• Translational invariance

$\Downarrow$

the coordinate system K is displaced to a new set of axes K'

$\forall\ |A\rangle_K \Rightarrow$ translated state $|A'\rangle \in K'$

↳ the properties of $|A'\rangle$ described by an observer in K' are identical to those of $|A\rangle$, as seen from K.

$$\boxed{\langle B'|A'\rangle = \langle B|A\rangle}$$

Transformation $\quad |A\rangle \xrightarrow{\text{Unitary}} |A'\rangle \quad \in K$

$$|A'\rangle = U|A\rangle = \sum_B |B\rangle \boxed{\langle B|U|A\rangle}$$

$$|A'\rangle_{K'} = \sum_B \boxed{U_{BA}}\, |B\rangle_K$$

• translational invariance: $\langle B'|T'|A'\rangle = \langle B|T|A\rangle$

↳ operator expressed in K

↳ operator described in K'

$$T' = U T U^{-1}$$

$$|A'\rangle = \sum_B |B\rangle\langle B|U|A\rangle$$

$$\langle B'|T'|A'\rangle = \langle B|T|A\rangle$$

$$\langle B|U^{-1}T'U|A\rangle = \langle B|T|A\rangle$$

$$\Rightarrow U^{-1}T'U = T \Rightarrow$$

$$T' = U T U^{-1}$$

$$U(a) = e^{-\frac{i}{\hbar}\vec{a}\cdot\vec{p}} \qquad \vec{a}: \text{displacement}$$

$$U(\delta a) = 1 - \frac{i}{\hbar}\delta\vec{a}\cdot\vec{p}$$

$\hookrightarrow$ $\vec{p}$: generator of infinitesimal translation

Time displacements

$$|A\rangle \xrightarrow{\text{unitary transformation}} |A'\rangle$$

$$\boxed{U(t_0) = e^{+\frac{i}{\hbar}H t_0}}$$

$H \rightarrow$ total energy

$$T' = T(t) = e^{\frac{i}{\hbar}Ht}\, T_{(t=0)}\, e^{-\frac{i}{\hbar}Ht} \qquad T' = UTU^{-1}$$

$\hookrightarrow$ Conservation of total momentum $\rightarrow$ expression of the commutability of spatial translations and time displacements

$\Downarrow$

translations = time-independent operations

Rotations