9

The shell correction method and the nuclear deformation energy

We have previously calculated the total energy of the atomic nucleus by use of the *'macroscopic' liquid-drop model.* In this model the energy is assumed to be a sum of a volume term, a surface term and a Coulomb term ($I = (N - Z)/A$):

$$E = -a_{\mathrm{v}} \left(1 - \kappa_{\mathrm{v}} I^2\right) A + a_{\mathrm{s}} \left(1 - \kappa_{\mathrm{s}} I^2\right) A^{2/3} B_{\mathrm{s}}(\mathrm{def}) + a_{\mathrm{C}} \frac{Z^2}{A^{1/3}} B_{\mathrm{C}}(\mathrm{def})$$

We have found that this model could reasonably well explain, in addition to the variation in nuclear mass, various phenomena associated with fission, e.g. why the heavier elements undergo spontaneous fission and, furthermore, the approximate heights of the fission barriers.

However, it is also apparent that many phenomena could not be understood in terms of this model. Thus it does not reproduce the detailed variation in fission barrier height with particle number or the two-peak character of the barriers in the actinide region. Neither could it explain why many nuclei are deformed and not spherical in their ground state. One might say that various nuclear properties are only explained *on the average* (where the average might be taken over particle number or alternatively over deformation) by the liquid-drop model.

To reproduce other aspects of nuclear structure such as ground state spins and energy spectra, it was found that a different description was necessary. In the preceding chapters, we have therefore introduced the *single-particle model.* In this connection we calculated single-particle energies e_ν as functions of the deformation parameters, $e_\nu = e_\nu\,(\varepsilon, \varepsilon_3, \varepsilon_4, \ldots)$. It is now tempting to consider the total energy of the nucleus (often referred to as the potential energy) obtained by the addition of the single-particle energies e_ν:

$$E_{\mathrm{sp}}\,(\varepsilon, \varepsilon_3, \varepsilon_4 \ldots) = \sum_\nu e_\nu\,(\varepsilon, \varepsilon_3, \varepsilon_4, \ldots)$$

There are some problems connected with this procedure, however. First the single-particle energy e_ν is a sum of a kinetic-energy contribution $\langle T_\nu \rangle$ and a potential-energy contribution $\langle V_\nu \rangle$, the latter representing the expectation value of a sum of all the two-particle interactions

$$V_\nu = \sum_\mu U_{\nu\mu}$$

As all the terms $\langle V_\nu \rangle$ are added, the problem arises of whether or not the interactions are counted twice. A second problem concerns the volume conservation condition, which is difficult to generalise to include the effects of the $\ell \cdot$ s-term and, in the modified oscillator model, also the ℓ^2-term.

The recipe of single-particle energy summation has been tried, however, and found to have fair success. The energy surface (i.e. the energy considered as a function of two variables, e.g. ε and ε_4, see fig. 9.3 below) given by the single-particle sum is found to give a lowest minimum usually somewhat removed from spherical shape. The equilibrium shapes of well-deformed nuclei can be directly related to a quadrupole moment Q_2 and in some cases a hexadecapole moment Q_4, which can also be obtained from experiment (optical spectroscopy, Coulomb excitation cross sections etc.). It turns out that at least Q_2 is in good agreement with data. When extended to larger distortions the energy surface should then also account for the fission barrier. For this application, however, the single-particle sum recipe is found to be inadequate.

One may note that the restoring energy introduced by the volume conservation condition is a term of very large magnitude, being roughly proportional to $(\varepsilon^2/9)$ times the total nuclear energy, or for $\varepsilon = 0.9$ of the order of 1000 MeV. As, among other things, the entire nuclear potential is not included in the volume conservation condition, 'small' corrections to the gross trends of the total energy are not unexpected. On the other hand, the vicinity of the spherical shape appears to be correctly reproduced as long as the corrrect level order is reproduced. Indeed the entire topological character of the energy surface may be obtained although the entire surface appears to be tilted.

Indeed a renormalisation of the energy surface appears to be called for and it is brought about by the introduction of the Strutinsky (1967) procedure. The basic idea behind this is the following. The average, long-range behaviour of nuclear binding energy as a function of the nuclear charge and size is well reproduced by the liquid-drop model. One then surmises that on the average this model also adequately describes deformation†. The relative success of the liquid-drop theory of fission may be taken as a warrant for

† This is, of course, what is conjectured in the original application of the model to the theory of fission in the classical paper by N. Bohr and J.A. Wheeler (1939).