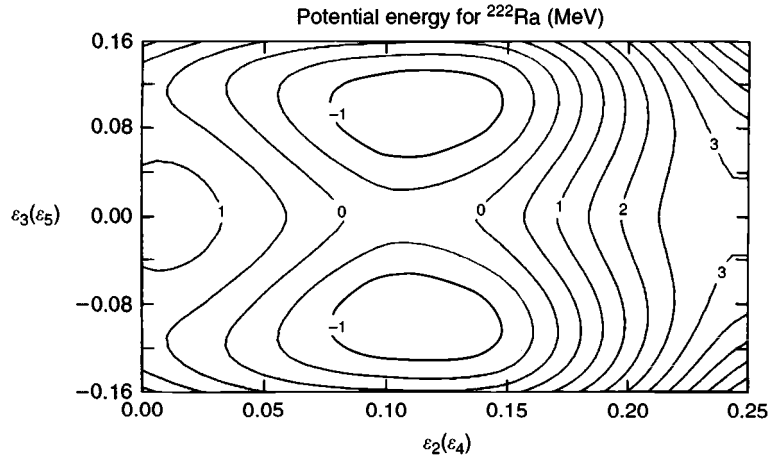

Fig. 9.10. Potential energy surface of ^{222}Ra calculated as a function of quadrupole ($\varepsilon_2(\varepsilon_4)$) and octopole ($\varepsilon_3(\varepsilon_5)$) deformation. A Woods–Saxon type single-particle potential has been used (more specifically, the folded-Yukawa potential). The energy gain due to quadrupole deformation is around 1.5 MeV and then an additional 1.2 MeV is gained because of octupole deformation (from Leander *et al.*, 1982).

parity, $0^+, 1^-, 2^+, 3^-, 4^+, \ldots$. Spectra of this kind have never been observed (at least not at very low spins), but in several nuclei, negative parity states are very low in energy. A very important consequence of the octupole deformation is that the centre of mass for the neutron distribution might be different from that of the proton distribution, i.e. the nucleus gets an intrinsic dipole moment with important consequences, in particular enhanced dipole radiation, see e.g. Leander *et al.* (1986). It seems clear that the concept of octupole deformation is very useful for our understanding of the spectroscopic properties of specific nuclei. This is so even though one could argue that it is probably difficult to find any nucleus that has acquired a shape of really permanent octupole deformation.

As indicated in chapter 5, it is also possible to do calculations within the more 'fundamental' self-consistent Hartree–Fock approach. These methods, although very time consuming, have also been applied to calculation of the fission barrier, where the two-peak character has been reproduced (Flocard *et al.*, 1974; Berger, Girod and Gogny, 1989). Also the absolute barrier heights are reproduced within a few MeV. The Strutinsky method can be formally derived from the Hartree–Fock equations (see Brack and Quentin (1981) for a review). From the calculational point of view, the Strutinsky prescription is much simpler and it has therefore made possible systematic calculations over large regions of the nuclear chart.

Exercises

9.1 The single-particle potential for a nucleus is assumed to be of harmonic oscillator type with the energy eigenvalues $e = \hbar\omega_0(N + 3/2)$. This potential is used for protons as well as neutrons and the nucleus is assumed to have equally many particles of each kind. The shells with $N \leq N^*$ are completely filled and the filling of the shell $N = N^* + 1$ is given by ρ $(0 \leq \rho \leq 1)$. The total number of particles is A and the total energy is E. It is now straightforward to determine $A = A(N^*, \rho)$ and $E = E(N^*, \rho)$. Elimination of N^* leads to $E = E(A, \rho)$. Carry through these calculations by expanding

$$N^* = \alpha A^{1/3} + \beta + \gamma A^{-1/3} + \ldots$$

and from this, with $\hbar\omega_0 = 41 \cdot A^{-1/3}$ MeV,

$$E = \alpha' A + \beta' A^{2/3} + \gamma' A^{1/3} + \ldots \quad \text{MeV}$$

Show that E can be split into one 'smooth part' $\tilde{E}$ and one shell correction part E_{shell} where

$$\tilde{E} = (41/8)\left[3(12)^{1/3}A + (18)^{1/3}A^{1/3}\right] \text{ MeV}$$

$$E_{\text{shell}} = (41/4)\,[12\rho(1-\rho) - 1]\,12^{-1/3}A^{1/3} \text{ MeV}$$

Sketch E_{shell} and compare it with the experimental shell effects shown in figs. 3.9 and 9.7. Comments!

9.2 For a simple estimate of the single-particle effect behind the asymmetric deformations one might consider the matrix elements of the $\rho^3 Y_{30}(\theta_t, \varphi_t)$ operator (instead of $\rho^2 Y_{30}(\theta_t, \varphi_t)$, which is used in the modified oscillator). Apply the operator method of chapter 8 to find the selection rules for N' and n_z' in the matrix element

$$\left\langle N n_z \Lambda \left| \rho^3 Y_{30} \right| N' n_z' \Lambda \right\rangle$$

Calculate the distance between the corresponding orbitals as functions of ε for a pure oscillator potential. Evaluate the matrix element for those orbitals that come closest together for large ε. Compare with the single-particle diagrams of the modified oscillator, figs. 8.5 and 9.5.

9.3 The smearing function of the Strutinsky shell correction method is defined in such a way that long-range variations are preserved. Thus, with a correction function, $f_{\text{corr}}(u) = a_0 + a_2 u^2 + \ldots a_m u^{2m}$, $(u = (e - e')/\gamma)$, a polynomial function $G(e)$ of order $2m + 1$ should