ROTATIONAL MOTION OF TRIAXIAL SHAPE IN UNFAVOURED-SIGNATURE STATES OF ODD-A NUCLEI

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In the region of the angular momentum where the intrinsic energy (namely, the odd-quasiparticle energy) plays a more important role than the collective rotational energy, we point out that the way of rotation in unfavoured-signature states of odd-A nuclei, which is clearly different from that in the usual cranking approximation, can be realized for triaxial intrinsic shape and leads to a peculiar signature dependence of $B(E2; I\to I-1)$ values.

The deviation of nuclear shape from axial symmetry is expected for high-spin states, though in the ground state of medium or heavy nuclei there has been no clear-cut evidence for the deviation. In order to conclude that a nucleus has a triaxial shape, we need to obtain various kinds of independent experimental evidence for the shape, since just one kind of data (for example, energy spectra) could be interpreted in many ways. The signature dependence of $B(E2; I\to I-1)$ values, which has been observed [1,2] in the transitions between the negative-parity yrast states of rare earth odd-Z nuclei, is regarded [3] as a clear evidence for the deviation (including the fluctuation) of nuclear shape from axial symmetry.

In the present letter I consider the case of collective rotation [#1] (i.e. $-60°\leqslant\gamma\leqslant 0°$) and take the yrast states of odd-A nuclei in which the odd particle is in a high-j orbit, such as the $h_{11/2}$ ($i_{13/2}$) orbit in odd-Z (odd-N) rare earth nuclei. Since the rotation considered here is collective, we assume, as is usually done, that the γ dependence of the moments of inertia is the same as that of the hydrodynamical moments of inertia. Then, in the region of the I and the γ values which are of physical interest before the first band-crossing the unfavoured-signature states are shown to have a way of rotation which is clearly different from the one in the usual cranking-model approximation. The way of rotation is specifically exhibited in the particular pattern of the signature dependence of $B(E2; I\to I-1)$ values, which are the reduced probabilities of the transitions between the states with the favoured and the unfavoured signature.

The signature of the favoured (unfavoured) states is defined as

$$\alpha_f=\tfrac{1}{2}(-1)^{j-1/2}\,,\quad \alpha_u=\tfrac{1}{2}(-1)^{j+1/2}\,, \tag{1}$$

where the angular momentum of the odd particle is expressed by j. We note here the relation between the angular momentum I and the signature α,

$$\alpha=I \bmod 2\,. \tag{2}$$

Expressing a quadrupole-deformed shape of nuclei by the two parameters [5] $Q_0=\tfrac{4}{5}ZR^2\beta\cos\gamma$ and $Q_2=-(4/5\sqrt{2})ZR^2\beta\sin\gamma$, the E2 operator quantized along the rotation axis is written as

$$\hat{Q}_\mu=D^2_{\mu 0}Q_0+(D^2_{\mu 2}+D^2_{\mu -2})Q_2\,. \tag{3}$$

In the limit of large angular momentum I one obtains the quadrupole operators [3,6]

$$\hat{Q}_0=-\tfrac{1}{2}Q_0+\sqrt{3/2}\,Q_2=-\tfrac{4}{5}ZR^2\beta\sin(\gamma+30°)$$

$$\text{for static moments}\,, \tag{4a}$$

[#1] We use the definition of γ-values, according to the "Lund convention" [4].

$$\hat{Q}_2 = -\sqrt{3/8}\, Q_0 - \tfrac{1}{2} Q_2$$
$$= -(4/5\sqrt{2}) Z R^2 \beta \cos(\gamma + 30°)$$
$$\text{for } \Delta I = \pm 2 \text{ transitions}, \tag{4b}$$

in which the rotation axis is taken as the 1-axis.

In the $\Delta I = 1$ E2 transitions between the yrast states of odd-A nuclei in which the odd particle is in a high-j orbit the E2 operator is written as [7,8]

$$\hat{Q}_1 \propto -\sqrt{3/2}\, Q_0 - Q_2 \propto \hat{Q}_2$$
$$\text{for } (\alpha_u, I) \rightarrow (\alpha_f, I-1) \text{ transitions},$$
$$\hat{Q}_1 \propto -\sqrt{3/2}\, Q_0 + 3 Q_2 \propto \hat{Q}_0$$
$$\text{for } (\alpha_f, I) \rightarrow (\alpha_u, I-1) \text{ transitions} \tag{5}$$

in the limit of large I values (or in the cranking approximation). Since in this limit the B(E2; $I \rightarrow I-1$) values are proportional to $\langle \hat{Q}_1 \rangle^2$, we see from (5) that they have a conspicuous signature dependence for the nuclear shape deviating from axial symmetry (i.e. $Q_2 \neq 0$) and that the signature dependence for positive γ values (where $Q_0 Q_2 < 0$) is out of phase to that for negative γ values (where $Q_0 Q_2 > 0$).

For finite values of I one has to calculate numerically B(E2; $I \rightarrow I-1$) values. A numerical example taken from ref. [3] which is calculated by using the particle-rotor model in which one quasiparticle in an $h_{11/2}$ orbit is coupled to quadrupole-deformed rotors is shown in fig. 1. It is observed that the zigzag phase of the signature dependence of the calculated B(E2; $I \rightarrow I-1$) values is the same for both positive and negative γ values. Namely, for $\gamma = +15°$ it is the same phase as that given by the cranking approximation (5), while the phase of the signature dependence for $\gamma = -15°$ is opposite to the one given by (5). The pattern of the signature dependence, which is observed experimentally in transitions between the $h_{11/2}$ yrast states of rare earth odd-Z nuclei, has always been the one exhibited in fig. 1 for both positive and negative γ values. Thus, assuming that a possible γ fluctuation has a minor effect, one could not determine the sign of the γ values of the configurations in those nuclei just by examining the observed signature dependence. Since for very large I values the signature dependence has to be given by (5) in consistency with the cranking approximation, at a certain I value the calculated signature dependence for $\gamma < 0$ changes the phase of the zigzag behaviour.

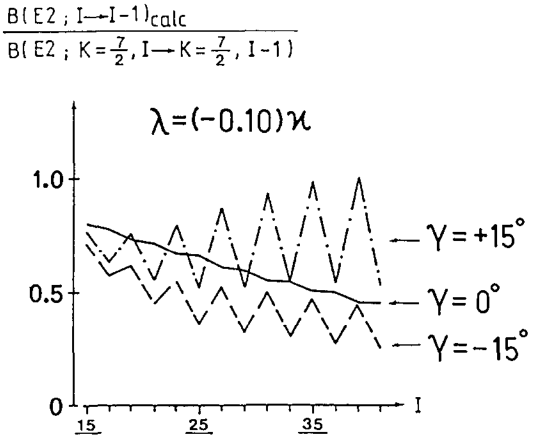

Fig. 1. Example of calculated B(E2) values of the transitions between the yrast states of odd-A nuclei, which are obtained by using the particle-rotor model in which one quasiparticle in an $h_{11/2}$ orbit is coupled to a quadrupole-deformed rotor. For convenience, the B(E2) values for an axially symmetric shape with $K=7/2$ are used as units. Used parameters are: $J_0\kappa/\hbar^2 = 72$ and $\Delta = 0.45\kappa$. The figure is taken from ref. [3].

The $h_{11/2}$ yrast states of rare earth odd-Z nuclei before the first band-crossing are expected to be collectively rotating states (i.e. $-60° < \gamma \leqslant 0°$). The expectation comes partly from [2,8–11] the analysis of the observed signature dependence in both the energies and the B(M1) values, and partly from the fact that the collective rotation is theoretically shown to be favoured [12] when the Fermi level is placed around the middle of the j shell. Namely, in the case of a single j shell the γ value favoured by a completely aligned quasiparticle is analytically obtained [13] and is given by the relation

$$-2\cos(\gamma - 60°) = \lambda/\kappa\,, \tag{6}$$

where the degree of shell filling is expressed by the Fermi level λ and the quadrupole-deformed one-particle potential is written as

$$V = \frac{\kappa}{j(j+1)} \{[3 j_3^2 - j(j+1)] \cos\gamma$$
$$+ \sqrt{3}(j_2^2 - j_1^2) \sin\gamma\}\,. \tag{7}$$

For example, eq. (6) gives $\gamma = -30°$ for $\lambda = 0.0$, for the case in which the Fermi level is placed exactly in the middle of the single-j shell. Though, in practice, a high-j quasiparticle is never completely aligned, the practically interesting range of γ values for a given λ value lies in the neighbourhood of the γ value given by the relation (6).

In order to understand the peculiar zigzag behaviour for a negative γ value, which is exhibited in fig. 1, we take a special example, with parameters $\lambda=0.0$ and $\gamma=-30°$, and use the particle-rotor model employed in refs. [3,13], namely, the model in which one $h_{11/2}$ quasiparticle is coupled to the rotor and angular momentum is treated as a good quantum number. In fig. 2 we show the calculated $B(E2)$ values, $B(M1)$ values and the expectation values of the intrinsic hamiltonian

$$H_{\text{intr}}=\sum_{\nu}(\epsilon_\nu-\lambda)a_\nu^+a_\nu + \tfrac{1}{2}\Delta\sum_{\mu,\nu}\delta(\bar{\mu},\nu)(a_\mu^+a_\nu^+ + a_\nu a_\mu)\,, \tag{8}$$

where ϵ_ν are the one-particle energies for a single particle moving in the potential V, eq. (7). In this particular example, the rotational hamiltonian is written as

$$H_{\text{rot}}=(3\hbar^2/8J_0)[R_1^2+4(R_2^2+R_3^2)]\,, \tag{9}$$

while the intrinsic hamiltonian is written as [13]

$$H_{\text{intr}}=\{3[\kappa/j(j+1)]^2(j_2^2-j_3^2)^2+\Delta^2\}^{1/2}\,. \tag{10}$$

Thus, the total hamiltonian is invariant under a rotation of $\pi/2$ about the 1-axis (i.e. the rotation $\mathscr{R}_1(\pi/2)$). That means the wave function of each state has components with either $R_1=0, \pm4, \pm8, ...$ or $R_1=\pm2, \pm6, \pm10, ...,$ in which the angular momentum of the core is expressed by $\boldsymbol{R}\equiv\boldsymbol{I}-\boldsymbol{j}$. ($R$ is not a good quantum number in the total hamiltonian.) The quantum number r_H of the yrast states, which is defined by

$$r_H\equiv\exp(\tfrac{1}{2}\pi i R_1) \tag{11}$$

$$=1\,,\quad \text{for } R_1=0, \pm4, \pm8, ...,$$

$$=-1 \quad \text{for } R_1=\pm2, \pm6, \pm10, ...,\tag{11a}$$

is written just above the I axis in fig. 2. Due to the presence of the r_H quantum number each E2 transition occurs either by the operator $\hat{Q}_0$ or by $\hat{Q}_2$. The operator which is relevant in each $\Delta I=1$ E2 transition is denoted at the top part of fig. 2. The M1 transitions between the states with different r_H quantum numbers are strictly forbidden, while the E2 transitions between the states with the same r_H quantum numbers are performed by the operator $\hat{Q}_0$ and thus vanish since $\hat{Q}_0=0$ for $\gamma=-30°$.

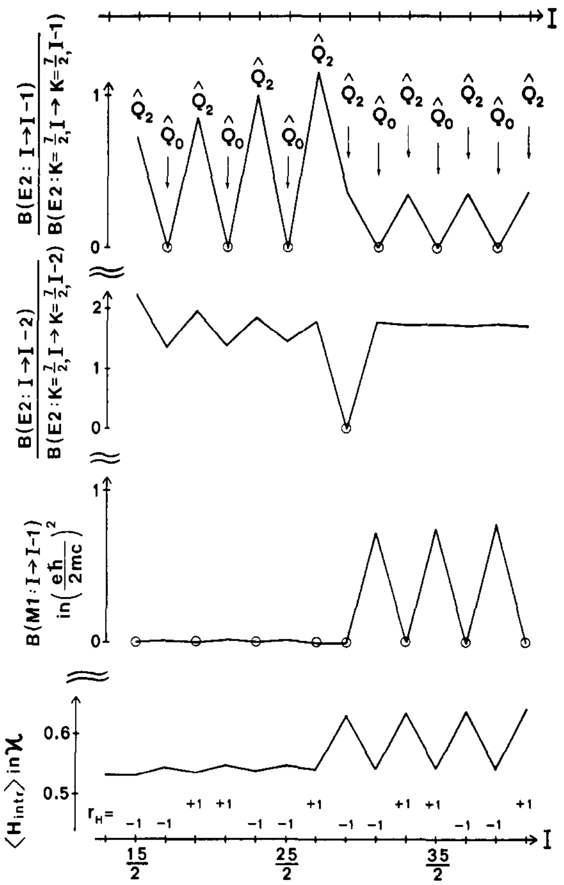

Fig. 2. Properties of the yrast states for the parameters ($\lambda=0.0$, $\gamma=-30°$) as a function of angular momentum I. A particle-rotor model with parameters $J_0\kappa/\hbar^2=72$ and $\Delta=0.45\kappa$ is used, in which one $h_{11/2}$ quasiparticle is coupled to the triaxial rotor. The points with open circles express the values, which vanish exactly due to the presence of the r_H quantum number. The r_H quantum number of each yrast state is denoted just above the I-axis at the bottom. The E2 operator, which is responsible for each $B(E2; I\to I-1)$ value, is denoted at the top. $B(E2)$ values are expressed, for convenience, in terms of those for an axially symmetric shape with $K=7/2$.

Now, an interesting observation in the example of fig. 2 is that for the yrast states with unfavoured signature ($\alpha_u=1/2$ and thus $I=1/2$ mod 2 in the present example with $j=11/2$) there is a band-crossing between $I=25/2$ and $29/2$. Due to the special symmetry present in this example there is no interaction between the two bands and thus the $B(\mathrm{E2}; I=29/2 \rightarrow I=25/2)$ value vanishes. For $I \leqslant 25/2$ the $\langle H_{\mathrm{intr}} \rangle$ value (as well as the quantities such as $\langle j_1^2 \rangle$ which are not shown in fig. 2) is more or less the same for both $\alpha=\alpha_{\mathrm{u}}$ and $\alpha=\alpha_{\mathrm{f}}$ states, while for $I \geqslant 29/2$ it is clearly different and, in fact, shows the same feature as that given by the cranking calculations. The zigzag behaviour of $B(\mathrm{E2}; I \rightarrow I-1)$ values changes the zigzag phase at $I=29/2$, and the behaviour for $I \geqslant 29/2$ is in agreement with the one given by the cranking description (5).

The feature of the structure of the two bands is illustrated in fig. 3, by using a simplified and semi-classical picture. The figure is sketched based on the calculated values such as $\langle R_1^2 \rangle$, $\langle R^2 \rangle$, $\langle I_1^2 \rangle$ and $\langle j_1^2 \rangle$. For the quadrupole-deformed shape, which has $\mathscr{R}_1(\pi)$, $\mathscr{R}_2(\pi)$ and $\mathscr{R}_3(\pi)$ symmetry, the core angular momentum R takes only even integers [5]. (Though R is not strictly a good quantum number in the total wave functions, in fig. 3 we sketch as if it were.) For the favoured states in which $(I-j)$ is an even integer the cheapest way to construct a given I value is to make $\boldsymbol{R}$ and $\boldsymbol{j}$ parallel to each other so that $I=R+j$. Namely, in this way one can achieve the minimum value for both the intrinsic energy $\langle H_{\mathrm{intr}} \rangle$ and the rotational energy $\langle H_{\mathrm{rot}} \rangle$. We note here that when the condition (6) is fulfilled it is possible to have $j_1=j$ at the minimum of the intrinsic energy. For the unfavoured states in which $(I-j)$ is an odd integer it is not possible that $\boldsymbol{R}$ and $\boldsymbol{j}$ are parallel to each other. One finds that for smaller I values the scheme in fig. 3a is energetically favourable in which the intrinsic energy is minimum at the cost of a larger rotational energy, while for larger I values the scheme in fig. 3b is favourable in which the rotational energy is minimum at the cost of a larger intrinsic energy.

The scheme in fig. 3b corresponds to the cranking approximation in which the system is cranked about the 1-axis. In contrast, the scheme for the unfavoured signature states in fig. 3a is quite different [#2] from the usual cranking picture, as is clearly seen, for example, from the fact that the collective rotation

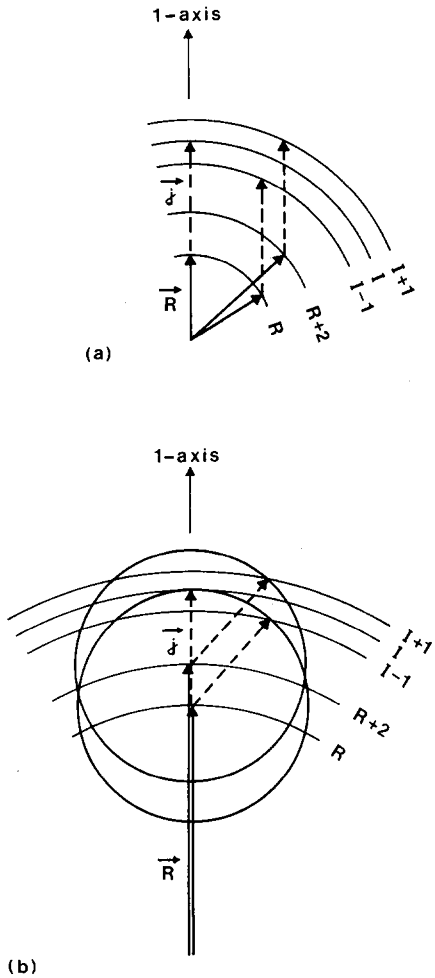

Fig. 3. Simplified and semi-classical illustration of the two ways of rotational motion, which are discussed in the text. (Note the role of the 1-axis in connection with H_{rot} and H_{intr}.) The angular momentum of favoured-signature states is expressed by I, while $I-1$ and $I+1$ denote that of unfavoured-signature states. The angular momentum of the core is expressed by R, which takes only even integers. For favoured-signature states the scheme in (a) and (b) is the same. For unfavoured-signature states the scheme (a) is energetically favourable for smaller angular momenta in which the intrinsic energy plays a more important role than the rotational energy, while the scheme (b) is favourable for larger angular momenta. The scheme (b) corresponds to the usual cranking approximation. (In a quantum-mechanical system the relation $j_1=j$, for example, does not mean that the vector $\boldsymbol{j}$ is exactly parallel to the 1-axis. However, in the present illustration it is sketched to be parallel.)

[#2] The one-dimensional cranking approximation to the rotation of the system with triaxial shape, as is usually formulated, has been questioned in, for example, ref. [14].

axis (i.e. the direction of the vector $\boldsymbol{R}$) is different for the states with different signatures. In fact, this scheme illustrates the structure of the unfavoured signature states with $I \leqslant 25/2$ in fig. 2. For example, in the scheme in fig. 3a we see the feature that the E2 transition from the state $|\alpha_u, I+1\rangle$ to $|\alpha_f, I\rangle$ is essentially made by the operator $\hat{Q}_0$ (i.e. $\Delta R_1 = 0$), while the E2 transition from $|\alpha_f, I\rangle$ to $|\alpha_u, I-1\rangle$ is done by $\hat{Q}_2$ (i.e. $\Delta R_1 = 2$). The extreme example of this feature is shown in the top part of fig. 2 for $I \leqslant 27/2$.

It should also be noted that for an axially symmetric shape the collective rotation can occur only about the axis perpendicular to the symmetry axis. Then, the way of rotation for unfavoured states which is shown in fig. 3a is not possible. Instead, only the scheme shown in fig. 3b is applicable, when we interpret the "1-axis" as one of the axes perpendicular to the symmetry axis.

The example ($\lambda = 0.0$, $\gamma = -30°$) shown in fig. 2 is an extreme case. However, we have found that for a given λ the main feature expressed in fig. 3a remains for the γ values in the neighbourhood of the γ value determined by (6), as far as I is not so large and we use realistic values of the parameters involved in H_{intr} and H_{rot} when we have an appreciable deviation of the nuclear shape from axial symmetry. For example, analyzing the signature dependence of $B(\text{E2}; I \to I-1)$ values for $\gamma = -15°$ shown in fig. 1, we find that the E2 transitions from $|\alpha_u, I+1\rangle$ to $|\alpha_f, I\rangle$ are predominantly governed by the operator $\hat{Q}_0$, while the E2 transitions from $|\alpha_f, I\rangle$ to $|\alpha_u, I-1\rangle$ are governed by $\hat{Q}_2$. Thus, we find that the picture in fig. 3a works rather well.

One of the features of the scheme in fig. 3a is the rather small $B(\text{M1}; I \to I-1)$ values, since the M1 operator in the high-j states is in essence proportional to $\boldsymbol{j}$ and the vector $\boldsymbol{j}$ is not really changing from the unfavoured states to the favoured states.

Furthermore, it is observed that the calculated $B(\text{M1})$ values in fig. 2 for $I \leqslant 27/2$ have the signature dependence which is out of phase to that for $I \geqslant 29/2$. For the collective rotation (i.e. $-60° \leqslant \gamma \leqslant 0°$) we know that the estimated signature splitting in energies is always such that $E'_{\alpha_u} > E'_{\alpha_f}$, writing energies in the rotating coordinate system by E'. Now, in the cranking approximation there is a definite relation [7,15] between the signature dependence in energies and $B(\text{M1})$ values: Namely, the $B(\text{M1}; \alpha_f, I \to \alpha_u, I-1)$ value is always larger (smaller) than the $B(\text{M1}; \alpha_u, I-1 \to \alpha_f, I-2)$ value, if $E'_{\alpha_{ui}}$ is larger (smaller) than E'_{α_f}. That means, in the region of $I \leqslant 27/2$ in the example of fig. 2 the cranking relation between the signature-dependence of energies and $B(\text{M1})$ values is broken. We have found that the breaking of this cranking relation seems to happen in a very limited region of the parameters (γ, λ, I), though the scheme in fig. 3a works in a larger region.

If one can experimentally find the angular momentum at which the zigzag behaviour of $B(\text{E2}; I \to I-1)$ values changes the phase, it gives experimental information on the competition between the intrinsic energy and the rotational energy.

In conclusion, we have pointed out that for not so large I values the way of rotation in unfavoured-signature states, which is clearly different from that in the cranking approximation, can be realized for a triaxial shape and leads to a peculiar signature dependence of $B(\text{E1}; I \to I-1)$ values. Though the numerical calculations were performed by using a particle-rotor model, our conclusion is rather general in the sense that this particular way of rotation is energetically favourable when the intrinsic energy plays a more important role than the rotational energy.

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