Rotational Motion of Deformed Nuclei

By V. A. KRUTOV

With 1 Figure

Abstract

Collective motion in the nucleus is defined as change of the density distribution of nuclear matter in time. On the basis of this definition the Hamiltonian of nuclear rotation is obtained with moments of inertia corresponding satisfactorily to experimental data. The theory is easily applied to nuclei with non-axial equilibrium shape. For the latter the parameters of non-axiality are considerably smaller than in the DAVYDOV-FILIPPOV model.

I. Introduction

The present description of the rotational motion of nuclei is based on one of two basic assumptions: either on the supposed analogy with the ideal liquid drop [1, 2] or on the assumption of INGLIS' "cranking-model" [3, 4].

The first assumption, involving the analogy between the collective nuclear motion and the surface vibrations of a liquid drop, is the starting point of the BOHR-MOTTELSON description [1]. The same analogy is used, in the form of the rotational Hamiltonian, in the DAVYDOV-FILIPPOV model as well [2]. In the other group of models [3, 4] the starting point of the calculations is INGLIS' supposition that the change of the intrinsic nuclear energy due to the rotation of the non-spherical potential is just the rotational energy.

In this paper a new approach to the description of the rotation of deformed nuclei is suggested[1]). This approach is based on a new definition of the concept of collective motion in the nucleus. In the present paper the rotational Hamiltonian of the nucleus (H_{rot}) is obtained and investigated. The problem of the interaction between the rotation and the intrinsic motion in our approach (the operator $H(\text{rot/in})$) is examined in the next paper [6] (further on referred to as II).

2. Energy of Rotation and Moments of Inertia

The only physical quantity that can be observed during the collective motion in the nucleus is the change of the density distribution[2]). Hence the

[1]) See also the author's reports at the XIVth Conference on Nuclear Spectroscopy [5], and at the XVIIth Conference as well [5a].

[2]) The motion of charge separately from mass isn't considered here. In case of rotation such a separation can have some effect on the renormalization of the moment of inertia.

collective motion may naturally be defined as change of the mass density distribution in time.

Any motion that doesn't change the mass density distribution contributes nothing to the energy of collective motion. Hence the counting of the mass density in the expression for the energy of collective motion must start from the minimal density, and this energy takes the form:

$$T = \frac{1}{2} \int \tilde{\varrho} \cdot V_c^2 \cdot d\boldsymbol{r} \tag{1}$$

where V_c is the velocity of transfer of equal density surfaces while $\tilde{\varrho}$ is the "moving density", defined as the difference between the actual density at the time t and the minimum value of the density:

$$\tilde{\varrho}(\boldsymbol{r}, t) = \varrho(\boldsymbol{r}, t) - \varrho_{\min}(\boldsymbol{r}). \tag{2}$$

In case of a continuous medium considered as an assembly of material points (quasi-continuum) one has $\varrho_{min} = 0$ and eq. (1) takes the usual form.

According to eq. (1) T is positive definite and a quadratic function of velocity. Now the Lagrangian $\mathscr{L}$ of a system is defined only up to the total time derivative of an arbitrary function of time and coordinates. As a matter of fact our definition of $\mathscr{L}$ differs from the conventional one by just such a derivative in case of rotation to which the author is limiting himself: our T, defined by eqs. (1)–(2), differs from the ordinary expression by a constant.

In view of the fact that some confusion exists in the notions about the deformation of a nucleus due to rotation let us explain the introduction of the rotational velocity. We divide (see II) the total motion into the rotation, the intrinsic motion, and their interaction, i.e., we extract the rotation from the total motion. Then the rotational velocity has to be defined as the velocity of the rotation of equal density surfaces in the form:

$$\boldsymbol{V}_c = \boldsymbol{\omega} \times \boldsymbol{r}. \tag{3}$$

This expression coincides with the velocity of solid body rotation as any body is rotating kinematically as a solid one if one considers only the rotation itself (the "pure rotation"). As the other forms of motion, excepting rotation, are components of the intrinsic motion the introduction of any additional deformations due to rotation has no sense in our approach. However, this doesn't mean that we consider the nucleus as being hard during the rotation. The possible deformations due to rotation are automatically taken into account as one of the rotational-intrinsic coupling effects (see II).

Let's now insert (3) into (1) and transform to the principal axes. If our body has an axial symmetry the moment of inertia for the rotation about the symmetry axis z' (i.e. the $J_{z'z'}$ component of the tensor) is vanishing on account of $\tilde{\varrho} = 0$. For the same reason all the three moments of inertia are equal to zero for the spherically symmetrical nucleus. Thus, owing to our definition of the collective motion the rotation about a symmetry axis always drops out.

The kinetic rotational energy for an axially symmetric nucleus is given by

$$T_{\text{rot}} = \frac{1}{2} (\omega_{x'}^2 + \omega_{y'}^2) \cdot J = \frac{1}{2} (\omega^2 - \omega_{z'}^2) \cdot J \tag{4}$$

where the effective moment of inertia is

$$J = \frac{1}{2} \int \tilde{\varrho} \, [r^2 + (z')^2] \, d\boldsymbol{r}. \tag{5}$$

From the point of view of classical analogy our body is similar to a hard infinitely thin needle with the mass distributed along its length according to ϱ.

3. Quantum mechanical description

We take the wave function of the nucleus considered as a system of N interacting nucleons to be:

$$\Psi_{IM\alpha}(\boldsymbol{r}_1 \cdots \boldsymbol{r}_N t) = \Psi_{IM\alpha}(\boldsymbol{r}_1 \cdots \boldsymbol{r}_N) \cdot \exp(-iEt) \tag{6a}$$

where I and M are, respectively, the value and the projection (onto a space-fixed z-axis) of the total angular momentum of the nucleus; α designates the rest of the quantum numbers of the given nuclear state. If the nucleus has an axis and a centre of symmetry the total angular momentum projection (K) onto the symmetry axis is a good quantum number and $\alpha \to K, \alpha$.

Let's extract the rotation of the nucleus as a whole from the total motion of the nucleus, that is we pass to the nucleus-fixed system (the interaction between rotation and intrinsic motion is not taken into consideration here; it will be studied in II):

$$\begin{aligned} &\Psi_{IMK\alpha}(\boldsymbol{r}_1 \cdots \boldsymbol{r}_N) \cdot \exp(-iEt) \\ &\quad = \frac{1 + \hat{R}}{2^{\frac{1}{2}} \cdot (1 + \delta_{K0})^{\frac{1}{2}}} \{\Phi^I_{MK}(\theta_i) \cdot \exp(-iE_{\text{rot}} \cdot t)\} \\ &\quad \times \{\psi_{K\alpha}(\boldsymbol{r}'_1 \cdots \boldsymbol{r}'_N) \cdot \exp(-iE_{\text{in}} \cdot t)\} \end{aligned} \tag{6b}$$

where $\hat{R}$ is the symmetrization operator; θ_i — the EULER angles; $\boldsymbol{r}'_1 \cdots \boldsymbol{r}'_N$ the nucleonic coordinates in the nucleus-fixed system; Φ^I_{MK} — the normalized symmetric-top wave functions:

$$\Phi^I_{MK} = \left(\frac{2I + 1}{8\pi^2}\right)^{\frac{1}{2}} \cdot D^I_{MK}(\theta_i). \tag{6c}$$

The state $\Psi_{IMK\alpha} \cdot \exp(-iEt)$ is a stationary one and the density matrix is independent of time. The intrinsic state $\psi_{K\alpha} \cdot \exp(-iE_{\text{in}} \cdot t)$ is stationary, too. However, it is defined in the nucleus-fixed system and the density

$$|\psi_{K\alpha} \cdot \exp(-iE_{\text{in}} t)|^2$$

is independent of time in the nucleus-fixed system only but not in the laboratory system.

It is natural to define the mass density in the nucleus-fixed system as

$$\varrho(\boldsymbol{r}') = \sum_i \int_{(i)} |\psi_{K\alpha}(\boldsymbol{r}'_1 \cdots \boldsymbol{r}'_N)|^2 \, d\boldsymbol{r}'_1 \cdots d\boldsymbol{r}'_N \tag{7}$$

where the symbol (i) signifies the absence of integration over the coordinates of the ith nucleon.

Eq. (7) determines the density of mass distribution in the nucleus[3]). It is clear from afore-said that in the laboratory (space-fixed) system this density is rotating.

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[3]) To make $\varrho(\boldsymbol{r}')$ correspond to the mass distribution one must multiply eq. (7) by M_A/N where M_A is the nuclear mass.

Substituting in (4) the corresponding operators for the angular velocity components $\left(\omega_i \to \frac{\hbar I_i}{J};\ i = x', y'\right)$ one obtains the rotational Hamiltonian of the nucleus:

$$H_{\text{rot}} = \frac{\hbar^2}{2 J_\alpha^{(0)}} (\boldsymbol{I}^2 - \boldsymbol{I}_{z'}^2) \tag{8}$$

where $\boldsymbol{I}$ and $\boldsymbol{I}_{z'}$ are the operators of the total nuclear angular momentum and its projection along the symmetry axis, respectively. Index α in $J_\alpha^{(0)}$ means a quantum number set $K\alpha$, index (0) signifies that $J_\alpha^{(0)}$ is taken in zero approximation in coupling H(rot/in) (see II).

The effective moment of inertia $J_\alpha^{(0)}$ in eq. (8) is determined according to eqs. (5) and (2) by the equilibrium distribution of the density in the intrinsic state $K\alpha$ and, generally speaking, it must be found from eqs. (5) and (2) by using eq. (7). In this sense our approach is rather microscopic: $\varrho(\boldsymbol{r}')$ must be found from the intrinsic motion of interacting nucleons (i.e., from average field and residual interactions). However, as the wave function of the nucleus is not well known, it is more natural at present to use for certain computations the model density distributions instead of eq. (7).

It is only for the sake of simplicity and definitness that we assumed the nuclei to possess axial symmetry. Our method does not imply any limitation at all. For nuclei with non-axial equilibrium shape we have:

$$H_{\text{rot}} = \frac{\hbar^2}{2} \sum_{\nu=1}^{3} \frac{I_\nu^2}{J_\nu} \tag{9}$$

where the axes x', y', z' are designated as 1, 2, 3; $J_\nu = (J_\alpha^{(0)})_\nu$ results from the ordinary formulae of rigid-body mechanics by writing $\tilde{\varrho}$ for ϱ, where $\tilde{\varrho}$ is defined by eq. (2).

In case of small deviations from axial symmetry (a case most important in practical applications to deformed nuclei as will be seen later on) it is convenient to express H_{rot} in the following form:

$$H_{\text{rot}} = H'_{\text{rot}} + H'_{\text{int}}; \tag{10a}$$

$$H'_{\text{rot}} = \frac{\hbar^2}{2J'} (\boldsymbol{I}^2 - \boldsymbol{I}_3^2) + \frac{\hbar^2}{2J_3} \cdot \boldsymbol{I}_3^2; \tag{10b}$$

$$H'_{\text{int}} = \delta J \cdot \frac{\hbar^2}{8J} \cdot (I_+^2 + I_-^2) \tag{10c}$$

where $J' = J \cdot \left(1 + \frac{1}{2}\,\delta J\right)^{-1}$; $\delta J = \frac{J - J_2}{J_2}$; $I_\pm = \boldsymbol{I}_1 \pm i\boldsymbol{I}_2$; J_1 is designated as J.

The small term H'_{int} has to do with the connection of rotation and intrinsic motion (see II). Hence we consider H'_{rot} as our Hamiltonian of nuclear rotation. Then

$$H'_{\text{rot}} \Phi^I_{KM} = E'_{\text{rot}} \Phi^I_{MK}; \tag{11a}$$

$$E'_{\text{rot}} = E_{IK\alpha} = \frac{\hbar^2}{2J'} [I(I+1) - K^2] + \frac{\hbar^2}{2J_3} \cdot K^2. \tag{11b}$$

As J' is considerably larger than J_3 the splitting of rotational levels with the same spin I but with different projections K is considerably greater than the splitting of levels with the same K but with different I. Therefore it is convenient to consider the states with the same K and α but different I as a rotational band $|IK\alpha\rangle$. In particular, for even-even nuclei there exists along side with the ground state rotational band $|IK = 0\alpha\rangle$ the band $|IK = 2\alpha\rangle$ (in DAVYDOV's papers it is called "anomalous"). Then from eq. (11b) one can easily find a simple relation between the energy of the level with any I of the band $K^{\pi\alpha} = 2^+$ and the energy of the levels $E_{I^\pi K\alpha} = E_{2^+0\alpha} = E_{2^+}(1)$ and $E_{2^+2\alpha} = E_{2^+}(2)$:

$$E_{I2\alpha} = \frac{I(I+1)-6}{6} \cdot E_{2^+}(1) + E_{2^+}(2). \tag{12}$$

Eq. (12) is fulfilled the better, the smaller the influence of $H(\text{rot/in})$ and H'_{int} (these terms are not taken into account in H'_{rot}); in other words, the greater the deformation, the smaller is the spin and the non-axiality (see II). A particular case of eq. (12) is the well-known rule

$$E_{3^+}(1) = E_{2^+}(1) + E_{2^+}(2).$$

4. Value of moments of inertia

Though computation according to eq. (5) is not complicated for any realistic distribution of mass, we shall, for the sake of simplicity, take $\varrho(\boldsymbol{r}') = \text{const}$ within the nucleus. Let the nucleus have the shape of a prolate ellipsoid of revolution with the mean radius R, the difference between the major and the minor semi-axes being equal to $c - a = \Delta R = R \cdot \delta$. Then according to eq. (5) we have:

$$J_\alpha^{(0)} = \frac{4}{5} M_A \cdot R^2 \cdot \delta \cdot \left(1 - \frac{7}{12}\delta + \frac{11}{18}\delta^2 - \cdots\right) \tag{13a}$$

or, in the so-called [1] "rigid body units" $((J_{\text{rig}})_{x'x'} = J_{\text{rig}})$:

$$J_\alpha^{(0)}/J_{\text{rig}} = 2\delta \cdot \left(1 - \frac{11}{12}\delta + \frac{5}{12}\delta^2 - \cdots\right). \tag{13b}$$

As it is seen from eqs. (13), the value, as well as the dependence of $J_\alpha^{(0)}$ on the deformation, are essentially different from the "hydrodynamic approximation" of the unified model [1] where $J/J_{\text{rig}} \approx \delta^2$. According to our treatment the rotational motion is of a vortical type, whereas in the unified model it is supposed to be potential [1]. However, it would be difficult to treat our collective motion from the point of view of the conventional classical analogies. For example, under the simplified assumption $\varrho(\boldsymbol{r}') = \text{const}$ and for the prolate form of density distribution, the effective moment of inertia according to eqs. (5) and (2) is numerically equal to the components $J_{x'x'}$ and $J_{y'y'}$ of the inertia tensor of a rigid ellipsoid with a hollow middle of spherical shape (with $r' < a$). However, the moment of inertia $J_{z'z'}$ of such a rigid body is also essentially different from zero, whereas in our approach $J_{z'z'}$ is equal to zero, the axially symmetric nucleus rotating, as we have already mentioned, like a hard needle which is infinitely thin[4]). For the oblate shape of density distribution and for

[4]) In [7] it's noted that the moment of inertia of a rigid ellipsoid with a cut out spherical middle corresponds well to the experimental data. It's also noted there, however, that such a formula cannot be proved. In fact, to say nothing of the physical absurdity of this analogy for the nucleus, while transforming the inertia tensor of such a body to the principal axes, one of the two similar components of the moment of inertia (the third component not being equal to zero) cannot be an effective moment of inertia.

$J_{x'x'}$ and $J_{y'y'}$ one must no longer cut out a sphere but an ellipsoid according to eq. (2). The use of such an obvious picture for illustrating eq. (2) is more difficult for non-axial nuclei and especially in case of diffuse density distribution.

Further, one can see from eqs. (13) that the dependence of $J_\alpha^{(0)}$ on δ in case of small deformations is linear, contrary to the conventional idea about the quadratic dependence. To exclude any possible misunderstanding as to the sign of the deformation, we shall express $J_\alpha^{(0)}$ as a function of the positive parameter β characterizing the general deviation of the nuclear shape from the spherical one (for the definition of β cf. ref. [1]).

For a nucleus having the form of a prolate ellipsoid of revolution we have:

$$J_\alpha^{(0)}/J_{\text{rig}} = \frac{3}{2} \cdot \left(\frac{5}{\pi}\right)^{\frac{1}{2}} \cdot \beta \cdot (1 - 0.66\beta + 0.15\beta^2 - \cdots) \tag{14a}$$

and for the oblate ellipsoid of revolution:

$$J_\alpha^{(0)}/J_{\text{rig}} = \frac{3}{2} \cdot \left(\frac{5}{\pi}\right)^{\frac{1}{2}} \cdot \beta \cdot (1 - 0.75\beta + 0.28\beta^2 - \cdots). \tag{14b}$$

The intrinsic quadrupole moments are as followes:

$$Q_0 = \pm \frac{3}{(5\pi)^{\frac{1}{2}}} \cdot Z \cdot R_e^2 \cdot \beta_e \cdot (1 \pm 0.36\beta_e + 0.3\beta_e^2 \pm \cdots) \tag{15}$$

where the upper sign refers to the prolate ellipsoid of revolution, and the lower to the oblate one, R_e and β_e being respectively values of the charge radius and the charge deformation ($R_e \approx R$, $\beta_e \approx \beta$).

From the given point of view the measurement of $J_\alpha^{(0)}$ can be an independent experiment for determining mass deformation in the same way as the measurement of quadrupole moments serves the aim of determining charge deformation. Then, at least for even-even nuclei (for odd and odd-odd nuclei considerable renormalization of the moments of inertia and the quadrupole moments may take place because of H (rot/in), see II), the accuracy of measuring β from J_α can be rather high because the experimental energy values necessary for the determination of J_α are known with very great precision (whereas the experimental values of Q_0 are determined only with 10–20% accuracy).

In fig. 1 the values of $J_\alpha^{(0)}/J_{\text{rig}}$ calculated from eq. (14a) are compared with the experimental values (for even-even nuclei). The latter have been

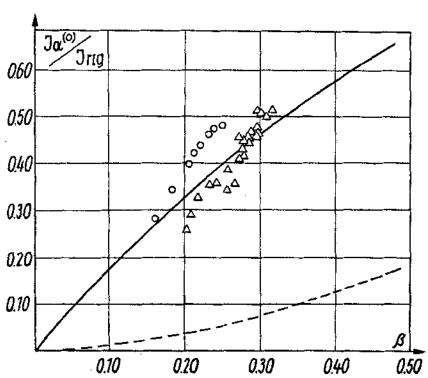

Fig. 1. Moments of inertia of even-even nuclei. The full drawn curve is calculated from eq. (14a). The experimental values for even-even nuclei in the region $152 \leq A \leq 186$ and $A \geq 224$ are plotted by triangles and circles respectively. The moments of inertia corresponding to the hydrodynamic approximation of the unified model are shown by the dotted curve.

found from the position of the first rotational levels and have been plotted according to the ordinary assumption $\beta = \beta_e$. The deformation parameters β_e have been found according to eq. (15) from Q_0. The experimental values Q_0 are taken from [8] and [9]. As one can see from Fig. 1 the agreement with the experiment is good enough (further precise definition see Sec. 5).

Using eq. (2) and assuming $\varrho(r') = \text{const}$, we have for the moments of inertia of the non-axial nucleus:

$$J_{\{{1 \atop 2}} = \frac{3}{2}\left(\frac{5}{\pi}\right)^{\frac{1}{2}} \cdot \beta \cdot \left(1 - \frac{7}{16}\sqrt{\frac{5}{\pi}}\beta \pm \frac{1}{\sqrt{3}}\gamma + \cdots\right) \tag{16a}$$

$$J_3 = \left(\frac{15}{\pi}\right)^{\frac{1}{2}} \cdot \gamma \cdot \beta \cdot \left(1 - \frac{1}{4}\sqrt{\frac{5}{\pi}}\beta + \cdots\right) \tag{16b}$$

where γ is the usual parameter of non-axiality (see, e.g., [1]). The moments of inertia in eqs. (16a, b) are in rigid sphere units $\left(J_{rs} = \frac{2}{5} M_A \cdot R^2\right)$.

We interpret (in the same way as DAVYDOV and FILIPPOV [2] do) the second excited state of deformed even-even nucleus with $I^{\pi\alpha} = 2^+$ as a rotational state. Then from eqs. (16) and (11b) we have:

$$\gamma \approx \frac{1}{\sqrt{3}} \frac{E_{2^+}(1)}{E_{2^+}(2)}. \tag{17}$$

Eq. (17) is rather exact (accuracy 5 to 10%) in the wide region of deformed nuclei ($150 < A < 190$, $A > 220$). At the same time the calculations according to eq. (17) are elementary.

It is worth noting that in our approach the violations of axial symmetry prove to be considerably smaller for the same nuclei than in the DAVYDOV-FILIPPOV model [2]. For example, according to eq. (17) $\gamma = 3°$ for Dy^{160} and $\gamma = 6°$ for Os^{186} instead of 12° and 18° according to [2]. Such a great difference in values of γ is natural, for in our approach J_3 is proportional to γ whereas it is proportional to γ^2 in the DAVYDOV-FILIPPOV model. In the latter the dependence of J_ν on γ is assumed to have the following form:

$$J_\nu = \frac{\hbar^2}{A}\sin^2\left(\gamma - 2\pi \cdot \frac{\nu}{3}\right).$$

Such a functional relationship corresponds to the model of surface oscillations of a liquid drop $\left(\text{with } A = \frac{\hbar^2}{4B\beta^2}\right)$.

One must also add that smaller values of the parameters of non-axiality are in better agreement with the calculations of the equilibrium nuclear shape [10].

We don't consider the electromagnetic transition probabilities in this work. Let's only note that, neglecting the connection between rotation and intrinsic motion, one can get for the reduced probabilities of electromagnetic $E2$ transitions between the rotational bands $|IK = 2\alpha\rangle$ and $|I'K = 0\alpha\rangle$ the following relation

$$B(E2|I \to I') = \frac{5}{32\pi} \cdot \frac{2I' + 1}{2I + 1} \cdot (22I'0|I2)^2 \cdot e^2 \cdot Q_0^2 \cdot \sin^2\gamma_e \tag{18}$$

where $(j_1 m_1 j_2 m_2 | j m)$ is the CLEBSCH-GORDAN coefficient, γ_e is the parameter of non-axiality for nuclear charge. Eq. (18) is justified for small values of γ_e. The probabilities ratios obtained in accordance with eq. (18) correspond to ALAGA's rules [11].

The detailed analysis of rotating non-axial nuclei in the framework of our approach (including consideration of large deviations from axial symmetry and of electromagnetic transitions) will be published in a separate paper.

5. Further precise definitions

1. In this paper we calculated the moments of inertia in zero approximation in the coupling H(rot/in). If H(rot/in) is taken into consideration it leads to a renormalization of the moments of inertia, and the experimental values of J_α are sure to include renormalization. The effects of H(rot/in) (renormalization including) are considered in greater detail in paper II. Here we only notice the following.

The renormalization of J_α due to H(rot/in) is rather essential for odd and odd-odd nuclei. This is the principal reason why the moments of inertia of the odd and odd-odd nuclei are considerably larger than those of the neighbouring even-even nuclei. The calculation of renormalization, effects of pairing correlations included, is being done by us at present. The corresponding formulae are given in paper II and the preliminary estimations show satisfactory agreement with experiment.

For even-even nuclei the renormalization is essential only for the two-quasi particle states and is small for the ground rotational band (see II.) $J_\alpha^{(0)}$ values of this band are given in the Figure.

2. When we plotted the experimental values of the moments of inertia (see Figure) we assumed, as usual, that mass deformation and charge deformation are equal. Certainly the possible difference between β and β_e is not great and cannot change the given results in any fundamental way. However, in view of the fact that the developed method opens the possibility for finding the difference between β and β_e derived from experimental values J_α and Q, the utilization of such a possibility seems to be very tempting. Certainly the final conclusion about values β and β_e can be drawn only after renormalization of J_α and Q because H(rot/in) will be taken into account. However, even now the results shown in the Figure give us the possibility to draw some preliminary conclusion as to the existence of a systematic exceeding of value β over β_e for the nuclei in the region $A > 224$.

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Leningrad (USSR), Leningrad State University, Research Physical Institute.

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