• Translational invariance

The coordinate system K is displaced to a new set of axes K'

$\forall\ |A\rangle_K \Rightarrow$ translated state $|A'\rangle \in K'$

↳ the properties of $|A'\rangle$ described by an observer in K' are identical to those of $|A\rangle$, as seen from K.

$$\boxed{\langle B'|A'\rangle = \langle B|A\rangle}$$

Transformation $\quad |A\rangle \xrightarrow{\text{Unitary}} |A'\rangle \quad \in K$

$$|A'\rangle = U|A\rangle = \sum_B |B\rangle \boxed{\langle B|U|A\rangle}$$

$$|A'\rangle_{K'} = \sum_B \boxed{U_{BA}}\, |B\rangle_K$$

• translational invariance: $\langle B'|T'|A'\rangle = \langle B|T|A\rangle$

↳ operator expressed in K

↳ operator described in K'

$$T' = U T U^{-1}$$

$$|A'\rangle = \sum_B |B\rangle\langle B|U|A\rangle$$

$$\langle B'|T'|A'\rangle = \langle B|T|A\rangle$$

$$\langle B|U^{-1}T'U|A\rangle = \langle B|T|A\rangle$$

$$\Rightarrow U^{-1}T'U = T \Rightarrow$$

$$T' = U T U^{-1}$$

$U(a) = e^{-\frac{i}{\hbar}\vec{a}\cdot\vec{p}}$ $\quad$ $\vec{a}$: displacement

$U(\delta a) = 1 - \frac{i}{\hbar}\delta\vec{a}\cdot\vec{p}$ $\rightarrow$ $\vec{p}$: generator of infinitesimal translations

Time displacements

$|A\rangle \xrightarrow{\text{unitary transformation}} |A'\rangle$

$$\boxed{U(t_0) = e^{+\frac{i}{\hbar}Ht_0}}$$

$H \rightarrow$ total energy

$$T' = T(t) = e^{\frac{i}{\hbar}Ht}\, T_{(t=0)}\, e^{-\frac{i}{\hbar}Ht} \qquad T' = UTU^{-1}$$

$\rightarrow$ Conservation of total momentum $\rightarrow$ expression of the commutability of spatial translations and time displacements

$\Downarrow$

translations = time-independent operations

Rotations

$\boxed{\text{Nuclear states}} \xrightarrow{\varphi}$ infinitesimal rotations

$\downarrow$

$\varphi \rightarrow$ angular momentum
$\rightarrow$ spherical tensor properties

- Finite rotations $\rightarrow$ intrinsic coordinate system

$\vec{\chi}$ $\rightarrow$ direction of the axis of rotation
$\rightarrow$ magnitude of the rotation angle

$$R(\vec{\chi}) = e^{-i\vec{\chi}\cdot\vec{I}}$$

$I \rightarrow$ total angular momentum

- Hamiltonian of a nucleus

$$H = \sum_k \frac{(\vec{p}_k)^2}{2M_k} + \underbrace{W}_{\text{interactions}}$$

$\hookrightarrow$ k-th particle

$$M = \sum_k M_k \quad \Rightarrow \quad H = H_{intr} + \frac{(\vec{P})^2}{2M}$$

$H_{intr} \rightarrow$ intrinsic Hamiltonian $\rightarrow$ Galilean invariant

$$R_{cm} = \frac{1}{M}\sum_k M_k \vec{r}_k \rightarrow \text{center-of-mass coordinate}$$

Dynamics of the system $\rightarrow$ intrinsic motion

$\hookrightarrow$ center-of-mass motion (system as a whole)

$$\frac{i}{\hbar}[H, R_{cm}] = \frac{1}{M}\vec{P}$$

$$H = \left(H_{intr}^2 + c^2\vec{P}^2\right)^{\frac{1}{2}}$$