

Evaluation of the Wobbling Motion in Even-Even Nuclei Within a Simple Rotor Model

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Nuclear Deformation

- Most of the nuclei are either *spherical* or *axially symmetric* in their ground-state.
- Deformation parameter β (Bohr, 1969): preserves axial symmetry

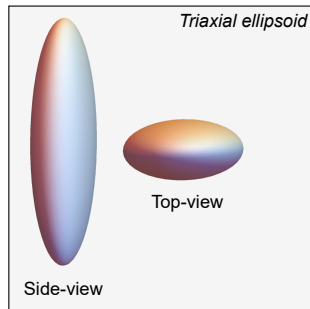
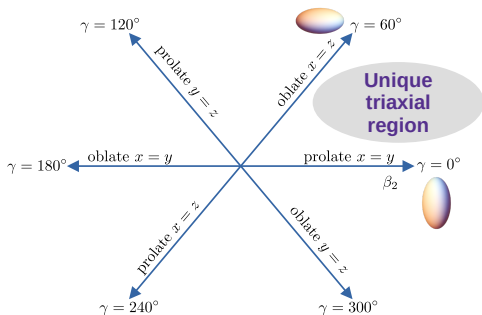


Figure 1: **spherical:** $\beta = 0$ **prolate:** $\beta > 0$ **oblate:** $\beta < 0$

Nuclear Triaxiality

Non-axial shapes

- Deviations from symmetric shapes can occur across the chart of nuclides → **triaxial nuclei**.
- The triaxiality parameter γ (Bohr, 1969): departure from axial symmetry



Fingerprints for Triaxiality

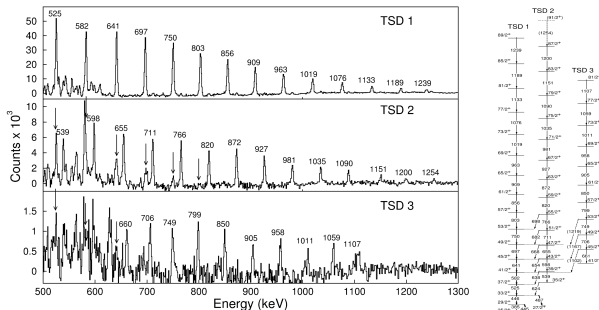
- Stable triaxial nuclei represent a real challenge for experimentalists and theoreticians
- Clear signatures for confirming stable triaxiality in nuclei
 - ① Chiral symmetry breaking (*Frauendorf, 1997*)
 - ② **Wobbling motion** (*Bohr & Mottelson, 1975*)

Wobbling Motion (WM)

- Unique to non-axial nuclei
- Predicted 50 years ago for even- A nuclei (i.e., the simple wobblers)
- First experimental evidence for ^{163}Lu (*Ødegård, 2001*)
- Currently confirmed wobblers $A \approx [100, 130, 160, 180]$.

Triaxial Rotor Energy

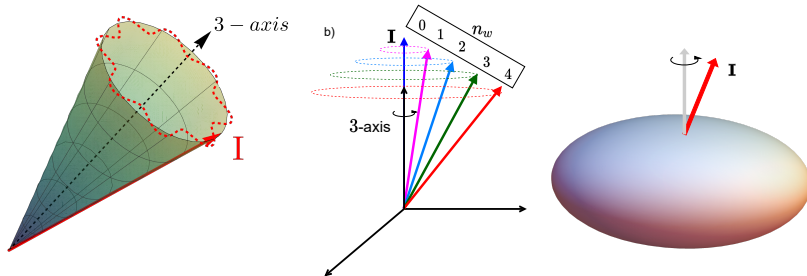
- Rigid body rotational energy: $E_{\text{rot}} \propto \frac{\hbar^2}{2\mathcal{J}_{\text{max}}} I(I+1)$
- A triaxial nucleus can rotate about any of the three axes \rightarrow *rich energy spectra*
- MOI anisotropy \rightarrow the *main rotation* around \mathcal{J}_{max} is disturbed by the other two axes \rightarrow **resulting motion of the rotating nucleus has an oscillating behavior**



Figures from Schönwaßer et al., 2001

Wobbling Motion

- Oscillatory character of $\mathbf{I} \rightarrow \mathbf{I}$ *disaligned* w.r.t. body-fixed axes
- The a.m. **precesses** and **wobbles** around the axis with \mathcal{I}_{\max}
- The precession of \mathbf{I} can increase by **tilting**
- Tilting by an energy quanta \sim *vibrational character* \rightarrow **wobbling phonon** $n_w = 0, 1, 2, \dots$

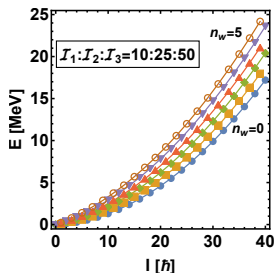
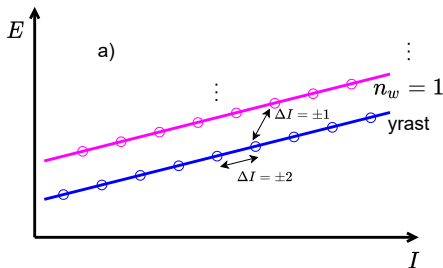


Wobbling Spectrum

Even-A Nuclei

- Employing the Harmonic Approximation (*Bohr, 1969*)
- \hat{H} composed of a *rotational* part and *harmonic oscillation* (i.e., wobbling) part:

$$\hat{H} = \frac{\hbar^2}{2\mathcal{J}_{\max}} I(I+1) + \hbar\omega_{\text{wob}} \left(n_w + \frac{1}{2} \right), \quad n_w = 0, 1, 2, \dots \quad (1)$$



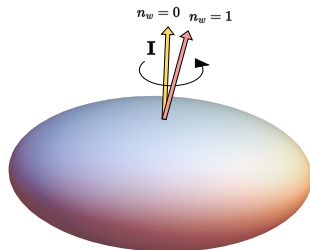
Energy spectrum - simple wobbling

- Employed an energy spectrum of harmonic type according to Eq. 1:

$$E_I = \frac{\hbar^2}{2\mathcal{J}_3} I(I+1) + \hbar\omega_{\text{wob}} \left(n_w + \frac{1}{2} \right)$$

- $\hbar\omega_{\text{wob}}$ - **wobbling frequency** - linear dependence on I (fixed MOI ordering $\mathcal{J}_3 > \mathcal{J}_{1,2}$)

$$\hbar\omega_{\text{wob}}(I) = 2f(\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3) \cdot I$$



- New experimental measurements show *potential* wobbling candidates in the $A \approx 130$ region
- Three even- A are studied with the *simple wobblers* formalism
 - ① ^{130}Ba (*Petrache et al. 2019*)
 - ② ^{134}Ce (*Petrache and Guo, 2016*)
 - ③ ^{136}Nd (*Lv et al., 2018*)
- Study the excited spectra: *theoretical model checks the data?*

Harmonic Approximation

- Reproduced the excited spectra for the wobbling bands
- Employ a *free parameter set*: $\mathcal{P} = [\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3]$
- Adopt a fitting procedure:

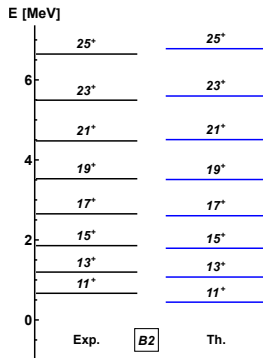
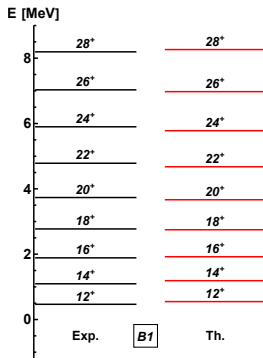
$$\chi^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)^2}{E_{\text{exp}}^{(i)}} \quad (2)$$

- $N_T \rightarrow$ total number of wobbling states within the nucleus

New Results for ^{130}Ba II

Results for ^{130}Ba PRELIMINARY!

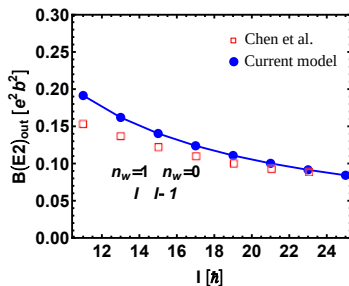
- $\mathcal{I}_1 : \mathcal{I}_2 : \mathcal{I}_3 \rightarrow 27 : 22 : \mathbf{43}$
- Maximal MOI is $\mathcal{I}_3 > \mathcal{I}_{1,2}$



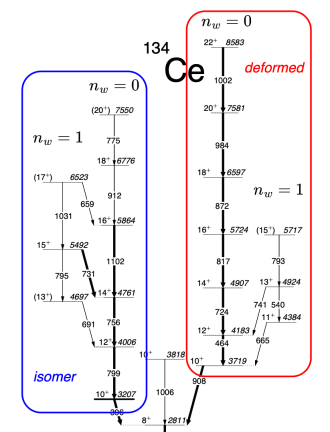
Electromagnetic Transitions ^{130}Ba

- In the harmonic approximation, the three MOI are used to determine $B(E2)_{\text{out}}$
- β_2 and γ are taken from Chen et al. $\rightarrow (\beta, \gamma) = 0.24, 21.5^\circ$
 \rightarrow used to calculate the quadrupole components Q_{20} and Q_{22}
- $B(E2)_{\text{out}}(I) = \frac{5}{16\pi} I^{-1} (\sqrt{3} Q_{20} \cdot f(\mathcal{J}) + \sqrt{2} Q_{22} \cdot g(\mathcal{J}))$

I	$B(E2)_{\text{out}}/B(E2)_{\text{in}}$		
	Th.	PRM*	Exp.
11	0.37	-	-
13	0.32	0.51	0.32
15	0.27	0.42	0.36
17	0.24	0.35	0.22
19	0.21	0.29	0.22
21	0.19	0.25	0.41
23	0.18	-	-
25	0.16	-	-



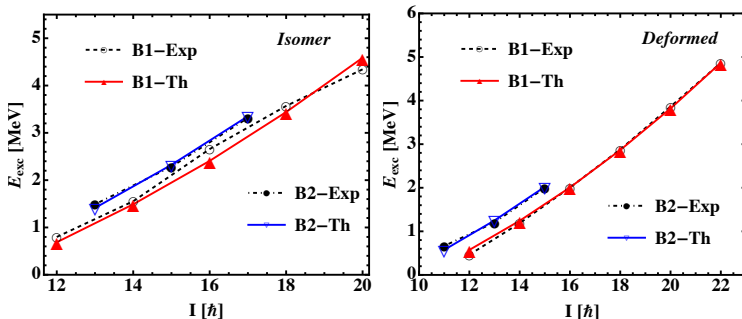
New Results for ^{134}Ce



- Petrache et al. found two sets of wobbling bands in ^{134}Ce
- Wobbling confirmed in odd-A ^{135}Pr by Matta et al. 2015 → *even-A neighbor also with wobbling character?*
- The *isomer structure* is based on a 10^+ level with lower quadrupole deformation but higher life-time ($t \approx 300\text{ns}$)

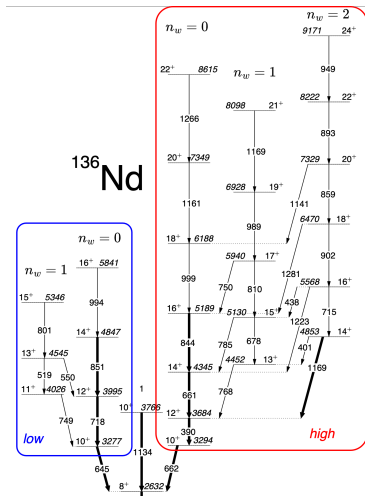
New Results for ^{134}Ce II

- Separate fitting procedures for the *isomer* and *deformed*
- Isomer: $(\beta, \gamma) = (0.14, -35^\circ)$, $E_{\text{RMS}} \approx 90$ keV
- Deformed: $(\beta, \gamma) = (0.22, 25^\circ)$, $E_{\text{RMS}} \approx 60$ keV



- isomer: $\mathcal{J}_1 : \mathcal{J}_2 : \mathcal{J}_3 \rightarrow 14 : 21 : \mathbf{34} \hbar^2 \text{MeV}^{-1}$
- deformed: $\mathcal{J}_1 : \mathcal{J}_2 : \mathcal{J}_3 \rightarrow 15 : 23 : \mathbf{42} \hbar^2 \text{MeV}^{-1}$

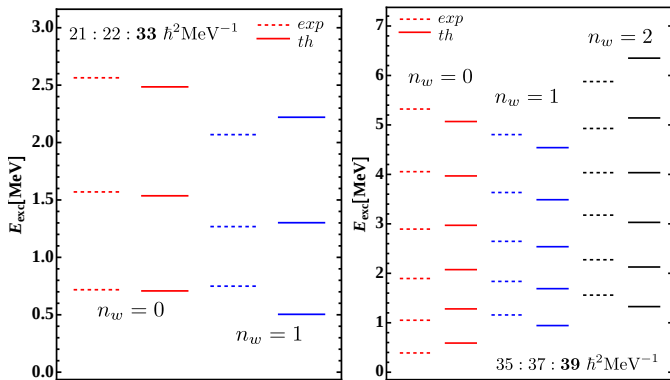
New Results for ^{136}Nd



- Lv et al. found two sets of wobbling bands in $^{136}\text{Nd} \rightarrow$ *worth investigating $A = 137$ neighbor nuclei?*
- low/high label is used to differentiate the energy of 10^+ state of each structure \rightarrow *similar 10^+ structures as for ^{134}Ce*
- The higher structure has two phonon excitations

New Results for ^{136}Nd II

- Separate fitting for each structure (low/high)
- Low: $(\beta, \gamma) = (0.15, -35^\circ)$, $E_{\text{RMS}} \approx 120$ keV
- High: $(\beta, \gamma) = (0.21, 25^\circ)$, $E_{\text{RMS}} \approx 145$ keV



Conclusions & Future Outlook

- New wobbling nuclei were investigated through a semi-classical formalism
- The harmonic approximation reproduces the experimental data of even- A wobbling nuclei
 - One wobbling structure for ^{130}Ba
 - Two wobbling structures for ^{134}Ce and ^{136}Nd
- Quality of the fit was reflected in the transition probabilities for ^{130}Ba
- Calculations were done for fixed deformation parameters
- + Employ spin-dependence for the moments of inertia
- + Find classical trajectories Poenaru R, Raduta A A, IJMPE 2021
- *Potential progress in triaxial deformation within a semi-classical picture*

Thank you for your attention!