Electromagnetic Transitions

In classical physics:

If a charge system is oscillating with energy E_0 it will radiate energy with a "transition rate" λ,

i.e. $E = E_0 e^{-\lambda t}$

By taking the Fourier transform of the radiation field as a function of time one can get the frequency spectrum of the emitted radiation:

$$I(\omega) = \frac{Const.}{(\omega - \omega_0)^2 + (\lambda/2)^2}$$

This spectrum has a Full Width at Half Maximum (FWHM) = λ

We get a similar result when we move to a quantum mechanical system.

Electromagnetic Transitions

A quantum mechanical system at initial energy $E_0 = \hbar\omega_0$ emits radiation.

The wave function of the initial state may be written

$$\Psi(\vec{r},t) = \psi(\vec{r})e^{-iE_0 t/\hbar}$$

But we know the probability of finding the nucleus in the initial state decreases with decay constant λ, i.e.

$$\left|\Psi(\vec{r},t)\right|^2 = \left|\Psi(\vec{r},0)\right|^2 e^{-\lambda t}$$

A time dependent state cannot be represented by a stationary state (with a fixed energy), but must include a distribution of energies.

This suggests that the state should really be

$$\Psi(\vec{r},t) = \psi(\vec{r})e^{-iE_0 t/\hbar - \lambda t/2} \quad = \int \underbrace{A(E)}_{\text{Probability Amplitude}} \underbrace{\psi(\vec{r})e^{-iEt/\hbar}}_{\text{State with energy } E} dE$$

Electromagnetic Transitions

The probability amplitude $A(E)$ of finding the particle in a state with energy E, is found by taking the Fourier transform,

i.e.

$$A(E) \propto \int_{-\infty}^{+\infty} e^{-iE_0 t/\hbar - \lambda t/2} e^{iEt/\hbar} dt$$

Since the state begins to decay at time $t = 0$, we can set the lower limit of the integration to zero. Then we find that the probability of finding the nucleus in a state with energy E is

$$P(E) = |A(E)|^2 = \frac{1}{4\pi^2} \frac{1}{(E - E_0)^2 + \Gamma^2/4}$$

where the FWHM of this distribution is $\Gamma = \hbar\lambda = \dfrac{\hbar}{\tau}$

This is called the natural width of the initial state.

λ = transition rate or decay constant

$\tau = \dfrac{1}{\lambda}$ = mean life time

Electromagnetic Transitions

The distribution is called a Lorentzian

$$P(E) \sim \frac{1}{(E-E_0)^2 + \Gamma^2/4}$$

This distribution is related to the Uncertainty Principle.

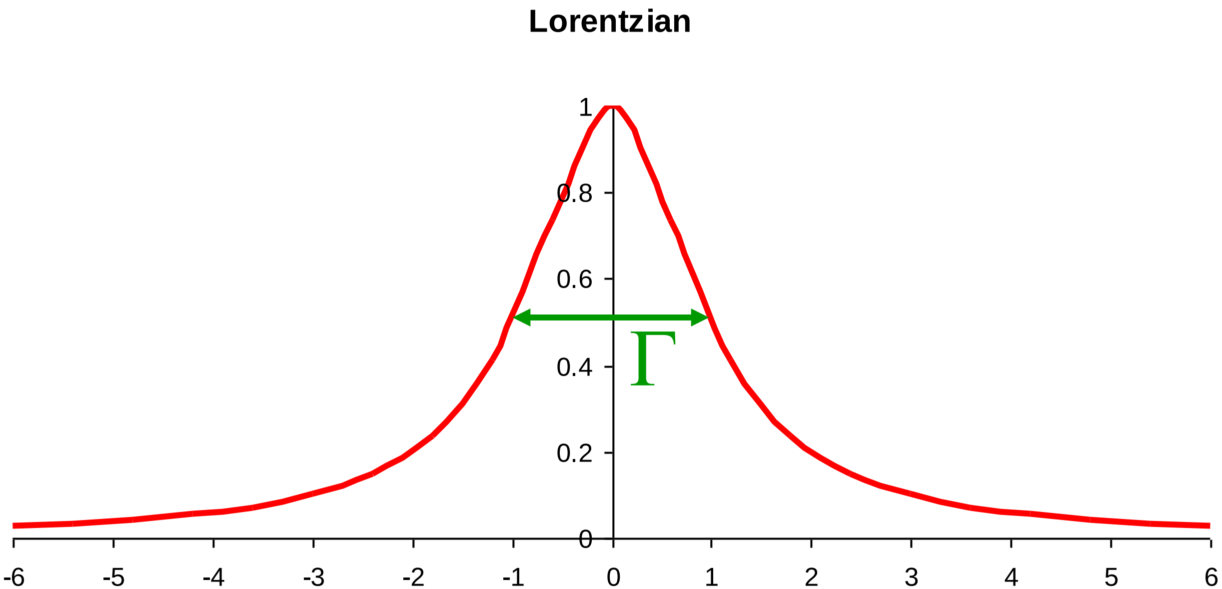
$$\Delta E \Delta t \sim \frac{\hbar}{2}$$

With $\Delta E \to \Gamma/2$

$\Delta t \to \tau$

Gives

$$\boxed{\Gamma = \hbar\lambda = \frac{\hbar}{\tau}}$$



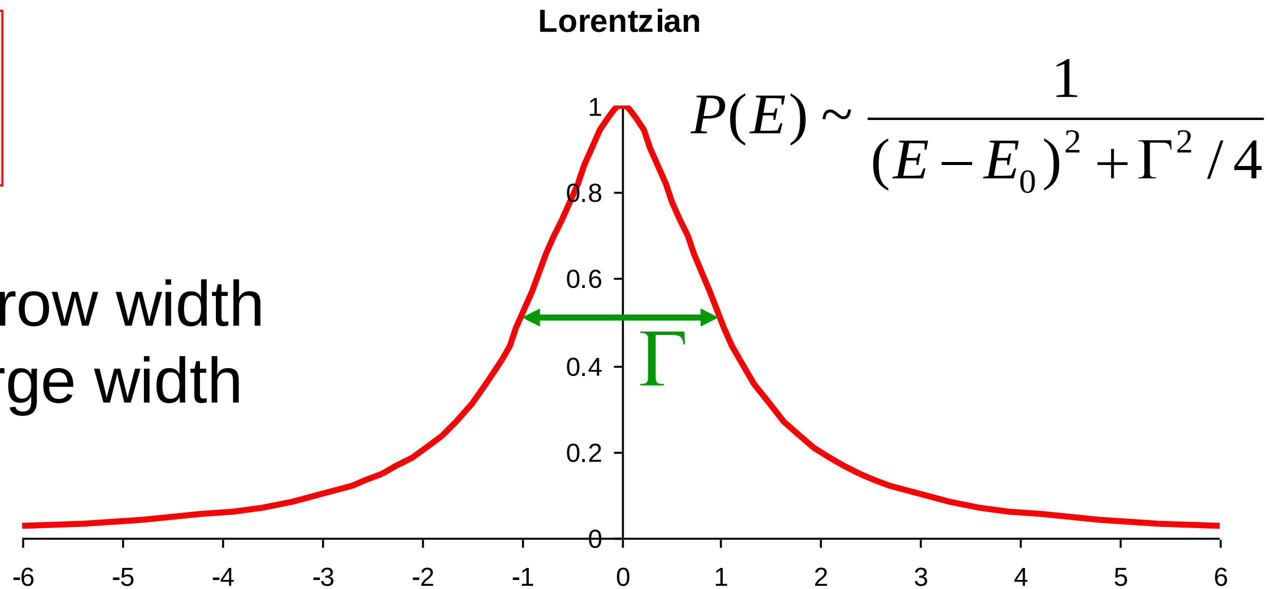
$(E - E_0)$ in $\Gamma/2$ units

This kind of line shape is appropriate for all kinds of unstable states, not just those that decay by EM radiation.

Electromagnetic Transitions

If we measure the energy of an unstable state, we will not get the same answer every time. We will get a distribution, or width, which is related to the lifetime of the state.

$$\boxed{\Gamma = \hbar\lambda = \frac{\hbar}{\tau}}$$



$$P(E) \sim \frac{1}{(E-E_0)^2 + \Gamma^2/4}$$

$$(E - E_0) \text{ in } \Gamma/2 \text{ units}$$

Long lifetime $\rightarrow$ narrow width
Short lifetime $\rightarrow$ large width

Many short lifetimes can only be measured by measuring the width of the state.

Electromagnetic Transitions

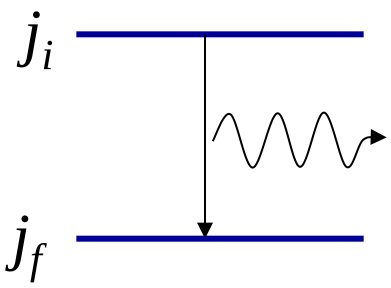
For γ-ray emission from nuclei, the transition rate λ depends on

- the type of radiation mechanism,
- the angular momentum carried away by the photon.

Suppose we have a transition

from a state with angular momentum quantum number j_i

to a state with angular momentum quantum number j_f.



The photon carries angular momentum $\vec{L}$

$$\vec{J}_i = \vec{J}_f + \vec{L}$$

The photon angular momentum quantum number, l, is

$$\left| j_i - j_f \right| \leq l \leq j_i + j_f$$

Electromagnetic Transitions

The radiation can be due to

An oscillating charge distribution – Electric radiation.

An oscillating current distribution – Magnetic radiation.

For Electric radiation

$$\lambda_E(lm) = \frac{2(l+1)}{\hbar\varepsilon_0 l[(2l+1)!!]^2}\left(\frac{\omega}{c}\right)^{2l+1}\left|Q_{lm}\right|^2$$

Double factorial $x!! = x(x-2)(x-4)...$

Q_{lm} is a Multipole Matrix Element between the initial state ψ_i and the final state ψ_f .

$$Q_{lm} = e\sum_{k=1}^{Z}\int r_k^l Y_{lm}^*(\theta_k,\varphi_k)\psi_i^*\psi_f \, dV$$

Electromagnetic Transitions

E.g. for $l = 1$, Q_{lm} becomes the matrix element for Dipole radiation.

It can be shown that a transition with $l = 0$ is not possible (from properties of the spherical harmonics).

i.e. the emitted photon must always have $l > 0$.

Similarly for Magnetic radiation.

$$\lambda_M(lm) = \frac{2(l+1)\mu_0}{\hbar l[(2l+1)!!]^2}\left(\frac{\omega}{c}\right)^{2l+1}\left|M_{lm}\right|^2$$

Again we must always have $l > 0$.

For both Electric and Magnetic radiation.

$l = 1$ Dipole radiation

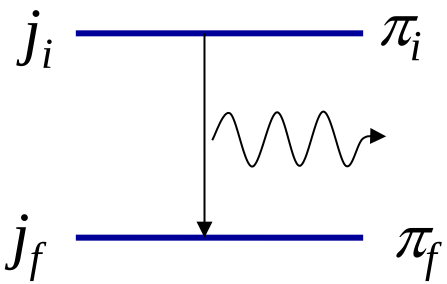
$l = 2$ Quadrupole radiation

$l = 3$ Octapole radiation

l 2^l – pole

Electromagnetic Transitions

Parity:



We may have a parity change

$$\pi_\gamma = \pi_i \pi_f$$

From the matrix elements it can be shown that:

Electric multipole transitions: $\pi_i \pi_f = (-1)^l$

Magnetic multipole transitions: $\pi_i \pi_f = (-1)^{l+1}$

Electromagnetic Transitions

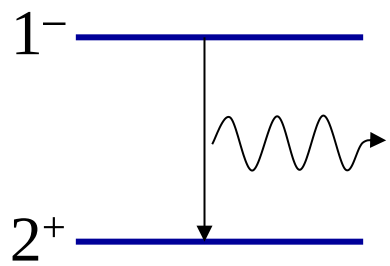
Summarizing:

Selection rules for gamma-ray transitions.

l	π_γ	**Radiation type**	**Label**
1	–1	Electric Dipole radiation	E1
1	+1	Magnetic Dipole radiation	M1
2	–1	Magnetic Quadrupole radiation	M2
2	+1	Electric Quadrupole radiation	E2
3	–1	Electric Octapole radiation	E3
3	+1	Magnetic Octapole radiation	M3

Electromagnetic Transitions

e.g.



$$\pi_\gamma = \pi_i \pi_f = (-1)(+1) = -1$$

$$|2-1| \le l \le 2+1$$

Therefore for the gamma ray: $l^\pi = 1^-, 2^-, \text{or } 3^-$

i.e. we can have an E1, M2, or E3 transition.

An E1 transition is most likely.

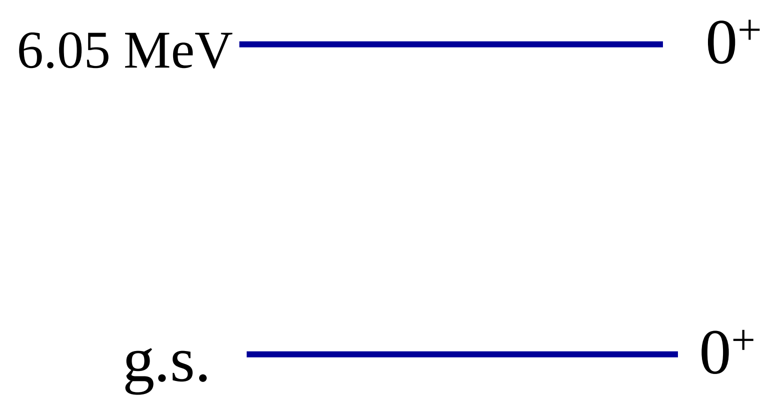
In General:

- Lowest order (lowest *l*) is the most probable (i.e. has larger transition rate λ)
- Electric transitions are more probable than Magnetic transitions.

Electromagnetic Transitions

e.g. First Excited state of ^{16}O

[By the way: The 6.05 MeV state is not simply described by the Shell Model.
It actually looks like a 4-particle excitation.
i.e. like an excited α-particle.]



This state cannot decay by photon emission
– would require $l = 0$.

How does it decay?

Perhaps by particle emission…

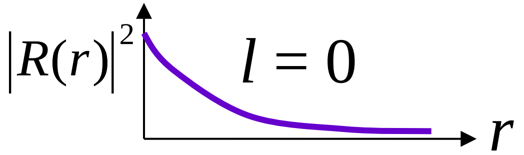
$$^{16}\mathrm{O}\,(6.05\,\mathrm{MeV}) \rightarrow {}^{15}\mathrm{O} + n$$
$$\rightarrow {}^{15}\mathrm{N} + p$$
$$\rightarrow {}^{12}\mathrm{C} + \alpha$$

All are not energetically possible. $Q < 0$

Electromagnetic Transitions

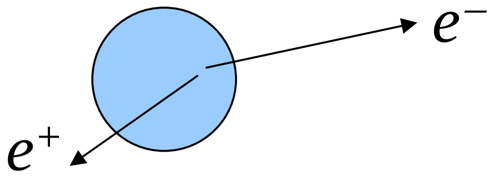
Internal Conversion (IC) is a possibility.

- K-shell atomic electron takes the 6.05 MeV and is ejected from the atom.
- An s-state electron has a significant probability of being found inside the nucleus.



Internal Pair Conversion (IPC) is another possibility.

- Here the 6.05 MeV is converted directly into an electron-positron pair.
- The IPC process becomes more important as the excitation energy increases.



Electromagnetic Transitions

^{16}O energy levels from NNDC

E_{level} (keV)	XREF	Jπ	$T_{1/2}$	E_γ (keV)	I_γ	γ mult.	Final level	
0.0	ABCDEF HIJKLMNOPQ	0+	STABLE					
6049.4 *10*	ABC EF IJK M P	0+	67 ps *5*	6048.2 *10*		[E0]	0.0	0+
6129.89 *4*	ABC EF HIJKL NOPQ	3−	18.4 ps *5*	6128.63 *4*	100	[E3]	0.0	0+
6917.1 *6*	ABC EF HI KLMNOPQ	2+	4.70 fs *13*	787.2 *6* 867.7 *12* 6915.5 *6*	≤0.008 0.027 *3* 100	[E1] [E2] [E2]	6129.89 6049.4 0.0	3− 0+ 0+
7116.85 *14*	AB EF HIJKLM OPQ	1−	8.3 fs *5*	986.93 *15* 1067.5 *10* 7115.15 *14*	0.070 *14* <6E-4 100	[E2] [E1] [E1]	6129.89 6049.4 0.0	3− 0+ 0+

Note: the 6.05 MeV state is labeled as E0 – this does not mean a photon with $l = 0$. The state must decay by other means.

Note the longer lifetimes for the 0^+ and 3^- states.

Electromagnetic Transitions

The actual calculation of the transition rates is complex since it involves calculating the matrix elements Q_{lm} and M_{lm}.

This is generally not possible since the wave functions of the nuclear states are not well known.

However, by making some broad assumptions we can get rough estimates of the transition rate that can give us some guidance in understanding the relative importance of each transition mechanism.

These assumptions are:

- that the initial and final states are single particle wave functions derived from a spherical potential, and that the final state is an s-state.
- that the radial wave functions are constants equal to $\sqrt{3}R_0^{-3/2}$ over the nuclear volume (of radius R_0) and zero outside the nucleus.

Electromagnetic Transitions

These broad estimates are known as the Weisskopf estimates.

The estimates are (E_γ is in MeV):

l	$\lambda(\mathrm{E}\text{-}l)$ (s^{-1})	$\lambda(\mathrm{M}\text{-}l)$ (s^{-1})
1	$1.02 \times 10^{14}\, A^{2/3} E_\gamma^{\,3}$	$3.15 \times 10^{13}\, E_\gamma^{\,3}$
2	$7.28 \times 10^{7}\, A^{4/3} E_\gamma^{\,5}$	$2.24 \times 10^{7}\, A^{2/3} E_\gamma^{\,5}$
3	$3.39 \times 10^{1}\, A^{2} E_\gamma^{\,7}$	$1.04 \times 10^{1}\, A^{4/3} E_\gamma^{\,7}$
4	$1.07 \times 10^{-5}\, A^{8/3} E_\gamma^{\,9}$	$3.27 \times 10^{-5}\, A^{2} E_\gamma^{\,9}$

Electromagnetic Transitions

The Weisskopf estimates should not be considered valuable for calculating precise values.

They do give a useful understanding of the relative transition rates for E and M radiation as a function of l.

From these we see:

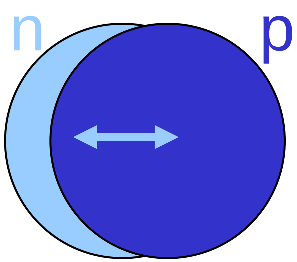
1. For a given transition energy, there is a substantial (orders of magnitude) decrease in decay rate with increasing l.
2. Electric transitions have transition rates that are typically two orders of magnitude higher than for the corresponding magnetic transition (due to stronger dependence on A).

Electromagnetic Transitions

There is a type of collective motion that can be excited in all nuclei.

This is when the protons and neutrons appear to oscillate against each other.

The protons move back and forth, so relative to the centre of mass, the neutrons move in the opposite direction. This motion is analogous to the electrons moving back and forth in a dipole antenna.



This is dipole oscillation that can be induced in any nucleus.
It is most easily induced by the absorption of an E1 photon.
e.g. if the ground state is 0^+ this state will be a 1^- state.

Electromagnetic Transitions

This dipole oscillation of the neutrons and protons moving in opposite directions has a very strong restoring force which is due to the nuclear force.

i.e. it acts like a very stiff spring.

Therefore the energy of the oscillation is quite high – typically of the order 15-20 MeV.

Therefore the transition rate $\lambda(\mathrm{E1})$ is large and so these states have very short lifetimes.

Therefore the natural width, Γ, of this state is large, of the order several MeV.

The corollary is that it is very easy for a photon to be absorbed into this state. So the cross section for absorption is quite large.

Therefore these states are known a Giant Dipole Resonances.

Electromagnetic Transitions

Because of their high energy, these states are most likely to decay by particle emission, of which neutron emission is commonly the dominant mode for heavy nuclei.

Therefore the Giant Dipole Resonance is easily seen in a photo-neutron cross sections.

e.g $^{90}\mathrm{Zr}(\gamma,\mathrm{n})$ Width: $\Gamma = \hbar\lambda \approx 4.0\,\mathrm{MeV}$

$$\Gamma = \hbar\lambda = \frac{\hbar}{\tau} \Rightarrow \tau = \frac{\hbar}{\Gamma}$$

$$\tau = \frac{6.58\times10^{-16}\ \mathrm{eV.s}}{4.0\times10^{6}\ \mathrm{eV}} = 1.6\times10^{-22}\ \mathrm{s}$$

Such a short lifetime indicates that the decay is via the strong interaction. i.e. by particle emission rather than by emitting a gamma ray (electromagnetic interaction).