Quantized Wobbling in Nuclei

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Abstract

Quantized wobbling-phonon excitations have recently been observed in ^{163}Lu, which is clear evidence for breaking the axial symmetry. Rotational motion, which is so far the best-established collective mode in nuclei, has thereby obtained a new dimension. Nuclei can rotate about any of the principal axes.

Nuclear Deformations

The most important deformation in the ground-states of nuclei is the axially-symmetric quadrupole deformation corresponding to either prolate or oblate shape. The difference between the radii along the longest and shortest axes is up to 30% of the averaged radius. Depending on the nuclear shell structure associated with the given numbers of neutrons and protons, other nuclear shapes are obtained for excited states, theoretically. When a nucleus rotates rapidly, the intrinsic shell structure is in some cases expected to stabilize some more exotic nuclear shapes, such as a much larger deformation (superdeformation, for example) or axially-asymmetric (triaxial) deformation. For years clear experimental evidence for the presence of the predicted nuclear triaxial shape has been sought. Here we are talking about a triaxial shape, in which the triaxiality is clearly established compared with the possible quantum-fluctuation about an axially-symmetric shape, and which is stable in a sufficiently wide region of angular momentum.

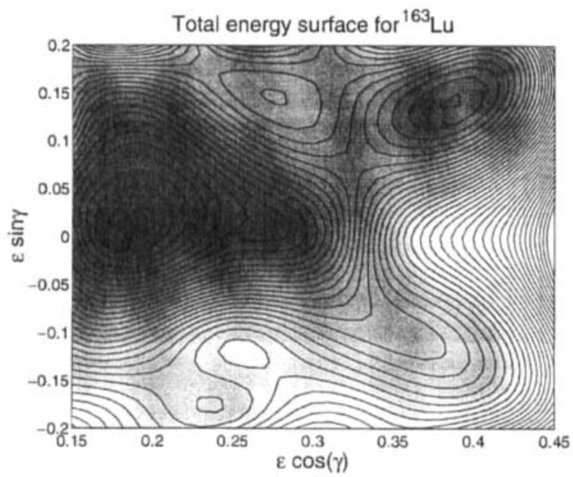

Figure 1. *Calculated potential energy surface [13] for ^{163}Lu for I = 57/2. The energy difference between contour lines is 0.1 MeV. The values of the shape parameters $(\varepsilon,\gamma) \sim (0.4,+20°)$ for the local triaxial minimum at $(\varepsilon\cos\gamma, \varepsilon\sin\gamma) \sim (0.38, 0.15)$ correspond to the axis ratios ~ 1.6:1.2:1 for the nuclear shape.*

Figure 1 shows an example of a calculated potential energy surface, where a local energy minimum is obtained at a parameter set of (ε,γ), which shows a large quadrupole deformation with clear triaxiality. The size of the deformation is proportional to ε, while γ expresses the degree of triaxiality.

Quantum Mechanical Rotation and Wobbling

As a quantum mechanical system, axially-symmetric deformed nuclei, like linear molecules, cannot rotate collectively about the symmetry axis. Any rotation about the axis perpendicular to the symmetry axis is equivalent, and the rotational energy is expressed in terms of the moment of inertia, $\mathfrak{J}$, about the perpendicular axis as

$$E(I) = \frac{\hbar^2}{2\mathfrak{J}} I(I+1).$$

The gamma-ray transition energy between rotational states with angular momenta I and I-2 is written as

$$E_\gamma = \frac{\hbar^2}{2\mathfrak{J}}(4I-2)$$

with the associated rotational frequency

$$\hbar\omega_{rot} \equiv \frac{dE}{dI} \approx \frac{E_\gamma}{2}.$$

Measured moments of inertia in the neighborhood of the ground states of even-even nuclei are known to be a factor 2–3 smaller than the rigid body moments of inertia estimated for a body of the same shape, due to the presence of pair correlation. The reduced transition probability B(E2,$I \rightarrow I-2$) of the E2 transition within a given rotational band is a strongly collective quantity, which is proportional to the square of the size of the quadrupole deformation and independent of I in the limit of $I >> 1$.

Nuclei with a triaxial shape can, however, rotate about any one of the principal axes, showing much richer spectra of collective rotation. The rotation about the axis with the largest moment of inertia is energetically most favorable. When a given intrinsic structure is preferred, a family of rotational bands can be built by transferring some angular momentum to the other two axes while freezing the intrinsic structure. This family of rotational bands is formulated in terms of phonon excitations. The classical analog of this wobbling mode is the spinning motion of an asymmetric top, but the motion in the nuclear system is quantized and expressed in terms of the wobbling phonon number. A family of rotational bands of wobbling excitations can be characterized by specific electromagnetic decay properties between them, which depend also on the triaxial shape as well as the intrinsic structure. This quantized wobbling phonon picture was proposed more than 25 years ago by A. Bohr and B. Mottelson [1]. The relation between rotational energy E_{rot}, rotational frequency ω_{rot}, wobbling frequency ω_w, angular momentum I, and moments of inertia $\Im_k$ is written as follows:

$$E_{rot} = \sum_k \left(\frac{\hbar^2}{2\Im_k}\right) I_k^2 \approx \frac{\hbar^2}{2\Im_x} I(I+1) + \hbar\omega_w(n_w + 1/2)$$

for $\Im_x >> \Im_y, \Im_z$, where $I(I+1) = I_x^2 + I_y^2 + I_z^2$ and $\hbar\omega_w = \hbar\omega_{rot}\sqrt{(\Im_x - \Im_y)(\Im_x - \Im_z)/\Im_y\Im_z}$ with $\hbar\omega_{rot} = I\hbar^2/\Im_x$. The wobbling motion is expressed in terms of a vibration with one variable and, thus, in the above expression of E_{rot}, the energy of the zero-point fluctuation appears as $(1/2)\hbar\omega_w$ and not as $(3/2)\hbar\omega_w$.

Experiments and Results

The study of the characteristic cascades of γ-transitions through rotational bands, and in particular the weaker γ-transitions between bands, requires effective and advanced detector arrays with high resolution and sensitivity. The aim is to simultaneously register as many as possible of the coincident γ-rays in order to establish the decay pattern. Almost two years ago a wobbling band, a one-phonon excitation, was found [2] in ^{163}Lu, and more recently even a two-phonon excitation was also established [3] in ^{163}Lu, built on the same intrinsic configuration. The high-spin states in ^{163}Lu were populated through the ^{139}La(^{29}Si,5n) reaction using the ^{29}Si beam energies of 152 and 157 MeV. At the higher beam energy in the second experiment the 5n exit channel is relatively more strongly populated at the expense of a less clean condition due to a higher number of open exit channels. The γ-rays from the reaction were studied with the EUROBALL IV detector array [4] at IReS in Strasbourg, where the beam was provided by the Vivitron accelerator. A multiplicity filter consisting of about 500 Bismuth-Germanat (BGO) crystals mounted in a spherical geometry inside the array of 239 high-purity Ge-detectors (some of which combined in Clusters (7) and Clovers (4)) was used to select data related to the longest γ-cascades from the highest angular momentum. In the experiments establishing the one- and two-phonon wobbling excitations about 2 and 6x10^9 high-fold coincidence events containing information on energy, relative time, detector identity, and γ-ray multiplicity were collected.

The data analysis requires careful calibrations and matching of detector responses in energy and time and corrections for Doppler shifts at different detector angles in order to obtain the best possible resolution and thereby sensitivity. The events are sorted into multidimensional (2D–4D) matrices for the study of coincidence relations from which the band structures are built and decay between bands is established. The electromagnetic properties of the γ-rays can be determined from mutual directional correlations to coincident γ-rays of known multipolarity. Linear polarization obtained from scattering between clover crystals is used to distinguish between electric and magnetic nature. These steps are necessary to determine spin and parity of the levels.

As results of these experiments, three rotational bands extending quite high in spin are shown in Figure 2. The labels $n_w = 1,2$ indicate their interpretation as one- and two-phonon excitations built on the $n_w = 0$ band. The configuration of the $n_w = 0$ band corresponds to the lowest proton orbital in the triaxial minimum $(\varepsilon,\gamma) \approx (0.4, 20°)$ with the intrinsic angular momentum $j = 13/2$ predominantly aligned along the axis with the largest moment of inertia, here the x-axis. The lowest spin in the $n_w = 0$ band is indeed I=13/2. Some of the important coincidence relations supporting the two-phonon excitation are illustrated by the double-gated spectra shown in Figure 3. For most of the $\Delta I = 1$, $n_w=1 \rightarrow n_w=0$ transitions it was possible to determine the electromagnetic composition of mixed E2 and M1 radiation, whereas only

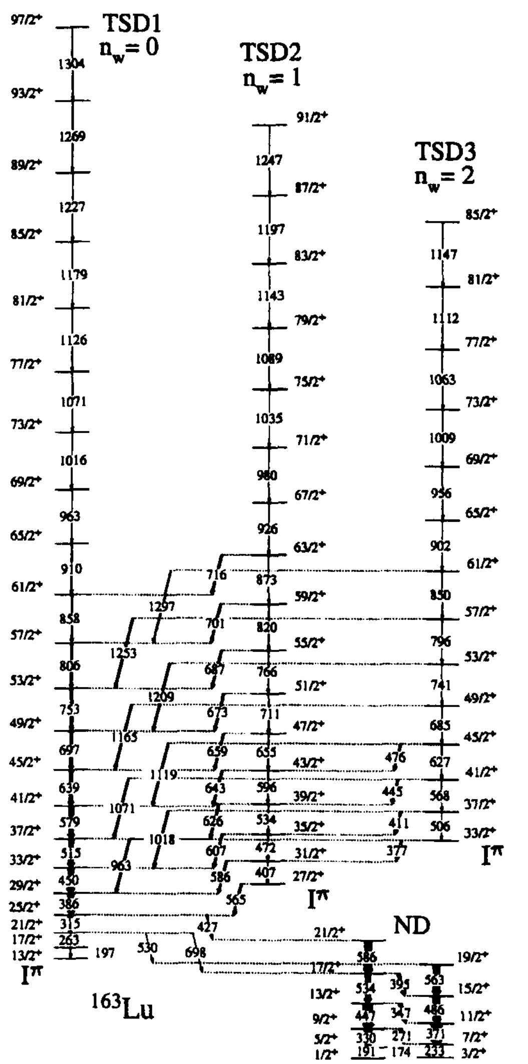

Figure 2. *Partial level scheme of ^{163}Lu showing the triaxial strongly deformed (TSD) rotational bands related to the wobbling excitations (n_w = 0,1,2) and some levels of normal deformed (ND) bands to which they decay. The full level scheme comprises ~ 20 ND bands and 3 additional weakly populated TSD bands which are not related to wobbling excitations. See [2],[3],[14].*

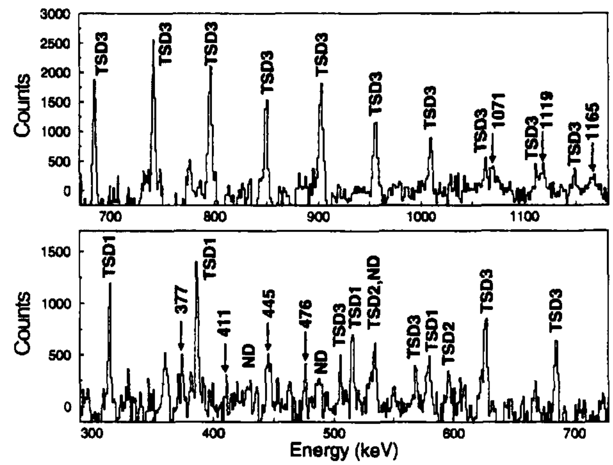

Figure 3. *Double coincidence γ-spectra illustrating the quality of data revealing the two-phonon wobbling excitation. See Figure 2. Upper part: Double gates on a sum of the 386, 450, and 515 keV transitions in TSD1 and a sum of the 685, 741, 796, and 850 keV transitions in TSD3, emphasizing the high-energy TSD3→TSD1 transitions. Lower part: Same as above, except for using only the 450 keV transition as gate in TSD1 and emphasizing the TSD3→TSD2 decay. The connecting transitions are marked by arrows.*

one of the n_w=2→n_w=1 transitions was uncontaminated and intense enough to allow a determination. From these compositions and the relative intensities of the out-of-band to in-band transitions the relative E2 strength can be extracted for transitions in the spin range 15<I<30. The excitation energy of the three bands relative to a rigid reference rotor are shown in Figure 4, which also illustrates the striking similarity of their dynamic moments of inertia $\Im^{(2)}/\hbar = dI/d\omega_{rot}$. This similarity is further amplified by the results from recent lifetime measurements showing almost identical transition quadrupole moments for the n_w = 0 and n_w = 1 bands [5].

Wobbling in the Presence of Alignment

The textbook example of wobbling motion [1] contains no angular momentum of the intrinsic motion. While the presence of high-j aligned particles favors a specific (triaxial) shape depending on the degree of the j-shell filling, the states with high-j aligned particles can easily appear in the neighborhood of the yrast line because of the relatively small rotational energy needed for building a given total angular momentum. In [6] the wobbling excitation of the collective rotational angular momentum of the core (***R*** in Figure 4) in the presence of a high-j aligned quasiparticle is described, and a unique pattern of electromagnetic transitions between the bands is obtained. It can also be shown [7] that in the presence of alignments the wobbling excitations may appear more easily at lower angular momentum. The intrinsic configuration of rotational bands in ^{163}Lu is represented by a

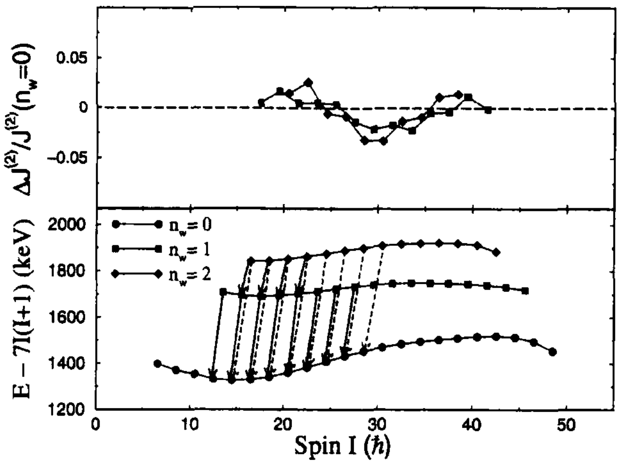

Figure 4. *Upper part: Relative difference in dynamic moment of inertia, $\Delta\Im^{(2)}$, of the bands with $n_w = 1,2$ compared to $\Im^{(2)}$ for the $n_w = 0$ band. Lower part: Energies relative to a rigid rotor for the three bands with $n_w = 0,1,2$. The solid and dashed arrows indicate observed $\Delta n_w = 1$ and $\Delta n_w = 2$ transitions, respectively.*

highly aligned quasiproton in the $i_{13/2}$ (l=6, j=13/2) shell, which is coupled to a triaxial rotor with a shape corresponding to the local energy minimum in Figure 1. For the triaxial shape, and in the relevant region of angular momenta, the spin of the $i_{13/2}$ quasiproton is highly aligned parallel to the axis of the largest moment of inertia. Then, the lowest excitations of the total system are built by a wobbling motion of the collective rotational angular momentum of the core (see Figure 5) while freezing the intrinsic configuration expressed by the aligned $i_{13/2}$ quasiproton [7],[8]. The calculations show that the electric quadrupole (E2) transitions between the bands with $\Delta n_w = 1$ are surprisingly large at high

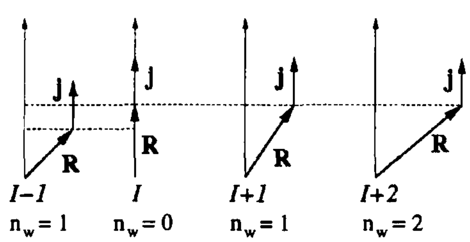

Figure 5. *Schematic coupling scheme, where the angular momentum of collective rotation of the core **R** produces wobbling excitations, while the proton angular momentum j is fully aligned along the axis of the largest moment of inertia. The total angular momentum **I** = j + **R**.*

spins, for which the B(E2, $I \rightarrow I-1$) values are approximately proportional to $1/I$ and not to $1/I^2$ as usually observed for ΔI=1 transitions in high-spin phenomena. The calculated transition strengths are determined almost exclusively by the amount of triaxiality, while the excitation energies or the wobbling frequencies depend on the moments of inertia used.

Comparison with Data

The measured electromagnetic transitions between the family of rotational bands shown in Figure 2 are in good agreement with the wobbling phonon picture and the calculated results. First, in ΔI = 1 transitions with $\Delta n_w = 1$ transitions the E2-strength at such high spins is unusually large and E2 is dominant over M1 in the decay. Moreover, the measured transitions are the stronger ones of the zigzag pattern expected for the ΔI = 1 transition strengths. Second, as seen in Figure 6, the quantal phonon rules for transition probability, B(E2,$n_w = 2 \rightarrow n_w = 1$) = 2•B(E2,$n_w = 1 \rightarrow n_w = 0$) and B(E2,$n_w = 2 \rightarrow n_w = 0$) = 0, are almost satisfied. The deviation from the phonon rules comes from the anharmonicity and is consistent with calculations. The calculated $B(E2)_{out}/B(E2)_{in}$ ratios plotted in Figure 6 are sensitive functions of γ ($\geq 15°$) and decrease as $1/I$ as I increases [7],[8], while the measured ratios show no I-dependence. This observed lack of I-dependence may indicate that the γ-value increases slightly with I in the present spin-region [7], which is in qualitative agreement with a gradual decrease observed in the transition quadrupole moments for both of the n_w=0 and n_w=1 bands [5].

As seen in Figure 4, the observed wobbling frequency does not really agree with the simple picture of harmonic vibrations.

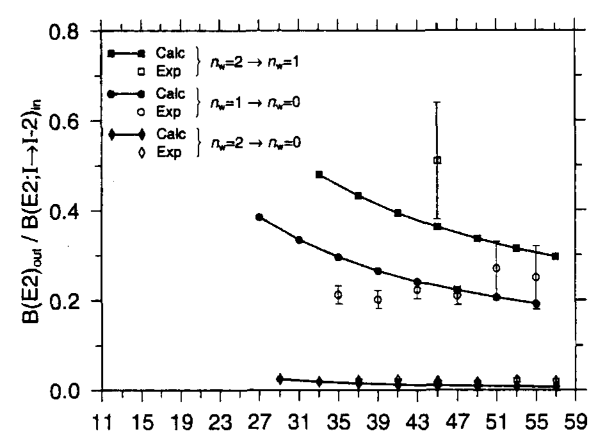

Figure 6. *Comparison of the measured [2],[3],[14] ratios of B(E2) values in ^{163}Lu with calculated ones. Calculated values are taken from [7], in which γ = 20° is used.*

The energy distance between the $n_w=2$ and $n_w=1$ bands is appreciably smaller than the one between the $n_w=1$ and $n_w=0$ bands. Using moments of inertia close to the rigid values, it is possible to reproduce the average wobbling frequency between the $n_w=1$ and $n_w=0$ bands [7]. However, the spin dependence of the moments of inertia has to be taken into account in order to obtain the observed dependence of the wobbling frequency on spins. An attempt to explain the spin-dependence in the wobbling frequency can be found in [9].

Conclusions

The two-phonon wobbling excitation observed in the nucleus ^{163}Lu is one of the most exotic properties of rotating nuclei. The observation has become possible thanks to the increased efficiency of the detection of gamma-rays with high resolution germanium spectrometers which have been developed and have become available within the last decade. Rotational bands, of which the level scheme, spin-alignments, and electromagnetic properties are very similar to the $n_w = 0,1$, and 2 bands in ^{163}Lu, have very recently been found also in the neighboring nuclei, ^{165}Lu [10] and ^{167}Lu [11]. Thus, the wobbling excitation in ^{163}Lu is not an isolated phenomenon.

The existence of the quantized wobbling mode shows that the nucleus can have a stable triaxial shape in the wide range of angular momentum, in which both the spherical and axial symmetry are broken. The rotational energies here depend on an interplay between three different moments of inertia. The observation of a two-phonon excitation, of which the relevant degree of freedom is collective rotation, amplifies the uniqueness of the present finding of nuclear wobbling. The purity of the wobbling mode may be studied by its interaction with coexisting multi-quasiparticle excitations in the triaxial well.

Triaxiality and wobbling may be found in other mass regions. Yet, the identification and further study of higher phonon excitations will require an experimental sensitivity level beyond what is available today. The development of the AGATA Ge-detector array [12] will provide a sensitivity level orders of magnitude higher than at EUROBALL IV. With AGATA and access, hopefully, to more intense stable ion beams in the future, it is most likely that other, even more fascinating collective modes of the atomic nucleus will be identified.

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