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ELECTROMAGNETIC RADIATION OF ROTATING NUCLEI

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Abstract: Based on the well-known concept of the rotor-plus-particle coupling an intrinsic representation of the radiative multipole operators is derived which enables one to calculate directly electromagnetic transition amplitudes with cranking states. In this way both the orientation of the intrinsic system in space and the structure of the quasiparticles can be properly taken into account. This method is then applied to M1 and E2 properties of a high-j one-quasiparticle orbital treated in cranking approximation and it is compared to the results of an equivalent rotor-plus-particle model. In considering a particular case of a three-quasiparticle configuration one can establish the semiclassical vector coupling scheme which has been successful in describing M1 enhancement effects as observed in several experiments.

1. Introduction

The cranked shell model (CSM) is quite successful in providing the appropriate classification of band structures in rotating axially [1]) and also triaxially [2]) deformed nuclei. Compared to the extensive studies of the rotational energies the theoretical analysis of the corresponding γ-transition probabilities is scarce and not in a satisfactory stage. This is due to the difficulty of how the angular momentum properties can be properly taken into account when using cranking theories. We remember the fact that cranking wave functions must be considered as wave packets involving a possibly broad distribution of various angular momenta. In particular, the orientation of the deformed nucleus relative to its axis of rotation is of great importance for the actual strength of the emitted multipole γ-radiation. In order to make the cranking concept applicable also to the calculation of radiation amplitudes it is necessary to recover information on the intrinsic angular momentum components that fix the orientation. The derivation of transition amplitudes by angular momentum projection [3]) or tensor algebraic techniques [4]) is possible but these methods might be in practice rather complicated and hardly transparent. We propose in this paper a direct method to obtain the radiation amplitudes which is inspired by the rotor model and which follows closely the intuitive basis of the CSM.

An important step to supply the full spectroscopoic information from cranking wave functions was started by Hamamoto and Sagawa [5]). These authors, however, treated the case of a simple geometry by assuming the total angular momentum to be parallel to the axis of the collective rotation. We shall demonstrate that this presumption is of restricted validity and that the declination of the nuclear spin direction with respect to the principal axis leads to observable effects in the γ-radiation. The content of the paper is as follows.

Taking the rotor-plus-particle model as a suitable analog for treating the angular momentum properly we formulate the cranking approximation for transition amplitudes of multipole operators.

In order to show the quality of the method we present a detailed comparison of the M1/E2 radiation probabilities calculated on the one hand with the rotor plus one-quasiparticle (Coriolis mixing approach) and on the other hand obtained with the states of a corresponding cranking model. In this way one can gain also more physical insight into the relation between the quasiparticle orbitals and emitted radiation.

The discussion of a more complex quasiparticle configuration shows that the intrinsic geometry of multiquasiparticle states may lead to enhancement effects due to the coherent addition of the momenta of the various quasiparticles.

2. Intrinsic representation of the transition operators

The intrinsic states provided by the cranking model cannot be used directly for calculating the matrix elements of the laboratory components of multipole operators. The radiation operators must be transformed into the intrinsic system. In the previous work [6]) an explicit transformation has been derived based on the fact that the Euler angles α, β, γ commonly used for defining the orientation of the intrinsic system can be expressed in terms of the intrinsic components I_x, I_y, I_z of the total angular momentum I. This transformation is founded on the classical limit $I \gg 1$.

Now we are going to propose another method which in this limit is equivalent but it is by construction quantum-mechanically correct for a rotor-plus-particle system. The concept is the following one. The calculation of the transition elements for a rotor-plus-particle system can be done by standard methods and the results are well known [7]). Remembering that within such a system the coupling of the collective and single-particle angular momentum (a.m.) is exactly taken into account we consider this system as the proper analog for constructing transition amplitudes for cranking states. This means in practice to add an a.m. dependent factor that is suggested from the rotor model to the intrinsic transition matrix elements. One may express this procedure also as an "a.m. projection by hand".

To work out this method the transition matrix elements of the rotor-plus-particle system are analysed in more detail. We confine ourselves to the axially symmetric

systems only. This is because the a.m. orientation which is the relevant geometrical information needed for the radiation amplitudes is not easy to define for triaxial systems. Similar problems exist for the a.m. projection technique of non-axially deformed shapes [3]).

In the rotor-plus-particle model the nuclear states $|IM\rangle$ are conventionally expressed in terms of the adiabatic basis [7]), i.e.

$$|IM\rangle = \sum_K c_K^{(I)} \chi_K D^I_{MK}(\alpha\beta\gamma) \frac{\sqrt{2I+1}}{8\pi^2}, \tag{1}$$

where I is the total a.m. and M and K denote its components onto the lab and intrinsic z-axis, respectively, the latter defining the symmetry axis. The Euler angles α, β, γ involved in the Wigner D-functions parametrize the various orientations of the deformed density distribution in lab space. χ_K is the intrinsic wave function for the particles outside the rotor where for simplicity only the K quantum number is specified suppressing other labels.

The coefficients $c_K^{(I)}$ mean the Coriolis mixing amplitudes resulting from the diagonalization of the rotor-plus-particle hamiltonian.

Using the usual transformation of a given multipole operator $(\mathrm{M}\lambda)$ to the intrinsic system, i.e.

$$(\mathrm{M}\lambda)^{\mathrm{lab}}_\mu = \sum_\kappa D^\lambda_{\mu\kappa}(\alpha, \beta, \gamma)(\mathrm{M}\lambda)_\kappa, \tag{2}$$

one can readily obtain any reduced transition matrix element [7])

$$\begin{aligned}\langle I' \| (\mathrm{M}\lambda)^{\mathrm{lab}} \| I\rangle &\equiv \sum_{\mu MM'} \langle IM, \lambda\mu \,|\, I'M'\rangle (2I'+1)^{-1/2} \langle I'M' | (\mathrm{M}\lambda)^{\mathrm{lab}}_\mu | IM\rangle \\ &= \sqrt{2I+1} \sum_{\kappa KK'} c_K^{(I')} c_K^{(I)} \langle IK, \lambda\kappa \,|\, I'K'\rangle \langle K' | (\mathrm{M}\lambda)_\kappa | K\rangle . \end{aligned} \tag{3}$$

In general the intrinsic components $(\mathrm{M}\lambda)_\kappa$ of the multipole operator split into a collective and a single-particle part (cf. eq. (35)).

In eq. (3) the Clebsch-Gordan coefficients $\langle IK, \lambda\kappa \,|\, I'K'\rangle$ stemming from the integration over the Euler angles emerge as the necessary geometrical factors containing the dependence of the multipole radiation on the orientation of the a.m. vector $\boldsymbol{I}$ relative to the symmetry axis (z). The a.m. vector $\boldsymbol{I}$ might be thought as still-standing in lab space because it is a constant of motion.

It is worth mentioning that the transition matrix elements found for a.m. projected cranking states [3]) appear in an analogous form, i.e. involve the same geometrical factors.

We propose, therefore, the following approximation for the calculation of transition amplitudes of cranking states suggested by eq. (3):

(i) Assign as usual the a.m. value $I(\omega_x)$ to the cranking states under consideration (cf. the prescription [1])) where the value of the rotational frequency ω_x is consistent with the required integer or half-integer I-values of the initial or final state.

(ii) Expand the cranking states in terms of a K-basis (I_z diagonal) thus obtaining coefficients $c_K^{(\omega_x)}$.

(iii) Insert into eq. (3) the corresponding cranking expressions, i.e. amplitudes and intrinsic transition matrix elements. In this way also the Clebsch-Gordan factor is well defined.

The above procedure (i)-(iii) is just the "a.m. projection by hand".

One may argue whether the distribution of the K-values of the cranking states does or does not sufficiently well describe that of the related nuclear states. Usually only the cranking energies are compared to the experimental findings [routhians [1,2])]. However, if the experimental band energies are well reproduced then the structure inherent the related cranking states should be taken as an additional model prediction to be checked. For this purpose the transition amplitudes are the proper observables.

It is useful to form from eq. (3) an analogous formal operator relation between the components of the multipole operator $(\mathrm{M}\lambda)^{\mathrm{lab}}$ and those of the intrinsic frame. This enables us to perform later on further simplifications and, in particular, it provides semiclassical compact formulae.

Suppose as usually the multipolarity λ as well as the spin difference $\Delta I = I' - I$ are given as characteristics of the transition. Note that in most applications only the values $\lambda = 1, 2$ and $\Delta I = 0, 1, 2$ shall be of practical importance. The desired relation (cf. eq. (3)) reads

$$(\mathrm{M}\lambda)^{\mathrm{lab}}_{\Delta I} = \sum_{\kappa=-\lambda}^{\lambda} (\mathrm{M}\lambda)_\kappa \langle \tilde{I}\tilde{K}, \lambda\kappa \,|\, \tilde{I} + \Delta I\tilde{K} + \kappa \rangle \,. \tag{4}$$

The "operator" $\langle \tilde{I}\tilde{K}, \lambda\kappa \,|\, \tilde{I} + \Delta I\tilde{K} + \kappa \rangle$ which is below denoted as angle operator is built by a formal substitution of the a.m. parameters by the quantities $\tilde{I}$ and $\tilde{K}$ into the *explicit analytical form* of the corresponding Clebsch-Gordan coefficients [cf. expressions (16), (24)]. The quantity $\tilde{K}$ is identical with the component I_z of the a.m. operator which defines also the intrinsic components $(\mathrm{M}\lambda)_\kappa$ of the multipole operator.

Further, the symbol $\tilde{I}$ denotes the "operator" of the a.m. value I in the only sense that it picks up the *assigned* I-value from the cranking state upon which it is "acting". Since such a substitution $I \to I(\omega_x)$ is not an operation in a strict mathematical sense we put the term operator in quotation marks. However, one may easily check that one can even obtain the "hermitean conjugated operator"

$$(\mathrm{M}\lambda)^{\mathrm{lab}}_{-\Delta I} = (-)^{\Delta I} (\mathrm{M}\lambda)^{\mathrm{lab}^+}_{\Delta I} = \sum_{\kappa=-\lambda}^{\lambda} \langle \tilde{I}\tilde{K}, \lambda\kappa \,|\, \tilde{I} + \Delta I\tilde{K} + \kappa \rangle (-)^\kappa (\mathrm{M}\lambda)_{-\kappa} \tag{5}$$

as formally suggested (note the ordering is important). We emphasize, the relation (4) called later on intrinsic representation of the transition operator has the only purpose to form the desired cranking matrix elements in a direct way. Writing the cranking states in terms of an $I_z \cong \tilde{K}$ diagonal basis η_K, i.e.

$$|\omega_x\rangle = \sum_k c_K^{(\omega_x)} \eta_K \,, \tag{6}$$

one obtains with eq. (4)

$$\langle\omega'_x|(\mathrm{M}\lambda)^{\mathrm{lab}}_{\Delta I}|\omega_x\rangle = \sum_{KK'\kappa} c^{(\omega'_x)}_{K'} c^{(\omega_x)}_{K} \langle\eta'_K|(\mathrm{M}\lambda)_\kappa|\eta_K\rangle \langle I(\omega_x) K\lambda\kappa \,|\, I(\omega_x)+\Delta I K'+\kappa\rangle\,, \quad (7)$$

which is for $I'(\omega_x) = I(\omega_x)+\Delta I$ the proposed cranking approximation described above for the reduced matrix element (3)

$$\langle\omega'_x|(\mathrm{M}\lambda)^{\mathrm{lab}}_{\Delta I}|\omega_x\rangle \approx \frac{\langle I'(\omega_x)\|(\mathrm{M}\lambda)^{\mathrm{lab}}\|I(\omega_x)\rangle}{\sqrt{2I+1}}\,. \quad (8)$$

Accordingly the cranking approximation of the reduced transition strength [7]) reads

$$B(\mathrm{M}\lambda, I\to I') \approx |\langle\omega_x|(\mathrm{M}\lambda)^{\mathrm{lab}}_{\Delta I}|\omega'_x\rangle|^2\,. \quad (9)$$

where the r.h.s. is given by eq. (7).

We mention that the matrix element (7) reduces to the expression applied in ref. [5]) when replacing the Clebsch-Gordan factor by unity and taking the term $\kappa = \Delta I$ only.

In order to see that eq. (4) can be indeed interpreted as a transformation from the lab to the intrinsic system we take the asymptotic form [8]) of the Clebsch-Gordan coefficient ($\|\tilde{I}\| \gg \lambda$)

$$\langle \tilde{I}\tilde{K}, \lambda\kappa \,|\, \tilde{I}+\Delta I\tilde{K}+\kappa\rangle \approx D^{\lambda}_{\Delta I\kappa}(0, \tilde{\beta}, 0) \quad (10)$$

with

$$\cos\tilde{\beta} = \tilde{K}/(\tilde{I}+\tfrac{1}{2}) \quad (11)$$

and obtain in this limit

$$(\mathrm{M}\lambda)^{\mathrm{lab}}_{\Delta I} = \sum_{\kappa} (\mathrm{M}\lambda)_\kappa D^{\lambda}_{\Delta I\kappa}(0, \tilde{\beta}, 0)\,, \quad (12)$$

i.e. the familiar transformation to an intrinsic system the orientation of which is described by the only essential angle part β connected with the intrinsic quantization axis (the angles α and γ are not observable because of a.m. fluctuations).

The expression (12) is in essence the same as described previously [6,9]) by using classical arguments.

3. Application of the method

In this section the method is specified to M1 and E2 properties and applied to a single j-shell quasiparticle in a rotating axially deformed field. In the case considered

one can compare with the standard rotor-plus-particle model in which by construction the angular momentum coupling is properly taken into account and for which the intrinsic wave functions (cf. eq. (1)) and the radiation amplitudes (3) can easily be obtained.

The basic cranking hamiltonian for a single j-shell reads

$$h'_{\mathrm{crank}} = h_{\mathrm{def}} - \omega_x j_x ,$$
$$h_{\mathrm{def}} = \sum_{k=-j}^{j} \alpha_k^+ \alpha_k = \sum \sqrt{(\kappa k^2 - \lambda_{\mathrm{F}})^2 + \Delta^2}\, \alpha_k^+ \alpha_k , \tag{13}$$

where k is the z-component of the single-particle a.m. The deformed s.p. hamiltonian involves a deformed Nilsson-like potential with the parameter κ that is proportional to the deformation. Further, the static pair field is taken into account and thus leading to the gap parameter Δ and the parameter λ_{F} characterizing the Fermi level.

The a.m. j_x in the cranking hamiltonian (13) is explicitly given by

$$j_x = \sum_{kk'} (u_k u_{k'} + v_k v_{k'}) (\sqrt{(j+k)(j-k+1)}\, \delta_{kk'-1}$$
$$+ \sqrt{(j-k)(j+k+1)}\, \delta_{kk'+1}) \alpha_k^+ + \alpha_{k'} , \tag{14}$$

where only the quasiparticle conserving terms are included in order to avoid possible complications of band crossings. Therefore, the usual pairing occupation factors $(u_k u_{k'} + v_k v_{k'})$ appear in eq. (14). The diagonalization of the cranking hamiltonian (13) provides us the wave functions $|\omega_x\rangle$ for given values of κ, λ_{F} and Δ later on used for the calculation of γ-transition amplitudes.

Now we consider a rotor plus particle system which describes an analogous physical situation with the only difference that the collective ($\boldsymbol{R}$) and the single-particle a.m. ($\boldsymbol{j}$) add up correctly to the total spin $\boldsymbol{I} = \boldsymbol{R} + \boldsymbol{j}$. In the corresponding cranking problem one introduces an external uniform rotation ω_x which affects the single-particle motion and there the a.m. conservation can be satisfied only in average (cf. eq. (21)).

The rotor hamiltonian is [7])

$$h_{\mathrm{rot}} = h_{\mathrm{def}} + A(R_x^2 + R_y^2) , \tag{15}$$

where $A = 1/(2J_0)$ is the inertial parameter of the collective motion.

In what follows we demonstrate for the one-quasiparticle case that the cranking approximation (7) provides us quite reasonable γ-transition probabilities when comparing with the rotor ones.

Let us study the $\Delta I = \pm 1$ M1 transitions in the rotor and cranking system in more detail.

The intrinsic representation (5) obtained from the explicit expressions [8]) of the Clebsch-Gordan coefficients is

$$
\begin{aligned}
(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=-1} &= -(\mathrm{M1})^{\mathrm{lab}+}_{\Delta I=1} \\
&= \sqrt{\frac{\tilde{I}}{2\tilde{I}+1}}\left\{\left[1-\left(\frac{I_z}{\tilde{I}}\right)^2\right]^{1/2}(\mathrm{M1})_0\right. \\
&\quad +\frac{1}{\sqrt{2}}\left[\left(1+\frac{I_z}{\tilde{I}}\right)\left(1+\frac{I_z+1}{\tilde{I}}\right)\right]^{1/2}(\mathrm{M1})_{+1} \\
&\quad \left.+\left[\left(1-\frac{I_z}{\tilde{I}}\right)\left(1-\frac{I_z+1}{\tilde{I}}\right)\right]^{1/2}(\mathrm{M1})_{-1}\right\} \qquad (16)
\end{aligned}
$$

with $(\mathrm{M1})_0=(\mathrm{M1})_z$ and $(\mathrm{M1})_{\pm1}=\mp\sqrt{\frac{1}{2}}[(\mathrm{M1})_x\pm i(\mathrm{M1})_y]$, $I_z=\tilde{K}$.

Note that all quantities on the r.h.s. are intrinsic ones.

From eq. (16) it is obvious that the evaluation of the transition matrix elements in cranking approximation takes essentially into account the information on the spin value $I=I(\omega_x)$ and the actual distribution of the K-values within the cranking states.

Eq. (16) is generally valid, i.e. it can be used also for more complex quasiparticle configurations implying several quasiparticles. For comparing with the previously obtained expressions [6]) we consider two approximations to eq. (16). Neglecting terms of the order $1/I$ one arrives at

$$
(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=-1}\approx\frac{1}{\sqrt{2}}\left\{\left[1-\left(\frac{I_z}{\tilde{I}}\right)^2\right]^{1/2}(\mathrm{M1})_z-\frac{I_z}{\tilde{I}}(\mathrm{M1})_x-i(\mathrm{M1})_y\right\} \qquad (17)
$$

that agrees with the classical expression [9]). The formula derived in [6]) has the correct K-dependence, i.e. the correct geometrical factors, but the spin factor in front of eq. (16) is approximated as $[I/(2I+1)]^{1/2}\approx\sqrt{\frac{1}{2}}$.

The formula used in [5]) is obtained in the extreme limit $I\to\infty$, i.e.

$$
(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=-1}\to\sqrt{\tfrac{1}{2}}[(\mathrm{M1})_z-i(\mathrm{M1})_y] \qquad (18)
$$

which obviously means to neglect all geometrical dependence.

One recognizes that the complete intrinsic representation (5) which for the M1 transitions reduces to eq. (16) combines properly the geometrical dependence and the intrinsic transition strength.

Returning now to the single j-shell quasiparticle we can obtain the M1 transition amplitudes for the rotor-plus-particle system employing eq. (3).

The calculation of the corresponding cranking amplitudes needs a special attention for the particular case of a single j-shell. To find the correct correspondence between the rotor and the cranking system we derive the basic relation $I=I(\omega_x)$ directly from the approximation necessary to bring the rotor hamiltonian (15) into the cranking form (13). Considering the collective energy term we obtain

$$
\boldsymbol{AR}^2=A(\boldsymbol{I}^2+\boldsymbol{j}^2-2I_xj_x-2I_yj_y-2I_zj_z)\,, \qquad I_z=j_z\,. \qquad (19)
$$

The terms $\boldsymbol{I}^2$ and $\boldsymbol{j}^2$ are in the rotor case constants of motion and can be omitted. The term $-2Aj_z^2$ is of minor importance as long as $|\kappa|\gg A$ which is typical for deformed nuclei and it can be neglected in cranking approximation. The axial symmetry of the deformed field allows one to choose the intrinsic x-axis within the

plane spanned by the a.m. vector $\boldsymbol{I}$ and the symmetry axis. In this way one may achieve (at least classically) $I_y = 0$. The optimal choice for comparing the rotor and the analogous cranking system is then given by the substitutions

$$2AI_x \to \omega_x, \qquad \text{i.e. } I_x^{\text{crank}} = J_0\omega_x, \qquad I_y \to 0. \tag{20}$$

The value of the a.m. follows to be here

$$I = I(\omega_x) = \sqrt{(J_0\omega_x)^2 + \langle j_z^2 \rangle}. \tag{21}$$

We emphasize that this relation is valid only for the single j-shell quasiparticle problem under study whereas in the cranked shell model one has [1])

$$I_x^{\text{CSM}} = J(\omega_x)\omega_x + i(\omega_x) \tag{22}$$

implying a contribution from the particle alignment, $i(\omega_x)$, which is related to the use of the canonical relation $\omega_x = \partial E/\partial I_x$ for defining the frequency.

It is worthwhile to introduce for a multipole transition $I \to I' = I + \Delta I$ the transitional spin

$$I_t = \tfrac{1}{2}(I + I') = I + \tfrac{1}{2}\Delta I. \tag{23}$$

With help of this definition one can reasonably approximate the transition between the cranking states $\omega_x(I) \to \omega_x(I')$ by that of the fixed frequency $\omega_t = \omega_x(I_t)$ denoted as transitional frequency. For that purpose we need to consider the transition amplitudes as a function of the continuous parameter ω_t. For stretched ($\Delta I = 2$) E2 transitions one may overestimate the B(E2) strength by some percent because the overlap $\langle \omega_x(I) | \omega_x(I-2) \rangle$ of the cranking states is smaller than 1. This effect is small for $\Delta I = 1$ transitions.

Since the $\Delta I = 1, 2$ electric quadrupole transition will also be compared we give the E2 operators for the axial deformed case. Taking into account only the collective contribution one gets the following explicit expressions (q_0 = intrinsic collective quadrupole moment)

$$(\text{E2})^{\text{lab}}_{\Delta I=0} = q_0 \frac{3I_z^2 - \tilde{I}(\tilde{I}+1)}{[(2\tilde{I}-1)\tilde{I}(\tilde{I}+1)(2\tilde{I}+3)]^{1/2}},$$

$$(\text{E2})^{\text{lab}}_{\Delta I=1} = q_0 I_z \left[\frac{3(\tilde{I}^2 - I_z^2)}{(\tilde{I}-1)(2\tilde{I}-1)\tilde{I}(\tilde{I}+1)}\right]^{1/2},$$

$$(\text{E2})^{\text{lab}}_{\Delta I=2} = q_0 \left[\frac{3((\tilde{I}-1)^2 - I_z^2)(\tilde{I}^2 - I_z^2)}{(2\tilde{I}-3)(2\tilde{I}-2)(2\tilde{I}-1)\tilde{I}}\right]^{1/2}. \tag{24}$$

According to the proposed procedure the a.m. value is implemented into eqs. (16), (24) by substituting $I \to I(\omega_t)$ as given by eq. (21).

The practical calculations have been performed with the following parameters: $\kappa = 0.2$ MeV, $\Delta = 1.0$ MeV, $A = 8.33$ keV. We considered a $j = \frac{13}{2}$ quasiparticle placed on the deformed orbitals $K = \frac{13}{2}, \frac{7}{2}$ and $\frac{1}{2}$, respectively by adjusting the Fermi level λ_F accordingly. The selection of these parameter values is representative for deformed nuclei.

Furthermore the magnetic dipole operator is taken for simplicity as

$$(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=\pm 1} = \sqrt{\frac{3}{4\pi}} j_{\Delta I=\pm 1} = \sqrt{\frac{3}{4\pi}} \sum_{kk'} (u_k u_{k'} + v_k v_{k'}) \langle k|j_{\pm 1}|k\rangle \alpha^+_k \alpha_{k'} \qquad (25)$$

defined in analogy to eq. (14) and implying the gyromagnetic factor equals 1.

4. Discussion of the intraband transitions

Our comparison between the rotor and the cranking model concerns the $\Delta I = 1$, 2 E2 transitions and the $|\Delta I| = 1$ M1 transitions within the yrast band (intraband transitions). The magnetic transitions show up in the high-spin region strong signature* dependence. As discussed in [5]) one has two types of $\Delta I = 1$ M1 transitions corresponding to the change of the signature $\Delta\alpha = 1$ or -1 that is parallel to the appearance of the signature staggering in the rotational band structure.

The cranking calculation of the M1 properties (cf. figs. 1-3) is performed by using the expression (7).

One may apply, however, the following approximations to the above mentioned calculations:

(i) Instead of the eq. (16) the approximation (17) is used.

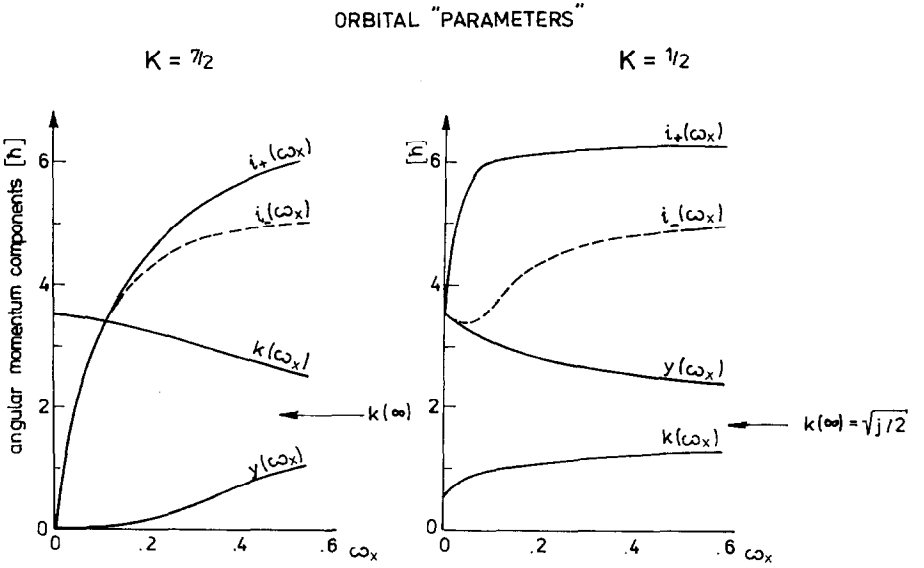

Fig. 1. Orbital parameters of a $j = \frac{13}{2}$ quasiparticle placing the Fermi level at $K = \frac{7}{2}$ and $K = \frac{1}{2}$, respectively. The parameters of the underlying cranking hamiltonian are $\kappa = 0.2$ MeV and $\Delta = 1$ MeV (cf. eq. (13)).

* We remember that the signature quantum number α is related to the reflexion symmetry of the deformed potential. For one quasiparticle states holds $\alpha = \frac{1}{2}$ for the sequence $I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}$, etc. and $\alpha = -\frac{1}{2}$ for the sequence $I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}$, etc.

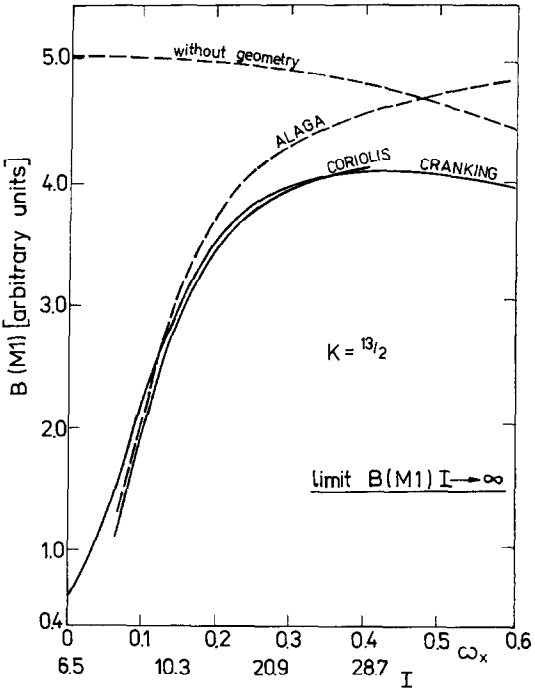

Fig. 2. $B(\mathrm{M1})$ values for a $j=\frac{13}{2}$ quasiparticle as a function of the transitional frequency ω_x placing the Fermi level at $K=\frac{13}{2}$. For comparison the respective spin value I is given at the abscissa. For simplicity the gyromagnetic factor is chosen to be $g=1$. The rotor calculation ("Coriolis") is performed using the rotational parameter $A=8.33$ keV. The Alaga value is obtained by taking $K(\omega_x)=$ constant $(\frac{13}{2})$ and neglecting all K-mixing. The dashed curve "without geometry" is calculated with cranking states but taking no care of the actual orientation of the intrinsic system in lab space.

(ii) Matrix elements of operator products like $I_z(\mathrm{M1})_x$ involved in the representation (17) are factorized in the following manner

$$\begin{aligned}\langle +|I_z(\mathrm{M1})_x|-\rangle &= \sum \langle +|I_z|-,\beta\rangle\langle -,\beta|(\mathrm{M1})_x|-\rangle \\ &\approx \langle +|I_z|-\rangle\langle -|(\mathrm{M1})_x|-\rangle\,, \end{aligned} \tag{26}$$

where $|\pm\rangle$ is the short-hand notation for the initial and final quasiparticle yrast state $|\omega_t, \alpha=\pm\frac{1}{2}\rangle$ and accordingly $|-,\beta\rangle$ denotes *all* quasiparticle states with $\alpha=-\frac{1}{2}$ including the yrast and excited states at $\omega_x=\omega_t$.

Our experience shows that the approximations (i) and (ii) are rather good. Because the approximations provide on the other hand a very transparent formula for the M1 properties we give the expressions explicitly. Taking the operator (25) one obtains then the following M1 transition amplitude

$$\langle \pm|(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=1}|\mp\rangle = \sqrt{\frac{3}{8\pi}}\left[\frac{J_0\omega_t - i^{\pm}(\omega_t)}{I(\omega_t)+\frac{1}{2}}k(\omega_t) \pm y(\omega_t)\right], \tag{27}$$

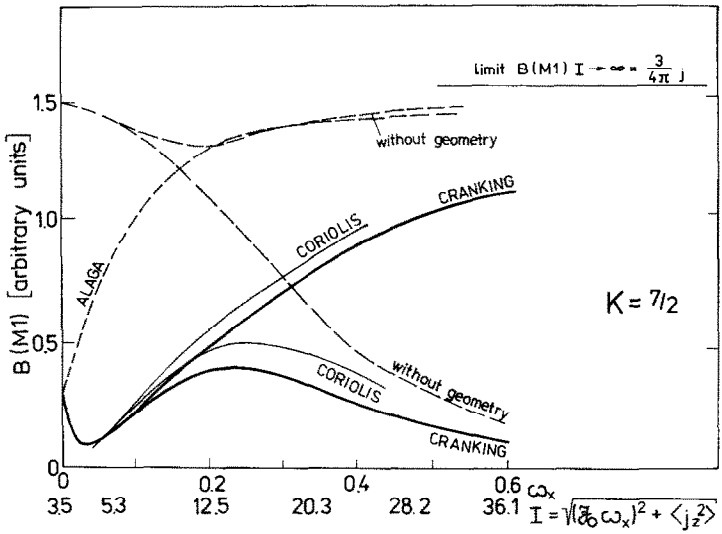

Fig. 3. The same as fig. 2 for $K=\frac{7}{2}$. One recognizes the onset of the double branching of the curves for $\omega_x \geq 0.1$ MeV which is due to the difference of the transitions $I \to I-1$ and $I \to I+1$ corresponding to the changes $\Delta\alpha = \pm 1$ in the signature. According to eq. (27) and fig. 1 this signature effect reflects the fact that the quasiparticle a.m. j runs out off the plane x, z with a finite component $y(\omega_x)$.

where $I(\omega_t)=\sqrt{(J_0\omega_t)^2+k^2(\omega_t)}$ (cf. eqs. (21), (23)). The quantities k, i and y are matrix elements of the quasiparticle angular momentum given by

$$k(\omega_t)=\langle +|j_z|-\rangle, \qquad i^{\pm}(\omega_t)=\langle \pm|j_x|\pm\rangle, \qquad y(\omega_t)=\langle +|ij_y|-\rangle \tag{28}$$

using again the notation $|\pm\rangle = |\omega_t, \alpha = \pm\frac{1}{2}\rangle$ for the initial and final cranking state. The two types $\Delta\alpha = \pm 1$ of M1 transitions belong to the changes of the spin $I \to I \pm 1$ when comparing with the rotor case.

The meaning of the quantities (28) is obvious: The function $i^{\pm}(\omega_t)$ represents the familiar single-particle alignment gained by the rotation.

The function $k(\omega_t)$ is the actual k-value at the frequency ω_t when the particle starts to be more and more aligned with respect to the cranking axis x. At $\omega_t = 0$ the cranking states $|\pm\rangle$ coincide with the combinations $\sqrt{\frac{1}{2}}(|K\rangle \pm |-K\rangle)$ of ordinary Nilsson states, and, one checks easily that the value of $k(\omega_t = 0) = K$ the latter being the K-quantum number.

The K-value is the effective component of the quasiparticle a.m. j onto the symmetry axis whereas $y(\omega_t)$ defines its component onto the y-axis which is by definition out off the plane spanned by the axis collective rotation (x) and the symmetry axis.

Hence the function $y(\omega_t)$ provides the additional information on the extension of the quasiparticle orbital in the y-direction. Making use of the commutator (cf. eq. (13))

$$[h'_{\text{crank}}, j_z] = -\omega_x[j_x, j_z] = i\omega_x j_y \tag{29}$$

one derives a relation between the function $k(\omega_t)$ and $y(\omega_t)$ when taking the matrix elements:

$$y(\omega_t) = \frac{\Delta e'}{\omega_t} k(\omega_t) , \tag{30}$$

where $\Delta e'$ is the energy difference of the signature pair of the cranking states $|\pm\rangle$, i.e. the well-known signature splitting. Such a relation has been previously discussed in ref. [5]) in connection to the signature staggering of M1 transitions. Hence one recognizes that the ratio of the "orbital parameters" $y(\omega_t)$ and $k(\omega_t)$ defining the intrinsic motion of the quasiparticle a.m. can be extracted from the energetics of the band structure.

According to eqs. (27), (30) one gets the M1 transition strength (cf. eq. (9)):

$$B(\mathrm{M1}, \Delta\alpha = \pm 1) = \langle \pm | (\mathrm{M1})^{\mathrm{lab}}_{\Delta I = 1} | \mp \rangle^2$$
$$= \frac{3}{8\pi} \left\{ k(\omega_t) \left[\frac{J_0 \omega_t - i^{\pm(\omega_t)}}{I(\omega_t) + \frac{1}{2}} \pm \frac{\Delta e'}{\omega_t} \right] \right\}^2 . \tag{31}$$

This formula enables one to extract information about the quasiparticle orbitals in rotating nuclei on an absolute scale by analysing systematically M1 transitions.

In fig. 1 the functions $k(\omega_t)$, $i(\omega_t)$ and $y(\omega_t)$ are shown placing the Fermi level on the $k = \frac{7}{2}, \frac{1}{2}$ states, respectively. The frequency interval $\omega_x = \omega_t$ corresponds to the spin region $I = \frac{3}{2}$ to $I = \frac{51}{2}$ when comparing with the rotor calculation.

Discussing the quasiparticle orbital in the context of semiclassical considerations similarly as done in [6,9]) then the orbital must be thought as an ellipse with the semiaxes $k(\omega_t)$ and $y(\omega_t)$ in z and y direction, respectively. The whole ellipse is located in the plane $x = i_t = \frac{1}{2}(i^+(\omega_t) + i^-(\omega_t))$ that is given by the particle alignments and it is centered on the x-axis. With this construction one can derive the M1 strength (31) also directly by purely classical means considering the quasiparticle angular momentum as a magnetic dipole with the moment $\boldsymbol{\mu} = \sqrt{3/4\pi}\, \boldsymbol{j}$. This has been extensively studied in [9]).

In figs. 2–4 the various $B(\mathrm{M1})$ values are plotted as a function of the frequency ω_x as obtained from the following versions:

(i) the rotor model;

(ii) the cranking approach;

(iii) the Alaga value, i.e. neglecting the $\Delta K = \pm 1$ mixing terms in eq. (16) and putting $k(\omega_t) = K = \text{constant}$;

(iv) the extreme limit $I \to \infty$ (eq. (18)) neglecting the geometrical dependence as done in [5]).

The qualitative features of the diagrams 2–4 can be easily understood in terms of the three basic elements i.e. actual K-value $k(\omega_t)$, alignment $i(\omega_t)$ and signature splitting $\Delta e'$ when considering the explicit formula (31).

In the classical region, i.e. for $\Delta e'/\omega_t \ll 1$, where there exists practically no signature staggering in the band structure,the M1 strength is increasing up to a maximum

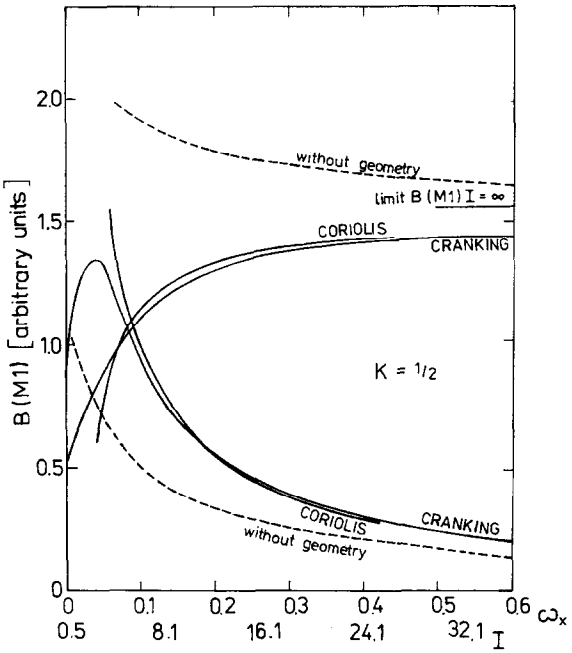

Fig. 4. The same as fig. 2 for $K=\frac{1}{2}$. The signature effect leads in the high-frequency region to a total cancellation of one of the possible M1 transitions $I \to I \pm 1$ (i.e. the transition with the higher energy difference).

value that is defined by the actual K-value. This is due to the geometrical factor $(I_x - j_x)/I$ which accounts for the angle between the total spin $\boldsymbol{I}$ and the quasiparticle angular momentum $\boldsymbol{j}$ (cf. fig. 5). Note that for the magnetic transition strength only the component $j_\perp$ perpendicular to the spin $\boldsymbol{I}$ is effective whereas the parallel component $j_\parallel$ contributes to the static magnetic moment. This feature is reflected in the Alaga value but not in the extreme limit $I \to \infty$ without geometrical dependence.

After the onset of the signature splitting the quasiparticle orbital being formerly a long ellipse in z-direction gets more and more broad in y-direction supposed the

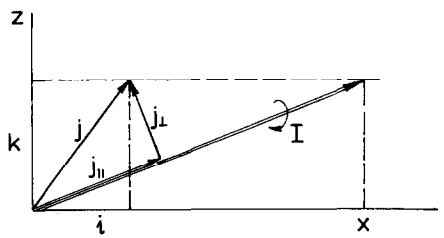

Fig. 5. Coupling scheme of a one-quasiparticle plus rotor (reference) system. The intraband M1 radiation is carried by the perpendicular component $j_\perp$ of the quasiparticle angular momentum $\boldsymbol{j}$. The intrinsic system is thought to rotate about the total a.m. vector $\boldsymbol{I}$ that is fixed in lab space.

K-value was large enough ($K > \sqrt{\frac{1}{2}j}$). In this case the $B(\mathrm{M1})$ value starts more and more staggering in accordance with the signature splitting of the band structure. This is made by the explicit signature dependence in the $B(\mathrm{M1})$ strength (31) that causes the double branching of the curves shown in figs. 3, 4.

In the case $K = \frac{1}{2}$ there exists already at $\omega_x = 0$ a finite signature splitting. In fact the elliptical orbital is then squeezed in z-direction and is broad in y-direction at the beginning. As seen in fig. 1 the actual K-value starting at $K = \frac{1}{2}$ is increasing with the frequency and reaches asymptotically $K(\omega_x = \infty) = \sqrt{\frac{1}{2}j}$ which is the value of the a.m. zero-point fluctuation when the particle becomes fully aligned.

One recognizes that the $B(\mathrm{M1})$ values of the rotor serving as an exact model can be rather well reproduced by the cranking calculation when using the intrinsic representation (16) of the transition operator. One notices that the Alaga value involves properly the geometrical aspects in the low-frequency region but it fails when the signature oscillations come into play. The extreme limit used in ref. [5])

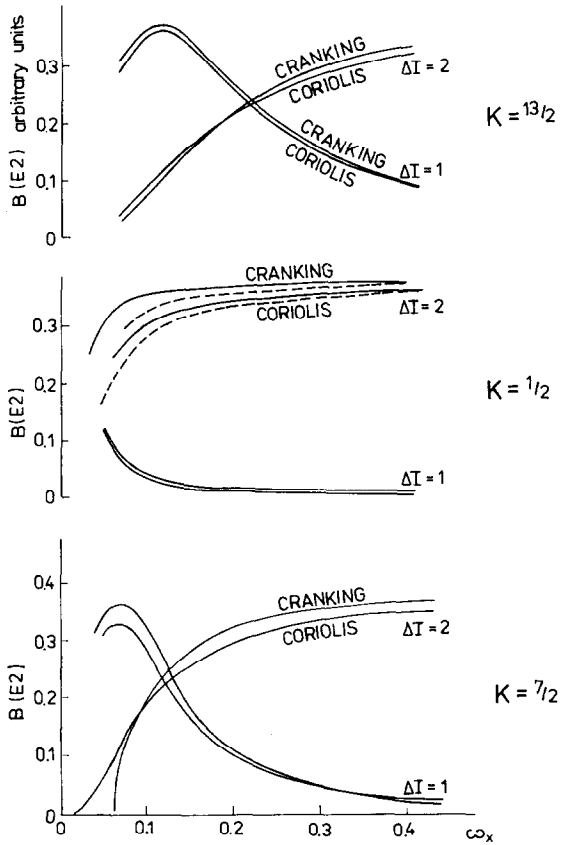

Fig. 6. $B(\mathrm{E2})$ values calculated within the cranking and Coriolis mixing approach as a function of the transitional frequency. The Fermi level is placed at $K = \frac{13}{2}, \frac{7}{2}$ and $\frac{1}{2}$, respectively.

neglects the geometry of the a.m. couplings and fails, therefore, in describing the proper trend of the M1 properties. This limit takes care only of the quantal character of the quasiparticle states which leads to the selection rule in the $B(\mathrm{M1})$ properties as obtained for $I \to \infty$ from eq. (31) as

$$B(\mathrm{M1}, \Delta\alpha = \pm 1) \to (3/8\pi)(1 - \Delta\alpha)j \tag{32}$$

in agreement with the one recognized by Hamamoto [5]).

In fig. 6 the $B(\mathrm{E2})$ values for the spin differences $\Delta I = 1$ and $\Delta I = 2$ are shown as found from the cranking model (cf. eq. (24)) and the rotor model. In all three cases the agreement is quite good.

5. Discussion of multi-quasiparticle configurations

The case of more complex configurations is of great practical importance when attempting to predict or to analyse electromagnetic transition properties in addition to the energetics of quasiparticle (qp) band structures in rotational nuclei. Basing on the one-body character of the transition operator one can generally expect that several quasiparticles can add up their various contributions to the transition amplitude coherently thus leading to enhancement or quenching effects. Further, we emphasize that similar to the 1 qp case studied in the previous section the transition amplitude is affected by both, the a.m. orientation and the intrinsic transition strength.

In this work we do not present a rigorous calculation of the transition amplitude for a multi-qp configuration. A realistic calculation for such a case can get numerically involved because of the necessary summations over intermediate states. In addition, we are aware of the deficiencies of the common cranking theory which may restrict the direct applicability of the proposed method. For instance, the definition of the qp configurations in the band crossing regions emerges as such a problem.

To show up the important role of the geometrical dependence in the transition amplitudes we study a specific qp configuration that consists of two rotation-aligned (x-axis) qp's coupled to a third one which is aligned to the symmetry (z-) axis so having a certain K-value. The interest in this particular 3-qp configuration the coupling scheme of which is illustrated in fig. 7 results from the striking enhancement of the M1 strength as observed [10–12]) for selected rotational bands in Kr, Tm and Lu isotopes where the underlying configurations change from 1 qp to 3 qp type.

To simplify our calculation we apply rigorously the same factorization as used in the 1 qp case (eq. (26)). We suppose that this approximation supplies at least qualitatively correct results.

The starting point is the expression (16) for the M1 operator, i.e.

$$(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=1} = -\sqrt{\tfrac{1}{2}}\{(\mathrm{M1})_z[1-(I_z/\tilde{I})^2]^{1/2} - (\mathrm{M1})_x(I_z/\tilde{I}) + i(\mathrm{M1})_y\}\,. \tag{33}$$

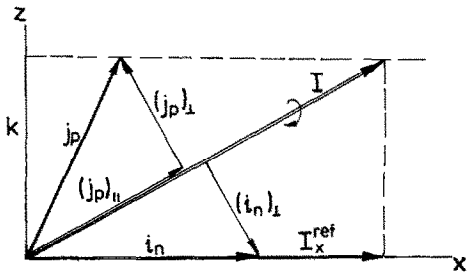

Fig. 7. Coupling scheme of a selected three-quasiparticle plus reference configuration. Two quasiparticles (n) are assumed to be rotation-aligned ($i_n = i_n^+ + i_n^-$) whereas the third one has still a sizeable K-value. The M1 radiation is then governed by the sum of the perpendicular magnetic moments, i.e. $\mu_\perp = g_p(j_p)_\perp + g_n(i_n)_\perp$.

Because the M1 operator (33) carries the signature parity $(-1)^{\Delta I} = -1$ the transitions take place between the two signature branches $\alpha = \pm\frac{1}{2}$ of the above mentioned configuration. The corresponding transition between the 3 qp cranking states is ascribed as

$$|1, -1, 2\rangle \xrightarrow{\text{M1}} |1, -1, -2\rangle , \tag{34}$$

where the states $(1, -1)$ refer to the signature pair of the rotation-aligned quasiparticles originating from a degenerate K-level at $\omega_x = 0$. The third quasiparticle changes in the transition the orbit 2 into the signature counterpart -2.

The M1 operator is decomposed in quasiparticle and collective terms

$$(\text{M1})^{\text{lab}}_{\Delta I=1} = \sqrt{\frac{3}{4\pi}} \left\{ \sum_q \tilde{g}_q j_\mu^{(q)} + g_R I_\mu \right\}^{\text{lab}} , \tag{35}$$

where $\tilde{g}_q \cong g_j^{(q)} - g_R$ means the effective gyromagnetic factor for the j-shell quasiparticle (q). The collective contribution vanishes for $\Delta I = \pm 1$ transitions.

The calculation of the matrix element $\langle 1, -1, 2|(\text{M1})^{\text{lab}}_{\Delta I=\pm 1}|1, -1, -2\rangle$ is simplified by a similar factorization as used in the one-quasiparticle case considered in sect. 4. It is based on the assumption that the rotation-aligned quasiparticles yield no sizable a.m. component onto the symmetry axis. This component stems essentially from the deformation-aligned third quasiparticle which brings in the K-value

$$k(\omega_x) = |\langle 2|j_z|-2\rangle|_{\omega_x} \tag{36}$$

into the value $I(\omega_x) = \sqrt{I_x(\omega_x) + k^2(\omega_x)}$ of the total angular momentum. The component I_x is obtained in accordance with the CSM [1]) as

$$\begin{aligned} I_x^{\pm}(\omega_x) &= J^{\text{ref}}\omega_x + \langle 1, -1, \pm 2|j_x|1, -1, \pm 2\rangle \\ &= J^{\text{ref}}\omega_x + i^{(1)} + i^{(-1)} + i^{(\pm 2)} , \end{aligned} \tag{37}$$

where $i^{(q)}$ is the respective alignment of the various quasiparticles in the configuration under consideration.

The z-component of the magnetic operator (33) yields the following matrix element

$$\langle 1,-1,2|(\mathrm{M1})_z[1-(I_z/\tilde{I})^2]^{1/2}|1,-1,-2\rangle \approx \langle 2|(\mathrm{M1})_z|-2\rangle\langle -2|[1-(I_z/\tilde{I})^2]^{1/2}|2\rangle$$
$$= \sqrt{\frac{3}{4\pi}}\tilde{g}^{(2)}k(\omega_x)\sqrt{1-\frac{k^2(\omega_x)}{I^2(\omega_x)}}, \qquad (38)$$

where eqs. (35), (36) have been used.

For the x-component one has

$$\langle 1,-1,2|(\mathrm{M1})_x\frac{I_z}{\tilde{I}}|1,-1,2\rangle \approx \sqrt{\frac{3}{4\pi}}[\tilde{g}^{(1)}(i^{(1)}+i^{(-1)})+\tilde{g}^{(2)}i^{(-2)}]\frac{k(\omega_x)}{I(\omega_x)}. \qquad (39)$$

The y-component can be exactly given as (cf. eq. (30))

$$\langle 1,-1,2|i(\mathrm{M1})_y|1,-1,-2\rangle = \sqrt{\frac{3}{4\pi}}g^{(2)}\langle 2|ij_y|-2\rangle$$
$$= \sqrt{\frac{3}{4\pi}}\tilde{g}^{(2)}y(\omega_x)$$
$$= \sqrt{\frac{3}{4\pi}}\tilde{g}^{(2)}\frac{\Delta e'}{\omega_x}k(\omega_x). \qquad (40)$$

Hence the total transition element becomes finally

$$\langle 1,-1,-2|(\mathrm{M1})^{\mathrm{lab}}_{\Delta I=\pm 1}|1,-1,-2\rangle$$
$$= \sqrt{\frac{3}{8\pi}}k(\omega_x)\left\{\tilde{g}^{(2)}\left[\sqrt{1-\left(\frac{k(\omega_x)}{I(\omega_x)}\right)^2}-\frac{i^{(\pm 2)}}{I(\omega_x)}\pm\frac{\Delta e'}{\omega_x}\right]-\tilde{g}^{(1)}\frac{i^{(1)}+i^{(-1)}}{I(\omega_x)}\right\}. \qquad (41)$$

Aside from the signature term the above formula agrees with the classical vector formula derived previously [7]) based on the coupling scheme shown in fig. 7. After extracting the "orbital parameters" $k(\omega_x)$, $i^{(q)}(\omega_x)$ and $\Delta e'(\omega_x)$ from an appropriate CSM calculation providing the routhians, $e'(\omega_x)$, the formula (41) may serve for the calculation of the corresponding M1 transition probabilities.

The M1 strength of a 1 qp configuration is given by putting consistently $i^{(1)} = i^{(-1)} = 0$ in eq. (41). According to the resulting expression (41) the presence of the rotation-aligned qp's gives rise to an additional magnetic moment along the x-axis. Hence, if the gyrofactors $\tilde{g}^{(1)}$ and $\tilde{g}^{(2)}$ have opposite sign then the aligned pair of qp's adds up constructively their contribution to the one of the odd-qp so leading to an enhanced transition amplitude compared to the case of one quasiparticle.

The observed behaviour of the M1 strength in Kr, Tm and Lu [refs. [10-12])] can be qualitatively interpreted in this way. There, the corresponding 3 qp configurations are indeed of the type that a pair of rotation-aligned qp's is coupled to an odd unlike qp. In fig. 8, for example, the $B(\mathrm{M1})$ values extracted from measured $B(\mathrm{M1})_{\Delta I=1}/B(\mathrm{E2})_{\Delta I=2}$ ratios for the $\frac{9}{2}^-[514]$ band in ^{165}Lu [ref. [11])] are displayed taking the rotor estimate for the $B(\mathrm{E2})$ values. In the lower panel of fig. 8 the respective alignment i and the frequency ω_x are represented as a function of the

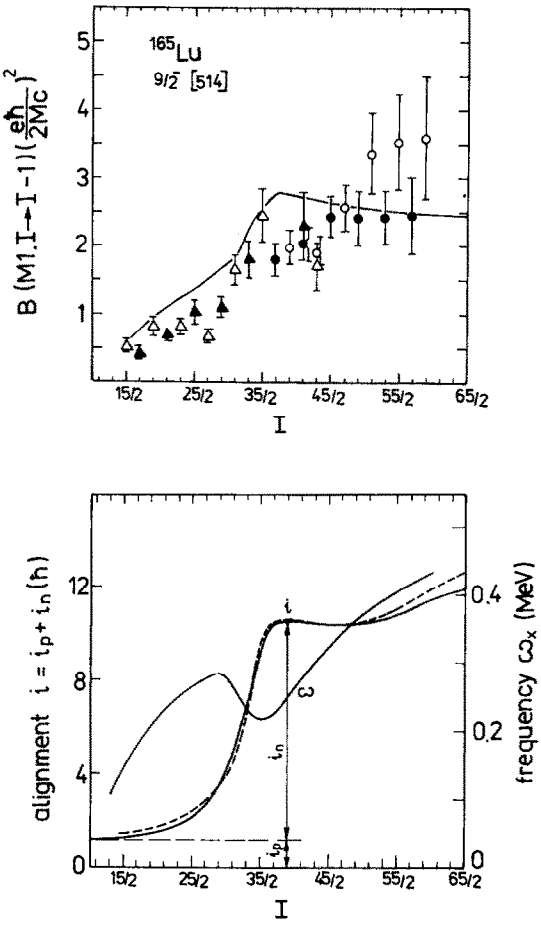

Fig. 8. Frequency ω_x, alignment $i = i_p + i_n$ and B(M1) values as a function of the spin I for the band $\frac{9}{2}^-[514]$ in ^{165}Lu [ref. [11])]. The theory (full line) is compared to the experimental B(M1) values (triangles and circles) as obtained by assuming a constant intrinsic quadrupole moment [11]) for this band.

a.m. I which have been constructed [11]) by means of the cranked shell model [1]) from the band energies [11]). For a qualitative estimate it is sufficient to adopt the K-value from the band head ($K = \frac{9}{2}$) and to neglect the small signature splitting for simplicity ($\Delta e' = 0$). Inserting all these "reasonable" parameters into eq. (41) yields the theoretical B(M1) values displayed in fig. 8 which obviously can qualitatively reproduce the experimental enhancement effect.

We mention that the band crossing region $I = \frac{27}{2}$ to $i = \frac{35}{2}$ might be excluded from the comparison because the amplitude (41) has been constructed for a pure configuration.

A more quantitative analysis in terms of eq. (41) for the considered case has shown [13]) that the size and signature oscillation of the observed B(M1) values are

grossly underestimated above the spin $I = \frac{45}{2}$ where the alignment starts to increase again (cf. fig. 8). Basing on a particle plus triaxial rotor model Hamamoto and Mottelson [14]) can describe the strong oscillation of the $B(M1)$ values in the high spin region but it is hard to reproduce the small signature splitting in the corresponding energies.

The result is considered to be promising with respect to more sophisticated calculations applying the proposed method.

6. Summary

A cranking approximation for calculating electromagnetic transition amplitudes is proposed which is suggested from the rotor-plus-particle system. The application to the case of a high-j quasiparticle demonstrates that the proposed method accounts for the orientation of the deformed nucleus as well as for the intrinsic structure of the states. The analysis of the M1 radiation allows one to determine the parameters of quasiparticle orbitals on an absolute scale. By a factorization approximation the semiclassical radiation formula for a three-quasiparticle configuration can be established which may be used to describe the enhancement effects experimentally found in several cases [10-12]).

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