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Anharmonic oscillator potentials in the prolate γ-rigid regime of the collective geometrical model

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Abstract. Based on higher order JWKB approximation, an analytical formula is derived for the energy spectrum of the prolate γ-rigid Bohr-Mottelson Hamiltonian with an oscillator potential and a quartic or sextic anharmonicity in β shape variable. The model is found to provide good description for near vibrational nuclei situated in vicinity of a closed neutron shell.

1. Introduction

An analytical expression for the energy spectrum of the ground and β bands was obtained in the axially symmetric γ-rigid regime of the Bohr Hamiltonian with a harmonic oscillator potential in the β shape variable amended with a higher order anharmonicity term. The anharmonic terms are considered of quartic [1] and sextic [2] types. The particular structure of the model space facilitates the exact separation of angular variables from the β variable, leading to a differential Schrödinger equation with an anharmonic potential and a centrifugal-like barrier. The Schrödinger equation for such potentials is not exactly solvable. Thus, the corresponding eigenvalues are approximated by an analytical formula derived on the basis of the JWKB approximation [3, 4], which depends on a single parameter except the scale. The experimental realization of the model is found in a variety of vibrational-like nuclei exhibiting some regularity regarding the order of the considered anharmonicity.

2. Prolate γ-rigid collective Hamiltonian

The Hamiltonian associated to a prolate γ-rigid nucleus is [1, 2, 5, 6, 7]:

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta} - \frac{\hat{\mathbf{I}}^2}{3\hbar^2\beta^2}\right] + U(\beta), \tag{1}$$

where $\hat{\mathbf{I}}$ is the angular momentum operator from the intrinsic frame of reference, while B is the mass parameter. The Schrödinger equation associated to such a Hamiltonian is solved by separating the β variable from the angular ones which is achieved through the factorization $\Psi(\beta,\theta_1,\theta_2) = F(\beta)Y_{IM}(\theta_1,\theta_2)$, where the angular factor state is a spherical harmonic function. With this, the Schrödinger equation is reduced to a second order differential equation in variable

β, which is brought to a canonical form through the change of function $F(\beta) = \frac{f(\beta)}{\beta}$:

$$\left[\frac{d^2}{d\beta^2} - \frac{I(I+1)}{3\beta^2} + 2(\varepsilon - u(\beta))\right] f(\beta) = 0, \tag{2}$$

where $\varepsilon = BE/\hbar^2$, $u(\beta) = BU(\beta)/\hbar^2$ denote the reduced energy and potential.

3. Quartic and sextic anharmonicities

The eigenvalue problem for potentials:

$$u(\beta) = \frac{1}{2}\alpha_1\beta^2 + \alpha_2\beta^{2m}, \;\; \alpha_1 \geqslant 0, \alpha_2 > 0 \text{ and } m = 2, 3, \tag{3}$$

has the following scaling property:

$$\varepsilon(\alpha_1, \alpha_2) = \alpha_2^{\frac{1}{m+1}}\varepsilon(\lambda_m, 1) = \alpha_2^{\frac{1}{m+1}} W^m(\lambda_m), \tag{4}$$

where $\lambda_m = \alpha_1\alpha_2^{\frac{m}{6}-1} \geqslant 0$. This follows from the change of variable $\beta' = \alpha_2^{\frac{1}{2(m+1)}}\beta$ in the differential equation (2). Similar scaling was used in Refs.[7, 8] in the case of a quasi exactly solvable sextic potential. The problem is then reduced to a radial Schrödinger equation with a modified centrifugal term for the scaled potential $\tilde{u}(\beta') = \frac{1}{2}\lambda_n\beta'^2 + \beta'^{2m}$. Adapting the procedure of Refs.[3, 4] one can write the eigenvalue W^m as function of λ_m, n - the β vibration quantum number and the intrinsic angular momentum I, as follows:

$$W^2_{nI}(\lambda_2) = (C_2N_{nI})^{\frac{4}{3}} \sum_{k=0}^{8} G^2_k(\lambda_2, I)\,(C_2N_{nI})^{-\frac{2k}{3}}\,, \tag{5}$$

$$W^3_{nI}(\lambda_3) = (C_3N_{nI})^{\frac{3}{2}} \sum_{k=0}^{5} G^3_k(\lambda_3, I)\,(C_3N_{nI})^{-k}\,, \tag{6}$$

where $N_{nI} = 2n+1+(\sqrt{1+4I(I+1)/3})/2$. The functions $G^m_k(\lambda_m, I)$ represent polynomials in λ_m of order k with angular momentum dependent coefficients and are given explicitly in [1, 2] together with the constants C_m.

The energy spectrum of a nucleus described by the Hamiltonian (1) with an oscillator potential with quartic or sextic anharmonicities is then given by:

$$E^m_{nI} = \frac{\hbar^2}{B}\alpha_2^{\frac{1}{m+1}}\left[W^m_{nI}(\lambda_m) - W^m_{00}(\lambda_m)\right]. \tag{7}$$

4. Numerical results

The energy formula depends up to an overall factor on a single free parameter which is bounded by the convergence radius λ^c_m of the considered approximation. The acting space of the model is then restricted to the interval $[0, \lambda^c_m]$ which put in terms of the ratios $R_{4/2}$ and $R_{0/2}$ lies between $[2.29, 2.00]$ and $[2.37, 1.81]$ for quartic and respectively $[2.34, 2.15]$ and $[2.56, 2.11]$ for sextic anharmonicities (See Figure 1). The upper limit corresponds to $\lambda_m = 0$ which is associated to the free parameter models $X(3)$-β^{2m}, while the lower limit is close to the $X(3)$-β^2 predictions [1].

As can be seen from Figure 2, the quartic model is experimentally realized for ^{100}Pd, ^{148}Sm, and ^{222}Th nuclei, while the sextic one describes their next heavier neighbours. The three isotopic pairs also have the same number of neutrons above the corresponding shell closure at magic numbers $N = 50, 82$ and 126.

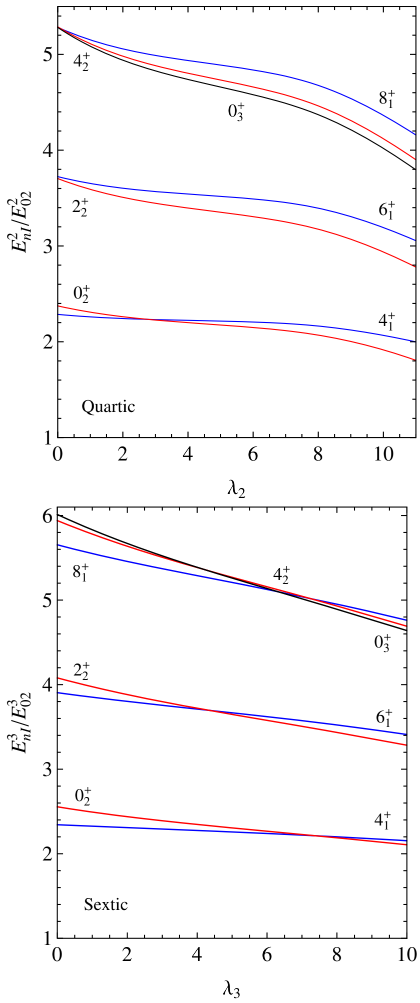

Figure 1. Theoretical low lying energy spectrum for ground and two β excited bands.

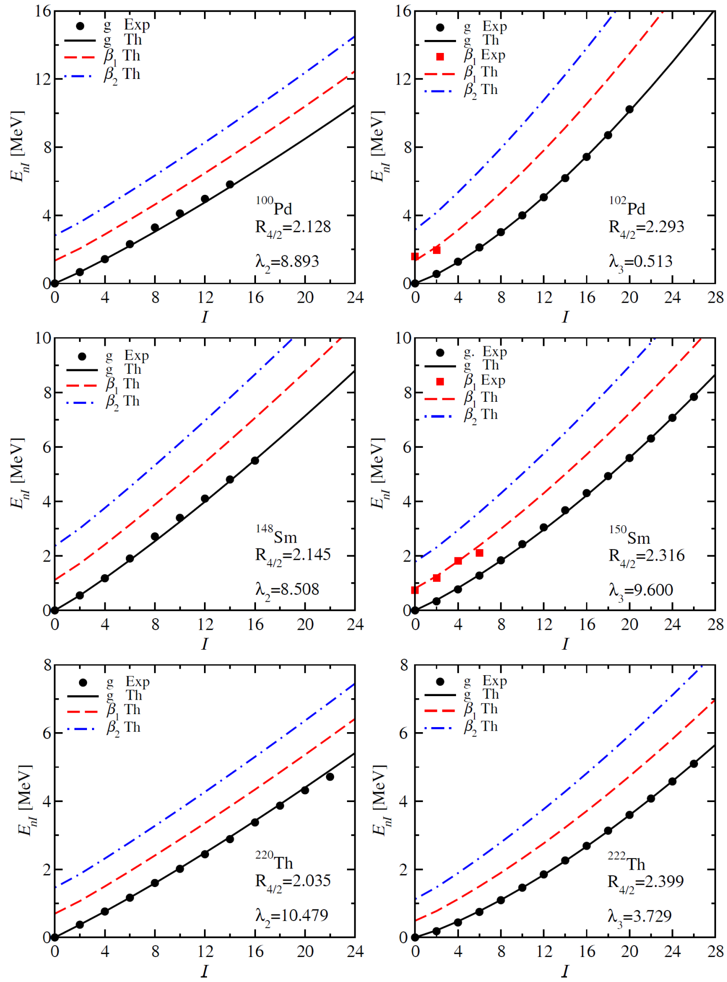

Figure 2. Quartic model description of the lighter isotopes of Pd, Gd and Th [9] in comparison to the heavier ones treated by means of a sextic term.

5. Outlook

The present approximate formulas can be easily translated to the case of four and five dimensional Bohr-Mottelson Hamiltonians with a similar separated potential for the β shape variable.

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