

There are some problems connected with this procedure, however. First the single-particle energy e_v is a sum of a kinetic-energy contribution $\langle T_v \rangle$ and a potential-energy contribution $\langle V_v \rangle$, the latter representing the expectation value of a sum of all the two-particle interactions

$$V_v = \sum_{\mu} U_{v\mu}$$

As all the terms $\langle V_v \rangle$ are added, the problem arises of whether or not the interactions are counted twice. A second problem concerns the volume conservation condition, which is difficult to generalise to include the effects of the $\ell \cdot s$ -term and, in the modified oscillator model, also the ℓ^2 -term.

The recipe of single-particle energy summation has been tried, however, and found to have fair success. The energy surface (i.e. the energy considered as a function of two variables, e.g. ε and ε_4 , see fig. 9.3 below) given by the single-particle sum is found to give a lowest minimum usually somewhat removed from spherical shape. The equilibrium shapes of well-deformed nuclei can be directly related to a quadrupole moment Q_2 and in some cases a hexadecapole moment Q_4 , which can also be obtained from experiment (optical spectroscopy, Coulomb excitation cross sections etc.). It turns out that at least Q_2 is in good agreement with data. When extended to larger distortions the energy surface should then also account for the fission barrier. For this application, however, the single-particle sum recipe is found to be inadequate.

One may note that the restoring energy introduced by the volume conservation condition is a term of very large magnitude, being roughly proportional to $(\varepsilon^2/9)$ times the total nuclear energy, or for $\varepsilon = 0.9$ of the order of 1000 MeV. As, among other things, the entire nuclear potential is not included in the volume conservation condition, 'small' corrections to the gross trends of the total energy are not unexpected. On the other hand, the vicinity of the spherical shape appears to be correctly reproduced as long as the correct level order is reproduced. Indeed the entire topological character of the energy surface may be obtained although the entire surface appears to be tilted.

Indeed a renormalisation of the energy surface appears to be called for and it is brought about by the introduction of the Strutinsky (1967) procedure. The basic idea behind this is the following. The average, long-range behaviour of nuclear binding energy as a function of the nuclear charge and size is well reproduced by the liquid-drop model. One then surmises that on the average this model also adequately describes deformation†. The relative success of the liquid-drop theory of fission may be taken as a warrant for

† This is, of course, what is conjectured in the original application of the model to the theory of fission in the classical paper by N. Bohr and J.A. Wheeler (1939).

this. One should therefore require that on the average – the average taken over so many nuclei that shell effects are averaged out – the total energy has the same distortion dependence as that of a liquid drop. This requirement can be enforced by subtracting out of the total energy an *averaged energy* and replacing the latter by the liquid-drop energy. The main problem consists of forming this average in a satisfactorily unique way.

To this end, Strutinsky (1967) first defines a smoothed level density $\tilde{g}(e)$ by smearing the calculated single-particle levels e_v over a range γ , where γ is an energy of the order of the shell spacing, $\hbar\omega_0$. Strutinsky thus considers a comparison between the actual discrete level structure g , where

$$g(e) = \sum_v \delta(e - e_v)$$

and a smeared level density $\tilde{g}(e)$ (see below). One then defines a shell energy as the difference

$$E_{\text{sh}} = 2\sum e_v - 2 \int e \tilde{g}(e) de$$

(the factor 2 coming from the double degeneracy of the deformed levels).

To obtain the smooth density $\tilde{g}(e)$ the discrete levels are associated with a smearing function,

$$\tilde{g}(e) = \frac{1}{\gamma \sqrt{\pi}} \sum_v \int de' f_{\text{corr}} \left(\frac{e - e'}{\gamma} \right) \delta(e' - e_v) \exp \left(-\frac{(e - e')^2}{\gamma^2} \right)$$

or

$$\tilde{g}(e) = \frac{1}{\gamma \sqrt{\pi}} \sum_v f_{\text{corr}} \left(\frac{e - e_v}{\gamma} \right) \exp \left(-\frac{(e - e_v)^2}{\gamma^2} \right)$$

The role of the exponential function is the obvious one of smearing. Through this we eliminate fluctuations of order $L \ll \hbar\omega_0$ by a choice of $\gamma \approx \hbar\omega_0$. In order not to disturb the long-range variations of order $L \gg \hbar\omega_0$, a correction function f_{corr} is introduced (cf. problem 9.3),

$$f_{\text{corr}}(u) = 1 + \left(\frac{1}{2} - u^2 \right) + \left(\frac{3}{8} - \frac{3}{2}u^2 + \frac{1}{2}u^4 \right) + \dots = \sum_{k=0}^n \frac{(-)^k}{(2k)!!2^k} H_{2k}(u)$$

where H_n are Hermite polynomials†.

† One can understand the occurrence of Hermite polynomials by the fact that the δ -function can be expanded as follows:

$$\delta(u) = \sum_{n=0}^{\infty} \frac{(-)^n}{\sqrt{\pi(2n)!!2^n}} H_{2n}(u) e^{-u^2}$$

If all the H_n polynomials up to $n = \infty$ were retained, then obviously the 'smeared' function $\tilde{g}(e)$ would be identical to the unsmeared one $g(e)$. It is thus by a proper break-off of the δ -function expansion that the desired $\tilde{g}(e)$ is obtained.

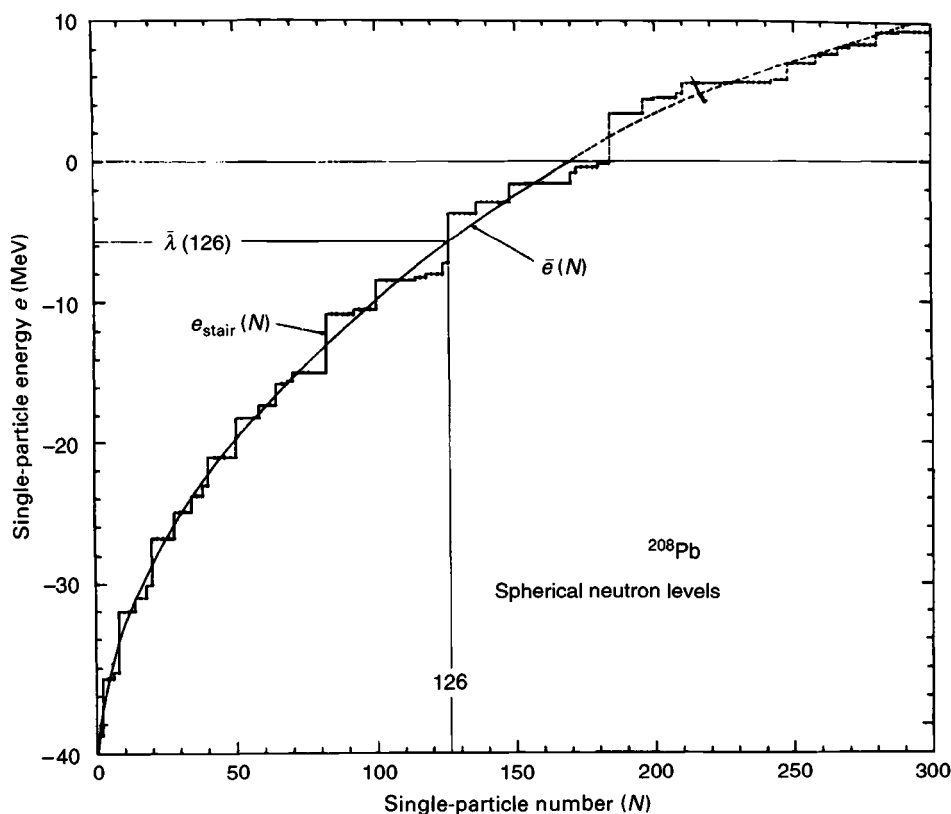


Fig. 9.1. Theoretical single-particle (neutron) energies in a Woods-Saxon type radial potential applicable to ^{208}Pb . The discrete energies define a 'staircase' function $e_{\text{stair}}(N)$. The smooth curve $\bar{e}(N)$ removes the local fluctuations but retains the long-range behaviour. The Fermi surface $\bar{\lambda}$ of the smooth distribution of levels is illustrated for 126 neutrons. The corresponding shell correction is given by the difference between the areas under the two curves up to $N = 126$. Note that a large section of the staircase curve is below the smooth one for N values just below $N = 126$ (from M. Bolsterli, E.O. Fiset, J.R. Nix and J.L. Norton, *Phys. Rev. C* **5** (1972) 1050).

The shell correction method is illustrated in fig. 9.1. One understands that the shell correction is negative when the sum of single-particle levels is below average, i.e. when there is a gap. It is positive and large where there is a high level density. To the shell energy, E_{sh} , which is defined independently for protons and neutrons, is to be added the liquid-drop energy. For an even nucleus the total energy is therefore given by the following expression:

$$E_{\text{tot}} = E_{\text{L.D.}} + E_{\text{sh}}(\text{prot}) + E_{\text{sh}}(\text{neutr})$$

For the method to be well defined, the results should not be too sensitive