CHAPTER 12

Electromagnetic Transitions in Complex Nuclei

12.1. Introduction

Until relatively few years ago the investigation of electromagnetic transitions between nuclear states played a small role in nuclear physics. The reasons for this were mainly technical. The recent developments of the scintillation counter, fast electronics, and the methods of investigating resonance scattering, together with theoretical calculations on internal conversion coefficients, angular correlation, and Coulomb excitation (Chapter 10) have made it possible to derive useful data for the understanding of nuclear structure from the study of radiative transitions. The data provide inferences of effectively two types. Measurement of internal conversion coefficients and of angular distributions and correlations lead to inferences on spin values and parities of excited states of nuclei. Measurements of lifetimes or widths of absorption lines, on the other hand, give values of the absolute squares of matrix elements of transition operators. If the interaction responsible for the electromagnetic transition arises wholly from the free charges and dipole moments of the nucleons, the measured matrix elements in question are off-diagonal elements of operators similar in structure to the magnetic dipole and electric quadrupole operators which were considered in connection with static moments of the ground states of nuclei (Chapter 3) and more explicitly in connection with Coulomb excitation (Chapter 10) and β-decay (Chapter 11).

Insofar as problems of nuclear structure are concerned, the study of radiative transitions and the study of β-decay play similar roles. The relative ease of detection of the β-particles and of the measurement of the slow transition rates stimulated the early study of the β-decay processes. Analysis of the rate of β-transitions has theoretical advantages also because the operators responsible for these are the most simple operators in the case of allowed transitions. Hence inferences from the measured rates to the properties of nuclear states are fairly immediate. Analysis of nuclear states by means of β-decay suffers, however, from two features: (a) the slow transition rate for the process means that for the most part only the ground states of the parent nuclei are involved, and (b) in the case of forbidden transitions, which form the majority of all transitions, matrix elements of several operators play a role. The radiative transitions connect states of a single nucleus. The operator responsible for the simplest transitions involves more complex properties of nuclear states than do the allowed β-decay operators. On the other hand, the radiative transition rates are probably easier to interpret than the rates of forbidden β-transitions. This applies at least to the more important class of transitions, the electric transitions. The situation may not be so simple as far as magnetic transitions are concerned.

An excited nucleus may lose its energy either by simple radiation or by internal conversion. In the latter process the excitation energy is transferred to an extra-nuclear electron. Internal conversion is of particular importance if the energy of the transition is low (< 1 Mev), and the charge of the nucleus is high, or if the radiative transition is inhibited by special selection rules (e.g. in $0 \to 0$ transitions, i.e. transitions between states of zero angular momentum). In addition, if the energy of the transition is large enough, de-excitation may occur through production of pairs. In general this process is considerably slower (about 10^{-3} times slower) than the radiative process and has little influence on the effective half-life of the excited state. However, the 6.06 mev $0 \to 0$ transition in O^{16} occurs almost wholly by pair production.

In the following the theories of radiative transition and of internal conversion are sketched in a qualitative manner. The measured matrix elements are then compared in a general way with theoretical estimates based on the single particle model.

12.2. Radiative Transitions

The theory of electromagnetic radiation is much older than that of β-decay and the literature contains good comprehensive treatments of the subject. In fact, as mentioned before, the theory of β-decay was largely patterned on the quantum theory of radiation. The following treatment, which is now the customary one, differs from that adopted for β-decay inasmuch as it uses spherical rather than plane waves as the basic set of states for the electromagnetic field. Such spherical waves have the advantage that they possess, in addition to a definite energy, also a definite total angular momentum, which will be called l, and even or odd parity. The total angular momentum is usually specified as the multipolarity of the radiation. Instead of a direct specification of parity the "electric" (E) or "magnetic" (M) character of the radiation is given. The parity of a radiation state is then defined by l and the character of the radiation (E or M). Electric radiation has parity $(-)^l$, while magnetic radiation has parity $(-)^{l+1}$. The radiation field has, for a given l and parity, $2l + 1$ different states characterized by $2l + 1$ different values ($\mu = -l, -l + 1, \cdots, l - 1, l$) of the z component of total angular momentum. The names "electric" and "magnetic" derive from the classical distribution of charges and currents which give rise to these fields. Expansion of the field into spherical rather than plane waves is preferable if only one particle (in this case a γ-quantum) is emitted, because energy, angular momentum, its z component μ, and parity completely specify the state. This is not the case if two particles are emitted. Use of spherical waves, in the case of the emission of two particles, would obscure the angular correlations between these.

The rate of radiative transitions is calculated, as that of β-decay, by the theory of transition probabilities. Since the state of the emitted photon is completely determined by its energy and character, the probability of emission is given by an expression analogous to that of (11.3) for β-decay:

$$\lambda(f, i; l, \mu, E) = \frac{2\pi}{\hbar} |(\Psi_f, eQ(l, \mu, E)\Psi_i)|^2 \rho. \tag{12.1}$$

Since the strength and form of the operator of electromagnetic interaction are well known, the indeterminate constant g_K of β-decay could be replaced by the charge of the electron and there is no need for the distinguishing marks K, m used in the case of β-decay. Because of the different choice of basic states, the momenta p and q of electron and neutrino are replaced by the characteristics of the emitted photon. The E in (12.1) indicates an electric transition but there is a similar expression, with M instead of the E, for magnetic transitions. The energy of the photon is given as the energy difference between initial and final states; ρ is again the number of states of the photon per unit energy interval.

The wave lengths of the γ-quanta are much longer than the radius of the nucleus. One can, therefore, expand the operator Q in (12.1) as function of the radius and discard all but the first term. This gives:

$$\lambda(f, i; l, \mu, E) = \frac{8\pi(l-1)!(l+1)!2^l}{[(2l+1)!]^2} k^{2l+1} |E_{l\mu} + E'_{l\mu}|^2, \tag{12.2}$$

and a similar expression for magnetic transitions in which E is replaced by M. The wave number k of the photon is its energy divided by $\hbar c$. The matrix elements E and M are:

$$E_{l\mu} = e \sum_{j=1}^{Z} (\Psi_f, r_j^l Y_{l\mu}(\theta_j, \phi_j)^* \Psi_i) \quad \text{and} \tag{12.3a}$$

$$M_{l\mu} = \frac{e\hbar}{Mc} \frac{1}{l+1} \sum_{j=1}^{Z} \sum_{\alpha} \left(\Psi_f, \frac{\partial}{\partial x_{j\alpha}} [r_j^l Y_{l\mu}(\theta_j, \phi_j)]^* \mathbf{L}_{j\alpha} \Psi_i\right). \tag{12.3b}$$

These expressions represent the interaction of the charge distribution and of the current due to the motion of the protons with the electromagnetic field. Hence, the summation is extended only over the coordinates of the protons. Alternately, a factor $\frac{1}{2}(1 - \tau_{j\zeta})$ could be introduced and the summation extended over all particles. M is the mass of the proton; r_j, θ_j, ϕ_j, its polar, and x_j, its rectangular coordinates: the summation over α is to be extended over the three coordinates x, y, z. The $\mathbf{L}_j$ is the operator of the angular momentum:

$$\mathbf{L}_j = -ir \times \text{grad}_j ; \tag{12.4}$$

the Y are normalized spherical harmonics. The interaction of the spin with the electromagnetic field is represented by the E' and M' terms. Only

$$M'_{l\mu} = \sum_{j=1}^{A} \sum_{\alpha} \mu_j \left(\Psi_f, \frac{\partial}{\partial x_{j\alpha}} [r_j^l Y_{l\mu}(\theta_j, \phi_j)]^* \sigma_{j\alpha} \Psi_i \right) \tag{12.3c}$$

is important; $E'_{l\mu}$ is in general small as compared with $E_{l\mu}$. The summation here extends over all particles; μ_j is the magnetic moment of particle j (that is 2.78 $e\hbar/2Mc$ for protons and -1.91 $e\hbar/2Mc$ for neutrons). For $l = 1$, $\mu = 0$, that is the z component of the dipole radiation, one obtains the expressions:

$$E_{10} = e\left(\frac{3}{4\pi}\right)^{1/2} \sum_{j=1}^{Z} (\Psi_f, z_j \Psi_i), \tag{12.4a}$$

$$M_{10} = \frac{ie\hbar}{2Mc} \left(\frac{3}{4\pi}\right)^{1/2} \sum_{j=1}^{Z} \left(\Psi_f, \left(y_j \frac{\partial}{\partial x_j} - x_j \frac{\partial}{\partial y_j}\right)\Psi_i\right), \tag{12.4b}$$

$$M'_{10} = \left(\frac{3}{4\pi}\right)^{1/2} \sum_{j=1}^{A} \mu_j (\Psi_f, \sigma_{jz} \Psi_i). \tag{12.4c}$$

The selection rules can be read off these expressions immediately, but they are independent of the assumption that the wave length of the emitted light is much longer than the radius of the nucleus. If the parities of Ψ_i and of Ψ_f are the same, only even electric multipole and odd magnetic multipole radiations are possible. If the parities of Ψ_i and of Ψ_f are opposite, the polarity of the electric radiation is odd; that of the magnetic radiation, even. In addition, the J values of Ψ_i and Ψ_f must form a vector triangle: $|J_i - J_f| \leq l \leq J_i + J_f$. Transition rates of the electric multipole radiations decrease with increasing l and the same is true of transition rates due to magnetic multipole radiations. Hence, if the selection rules admit an electric l-pole radiation, transition rates due to higher electric multipoles can be, in general, disregarded, and the same is true of the magnetic multipoles. In general, the magnetic l-pole should give a lower rate than the electric l-pole; in the case of dipole radiation the two are often of the same order of magnitude. It is more usual, however, to consider magnetic l-pole and electric $(l + 1)$-pole to be of the same order of magnitude; they also can occur in the same transition and often compete with each other. If J_i and J_f differ by several units, the multipole radiation will be of high order and the transition slow. This is the explanation of nuclear isomerism, suggested by v. Weizsäcker (Chapter 1).

Expressions for radiative transition rates are used, principally, to test the accuracy of wave functions furnished by the various models. They are given already in terms of the wave functions with two-valued spin; this accounts for the fact that they are rather artificially divided into two parts: one due to charges, the other due to spin. There is general agreement that the expressions for the electric transition rates are sufficiently accurate. The rate of the magnetic transitions, on the other hand, may be influenced by the currents which are not due to motion of the nucleons but to the mesons which transmit the nuclear interaction. These currents are held responsible also for the magnetic moment of the neutron and the anomalous magnitude of the magnetic moment of the proton. The use of the observed moments μ_i in (12.3c) therefore already partially accounts for these "exchange currents". It is to be expected, however, that the meson current associated with the free nucleons is modified by the proximity of other nucleons so that appreciable corrections to the transition rates may result. The magnetic transition rates will be more strongly influenced than the electric transition rates because the average velocity of the mesons is higher than that of the nucleons and the magnetic transition rates depend on the current distribution while the electric rates depend on the charge distribution. For this last reason also, $E'_{l\mu}$ is in general much smaller than $E_{l\mu}$, and the electric transition rates are practically independent of the magnetic moments μ_i. The fact that the static magnetic moments of complex nuclei can be calculated, under the assumption of the additivity of the moments, with an accuracy of about 0.2 nuclear magnetons from sufficiently accurate wave functions indicates that the corrections to the transition rates may not be as large as might have been feared. This may be a less valid conclusion for the high-multipole moments than for the low ones. The selection rules are unaffected by the exchange moments.

12.3. Single Particle Matrix Elements

The expressions for the radiative transition rates can be evaluated most easily in the independent particle models for states which differ in the quantum numbers of a single nucleon. Under somewhat schematic assumptions for the radial dependence of the wave function of this single nucleon, one obtains:

$$\lambda(l, E) = \frac{18}{l+3}\frac{(l-1)!(l+1)!}{[(2l+1)!]^2}(2kR)^{2l}kc, \qquad (12.5a)$$

$$\lambda(l, M) = \frac{180}{l+3}\frac{(l-1)!(l+1)!}{[(2l+1)!]^2}(2kR)^{2l}\frac{e^2}{Mc^2R}\frac{\hbar k}{MR}. \qquad (12.5b)$$

R is the nuclear radius. These rates are called Weisskopf units, and actual transition probabilities are conveniently expressed in terms of these units.

The expressions (12.5) are not the rates expected on the basis of the independent particle pictures; they are rates in terms of which the actual transition rates can be conveniently measured. Correction factors must be applied even if there is only a single particle outside of closed shells. These correction factors are less important for magnetic than for electric transitions because the neutron's and the proton's magnetic moments are of the same order of magnitude. They generate, therefore, magnetic fields of similar magnitude and (12.5b) give the corresponding transition rate.

If there is a single proton outside of closed shells, this proton and the rest of the nucleus will move about their common center of mass. Since the rest of the nucleus (the "core") also has a positive charge, and since proton and core are always on opposite sides of the center of mass, the dipole moment of the system will be decreased. This will lead to a decrease of the transition rate by a factor $(1 - Z/A)^2 = (N/A)^2$. This factor can be obtained directly from (12.4a) but follows more easily from the remark that two bodies, moving about their common center of mass and having the same charge-to-mass ratio, do not create any dipole field. If the extra particle outside of the closed shells is a neutron, it is the core which creates the dipole field and the transition rate is given by (12.5a) multiplied by $(Z/A)^2$. For higher electric multipole radiations the correction factor for the proton is negligible; that for the neutron is Z^2/A^{2l}, a very small number.

If there are several particles outside of closed shells, the transition rate will vanish under the assumption of the independent particle model if more than one of the orbits of initial and final states are different: the operators in (12.3) change the state of only one particle. There is a similar selection rule in atomic spectroscopy which is, in general, quite well obeyed. Even if initial and final configurations differ in only one orbit, the calculated rate will be, as a rule, well below one Weisskopf unit because the wave functions will contain many terms. This is apparent already from (7.1) and (7.2) even though these expressions apply for the case of only two particles. Each term of Ψ_i is connected by the operators of (12.3) with only those terms of Ψ_f in which all factors except one are also factors of the term in Ψ_i. This, and the possibility of cancellation of the terms—of which there still will be many—reduces the calculated transition rate below one Weisskopf unit unless there are phase relations between the contributions of the various terms.

The experimental material bears out these conclusions only partially. Dipole transitions, on the whole, conform to the rules given. They are usually only a fraction of a Weisskopf unit and their magnitude shows considerable spread—a factor of about five or ten in either direction from the average of about a tenth of a Weisskopf unit. There are several very weak transitions which *may* be interpreted as occurring between configurations which differ by more than one orbit. There is, however, no clear evidence for this last point.

Many of the higher multipole transition rates are surprisingly high. For E-2 transitions the collective model predicts such high rates and these were mentioned before as supporting the collective model. However, surprisingly high rates are observed also in regions for which the collective model is not usually considered to be appropriate. Thus, the E-2 transition from the 0.87 mev excited state of O^{17} to the normal state proceeds at the rate of one Weisskopf unit. There is only one particle outside of closed shells in this case and this is a neutron. The Z^2/A^{2l} factor is, therefore, less than 10^{-3} and the discrepancy is quite flagrant. On the whole, one gains the impression that the neutrons radiate about as strongly as the protons—just as they are responsible for quadrupole moments of similar magnitude. Another surprising fact is the lack of spread in the strengths of M-4 transitions, most of which proceed at the rate of about two Weisskopf units. As yet no fully satisfactory explanation has been found for these facts.