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Title: “Everyone ate all the baguettes”
Topics: Context-free grammar, predicate logic, unintended consequences

One night in a dream, I was speaking to Gottlob Frege, the inventor of predicate logic. We had gone to a friend’s house to eat soup and baguettes, but when we arrived there were no baguettes. Mr. Frege said, quite reasonably, “Someone ate all the baguettes.”

It was, of course, Algernon Monkrief, from The Importance of Being Earnest.

[picture: someone eats all the baguettes]

I said, “Perhaps everyone ate all the baguettes,” because in logic you can say things like that. Frege said that was absurd, and I woke up.

[picture: everyone eats all the baguettes]

There is a linguistic idea of context-free grammar, which comes from the fact that English sentences needn’t be said in context. You can just say a sentence out of nowhere, and people have to figure out what you mean. For example, the sentences “colorless green thoughts sleep furiously” and “more people have been to Russia than I have” don’t make sense without context, but are still grammatically correct. At a glance, “everyone ate all the baguettes” seems to be one of those.

Recently, I’ve been learning predicate logic from Mr. Frege, which allows us to decompose the statement. In a later dream, he told me the translation:

> “For EVERY human h, for EVERY baguette b, h ate b.
> or
> (ALL x)(ALL y)[ IF (x is human AND y is a baguette) THEN (x ate y)]

“There’s nothing wrong with this as such,” he said, “but no 2 people can eat the same baguette. If you eat half and I eat half, we’re each eating half a baguette, which is a different thing.”

“What if,” I asked, “you ate a baguette, and thousands of years later, all the atoms of it found their way into a wheat field, were made into flower, cooked, and turned into a brand new baguette for me to eat?”

“You said it yourself,” answered Frege. “That is a new baguette. So we add:

> “For EVERY baguette b, if there EXISTS a human h where h ate b, there does NOT EXIST a human i where i ate b.
> or
> (ALL x)(IF (x is a baguette AND (EXISTS y) (y is human AND y ate x)) THEN NOT (EXISTS z) (z is human AND z ate x))

“So it’s a contradiction,” I said.

“Wrong again! Remember, every human must have eaten every baguette. But for a given baguette, no 2 humans can have eaten it. Do you see the resolution?”

I said, “there is only a single human?”

“That is one resolution,” said Gottlob, “but when you made the original statement, I could clearly see a dozen people eating soup. So I knew you couldn’t have meant that. No, the correct solution is that *there were never any baguettes to begin with*.”

“Of course!” I said. “Everyone must have eaten all of them, but no 2 people could have eaten any. Because there weren’t any!”

Gottlob said, “Yes. However, there most certainly were. I know my friend quite well, and it would take a serious emergency for him to not prepare baguettes. So when you said everyone ate all the baguettes, I understood you to mean there had been a serious emergency, which I said was absurd. Do you understand?”

I did. My statement had implied something I did not intend. Logic is very dangerous in this way, because it’s not always obvious when statements will lead to absurdities. Here is a historical example:

In the 1870s, Georg Cantor invented a branch of mathematics called set theory. In set theory, there are objects called sets that contain things. They can contain other objects, other sets, or even themselves. They can even be defined in funky ways: for example, you might have the set of every number, or every baguette, or even the set of all sets. Mathematicians used set theory for many things over dozens of years, but in 1901 Bertrand Russel discovered that at the heart of set theory was a contradiction.

Take R, the set of all sets which do not contain themselves. It contains a set which has only a tree, but not a set which has both a tree and itself. Indeed, *every* set that doesn’t have itself inside it must be contained within R.
Here is the question: does R contain itself?

> If it does contain itself, then it cannot contain itself.
> If it does not contain itself, it must.

This is a contradiction, so by the principle of explosion (look it up!), it actually implies *all* statements. Since it came from Cantor’s set theory, that means his theory implies *all* statements too, which makes it useless as a form of math. Set theory today is quite strict on what is allowed to be a set, so it doesn’t have any contradictions. No known ones, anyway.