4-4 COUPLING BETWEEN ROTATIONAL AND INTRINSIC MOTION FOR AXIALLY SYMMETRIC NUCLEI

The phenomenological analysis of matrix elements in the rotational coupling scheme, considered in Sec. 4-3, is based on the symmetry of the deformation and on the adiabatic condition that makes it possible to represent the matrix elements in terms of an expansion in powers of the rotational angular momentum. In such a description, the coupling between rotational and intrinsic motion resulting from the Coriolis and centrifugal forces does not appear explicitly, but manifests itself in the I dependence of the effective operators. These coupling effects can also be viewed in terms of the admixing of different rotational bands. In the present section, we shall consider such band mixing involving the low-lying intrinsic excitations. By further development of this approach, one may attempt to derive all the I-dependent nuclear properties on the basis of the interaction between rotation and intrinsic motion. (The cranking model considered on pp. 75ff. is an example of such an extension.)

Coriolis interaction

The Hamiltonian describing the coupling between a set of bands, labeled by $i=1,2,\ldots,n$, can be written

$$H = H_0 + H_c \tag{4-195}$$

where H_0 is diagonal in i, while H_c connects states belonging to different bands (but having the same value of I). In the representation (4-195), the Hamiltonian is assumed to be diagonalized with respect to all modes of excitation other than those explicitly under consideration, and the operators therefore include the I-dependent terms associated with all couplings other than those described by H_c.

The coupling H_c expresses the nondiagonal effects of the rotation within the space of states i, and the lowest-order coupling term is proportional to the rotational angular momentum

$$H_c = h_{+1} I_- + \mathscr{R}\text{-conj.} \tag{4-196}$$

where h_{+1} $(=h_{\Delta K=+1})$ is an intrinsic operator (a matrix in the space i). The interaction (4-196) gives rise to a coupling between bands with $\Delta K=1$, as well as between two bands with $K=1/2$. The diagonal effect of the

interaction (4-196), in a $K=1/2$ band, gives rise to a signature-dependent term in the rotational energy (see the decoupling term in Eq. (4-61)).

If the intrinsic degrees of freedom i can be described in terms of one or a few particles moving in an average potential produced by the rest of the nucleons, and if it is assumed that this potential is independent of the rotational motion, the coupling H_c is the Coriolis interaction (see the discussion of the particle-rotor model in Appendix 4A), and we have

$$h_{+1} = -\frac{\hbar^2}{2\mathscr{I}_0} J_+ \tag{4-197}$$

where $\mathscr{I}_0$ is the moment of inertia of the residual nucleus, while $J_+ = J_1 + iJ_2$ is the component of the total angular momentum of the particles considered.

First-order effects of Coriolis coupling

The coupling (4-196) between two bands with $\Delta K = 1$ gives rise to a mixing of these bands,

$$\begin{aligned} |\hat{K}I\rangle &\approx |KI\rangle - c(I)|K+1, I\rangle \\ |\hat{K}+1, I\rangle &\approx |K+1, I\rangle + c(I)|KI\rangle \end{aligned} \tag{4-198}$$

where the caret denotes the coupled (or renormalized) states; the first-order amplitude of admixture is

$$c(I) = \langle K+1|\varepsilon_{+1}|K\rangle (I-K)^{1/2}(I+K+1)^{1/2} \begin{cases} \sqrt{2} & K=0 \\ 1 & K\neq 0 \end{cases} \tag{4-199}$$

$$\langle K_2|\varepsilon_{+1}|K_1\rangle \equiv \frac{\langle K_2|h_{+1}|K_1\rangle}{E(K_2) - E(K_1)} \qquad (K_2 = K_1 + 1)$$

In the energy denominator of the last expression, we ignore rotational energies as compared with intrinsic energies. The above expressions also apply to the mixing of two $K=1/2$ bands, if the intrinsic state $|K_1\rangle$ is replaced by the time-reversed $|\bar{K}_1\rangle$ and if the signature factor $(-1)^{I_1+K_1}$ is added to $c(I)$.

The perturbation treatment of the Coriolis coupling is based on the smallness of the quantity ε_{+1}. Since the intrinsic excitation energies in heavy nuclei are of the order of 1 MeV, the expression (4-197) for h_{+1} together with Eq. (4-199) yields values for the matrix elements of ε_{+1} that are typically of the order of a few percent. (Empirical values of ε_{+1} are derived from the analysis of the spectra of ^{175}Lu (p. 154) and ^{235}U (p. 273).)

The band mixing (4-198) leads to modifications in the various matrix elements involving the coupled bands (Kerman, 1956). Compact expressions for these effects are obtained by describing the band mixing in terms of a canonical transformation

$$\begin{aligned} |\hat{K}I\rangle &= \exp\{-iS\}|KI\rangle \\ &\approx |KI\rangle - iS|KI\rangle \end{aligned} \tag{4-200}$$

where the operator S is linear in the matrix ε_{+1} defined by Eq. (4-199)

$$S = -i(\varepsilon_{+1}I_- + \mathscr{R}\text{-conj.}) \tag{4-201}$$

The transformation $\exp\{iS\}$ diagonalizes the Hamiltonian (4-195), to leading order in the coupling (see the discussion in Appendix 4A, pp. 206ff.).

The matrix elements of an operator $\mathscr{M}$ between the perturbed states can be obtained by evaluating the transformed operator

$$\begin{aligned} \exp\{iS\}\ \mathscr{M}\exp\{-iS\} &= \mathscr{M} + i[S, \mathscr{M}] - \tfrac{1}{2}[S,[S, \mathscr{M}]] + \cdots \\ &= \mathscr{M} + \delta\mathscr{M} \end{aligned} \tag{4-202}$$

in the unperturbed basis. For a tensor operator $\mathscr{M}(\lambda,\mu)$, the transformation (4-90) to intrinsic axes yields, to first order in S,

$$\begin{aligned} \delta\mathscr{M}(\lambda\mu) &\approx i[S,\ \mathscr{M}(\lambda\mu)] \\ &= \left[(\varepsilon_{+1}I_- + \mathscr{R}\text{-conj.}),\ \sum_\nu \mathscr{M}(\lambda\nu)\mathscr{D}^\lambda_{\mu\nu}\right] \\ &= \sum_\nu \tfrac{1}{2}\{\varepsilon_{+1},\ \mathscr{M}(\lambda\nu)\}\left[I_-,\mathscr{D}^\lambda_{\mu\nu}\right] + \tfrac{1}{2}[\varepsilon_{+1},\ \mathscr{M}(\lambda\nu)]\left\{I_-,\mathscr{D}^\lambda_{\mu\nu}\right\} + \mathscr{R}\text{-conj.} \end{aligned} \tag{4-203}$$

where the intrinsic moments $\mathscr{M}(\lambda\nu)$ have been assumed to be I independent. We have employed the identity (4A-26) for commutators of products and have used the conventional notation for anticommutators ($\{a,b\} = ab + ba$).

In the first term of Eq. (4-203), the commutator of I_- and $\mathscr{D}^\lambda_{\mu\nu}$ is proportional to $\mathscr{D}^\lambda_{\mu,\nu+1}$ (see Eq. (1A-91)). This term, therefore, gives rise to an I-independent renormalization of the intrinsic moments, and does not lead to a modification of the leading-order intensity relations. The second term in Eq. (4-203) introduces effective moments that are linear in $I_\pm$ and leads to generalized intensity relations of the type discussed in Sec. 4-3.

For dipole moments, the various terms occurring in Eq. (4-203) are discussed in Appendix 4A (see pp. 208ff.). The coupling partly leads to a

renormalization of the leading-order terms, such as the parameters g_K, g_R, and b that characterize $M1$ transitions within a band and partly introduces new terms linear in $I_\pm$, which contribute to matrix elements with $\Delta K = 0$, ± 1, and ± 2. (Coriolis effects in $M1$ transitions between bands in ^{175}Lu are considered on pp. 156ff. The I-dependent terms in the $E1$-transition moments observed in ^{177}Hf may also be attributed to the effect of first-order Coriolis couplings; see p. 110.)

The coupling between two bands has an especially large effect on the $E2$ transitions between the bands, since the admixed components in the wave function contribute with the collective matrix element proportional to Q_0. The resulting contribution to the $E2$-transition moment is contained in the terms in Eq. (4-203) involving $\mathcal{M}(E2, \nu = 0) = (5/(16\pi))^{1/2} e Q_0$. If we neglect the difference between the Q_0 values for the two bands, the commutator of ε_{+1} and $\mathcal{M}(E2, \nu = 0)$ vanishes, and the induced moment $\delta\,\mathcal{M}(E2)$ gives an I-independent renormalization of the intrinsic transition moment $\mathcal{M}(E2, \nu = 1)$, without any effect on the relative transition probabilities. For low-frequency $E2$ transitions ($\Delta E \lesssim 1$ MeV), the induced moment is large compared with the unperturbed moment and implies a strong enhancement of the absolute transition rates. (See the estimate in Chapter 5, pp. 281ff., and the example discussed on pp. 154ff.)

For $E2$ transitions within a band, the second term in Eq. (4-203) gives rise to deviations from the leading-order intensity relations (provided $K \neq 0$), but the effects are expected to be small, since they involve the intrinsic transition moments $\mathcal{M}(E2, \nu = 1)$ compared with the collective moments $\mathcal{M}(E2, \nu = 0)$ in the leading-order term. (See the example discussed on pp. 130ff.)

In a number of cases, it has been possible to estimate the band-mixing amplitude $\langle K+1|\varepsilon_{+1}|K\rangle$ on the basis of several different observed contributions to the transitions between the bands, and thus to test the consistency of the analysis. (See the example (^{175}Lu) discussed on pp. 154ff. The interpretation of the observed matrix elements of h_{+1} in terms of the intrinsic one-particle configurations is discussed in Sec. 5-3d.)

Second-order contributions to rotational energy

The coupling (4-196) produces a repulsion between the interacting bands resulting in the energy shifts

$$\begin{aligned} \delta E(K_1 I) &= -\delta E(K_2 = K_1 + 1, I) \\ &= -\frac{\langle K_2|h_{+1}(q)|K_1\rangle^2}{E(K_2) - E(K_1)} (I(I+1) - K_1(K_1+1)) \end{aligned} \tag{4-204}$$

The term proportional to $I(I+1)$ amounts to a renormalization of the A coefficient in the rotational energy (see Eq. (4-46)), and corresponds to a contribution to the moment of inertia

$$\delta\mathscr{I}(K_1)\approx-\delta\mathscr{I}(K_2)\approx 2\hbar^2\frac{\langle K_2|J_1|K_1\rangle^2}{E(K_2)-E(K_1)} \tag{4-205}$$

In the derivation of Eq. (4-205), we have assumed $\delta\mathscr{I}\ll\mathscr{I}_0$ and have employed the relation (4-197) for h_{+1} (note that $J_1=\frac{1}{2}(J_+ + J_-)$). The contribution of the term (4-205) to the difference of the moments of inertia of even-even and odd-A nuclei is discussed in Sec. 5-3d, pp. 251ff.

The expression (4-205) is the same as that obtained by considering particles moving in an external potential rotated with a fixed frequency (the cranking model; see pp. 75ff.). The above derivation applies to a single degree of freedom; if several degrees of freedom together contribute a significant fraction of the total moment of inertia, the results (4-204) and (4-205) differ. The analysis based on the cranking model corresponds to adding the contributions (4-205) to obtain the resultant moment $\mathscr{I}$ (see Eq. (4-110)), and this is expected to be the correct answer as long as no single degree of freedom contributes a significant fraction of the total. This result may be obtained by treating the coupling between the intrinsic degrees of freedom produced by the "recoil term", as illustrated by the analysis of the particle-rotor model in Appendix 4A, pp. 205ff. (The early discussion of Casimir, 1931, established the result for the inertial effect of a particle bound harmonically to a rotor.)

Effective couplings with $\Delta K=2$

When the spectrum involves elementary modes of excitation with $\Delta K=2$, significant rotational coupling effects may be described by an effective coupling of the form

$$H_c=h_{+2}I_-^2+\mathscr{R}\text{-conj.} \tag{4-206}$$

which produces a direct mixing of the bands. Coupling effects that are of second order in the rotational angular momentum arise from the Coriolis coupling (4-196) acting in second order through intermediate states with $\Delta K=1$. These effects can be represented by the direct coupling (4-206), provided the energies of the $\Delta K=1$ excitations are large compared with the energy of the $\Delta K=2$ mode.

A term such as (4-206) that is of second order in I may be considered as part of the rotational energy, expressing the dependence of the moment