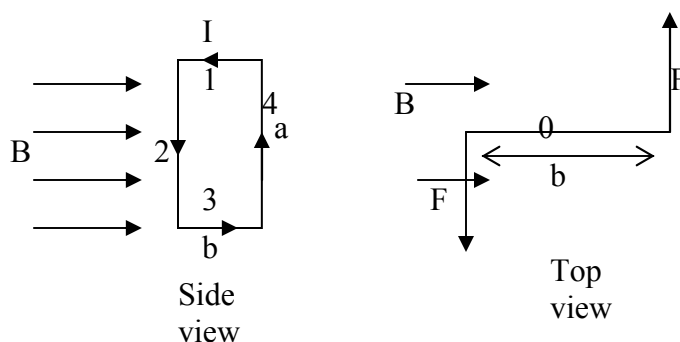


Torque on a Current Loop

We showed that the net magnetic force on a closed current containing loop is zero when it is in a uniform magnetic field. A non-uniform magnetic field can produce a net force, but it is more complex and we will not study it in this course. Here we will look at the torque on the current loop in a uniform magnetic field and show that it will tend to orient the loop in the magnetic field. This will be important in numerous applications including the electric motor and NMR as we will see.

Let's start out with a rectangular loop with dimensions $a \times b$ in a uniform magnetic field B as shown.



Using our basic equation for the magnetic force on a straight wire with current I in a magnetic field B perpendicular to the current, $F_B = ILB$, we have for each segment:

$$F_B \text{ on 1 and 3} = 0$$

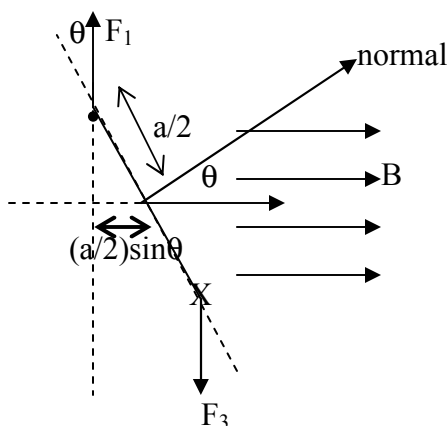
$$F_{B2} = F_{B4} = IaB, \text{ but in opposite directions as shown.}$$

Therefore the net force is zero, but there is a net torque, given about the center point O by

$$\tau_{\text{about } O} = F_{B2}(b/2) + F_{B4}(b/2) = IaB(b) = IAB,$$

where A is the area of the loop, $A = ab$. This is the maximum possible torque of the loop; other orientations of the loop will produce torques of smaller magnitude.

If the loop has its normal not perpendicular to the B field (as was the case above), but rather oriented at some angle θ with respect to the B field, then we have to re-do the calculation from above. Using the top view shown (see also the pictures in the text on p. 676)



The torque about the center of the coil is given by $(IbB)(a/2 \sin \theta) + (IbB)(a/2 \sin \theta) = IAB \sin \theta$

where θ is the angle between the normal (direction of the vector representing the area) and magnetic field. This can be re-written as

$$\tau = I\vec{A} \times \vec{B}$$

where the direction of \vec{A} is along the normal to its area using a right hand rule (right hand fingers curl around the circuit loop along I and your thumb points along the normal)

We introduce the magnetic dipole moment

$$\vec{\mu} = I\vec{A}, \text{ with units of A}\cdot\text{m}^2, \text{ so that}$$

the torque on the current loop can be re-written in a final form as

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

If the coil is made of N loops, each with the same area and current flowing, then the total torque on the coil is given by the same equation, but the magnetic dipole moment (or simply magnetic moment) of the coil is N times that of a single loop:

$$\vec{\mu}_{coil} = N\vec{\mu}_{loop}.$$

For our first application of these new ideas, consider the NMR experiment. The nucleus of atoms has a net positive charge and also has a nuclear spin, an effective magnetic dipole moment of the nucleus due to its spinning protons. This spin is an intrinsic property of the protons in the nucleus. Ordinarily the magnetic dipole moment of any nucleus is randomly oriented in space. Quantum mechanics tells us that when placed in a magnetic field the nucleus will experience a torque due to the interaction of the magnetic dipole with the external magnetic field and this will result in two possible orientations of the nuclear magnetic dipole, said to be spin up and spin down. In an NMR experiment, a very strong magnetic field is used to create this orientation. Next a radio wave signal is used to induce spin flips between the up and down situations. The energy difference between these two orientations is given from the expression for the energy of a magnetic dipole in a B field, $U = -\vec{\mu} \cdot \vec{B}$, so that the difference between a spin up and down nucleus has an energy $\Delta U = 2\mu_B B$. The RF signal must match this energy difference to be absorbed – a condition known as a resonance (hence the R in NMR). A spin flip to a higher energy from a lower energy, eventually results in the re-emission of this energy difference in the form of a photon, or packet of energy, at Radio Frequencies (RF), which can be detected and analyzed. The component of μ along B is a constant, but the local B field at the site of the nucleus may depend on its local environment, since the other atoms nearby may produce small B fields themselves due to their circulating currents. NMR is so sensitive to this that it can measure a signal that tells the numbers of nuclei from atoms in different local environments. The signal looks like a series of peaks at different RF energies, each different energy corresponding to a different type of local environment and the size of the peak telling you how many of these particular nuclei there are. This information can be used to determine the structure of a molecule, or using a more sophisticated imaging system, can be used to see inside the human body.