

Tilted Rotation and Wobbling Motion in Nuclei

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Abstract

The self-consistent harmonic oscillator model including the three-dimensional cranking term is extended to describe collective excitations in the random phase approximation. It is found that quadrupole collective excitations associated with wobbling motion in rotating nuclei lead to the appearance of two- or three-dimensional rotation.

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Nuclear states grouped into $\Delta I = 2$ sequences are remarkably well described in terms of the principal axis rotation which is the basis of the cranking model [1]. The principal axis cranking rotation was intuitively justified by a classical rigid body rotation which is favorable for a uniform rotation around the long or the short principal axis. However, if the classical body is not rigid, it may also uniformly rotate about an axis which does not coincide with the principal axes of the density distribution. In fact, more than a century ago Riemann [2] pointed out that such a situation could occur in the ellipsoidal self-gravitating fluid. The physical mechanism behind it is the vortex motion in a macroscopic system.

Numerous experimental observations implying $\Delta I = 1$ sequences in near spherical nuclei [3] raised the question about a new type of rotation and its physical nature in a finite quantum many body system. For particular nuclei the proton and neutron spin vectors may be oriented along different axes. Consequently, the total angular momentum could lie at an angle different from one of the principal axes, i.e. at a tilted angle. It seems shell effects are the driving forces in this case which can be described within the Tilted Axis Cranking Model [4] which assumes a two-dimensional rotation. Other intriguing examples are the rotational bands observed in Hf , W and Os ($Z = 72 - 76$) nuclei with $A \sim 180$ [5, 6, 7]. These bands are characterized by high K-values, where K is the angular momentum projection on

the body-fixed symmetry axis [8]. At finite rotational frequency a competition could occur between states with large collective angular momentum oriented perpendicular to the symmetry axis and high-K states. Such competition gives rise to the new backbending phenomenon which is expected to be caused by the crossing of the ground and tilted rotational bands [4]. The one-dimensional cranking model based on the signature concept (the invariance of the wave function under rotation by π around the rotational axis) fails in this situation, since signature is not conserved for tilted rotations. Furthermore, according to the standard Alaga rules [1, 8], the quadrupole B(E2) transitions to the ground band are forbidden due to the K selection rules ($\Delta K > 2$). However, it is observed that some K-isomer states do decay via quadrupole transitions to the ground state. It was suggested that either fluctuations in the orientation of the angular momentum or shape fluctuations [6] or tilting degrees of freedom in the rotation axis [5] are responsible for this phenomenon. It was also observed that the $\Delta I = 2$ sequence of the rotational states has been superseded by $\Delta I = 1$ transitions in the yrast rotational band at high rotational frequency. In the present paper we considerably extend a previous study [9] and demonstrate that tilted rotations naturally occur beyond a critical rotational frequency as an instability of collective vibrations caused by the wobbling motion.

At high spin, the dynamical fluctuations of the shape and angular momentum vector can be described by the cranking random phase approximation (CRPA) as formulated in [10, 11] for a one-dimensional rotation. This approach is the main theoretical tool used for the analysis of collective excitations in rotating nuclei (see [12, 13, 14] and references therein). To understand the main features of the formation of tilted bands, we assume the angular momentum and shape fluctuations to be dominant. For comparison with experimental data pairing vibrations [13, 15] should be taken into account, but this degree of freedom does not change our main conclusions; it involves tedious calculations to be postponed for future publication. The CRPA and many RPA calculations for non-rotating nuclei suffer from the inconsistency between the basis generated by the mean field and the residual two-body interaction. To facilitate analytical and numerical results and to avoid spurious solutions due to such inconsistencies, we base our analysis upon the three-dimensional harmonic oscillator model.

We start with the single-particle Routhian

$$H_{\Omega} = H_0 - \vec{\Omega} \cdot \vec{L} \equiv \sum_{j=1}^N \left(\frac{1}{2m} \vec{p}_j^2 + \frac{m}{2} (\omega_x^2 x_j^2 + \omega_y^2 y_j^2 + \omega_z^2 z_j^2) - \vec{\Omega} \cdot \vec{l}_j \right) \quad (1)$$

where the rotational vector $\vec{\Omega}$ of the cranking term has the components $(\Omega_x, \Omega_y, \Omega_z) = \Omega(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. The Hamiltonian, Eq.(1), represents the simplest mean field Hamiltonian which reproduces quite well the main properties of rotating nuclei [8, 16]. The spectrum of this Hamiltonian, i.e. the eigenmodes, is obtained by solving a third order polynomial in E^2 . The normal mode operators a_k^\dagger and a_k , $k = 1, 2, 3$ ($[a_k, a_l^\dagger] = \delta_{k,l}$) are linear transformations in the coordinates q_i and momenta p_i , ($i = x, y, z$) with complex coefficients. In this way we obtain $H_{\Omega} = \sum_{j=1}^N \sum_{i=1,2,3} E_i(n_i + 1/2)_j$ with $n_i = a_i^\dagger a_i$. The normal modes are filled from the bottom which gives the ground state energy in the rotating frame $E_{tot} = E_1 \sum_1 + E_2 \sum_2 + E_3 \sum_3$ where $\sum_k = \langle \sum_j^N (n_k + 1/2)_j \rangle$ and the configuration is determined by $\sum_1 \leq \sum_2 \leq \sum_3$.

The minimization of the Hamiltonian Eq.(1) with respect to the three frequencies ω_i , subject to the volume conservation condition $\omega_x \omega_y \omega_z = \omega_0^3$, yields the self-consistency relation

$$\omega_x^2 \langle x^2 \rangle = \omega_y^2 \langle y^2 \rangle = \omega_z^2 \langle z^2 \rangle. \quad (2)$$

These energy minima and their corresponding values for ω_i depend on the rotational vector $\vec{\Omega}$. For given Ω we search for tilted solutions by seeking the minimum in the $\theta - \phi$ -plane [9]. Three major results are reported here:

1. starting from oblate, prolate or tri-axial shapes, for sufficiently high values of Ω a local minimum, which is characterized by an oblate shape with the symmetry axis coinciding with the rotational axis, ($\theta = 90^\circ$, $\phi = 0^\circ$) is *always* obtained;
2. increasing further the value of Ω this local minimum shifts away from $\theta = 90^\circ$ to $\theta < 90^\circ$ and possibly $\phi > 0^\circ$ thus giving rise to a two- or three-dimensional rotation;
3. only when starting from tri-axial, near oblate, nuclei is this specific local minimum a global and stable minimum. For prolate nuclei, this

local minimum is attained only for values of Ω so large that other (possibly unphysical) minima *with different configurations* occur at a considerably lower energy, see Fig.1. When starting from the outset with an oblate nucleus, the global minimum is obtained for the rotational axis perpendicular to the symmetry axis; the specific local minimum referred to above is then the second lowest minimum with the *same* configuration.

We first focus our attention to tri-axial near oblate nuclei. Here we encounter two critical values. At $\Omega_{\text{cr}}^{(1)}$ there is a shape transition from tri-axial to axial (oblate) symmetry. This axial shape remains unchanged for a range $\Omega_{\text{cr}}^{(1)} \leq \Omega \leq \Omega_{\text{cr}}^{(2)}$ where $\Omega_{\text{cr}}^{(2)}$ signifies the onset of a tilted rotation where the rotational axis no longer coincides with a principal axis. Note that for $0 \leq \Omega \leq \Omega_{\text{cr}}^{(2)}$ the rotational axis is aligned to a principal axis which becomes the symmetry axis for $\Omega \geq \Omega_{\text{cr}}^{(1)}$. At this point there is a confluence in the energy contours $E(\omega_y, \omega_z)$ of two minima into one at $\omega_{\perp} = \omega_y = \omega_z$. In contrast, at the larger value $\Omega = \Omega_{\text{cr}}^{(2)}$ the minimum at $\theta = 90^\circ, \phi = 0^\circ$ bifurcates into two minima as illustrated in Fig.2. The two minima occurring at $\theta = 75^\circ$ and $\theta = 105^\circ$ ($\phi = 0^\circ$) appear at first glance physically equivalent. However, they are not as can be seen when Ω is increased further in which case they move into positions with different values of ϕ (and θ) and only one is the global minimum. In fact, the directions of $\vec{\Omega}$ play a different role in the different octants [17]. We interpret the rotation for $\Omega_{\text{cr}}^{(1)} \leq \Omega \leq \Omega_{\text{cr}}^{(2)}$ as a K-isomer. For the tilted rotation the symmetric shape no longer prevails. While these findings are obtained from the numerical minimization procedure we now further substantiate our analysis by an analytical procedure. Using the fact that the first critical rotational frequency is associated with a one-dimensional rotation, we exploit in Eq.(2) that at the transition point $\omega_y = \omega_z = \omega_{\perp}$ and obtain a third order equation

$$u^3 - \frac{u}{2} + \frac{1}{2} \frac{r-1}{r+1} = 0 \quad (3)$$

where $u = \Omega/\omega_{\perp}$ and $r = \Sigma_3/\Sigma_2$. From the discriminant of Eq.(3) we obtain the critical value $r_{\text{cr}} = (\sqrt{27} + \sqrt{2})(\sqrt{27} - \sqrt{2})$. It was shown in [18] that a prolate system becomes eventually oblate with the rotational axis coinciding with the symmetry axis, if the initial deformation obeys $r \leq r_{\text{cr}}$. However, according to our results, this case is not a global energy minimum,

in fact the global minimum is unphysical. In general, for $r < r_{cr}$ we obtain three solutions

$$u = \sqrt{\frac{2}{3}} \cos\left(\frac{\chi + 2\pi n}{3}\right) \quad n = 0, 1, 2 \quad (4)$$

$$\cos \chi = 3\sqrt{\frac{3}{2}} \frac{1-r}{1+r} \quad (5)$$

These values of the rotational frequency correspond to three bifurcation points. Below we demonstrate that one of these solutions is the critical point $\Omega_{cr}^{(1)}$ where the lowest vibrational frequency tends to zero.

The variations in the one-body potential around the equilibrium deformation determine the effective quadrupole-quadrupole interaction [19]. The total Hamiltonian can be presented as

$$H = H_\Omega - \frac{\kappa}{2} \sum_{\mu=-2}^2 Q_\mu^\dagger Q_\mu \equiv \tilde{H} - \vec{\Omega} \cdot \vec{L} \quad (6)$$

While the mean field Hamiltonian Eq.(1) breaks the rotational symmetry, the Hamiltonian \tilde{H} in the total Hamiltonian Eq.(6) fulfills the commutation rules $[\tilde{H}, L_i] = 0$, $i = x, y, z$. Note that the self-consistent condition Eq.(2) fixes the quadrupole strength constant κ in the RPA calculations. We solve the RPA equation of motion for the general coordinates \mathcal{X}_λ and momenta \mathcal{P}_λ (see for details [12])

$$[H, \mathcal{X}_\lambda] = -i\omega_\lambda \mathcal{P}_\lambda, \quad [H, \mathcal{P}_\lambda] = i\omega_\lambda \mathcal{X}_\lambda, \quad [\mathcal{X}_\lambda, \mathcal{P}_{\lambda'}] = i\delta_{\lambda, \lambda'} \quad . \quad (7)$$

Here ω_λ is the RPA eigenfrequency in the rotating frame and the associated phonon $O_\lambda^\dagger = (\mathcal{X}_\lambda - i\mathcal{P}_\lambda)/\sqrt{2}$. In contrast to the CRPA approach, the phonon is in the present model a superposition of different signature phonons. The degree of the mixture depends on the tilted angle: the signature and $|K|$ are good quantum numbers, respectively, for rotations perpendicular and parallel to the symmetry axis. The non-zero solutions appear in pairs $\pm\hbar\omega_\lambda$, we choose solutions with positive norm.

Let us first focus on the RPA solution which leads to the transition from the non-axial to the oblate regime of rotation. Since at this transition point the minimal solution corresponds to a one-dimensional rotation ($\vec{\Omega} \equiv (\Omega, 0, 0)$)

around the symmetry axis, the angular momentum becomes a good quantum number. The RPA states can be characterized by the projection of the angular momentum because $[L_x, O_\lambda^\dagger] = \lambda O_\lambda^\dagger$. Consequently, we obtain

$$[H, O_\lambda^\dagger] = [\tilde{H} - \Omega L_x, O_\lambda^\dagger] = (\tilde{\omega}_\lambda - \lambda\Omega)O_\lambda^\dagger \equiv \omega_\lambda O_\lambda^\dagger \quad (8)$$

From Eq.(8) it follows that at the rotational frequency $\Omega_{cr} = \tilde{\omega}_\lambda/\lambda$ one of the RPA frequency should be equal zero. The solution of the RPA equation for the positive signature quadrupole phonons with the largest projection $\lambda = -2$ gives

$$\omega_{\lambda=-2} = -\omega_\perp \sqrt{\frac{2}{3}} \left(\cos \frac{\chi + \pi}{3} - \sqrt{3} \sin \frac{\chi + \pi}{3} \right) + 2\Omega. \quad (9)$$

This mode corresponds to the quadrupole de-excitation which leads to the state with two units angular momentum less then the vacuum state (K-isomer state). From Eq.(9) we obtain the first critical value of the rotational frequency at which the transition from the non-collective rotation to the non-axial collective rotation takes place

$$\Omega_{cr}^{(1)} = \frac{\omega_\perp}{2} \sqrt{\frac{2}{3}} \left(\cos \frac{\chi + \pi}{3} - \sqrt{3} \sin \frac{\chi + \pi}{3} \right) \quad (10)$$

Therefore, the positive signature quadrupole de-excitation leads from the K-isomer state to the yrast state with non-axial quadrupole shape.

Quantum excitations describing the wobbling motion correspond to the negative signature quadrupole phonons[10, 11, 20]. These excitations are connected to the yrast line by means of the quadrupole transitions which carry one unit of the angular momentum. Similar to the positive signature phonons, the negative signature phonons can have zero excitation energy at particular rotational frequency. Solution of the RPA equation for the negative signature phonons with $\lambda = -1$ for the oblate rotation regime leads to the result

$$\omega_{\lambda=-1} = -\sqrt{\frac{\omega_x^2 + \omega_\perp^2}{3}} \left(\cos \frac{\psi}{3} - \sqrt{3} \sin \frac{\psi}{3} \right) + \Omega \quad (11)$$

$$\cos \psi = 3\sqrt{3} \frac{\omega_x^2 \omega_\perp}{(\omega_x^2 + \omega_\perp^2)^{3/2}} \frac{r-1}{1+r} \quad (12)$$

The condition $\omega_{\lambda=-1} = 0$ yields the second critical frequency

$$\Omega_{\text{cr}}^{(2)} = \sqrt{\frac{\omega_x^2 + \omega_{\perp}^2}{3}} \left(\cos \frac{\psi}{3} - \sqrt{3} \sin \frac{\psi}{3} \right). \quad (13)$$

Beyond this rotational frequency the mean field solution corresponds to the stable tilted rotation. We recall that the expressions given in Eqs.(10) and (13) coincide exactly with the transitional points found in the numerical minimization procedure. Note that, in contrast to the familiar phase transition from spherical to deformed shape owing to the variation of a strength parameter [1], it is here the rotation that leads to a phase transition from tri-axial to oblate shape and then to the tilted rotation associated again with a non-axial shape for the same strength parameter of the residual quadrupole-quadrupole interaction.

According to our analysis, only nuclei near to oblate shape, when non-rotational, exhibit at a certain rotational frequency tilted rotation which, for lesser rotational speed, leads to the high K-isomer states. These states decay through non-axial shapes to the ground states. Nuclei which are oblate when non-rotational also can have this behavior which corresponds to the second (local) minimum discussed above. However, they also have states of good signature with an even lower energy for the same rotational speed and the same configuration. In contrast, for prolate nuclei the local minimum with these characteristics occurs only at a value Ω so large that the global minimum occurs for an absurdly unphysical configuration; for this reason it is doubtful to attach particular physical significance to the local minimum.

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References

- [1] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, Berlin, 1980).
- [2] B. Riemann, Abh. Kön. Ges. Wiss. (Göttingen), **9**,1 (1860).
- [3] R.M. Clark *et al*, Phys.Rev.Lett. **82**, 3220 (1999); H. Schnare *et al*, Phys.Rev.Lett. **82**, 4408 (1999); D.G. Jenkins *et al*, Phys.Rev.Lett. **83**, 500 (1999); D.G. Jenkins *et al*, Phys.Rev. **C58**, 2703 (1998); O. Vogel *et al*, Phys.Rev. **C56**, 1338 (1998); R.M. Clark, J.Phys.G: Nucl.Phys. **25**, 695 (1999).
- [4] S. Frauendorf, Nucl.Phys. **A557**, 259c (1993); Rev.Mod.Phys., submitted.
- [5] P.M. Walker *et al*, Phys.Lett. **B309**, 14 (1993).
- [6] B. Crowell *et al*, Phys.Rev.Lett. **72**, 1164 (1994).
- [7] T. Kutsarova *et al*, Nucl.Phys. **A587**, 111 (1995); N.L. Gjørup *et al*, Nucl.Phys. **A582**, 369 (1995); C.J. Pearson *et al*, Phys.Rev.Lett. **79**, 605 (1997); C.S. Purry *et al*, Nucl.Phys. **A632**, 229 (1998).
- [8] A. Bohr and B.R. Mottelson, *Nuclear Structure* Vol. II , (Benjamin, New York, 1975).
- [9] W.D. Heiss and R.G. Nazmitdinov, Phys.Lett. **B 397**,1 (1997)
- [10] E.R. Marshalek, Nucl.Phys. **A266**, 317 (1976).
- [11] D. Janssen and I.N. Mikhailov, Nucl.Phys. **A318**, 390 (1979).
- [12] J. Kvasil and R.G. Nazmitdinov, Sov.J.Part.Nucl. **17**, 265 (1986).
- [13] Y.R. Shimizu, J.D. Garrett, R.A. Broglia, M. Gallardo and E. Vigezzi, Rev. Mod. Phys. **61**, 131 (1989).
- [14] T. Nakatsukasa, K. Matsuyanagi, S. Mizutori and Y.R. Shimizu, Phys.Rev. **C53**, 2213 (1996); G. Hackman *et al*, Phys.Rev. **C57**, R1056 (1998).

- [15] F. Döna, D. Almed and R.G. Nazmitdinov, Phys.Rev.Lett. **83**, 280 (1999); D. Almed, F. Döna, S. Frauendorf and R.G. Nazmitdinov, Physica Scripta, in press.
- [16] S.G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure* (Cambridge University Press, Cambridge, 1995).
- [17] V.I. Dimitrov, S. Frauendorf and F. Döna, Phys. Rev. Lett. **84**, 5732 (2000).
- [18] T. Troudet and R. Arvieu, Ann.of Phys. (N.Y.) **134**, 1 (1981).
- [19] T. Kishimoto *et al*, Phys.Rev.Lett. **35**, 552 (1975); H. Sakamoto and T. Kishimoto, Nucl.Phys. **A501**, 205 (1989) and references therein.
- [20] D. Janssen, I.N. Mikhailov, R.G. Nazmitdinov, B. Nerlo-Pomorska, K. Pomorski and R.Kh. Safarov, Phys. Lett. **79B**, 347 (1978).

Figure Captions

Fig.1 Energy contours in the $\omega_x - \omega_y$ -plane for a rotational speed where the onset of an oblate shape has just taken place and corresponds to the local minimum shown in the top. However, for the nucleus considered ($N = 44$, tri-axial near prolate) a much deeper minimum has developed as an instability as seen in the top right corner of the bottom picture. The section of the top illustration is indicated in the bottom part.

Fig.2 Energy minimum in the $\phi - \theta$ -plane just before (top) and just after (bottom) the onset of tilted rotation. Note the local maximum developing at the centre. The example chosen is $N = 36$, a tri-axial near oblate nucleus.



