SIGNATURE INVERSION AT LOW SPINS OF ODD–ODD NUCLEI: TRIAXIAL SHAPE?

Ikuko HAMAMOTO

Department of Mathematical Physics, Lund Institute of Technology, P.O. Box 118, S-221 00 Lund, Sweden

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It is shown that the signature inversion in the rotational spectra of the high-j-configuration bands, which has been observed at low spins of some odd–odd nuclei, can well be consistent with an axially symmetric shape. Thus, the observed signature inversion may not be taken as evidence for a triaxial shape.

Expressing the routhian (i.e. the energy referring to the rotating coordinate system) of the states with signature α by $E'(\alpha)$, the relation

$$E'(\alpha_u) < E'(\alpha_f) \tag{1}$$

for a given rotational frequency is called "anomalous" signature splitting, in which α_f (α_u) denotes the favoured (unfavoured) signature.

The statement on lower-spin states (i.e. one-quasiparticle states) of odd-A nuclei is as follows: Using not only the cranking model [1,2] but also the particle-rotor model [3] in which the angular momentum is treated as a good quantum number, the relation (1) is expected only for some particular shell-fillings, only for smaller rotational frequency and only in the region of either $\gamma > 0°$ or $\gamma < -60°$ (in the Lund convention of γ). In short, a clear observation of the relation (1) at low spins of odd-A nuclei can be used as an indication of a deviation of the nuclear shape from axial symmetry. Experimentally, the relation (1) has never been observed in odd-A nuclei, especially for the high-j configurations in which an unambiguous analysis can be carried out.

In contrast, the relation (1) is observed at very low spins of some medium-heavy odd–odd nuclei, at the band with the "high-j" configuration in which both odd-proton and odd-neutron occupy high-j orbits. However, most of such observation was done in nuclei with small (or very soft) quadrupole deformation. In fig. 1 we choose to show the observed energies of the high-j yrast configuration in two "well-deformed" nuclei, $^{156}_{65}$Tb and $^{160}_{67}$Ho, which show the relation (1) for very low spins. OBserved energy differences (and not routhians) are plotted as a function of angular momentum, since energies and angu-

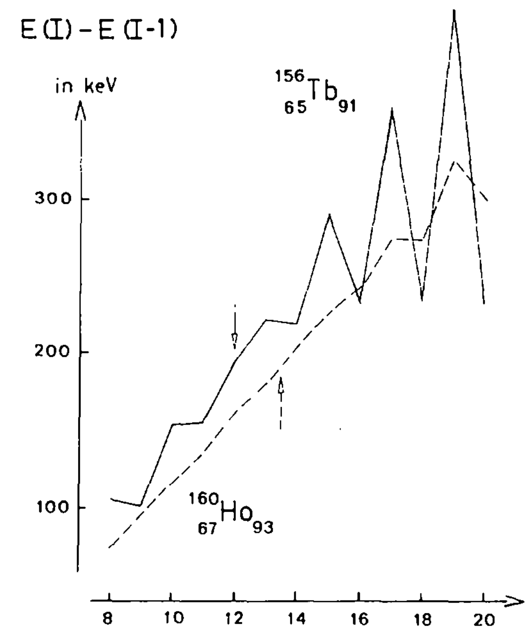

Fig. 1. Observed energies of the high-j ($j_p = h_{11/2}$ and $j_n = i_{13/2}$) yrast band in nuclei ^{156}Tb and ^{160}Ho. The point at which the phase of the signature-dependence changes is marked by an arrow. The data are taken from ref. [4] (^{156}Tb) and refs. [5,6] (^{160}Ho).

lar momenta are directly obtained from measurements and since the procedure of obtaining the rotational frequency from measured energies is not uniquely defined in the case that the structure of energy spectra is clearly changing as a function of angular momentum. By an arrow in fig. 1 we point out the angular momentum at which the phase of the signature dependence (namely, the zigzag behaviour) changes. Since the signature dependence of the ^{160}Ho spectra around the arrow point is so small, in the following we shall take the ^{156}Tb spectra as a serious numerical example. In the high-j configuration of odd–odd nuclei the favoured signature is defined as

$$\alpha_f = \tfrac{1}{2}(-1)^{j_p - 1/2} + \tfrac{1}{2}(-1)^{j_n - 1/2}. \tag{2}$$

Since we have $j_p = h_{11/2}$ and $j_n = i_{13/2}$ in the example of fig. 1, we have $\alpha_f = 0$. That means, the favoured-signature states have even-integer spins. Then, the zigzag behaviour observed at $I \gtrsim 14$ of ^{156}Tb is a normal signature splitting, while the one at $I \lesssim 12$ is an anomalous signature splitting.

Applying a cranking-model calculation, it was proposed in ref. [1] that the anomalous signature splitting and the subsequent signature inversion observed at very low spins of some odd–odd nuclei are due to a $\gamma > 0$ triaxial deformation. Following the proposal, for example, in ref. [6] a small triaxial deformation $\gamma = +5°$ was concluded on for the high-j configuration in ^{160}Ho. Though the proposal was attractive, there come immediately many questions such as: (a) For such a small triaxiality and low angular momentum the rotation expressed by the assumed one-dimensional cranking could hardly be realistic [7,8], in order to construct a given angular-momentum state; (b) Especially, a cranking model may not be applicable to the region of such low angular momenta as $I < j_p + j_n$. This is analogous to the inapplicability [9] of the cranking model for $I < j$ in odd-A nuclei, since for $I \lesssim j$ the system has to be cranked with a negative frequency so as to cancel partly the magnitude of the angular momentum of the odd particle, j.

In the present letter we show the anomalous signature splitting of the high-j configuration in odd–odd nuclei can occur at very low spins such as $I \lesssim j_p + j_n$, even for an axially symmetric shape, if the coupling of the angular momenta of the odd particles and the core is properly taken into account. In fig. 2a the yrast energies calculated by using a particle-rotor model [11] in which one $j_p = \frac{11}{2}$ quasiproton and one $j_n = \frac{13}{2}$ quasineutron are coupled to an axially symmetric rotor. The model hamiltonian is written as

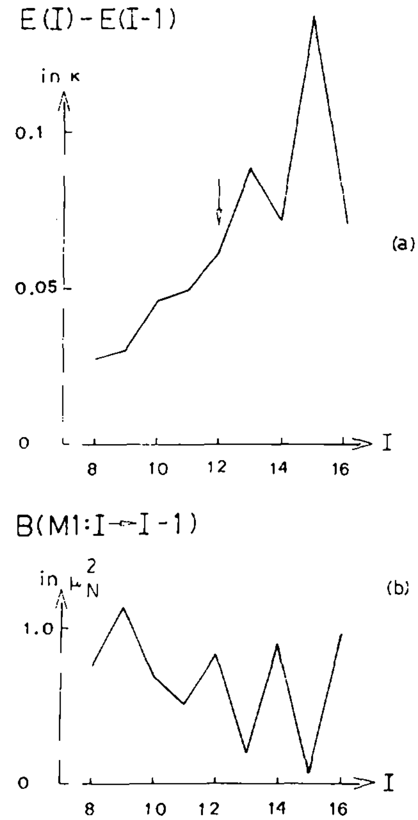

Fig. 2. (a) Yrast energies calculated by using a particle-rotor model in which one $j_p = h_{11/2}$ quasiproton and one $j_n = i_{13/2}$ quasineutron are coupled to an axially symmetric rotor. Used parameters are $\Delta_p/\kappa = \Delta_n/\kappa = 0.30$, $\lambda_p/\kappa = -0.45$, $\lambda_n/\kappa = -0.90$ and $\theta\kappa = 122$, where κ is an energy unit which is proportional to the Y_{20} deformation [10]. A Coriolis reduction of 0.8 is used. (b) Calculated B(M1) values using the wave functions of the states shown in fig. 2a. Used g-factors are $g_s(p) = 3.91$, $g_\ell(p) = 1.0$, $g_s(n) = -2.67$, $g_\ell(n) = 0.0$ and $g_R = 0.42$.

$$H = \sum_{\Omega}^{(p)} E_\Omega \alpha_\Omega^\dagger \alpha_\Omega + \sum_{\Omega}^{(n)} E_\Omega \beta_\Omega^\dagger \beta_\Omega + \theta^{-1} \sum_{k=1,2} (I_k - j_{nk} - j_{pk})^2, \tag{3}$$

where the BCS quasiparticle energies are written as

$$E_\Omega = \sqrt{[\epsilon(\Omega) - \lambda]^2 + \Delta^2}. \tag{4}$$

The parameters used in the calculation of fig. 2a are given in the figure caption and are chosen to be appropriate for the yrast high-j configuration in ^{156}Tb. It is stressed that the numerical calculation presented in fig. 2a is taken to be schematic using I-independent parameters, since the main purpose here is to show the presence of the anomalous signature splitting for $I \lesssim j_p + j_n = 12$. One sees an anomalous signature splitting for $I \lesssim j_p + j_n$ and a normal one for $I \gtrsim j_p + j_n$, just exactly analogous to the observed spectra of ^{156}Tb shown in fig. 1. This anomalous signature splitting may be obtained for some particular shell fillings of the particle (proton in the present example), which is responsible for the signature splitting of the spectra. With an almost perfectly aligned odd neutron the Fermi level of the $j_p = \frac{11}{2}$ proton, $\lambda_p \approx \epsilon(\Omega = \frac{5}{2})$, is most favourable to see a realization of the anomalous signature splitting in the region of $I \lesssim j_p + j_n$, since for $\lambda_p \approx \epsilon(\Omega = \frac{9}{2})$ the signature splitting is anyway too small and for $\lambda_p \approx \epsilon(\Omega = \frac{1}{2})$ the Coriolis coupling is too strong. In order to see more intuitively when the anomalous signature splitting is expected to occur, in the following I illustrate the situation using an especially simple model.

Let us take a case, in which the odd neutron is in the $j_n = \frac{1}{2}$ shell and the odd proton in the $j_p = \frac{5}{2}$ shell. In fig. 3 a qualitative feature of the calculated spectra is sketched, taking three cases of the proton shell filling..For $\lambda_p \approx \epsilon(\Omega = \frac{3}{2})$ the $I = 2$ state is degenerate with the $I = 1$ state. In this example we have $\alpha_f = 1$ and, thus, the normal signature splitting means that the sequence of the I = odd states shifts downward compared with that of the I = even states. Therefore, the degeneracy of the $I = 2$ state with the $I = 1$ state corresponds to the anomalous signature splitting. In the following we try to illustrate at which λ_p values the anomalous signature splitting occurs: For the case of $j_n = \frac{1}{2}$ the eigenstates of the hamiltonian (3) have the angular momentum $|\boldsymbol{I} - \boldsymbol{j}_n|$ as a good quantum number. For a given I value (except for the $I = 0$ case) there are two possible values of $|\boldsymbol{I} - \boldsymbol{j}_n|$; for example, $\frac{1}{2}$ and $\frac{3}{2}$ for $I = 1$, $\frac{3}{2}$ and $\frac{5}{2}$ for $I = 2$, and so on. The $|\boldsymbol{I} - \boldsymbol{j}_n|$ values of the yrast states are shown in the spectra of fig. 3. In fact, the present eigenvalue problem of odd–odd nuclei with $j_n = \frac{1}{2}$ is nearly the same as the particle-rotor calculation of odd-A nuclei with the total angular momentum, $|\boldsymbol{I} - \boldsymbol{j}_n|$, in which one odd-$j_p$ particle is coupled to an axially symmetric core. The only difference is that in the present case the problem is solved for a given I, while in the corresponding odd-A nuclei one solves a problem for a given $|\boldsymbol{I} - \boldsymbol{j}_n|$. Now, both experimentally and theoretically, we know how the yrast spectra of odd-A nuclei with one quasiparticle in a $j = \frac{5}{2}$ shell depend on the shell filling: Namely, assuming the Coriolis coupling is not extremely strong, we have a sequence of the yrast states with angular momenta $\frac{1}{2}$, $\frac{5}{2}$, $\frac{9}{2}$, ... for $\lambda \approx \epsilon(\Omega = \frac{1}{2})$, a sequence with angular momenta $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, ... for $\lambda \approx \epsilon(\Omega = \frac{3}{2})$, and a sequence with angular momenta $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, ... for $\lambda \approx \epsilon(\Omega = \frac{5}{2})$. From the above consideration one could immediately obtain the depen-

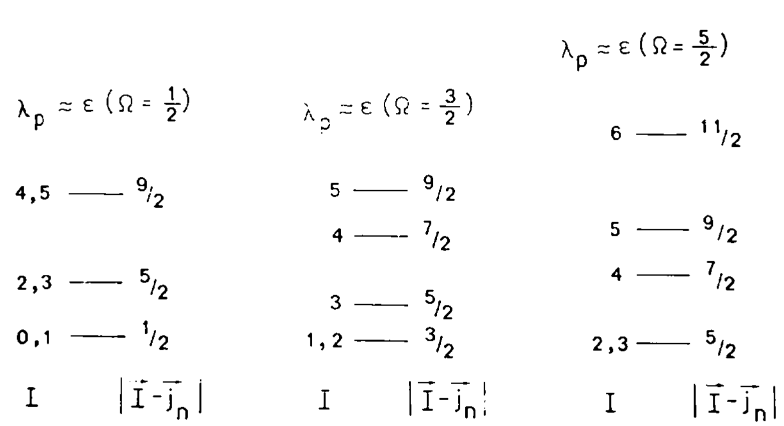

Fig. 3. Illustration of low-lying yrast spectra of odd–odd nuclei, in which one quasiproton is in a $j_p = \frac{5}{2}$ shell and one quasineutron is in a $j_n = \frac{1}{2}$ shell. Three different proton-shell fillings are chosen. The total angular momentum is expressed by I. The eigenstates of the hamiltonian are also the eigenstates of $\boldsymbol{I} - \boldsymbol{j}_n$, the magnitudes of which are shown in the figure.

dence of the yrast signature splitting of odd–odd nuclei on the proton shell fillings. The dependence is exactly the same as the one shown in fig. 3. For example, for $\lambda \approx \epsilon(\Omega=\frac{3}{2})$ the lowest eigenvalues of the $I=2$ and $I=1$ states are equal since both states have a quantum number of $|\boldsymbol{I}-\boldsymbol{j}_n|=\frac{3}{2}$. They are the yrast states, since from our knowledge of odd-A nuclei with one $j=\frac{5}{2}$ odd particle we know that the lowest-lying $I=2$ state (the $I=1$ state) which has $|\boldsymbol{I}-\boldsymbol{j}_n|=\frac{5}{2}$ ($|\boldsymbol{I}-\boldsymbol{j}_n|=\frac{1}{2}$ lies higher than the one with $|\boldsymbol{I}-\boldsymbol{j}_n|=\frac{3}{2}$. In other words, if the proton ($j_p=\frac{5}{2}$) shell is about half-filled, the anomalous signature splitting in the odd–odd nucleus occurs for $I \lesssim j_p+j_n=3$. Note that the important condition here is that $\Omega=\frac{3}{2}$ is equal to "(j_p-1) mod 2".

In fig. 2b we show, for reference, the B(M1) values calculated by using the wave functions of the states shown in fig. 2a, since we are aware of the presence of some recent interesting measurements [12] of electromagnetic properties of high-j configurations in odd–odd nuclei. Such measurements and the interpretation of the data will further elucidate the structure of low-spin states in odd–odd nuclei.

In conclusion, we state that the occurrence of the signature inversion in well-deformed odd–odd nuclei for $I \lesssim j_p+j_n$ can well be consistent with an axially symmetric shape. Thus, it may not be taken as an indication of the deviation of nuclear shape from axial symmetry.

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