

• Translational invariance

- ↓
- the coordinate system  $K$  is displaced to a new set of axes  $K'$
- $|A\rangle_K \Rightarrow$  translated state  $|A'\rangle \in K'$
- the properties of  $|A'\rangle$  described by an observer in  $K'$  are identical to those of  $|A\rangle$ , as seen from  $K$ .

$$\langle B'|A'\rangle = \langle B|A\rangle$$

Transformation

$$|A\rangle \xrightarrow{\text{Unitary}} |A'\rangle \in K$$

$$|A'\rangle = U|A\rangle = \sum_B |B\rangle \langle B|U|A\rangle$$

$$|A'\rangle_{K'} = \sum_B U_{BA} |B\rangle_K$$

• translational invariance:  $\langle B'|T'|A'\rangle = \langle B|T|A\rangle$

$\hookrightarrow$  operator expressed in  $K$   
 $\hookrightarrow$  operator described in  $K'$

$$T' = U T U^{-1}$$

$$|A'\rangle = \sum_B |B\rangle \langle B|U|A\rangle$$

$$\langle B'|T'|A'\rangle = \langle B|T|A\rangle$$

$$\langle B|U^{-1}T'U|A\rangle = \langle B|T|A\rangle$$

$$\Rightarrow U^{-1}T'U = T \Rightarrow$$

$$T' = U T U^{-1}$$

$$V(a) = e^{-\frac{i}{\hbar} \vec{a} \cdot \vec{P}}$$

$\vec{a}$ : displacement

$$V(\delta_a) = 1 - \frac{1}{\hbar} \delta \vec{a} \cdot \vec{P} \rightarrow \vec{P} : \text{generator of infinitesimal translation}$$

### Time displacements

$$\begin{array}{ccc} |A\rangle & \xrightarrow{\text{unitary transformation}} & |A'\rangle \\ \boxed{V(t_0) = e^{\frac{i}{\hbar} H t_0}} & & \end{array}$$

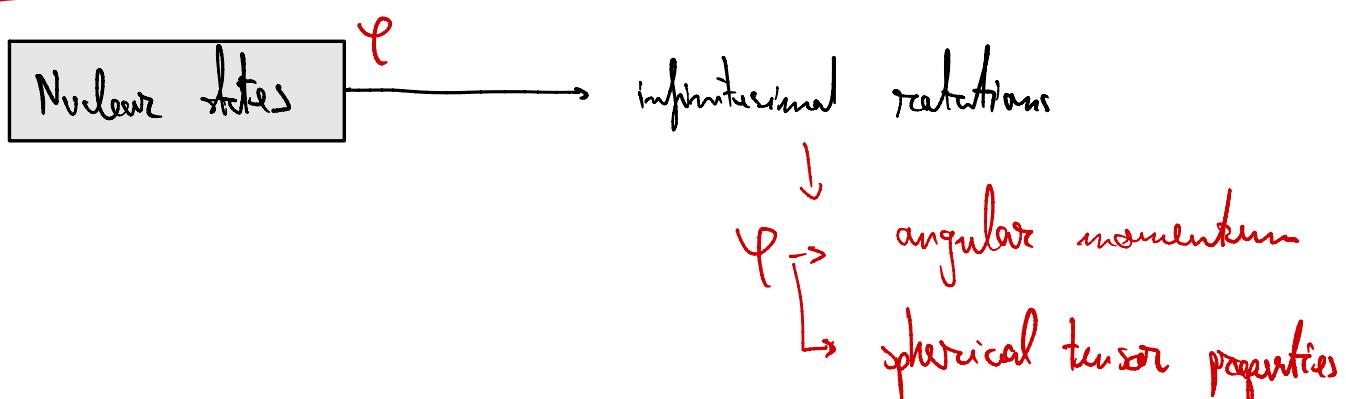
$H \rightarrow$  total energy

$$T' = T(+)= e^{\frac{i}{\hbar} H t} \quad T_{(t=0)} \quad e^{-\frac{i}{\hbar} H t} \quad T' = U T U^{-1}$$

↳ Conservation of total momentum  $\rightarrow$  expression of the commutability  
 of spatial translations and time displacements

$\Downarrow$   
 translations = time-independent operations

### Rotations



- Finite rotations  $\rightarrow$  intrinsic coordinate system

$\vec{x}$  ↳ direction of the axis of rotation  
 ↳ magnitude of the rotation angle

$$R(\vec{x}) = e^{-i\vec{x} \cdot \vec{I}}$$

$\vec{I}$  → total angular momentum

- Hamiltonian of a nucleus

$$H = \sum_k \frac{(\vec{p}_k)^2}{2M_k} + V$$

↓  
 k-th particle  
 interactions

$$M = \sum_k M_k \quad \Rightarrow \quad H = H_{\text{int}} + \frac{(\vec{P})^2}{2M}$$

↓  
 intrinsic Hamiltonian → Galilean invariant

$$R_{\text{cm}} = \frac{1}{M} \sum_k M_k \vec{r}_k \rightarrow \text{center-of-mass coordinate}$$

Dynamics of the system

- intrinsic motion
- center-of-mass motion  
(system as a whole)

$$\frac{i}{\hbar} [H, R_{\text{cm}}] = \frac{1}{M} \vec{P}$$

$$H = \left( H_{\text{int}} + c^2 \vec{P}^2 \right) \frac{1}{2}$$