4-2a Degrees of Freedom Associated with Spatial Rotations

Rotational motion in two dimensions (rotation about a fixed axis) has a very simple structure. The orientation is characterized by the azimuthal angle ϕ, and the state of motion by the eigenvalue M of the conjugate angular momentum. The associated rotational wave function is

$$\varphi_M(\phi) = (2\pi)^{-1/2} \exp\{iM\phi\} \tag{4-5}$$

The orientation of a body in three-dimensional space involves three angular variables, such as the Euler angles, $\omega = \phi, \theta, \psi$ (see Fig. 1A-1, Vol. I, p. 76), and three quantum numbers are needed in order to specify the state of motion. The total angular momentum I and its component $M = I_z$ on a space-fixed axis provide two of these quantum numbers; the third may be obtained by considering the components of $\mathbf{I}$ with respect to an intrinsic (or body-fixed) coordinate system with orientation ω (see Sec. 1A-6a). The

→ 2D - rotation

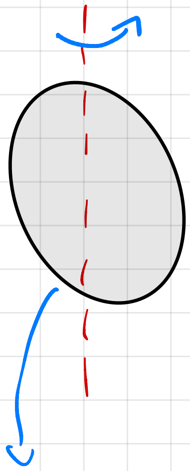
rotation about a fixed axis

→ orientation

2D - rotation → azimuthal angle ϕ

→ M → eigenvalue of the conjugate angular momentum

→ state of motion



$$\hookrightarrow \varphi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{iM\phi}$$

$\omega \equiv$ orientation

3D - rotation

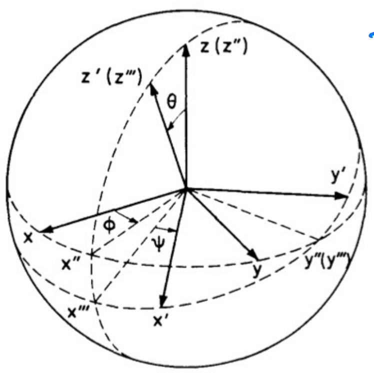


Figure 1A-1 Euler angles. The rotation from $\mathscr{K}(x, y, z)$ to $\mathscr{K}'(x', y', z')$ can be decomposed into three parts: a rotation by ϕ about the z axis to $\mathscr{K}''(x'', y'', z'')$, a rotation of θ about the new y axis (y'') to $\mathscr{K}'''(x''', y''', z''')$, and finally a rotation of ψ about the new z axis (z'''). It is seen that the Euler angles (ϕ, θ, ψ) are so defined that (θ, ϕ) are the polar angles of z' in $\mathscr{K}$, while (θ, $\pi - \psi$) are the polar angles of z in $\mathscr{K}'$. The Euler angles are, collectively, denoted by ω.