

A Systematic Description of the Wobbling Motion in Odd-Mass Nuclei Within a Semi-Classical Formalism

PhD Candidate

Robert Poenaru^{1,2}

Scientific Supervisor

Prof. Dr. Em. A. A. Raduta²

¹Doctoral School of Physics, UB

²Department of Theoretical Physics, IFIN-HH

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Aim



Research Objectives

- Extend the current interpretation of the **nuclear triaxiality** in the context of its unique fingerprint: **Wobbling Motion**
- Adopt a framework that is as close as possible to **classical physics**.
- Provide new formalisms for the phenomena related to **nuclear deformation**.

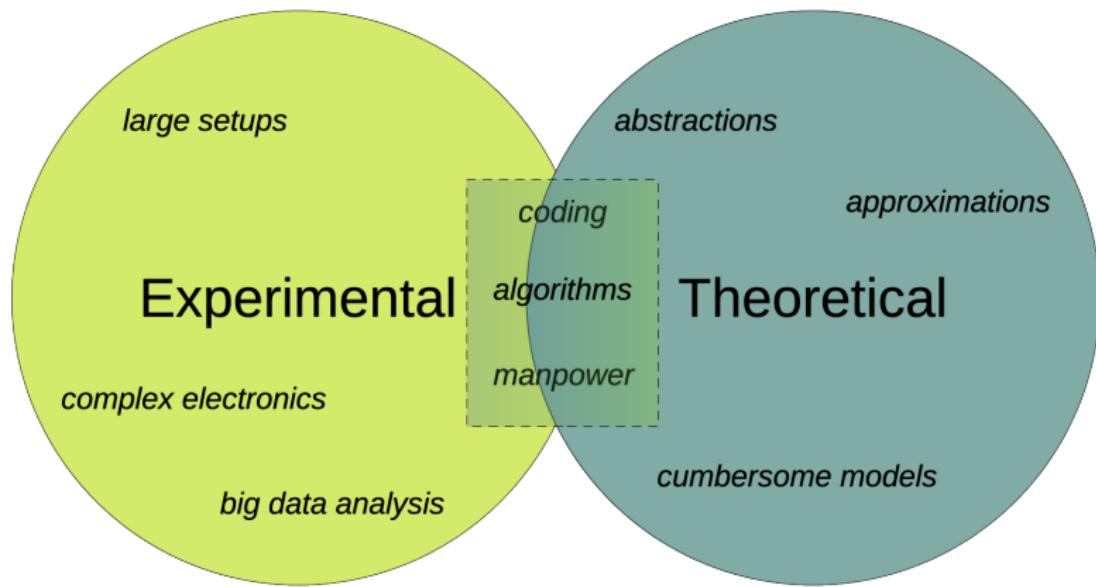


Objectives exclusive to the thesis

- Give the reader enough context towards a better understanding of the underlying concepts, methods, and results.
- Create a completely *open-source* project.

Motivation

- **Nuclear Triaxiality** has become a *hot topic* within the scientific community.
- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge.



Nuclear Deformation

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta, \varphi) = R_0 \left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \varphi) \right)$$

Quadrupole deformations $\lambda = 2$

- **For us:** Most relevant modes are the **quadrupole vibrations** $\lambda = 2$
 \implies Play a crucial role in the rotational spectra of nuclei:
- *Bohr, 1969:* Coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (**eccentricity**) and γ (**triaxiality**).

Axial shapes

Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state (*Budaca, 2018*).
- Moments of inertia: $\mathcal{I}_{1,2,3}$: two are equal, one is different.

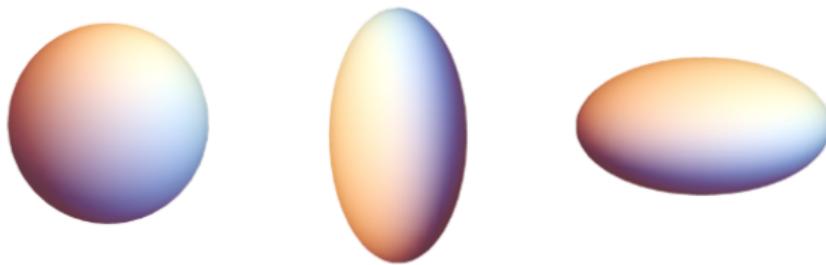
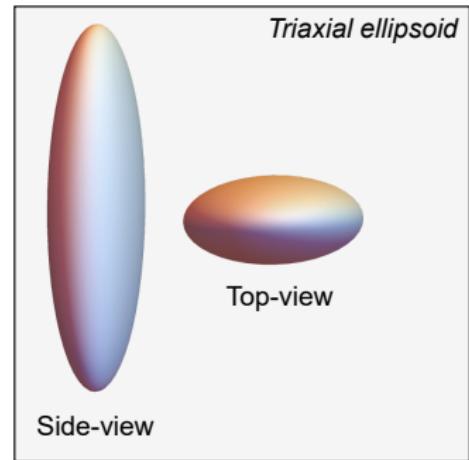
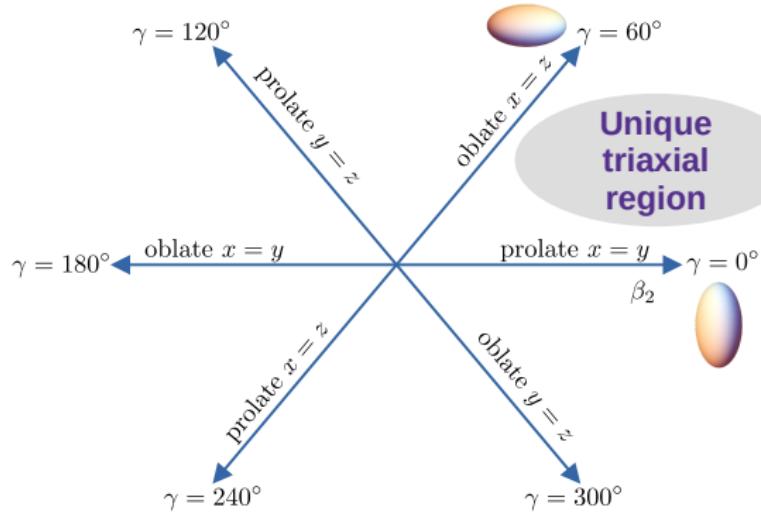


Figure: **spherical**: $\beta_2 = 0$ **prolate**: $\beta_2 > 0$ **oblate**: $\beta_2 < 0$. ($\gamma = 0^\circ$).

Non-axial shapes

- The triaxiality parameter $\gamma \neq 0^\circ$: departure from axial symmetry.
- Moments of inertia: $I_1 \neq I_2 \neq I_3$.



Fingerprints of Triaxiality

Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
 - ① **Wobbling Motion** - WM (*Bohr and Mottelson, 1970s*)
 - ② Chiral Motion - χ M (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

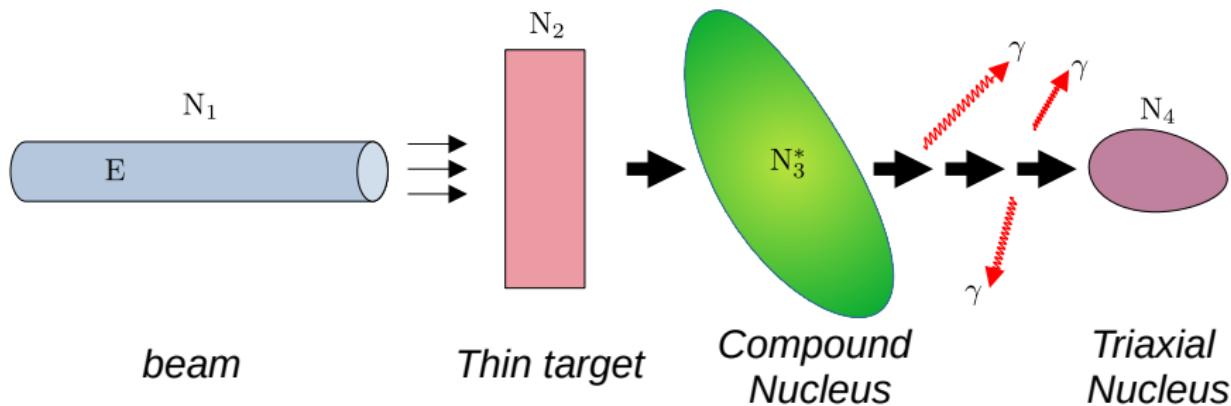
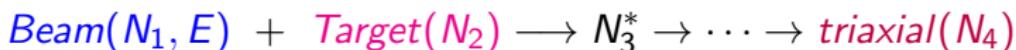
Goal

Describe the elusive character of Wobbling Motion in the context of nuclear triaxiality.

Q Probing triaxiality in nuclei

Triaxial nuclei can be observed/obtained in several experiments:

- Nuclear fission: $A \rightarrow B + C$
- Nuclear fusion: $X + Y \rightarrow Z$
- **Fusion-evaporation reactions:** Long-lived + enhanced deformation



Q Nuclear facilities



Figure: Gammasphere detector,
ANL-ATLAS USA. *Source:*
aps.org

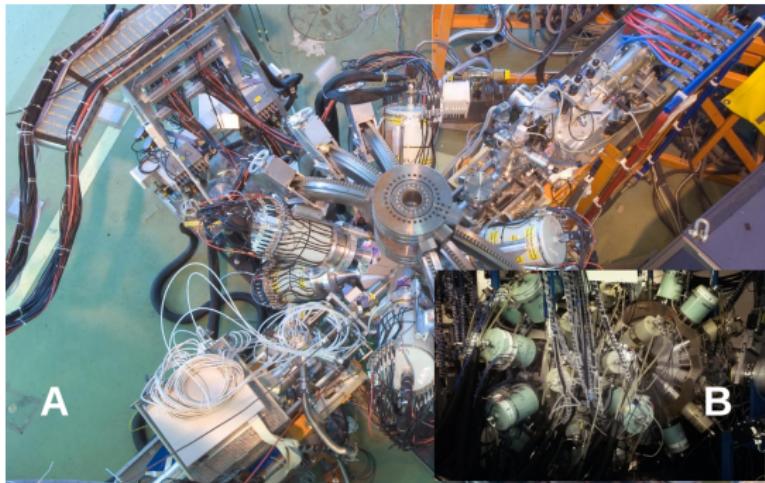
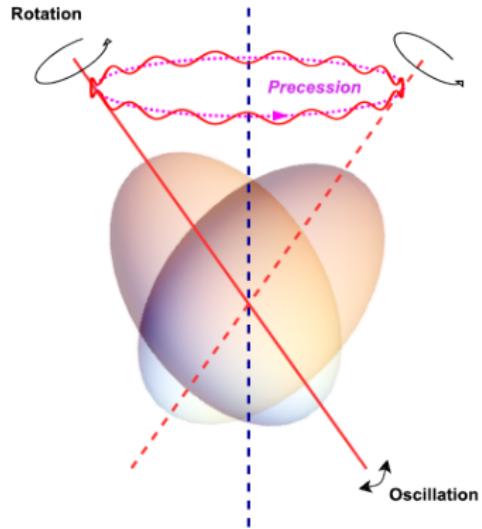
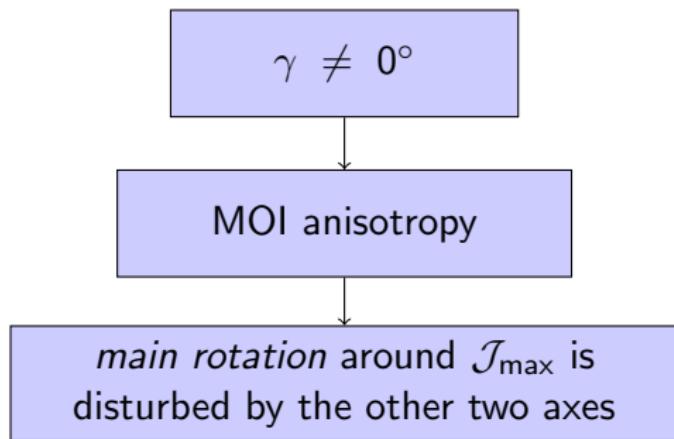


Figure: a) IDS detector, CERN. *Source:*
isodel.web.cern.ch b) JUROGAM II, Finland.
Source: twitter.com

Wobbling Motion



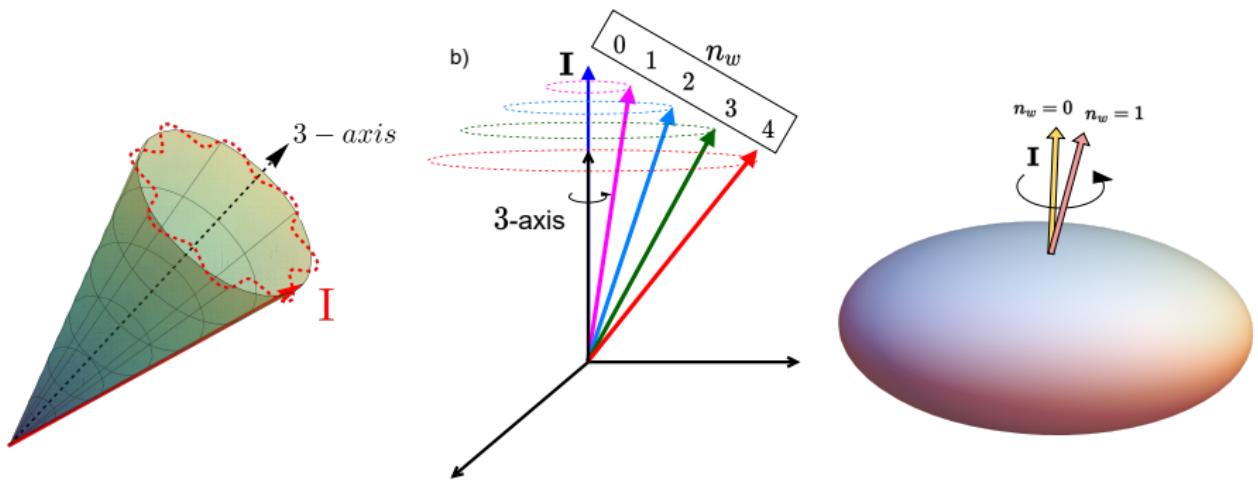
Wobbling Effect

- The **total angular momentum** of the nucleus **precesses** and **oscillates** around \mathcal{J}_{\max} .

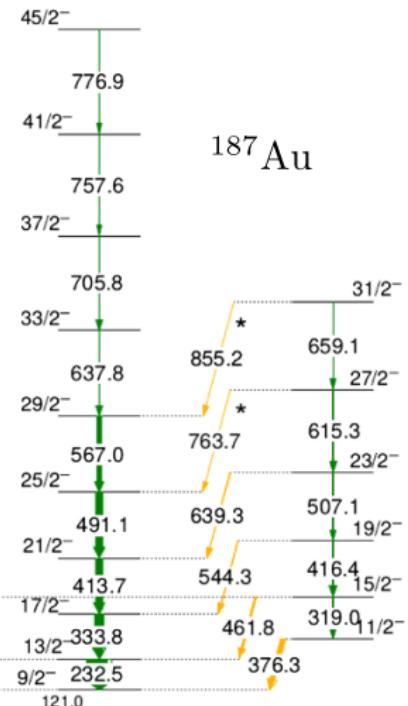
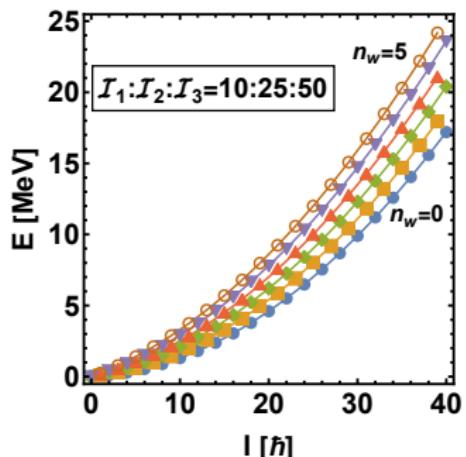
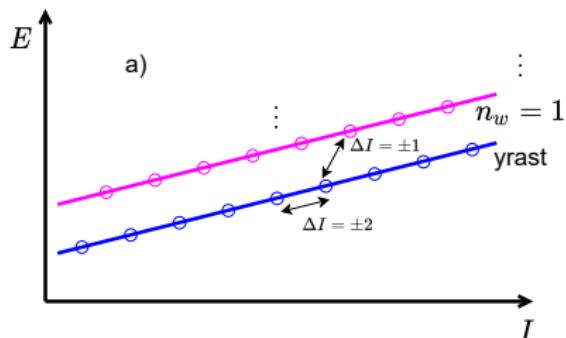
Wobbling Motion

Harmonic oscillation

- Precession of \mathbf{I} is affected by **rotational frequency** and/or **tilting**
- Tilting only by "specific" amount \rightarrow **harmonic character** \rightarrow **wobbling phonon**: $n_w = 0, 1, 2, \dots$



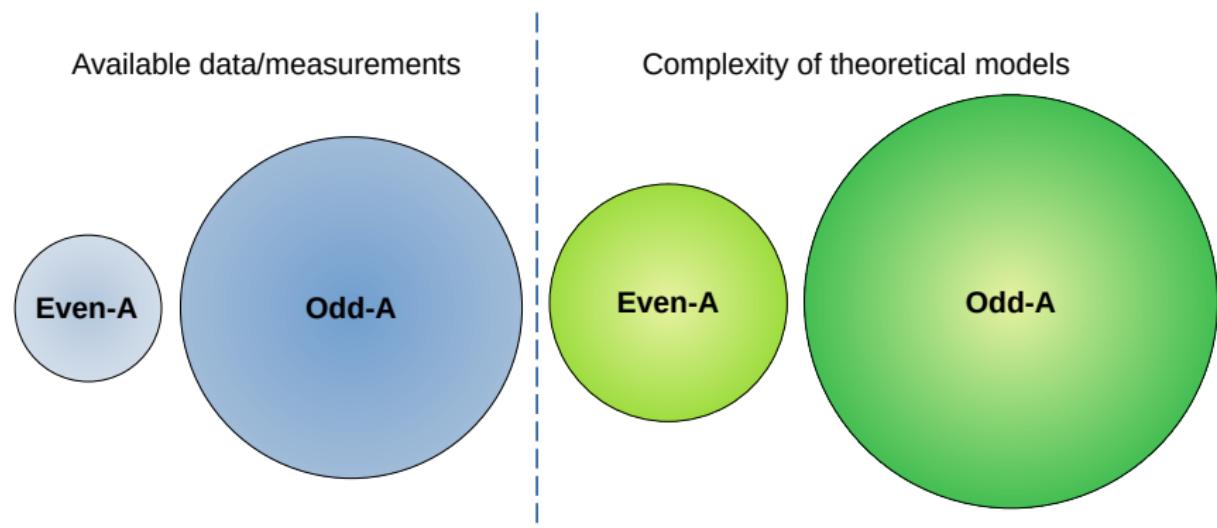
Wobbling Motion II



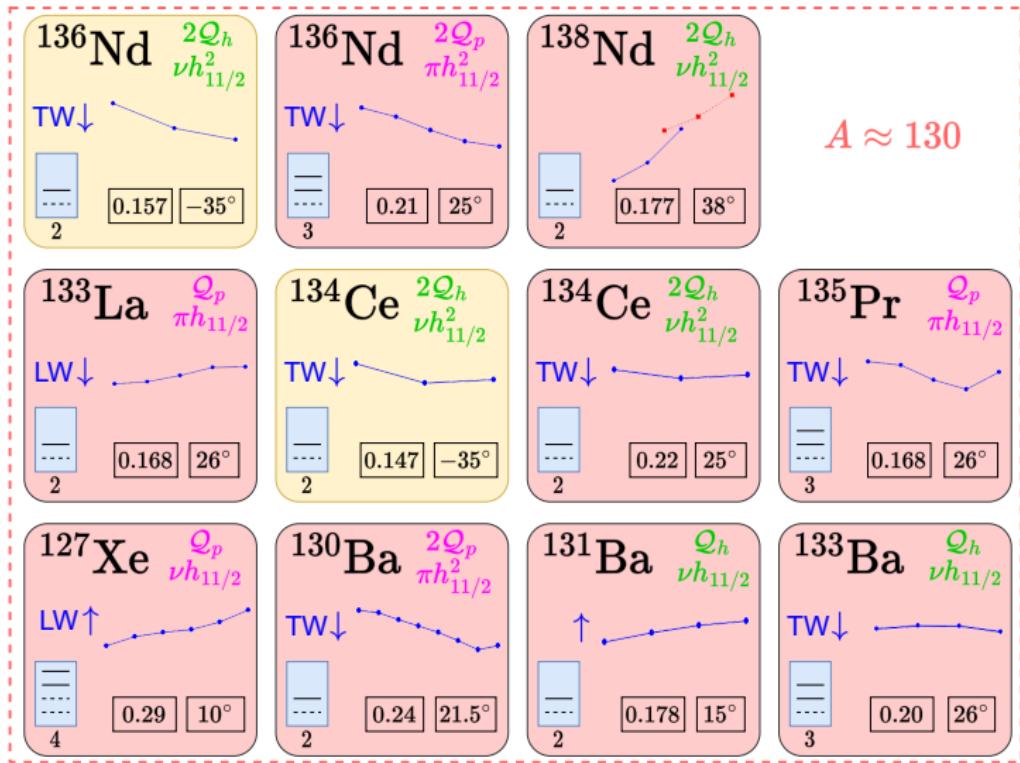
Sensharma, 2020.

Even- A vs. Odd- A Picture

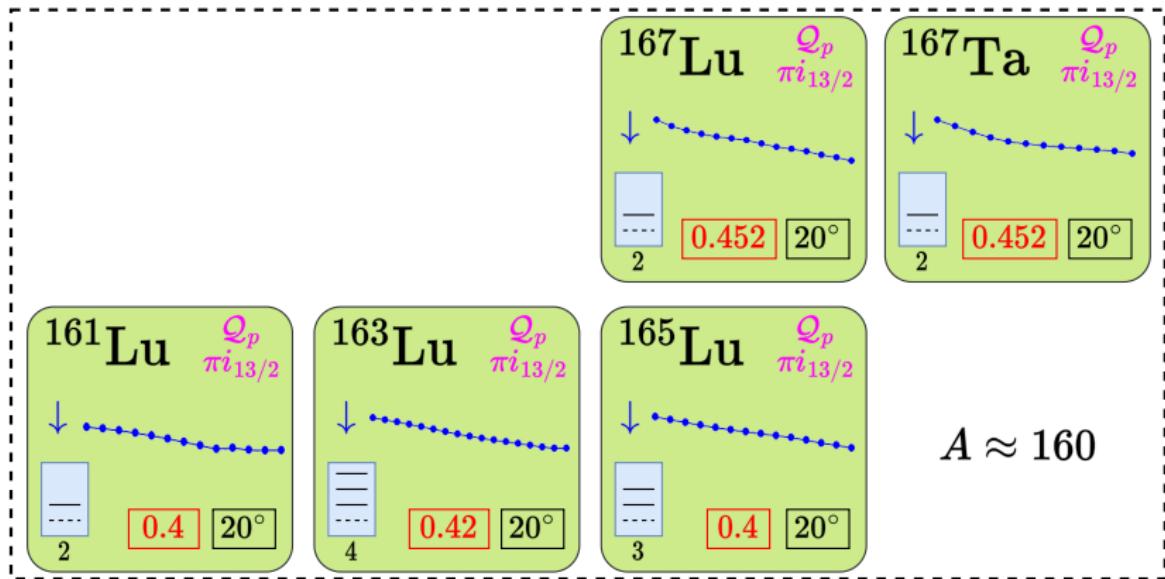
- Predicted for even- A nuclei more than 50 years ago.
- First experimental evidence for **nuclear wobbling motion**: ^{163}Lu (*Ødegård, 2001*).
- Current mass-regions for wobblers: $A = 130, 160, 180$.



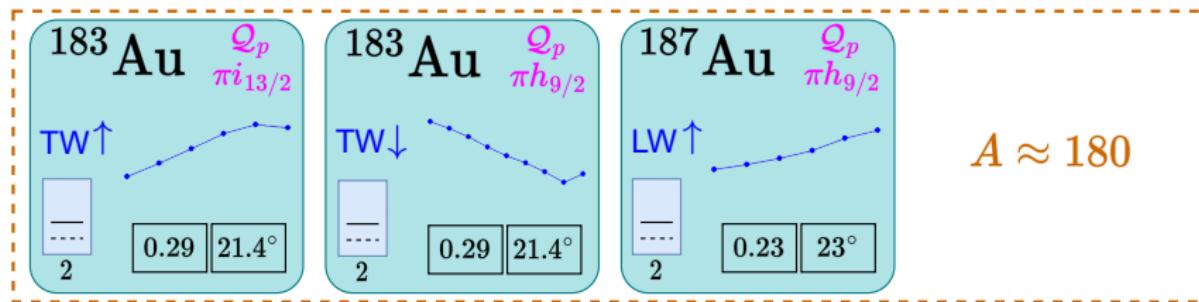
Wobblers in the A=130 mass region



Wobblers in the A=160 mass region



Wobblers in the A=180 mass region



All diagrams and their data sources available in Chapter 3, Section 3.3.5
Presented at the Annual Meeting, FFUB, 2022.

Wobbling Motion in ^{130}Ba

Q Experimental measurements show **two** wobbling bands (*Petrache et. al. 2019*).

Harmonic formalism

Harmonic Approximation (*Bohr & Mottelson, 1969*):

$$E_{I,n_w} = A_3 I(I+1) + \hbar\omega_w \left(n_w + \frac{1}{2} \right),$$
$$A_3 = (2\mathcal{I}_3)^{-1}.$$

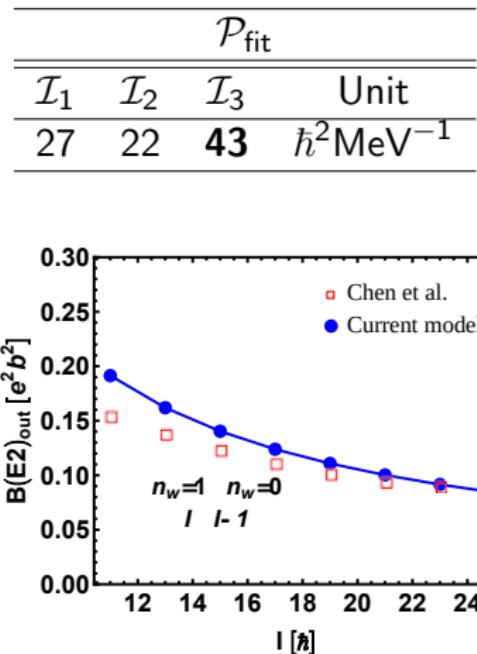
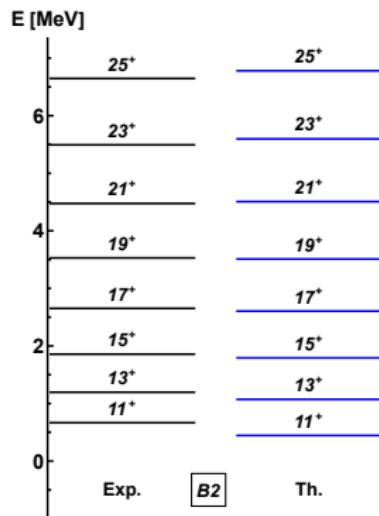
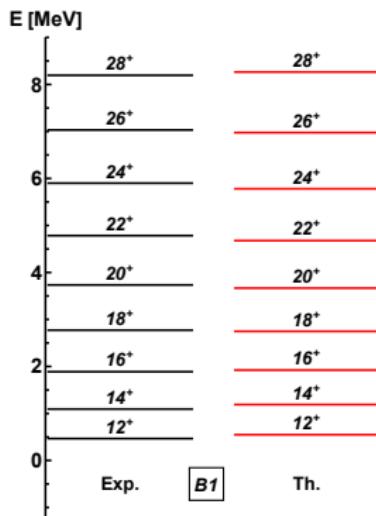
(rotational term + wobbling frequency)



GALILEO, LNL, Source: lnl.infn.it

Fusion evaporation: ^{13}C beam of
 $E = 65$ MeV and ^{122}Sn target.

Results for ^{130}Ba



Full description: Chapter 3 (Section 3.1.2)

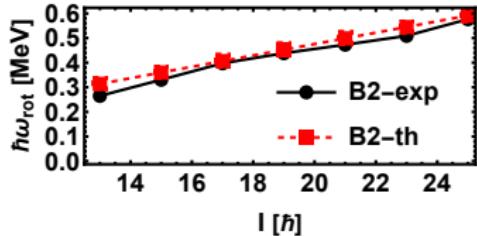
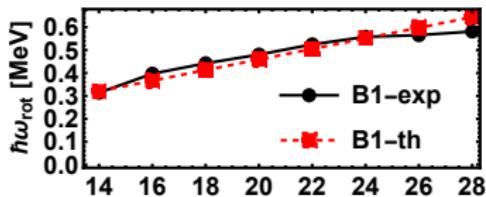
Results presented at the international conference NSP-2022, Turkey.

Results for $^{130}\text{Ba II}$

Excitation energies vs. Wobbling Energies:

$$E_{\text{wob}}(I_{\text{even}}) = E_{I,n} - E_{I,0} ,$$

$$E_{\text{wob}}(I_{\text{odd}}) = E_{I,n} - \frac{1}{2} (E_{I-1,0} + E_{I+1,0})$$



Results presented at the international conference NSP-2022, Turkey.

Starting Point

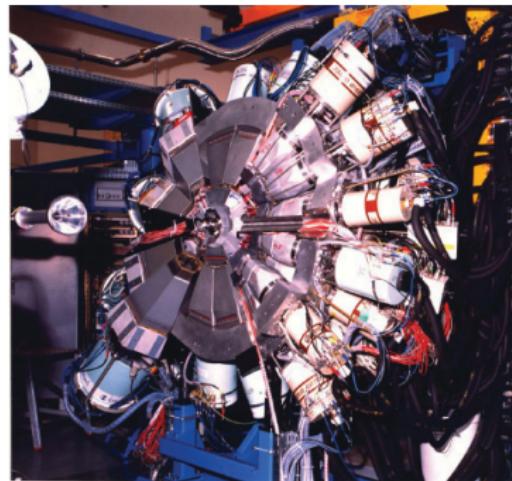
- A. A. Raduta, R. Poenaru, L. Gr. Ixaru, PRC, 2017 + ■ A. A. Raduta, R. Poenaru, Al. H. Raduta, JPG, 2018 → W_0 in the thesis.

Framework

- First semi-classical description for the ^{163}Lu , using the **Particle-Rotor-Model** (*Hamamoto, 2002.*) for an odd-mass nucleus in the $A \approx 160$ region.

PRM

An odd-nucleon moving in a quadrupole deformed mean field generated by an even-even triaxial core.

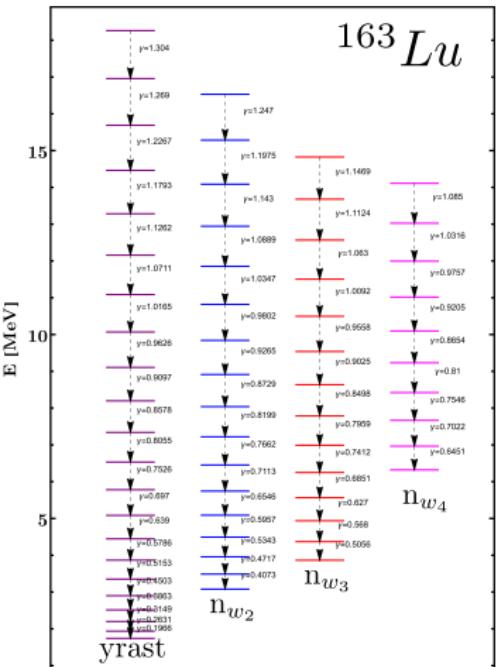
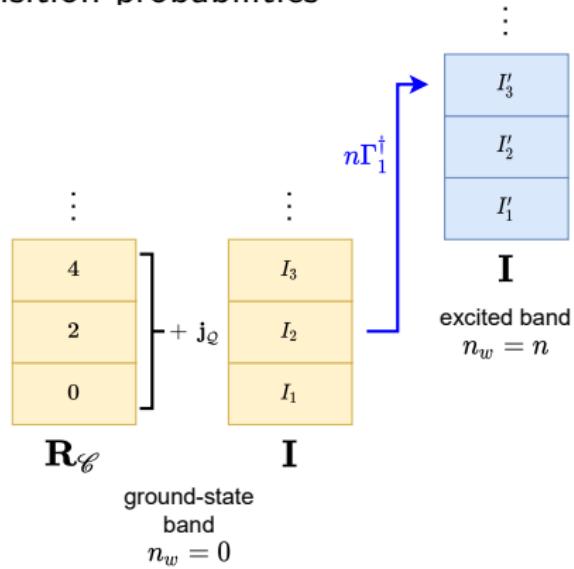


Euroball IV, Strasbourg, Source:
technology.i.stfc.ac.uk

Fusion evaporation: ^{29}Si beam of $E = 152$ MeV and ^{139}La target.

Overview of W_0

- Time-Dependent Variational Principle applied on the PRM Hamiltonian
- **Phonon operators** → energies + transition probabilities



Overview of W_0 II

Model characteristics

- + Numerical data consistent with other work
- TSD4: three-phonon wobbling band (disagreement with Jensen et al.)
- Adopted rigid-body MOIs ("unpleasant" choice to the referees)
- Deformation parameters β and γ taken from literature

Onset of a redesign → start of a new research project

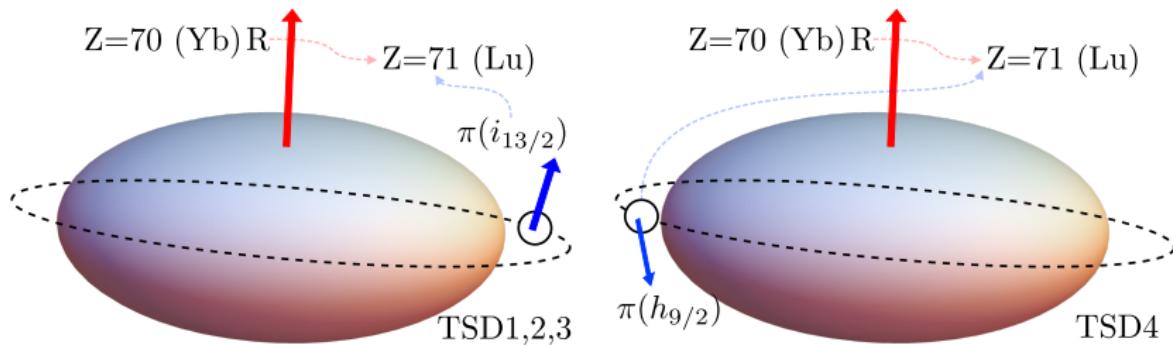
Two new models developed presented in Chapter 4 (W_1) and 5 (W_2)

Fresh-up: \mathbf{W}_1

Particle-Rotor Model Hamiltonian for an odd- A nucleus:

$$\hat{H} = \hat{H}_{\text{rot}} + \hat{H}_{\text{sp}}, \quad \hat{H}_{\text{rot}} = \sum_{k=1}^3 A_k (\hat{l}_k - \hat{j}_k)^2,$$

$$\hat{H}_{\text{sp}} = \epsilon_j + \frac{\nu}{j(j+1)} \left[\cos \gamma (3\hat{j}_3^3 - \mathbf{j}^2) - \sqrt{3} \sin \gamma (\hat{j}_1^2 - \hat{j}_2^2) \right]$$



A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

Variational Principle + Eqs. of Motion

Time-Dependent Variational Equation

$$\delta \int_0^t \langle \Psi_{IM;j} | \hat{H} - i \frac{\partial}{\partial t'} | \Psi_{IM;j} \rangle dt' = 0$$

$$\Psi_{\text{trial}} \equiv |\Psi_{IM;j}\rangle = \mathcal{N} e^{z\hat{I}_-} \cdot e^{s\hat{j}_-} |IMI\rangle \otimes |jj\rangle$$

- $(z, r, \varphi, \hat{I}, |IMI\rangle)$ - core (**R**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$ - single-particle (**j**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$ - single-particle (**j**)
- $\{z, s\}$ → phase space coordinates

Constant of motion:

$$\mathcal{H} \equiv \langle \Psi_{IM;j} | \hat{H} | \Psi_{IM;j} \rangle$$

Canonical equations of motion:

$$\mathcal{S}_1 : \frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi}, \quad \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r}$$

$$\mathcal{S}_2 : \frac{\partial \mathcal{H}}{\partial f} = \dot{\psi}, \quad \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{f}$$

The two sets of Hamilton equations are the semi-classical description of the initial quantal \hat{H} .

Wobbling frequency

Solving \mathcal{S}_1 and \mathcal{S}_2 leads to the algebraic equation:

$$\Omega^4 + B\Omega^2 + C = 0$$

four solutions \rightarrow **only two are real**:

$$\Omega_{1,2} = \left[\frac{1}{2} \left(-B \mp \sqrt{B^2 - 4C} \right) \right]^{1/2}$$

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

- Ω_1 : wobbling frequency of the even-*A* core **R**
- Ω_2 : wobbling frequency of the odd-nucleon **j**
- **Two wobbling phonon numbers: n_{w_1} and n_{w_2}**

Energy spectrum

Spectra of odd-A nuclei within W_1

$$E_{I,n_1,n_2} = \epsilon_j + \mathcal{H}_{\min}^I + \hbar\Omega_1^I \left(n_{w_1} + \frac{1}{2} \right) + \hbar\Omega_2^I \left(n_{w_2} + \frac{1}{2} \right)$$

- Phonon factor:

$$\mathcal{F}_{n_{w_1} n_{w_2}}^I = \hbar\Omega_1^I \left(n_{w_1} + \frac{1}{2} \right) + \hbar\Omega_2^I \left(n_{w_2} + \frac{1}{2} \right)$$

- \mathcal{H}_{\min}^I is the Classical Energy Function taken in its minimum point:
 $p_0 = (0, I; 0, j)$.

A new interpretation for TSD1 and TSD2

Previous models

$TSD1$ = zero-phonon wobbling band

$TSD2$ = one-phonon wobbling band...

Redefinition

$TSD1$ and $TSD2$ are **Signature Partner Bands** (In favor of Ultimate Cranker calculations, *Jensen, 2004*).

$$\left(\alpha_{fav} = +\frac{1}{2} \right) : \{ TSD1 \} \equiv \{ [0^+, 2^+, 4^+, \dots] \otimes [J^\pi = 13/2^+] \}$$

$$\left(\alpha_{unfav} = -\frac{1}{2} \right) : \{ TSD2 \} \equiv \{ [1^+, 3^+, 5^+, \dots] \otimes [J^\pi = 13/2^+] \}$$

$TSD4$: **ground-state wobbling band**, $\pi(h_{9/2})$ configuration.

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

A new band structure for ^{163}Lu

Band	Spins	π	α	$\pi(I_j)$	$\mathbf{W}_0: \mathcal{C} + \mathcal{Q}_p$	$\mathbf{W}_1: \mathcal{C} + \mathcal{Q}_p$
TSD1	$13/2, 17/2 \dots 97/2$	+	+1/2	$\pi(i_{13/2})$	$0^+, 2^+, 4^+, \dots$	$0^+, 2^+, 4^+, \dots$
TSD2	$27/2, 31/2 \dots 91/2$	+	-1/2	$\pi(i_{13/2})$	$\text{TSD1} + 1\Gamma^\dagger$	$1^+, 3^+, 5^+, \dots$
TSD3	$33/2, 37/2 \dots 85/2$	+	+1/2	$\pi(i_{13/2})$	$\text{TSD1} + 2\Gamma^\dagger$	$\text{TSD2} + \Gamma^\dagger$
TSD4	$47/2, 51/2 \dots 83/2$	-	-1/2	$\pi(h_{9/2})$	$\text{TSD1} + 3\Gamma^\dagger$	$1^+, 3^+, 5^+, \dots$

Bands	n_{w_1}	n_{w_2}	$\mathcal{F}_{n_{w_1} n_{w_2}}^I$	I_0	I_t	\mathcal{Q}
TSD1	0	0	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$13/2^+$	$97/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
TSD2	0	0	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$27/2^+$	$91/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
TSD3	1	0	$\mathcal{F}_{10}^{I-1} = \frac{3}{2} \Omega_1^{I-1} + \frac{1}{2} \Omega_2^{I-1}$	$33/2^+$	$85/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
TSD4	0	0	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$47/2^-$	$83/2^-$	$j^\pi = 9/2^- \stackrel{\text{not}}{\equiv} \mathcal{Q}_2$

A.A. Raduta, R. Poenaru, C.M. Raduta, Journal of Physics G 47, 2020.

Extension to $A \approx 160$ mass region

The model was further applied to the other Lu isotopes where wobbling motion has been observed.

^{161}Lu Bands	Spins	\mathcal{Q}	\mathcal{C}	(n_{w_1}, n_{w_2})	I_b
TSD1	$21/2^+, 25/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$4^+, 6^+, 8^+ \dots$	$(0, 0)$	
TSD2	$31/2^+, 35/2^+, \dots, 79/2^+$	$j^\pi = 13/2^+$	$9^+, 11^+, 13^+ \dots$	$(0, 0)$	$21/2$

^{165}Lu Bands	Spins	\mathcal{Q}	\mathcal{C}	(n_{w_1}, n_{w_2})	I_b
TSD1	$25/2^+, 29/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \dots$	$(0, 0)$	
TSD2	$35/2^+, 39/2^+, \dots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \dots$	$(0, 0)$	$25/2$
TSD3	$41/2^+, 45/2^+, \dots, 81/2^+$	$j^\pi = 13/2^+$	$\text{TSD2} + \Gamma^\dagger$	$(1, 0)$	

^{167}Lu Bands	Spins	\mathcal{Q}	\mathcal{C}	(n_{w_1}, n_{w_2})	I_b
TSD1	$25/2^+, 29/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \dots$	$(0, 0)$	
TSD2	$35/2^+, 39/2^+, \dots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \dots$	$(0, 0)$	$25/2$

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

W₁ — Numerical Results

- Free parameters in the model $\rightarrow \mathcal{P}_{\text{fit}} = [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, V, \gamma]$.
- V : single-particle potential strength $\propto \beta_2$ (*Tanabe, 20017*)

Fitting procedure

$$\chi^2 = \frac{1}{N_T} \sum_i \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)} \right)^2}{E_{\text{exp}}^{(i)}}$$

¹⁶³Lu-TSD4: separate fitting procedure (different nucleon configuration)

Isotope	Bands	\mathcal{I}_1 [\hbar^2/MeV]	\mathcal{I}_2 [\hbar^2/MeV]	\mathcal{I}_3 [\hbar^2/MeV]	V [MeV]	γ [°]	n.o.s	E_{rms} [MeV]
¹⁶¹ Lu	TSD1-2	87.555	2.773	22.744	2.933	20	29	0.168
¹⁶³ Lu	TSD1-3	63.2	20	10	3.1	17	52	0.264
	TSD4	67	34.5	50	0.7	17	10	0.057
¹⁶⁵ Lu	TSD1-3	77.295	16.184	4.399	1.673	20	42	0.125
¹⁶⁷ Lu	TSD1-2	87.032	10.895	3.758	8.167	19.48	30	0.165

Graphical representation for MOIs and V

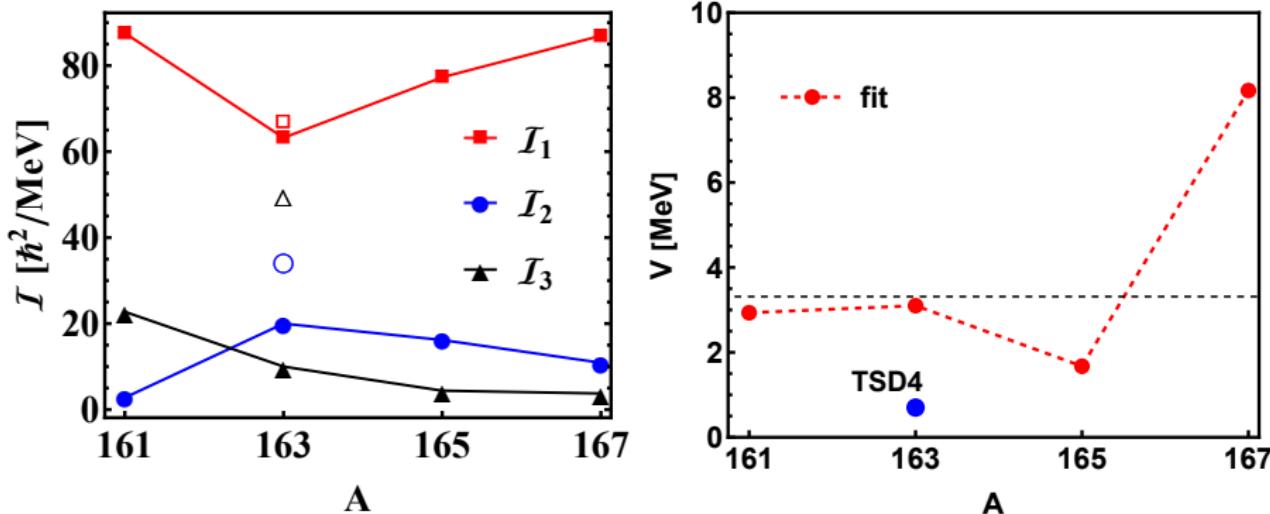


Figure: Fitted values for the MOIs and the single-particle potential strength.

$V \approx 3 \text{ MeV} \rightarrow$ agreement with other calculations (Tanabe, 2017)

Thank you for your attention ❤