

UNIVERSITY OF BUCHAREST

**Semiclassical and Boson  
Descriptions of the Wobbling  
Motion in Odd-A Nuclei**

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# *Abstract*

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# Chapter 1

## Introduction

Ground-state nuclear shapes with spherical symmetry or axial symmetry are predominant across the chart of nuclides. Near closed shells, the deformation is indeed sufficient that models based on spherical symmetries can be used to describe nuclear properties (e.g., energies, quadrupole moments, and so on). Besides the spherical and axially-symmetric shapes, the existence of triaxial nuclear deformation was theoretically predicted a long time ago [1]. The rigid triaxiality of nuclei is defined by the asymmetry parameter  $\gamma$ , giving rise to unique quantum phenomena (this parameter will be characterized later on). The quantum mechanical properties of the rigid triaxial shapes drew a lot of attention within the nuclear physics community lately, since the description of nuclear properties for the deformed nuclei represents a great challenge from both an experimental and a theoretical standpoint (e.g., great progress for the experimental evidence of strong nuclear deformation has only been possible after the 2000s). It is worth mentioning that some experiments concerning alpha- $\alpha$  particle reactions induced in heavy nuclei in the early 1960s (e.g., [2]) helped to produce decent amount of data related to the rotational in the high-spin region ( $\geq 20\hbar$ ). Within the experimental studies made by Morinaga et. al., the alpha reactions which were induced in the nuclei were generated by the formation of a so-called *compound nucleus*. This system may exist at a large spin value due to the absorption of the angular momentum from the incident particle (i.e., spin values up to  $\approx 25 \hbar$  can be obtained from a 50 MeV alpha particle energy - relative to the target nucleus [2]).

The physics of *high-spin* states have been studied from the early 1950s, with the major breakthrough on the theoretical side made by Bohr and Mottelson [1].

The elusive properties of nuclear rotation were described in terms of the rotational degrees of freedom associated with other nuclear degrees of freedom (e.g., particle-vibration, quadrupole-quadrupole, parity, and so on).

# Chapter 2

## Deformed Nuclei

### 2.1 Nuclear deformation

Most of the nuclei across the nuclide chart are spherical or symmetric in their ground state. Moreover, for the axially symmetric nuclei (i.e, either *oblate* or *prolate*), there is a prolate over oblate dominance.

The spherical shell model only describes nuclei near the closed shells. On the other side, for the nuclei that lie far from closed shells, a deformed potential must be employed.

In the case of even-even nuclei, unique band structures resulting from the vibrations and rotations of the nuclear surface (as proposed by Bohr and Mottelson [1] in the *Geometric Collective Model* - GCM) appear in the energy range 0-2 MeV.

Within the GCM, the nucleus is described as a classical charged liquid drop. For the low-lying energy spectrum, usually, the compression of nuclear matter and the nuclear skin thickness are neglected. This results in the final picture of a liquid drop with a constant nuclear density and a sharp surface [3].

#### 2.1.1 Collective coordinates

The nuclear surface can be described via an expansion of the spherical harmonic functions with some time-dependent parameters as *expansion coefficients*. The

expression of the nuclear shape is shown below [3]:

$$R(\theta, \varphi, t) = R_0 \left( 1 + \sum_{\lambda=0}^{\infty} \sum_{-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y_{\lambda}^{\mu}(\theta, \varphi) \right) . \quad (2.1)$$

In 2.1,  $R$  denotes the nuclear radius as a function of the spherical coordinates  $\theta, \varphi$  expressing the direction, and the time  $t$ , while  $R_0$  is the radius of the spherical nucleus when all the expansion coefficients vanish. It is worth mentioning that the expansion coefficients  $\alpha_{\lambda\mu}$  act as *collective coordinates* since the time-dependent amplitudes describe the vibrations of the nuclear surface.

### 2.1.2 Nuclear radius under rotation

To get a grasp at the physical meaning behind the deformation parameters that are used to describe the nuclear surface, it is instructive to see what happens when the system undergoes a rotation transformation.

The function  $R(\theta, \varphi)$  describes the original (non-rotated) nuclear shape. Rotating the system will result in the change of the angular coordinates  $(\theta, \varphi)$  to  $(\theta', \varphi')$ , which will correspond to a new function  $R'(\theta', \varphi')$ . Moreover, both nuclear surfaces (i.e., the non-rotated and the rotated one) must hold the equality:

$$R'(\theta', \varphi') = R(\theta, \varphi) \quad (2.2)$$

The rotational invariance of  $R$  employs that  $R'(\theta, \varphi)$  must have the same functional form, but the expansion coefficients  $\alpha_{\lambda\mu}$  must be rotated, meaning:

$$\sum_{\lambda\mu} \alpha'_{\lambda\mu} Y'_{\lambda\mu}(\theta, \varphi) = \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) . \quad (2.3)$$

Note that in Eq. 2.3, the spherical harmonics  $Y'_{\lambda\mu}$  are obtained via the usual rotation matrices. Finally, the invariance of Eq. 2.1 is achieved if the set of parameters  $\alpha_{\lambda\mu}$  transform similarly to a *spherical tensor with angular momentum*  $\lambda$  [4], that is:

$$\alpha'_{\lambda\mu} = \sum_{\mu'} \mathcal{D}_{\mu\mu'}^{(\lambda)} \alpha_{\lambda\mu'} . \quad (2.4)$$



Besides the spherical tensor character, the collective coordinates also have the following properties (emerging from Eq. 2.1):

- Complex Conjugation.

$$Y_{\lambda\mu}^*(\theta, \varphi) = (-1)^\mu Y_{\lambda-\mu}(\theta, \varphi), \quad (2.5)$$

$$\alpha_{\lambda\mu}^* = (-1)^\mu \alpha_{\lambda-\mu} . \quad (2.6)$$

- Parity - the coordinates  $\alpha_{\lambda\mu}$  must undergo the same change of sign under a parity transformation as the spherical harmonics, in order to keep the invariance of the nuclear surface.

$$(r, \theta, \varphi) \xrightarrow{P} (r, \pi - \theta, \pi + \varphi) ,$$

$$Y_{\lambda\mu}(\theta, \varphi) \xrightarrow{P} Y_{\lambda\mu}(\pi - \theta, \pi + \varphi) = (-1)^\lambda Y_{\lambda\mu}(\theta, \varphi) .$$

Therefore, the parity of the expansion coefficients are:

$$\pi(\alpha_{\lambda\mu}) = (-1)^\lambda \quad (2.7)$$

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