Parity Partner Bands and the Wobbling Motion in 163Lu

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Presentation Overview

Motivation

Motivation

Nuclear Triaxiality has become a *hot topic* within the scientific community.

- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge
- Experimental side: large setups, complex electronics,
- Theoretical side: cumbersome models, approximations, abstractions...

Nuclear facilities

Figure: Gammasphere detector, ANL-ATLAS USA. *Source: aps.org*

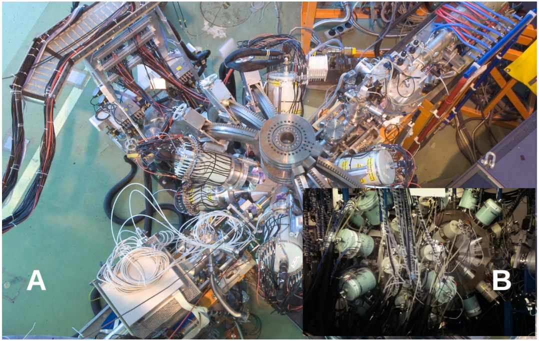


Figure: a) IDS detector, CERN. *Source: isolde.web.cern.ch* b) JUROGAM II, Finland. *Source: twitter.com*

Fingerprints of Triaxiality

Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
  1. Wobbling Motion WM (*Bohr and Mottelson, 1950s*)
  2. Chiral Motion χM (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

Experimental observations

First experimental evidence for **nuclear wobbling motion** in 2001.

Goal

Describe the elusive character of nuclear triaxiality through theoretical models on Wobbling Motion.

Theoretical Models

Theoretical Models for the study of triaxial nuclei

1. *Harmonic Approximation* (Bohr and Mottelson, 1975)
2. Triaxial-Rotor-Model (Davydov and Filippov, 1958)
3. Boson-approximations (Tanabe, 1971)
4. Particle Rotor Model (Hamamoto, 2002)
5. Collective Hamiltonians (Chen, 2014)

Others: RPA, Mean-Field Theories, GCM+AMP...

More recent work on wobbling motion..

- RPA for odd-mass nuclei, Raduta et al (PRC, 2017)
- Tilted-axis wobbling, Budaca (PRC, 2018)
- Tilted-Precession (TiP), Lawrie et. al. (PRC, 2020)
- PRM+HA for ^{163}Lu, R. Poenaru (IJMPE, 2021)

Nuclear Shapes (in the context of WM)

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta,\varphi;t)=R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}(t)Y_\lambda^\mu(\theta,\varphi)\right) \quad (1)$$

Quadrupole radius - pure quadrupole deformations

- Most relevant modes are the **quadrupole vibrations** $\lambda=2 \implies$ *Play a crucial role in the rotational spectra of nuclei*

$$R(\theta,\varphi)=R_0\left(1+\sum_{\mu}\alpha_{2\mu}Y_2^\mu(\theta,\varphi)\right)\ , \quad (2)$$

Nuclear Shapes II

Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state.

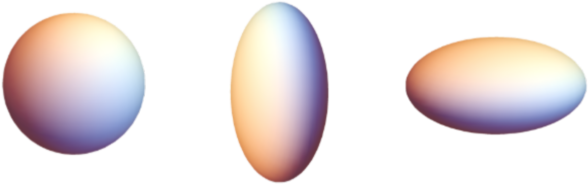
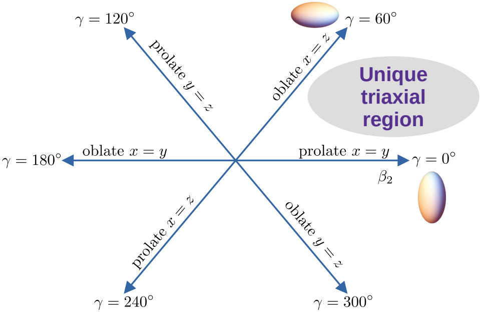


Figure: **spherical:** $\beta_2 = 0$ **prolate:** $\beta_2 > 0$ **oblate:** $\beta_2 < 0$

Nuclear Shapes III

Non-axial shapes

- Coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (*eccentricity*) and γ (**triaxiality**).
- The triaxiality parameter γ (*Bohr, 1969*): departure from axial symmetry



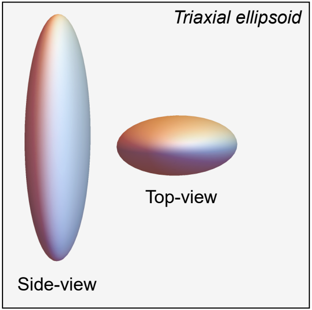


Figure: Source: *R. Poenaru, PhD Thesis (WIP)*

Wobbling Motion

Characteristics

- MOI anisotropy $\to$ the *main rotation* around $\mathcal{J}_{\text{max}}$ is disturbed by the other two axes $\to$ **resulting motion of the rotating nucleus has an oscillating behavior**
- The **total angular momentum** (a.m.) **precesses** and **wobbles** around $\mathcal{J}_{\text{max}}$
- Tilting by an energy quanta $\sim$ *vibrational character* $\to$ **wobbling phonon** $n_w = 0, 1, 2 \ldots$
- Currently confirmed wobblers $A \approx [100, 130, 160, 180]$.

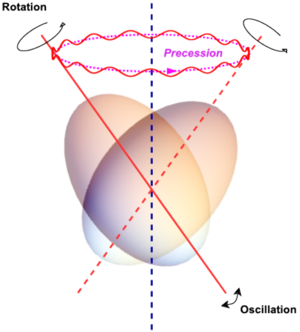


Figure: Geometrical interpretation of a wobbling triaxial nucleus: *rotation* + *precession* + *oscillation*. *R. Poenaru, PhD Thesis (WIP)*

Simple wobbler - Spectra

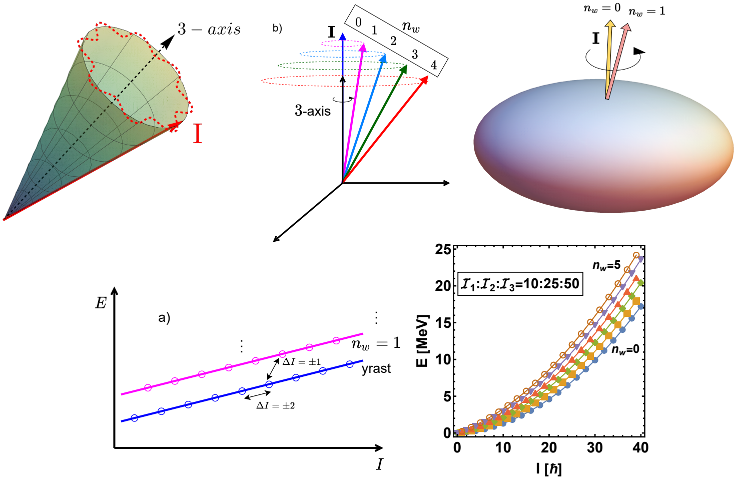


Figure: precession of **I** → wobbling excitations → tilting of **I** → tilting of **I**. *R. Poenaru, PhD Thesis (WIP)*

WM in odd-mass nuclei

Triaxial Rotor + Particle Model (PRM)

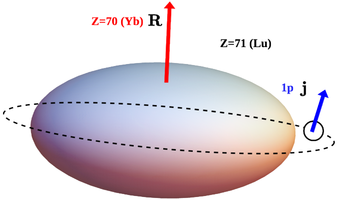
- The WM in odd-mass nuclei can be described through the **Particle - Rotor Model** (Hamamoto, 2002).
- System is described by the interaction between an even-even triaxial core + single nucleon
- Nucleon is moving in a *quadrupole deformed mean field* generated by the core

$$
\begin{aligned}
H &= H_{\text{rot}} + H_s p = \\
&= \sum_{k=1,2,3} A_k \left(\hat{I}_k - \hat{j}_k\right)^2 + \\
&+ \frac{V}{j(j+1)} \left[\left(3\hat{j}_3^2 - \mathbf{j}^2\right) \cos\gamma - \sqrt{3}\left(\hat{j}_1^2 - \hat{j}_2^2\right) \sin\gamma \right] + \epsilon_j
\end{aligned} \tag{3}
$$

WM in 163-Lu - old vs. new pictures

^{163}Lu - old picture

- Accepted in literature (Hamamoto et. al. 2003, Raduta et. al. 2017).
- Nucleus has **four** known wobbling bands: $n_w = 0, 1, 2, 3$.
- TSD1-3: $\mathbf{R}^+ + j^+ = \pi(i_{13/2})$. TSD4: $\mathbf{R}^+ + j^- = \pi(h_{9/2})$.



^{163}Lu - new picture

- All bands described by a *unique single particle*: $j'^+ = \pi(i_{13/2})$.
- TSD1: $(\mathbf{R}^+ = 0, 2, 4) + j'^+$. TSD1 and 2: *signature partner bands* (Raduta et. al. 2020).
- TSD2: $(\mathbf{R}^+ = 1, 3, 5) + j'^+$.
- TSD3: one-phonon wobbling band ($n_w = 1$), built on TSD2.
- TSD4: $(\mathbf{R}^- = 1, 3, 5) + j'^+$. TSD2 and 4 are **parity partner bands** (Poenaru et. al. 2021).

Theoretical model - semiclassical approach

Algorithm

- Apply the *variational principle* (VP) for the initial Hamiltonian using the trial function:

$$|\Psi_{Ij;M}\rangle = \mathcal{N} e^{z\hat{I}_-} e^{s\hat{j}_-} |IMI\rangle \otimes |jj\rangle \tag{4}$$

- $(z, \hat{I}, |IMI\rangle)$ - core states (for the even-even core)
- $(s, \hat{j}, |jj\rangle)$ - single-particle states (for the proton)

Equations of motion

The VP principle will lead to a system of *equations of motion*:

$$\frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi}\ , \ \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r}\ , \ \frac{\partial \mathcal{H}}{\partial t} = \dot{\psi}\ , \ \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{t}\ . \tag{5}$$

Solve: $\Omega^4 + B\Omega^2 + C = 0 \longrightarrow$ obtain two $\Omega \equiv$ wobbling frequencies Ω_1 (core) and Ω_2 (odd-particle).

Excitation energies

Energy scheme

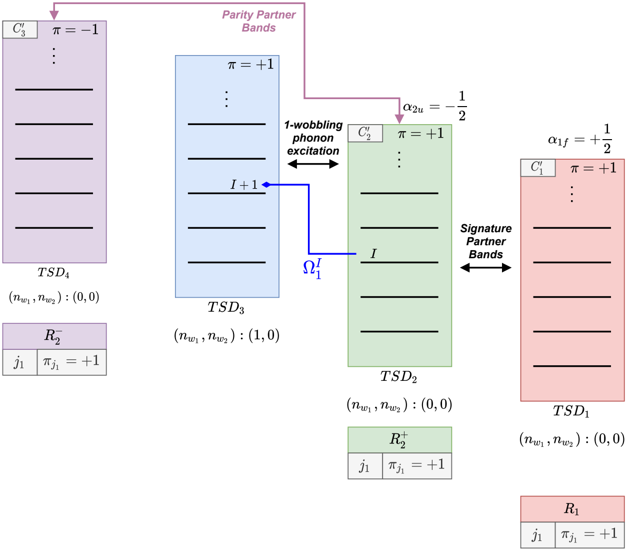
- From the equations of motion and the two solutions for Ω, the spectrum becomes:

$$\begin{aligned} E_I^{\mathrm{TSD1}} =& \epsilon_j + \mathcal{H}_{\mathrm{min}}^{(I,j)} + \mathcal{F}_{00}^I,\ I = R + j,\ R = 0, 2, 4, \ldots \\ E_I^{\mathrm{TSD2}} =& \epsilon_{j_1} + \mathcal{H}_{\mathrm{min}}^{(I,j)} + \mathcal{F}_{00}^I,\ I = R + j,\ R = 0, 2, 4, \ldots \\ E_I^{\mathrm{TSD3}} =& \epsilon_j + \mathcal{H}_{\mathrm{min}}^{(I-1,j)} + \mathcal{F}_{10}^{I-1},\ \text{one-phonon wobbling band} \\ E_I^{\mathrm{TSD4}} =& \epsilon_{j_2} + \mathcal{H}_{\mathrm{min}}^{(I,j)} + \mathcal{F}_{00}^I,\ I = R + j,\ R = 0, 2, 4, \ldots \end{aligned} \tag{6}$$

- Phonon term (contains the wobbling frequencies Ω, R. Poenaru, IJMPE 2021):

$$\mathcal{F}_{n_{w_1} n_{w_2}}^I = \Omega_1^I \left(n_{w_1} + \frac{1}{2}\right) + \Omega_2^I \left(n_{w_2} + \frac{1}{2}\right) \tag{7}$$

Schematic representation of the new picture



Results

Fitting method

Apply fitting on a model with four free parameters: 3 moments of inertia $\mathcal{I}_{1,2,3}$, and the single-particle potential strength V.

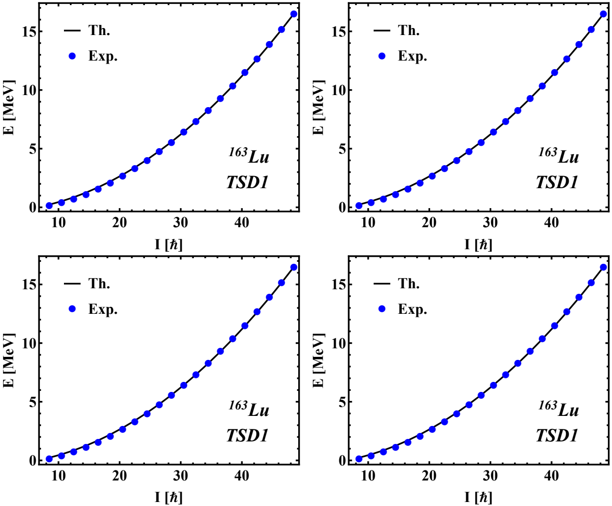
$$\chi^2 = \frac{1}{N_T}\sum_i \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)}{E_{\text{exp}}^{(i)}} \tag{8}$$

Table: The obtained parameter set:

$\mathcal{I}_1$ $[\hbar^2/\text{MeV}]$	$\mathcal{I}_2$ $[\hbar^2/\text{MeV}]$	$\mathcal{I}_3$ $[\hbar^2/\text{MeV}]$	γ [deg.]	V [MeV]
72	15	7	22	2.1

Full work published in three papers: one in IJMPE 2021, two in RJP 2021.

Excitation energies



Moments of Inertia

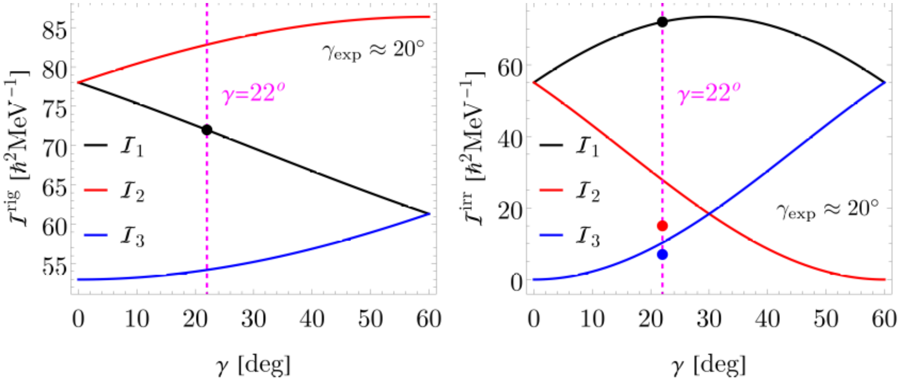


Figure: The rigid (**left**) and hydrodynamical (**right**) MOI as function of γ.

Conclusions

- Wobbling motion has been described in the context of triaxial nuclei.
- The particle + rotor model was implemented in a *new picture*, to describe the band structure of ^{163}Lu.
  - Variational principle applied to H
  - semi-classical procedure to obtain equations of motion
  - two wobbling frequencies emerge in the model
- The four triaxial bands were numerically evaluated in terms of their excitation energies trough a fitting procedure.
- Moments of inertia show *irrotational character* of the triaxial nucleus.
- Agreement with experimental data was obtained

Thank you for your attention !