```
In[171]:= proc02[10, 0.5]
λ[1]=28.525 ; λ[1]=1.0
λ[2]=20.6939 ; λ[2]=0.725466
λ[3]=15.0612 ; λ[3]=0.528
λ[4]=10.7081 ; λ[4]=0.375395
λ[5]=7.27744 ; λ[5]=0.255125
λ[6]=4.58491 ; λ[6]=0.160733
λ[7]=2.52251 ; λ[7]=0.0884318
λ[8]=1.02294 ; λ[8]=0.0358614
λ[9]=-0.439808 ; λ[9]=-0.0154184
λ[10]=0.0438675 ; λ[10]=0.00153786
```

→ normalized eigenvalues

eigenvalues

$N = 10 \,,\; \frac{V}{\varepsilon} = \frac{1}{2}$

10x10 matrix for $\hat{H}$

↳ $|0\rangle, |2\rangle, \ldots$ bosonic states

```
In[197]:= spectrum02[n_, id_] := Table[{q, normal[sols02[
```

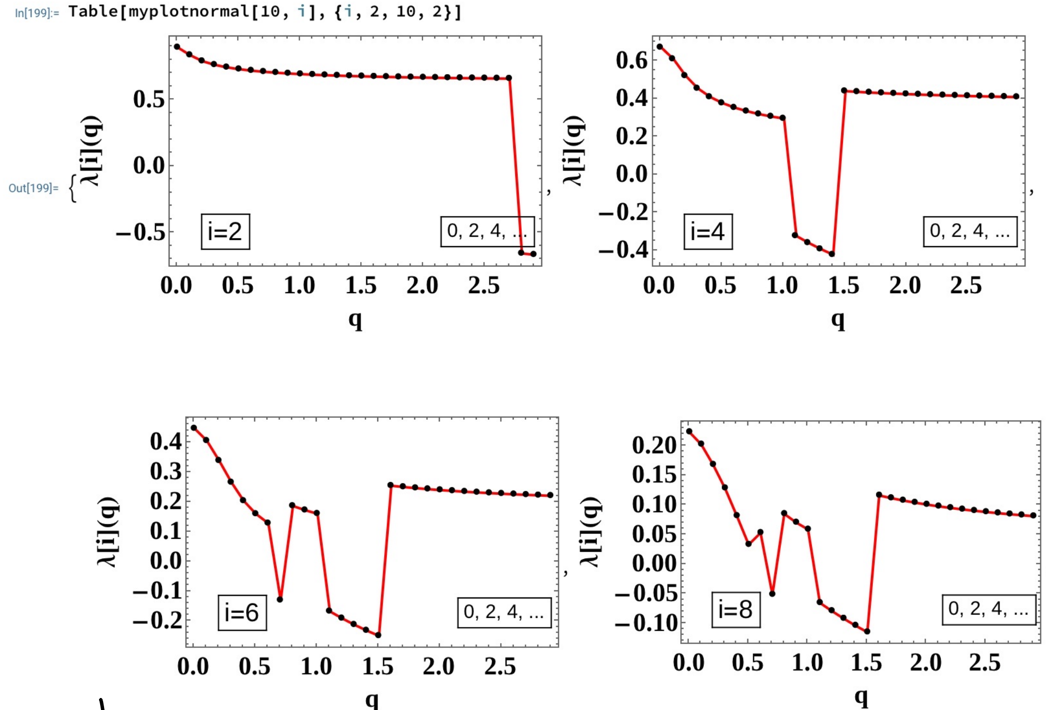
$N = 10 \,,\; \Lambda = \{\lambda_1, \ldots, \lambda_{10}\}$

$\lambda = f\left(\frac{V}{\varepsilon}\right)$

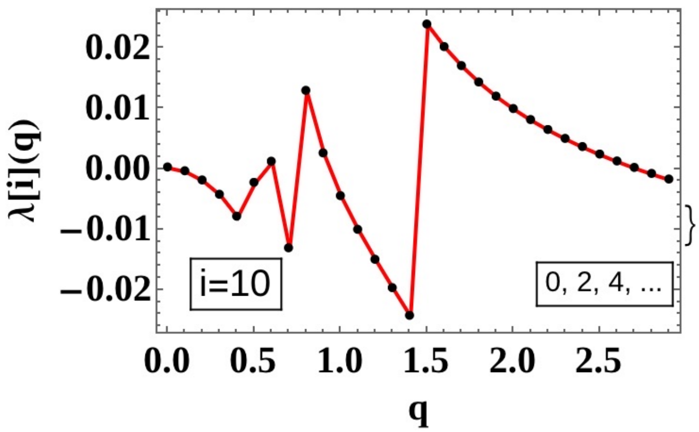
$\frac{V}{\varepsilon} \overset{\text{not}}{=} q$

```
In[199]:= Table[myplotnormal[10, i], {i, 2, 10, 2}]
```

Out[199]=



↓ normalized eigenvalues as function of $q = \frac{V}{\varepsilon}$



$\longrightarrow \lambda_{10}$ normalized

The Hamiltonian in Matrix form:

```
In[143]:= Hmat02[6, e, v] // MatrixForm
Out[143]//MatrixForm=
```

$$\begin{pmatrix} 0 & -\sqrt{2}\,v & 0 & 0 & 0 & 0 \\ -\sqrt{2}\,v & 2\,e & -2\sqrt{3}\,v & 0 & 0 & 0 \\ 0 & -2\sqrt{3}\,v & 4\,e & -\sqrt{30}\,v & 0 & 0 \\ 0 & 0 & -\sqrt{30}\,v & 6\,e & -2\sqrt{14}\,v & 0 \\ 0 & 0 & 0 & -2\sqrt{14}\,v & 8\,e & -3\sqrt{10}\,v \\ 0 & 0 & 0 & 0 & -3\sqrt{10}\,v & 10\,e \end{pmatrix}$$

$\rightarrow$ Matrix using (ε, V)

$N=6$; $|0\rangle, |2\rangle \ldots$

Even „k" bosonic states

```
In[144]:= Hmat02[6, e, v] / e // MatrixForm
Out[144]//MatrixForm=
```

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}\,v}{e} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}\,v}{e} & 2 & -\frac{2\sqrt{3}\,v}{e} & 0 & 0 & 0 \\ 0 & -\frac{2\sqrt{3}\,v}{e} & 4 & -\frac{\sqrt{30}\,v}{e} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{30}\,v}{e} & 6 & -\frac{2\sqrt{14}\,v}{e} & 0 \\ 0 & 0 & 0 & -\frac{2\sqrt{14}\,v}{e} & 8 & -\frac{3\sqrt{10}\,v}{e} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{10}\,v}{e} & 10 \end{pmatrix}$$

$\longrightarrow$ Matrix using $q = \frac{V}{\varepsilon}$

$N=6$; $|0\rangle\ |2\rangle \ldots$

$$H_{00} = \langle 0|H|0\rangle = \langle 0|\, \varepsilon b^{+}b - v b^{+}b^{+} - v b b\, |0\rangle = 0$$

$\hookrightarrow$ first element of matrix $\hat{H}$ is null

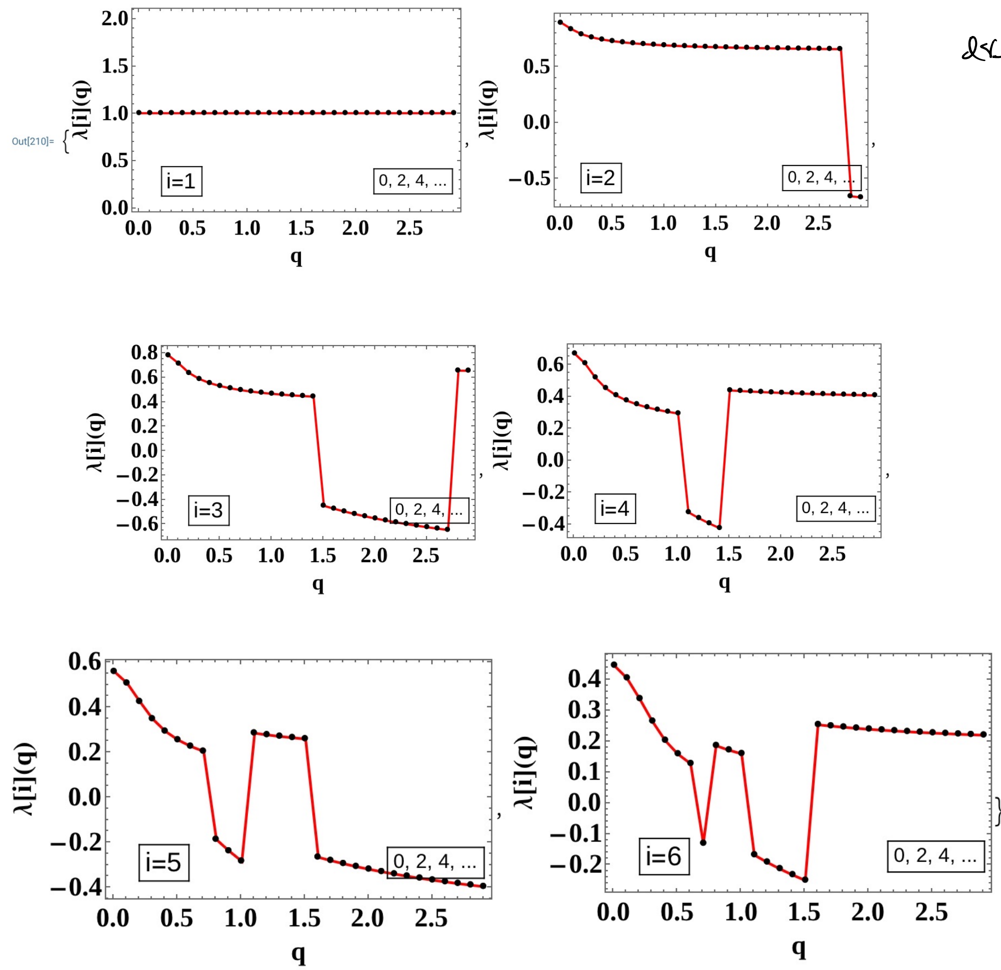
First six solutions $(\lambda_1 \dots \lambda_6)$ for $N = 10$ truncation order

↓

even states

```
In[210]:= Table[myplotnormal[10, i], {i, 1, 6, 1}]
```

Out[210]=

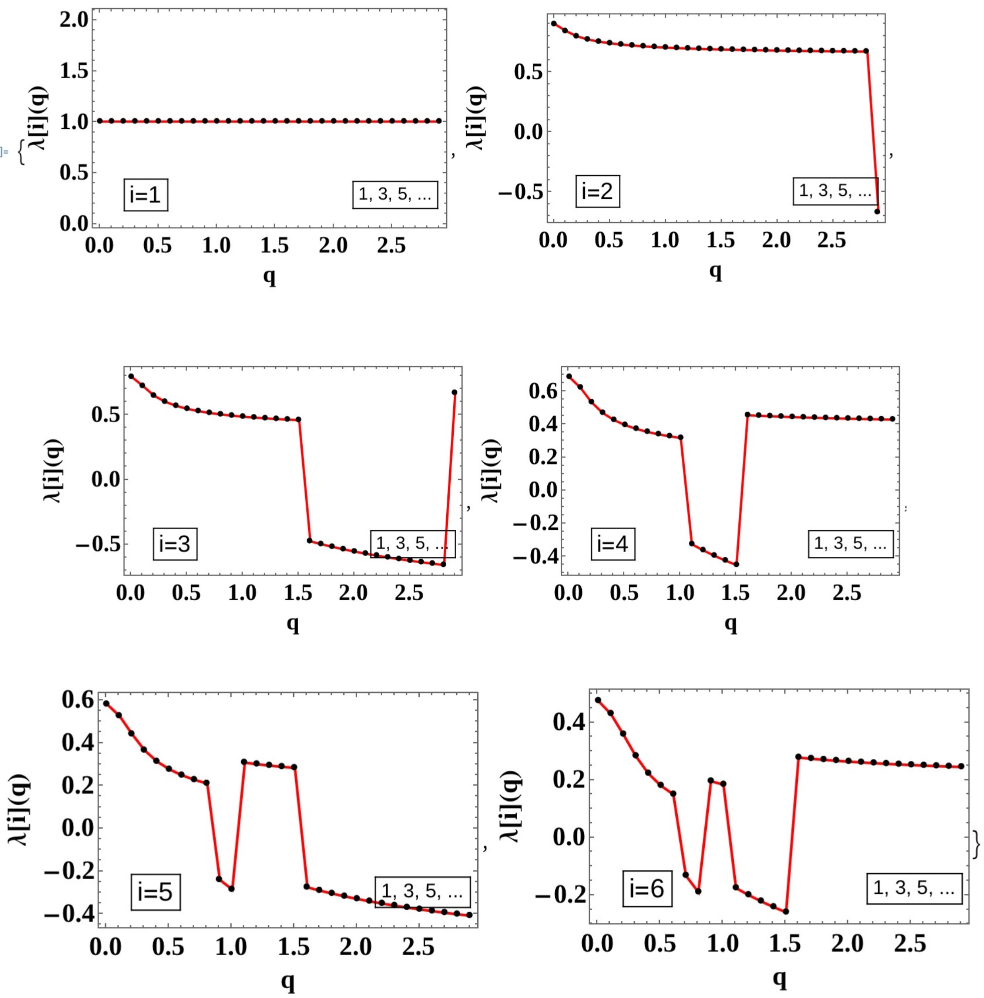


↳ working with normalized λ_i : $\lambda_i = \dfrac{\lambda_i}{\lambda_1}$

First six solutions $\lambda_1 \dots \lambda_6$ (normalized) for odd bosonic states

```
In[220]:= Table[myplotnormal13[10, i], {i, 1, 6, 1}]
```

Out[220]=



The normalized eigenvalues for $N = 10$, $q = \frac{1}{2}$
$|1\rangle, |3\rangle \dots$ odd bosonic states

$N = 10$, $q = \frac{1}{2}$; odd states

In[217]:= **proc13[10, 0.5]**

Normalized

```
λ[1]=30.3064 ; λ[1]=1.0
λ[2]=22.291 ; λ[2]=0.735521
λ[3]=16.4908 ; λ[3]=0.544137
λ[4]=11.9748 ; λ[4]=0.395125
λ[5]=8.38035 ; λ[5]=0.276521
λ[6]=5.51992 ; λ[6]=0.182137
λ[7]=3.28288 ; λ[7]=0.108323
λ[8]=1.59941 ; λ[8]=0.0527747
λ[9]=0.424482 ; λ[9]=0.0140063
λ[10]=-0.270127 ; λ[10]=-0.0089132
```