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Properties of signature partner superdeformed bands in mercury nuclei

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Abstract

The properties of superdeformed (SD) bands of five pairs signature partners in mercury nuclei have been systematically analyzed in framework of four parameters formula including higher order terms of Bohr-Mottelson collective rotational energies. The level spins and the model parameters are determined by fitting procedure using a computer simulated search program in order to obtain minimum root mean square deviations between the calculated and the experimental transition energies.

The best fitted parameters have been used to calculate the transition energies E_γ, the rotational frequencies $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia. The calculated results agree excellently with the experimental data. $J^{(2)}$ is significantly larger than $J^{(1)}$ for all values of $\hbar\omega$. Also $J^{(2)}$ show a smooth increase with increasing $\hbar\omega$. The appearance of $\Delta I = 1$ and $\Delta I = 2$ staggering in γ-ray transition energies have been examined by using the five-points formula representing the finite difference approximation to the fourth derivative of the γ-ray transition energies at a given spin. The signature partners in Hg nuclei show large amplitude staggering. Also to appear the $\Delta I = 1$ staggering, the transition energies relative to a rigid rotor with a moment of inertia J = 128.219 $\hbar^2\ MeV^{-1}$ are plotted against spins for each signature partner pairs. The difference in transition energies between transitions in the two SD bands ^{191}Hg(SD3)and ^{193}Hg(SD3) are small, therefore, these two bands have been considered as identical bands.

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1.Introduction

In recent years, many superdeformed (SD) bands have been found in several mass regions [1-3]. The A ~190 mass region is special interest, more than 85 SD bands have now been observed in this mass region alone. The SD bands in this mass region were observed down to quite low spin and the behavior of the dynamical moment of inertia for all SD bands are very similar to each other, because the high-N intruder orbital configurations change very little throughout the region [4]. Dynamical moment of inertia show smooth rise as rotational frequency increases, this rise results mainly from the alignment of the angular momentum of paired nucleons in high-N intruder orbitals and from the gradual disappearance of pairing correlations with the collective motion [4,5]. Many microscopic calculations with different treatments of the pairing interaction have been made in an attempt to account for the increase of dynamical moment of inertia with collective rotation [6, 7].

For most SD bands, the spins have not been determined and only dynamical moment of inertia can be extracted from experimental transition energies. Fortunately, because of the regular behavior of transition energies, their spins have been consistently and reliably predicated by various approaches [8-19]. Moreover, the spins of some SD bands have been established experimentally [20].

One of the most striking properties of SD bands is the existence of identical bands (IB's) or twin bands [21-23], that is nearly identical transition energies of the emitted gamma radiation in bands belonging to neighboring nuclei with different mass numbers. Several groups have been tried to understand this phenomenon [16, 19, 24-28].

It has been demonstrated that rotational sequences in some SD nuclei with nuclear spins differing by two may split into two branches [29, 30]. This phenomenon is called ΔI = 2 staggering or ΔI = 4 bifunction in the γ-ray transition energies. Thus, the SD band can be viewed as two sequences of states in which spins differing by $4\hbar$ from level to level and a small energy displacement occur between the two states. Several theoretical proposal for the possible explanation of this ΔI = 2 staggering phenomenon wear made [16, 28, 31-35].

There is another staggering phenomenon, the ΔI = 1 staggeringor signature splitting in SD nuclei. It was seen that whelming majority of SD bands observed in odd-A nuclei in the A~ 190 regions are signature partners [36-39]. Most of these signature partners show large amplitude ΔI = 1 staggeringand the band head moments of inertia of each pair are almost identical.

In this paper the identical bands and the ΔI = 1, ΔI = 2 staggering phenomenon in signature partner superdeformed bands in mercury nuclei have been investigated in framework of the extended Bohr-Mottelson model. The paper is arranged as follows: following this introduction, in section 2 a four parameter rotational energy originating from Bohr-Mottelson collective rotational model is suggested to describe the superdeformed rotational bands in mercury nuclei. The transitional frequency, the kinematic and dynamic moments of inertia are extracted. Section 3 concerns the origin of the ΔI = 2 staggering superdeformednuclei.The deviation of the γ-ray energies from smooth reference representing the finite difference approximation to the fourth order derivative of the γ-ray transition energies at a given spin is considered. The ΔI = 1 staggering or signature splitting in signature partner pairs in mercury nuclei is proposed in section 4. Numerical calculation are performed and discussed in section 5. Finally conclusion remarks are given in section 6.

1.Model for Superdeformed Rotational Bands (SDRB's)

The occurrence of rotational spectra is a general characteristic of nuclei possessing a nonspherical equilibrium shape. For such nuclei it is possible to separate between a collective rotational motion and the nucleonic or intrinsic motion for fixed nuclear orientation. The rotational spectra become simple if the nuclear shape possesses axial symmetry. Since there can be no collective rotations about a symmetry axis, the component of the total angular momentum along the nuclear symmetry axis k is a constant for each rotational band and represents an intrinsic angular momentum. For a rigid rotor nucleus the rotational spectrum has the general form

$$E(I) = \frac{\hbar^2}{2J}[I(I+1) - k(k+1)] \qquad (1)$$

where J represents the effective moment of inertia about an axis perpendicular to the nuclear symmetry axis (kinematic moment of inertia). More generally, for axial symmetry nuclei the rotational energy can be expressed as an expansion in powers of $\hat{I}^2 = I(I+1)$ as:

$$E(I) = A\,\hat{I}^2 + B\hat{I}^4 + C\hat{I}^6 + D\hat{I}^8 \qquad (2)$$

where B, C and D are corresponding higher order inertial parameters.

The rotational frequency $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia associated with A, B, C, D formula (2) are:

$$\hbar\omega = \frac{dE(I)}{d\hat{I}}$$

$$= 2A\hat{I} + 4B\hat{I}^3 + 6C\hat{I}^5 + 8D\hat{I}^7 \qquad (3)$$

$$\frac{J^{(1)}}{\hbar^2} = \left[\frac{1}{\hat{I}}\frac{dE}{d\hat{I}}\right]^{-1} = \frac{\hat{I}}{\hbar\omega}$$

$$= \left[2A + 4B\hat{I}^2 + 6C\hat{I}^4 + 8D\hat{I}^6\right]^{-1}$$

$$= \frac{1}{2}\left[A^{-1} - 2BA^{-2}\hat{I}^2 + (4BA^{-3} - 3CA^{-2})\hat{I}^4 + \cdots\right] \quad (4)$$

$$\frac{J^{(2)}}{\hbar^2} = \left[\frac{d^2E}{d\hat{I}^2}\right]^{-1}$$

$$= \left[2A + 12B\hat{I}^2 + 30C\hat{I}^4 + 56D\hat{I}^6\right]^{-1}$$

$$= \frac{1}{2}\left[A^{-1} - 6BA^{-2}\hat{I}^2 + (36B^2A^{-3} - 15CA^{-2})\hat{I}^4 + \cdots\right] \quad (5)$$

If we truncate the expression (2) at the second term only, the band head moment of inertia is given by

$$J_o = \frac{1}{2A} \quad (6)$$

In SDRB's the experimentally determined quantities are the γ-ray transition energies between levels differing by two units of angular momentum, then one can write

$$E_{\gamma 2}(I) = E(I) - E(I-2)$$

$$= (2I-1)[2A + 4B(I^2 - I + 1) + 2C(3I^4 - 6I^3 + 12I^2 - 10I + 4) + 8D(I^6 - 3I^5 + 10I^4 - 15I^3 + 15I^2 - 8I+2 \quad (7)$$

Experimentally, the γ-ray transition energies are commonly translated into values of rotational frequency $\hbar\omega$ anddynamical moment of inertia $J^{(2)}$ as:

$$\hbar\omega = \frac{1}{4}\left[E_\gamma(I+2) + E_\gamma(I)\right] \quad (8)$$

$$\frac{J^{(2)}}{\hbar^2} = \frac{4}{E_\gamma(I+2) + E_\gamma(I)} \quad (9)$$

And if the bandhead spin is determined theoretically, the kinematic moment of inertia $J^{(1)}$ can be extracted by using the experimental γ-ray transition energies as:

$$\frac{J^{(1)}}{\hbar^2} = \frac{2I-1}{E_\gamma(I)} \quad (10)$$

It is seen that, while the extracted $J^{(1)}$ depends on the spin I proposition $J^{(2)}$ does not.

1.The ΔI = 2 Staggering in SD Bands

Some SD rotational bands show unexpectedΔI = 2staggering effects in the transition energies (a zigzag behavior) as a function of rotational frequency or spin. The ΔI = 2rotational bands are perturbed and two ΔI = 4 rotational sequences emerge with energy splitting. This is commonly called ΔI = 4 bifunction, because the SD energy levels consequently separated into two spin sequences with spin values I, I+4, I+8,….. and I+2, I+6, I+10,….. respectively.

Using the finite difference approximation to the fourth derivative of the γ-ray transition energies at a given spin in ΔI = 2 (d^4E_γ/dI^4) , yield

$$\Delta^4 E_\gamma(I) = \frac{1}{16}\left[E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)\right] \quad (11)$$

The formula includes five consecutive E_γ values, and is called five-point formula [30] for ΔE_γ.

1.Signature Splitting in SD Bands

Signature is a quantum number specifically appearing in a deformed intrinsic system. It is related to the invariance of a system with quadrupole deformation under a rotation of 180° around a principle axis. For an odd-A nuclei the signature quantum number can take two different values $\alpha = (-1)^{I-1}/2$. In SDRB's, two rotational bands with sequence of levels differing in spin by $1\hbar$is now divided into two branches, each consisting of levels differing in spin by $2\hbar$ and classified by the signature quantum number $\alpha = \pm 1/2$ respectively. The energetically favored branch is formed by those spin I states that satisfyI - j = even, where j is the total angular momentum of corresponding single particle state. For even-even nuclei $\alpha = 0 \; or \; 1$.

An interesting phenomenon is theΔI = 1 signature splitting in SD bands. In a plot of signature partner depending on the transition energies versus spin, a staggering or zigzag pattern can be seen. These irregularities are attributed to the decoupling effect. To explore more clearly the ΔI = 1 staggering in SD signature partners, we may use the five point formula used in ΔI = 2 staggering and remember that$E_\gamma(I)$ denotes the dipole transition energy (the transition energy from a spin state with I to (I - 1))

$$E_\gamma(I) = E(I) - E(I-1)$$

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$= 2I[A + 2BI^2 + C(3I^4 + I^2) + 4D(I^6 + I^4)]$ (12)

The ΔI = 1 staggering parameter becomes

$\Delta^{uf} E_\gamma(I) = \frac{1}{16}\left[E_\gamma(I+2) - 4E_\gamma(I+1) + 6E_\gamma(I) - 4E_\gamma(I-1) + E_\gamma(I-2)\right]$(13)

If the rotational energy follows the pure rotator low A I(I + 1), then $\Delta^{uf} E_\gamma(I)$ is constant quantity A.

However, in most SD signature partners $\Delta E_\gamma^{uf}(I)$ show a zigzag pattern. Another way to explore the ΔI = 1 signature splitting is transition energies relative to the rigid rotor with constant moment of inertia if plotted as a function of angular momentum I.

1. Numerical Calculations and Discussions

Oursignature partner pairs in Hg nuclei include five pairs namely $^{191}Hg\ (SD2, SD3)$, $^{193}Hg\ (SD1, SD2)$, $^{193}Hg\ (SD3, SD4)$, $^{195}Hg\ (SD3, SD5)$ and $^{194}Hg\ (SD2, SD3)$. The expansion parameters A, B, C and D of the theoretical transition energies and the bandhead spin I_0 for each band have been calculated by best fitting procedure to the observed experimental transition energies by using a computer simulation search program. To parameterize the spins, we assumed various values for the bandhead spin I_0 for each SD band. The fitting procedure was repeated with spin I_0 fixed at the nearest half integer. The quality of the fit is indicated by the root mean square (rms) derivation χ given by

$$\chi = \left[\frac{1}{N}\sum_{I_i}\left|\frac{E_\gamma^{exp}(I_i) - E_\gamma^{theor.}(I_i)}{\Delta E_\gamma^{exp}(I_i)}\right|^2\right]^{1/2}$$

where N is the number of the data points entering into the fitting procedure and $\Delta E_\gamma^{exp}(I_i)$is the experimental error of the γ-transition energies. The optimized four model parameters for each band resulted from the fitting γ-ray energies E_γ, the rotational frequency $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia for the studied five signature parameter points. The agreement between calculated and observed ones are excellent. The experimental transition energies are taken from Ref[1]. Table (1) lists the optimized model parameters A, B, C, D, the band head spin proposition I_0, the bandhead moment of inertia I_0 and the lawest transition energies $E_\gamma(I_0 + 2 \longrightarrow I_0)$ for each SD band. Figure (1) illustrate the behavior of the kinematic moment of inertia $J^{(1)}$(dashed curve) and the dynamic moment of inertia $J^{(2)}$ (solid curve) as a function of rotational frequency $\hbar\omega$. It is seen that the agreement between theory and experiment (closed circles with error bars) is excellent. Both moments of inertia $J^{(1)}$ and $J^{(2)}$ shows a smooth and similar increase with increasing $\hbar\omega$, which can be understood as the gradual alignment of angular momentum of a pair of two intruder nucleons in high orbit. The $J^{(2)}$ moment of inertia in significantly larger than $J^{(1)}$ over a large rotational frequency range.

The signature splitting (or the ΔI = 1 staggering) $\Delta E_\gamma^{uf}(I_i)$ in SD odd-A signature partners in ^{191}Hg, ^{193}Hg and ^{195}Hgnuclei and the signature partner pair in even-even nuclei^{194}Hg have been extracted and plotted versus spin I in Figure (2) using the five point formula. It is shown that, the signature partner pairs in Hg nuclei exhibit a large amplitude ΔI = 1 staggering. Another ΔI = 1 staggering happen in the transition energies $E_\gamma(I)$ after subtracting a rigid rotor reference, when plotted versus spin. The results shown in Figure (3) with rotor reference having moment of inertia $J = 128.219\ \hbar^2 MeV^{-1}$.

Another result of the present work is the appearance of a ΔI = 2 staggering effect in the γ-ray transition energies in$^{194}Hg\ (SD2, SD3)$. The theoretical staggering parameter $\Delta^4 E_\gamma(I)$ has been calculated by using the five-point formula which includes five consecutive transition energies and illustrated in Figure (4) as a function of rotational frequencies$\hbar\omega$. A significant anomalous staggering has been observed. The difference in a γ-ray energies ΔE_γ between

transitions in the two SD band $^{191}Hg\ (SD3)$ and $^{193}Hg(SD3)$ are small (less than 2.8KeV). Therefore, there twobands have been considered as identical bands. This fact was interpreted in terms of pseudo spin alignment[22].

Table (1) lists the optimized model parameters A, B, C, D, the bandhead spin proposition I_0, the bandhead moment of inertia I_0 and the lawest transition energies $E_\gamma(I_0+2 \longrightarrow I_0)$ for each SD band.

SD Band	$E_\gamma(I_0+2\rightarrow I_0)$ (KeV)	A (KeV)	B (KeV) *10^-4	C (KeV) *10^-8	D (KeV)	I_0 (h)	J_0 ($\hbar^2 MeV^{-1}$)
1	252.4	5.31491	-2.2632	3.4863	-6.7132*10^-12	10.5	94.0748
1	272.0	5.32766	-2.7036	4.9998	-1.1557*10^-11	11.5	93.8495
1	233.2	5.60630	-10.785	1.2642	-1.8524*10^-9	9.5	89.1851
1	254.0	5.38012	-3.0426	6.6720	-1.8288*10^-11	10.5	92.9345
1	233.5	5.38273	-3.0969	6.9656	-1.9584*10^-11	9.5	92.8894
1	291.0	5.39564	-3.6995	9.9895	-3.3714*10^-11	12.5	94.2199
1	244.0	5.83561	-3.1841	2.9784	-3.4826*10^-12	8.5	85.6798
1	341.9	5.12356	-1.6010	1.9300	-2.9082*10^-12	15	97.5880
1	200.79	5.2633	-1.6345	10.686	-1.0480*10^-10	8	94.9974
1	222.0	5.2805	-1.7116	11.718	-1.2034*10^-10	9	94.6880

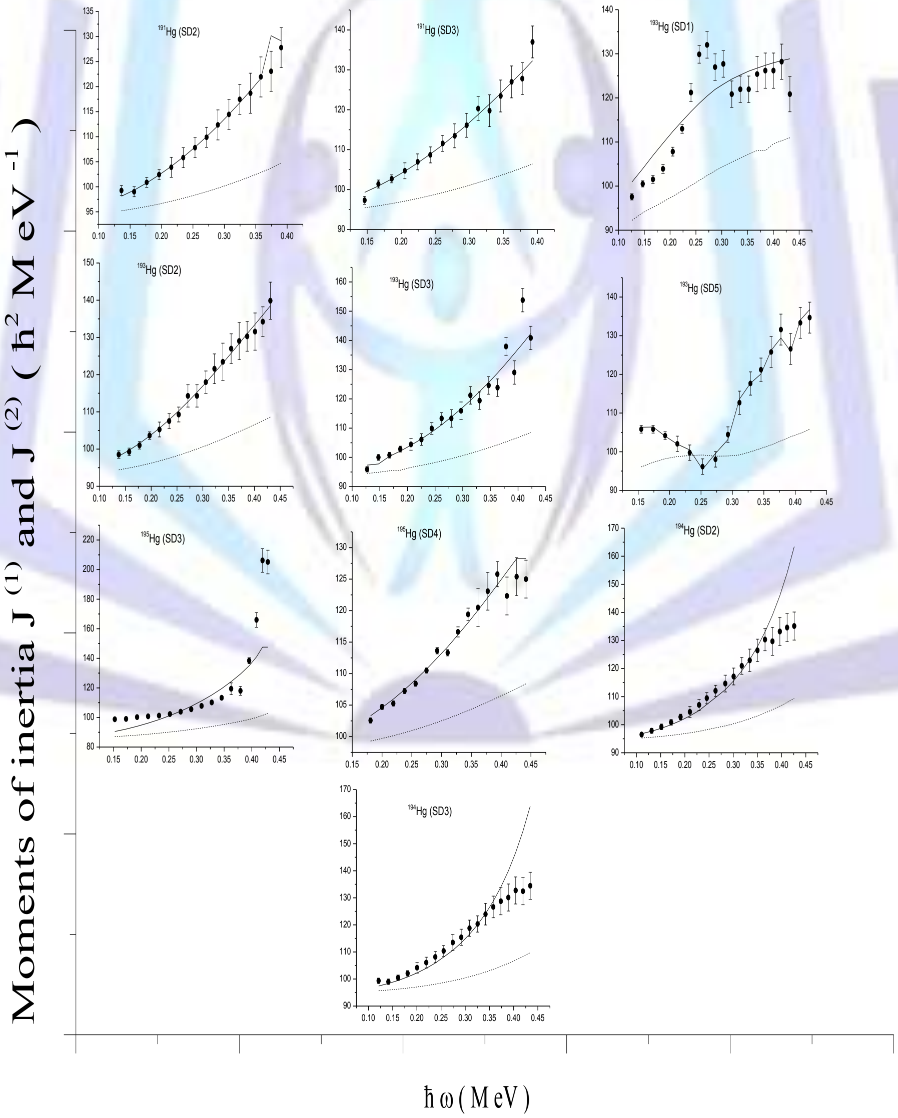

Figure (1) The calculated results of the kinematic moment of inertia $J^{(1)}$ (dashed curve) and the dynamic moment of inertia $J^{(2)}$ (solid curve) as a function of rotational frequency $\hbar\omega$ for SD signature partner pairs in Hg nuclei comparison with experimental data (closed circles with error bars).

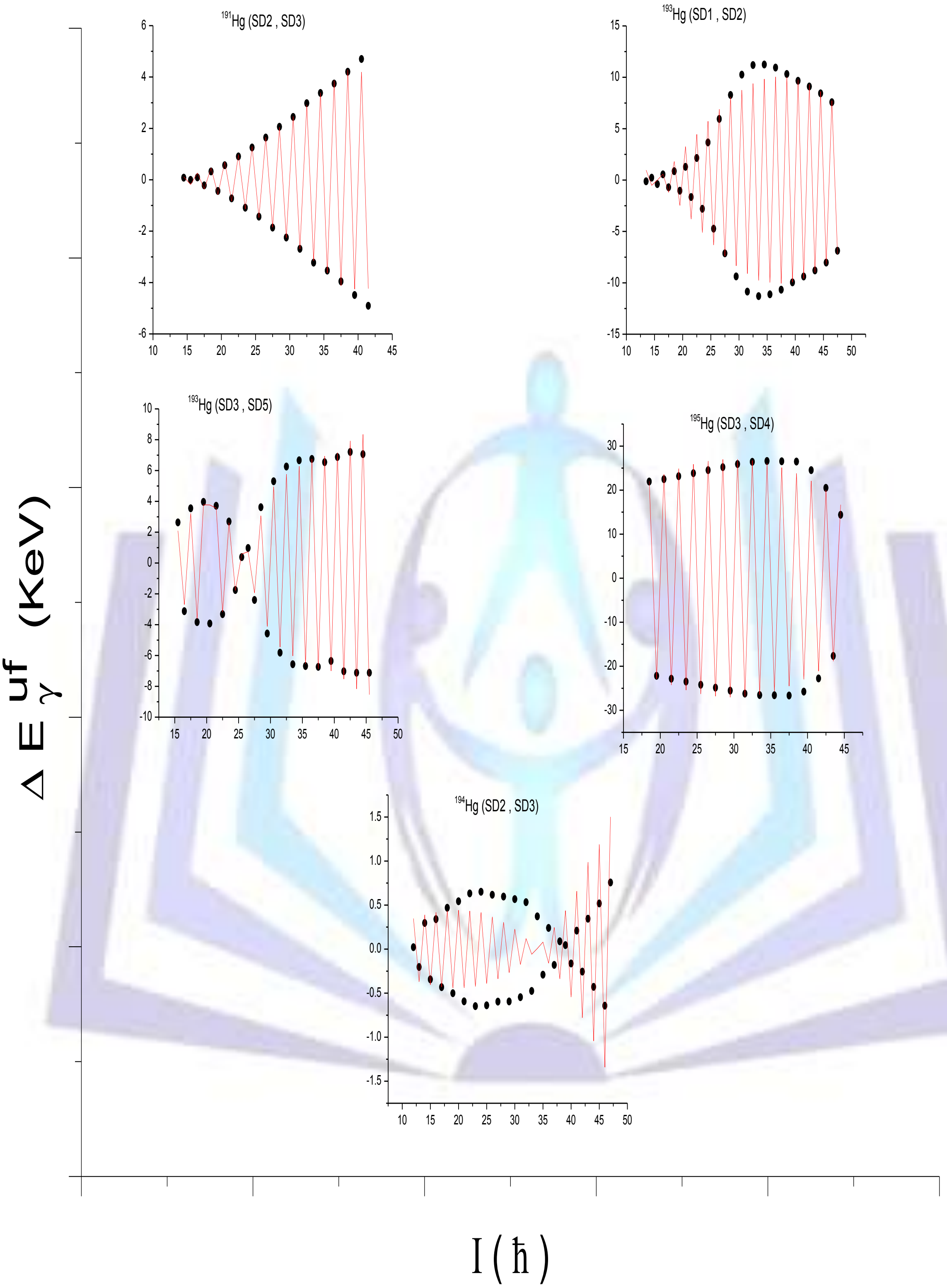

Figure (2). The calculated staggering parameter $\Delta E_{\gamma}^{uf}(I)$ (solid curves) as a function of spin I for the signature partner pairs in Hg nuclei. The experimental values are represented by dots.

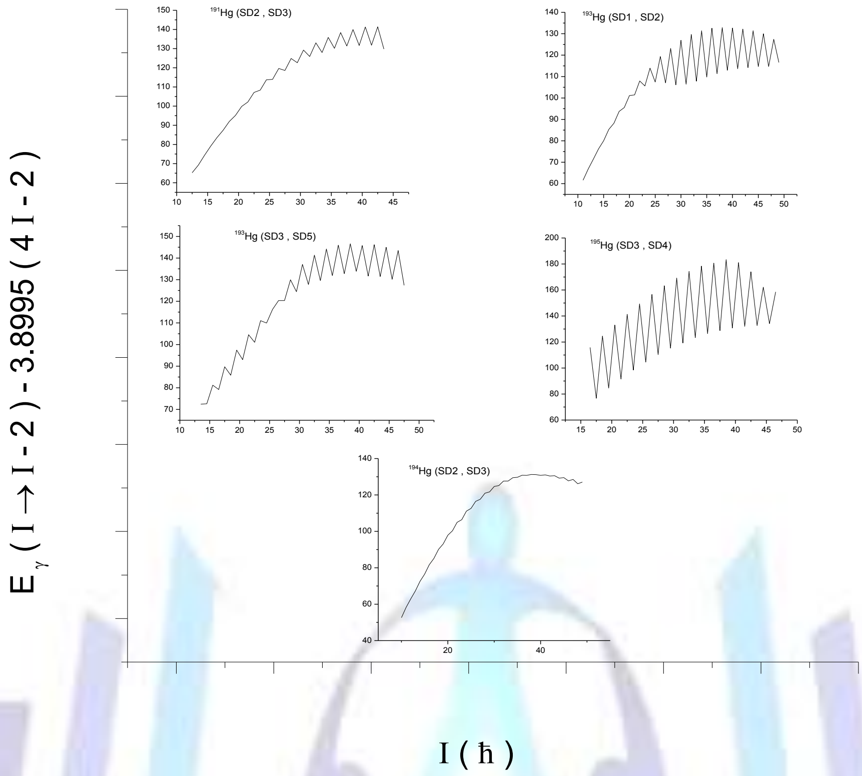

Figure (3). The ΔI = 1 staggering in the calculated transition energies minus rigid rotor reference with a moment of inertia J = 128.219 $\hbar^2\ MeV^{-1}$ as a function of spin I for the signature partner pairs in Hg nuclei.

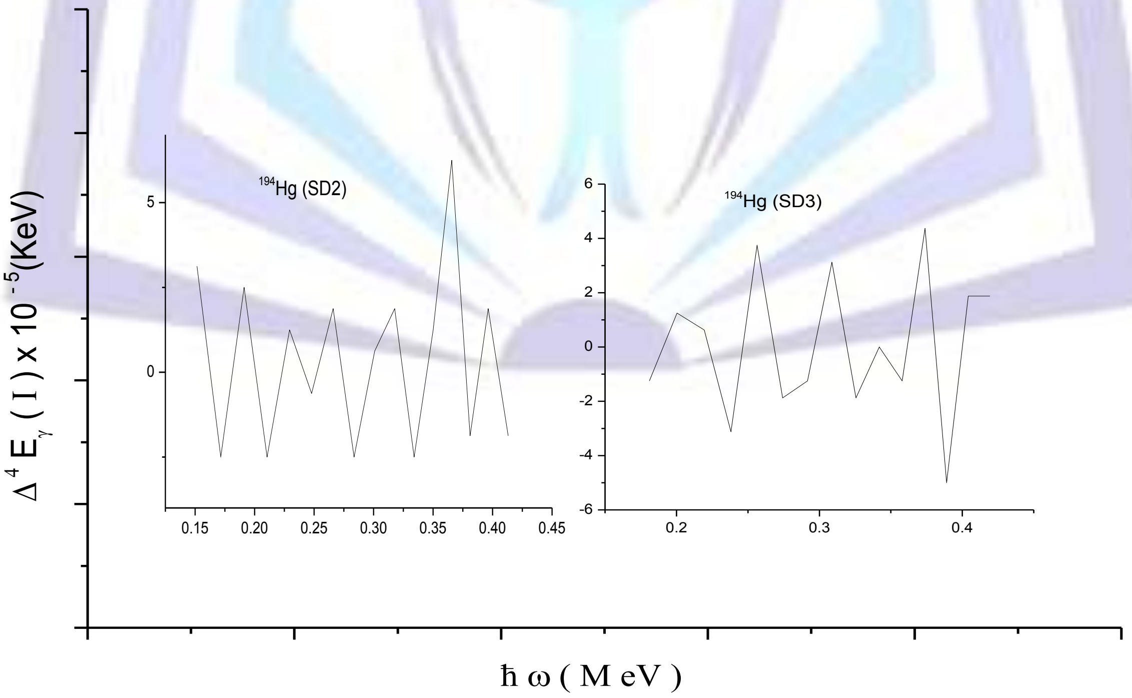

Figure (4). The calculated ΔI = 2 energy staggering parameter $\Delta^4 E_\gamma(I)$ plotted as a function of the rotational frequency $\hbar\omega$ for the signature partner pairs ^{194}Hg (SD2, SD3).

Conclusion

We showed in this paper that nuclearsuperdeformedrotational bands of signaturepartner pair in mercury nuclei can be described with the modified higher order terms of Bohr- Mottelson formula which connected directly the energy with the unknown spin. For each superdeformed band the bandhead spin is determined and the model parameters are fitted to reproduce the observed experimental γ-ray transitionenergies. Using the adopted best optimized model parameters and the determined spins, the theoreticaltransition energies, the rotationalfrequency, the kinematic and dynamic moments of inertia have been calculated. The calculated results agree very well with the experimentalones. By performing the staggeringparameter analysis for each band and using the five–point formula which includes five consecutive transition energies,we found $\Delta I = 1$ staggering in all the considered five signature partner pairs and $\Delta I = 2$ staggering in the signaturepartner pair^{194}Hg (SD3,SD4) when plotting the staggering parameter against the rotational frequency or spin . Most of these superdeformed rotational bands show large significant staggering. We noticed that transition energies in ^{191}Hg(SD3)is identical to ^{193}Hg (SD3).

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