A Novel Approach for the Semi-Classical Description of the Wobbling Properties in Odd-A Nuclei

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Outline

Nuclear shapes

Nuclear shapes: most of the nuclei are spherical or axially symmetric in the ground state.

Figure 1: **Spherical:** $\beta_2 = 0$; **Prolate:** $\beta_2 > 0$; **Oblate:** $\beta_2 < 0$

Triaxial shapes

There are also deviations from *axial symmetric shapes* $\rightarrow$ **triaxial shapes** (e.g. no symmetry axis). The three PA's have different lengths.

Wobbling motion

Wobbling motion (WM)

- Uniquely associated to triaxial structures.
- It was theoretically predicted by Bohr and Mottelson more than 50 years ago (for the even-A case).
- This motion has an oscillating behavior.
- Spectrum: $E = E_{\text{rot}} + E_{\text{wob}}\left(n_w + \frac{1}{2}\right)$.

Experimental evidence

- Experimentally confirmed for ^{163}Lu in 2001.
- In present, few wobblers are experimentally confirmed in the mass regions: $A \approx 130, 160, 180$.

Wobbling motion

Triaxial nuclei

A triaxial nucleus can rotate about any of the three axes.

The rotational angular momentum is NOT aligned along any of the body-fixed axes: it **precesses** and **wobbles** around the axes with the largest MOI.

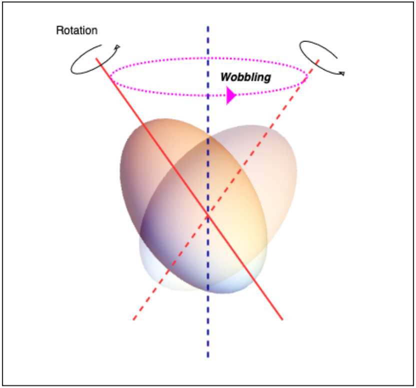

Figure 2: Schematic representation for the nuclear wobbling motion.

Wobbling Bands

Wobbling bands

Sequences of $\Delta I = 2\hbar$ rotational bands that are built on different *wobbling phonon excitations*.

Experimental evidence

^{163}Lu has **four** such wobbling bands, with $n_w = 0, 1, 2, 3$ wobbling phonon numbers, respectively.

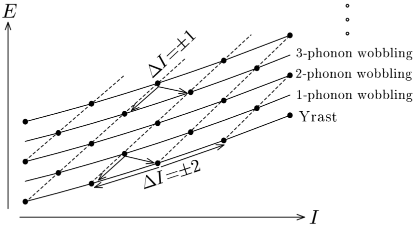

Figure 3: Rotational-band structures of the wobbling motion.

Theoretical framework

The odd-mass system consists of an even-even core (described by a triaxial rotor Hamiltonian H_{rot}) and a single j-shell nucleon described by its single-particle Hamiltonian H_{sp}.

Total system:

$$H = H_{\text{rot}} + H_{\text{sp}} = \tag{1}$$

$$\sum_{k=1,2,3} A_k(I_k - j_k)^2 + \epsilon_j + \frac{V}{j(j+1)}\left[\cos\gamma(3j_3^2 - \mathbf{j}^2) - \sqrt{3}\sin\gamma(j_1^2 - j_2^2)\right] \tag{2}$$

Solving the Hamiltonian in semi-classical approach for ^{163}Lu: *R. Poenaru and A. A. Raduta, International Journal of Modern Physics E, 2150033,* **2021**.

For ^{135}Pr: *A. A. Raduta and R. Poenaru, Journal of Physics G, 015106,* **2020**.

Energy function - Semiclassical approach

The eigenvalues for $\hat{H} = H_{rot} + H_{sp}$ are obtained by solving the *variational principle*:

$$\delta \int_0^t \langle \Psi_{IjM} | \hat{H} - i \frac{\partial}{\partial t'} | \Psi_{IjM} \rangle \, dt' = 0. \quad (3)$$

The trial function $|\Psi_{IjM}\rangle$ is a direct product $|\psi\rangle_{\text{core}} \otimes |\phi\rangle_{\text{s.p.}}$. From Eq. 3, one obtains the *classical equations of motion* in the canonical form:

$$\frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi} \; ; \; \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r} \quad (4)$$

$$\frac{\partial \mathcal{H}}{\partial t} = \dot{\psi} \; ; \; \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{t} \quad (5)$$

Semiclassical wobbling energies

Harmonic Frequencies

- They represent the harmonic-like behavior of the wobbling motion in odd-A nuclei.
- In our model, the pair of frequencies (Ω_1^I, Ω_2^I) is associated with the rotational motion of the core and the odd particle, respectively.

Energy spectrum

$$E_I = \epsilon_j + \mathcal{H}_{\text{min}}(I) + \left[\Omega_1^I \left(n'_{w_1} + \frac{1}{2}\right) + \Omega_2^I \left(n'_{w_2} + \frac{1}{2}\right)\right] \tag{6}$$

Terms: single particle energy, energy in the minimum point and [harmonic-like motion].

Characteristics of the model

Existing interpretation of WM in ^{163}Lu

- $n_{w_1} = 0,\ n_{w_2} = 1,\ n_{w_3} = 2,\ n_{w_4} = 3$
- TSD1-3: rotor couples with $j = 13/2$ proton. TSD4: rotor couples with $j = 9/2$ proton.
- The moments of inertia for band TSD4 differ than TSD1-3

New (current) approach

- TSD1 and TSD2 are *signature partner bands* $\rightarrow n_{w_1} = 0,\ n_{w_2} = 0$
- TSD3 is one-phonon wobbling band built on top of TSD2. $n_{w_3} = 1$
- Same $j = 13/2$ proton occurs in the particle-rotor coupling for all four bands.
- TSD2 and TSD4 are *parity partner bands* $(n_{w_2} = 0,\ n_{w_4} = 0)$. Same structure but opposite parity $\rightarrow$ The triaxial rotor admits **wave-functions** of both parities.

Excitation energies I

R. Poenaru, A. A. Raduta, Romanian Journal of Physics 66, 308, **2021***.*

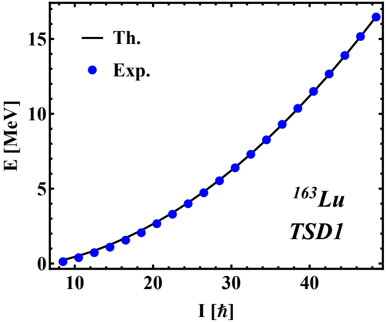

Figure 4: The excitation energies for the band TSD1.

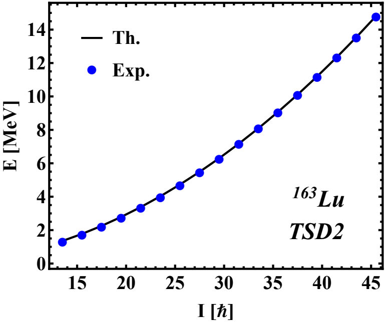

Figure 5: The excitation energies for the band TSD2.

Parameters: $\mathcal{I}_1 = 72$, $\mathcal{I}_2 = 15$, $\mathcal{I}_3 = 7\ \hbar^2/\text{MeV}$. $\gamma = 22^\circ$. $V = 2.1$ MeV.

Excitation energies II

R. Poenaru, A. A. Raduta, Romanian Journal of Physics 66, 308, **2021**.

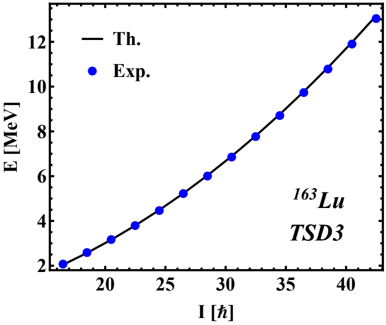

Figure 6: The excitation energies for the band TSD3.

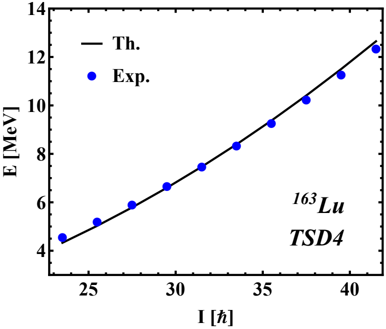

Figure 7: The excitation energies for the band TSD4.

Wobbling energies + frequencies

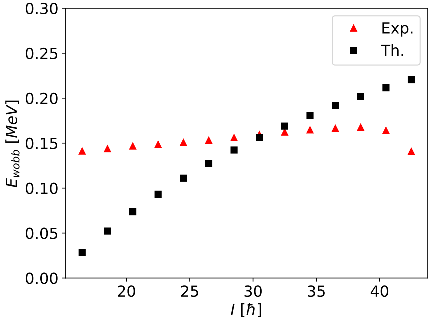

Figure 8: The wobbling energies for ^{163}Lu given as formula below. E_1 belong to TSD_3, while E_0 correspond to TSD_2.

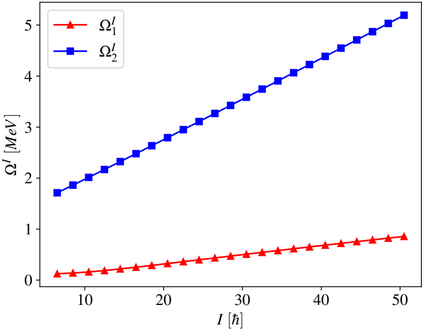

Figure 9: The wobbling frequencies as function of total angular momentum.

$$E_{\text{wob}} = E_1(I) - \frac{1}{2}\left(E_0(I+1) + E_0(I-1)\right)$$

Classical energy function - contour plots I

Spin: **Quantal** $\rightarrow$ **classical**. $I = I_1, I_2, I_3 \rightarrow I = x_1, x_2, x_3$.
Spherical coordinates $\rightarrow x_k = f(I_0, \theta, \varphi)$.
Express the $\mathcal{H}$ (average of the Hamiltonian on the trial function) in terms of (θ, φ).

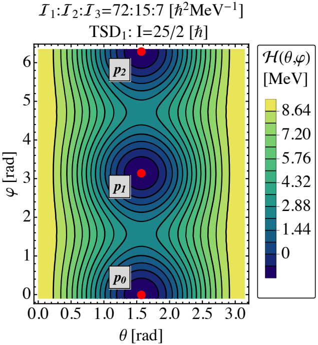

Figure 10: TSD1

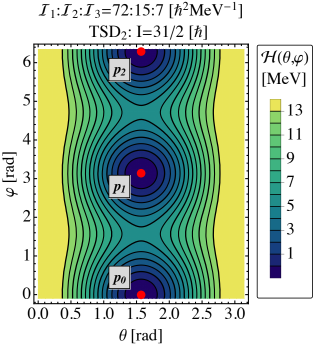

Figure 11: TSD2

Classical energy function - contour plots II

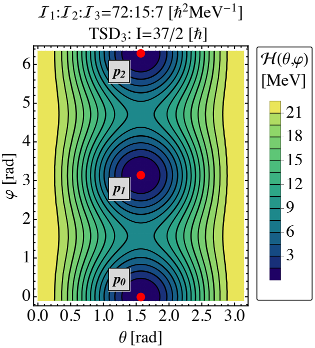

Figure 12: TSD3

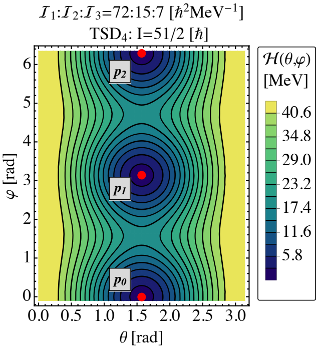

Figure 13: TSD4

As the energy increases, the trajectories go around all minima.The lack of localization indicates unstable wobbling motion.

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Classical trajectories I

3D representation of the angular momentum operator $\rightarrow$ Angular momentum sphere.
3D representation of $\mathcal{H}$ as a function of x_1, x_2, x_3. (the energy within a.m. space)

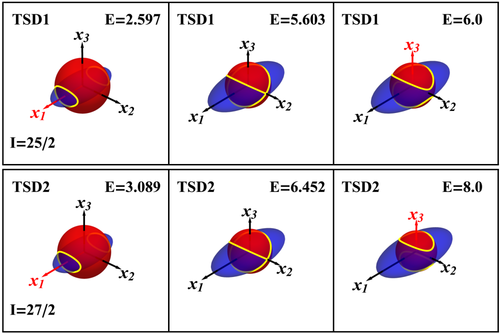

Figure 14: Intersection lines marked by yellow color represent the trajectories.

Classical trajectories II

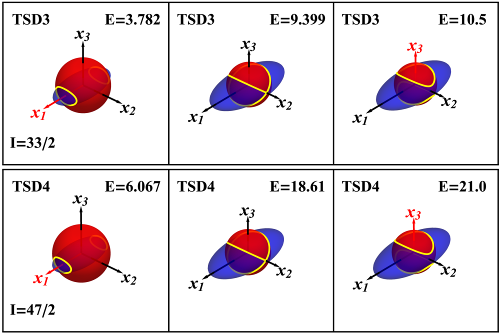

Figure 15: Intersection lines marked by yellow color represent the trajectories.

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Conclusions

- Wobbling motion in odd-mass has been described through a semi-classical approach based on the particle+rotor model
- Initial quantal Hamiltonian has been dequantized via the TDVE
- Analytical expression of the excitation energy was obtained, with the pair of wobbling frequencies
- The odd ^{163}Lu nucleus has been considered as a case study.
- Redefinition of its band structure through:
  - *signature partner bands* for TSD1 and TSD2
  - *parity partner bands* for TSD2 and TSD4
- Calculation of the excitation energies, wobbling energies, and wobbling frequencies
- Graphical representation of the classical energy function:
  - Contour-plot: stability of wobbling motion
  - 3D: determination of the classical trajectories of the wobbling motion

Thank you for your attention!