the rotation aligned coupling scheme disadvantageous at large deformations. The dependence of the energy splitting, $e_\Omega - e_{\frac{1}{2}}$, on ε can be calculated from the first order expression given above or can be extracted from a figure like fig. 11.6.

We also must find a value for $\hbar^2/2\mathscr{J}$. An empirical relation (Grodzins, 1962) for the rotational 2^+ energies of even nuclei is $E_{2+} \approx 1225/(A^{7/3}\beta^2)$ MeV. With β transformed to ε we get $E_{2+} \simeq 1100/(A^{7/3}\varepsilon^2)$ MeV, i.e. for $A \simeq 160$:

$$\frac{\hbar^2}{2\mathscr{J}} = \frac{E_{2+}}{6} \approx \frac{1.3}{\varepsilon^2} \quad \text{keV}$$

The two energy expressions corresponding to the deformation aligned (strongly coupled) and rotation aligned coupling schemes are compared in fig. 11.7. This figure should mainly be taken to show the trends. With increasing spin I, increasing particle spin j and decreasing deformation, the rotation aligned coupling scheme becomes more favoured. This is especially the case when the Fermi level is in the region of low-Ω orbitals of a high-j shell.

In fig. 11.7, we only plot the so-called favoured states, $I = j, j+2, j+4, \ldots$. In the pure rotation aligned case, the total nuclear wave function is symmetric with respect to an R_1-rotation and, in a similar way as for an even nucleus, the wave function for the intermediate spin states, $j+1, j+3, \ldots$ disappears. Full alignment is hardly realised in any nucleus. Experimentally, one thus often observes also the $j+1, j+3, \ldots$ states but they come relatively higher in energy than the $j, j+2, \ldots$ states.

In the rotation aligned case, the spin projection on the rotation axis, α, equals j and the rotational energy can be written

$$\begin{aligned} E_{\text{rot}} &= \frac{\hbar^2}{2\mathscr{J}}[I(I+1) + j(j+1) - 2I\alpha] = \frac{\hbar^2}{2\mathscr{J}}[(I-\alpha)(I-\alpha+1) + 2\alpha] \\ &= \frac{\hbar^2}{2\mathscr{J}} R(R+1) + \text{constant} \end{aligned}$$

where $R = I - \alpha$ describes the collective rotation. Thus, the energy spacings in a rotation aligned spectrum of an odd nucleus should be the same as in neighbouring even nuclei. This is nicely illustrated in fig. 11.8. With 57 protons in the La nuclei, the Fermi level is situated around the [550 1/2] orbital with $j \approx 11/2$ (fig. 11.5) and rotation aligned bands starting with $I = 11/2$ are formed.

The breaking of the deformation aligned coupling scheme is generally referred to as decoupling. The wave function of the particle is then distributed

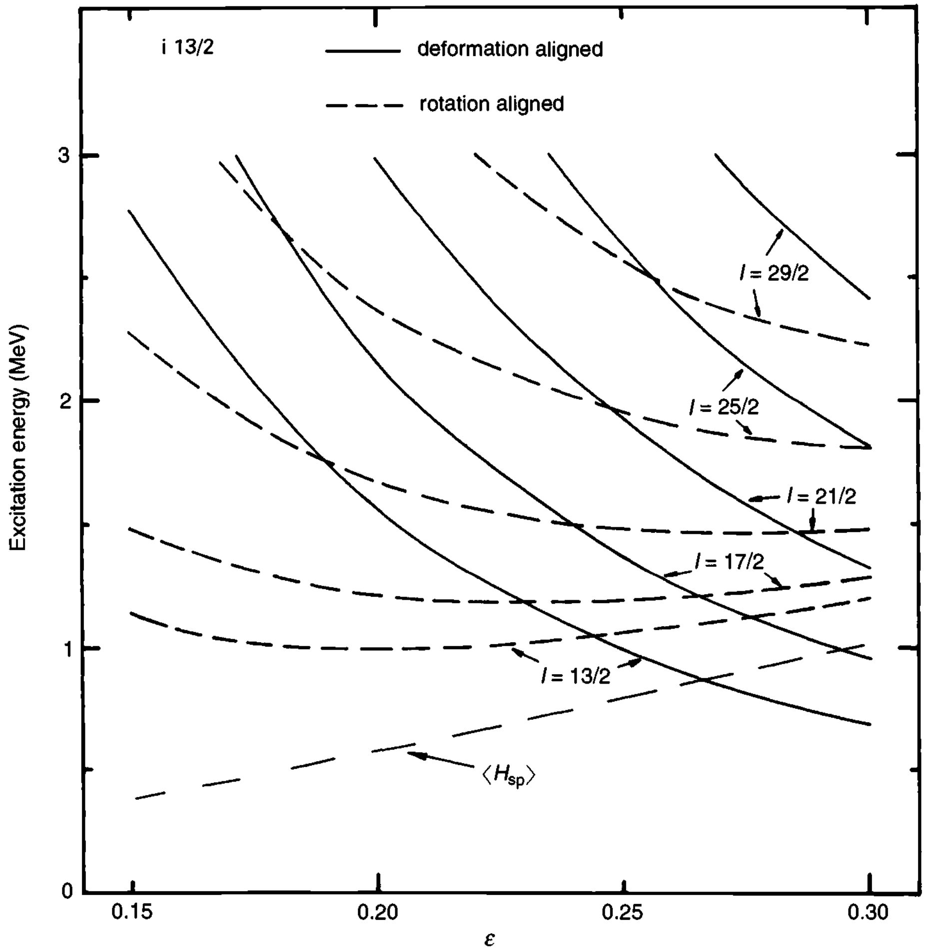

Fig. 11.7. With the Fermi level on the $\Omega = \frac{1}{2}$ state in an $i_{13/2}$ shell, the spectra of the favoured states in the two coupling schemes, deformation alignment and rotation alignment, are shown as functions of ε. The quantity $\langle H_{sp} \rangle$ is the energy required when the wave function of the odd particle is redistributed over the $i_{13/2}$ orbitals to get its spin vector aligned with the axis of rotation. One notes that small deformations and high spins tend to favour the rotation aligned scheme. If the particle–rotor Hamiltonian is fully diagonalised, a situation between the two simple models of the figure will result. However, many experimental spectra can be quite accurately described by one or the other of the two extremes.

over several 'deformed orbitals' and the spin of the particle is largely aligned along the collective rotation vector, **R**. In the idealised situation described in the present section, this alignment is complete. In the strong coupling scheme, the mixing of the $\Omega = \pm\frac{1}{2}$ orbitals correspond to a partial alignment. As the particle wave function is equally distributed over the $\Omega = \frac{1}{2}$ and the

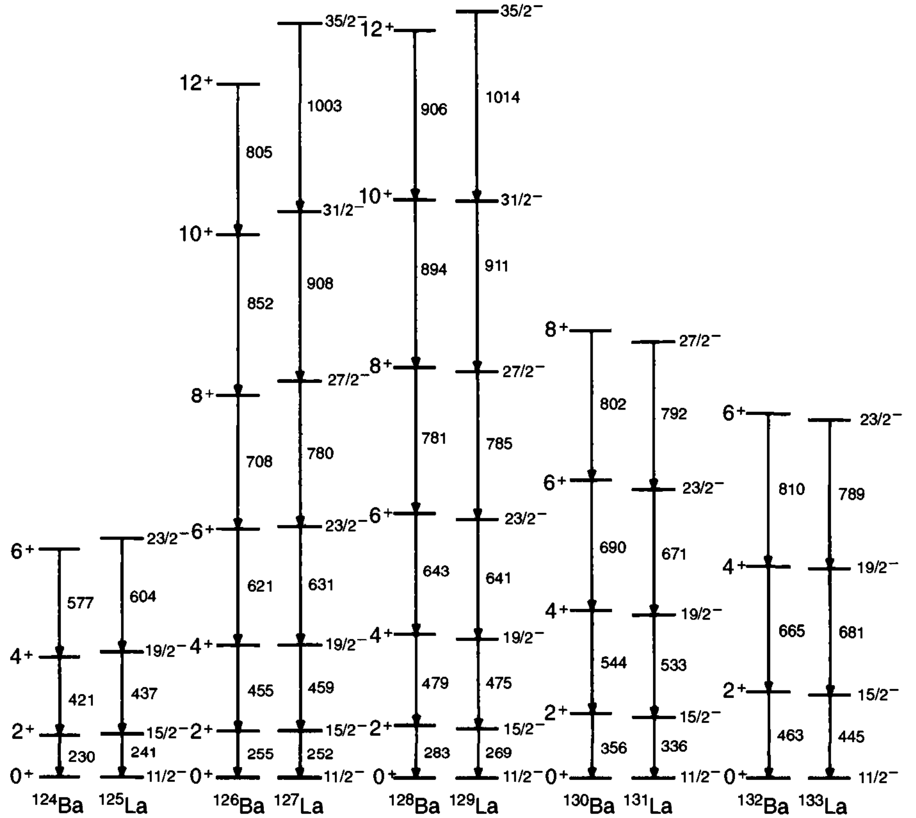

Fig. 11.8. A comparison of the rotational bands based on the $h_{11/2}$ proton orbital in the odd-mass $_{57}$La isotopes with the ground state bands of the neighbouring $_{56}$Ba nuclei. The very similar features of the neighbouring spectra suggest that the spin of the odd nuclei is obtained by simple addition of the particle spin and the collective spin, i.e. that these two spin vectors are aligned (from R.M. Lieder and H. Ryde, *Adv. Nucl. Phys.*, eds. M. Baranger and E. Vogt (Plenum Publ. Corp., New York) vol. 10 (1978) p. 1).

$\Omega = -\frac{1}{2}$ orbitals (the v and $\bar{v}$ orbitals), the alignment is easily calculated as

$$\begin{aligned}\langle j_1 \rangle &= \left\langle \frac{1}{\sqrt{2}}(\phi_v + \phi_{\bar{v}}) \,|j_1|\, \frac{1}{\sqrt{2}}(\phi_v + \phi_{\bar{v}}) \right\rangle \\ &= \frac{1}{4}\left(\langle \phi_v \,|j_+|\, \phi_{\bar{v}} \rangle + \langle \phi_{\bar{v}} \,|j_-|\, \phi_v \rangle \right) = \frac{1}{2}a\end{aligned}$$

For an orbital $|j\Omega\rangle = |13/2\ 1/2\rangle$, it was found from the equation above that the decoupling factor $a = -7$, i.e. $|\langle j_1 \rangle| = 3.5$ in the strong coupling approximation. This should be compared with $\langle j_1 \rangle = \alpha = 6.5$ for full alignment in a $j = 13/2$ shell.

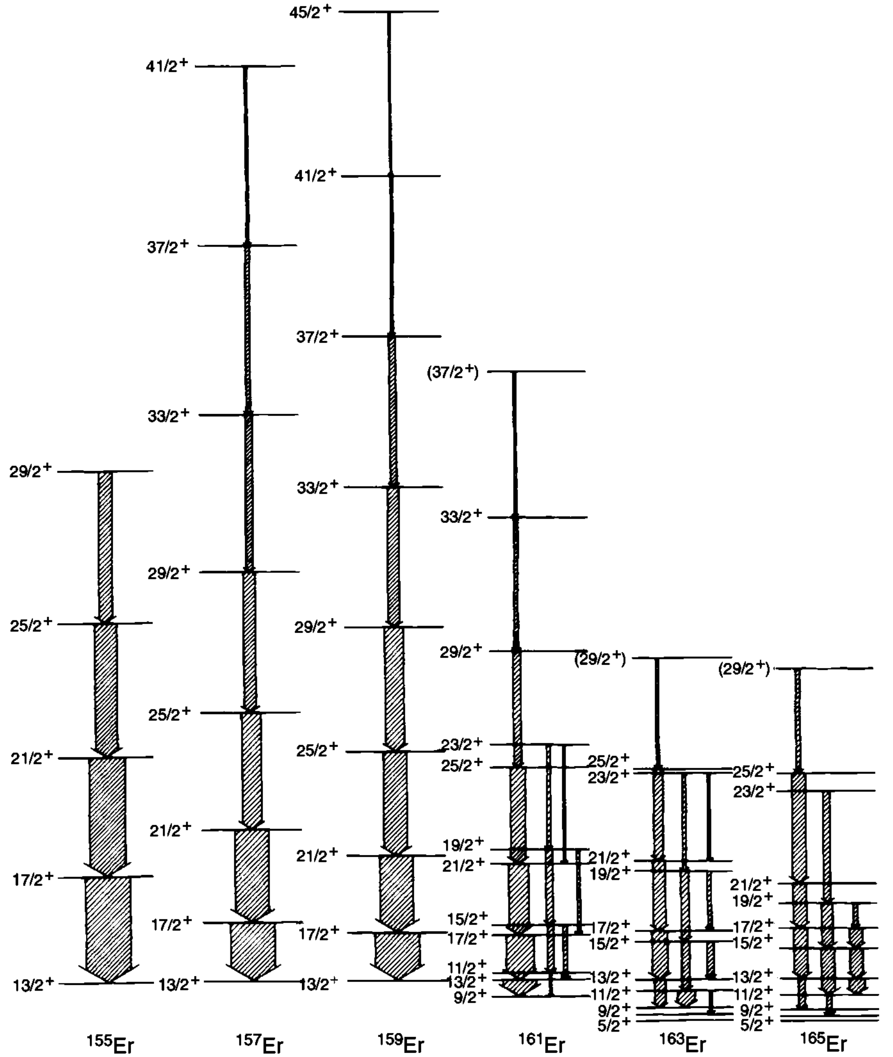

Fig. 11.9. Observed rotational bands based on the $i_{13/2}$ neutron orbitals in odd-mass $_{68}$Er isotopes. One observes a gradual change from a rotation aligned spectrum in ^{155}Er and ^{157}Er, to a deformation aligned spectrum for the lower spin states in ^{165}Er (from R.M. Lieder and H. Ryde, *Adv. Nucl. Phys.*, eds. M. Baranger and E. Vogt (Plenum Publ. Corp., New York) vol. 10 (1978) p. 1.).

The transition between the two coupling schemes is illustrated in fig. 11.9. The positive-parity spectra of the odd $_{68}$Er isotopes with $N = 89$–97 are shown. These isotopes change from being weakly deformed with the Fermi level around the $i_{13/2}$, $\Omega = \frac{1}{2}$ orbital for small N to larger deformations with the Fermi level higher up in the $i_{13/2}$ shell with increasing N. Consequently,