Boolean Solving

Sophie Tourret & Pascal Fontaine & Christophe Ringeissen

Univ. of Lorraine, CNRS, Inria, LORIA

Procédures de décision et vérification de programmes: Lecture 5

Introduction (1/2)

- SAT: NP-complete (NPC) problem

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers
  (or $P = NP$)

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers
  (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Today's lecture

- ▶ history: truth tables, DPLL (Davis, Putnam, Logemann, Loveland)

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Today's lecture

- ▶ history: truth tables, DPLL (Davis, Putnam, Logemann, Loveland)
- ▶ now: CDCL (Conflict Driven Clause Learning)

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers
  (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Today's lecture

- ▶ history: truth tables, DPLL (Davis, Putnam, Logemann, Loveland)
- ▶ now: CDCL (Conflict Driven Clause Learning)
  - ▶ Boolean propagation

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers (or $\mathrm{P} = \mathrm{NP}$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Today's lecture

- ▶ history: truth tables, DPLL (Davis, Putnam, Logemann, Loveland)
- ▶ now: CDCL (Conflict Driven Clause Learning)
  - ▶ Boolean propagation
  - ▶ Clause representation

Introduction (1/2)

- ▶ SAT: NP-complete (NPC) problem
- ▶ No efficient solution for all cases on sequential computers
  (or $P = NP$)
- ▶ Truth tables: how many lines for n variables?
- ▶ Many problems require many variables
- ▶ Nowadays: 10^6 Boolean variables, 10^7 clauses ($2^{(10^6)} = 10^{301029}$)
- ▶ SAT4J, MiniSAT, Glucose, Crypto-MiniSAT, PicoSAT,...

Today's lecture

- ▶ history: truth tables, DPLL (Davis, Putnam, Logemann, Loveland)
- ▶ now: CDCL (Conflict Driven Clause Learning)
  - ▶ Boolean propagation
  - ▶ Clause representation
  - ▶ Conflict analysis and non-chronological backtracking

Introduction (2/2)

Satisfiability checking for profit:

▶ Verification (model checking, formal proofs,. . .)

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,...)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,...)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)
- ▶ Configuration selection

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)
- ▶ Configuration selection
- ▶ Circuit placement on silicon die

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)
- ▶ Configuration selection
- ▶ Circuit placement on silicon die
- ▶ . . .

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)
- ▶ Configuration selection
- ▶ Circuit placement on silicon die
- ▶ . . .

Introduction (2/2)

Satisfiability checking for profit:

- ▶ Verification (model checking, formal proofs,. . .)
- ▶ Maths (e.g. Erdős discrepancy problem)
- ▶ Planning problems
- ▶ Dependency checking (e.g. Eclipse)
- ▶ Configuration selection
- ▶ Circuit placement on silicon die
- ▶ . . .

> Propositional satisfiability checking problem (SAT) is NPC.
> Any NPC problem can be translated polynomially into SAT

Prerequisites / Notations

▶ Prerequisite: knowledge of propositional and first-order logic

Prerequisites / Notations

- ▶ Prerequisite: knowledge of propositional and first-order logic
- ▶ Boolean logic:

Prerequisites / Notations

- Prerequisite: knowledge of propositional and first-order logic
- Boolean logic:
  - Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$

Prerequisites / Notations

▶ Prerequisite: knowledge of propositional and first-order logic
▶ Boolean logic:
  ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  ▶ clause (e.g. $a \vee \neg b \vee c$)

Prerequisites / Notations

▶ Prerequisite: knowledge of propositional and first-order logic
▶ Boolean logic:
  ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  ▶ clause (e.g. $a \vee \neg b \vee c$)
  ▶ cube (e.g. $a \wedge \neg b \wedge c$)

Prerequisites / Notations

- ▶ Prerequisite: knowledge of propositional and first-order logic
- ▶ Boolean logic:
  - ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  - ▶ clause (e.g. $a \vee \neg b \vee c$)
  - ▶ cube (e.g. $a \wedge \neg b \wedge c$)
  - ▶ formula ($\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\equiv$, $\dots$)

Prerequisites / Notations

- Prerequisite: knowledge of propositional and first-order logic
- Boolean logic:
  - Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  - clause (e.g. $a \lor \neg b \lor c$)
  - cube (e.g. $a \land \neg b \land c$)
  - formula ($\neg$, $\land$, $\lor$, $\Rightarrow$, $\equiv$, ...)
  - interpretation, (un)satisfiable formula, valid formula

Prerequisites / Notations

- ▶ Prerequisite: knowledge of propositional and first-order logic
- ▶ Boolean logic:
  - ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  - ▶ clause (e.g. $a \vee \neg b \vee c$)
  - ▶ cube (e.g. $a \wedge \neg b \wedge c$)
  - ▶ formula ($\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\equiv$, . . .)
  - ▶ interpretation, (un)satisfiable formula, valid formula
- ▶ Unit clause: clause with one literal only

Prerequisites / Notations

- ▶ Prerequisite: knowledge of propositional and first-order logic
- ▶ Boolean logic:
  - ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  - ▶ clause (e.g. $a \vee \neg b \vee c$)
  - ▶ cube (e.g. $a \wedge \neg b \wedge c$)
  - ▶ formula ($\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\equiv$, ...)
  - ▶ interpretation, (un)satisfiable formula, valid formula
- ▶ Unit clause: clause with one literal only
- ▶ Empty clause: $\square$, unsatisfiable

Prerequisites / Notations

▶ Prerequisite: knowledge of propositional and first-order logic
▶ Boolean logic:
  ▶ Boolean (propositional) variable (v), literal (e.g. v, $\neg v$)
    $\overline{\neg v} = v$, $\overline{v} = \neg v$
  ▶ clause (e.g. $a \vee \neg b \vee c$)
  ▶ cube (e.g. $a \wedge \neg b \wedge c$)
  ▶ formula ($\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\equiv$, ...)
  ▶ interpretation, (un)satisfiable formula, valid formula
▶ Unit clause: clause with one literal only
▶ Empty clause: $\square$, unsatisfiable
▶ Resolution rule:

$$\frac{C \vee \ell \quad C' \vee \overline{\ell}}{C \vee C'}$$

A propositional problem (1/2)

You are chief of protocol for the embassy ball. The crown prince instructs you either to invite Peru or to exclude Qatar . The queen asks you to invite either Qatar or Romania or both. The king, in a spiteful mood, wants to snub either Romania or Peru or both. Is there a guest list that will satisfy the whims of the entire royal family?

A propositional problem (1/2)

You are chief of protocol for the embassy ball. The crown prince instructs you either to invite Peru or to exclude Qatar . The queen asks you to invite either Qatar or Romania or both. The king, in a spiteful mood, wants to snub either Romania or Peru or both. Is there a guest list that will satisfy the whims of the entire royal family?

$$(P \vee \neg Q)$$

A propositional problem (1/2)

You are chief of protocol for the embassy ball. The crown prince instructs you either to invite Peru or to exclude Qatar . The queen asks you to invite either Qatar or Romania or both. The king, in a spiteful mood, wants to snub either Romania or Peru or both. Is there a guest list that will satisfy the whims of the entire royal family?

$$(P \vee \neg Q) \wedge (Q \vee R)$$

A propositional problem (1/2)

You are chief of protocol for the embassy ball. The crown prince instructs you either to invite Peru or to exclude Qatar . The queen asks you to invite either Qatar or Romania or both. The king, in a spiteful mood, wants to snub either Romania or Peru or both. Is there a guest list that will satisfy the whims of the entire royal family?

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

P	Q	R	φ

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

P	Q	R	φ
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$$P \vee \neg Q$$
$$Q \vee R$$
$$\neg R \vee \neg P$$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

DIMACS format

Vars $\to$ numbers: $P \to 1$, $Q \to 2$, $R \to 3$

Literals: $P \to 1$, $\neg P \to -1$

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$$P \vee \neg Q$$
$$Q \vee R$$
$$\neg R \vee \neg P$$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

```
                p cnf 3 3
P ∨ ¬Q          1 -2 0
Q ∨ R           2 3 0
¬R ∨ ¬P         -3 -1 0
```

A propositional problem (2/2)

$$\varphi =_{\mathrm{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

```
                    p cnf 3 3
    P ∨ ¬Q          1 -2 0
    Q ∨ R           2 3 0
    ¬R ∨ ¬P         -3 -1 0
```

Solution: -1 -2 3 $\rightarrow \neg P \wedge \neg Q \wedge R$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

DIMACS format

Vars $\to$ numbers: $P \to 1$, $Q \to 2$, $R \to 3$

Literals: $P \to 1$, $\neg P \to -1$

```
                      p cnf 3 3
P ∨ ¬Q                1 -2 0
Q ∨ R                 2 3 0
¬R ∨ ¬P               -3 -1 0
```

Solution: -1 -2 3 $\to \neg P \wedge \neg Q \wedge R$

Another solution? Add clause

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

```
                    p cnf 3 3
    P ∨ ¬Q          1 -2 0
    Q ∨ R           2 3 0
    ¬R ∨ ¬P         -3 -1 0
    P ∨ Q ∨ ¬R      1 2 -3 0
```

Solution: -1 -2 3 $\rightarrow \neg P \wedge \neg Q \wedge R$

Another solution? Add clause

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

```
              p cnf 3 3
P ∨ ¬Q        1 -2 0
Q ∨ R         2 3 0
¬R ∨ ¬P       -3 -1 0
P ∨ Q ∨ ¬R    1 2 -3 0
```

Solution: -1 -2 3 $\rightarrow \neg P \wedge \neg Q \wedge R$

Another solution? Add clause

Solution: 1 2 -3 $\rightarrow P \wedge Q \wedge \neg R$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

```
                    p cnf 3 3
```
$P \vee \neg Q$	`1 -2 0`
$Q \vee R$	`2 3 0`
$\neg R \vee \neg P$	`-3 -1 0`
$P \vee Q \vee \neg R$	`1 2 -3 0`
$\neg P \vee \neg Q \vee R$	`-1 -2 3 0`

Solution: -1 -2 3 $\rightarrow \neg P \wedge \neg Q \wedge R$

Another solution? Add clause

Solution: 1 2 -3 $\rightarrow P \wedge Q \wedge \neg R$

A propositional problem (2/2)

$$\varphi =_{\text{def}} (P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Truth table for φ

SAT checker

P	Q	R	φ
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

DIMACS format

Vars $\rightarrow$ numbers: $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

Literals: $P \rightarrow 1$, $\neg P \rightarrow -1$

	p cnf 3 3
$P \vee \neg Q$	1 -2 0
$Q \vee R$	2 3 0
$\neg R \vee \neg P$	-3 -1 0
$P \vee Q \vee \neg R$	1 2 -3 0
$\neg P \vee \neg Q \vee R$	-1 -2 3 0

Solution: -1 -2 3 $\rightarrow \neg P \wedge \neg Q \wedge R$

Another solution? Add clause

Solution: 1 2 -3 $\rightarrow P \wedge Q \wedge \neg R$

No more solutions

DPLL: rule-based view
Davis, Putnam, Logemann, Loveland

Let S be a set of clauses

$$\text{Unit Resolution} \qquad \frac{S \cup \{\ell, C \vee \overline{\ell}\}}{S \cup \{\ell, C\}}$$

$$\left(\text{Unit Subsumption} \qquad \frac{S \cup \{\ell, C \vee \ell\}}{S \cup \{\ell\}}\right)$$

$$\text{Splitting} \qquad \frac{S}{S \cup \{v\} \mid S \cup \{\neg v\}} \qquad \text{if } v \text{ is a variable occurring in } S$$

▶ Failed branch: a trivial contradiction $\{\ldots, v, \ldots, \neg v \ldots\}$

▶ Successful branch: not failed, only unit clauses (with unit sub.)

DPLL: rule-based view
Davis, Putnam, Logemann, Loveland

Let S be a set of clauses

Unit Resolution
$$\frac{S \cup \{\ell, C \vee \overline{\ell}\}}{S \cup \{\ell, C\}}$$

$\Big($ Unit Subsumption
$$\frac{S \cup \{\ell, C \vee \ell\}}{S \cup \{\ell\}} \Big)$$

Splitting
$$\frac{S}{S \cup \{v\} \mid S \cup \{\neg v\}} \quad \text{if } v \text{ is a variable occurring in } S$$

- ▶ Failed branch: a trivial contradiction $\{\ldots, v, \ldots, \neg v \ldots\}$
- ▶ Successful branch: not failed, only unit clauses (with unit sub.)

Exercise: explain how a Boolean model can be extracted from the application of these rules

DPLL: rule-based view

Davis, Putnam, Logemann, Loveland

Let S be a set of clauses

$$\textit{Unit Resolution} \qquad \frac{S \cup \{\ell, C \vee \overline{\ell}\}}{S \cup \{\ell, C\}}$$

$$\left(\textit{Unit Subsumption} \qquad \frac{S \cup \{\ell, C \vee \ell\}}{S \cup \{\ell\}}\right)$$

$$\textit{Splitting} \qquad \frac{S}{S \cup \{v\} \mid S \cup \{\neg v\}} \qquad \text{if } v \text{ is a variable occurring in } S$$

- ▶ Failed branch: a trivial contradiction $\{\ldots, v, \ldots, \neg v \ldots\}$
- ▶ Successful branch: not failed, only unit clauses (with unit sub.)

Exercise: explain how a Boolean model can be extracted from the application of these rules
(Hint: think of derivation trees and collect unit clauses...)

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

$$\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}$$

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

DPLL: example

$$(P \lor \neg Q) \land (Q \lor R) \land (\neg R \lor \neg P)$$

Already a set of clauses:

$$Split \quad \frac{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P\}}{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\} \mid \{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, \neg P\}}$$

And then only *Unit Resolution* rules

$$\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\}$$

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\{P \vee \neg Q, Q \vee R, \neg R, P\}}$

DPLL: example

$$(P \lor \neg Q) \land (Q \lor R) \land (\neg R \lor \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P\}}{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\} \mid \{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$$\dfrac{\dfrac{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\}}{\{P \lor \neg Q, Q \lor R, \neg R, P\}}}{\{P \lor \neg Q, Q, \neg R, P\}}$$

DPLL: example

$$(P \lor \neg Q) \land (Q \lor R) \land (\neg R \lor \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P\}}{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\} \mid \{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$\dfrac{\dfrac{\{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, P\}}{\{P \lor \neg Q, Q \lor R, \neg R, P\}}}{\{P \lor \neg Q, Q, \neg R, P\}}$ $\qquad\qquad \{P \lor \neg Q, Q \lor R, \neg R \lor \neg P, \neg P\}$

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$$\dfrac{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\{P \vee \neg Q, Q \vee R, \neg R, P\}}}{\{P \vee \neg Q, Q, \neg R, P\}} \qquad \dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\{\neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$$

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R, P\}}{\{P \vee \neg Q, Q, \neg R, P\}}}$
$\qquad$
$\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\dfrac{\{\neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\{\neg Q, R, \neg R \vee \neg P, \neg P\}}}$

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$\dfrac{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\{P \vee \neg Q, Q \vee R, \neg R, P\}}}{\{P \vee \neg Q, Q, \neg R, P\}}$
$\qquad$
$\dfrac{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\{\neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}}{\{\neg Q, R, \neg R \vee \neg P, \neg P\}}$

Remarks :

- ▶ Same two models again

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split $\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$

And then only *Unit Resolution* rules

$$\dfrac{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\{P \vee \neg Q, Q \vee R, \neg R, P\}}}{\{P \vee \neg Q, Q, \neg R, P\}} \qquad \dfrac{\dfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\{\neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}}{\{\neg Q, R, \neg R \vee \neg P, \neg P\}}$$

Remarks :

▶ Same two models again

▶ Satisfiability procedure: find one model (and stop)

DPLL: example

$$(P \vee \neg Q) \wedge (Q \vee R) \wedge (\neg R \vee \neg P)$$

Already a set of clauses:

Split
$$\cfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}}{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\} \mid \{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}$$

And then only *Unit Resolution* rules

$$\cfrac{\cfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, P\}}{\{P \vee \neg Q, Q \vee R, \neg R, P\}}}{\{P \vee \neg Q, Q, \neg R, P\}}$$

$$\cfrac{\cfrac{\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}{\{\neg Q, Q \vee R, \neg R \vee \neg P, \neg P\}}}{\{\neg Q, R, \neg R \vee \neg P, \neg P\}}$$

Remarks :

▶ Same two models again

▶ Satisfiability procedure: find one model (and stop)

▶ Much less sensitive to the number of variables than truth tables

DPLL: exercises

- $P \vee Q, \neg P \vee Q, \neg R \vee \neg Q, R \vee \neg Q$
- $P \vee Q \vee R, \neg P \vee \neg Q \vee \neg R, \neg P \vee Q \vee R, \neg Q \vee R, Q \vee \neg R$
- $\neg Q \vee P, \neg P \vee \neg Q, Q \vee R, \neg Q \vee \neg R, \neg P \vee \neg R, P \vee \neg R$

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, . . .

Boolean formulas, CNF, DNF

- Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, ...
- Conjunctive Normal Form (CNF): conjunction of clauses

Boolean formulas, CNF, DNF

▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, ...
▶ Conjunctive Normal Form (CNF): conjunction of clauses
▶ Disjunctive Normal Form (DNF): disjunction of cubes

Boolean formulas, CNF, DNF

▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, ...
▶ Conjunctive Normal Form (CNF): conjunction of clauses
▶ Disjunctive Normal Form (DNF): disjunction of cubes

Boolean formulas, CNF, DNF

▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, …

▶ Conjunctive Normal Form (CNF): conjunction of clauses

▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM

Every formula is logically equivalent to a CNF (DNF)

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg, \wedge, \vee, \Rightarrow, \ldots$
- ▶ Conjunctive Normal Form (CNF): conjunction of clauses
- ▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM

Every formula is logically equivalent to a CNF (DNF)

Remark :

- ▶ Converting to DNF, then finding one satisfiable cube is a trivial satisfiability procedure

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, ...
- ▶ Conjunctive Normal Form (CNF): conjunction of clauses
- ▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM

Every formula is logically equivalent to a CNF (DNF)

Remark :

- ▶ Converting to DNF, then finding one satisfiable cube is a trivial satisfiability procedure
- ▶ checking the satisfiability of a cube is linear

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, ...
- ▶ Conjunctive Normal Form (CNF): conjunction of clauses
- ▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM

Every formula is logically equivalent to a CNF (DNF)

Remark :

- ▶ Converting to DNF, then finding one satisfiable cube is a trivial satisfiability procedure
- ▶ checking the satisfiability of a cube is linear
- ▶ so DNF conversion cannot be efficient (i.e. polynomial) or $\mathrm{P} = \mathrm{NP}$

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg$, $\wedge$, $\vee$, $\Rightarrow$, . . .
- ▶ Conjunctive Normal Form (CNF): conjunction of clauses
- ▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM
Every formula is logically equivalent to a CNF (DNF)

Remark :

- ▶ Converting to DNF, then finding one satisfiable cube is a trivial satisfiability procedure
- ▶ checking the satisfiability of a cube is linear
- ▶ so DNF conversion cannot be efficient (i.e. polynomial) or $\mathrm{P} = \mathrm{NP}$
- ▶ computing DNF of formula: negation of CNF of negation of formula

Boolean formulas, CNF, DNF

- ▶ Boolean formulas: built with variables $\neg, \wedge, \vee, \Rightarrow, \ldots$
- ▶ Conjunctive Normal Form (CNF): conjunction of clauses
- ▶ Disjunctive Normal Form (DNF): disjunction of cubes

THEOREM

Every formula is logically equivalent to a CNF (DNF)

Remark :

- ▶ Converting to DNF, then finding one satisfiable cube is a trivial satisfiability procedure
- ▶ checking the satisfiability of a cube is linear
- ▶ so DNF conversion cannot be efficient (i.e. polynomial) or $\mathrm{P} = \mathrm{NP}$
- ▶ computing DNF of formula: negation of CNF of negation of formula
- ▶ so CNF conversion cannot be efficient

Efficient computation of CNF

Consider
$$\Phi = (a_1 \wedge \cdots \wedge a_m) \vee (b_1 \wedge \cdots \wedge b_n)$$

Equivalent CNF:
$$\bigwedge_{i=1}^{m} \bigwedge_{j=1}^{n} (a_i \vee b_j)$$

Equisatisfiable CNF:

$$(X \vee Y) \wedge (X \Leftrightarrow a_1 \wedge \cdots \wedge a_m) \wedge (Y \Leftrightarrow b_1 \wedge \cdots \wedge b_n)$$

where $(X \Leftrightarrow a_1 \wedge \cdots \wedge a_m)$ can be represented as a conjunction of clauses

Efficient computation of CNF

Consider

$$\Phi = (a_1 \wedge \cdots \wedge a_m) \vee (b_1 \wedge \cdots \wedge b_n)$$

Equivalent CNF:

$$\bigwedge_{i=1}^{m} \bigwedge_{j=1}^{n} (a_i \vee b_j)$$

Equisatisfiable CNF:

$$(X \vee Y) \wedge (X \Leftrightarrow a_1 \wedge \cdots \wedge a_m) \wedge (Y \Leftrightarrow b_1 \wedge \cdots \wedge b_n)$$

where $(X \Leftrightarrow a_1 \wedge \cdots \wedge a_m)$ can be represented as a conjunction of clauses (Exercise next slide).

Efficient computation of CNF

Consider
$$\Phi = (a_1 \wedge \cdots \wedge a_m) \vee (b_1 \wedge \cdots \wedge b_n)$$

Equivalent CNF:
$$\bigwedge_{i=1}^{m} \bigwedge_{j=1}^{n} (a_i \vee b_j)$$

Equisatisfiable CNF:

$$(X \vee Y) \wedge (X \Leftrightarrow a_1 \wedge \cdots \wedge a_m) \wedge (Y \Leftrightarrow b_1 \wedge \cdots \wedge b_n)$$

where $(X \Leftrightarrow a_1 \wedge \cdots \wedge a_m)$ can be represented as a conjunction of clauses (Exercise next slide).

> ### THEOREM (Tseitin transformation)
> *Every formula can be transformed in linear time into an equisatisfiable CNF*

Computing CNFs: exercises

- $(X \Leftrightarrow a_1 \wedge \cdots \wedge a_m)$
- $(p \Rightarrow q) \equiv (p \Rightarrow r)$
- $(p \wedge q) \vee (r \wedge s) \vee (\neg q \wedge (p \vee t))$

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

Stack of assigned literals:

- ▶ some *decided*, others *propagated*

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

Stack of assigned literals:

- ▶ some *decided*, others *propagated*
- ▶ *level*: number of decided literals on stack

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

Stack of assigned literals:

- ▶ some *decided*, others *propagated*
- ▶ *level*: number of decided literals on stack
- ▶ some clause become unit
  all literals but one assigned to false, last literal not assigned
  $\implies$ new propagated literal on the stack

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

Stack of assigned literals:

- ▶ some *decided*, others *propagated*
- ▶ *level*: number of decided literals on stack
- ▶ some clause become unit
  all literals but one assigned to false, last literal not assigned
  $\implies$ new propagated literal on the stack
- ▶ clause may be falsified: all literals assigned to false
  $\implies$ backtrack the last decision

DPLL: from rules to algorithm

From rules to algorithm:

- ▶ a way to enumerate splittings
- ▶ some splittings may lead to closed branches (clause unsatisfied / empty clause derived)
- ▶ backtracking
- ▶ avoid copying sets of clauses and modifying clauses

Stack of assigned literals:

- ▶ some *decided*, others *propagated*
- ▶ *level*: number of decided literals on stack
- ▶ some clause become unit
  all literals but one assigned to false, last literal not assigned
  $\implies$ new propagated literal on the stack
- ▶ clause may be falsified: all literals assigned to false
  $\implies$ backtrack the last decision
- ▶ never mind about satisfied clause

DPLL: algorithmic view

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:         BACKTRACK()
```

DPLL: algorithmic view

▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns $\bot$ iff unsatisfied clause

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:         BACKTRACK()
```

DPLL: algorithmic view

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:         BACKTRACK()
```

- ▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns ⊥ iff unsatisfied clause
- ▶ DECIDE: choses one non assigned literal, push on stack. Returns ⊥ iff no literal

DPLL: algorithmic view

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:        BACKTRACK()
```

- ▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns ⊥ iff unsatisfied clause

- ▶ DECIDE: choses one non assigned literal, push on stack. Returns ⊥ iff no literal

- ▶ BACKTRACK: backtrack (pop literals from stack) until the last decision and add the opposite literal *as propagated*

DPLL: algorithmic view

- ► PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns $\bot$ iff unsatisfied clause
- ► DECIDE: choses one non assigned literal, push on stack. Returns $\bot$ iff no literal

```
 1: procedure SAT(C)
 2:     while ⊤ do
 3:         if PROPAGATE() then
 4:             if ¬DECIDE() then
 5:                 return SAT
 6:             continue
 7:         if level = 0 then
 8:             return UNSAT

10:         BACKTRACK()
```

- ► BACKTRACK: backtrack (pop literals from stack) until the last decision and add the opposite literal *as propagated*

Write successive stacks for runs on

- ► $\{P \vee \neg Q, Q \vee R, \neg R \vee \neg P\}$
- ► $\{a \vee b, \neg b \vee c \vee d, \neg b \vee e, \neg d \vee \neg e \vee f, a \vee c \vee f, \neg a \vee g, \neg g \vee b, \neg h \vee j, \neg i \vee k\}$

DPLL: abstract view

Rules handle a data-structure $M \parallel F$ where M is a partial assignment of Boolean variables, and F is a set of clauses

Propagate $\quad M \parallel F, C \vee \ell \quad \vdash \quad M\, \ell \parallel F, C \vee \ell$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ if $M \models \neg C, \ell$ undefined in M

Decide $\qquad M \parallel F \qquad\qquad \vdash \quad M\, \ell^d \parallel F$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ if ℓ or $\bar{\ell}$ in F, ℓ undefined in M

Fail $\qquad\quad M \parallel F, C \qquad \vdash \quad \perp$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ if $M \models \neg C,$ no decision literals in M

Backtrack $\quad M\, \ell^d\, N \parallel F, C \quad \vdash \quad M\, \bar{\ell} \parallel F, C$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ if $\begin{cases} M\, \ell^d\, N \models \neg C \\ \text{no decision literals in } N \end{cases}$

DPLL: algorithmic view

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:        BACKTRACK()
```

- ▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns ⊥ iff unsatisfied clause
- ▶ DECIDE: choses one non assigned literal, push on stack. Returns ⊥ iff no literal

- ▶ BACKTRACK: backtrack (pop literals from stack) until the last decision and add the opposite literal *as propagated*

Towards CDCL
Conflict Driven Clause Learning

BACKTRACK

▶ depending on decisions, the same dead end may be tried again and again

Towards CDCL
Conflict Driven Clause Learning

BACKTRACK

- depending on decisions, the same dead end may be tried again and again
- would be much better to remember the very reason why conflict: new clause

Towards CDCL
Conflict Driven Clause Learning

BACKTRACK

▶ depending on decisions, the same dead end may be tried again and again

▶ would be much better to remember the very reason why conflict: new clause

▶ then forget about backtracking and changing decision. Just add clause, backtrack to when it is propagating

DPLL: algorithmic view

```
1: procedure SAT(C)
2:     while ⊤ do
3:         if PROPAGATE() then
4:             if ¬DECIDE() then
5:                 return SAT
6:             continue
7:         if level = 0 then
8:             return UNSAT

10:        BACKTRACK()
```

- ▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns ⊥ iff unsatisfied clause

- ▶ DECIDE: choses one non assigned literal, push on stack. Returns ⊥ iff no literal

- ▶ BACKTRACK: backtrack (pop literals from stack) until the last decision and add the opposite literal *as propagated*

CDCL: algorithmic view

```
 1: procedure SAT(C)
 2:     while ⊤ do
 3:         if PROPAGATE() then
 4:             if ¬DECIDE() then
 5:                 return SAT
 6:             continue
 7:         if level = 0 then
 8:             return UNSAT
 9:         ANALYSE()
10:         BACKTRACK()
```

- ▶ PROPAGATE: find unit clauses repeatedly and push literals on the stack. Returns ⊥ iff unsatisfied clause

- ▶ DECIDE: choses one non assigned literal, push on stack. Returns ⊥ iff no literal

- ▶ ANALYSE: analyse the conflict from propagate, create conflict clause, add it in the set of clauses

- ▶ BACKTRACK: backtrack (eliminate literals from stack) until conflict clause is unit

Conflict analysis, example (1/5)

$$C_1 : \quad x_1 \vee x_2$$
$$C_2 : \neg x_2 \vee \neg x_3$$
$$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$$
$$C_4 : \quad x_3 \vee x_5 \vee x_6$$
$$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$$
$$C_6 : \neg x_4 \vee x_8 \vee x_9$$
$$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$$
$$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$$
$$C_9 : \quad x_{12} \vee \neg x_{13}$$
$$C_{10} : x_7 \vee x_{12} \vee x_{14}$$
$$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$$
$$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$$
$$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$$

Conflict analysis, example (1/5)

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- ▶ No unit clause: decide $\neg x_1$

$\neg x_1$

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- ▶ No unit clause: decide $\neg x_1$
- ▶ C_1: propagate x_2

$\neg x_1$
x_2 / C_1

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$

Conflict analysis, example (1/5)

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- ▶ No unit clause: decide $\neg x_1$
- ▶ C_1: propagate x_2
- ▶ C_2: propagate $\neg x_3$
- ▶ No unit clause: decide x_4

$\neg x_1$
$\quad x_2 / C_1$
$\neg x_3 / C_2$
$\quad x_4$

Conflict analysis, example (1/5)

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4

Conflict analysis, example (1/5)

$C_1 : x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : x_3 \lor x_5 \lor x_6$
$C_5 : x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$

Conflict analysis, example (1/5)

$C_1 :\ x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 :\ x_3 \lor x_5 \lor x_6$
$C_5 :\ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 :\ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 :\ x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- ▶ No unit clause: decide $\neg x_1$
- ▶ C_1: propagate x_2
- ▶ C_2: propagate $\neg x_3$
- ▶ No unit clause: decide x_4
- ▶ C_3: propagate $\neg x_5$
- ▶ C_4: propagate x_6
- ▶ No unit clause: decide $\neg x_7$
- ▶ C_5: propagate $\neg x_8$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$

Conflict analysis, example (1/5)

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- ▶ No unit clause: decide $\neg x_1$
- ▶ C_1: propagate x_2
- ▶ C_2: propagate $\neg x_3$
- ▶ No unit clause: decide x_4
- ▶ C_3: propagate $\neg x_5$
- ▶ C_4: propagate x_6
- ▶ No unit clause: decide $\neg x_7$
- ▶ C_5: propagate $\neg x_8$
- ▶ C_6: propagate x_9
- ▶ No unit clause: decide $\neg x_{10}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9
- No unit clause: decide $\neg x_{10}$
- C_7: propagate x_{11}

$\neg x_1$
$\quad x_2/C_1$
$\neg x_3/C_2$
$\quad x_4$
$\neg x_5/C_3$
$\quad x_6/C_4$
$\neg x_7$
$\neg x_8/C_5$
$\quad x_9/C_6$
$\neg x_{10}$
$\quad x_{11}/C_7$

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9
- No unit clause: decide $\neg x_{10}$
- C_7: propagate x_{11}
- C_8: propagate $\neg x_{12}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$

Conflict analysis, example (1/5)

$C_1 :\ x_1 \lor x_2$
$C_2 : \lnot x_2 \lor \lnot x_3$
$C_3 : \lnot x_2 \lor \lnot x_4 \lor \lnot x_5$
$C_4 :\ x_3 \lor x_5 \lor x_6$
$C_5 :\ x_7 \lor \lnot x_6 \lor \lnot x_8$
$C_6 : \lnot x_4 \lor x_8 \lor x_9$
$C_7 :\ x_{10} \lor \lnot x_9 \lor x_{11}$
$C_8 : \lnot x_{11} \lor x_8 \lor \lnot x_{12}$
$C_9 :\ x_{12} \lor \lnot x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \lnot x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\ x_{13} \lor \lnot x_{14} \lor \lnot x_{16}$
$C_{13} : \lnot x_{15} \lor \lnot x_{14} \lor x_{16}$

- No unit clause: decide $\lnot x_1$
- C_1: propagate x_2
- C_2: propagate $\lnot x_3$
- No unit clause: decide x_4
- C_3: propagate $\lnot x_5$
- C_4: propagate x_6
- No unit clause: decide $\lnot x_7$
- C_5: propagate $\lnot x_8$
- C_6: propagate x_9
- No unit clause: decide $\lnot x_{10}$
- C_7: propagate x_{11}
- C_8: propagate $\lnot x_{12}$
- C_9: propagate $\lnot x_{13}$

$\lnot x_1$
x_2/C_1
$\lnot x_3/C_2$
x_4
$\lnot x_5/C_3$
x_6/C_4
$\lnot x_7$
$\lnot x_8/C_5$
x_9/C_6
$\lnot x_{10}$
x_{11}/C_7
$\lnot x_{12}/C_8$
$\lnot x_{13}/C_9$

Conflict analysis, example (1/5)

$C_1 : x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : x_3 \lor x_5 \lor x_6$
$C_5 : x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : x_{13} \lor \neg x_{14} \lor \neg x_{16}$
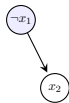$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9
- No unit clause: decide $\neg x_{10}$
- C_7: propagate x_{11}
- C_8: propagate $\neg x_{12}$
- C_9: propagate $\neg x_{13}$
- C_{10}: propagate x_{14}

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9
- No unit clause: decide $\neg x_{10}$
- C_7: propagate x_{11}
- C_8: propagate $\neg x_{12}$
- C_9: propagate $\neg x_{13}$
- C_{10}: propagate x_{14}
- C_{11}: propagate x_{15}

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}

Conflict analysis, example (1/5)

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- No unit clause: decide $\neg x_1$
- C_1: propagate x_2
- C_2: propagate $\neg x_3$
- No unit clause: decide x_4
- C_3: propagate $\neg x_5$
- C_4: propagate x_6
- No unit clause: decide $\neg x_7$
- C_5: propagate $\neg x_8$
- C_6: propagate x_9
- No unit clause: decide $\neg x_{10}$
- C_7: propagate x_{11}
- C_8: propagate $\neg x_{12}$
- C_9: propagate $\neg x_{13}$
- C_{10}: propagate x_{14}
- C_{11}: propagate x_{15}
- C_{12}: propagate $\neg x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$

Conflict analysis, example (1/5)

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

- ▶ No unit clause: decide $\neg x_1$
- ▶ C_1: propagate x_2
- ▶ C_2: propagate $\neg x_3$
- ▶ No unit clause: decide x_4
- ▶ C_3: propagate $\neg x_5$
- ▶ C_4: propagate x_6
- ▶ No unit clause: decide $\neg x_7$
- ▶ C_5: propagate $\neg x_8$
- ▶ C_6: propagate x_9
- ▶ No unit clause: decide $\neg x_{10}$
- ▶ C_7: propagate x_{11}
- ▶ C_8: propagate $\neg x_{12}$
- ▶ C_9: propagate $\neg x_{13}$
- ▶ C_{10}: propagate x_{14}
- ▶ C_{11}: propagate x_{15}
- ▶ C_{12}: propagate $\neg x_{16}$
- ▶ C_{13}: propagate x_{16}
- ▶ Conflict

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
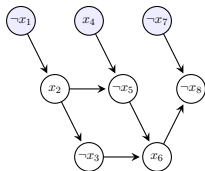$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Conflict graph

$C_1: \quad x_1 \lor x_2$

$C_2: \neg x_2 \lor \neg x_3$

$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$

$C_4: \quad x_3 \lor x_5 \lor x_6$

$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$

$C_6: \neg x_4 \lor x_8 \lor x_9$

$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$

$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$

$C_9: \quad x_{12} \lor \neg x_{13}$

$C_{10}: x_7 \lor x_{12} \lor x_{14}$

$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$

$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$

$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$

$\boxed{\neg x_1}$

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
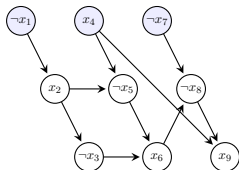$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2 / C_1

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
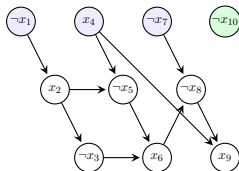$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
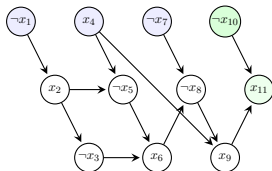$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4

Conflict analysis, example (2/5)

Conflict graph

$C_1 :\quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 :\quad x_3 \vee x_5 \vee x_6$
$C_5 :\quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 :\quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 :\quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} :\quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
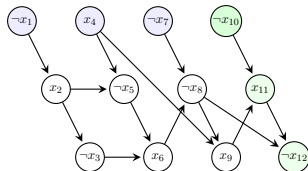$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$

Conflict graph

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
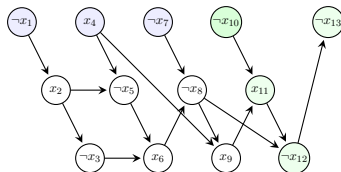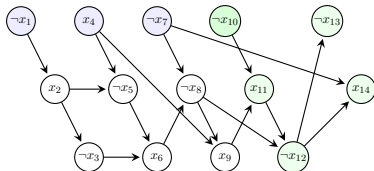$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$

Conflict analysis, example (2/5)

Conflict graph

$C_1 :\ \ x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 :\ \ x_3 \lor x_5 \lor x_6$
$C_5 :\ \ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 :\ \ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 :\ \ x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\ \ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$

Conflict analysis, example (2/5)

Conflict graph

$C_1:$ $x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4:$ $x_3 \lor x_5 \lor x_6$
$C_5:$ $x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7:$ $x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9:$ $x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}:$ $x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$
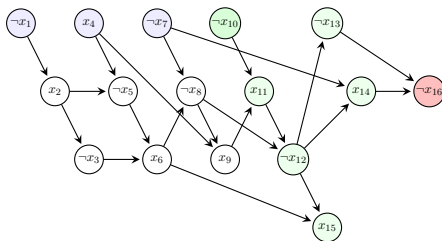

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
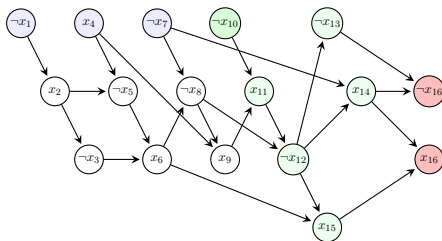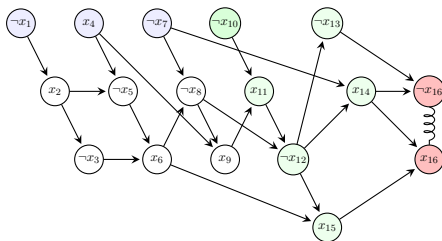$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$

Conflict analysis, example (2/5)

Conflict graph

$C_1 :\ \ x_1 \lor x_2$
$C_2 : \lnot x_2 \lor \lnot x_3$
$C_3 : \lnot x_2 \lor \lnot x_4 \lor \lnot x_5$
$C_4 :\ \ x_3 \lor x_5 \lor x_6$
$C_5 :\ \ x_7 \lor \lnot x_6 \lor \lnot x_8$
$C_6 : \lnot x_4 \lor x_8 \lor x_9$
$C_7 :\ \ x_{10} \lor \lnot x_9 \lor x_{11}$
$C_8 : \lnot x_{11} \lor x_8 \lor \lnot x_{12}$
$C_9 :\ \ x_{12} \lor \lnot x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \lnot x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\ \ x_{13} \lor \lnot x_{14} \lor \lnot x_{16}$
$C_{13} : \lnot x_{15} \lor \lnot x_{14} \lor x_{16}$

$\lnot x_1$
x_2/C_1
$\lnot x_3/C_2$
x_4
$\lnot x_5/C_3$
x_6/C_4
$\lnot x_7$
$\lnot x_8/C_5$
x_9/C_6
$\lnot x_{10}$
x_{11}/C_7
$\lnot x_{12}/C_8$
$\lnot x_{13}/C_9$

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$
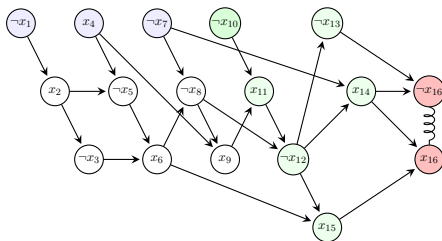

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \lor x_2$
$C_2: \lnot x_2 \lor \lnot x_3$
$C_3: \lnot x_2 \lor \lnot x_4 \lor \lnot x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \lnot x_6 \lor \lnot x_8$
$C_6: \lnot x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \lnot x_9 \lor x_{11}$
$C_8: \lnot x_{11} \lor x_8 \lor \lnot x_{12}$
$C_9: \quad x_{12} \lor \lnot x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \lnot x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \lnot x_{14} \lor \lnot x_{16}$
$C_{13}: \lnot x_{15} \lor \lnot x_{14} \lor x_{16}$

$\lnot x_1$
x_2/C_1
$\lnot x_3/C_2$
x_4
$\lnot x_5/C_3$
x_6/C_4
$\lnot x_7$
$\lnot x_8/C_5$
x_9/C_6
$\lnot x_{10}$
x_{11}/C_7
$\lnot x_{12}/C_8$
$\lnot x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}

Conflict analysis, example (2/5)

Conflict graph

$C_1 :\ \ x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 :\ \ x_3 \lor x_5 \lor x_6$
$C_5 :\ \ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 :\ \ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 :\ \ x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\ \ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
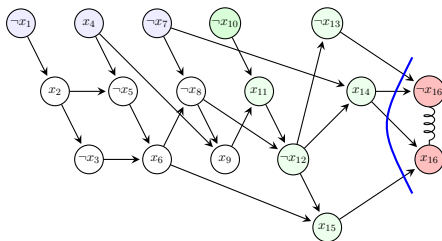$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
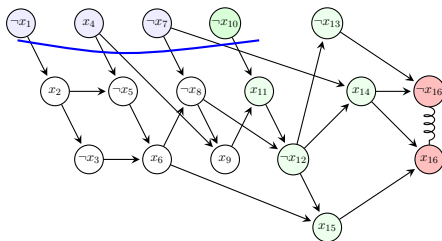$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause
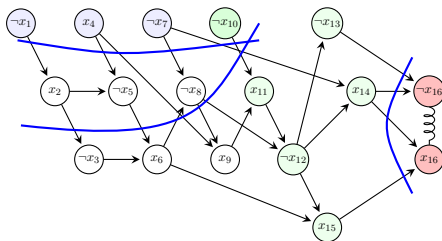
$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)
Conflict graph

C_1 : $x_1 \lor x_2$
C_2 : $\neg x_2 \lor \neg x_3$
C_3 : $\neg x_2 \lor \neg x_4 \lor \neg x_5$
C_4 : $x_3 \lor x_5 \lor x_6$
C_5 : $x_7 \lor \neg x_6 \lor \neg x_8$
C_6 : $\neg x_4 \lor x_8 \lor x_9$
C_7 : $x_{10} \lor \neg x_9 \lor x_{11}$
C_8 : $\neg x_{11} \lor x_8 \lor \neg x_{12}$
C_9 : $x_{12} \lor \neg x_{13}$
C_{10} : $x_7 \lor x_{12} \lor x_{14}$
C_{11} : $\neg x_6 \lor x_{12} \lor x_{15}$
C_{12} : $x_{13} \lor \neg x_{14} \lor \neg x_{16}$
C_{13} : $\neg x_{15} \lor \neg x_{14} \lor x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause
$x_{13} \lor \neg x_{14} \lor \neg x_{15}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause

$x_1 \vee \neg x_4 \vee x_7 \vee x_{10}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1:\quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4:\quad x_3 \vee x_5 \vee x_6$
$C_5:\quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7:\quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9:\quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}:\quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause

$$\neg x_2 \vee \neg x_4 \vee x_5 \vee x_7 \vee x_8 \vee x_{10}$$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
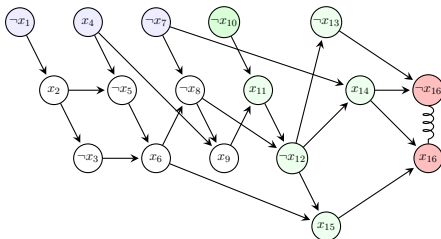$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause
$x_{13} \vee \neg x_{14} \vee \neg x_{15}$
$x_1 \vee \neg x_4 \vee x_7 \vee x_{10}$
$\neg x_2 \vee \neg x_4 \vee x_5 \vee x_7 \vee x_8 \vee x_{10}$
Which one to choose?

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (2/5)

Conflict graph

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
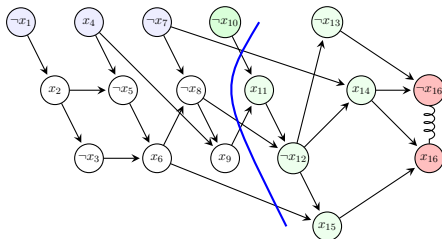$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

Every line separating the conflict from the decisions
defines a logical consequent clause
$x_{13} \vee \neg x_{14} \vee \neg x_{15}$
$x_1 \vee \neg x_4 \vee x_7 \vee x_{10}$
$\neg x_2 \vee \neg x_4 \vee x_5 \vee x_7 \vee x_8 \vee x_{10}$
Which one to choose?
UIP: unique implication point: just ONE green
One variable at conflicting decision level

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (3/5)
Conflict graph, UIP

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
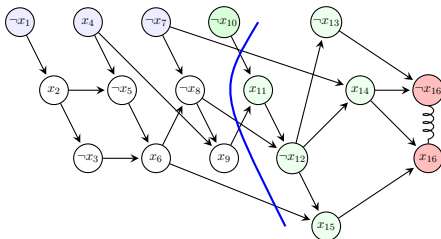$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$
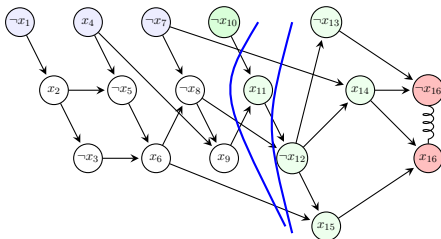

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (3/5)

Conflict graph, UIP

$C_1: \ x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \ x_3 \lor x_5 \lor x_6$
$C_5: \ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \ x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (3/5)

Conflict graph, UIP

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
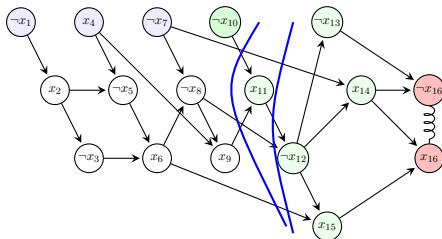$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \lor x_7 \lor x_8 \lor \neg x_9 \lor x_{10}$

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

Conflict analysis, example (3/5)

Conflict graph, UIP

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
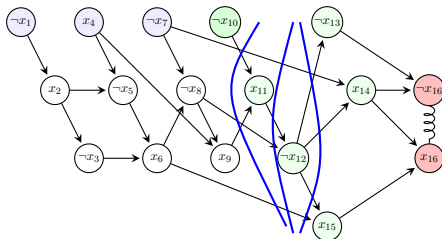$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_9 \vee x_{10}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict graph, UIP

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
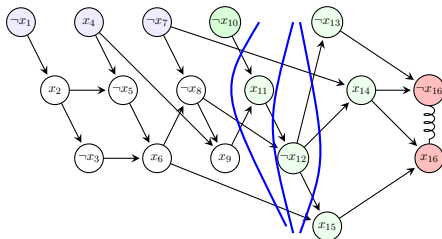$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \lor x_7 \lor x_8 \lor \neg x_9 \lor x_{10}$
$\neg x_6 \lor x_7 \lor x_8 \lor \neg x_{11}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict graph, UIP

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
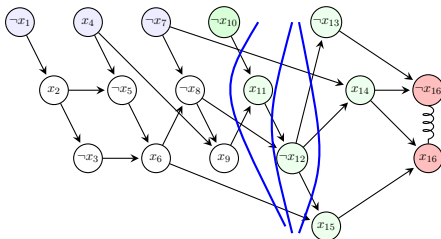$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_9 \vee x_{10}$
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_{11}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict graph, UIP

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_9 \vee x_{10}$
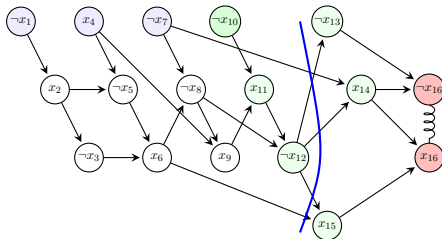$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_{11}$
$\neg x_6 \vee x_7 \vee x_{12}$

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict graph, UIP

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: unique implication point: just ONE green
One variable at the conflicting decision level
There are several of them
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_9 \vee x_{10}$
$\neg x_6 \vee x_7 \vee x_8 \vee \neg x_{11}$
$\neg x_6 \vee x_7 \vee x_{12}$
Take the FUIP (first unique implication point):
closest to conflict

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \vee x_7 \vee x_{12}$

$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \vee x_7 \vee x_{12}$
Resolve 2 clauses with conflicting variable

$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

C_1 : $x_1 \lor x_2$
C_2 : $\neg x_2 \lor \neg x_3$
C_3 : $\neg x_2 \lor \neg x_4 \lor \neg x_5$
C_4 : $x_3 \lor x_5 \lor x_6$
C_5 : $x_7 \lor \neg x_6 \lor \neg x_8$
C_6 : $\neg x_4 \lor x_8 \lor x_9$
C_7 : $x_{10} \lor \neg x_9 \lor x_{11}$
C_8 : $\neg x_{11} \lor x_8 \lor \neg x_{12}$
C_9 : $x_{12} \lor \neg x_{13}$
C_{10} : $x_7 \lor x_{12} \lor x_{14}$
C_{11} : $\neg x_6 \lor x_{12} \lor x_{15}$
C_{12} : $x_{13} \lor \neg x_{14} \lor \neg x_{16}$
C_{13} : $\neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

UIP: One variable at conflicting decision level
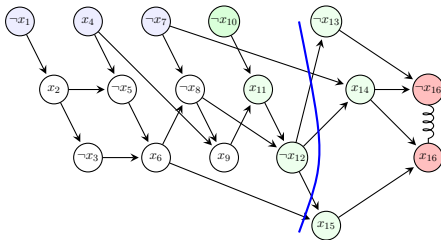FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
Resolve 2 clauses with conflicting variable

$$\frac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \quad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}$$

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
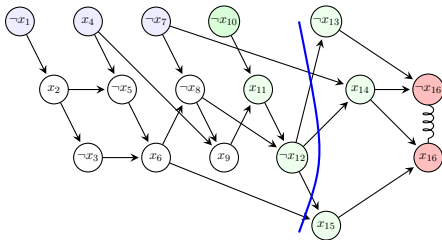Resolve 2 clauses with conflicting variable
Repeatedly eliminate latest "green" var., not last one

$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

$$\frac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \quad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}$$

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

C_1 : $x_1 \lor x_2$
C_2 : $\neg x_2 \lor \neg x_3$
C_3 : $\neg x_2 \lor \neg x_4 \lor \neg x_5$
C_4 : $x_3 \lor x_5 \lor x_6$
C_5 : $x_7 \lor \neg x_6 \lor \neg x_8$
C_6 : $\neg x_4 \lor x_8 \lor x_9$
C_7 : $x_{10} \lor \neg x_9 \lor x_{11}$
C_8 : $\neg x_{11} \lor x_8 \lor \neg x_{12}$
C_9 : $x_{12} \lor \neg x_{13}$
C_{10} : $x_7 \lor x_{12} \lor x_{14}$
C_{11} : $\neg x_6 \lor x_{12} \lor x_{15}$
C_{12} : $x_{13} \lor \neg x_{14} \lor \neg x_{16}$
C_{13} : $\neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
Resolve 2 clauses with conflicting variable
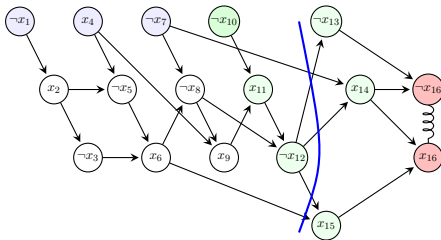Repeatedly eliminate latest "green" var., not last one
Eliminate a variable? Resolve with propagating clause

$$\frac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \quad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}$$

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
Resolve 2 clauses with conflicting variable
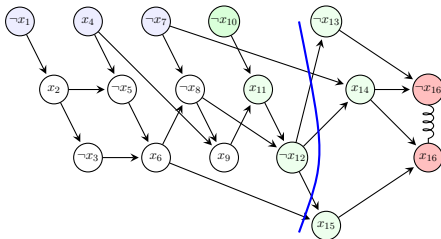Repeatedly eliminate latest "green" var., not last one
Eliminate a variable? Resolve with propagating clause

$$\cfrac{\neg x_6 \lor x_{12} \lor x_{15} \qquad \cfrac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \qquad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}}{\neg x_6 \lor x_{12} \lor x_{13} \lor \neg x_{14}}$$

Conflict analysis, example (4/5)

Conflict graph, computing FUIP

$C_1:\ x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4:\ x_3 \lor x_5 \lor x_6$
$C_5:\ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7:\ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9:\ x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}:\ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
Resolve 2 clauses with conflicting variable
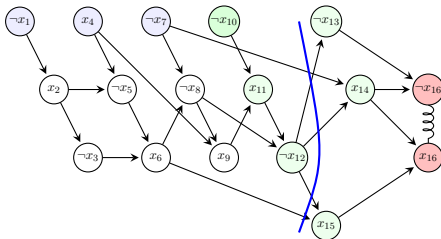Repeatedly eliminate latest "green" var., not last one
Eliminate a variable? Resolve with propagating clause

$$
\cfrac{x_7 \lor x_{12} \lor x_{14} \quad \cfrac{\neg x_6 \lor x_{12} \lor x_{15} \quad \cfrac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \quad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}}{\neg x_6 \lor x_{12} \lor x_{13} \lor \neg x_{14}}}{\neg x_6 \lor x_7 \lor x_{12} \lor x_{13}}
$$

Conflict graph, computing FUIP

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

UIP: One variable at conflicting decision level
FUIP, closest to conflict: $\neg x_6 \lor x_7 \lor x_{12}$
Resolve 2 clauses with conflicting variable
Repeatedly eliminate latest "green" var., not last one
Eliminate a variable? Resolve with propagating clause

$$\cfrac{\cfrac{\cfrac{x_{13} \lor \neg x_{14} \lor \neg x_{16} \quad \neg x_{15} \lor \neg x_{14} \lor x_{16}}{x_{13} \lor \neg x_{14} \lor \neg x_{15}}}{\cfrac{\neg x_6 \lor x_{12} \lor x_{15} \quad}{\neg x_6 \lor x_{12} \lor x_{13} \lor \neg x_{14}}}}{\cfrac{\cfrac{x_7 \lor x_{12} \lor x_{14} \quad}{\neg x_6 \lor x_7 \lor x_{12} \lor x_{13}}}{\cfrac{x_{12} \lor \neg x_{13} \quad}{x_7 \lor \neg x_6 \lor x_{12}}}}$$

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$

The whole picture

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$

$C_1 : \ x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \ x_3 \lor x_5 \lor x_6$
$C_5 : \ x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \ x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \ x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \ x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$

$C_1 : \quad x_1 \lor x_2$
$C_2 : \lnot x_2 \lor \lnot x_3$
$C_3 : \lnot x_2 \lor \lnot x_4 \lor \lnot x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \lnot x_6 \lor \lnot x_8$
$C_6 : \lnot x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \lnot x_9 \lor x_{11}$
$C_8 : \lnot x_{11} \lor x_8 \lor \lnot x_{12}$
$C_9 : \quad x_{12} \lor \lnot x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \lnot x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \lnot x_{14} \lor \lnot x_{16}$
$C_{13} : \lnot x_{15} \lor \lnot x_{14} \lor x_{16}$

▶ Decide and propagate

$\lnot x_1$
x_2 / C_1
$\lnot x_3 / C_2$
x_4
$\lnot x_5 / C_3$
x_6 / C_4

Conflict analysis, example (5/5)

The whole picture

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$

$C_1:\quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4:\quad x_3 \lor x_5 \lor x_6$
$C_5:\quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7:\quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9:\quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}:\quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$

Conflict analysis, example (5/5)
The whole picture

$C_1 :\quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 :\quad x_3 \lor x_5 \lor x_6$
$C_5 :\quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 :\quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 :\quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} :\quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7

$C_1 : \quad x_1 \lor x_2$

$C_2 : \neg x_2 \lor \neg x_3$

$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$

$C_4 : \quad x_3 \lor x_5 \lor x_6$

$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$

$C_6 : \neg x_4 \lor x_8 \lor x_9$

$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$

$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$

$C_9 : \quad x_{12} \lor \neg x_{13}$

$C_{10} : x_7 \lor x_{12} \lor x_{14}$

$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$

$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$

$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$

x_2/C_1

$\neg x_3/C_2$

x_4

$\neg x_5/C_3$

x_6/C_4

$\neg x_7$

$\neg x_8/C_5$

x_9/C_6

$\neg x_{10}$

x_{11}/C_7

$\neg x_{12}/C_8$

Conflict analysis, example (5/5)

The whole picture

$C_1: \quad x_1 \lor x_2$
$C_2: \neg x_2 \lor \neg x_3$
$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4: \quad x_3 \lor x_5 \lor x_6$
$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6: \neg x_4 \lor x_8 \lor x_9$
$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9: \quad x_{12} \lor \neg x_{13}$
$C_{10}: x_7 \lor x_{12} \lor x_{14}$
$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$

$C_1 :\quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 :\quad x_3 \vee x_5 \vee x_6$
$C_5 :\quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 :\quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 :\quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} :\quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

► Decide and propagate

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$

Conflict analysis, example (5/5)

The whole picture

$C_1 :\ \ x_1 \vee x_2$
$C_2 :\ \neg x_2 \vee \neg x_3$
$C_3 :\ \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 :\ \ x_3 \vee x_5 \vee x_6$
$C_5 :\ \ x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 :\ \neg x_4 \vee x_8 \vee x_9$
$C_7 :\ \ x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 :\ \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 :\ \ x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} :\ \ x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

▶ Decide and propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

► Decide and propagate

► Until conflict

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
$\neg x_{10}$
x_{11}/C_7
$\neg x_{12}/C_8$
$\neg x_{13}/C_9$
x_{14}/C_{10}
x_{15}/C_{11}
$\neg x_{16}/C_{12}$
x_{16}/C_{13}

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- Decide and propagate
- Until conflict
- Analyse (compute FUIP)

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

The whole picture

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$

- ▶ Decide and propagate
- ▶ Until conflict
- ▶ Analyse (compute FUIP)
- ▶ Add clause

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

$C_1: \quad x_1 \lor x_2$

$C_2: \neg x_2 \lor \neg x_3$

$C_3: \neg x_2 \lor \neg x_4 \lor \neg x_5$

$C_4: \quad x_3 \lor x_5 \lor x_6$

$C_5: \quad x_7 \lor \neg x_6 \lor \neg x_8$

$C_6: \neg x_4 \lor x_8 \lor x_9$

$C_7: \quad x_{10} \lor \neg x_9 \lor x_{11}$

$C_8: \neg x_{11} \lor x_8 \lor \neg x_{12}$

$C_9: \quad x_{12} \lor \neg x_{13}$

$C_{10}: x_7 \lor x_{12} \lor x_{14}$

$C_{11}: \neg x_6 \lor x_{12} \lor x_{15}$

$C_{12}: \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$

$C_{13}: \neg x_{15} \lor \neg x_{14} \lor x_{16}$

$C_1': x_7 \lor \neg x_6 \lor x_{12}$

▶ Decide and propagate

▶ Until conflict

▶ Analyse (compute FUIP)

▶ Add clause

$\neg x_1$

x_2/C_1

$\neg x_3/C_2$

x_4

$\neg x_5/C_3$

x_6/C_4

$\neg x_7$

$\neg x_8/C_5$

x_9/C_6

$\neg x_{10}$

x_{11}/C_7

$\neg x_{12}/C_8$

$\neg x_{13}/C_9$

x_{14}/C_{10}

x_{15}/C_{11}

$\neg x_{16}/C_{12}$

x_{16}/C_{13}

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$
$C_1' : x_7 \vee \neg x_6 \vee x_{12}$

- ▶ Decide and propagate
- ▶ Until conflict
- ▶ Analyse (compute FUIP)
- ▶ Add clause
- ▶ Backtrack to the point where the clause is propagating
  Often more than just one level

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6
$\neg x_{10}$
x_{11} / C_7
$\neg x_{12} / C_8$
$\neg x_{13} / C_9$
x_{14} / C_{10}
x_{15} / C_{11}
$\neg x_{16} / C_{12}$
x_{16} / C_{13}

Conflict analysis, example (5/5)
The whole picture

$C_1: \quad x_1 \vee x_2$
$C_2: \neg x_2 \vee \neg x_3$
$C_3: \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4: \quad x_3 \vee x_5 \vee x_6$
$C_5: \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6: \neg x_4 \vee x_8 \vee x_9$
$C_7: \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8: \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9: \quad x_{12} \vee \neg x_{13}$
$C_{10}: x_7 \vee x_{12} \vee x_{14}$
$C_{11}: \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12}: \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13}: \neg x_{15} \vee \neg x_{14} \vee x_{16}$
$C_1': x_7 \vee \neg x_6 \vee x_{12}$

- ▶ Decide and propagate
- ▶ Until conflict
- ▶ Analyse (compute FUIP)
- ▶ Add clause
- ▶ Backtrack to the point where the clause is propagating
  Often more than just one level

$\neg x_1$
x_2 / C_1
$\neg x_3 / C_2$
x_4
$\neg x_5 / C_3$
x_6 / C_4
$\neg x_7$
$\neg x_8 / C_5$
x_9 / C_6

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$
$C_1' : x_7 \vee \neg x_6 \vee x_{12}$

- ▶ Decide and propagate
- ▶ Until conflict
- ▶ Analyse (compute FUIP)
- ▶ Add clause
- ▶ Backtrack to the point where the clause is propagating
  Often more than just one level
- ▶ Propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \lor x_2$
$C_2 : \neg x_2 \lor \neg x_3$
$C_3 : \neg x_2 \lor \neg x_4 \lor \neg x_5$
$C_4 : \quad x_3 \lor x_5 \lor x_6$
$C_5 : \quad x_7 \lor \neg x_6 \lor \neg x_8$
$C_6 : \neg x_4 \lor x_8 \lor x_9$
$C_7 : \quad x_{10} \lor \neg x_9 \lor x_{11}$
$C_8 : \neg x_{11} \lor x_8 \lor \neg x_{12}$
$C_9 : \quad x_{12} \lor \neg x_{13}$
$C_{10} : x_7 \lor x_{12} \lor x_{14}$
$C_{11} : \neg x_6 \lor x_{12} \lor x_{15}$
$C_{12} : \quad x_{13} \lor \neg x_{14} \lor \neg x_{16}$
$C_{13} : \neg x_{15} \lor \neg x_{14} \lor x_{16}$
$C_1' : x_7 \lor \neg x_6 \lor x_{12}$

- ▶ Decide and propagate
- ▶ Until conflict
- ▶ Analyse (compute FUIP)
- ▶ Add clause
- ▶ Backtrack to the point where the clause is propagating
  Often more than just one level
- ▶ Propagate

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
x_{12}/C_1'

Conflict analysis, example (5/5)

The whole picture

$C_1 : \quad x_1 \vee x_2$
$C_2 : \neg x_2 \vee \neg x_3$
$C_3 : \neg x_2 \vee \neg x_4 \vee \neg x_5$
$C_4 : \quad x_3 \vee x_5 \vee x_6$
$C_5 : \quad x_7 \vee \neg x_6 \vee \neg x_8$
$C_6 : \neg x_4 \vee x_8 \vee x_9$
$C_7 : \quad x_{10} \vee \neg x_9 \vee x_{11}$
$C_8 : \neg x_{11} \vee x_8 \vee \neg x_{12}$
$C_9 : \quad x_{12} \vee \neg x_{13}$
$C_{10} : x_7 \vee x_{12} \vee x_{14}$
$C_{11} : \neg x_6 \vee x_{12} \vee x_{15}$
$C_{12} : \quad x_{13} \vee \neg x_{14} \vee \neg x_{16}$
$C_{13} : \neg x_{15} \vee \neg x_{14} \vee x_{16}$
$C_1' : x_7 \vee \neg x_6 \vee x_{12}$

▶ Decide and propagate

▶ Until conflict

▶ Analyse (compute FUIP)

▶ Add clause

▶ Backtrack to the point where the clause is propagating
Often more than just one level

▶ Propagate

▶ Decide and propagate, until conflict, analyse,...

$\neg x_1$
x_2/C_1
$\neg x_3/C_2$
x_4
$\neg x_5/C_3$
x_6/C_4
$\neg x_7$
$\neg x_8/C_5$
x_9/C_6
x_{12}/C_1'

CDCL: FUIP and more practical aspects

- ▶ It turns out that FUIP works best,
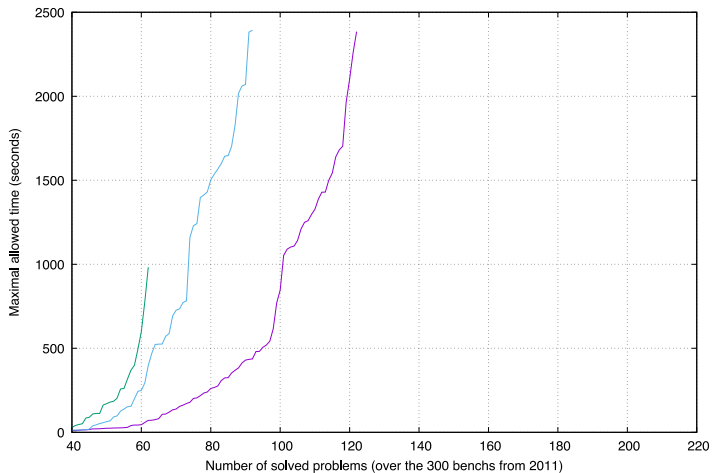 but this is an experimental observation

CDCL: FUIP and more practical aspects

- ▶ It turns out that FUIP works best,
  but this is an experimental observation
- ▶ Heuristics for decisions

CDCL: FUIP and more practical aspects

- It turns out that FUIP works best,
  but this is an experimental observation
- Heuristics for decisions
- Clause removal (and heuristics to evaluate the ones to remove)

CDCL: FUIP and more practical aspects

- ▶ It turns out that FUIP works best,
  but this is an experimental observation
- ▶ Heuristics for decisions
- ▶ Clause removal (and heuristics to evaluate the ones to
  remove)
- ▶ Conflict clause minimization

CDCL: FUIP and more practical aspects

- ▶ It turns out that FUIP works best,
  but this is an experimental observation
- ▶ Heuristics for decisions
- ▶ Clause removal (and heuristics to evaluate the ones to remove)
- ▶ Conflict clause minimization
- ▶ Restarting

CDCL: FUIP and more practical aspects

- ▶ It turns out that FUIP works best, but this is an experimental observation
- ▶ Heuristics for decisions
- ▶ Clause removal (and heuristics to evaluate the ones to remove)
- ▶ Conflict clause minimization
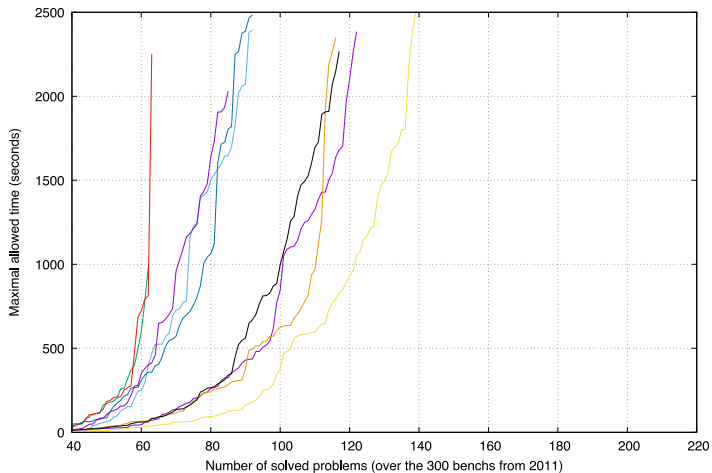- ▶ Restarting
- ▶ Preprocessing

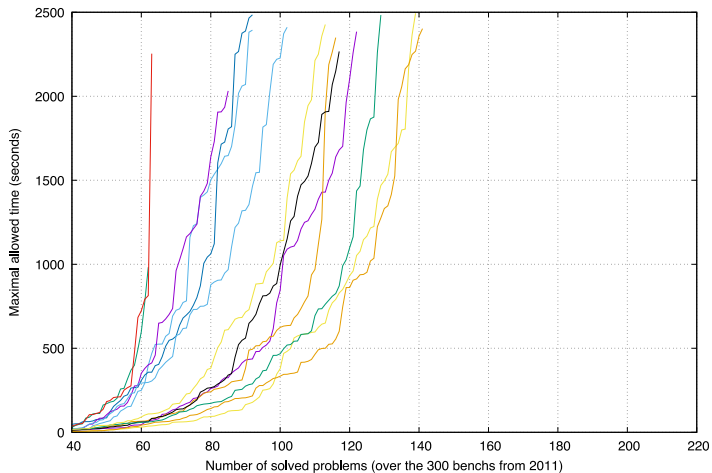# CDCL: evolution of solvers

2002

[Source: Laurent Simon]

CDCL: evolution of solvers

2003

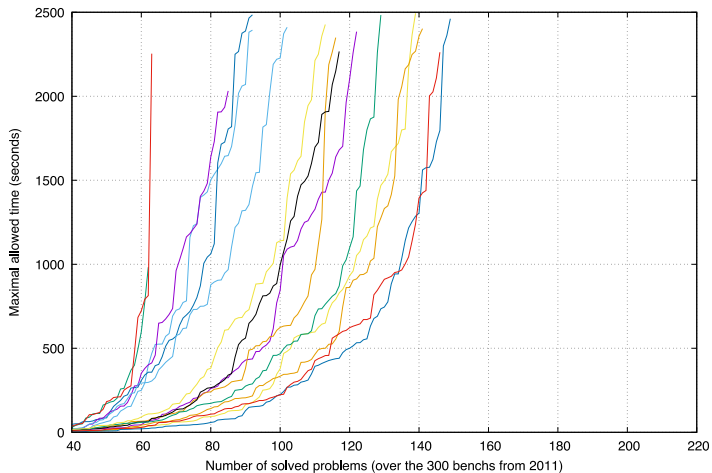[Source: Laurent Simon]

CDCL: evolution of solvers
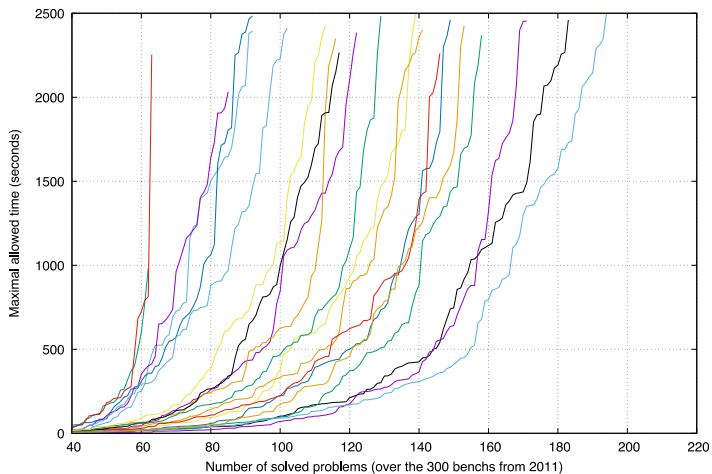

2005

[Source: Laurent Simon]

CDCL: evolution of solvers

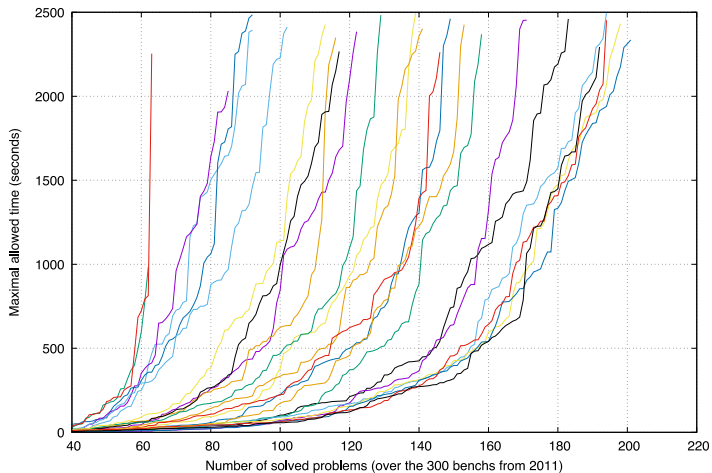2007

[Source: Laurent Simon]

CDCL: evolution of solvers
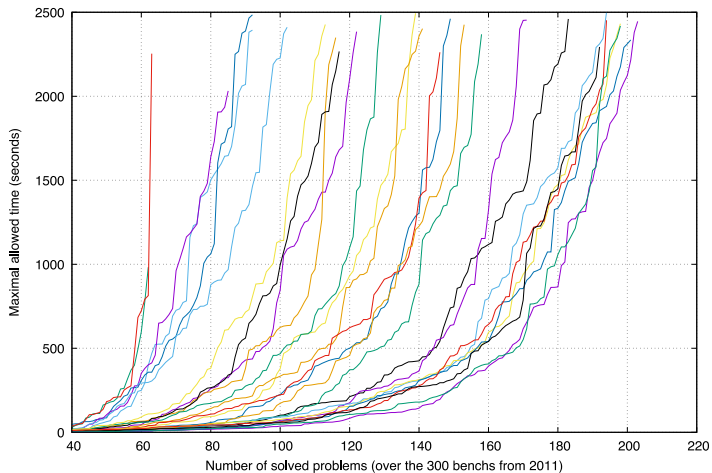

2009

[Source: Laurent Simon]

CDCL: evolution of solvers

2011

[Source: Laurent Simon]

CDCL: evolution of solvers
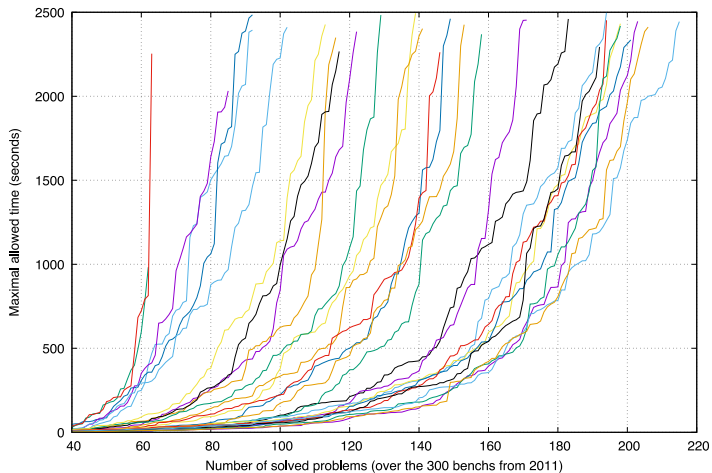

2014

[Source: Laurent Simon]

CDCL: evolution of solvers

2016

[Source: Laurent Simon]

CDCL: evolution of solvers

Winners

[Source: Laurent Simon]

Conclusion

- only a brief glimpse on the practical aspects of SAT solving

Conclusion

- only a brief glimpse on the practical aspects of SAT solving
- many more tricks, hacks, techniques, heuristics, and even theories

Conclusion

- only a brief glimpse on the practical aspects of SAT solving
- many more tricks, hacks, techniques, heuristics, and even theories
- a good SAT solver: good set of those tricks, mixed in the right amount and the right order, low level optimization

Conclusion

- only a brief glimpse on the practical aspects of SAT solving
- many more tricks, hacks, techniques, heuristics, and even theories
- a good SAT solver: good set of those tricks, mixed in the right amount and the right order, low level optimization
- oldest logic problem, still very active research subject (particularly since mid 90s)

Conclusion

- only a brief glimpse on the practical aspects of SAT solving
- many more tricks, hacks, techniques, heuristics, and even theories
- a good SAT solver: good set of those tricks, mixed in the right amount and the right order, low level optimization
- oldest logic problem, still very active research subject (particularly since mid 90s)

Conclusion

- only a brief glimpse on the practical aspects of SAT solving
- many more tricks, hacks, techniques, heuristics, and even theories
- a good SAT solver: good set of those tricks, mixed in the right amount and the right order, low level optimization
- oldest logic problem, still very active research subject (particularly since mid 90s)

Laurent Simon (Glucose)

We know how to built efficient SAT Solvers but we can hardly explain their power

SAT Solving: further reading/learning

- Eén, Sörensson: MiniSAT 2011
- Marijn Heule:
  `http://www.sc-square.org/CSA/school/lectures.html`
- Armin Biere, Marijn Heule, Hans van Maaren and Toby Walsh editors, Handbook on Satisfiability. IOS Press, February 2009.
- SAT/SMT/AR summer schools

SAT solvers input format: DIMACS

- input: CNF. File extension `.cnf`
    - Boolean variable: number ≥ 1
    - literal either positive (represented by positive number)
                      negative (represented by negative number)
    - clause: series of numbers seperated by spaces, terminated by 0
    - cnf: series of clauses
    - file starts with `p cnf X Y` (X variables, Y clauses)
    - comments start by `c`

> ### EXEMPLE
>
> $p \vee q$
> $p \vee r$
> $\neg q \vee \neg r$
> $\neg p$

SAT solvers input format: DIMACS

- input: CNF. File extension `.cnf`
    - Boolean variable: number ≥ 1
    - literal either positive (represented by positive number)
      negative (represented by negative number)
    - clause: series of numbers seperated by spaces, terminated by 0
    - cnf: series of clauses
    - file starts with `p cnf X Y` (X variables, Y clauses)
    - comments start by `c`

EXEMPLE

$p \longrightarrow 1$, $q \longrightarrow 2$, $r \longrightarrow 3$

$p \vee q$
$p \vee r$
$\neg q \vee \neg r$
$\neg p$

SAT solvers input format: DIMACS

- input: CNF. File extension `.cnf`
    - Boolean variable: number ≥ 1
    - literal either positive (represented by positive number)
              negative (represented by negative number)
    - clause: series of numbers seperated by spaces, terminated by 0
    - cnf: series of clauses
    - file starts with `p cnf X Y` (X variables, Y clauses)
    - comments start by `c`

EXEMPLE

$p \longrightarrow 1$, $q \longrightarrow 2$, $r \longrightarrow 3$

$p \vee q$	$\longrightarrow$	`1 2 0`
$p \vee r$	$\longrightarrow$	`1 3 0`
$\neg q \vee \neg r$	$\longrightarrow$	`-2 -3 0`
$\neg p$	$\longrightarrow$	`-1 0`

SAT solvers input format: DIMACS

- input: CNF. File extension `.cnf`
    - Boolean variable: number ≥ 1
    - literal either positive (represented by positive number)
      negative (represented by negative number)
    - clause: series of numbers seperated by spaces, terminated by 0
    - cnf: series of clauses
    - file starts with `p cnf X Y` (X variables, Y clauses)
    - comments start by `c`

EXEMPLE

```
p ⟶ 1, q ⟶ 2, r ⟶ 3
                p cnf 3 4
 p ∨ q      ⟶   1 2 0
 p ∨ r      ⟶   1 3 0
 ¬q ∨ ¬r    ⟶   -2 -3 0
 ¬p         ⟶   -1 0
```

Sudoku (1/3)

at line i, column j, is number x ($p_{i,j,x}$)

Sudoku (1/3)

at line i, column j, is number x $(p_{i,j,x})$

▶ Number 1 is at line 1, column 1

Sudoku (1/3)

1			
			3
		2	
	2		

at line i, column j, is number x ($p_{i,j,x}$)

▶ Number 1 is at line 1, column 1

$p_{1,1,1}$

Sudoku (1/3)

at line i, column j, is number x ($p_{i,j,x}$)

▶ Number 1 is at line 1, column 1

$p_{1,1,1}$

▶ Number 3 is at line 2, column 4

Sudoku (1/3)

at line i, column j, is number x $(p_{i,j,x})$

▶ Number 1 is at line 1, column 1

$p_{1,1,1}$

▶ Number 3 is at line 2, column 4

$p_{2,4,3}$

Sudoku (1/3)

at line i, column j, is number x ($p_{i,j,x}$)

▶ Number 1 is at line 1, column 1

$p_{1,1,1}$

▶ Number 3 is at line 2, column 4

$p_{2,4,3}$

▶ At location $(1, 2)$, there is at most one number

Sudoku (1/3)

at line i, column j, is number x ($p_{i,j,x}$)

- ▶ Number 1 is at line 1, column 1

  $p_{1,1,1}$

- ▶ Number 3 is at line 2, column 4

  $p_{2,4,3}$

- ▶ At location $(1, 2)$, there is at most one number

  $\neg p_{1,2,1} \vee \neg p_{1,2,2}$

  $\neg p_{1,2,1} \vee \neg p_{1,2,3}$

  $\neg p_{1,2,1} \vee \neg p_{1,2,4}$

  $\neg p_{1,2,2} \vee \neg p_{1,2,3}$

  $\neg p_{1,2,2} \vee \neg p_{1,2,4}$

  $\neg p_{1,2,3} \vee \neg p_{1,2,4}$

Sudoku (2/3)

at line i, column j, is number x ($p_{i,j,x}$)

at line i, column j, is number x ($p_{i,j,x}$)

▶ At location $(1, 2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)

Sudoku (2/3)

at line i, column j, is number x $(p_{i,j,x})$

▶ At location $(1, 2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)
$p_{1,2,1} \lor p_{1,2,2} \lor p_{1,2,3} \lor p_{1,2,4}$

Sudoku (2/3)

1			
			3
		2	
	2		

at line i, column j, is number x ($p_{i,j,x}$)

▶ At location $(1,2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)
$p_{1,2,1} \vee p_{1,2,2} \vee p_{1,2,3} \vee p_{1,2,4}$

▶ Number 1 should be somewhere at line 2 (repeat $\forall$ number, line...)

Sudoku (2/3)

1			
			3
		2	
	2		

at line i, column j, is number x ($p_{i,j,x}$)

▶ At location $(1,2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)
$p_{1,2,1} \vee p_{1,2,2} \vee p_{1,2,3} \vee p_{1,2,4}$

▶ Number 1 should be somewhere at line 2 (repeat $\forall$ number, line...)
$p_{2,1,1} \vee p_{2,2,1} \vee p_{2,3,1} \vee p_{2,4,1}$

Sudoku (2/3)

at line i, column j, is number x ($p_{i,j,x}$)

▶ At location $(1, 2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)
$p_{1,2,1} \lor p_{1,2,2} \lor p_{1,2,3} \lor p_{1,2,4}$

▶ Number 1 should be somewhere at line 2 (repeat $\forall$ number, line. . .)
$p_{2,1,1} \lor p_{2,2,1} \lor p_{2,3,1} \lor p_{2,4,1}$

▶ Number 1 should be at most once at line 1

Sudoku (2/3)

1			
			3
		2	
	2		

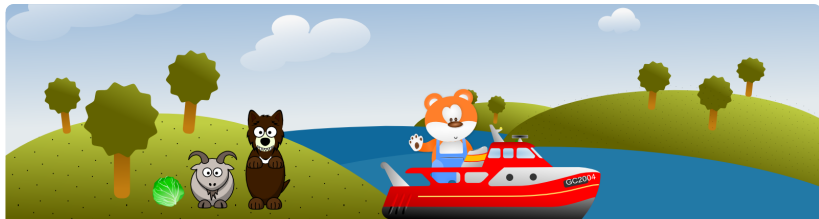at line i, column j, is number x $(p_{i,j,x})$

▶ At location $(1,2)$, there is either 1, 2, 3, or 4 (repeat $\forall$ location)
$p_{1,2,1} \vee p_{1,2,2} \vee p_{1,2,3} \vee p_{1,2,4}$

▶ Number 1 should be somewhere at line 2 (repeat $\forall$ number, line...)
$p_{2,1,1} \vee p_{2,2,1} \vee p_{2,3,1} \vee p_{2,4,1}$

▶ Number 1 should be at most once at line 1
$\neg p_{1,1,1} \vee \neg p_{1,2,1}$
$\neg p_{1,1,1} \vee \neg p_{1,3,1}$
$\neg p_{1,1,1} \vee \neg p_{1,4,1}$
$\neg p_{1,2,1} \vee \neg p_{1,3,1}$
$\neg p_{1,2,1} \vee \neg p_{1,4,1}$
$\neg p_{1,3,1} \vee \neg p_{1,4,1}$

Sudoku (3/3)

Demo / Practical session

▶ https://members.loria.fr/PFontaine/sudoku-pack.zip

The Wolf, the Goat, and the Cabbage (1/5)

A farmer wants to cross a river in his small boat, with a wolf, a goat and a cabbage. He should make sure:

- ▶ to only take one animal or object with him, the boat being so small
- ▶ not to leave the wolf and the goat alone (or no more goat)
- ▶ not to leave the goat and the cabbage alone (or no more cabbage)

Is this possible? With how many crossings?

Use logic to encode the problem.

The Wolf, the Goat, and the Cabbage (2/5)

Use logic to encode the problem.
Four variables (that can be true or false):

- f farmer
- w wolf
- g goat
- c cabbage

E.g. f is true if f is on the left side, false if on the right side

- We start with
- We want to finish with

The Wolf, the Goat, and the Cabbage (2/5)

Use logic to encode the problem.
Four variables (that can be true or false):

- f farmer
- w wolf
- g goat
- c cabbage

E.g. f is true if f is on the left side, false if on the right side

- We start with $init =_{\mathrm{def}} f \wedge w \wedge g \wedge c$
- We want to finish with

The Wolf, the Goat, and the Cabbage (2/5)

Use logic to encode the problem.
Four variables (that can be true or false):

- f farmer
- w wolf
- g goat
- c cabbage

E.g. f is true if f is on the left side, false if on the right side

- We start with $init =_{\text{def}} f \wedge w \wedge g \wedge c$
- We want to finish with $fin =_{\text{def}} \neg f \wedge \neg w \wedge \neg g \wedge \neg c$

How to express there is some danger

How to express there is some danger

A state is *dangerous* if the wolf and the goat (or the goat and the cabbage) are on one bank, and the farmer on the other

Formally:

How to express there is some danger

A state is *dangerous* if the wolf and the goat (or the goat and the cabbage) are on one bank, and the farmer on the other

Formally:

$$\textit{danger} \quad =_{\text{def}} \quad \begin{aligned} &\big((w \equiv g) \wedge (w \equiv \neg f)\big) \\ \vee \ &\big((g \equiv c) \wedge (g \equiv \neg f)\big) \end{aligned}$$

To find out if it is possible to find a solution with n crossing, we will use $n + 1$ copies of the variables f_i, w_i, g_i, c_i.

First, let's write the formula corresponding the i-th crossing of the farmer.

$$cross_i \quad =_{\text{def}} \quad \wedge \Big(\begin{array}{c} \\ \vee \\ \vee \end{array} \qquad\qquad \Big)$$

The Wolf, the Goat, and the Cabbage (4/5)

To find out if it is possible to find a solution with n crossing, we will use $n+1$ copies of the variables f_i, w_i, g_i, c_i.

First, let's write the formula corresponding the i-th crossing of the farmer.

▶ The farmer is crossing so f_{i+1} and $\neg f_i$ should be different

$$cross_i \quad =_{\text{def}} \quad \wedge\Big(\begin{array}{c} \\ \vee \\ \vee \end{array} \qquad\qquad\qquad \Big)$$

The Wolf, the Goat, and the Cabbage (4/5)

To find out if it is possible to find a solution with n crossing, we will use $n + 1$ copies of the variables f_i, w_i, g_i, c_i.

First, let's write the formula corresponding the i-th crossing of the farmer.

▶ The farmer is crossing so f_{i+1} and $\neg f_i$ should be different

$$cross_i \quad =_{\text{def}} \quad \wedge \Big(\begin{array}{l} (f_{i+1} \equiv \neg f_i) \\ \\ \vee \\ \vee \end{array} \Big)$$

The Wolf, the Goat, and the Cabbage (4/5)

To find out if it is possible to find a solution with n crossing, we will use $n + 1$ copies of the variables f_i, w_i, g_i, c_i.

First, let's write the formula corresponding the i-th crossing of the farmer.

- The farmer is crossing so f_{i+1} and $\neg f_i$ should be different
- Only one animal/object is changing bank; two (or more) variables among w, g, c should stay the same

$$\text{cross}_i \quad =_{\text{def}} \qquad (f_{i+1} \equiv \neg f_i)$$
$$\wedge \Big($$
$$\vee$$
$$\vee \qquad\qquad\qquad\qquad \Big)$$

The Wolf, the Goat, and the Cabbage (4/5)

To find out if it is possible to find a solution with n crossing, we will use $n+1$ copies of the variables f_i, w_i, g_i, c_i.

First, let's write the formula corresponding the i-th crossing of the farmer.

- The farmer is crossing so f_{i+1} and $\neg f_i$ should be different
- Only one animal/object is changing bank; two (or more) variables among w, g, c should stay the same

$$
\begin{aligned}
\textit{cross}_i \quad =_{\text{def}} \quad & \left(f_{i+1} \equiv \neg f_i \right) \\
\wedge \Big(\quad & \left((w_{i+1} \equiv w_i) \wedge (g_{i+1} \equiv g_i) \right) \\
\vee \quad & \left((w_{i+1} \equiv w_i) \wedge (c_{i+1} \equiv c_i) \right) \\
\vee \quad & \left((g_{i+1} \equiv g_i) \wedge (c_{i+1} \equiv c_i) \right) \Big)
\end{aligned}
$$

We want to write a formula to encode solutions in n crossings

$\wedge$

$\wedge$

$\wedge$

We want to write a formula to encode solutions in n crossings

► Starting and ending state have been defined

$$init_1 \wedge fin_{n+1}$$
$$\wedge$$
$$\wedge$$

The Wolf, the Goat, and the Cabbage (5/5)

We want to write a formula to encode solutions in n crossings

- ▶ Starting and ending state have been defined
- ▶ Two successive states should correspond to one crossing

$$
\begin{aligned}
& \textit{init}_1 \wedge \textit{fin}_{n+1} \\
\wedge \quad & \textit{cross}_1 \wedge \textit{cross}_2 \wedge \ldots \wedge \textit{cross}_n \\
\wedge \quad &
\end{aligned}
$$

The Wolf, the Goat, and the Cabbage (5/5)

We want to write a formula to encode solutions in n crossings

- ▶ Starting and ending state have been defined
- ▶ Two successive states should correspond to one crossing
- ▶ No state should be dangerous

$$init_1 \wedge fin_{n+1}$$
$$\wedge \quad cross_1 \wedge cross_2 \wedge \ldots \wedge cross_n$$
$$\wedge \quad \neg danger_1 \wedge \neg danger_2 \wedge \ldots \wedge \neg danger_{n+1}$$

The Wolf, the Goat, and the Cabbage (5/5)

We want to write a formula to encode solutions in n crossings

- ▶ Starting and ending state have been defined
- ▶ Two successive states should correspond to one crossing
- ▶ No state should be dangerous

$$\begin{aligned}
& \mathit{init}_1 \wedge \mathit{fin}_{n+1} \\
\wedge \quad & \mathit{cross}_1 \wedge \mathit{cross}_2 \wedge \ldots \wedge \mathit{cross}_n \\
\wedge \quad & \neg\mathit{danger}_1 \wedge \neg\mathit{danger}_2 \wedge \ldots \wedge \neg\mathit{danger}_{n+1}
\end{aligned}$$

A SAT solver can find out that there is no solution in 4 traversals, but that 6 traversals are enough.