

and

$$(\ell + \mathbf{s})^2 = \mathbf{j}^2$$

i.e.

$$\ell \cdot \mathbf{s} = \frac{1}{2} (\mathbf{j}^2 - \ell^2 - \mathbf{s}^2)$$

We shall now assume that it is possible to construct wave functions for which  $\ell$ ,  $\mathbf{j}$  and  $\mathbf{s}$  are good quantum numbers and  $\ell \cdot \mathbf{s}$  therefore an eigenoperator. Then (expressing  $\ell$  and  $\mathbf{s}$  in convenient units of  $\hbar$ ):

$$\ell \cdot \mathbf{s} \rightarrow \frac{1}{2} [j(j+1) - \ell(\ell+1) - s(s+1)]$$

We have thus

$$2 \langle \ell \cdot \mathbf{s} \rangle = \begin{cases} \ell & \text{for } j = \ell + \frac{1}{2} \\ -\ell - 1 & \text{for } j = \ell - \frac{1}{2} \end{cases}$$

(expressing as before  $\ell$  and  $\mathbf{s}$  in units of  $\hbar$ ). As we shall see later, out of the  $2 \cdot (2\ell + 1)$  states with a given  $\ell$  we form one group of states ( $2j + 1 = 2\ell + 2$  of them) with  $j = \ell + \frac{1}{2}$  and one with  $j = \ell - \frac{1}{2}$  ( $2j + 1 = 2\ell$  of them). These two groups separate energy-wise. E.g., the g-shell with a degeneracy  $2 \cdot (2\ell + 1) = 18$ , splits up into  $g_{9/2}$  (degeneracy 10) and  $g_{7/2}$  (degeneracy 8), as is shown in fig. 6.3 below.

## 6.2 'Realistic' nuclear one-body potentials

With the addition of the spin-orbit term, the Woods-Saxon (WS) potential takes the form

$$V_{\text{WS}} = V(r) + V_{\text{LS}} + V_{\text{C}}$$

with

$$V(r) = -\frac{V_0}{1 + \exp[(r - R)/a]}$$

and

$$V_{\text{LS}} = \lambda \frac{1}{r} \frac{\partial V_{\text{SO}}(r)}{\partial r} \ell \cdot \mathbf{s}$$

where we have used the notation  $V_{\text{SO}}$  to indicate that one might choose the radial function entering into  $V_{\text{LS}}$  somewhat different from the central potential,  $V(r)$ . The Coulomb potential,  $V_{\text{C}}$ , enters only for protons and is generated by a charge  $(Z - 1)e$ , which is uniformly distributed (or possibly with a diffuse surface) inside the nuclear volume.

In practical applications the Woods-Saxon potential has the disadvantage

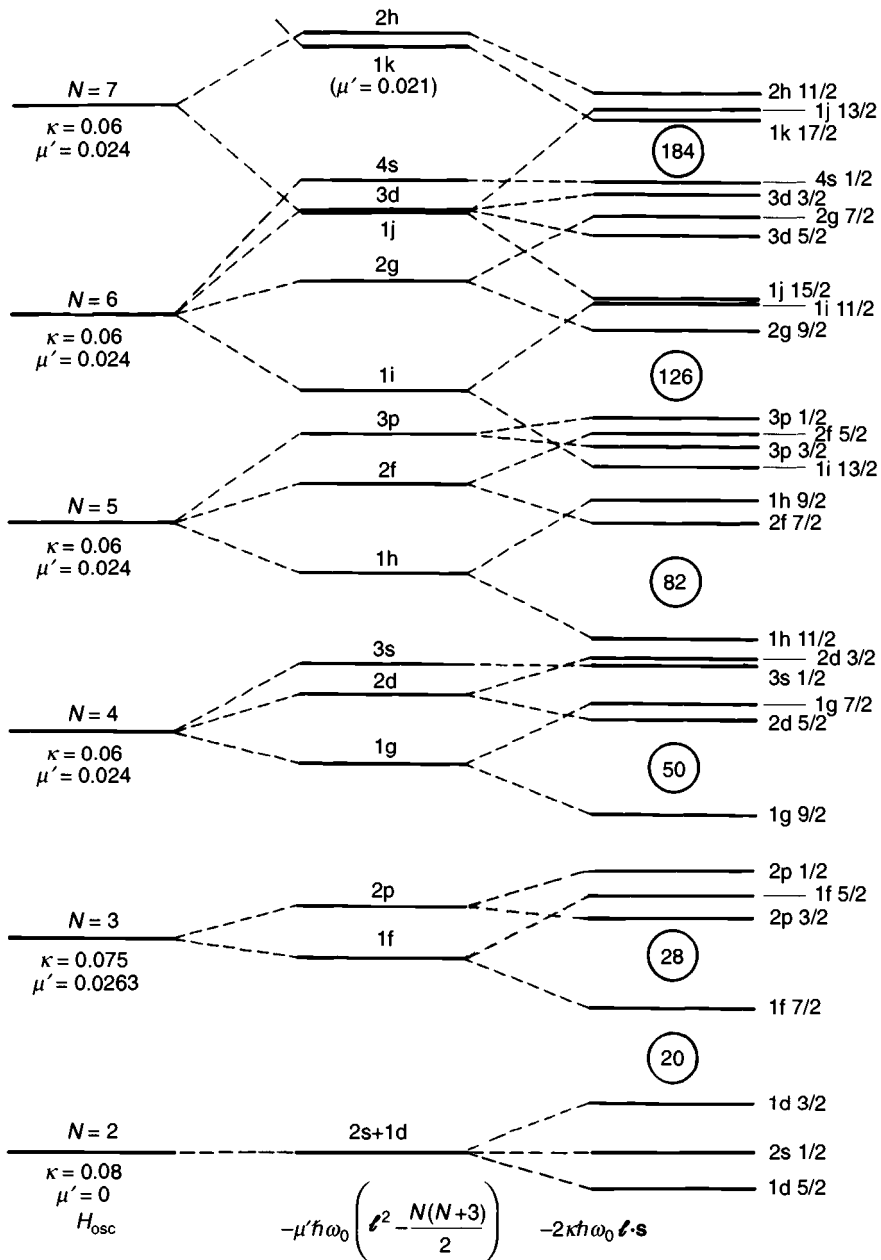


Fig. 6.3. To the left in the figure the pure oscillator shells are exhibited. In the middle graph the effects of an  $\ell^2$ -term proportional to  $\mu'$  are shown. Finally on the right in the figure the added effect of the  $\ell \cdot \mathbf{s}$  term is included. The  $\kappa$ - and  $\mu'$ -values employed, which are different for the different  $N$ -shells, are shown on the left.

that it cannot be solved analytically. This is contrary to the so-called modified oscillator (MO) potential (Nilsson 1955, Gustafsson *et al.* 1967)

$$V_{\text{MO}} = \frac{1}{2}\hbar\omega_0\rho^2 - \kappa\hbar\omega_0 \left[ 2\ell \cdot \mathbf{s} + \mu \left( \ell^2 - \langle \ell^2 \rangle_N \right) \right]; \quad \rho = \left( \frac{M\omega_0}{\hbar} \right)^{1/2} r$$

where, as already suggested in this chapter, the last term in parentheses has the effect of interpolating between the oscillator and the square well and thus reproducing effectively the Woods–Saxon radial shape (in fact, for  $\kappa\mu = \mu' = 0.04$ , the energy levels order of the infinite square well potential is nearly exactly reproduced, see e.g. fig. 27 of Ragnarsson, Nilsson and Sheline, 1978). The  $\ell^2$ -term alone would result in a general compression of the shells. To avoid this undesired feature, the average value of  $\langle \ell^2 \rangle$ , taken over each  $N$ -shell, is subtracted. This average, which is calculated in problem 6.7, takes the value

$$\langle \ell^2 \rangle_N = \frac{N(N+3)}{2}$$

Once the analytic form of the potential is chosen, it remains to fix the parameters. For the Woods–Saxon potential, they have a simple physical meaning. Thus, the nuclear radius  $R$ , the diffuseness depth  $a$  and the potential depth  $V_0$  should have values in reasonable agreement with Hartree–Fock potentials such as those exhibited in fig. 5.3. The radial function  $V_{\text{SO}}(r)$ , which enters into  $V_{\text{LS}}$ , is generally parametrised in a similar way to  $V(r)$ . Thus, together with the coupling strength  $\lambda$ , another four parameters are added. Furthermore, there is one potential for protons and another for neutrons and in principle also one potential for each combination of  $Z$  and  $N$ .

In practice, the same potential is used for some limited region of nuclei and furthermore, some of the parameters are more or less arbitrarily put equal. For example,  $V_{\text{SO}}(r)$  is often chosen identical to  $V(r)$ , the same diffuseness depth  $a$  and/or spin–orbit coupling strength  $\lambda$  is used for protons and neutrons, etc. The rather small number of remaining free parameters might then be varied within reasonable limits in order to describe different nuclear properties (mainly excitation spectra, see below) as accurately as possible. It then also seems reasonable that the different parameters should vary smoothly, or maybe even stay constant, when considered as functions of mass number (or proton and neutron number). A recent fit of the Woods–Saxon parameters ('universal parameters') is briefly described by Nazarewicz *et al.* (1985). Let us also mention the folded Yukawa potential, which is parametrised in a different way to the Woods–Saxon potential but which appears very similar in practical applications. Some standard parameters for

the folded Yukawa potential have been published e.g. by Möller and Nix (1981).

In the modified oscillator potential there are basically three parameters,  $\omega_0$ ,  $\kappa$  and  $\mu$  for each kind of nucleon. If we for the moment neglect the neutron–proton differences of the three parameters,  $\omega_0$  is used to determine the radius of the resulting matter distribution (from the wave functions of the occupied orbitals),  $\mu$  or rather  $\mu' = \kappa\mu$  can be viewed as simulating the surface diffuseness depth, and  $\kappa$  is the spin–orbit coupling strength.

To illustrate further the effect of the  $\ell \cdot s$ - and  $\ell^2$ -terms, we show on the left in fig. 6.3 the pure harmonic oscillator levels. Next is shown how the term  $-\mu'\hbar\omega_0[\ell^2 - N(N+3)/2]$  energetically favours high- $\ell$  subshells. Finally, in the third column of levels a spin–orbit term  $-2\kappa\hbar\omega_0\ell \cdot s$  has been added. The values of  $\kappa$  and  $\mu'$  are roughly those that fit the neutrons in the  $^{208}_{82}\text{Pb}_{126}$  region of nuclei as to level order etc. To make the same diagram applicable with somewhat improved accuracy in the lighter-element region, smaller  $\mu'$ -values and somewhat larger  $\kappa$ -values have been used in the plot for the lower shells, as indicated in the figure. (This is roughly the same fit as made by Nilsson (1955).)

In the applications of the following chapters, we will mainly concentrate on the modified oscillator (MO) potential. This is so because the physical effects we want to illustrate come out in a similar way in all reasonable potentials (or Hartree–Fock calculations) and then we want to add as few calculational difficulties as possible. To conclude this chapter, we will discuss the parameters of the MO potential and the experimental information used for their determination in some detail.

### 6.3 The nuclear volume parameter

Let us first consider the parameter  $\omega_0$  of the MO (modified oscillator) potential. The radial coordinate of the nuclear wave function is  $(M\omega_0/\hbar)^{1/2} \cdot r$ . The characteristic length is thus  $(\hbar/M\omega_0)^{1/2}$ . From the wave functions we may calculate a total nuclear density as a sum of all the single-particle densities from which the average radius could be further studied and compared with experiments. A simply accessible quantity for oscillator wave functions (or for any wave function that is given in an oscillator basis) is, however

$$\langle r_i^2 \rangle = \left( N_i + \frac{3}{2} \right) \frac{\hbar}{M\omega_0}$$

(see problem 6.8).

In terms of this we have already in preceding chapters defined an average