alternative to the energy interval, $S$, one could consider a fixed number of orbitals above and below the Fermi surface. With $\rho \propto A$, this number should be proportional to $A^{1/2}$ or equivalently to $N^{1/2}$ for neutrons and $Z^{1/2}$ for protons. A standard procedure in modified oscillator calculations is to consider $(15N)^{1/2}$ and $(15Z)^{1/2}$ orbitals, respectively, above and below the Fermi surface (Nilsson *et al.*, 1969). The following estimate is then obtained for the coupling constant:

$$G = -\left[\rho \ln\left(\frac{\Delta}{2S}\right)\right]^{-1} = -\left[\rho \ln\left(\frac{12/\sqrt{A}}{2(15 \cdot A/2)^{1/2}} \cdot \rho\right)\right]^{-1} \simeq \frac{1}{A} \cdot 18.7 \text{ MeV}$$

where the value of $\rho$ according to the harmonic oscillator expression has been inserted.

With $N \neq Z$, one could furthermore expect that $G_n \neq G_p$. As $\Delta_n \simeq \Delta_p$, we should from the uniform model formula require that the same cut-off in energy is made for neutrons and protons and furthermore that $\rho_n G_n = \rho_p G_p$. In a similar way as in problem 6.12, we consider only first order terms in the small parameter $x = (N - Z)/A$ to obtain

$$G\begin{pmatrix} n \\ p \end{pmatrix} = \frac{G_0}{A}\left[1 \mp \frac{1}{3}\left(\frac{N-Z}{A}\right)\right]$$

where $G_0$ is a constant, which is common for protons and neutrons. One furthermore finds that it is approximately correct to make the cut-off as suggested above with the number of 'paired orbitals' proportional to $\sqrt{N}$ and $\sqrt{Z}$, respectively.

In realistic calculations, one should determine the pairing constants somewhat more carefully. A possibility is to assume a pairing matrix element

$$G\begin{pmatrix} n \\ p \end{pmatrix} = \left(g_0 \mp g_1 \frac{N-Z}{A}\right) A^{-1} \text{ MeV}$$

where the general dependence on neutron and proton number has been taken from the uniform model expression. With $(15N)^{1/2}$ or $(15Z)^{1/2}$ orbitals considered above and below the Fermi surface, the constants $g_0$ and $g_1$ have been determined from a fit of the $\Delta$-value at the calculated ground state deformation to the odd–even mass differences for a number of nuclei in the $A = 150$–$250$ region (Nilsson *et al.*, 1969). The quality of the fit is illustrated for rare-earth nuclei in fig. 14.4 where the constants

$$g_0 = 19.2 \text{ MeV}; \qquad g_1 = 7.4 \text{ MeV}$$

have been used.

When calculating the nuclear energy by the shell correction method, the

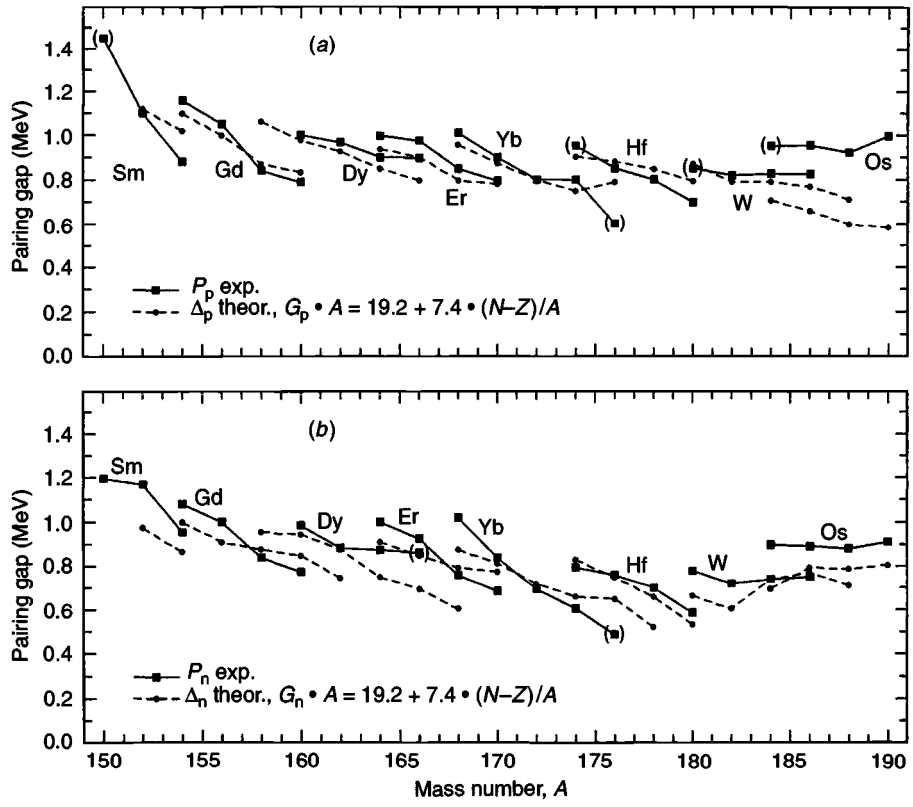


Fig. 14.4. a) Calculated pairing gap $\Delta_p$ for nuclei in the rare-earth region compared with odd–even mass differences, $P_p$. The latter were extracted from measured masses of even-$Z$ and odd-$Z$ isotones using the difference formula illustrated in problem 3.5. The theoretical pairing gaps were extracted at the calculated equilibrium deformations with the parameters given in the text (from Nilsson *et al*, 1969). b) Same as part a) but for neutrons instead, i.e. pairing gaps $\Delta_n$ compared with odd–even mass differences extracted from measured masses of odd-$N$ and even-$N$ isotopes.

parameters of the liquid-drop energy are fitted to reproduce the average trends. Thus, one must assume that the average pairing energy is also accounted for. This means that we should only consider the variation of the pairing energy around an average value, $\delta E_{\text{pair}} = E_{\text{pair}} - \langle E_{\text{pair}} \rangle$. The average value can be estimated from the uniform model. It comes out as

$$\langle E_{\text{pair}} \rangle = -\frac{1}{2}\left(\rho_n \Delta_n^2 + \rho_p \Delta_p^2\right) \approx -(1.15 + 1.15)\ \text{MeV} = -2.3\ \text{MeV}$$

independently of mass number, $A$. In calculating this value, we have used the harmonic oscillator estimate for the level density and the empirical

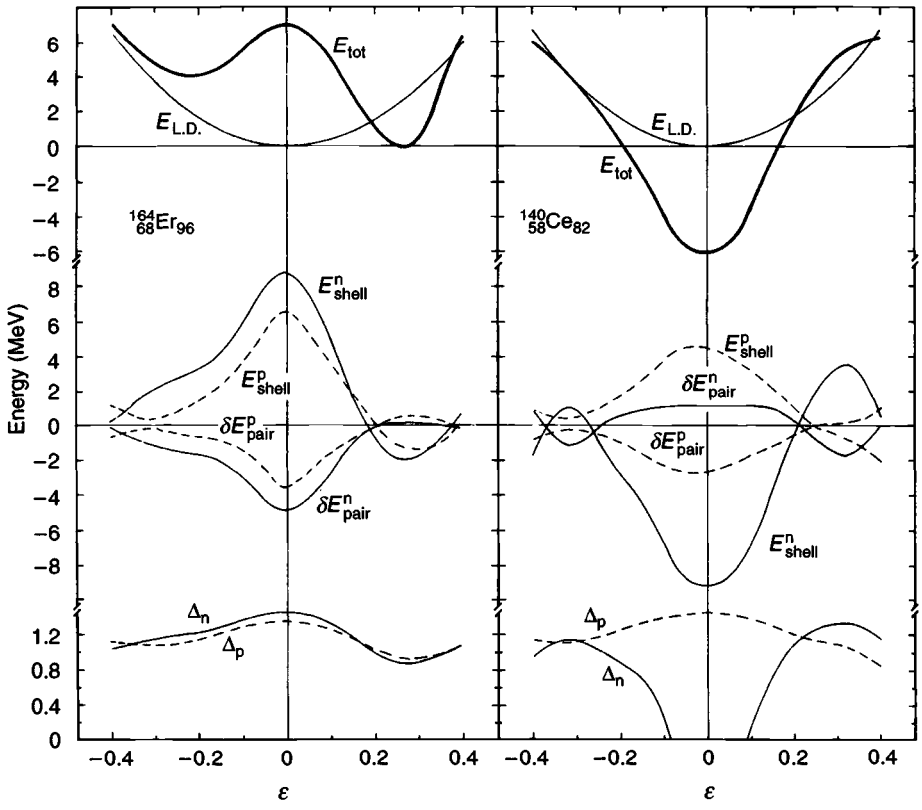


Fig. 14.5. The different energy contributions for protons (p) and neutrons (n) which build up the total energy in the shell correction approach are exhibited as a function of quadrupole deformation for $^{164}$Er and $^{140}$Ce. The shell energies $E_{shell}$ were calculated by the Strutinsky method. The BCS formalism was used to calculate the pairing energies $E_{pair}$ (where the average value, −1.15 MeV in the present approach, has been subtracted) and the pairing gap. The total energy $E_{tot}$ is obtained as the sum of the liquid-drop energy, $E_{L.D.}$, the shell and the pairing energies.

value, $12/\sqrt{A}$ MeV, for the pairing gap. It is also possible to calculate the level density and especially its variation with energy, $\rho(e)$, by the Strutinsky method (chapter 9). The average pairing energy is then calculated numerically leading to a more accurate and systematic method to obtain $\langle E_{pair} \rangle$ (Brack *et al.*, 1972).

In fig. 14.5, the variation of the pairing energy and its relation to the shell correction energy are illustrated as functions of quadrupole deformation, $\varepsilon$, for the nuclei $^{164}$Er and $^{140}$Ce. The energies were calculated from the single-particle orbitals of figs. 11.5 and 11.6. Let us first consider the case of $^{164}$Er. From the single-particle diagram, the level density around the Fermi surface