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Collective Excitations and a Backbending Phenomenon in ^{156}Dy*

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Abstract—We propose a self-consistent practical method to study collective excitations in rotating nuclei within the cranking + random phase approximation approach. It consists in solving the cranking Hartree–Bogolyubov equations for the modified Nilsson potential + monopole pairing forces. Further, the mean field results are used to construct collective vibrations treated in the random phase approximation (RPA). Special attention is paid to fulfill all conservation laws in the RPA to separate spurious and physical solutions. We demonstrate that the backbending in ^{156}Dy can be explained as a result of the disappearance of collective γ vibrations of the positive signature in the rotating frame. © *2004 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

The backbending phenomenon has been a subject of interest in nuclear physics for more than three decades [1]. It is widely accepted that the phenomenon is caused by the rotational alignment of angular momenta of a nucleon pair occupying a high-j intruder orbital near the Fermi surface. The alignment breaks a Cooper pair and leads to a sudden increase in the kinematical moment of inertia $\mathcal{J} = I/\Omega$ along the yrast level sequence as a function of rotational frequency Ω.

The mainstream of microscopic analysis of the backbending has been carried out in terms of various cranking Hartree–Fock–Bogolyubov (CHFB) calculations. In medium-heavy and heavy nuclei, the studies are based on the pairing plus quadrupole–quadrupole interaction (cf. [2]) and on effective forces such as the Gogny [3] and the Skyrme interactions [4]. It should be noted that there also exist a few calculations of the backbending in lighter nuclei such as ^{48,50}Cr in the shell model framework [5]. In this paper, we attempt to shed light upon the role of collective excitations in the backbending phenomenon.

The application of the shell model to heavy nuclei loses it feasibility due to a large number of relevant basis states. The CHFB approach, being a mean field theory, violates fundamental symmetries such as the particle number and the angular momentum conservation laws. However, a correct description of the backbending requires dealing with states that have good quantum numbers. The RPA is one of the effective methods to restore broken symmetries. In the case of broken symmetries, the RPA separates out the collective excitation associated with each broken symmetry as a zero-energy or *spurious* RPA mode, with an inertial parameter fixed by the approximation (cf. [6]). Furthermore, the RPA provides a consistent way to take into account quantum fluctuations around the mean field solution. In fact, the contribution of quantum fluctuations to the total energy modifies the moment of inertia [7, 8] and, consequently, the analysis of this contribution could bring new insight into the backbending phenomenon. It should be emphasized that it is quite important to achieve self-consistency between the mean field and the single-particle orbitals and total energy minimization at the stage of the application of the RPA. In addition, for detailed analysis of experimental data that manifest a complex interplay between single-particle and collective dynamics, it is important to start with a reliable single-particle basis.

The practical application of the RPA for nonseparable effective forces such as the Gogny or Skyrme interactions in rotating nuclei requires too large a configuration space. Moreover, different parametrizations of effective forces do not provide a well-established single-particle spectrum. A self-consistent mean field based upon phenomenological cranking Nilsson or Saxon–Woods potentials with pairing forces is still quite competitive from the above point of view. These potentials allow one to construct also a self-consistent residual interaction neglected at the mean field level. The RPA with separable multipole–multipole interaction based on these phenomenological potentials is an effective tool for the analysis

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of low-lying collective excitations of rotating nuclei (cf. [9, 10]). In this paper, we propose that a shape transition, when one of the low-lying collective modes disappears, could lead to the backbending. Our goal is to clarify this issue by using the RPA with separable interaction and comparing the result of our analysis with existing experimental data for ^{156}Dy.

The paper is organized as follows. In Section 2, we introduce our mean field and RPA model. Section 3 is devoted to analysis of experimental data within the framework of our model. The conclusions are drawn in Section 4.

2. THE CRANKING + RPA MODEL

2.1. The Effective Mean Field

We start with the cranking Hamiltonian in the form

$$H_\Omega = H - \sum_{\tau=N,P} \lambda_\tau \hat{N}_\tau - \hbar\Omega I_1 + H_{\rm int}. \quad (1)$$

Here, $H = H_0 + H_{\rm add}$, where $H_0 = \sum_i^A h_0(i)$ is a spherical Nilsson Hamiltonian with

$$h_0 = \frac{\mathbf{p}^2}{2m} + \frac{m}{2}\omega_0^2 r^2 - \hbar\omega_{00}[2\kappa \mathbf{l}\cdot\mathbf{s} + \kappa\mu(\mathbf{l}^2 - \langle \mathbf{l}^2\rangle_N)]. \quad (2)$$

Here, κ and μ are the Nilsson parameters [11], and $\omega_0 = \omega_0(\beta,\gamma)$ and $\hbar\omega_{00} = 41A^{-1/3}$ MeV are oscillator frequencies (β and γ are quadrupole deformation parameters defined below). In the cranking model with the standard Nilsson potential, the value of the moment of inertia is largely overestimated due to the presence of the velocity dependent $\mathbf{l}^2$ term. This shortcoming can be overcome by introducing the additional term $H_{\rm add} = \sum_i^A h_{\rm add}(i)$ with

$$h_{\rm add} = -(\boldsymbol{\nabla}_p V)\cdot m\boldsymbol{\Omega}\times\mathbf{r} = -\Omega m\left(x_2\frac{\partial}{\partial p_3} - x_3\frac{\partial}{\partial p_2}\right)V(\mathbf{r},\mathbf{p}) = \Omega m\omega_{00}\kappa\left[2\left(r^2 s_1 - x_1 \mathbf{r}\cdot\mathbf{s}\right) + \mu\left(2r^2 - \frac{\hbar}{m\omega_{00}}\left(N+\frac{3}{2}\right)\right)l_1\right]. \quad (3)$$

This term restores the local Galilean invariance of the Nilsson potential in the rotating frame (see [12] for details). As a result, the kinematical and dynamical moment of inertia calculated in the mean field approximation become much closer to the experimental values.

Since in ^{156}Dy the backbending occurs in the positive parity yrast band, it is enough to consider only the positive parity residual interaction. The interaction is taken in a separable form

$$H_{\rm int} = -\sum_\tau G_\tau P_\tau^+ P_\tau - \frac{1}{2}\kappa\left[\sum_{r,m} Q_m^{(r)2} + M^{(+)2}\right]. \quad (4)$$

Here, τ = neutron or proton particle number; $P^+ = \sum_k c_k^+ c_{\bar{k}}^+$; and c_k^+ and c_k are creation and annihilation single-particle operators, respectively. The index k labels a complete set of oscillator quantum numbers ($|k\rangle = |Nljm\rangle$) and the index $\bar{k}$ denotes the time-conjugated state. The index r indicates the signature quantum number ($r = \pm 1$) (see [9]). The quadrupole operators Q_m are defined by

$$\hat{Q}_m^{(r)} = \frac{i^{2+m+(r+3)/2}}{\sqrt{2(1+\delta_{m0})}}\left(\hat{Q}_{2m} + (-1)^{(r+3)/2}\hat{Q}_{2-m}\right), \quad m = 0,1,2. \quad (5)$$

The monopole interaction is defined by the positive signature operator $M^{(+)} = r^2 Y_0$. Single-particle matrix elements of any one-body Hermitian operator $F = \sum_{kl} f_{kl} c_k^+ c_l$ (P,Q,M) are determined by the signature, time-reversal, and Hermitian conjugation properties of the operator (cf. [9]). The choice of the constants G and κ will be discussed below.

Using the generalized Bogolyubov transformation for quasiparticles (for example, for the positive signature quasiparticle, we have $\alpha_i^+ = \sum_k \mathcal{U}_{ki} c_k^+ + \mathcal{V}_{\bar{k}i} c_{\bar{k}}$) and the variational principle (see details in [9]), we obtain the Hartree–Bogolyubov (HB) equations for the positive signature quasiparticle energies ε_i,

$$\begin{pmatrix} h(1) & \triangle \\ \triangle & h(2)\end{pmatrix}\begin{pmatrix}\mathcal{U}_i \\ \mathcal{V}_i\end{pmatrix} = \varepsilon_i \begin{pmatrix}\mathcal{U}_i \\ \mathcal{V}_i\end{pmatrix}. \quad (6)$$

It is sufficient to solve the HB equations only for the positive signature, since the negative signature eigenvalues and eigenvectors are obtained from the positive ones.

For the present discussion, we need to know only the structure of matrix elements $h(1)$ and Δ, which are the following:

$$(h(1))_{kl} = \delta_{kl}\varepsilon_{kl}^0 + \langle k|h_{\rm add}|l\rangle - \hbar\Omega\langle k|I_1|l\rangle - \kappa(\langle M\rangle\langle k|\hat{M}|l\rangle + \langle Q_0\rangle\langle k|\hat{Q}_0|l\rangle + \langle Q_2^{(+)}\rangle\langle k|\hat{Q}_2^{(+)}|l\rangle), \quad (7)$$

$$\Delta_{kl} = -\delta_{kl} G_\tau \langle P_\tau\rangle. \quad (8)$$

Here, ε_{kl}^0 are eigenstates of the spherical Nilsson Hamiltonian, and $\langle\ldots\rangle$ means the averaging over the HB vacuum state of the rotating system.

We define the quadrupole deformation β, γ using the parametrization

$$m\omega_0^2\beta\cos\gamma = \kappa\langle Q_0\rangle, \qquad (9)$$
$$m\omega_0^2\beta\sin\gamma = -\kappa\langle Q_2^{(+)}\rangle.$$

It results in the following definition of oscillator frequencies along the principal axes:

$$\omega_i^2 = \omega_0^2\left[1 - 2\beta\sqrt{\frac{5}{4\pi}}\cos\left(\gamma - \frac{2\pi}{3}i\right)\right], \qquad (10)$$
$$i = 1, 2, 3.$$

In addition, we use the volume conservation constraint $\omega_1\omega_2\omega_3 = \omega_{00}^3$. We recall that, in the limit of the harmonic oscillator, the use of the double stretched coordinate representation allows one to represent a three-dimensional oscillator determined by the frequencies [Eq. (10)] as a spherical-oscillator-type potential again, i.e., $x_i'' = (\omega_i/\omega_0)x_i \Rightarrow V_{\text{def.osc}}(\mathbf{r}'') = m\omega_0^2 r''^2/2$.

2.2. Collective Excitations

We introduce the following boson-like operators: $b_{k\bar{l}}^+ = \alpha_k^+\alpha_{\bar{l}}^+$, $b_{kl}^+ a = \alpha_k^+\alpha_l^+$, $b_{\bar{k}\bar{l}}^+ = \alpha_{\bar{k}}^+\alpha_{\bar{l}}^+$. These two-quasiparticle operators are treated in the quasiboson approximation (QBA) as elementary bosons; i.e., all commutators between them are approximated by their expectation values with the uncorrelated HB vacuum. The first equality introduces the positive signature boson, while the other two determine the negative signature ones. The corresponding commutation relations can be found in [9]. The positive and negative boson spaces are not mixed, since the corresponding operators commute. In this approximation, any single-particle operator F can be expressed as $F = \langle F\rangle + F^{(1)} + F^{(2)}$, where the second and third terms are linear and bilinear order terms in the boson expansion. We recall that, in the QBA, one includes all second order terms in the boson Hamiltonian such that $(F - \langle F\rangle)^2 = F^{(1)}F^{(1)}$.

Although in the double stretched representation the residual interaction formally has the same form as in (4), its structure is different. For example, the operators Q_0'' and $Q_2''^{(+)}$ expressed via nonstretched coordinates have the following forms:

$$Q_0'' = \left(1 - \frac{1}{2}\sqrt{\frac{5}{\pi}}\beta\cos\gamma\right)Q_0 \qquad (11)$$
$$- \frac{5}{4\pi}\beta\cos\gamma M^{(+)} + \frac{1}{2}\sqrt{\frac{5}{\pi}}\beta\sin\gamma Q_2^{(+)},$$
$$Q_2''^{(+)} = \left(1 + \frac{1}{2}\sqrt{\frac{5}{\pi}}\beta\cos\gamma\right)Q_2^{(+)}$$
$$- \frac{5}{4\pi}\beta\cos\gamma M^{(+)} + \frac{1}{2}\sqrt{\frac{5}{\pi}}\beta\sin\gamma Q_0.$$

The RPA Hamiltonian is diagonalized by solving the equations of motion

$$[\hat{H}_\Omega, \hat{P}_\lambda]_{\text{RPA}} = i\hbar\omega_\nu^2\hat{X}_\lambda, \qquad (12)$$
$$[\hat{H}_\Omega, \hat{X}_\lambda]_{\text{RPA}} = -i\hbar\hat{P}_\lambda, \quad [\hat{X}_\Omega, \hat{P}_\lambda]_{\text{RPA}} = i\hbar\delta_{\lambda\lambda'}$$

for each signature separately. Here, $\hat{X}_\lambda$ and $\hat{P}_\lambda$ are generalized coordinates and linear momenta of a given intrinsic state with the energy $\hbar\omega_\lambda$. The equations of motion (12) lead to the determinant of the secular equations $\mathcal{F}(\omega_\lambda) = \det(\mathbf{R} - 1/2\kappa)$, which is the fifth and the second order for the positive and negative signature, respectively (cf. [9]).

The zeros of the function $\mathcal{F}(\omega_\lambda) = 0$ yield the RPA eigenfrequencies ω_λ. Since the mean field violates the rotational invariance and particle number conservation law, among the RPA eigenfrequencies, there exist few spurious solutions. Introducing the operator $\Gamma^+ = (\hat{I}_2^{(1)} - i\hat{I}_3^{(1)})/\sqrt{2\langle I_1\rangle}$, we can separate the "spurious" rotational mode $\omega_\lambda = \Omega$ from the vibrational ones of the negative signature part of the full Hamiltonian,

$$H(r = -1) \qquad (13)$$
$$= 1/2\sum_{\lambda(\omega_\lambda\neq 0,\Omega)}(\hat{P}_\lambda^2 + \hbar^2\omega_\lambda^2\hat{X}_\lambda^2) + \hbar\Omega(\Gamma^+\Gamma + 1/2).$$

The other solutions with zero frequency are associated with the rotation around the first axis and the particle number conservation law, since

$$[\hat{H}, \hat{I}_1] = [\hat{H}, \hat{N}_\tau] = 0, \quad \tau = n, p. \qquad (14)$$

The first equation allows one to determine the Thouless–Valatin moment of inertia, while the second one gives the nucleus mass [13, 14]. As a result, for the positive signature RPA solutions, we have

$$H(r = +1) \qquad (15)$$
$$= \frac{1}{2}\sum_{\lambda(\omega_\lambda\neq 0)}\left(\hat{P}_\nu^2 + \hbar^2\omega_\lambda^2\hat{X}_\lambda^2\right)$$
$$+ \frac{1}{2}g_{I_1}\hat{I}_1^{(1)^2} + \frac{1}{2}\sum_\tau g_{N_\tau}\hat{N}_\tau^{(1)^2},$$

where $1/g_{N_\tau}$ and $1/g_{I_1}$ are the mass parameters for neutrons or protons and the Thouless–Valatin moment of inertia, respectively.

3. NUMERICAL RESULTS

3.1. Mean Field Solutions

For a three-dimensional harmonic oscillator, the variational principle with the volume conservation

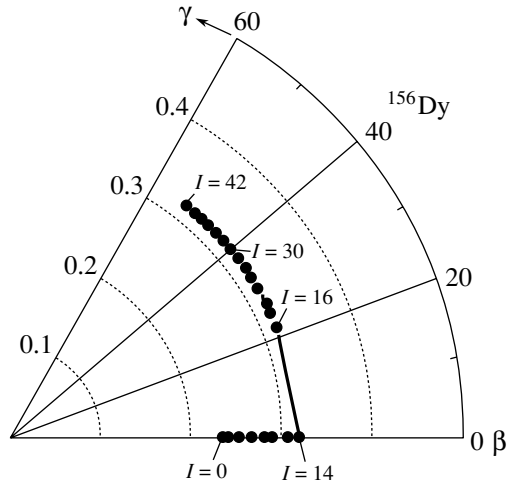

Fig. 1. The experimental dependence of quadrupole deformation parameters β and γ on the angular momentum I. These values were taken from [20], where they were determined from $E2$ transition probabilities along the yrast line.

condition leads to the self-consistent condition $\omega_1^2\langle x_1^2\rangle = \omega_2^2\langle x_2^2\rangle = \omega_3^2\langle x_3^2\rangle$ [15]. This condition fixes the self-consistent constant $\kappa_2 = (4\pi/5)(m\omega_0^2/\langle r''^2\rangle)$ for a residual interaction (quadrupole and monopole terms) in the oscillator potential [16].

At the minimum of the mean field, the expectation values of the operators Q_0 and $Q_2^{(+)}$ expressed in the double stretched coordinates [Eqs. (11)] are zeros,

$$\langle Q_0''\rangle = \langle Q_2''^{(+)}\rangle = 0. \tag{16}$$

We propose to use Eqs. (16) as the self-consistent HF conditions for equilibrium deformations β and γ in the yrast state $|\ \rangle$. Since the residual interaction consists of such a type of operators, with these conditions, we will fulfill the requirement that the residual interaction does not change the equilibrium deformation. The spin–orbit and l^2 terms would break these conditions; however, the violation is very weak.

In order to avoid solving highly nonlinear mean field equations, we use the experimental values of deformation parameters β and γ (see Fig. 1). These values are the input parameters for Eqs. (9), (10) and the HB equations (6). For the pairing field $\Delta_{kl} = \delta_{kl}\Delta_\tau(\Omega)$ in Eq. (6), we assume the phenomenological dependence of the pairing gap Δ_τ ($\tau = n, p$) on the rotational frequency Ω introduced in [17],

$$\Delta_\tau(\Omega) = \begin{cases} \Delta_\tau(0)\left[1 - \frac{1}{2}\left(\frac{\Omega}{\Omega_c}\right)^2\right] & \text{for } \Omega < \Omega_c, \\ \Delta_\tau(0)\frac{1}{2}\left(\frac{\Omega}{\Omega_c}\right)^2 & \text{for } \Omega > \Omega_c. \end{cases} \tag{17}$$

Here, $\Omega_c = 0.32$ MeV is a rotational frequency where the first band crossing (which is approximately the same for protons and neutrons) occurs for ^{156}Dy. The values of pairing gaps $\Delta_n(0) = 0.857$ MeV, $\Delta_p(0) = 0.879$ MeV (for ^{156}Dy) are obtained from the odd–even mass difference (see also [18]). The Nilsson parameters κ and μ are taken from systematic analysis for all deformed nuclei [11]. The calculations have been done for all shells up to $N = 8$. This configuration space exhausted 97% of the energy-weighted sum rule for quadrupole transitions.

To check the self-consistency of our mean field solutions, we calculate Eqs. (16) when all terms are included in the HB equations (6). In fact, the double stretched quadrupole moments are approximately zero for all values of the rotational frequency. The variation of the deformation parameters β and γ around the experimental (equilibrium) values results in a strong deviation of the double stretched quadrupole moments from zero values. We found also that the experimental values of β and γ correspond to the minimum of the total energy, but this minimum is very shallow for higher rotational frequencies.

3.2. Low-Lying Excited States

In order to analyze the low-lying excited states, we construct the Routhian function for each rotational band ($\nu =$ yrast, $\beta, \gamma, \ldots$),

$$R_\nu(\Omega) = E_\nu(\Omega) - \Omega I_1(\Omega). \tag{18}$$

Next, we define the experimental excitation energy in the rotating frame [19],

$$\omega_\nu(\Omega) = R_\nu(\Omega) - R_{\rm yr}(\Omega). \tag{19}$$

The experimental excitation energy [Eq. (19)] can be directly compared with the corresponding solutions $\hbar\omega_\lambda$ of the RPA secular equation. Experimental Routhianes R_ν have been obtained in [20] using experimental data from [21].

It is important to hold self-consistency in the RPA as well as in the mean field. In the pure oscillator, the self-consistent constant κ_2 warrants the fulfillment all conservation laws in the RPA for rotating nuclei [7]. We propose the following recipe to recover lost self-consistency for realistic calculations: we define our constants from the requirement of the fulfillment of the conservation laws Eqs. (14) and the separation of the rotational mode in the RPA. In other words, we force the separation between the spurious and the physical solutions. Otherwise, the physical solutions would be mixed with the spurious one.

We compare the positive signature experimental and calculated lowest excitation energies in Fig. 2. The lowest excited rotational bands are γ and β

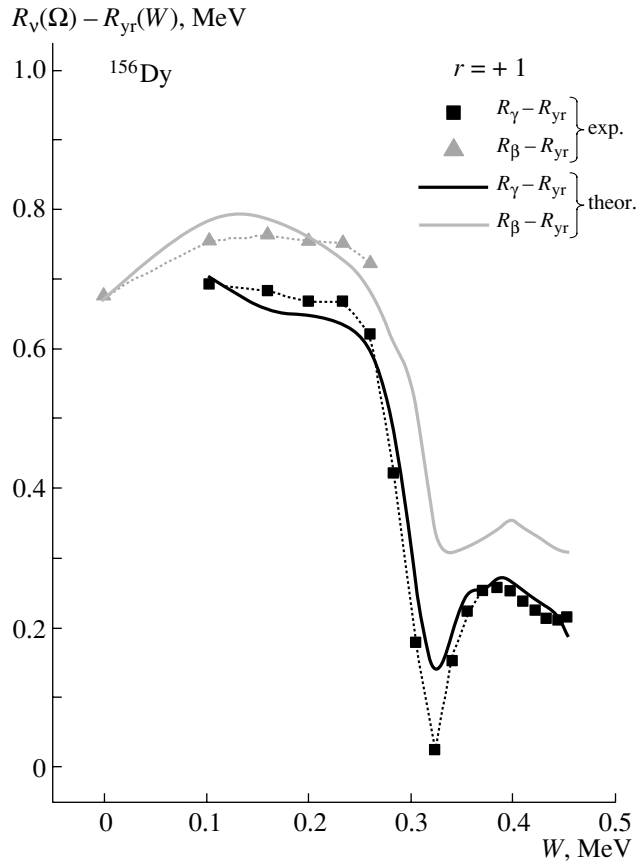

Fig. 2. The comparison of the lowest two experimental and calculated Routhians of positive signature in the dependence on rotational frequency Ω. The low-spin part of the lowest rotational band can be identified with the γ band and the second lowest band starts as β band. Experimental Routhians were obtained in [20] using the data from [21].

bands. The results demonstrate excellent agreement between theory and experiment. The lowest γ-vibrational frequency calculated for the positive signature states (even spins) becomes zero at $\Omega \sim 0.32$ MeV. Near this rotational frequency, backbending occurs in the considered case. The negative signature states describe quite well the γ-vibrational states with odd spins at low rotational frequencies. At higher rotational frequencies, these excitations correspond to wobbling excitations of the three axially deformed systems.

4. SUMMARY

Although the cranking + RPA approach has been well known for a long time, a reliable practical scheme for the analysis of vibrational states in rotating nuclei is still missing. In this paper, we further developed the effective approach [9] to describe numerically, at the same level of accuracy, both the yrast properties and the low-lying excitations. The approach is based on the cranking Nilsson model + monopole pairing forces to study mean field properties and the RPA to analyze the vibrational states. The key point of our approach is to hold the self-consistency at the mean field level and to restore mean field broken symmetries at the RPA. To this aim, we proposed the self-consistency conditions that should be fulfilled at the mean field level and in the RPA.

We solve the RPA equations of motion and obtain remarkable agreement with available experimental data for ^{156}Dy for low-lying excitations. We found that the disappearance of the positive signature γ-vibrational excitations in the rotating frame leads to backbending in this nucleus. It is a *novel* mechanism, in contrast to the standard explanation of the backbending phenomenon as a result of two-quasiparticle rotational alignment. The analysis of the backbending, based on this mechanism, in other nuclei is in progress. At low-rotational frequencies, we describe the low-lying states that belong to the γ band with odd spins as the negative signature γ-vibrational RPA excitations. We predict that, with the increase in the rotational frequency, these states can be associated with the wobbling excitations.

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REFERENCES

1. Z. Szymanski, *Fast Nuclear Rotation* (Clarendon, Oxford, 1980).
2. K. Enami, K. Tanabe, and N. Yoshinaga, Phys. Rev. C **59**, 135 (1999).
3. E. Carote, J. L. Egido, and L. M. Robledo, Phys. Rev. Lett. **80**, 4398 (1998).
4. P. Bonche, P. H. Heenen, and H. C. Flocard, Nucl. Phys. A **467**, 115 (1987).
5. E. Caurier *et al.*, Phys. Rev. Lett. **75**, 2466 (1995); G. Martinez-Pinedo *et al.*, Phys. Rev. C **54**, R2150 (1996).
6. P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
7. R. G. Nazmitdinov, D. Almehed, and F. Dönau, Phys. Rev. C **65**, 041307(R) (2002).
8. D. Almehed, F. Dönau, and R. G. Nazmitdinov, J. Phys. G **29**, 2193 (2003).
9. J. Kvasil and R. G. Nazmitdinov, Sov. J. Part. Nucl. **17**, 265 (1986).
10. Y. R. Shimizu *et al.*, Rev. Mod. Phys. **61**, 131 (1989).

11. S. G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure* (Cambridge Univ. Press, Cambridge, 1995).
12. T. Nakatsukasa *et al.*, Phys. Rev. C **53**, 2213 (1996).
13. D. J. Thouless and J. G. Valatin, Nucl. Phys. **31**, 211 (1962).
14. E. R. Marshalek and J. Weneser, Ann. Phys. (N.Y.) **53**, 569 (1969).
15. T. Troudet and R. Arvieu, Ann. Phys. (N.Y.) **134**, 1 (1981); E. R. Marshalek and R. G. Nazmitdinov, Phys. Lett. B **300**, 199 (1993).
16. H. Sakamoto and T. Kishimoto, Nucl. Phys. A **501**, 205 (1989).
17. R. Wyss, W. Satula, W. Nazarewicz, and A. Johnson, Nucl. Phys. A **511**, 324 (1990).
18. http://t2.lanl.gov/data/map.html.
19. R. G. Nazmitdinov, Sov. J. Nucl. Phys. **46**, 412 (1987).
20. R. Ch. Safarov and A. S. Sitdikov, Izv. Ross. Akad. Nauk, Ser. Fiz. **63**, 162 (1999).
21. http://www.nndc.bnl.gov.