orbital (the minus sign results because the particle is taken away, i.e. a hole excitation, when going from ^{152}Dy to ^{151}Tb). Indeed, in the single-particle diagram, the orbital [301 1/2] has essentially the desired properties even though the decoupling factor comes out somewhat smaller than $a = 1$ in most calculations using different potentials.

It was the observation (Byrski *et al.*, 1990) of the identical bands in ^{152}Dy and ^{151}Tb (and a similar identity between the yrast band in ^{151}Tb and one excited band in ^{150}Gd) that really focused attention on such bands (see e.g. Stephens *et al.*, 1990) and started a lot of theoretical investigations. This was so even though the identical bands in ^{152}Dy and ^{153}Dy were already known, as were also two identical superdeformed bands in the $A = 190$ region, the latter, however, over a range of fewer transitions. In the best cases, the identity of the transition energies is good within 1–2 keV extending over about 15 transitions with energies about 600–1600 keV. These numbers might, however, give the impression that the identity is even more strange than it really is. We must remember that all the superdeformed bands are very regular and follow essentially the same curve in an I versus E_γ diagram. This is illustrated in fig. 12.23 where we also give the transition energies for ^{151}Dy. The superdeformed band in ^{151}Dy is understood as being formed when one $N = 7$ neutron is removed from ^{152}Dy. Considering that these bands are found in neighbour nuclei, they are unusually different. This indicates that, if a more inert orbital is either empty or filled in two bands in neighbouring nuclei, the bands by necessity have to be rather similar. Even so, the extreme identity observed appears very strange. Furthermore, in view of the accuracy obtained in nuclear calculations in general, it is indeed surprising that the simple theories discussed here seem to describe the experimental situation so well. One would expect that different correlations not accounted for, especially pairing, would make the very detailed comparison between theory and experiment impossible. The models introduced here seem to be useful mainly in the Dy/Gd region. The superdeformed bands in the Hg/Pb region could not really be described within this scheme assuming pure single-particle degrees of freedom with no pairing. Different ideas on how the identical bands in Hg/Pb nuclei could be understood have been published e.g. by Stephens *et al.* (1990) and Azaiez *et al.* (1991). There is however no established understanding of these bands, see e.g. Baktash *et al.* (1993).

If the mechanisms for creating identical bands discussed here are qualitatively correct, it should also be possible to describe differences between bands that are not identical. One might say that to invent a theory which gives identical bands, or even bands that differ by some smooth quantity, is not

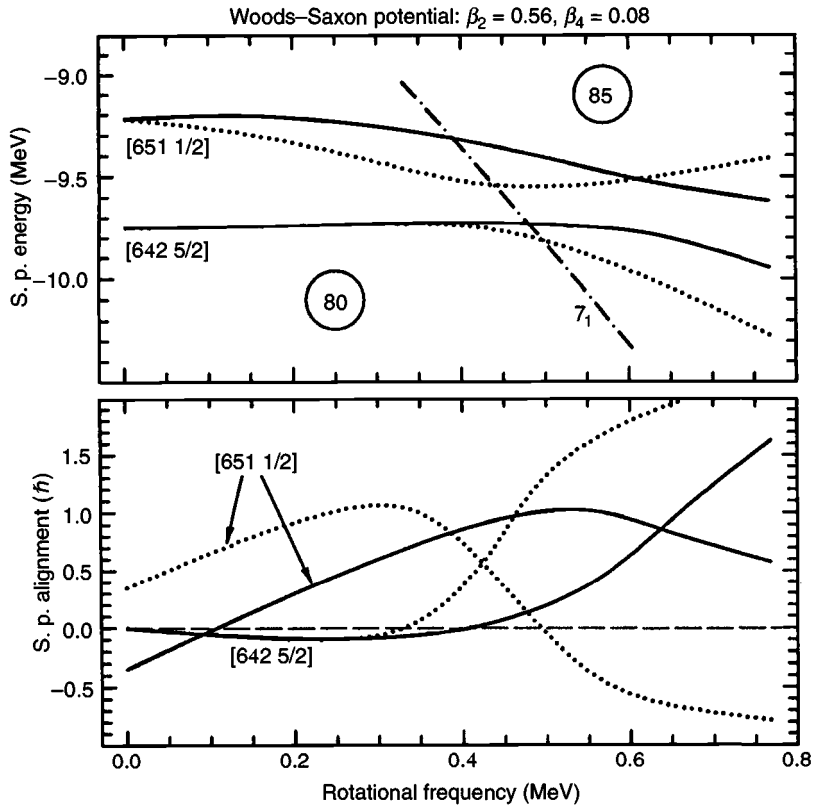

Fig. 12.25. The orbitals calculated in the Woods–Saxon potential at superdeformation in the $N = 80$–85 region are drawn versus rotational frequency ω in the upper figure while the alignments of the orbitals labelled by asymptotic quantum numbers [651 1/2] and [642 5/2] are shown in the lower figure.

so difficult. A better test of some theory is its ability to describe non-smooth differences between the observed quantities. Then, the superdeformed bands observed in $^{146-149}$Gd are especially favourable because some of these bands show a band-crossing while others do not.

When searching for orbitals at 2 : 1 deformations and neutron numbers $N = 82$–85, which could give rise to an observable crossing, the only reasonable candidates seem to be the $N = 6$ orbitals [642 5/2] and [651 1/2]. As should be evident from fig. 8.3, drawn for somewhat smaller deformations, these orbitals come close together for $\varepsilon = 0.5$–0.55. Other crossings occur between orbitals from different N-shells and appear to interact much less than observed in experimental bands. The details of this single-particle crossing are illustrated in fig. 12.25, as calculated in the Woods–Saxon

model. The single-particle orbitals are drawn in the upper figure where a very small energy interval is considered so that only the two signature branches of these two orbitals together with the lowest $N = 7$ orbital are seen (cf. fig. 12.16, which shows a much larger energy interval but where the crossings are drawn somewhat schematically. Consider e.g. the crossing between the [532 5/2] and [541 1/2] orbitals around $Z = 60$ in fig. 12.16 which in many ways is similar to the crossing in fig. 12.25). In the lower part of fig. 12.25, the alignments $\langle j_x \rangle$ of the orbitals are drawn. These alignments are proportional to the slopes in the upper figure.

In a way analogous to the identical bands, we now consider (Haas *et al.*, 1993) the differences between the transition energies of two bands in neighbouring nuclei with one orbital either filled or empty. For the orbitals of fig. 12.25, the calculated differences are drawn in the lower part of fig. 12.26. It is evident that this figure has the same structure as the alignments in fig. 12.25 and it is straightforward to see which orbital is either empty or filled when comparing two bands. The differences when comparing the two figures arise mainly from the fact that the Woods–Saxon potential has been used in one figure and the modified oscillator in the other. Furthermore, in fig. 12.26, we compare rotational bands which have been minimised in deformation independently while in fig. 12.25, the single-particle alignment $\langle j_x \rangle$ is shown. The comparison shows that, in the present formalism corresponding to single-particle motion in a mean field, it is the alignment of the specific orbitals which is the important factor and that e.g. deformation changes between different bands will only lead to minor corrections.

In the upper panel of fig. 12.26, the differences in transition energies between the observed bands are drawn. The large similarity between experiment and theory in fig. 12.26 seems to be very strong evidence that our interpretation of which orbitals are active is really correct. In drawing the experimental figure, one has to make specific assumptions about the relative spins but now it seems possible to turn the argument around, claiming that the good agreement between theory and experiment means that we have determined these relative spins. This would mean that, if it becomes possible to measure the spin values in one superdeformed band, we might extract the spins also for the bands in neighbouring nuclei. Indeed, in a recent paper (Atac *et al.*, 1993), it has been claimed that the spins in the superdeformed band of ^{143}Eu have been measured. At present, very few superdeformed bands are known in neighbouring nuclei of ^{143}Eu so it does not seem possible to carry out a similar analysis around ^{143}Eu as for $^{146-149}$Gd.

The cases we have chosen in fig. 12.26 are not really typical but more

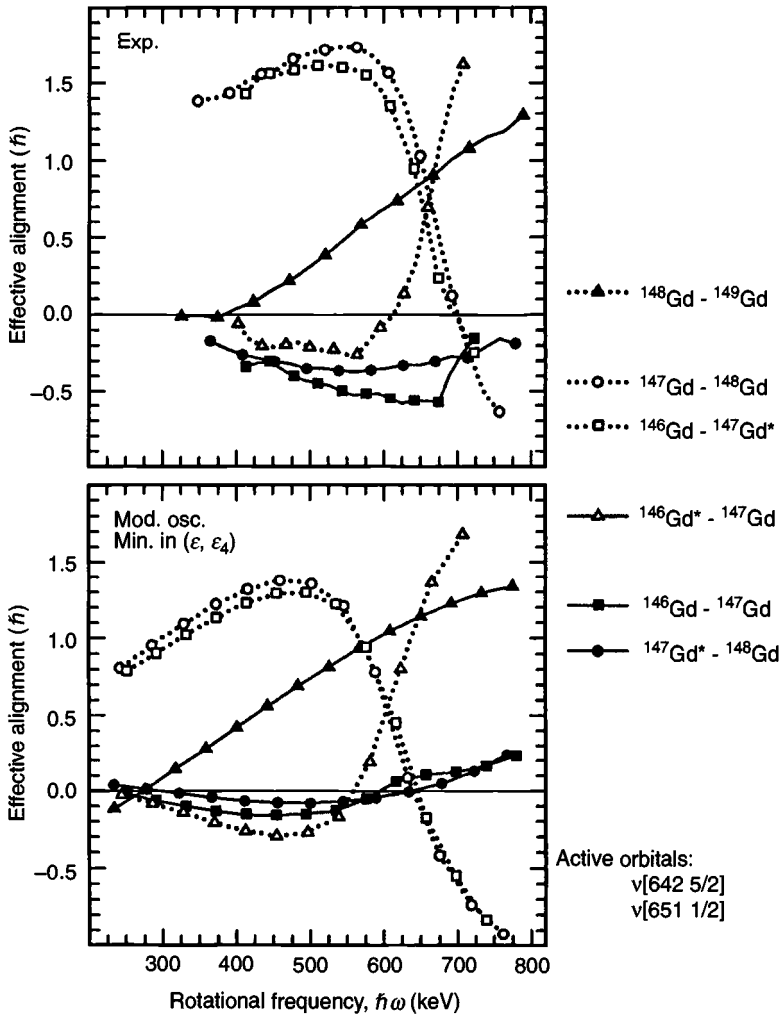

Fig. 12.26. The upper figure shows the relative transition energies for superdeformed bands in $^{146-149}$Gd where an asterisk indicates the 'second' band in that nucleus. The differences are plotted as (effective) alignments extracted as indicated in the lower right panel of fig. 12.24 ($\omega = E_\gamma/2$). Effective alignments extracted in the same way from rotational bands calculated in the modified oscillator are shown in the lower figure. A comparison with fig. 12.25 shows that it is the orbital that is labelled by [651 1/2] at $\omega = 0$ and has $\langle j_x \rangle > 0$ (the signature $\alpha = -1/2$ branch), which is being filled in the calculations when going from ^{147}Gd to ^{148}Gd or from ^{146}Gd to ^{147}Gd* etc. A comparison between the upper and lower figure strongly suggests a one to one correspondence between those orbitals used in the calculations and those active in the observed bands.