Furthermore, we show the energy

$$E = \frac{\hbar^2}{2\mathscr{J}_{\text{rig}}} I^2$$

where $\mathscr{J}_{\text{rig}}$ is taken as a constant, namely the rigid moment of inertia of the harmonic oscillator at the $I = 0$ deformation (see problem 12.4). Except for the 2^+ energy, the calculated yrast line of the modified oscillator potential (Ragnarsson *et al.*, 1981) is in quite good agreement with experiment. It is especially satisfying that the relatively low energy of the $I = 8$ state is reproduced. This is in contrast to the harmonic oscillator calculations, which give a much too large E_{8+} to E_{6+} spacing. The equilibrium shapes in the two models are also shown in fig. 12.7. The main feature is that the shape remains essentially prolate up to the $I = 6$ state and that a large change in deformation occurs between $I = 6$ and $I = 8$.

If one goes to higher spins than $I = 8$ for ^{20}Ne, one expects an increase of the deformation. In the macroscopic description, this is understood as a result of the centrifugal forces. In the microscopic harmonic oscillator model, particles must be excited to higher shells to get spins above $I = 8$. This naturally leads to larger deformations.

The case of ^{20}Ne is particularly simple because of the axial symmetry ($\Sigma_1 = \Sigma_2$). For a triaxial configuration, there is the possibility to rotate around each of the three principal axis. Three different bands are thus formed but the stability and physical significance of the higher ones is unclear.

The ground state of ^{24}Mg with $\Sigma_1 = 16, \Sigma_2 = 20$ and $\Sigma_3 = 28$ is an example of a configuration that is triaxial in the harmonic oscillator approximation (problem 12.5). In the modified oscillator, however, the ground state comes out as essentially prolate (being soft towards γ-deformations) (Ragnarsson *et al.*, 1981; Sheline *et al.*, 1988). The shape evolution with increasing spin is then essentially the same as in the ground band of ^{20}Ne. Thus, figs. 12.4 and 12.6 can also be used for a qualitative understanding of ^{24}Mg. Note, however, the high level density at the terminating 12^+ state, indicating a competition between different configurations. This is in contrast to ^{20}Ne where the terminating 8^+ state is calculated to be energetically very favoured compared with other states of similar spin.

12.5 The shell correction method for $I \neq 0$

When the ground state potential energy has been calculated at some fixed deformation it should be possible to get the I-dependence simply by adding

the rotational energy as extracted from the cranking model. Thus, for a prescribed spin I_0, the frequency ω_0 is determined so that

$$I_0 = \sum_{\substack{i \\ \text{occ}}} \langle j_x \rangle_i$$

Then the excitation energy is obtained as

$$E_{\text{exc}} = \sum_{\substack{i \\ \text{occ}}} e_i|_{\omega=\omega_0} - \sum_{\substack{i \\ \text{occ}}} e_i|_{\omega=0}$$

For example, in fig. 12.5, the Strutinsky shell correction method has been applied to calculate the $I = 0$ energy surface while the I-dependence at a fixed deformation has been calculated according to the above formulae. In practice, in each mesh point in deformation space, the cranking Hamiltonian is diagonalised for a number of ω-values. Subsequently, ω_0 and then E_{exc} are obtained from interpolation. In the energy surface of fig. 12.5, the energy has also been minimised with respect to ε_4 deformations.

Very often, however, simple summation to obtain E_{exc} might lead to undesired features. In general, this is caused by deficiencies in the single-particle potential so that the average behaviour of E_{exc} is unrealistic. For example, the (unphysical) ℓ^2-term in the modified oscillator potential corresponds to a velocity-dependence and leads to an average moment of inertia considerably larger than $\mathscr{J}_{\text{rig}}$. Similarly, in some parametrisations of the Woods–Saxon potential, the radius parameter is different from experimentally observed nuclear radii and, with $\mathscr{J} \propto r^2$, this might have rather drastic effects.

It is expected, however, that the *fluctuations* are more accurately described by the sums, cf. chapter 9. Therefore, it appears reasonable to retain only these fluctuations with the average behaviour governed by the rigid body moment of inertia. To this end we define (see e.g. Andersson *et al.*, 1976) a spin-dependent shell correction energy

$$E_{\text{sh}}(I_0) = \Sigma e_i \Big|_{I=I_0} - \widetilde{\Sigma e_i}\Big|_{\tilde{I}=I_0}$$

where the smoothed single-particle sums (indicated by '$\sim$') are calculated from a Strutinsky procedure essentially the same as that described in chapter 9. Subsequently, the total energy is calculated as the sum of the rotating liquid-drop energy and the shell energy,

$$E_{\text{tot}}(\bar{\varepsilon}, I) = E_{\text{L.D.}}(\bar{\varepsilon}, I = 0) + \frac{\hbar^2}{2\mathscr{J}_{\text{rig}}(\bar{\varepsilon})} I^2 + E_{\text{sh}}(\bar{\varepsilon}, I)$$

where $\bar{\varepsilon}$ is a shorthand notation for the deformation, $\bar{\varepsilon} = (\varepsilon, \gamma, \varepsilon_4, \ldots)$.

In the definition of the shell energy, all quantities should be evaluated at the same spin I_0, i.e. the smoothed single-particle energy sum should be calculated at an ω-value giving a smoothed spin $\widetilde{\Sigma m_i} = I_0$. Thus, the ω-values in the discrete sum and in the smoothed sum are generally different and it becomes difficult to get any feeling for the variation of E_{sh} from an inspection of a single-particle diagram. However, it can be shown that the quantity

$$E_{\text{quasi-sh}}(\omega) = \Sigma e_i^{\omega} - \widetilde{\Sigma e_i^{\omega}}$$

with all quantities calculated at the same ω is numerically very similar to E_{sh}. An elementary discussion of this is given in Ragnarsson *et al.* (1978). The quantity $E_{\text{quasi-sh}}$ is defined exactly analogous to the static shell energy discussed in chapter 9. Thus, ω enters very much as a deformation parameter and we can take over all our experience from the static case; specifically that gaps in the single-particle spectrum give a favoured (negative) shell energy while a large level density leads to a positive shell energy, i.e. an unfavoured configuration.

12.6 Competition between collective and single-particle degrees of freedom in medium-heavy nuclei

We will now turn to heavier nuclei where, as seen in fig. 11.2, the moment of inertia extracted from the measured 2^+ to 0^+ energy spacing is less than 50% of the calculated rigid body value. We have already pointed out that the low value is due to the pairing correlations (the pairing correlations are less important in a light nucleus like ^{20}Ne). With increasing spin, the experimental moment of inertia becomes larger (fig. 11.13) and for the deformed rare-earth nuclei, it comes close to the rigid body value in the $I = 20$–30 region. This suggests that the pairing correlations are rather unimportant at these spins and the same conclusion is also reached from more fundamental theoretical considerations. The cranking model in the form in which we applied it to ^{20}Ne, with independent particles in a rotating potential, should then be applicable to heavy nuclei at high enough spins, let's say $I \geq 30$. For such high spins, the approximation of identifying the total spin with the projection on the rotation axis should also be quite accurate. The result from ^{20}Ne that the model seems to describe the spectrum quite reasonably all the way down to $I = 0$ or at least $I = 2$ is in some ways surprising. Indeed, the application of a rotating independent particle model to the $I = 0, 2, \ldots$ states of ^{20}Ne can hardly be justified theoretically.

Calculated potential energy surfaces for ^{160}Yb at different spin values