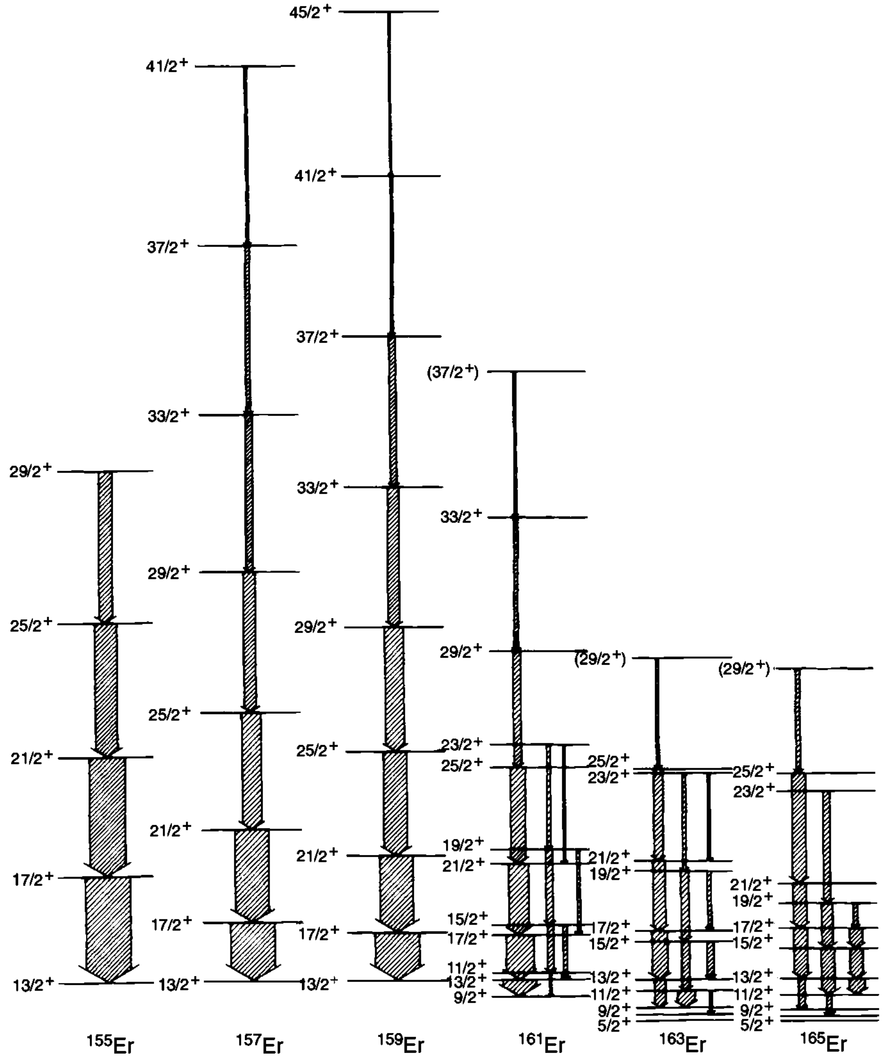

Fig. 11.9. Observed rotational bands based on the $i_{13/2}$ neutron orbitals in odd-mass $_{68}$Er isotopes. One observes a gradual change from a rotation aligned spectrum in ^{155}Er and ^{157}Er, to a deformation aligned spectrum for the lower spin states in ^{165}Er (from R.M. Lieder and H. Ryde, *Adv. Nucl. Phys.*, eds. M. Baranger and E. Vogt (Plenum Publ. Corp., New York) vol. 10 (1978) p. 1.).

The transition between the two coupling schemes is illustrated in fig. 11.9. The positive-parity spectra of the odd $_{68}$Er isotopes with $N = 89$–97 are shown. These isotopes change from being weakly deformed with the Fermi level around the $i_{13/2}$, $\Omega = \frac{1}{2}$ orbital for small N to larger deformations with the Fermi level higher up in the $i_{13/2}$ shell with increasing N. Consequently, the spectrum is essentially decoupled for $N = 89$–91, intermediate for $N = 93$ and strongly coupled for $N = 95, 97$. Note, however, that also for these latter isotopes the high spin favoured states, $I = 17/2,\ 21/2,\ 25/2, \ldots$ come relatively lower in energy than the unfavoured $15/2,\ 19/2,\ 23/2$ states. Thus, as expected, the rotation aligned coupling scheme becomes more important with increasing spin.

Here we have only discussed the two extreme coupling schemes. It should, however, be evident that it is straightforward to diagonalise the full particle–rotor Hamiltonian and thus to describe intermediate situations as for example the spectra of ^{161}Er and ^{163}Er shown in fig. 11.9. Furthermore, only axially symmetric shapes have been considered. For the generalisation of the particle–rotor Hamiltonian to non-axial shapes, we refer to Larsson, Leander and Ragnarsson (1978) for a derivation along the lines presented here or to Meyer-ter-Vehn (1975) for a somewhat different derivation.

11.3 Two-particle excitations and back-bending

The collective angular momentum vector, $\mathbf{R}$, is built from small contributions of all the paired nucleons. None of the wave functions is then strongly disturbed. For particles in low-Ω high-j orbitals one must, however, expect tendencies, not only for odd nucleons but also for paired nucleons, to align their spin vectors along the collective spin vector (Stephens and Simon, 1972). The maximal aligned spin for the two nucleons in a pure j-shell is then $\alpha_1 = j$ and $\alpha_2 = j - 1$, respectively, leading to a total aligned spin of $\alpha = \alpha_1 + \alpha_2 = 2j - 1$. With $R = I - \alpha$, the collective rotational energy is given by (cf. preceding section):

$$E_{\mathrm{rot}} = \frac{\hbar^2}{2\mathscr{J}} R(R+1) = \frac{\hbar^2}{2\mathscr{J}} (I-\alpha)(I-\alpha+1)$$

The alignment is however accompanied by the breaking of one pair leading to a configuration with 'two odd particles'. A rough estimate is therefore that the energy cost for breaking the pairs is approximately twice the odd–even mass difference, 2Δ (see chapter 14). Compared to this, the energy cost for redistributing the particle wave function over the different orbitals, as discussed in the preceding section, can be neglected. Furthermore, the pairing correlations will tend to decrease this energy.

In the present approximation, we thus get for the band with 'two aligned spins'

$$E \approx 2\Delta + \frac{\hbar^2}{2\mathscr{J}} (I-\alpha)(I-\alpha+1); \quad I \geq \alpha$$

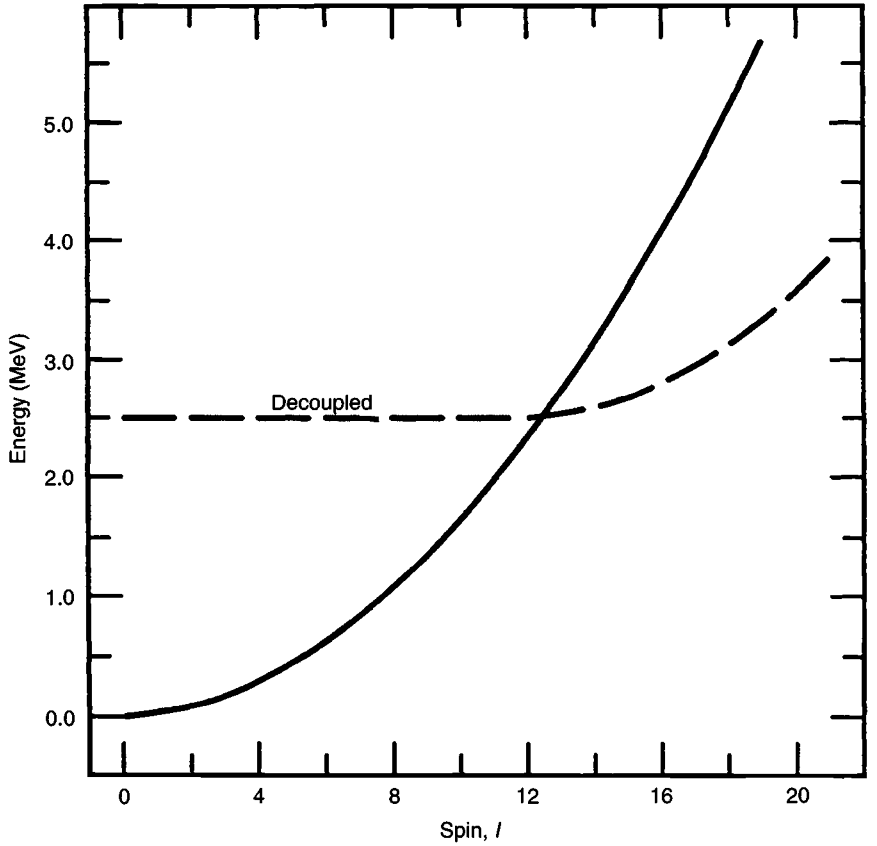

Fig. 11.10. Simple estimates are shown of the ground-band energy in an even–even $A \simeq 160$ nucleus (solid line) and the energy of a decoupled band based on two 'aligned' $i_{13/2}$ particles (dashed line) (from F.S. Stephens, *Proc. 4th Summer School on Nuclear Physics*, Rudziska, Poland, 1972, p. 190).

Total spin values smaller than the largest possible aligned spin can be obtained by a partial alignment with no collective rotation. Provided one pair is broken, this should lead to an energy $E \approx 2\Delta$. The resulting 'aligned' band is compared with the ground band in fig. 11.10.

The states having lowest possible energy for given spin are referred to as the yrast states. A typical yrast line for an $A \simeq 160$ nucleus is sketched in fig. 11.11. In this figure is also shown how the yrast levels can be studied. If two nuclei, e.g. $^{40}_{18}Ar_{22}$ and $^{124}_{52}Te_{72}$ collide in a non-central collision, a compound nucleus having a large excitation energy and a large angular momentum might be formed. By emission of e.g. four neutrons, which each carry away about 8 MeV of excitation energy (i.e. the neutron binding energy) a point some few MeV above the yrast line is reached.