A Systematic Description of the Wobbling Motion in Odd-Mass Nuclei Within a Semi-Classical Formalism

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TOC

Aim

Research Objectives

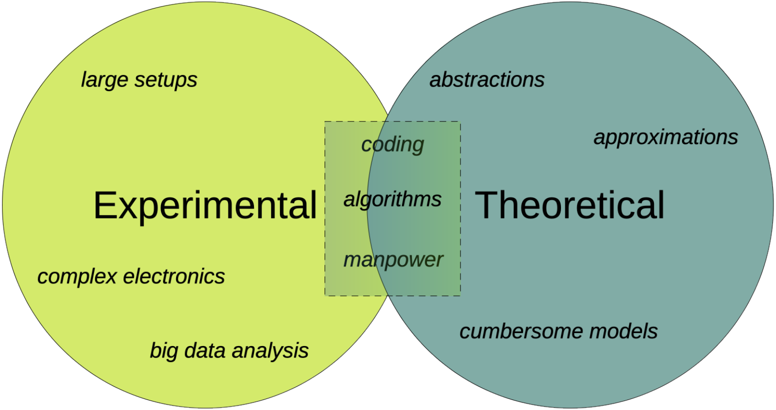
- Extend the current interpretation of the **nuclear triaxiality** in the context of its unique fingerprint: **Wobbling Motion**
- Adopt a framework that is as close as possible to **classical physics**.
- Provide new formalisms for the phenomena related to **nuclear deformation**.

Objectives exclusive to the thesis

- Give the reader enough context towards a better understanding of the underlying concepts, methods, and results.
- create a completely *open-source* project.

Motivation

- **Nuclear Triaxiality** has become a *hot topic* within the scientific community.
- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge.



Nuclear Deformation

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta,\varphi) = R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}Y_{\lambda}^{\mu}(\theta,\varphi)\right) \tag{1}$$

Quadrupole deformations $\lambda = 2$

- For us: Most relevant modes are the **quadrupole vibrations** $\lambda = 2$ $\Longrightarrow$ *Play a crucial role in the rotational spectra of nuclei:*
- *Bohr, 1969*: Coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (**eccentricity**) and γ (**triaxiality**).

Axial shapes

Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state *(Budaca, 2018)*.
- Moments of inertia: $\mathcal{I}_{1,2,3}$: two are equal, one is different.

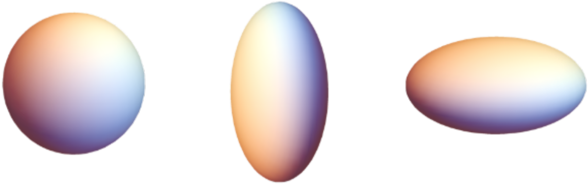
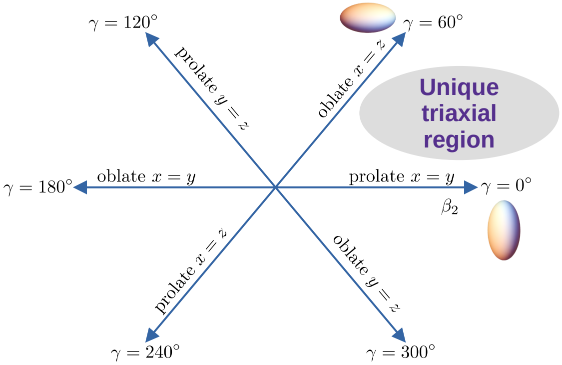
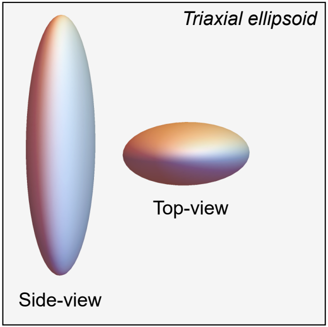


Figure: **spherical:** $\beta_2 = 0$ **prolate:** $\beta_2 > 0$ **oblate:** $\beta_2 < 0$. $(\gamma = 0°)$.

Non-axial shapes

- The triaxiality parameter $\gamma \neq 0^\circ$: departure from axial symmetry.
- Moments of inertia: $\mathcal{I}_1 \neq \mathcal{I}_2 \neq \mathcal{I}_3$.





Fingerprints of Triaxiality

Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
  1. **Wobbling Motion** - WM (*Bohr and Mottelson, 1970s*)
  2. Chiral Motion - χM (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

Goal

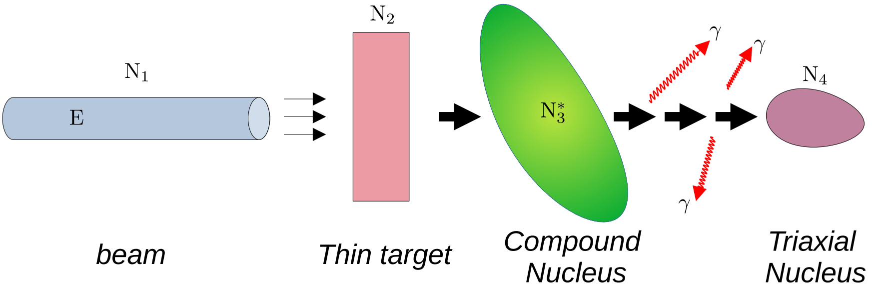
Describe the elusive character of Wobbling Motion in the context of nuclear triaxiality.

Probing triaxiality in nuclei

Triaxial nuclei can be observed/obtained in several experiments:

- Nuclear fission: $A \rightarrow B + C$
- Nuclear fusion: $X + Y \rightarrow Z$
- **Fusion-evaporation reactions**: Long-lived + enhanced deformation

$$Beam(N_1, E) + Target(N_2) \longrightarrow N_3^* \rightarrow \cdots \rightarrow triaxial(N_4)$$



Nuclear facilities

Figure: Gammasphere detector, ANL-ATLAS USA. *Source: aps.org*

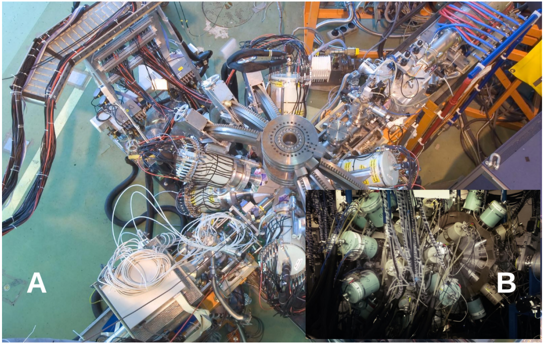


Figure: a) IDS detector, CERN. *Source: isolde.web.cern.ch* b) JUROGAM II, Finland. *Source: twitter.com*

Wobbling Motion

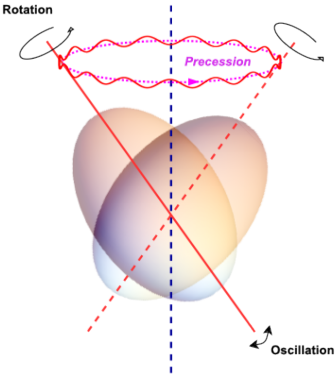
$\gamma \neq 0^\circ$

↓

MOI anisotropy

↓

main rotation around $\mathcal{J}_{\text{max}}$ is disturbed by the other two axes



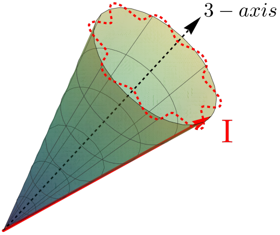
Wobbling Effect

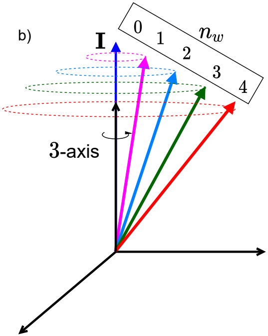
- The **total angular momentum** of the nucleus **precesses** and **oscillates** around $\mathcal{J}_{\text{max}}$.

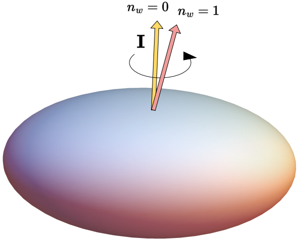
Wobbling Motion

Harmonic oscillation

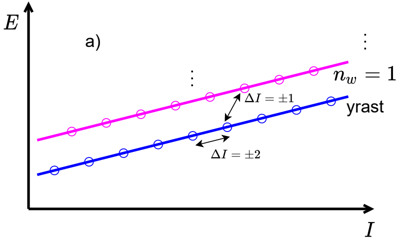
- Precession of **I** is affected by **rotational frequency** and/or **tilting**
- Tilting only by "specific" amount → **harmonic character** → **wobbling phonon**: $n_w = 0, 1, 2, \dots$.

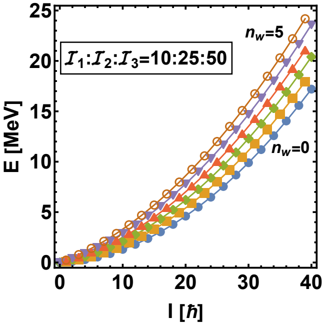


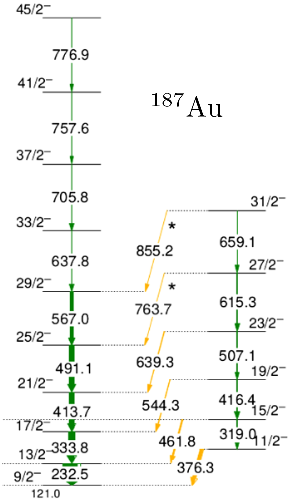




Wobbling Motion II



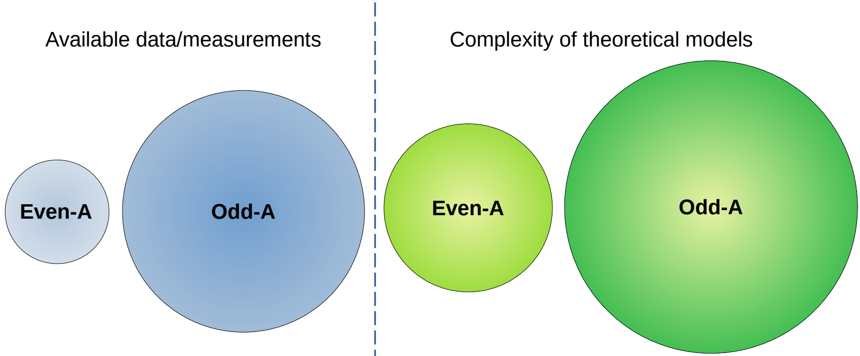




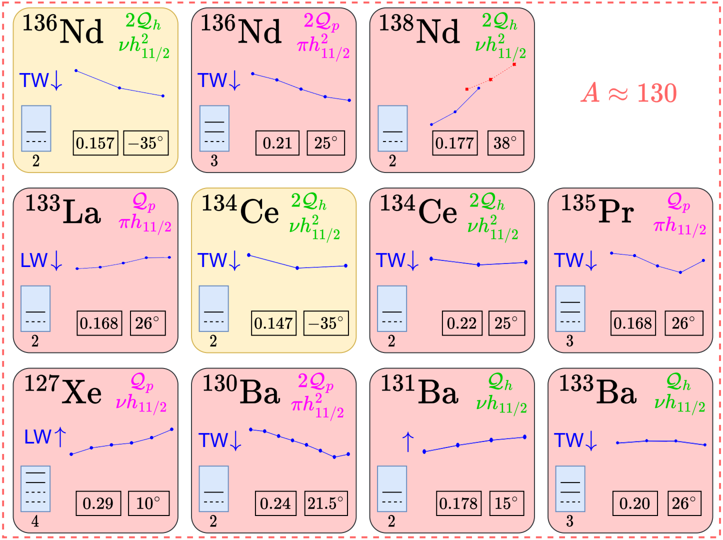
Sensharma, 2020.

Even-A vs. Odd-A Picture

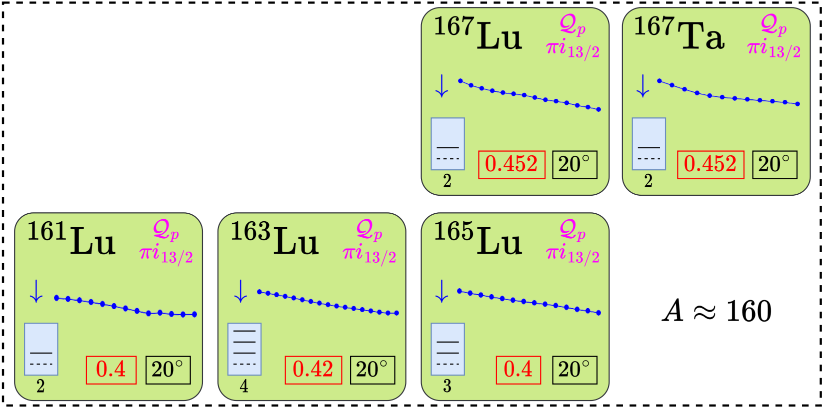
- Predicted for even-A nuclei more than 50 years ago.
- First experimental evidence for **nuclear wobbling motion**: ^{163}Lu (*Ødegård, 2001*).
- Current mass-regions for wobblers: $A = 130, 160, 180$.



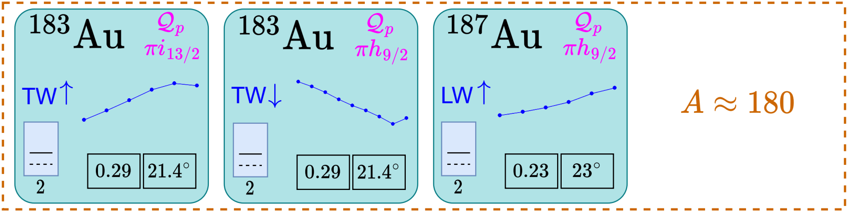
Wobblers in the A=130 mass region



Wobblers in the A=160 mass region



Wobblers in the A=180 mass region



All diagrams and their data sources available in Chapter 3, Section 3.3.5

Presented at the Annual Meeting, FFUB, 2022.

Wobbling Motion in ^{130}Ba

- 🔍 Experimental measurements show **two** wobbling bands (*Petrache et. al. 2019*).

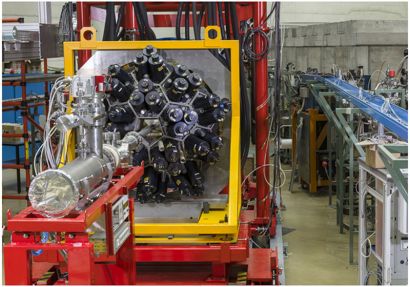
Harmonic formalism

Applied the Harmonic Approximation *Bohr & Mottelson, 1969*.

$$E_{I,n_w} = A_3 I(I+1) + \hbar\omega_w \left(n_w + \frac{1}{2}\right)$$

$$A_3 = (2\mathcal{I}_3)^{-1}$$

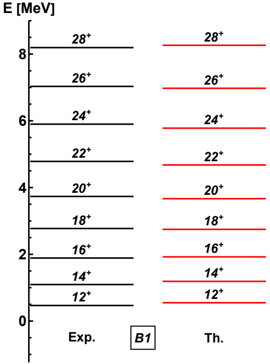
(rotational term + wobbling frequency)

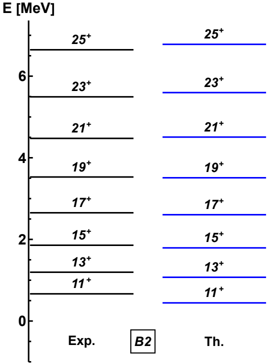


GALILEO, LNL, *Source: lnl.infn.it*

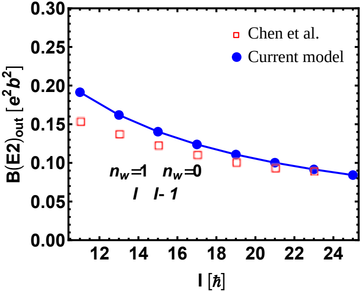
Fusion evaporation: ^{13}C beam of $E = 65$ MeV and ^{122}Sn target.

Results for ^{130}Ba





$\mathcal{P}_{\text{fit}}$			
$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	Unit
27	22	**43**	$\hbar^2\text{MeV}^{-1}$



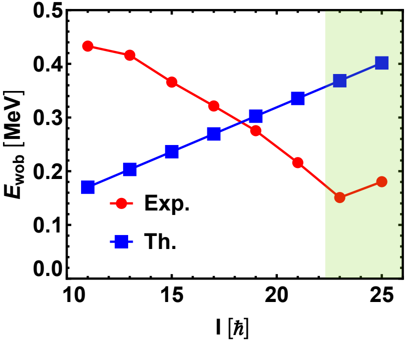
Full description: Chapter 3 (Section 3.1.2)

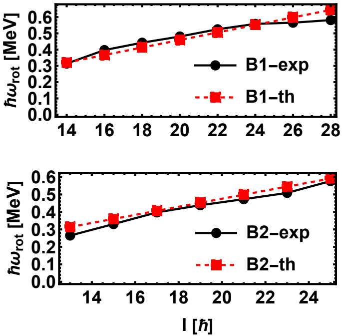
Results presented at the international conference NSP-2022, Turkey.

Results for ^{130}Ba II

Excitation energies vs. **Wobbling Energies**:

$$E_{\text{wob}}(I_{\text{even}}) = E_{I,n} - E_{I,0} \ ,$$
$$E_{\text{wob}}(I_{\text{odd}}) = E_{I,n} - \frac{1}{2}\left(E_{I-1,0} + E_{I+1,0}\right)$$





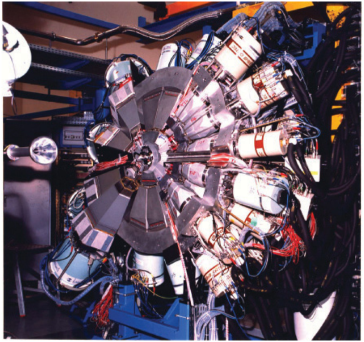
Results presented at the international conference NSP-2022, Turkey.

The "seed"

A. A. Raduta, **R. Poenaru**, L. Gr. Ixaru, PRC, 2017 + A. A. Raduta, **R. Poenaru**, Al. H. Raduta, JPG, 2018→$\mathbf{W}_0$ in the thesis.

Framework

- **First semi-classical description** for the ^{163}Lu, using the Particle-Rotor-Model (*Hamamoto, 2002.*) for an odd-mass nucleus in the $A \approx 160$ region.

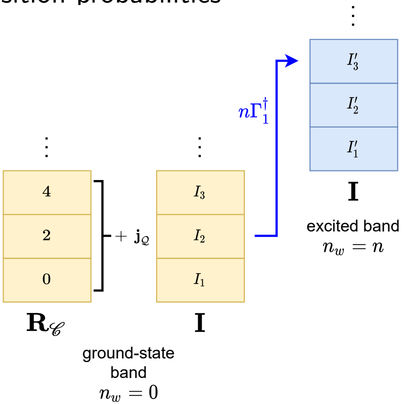


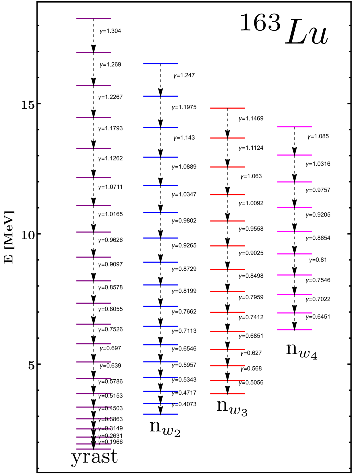
Euroball IV, Strasbourg, *Source: technologysi.stfc.ac.uk*

Fusion evaporation: ^{29}Si beam of $E = 152$ MeV and ^{139}La target.

Overview

- Time-Dependent Variational Principle applied on the PRM Hamiltonian
- **Phonon operators** $\rightarrow$ energies + transition probabilities





$\mathbf{W}_0$ Drawbacks

Thank you for your attention ❤