DESCRIPTION OF THE WOBBLING MOTION THROUGH A BOSON METHOD

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Outline

Nuclear Deformation

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta,\varphi;t) = R_0 \left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y_{\lambda}^{\mu}(\theta,\varphi)\right) \tag{1}$$

- The $\alpha_{\lambda\mu}$ are collective coordinates $\Longrightarrow$ *vibrations of the nucleus*.
- Y_{λ}^{μ} are the spherical harmonics.

Nuclear shapes

Most nuclei are spherical or axially symmetric in the ground state.

Figure 1: **Spherical:** $\beta_2 = 0$; **Prolate:** $\beta_2 > 0$; **Oblate:** $\beta_2 < 0$

Quadrupole deformations

- Most relevant vibrational degrees of freedom in nuclei.
- Play a crucial role in the rotational spectra of nuclei.

Quadrupole radius

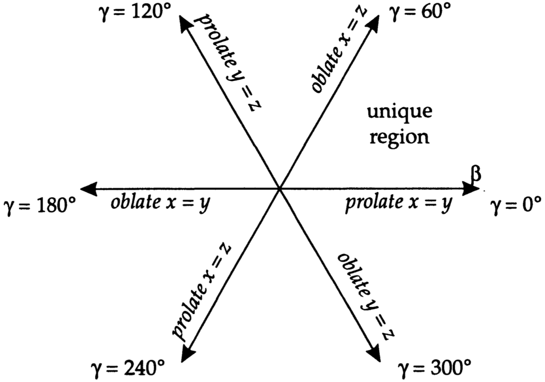
For pure quadrupole deformations:

$$R(\theta,\varphi)=R_0\left(1+\sum_{\mu}\alpha_{2\mu}Y_2^{\mu}(\theta,\varphi)\right) \ , \tag{2}$$

Using A. Bohr's description, the coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (*eccentricity*) and γ (**triaxiality**).

Nuclear triaxiality

- Besides the axially symmetric shapes (i.e., spherical, prolate, and oblate), nuclei can be **triaxial** $\implies$ lack of symmetry along any of the principal axes.
- The asymmetry is given by the non-zero value of γ.



Triaxial ellpsoid

Schematic example with a triaxial ellipsoid $(\gamma \neq 0)$ $\beta_2 > 0$.

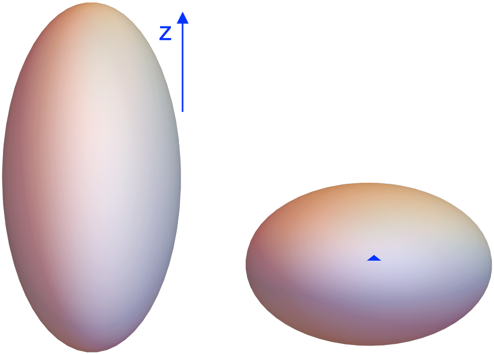


Figure 2: **Left:** side-view. **Right:** top view.

Nuclear triaxiality

- Probing triaxiality experimentally is a real challenge (e.g., large and complex detector setups).
- Only two fingerprints known so far: **chiral motion** (Frauendorf, 1997) and **wobbling motion** (Bohr and Mottelson, 1975).

Wobbling Motion (WM)

- Collective effect $\rightarrow$ *unique* to triaxial nuclei.
- Predicted almost 50 years ago, first experimental confirmation: in 2001 (Odegard et al.) for ^{163}Lu.
- In present, few wobblers are experimentally confirmed in the mass regions: $A \approx 130, 160, 180$ $\rightarrow$ A list of all known wobblers will be available in my PhD thesis (Chapter 3).

Wobbling motion

Triaxial nuclei

A triaxial nucleus can rotate about any of the three axes.

The rotational angular momentum (a.m.) is NOT aligned along any of the body-fixed axes $\Longrightarrow$ **precesses** and **wobbles** around the axes with the largest MOI.

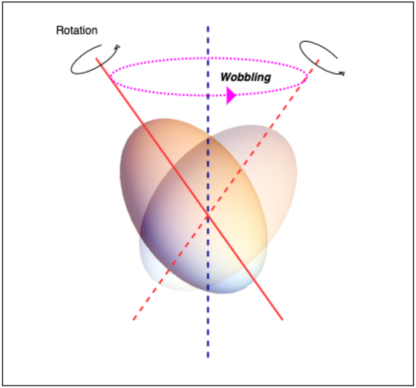


Figure 3: Schematic representation for the nuclear wobbling motion.

Wobbling Bands

Wobbling bands

Sequences of $\Delta I = 2\hbar$ rotational bands that are built on different *wobbling phonon excitations*.

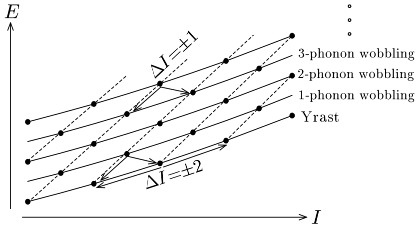


Figure 4: Rotational-band structures of the wobbling motion.

- For ^{163}Lu: $n_w = 0, 1, 2, 3$ wobbling phonon numbers, respectively.
- Nuclei have **large** quadrupole moments
- **Strong E2** character for the electro-magnetic transitions.

Even-Even vs. Even-Odd Nuclei

Theoretical frameworks for **even-mass** nuclei

1. *Harmonic Approximation(s)* (Bohr and Mottelson, 1975)
2. Triaxial-Rotor-Model (Davydov and Filippov, 1958)
3. Boson-approximations (Tanabe, 1971)

Theoretical frameworks for **odd-mass** nuclei

1. Particle Rotor Model (Hamamoto, 2002)
2. Tilted-axis wobbling (Frauendorf and Meng, 1997)
3. RPA, Mean-Field Theories, GCM+AMP...

Recent work on wobbling motion

- RPA for ^{163}Lu, Raduta et al (PRC, 2017)
- Tilted-axis wobbling for ^{135}Pr, R. Budaca (PRC, 2018)
- PRM for ^{163}Lu, R. Poenaru (IJMPE, 2021)

Theoretical formalism - Even-Odd Nuclei

Description of WM for an even-odd nucleus

- one single-particle (nucleon) *coupled* to an *even-even* triaxial core.
- the nucleon is moving in a quadrupole deformed mean-field generated by the core
- particle + rotor coupling drives the entire system to large (and stable) deformations ($\epsilon \sim 0.2 - 0.4$).

Hamiltonian:

$$\hat{H}_{\text{rot}} = \sum_{k=1}^{3} A_k \left(\hat{I}_k - \hat{j}_k\right)^2 \quad . \tag{3}$$

$A_k \rightarrow$ inertia parameters: $A_k = (2\mathcal{I}_k)^{-1}$.

Theoretical Formalism - Rotational Hamiltonian

Expanding $\hat{I}_2$ up to first order (particle is *rigidly coupled* to the core):

$$\hat{I}_2 = I\left(1 - \frac{1}{2}\frac{\hat{I}_1^2 + \hat{I}_3^2}{I^2}\right) , \tag{4}$$

can help re-write the initial Hamiltonian.

$$\hat{H}_{\text{rot}} = AH' + H_{sp} + \text{Spin-term} \tag{5}$$

with:

$$H' = \hat{I}_2^2 + u\hat{I}_3^2 + 2v_0\hat{I}_1 , \tag{6}$$

$$H_{sp} = \sum_{k=1}^{3} A_k \hat{j}_k^2 , \tag{7}$$

$$\text{Spin-term} = A_1 I^2 - A_2 j_2 I . \tag{8}$$

Rotational Hamiltonian

H' looks like the Hamiltonian for a triaxial rigid rotator + constrained (cranked) to move along the 1-axis. The a.m. algebra is defined as:

$$\hat{I}_{\pm} = \hat{I}_2 \pm i\hat{I}_3 \ , \ \hat{I}_0 = \hat{I}_1 \ , \tag{9}$$

$$\left[\hat{I}_{-}, \hat{I}_{+}\right] = 2\hat{I}_0 \ , \ \left[\hat{I}_{\mp}, \hat{I}_0\right] = \mp\hat{I}_{\mp} \ . \tag{10}$$

With this angular momentum algebra, H' becomes:

$$H' = a\left(\hat{I}_{+}^2 + \hat{I}_{-}^2\right) + b\left(\hat{I}_{+}\hat{I}_{-} + \hat{I}_{-}\hat{I}_{+}\right) + c\hat{I}_0. \tag{11}$$