Evaluation of the Wobbling Motion in Even-Even Nuclei Within a Simple Rotor Model

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Nuclear Deformation

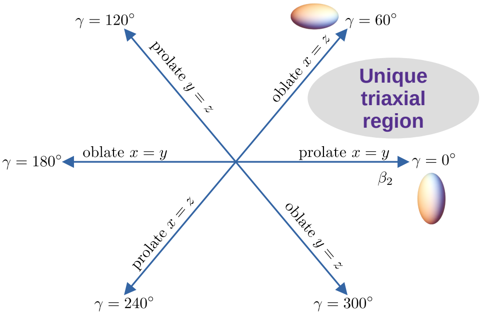
- Most of the nuclei are either *spherical* or *axially symmetric* in their ground-state.
- Deformation parameter β (*Bohr, 1969*): preserves axial symmetry

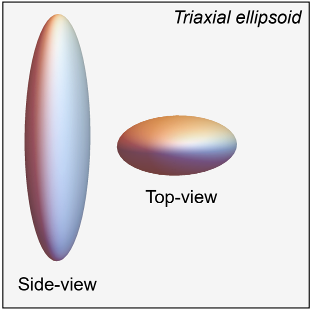
Figure 1: **spherical:** $\beta = 0$ **prolate:** $\beta > 0$ **oblate:** $\beta < 0$

Nuclear Triaxiality

Non-axial shapes

- Deviations from symmetric shapes can occur across the chart of nuclides → **triaxial nuclei**.
- The triaxiality parameter γ (*Bohr, 1969*): departure from axial symmetry





Fingerprints for Triaxiality

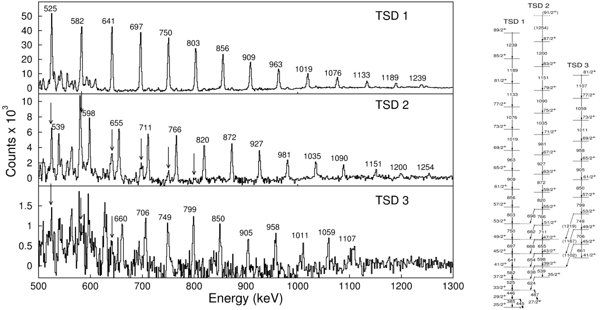
- Stable triaxial nuclei represent a real challenge for experimentalists and theoreticians
- Clear signatures for confirming stable triaxiality in nuclei
    1. Chiral symmetry breaking (*Frauendorf, 1997*)
    2. **Wobbling motion** (*Bohr & Mottelson, 1975*)

Wobbling Motion (WM)

- Unique to non-axial nuclei
- Predicted 50 years ago for even-A nuclei (i.e., the simple wobbler)
- First experimental evidence for ^{163}Lu (*Ødegård*, 2001)
- Currently confirmed wobblers $A \approx [100, 130, 160, 180]$.

Triaxial Rotor Energy

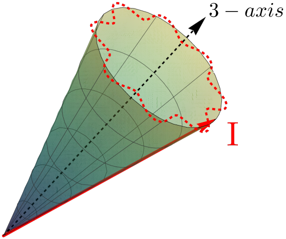
- Rigid body rotational energy: $E_{\mathrm{rot}} \propto \frac{\hbar^2}{2\mathcal{J}_{\mathrm{max}}} I(I+1)$
- A triaxial nucleus can rotate about any of the three axes $\rightarrow$ *rich energy spectra*
- MOI anisotropy $\rightarrow$ the *main rotation* around $\mathcal{J}_{\mathrm{max}}$ is disturbed by the other two axes $\rightarrow$ **resulting motion of the rotating nucleus has an oscillating behavior**

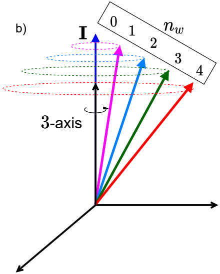


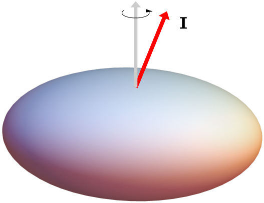
Figures from Schönwaßer et al., 2001

Wobbling Motion

- Oscillatory character of **I** $\rightarrow$ **I** *disaligned* w.r.t. body-fixed axes
- The a.m. **precesses** and **wobbles** around the axis with $\mathcal{J}_{\text{max}}$
- The precession of **I** can increase by **tilting**
- Tilting by an energy quanta $\sim$ *vibrational character* $\rightarrow$ **wobbling phonon** $n_w = 0, 1, 2...$





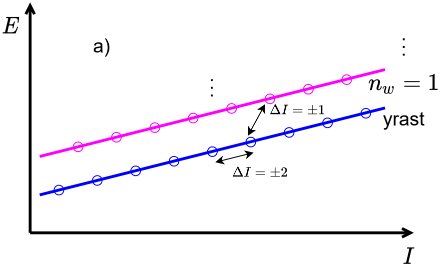


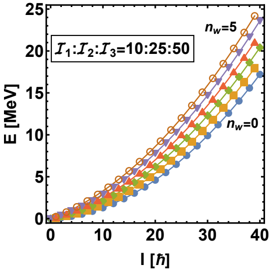
Wobbling Spectrum

Even-A Nuclei

- Employing the Harmonic Approximation *(Bohr, 1969)*
- $\hat{H}$ composed of a *rotational* part and *harmonic oscillation* (i.e., wobbling) part:

$$\hat{H} = \frac{\hbar^2}{2\mathcal{J}_{\text{max}}} I(I+1) + \hbar\omega_{\text{wob}}\left(n_w + \frac{1}{2}\right), n_w = 0, 1, 2, \ldots \quad (1)$$





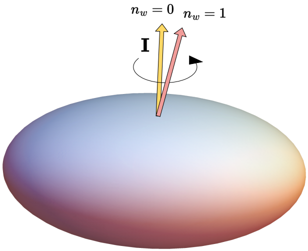
Energy spectrum - simple wobbling

- Employed an energy spectrum of harmonic type according to Eq. 1:

$$E_I = \frac{\hbar^2}{2\mathcal{J}_3} I(I+1) + \hbar\omega_{\text{wob}} \left(n_w + \frac{1}{2}\right)$$

- $\hbar\omega_{\text{wob}}$ - **wobbling frequency** - linear dependence on I (fixed MOI ordering $\mathcal{J}_3 > \mathcal{J}_{1,2}$)

$$\hbar\omega_{\text{wob}}(I) = 2f(\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3) \cdot I$$



New Experimental Findings

- New experimental measurements show *potential* wobbling candidates in the $A \approx 130$ region
- Three even-A are studied with the *simple wobbler* formalism
    1. ^{130}Ba (*Petrache et al. 2019*)
    2. ^{134}Ce (*Petrache and Guo, 2016*)
    3. ^{136}Nd (*Lv et al., 2018*)
- Study the excited spectra: *theoretical model checks the data?*

Model

Harmonic Approximation

- Reproduced the excited spectra for the wobbling bands
- Employ a *free parameter set*: $\mathcal{P} = [\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3]$
- Adopt a fitting procedure:

$$\chi^2 = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)^2}{E_{\text{exp}}^{(i)}} \tag{2}$$

- N_T $\rightarrow$ total number of wobbling states within the nucleus

$Z = 56$

Recent findings for even-even nuclei

- Two wobbling bands have been identified experimentally in ^{130}**Ba** (*Petrache et al., 2019*)
- DFT+PRM description of the wobbling motion described the excited spectra (*Chen et al., 2019*)

    - Reproduced experimental energies
    - Obtained deformation parameters self-consistently
    - Stable triaxiality for $\beta = 0.24$ and $\gamma = 21.5°$

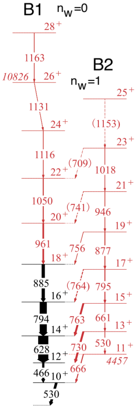
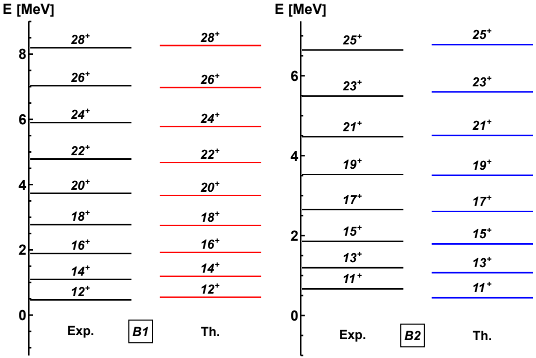


Figure from Petrache et al., 2019

New Results for 130Ba II

Results for ^{130}Ba **PRELIMINARY!**

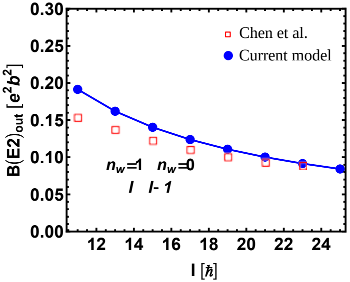
- $\mathcal{J}_1 : \mathcal{J}_2 : \mathcal{J}_3 \rightarrow 27 : 22 : \mathbf{43}$
- Maximal MOI is $\mathcal{J}_3 > \mathcal{J}_{1,2}$



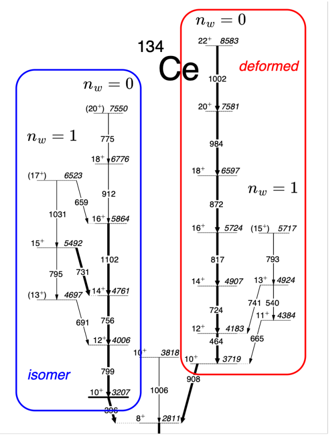
Electromagnetic Transitions 130Ba

- In the harmonic approximation, the three MOI are used to determine $B(E2)_{\text{out}}$
- β_2 and γ are taken from Chen et al. $\rightarrow (\beta,\gamma) = 0.24, 21.5^\circ$ $\rightarrow$ used to calculate the quadrupole components Q_20 and Q_22
- $B(E2)_{\text{out}}(I) = \frac{5}{16\pi} I^{-1}\left(\sqrt{3}Q_{20}\cdot f(\mathcal{J}) + \sqrt{2}Q_{22}\cdot g(\mathcal{J})\right)$

I	$B(E2)_{\text{out}}/B(E2)_{\text{in}}$		
	Th.	PRM*	Exp.
11	0.37	-	-
13	0.32	0.51	0.32
15	0.27	0.42	0.36
17	0.24	0.35	0.22
19	0.21	0.29	0.22
21	0.19	0.25	0.41
23	0.18	-	-
25	0.16	-	-



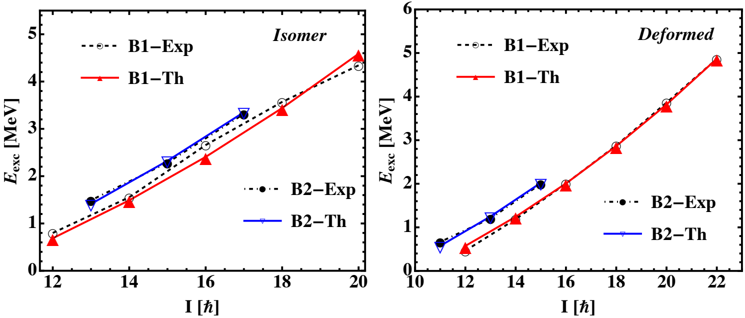
New Results for 134Ce



- Petrache et al. found two sets of wobbling bands in ^{134}Ce, $Z = 55$
- Wobbling confirmed in odd-A ^{135}Pr by Matta et al. 2015 $\rightarrow$ *even-A neighbor also with wobbling character?*
- The *isomer structure* is based on a 10^+ level with lower quadrupole deformation but higher life-time ($t \approx 300$ns)

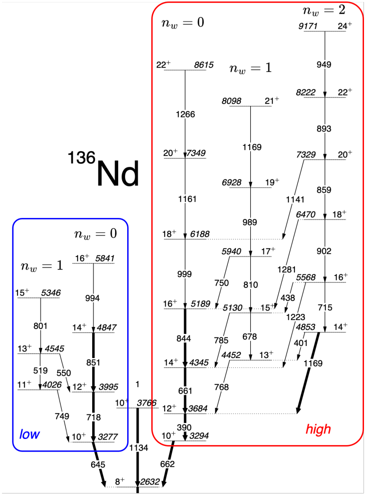
New Results for 134Ce II

- Separate fitting procedures for the *isomer* and *deformed*
- Isomer: $(\beta, \gamma) = (0.14, -35^\circ)$, $E_{\text{RMS}} \approx 90$ keV
- Deformed: $(\beta, \gamma) = (0.22, 25^\circ)$, $E_{\text{RMS}} \approx 60$ keV



- isomer: $\mathcal{J}_1 : \mathcal{J}_2 : \mathcal{J}_3 \to 14 : 21 : \mathbf{34}\ \hbar^2\text{MeV}^{-1}$
- deformed: $\mathcal{J}_1 : \mathcal{J}_2 : \mathcal{J}_3 \to 15 : 23 : \mathbf{42}\ \hbar^2\text{MeV}^{-1}$

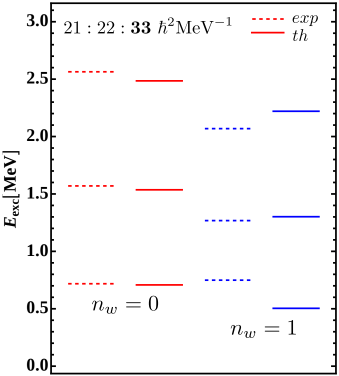
New Results for 136Nd

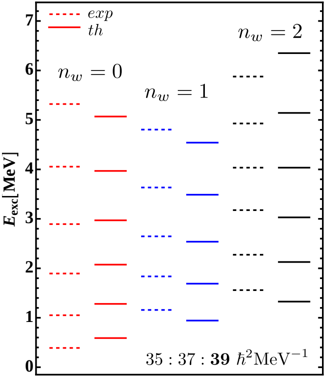


- Lv et al. found two sets of wobbling bands in ^{136}Nd, $Z = 60$, $\rightarrow$ *worth investigating* $A = 137$ *neighbor nuclei ?*
- low/high label is used to differentiate the energy of 10^+ state of each structure $\rightarrow$ *similar* 10^+ *structures as for* ^{134}*Ce*
- The higher structure has two phonon excitations

New Results for 136Nd II

- Separate fitting for each structure (low/high)
- Low: $(\beta, \gamma) = (0.15, -35^\circ)$, $E_{\text{RMS}} \approx 120$ keV
- High: $(\beta, \gamma) = (0.21, 25^\circ)$, $E_{\text{RMS}} \approx 145$ keV





Moments of inertia

- ^{130}Ba

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$
27	22	**43**

- ^{134}Ce - isomeric structure

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$
14	21	**34**

- ^{134}Ce - high deformed structure

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$
15	23	**42**

- ^{136}Nd - lower 10^+

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$
21	22	**33**

- ^{136}Nd - higher 10^+

$\mathcal{J}_1^{\text{fit}}$	$\mathcal{J}_2^{\text{fit}}$	$\mathcal{J}_3^{\text{fit}}$
35	37	**39**

Worth investigating the reversing of $\mathcal{J}_1$ and $\mathcal{J}_2$ identified at A=130 Raduta A A, Poenaru R, Phys Rev C, 2020

Conclusions & Future Outlook

- New wobbling nuclei were investigated through a semi-classical formalism
- The harmonic approximation reproduces the experimental data of even-A wobbling nuclei
  - One wobbling structure for ^{130}Ba
  - Two wobbling structures for ^{134}Ce and ^{136}Nd
- Quality of the fit was reflected in the transition probabilities for ^{130}Ba
- Calculations were done for fixed deformation parameters
- $+$ Employ spin-dependence for the moments of inertia
- $+$ Find classical trajectories *Poenaru R, Raduta A A, IJMPE 2021*
- *Potential progress in triaxial deformation within a semi-classical picture*

Thank you for your attention!