one is necessary in order to reproduce a signature inversion in a $(\pi h_{11/2})^1(\nu i_{13/2})^2$ band. It should be examined whether V_{pn} (eq. (4)) is so unreasonably strong or not.

By expressing the angular part of the delta function with spherical harmonics, V_{pn} can be expanded as

$$V_{\text{pn}} = V_0 \sum_l Y_l(\hat{r}_{\text{p}}) \cdot Y_l(\hat{r}_{\text{n}}) + V_1 \sum_l Y_l(\hat{r}_{\text{p}}) \cdot Y_l(\hat{r}_{\text{n}}) \boldsymbol{\sigma}_{\text{p}} \cdot \boldsymbol{\sigma}_{\text{n}}, \quad (5)$$

where $V_0 = -11.2$ MeV and $V_1 = -1.2$ MeV (estimated for $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations). The transferred angular momentum k between the proton and the neutron can take on l (l and $l \pm 1$) for the terms in the first (second) summation. As far as $(j_{\text{p}})^1(j_{\text{n}})^1$ -type configurations concern, the two-body matrix elements of the right-hand side of eq. (5) vanish for terms with odd l (due to parity conservation) and for contributions with $k=l$ from the terms in the second summation[23]. It follows that an interaction with $k=2$ is contained only in the spin-independent part of the interaction and its strength is -11.2 MeV.

An estimate of the strength of the $Q_{\text{p}} \cdot Q_{\text{n}}$ force is found in ref.[7], which gives,

$$V_{\text{QQ}} = -7.7 \text{ [MeV]} \times Y_2(\hat{r}_{\text{p}}) \cdot Y_2(\hat{r}_{\text{n}}), \quad (6)$$

for $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations in $A=124$ nuclei. (We used eqs.(6-78,127,380) of ref. [7]. The strength is $(X_0 - X_1)\langle r_{\text{p}}^2 \rangle \langle r_{\text{n}}^2 \rangle$.) Hence, concerning the $k=2$ component, V_{pn} is stronger than V_{QQ} only by a factor of 1.5.

³ In refs. [12, 13, 19], a different factor $(\pi b^3/2^{1/2})^{1/2}$ is printed instead of $(\pi/2)^{1/2}b^3$ of eq. (4). However, eq. (4) is the correct definition in the sense that it is consistent with the particle-rotor-model code of Semmes and Ragnarsson and that the code reproduces[20] the matrix elements tabulated in ref. [19]. Therefore all the numerical results in refs. [12, 13, 14, 19] are completely correct if the definition is changed to eq.(4).

V_{pn} has, however, other multipole components. To see their effects, we have plotted the two-body matrix elements,

$$g_J = \langle (\pi h_{11/2})^1 (\nu h_{11/2})^1; JM | V_{\text{pn}} | (\pi h_{11/2})^1 (\nu h_{11/2})^1; JM \rangle, \quad (7)$$

in fig. 2. In the figure, $g_J^{(0)}$ ($g_J^{(1)}$) denotes the part of g_J proportional to u_0 (u_1). $g_J^{(\text{QQ})}$ is the matrix element of V_{QQ} (eq. (6)). The range of fluctuation in g_J ($g_J^{(\text{QQ})}$) versus J is 3.4 MeV (1.1 MeV). In this respect, the former is stronger than the latter by a factor of 3.0.

From the obtained enhancement factor of 1.5-3.0, V_{pn} does not seem unreasonably strong. The difference of the strength between V_{pn} of Semmes and Ragnarsson and $\sim 10 \times V_{\text{QQ}}$ of Hamamoto may originate in the $k \neq 2$ multipole interactions of the former force or in the difference of configuration.

3.2 The effect of the spin-dependent interaction

Let us show that the spin-dependent term does not have strong influence on the bands to be studied in this paper. It has been known that the proton-neutron interaction prefers spin-triplet state to singlet one. In deformed nuclei, it is realized as an empirical law called the GM rule [21]. The GM splitting is the difference of energy between states with $K_>$ and $K_<$ (see sect. 2.2 for the definitions). It is estimated most simply with the difference in the diagonal matrix element of the spin-dependent interaction between stretched and antistretched unperturbed configurations. For a product state of a proton and a neutron, $|(l\frac{1}{2})j\Omega_p\rangle_p |(l\frac{1}{2})j\Omega_n\rangle_n$, the probabilities of spin singlet and triplet components are,

$$\text{Prob.}(S=1) = 1 - \text{Prob.}(S=0) = \frac{3}{4} + \frac{\Omega_p \Omega_n}{4j^2}. \quad (8)$$

In case of ^{124}Cs , by using $j = \frac{11}{2}$, $|\Omega_p| = \frac{1}{2}$, and $|\Omega_n| = \frac{7}{2}$, one finds only 3 % difference of the probabilities between configurations with $\Omega_p \Omega_n > 0$ and < 0 . Therefore the GM splitting is very small. The off-diagonal effects are studied in sect. 5 (see fig. 3).

4 The particle-triaxial-rotor model for odd-odd nuclei

The hamiltonian of our model is expressed as,

$$\hat{H} = \sum_{\kappa=1}^3 \frac{\hbar^2}{2\mathcal{I}_{\kappa}} (\hat{I}_{\kappa} - \hat{j}_{p\kappa} - \hat{j}_{n\kappa})^2 + \hat{h}_{\text{Nilsson}} + V_{\text{pn}}, \quad (9)$$

where

$$\mathcal{I}_{\kappa} = \frac{4}{3} \mathcal{I}_0 \sin^2(\gamma \mp \frac{2}{3}\pi\kappa) \quad \text{for} \begin{cases} \gamma \leq 0 & (\text{irrotational flow}), \\ \gamma > 0 & (\gamma\text{-reversed}). \end{cases} \quad (10)$$

h_{Nilsson} is a deformed single-particle potential, which depends on γ in such a way that $\Delta\omega_\kappa \propto \cos(\gamma + \frac{2}{3}\pi\kappa)$. V_{pn} is defined by eq. (4). The subspace in which we diagonalize \hat{H} is,

$$(1\text{qp in } \pi h_{11/2}) \times (1\text{qp in } \nu h_{11/2}) \times (|K_{\text{rotor}}| = 0, 2, 4, 6). \quad (11)$$

In eq. (11), “ $h_{11/2}$ ” stands for those Nilsson orbitals which are continuously transformed to the $h_{11/2}$ spherical orbitals as $\epsilon_2 \rightarrow 0$. Orbitals with $\Omega = \frac{11}{2}$ are not included.

The first term in the right-hand side of eq. (9) is divided into three parts in the usual way[24], *i.e.*, the strong-coupling rotational energy, the Coriolis interaction, and the recoil term. In sect. 5.2, we use a Coriolis attenuation factor ρ of 0.7, which should be multiplied to all the off-diagonal matrix elements of the Coriolis interaction. Concerning the recoil term, operators $\hat{\mathbf{j}}_{\text{p}}^2$ and $\hat{\mathbf{j}}_{\text{n}}^2$ are treated as one-body operators to avoid a double counting of a rotor’s contribution[25].

The distinction between negative and positive values of γ in the definition (10) is introduced just for the sake of convenience[6]. As for the γ -dependence of the moment of inertia \mathcal{I}_κ , that of irrotational flow is usually assumed. Sometimes the γ -reversed moment of inertia is also employed, which are obtained by exchanging \mathcal{I}_x and \mathcal{I}_y by hand. The definition (10) is adopted so that the distinction between the irrotational-flow moment of inertia and the γ -reversed one can be made simply by means of the sign of γ . Incidentally, in this paper, a “negative- γ (positive- γ) rotor” means a rotor having an irrotational-flow (γ -reversed) moment of inertia.

We employ the triaxial rotor model not because we believe in rigid triaxial shapes but because it is a very convenient model which can emulate the effects of fluctuation and vibration of γ around an axial shape. Indeed, a triaxial rotor and a γ -vibrational axial rotor are indistinguishable to the first order in γ . For more explanations of their practical equivalence, see ref. [26] and references therein.

The parameters of the Nilsson potential are taken from table 1 of ref.[27]. The pairing-active space is restricted to $(15Z)^{1/2}$ proton and $(15N)^{1/2}$ neutron orbital pairs below and above each fermi level. The interaction strengths are obtained by multiplying 0.95 to the standard strengths[28] in order to take into account the blocking effect. The stretched coordinates are used.

More explanations of the model are given in refs.[12, 13]. Ref. [29] is about a simpler version of the model for 1qp bands of odd- A nuclei, originally introduced in ref. [30], and can be of some help to understand the present model.

The size of quadrupole deformation ϵ_2 is taken from ref. [14] for ^{120}Cs and from ref. [5] for ^{124}Cs . A Nilsson-Strutinski calculation results in the same ϵ_2 for ^{124}Cs [31]. For other nuclei, it is determined by averaging over neighboring even-even nuclei the values of β derived from $B(E2) \uparrow$ [32]. When no data are available, extrapolations are done. Adopted values are given in the second column of table 1.

For the triaxiality parameter γ , we tried determining it by equating $E(2_\gamma^+)/E(2_{\text{gr}}^+)$ of a triaxial rotor[17] to the average of experimental values over neighboring even-even nuclei[33]. For ^{124}Cs , we have obtained $|\gamma| = 23^\circ$, which turns out to be a very appropriate value concerning signature splitting. Considering that this prescription to determine γ is also successful for odd- A nuclei[34], we adopt it for other nuclei, too. The third column of table 1 presents the resulting values.

As a parameter to specify the moment of inertia, we use $E_c(2^+)$, which stands for the smallest excitation energy of the bare rotor, instead of \mathcal{I}_0 . In determining this parameter one has to keep in mind that the moment of inertia of a real nucleus is not a constant but usually increases versus spin. As we are interested in signature inversions, which take place at $I < I_{\text{inv}} \cong 16$ (for ^{124}Cs), we determine $E_c(2^+)$ so as to fit $\Delta E/\Delta I$ at $I = I_{\text{inv}}$, where E is the eigenvalue of \hat{H} . The practical procedure of the fitting is as follows: For $^{120-128}\text{Cs}$ and $^{124-130}\text{La}$, we adjust $E_c(2^+)$ for each nucleus so that the calculated $E(I = 17) - E(I = 15)$ agrees with the experimental energy difference. For ^{132}La , ^{134}Pr , and ^{136}Pm , $E(I = 13) - E(I = 11)$ is used for the fitting because no levels have been observed for $I \geq 15$ for these nuclei. The values in the fourth column of table 1 are determined for $\rho = 0.7$ and used in section 5.2.

5 The results of calculations

In this section, we employ residual pn interactions and γ -deformations and examine the resulting signature splittings and electro-magnetic transition amplitudes.

We choose to define the signature splitting as follows: For each even value of I , we interpolate the odd- I sequence of the spectrum to the even value of I and subtract the interpolated energy from the even- I energy level⁴. Negative values of the splitting mean signature inversion. In this way we can treat the signature splitting independent of the slope of the spectrum, $\Delta E/\Delta I$ ($\Delta I=2$), and concentrate ourselves on the former quantity. Indeed, the signature splittings contain all the significant information of the spectra calculated in subsect. 5.2 since we use a parameter to fit ΔE ($\Delta I=2$).

5.1 Effects of pn interaction and γ -deformation

In this subsection, we try various combinations of two factors, *i.e.*, pn force and triaxiality. A nucleus ^{124}Cs seems most appropriate to this intensive study, since the experimental spin-assignments for the nucleus are very reliable[5]. $E_c(2^+)$ is fixed at 0.15 MeV for ^{120}Cs and

⁴ For each even spin I_0 , we choose the nearest n odd spins to I_0 and construct a polynomial in I of degree $n - 1$ which passes through the n levels for the n spins (the Lagrange interpolation formula). It has been confirmed that the result is practically the same for $n=4, 6$, and 8 . We have observed no artificial ripples due to our specific choice of the interpolation method. We adopt $n=6$ in this paper.

0.20 MeV for ^{124}Cs . The Coriolis attenuation is not introduced (*i.e.*, $\rho=1$)⁵.

First, we try changing u_0 while keeping axial symmetry. The value of u_1 is fixed at -0.8 MeV. The results for ^{120}Cs and ^{124}Cs are shown in fig. 3. The solid circles connected with dashed curve designate the experimental signature splitting. The dots connected with solid curves are the calculated points for which different values of u_0 (0, -1.8 , -3.6 , -5.4 , -7.2 , -9.0 , and -10.8 [MeV]) are used. For ^{120}Cs , I_{inv} coincides with the experimental value when $u_0 = -7.2$ MeV. The slope is, however, twice as small as the experimental trend. (This is the same strength as proposed by Semmes and Ragnarsson. Our results seem in agreement with fig. 7d of ref. [14].) For ^{124}Cs , I_{inv} as well as the slope are reproduced using a different strength $u_0 = -5.4$ MeV. It is not preferable that the interaction strength changes so widely between neighboring nuclei.

While all the solid curves in the figure are calculated with $u_1 = -0.8$ MeV, the plus marks connected with a dot-dash curve for each nucleus are obtained without any pn interactions ($u_0 = u_1 = 0$). By comparing the curve with the top solid curve, which is calculated with $u_0 = 0$ and $u_1 = -0.8$ MeV, one can see that a spin-dependent interaction favoring $S=1$ enhances the normal-sign signature splitting, opposite to the signature-inverting effect of spin-independent attractive interactions.

In fig. 4, we tried introducing γ -deformations while turning off the pn interaction ($u_0 = u_1 = 0$). With an axial rotor (0°), the inversion does not occur. A negative- γ rotor (-23°) does not change the result very much. (See eq. (10) for the distinction between “negative- γ ” and “positive- γ ” rotors.) For positive- γ rotors ($+18^\circ, +23^\circ$), the signature splitting is sensitive to the value of γ . With $\gamma = +18^\circ$, I_{inv} as well as the slope are reproduced. As remarked in the introduction, however, it is unlikely that real nucleus behaves like a positive- γ rotor.

In fig. 5, we show the results of calculations in which γ -deformations as well as V_{pn} of Semmes and Ragnarsson ($u_0 = -7.2$ MeV, $u_1 = -0.8$ MeV) are taken into account. With a positive- γ rotor ($+23^\circ$), the effect to invert the signature splitting is too strong, which is a natural result since both factors cooperate to cause the inversion. With a negative- γ rotor (-23°), however, the agreement with the experiment can be very good.

We imagine that a negative- γ rotor plays the following role: At low spins, the fluctuation in the orientation of the rotation axis is so large that neither negative- γ nor positive- γ rotation is realized. Consequently, the static mean-field effect (such as considered in the cranking model for negative- γ rotation) on the signature dependence of single-particle energy

⁵ Although $E_c(2^+)$ is fixed, obtained spectrum is satisfactory for the calculations given in fig. 3: $\Delta E \equiv E(I=17) - E(I=15)$ is 90-100% of the experimental values. For the calculations given in figs. 4 and 5, ΔE is rather small for $\gamma \neq 0^\circ$: It amounts to $\sim 100\%$, $70\text{--}80\%$, and $60\text{--}70\%$ of the experimental value for $\gamma = 0^\circ$, -23° , and $+23^\circ$, respectively. To see the influence of such discrepancy, we have also done calculations adjusting $E_c(2^+)$ for each value of γ to fit ΔE . The result supports the conclusion of this subsection, provided $\rho \sim 0.7$ (see sect. 5.2).

is small⁶. At high spins, the rotation axis is confined in the vicinity of the intermediate-length axis because the excitation energy for wobbling motion is proportional to I [7]. This rotation corresponds to a negative- γ rotation of the cranking model and contributes to the normal-sign signature splitting. Therefore, a negative- γ rotor is expected to promote the restoration of the normal-sign signature splitting at high spins without hindering the signature inversion at low spins.

We now show that the combination of V_{pn} and a negative- γ rotor is not only physically most reasonable but also supported by the experimental $B(\text{M1}; I \rightarrow I-1)/B(\text{E2}; I \rightarrow I-2)$ ratio. In fig. 6, this ratio is plotted as a function of I . Calculations are done for three cases, *i.e.*, (1) when only a pn interaction is taken into account (dotted lines, corresponding to the curve using $u_0 = -5.4$ MeV in the right-hand portion of fig. 3), (2) when only a positive- γ rotor is employed (dashed lines, corresponding to the curve calculated with $\gamma = +18^\circ$ in fig. 4), and (3) when the pn interaction V_{pn} is combined with a negative- γ rotor (solid lines, corresponding to the curve using $\gamma = -23^\circ$ in fig. 5). In the left-hand portion, where the ratios for odd values of I are plotted, the solid line reproduces best the decrease of the ratio versus I (except for $I=19$). In the right-hand portion, which shows the ratios for even I , the sudden drop in the ratio is reproduced only by the solid line. (The drop is attributed to the $B(\text{M1})$ value while $B(\text{E2})$ value increases continuously.)

5.2 A systematic calculation for $A \sim 130$ nuclei

We have seen that the combination of a pn interaction and a negative- γ rotor makes the most successful model to reproduce signature-dependent quantities. In this subsection we present signature splittings calculated by this method for some Cs and La isotopes and $N=75$ isotones.

Employed values for the parameters (ϵ_2 , γ , and $E_c(2^+)$) are given in table 1. We use V_{pn} of Semmes and Ragnarsson ($u_0 = -7.2$ MeV, $u_1 = -0.8$ MeV). The Coriolis attenuation factor of $\rho=0.7$ is assumed for all the nuclei, the necessity of which is explained soon.

As explained in sect. 4, we choose to determine $E_c(2^+)$ so that ΔE for $\Delta I=2$ is reproduced around I_{inv} . In fig. 7, resulting spectra for $^{120,124}\text{Cs}$ are shown. The solid (dashed) curves connect smoothly the calculated favored (unfavored) signature levels. Experimental favored (unfavored) signature levels are denoted by solid (open) circles. The agreements with experiment are quite excellent for wide ranges of I centering on $I = 16$. Equally excellent agreements are achieved for other nuclei, too.

For the calculations in this section, we introduce the Coriolis attenuation factor because the absolute values of the signature splittings are too large when $E_c(2^+)$ is adjusted to

⁶ At *very* low spins, the angular-momentum-coupling scheme between a particle and a rotor is not so simple as in this argument[35, 36]. It seems related to the discussion in sect. 2.1. It may be the reason why, in our calculations, negative- γ rotors as well as positive- γ ones seem to enhance the inversions.

reproduce the $\Delta I=2$ transition energy. The attenuation factor stands for many ingredients which are not taken into account in the particle-rotor model, some of which may cooperate to weaken the Coriolis interaction. In fig. 8, signature splittings calculated with various values of ρ are shown. We have adjusted $E_c(2^+)$ for each value of ρ to reproduce $E(I=17)-E(I=15)$ ⁷. From this figure, one can see that I_{inv} as well as the slope are reproduced excellently with $\rho = 0.7$. We use the same value of ρ for other nuclei, too.

The left-hand portion of fig. 9 shows experimental signature splittings of some Cs isotopes. As for $^{120,124,126}\text{Cs}$, the behaviors of signature splittings are rather similar to one another. The curve for ^{128}Cs seems to have the opposite sign to other curves. The right-hand portion gives the calculated splittings. As for ^{120}Cs , the relatively large slope of the splitting is reproduced. Although the agreement is very good for $I \leq 12$ and $I \geq 20$, there is a bump around $I \sim 15$ in the calculation and hence I_{inv} is smaller than experiment by $\Delta I = -1.8$. Concerning ^{124}Cs , the agreement is quite satisfactory. For ^{126}Cs , the calculated I_{inv} is larger than the experimental one by $\Delta I = 2.0$. The abrupt reversion of the sign of the splitting between $A = 126$ and $A = 128$ is not reproduced by our calculation. The agreements of I_{inv} between the experiments and the calculations would become remarkably excellent if the experimental spin-assignments could be changed by -2 , $+2$, and $+1$ for $^{120,126,128}\text{Cs}$, respectively.

In fig. 10, some La isotopes are studied. One can see a common feature between the experiments and the calculations that I_{inv} increases as A increases. However, while in experiment signature splittings of these isotopes seem to have the opposite sign to those of $^{120-126}\text{Cs}$, in our calculation such opposite-sign behaviors between $\Delta Z=2$ neighbors are not reproduced at all. This systematic discrepancy in the sign might signify that all the experimental spin-assignments were incorrect by odd values of ΔI or it would indicate the necessity of a more elaborate model.

The signature splittings of three $N=75$ isotones are shown in fig. 11. In this case, the correspondence between the experiments and the calculations is not very clear, partly because the observed rotational sequences are very short.

6 Summary

In this paper, we have applied a particle-triaxial-rotor model to odd-odd $A \sim 130$ nuclei to investigate the signature dependence of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ bands.

In sect. 2, we have explained two deficiencies of the cranking model and questioned its applicability to signature-inversion phenomena in odd-odd nuclei. One is that there is not

⁷ Because of this adjustment, the “rotational frequency” $\omega_{\text{rot}} = dE/dI$, which is the strength of the Coriolis field in the cranking model, is constant despite the change in ρ . However, as intended, the Coriolis interaction of our model is attenuated: It is proportional to a ratio $\rho/E_c(2^+)$, which decreases as ρ decreases ($E_c(2^+)=0.28$ MeV for $\rho = 1$ and $E_c(2^+)=0.19$ MeV for $\rho = 0.6$).

always clear correspondence between the signature quantum number and the parity of the total angular momentum. The other is that single-particle signatures are not conserved in multi-qp configurations. We have also discussed about the signature quartette and how its members are grouped into rotational bands.

In sect. 3, we have compared the zero-range residual interaction of Semmes and Ragnarsson with a $Q_p \cdot Q_n$ force having a standard strength as for $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations. The former has turned out stronger than the latter by a factor of 1.5-3.0 but not unreasonably strong. The spin-dependent part of the force has also been discussed.

The model and the methods to determine the parameters have been described in sect. 4.

In sect. 5.1, we have tried various combinations of the residual interactions and the rotors in order to reproduce the signature splitting and the B(M1)/B(E2) ratio for ^{124}Cs . We have found that the best result is obtained when the interaction of Semmes and Ragnarsson is combined with a triaxial rotor with irrotational-flow moment of inertia. The residual interaction gives rise to the signature inversion. The triaxial rotor does not deteriorate but rather enhance the inversion at low spins, while it promotes the restoration of the normal-sign splitting at high spins.

In sect. 5.2, we have performed a systematic calculation of the signature splittings for $^{120,124,126,128}\text{Cs}$, $^{124,126,128,130}\text{La}$, $^{132}\text{La}_{75}$, $^{134}\text{Pr}_{75}$, and $^{136}\text{Pm}_{75}$ following the prescription given in sect. 5.1. We need the Coriolis attenuation factor of 0.7 to reproduce both the $\Delta I=2$ transition energy and the signature splitting. The agreement with experiment is good for Cs isotopes except ^{128}Cs . For La isotopes and ^{128}Cs , the experimental splittings are of the opposite sign to those of $^{120-126}\text{Cs}$. Our calculation do not produce such opposite-sign behaviors between $\Delta Z=2$ or $\Delta N=2$ nuclei at all. For $N=75$ isotones, the correspondence is not clear between the experiments and the calculations.

As an indispensable step to clarify the origin of the opposite-sign signature splittings between experiment and theory, we hope that more reliable experimental spin-assignments are conducted for nuclei other than ^{124}Cs .

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References

- [1] S. Frauendorf and F.R. May, Phys. Lett. **B125** (1983), 245.
- [2] R. Bengtsson, H. Frisk, F.R. May, and J.A. Pinston, Nucl. Phys. **A415** (1984) 189.
- [3] N. Onishi and N. Tajima, Progr. Theor. Phys. **80** (1988) 130.
- [4] G. Andersson, S.E. Larsson, G. Leander, P. Möller, S.G. Nilsson, I. Ragnarsson, S. Åberg, R. Bengtsson, J. Dudek, B. Nerlo-Pomorska, K. Pomorski, and Z. Szymański, Nucl. Phys. **A268** (1976) 205.
- [5] T. Komatsubara, K. Furuno, T. Hosoda, J. Mukai, T. Hayakawa, T. Morikawa, Y. Iwata, N. Kato, J. Espino, J. Gascon, N. Gjorup, G.B. Hagemann, H.J. Jensen, D. Jerrestam, J. Nyberg, G. Sletten, B. Cederwall, and P.O. Tjom, Conf. rapidly rotating nuclear structure, Tokyo, October 1992, Nucl.Phys. **A557** (1993) 419c.
- [6] I. Hamamoto and B.R. Mottelson, Phys. Lett. **B132** (1983) 7.
- [7] A. Bohr and B.R. Mottelson, *Nuclear Structure*, vol. 2 (Benjamin, New York, 1975).
- [8] J. Helgesson and I. Hamamoto, Phys.Scripta. **40** (1989) 595.
- [9] N. Tajima, Nucl. Phys. **A520** (1990) 317c.
- [10] and sign.inv in 3qp band I. Hamamoto, Phys. Lett. **B179** (1986) 327.
- [11] signature inversion using gamma-reversed m.o.i A. Ikeda and T. Shimano, Phys. Rev. Lett. **63** (1989) 139.
- [12] P.B. Semmes and I. Ragnarsson, Conf. high spin physics and gamma-soft nuclei, Pittsburg, September 1990 (World Scientific, Singapore, 1991) p.500.
- [13] P.B. Semmes and I. Ragnarsson, Conf. future directions in nuclear physics with 4 π detection systems of the new generation, Strasbourg, March 1991 (AIP, New York, 1992) p.566.
- [14] B. Cederwall, F. Lidén, A. Johnson, L. Hildingsson, R. Wyss, B. Fant, S. Juutinen, P. Ahonen, S. Mitarai, J. Mukai, J. Nyberg, I. Ragnarsson, and P. Semmes, Nucl. Phys. **A542** (1992) 454.
- [15] M. Matsuzaki, Phys. Lett. **B269** (1991) 23.
- [16] N. Yoshida, H. Sagawa, and T. Otsuka, Nucl.Phys. **A**, in press.
- [17] A.S. Davydov and G.F. Filippov, Nucl.Phys. **8** (1958) 237.

- [18] S. Frauendorf, Nucl.Phys. **A557** (1993) 259c.
- [19] J.P. Boisson and R. Piepenbring, Phys.Rep. **26** (1976) 99.
- [20] P. Semmes, private communication.
- [21] C.J. Gallagher and S.A. Moszkowski, Phys.Rev. **111** (1958) 1282.
- [22] A. de Shalit and H. Feshbach, Theoretical Nuclear Physics (John Wiley and Sons, 1974), p.310.
- [23] A. de Shalit and I. Talmi, Nuclear Shell Theory, (Academic Press, New York, 1963), pp.213-214.
- [24] P. Ring and P. Schuck, The nuclear many-body problem (Springer, New York, 1980)
- [25] L. Bennour, J. Libert, M. Meyer, and P. Quentin, Nucl. Phys. **A465** (1987) 35, see pp. 73-74.
- [26] N. Tajima and N. Onishi, Nucl. Phys. **A491** (1989) 179.
- [27] T. Bengtsson and I. Ragnarsson, Nucl.Phys. **A436** (1985) 14.
- [28] S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I. Lamm, P. Möller, and B. Nilsson, Nucl.Phys. **A131** (1969) 1.
- [29] I. Ragnarsson and P. Semmes, Hyp.Int. **43** (1988) 425.
- [30] J. Meyer-ter-Vehn, F.S. Stephens, and R.M. Diamond, Phys. Rev. Lett. **32** (1974) 1383.
- [31] Y.R. Shimizu, private communication.
- [32] S. Raman, C.H. Malarkey, W.T. Milner, C.W. Nestor, Jr., and P.H. Stelson, Atomic Data and Nucl. Data Tables, **36** (1987) 1.
- [33] M. Sakai, Atomic Data and Nucl. Data tables **31** (1984) 399.
- [34] J. Meyer-ter-Vehn, Nucl. Phys. **A249** (1975) 111,141.
- [35] I. Hamamoto, Phys. Lett. **B193** (1987) 399.
- [36] A. Ikeda and S. Åberg, Nucl.Phys. **A480** (1988) 85.
- [37] T. Komatsubara, K. Furuno, T. Hosoda, J. Espino, J. Gascon, G.B. Hagemann, Y. Iwata, D. Jerrestam, N. Kato, T. Morikawa, J. Nyberg, G. Sletten, and P.O. Tjom, Z. Phys. **A335** (1990) 113.

- [38] E.S. Paul, D.B. Fossan, Y. Liang, R. Ma, and N. Xu, Phys.Rev. **C40** (1989) 619.
- [39] N. Xu, Y. Liang, R. Ma, E.S. Paul, D.B. Fossan, and H.M. Latvakoski, Phys.Rev. **C41** (1990) 2681.
- [40] B.M. Nyakó, J. Gizon, D. Barnéoud, A. Gizon, M. Józsa, W. Klamra, F.A. Beck, and J.C. Merdinger, Z.Phys. **A332** (1989) 235.
- [41] M.J. Godfrey, Y. He, I. Jenkins, A. Kirwan, P.J. Nolan, D.J. Thornley, S.M. Mullins, and R. Wadsworth, J.Phys. **G15** (1989) 487.
- [42] J.R.B. Oliveira, L.G.R. Emediato, M.A. Rizzutto, R.V. Ribas, W.A. Seale, M.N. Rao, N.H. Medina, S. Botelho, and E.W. Cybulska, Phys.Rev. **C39** (1989) 2250.
- [43] C.W. Beausang, L. Hildingsson, E.S. Paul, W.F. Piel, Jr., N. Xu, and D.B. Fossan, Phys.Rev. **C36** (1987) 1810.

TABLE

TABLE 1. Adopted parameters for each nucleus. In sect. 5.1, we use $E_c(2^+) = 0.15$ MeV for ^{120}Cs and 0.20 MeV for ^{124}Cs with $\rho = 1$. In sect. 5.2, $\rho = 0.7$ is employed.

	ϵ_2	γ	$E_c(2^+)$ [MeV]
^{120}Cs	0.25	-23°	0.159
^{124}Cs	0.22	-23°	0.206
^{126}Cs	0.244	-24°	0.217
^{128}Cs	0.210	-24°	0.182
^{124}La	0.31	-17°	0.182
^{126}La	0.288	-19°	0.191
^{128}La	0.268	-21°	0.200
^{130}La	0.246	-23°	0.210
$^{132}\text{La}_{75}$	0.217	-25°	0.179
$^{134}\text{Pr}_{75}$	0.225	-25°	0.206
$^{136}\text{Pm}_{75}$	0.225	-25°	0.224

FIGURE CAPTIONS

- Fig. 1. Possible band structures of a signature quartette for $(j_p)^1(j_n)^1$ -type configurations (when $j_p - j_n$ is even). RA (DA) means that the orbital is rotation- (deformation-) aligned. f (u) stands for the wavefunction for a favored- (unfavored-) signature orbital. In the figure, we have omitted the subscripts p and n used in the text. Instead, the first (second) symbol specifies the proton (neutron) orbital. For example, fu in a box labeled by (RA,DA) means a configuration in which a proton is in a favored-signature rotation-aligned orbital and a neutron is in an unfavored-signature deformation-aligned orbital.
- Fig. 2. Two-body matrix elements of the residual interaction proposed by Semmes and Ragnarsson for $(\pi h_{11/2})^1(\nu h_{11/2})^1$ configurations (g_J). The contributions from its spin dependent and independent terms are also shown ($g_J^{(1)}$ and $g_J^{(0)}$, respectively). The matrix elements of a $Q_p \cdot Q_n$ force with a standard strength are included for the sake of comparison ($g_J^{(QQ)}$).
- Fig. 3. Signature splitting of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{120}Cs (left-hand portion) and ^{124}Cs (right-hand portion) calculated with various values of u_0 . Axial rotors are used. See text for explanations.
- Fig. 4. Signature splitting of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{124}Cs calculated without V_{pn} . Solid and dot-dash curves are the results using different values of γ . The experimental values are expressed by solid circles connected with a dashed curve.
- Fig. 5. Same as in fig. 4 but calculations are done using V_{pn} of Semmes and Ragnarsson.
- Fig. 6. $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ ratio of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{124}Cs . The ratios for odd (even) values of I are given in the left-hand (right-hand) portion. The experimental values are taken from fig. 2 (c) of ref. [37] and expressed by dots with error bars. The dotted lines are calculated with $\gamma = 0^\circ$ and $u_0 = -5.4$ MeV. The dashed lines are calculated with $\gamma = +18^\circ$ and $u_0 = u_1 = 0$. The solid lines are calculated with $\gamma = -23^\circ$ and $u_0 = -7.2$ MeV. See text for explanations.
- Fig. 7. Rotational spectrum of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{120}Cs and ^{124}Cs . The experimental energy levels of favored (unfavored) signature are designated with solid (open) circles. Solid (dashed) curves pass through the calculated levels of favored (unfavored) signature. Used parameters are $\gamma = -23^\circ$, $u_0 = -7.2$ MeV, $u_1 = -0.8$ MeV, and $\rho = 0.7$ for both nuclei. The parameter $E_c(2^+)$ has been chosen to fit experimental

$E(I = 17) - E(I = 15)$. A constant energy is added to the calculated levels for each nucleus so that $E(I = 16)$ is equal to the experimental value.

Fig. 8. Effects of the Coriolis attenuation factor ρ on the signature splitting of the $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{124}Cs . Except for ρ , the same parameters as in fig. 7 are used. Experimental splittings are represented by solid circles connected with a dashed curve. See text for explanations.

Fig. 9. Experimental (left-hand side) and calculated (right-hand side) signature splittings of $(\pi h_{11/2})^1(\nu h_{11/2})^1$ bands in some Cs isotopes. Experimental data are taken from ref. [14] for ^{120}Cs , from ref. [5] for $^{124,126}\text{Cs}$, and from ref. [38] for ^{128}Cs . No observed bands are ascribed to this configuration for ^{122}Cs [39].

Fig. 10. Experimental and calculated signature splittings of $(\pi h_{11/2})^1(\nu h_{11/2})^1$ bands in some La isotopes. Experimental data are taken from ref. [5] for ^{124}La , from ref. [40] for ^{126}La , and from ref. [41] for $^{128,130}\text{La}$.

Fig. 11. Experimental and calculated signature splittings of $(\pi h_{11/2})^1(\nu h_{11/2})^1$ bands in some $N=75$ isotones. Experimental data are taken from ref. [42] for ^{132}La and from ref. [43] for ^{134}Pr and ^{136}Pm .