

# Evaluation of the Wobbling Motion in Even-Even Nuclei Within a Simple Rotor Model

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# Nuclear Deformation

- Most of the nuclei are either *spherical* or *axially symmetric* in their ground-state.
- Deformation parameter  $\beta$  (Bohr, 1969): preserves axial symmetry

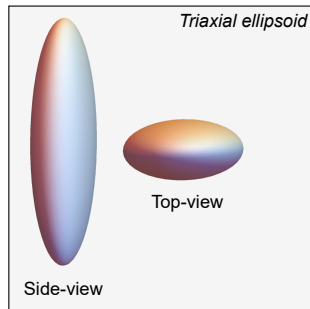
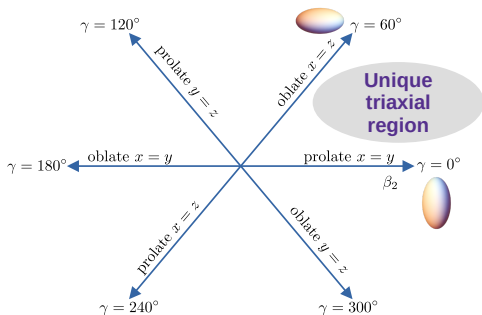


Figure 1: **spherical:**  $\beta = 0$  **prolate:**  $\beta > 0$  **oblate:**  $\beta < 0$

# Nuclear Triaxiality

## Non-axial shapes

- Deviations from symmetric shapes can occur across the chart of nuclides → **triaxial nuclei**.
- The triaxiality parameter  $\gamma$  (*Bohr, 1969*): departure from axial symmetry



# Fingerprints for Triaxiality

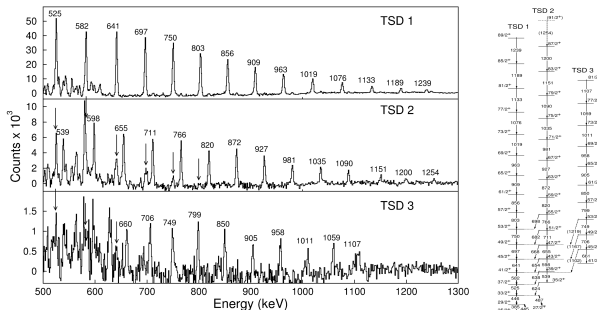
- Stable triaxial nuclei represent a real challenge for experimentalists and theoreticians
- Clear signatures for confirming stable triaxiality in nuclei
  - ① Chiral symmetry breaking (*Frauendorf, 1997*)
  - ② **Wobbling motion** (*Bohr & Mottelson, 1975*)

## Wobbling Motion (WM)

- Unique to non-axial nuclei
- Predicted 50 years ago for even- $A$  nuclei (i.e., the simple wobblers)
- First experimental evidence for  $^{163}\text{Lu}$  (*Ødegård, 2001*)
- Currently confirmed wobblers  $A \approx [100, 130, 160, 180]$ .

# Triaxial Rotor Energy

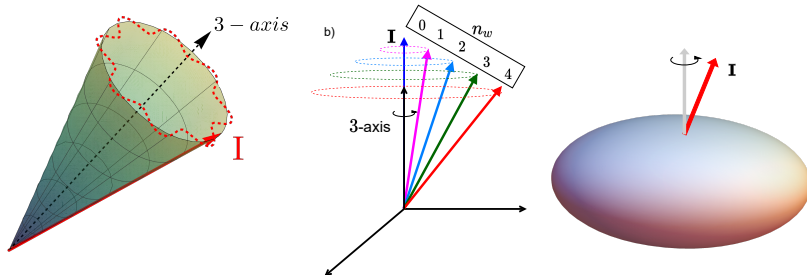
- Rigid body rotational energy:  $E_{\text{rot}} \propto \frac{\hbar^2}{2\mathcal{J}_{\text{max}}} I(I+1)$
- A triaxial nucleus can rotate about any of the three axes  $\rightarrow$  *rich energy spectra spectra*
- MOI anisotropy  $\rightarrow$  the *main rotation* around  $\mathcal{J}_{\text{max}}$  is disturbed by the other two axes  $\rightarrow$  **resulting motion of the rotating nucleus has an oscillating behavior**



Figures from Schönwaßer et al., 2001

# Wobbling Motion

- Oscillatory character of  $\mathbf{I} \rightarrow \mathbf{I}$  *disaligned* w.r.t. body-fixed axes
- The a.m. **precesses** and **wobbles** around the axis with  $\mathcal{I}_{\max}$
- The precession of  $\mathbf{I}$  can increase by **tilting**
- Tilting by an energy quanta  $\sim$  *vibrational character*  $\rightarrow$  **wobbling phonon**  $n_w = 0, 1, 2, \dots$

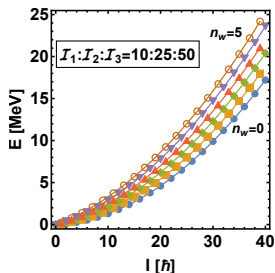
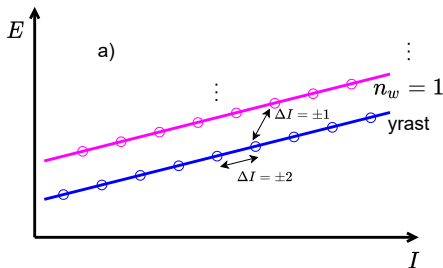


# Wobbling Spectrum

## Even-A Nuclei

- Employing the Harmonic Approximation (*Bohr, 1969*)
- $\hat{H}$  composed of a *rotational* part and *harmonic oscillation* (i.e., wobbling) part:

$$\hat{H} = \frac{\hbar^2}{2\mathcal{J}_{\max}} I(I+1) + \hbar\omega_{\text{wob}} \left( n_w + \frac{1}{2} \right), \quad n_w = 0, 1, 2, \dots \quad (1)$$





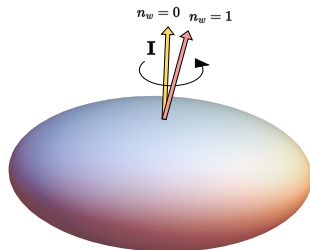
# Energy spectrum - simple wobbling

- Employed an energy spectrum of harmonic type according to Eq. 1:

$$E_I = \frac{\hbar^2}{2\mathcal{J}_3} I(I+1) + \hbar\omega_{\text{wob}} \left( n_w + \frac{1}{2} \right)$$

- $\hbar\omega_{\text{wob}}$  - **wobbling frequency** - linear dependence on  $I$  (fixed MOI ordering  $\mathcal{J}_3 > \mathcal{J}_{1,2}$ )

$$\hbar\omega_{\text{wob}}(I) = 2f(\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3) \cdot I$$



- New experimental measurements show *potential* wobbling candidates in the  $A \approx 130$  region
- Three even- $A$  are studied with the *simple wobblers* formalism
  - ①  $^{130}\text{Ba}$  (*Petrache et al. 2019*)
  - ②  $^{134}\text{Ce}$  (*Petrache and Guo, 2016*)
  - ③  $^{136}\text{Nd}$  (*Lv et al., 2018*)
- Study the excited spectra: *theoretical model checks the data?*

## Harmonic Approximation

- Reproduced the excited spectra for the wobbling bands
- Employ a *free parameter set*:  $\mathcal{P} = [\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3]$
- Adopt a fitting procedure:

$$\chi^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)^2}{E_{\text{exp}}^{(i)}} \quad (2)$$

- $N_T \rightarrow$  total number of wobbling states within the nucleus

# New Results for $^{130}\text{Ba}$

## Recent findings for even-even nuclei

- Two wobbling bands have been identified experimentally in  $^{130}\text{Ba}$  (*Petrache et al., 2019*)
- DFT+PRM description of the wobbling motion described the excited spectra (*Chen et al., 2019*)
- Stable triaxiality for  $\beta = 0.24$  and  $\gamma = 21.5^\circ$

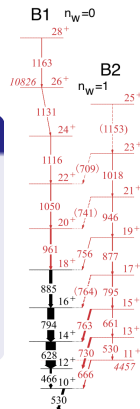


Figure from Petrache et al., 2019