

A Systematic Description of the Wobbling Motion in Odd-Mass Nuclei Within a Semi-Classical Formalism

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TOC

1 Aim and Motivation

2 Nuclear Shapes

3 Triaxiality and Wobbling Motion

- Even-A case study

Aim



Research Objectives

- Extend the current interpretation of the **nuclear triaxiality** in the context of its unique fingerprint: **Wobbling Motion**
- Adopt a framework that is as close as possible to **classical physics**.
- Provide new formalisms for the phenomena related to **nuclear deformation**.

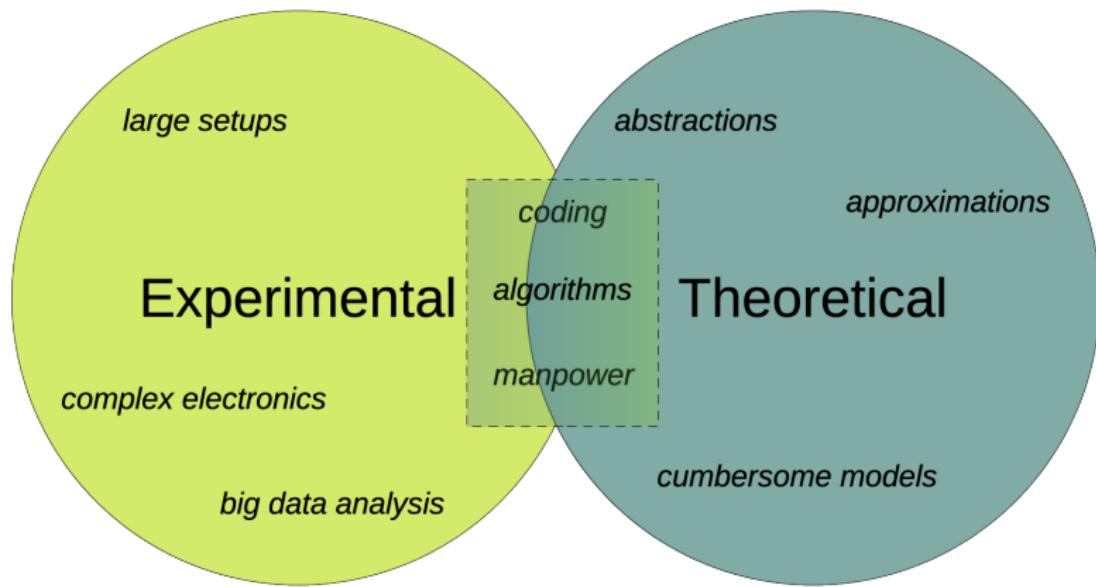


Objectives exclusive to the thesis

- Give the reader enough context towards a better understanding of the underlying concepts, methods, and results.
- Create a completely *open-source* project.

Motivation

- **Nuclear Triaxiality** has become a *hot topic* within the scientific community.
- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge.



Nuclear Deformation

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta, \varphi) = R_0 \left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \varphi) \right) \quad (1)$$

Quadrupole deformations $\lambda = 2$

- **For us:** Most relevant modes are the **quadrupole vibrations** $\lambda = 2$
 \implies Play a crucial role in the rotational spectra of nuclei:
- *Bohr, 1969:* Coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (**eccentricity**) and γ (**triaxiality**).

Axial shapes

Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state (*Budaca, 2018*).
- Moments of inertia: $\mathcal{I}_{1,2,3}$: two are equal, one is different.

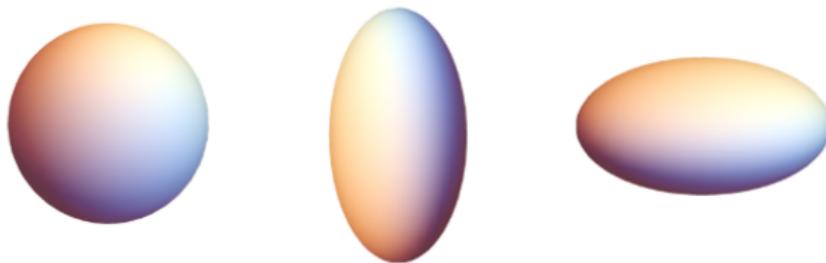
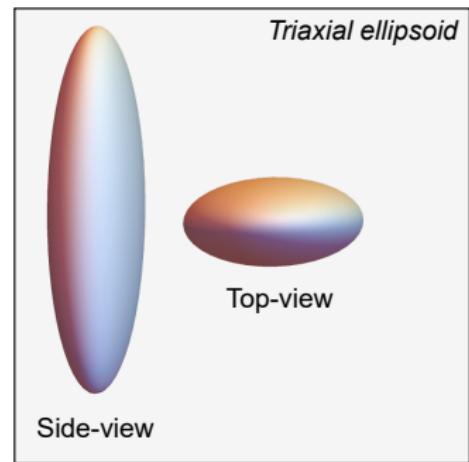
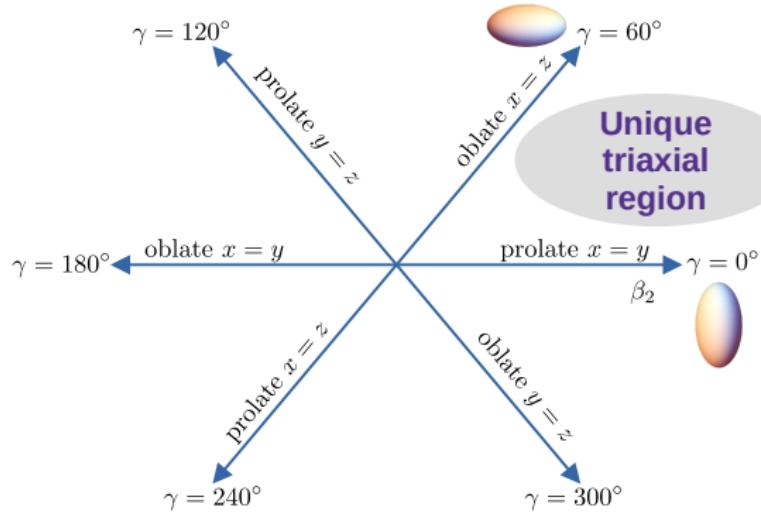


Figure: **spherical**: $\beta_2 = 0$ **prolate**: $\beta_2 > 0$ **oblate**: $\beta_2 < 0$. ($\gamma = 0^\circ$).

Non-axial shapes

- The triaxiality parameter $\gamma \neq 0^\circ$: departure from axial symmetry.
- Moments of inertia: $I_1 \neq I_2 \neq I_3$.



Fingerprints of Triaxiality

Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
 - ① **Wobbling Motion** - WM (*Bohr and Mottelson, 1970s*)
 - ② Chiral Motion - χ M (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

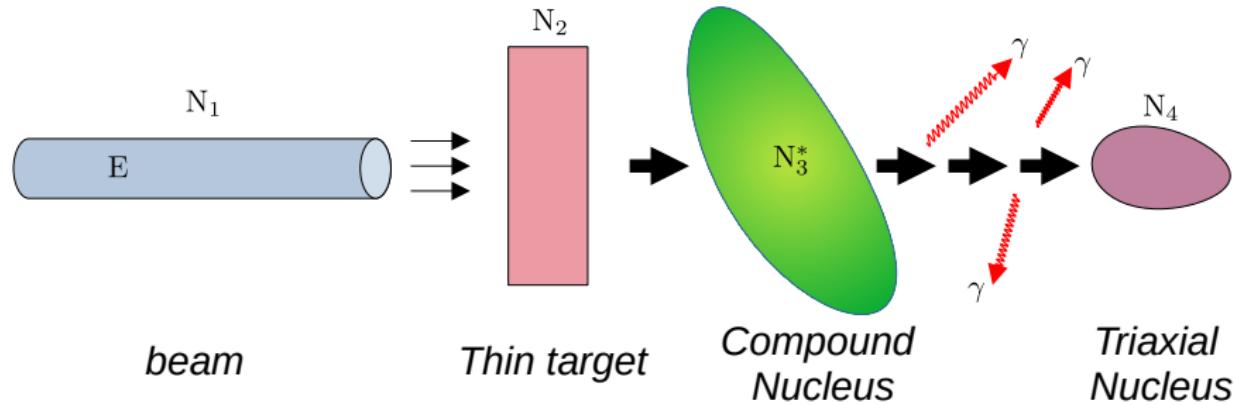
Goal

Describe the elusive character of Wobbling Motion in the context of nuclear triaxiality.

Q Probing triaxiality in nuclei

Triaxial nuclei can be observed/obtained in several experiments:

- Nuclear fission: $A \rightarrow B + C$
- Nuclear fusion: $X + Y \rightarrow Z$
- **Fusion-evaporation reactions:** Long-lived + enhanced deformation



Q Nuclear facilities



Figure: Gammasphere detector, ANL-ATLAS USA. *Source: aps.org*

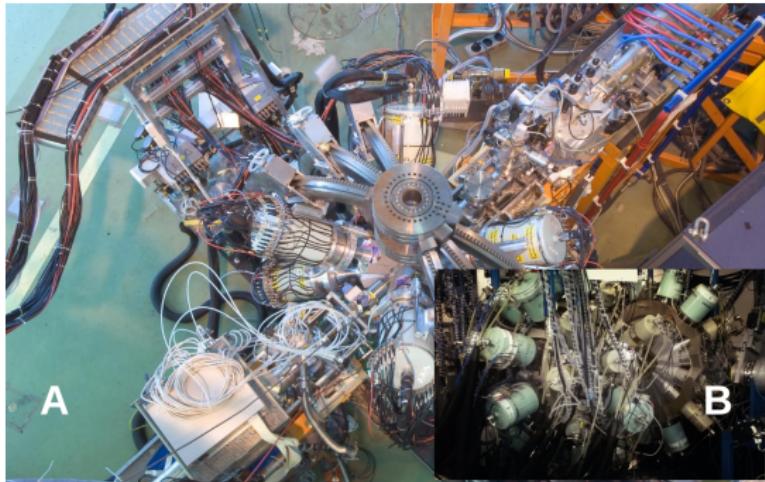
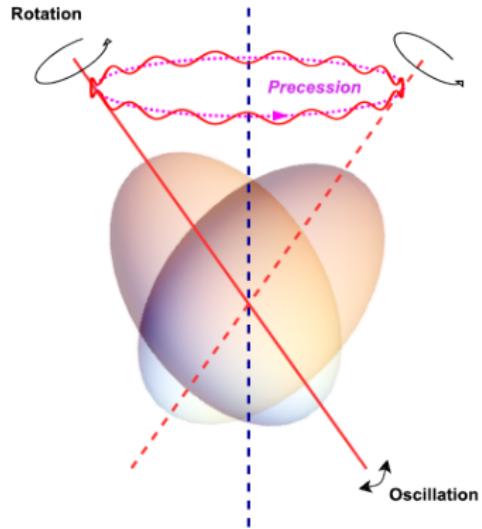
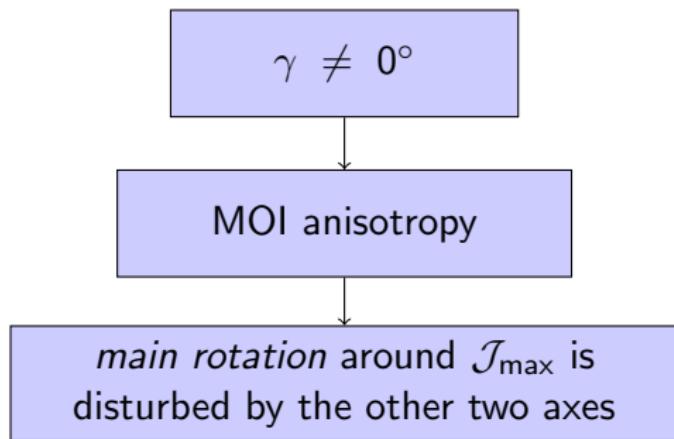


Figure: a) IDS detector, CERN. *Source: isolde.web.cern.ch* b) JUROGAM II, Finland. *Source: twitter.com*

Wobbling Motion



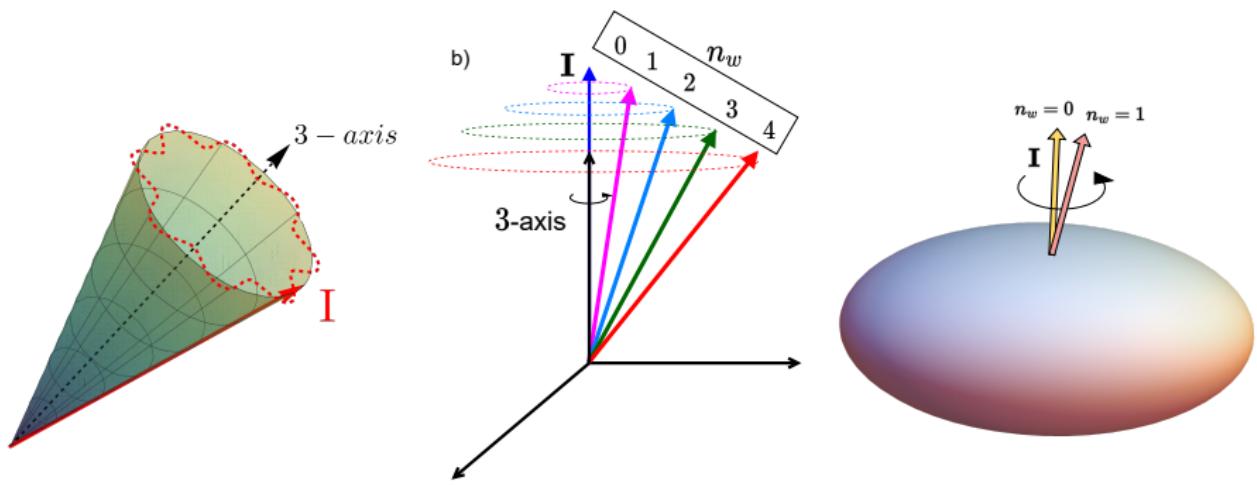
Wobbling Effect

- The **total angular momentum** of the nucleus **precesses** and **oscillates** around \mathcal{J}_{\max} .

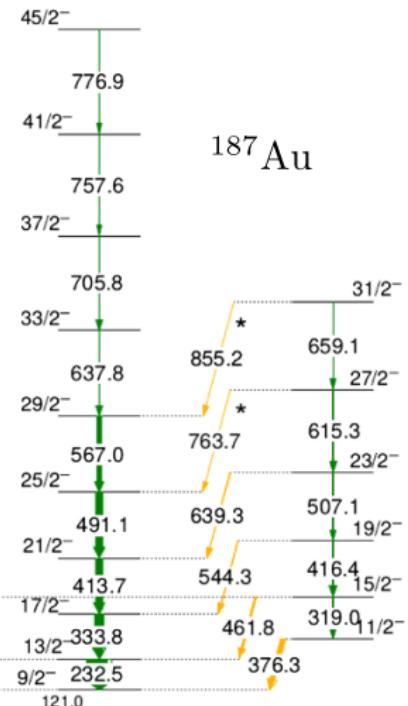
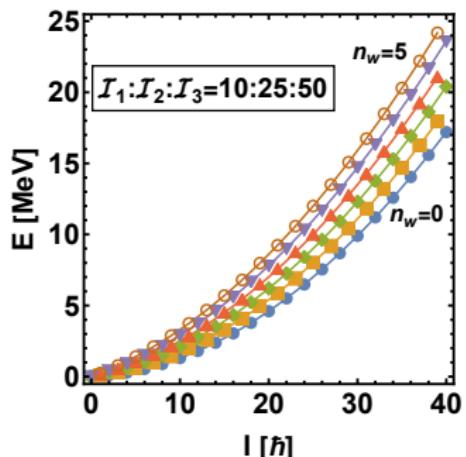
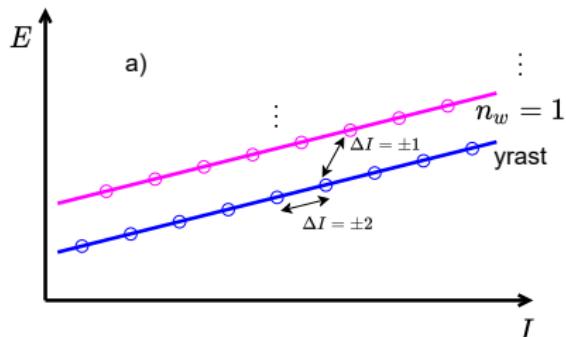
Wobbling Motion

Harmonic oscillation

- Precession of \mathbf{I} is affected by **rotational frequency** and/or **tilting**
 - Tilting only by "specific" amount \rightarrow **harmonic character** \rightarrow **wobbling phonon**: $n_w = 0, 1, 2, \dots$



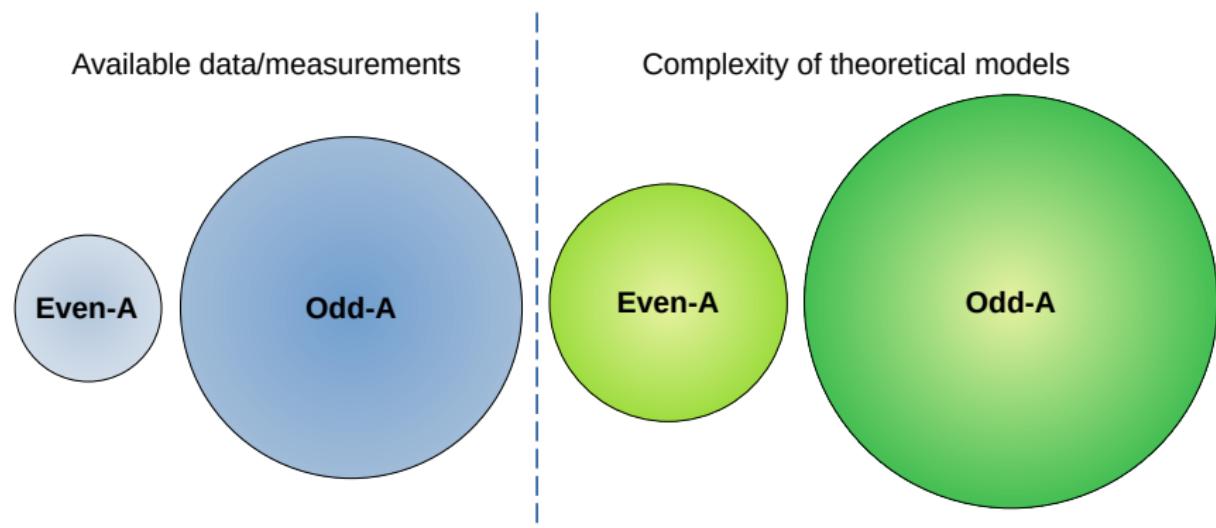
Wobbling Motion II



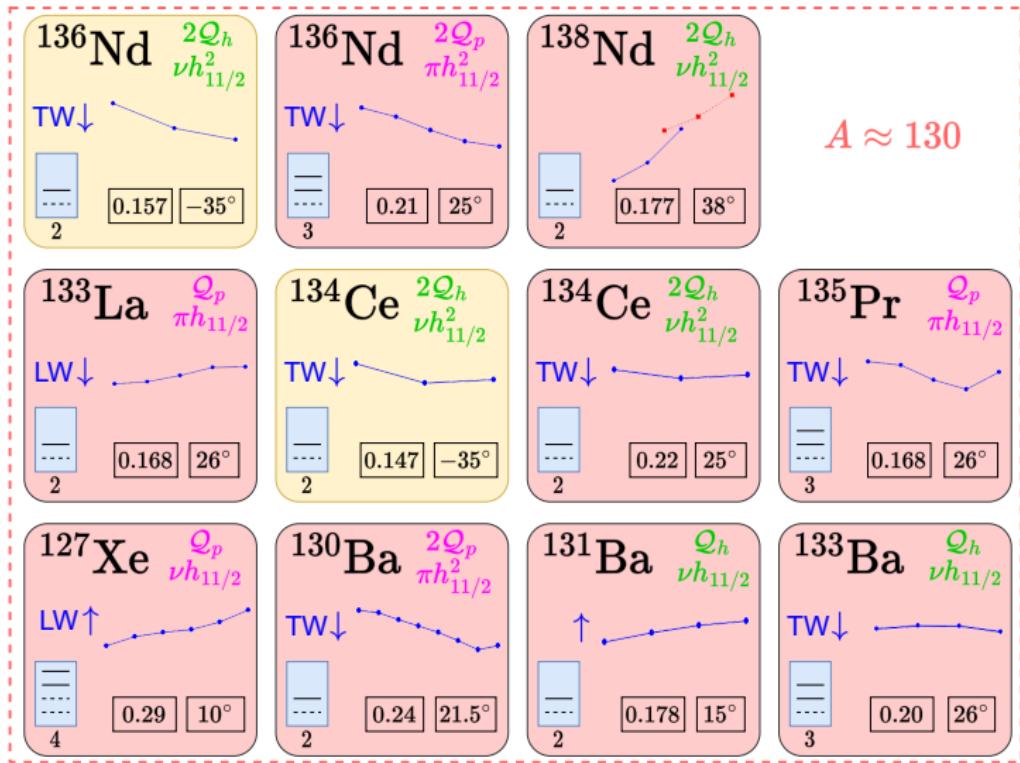
Sensharma, 2020.

Even- A vs. Odd- A Picture

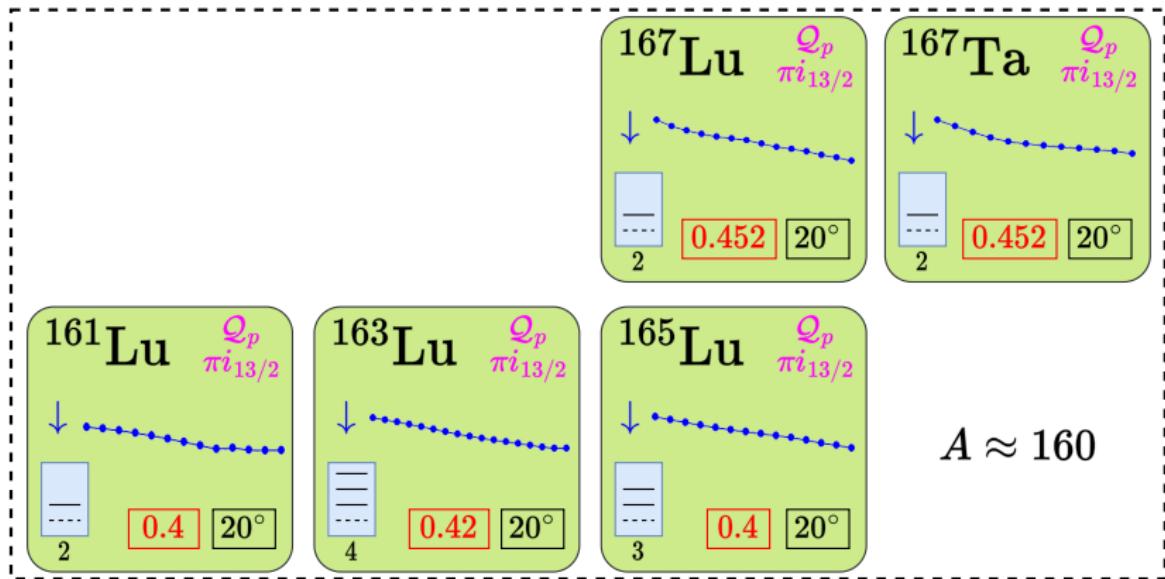
- Predicted for even- A nuclei more than 50 years ago.
- First experimental evidence for **nuclear wobbling motion**: ^{163}Lu (*Ødegård, 2001*).
- Current mass-regions for wobblers: $A = 130, 160, 180$.



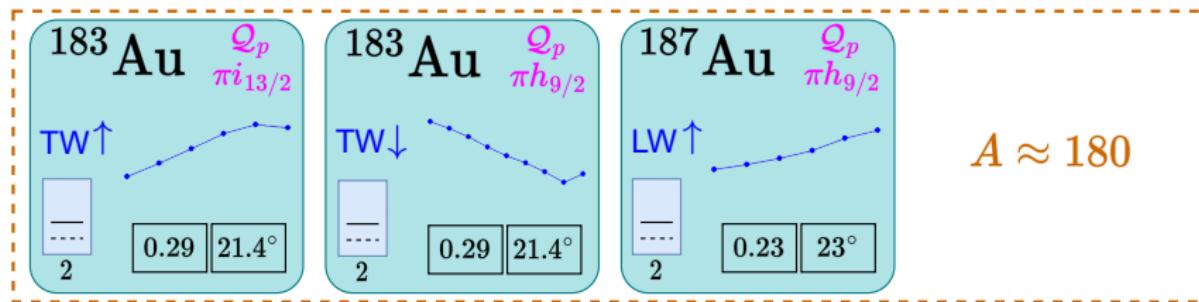
Wobblers in the A=130 mass region



Wobblers in the A=160 mass region



Wobblers in the A=180 mass region



All diagrams and their data sources available in Chapter 3, Section 3.3.5
Presented at the Annual Meeting, FFUB, 2022.

Wobbling Motion in ^{130}Ba

- Q Experimental measurements show **two** wobbling bands (*Petrache et. al. 2019*).

Harmonic formalism

Applied the Harmonic Approximation
Bohr & Mottelson, 1969.

$$E_{I,n_w} = A_3 I(I+1) + \hbar\omega_w \left(n_w + \frac{1}{2} \right)$$

$$A_3 = (2\mathcal{I}_3)^{-1}$$

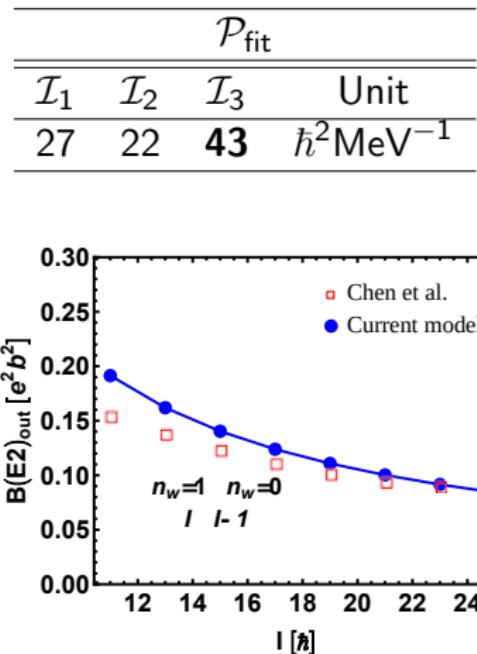
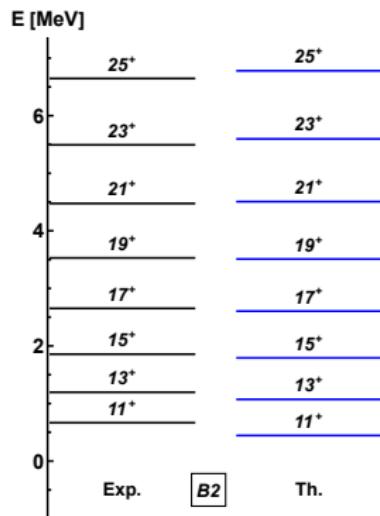
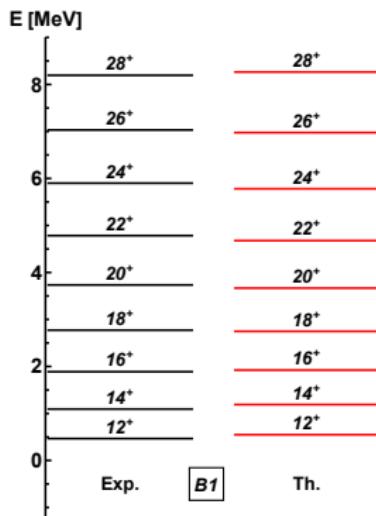
(rotational term + wobbling frequency)



Figure: GALILEO, Source: lnl.infn.it

Fusion evaporation: ^{13}C beam of $E = 65$ MeV and ^{122}Sn target.

Results for ^{130}Ba



Full model description implementation: Chapter 3 (Section 3.1.2)

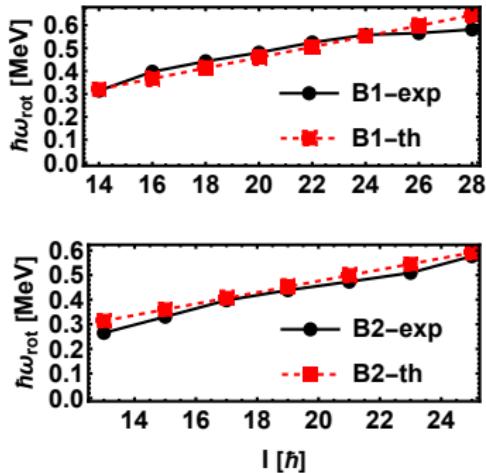
Results presented at the international conference NSP-2022, Turkey.

Results for $^{130}\text{Ba II}$

Excitation energies vs. Wobbling Energies:

$$E_{\text{wob}}(I_{\text{even}}) = E_{I,n} - E_{I,0} ,$$

$$E_{\text{wob}}(I_{\text{odd}}) = E_{I,n} - \frac{1}{2} (E_{I-1,0} + E_{I+1,0})$$



Results presented at the international conference NSP-2022, Turkey.

Thank you for your attention ❤