A Systematic Description of the Wobbling Motion in Odd-Mass Nuclei Within a Semi-Classical Formalism

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TOC

Aim

Research Objectives

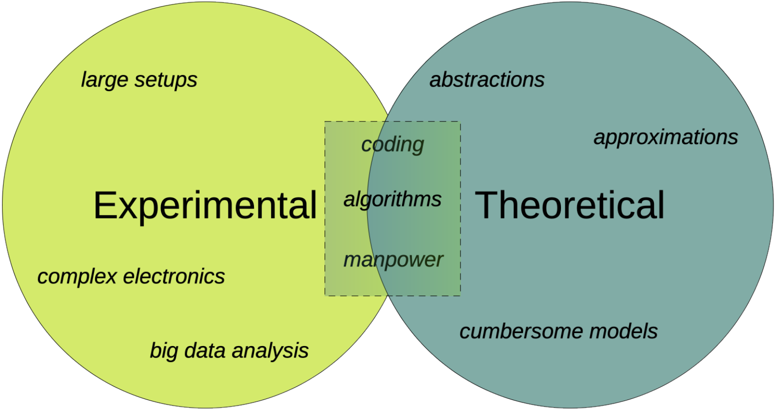
- Extend the current interpretation of the **nuclear triaxiality** in the context of its unique fingerprint: **Wobbling Motion**
- Adopt a framework that is as close as possible to **classical physics**.
- Provide new formalisms for the phenomena related to **nuclear deformation**.

Objectives exclusive to the thesis

- Give the reader enough context towards a better understanding of the underlying concepts, methods, and results.
- create a completely *open-source* project.

Motivation

- **Nuclear Triaxiality** has become a *hot topic* within the scientific community.
- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge.



Nuclear Deformation

Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta,\varphi)=R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}Y_{\lambda}^{\mu}(\theta,\varphi)\right)$$

Quadrupole deformations $\lambda=2$

- For us: Most relevant modes are the **quadrupole vibrations** $\lambda=2$ $\Longrightarrow$ *Play a crucial role in the rotational spectra of nuclei:*
- *Bohr, 1969*: Coordinates $\alpha_{2\mu}$ can be reduced to only two *deformation parameters*: β_2 (**eccentricity**) and γ (**triaxiality**).

Axial shapes

Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state *(Budaca, 2018)*.
- Moments of inertia: $\mathcal{I}_{1,2,3}$: two are equal, one is different.

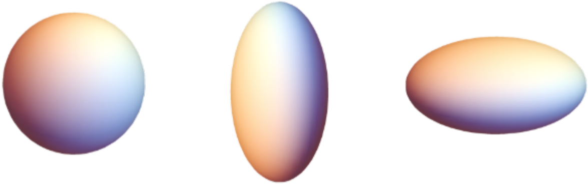
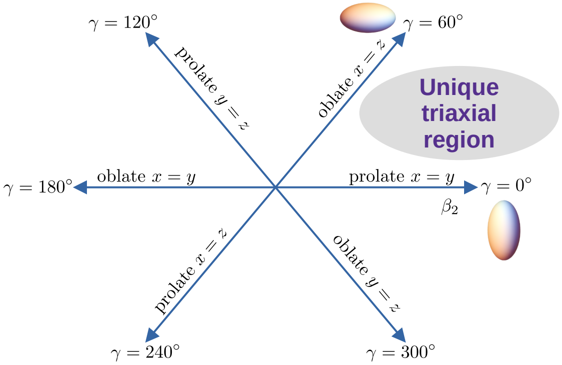
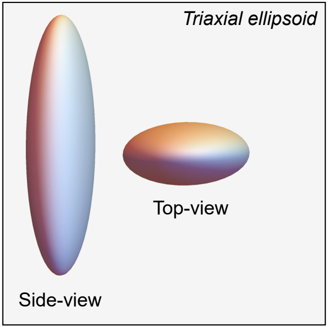


Figure: **spherical:** $\beta_2 = 0$ **prolate:** $\beta_2 > 0$ **oblate:** $\beta_2 < 0$. $(\gamma = 0°)$.

Non-axial shapes

- The triaxiality parameter $\gamma \neq 0°$: departure from axial symmetry.
- Moments of inertia: $\mathcal{I}_1 \neq \mathcal{I}_2 \neq \mathcal{I}_3$.





Fingerprints of Triaxiality

Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
  1. **Wobbling Motion** - WM (*Bohr and Mottelson, 1970s*)
  2. Chiral Motion - χM (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

Goal

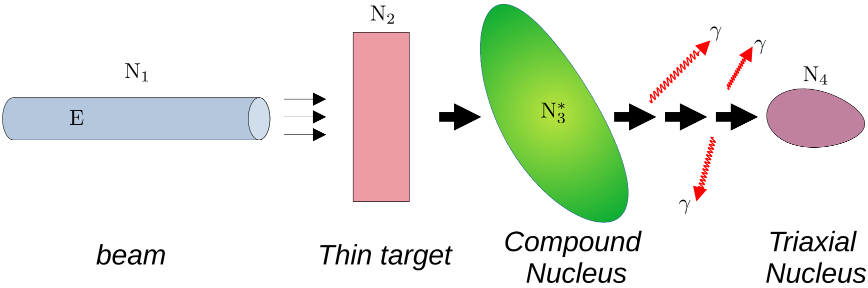
Describe the elusive character of Wobbling Motion in the context of nuclear triaxiality.

Probing triaxiality in nuclei

Triaxial nuclei can be observed/obtained in several experiments:

- Nuclear fission: $A \rightarrow B + C$
- Nuclear fusion: $X + Y \rightarrow Z$
- **Fusion-evaporation reactions**: Long-lived + enhanced deformation

$$Beam(N_1, E) + Target(N_2) \longrightarrow N_3^* \rightarrow \cdots \rightarrow triaxial(N_4)$$



Nuclear facilities

Figure: Gammasphere detector, ANL-ATLAS USA. *Source: aps.org*

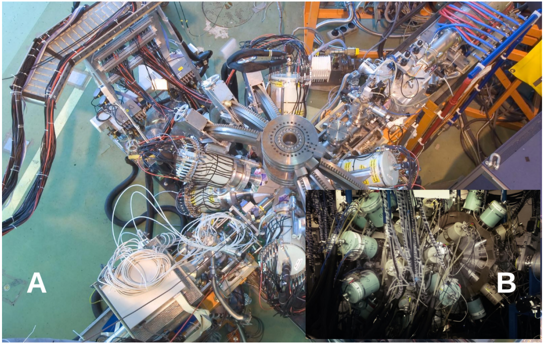


Figure: a) IDS detector, CERN. *Source: isolde.web.cern.ch* b) JUROGAM II, Finland. *Source: twitter.com*

Wobbling Motion

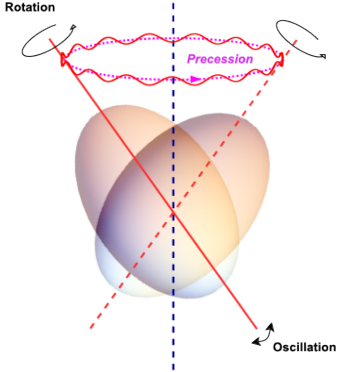
$\gamma \neq 0^\circ$

↓

MOI anisotropy

↓

main rotation around $\mathcal{J}_{\text{max}}$ is disturbed by the other two axes



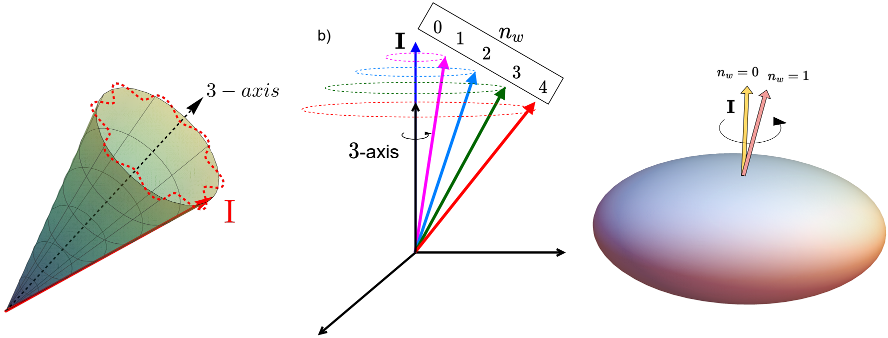
Wobbling Effect

- The **total angular momentum** of the nucleus **precesses** and **oscillates** around $\mathcal{J}_{\text{max}}$.

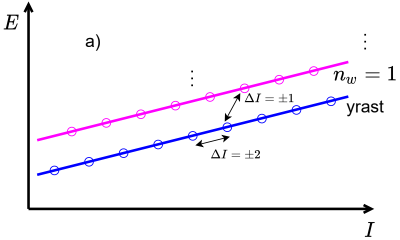
Wobbling Motion

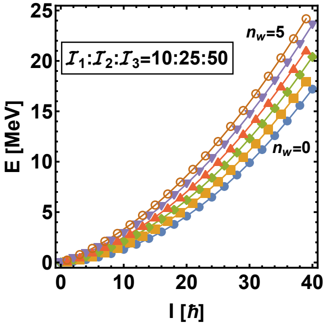
Harmonic oscillation

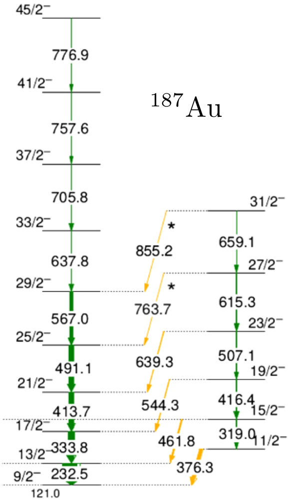
- Precession of **I** is affected by **rotational frequency** and/or **tilting**
- Tilting only by "specific" amount → **harmonic character** → **wobbling phonon**: $n_w = 0, 1, 2, \ldots$.



Wobbling Motion II



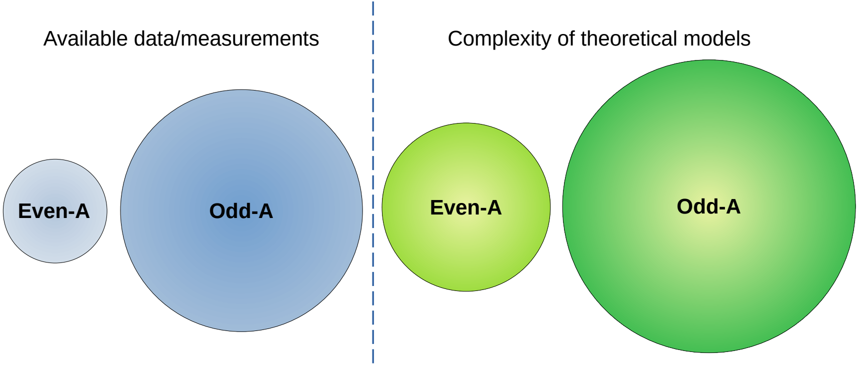




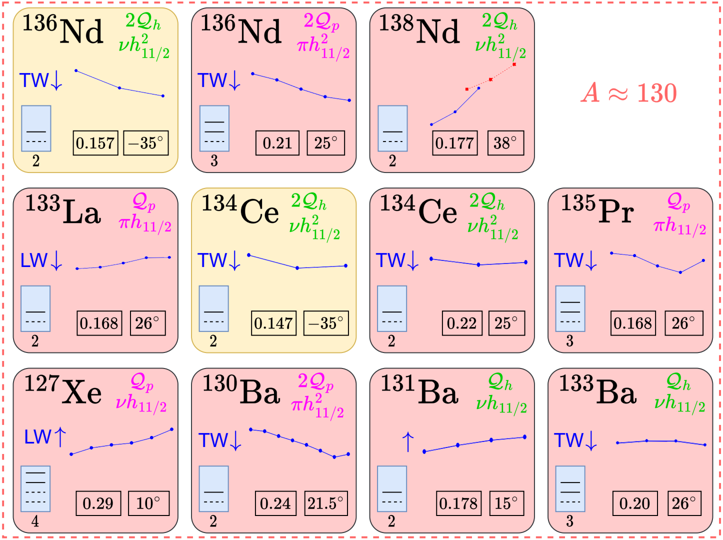
Sensharma, 2020.

Even-A vs. Odd-A Picture

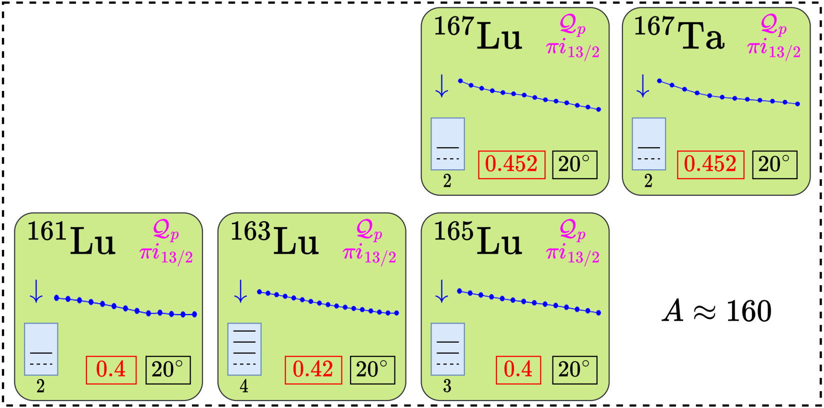
- Predicted for even-A nuclei more than 50 years ago.
- First experimental evidence for **nuclear wobbling motion**: ^{163}Lu (*Ødegård, 2001*).
- Current mass-regions for wobblers: $A = 130, 160, 180$.



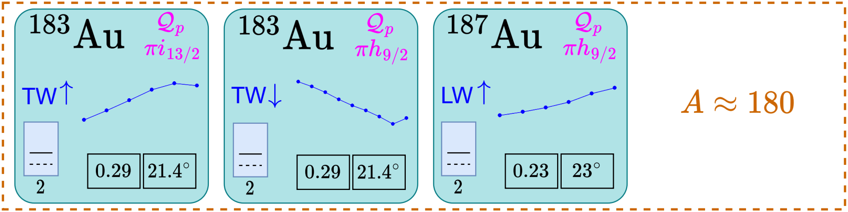
Wobblers in the A=130 mass region



Wobblers in the A=160 mass region



Wobblers in the A=180 mass region



All diagrams and their data sources available in Chapter 3, Section 3.3.5

Presented at the Annual Meeting, FFUB, 2022.

Wobbling Motion in ^{130}Ba

Experimental measurements show **two** wobbling bands (*Petrache et. al. 2019*).

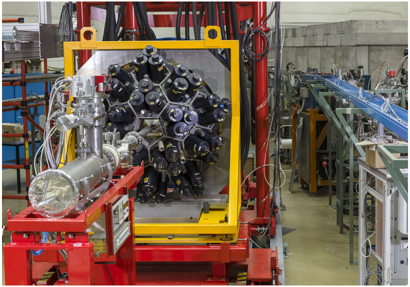
Harmonic formalism

Harmonic Approximation (*Bohr & Mottelson, 1969*):

$$E_{I,n_w} = A_3 I(I+1) + \hbar\omega_w\left(n_w+\frac{1}{2}\right),$$

$$A_3 = (2\mathcal{I}_3)^{-1}.$$

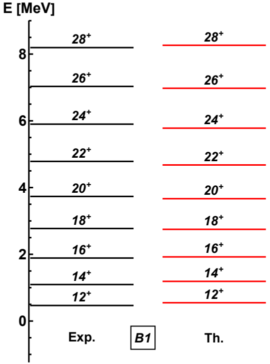
(rotational term + wobbling frequency)

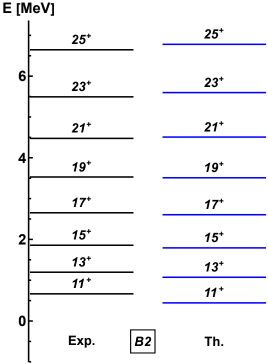


GALILEO, LNL, *Source: lnl.infn.it*

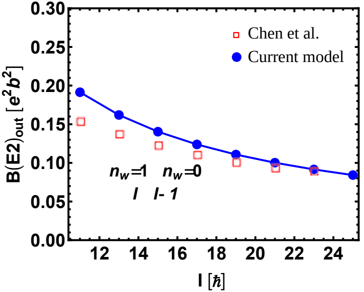
Fusion evaporation: ^{13}C beam of $E=65$ MeV and ^{122}Sn target.

Results for ^{130}Ba





$\mathcal{P}_{\text{fit}}$			
$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	Unit
27	22	**43**	$\hbar^2\text{MeV}^{-1}$



Full description: Chapter 3 (Section 3.1.2)

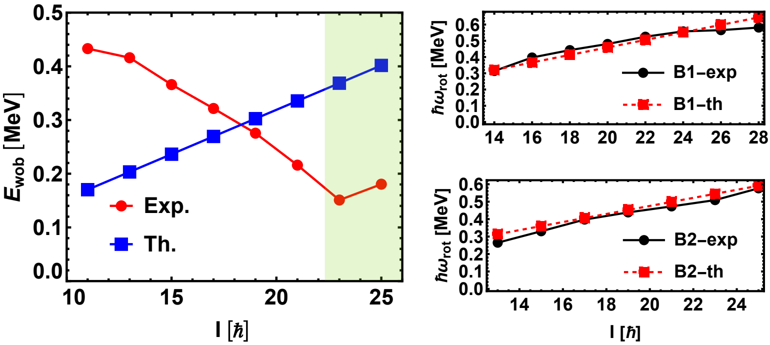
Results presented at the international conference NSP-2022, Turkey.

Results for ^{130}Ba II

Excitation energies vs. **Wobbling Energies**:

$$E_{\text{wob}}(I_{\text{even}}) = E_{I,n} - E_{I,0} \ ,$$

$$E_{\text{wob}}(I_{\text{odd}}) = E_{I,n} - \frac{1}{2}\left(E_{I-1,0} + E_{I+1,0}\right)$$



Results presented at the international conference NSP-2022, Turkey.

Starting Point

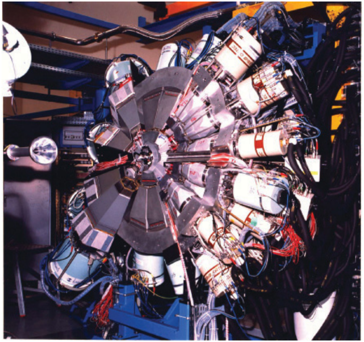
A. A. Raduta, **R. Poenaru**, L. Gr. Ixaru, PRC, 2017 + A. A. Raduta, **R. Poenaru**, Al. H. Raduta, JPG, 2018→$\mathbf{W}_0$ in the thesis.

Framework

- **First semi-classical description** for the ^{163}Lu, using the **Particle-Rotor-Model** (*Hamamoto, 2002.*) for an odd-mass nucleus in the $A \approx 160$ region.

PRM

An odd-nucleon moving in a quadrupole deformed mean field generated by an even-even triaxial core.

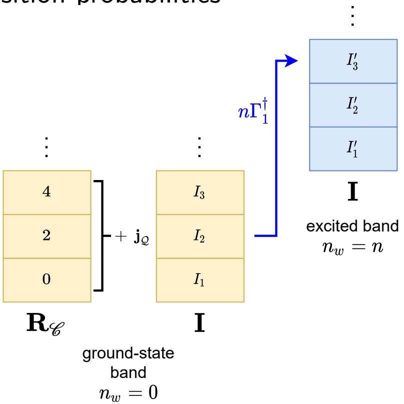


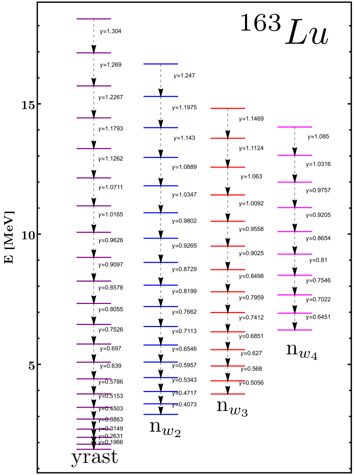
Euroball IV, Strasbourg, *Source: technologysi.stfc.ac.uk*

Fusion evaporation: ^{29}Si beam of $E = 152$ MeV and ^{139}La target.

Overview of $\mathbf{W}_0$

- Time-Dependent Variational Principle applied on the PRM Hamiltonian
- **Phonon operators** $\rightarrow$ energies + transition probabilities





Overview of $\mathbf{W}_0$ II

Model characteristics

- **+** Numerical data consistent with other work
- **−** TSD4: three-phonon wobbling band (disagreement with Jensen et al.)
- **−** Adopted rigid-body MOIs (”unpleasant” choice to the referees)
- **−** Deformation parameters β and γ taken from literature

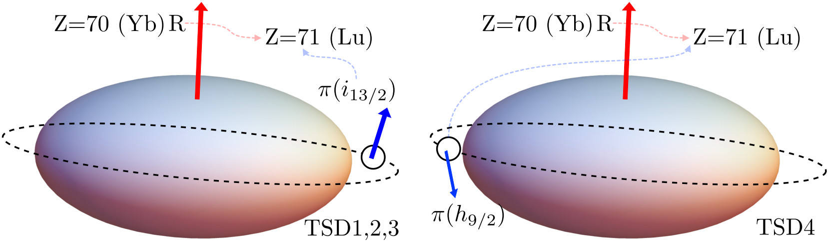
Onset of a redesign $\longrightarrow$ start of a new research project
Two new models developed presented in Chapter 4 ($\mathbf{W}_1$) and 5 ($\mathbf{W}_2$)

Fresh-up: $\mathbf{W}_1$

Particle-Rotor Model Hamiltonian for an odd-A nucleus:

$$\hat{H} = \hat{H}_{\mathrm{rot}} + \hat{H}_{\mathrm{sp}}\ ,\ \hat{H}_{\mathrm{rot}} = \sum_{k=1}^{3} A_k(\hat{I}_k - \hat{j}_k)^2,$$

$$\hat{H}_{\mathrm{sp}} = \epsilon_j + \frac{V}{j(j+1)}\left[\cos\gamma\left(3\hat{j}_3^2 - \mathbf{j}^2\right) - \sqrt{3}\sin\gamma\left(\hat{j}_1^2 - \hat{j}_2^2\right)\right]$$



A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

Variational Principle + Eqs. of Motion

Time-Dependent Varational Equation

$$\delta \int_0^t \langle \Psi_{IM;j} | \hat{H} - i\frac{\partial}{\partial t'} | \Psi_{IM;j} \rangle \, dt' = 0$$

$$\Psi_{\text{trial}} \equiv | \Psi_{IM;j} \rangle = \mathcal{N} e^{z\hat{I}_-} \cdot e^{s\hat{j}_-} | IMI \rangle \otimes | jj \rangle$$

- $(z, r, \varphi, \hat{I}, |IMI\rangle)$ - core (**R**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$ - single-particle (**j**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$ - single-particle (**j**)
- $\{z, s\} \rightarrow$ **phase space coordinates**

Constant of motion:

$$\mathcal{H} \equiv \langle \Psi_{IM;j} | \hat{H} | \Psi_{IM;j} \rangle$$

Canonical equations of motion:

$$\mathcal{S}_1 : \frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi} \, , \quad \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r}$$

$$\mathcal{S}_2 : \frac{\partial \mathcal{H}}{\partial f} = \dot{\psi} \, , \quad \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{f}$$

The two sets of Hamilton equations are **the semi-classical description** of the initial quantal $\hat{H}$.

Wobbling frequency

Solving $\mathcal{S}_1$ and $\mathcal{S}_2$ leads to the algebraic equation:

$$\Omega^4 + B\Omega^2 + C = 0$$

four solutions $\longrightarrow$ **only two are real**:

$$\Omega_{1,2} = \left[\frac{1}{2}\left(-B \mp \sqrt{B^2 - 4C}\right)\right]^{1/2}$$

A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

- Ω_1: wobbling frequency of the even-A core **R**
- Ω_2: wobbling frequency of the odd-nucleon **j**
- **Two wobbling phonon numbers:** n_{w_1} **and** n_{w_2}

Energy spectrum

Spectra of odd-A nuclei within $\mathbf{W}_1$

$$E_{I,n_1,n_2} = \epsilon_j + \mathcal{H}^I_{\text{min}} + \hbar\Omega^I_1\left(n_{w_1} + \frac{1}{2}\right) + \hbar\Omega^I_2\left(n_{w_2} + \frac{1}{2}\right)$$

- Phonon factor:

$$\mathcal{F}^I_{n_{w_1}n_{w_2}} = \hbar\Omega^I_1\left(n_{w_1} + \frac{1}{2}\right) + \hbar\Omega^I_2\left(n_{w_2} + \frac{1}{2}\right)$$

- $\mathcal{H}^I_{\text{min}}$ is the Classical Energy Function taken in its minimum point: $p_0 = (0, I; 0, j)$.

A new interpretation for TSD1 and TSD2

Previous models

$TSD1 =$ zero-phonon wobbling band
$TSD2 =$ one-phonon wobbling band...

Redefinition

$TSD1$ and $TSD2$ are **Signature Partner Bands** (In favor of Ultimate Cranker calculations, *Jensen, 2004*).

$$\left(\alpha_{fav} = +\frac{1}{2}\right) : \{TSD1\} \equiv \left\{ \left[0^+, 2^+, 4^+, \ldots\right] \otimes [j^\pi = 13/2^+] \right\}$$
$$\left(\alpha_{unfav} = -\frac{1}{2}\right) : \{TSD2\} \equiv \left\{ \left[1^+, 3^+, 5^+, \ldots\right] \otimes [j^\pi = 13/2^+] \right\}$$

$TSD4$: **ground-state wobbling band**, $\pi(h_{9/2})$ configuration.

A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

A new band structure for ^{163}Lu

Band	Spins	π	α	$\pi(l_j)$	$\mathbf{W_0}$: $\mathscr{C}+\mathcal{Q}_p$	$\mathbf{W_1}$: $\mathscr{C}+\mathcal{Q}_p$
TSD1	$13/2, 17/2 \ldots 97/2$	$+$	$+1/2$	$\pi(i_{13/2})$	$0^+, 2^+, 4^+, \ldots$	$0^+, 2^+, 4^+, \ldots$
TSD2	$27/2, 31/2 \ldots 91/2$	$+$	$-1/2$	$\pi(i_{13/2})$	$\mathrm{TSD1} + 1\Gamma^\dagger$	$1^+, 3^+, 5^+, \ldots$
TSD3	$33/2, 37/2 \ldots 85/2$	$+$	$+1/2$	$\pi(i_{13/2})$	$\mathrm{TSD1} + 2\Gamma^\dagger$	$\mathrm{TSD2} + \Gamma^\dagger$
TSD4	$47/2, 51/2 \ldots 83/2$	$-$	$-1/2$	$\pi(h_{9/2})$	$\mathrm{TSD1} + 3\Gamma^\dagger$	$1^+, 3^+, 5^+, \ldots$

Bands	n_{w_1}	n_{w_2}	$\mathcal{F}^I_{n_{w_1} n_{w_2}}$	I_0	I_t	$\mathcal{Q}$
TSD1	0	0	$\mathcal{F}^I_{00} = \frac{1}{2}\left(\Omega^I_1 + \Omega^I_2\right)$	$13/2^+$	$97/2^+$	$j^\pi = 13/2^+ \overset{not}{\equiv} \mathcal{Q}_1$
TSD2	**0**	**0**	$\mathcal{F}^I_{00} = \frac{1}{2}\left(\Omega^I_1 + \Omega^I_2\right)$	$27/2^+$	$91/2^+$	$j^\pi = 13/2^+ \overset{not}{\equiv} \mathcal{Q}_1$
TSD3	1	0	$\mathcal{F}^{I-1}_{10} = \frac{3}{2}\Omega^{I-1}_1 + \frac{1}{2}\Omega^{I-1}_2$	$33/2^+$	$85/2^+$	$j^\pi = 13/2^+ \overset{not}{\equiv} \mathcal{Q}_1$
TSD4	**0**	**0**	$\mathcal{F}^I_{00} = \frac{1}{2}\left(\Omega^I_1 + \Omega^I_2\right)$	$47/2^-$	$83/2^-$	$j^\pi = 9/2^- \overset{not}{\equiv} \mathcal{Q}_2$

A.A. Raduta, **R. Poenaru**, C.M. Raduta, Journal of Physics G 47, 2020.

Extension to $A \approx 160$ mass region

The model was further applied to the other Lu isotopes where wobbling motion has been observed.

^{161}Lu Bands	Spins	$\mathcal{Q}$	$\mathscr{C}$	(n_{w_1}, n_{w_2})	I_b
TSD1	$21/2^+, 25/2^+, \ldots, 89/2^+$	$j^\pi = 13/2^+$	$4^+, 6^+, 8^+ \ldots$	$(0,0)$	$21/2$
TSD2	$31/2^+, 35/2^+, \ldots, 79/2^+$	$j^\pi = 13/2^+$	$9^+, 11^+, 13^+ \ldots$	$(0,0)$	

^{165}Lu Bands	Spins	$\mathcal{Q}$	$\mathscr{C}$	(n_{w_1}, n_{w_2})	I_b
TSD1	$25/2^+, 29/2^+, \ldots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \ldots$	$(0,0)$	$25/2$
TSD2	$35/2^+, 39/2^+, \ldots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \ldots$	$(0,0)$	
TSD3	$41/2^+, 45/2^+, \ldots, 81/2^+$	$j^\pi = 13/2^+$	$\text{TSD2} + \Gamma^\dagger$	$(1,0)$	

^{167}Lu Bands	Spins	$\mathcal{Q}$	$\mathscr{C}$	(n_{w_1}, n_{w_2})	I_b
TSD1	$25/2^+, 29/2^+, \ldots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \ldots$	$(0,0)$	$25/2$
TSD2	$35/2^+, 39/2^+, \ldots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \ldots$	$(0,0)$	

A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

$\mathbf{W}_1$ — Numerical Results

- Free parameters in the model $\rightarrow \mathcal{P}_{\text{fit}} = [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, V, \gamma]$.
- V: single-particle potential strength $\propto \beta_2$ (*Tanabe, 20017*)

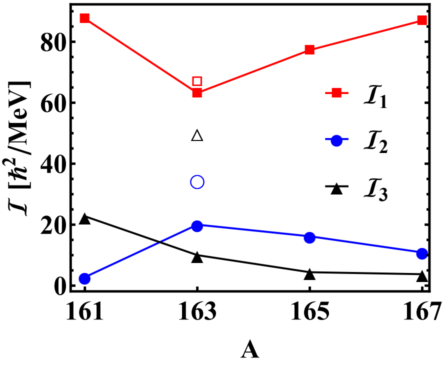
Fitting procedure

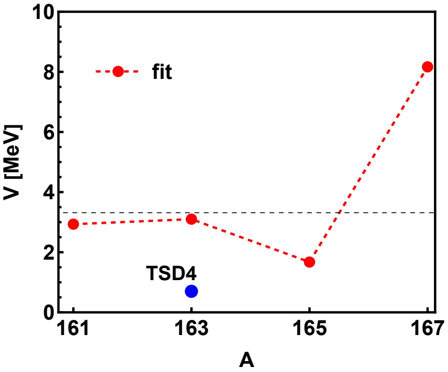
$$\chi^2 = \frac{1}{N_T} \sum_i \frac{\left(E_{\text{exp}}^{(i)} - E_{\text{th}}^{(i)}\right)^2}{E_{\text{exp}}^{(i)}}$$

^{163}Lu-TSD4: separate fitting procedure (different nucleon configuration)

Isotope	Bands	$\mathcal{I}_1$ $[\hbar^2/\text{MeV}]$	$\mathcal{I}_2$ $[\hbar^2/\text{MeV}]$	$\mathcal{I}_3$ $[\hbar^2/\text{MeV}]$	V [MeV]	γ [°]	n.o.s	E_{rms} [MeV]
^{161}Lu	TSD1-2	87.555	2.773	22.744	2.933	20	29	0.168
^{163}Lu	TSD1-3	63.2	20	10	3.1	17	52	0.264
	TSD4	67	34.5	50	0.7	17	10	0.057
^{165}Lu	TSD1-3	77.295	16.184	4.399	1.673	20	42	0.125
^{167}Lu	TSD1-2	87.032	10.895	3.758	8.167	19.48	30	0.165

Graphical representation for MOIs and V

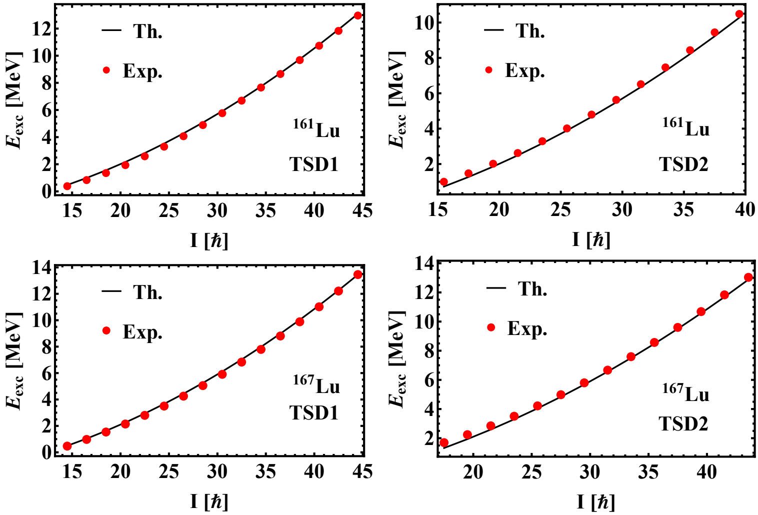




$V \approx 3$ MeV $\rightarrow$ agreement with other calculations (*Tanabe, 2017*)

A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

Excitation Energies — ^{161}Lu + ^{167}Lu



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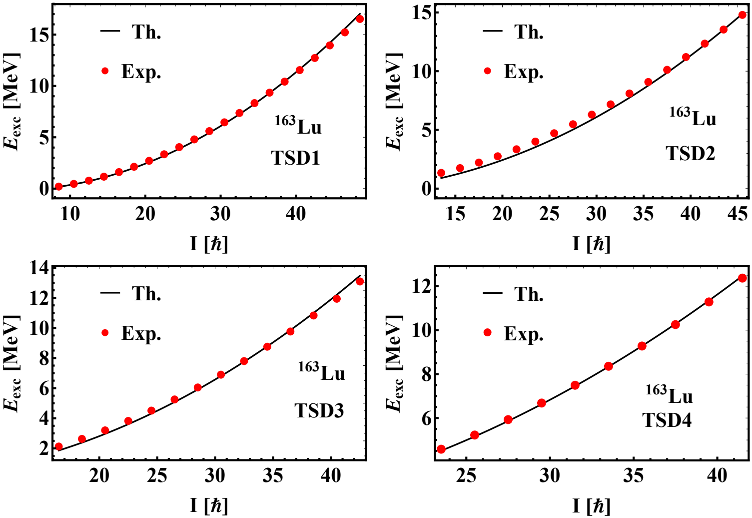
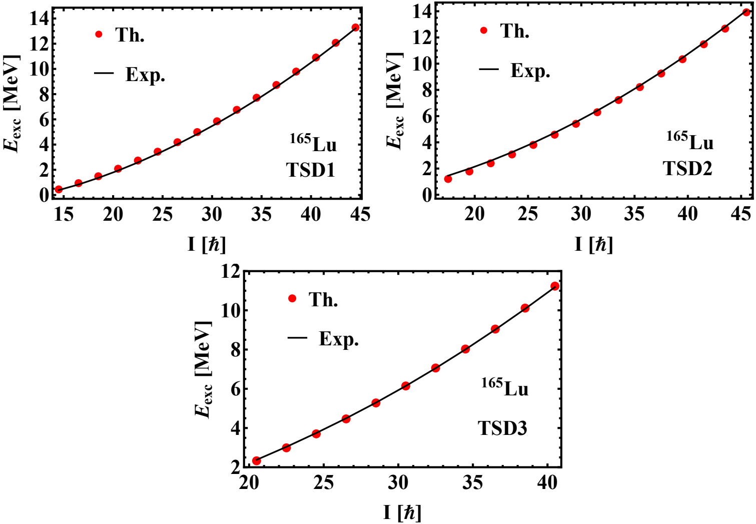
Excitation Energies — ^{163}Lu



A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

Excitation Energies — ^{165}Lu



A. A. Raduta, **R. Poenaru**, C. M. Raduta, Phys. Rev. C 101, 2020.

Thank you for your attention ❤