EFFECT OF THE CORIOLIS AND CENTRIFUGAL FORCES FOR NUCLEI WITH A STABLE OCTUPOLE DEFORMATION ☆

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Effects of the Coriolis and centrifugal forces for nuclei with a stable octupole deformation are examined in the frame of a schematic collective model. It is found that these effects are by no means attenuated with a rise of the octupole deformation. Taking them into account seems to allow for a consistent description of a strong anharmonization and differences in the moments of inertia of the positive- and negative-parity bands.

The discussion on the nature of the negative-parity bands lying exceptionally low in some even–even isotopes in the radium region is still continuing. Three alternative interpretations in the frame of collective model have been proposed so far. The oldest one, reinforced recently again, considers these excitations as the octupole vibrations around a quadrupole deformed shape [1]. According to the other one these bands are associated with octupole deformed isomers [2]. The newest interpretation, looking most complete, proposes introducing a stable octupole deformation in the ground state [3]. The latter seems to be also supported by recent theoretical investigations [4,5]. The main objection against it is the suggestion of Peker et al. [1] that the strong Coriolis coupling effects do not exist for nuclei with a stable octupole deformation and therefore the moments of inertia of the ground-state band and the negative-parity band should be close to each other, which is not the case for nuclei in question.

The aim of the present note is to examine how strong the effects of the Coriolis and centrifugal forces are, in fact, in the case of stable octupole deformation. We assume that the collective model is relevant for the description of the excitations in question (however, of ref. [6]). Then, for axially symmetric, both quadrupole and octupole deformed nuclei, the collective hamiltonian takes the form [7]:

$$\mathcal{H} = \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_c, \tag{1}$$

where $\mathcal{H}_2$ is the hamiltonian of the quadrupole vibrations which is, however, irrelevant in the present problem.

$$\mathcal{H}_3 = (1/2\mathcal{I})(\vec{\mathcal{L}}^2 - \mathcal{L}_z^2) + \sum_{k=0}^{3} \mathcal{H}_{3k} \tag{2}$$

describes rotations and the octupole anisotropic vibrations.

$$\mathcal{H}_{30} = \omega_0 \hbar [-\kappa_0^{-2}\, \mathrm{d}^2/\mathrm{d}\beta_0^2 + \tfrac{1}{4}\kappa_0^2 (|\beta_0| - a_0)^2] \tag{3}$$

represents the $K = 0$ octupole vibration in the double oscillator potential,

$$\mathcal{H}_{3k} = \omega_k \hbar [-\kappa_k^{-2} \beta_k^{-1} (\partial/\partial\beta_k)(\beta_k \partial/\partial\beta_k) - \kappa_k^{-2} \beta_k^{-2}\, \partial^2/\partial\varphi_k^2 + \tfrac{1}{4}\kappa_k^2 \beta_k^2] \tag{4}$$

are the hamiltonians of the $K \neq 0$ octupole harmonic two-dimensional vibrations. The collective variables β_0 and $(\pm 1)^k \beta_k \exp(\pm \mathrm{i}\varphi_k)/\sqrt{2}$ are the intrinsic spherical 0 and $\pm k$ ($k = 1,2,3$) components, respectively, of the octupole tensor α_3. Finally

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$$\mathcal{H}_c = -(1/2\mathcal{I})(\mathcal{L}_+^{(3)}\mathcal{L}_-^{(3)} + \mathcal{L}_-^{(3)}\mathcal{L}_+^{(3)} - 2\mathcal{L}_+\mathcal{L}_-^{(3)} - 2\mathcal{L}_-\mathcal{L}_+^{(3)}) \quad (5)$$

represents the Coriolis and centrifugal forces, $\mathcal{L}_\kappa$ ($\kappa = z, +, -$) and

$$\mathcal{L}_z^{(3)} = -i\hbar \sum_{k=1}^{3} k\partial/\partial\varphi_k,$$

$$\mathcal{L}_\pm^{(3)} = \tfrac{1}{2}\hbar \sum_{k=0}^{2} [\tfrac{1}{2}(4+k)(3-k)(1+\delta_{k0})]^{1/2} \times \exp[\pm i(\varphi_{k+1} - \varphi_k)] \times \{\beta_k [\partial/\partial\beta_{k+1} \pm (i/\beta_{k+1})\partial/\partial\varphi_{k+1}] - \beta_{k+1}[\partial/\partial\beta_k \pm (i/\beta_k)\partial/\partial\varphi_k]\} \quad (6)$$

are the intrinsic spherical components of the total and the partial octupole angular momentum. To demonstrate our point, we take a primitive octupole deformation potential and neglect all the possible couplings between the quadrupole and octupole modes.

The experimental data available so far are too scarce to allow for fitting the parameters of the model for any of the nuclei in question. The position of the $K \neq 0$ vibrational levels and B(E3) values are needed to do this. Therefore we examine general trends in the Coriolis and centrifugal effects as function of increasing octupole deformation and choose, rather arbitrarily, some reasonable values of the other parameters. These trends do not necessarily correspond to those in the isotope chains where perhaps not only deformation changes. With changes of parameters the effects in question can be either intensified or reduced but the general view remains the same.

We diagonalize $\mathcal{H}$ within the basis of eigenfunctions of $\mathcal{H}_3$. The unperturbed energy levels (the eigenvalues $\mathcal{H}_3$) are labelled by the following quantum numbers: L the angular momentum, K: its projection on the intrinsic symmetry axis, ν_k ($k = 0, 1, 2, 3$) the oscillator numbers for the corresponding four octupole vibrations. Only L and P, the parity, remain as good quantum numbers for the exact (perturbed) energy levels E_L^P. The following values of the parameters are used in the calculation: $\hbar\omega_0 = 0.5$ MeV, $\hbar\omega_1 = 1.0$ MeV, $\hbar\omega_2 = 1.5$ MeV, $\hbar\omega_3 = 2.0$ MeV, $\kappa_0 = 12$, $\kappa_1 = 15$, $\kappa_2 = 20$, $\kappa_3 = 25$, $\mathcal{I} = 45\,\hbar^2$/MeV. Fixing the above values of $\mathcal{I}$, ω_0, ω_1 and ω_1 and κ_0 we have observed the ^{226}Ra data [3,5] just to have very rough estimate of the lowest E_2^+, E_1^- ($K = 0$), E_1^- ($K = 1$) levels and B(E3; $0^+ \to 3^-$) values. Correspondingly, some higher values for ω_2, ω_3, κ_1, κ_2 and κ_3 are taken to describe reasonably the hypothetical $K \neq 0$ octupole vibrations. The basis is truncated in such a way that the states with the vibrational excitation energy not bigger than $8\,\hbar\omega_0 = 4$ MeV are taken into account only.

As is seen from eq. (6) the matrix elements of the Coriolis hamiltonian depend on the octupole deformation a_0 through the one-dimensional matrix elements of β_0 and $d/d\beta_0$ within the eigenfunctions of $\mathcal{H}_{30}$. Fig. 1, where some of such matrix elements for $\nu_0 = 0$, 1, 2 are drawn as functions of he deformation parameter $x_0 = \kappa_0 a_0$, shows that although the matrix element of $d/d\beta_0$ within the ground and the first excited states tends to zero for increasing deformation, the matrix element of β_0 increases proportional to the deformation [‡1]. This tendency is due to the double degeneration of the levels in a double-well potential for large deformations and does not much depend on its particular shape. Generally, all the matrix elements of β_0 within the states ν_0 and $\nu_0' = \nu_0 + 1$ for $\nu_0 = 0, 2, \ldots$ increase indefinitely when the deformation increases. As such elements occur in the most important Coriolis

‡1 The deformation dependence of the matrix elements of β_0 shown in fig. 3 of ref. [7] is unfortunately incorrect.

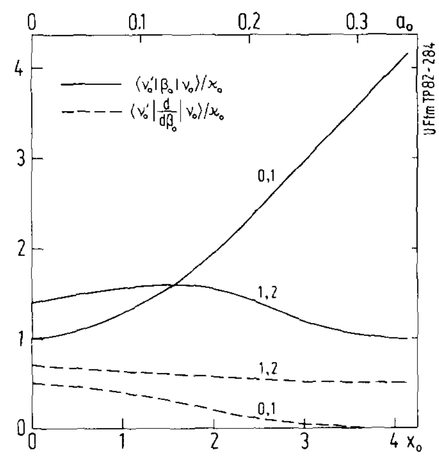

Fig. 1. Matrix elements $\langle\nu_0'|\beta_0|\nu_0\rangle$ (solid lines) and $\langle\nu_0'|d/d\beta_0|\nu_0\rangle$ (broken lines) within the three lowest double oscillator wave functions as functions of the octupole static deformation. The labels on the curves refer to the values of ν_0, ν_0'.

matrix elements we expect an intensification of the Coriolis coupling effects rather than an attenuation with the rise of the octupole deformation. This is indeed the case: the Coriolis force turns the rotational bands upside down for large values of x_0. But it would be senseless to consider the Coriolis interaction alone without the centrifugal forces which, in fact, restore – due to the positive definiteness of the kinetic rotational energy – a proper order of states. As a matter of fact, results of the present calculation show that the role of the perturbation $\mathcal{H}_c$ is not too small also for large deformations. Let us briefly present its main effects. Fig. 2 shows that the perturbed energy E_0^+/E_1^- (a measure of anharmonicity) of levels with the asymptotic numbers $\nu_0 = 2, KL^P = 00^+$ and $\nu_0 = 1$, $KL^P = 01^-$, respectively, is smaller than the unperturbed one for all deformations. Also, the reduced probability of the E3 transition from the ground 0^+ state to the lowest 3^- state becomes, excluding small deformations, smaller, as is seen from fig. 3. The E3 operator

$$\mathcal{M}(\mathrm{E3}, m) = (3Ze/4\pi)\, A r_0^3\, \alpha_{3m} \tag{7}$$

is used for simplicity in the calculation [7]. Indeed, the effective moment of inertia of the lowest negative-parity band

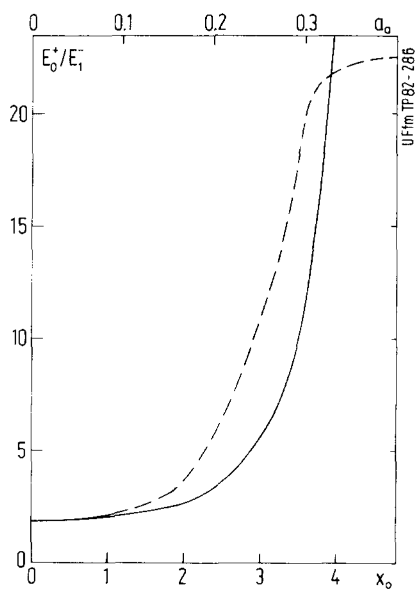

Fig. 2. The unperturbed (broken line) and perturbed (solid line) energy ratios E_0^+/E_1^- as functions of the octupole static deformation. Notice that the rotational energies are included.

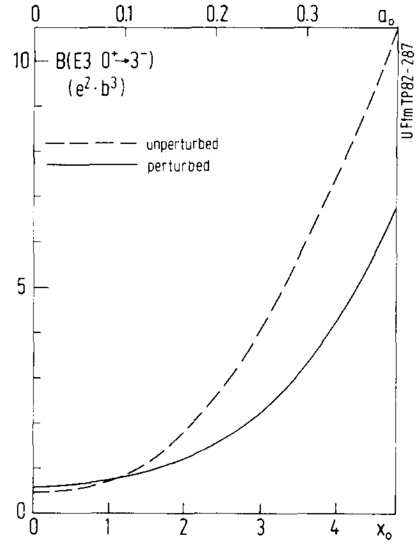

Fig. 3. The unperturbed (broken line) and perturbed (solid line) $B(\mathrm{E3}; 0^+ \to 3^-)$ values as functions of the octupole static deformation. The values $Z = 88, A = 226$ are used in the calculation.

$$\mathcal{J}^- = 5\hbar^2/(E_3^- - E_1^-) \tag{8}$$

decreases somewhat when the deformation increases, but it never approaches the unperturbed value $\mathcal{J}$ and increases again for large deformations. The effective moment of inertia of the ground-state band

$$\mathcal{J}^+ = 3\hbar^2/(E_2^+ - E_0^+), \tag{9}$$

being practically constant for small deformations, begins to increase rapidly for large deformations. The two achieve really values close to each other but far from the unperturbed value in the large deformation limit. This is seen in fig. 4. The effect of Coriolis coupling in the lowest positive- and negative-parity bands is usually estimated [8] by the quotient

$$R_\mathrm{L} = (E_{L+3}^- - E_{L+1}^-)/(E_L^+ - E_{L-2}^+), \tag{10}$$

which is equal to

$$R_\mathrm{L} = (2L+5)/(2L-1), \tag{11}$$

when the effective moments of inertia of both bands are equal to each other. Fig. 5 shows the values of R_L for a few values of the deformation parameter x_0. It can be seen that the picture given by Zimmermann [9] is retained on condition that the upper bounds given by eq. (11) are reached only for really very large defor-

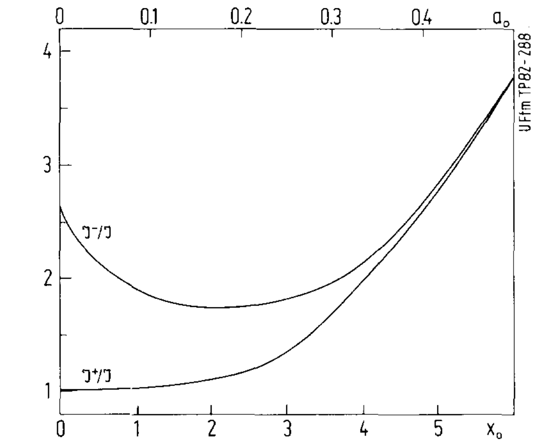

Fig. 4. The effective moments of inertia of the lowest positive- and negative-parity bands, $\mathcal{J}^+$ and $\mathcal{J}^-$, in units of the unperturbed moment $\mathcal{J}$ as functions of the octupole static deformation.

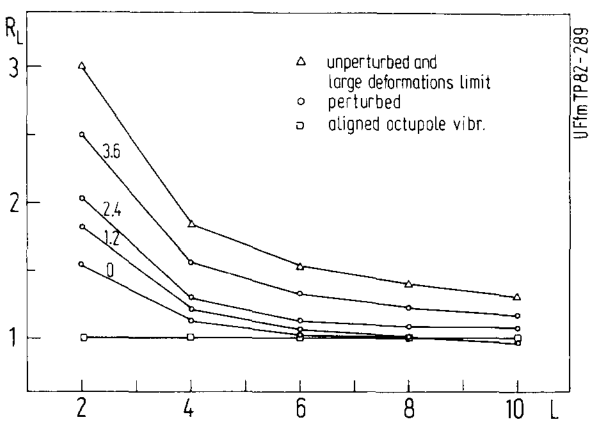

Fig. 5. The ratio R_L (defined in the text) for a few values of the deformation parameter x_0 indicated on the broken lines.

mations. More detailed results of the calculation will be published elsewhere [10].

We conclude that the Coriolis and centrifugal effects allow for a constant description of experimental data for nuclei suspected of a stable octupole deformation. In particular, a strong anharmonicity and a difference in the moments of inertia of the positive- and negative-parity bands can be explained at the same time. However, to fit more or less uniquely the parameters of the model more comprehensive data are needed. The unperturbed model underestimates the value of the octupole deformation.

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