

Figure 2.26. Schematic variation of the energy with deformation for a nucleus with a second minimum. The dashed line corresponds to the liquid drop barrier.

higher than the first minimum, which gives rise to the so-called shape isomeric states [St 66]. Qualitatively, the energy as a function of deformation is shown in Fig. 2.26.

2.9 Shell Corrections to the Liquid Drop Model and the Strutinski Method

2.9.1 Introduction

Up to now we have studied two quite different descriptions of the atomic nucleus. The *liquid drop model* (LDM) assumes that the nucleons produce a spatially uniform density distribution in the nucleus with a sharp edge at the surface. It is able to reproduce the overall features of the nucleus, that is, most properties that depend only in a smooth way on the nucleon number, as, for example, in Chapter 1, the A dependence of the nuclear binding energy (Fig. 1.2). On the other hand, there is the *shell model*. It assumes a quantized independent particle motion in an average potential to be valid, and we have seen in Section 2.3 and 2.8 that this model reproduces nicely those particular nuclear properties in which only the nucleons in the vicinity of the Fermi surface are involved.

Phenomenological shell models (in contrast to Hartree-Fock calculations; see Chap. 5), however, fail to correctly reproduce properties of the nucleus in which *all* nucleons contribute (the so-called bulk properties), like, for instance, the total binding energy. Strutinski [St 67, 68] invented a very elegant method* to reconcile both phenomenological descriptions of the nucleus which eliminates their defects but keeps their qualities. This is the *Strutinski shell correction procedure*. It is able to reproduce not only the

* Investigations on a similar line have been carried out by Myers and Swiatecki [MS 66].

experimental ground state energies of nuclei, but also their dependence on deformation parameters. This method was used extensively for the calculation of energy surfaces in the fission process.

2.9.2 Basic Ideas of the Strutinski Averaging Method

As we discussed in Chapter 1, the nuclear binding energies E as a function of A have a smooth part E_{LDM} well represented by the Bethe-Weizsäcker mass formula (1.4) (Fig. 1.2) and, in addition, an oscillatory part E_{osc} defined by

$$E = E_{\text{osc}} + E_{\text{LDM}}. \quad (2.100)$$

Similar oscillations would occur if we could calculate the exact energy of the many-body system as a function of the deformation and compare it with the corresponding LDM value. These oscillations are due to the occurrence of shell closures, that is, they have maxima at the magic numbers, as we mentioned at the beginning of Section 2.2 (Fig. 2.2). Therefore, their origin is entirely of a quantum mechanical nature. As a matter of fact, if we calculate the binding energies in the shell model [for example, in the Nilsson model; see Eq. (2.90)], one finds such oscillations. Only the corresponding average part is wrong. Furthermore, as we explained at the end of the last chapter, these oscillations are due to a grouping of levels into bunches—the shells. In fact, it is clear that for a smooth distribution of levels, the binding energy per particle depends in a smooth way on the position of the Fermi level, whereas for a shell-like distribution of the levels there is also an oscillatory behavior superimposed on top of it. We demonstrate this in Fig. 2.27, where we compare two such level densities whose average densities are equal.

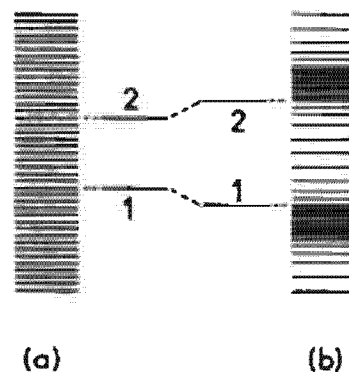


Figure 2.27. Comparison of an equally spaced level density to a schematic shell model level density. For the Fermi level (1), the binding in (b) is stronger than in (a), whereas for (2) the opposite is true.

We see that if the Fermi level is situated just above a shell we get more binding than on the average, whereas if it is placed just below we get less

binding than on the average. This picture is certainly very schematic, since, for example, the real average level density is not a constant; this however, is a smooth effect and should not influence the previous conclusions. The binding energies corresponding to the shell distribution (b) oscillate around the ones given by the "average" level density (a). Therefore, the average level density is responsible for the average behavior of the binding energies. It is this average part which has the wrong value in the phenomenological shell model. It was the decisive idea of Strutinski to calculate only the fluctuating part E_{osc} of the total energy E in Eq. (2.100) within the shell model and to take the rest, E_{LDM} , from the liquid drop model. This procedure contains the assumption that the fluctuating part E_{osc} is well approximated by the fluctuating part of the shell model energy. At the end of this chapter we will discuss the way we can justify this assumption. There then remains only the problem of how to divide up the shell model energy into an oscillating part, E_{osc} , and a smoothly varying part, \tilde{E}_{sh} .

$$E_{\text{sh}} = \sum_{i=1}^A \epsilon_i = E_{\text{osc}} + \tilde{E}_{\text{sh}}. \quad (2.101)$$

[Expression (2.101) does not take into account any effect coming from the two-body interaction as, for example, in Eq. (2.98). We will see that in the end this is of no importance.]

The decomposition (2.101) is a problem which has to be solved completely within the shell model. Therefore, it is useful to introduce the concept of the level density $g(\epsilon)$ by defining $g(\epsilon) \cdot d\epsilon$ as the number of levels in the energy interval between ϵ and $\epsilon + d\epsilon$. In the shell model the level density is given by

$$g(\epsilon) = \sum_i \delta(\epsilon - \epsilon_i). \quad (2.102)$$

If we know $g(\epsilon)$, we can calculate the particle number

$$A = \int_{-\infty}^{\lambda} g(\epsilon) d\epsilon \quad (2.103)$$

with a properly chosen Fermi energy λ .

In the shell model, λ is not defined uniquely by Eq. (2.103). It can be arbitrarily chosen to be between the last filled and the first unfilled level. For the shell model energy, we get

$$E_{\text{sh}} = \int_{-\infty}^{\lambda} \epsilon g(\epsilon) d\epsilon. \quad (2.104)$$

The shell model levels are grouped into bunches with an average distance of $\hbar\omega_0 \simeq 41A^{-1/3}$ (MeV) [see Eq. (2.12)]. Therefore, the level density g shows oscillations with roughly this frequency.

Since, as we have seen in Fig. 2.27, the fluctuations in the shell model energy E_{sh} are due to these oscillations, it is obvious that we can calculate the smooth part \tilde{E}_{sh} in Eq. (2.101) by introducing a continuous function $\tilde{g}(\epsilon)$, which represents the smooth part of the level density $g(\epsilon)$. It should

have the mean functional behavior of g , but must not contain oscillations with a frequency $\simeq \hbar\omega_0$. In a real nucleus the level density is by far more complex than in our simple picture of Fig. 2.27. Its increase, for instance, with increasing energy is in general nonlinear and certainly far from being a constant. Therefore, one has to give a well-defined mathematical prescription of how one can extract from a given level density its mean part. This will be explained in the next section. For the moment, we shall assume that we are given the average part $\tilde{g}(\epsilon)$ of the shell model level density $g(\epsilon)$. Then we can again calculate a corresponding Fermi energy $\tilde{\lambda}$ by the condition

$$A = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) d\epsilon. \quad (2.105)$$

Since \tilde{g} is continuous, $\tilde{\lambda}$ is well defined by this implicit equation and is usually different from λ . For the smooth part of the energy we finally get

$$\tilde{E}_{\text{sh}} = \int_{-\infty}^{\tilde{\lambda}} \epsilon \tilde{g}(\epsilon) d\epsilon. \quad (2.106)$$

The total energy E of the system is therefore given by

$$E = E_{\text{LDM}} + E_{\text{osc}} = E_{\text{LDM}} + E_{\text{sh}} - \tilde{E}_{\text{sh}}. \quad (2.107)$$

As we have seen in Chapter 1, stable liquid drops are always spherical. Because of the additional term E_{osc} , it can happen that in some region of the periodic table the "Strutinski averaged energy" (2.107) has its minimum at finite values of the deformation. However, before we go on to the general discussion of the Strutinski method, let us show how one can define appropriately the average part of the level density.

2.9.3 Determination of the Average Level Density

In this section we have to deal with the problem of how to define in an appropriate mathematical way an average level density, if we are given a shell model density in an infinite three-dimensional well (the restriction to infinite potentials is not essential and the following considerations can be generalized for finite potentials [SI 75]).

$$g(\epsilon) = \sum_{i=1}^{\infty} \delta(\epsilon - \epsilon_i). \quad (2.108)$$

The difficulty comes from the fact that for a three-dimensional potential the level density increases with energy in a nonlinear way. This rise, however, does not go smoothly, but the levels are grouped in bunches roughly $\hbar\omega_0$ apart. If we imagine, for convenience of presentation, the level density smeared out with a Gaussian of width $\ll \hbar\omega_0$, then we get, schematically, the picture in Fig. 2.28. In which way, then, can the average part, and the oscillating part of such a density distribution, be separated?

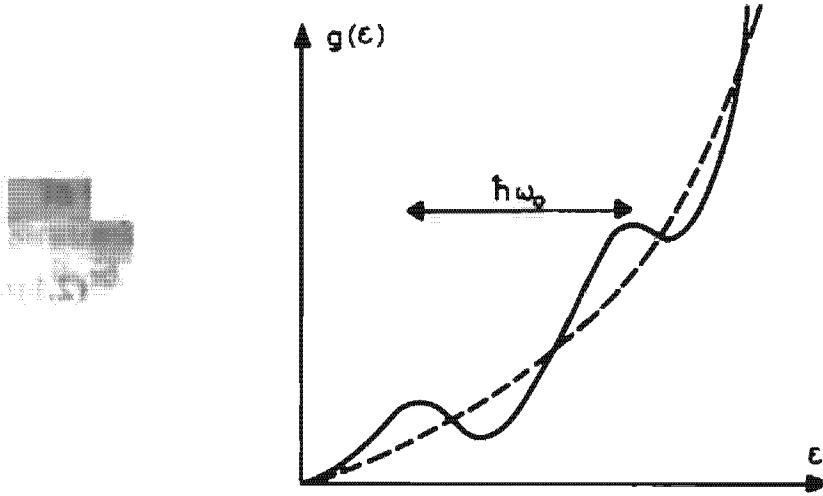


Figure 2.28. Schematic representation of the level density in an infinite three-dimensional potential.

As usual, we will think of the average density as given by a folding procedure

$$\tilde{g}(\epsilon) = \frac{1}{\gamma} \int_{-\infty}^{+\infty} g(\epsilon') f\left(\frac{\epsilon' - \epsilon}{\gamma}\right) d\epsilon', \quad (2.109)$$

and as a straightforward possibility we could think of f as being a Gaussian with $\gamma \simeq \hbar\omega_0$. However, this is not sufficient, because an appropriate averaging procedure should leave the averaged level density $\tilde{g}(\epsilon)$ unchanged if averaged again with the same procedure, that is,

$$\tilde{g}(\epsilon) = \frac{1}{\gamma} \int_{-\infty}^{+\infty} \tilde{g}(\epsilon') f\left(\frac{\epsilon' - \epsilon}{\gamma}\right) d\epsilon'. \quad (2.110)$$

For f a Gaussian and g not a constant, condition (2.110) is clearly not fulfilled. In general, it will be difficult to do this for a given \tilde{g} exactly, and it is not our intention to fulfill Eq. (2.110) for an arbitrary \tilde{g} , since then f would have to be a δ -function. Equation (2.110) will only be required to be fulfilled for smooth functions \tilde{g} . Since the essential contributions to the integral (2.110) come from the vicinity of the point $\epsilon' = \epsilon$, we suppose that $\tilde{g}(\epsilon)$ can be represented locally by a polynomial of degree $2M$ (usually $2M = 2, 4$, or 6). The condition (2.110) can then be fulfilled if one constructs f in the following way:

$$f(x) = P(x)w(x), \quad (2.111)$$

where $P(x)$ is an even polynomial of degree $2M$ and $w(x)$ is a weighting function like a Gaussian or a Lorentzian, for example.

We can check that Eq. (2.110) is satisfied in this approximation by [AS 65]

$$P(x) = \sum_{n=0}^M Q_{2n}(x) Q_{2n}(0), \quad (2.112)$$

where the set of orthogonal polynomials $Q_m(x)$ is determined by the condition

$$\int_{-\infty}^{\infty} w(x) Q_m(x) Q_{m'}(x) dx = \delta_{mm'}. \quad (2.113)$$

If we take for w a Gaussian

$$w(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}, \quad (2.114)$$

then $P(x)$ is given by a generalized Laguerre Polynomial [AS 65, Chap. 22]

$$P(x) = \sum_{n=0}^M (2^{2n} \cdot (2n)!)^{-1} \cdot H_{2n}(x) H_{2n}(0) = L_M^{1/2}(x^2), \quad (2.115)$$

where $H_{2n}(x)$ are the Hermite Polynomials. Thus we have explicitly constructed an averaging function f satisfying condition (2.110) for $\tilde{g}(\epsilon)$ which is locally approximated by a polynomial of degree $2M$. For practical applications, we give the coefficients of the polynomials $L_M^{1/2}(x^2)$ for $M=0, 2, 4, 6$ (Table 2.5):

$$L_M^{1/2}(x^2) = \sum_{n=0}^M a_{2n} x^{2n}. \quad (2.116)$$

Table 2.5 Coefficients of the four lowest Laguerre Polynomials

M	a_0	a_2	a_4	a_6
0	1	—	—	—
1	3/2	-1	—	—
2	15/8	-5/2	1/2	—
3	35/16	-35/8	7/4	-1/6

There remains now the question of how to determine the precise values of γ and M . In general, the results will depend on these parameters. Since we want to take out of $g(\epsilon)$ the oscillation with the approximate frequency $\hbar\omega_0$, our method will only be meaningful if we can find a certain interval of reasonable γ -values ($\gamma \simeq 1-1.5 \hbar\omega_0$) and corresponding M -values within which this dependence is practically negligible. We can say, then, that our results are independent of the averaging procedure (as should of course be the case). More precisely, this means that the averaged energy must show a "plateau" as a function of γ for fixed M within which the "plateau condition" [BP 73]

$$\frac{\partial \tilde{E}_{\text{av}}}{\partial \gamma} = 0 \quad (2.117)$$

is valid. Certainly for an arbitrary distribution of single-particle levels there will be no such plateau. But for the physically interesting distributions there always exists a certain bunching of levels with a frequency of roughly

$\hbar\omega_0$, so that we can expect that there is a plateau. (In practical calculations with finite potentials of the Woods–Saxon type, there are sometimes difficulties in finding a plateau. However, special methods have been developed to deal with such problems [BP 73, SI 75]).

In Chapter 13, we will show that there is a close connection between the Strutinski smoothing procedure and the semiclassical (Thomas–Fermi) method. This is not surprising, since it is known from the Thomas–Fermi theory of atoms that this method gives the relevant average quantities. We therefore urge the reader to consult Chapter 13 for a deeper understanding of this subject.

2.9.4 Strutinski's Shell Correction Energy

Having defined γ and the averaging function $f(x)$, we get the smooth level density from Eqs. (2.108) and (2.109).

$$\tilde{g}(\epsilon) = \sum_i \frac{1}{\gamma} f\left(\frac{\epsilon' - \epsilon_i}{\gamma}\right) \quad (2.118)$$

and the smooth part of the ground state energy in the shell model:

$$\tilde{E}_{\text{sh}} = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) \epsilon d\epsilon. \quad (2.119)$$

The chemical potential $\tilde{\lambda}$ is determined by the condition (2.105)

$$A = \sum_i \tilde{n}_i = \sum_i \int_{-\infty}^{t_i} f(x) dx; \quad t_i = \frac{\tilde{\lambda} - \epsilon_i}{\gamma}. \quad (2.120)$$

The \tilde{n}_i can be considered as generalized occupation numbers. Therefore, for fixed A , Eq. (2.120) allows the determination of $\tilde{\lambda}$ through an iteration procedure. It is convenient to rearrange Eq. (2.119) a little:

$$\tilde{E}_{\text{sh}} = \sum_i \epsilon_i \tilde{n}_i + F, \quad (2.121)$$

where F is then given by

$$F = \sum_i \int_{-\infty}^{\tilde{\lambda}} (\epsilon - \epsilon_i) \frac{1}{\gamma} f\left(\frac{\epsilon - \epsilon_i}{\gamma}\right) d\epsilon = \gamma \sum_i \int_{-\infty}^{t_i} x f(x) dx. \quad (2.122)$$

With this equation and Eq. (2.121), we get for the plateau condition (2.117)

$$\begin{aligned} \frac{\partial \tilde{E}_{\text{sh}}}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \sum_i \epsilon_i \tilde{n}_i + \frac{1}{\gamma} F + \gamma \sum_i \frac{\partial t_i}{\partial \gamma} t_i f(t_i) \\ &= \frac{1}{\gamma} F + \tilde{\lambda} \frac{\partial}{\partial \gamma} \sum_i \tilde{n}_i \\ &= \frac{1}{\gamma} F + \tilde{\lambda} \frac{\partial}{\partial \gamma} A = \frac{1}{\gamma} F. \end{aligned} \quad (2.123)$$

The last equality follows because the particle number A is a constant. Therefore, we find that the plateau condition (2.117) is equivalent to the vanishing of F .

Introducing the shell model occupation numbers

$$\begin{aligned} n_i &= 0 & \text{for } \epsilon_i > \epsilon_F, \\ n_i &= 1 & \text{for } \epsilon_i < \epsilon_F, \end{aligned} \quad (2.124)$$

we find for the total shell correction:

$$E_{\text{osc}} = \sum_i \epsilon_i (n_i - \bar{n}_i) = \sum_i \epsilon_i \delta n_i. \quad (2.125)$$

Figure 2.29 shows the quantities δn_i for the actual case of a deformed Woods-Saxon potential and a Gaussian average with $M=2$ and a width of $\gamma=6.6$ MeV.

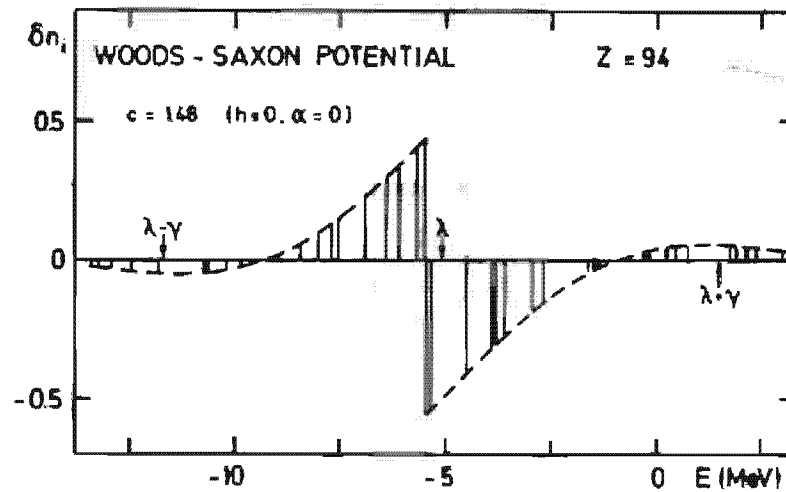


Figure 2.29. Deviations from the shell model occupation numbers resulting from a Strutinski averaging procedure for a Woods-Saxon potential. (From [BP 73].)

It is important to notice that the quantities \bar{n}_i are not actually occupation probabilities, as they can in fact have negative values. On the other hand, the \bar{n}_i behave like real occupation numbers and they indicate, for instance, how far we have to smear out the sharp Fermi surface of the shell model to get the smooth part of the ground state energy.

Having calculated the values δn_i , we are able to calculate the shell corrections to the density in configuration space [Di 71]:

$$\delta \rho_{\text{osc}}(\mathbf{r}, \mathbf{r}') = \sum_i^A \phi_i(\mathbf{r}) \phi_i^*(\mathbf{r}') \delta n_i. \quad (2.126)$$

This expression allows the calculation of averaged expectation values for all single-particle operators.

To give a definite example we will present the results of such a Strutinski calculation for the harmonic oscillator, which is naturally an over-idealized

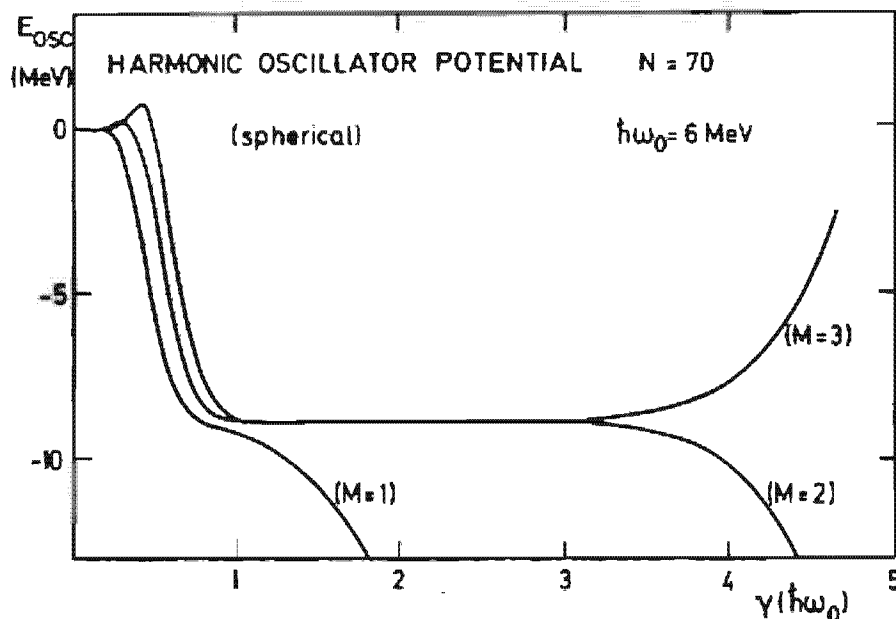


Figure 2.30. Shell correction energy for the harmonic oscillator as a function of γ and M . (From [BP 73].)

system, but which has the advantage of being rather instructive. In Fig. 2.30, the shell correction E_{osc} is shown calculated with a Gaussian weighting as a function of γ and M . We see that for $M \geq 2$ a well pronounced plateau develops at $E_{sh} = -8.91$ MeV.

It can also be shown that a plateau develops at precisely the same energy if we had taken $w = 1/2 \cosh^2 x$ instead of a Gaussian for the weight function.

From the behavior of the values δn_i shown in Fig. 2.29 we conclude that only the shells in the neighborhood of the Fermi surface are important for the calculation of the shell correction. It is therefore usually sufficient to include three major shells in the averaging procedure. Only energies in the neighborhood of the Fermi surface are described well by the shell model, and it is exactly these which go into the shell corrected liquid drop energy. This is quite gratifying, since the phenomenological models are adjusted so as to reproduce the single-particle levels close to the Fermi level.

For the sake of completeness we have to mention that pairing correlations play an important role in heavy nuclei. They are usually treated in the BCS-model (see Chaps. 6 and 7). Since they are closely connected with the level density at the Fermi surface, one also observes oscillations in the pairing energy P . The liquid drop energy is adjusted to experimental masses. Therefore, it already contains the smooth part of the exact pairing energy, and we have to add only the oscillating part P_{osc} . It is obtained using the same philosophy as the difference between the pairing energy P_{BCS} in the BSC-model and its smooth part \tilde{P}_{BCS} . The total energy then has the form

$$E = E_{LDM} + E_{sh} - \tilde{E}_{sh} + P_{BCS} - \tilde{P}_{BCS}. \quad (2.127)$$

The smooth part of the pairing energy \bar{P}_{BCS} can be calculated with similar methods as presented here. For details the reader is referred to the literature [BDJ 72]. Another way to obtain the average properties for pairing correlations would be the extension of the Thomas–Fermi method to superfluid systems (see Sec. 13.2.6)

2.9.5 Shell Corrections and the Hartree–Fock Method

As we will explain in Chapter 5, there exists a theory which allows one to calculate the shell model potential microscopically. This theory is the Hartree–Fock method and its generalizations [HFB; see Chapter 7]. Modern calculations of this type give quite good and detailed agreement of the binding energies over the whole mass region, indicating that in those calculations the bulk properties are also well described. Nevertheless, one can still try to separate the smooth part and the oscillating part of the quantities calculated in this theory. The smooth part should correspond exactly to the liquid drop results, and it has been shown that this is in fact the case [BQ 75a, b]. As an example, Fig. 2.31 shows the deformation energy of the nucleus ^{168}Yb as a function of the quadrupole moment Q_2 .

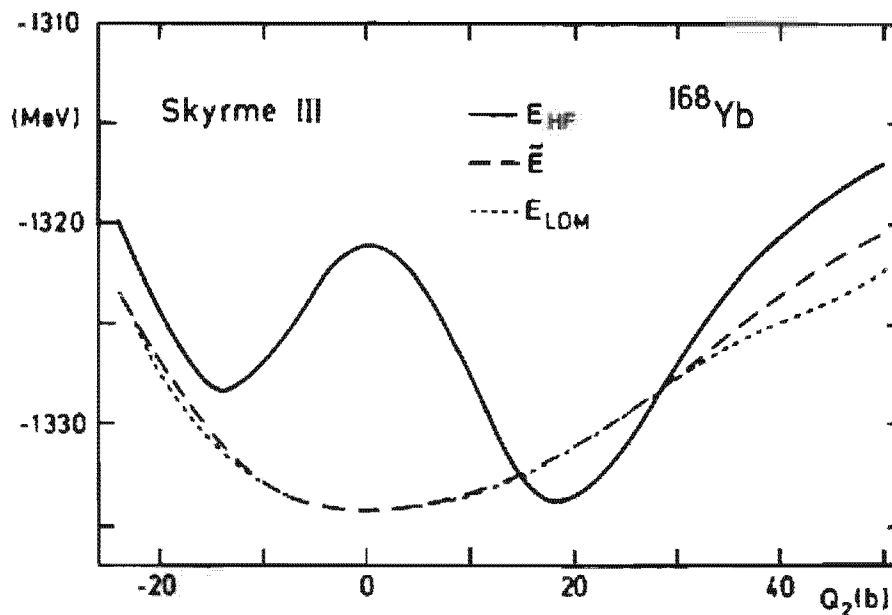


Figure 2.31. Deformation energies of the nucleus ^{168}Yb calculated with the force Skyrme III (see Chap. 4). E_{HF} is the exact HF-energy, \bar{E} the corresponding averaged part [Eq. (2.132)] and E_{LDM} is the liquid drop energy on the same path in the deformation coordinates Q_2 (quadrupole moment) and Q_4 (hexadecupole moment). (From [BQ 75b].)

To show that the Strutinski procedure based on a shell model potential gives, to a high degree of accuracy, the same results as a complicated Hartree–Fock

calculation, we now have only to prove that the oscillating part of the ground state energy in the Hartree–Fock theory is actually the same as the one derived in Eq. (2.125) from a shell model calculation. This we will do in the following, and we advise the reader who is not familiar with Hartree–Fock theory to return to the following somewhat subtle considerations after having progressed to the microscopic theories and, in particular, after having studied Chapter 5.

The Hartree–Fock ground state energy is, in a representation-independent way, given by* [see Eq. (5.28)]:

$$E_0^{\text{HF}} = \text{Tr}(\epsilon \rho) + \frac{1}{2} \text{Tr}_1 \text{Tr}_1 \rho \bar{\epsilon} \rho, \quad (2.128)$$

where ρ is the self-consistent single-particle density and $H = \epsilon + V$ is the total Hamiltonian of the system with ϵ and V the kinetic and potential energies, respectively. The Hartree–Fock part h^{HF} of the Hamiltonian is a one-particle operator, and is defined by

$$h^{\text{HF}} = \epsilon + \text{Tr}_1 \bar{\epsilon} \rho. \quad (2.129)$$

Now we can divide the density of ρ in the sense of Strutinski (Sec. 2.9.3) into a smooth and an oscillating part:

$$\rho = \bar{\rho} + \delta\rho, \quad (2.130)$$

where $\delta\rho$ is defined by analogy with Eq. (2.126) for HF-single-particle energies.

Inserting this into Eq. (2.128) and collecting terms of different order in $\delta\rho$, we get

$$E_0^{\text{HF}} = \bar{E} + \text{Tr} \bar{h}^{\text{HF}} \delta\rho + O(\delta\rho^2), \quad (2.131)$$

where

$$\bar{E} = \text{Tr}(\epsilon \bar{\rho}) + \frac{1}{2} \text{Tr}_1 \text{Tr}_1 \bar{\rho} \bar{\epsilon} \bar{\rho} \quad (2.132)$$

and

$$\bar{h}^{\text{HF}} = \epsilon + \text{Tr}_1 \bar{\epsilon} \bar{\rho}. \quad (2.133)$$

Therefore, if the phenomenological shell model potential gives the same single-particle spectrum in the vicinity of the Fermi surface as the averaged Hartree–Fock potential, \bar{h}^{HF} , then the definitions of the oscillating part of the ground state energy, as given by the second term on the r.h.s. of Eq. (2.131) and by Eq. (2.125), agree if second-order terms in $\delta\rho$ can be neglected. These have been checked numerically and indeed turn out to be small corrections [BKS 72, BDJ 72, BQ 75a].

If the phenomenological single-particle spectrum does not give exactly the same single-particle spectrum as the operator \bar{h}^{HF} , the above statement nevertheless stays true, because small deviations again give contributions of second order in Eq. (2.131). Equation (2.131) is also called the *Strutinski energy theorem* [St 68, St 74]. It states that all shell effects of first order in $\delta\rho$ are taken into account by summing up single-particle energies of an averaged single-particle Hamiltonian.

In this sense, the Strutinski procedure provides a method which reproduces microscopic results in an optimal way using phenomenological models. Needless to say, the latter are much easier to handle for realistic calculations.

* The definition of Tr_1 is given in Eq. (E.19).

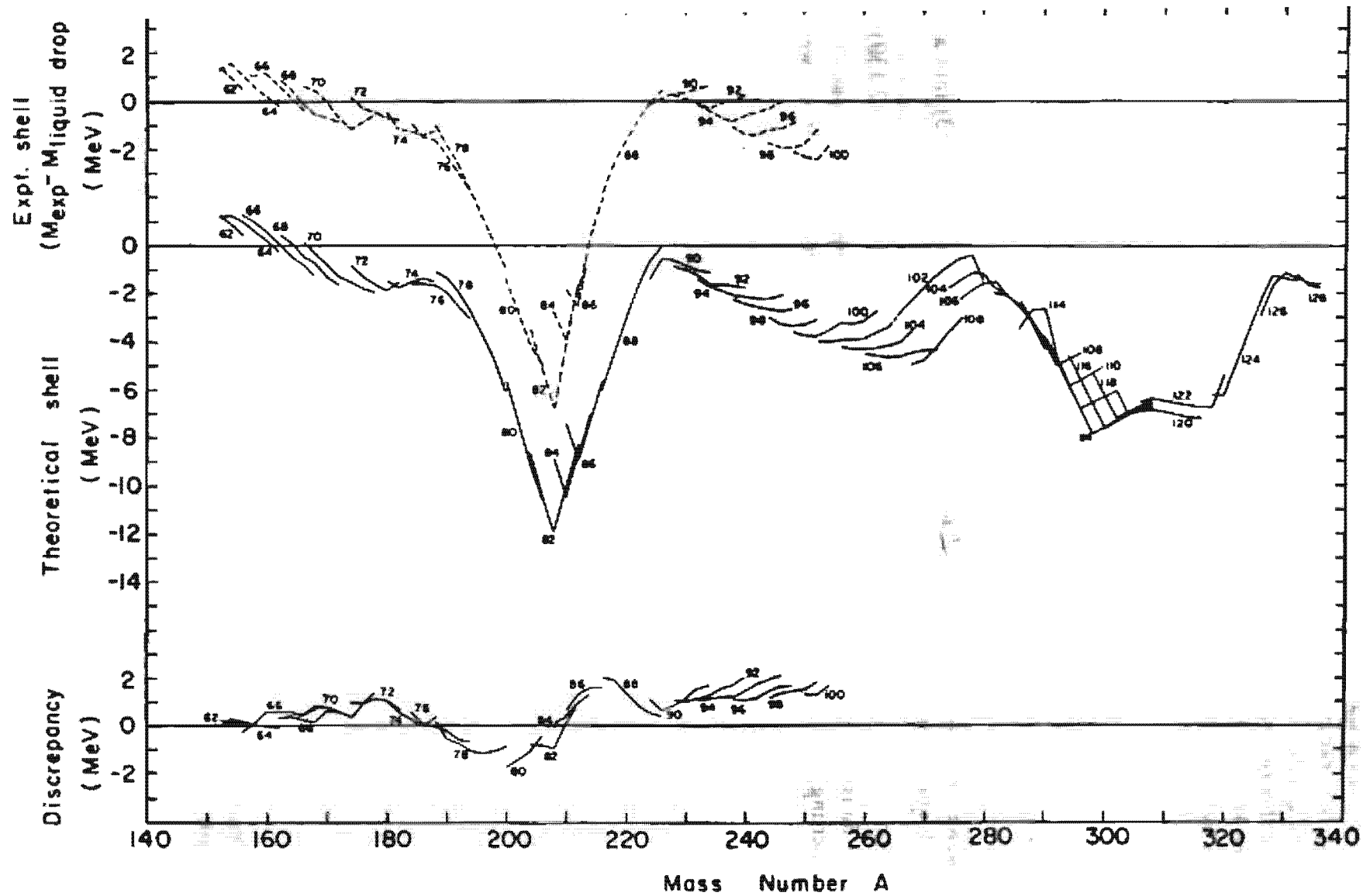


Figure 2.32. Comparison of theoretical mass corrections with experimental values. (From [NTT 69].)

2.9.6 Some Applications

The combination of the shell model with the liquid drop model as provided by Strutinski's theory has, for instance, been applied to the calculation of nuclear masses [NTT 69]. Figure 2.32 shows the theoretical mass corrections to the liquid drop model and compares them with the experimental data. The overall agreement is very good; the oscillating structure in the vicinity of the double magic nucleus ^{208}Pb is especially well reproduced. However, we must realize that these shell corrections were calculated with Nilsson's single-particle energies, which do not reproduce the experimental single-particle level scheme of ^{208}Pb (see Fig. 2.15) very well. In fact, the shell correction calculated with these experimental levels is 5 MeV off in ^{208}Pb . For a discussion of this discrepancy, see [BDJ 72].

Another application of the Strutinski method is the calculation of energy versus deformation curves. As we mentioned briefly at the end of Section 2.8, the calculation of the energy as a function of not only the quadrupole deformations but also octupole, hexadecupole, and higher deformations gives rise to multidimensional energy surfaces. As in the shell model, we find deformed minima in the rare earth nuclei as well as in the actinides, and sometimes second minima for very large deformations (fission isomers). It turns out that the ground state minima are reproduced roughly at the same deformations as in the pure Nilsson model. However, at large deformations, the deformation surfaces look different (for a detailed investigation, see [BDJ 72]). In these surfaces one looks for the path which goes from the equilibrium deformation past the lowest barrier and out to the region of deformation in which fission occurs. The knowledge of this path and its actual shape for a nucleus then allows an estimate of its fission lifetimes [ADD 70, JNS 70, BFN 72, Ni 72, BDJ 72, Pa 73, LP 73, MN 73, RNS 78]. In particular the stabilities of superheavy nuclei have also been investigated in this way (see [Ni 72, NN 74, BN 77] and references given there).