chiralPotential

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1 Elliptic potential for the triaxial nucleus ^{163}Lu

2 The chiral potential V_{chiral}

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This project is calculating the triaxial potential for the odd-A nucleus ^{163}Pr.

Code

2.1.1 Import modules

[41]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy import special as sp
from matplotlib import rc
```

2.1.2 Define the single-particle angular momentum components

$$\mathbf{j} = \{j_1, j_2, j_3\}$$

with

$$j_1 = j\cos\theta$$

and

$$j_2 = j\sin\theta$$

[42]:
```
def j1(j,theta):
    degrees=theta*np.pi/180.0
    return j*np.cos(degrees)
def j2(j,theta):
    degrees=theta*np.pi/180.0
    return j*np.sin(degrees)
```

2.1.3 Define the elliptic variables A, u, v_0, k

[43]:
```
def aFct(spin,j,theta,a1,a2):
    term1=1-j2(j,theta)/spin
    term2=a2*term1-a1
    return term2

def uFct(spin,j,theta,a1,a2,a3):
    term1=a3-a1
    term2=aFct(spin,j,theta,a1,a2)
    return term1/term2

def v0Fct(spin,j,theta,a1,a2,a3):
    term1=a1*j1(j,theta)
    return -term1/aFct(spin,j,theta,a1,a2)

def kFct(spin,j,theta,a1,a2,a3):
    term=uFct(spin,j,theta,a1,a2,a3)
    return np.sqrt(term)
```

3 THE JACOBI AMPLITUDE CALCULUS

3.1 Define the linear k argument which enters in the elliptic function

[44]:
```
def fiVar(u,k):
    return sp.ellipj(u,k)
```

3.2 Define the quadratic k argument which enters in the elliptic function

[45]:
```
def fiVar_sq(u,k):
    ksq=np.power(k,2)
    return sp.ellipj(u,ksq)
```

3.3 Test the elliptic variables

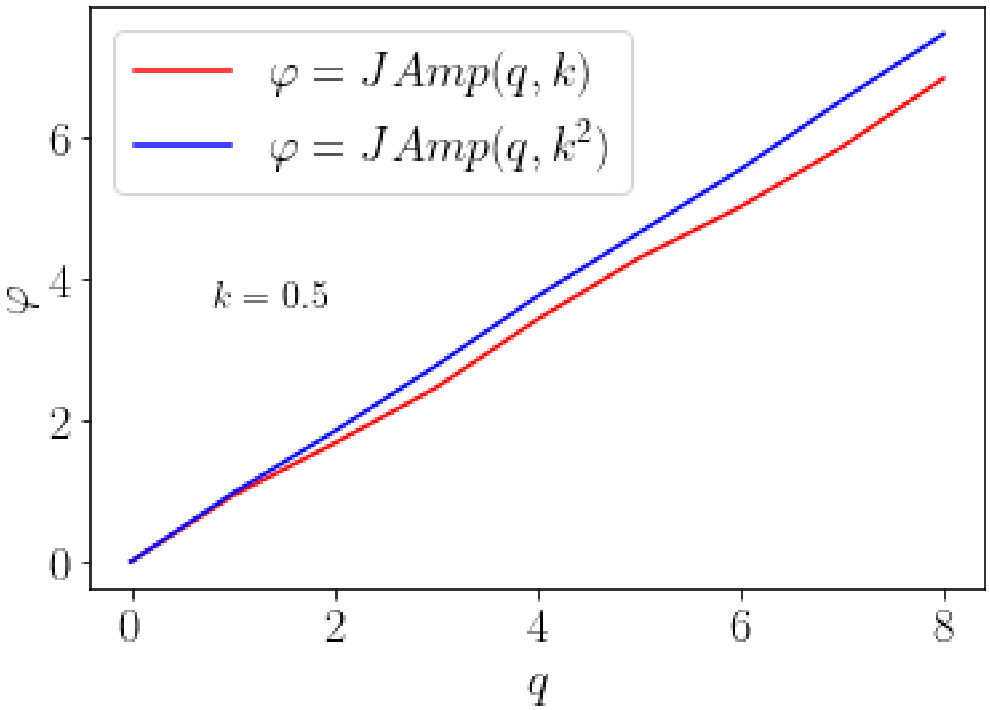
[46]:
```
def printer(u,k):
    ksquared=np.power(k,2)
    fipair=(fiVar(u,k)[3],fiVar(u,ksquared)[3])
    return fipair

qArray=[]
kArray=[]
ksqArray=[]
for i in np.arange(0,9,1.0):
    print(i,printer(i,0.5))
    qArray.append(i)
```

```
    kArray.append(printer(i,0.5)[0])
    ksqArray.append(printer(i,0.5)[1])
```

```
0.0 (0.0, 0.0)
1.0 (0.9323150798838539, 0.9660310526366139)
2.0 (1.6741639220482394, 1.8440491178856986)
3.0 (2.4600021012296027, 2.7721668994372144)
4.0 (3.431410753482452, 3.7605670271403726)
5.0 (4.304620857222439, 4.6628013359701725)
6.0 (5.026814633689735, 5.555423639554034)
7.0 (5.872672281436279, 6.539486453212575)
8.0 (6.851503073148641, 7.4798994265728345)
```

[47]:

```
# plt.rcParams["font.family"] = "Times New Roman"
plt.rcParams.update({'font.size': 18})
## for Palatino and other serif fonts use:
plt.rc('font',**{'family':'serif','serif':['Times New Roman']})
rc('text', usetex=True)
plt.rc('xtick', labelsize=18)
plt.rc('ytick', labelsize=18)
fig, ax=plt.subplots()
ax.plot(qArray,kArray,'r-',label='$\\varphi=JAmp(q,k)$')
ax.plot(qArray,ksqArray,'b-',label='$\\varphi=JAmp(q,k^2)$')
plt.xlabel('$q$')
plt.ylabel('$\\varphi$')
ax.text(0.2, 0.5, '$k=0.5$',␣
↪horizontalalignment='center',verticalalignment='center', transform=ax.
↪transAxes,fontsize=14)
plt.legend(loc='best')
plt.savefig('jacobiAmpl.pdf',bbox_inches='tight')
plt.show()
```

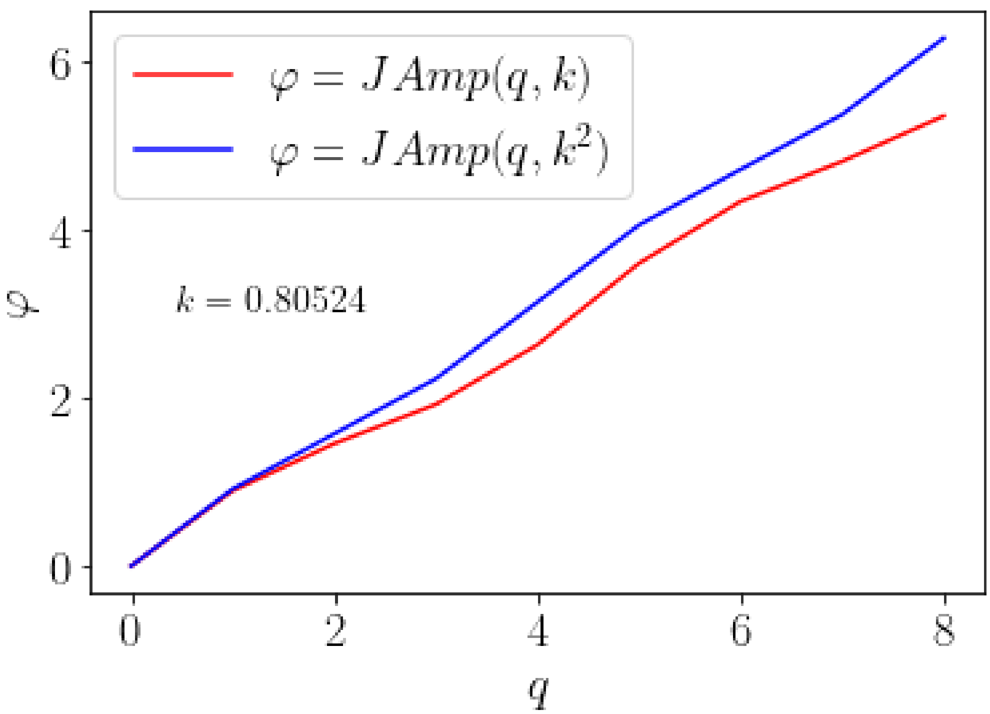


[48]:
```
qArray=[]
kArray=[]
ksqArray=[]
for i in np.arange(0,9,1.0):
    myk=kFct(45/2,13/2,210,1/(2*20),1/(2*100),1/(2*40))
    result=printer(i,myk)
    qArray.append(i)
    kArray.append(result[0])
    ksqArray.append(result[1])
    print(result)
```

```
(0.0, 0.0)
(0.8915386362062283, 0.9124321203112075)
(1.45080089258313, 1.5674613496127106)
(1.9163501357976978, 2.220508522901667)
(2.622629327497944, 3.130344007288082)
(3.59047820203022, 4.045333057637743)
(4.324712038259995, 4.7023837752157505)
(4.797606381658718, 5.353488687660488)
(5.337582832500058, 6.260688937376585)
```

[49]:
```
# plt.rcParams["font.family"] = "Times New Roman"
plt.rcParams.update({'font.size': 18})
## for Palatino and other serif fonts use:
```

```
plt.rc('font',**{'family':'serif','serif':['Times New Roman']})
rc('text', usetex=True)
plt.rc('xtick', labelsize=18)
plt.rc('ytick', labelsize=18)
fig, ax=plt.subplots()
ax.plot(qArray,kArray,'r-',label='$\\varphi=JAmp(q,k)$')
ax.plot(qArray,ksqArray,'b-',label='$\\varphi=JAmp(q,k^2)$')
plt.xlabel('$q$')
plt.ylabel('$\\varphi$')
ax.text(0.2, 0.5, '$k=0.80524$',␣
↪horizontalalignment='center',verticalalignment='center', transform=ax.
↪transAxes,fontsize=14)
plt.legend(loc='best')
plt.savefig('jacobiAmpl_customk.pdf',bbox_inches='tight')
plt.show()
```



3.4 Generate the elliptic coordinates s, c, d

[50]:
```
def ellipticArg(q,spin,theta,paramSet):
    #paramSet is a tuple of the form (SPIN,j,A1,A2,A3)
    #spin=paramSet[0]
    oddSpin=paramSet[1]
    a1=paramSet[2]
```

```
    a2=paramSet[3]
    a3=paramSet[4]
    currentK=kFct(spin,oddSpin,theta,a1,a2,a3)
    #work with linear k
    fi=fiVar(q,currentK)
    #work with quadratic k
    fi2=fiVar_sq(q,currentK)
    resultTuple=(fi,fi2)
    return resultTuple[1]

def sVar(q,spin,theta,paramSet):
    fi=ellipticArg(q,spin,theta,paramSet)
    return fi[0]

def cVar(q,spin,theta,paramSet):
    fi=ellipticArg(q,spin,theta,paramSet)
    return fi[1]

def dVar(q,spin,theta,paramSet):
    fi=ellipticArg(q,spin,theta,paramSet)
    return fi[2]
```

3.5 Defining the parameter set - according to DRAFT

pSet=$\{I, j, A_1, A_2, A_3\}$

[51]:
```
pSet=[45/2,13/2,1/(2*20),1/(2*100),1/(2*40)]
print(pSet)
```

```
[22.5, 6.5, 0.025, 0.005, 0.0125]
```

4 Defining the triaxial rotor potential $V_\theta(q)$

[52]:
```
def rotorPotential(q,spin,theta,paramSet):
    #paramSet is a tuple of the form (SPIN,j,A1,A2,A3)
    #spin=paramSet[0]
    oddSpin=paramSet[1]
    a1=paramSet[2]
    a2=paramSet[3]
    a3=paramSet[4]
    v0=v0Fct(spin,oddSpin,theta,a1,a2,a3)
    k=kFct(spin,oddSpin,theta,a1,a2,a3)
    s=sVar(q,spin,theta,paramSet)
    c=cVar(q,spin,theta,paramSet)
    d=dVar(q,spin,theta,paramSet)
    #define the potential sub-terms
    t1=spin*(spin+1.0)*np.power(k,2)+np.power(v0,2)
```

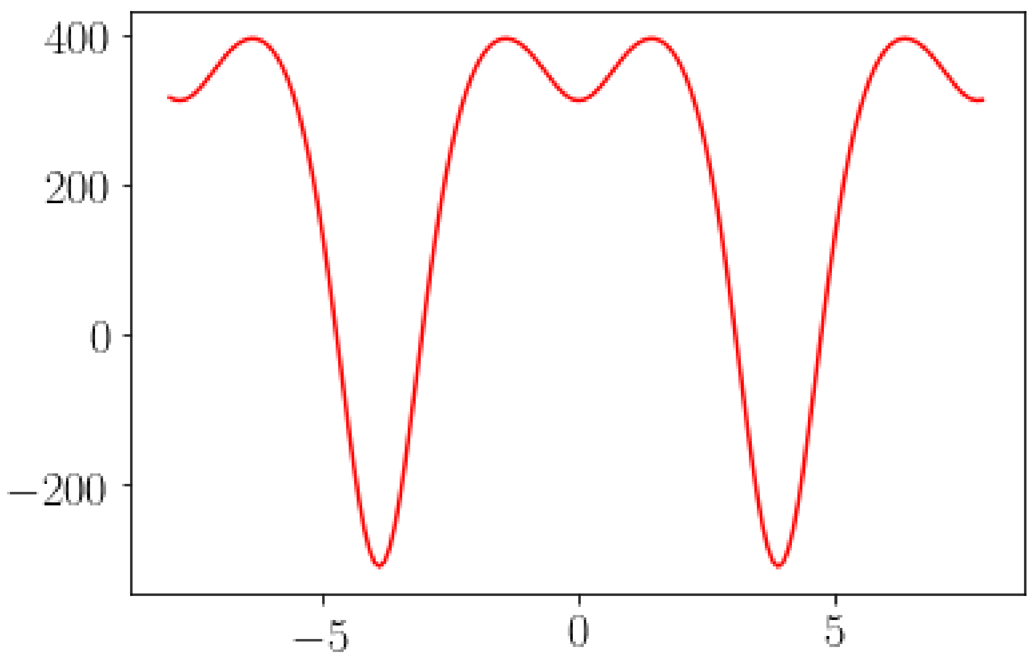
```
    t2=(2.0*spin+1.0)*v0*c*d
    t3=t1*np.power(s,2)
    vRotor=t3+t2
    return vRotor
```

4.1 Rotor potential - graphical representation

[53]:
```
qTable=[]
vTable=[]
for i in np.arange(-8,8,0.1):
    qTable.append(i)
    vTable.append(rotorPotential(i,45/2,30,pSet))
```

[54]:
```
plt.plot(qTable,vTable,'r-')
```

[54]: [<matplotlib.lines.Line2D at 0x133732110>]



5 The chiral potential definition

$V_{\text{chiral}} = \{V_{\text{s}}, V_{\text{a}}\}$

$V_s \rightarrow$ **symmetric potential** $V_{\text{symmetric}} = \frac{V_{\theta_1}+V_{\theta_2}}{2}$

$V_a \rightarrow$ **anti-symmetric potential** $V_{\text{antisymmetric}} = \frac{V_{\theta_1}-V_{\theta_2}}{2}$

5.1 The potentials V_θ depend on the set of angles $\{\theta_1,\theta_2\}$

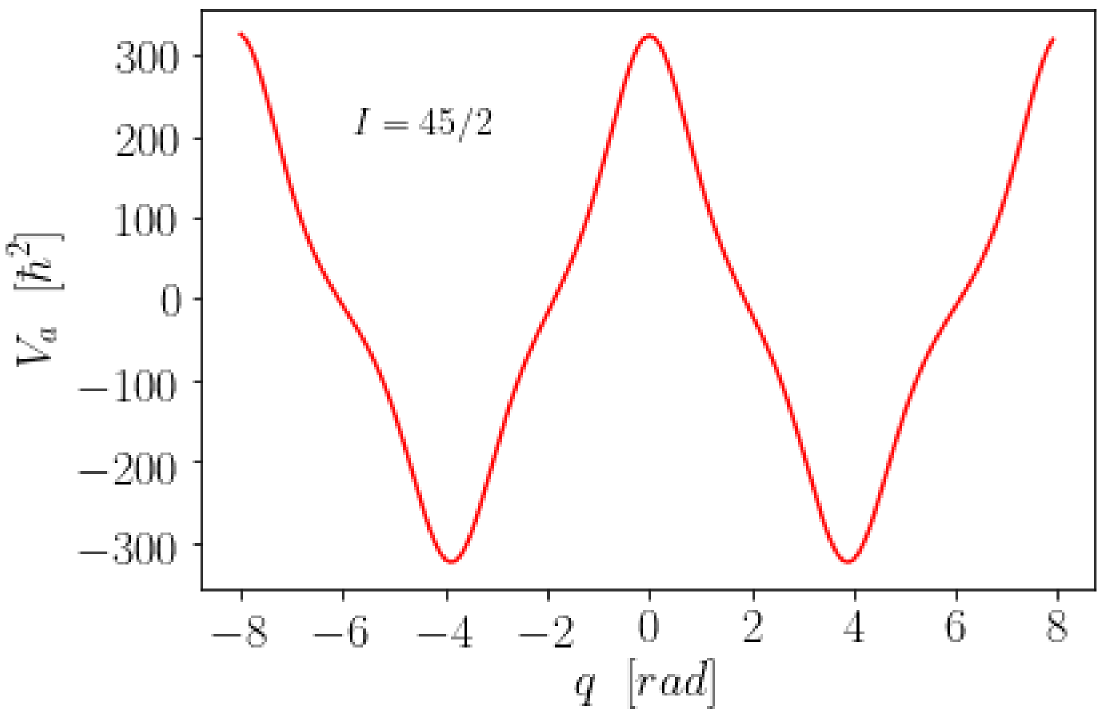
in this case $\theta_1 = 30^o$ and $\theta_2 = 210^o$

[55]:
```
def chiralPotential(q,spin,theta1,theta2,paramSet):
    potential1=rotorPotential(q,spin,theta1,paramSet)
    potential2=rotorPotential(q,spin,theta2,paramSet)
    #the symmetric potential
    symmetricTerm=0.5*(potential1+potential2)
    antiTerm=0.5*(potential1-potential2)
    chiralTuple=(symmetricTerm, antiTerm)
    return chiralTuple

def vSym(q,spin,theta1,theta2,paramSet):
    return chiralPotential(q,spin,theta1,theta2,paramSet)[0]

def vAsym(q,spin,theta1,theta2,paramSet):
    return chiralPotential(q,spin,theta1,theta2,paramSet)[1]
```

[56]:
```
vaTable=[]
vsTable=[]
for i in qTable:
    myspin=45/2
    theta1=30
    theta2=210
    vaTable.append(vAsym(i,myspin,theta1,theta2,pSet))
    vsTable.append(vSym(i,myspin,theta1,theta2,pSet))
```

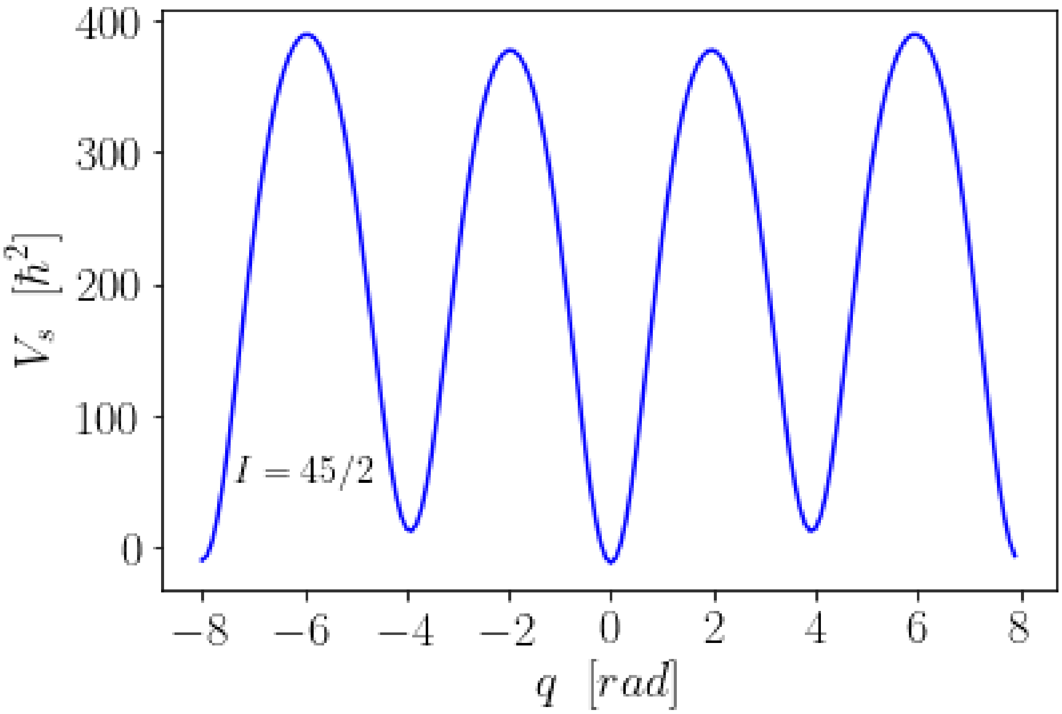
5.2 Calculus for the anti-symmetric potential

[63]:
```
fig, ax=plt.subplots()
ax.plot(qTable,vaTable,'r-')
#plt.title('The antisymmetric potential $V_a(q)$')
plt.xlabel('$q\ \  [rad]$')
plt.ylabel('$V_a\ \  [\hbar^2]$')
ax.text(0.25, 0.8, '$I=45/2$',␣
 ↪horizontalalignment='center',verticalalignment='center', transform=ax.
 ↪transAxes,fontsize=14)
ax.set_xticks([-8,-6,-4,-2,0,2,4,6,8])
ax.set_yticks([-300,-200,-100,0,100,200,300])
plt.savefig('antiSymmetricPotential.pdf',bbox_inches='tight')
plt.show()
```



5.3 Calculus for the symmetric potential

[65]:
```
fig, ax=plt.subplots()
ax.plot(qTable,vsTable,'b-')
#plt.title('The symmetric potential $V_s(q)$')
plt.xlabel('$q\ \  [rad]$')
plt.ylabel('$V_s\ \  [\hbar^2]$')
ax.text(0.16, 0.2, '$I=45/2$',␣
 ↪horizontalalignment='center',verticalalignment='center', transform=ax.
 ↪transAxes,fontsize=14)
ax.set_xticks([-8,-6,-4,-2,0,2,4,6,8])
ax.set_yticks([0,100,200,300,400])
plt.savefig('symetricPotential.pdf',bbox_inches='tight')
plt.show()
```


400
300
200
100
0
V_s [$\hbar^2$]
$I = 45/2$
−8 −6 −4 −2 0 2 4 6 8
q [rad]

```
[ ]:
```