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BAND CROSSING IN THE CRANKING MODEL

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Abstract: It is shown that the result obtained from the cranking model suffers from a large inaccuracy in the description of nuclei, at the rotational frequency where a band crossing occurs. We develop a more reliable picture of the band crossing, which stresses the importance of examining the behaviour of excited states as well as the ground state in the cranking model.

1. Introduction

The Hartree-Fock-Bogoliubov cranking (HFBC) model has been recently investigated [1-3]) in order to understand the properties of nuclei in the back-bending region. When the HFBC model is applied to the region of rotational angular frequency (ω) where a band crossing occurs, the calculated result, in general, may not be reliable. The reason is that when the band crossing [4]) between the two bands, which have very different values of spin alignment from each other, occurs, the cranking model considers the mixing of the two bands as crossing at a given ω, whereas the two bands should interact at a given I for which the corresponding values of ω of the two bands are very different. It seems that any available calculated result which has been so far published by using the HFBC model suffers from this shortcoming of the model, mainly because the seriousness of this shortcoming was not recognized. Namely, the zero-quasiparticle yrast states obtained from the HFBC equations at a given ω around the band crossing is presumably a result of this strange admixture of the two intrinsic configurations which have a very different spin alignment from each other. Even in the case where some calculated result [3]) shows that in the back-bending region of some nuclei the pairing gap goes to zero [5]), the result obtained from the cranking model may express a spurious effect due to the above shortcoming; the calculated pairing gap may go to zero for a different value of I when this spurious band mixing is carried out. When the self-consistent calculation is performed by allowing many deformation parameters to vary, some essential difficulties which are inherent in the model are often difficult to see, since it is not clear whether the feature of the calculated result comes from changing of the deformation parameters or from a shortcoming in the model.

In the present paper we investigate a possible result caused by this shortcoming which is inherent in the cranking model, describe a way of avoiding it, and discuss the picture of the band crossing obtained. Having in mind the neutron shell in rare-

earth nuclei, we investigate the properties of doubly even nuclei and the effect of spin alignment of the particles which are in the $i_{\frac{13}{2}}$ orbit. Namely, the particles in the $i_{\frac{13}{2}}$ orbit are considered to express the intrinsic degree of freedom and are explicitly treated. In order to see a shortcoming of the cranking model as clearly as possible, the self-consistency requirement of various deformation parameters is not carried out in the present paper. First, the Hartree-Fock-Bogoliubov cranking Hamiltonian for given deformation parameters is solved. We clearly see a strange result when a band crossing (or gapless superconductivity) occurs. Then we use a method of choosing a set of the two basis states for which the interaction between them is ω-independent. Using the two basis states, we diagonalize the particle-rotor model Hamiltonian. When the interaction between the two basis states is ω-independent, we can carry out the diagonalization of the Hamiltonian for a given I. The result of the diagonalization can be considered to express the crossing of the two physical bands, which have very different spin alignments from each other.

In sect. 2 we show a shortcoming of the cranking model in the description of the crossing of the bands having very different spin alignments from each other. In sect. 3 we describe a more reliable way of treating the band crossing and compare the result with that of the cranking model obtained in sect. 2. The conclusion and discussion are given in sect. 4.

2. A shortcoming of the cranking model

Our cranking Hamiltonian

$$\begin{aligned} H^{\mathrm{HFBC}} &\equiv H_{\mathrm{intr}} - \omega J_x \\ &= \sum_{\alpha} (\varepsilon_{\alpha} - \lambda) a_{\alpha}^{+} a_{\alpha} + \tfrac{1}{2}\Delta \sum_{\alpha\beta} \delta(\bar{\alpha}, \beta)(a_{\alpha}^{+} a_{\beta}^{+} + a_{\beta} a_{\alpha}) - \omega \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle a_{\alpha}^{+} a_{\beta}, \end{aligned} \tag{1}$$

consists of the single-particle energy term, the pairing potential term and a term which expresses the Coriolis and centrifugal force of the rotating coordinate system. In our present case the quantum number α represents $\{j = i_{\frac{13}{2}}, m\}$ where $m = \frac{13}{2}, \frac{11}{2}, \ldots -\frac{13}{2}$.

The single-particle energy term is assumed to be

$$\varepsilon_{\alpha} = \kappa \frac{3m^2 - j(j+1)}{j(j+1)}. \tag{2}$$

The magnitude of κ in a realistic case may be estimated [6]) in a way such as the following: The nuclear one-body potential $V(r, \theta)$ may be written as

$$V(r, \theta) = V_0 f(r) + V_2(r) P_2(\cos\theta). \tag{3}$$

Then

$$\begin{aligned} \langle V_2(r) \rangle &\approx -\tfrac{2}{3}\delta \left\langle R_0 V_0 \frac{\mathrm{d}f}{\mathrm{d}r} \right\rangle \\ &\approx -\tfrac{2}{3}\delta(50)\ \mathrm{MeV} \\ &= -\tfrac{100}{3}\delta\ \mathrm{MeV}. \end{aligned} \tag{4}$$

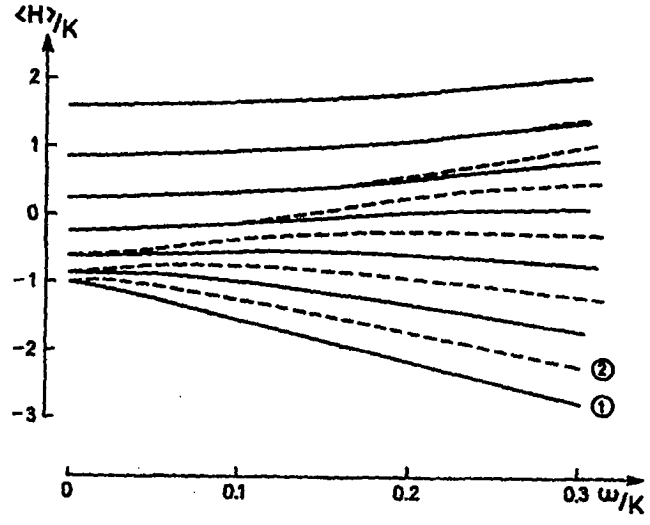

Fig. 1a. The result of the diagonalization of the single-particle Hamiltonian (6) for $\alpha \equiv \{j = i_{\frac{1}{2}}, m\}$. The solid lines are from the symmetric space ($|\alpha\rangle + |-\alpha\rangle$), while the dotted lines are from the anti-symmetric space ($|\alpha\rangle - |-\alpha\rangle$).

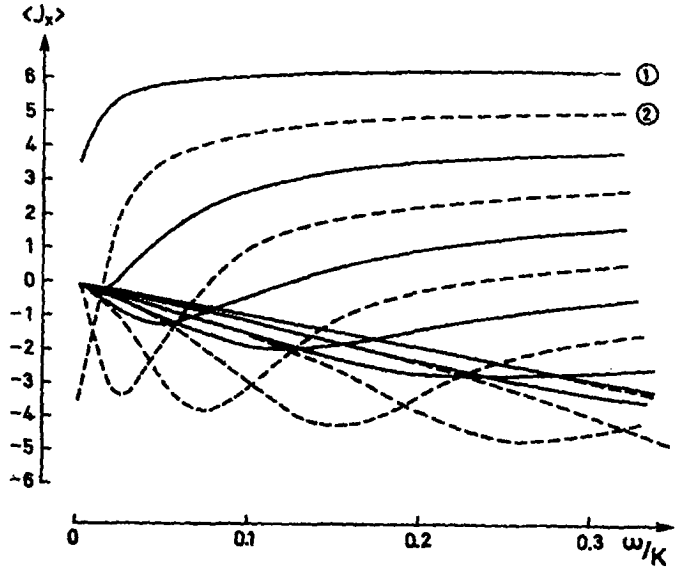

Fig. 1b. The expectation values of the operator J_x in the single-particle states shown in fig. 1a. See caption to fig. 1a.

From eqs. (2)–(4) one gets

$$\kappa = -\tfrac{1}{4}\langle V_2(r)\rangle$$
$$\approx \tfrac{2}{3}\delta \text{ MeV}. \tag{5}$$

In the following we will use κ as an energy unit, which is 2–2.5 MeV in a practical case.

First, it is instructive to see the properties of the solutions of the Hamiltonian without the pairing potential term in (1):

$$H = \sum_{\alpha} \varepsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} - \omega \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle a_{\alpha}^{+} a_{\beta}. \tag{6}$$

The result of the diagonalization of this Hamiltonian (6) is shown in fig. 1a, while the expectation value of the operator j_x in the resulting single-particle states is shown in fig. 1b. In the diagonalization procedure, the $(2j+1)$-dimensional space is separated [2]) into the space containing the symmetric combination $|\alpha\rangle + |-\alpha\rangle$ and the space containing the anti-symmetric combination $|\alpha\rangle - |-\alpha\rangle$, where α expresses $\{j, m\}$ while $-\alpha$ means $\{j, -m\}$. These two spaces do not couple each other and both have the dimension $j+\frac{1}{2}$. In fig. 1 the solutions of the symmetric space are shown by the solid line, while those of the anti-symmetric space are shown by the dotted line. We see that the single-particle states, which are energetically lowest lying, have a large spin alignment immediately after the rotation is switched on. This immediate alignment is a characteristic feature of the large j-orbit such as $j = i_{\frac{13}{2}}$.

By carrying out a Bogoliubov transformation [7])

$$a_{\alpha} = \sum_{i} (A_{\alpha}^{i} b_i + B_{\alpha}^{i} b_i^{+}),$$
$$b_i^{+} = \sum_{\alpha} (A_{\alpha}^{i} a_{\alpha}^{+} + B_{\alpha}^{i} a_{\alpha}), \tag{7}$$

and requiring that the coefficients of the $b_i^{+} b_j^{+}$ (or $b_j b_i$) term should vanish and that the $b_i^{+} b_j$ term should be diagonal, one gets the set of Hartree-Fock-Bogoliubov equations

$$(\varepsilon_{\alpha} - \lambda) A_{\alpha}^{i} + \Delta B_{\alpha}^{i} - \omega \sum_{\beta} \langle \alpha | j_x | \beta \rangle A_{\beta}^{i} = E_i A_{\alpha}^{i},$$
$$-(\varepsilon_{\alpha} - \lambda) B_{\alpha}^{i} - \Delta A_{\alpha}^{i} + \omega \sum_{\beta} \langle \beta | j_x | \alpha \rangle B_{\beta}^{i} = E_i B_{\alpha}^{i} \tag{8}$$

The set in eq. (8) can be written in the form [2])

$$E_i(A_{\alpha}^{i} \pm A_{-\alpha}^{i}) = (\varepsilon_{\alpha} - \lambda)(A_{\alpha}^{i} \pm A_{-\alpha}^{i}) \mp \Delta s_{\alpha}(B_{\alpha}^{i} \mp B_{-\alpha}^{i}) - \omega \sum_{\beta} \langle \alpha | j_x | \beta \rangle (A_{\beta}^{i} \pm A_{-\beta}^{i}),$$
$$E_i(B_{\alpha}^{i} \pm B_{-\alpha}^{i}) = -(\varepsilon_{\alpha} - \lambda)(B_{\alpha}^{i} \pm B_{-\alpha}^{i}) \pm \Delta s_{\alpha}(A_{\alpha}^{i} \mp A_{-\alpha}^{i}) + \omega \sum_{\beta} \langle \alpha | j_x | \beta \rangle (B_{\beta}^{i} \pm B_{-\beta}^{i}), \tag{9}$$

where

$$s_{\alpha} \equiv (-1)^{j+m}.$$

Thus, the equations for a set (a) in which $A^l_\alpha = A^l_{-\alpha}$ and $B^l_\alpha = -B^l_{-\alpha}$ do not couple with the equations for a set (b) in which $A^l_\alpha = -A^l_{-\alpha}$ and $B^l_\alpha = B^l_{-\alpha}$. The dimension of the matrix to be diagonalized is $2j+1$ for each set. For a given solution in a set (a) there is always the corresponding solution in a set (b) which is obtained by exchanging A^l_α with B^l_α and changing the sign of E_l. Thus, practically, it is sufficient to solve only the equations for a set (a).

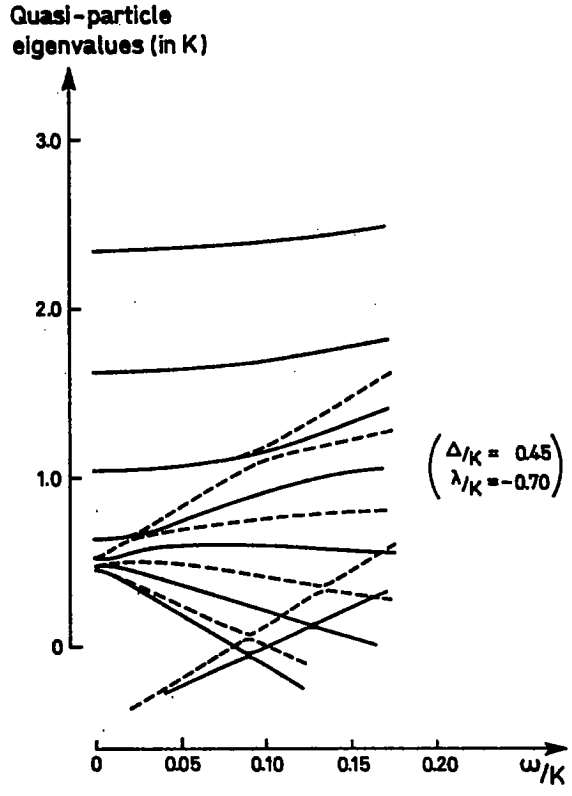

Fig. 2. The quasi-particle eigenvalues E_l obtained from solving (9). The solid lines express the eigenvalues for the set (a), while the dotted lines show those for the set (b). See text.

Some of the quasi-particle eigenvalues E_l are shown in fig. 2. The eigenvalues obtained from the set (a) are shown by a solid line, while those from the set (b) are drawn by a dotted line. For a certain value of ω (e.g., $0.08 \lesssim \omega/\kappa \lesssim 0.12$ in the case of fig. 2) the situation occurs in which the set (a) has $(j+\frac{1}{2})+1$ negative eigenvalues. That is, one eigenvalue in set (a) which was positive at $\omega = 0$ becomes negative in this region of ω. This situation is often called gapless superconductivity. However, as is seen from the consideration of the symmetry, the situation in which both sets (a) and (b) have the $(j+\frac{1}{2})+1$ negative eigenvalues will never occur.

Since, as a total, we have the $2(2j+1)$ quasi-particle eigenvalues, we have to choose the $2j+1$ "physical" quasi-particle orbits. For $\omega = 0$, it is clear that the $2j+1$ "physical" quasi-particle orbits should consist of the $j+\frac{1}{2}$ quasi-particle orbits in set (a) and the $j+\frac{1}{2}$ quasi-particle orbits in set (b), all of which have positive quasi-particle eigenvalues E_l. Or, in other words, the "physical" vacuum state for $\omega = 0$ is obtained by filling the lowest-lying $j+\frac{1}{2}$ quasi-particle orbits both in set (a) and in set (b), all of which have negative quasi-particle eigenvalues E_l. We define the "physical" vacuum state for any ω as the state obtained by filling the quasi-particle orbits in this way. Then, it is seen that the energetically lowest-lying two-quasi-particle state in the region of ω in which we are now interested is constructed by filling the lowest-lying $(j+\frac{1}{2})+1$ quasi-particle in both set (a) and set (b). This two-quasi-particle state, which we call the two-quasi-particle (s-a) state in the following, has a large spin alignment for $\omega/\kappa \lesssim 0.08$, as is already seen from the large negative slopes of the lowest-lying two positive quasi-particle eigenvalues in fig. 2.

Considering that we are now explicitly treating only the particles in the $i_{\frac{13}{2}}$ orbit, we consider $\langle H_{\text{intr}}\rangle + \frac{1}{2}\mathcal{I}_c\,\omega^2$ as a total nuclear energy and plot it as a function of the angular momentum $I \equiv \omega\mathcal{I}_c + \langle J_x\rangle_{i_{\frac{13}{2}}}$. Here the moment of inertia of the core is expressed by $\mathcal{I}_c$, which is a ω-independent parameter in the present model. The

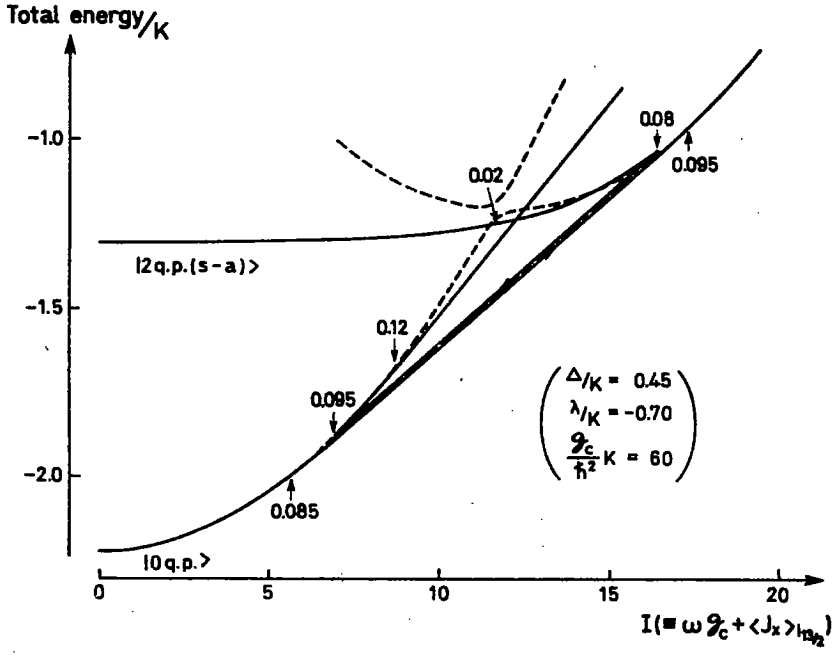

Fig. 3. The total nuclear energy as a function of I in the cranking model. The arrows on the straight line show explicitly the direction of increasing ω. The ω/κ value is shown by figures at several points, for the states $|0\text{q.p.}\rangle$ and $|2\text{q.p.(s-a)}\rangle$. The dotted lines are the result of the diagonalization performed in sect. 3 and are taken from fig. 7 after shifting the total energies by about 0.04κ. See text.

expectation value of H_{intr} in (1), $\langle H_{\text{intr}} \rangle$, or the expectation value of J_x, $\langle J_x \rangle_{I_{\frac{1}{2}}}$, is calculated by using the wave function of the quasi-particle vacuum state and the two-quasi-particle (s-a) states described above.

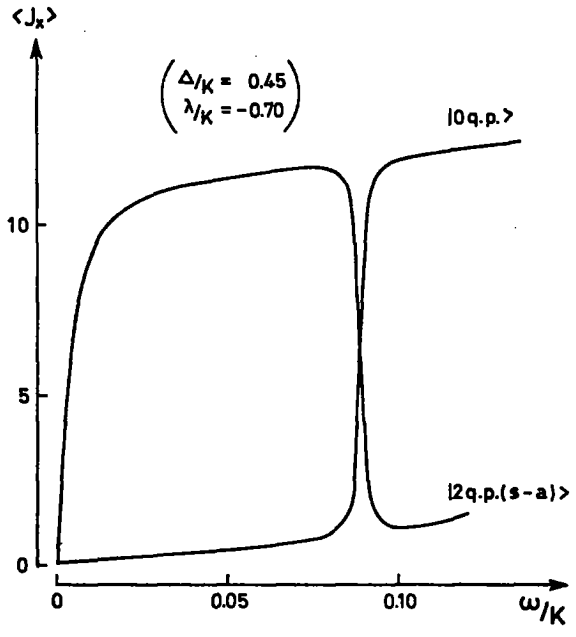

Fig. 4. The expectation values $\langle J_x \rangle$ as a function of ω.

A typical example of the calculated result is shown in fig. 3. The arrow in fig. 3 shows the increasing direction of ω. From fig. 2 one can see that the exchange of the characters between the quasi-particle vacuum state and the two-quasi-particle (s-a) state is effective around $\omega/\kappa \approx 0.09$. For $7 \lesssim I \lesssim 17$ the energy of the quasi-particle vacuum state in fig. 3 is a straight line which corresponds to the region of $0.085 \lesssim \omega/\kappa \lesssim 0.095$, while for the same region of ω/κ and I the energy of the two-quasi-particle (s-a) state is a straight line which is a decreasing function of ω. It is seen that in this region of ω/κ the two bands interact with one another and exchange their properties. The expectation values $\langle J_x \rangle_{I_{\frac{1}{2}}}$ are plotted in fig. 4. In this region of ω/κ the two-quasi-particle (s-a) state loses the spin alignment so rapidly that the total angular momentum I is a decreasing function of ω although $\omega\mathcal{J}_0$ is an increasing function of ω. It is clear that this band mixing indicated by the straight line in fig. 3 is a strange mixing between the two bands which have very different angular momentum from each other.

The fact that the part of the straight line in the total nuclear energy curve is not reliable can be seen, for example, by estimating the angular momentum fluctuation

$\langle J_x^2\rangle - \langle J_x\rangle^2$, which is shown in fig. 5. It is a condition for the validity of the cranking model to have a constant fluctuation, so that the width of the wave packet in the J_x space should not depend strongly on ω. The angular momentum fluctuation of both two bands becomes enormous in the region of $0.085 \lesssim \omega/\kappa \lesssim 0.095$. Thus, this enormous fluctuation can be taken as a direct evidence of the breakdown of the validity of the cranking model in this region of ω.

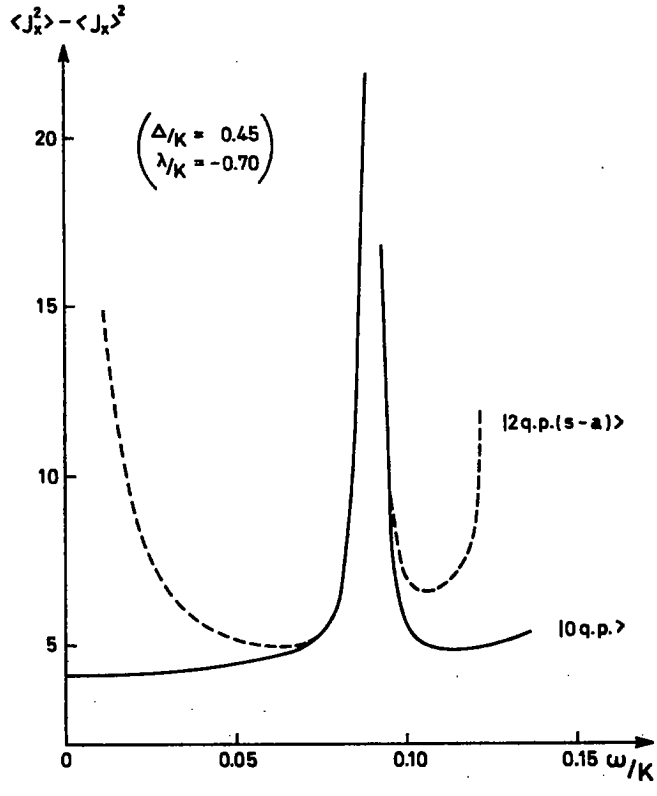

Fig. 5. The estimated angular momentum fluctuation, $\langle J_x^2\rangle - \langle J_x\rangle^2$. The solid line is for the state $|0\text{q.p.}\rangle$, while the dotted line is for the state $|2\text{q.p.(s-a)}\rangle$.

From fig. 5 we see also that the total energy curve of the two-quasi-particle (s.a.) state in the region of both $\omega/\kappa \lesssim 0.02$ and $\omega/\kappa \gtrsim 0.12$, which is again a straight line in fig. 3, cannot be trustworthy. This straight line shows again an exchange of the characters between the two-quasi-particle (s-a) state with other states, due to a spurious mixing. In the present paper we will not discuss this mixing.

It is noted that, if one solves automatically the Hartree-Fock-Bogoliubov cranking model Hamiltonian without noticing this strange mixing of the two bands, the solution (or the energy spectra) obtained would certainly be lying on this rather "unphysical" straight line.

3. A more exact treatment of the band crossing

In the present section we try to avoid the short-coming inherent in the cranking model described in the previous section, and get a better description of the picture of the band crossing.

For this purpose, first, we fix the space spanned by the two states [i.e. the quasi-particle vacuum state and the two-quasi-particle (s-a) state] for a given $\omega = \omega^{(0)}$ which is around the rotational frequency at which the band crossing occurs.

In terms of the $2j+1$ "physical" quasi-particle operators (b_i^+, b_i), the Hamiltonian (1) can be written as

$$\begin{aligned} H^{\mathrm{HFBC}} &= \sum_\alpha (\varepsilon_\alpha - \lambda) \sum_i (B_\alpha^i)^2 + \Delta \sum_\alpha \sum_i A_\alpha^i B_\alpha^i - \omega \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle \sum_i B_\alpha^i B_\beta^i \\ &+ \sum_{ij} b_i^+ b_j \{ \sum_\alpha (\varepsilon_\alpha - \lambda)(A_\alpha^i A_\alpha^j - B_\alpha^j B_\alpha^i) \\ &+ \tfrac{1}{2}\Delta \sum_{\alpha\beta} \delta(\bar{\alpha}, \beta)(A_\alpha^i B_\beta^j - B_\alpha^j A_\beta^i - A_\beta^j B_\alpha^i + B_\beta^i A_\alpha^j) - \omega \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle (A_\alpha^i A_\beta^j - B_\alpha^j B_\beta^i) \} \\ &+ \sum_{ij} (b_i^+ b_j^+ + b_j b_i) \{ \sum_\alpha (\varepsilon_\alpha - \lambda) A_\alpha^i B_\alpha^j \\ &\qquad + \tfrac{1}{2}\Delta \sum_{\alpha\beta} \delta(\bar{\alpha}, \beta)(A_\alpha^i A_\beta^j + B_\beta^i B_\alpha^j) - \omega \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle A_\alpha^i B_\beta^j \}. \end{aligned} \tag{10}$$

If by the index $m = 1$ $[m = 2]$ we express the "physical" quasi-particle operator which has the lowest eigenvalue in the set (a) [(b)], and write the part of H^{HFBC} which contains only the operators b_m^+ and b_m with $m = 1, 2$, then

$$\begin{aligned} (H^{\mathrm{HFBC}})_m &= \sum_{mm'} (b_m^+ b_{m'}^+ + b_{m'} b_m)(\omega^{(0)} - \omega) \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle A_\alpha^m B_\beta^{m'} \\ &+ \sum_{mm'} b_m^+ b_{m'} [E_m^{(0)} \delta(m, m') + (\omega^{(0)} - \omega) \sum_{\alpha\beta} \langle \alpha | j_x | \beta \rangle (A_\alpha^m A_\beta^{m'} - B_\alpha^{m'} B_\beta^m)] \\ &\qquad (m, m' = 1, 2), \end{aligned} \tag{11}$$

where both the operators b_m^+ etc. and the amplitudes A_α^m etc. should be interpreted as those determined at $\omega = \omega^{(0)}$ by solving the Hartree-Fock-Bogoliubov equations (9). In writing the expression (11) we made the assumption that in the small region of ω, which we are now interested in, the ω-dependence of the operators A_α^m, etc., which are the solution of the Hartree-Fock-Bogoliubov equations is negligible. That is, the assumption that in the small region of ω the space spanned by the two states [i.e. the quasi-particle vacuum state and the two-quasi-particle (s-a) state] is the same as the corresponding space determined at $\omega = \omega^{(0)}$. Although this assumption is certainly an approximation, it may not be so bad for the present purpose in which we want to treat the band crossing problem quantitatively.

Now, because of the symmetry expressed by eq. (9), there is no interaction between the "physical" quasi-particles $m = 1$ and $m = 2$. Instead, the interaction which is responsible for the band crossing can be expressed as an interaction between

the "physical" quasi-particle with $m = 1$ and the "unphysical" quasi-particle of which the creation (annihilation) operator is equal to the annihilation (creation) operator of the "physical" quasi-particle with $m = 2$. The role played by this "unphysical" quasi-particle, which will be denoted by $m = 2'$ in the following, can be easily seen by looking at fig. 2 around $\omega/\kappa = 0.09$. Since in our approximation the Hamiltonian (11) contains only the linear dependence on ω, we can get a representation in which the ω-dependence of the non-diagonal terms should vanish, by using the transformation

$$b^+_{m=1} = \sum_{i=1,2} D^i_{m=1} c^+_i,$$

$$b^+_{m=2'} \equiv b_{m=2} = \sum_{i=1,2} D^i_{m=2'} c^+_i. \tag{12}$$

In order to get the new representation (c^+_i and c_i), the transformation coefficients D^i_m are determined by the requirement that the coefficient of the term $\omega c^+_i c_j$ should be equal to $e_i\delta(i,j)$. Then, for example, the part of $(H^{\mathrm{HFBC}})_m$ which contains the term $c^+_i c_i$ $(i = 1, 2)$ can be written as

$$\sum_i c^+_i c_i \{(D^i_1)^2[E^{(0)}_1 + \omega^{(0)} \sum_{\alpha\beta} \langle\alpha|j_x|\beta\rangle (A^1_\alpha A^1_\beta - B^1_\alpha B^1_\beta)]$$

$$-(D^i_{2'})^2[E^{(0)}_2 + \omega^{(0)} \sum_{\alpha\beta} \langle\alpha|j_x|\beta\rangle (A^2_\alpha A^2_\beta - B^2_\alpha B^2_\beta)]$$

$$+2D^i_1 D^i_{2'} \omega^{(0)} \sum_{\alpha\beta} \langle\alpha|j_x|\beta\rangle (A^1_\alpha B^2_\beta - A^2_\alpha B^1_\beta) + \omega e_i\} \equiv \sum_i \mathscr{E}_i c^+_i c_i. \tag{13}$$

The calculated value $\mathscr{E}_i$ as a function of ω is shown in fig. 6 by the two straight lines, which fits nicely the asymptotic behaviour of the two relevant curves. This fitting for a remarkable wide range of the ω-value shows that the assumption made in writing the expression (11) was a good approximation at least for the present discussion of the energies of the crossing bands.

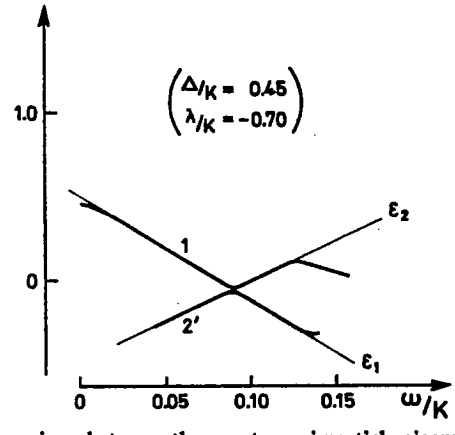

Fig. 6. Comparison between the exact quasi-particle eigenvalues with $m = 1, 2'$ and the values $\mathscr{E}_i$ which appear in expression (13).

Now, the two independent new basis states are

$$|0\rangle, \qquad \alpha_1^+\alpha_2^+|0\rangle, \tag{14}$$

by defining

$$\alpha_1^+ \equiv c_1^+,$$
$$\alpha_2^+ \equiv c_2, \tag{15}$$

where the vacuum state $|0\rangle$ was defined as

$$\alpha_1|0\rangle = 0, \qquad \alpha_2|0\rangle = 0,$$
$$b_i|0\rangle = 0 \qquad \text{for } i = 3, 4, \ldots, 2j+1. \tag{16}$$

It should be noted that the interaction between the two basis states (14) is ω-independent.

In this stage we introduce the particle-rotor Hamiltonian

$$H^{\mathrm{PR}} \equiv H_{\mathrm{intr}} + \frac{\hbar^2}{2\mathscr{I}_0}(I-J_x)^2, \tag{17}$$

where H_{intr} is defined in (1). The diagonalization of the Hamiltonian (17) is carried out in the space of the two basis states in (14). The non-diagonal matrix element of J_x vanishes, which corresponds to the fact that the interaction between the two states in (14) is I-independent. The matrix elements of J_x^2 should be explicitly calculated and they should not be replaced by the square of the matrix elements of J_x.

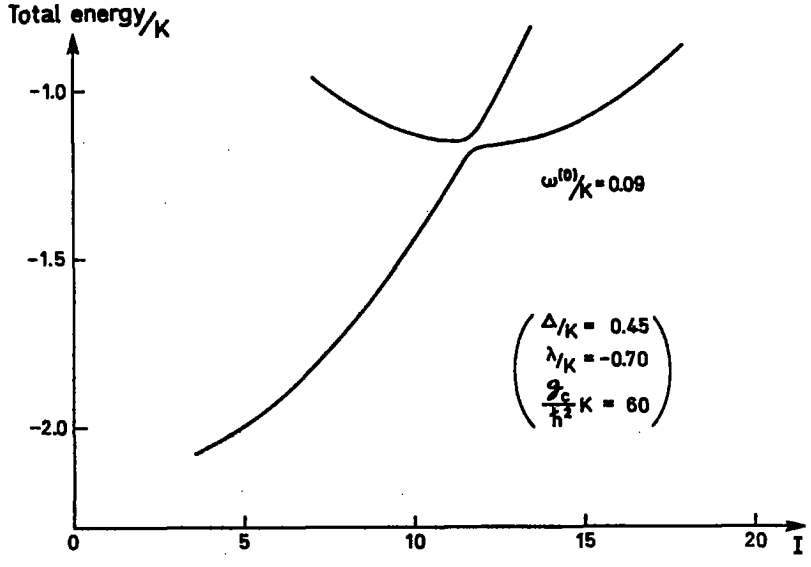

Fig. 7. The result of the diagonalization performed in sect. 3, which is expected to express the crossing of the two physical bands, having very different spin alignment from each other.

An example of the result of the diagonalization is shown in fig. 7, where $\omega^{(0)}/\kappa = 0.09$ was chosen. For reference, we show here the related matrices before the diagonalization:

$$(H_{\mathrm{intr}}) = \begin{pmatrix} -1.198 & 0.012 \\ 0.012 & -2.184 \end{pmatrix}, \qquad (J_x) = \begin{pmatrix} 11.888 & 0 \\ 0 & 0.814 \end{pmatrix},$$

$$(\langle J_x^2\rangle - \langle J_x\rangle^2) = \begin{pmatrix} 4.476 & 1.127 \\ 1.127 & 5.190 \end{pmatrix},$$

where the matrix elements of H_{intr} are expressed in the unit of κ. In fig. 7 we can see that the picture of the band crossing is nicely drawn. In addition, the interaction matrix element between the two crossing bands is seen to be very small. This smallness of the interaction matrix element is a feature of the lowest-lying band crossing in the yrast line, where the two bands which have very different spin alignments from each other cross. For a set of reasonable parameters (Δ, λ, κ, $\mathscr{I}_0$) corresponding to realistic cases, the magnitude of the calculated interaction matrix element in the present model is hardly larger than 100 keV.

The calculated energy curves drawn in fig. 7 are found to be rather insensitive to the $\omega^{(0)}$ value chosen. In fact, as can be guessed also from the graph in fig. 6, the variation of the $\omega^{(0)}/\kappa$ value between 0.03 and 0.12 gives almost negligible difference to the energy curves in fig. 7. This insensitivity may be a feature of the energy curves and cannot always work for other quantities, since the properties of actual nuclear states are continuously changing as a function of ω.

The curves in fig. 7 are drawn as dotted lines in fig. 3 after adding about 0.04κ to the total energies of both bands. The shifting of the total energies by this amount is reasonable when we want to compare the total energies shown in figs. 3 and 7, since in fig. 3 the quantity $\langle H_{\mathrm{intr}}\rangle + \frac{1}{2}\mathscr{I}_0(I - \langle J_x\rangle_{\frac{1}{2}})^2/\mathscr{I}_0)^2$ was plotted. Namely, a reliable part of the curves in fig. 3 would correspond to the curves in fig. 7, if the relation $\langle J_x^2\rangle = \langle J_x\rangle^2$ holds, which is not true in the model used in fig. 7.

The comparison shown in fig. 3 between the solid lines obtained from the cranking model and the dotted lines obtained in the present section indicated clearly that the part of the straight line in fig. 3 is meaningless. From the comparison we can also see that it is important to look at not only the ground state but also the behaviour of the excited states in the Hartree-Fock-Bogoliubov cranking model.

4. Conclusion and discussion

We have pointed out that the result of the Hartree-Fock-Bogoliubov cranking model contains a large inaccuracy in the description of the nuclear system at the rotational frequency where the two bands cross, which have very different spin-alignments from each other. This inaccuracy comes from the fact that the two bands should interact for a given angular momentum I while the cranking model considers

the mixing of the two bands for a given ω. The angular momenta which correspond to a given ω are very different in the two bands which have very different spin alignment from each other. This inaccuracy can be easily seen if one estimates the fluctuation of the angular momentum, $\langle J_x^2\rangle - \langle J_x\rangle^2$, as a function of I, and it appears as a straight line in the graph of the total energy as a function of I, if the self-consistency calculation of the deformation parameters is not carried out.

In sect. 3 we have tried to choose the two new basis states which are eigenstates of the Hamiltonian $H^{\mathrm{HFBC}} - h$ where h is ω-independent. By using these new basis states we have diagonalized the Hamiltonian of the particle-rotor model. We have got a clear picture of the band crossing which is in good agreement with the result of the cranking model in the region outside of the straight line, and which shows the spuriousness of the part of the straihgt line obtained by the cranking model shown in fig. 3.

When the set of the parameters which corresponds approximately to the case calculated by Stephens and Simon [4]) is chosen, we have found that the magnitude of the interaction between the two bands (i.e. the ground band and the excited band which has a large spin alignment) is estimated to be less than 100 keV, which is surprisingly small compared with the calculated result of ref. [4]). In the present model the contributions to the interaction matrix element come from both H_{intr} and $(\hbar^2/2\mathscr{I}_0)J_x^2$. The magnitudes of the two contributions are usually comparable for a set of realistic parameters $(\Delta, \lambda, \kappa, \mathscr{I}_0)$. In ref. [4]) the contribution coming from the recoil term, $(\hbar^2/2\mathscr{I}_0)J_x^2$, was not taken into account. Further, the basis states in ref. [4]) were chosen by using the BCS quasi-particle bases which are eigenstates for $\omega = 0$, although, for example, the influence of the inclusion of some BCS four-quasi-particle states was estimated. We feel that the difference of the magnitude of the interaction obtained in ref. [4]) from the one in our calculation may come partly from the choice of those basis states in ref. [4]).

The next interesting band crossing along the yrast line will occur around $I \gtrsim 30$ for a set of the parameters in fig. 3. This band crossing involves the intrinsic state which has spin-alignment $\langle J_x\rangle \approx \frac{13}{2}+\frac{11}{2}+\frac{9}{2}+\frac{7}{2} = 20$.

In the present paper we have not carried out the self-consistency calculation of the deformation parameters, since we wanted to point out the shortcoming of the cranking model as clearly as possible. The variation of the shape-deformation parameters may not be so important for the yrast states with $I \lesssim 30$. However, the variation of the pairing gap (namely, the deformation of the pairing potential) along the band-crossing line would presumably be more interesting and important. This dependence of the pairing gap on ω (or I) will be published in a forthcoming paper.

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Note added in proof: The author was informed by F. S. Stephens that the numerical calculations given in ref. [4]) contained an error and that a correct calculation performed along the lines of ref. [4]) gave a very small interaction matrix element between the two bands (namely, the ground band and the excited band which has a large spin alignment). The smallness is consistent with the result of the present paper.

References

1) P. C. Bhargava and D. J. Thouless, Nucl. Phys. A215 (1973) 515;
A. Faessler, K. R. Sandhya Devi, F. Grümmer, K. W. Schmid and R. R. Hilton, Nucl. Phys. A256 (1976) 106;
B. Banerjee, H. J. Mang and P. Ring, Nucl. Phys. A215 (1973) 366
2) A. L. Goodman, Nucl. Phys. A230 (1974) 466
3) A. L. Goodman, preprint
4) F. S. Stephens and R. S. Simon, Nucl. Phys. A183 (1972) 257
5) B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5 (1960) 511
6) A. Bohr and B. R. Mottelson, Nuclear structure, vol. 2, (Benjamin, New York, 1975)
7) N. N. Bogoliubov, Sov. Phys. Usp. 2 (1959) 236