Partial Fractions

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9) $\frac{14x-41}{x^2-5x+4}$

factor denominator

$$\frac{14x-41}{x^2-5x+4} = \frac{14x-41}{(x-4)(x-1)}$$

begin decomposing

$$\ldots = \frac{n_1}{x-4} + \frac{n_2}{x-1}$$

multiply both sides by the original denominator

$$\ldots = n_1(x-1) + n_2(x-4)$$

$$14x - 41 = n_1 x - n_1 + n_2 x - 4n_2$$

set up matrix equations

$$14 = n_1 + n_2$$

$$-41 = -n_1 - 4n_2$$

$$\left(\begin{array}{cc|c} 1 & 1 & 14 \\ -1 & -4 & -41 \end{array}\right)$$

rref

$$\left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 9 \end{array}\right)$$

$$\ldots = \frac{5}{x-4} + \frac{9}{x-1}$$

10) $\frac{48-8x}{x^3-8x^2+16x}$

factor

$$\ldots = \frac{-8(x-6)}{x(x-4)^2}$$

begin decomposing

$$\ldots = \frac{n_1}{x} + \frac{n_2}{x-4} + \frac{n_3}{(x-4)^2}$$

multiply both sides by the original denominator

$$\ldots = n_1(x-4)^2 + n_2x(x-4) + n_3x$$

distribute

$$0x^2 - 8x + 48 = n_1x^2 - 8n_1x + 16n_1 + n_2x^2 - 4n_2x + n_3x$$

group by x to set up matrix

$$0 = n_1 + n_2$$

$$-8 = -8n_1 - 4n_2 + n_3$$

$$48 = 16n_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -8 & -4 & 1 & -8 \\ 16 & 0 & 0 & 48 \end{array}\right)$$

rref

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\ldots = \frac{3}{x} + \frac{-3}{x-4} + \frac{4}{(x-4)^2}$$

11) $\frac{5x^2+7x+11}{x^3+4x^2+5x+20}$

factor

$$\ldots = \frac{5x^2+7x+11}{(x+4)(x^2+5)}$$

begin decomposing (note: x^2+5 is a special case)

$$\ldots = \frac{n_1}{x+4} + \frac{n_2x+n_3}{x^2+5}$$

multiply both sides by the original denominator

$$\ldots = n_1(x^2+5) + (n_2x+n_3)(x+4)$$

$$\ldots = n_1x^2 + 5n_1 + n_2x^2 + 4n_2x + n_3x + 4n_3$$

group by x to set up matrix equations

$$5 = n_1 + n_2$$

$$7 = 4n_2 + n_3$$

$$11 = 5n_1 + 4n_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 4 & 1 & 7 \\ 5 & 0 & 4 & 11 \end{array}\right)$$

rref

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\ldots = \frac{3}{x+4} + \frac{2x-1}{x^2+5}$$

12) $\frac{6x^3-5x^2+14x-19}{(x^2+3)^3}$

already factored

check for special cases. begin decomposing. (hint: use binomial expansion)

$$\ldots = \frac{n_1 x}{(x^2+3)^3} + \frac{n_2 x}{(x^2+3)^2} + \frac{n_3}{(x^2+3)^3} + \frac{n_4}{(x^2+3)^2}$$

multiply both sides by the original denominator

$$\ldots = n_1 x + n_2 x(x^2+3) + n_3 + n_4(x^2+3)$$

distribute

$$\ldots = n_1 x + n_2 x^3 + 3n_2 x + n_3 + n_4 x^2 + 3n_4$$

group by x to set up matrix equations

$$6 = n_2$$

$$-5 = n_4$$

$$14 = n_1 + 3n_2$$

$$-19 = n_3 + 3n_4$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 & -5 \\ 1 & 3 & 0 & 0 & 14 \\ 0 & 0 & 1 & 3 & -19 \end{array}\right)$$

rref

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right)$$

$$\ldots = \frac{-4x}{(x^2+3)^3} + \frac{6x}{(x^2+3)^2} + \frac{-4}{(x^2+3)^3} + \frac{-5}{(x^2+3)^2}$$