

# A Systematic Description of the Wobbling Motion in Odd-Mass Nuclei Within a Semi-Classical Formalism

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  - Even-A case study
- 4 Wobbling Motion in Odd-A
  - Old formalism
  - Redefinition of the Odd-A framework

# Aim



## Research Objectives

- Extend the current interpretation of the **nuclear triaxiality** in the context of its unique fingerprint: **Wobbling Motion**
- Adopt a framework that is as close as possible to **classical physics**.
- Provide new formalisms for the phenomena related to **nuclear deformation**.

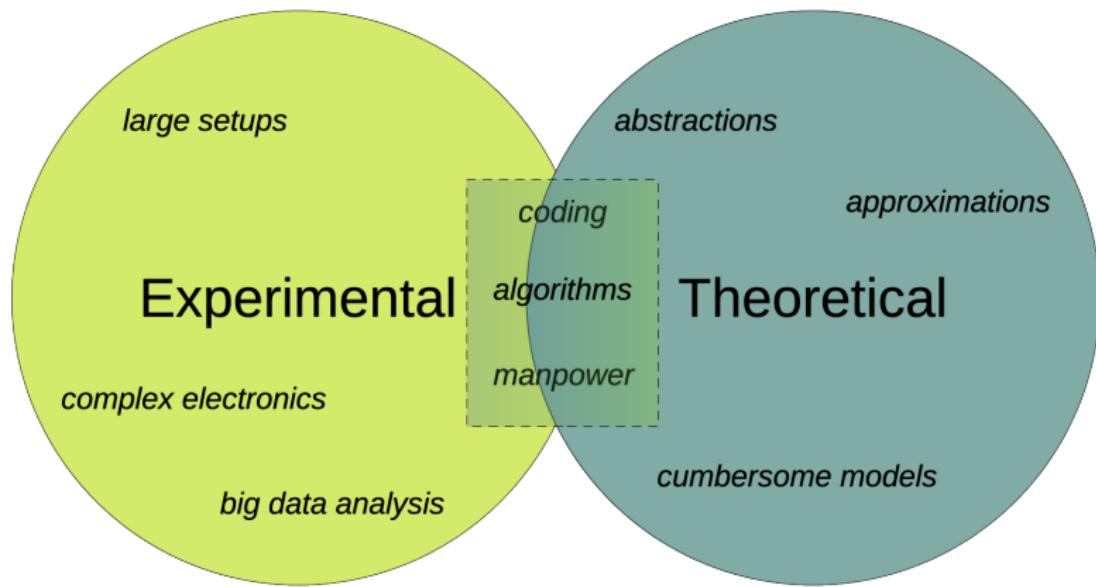


## Objectives exclusive to the thesis

- Give the reader enough context towards a better understanding of the underlying concepts, methods, and results.
- Create a completely *open-source* project.

# Motivation

- **Nuclear Triaxiality** has become a *hot topic* within the scientific community.
- Identifying nuclei with triaxial deformations represents a real **experimental** and **theoretical** challenge.



# Nuclear Deformation

## Nuclear Radius

The **shape** of the nucleus is most generally described in terms of the *nuclear radius*:

$$R(\theta, \varphi) = R_0 \left( 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \varphi) \right)$$

## Quadrupole deformations $\lambda = 2$

- **For us:** Most relevant modes are the **quadrupole vibrations**  $\lambda = 2$   
 $\implies$  Play a crucial role in the rotational spectra of nuclei:
- *Bohr, 1969:* Coordinates  $\alpha_{2\mu}$  can be reduced to only two *deformation parameters*:  $\beta_2$  (**eccentricity**) and  $\gamma$  (**triaxiality**).

# Axial shapes

## Collective coordinates

- Most of the nuclei are either **spherical** or **axially symmetric** in their ground-state (*Budaca, 2018*).
- Moments of inertia:  $\mathcal{I}_{1,2,3}$ : two are equal, one is different.

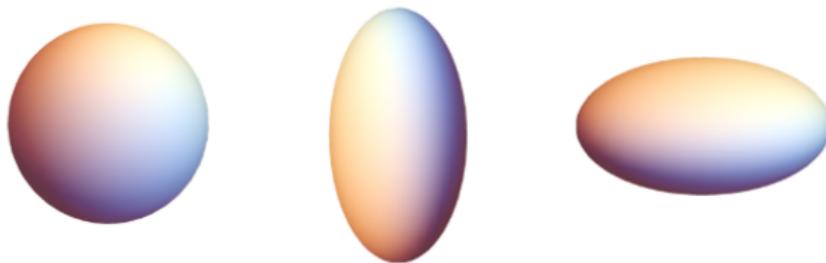
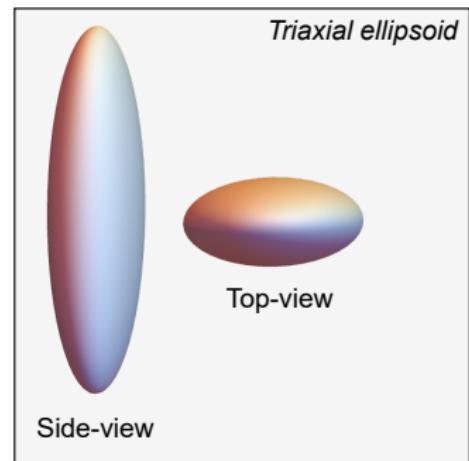
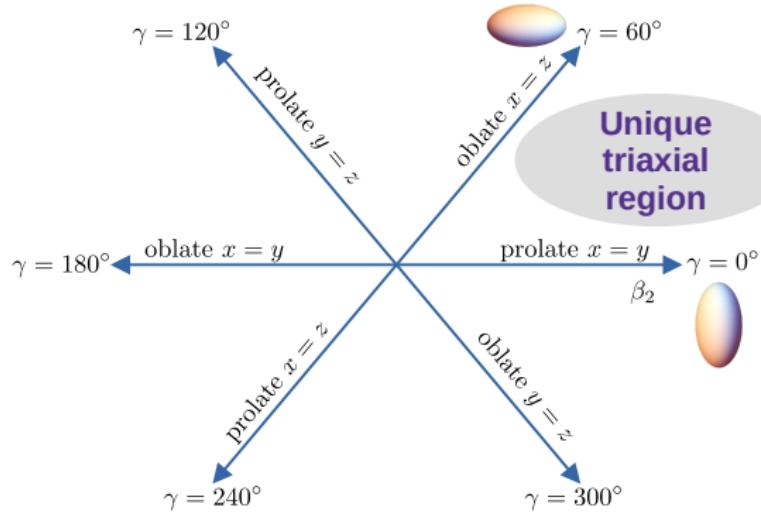


Figure: **spherical**:  $\beta_2 = 0$  **prolate**:  $\beta_2 > 0$  **oblate**:  $\beta_2 < 0$ . ( $\gamma = 0^\circ$ ).

# Non-axial shapes

- The triaxiality parameter  $\gamma \neq 0^\circ$ : departure from axial symmetry.
- Moments of inertia:  $I_1 \neq I_2 \neq I_3$ .



# Fingerprints of Triaxiality

## Evidence

- Currently, there are **only two** well-established phenomena uniquely attributed to triaxial deformation.
  - ① **Wobbling Motion** - WM (*Bohr and Mottelson, 1970s*)
  - ② Chiral Motion -  $\chi$ M (*Frauendorf, 1997*)
- These two can be measured/detected experimentally.

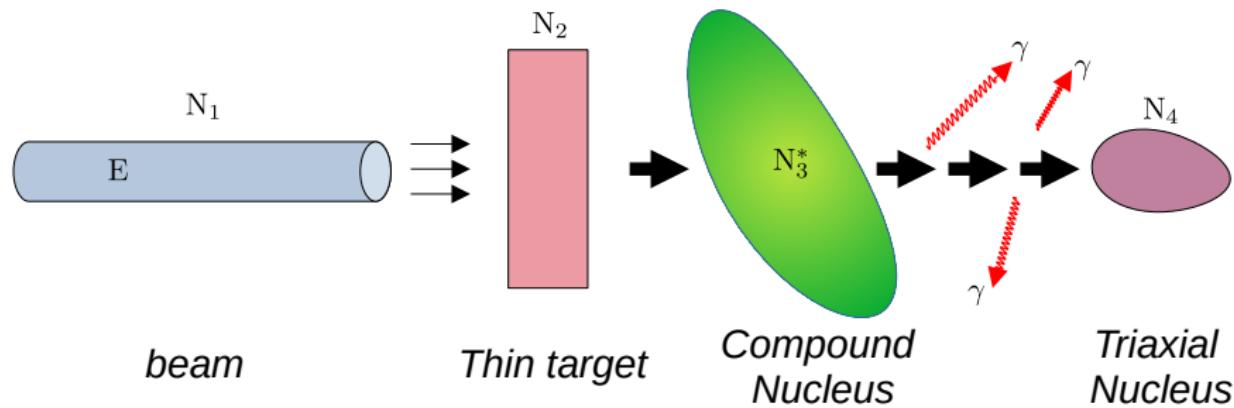
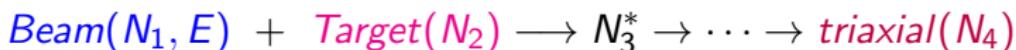
## Goal

**Describe the elusive character of Wobbling Motion in the context of nuclear triaxiality.**

# Q Probing triaxiality in nuclei

Triaxial nuclei can be observed/obtained in several experiments:

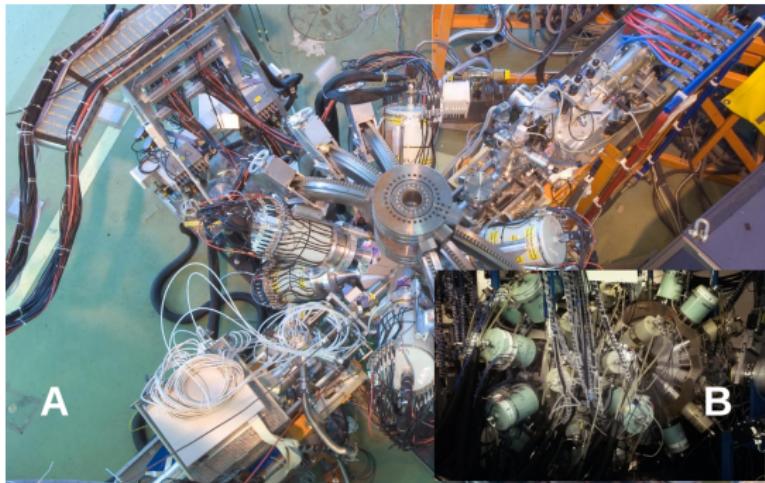
- Nuclear fission:  $A \rightarrow B + C$
- Nuclear fusion:  $X + Y \rightarrow Z$
- **Fusion-evaporation reactions:** Long-lived + enhanced deformation



# Q Nuclear facilities

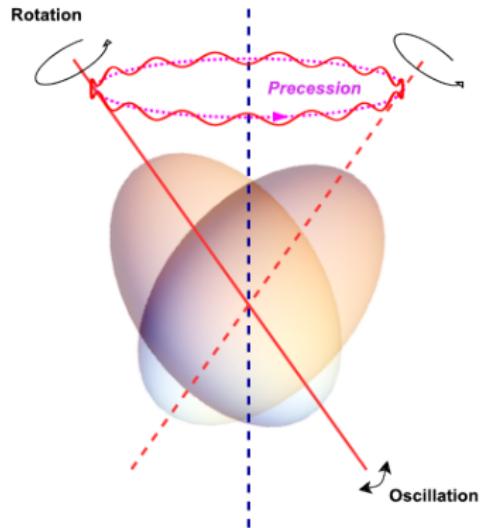
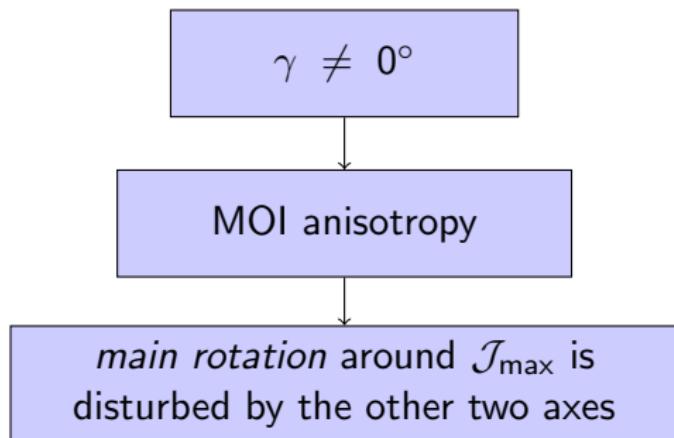


**Figure:** Gammasphere detector,  
ANL-ATLAS USA. *Source:*  
[aps.org](http://aps.org)



**Figure:** a) IDS detector, CERN. *Source:*  
[isodel.web.cern.ch](http://isodel.web.cern.ch) b) JUROGAM II, Finland.  
*Source:* [twitter.com](https://twitter.com)

# Wobbling Motion



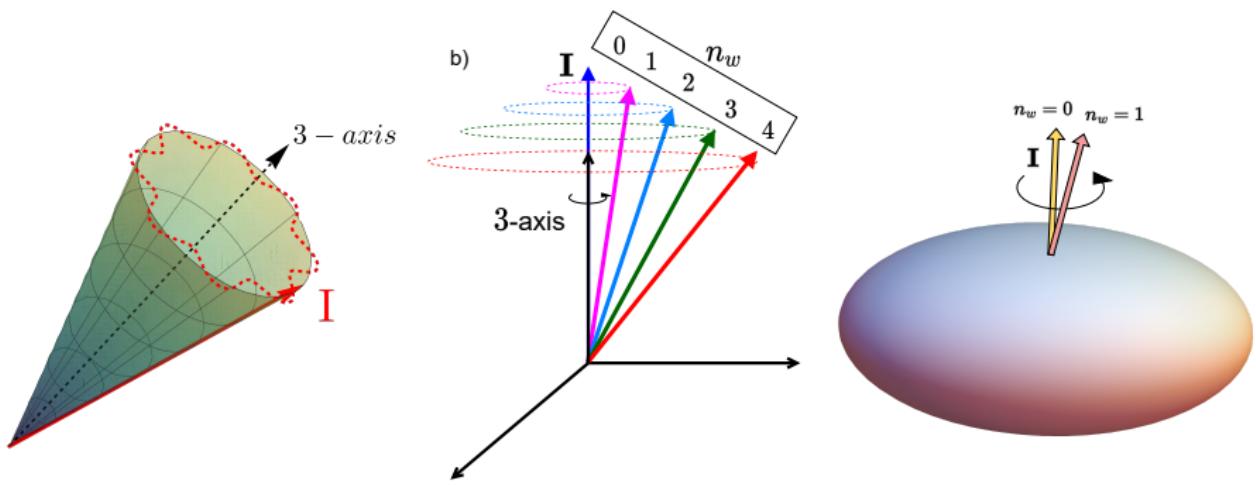
## Wobbling Effect

- The **total angular momentum** of the nucleus **precesses** and **oscillates** around  $\mathcal{J}_{\max}$ .

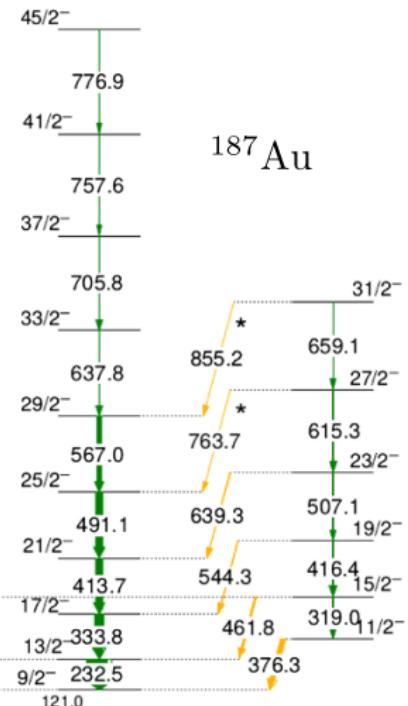
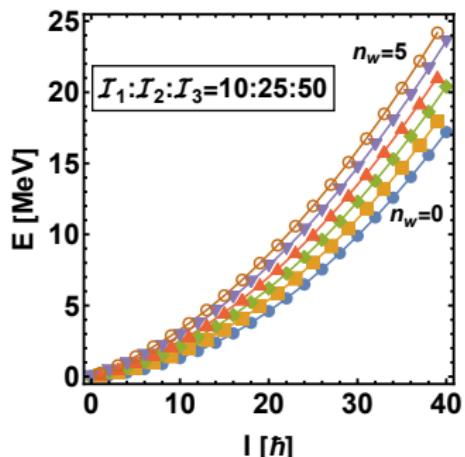
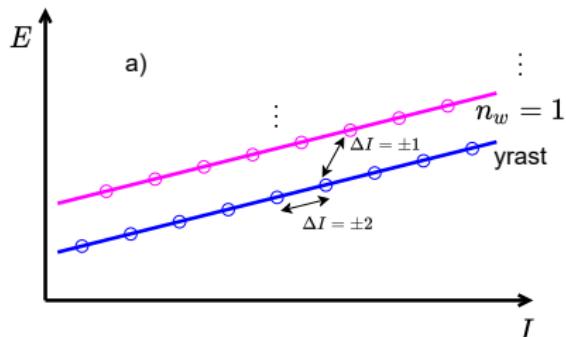
# Wobbling Motion

## Harmonic oscillation

- Precession of  $\mathbf{I}$  is affected by **rotational frequency** and/or **tilting**
- Tilting only by "specific" amount  $\rightarrow$  **harmonic character**  $\rightarrow$  **wobbling phonon**:  $n_w = 0, 1, 2, \dots$



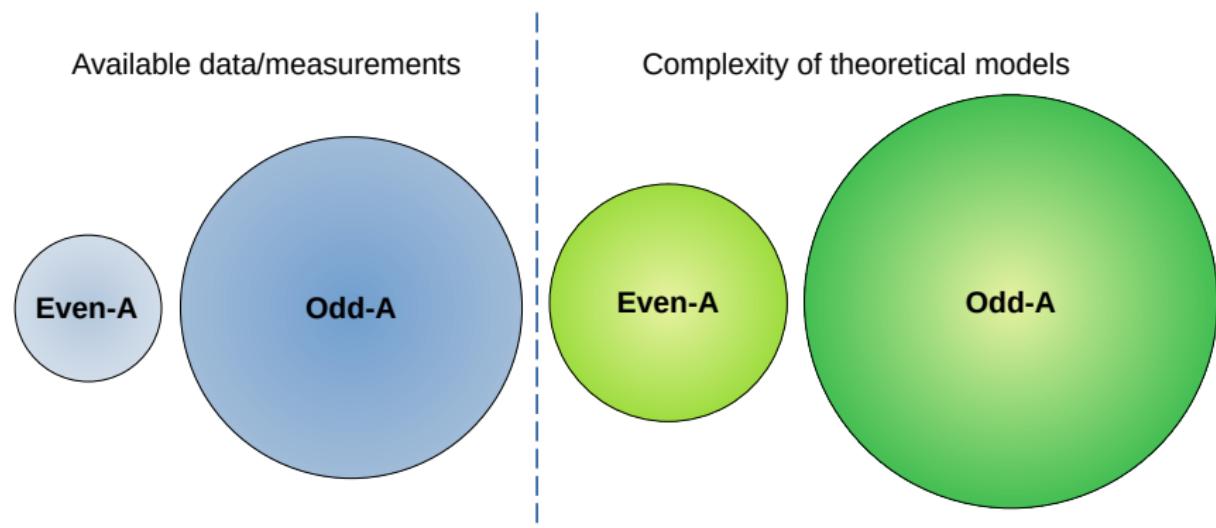
# Wobbling Motion II



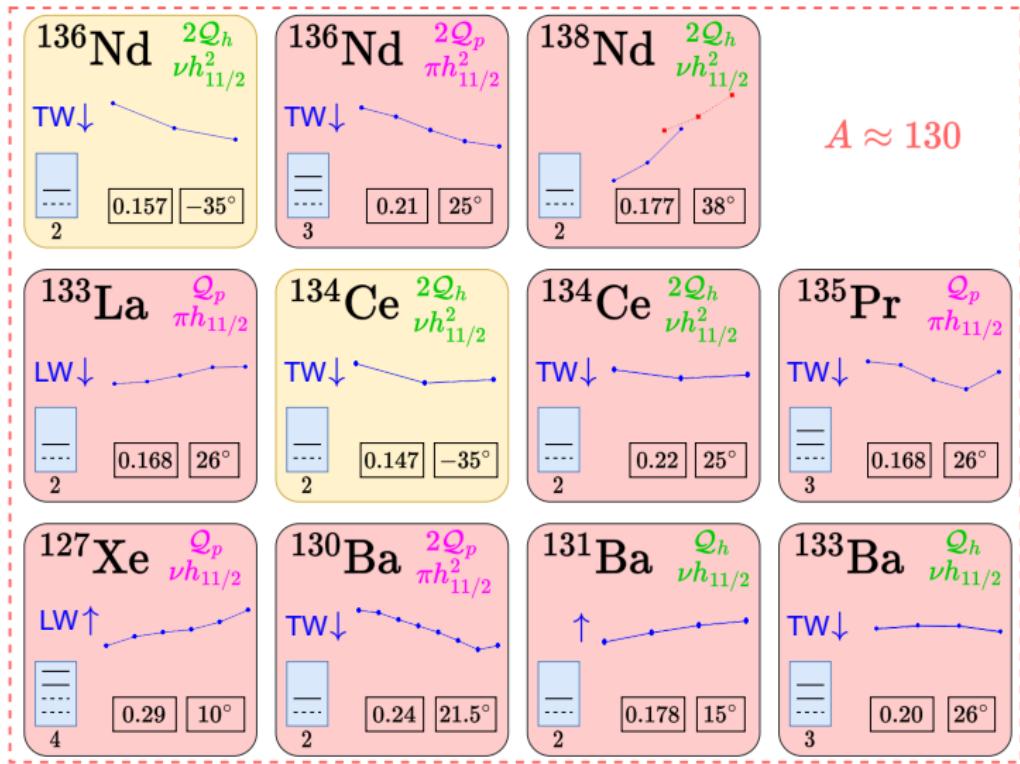
Sensharma, 2020.

# Even- $A$ vs. Odd- $A$ Picture

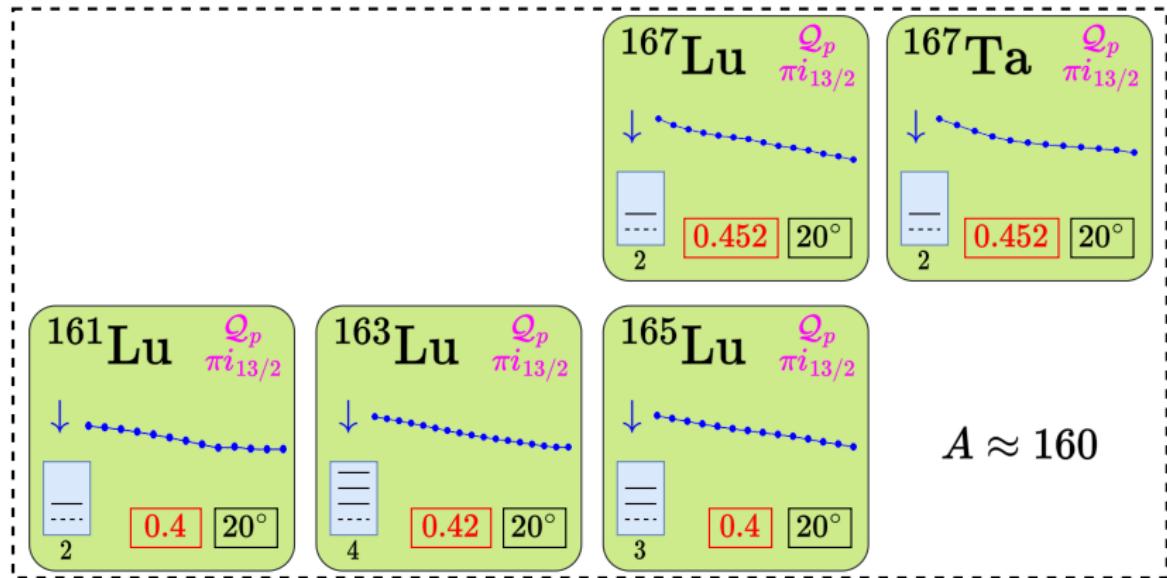
- Predicted for even- $A$  nuclei more than 50 years ago.
- First experimental evidence for **nuclear wobbling motion**:  $^{163}\text{Lu}$  (*Ødegård, 2001*).
- Current mass-regions for wobblers:  $A = 130, 160, 180$ .



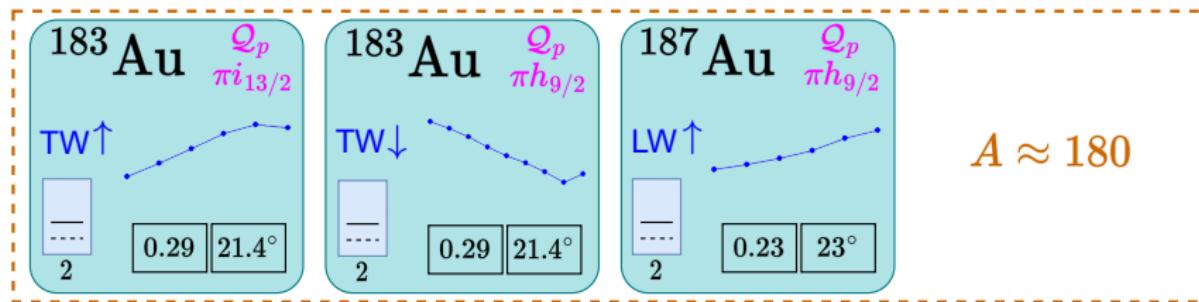
# Wobblers in the A=130 mass region



# Wobblers in the A=160 mass region



# Wobblers in the A=180 mass region



All diagrams and their data sources available in Chapter 3, Section 3.3.5  
**Presented at the Annual Meeting, FFUB, 2022.**

# Wobbling Motion in $^{130}\text{Ba}$

**Q** Experimental measurements show **two** wobbling bands (*Petrache et. al. 2019*).

## Harmonic formalism

**Harmonic Approximation** (*Bohr & Mottelson, 1969*):

$$E_{I,n_w} = A_3 I(I+1) + \hbar\omega_w \left( n_w + \frac{1}{2} \right),$$

$$A_3 = (2\mathcal{I}_3)^{-1}.$$

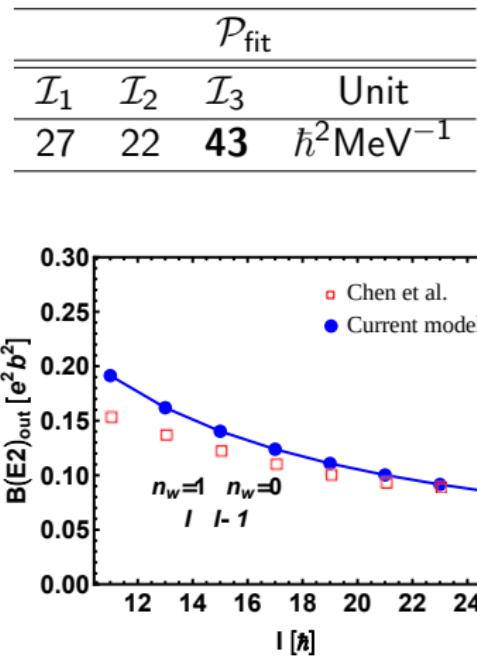
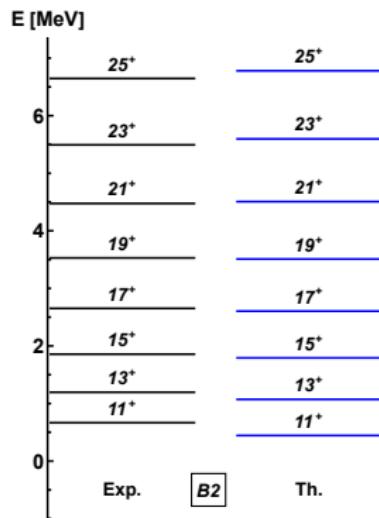
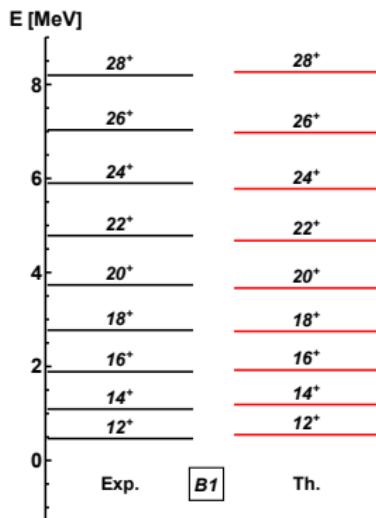
(rotational term + wobbling frequency)



GALILEO, LNL, Source: [lnl.infn.it](http://lnl.infn.it)

Fusion evaporation:  $^{13}\text{C}$  beam of  
 $E = 65$  MeV and  $^{122}\text{Sn}$  target.

# Results for $^{130}\text{Ba}$



Full description: Chapter 3 (Section 3.1.2)

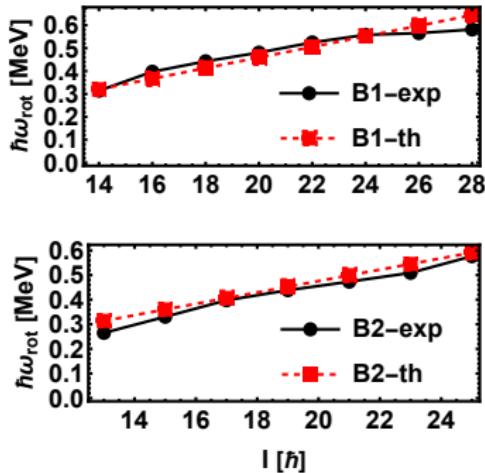
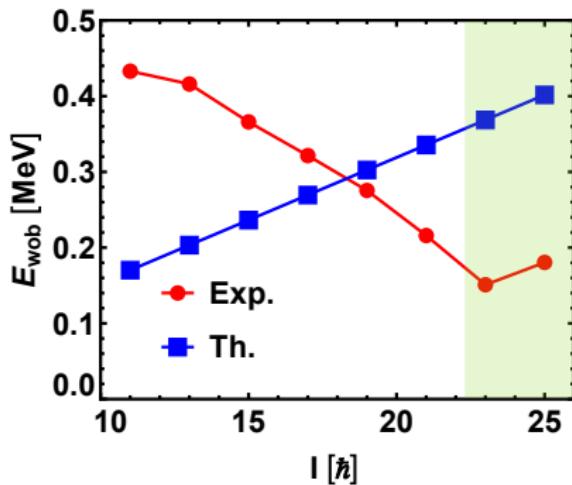
Results presented at the international conference NSP-2022, Turkey.

# Results for $^{130}\text{Ba II}$

## Excitation energies vs. Wobbling Energies:

$$E_{\text{wob}}(I_{\text{even}}) = E_{I,n} - E_{I,0} ,$$

$$E_{\text{wob}}(I_{\text{odd}}) = E_{I,n} - \frac{1}{2} (E_{I-1,0} + E_{I+1,0})$$



Results presented at the international conference NSP-2022, Turkey.

# Starting Point

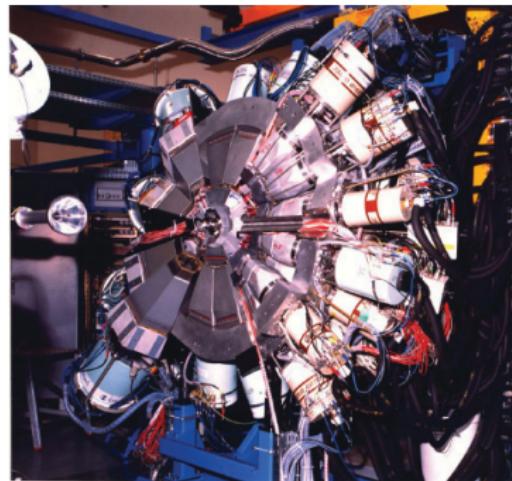
- A. A. Raduta, R. Poenaru, L. Gr. Ixaru, PRC, 2017 + ■ A. A. Raduta, R. Poenaru, Al. H. Raduta, JPG, 2018 →  $W_0$  in the thesis.

## Framework

- First semi-classical description for the  $^{163}\text{Lu}$ , using the **Particle-Rotor-Model** (*Hamamoto, 2002.*) for an odd-mass nucleus in the  $A \approx 160$  region.

## PRM

An odd-nucleon moving in a quadrupole deformed mean field generated by an even-even triaxial core.

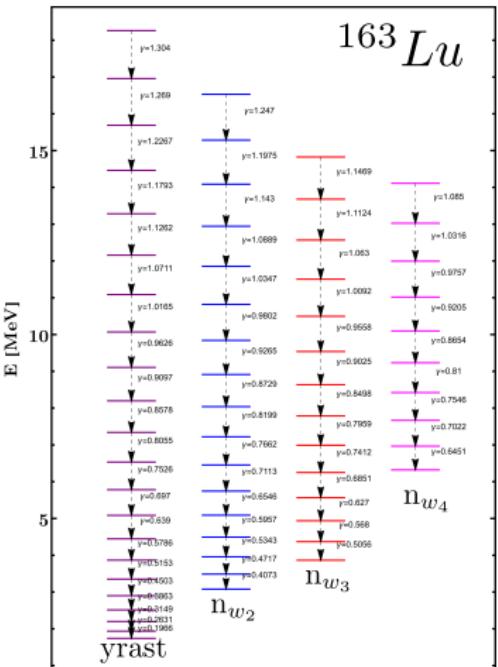
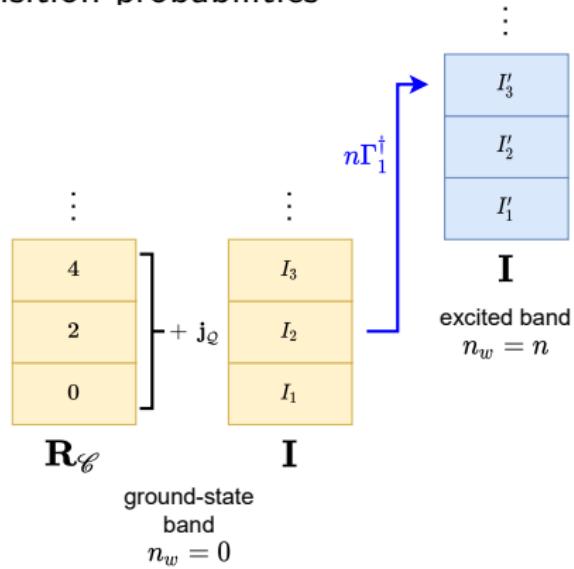


Euroball IV, Strasbourg, Source:  
[technology.i.stfc.ac.uk](http://technology.i.stfc.ac.uk)

Fusion evaporation:  $^{29}\text{Si}$  beam of  $E = 152$  MeV and  $^{139}\text{La}$  target.

# Overview of $W_0$

- Time-Dependent Variational Principle applied on the PRM Hamiltonian
- **Phonon operators** → energies + transition probabilities



# Overview of $W_0$ II

## Model characteristics

- + Numerical data consistent with other work
- TSD4: three-phonon wobbling band (disagreement with Jensen et al.)
- Adopted rigid-body MOIs ("unpleasant" choice to the referees)
- Deformation parameters  $\beta$  and  $\gamma$  taken from literature

**Onset of a redesign → start of a new research project**

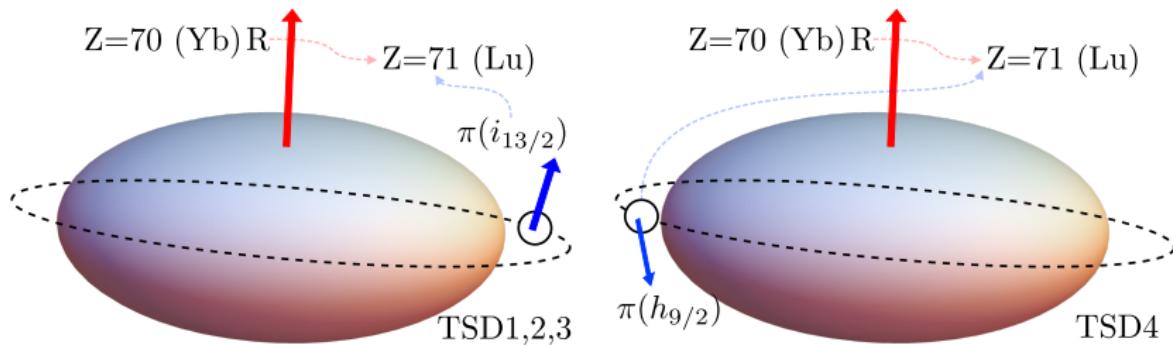
**Two new models developed presented in Chapter 4 ( $W_1$ ) and 5 ( $W_2$ )**

# Fresh-up: $\mathbf{W}_1$

Particle-Rotor Model Hamiltonian for an odd- $A$  nucleus:

$$\hat{H} = \hat{H}_{\text{rot}} + \hat{H}_{\text{sp}}, \quad \hat{H}_{\text{rot}} = \sum_{k=1}^3 A_k (\hat{l}_k - \hat{j}_k)^2,$$

$$\hat{H}_{\text{sp}} = \epsilon_j + \frac{\nu}{j(j+1)} \left[ \cos \gamma (3\hat{j}_3^3 - \mathbf{j}^2) - \sqrt{3} \sin \gamma (\hat{j}_1^2 - \hat{j}_2^2) \right]$$



A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

# Variational Principle + Eqs. of Motion

## Time-Dependent Variational Equation

$$\delta \int_0^t \langle \Psi_{IM;j} | \hat{H} - i \frac{\partial}{\partial t'} | \Psi_{IM;j} \rangle dt' = 0$$

$$\Psi_{\text{trial}} \equiv |\Psi_{IM;j}\rangle = \mathcal{N} e^{z\hat{I}_-} \cdot e^{s\hat{j}_-} |IMI\rangle \otimes |jj\rangle$$

- $(z, r, \varphi, \hat{I}, |IMI\rangle)$  - core (**R**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$  - single-particle (**j**)
- $(s, f, \psi, \hat{j}, |jj\rangle)$  - single-particle (**j**)
- $\{z, s\}$  → phase space coordinates

## Constant of motion:

$$\mathcal{H} \equiv \langle \Psi_{IM;j} | \hat{H} | \Psi_{IM;j} \rangle$$

## Canonical equations of motion:

$$\mathcal{S}_1 : \frac{\partial \mathcal{H}}{\partial r} = \dot{\varphi}, \quad \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{r}$$

$$\mathcal{S}_2 : \frac{\partial \mathcal{H}}{\partial f} = \dot{\psi}, \quad \frac{\partial \mathcal{H}}{\partial \psi} = -\dot{f}$$

The two sets of Hamilton equations are the semi-classical description of the initial quantal  $\hat{H}$ .

# Wobbling frequency

Solving  $\mathcal{S}_1$  and  $\mathcal{S}_2$  leads to the algebraic equation:

$$\Omega^4 + B\Omega^2 + C = 0$$

four solutions  $\rightarrow$  **only two are real**:

$$\Omega_{1,2} = \left[ \frac{1}{2} \left( -B \mp \sqrt{B^2 - 4C} \right) \right]^{1/2}$$

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

- $\Omega_1$ : wobbling frequency of the even-*A* core **R**
- $\Omega_2$ : wobbling frequency of the odd-nucleon **j**
- **Two wobbling phonon numbers:  $n_{w_1}$  and  $n_{w_2}$**

# Energy spectrum

## Spectra of odd-A nuclei within $W_1$

$$E_{I,n_1,n_2} = \epsilon_j + \mathcal{H}_{\min}^I + \hbar\Omega_1^I \left( n_{w_1} + \frac{1}{2} \right) + \hbar\Omega_2^I \left( n_{w_2} + \frac{1}{2} \right)$$

- Phonon factor:

$$\mathcal{F}_{n_{w_1} n_{w_2}}^I = \hbar\Omega_1^I \left( n_{w_1} + \frac{1}{2} \right) + \hbar\Omega_2^I \left( n_{w_2} + \frac{1}{2} \right)$$

- $\mathcal{H}_{\min}^I$  is the Classical Energy Function taken in its minimum point:  
 $p_0 = (0, I; 0, j)$ .

# A new interpretation for TSD1 and TSD2

## Previous models

$TSD1$  = zero-phonon wobbling band

$TSD2$  = one-phonon wobbling band...

## Redefinition

$TSD1$  and  $TSD2$  are **Signature Partner Bands** (In favor of Ultimate Cranker calculations, *Jensen, 2004*).

$$\left( \alpha_{fav} = +\frac{1}{2} \right) : \{ TSD1 \} \equiv \{ [0^+, 2^+, 4^+, \dots] \otimes [J^\pi = 13/2^+] \}$$

$$\left( \alpha_{unfav} = -\frac{1}{2} \right) : \{ TSD2 \} \equiv \{ [1^+, 3^+, 5^+, \dots] \otimes [J^\pi = 13/2^+] \}$$

$TSD4$ : **ground-state wobbling band**,  $\pi(h_{9/2})$  configuration.

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

# A new band structure for $^{163}\text{Lu}$

Band	Spins	$\pi$	$\alpha$	$\pi(I_j)$	$\mathbf{W}_0: \mathcal{C} + \mathcal{Q}_p$	$\mathbf{W}_1: \mathcal{C} + \mathcal{Q}_p$
TSD1	$13/2, 17/2 \dots 97/2$	+	+1/2	$\pi(i_{13/2})$	$0^+, 2^+, 4^+, \dots$	$0^+, 2^+, 4^+, \dots$
TSD2	$27/2, 31/2 \dots 91/2$	+	-1/2	$\pi(i_{13/2})$	$\text{TSD1} + 1\Gamma^\dagger$	$1^+, 3^+, 5^+, \dots$
TSD3	$33/2, 37/2 \dots 85/2$	+	+1/2	$\pi(i_{13/2})$	$\text{TSD1} + 2\Gamma^\dagger$	$\text{TSD2} + \Gamma^\dagger$
TSD4	$47/2, 51/2 \dots 83/2$	-	-1/2	$\pi(h_{9/2})$	$\text{TSD1} + 3\Gamma^\dagger$	$1^+, 3^+, 5^+, \dots$

Bands	$n_{w_1}$	$n_{w_2}$	$\mathcal{F}_{n_{w_1} n_{w_2}}^I$	$I_0$	$I_t$	$\mathcal{Q}$
TSD1	0	0	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$13/2^+$	$97/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
TSD2	<b>0</b>	<b>0</b>	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$27/2^+$	$91/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
TSD3	1	0	$\mathcal{F}_{10}^{I-1} = \frac{3}{2} \Omega_1^{I-1} + \frac{1}{2} \Omega_2^{I-1}$	$33/2^+$	$85/2^+$	$j^\pi = 13/2^+ \stackrel{\text{not}}{\equiv} \mathcal{Q}_1$
<b>TSD4</b>	<b>0</b>	<b>0</b>	$\mathcal{F}_{00}^I = \frac{1}{2} (\Omega_1^I + \Omega_2^I)$	$47/2^-$	$83/2^-$	$j^\pi = 9/2^- \stackrel{\text{not}}{\equiv} \mathcal{Q}_2$

A.A. Raduta, R. Poenaru, C.M. Raduta, Journal of Physics G 47, 2020.

# Extension to $A \approx 160$ mass region

The model was further applied to the other Lu isotopes where wobbling motion has been observed.

$^{161}\text{Lu}$ Bands	Spins	$\mathcal{Q}$	$\mathcal{C}$	$(n_{w_1}, n_{w_2})$	$I_b$
TSD1	$21/2^+, 25/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$4^+, 6^+, 8^+ \dots$	$(0, 0)$	
TSD2	$31/2^+, 35/2^+, \dots, 79/2^+$	$j^\pi = 13/2^+$	$9^+, 11^+, 13^+ \dots$	$(0, 0)$	$21/2$

$^{165}\text{Lu}$ Bands	Spins	$\mathcal{Q}$	$\mathcal{C}$	$(n_{w_1}, n_{w_2})$	$I_b$
TSD1	$25/2^+, 29/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \dots$	$(0, 0)$	
TSD2	$35/2^+, 39/2^+, \dots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \dots$	$(0, 0)$	$25/2$
TSD3	$41/2^+, 45/2^+, \dots, 81/2^+$	$j^\pi = 13/2^+$	$\text{TSD2} + \Gamma^\dagger$	$(1, 0)$	

$^{167}\text{Lu}$ Bands	Spins	$\mathcal{Q}$	$\mathcal{C}$	$(n_{w_1}, n_{w_2})$	$I_b$
TSD1	$25/2^+, 29/2^+, \dots, 89/2^+$	$j^\pi = 13/2^+$	$6^+, 8^+, 10^+ \dots$	$(0, 0)$	
TSD2	$35/2^+, 39/2^+, \dots, 91/2^+$	$j^\pi = 13/2^+$	$11^+, 13^+, 15^+ \dots$	$(0, 0)$	$25/2$

A. A. Raduta, R. Poenaru, C. M. Raduta, Phys. Rev. C 101, 2020.

# W<sub>1</sub> — Numerical Results

Thank you for your attention ❤