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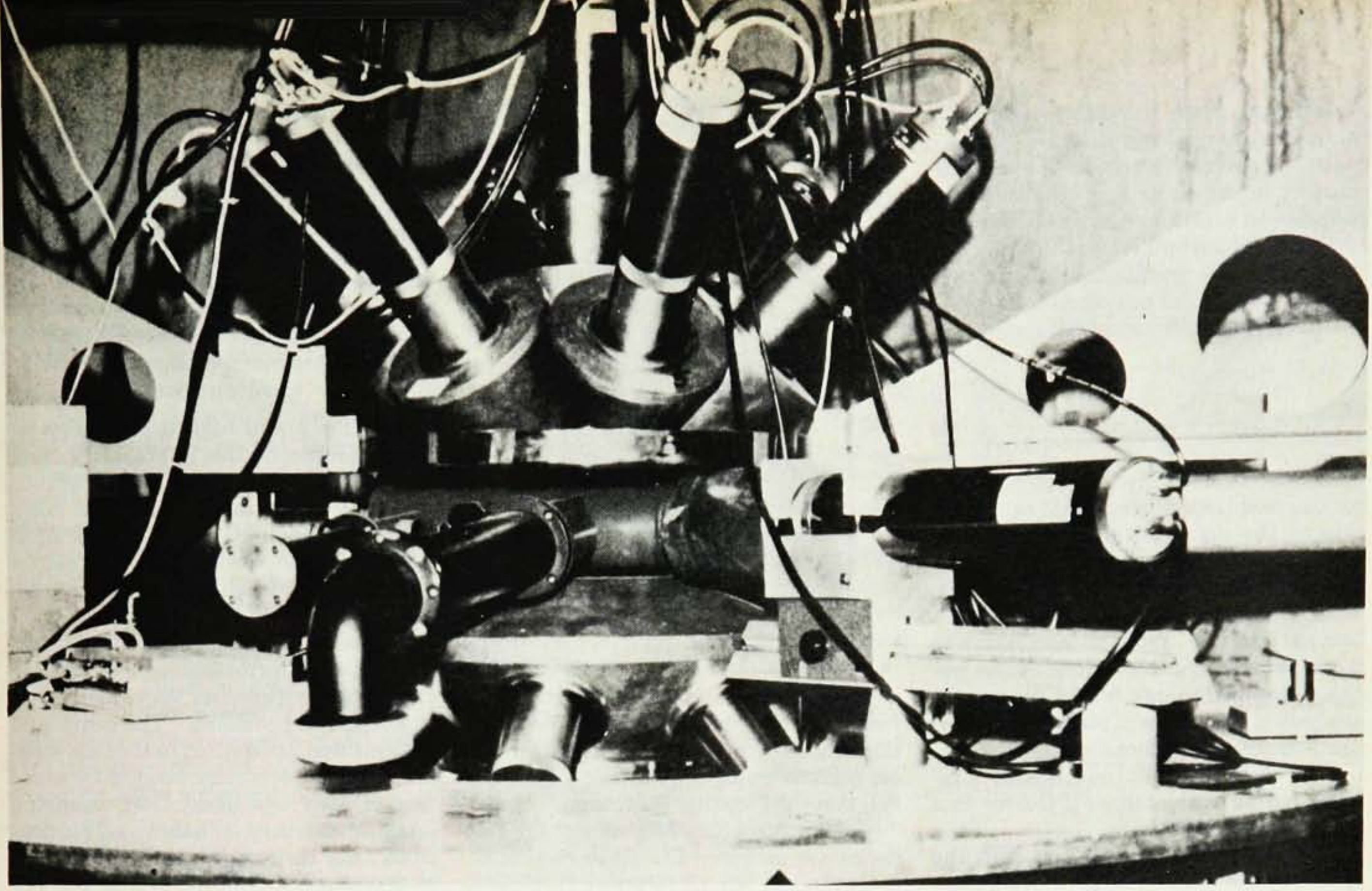

Physics of rapidly rotating nuclei

The study of nuclei with high angular momenta illuminates nuclear shell structure, the collective, often superfluid, motion of nucleons, and elastic deformation of nuclei.

Aage Bohr and Ben R. Mottelson

In recent years, a new field of nuclear research has been opened through the possibility of studying nuclei with very large values of the angular momentum. This development has been closely associated with the study of heavy-ion reactions, since collisions between two heavy nuclei are especially effective in producing metastable compound systems with large angular momenta. The study of such rapidly rotating nuclear systems provides the opportunity for exploring new aspects of nuclear dynamics. In figure 1 we show a multiple-coincidence spectrometer for studying gamma-ray cascades from the decay of these high-angular-momentum states. It comprises eighteen NaI detectors arranged around an on-line target; seven of the detectors and their photomultipliers are visible in the photo. The arrangement gives high sensitivity to gamma-ray cascades, which in some cases exceed thirty consecutive gamma rays. The spectrometer was constructed at the Niels Bohr Institute and is being used in a Copenhagen–Darmstadt collaboration.

The "porcupine" multiple-coincidence spectrometer in position at the heavy-ion linear accelerator in Darmstadt, Germany, as described in the text. Figure 1

Stability

The first problem that presents itself in this new field concerns the limits of stability for rotating nuclear systems. For sufficiently high frequency of rotation, the particles at the nuclear surface become unbound and are directly ejected by the centrifugal forces. However, for heavy nuclei a more severe limit to stability results from the possibility of collective deformations leading to fission into two fragments.

Such collective rotational instabilities are well-known from classical macroscopic systems. Thus, if a spherical body is set into rotation, it tends to acquire an oblate shape, as does the rotating Earth, for example. For sufficiently fast rotation, the axially symmetric shape may become instable, as first pointed out by K. G. J. Jacobi for rotating stars. This instability leads towards an elongated shape that eventually becomes unstable with respect to fission. (At the opposite end of the physicists' hierarchy of sizes, one encounters similar problems in the analysis of the shape of hadrons as a function of angular momentum. In these systems, fission is impeded by "quark confinement," that is, the apparent requirement that a break-up into two separate fragments leads to the creation of a new quark–antiquark pair. Thus, it has been conjectured that hadrons of high angular

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momentum may develop string-like shapes before disintegrating.)

For the nucleus, the model of a rotating charged liquid drop provides valuable guidance in exploring the rotational distortions and instability problems. For this model, the equilibrium shape as a function of angular momentum $\hbar I$ is obtained as the minimum of the energy

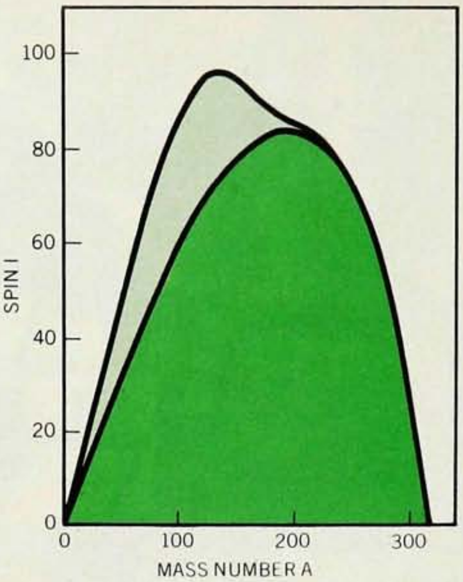
$$\mathcal{E}(\alpha, I) = \mathcal{E}_{\text{surf}}(\alpha) + \mathcal{E}_{\text{Coul}}(\alpha) + \frac{\hbar^2 I^2}{2\,\mathcal{J}(\alpha)} \quad (1)$$

where the first two terms represent the surface and Coulomb energies as a function of the deformation parameters α, while the last term is the rotational energy described by the moment of inertia $\mathcal{J}$, which is also a function of the deformation parameters. We shall later return to more detailed considerations of the nuclear rotational energy, but for the present analysis of the limits of stability, one expects to obtain a rather good approximation by assuming that the moment of inertia can be evaluated as if the nucleus rotated like a rigid body.

The parameters that enter into the different terms in equation 1 are known from the nuclear binding energies and the charge and mass distribution in the nucleus. Following the equilibrium shape as a function of angular momentum, one arrives at a critical value $I_{\max}$ at which the system is no longer stable with respect to fission. The value of $I_{\max}$ as a function of nuclear mass number is illustrated in figure 2. One sees that initially $I_{\max}$ increases with the size of the nucleus and reaches a maximum value of about 100 $\hbar$ for medium heavy nuclei. For still heavier nuclei, the disruptive Coulomb forces lead to a decrease in $I_{\max}$ until atomic numbers around 300, when the nucleus becomes unstable even in the absence of rotation.

Experimental evidence obtained from heavy-ion reactions has made it possible to test these stability limits. Thus, for example, a measurement of the cross section for fusion of the two colliding nuclei determines the maximum impact parameter $b_{\max}$ for such a reaction. The value of $b_{\max}$ together with the momentum p of the incoming projectile give the maximum value of the angular momentum ($\hbar I_{\max} = b_{\max}\, p$) compatible with the formation of a compound system. The experiments and their interpretation still involve points of uncertainty, but the evidence tends to confirm the approximate validity of the estimates obtained from figure 2.

The scope of the new field of high angular momentum studies is illustrated schematically by the phase diagram in figure 3, in which the two axes represent excitation energy and angular momentum. In nuclear physics, the lowest energy state for given angular momentum is called the "yrast" state. (This neologism[2] is based on the superlative form of the Nordic word *yr* meaning "dizzy." Thus, literally the yrast state is the most dizzy for a given energy.) Particle physicists call this the "leading trajectory."



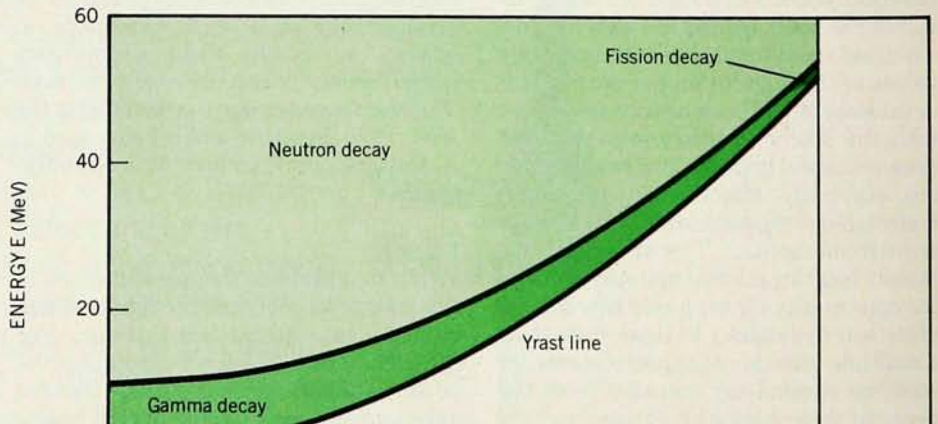
Fission stability of rotating liquid drop. The figure is taken from the theoretical analysis of reference 1, and shows the value $I_{\max}$ of the angular momentum for which the fission barrier vanishes. The value of $I_{\max}$ depends on both neutron and proton number, but for simplicity the figure refers to the beta-stable nuclei for different values of the total mass number A. The equilibrium shape of the nucleus is oblate in the dark-colored region and of triaxial symmetry between the two curves. Figure 2

Until a few years ago, almost all evidence was confined to a small section at the extreme left of figure 3 ($I \lesssim 10$). Thus, the high angular momentum studies opened up by the heavy-ion reactions can be seen as adding a whole new dimension to nuclear physics, concerned with the effect on the various nuclear properties of the large centrifugal stresses produced by the rotational motion.

Yrast spectra

Special interest attaches to the region near the yrast line, where almost all the excitation energy is tied up in the single degree of freedom associated with the rotation. In the yrast region, the nucleus is therefore "cold," despite the fact that the total excitation energy may be very high, and one expects to find the small level density and high degree of order characteristic of the approach to zero temperature. The exploration of quantal spectra in the yrast region thus holds promise of yielding quantitative information on the state of nuclear matter under the novel conditions of rapid rotation.

It should be emphasized that the extraction of information concerning the yrast spectra presents very demanding experimental problems. The compound systems that are formed in the heavy-ion reactions are typically excited by many tens of MeV and decay first through a series of neutron emissions. The neutrons only carry a few units of angular momentum, and most of the angular momentum therefore remains in the system at the end of the neutron cascade, when the system has cooled to within several MeV of the yrast line. At this point, the subsequent cooling and removal of angular momentum is accomplished by gamma emission. Each gamma quantum carries one or two units of angular momentum, and the reactions are characterized by a very high multiplicity of gamma rays. The photon cascade may proceed along a multitude of paths and thus one is faced with a gamma-spectrum of enormous complexity. Nevertheless, the ingenuity and resourcefulness of the experimenters has led to spectacular progress in this new spectroscopy, and it



Phase diagram in the energy–angular-momentum plane. The figure refers to a medium-heavy nucleus (A = 150), and the physical region, bordered by the yrast lines and $I_{\max}$, is divided according to the dominant decay modes of the system represented. Beyond $I_{\max}$ the nucleus is unstable with respect to fission; it effectively is spinning fast enough to throw off pieces. Figure 3 has already been possible to identify individual levels with spin numbers I all the way up to about 35.

The nuclear spectra have a great richness that reflects the fact that the elementary modes of excitation comprise both single-particle degrees of freedom as well as collective vibrations and rotations. Thus, in the well-studied low-energy spectra, the yrast configurations in some nuclei correspond to rearrangements of the orbits of single nucleons, while other nuclei accommodate the angular momentum by collective motion giving rise to regular sequences of levels of either vibrational or rotational character. One of the main themes in the exploration of the yrast spectra may be expected to be the new light that these studies may shed on the interplay between the collective and individual particle degrees of freedom in the nuclear many-body systems. We shall in the following discuss a few examples of current studies of yrast spectra, particularly of the rare-earth nuclei which may give an impression of the way these issues are being approached.[3]

Bandcrossing

The yrast spectra have an especially simple character for the rather large class of nuclei that, in their ground states, have a stable shape deviating significantly from spherical symmetry. For these deformed nuclei, the yrast spectra consist of rotational bands characterized by quantitative regularities in the energies and other properties of successive levels. The pursuit of these bands to higher angular momenta has led to the discovery of a striking new phenomenon[4] illustrated in figure 4. In the nucleus erbium-164, as the figure shows, an excited band (the S-band) crosses the ground-state band (g-band) rather sharply at a spin of about 16, and for higher values of I it is this new band that defines the yrast sequence. A similar phenomenon is found to occur for most nuclei in this region of the periodic system.

The occurrence of this band-crossing is remarkable in view of the energy gap of about 2 MeV that characterizes the excitations in the even–even nucleus at low values of the angular momentum. This gap is associated with the correlation of nucleons into pairs that form a superfluid condensate in the nuclear ground state analogous to that of the electron pairs in a superconductor. The binding energy of the pairs appears directly in the energy gap that separates the fully paired ground state of an even–even nucleus from the lowest excited states involving two quasi-particles, obtained from the breaking of a pair. The pairwise binding of the nucleons is also observed in the systematic odd–even difference in the nuclear masses, amounting to half the binding energy of a pair. (This mass difference between odd and even nuclei was recognized at a very early stage in the history of nuclear physics and is, for example, responsible for the preponderance of the even nuclei in the cosmic abundances of nuclear species.)

The S-band can be identified as a two-quasi-particle excitation with special properties. As we shall see, this interpretation can be directly tested by a comparison with single-quasi-particle excitations that are observed as low-lying bands in the neighboring odd-A nuclei.

In the analysis of relationships in these spectra, a quantity of central significance is the rotational frequency. The experimental data directly provide information on the properties of a band as a function of the angular momentum. The angular momentum is a symmetry quantum number associated with the invariance with respect to the orientation of the frame of reference. However, for a system that exhibits collective rotation, important dynamical variables are the orientation and frequency of whatever it is that is going around. For a rotational band, (that is, a sequence of levels whose energies $\mathcal{E}$ and other properties vary smoothly with angular momentum I) the frequency ω can be obtained from the canonical relation

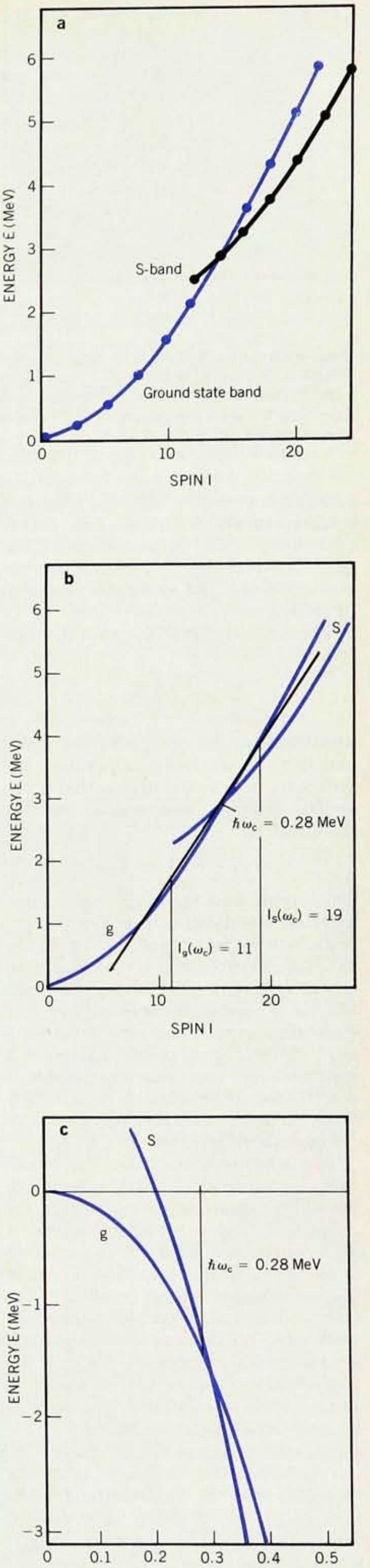
$$\hbar\omega = \frac{\partial \mathcal{E}}{\partial I} \approx \frac{1}{2}[\mathcal{E}(I+1) - \mathcal{E}(I-1)] \tag{2}$$

The conditions under which a rotational frequency can be meaningfully defined for a many-body system, such as the nucleus, are related to the general question of the possibility of separating a single degree of freedom (collective rotation) from the rest of the many-body dynamics. One should also bear in mind the possibility that the systems may exhibit several frequencies associated with rotations about different axes. However, the empirical spectrum in figure 4a leads to a unique identification of rotational sequences that are very smooth functions of I and therefore define a collective frequency ω with high accuracy.

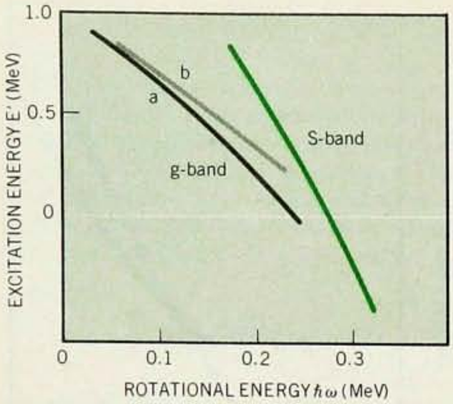
Having obtained the rotational frequency, let us go into a coordinate frame rotating with frequency ω, where the energy is given by the Legendre transform

$$\mathcal{E}'(\omega) = \mathcal{E}(I(\omega)) - \omega\hbar\, I(\omega) \tag{3}$$

In classical mechanics $\mathcal{E}'$ is sometimes referred to as the "Routhian;" it is the Hamiltonian describing the motion in the



Bandcrossing in erbium-164. Part a of the figure shows yrast levels in Er^{164}, an energy spectrum taken from references 5 and 6. The data are analyzed in parts **b** and **c**; in part **c** the plot is in terms of rotational frequency ω and energy $\mathcal{E}'$ in the rotating coordinate system. The frequency ω_c at which the g-band and the S-band cross in the $\mathcal{E}',\omega$ diagram appears in the $\mathcal{E},I$ diagram as the slope of the common tangent of the two bands. Figure 4



Quasi-particle excitations with respect to the ground-state band of erbium-164. The bands a and b refer to the nucleus Er^{165} (data from reference 8), while the S-band of Er^{164} is that given in figure 4c. Figure 5

rotating coordinate system. The last term represents the Coriolis and centrifugal forces acting in that frame. The spectrum of figure 4a, replotted for the rotating coordinate system, is shown in figure 4c.

The derivative of $\mathscr{E}'(\omega)$ obeys the relation

$$\frac{\partial \mathscr{E}'}{\partial \omega} = -\hbar I(\omega) \qquad (4)$$

which follows directly from equations 2 and 3. For an excited configuration, the difference in slope, relative to that of the ground state (for the same ω), determines the quantity

$$i(\omega) = I(\omega) - I_g(\omega) \qquad (5)$$

which represents the extra angular momentum associated with the excitation, and which is therefore referred to as the "alignment" (of angular momentum) characterizing this excitation. It is seen that the S-band is characterized by a very large alignment, amounting to about 8 units at the crossing point. Indeed, it is just this large alignment that makes it possible for the two-quasi-particle excitation to overcome the energy gap associated with the pairing.[7]

The behavior of the individual quasi-particles can be studied in the spectra of the odd-A nuclei. In the region of nuclei considered, the low-lying bands in odd-A nuclei can be associated with the different orbits of a single quasi-particle moving in the non-spherical average potential. The energy spectra of the odd-neutron nuclei systematically exhibit a pair of bands with especially large alignment. We show the energies of these quasi-particles in Er^{165} as a function of rotational frequency in figure 5 (energies are measured with respect to the g-band of Er^{164}, considered as the quasi-particle vacuum). For low frequency, the one-quasi-particle bands have an excitation energy very close to 1 MeV with respect to the vacuum state, corresponding to the odd–even mass difference. However, the large alignment in these bands implies that the energy rapidly decreases and becomes negative for a frequency of about 300 keV.

Figure 5 also includes the energy of the S-band in Er^{164}, which is seen to be well represented as the sum of the energies of the two single-quasi-particle bands. Thus, one sees that the key to understanding the band crossing is the physical mechanism that leads to large alignment of the special orbits involved.

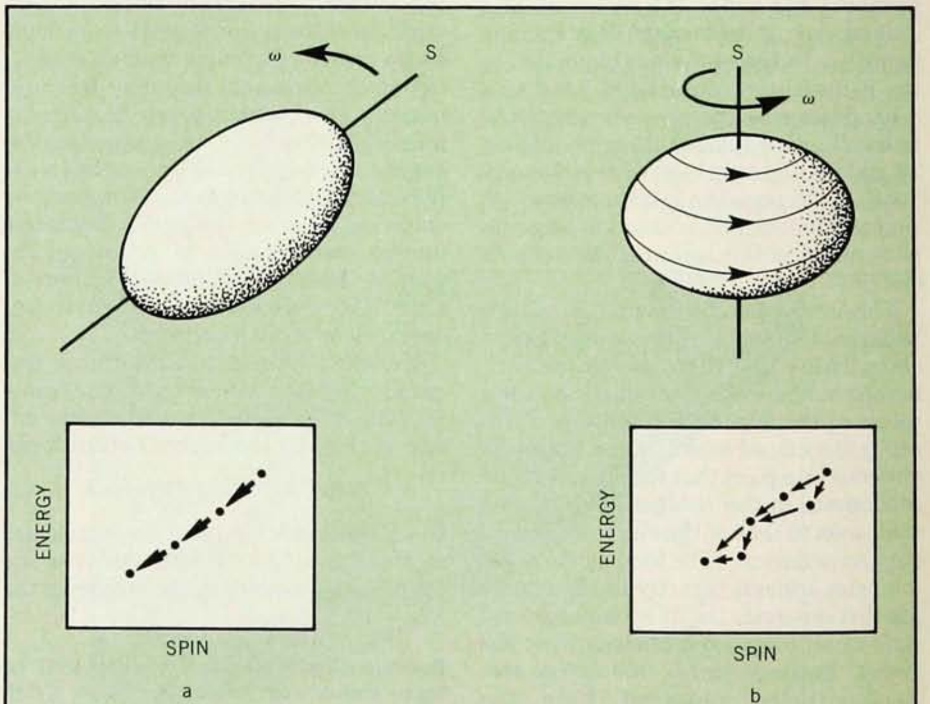
The motion of the individual quasi-particles in the rotating coordinate system is described by the Hamiltonian (analogous to the energy expression given in equation 3)

$$H'_{sp} = H_{sp} - \hbar\omega j_1 \qquad (6)$$

The first term, H_{sp}, describes the motion in the static (deformed) nucleus, and j_1 is the component of the angular momentum of the particle along the axis of the collective rotation. The general tendency to alignment can be identified in the effect of the Coriolis force (last term in equation 6), which favors orbits with large values of $\langle j_1 \rangle$. The special orbits that exhibit the large alignment indeed correspond to particle motion with especially large orbital angular momentum. To a good approximation they can be thought of as having $l = 6$ and $j = 13/2$ ($i_{13/2}$ orbits in the standard notation); because of the large spin–orbit coupling, these orbits are widely separated from other orbits of the same parity with which they would otherwise be mixed by the quadrupole deformation of the nuclear potential.

The nuclei concerned have a prolate shape and, in the absence of rotation, the lowest energy state for the particle has the angular-momentum vector $\mathbf{j}$ approximately in the plane perpendicular to the symmetry axis. For this lowest state, even a very small rotational frequency ω is sufficient for complete alignment of the angular momentum along the axis of rotation ($j_1 = j$). Such a rapid alignment is therefore expected for a configuration with a single particle in the $i_{13/2}$ shell. However, as the shell is occupied with increasing numbers of, particles, the alignment of the last filled orbits (those at the Fermi level) decreases. Indeed, for a filled shell, the total alignment vanishes, since such a configuration has zero total angular momentum. For a half-filled shell, which corresponds approximately to the erbium isotopes considered, the alignment of the particles at the Fermi surface is very small.

The effects of the Coriolis force are profoundly modified by the pair correlations. In the pairs that form the superfluid condensate, referred to above, the angular momenta of the two particles compensate each other (time-reversed states). Such pairs are much less affected by the Coriolis forces, and the angular momentum carried by the condensate is therefore strongly reduced. (This effect is directly observed in the moments of inertia of the ground state bands, which are found to be a factor 2–3 smaller than



Two ways for the nucleus to accommodate angular momentum. These two contrasting types of rotational motion are described in the text. The lower parts of the figure illustrate schematically yrast spectra and transitions corresponding to the two cases. In the first case (a) the nucleus accomodates additional angular momentum by a collective rotation, resulting in a simple yrast spectrum; in (b) the nucleus changes its spin by rearranging individual nucleon orbits, resulting in a complex spectrum. Figure 6 the rigid-body value expected in the absence of pair correlations.) The $i_{13/2}$ particles in the condensate are therefore much less aligned than they would be in the unpaired system, and this opens the possibility for the particles in broken pairs (the quasi-particles) to occupy orbits with large alignment, even when the Fermi level lies in the middle of the shell. Indeed, quasi-particle energies obtained from the Hamiltonian of equation 6 with inclusion of the pairing effect account rather well for the alignment observed in the orbits **a** and **b** in figure 5.

We have in this discussion focussed on a single special feature of the observed yrast spectra, but it should be clear that this phenomenon is part of a much broader set of problems that involve the detailed study of the one-particle motion as a function of the rotational frequency. In this connection, one may expect rather major rotationally induced changes in the parameters describing the average nuclear potential (deformation, pairing and so on), as well as the occurrence of new components in the potential reflecting the violation of time-reversal symmetry in the rotating coordinate system.

Phase transition

At rotational frequencies somewhat greater than those that have been reached so far in the yrast spectroscopy, one expects a phase transition from the superfluid to the normal phase (disappearance of pair correlations). In fact, as already mentioned, the pair correlations lead to a reduction in the moment of inertia and hence to an increase in the rotational energy for a given angular momentum. For increasing values of the angular momentum, this excess in the rotational energy drives the system towards a reduced pair correlation, and eventually a transition takes place to uncorrelated single-particle motion. This phase transition is analogous to that of a superconductor in a magnetic field. In a finite system such as the nucleus, one does not, of course, expect a transition with sharp singularities as in macroscopic systems. However, the finite nuclear system provides the possibility of studying such a phase transition in terms of the individual quantum states.

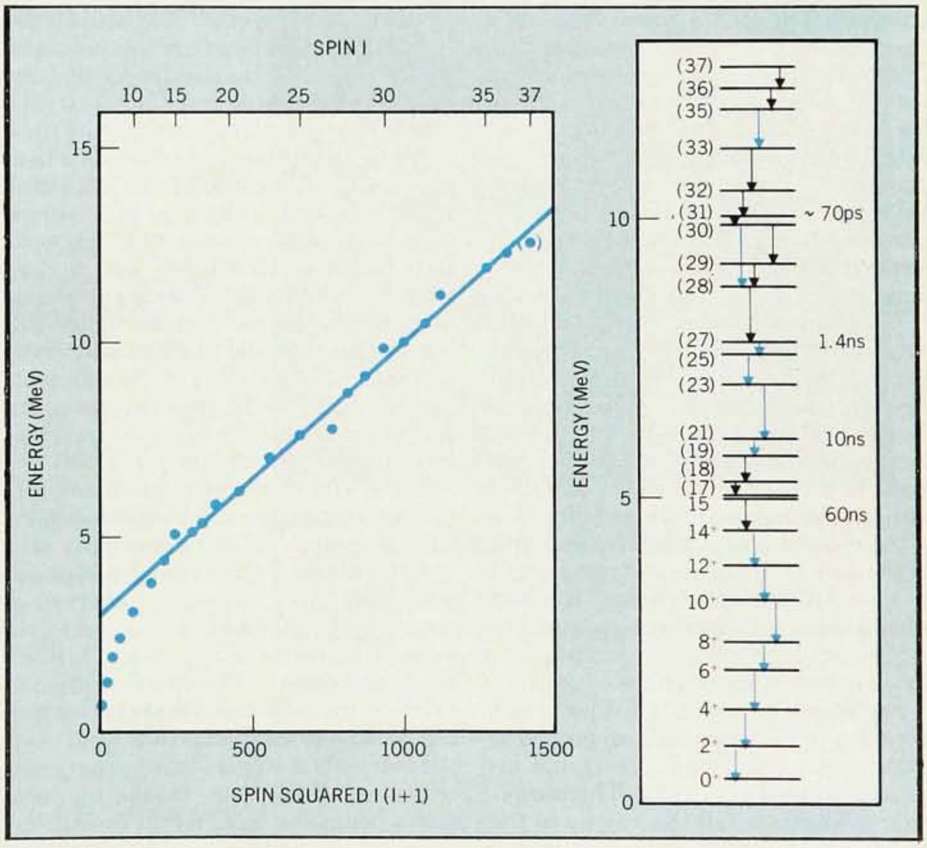
The band-crossing effect discussed above may be seen as the first evidence foreshadowing the impending phase transition. Indeed, the occurrence of quasi-particle excitations with vanishing energy corresponds to the phenomenon of "gapless" superconductivity[9] that occurs in certain types of superconductors in the presence of time-reversal-violating perturbations with a strength close to the critical value associated with the phase transition. In the nucleus, the gaplessness occurs relatively early as a function of the strength of the time-reversal-violating perturbation (Coriolis force) because of the occurrence of a special class of single-particle orbits that have retained their large total angular momentum even in the presence of the deformation of the nuclear potential. This feature of the one-particle spectrum is thus connected with the fact that the nuclear equilibrium shapes are relatively close to spherical symmetry and would be destroyed if the nuclear deformations were larger by a factor of 2 or 3.

Nuclear angular momentum

The strong particle alignment encountered in the bandcrossing phenomena represents a component in the total angular momentum of a completely different character from that of the collective rotation. The alignment occurs when the Coriolis forces acting on the special orbits involved can to a large extent overcome the effect of the deformation. Thus, an extreme situation in which all of the angular momentum is in the form of alignment occurs when the angular momentum of the nucleus is directed along a symmetry axis of the nuclear shape.

In such a situation one deals with a form of "rotational" motion that is radically different from the collective rotation, and the contrast of these two ways for the nucleus to accommodate angular momentum is illustrated in figure 6. The basis for the collective rotation is the existence of a deformation with respect to the axis of rotation, which makes it possible to define an orientation of the system as a whole. The left part of the figure shows a prolate nucleus rotating about an axis perpendicular to the symmetry axis, a situation encountered for a large class of nuclei. An increase in the rotational angular momentum involves small increments associated with the orbital motion of many different particles and thus leads to a rotational band structure with a high degree of regularity. The states within a band are connected by large collective quadrupole transition matrix elements whose magnitude reflects the strong time-dependent electric field generated by the rotation of a charged non-spherical body, which has a large electric quadrupole moment.

On the right in figure 6 we show the contrasting case of a nucleus whose angular momentum is in the direction of the symmetry axis; the shape is assumed to be oblate. For this type of rotation, the average density and potential remain static, and each particle moves in an orbit with component m along the symmetry axis. The total angular momentum along the axis is the sum of the quantized contributions of the individual particles. In this situation, an increase in the total angular momentum is achieved by a rearrangement in the occupation of the single-particle orbits. Thus, the energies



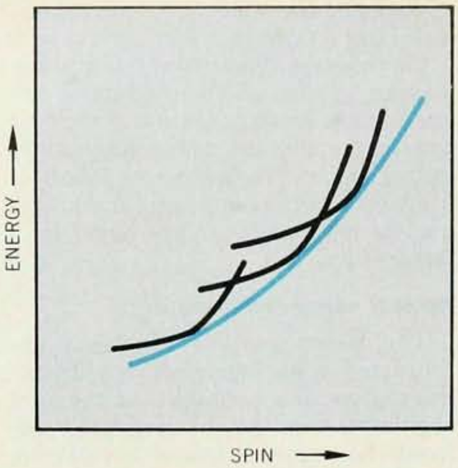
Yrast spectra for dysprosium-152. The data are taken from reference 11, where the yrast levels were established from spectroscopic studies of the gamma rays following the reaction $Sn^{124}(S^{32}, 4n)Dy^{152}$. The multipolarities of the gamma transitions have been determined from the angular distributions; the black arrows indicate either electric or magnetic dipole transitions (E1 or M1); colored arrows show quadrupole transitions (E2). Figure 7 on the yrast line may exhibit considerable irregularity, and the transitions are at most of one-particle strength and may suffer considerable hindrance. The average energy considered as a function of the angular momentum along the yrast line defines a frequency by means of equation 2 and a moment of inertia $\mathcal{I} = \hbar I/\omega$. In the absence of shell structure or pair correlations, this statistically defined moment of inertia is expected to have the classical value corresponding to rigid rotation.

In the nuclear spectra, one encounters the build-up of angular momentum by particle alignment in several different situations. For example, this coupling scheme is found for configurations with only a few particles (or holes) outside closed shells, for which the average potential is approximately spherical.[10]

Another example, where the angular momentum is generated along a symmetry axis is shown in figure 7. The lowest states in the spectrum of dysprosium-152 correspond to quadrupole vibrations about an approximately spherical equilibrium shape. The lifetimes for gamma decay of the vibrational states are of the order of 10 picosec. However, for the higher angular momenta, the irregularity in the yrast spectra and the frequent occurrence of relatively delayed transitions are characteristic of the coupling scheme based on the alignment of the angular momenta of individual particles. The average energy of the yrast states as a function of I^2 is seen to define a statistical moment of inertia that is comparable to the rigid body value. The slight excess of the observed moment as compared to that for a rigid sphere ($2\mathcal{I}_{rig}/\hbar^2$, or 142 MeV^{-1}) may reflect a small oblate deformation, but it is as yet difficult to draw any definite conclusion in view of the possible effects of shell structure and pair correlations.

The aligned-particle coupling scheme may be encountered for higher spins and larger deformations either as a result of shell structure effects favoring such a coupling, or as a result of the macroscopic centrifugal effect referred to earlier, which tends to produce an oblate deformation with angular momentum in the direction of the symmetry axis, which corresponds to the axis with the largest moment of inertia. An extensive experimental effort is being made to find such yrast structures at the very high angular momenta produced in heavy-ion reactions.

The identification of individual yrast states has in special cases been pushed to spins of about 35 (see figure 7), but information about states of still higher angular momentum (all the way up to the fission instability) is contained in the unresolved gamma spectra following heavy-ion reactions. Impressive progress has been achieved in extracting information from these spectra and, although the evidence is at the present tentative, there are indications that the patterns corresponding to both the collective and particle aligned yrast spectra are being encountered in different regions of nuclei.[12,13]



Interplay of particle alignment and collective rotation. The individual rotational bands are drawn as black lines, while the envelope of the bands is indicated in color. The slope of the lines gives the rotational frequency (see equation 2) and the curvature gives the (reciprocal) moment of inertia. Figure 8

The two modes of organizing angular momenta in the nuclear many-body system (illustrated in figure 6) represent limiting cases, and in general one expects an interweaving of these two mechanisms. The band-crossing effect discussed above is characteristic of such an intermediate situation in which the angular momentum of the nucleus can increase either by collective rotation (along a band) or by particle alignment (involving transition to a new band). A segment of an yrast spectrum involving a series of band crossings is schematically represented in figure 8. In the situation illustrated, one can distinguish between the collective moment of inertia determined from the individual bands, and the total moment of inertia defined by the envelope of the different bands comprising the yrast sequence. In this way, it would be possible to divide the total angular momentum into a part associated with collective rotation and the complementary part resulting from particle alignment. The limit of pure collective rotation corresponds to a situation where the yrast sequence comprises a single rotational band, with a collective moment of inertia indistinguishable from the total moment. Towards the opposite extreme, the collective moment of inertia can become so small that each band contributes only a single state to the yrast sequence, which thus breaks up into states belonging to different configurations (as in figure 7). The tentative character of this picture must be emphasized, since it is at present not clear under what conditions it may be possible to divide the total angular momentum unambiguously into the two separate parts. Important questions are associated with the interactions between bands based on different particle configurations and the limitations on collective rotational motion when the system approaches symmetry about the axis of rotation.

We hope that the points discussed above may give an indication of the flavor of the problems encountered in the exploration of the yrast spectra. These questions concern partly the global structure of the yrast line reflecting the dependence on rotational frequency of such bulk properties as the nuclear deformation, superfluidity and the systematic features of the nuclear shell structure. The study of the quantal structure of the yrast spectra not only opens the possibility for providing quite precise answers to these questions but also for penetrating into new dynamical problems associated with the interplay of single-particle motion and collective rotation.

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