Economics Needs A Scientific Revolution

Mark Alexiuk

Physics Department Colloquium

University of Winnipeg

March 15, 2024

- The UW mascot is Wes Lee Coyote
- Photo by Alan Emery on Unsplash

Abstract and Overview

Jean-Philippe Bouchaud, a professor of physics and head of research for a global asset management company, has called for change, a scientific revolution no less, in economics.

Two of his articles are reviewed.

His call to action is examined with examples from finance and reference to physics.

- Make realistic models!
- Measure more things!
- Use numerical simulations!

Bio

Mark is employed as a senior developer with TMX Vettafi.

He has worked at various startups (in Winnipeg) as a Research Scientist, Engineering Manager and CTO.

He graduated with a PhD in electrical and computer engineering (from that other university) with a focus on exploratory data analysis in bioinformatics.

Mark has roots in Manitoba.

NRC IBD

National Research Council
Institure for Biodiagnostics

Research and develop noninvasive medical diagnostic technologies to increase prospects for prevention, earlier diagnosis, improved treatment and prognosis of diseases.

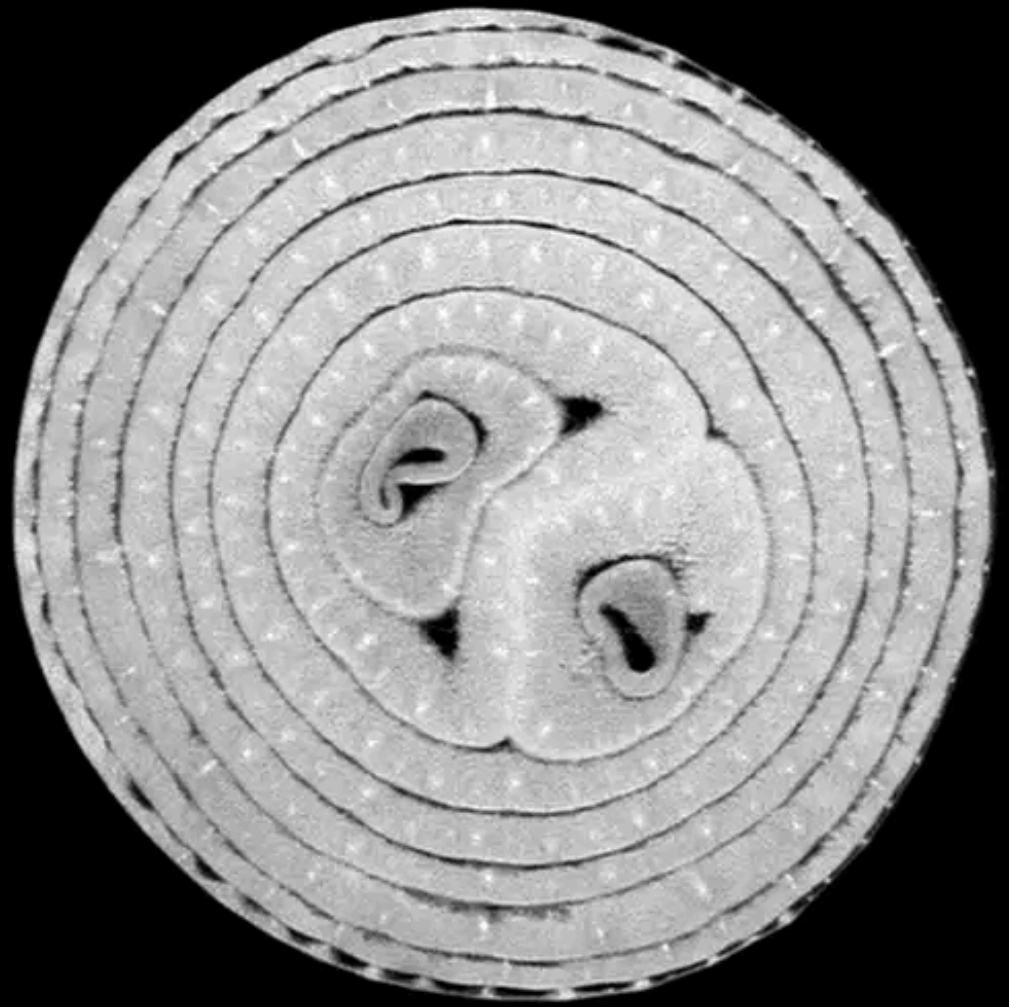
435 Ellice Avenue, Winnipeg, Manitoba

IMRIS

- founded in 2005
- spun out of intra-operative imaging research at NRC IBD.
- Wpg Free Press 1993: "Everyone thought Dr John Saunders was crazy."

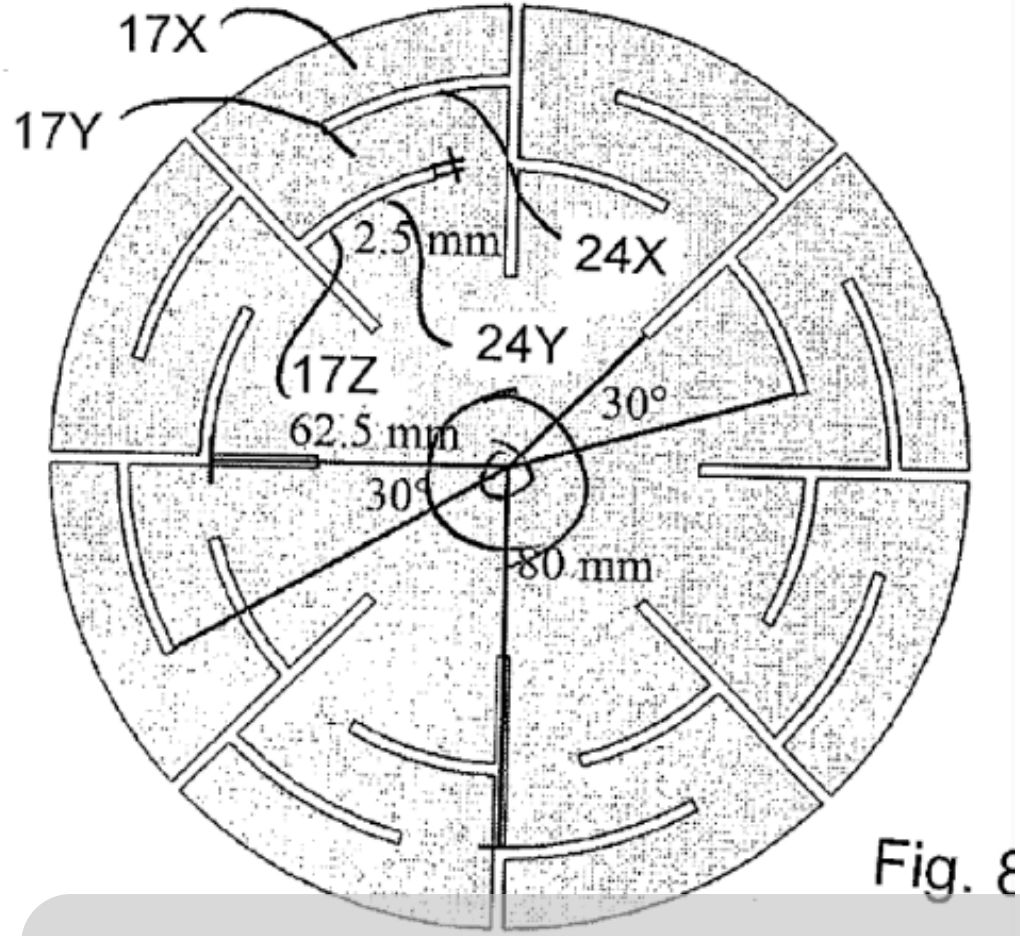
Worked at a medical device company for few years.

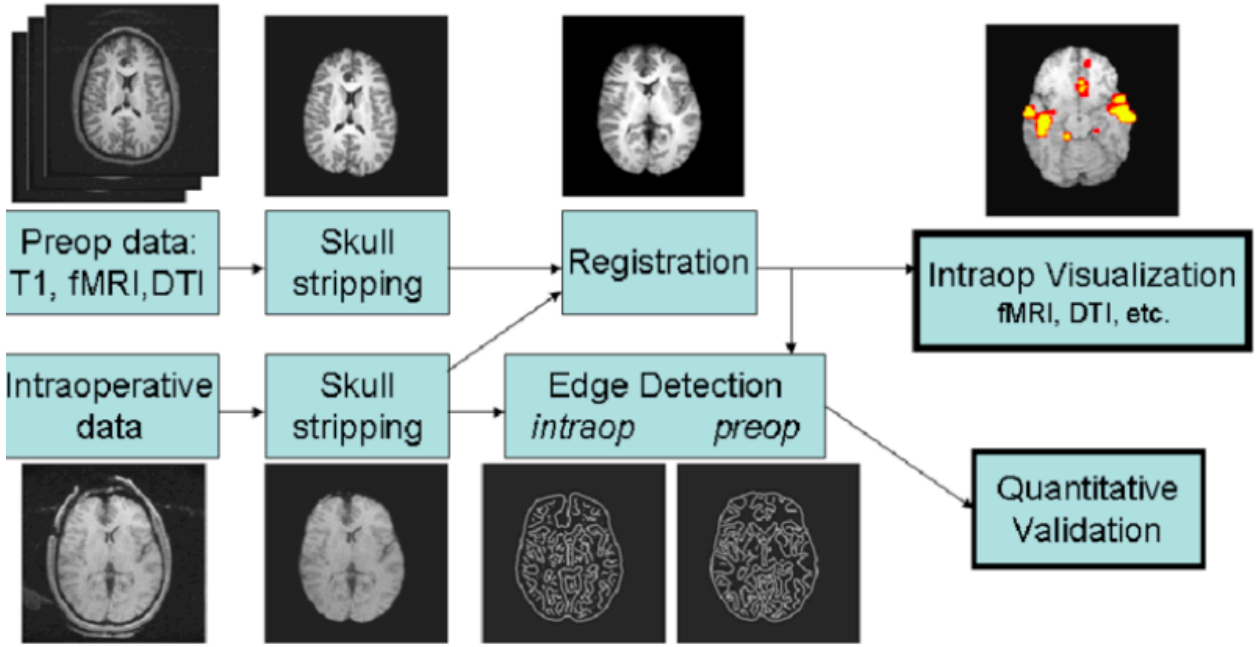
We put some fun thigs in the MRI scanners



Yes, we mostly had permission.

Left: Figure from patent for MRI compatible electrodes. M.Lang et al





Met, collaborated and published with folks from different fields.

Right: Figure from intra-operative image registration. Hans et al

Siemens MRI Factory in Erlangen, Germany

Went on a trip.

factory floor

assembly lines

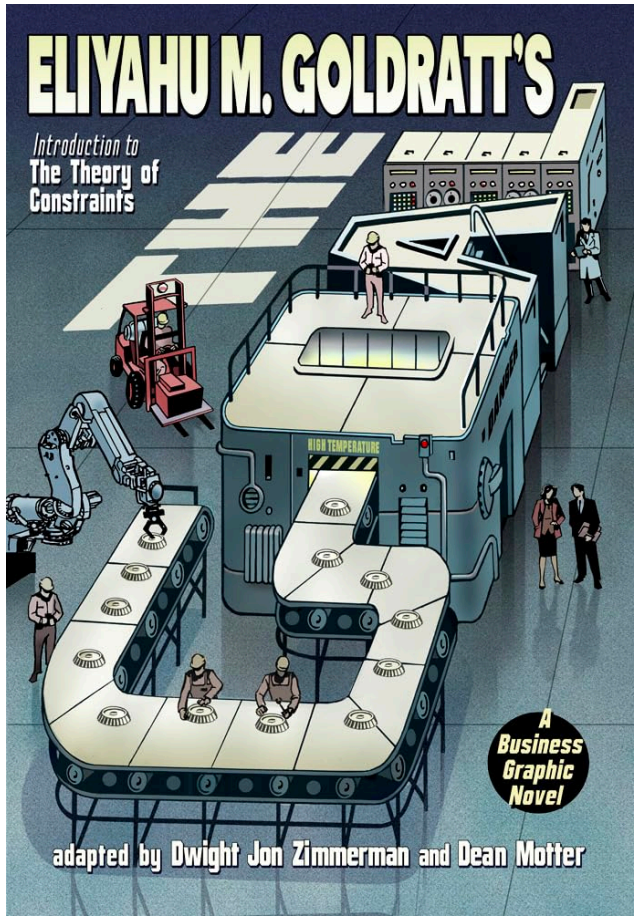
Saw the assembly lines.

Manufacturing and the Theory of Constraints

A constraint limits a system from achieving higher performance versus it's goal.

The Goal

- published in 1984
- is one of 3 books that Jeff Bezos recommends Amazon top executives read
- is part of many management programs
- spoiler: old Physics professor is hero



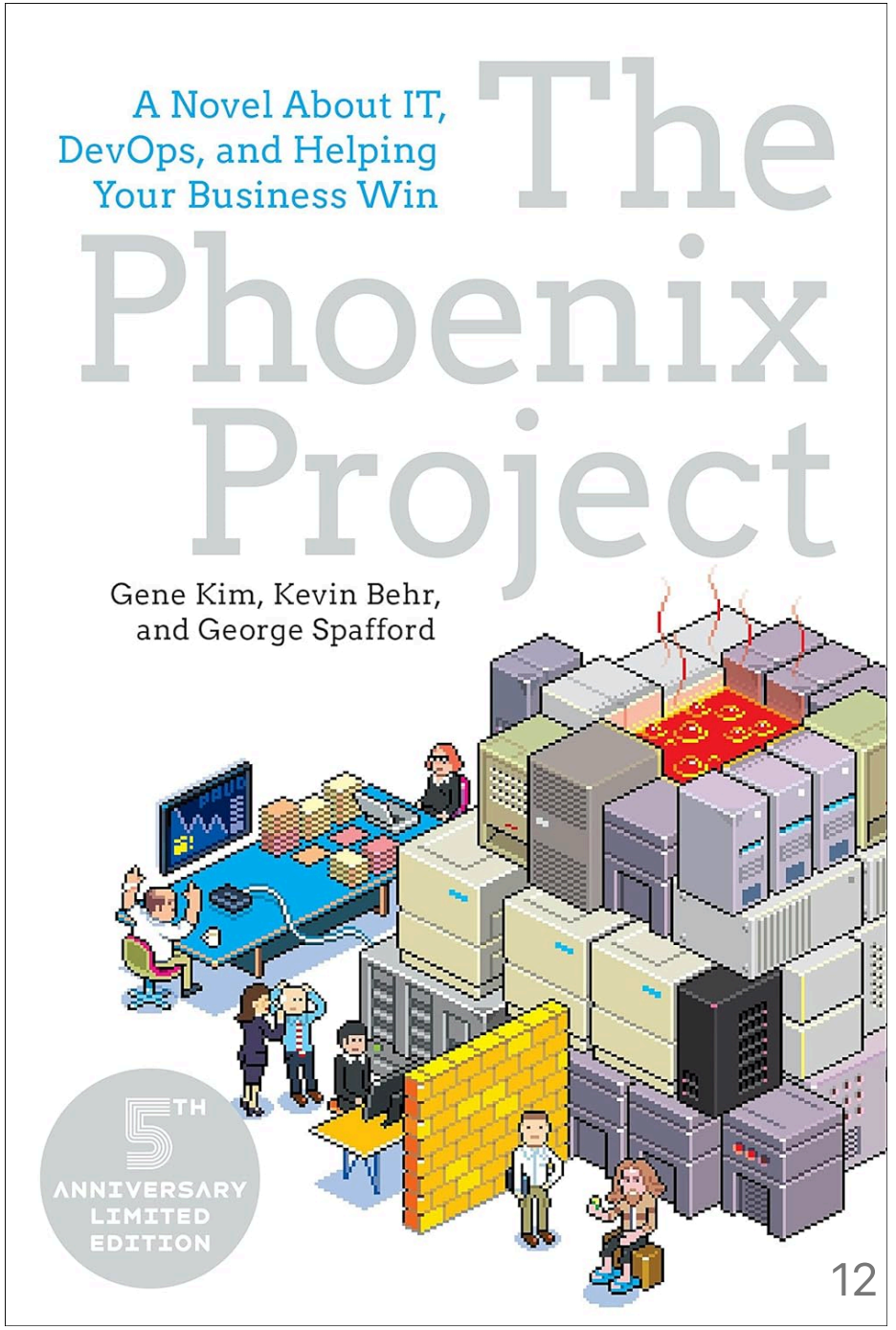
I could not put this book down when I first read it. And then later in life learned it inspired authors of another book I love.

Going from Manufacturing to Software...

The Phoenix Project

The IT death spiral is related to feedback loops between:

- lack of rigor and discipline
- silos and conflicting goals
- no visibility on workflow
- rushed work creates quality issues
- constant firefighting



Being able to take needless work out of the system is more important than being able to put more work into the system.

Any improvements made anywhere besides the bottleneck are an illusion.

Debt that is technical in nature also bears interest. The compounding interest of technical debt is proportional to the amount of unplanned work.

impact of the manufacturing revolution

before

- average plant order lead time was 6 weeks
- fewer than 70% of orders shipped on time

after

- average product lead times are less than three weeks
- more than 95% of orders shipped on time

The ever accelerating trend towards faster, cheaper, low risk delivery of software

	1970s–1980s	1990s	2000s–Present
Era	Mainframes	Client/Server	Commoditization and Cloud
Representative technology of era	COBOL, DB2 on MVS, etc.	C++, Oracle, Solaris, etc.	Java, MySQL, Red Hat, Ruby on Rails, PHP, etc.
Cycle time	1–5 years	3–12 months	2–12 weeks
Cost	$1M–$100M	$100k–$10M	$10k–$1M
At risk	The whole company	A product line or division	A product feature
Cost of failure	Bankruptcy, sell the company, massive layoffs	Revenue miss, CIO's job	Negligible

Marc Andreesen - Software will eat the world.

Dare to Compare

Software Metric	Improvement with DevOps
Code and Change Deployments	30x more frequent
Code and Change Deployment Lead Time	200x faster
Production Deployments	60x higher change success rate
Mean Time to Restore Services	168x faster
Productivity, Market Share, and Profitability Goals	2x more likely to exceed
Market Capitalization Growth	50% higher over 3 years

Source: The Phoenix Project

cost of the IT death spiral

human

- When people are trapped in the downward spiral for years...
- burnout, feelings of fatigue, cynicism, and even hopelessness and despair.

financial [1]

- 5% of the world wide gross domestic product ($3.1 trillion) was spent on IT (hardware, services and telecom).
- Estimate that 50% of $3.1 trillion was spent on operating costs and maintaining existing systems,
- one third of that 50% was spent on urgent and unplanned work or rework
- approximately $520 billion was wasted.

1. IDC and Gartner estimate for 2011

The opportunity

- halve the waste created by IT death spirals
- redeploy that human potential into something that has 5 times the value[1]

The reward

- healthier and happier employees
- create $2.6 trillion of value per year.

1. A low estimate.

phys·ics /ˈfiziks/

- the branch of science concerned with the nature and properties of **matter and energy**.

fi·nance /fəˈnæns/

- a field within accounting and economics that focuses on the **management of money** at personal, corporate and public levels.
- Its main aim is to study the **production, distribution and consumption of money, goods, assets and services**.

economics /ˌiːkəˈnɒmɪks/

- is the science of **choice in the context of scarcity**.

Source: Dictionary

When you have to make a choice, the better the information, the better the decision.

Someone must have said this.

Information is the resolution of uncertainty.

Claude Shannon

In science... novelty emerges only with difficulty, manifested by resistance, against a background provided by expectation.

Thomas Kuhn

We want to be a tech company with a banking license.

Ralph Hamers, 2017, CEO ING

Economics needs a scientific revolution

- by Jean-Philippe Bouchaud
- head of research, Capital Fund Management
- physics professor, École Polytechnique, France.
- October 30, 2008

Physics

- is the science of the structure of matter and interactions

- Law of ideal gases emerges from the chaotic motion of individual molecules.

Economics

- is the science of choice in context of scarcity.

- Statistical regularities should emerge in the behaviour of large populations.

Flagship achievements

Physics

- Rockets fly to the Moon
- Energy is extracted from minute changes of atomic mass.

Economics

- Recurrent inability to predict and avert crises

Classical economics has strong assumptions...that are treated like axioms:

- **The rationality of economic agents**
  - Every economic agent acts to maximize their profits
- **The *invisible hand***
  - Agents, in the pursuit of their own profit, are led to do what is best for society as a whole
- **Market efficiency**
  - Market prices faithfully reflect all known information about assets.

But physicists have learned to be suspicious of axioms.

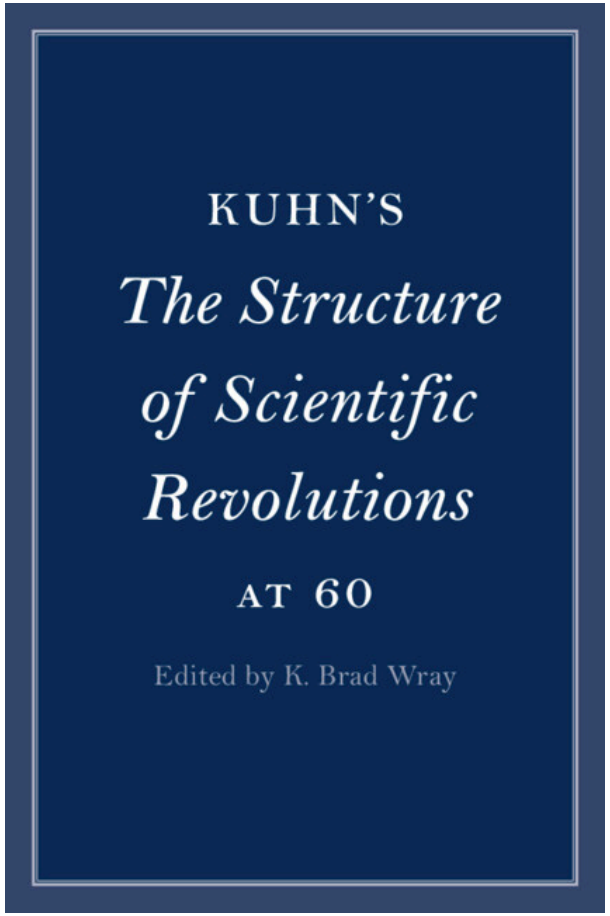
If

an empirical observation

is incompatible with a model,

then

the model must be trashed or amended.[1]



1. Even if it is conceptually beautiful or mathematically convenient.

The reality...

- markets are not efficient
- humans tend to be over-focused in the short-term and blind in the long-term,
- errors get amplified, ultimately leading to collective irrationality, panic and crashes.

Free markets are wild markets.

BROOKLYN DAILY EAGLE

And Complete Long Island News

LATE NEWS

89th YEAR—No. 295. ★ NEW YORK CITY, THURSDAY, OCTOBER 24, 1929. ★ 32 PAGES THREE CENTS

WALL ST. IN PANIC AS STOCKS CRASH

Attempt Made to Kill Italy's Crown Prince

ASSASSIN CAUGHT IN BRUSSELS MOB; PRINCE UNHURT

Royal Suitor Was About to Lay Wreath on Unknown Soldiers' Tomb.

Princess, Deeply Moved, Falls Into Fiance's Arms and Kisses Him

Hollywood Fire Destroys Films Worth Millions

ATTEMPT MADE ON LIFE

Crown Prince Humbert.

Consolidated Studios Are Swept by Flames Fatal to One—Master Pictures Burned Include Many New Talkie Productions.

FEAR 52 PERISHED IN LAKE MICHIGAN; FERRY IS MISSING

Wreckage Picked Up Indicates Craft Went Down With All Aboard.

PIECE OF PLANE LIKE DITEMAN'S IS FOUND AT SEA

Black and Orange Wreckage Indicates Daring Flier Went to Death.

High Duty Group Gave $700,000 to Coolidge Drive

Grundy Agrees Rates Went Up Due to His Activities as Propagandist. Favors More Lobbying to 'Carry Out Voters' Wish.'

By HENRY SUYDAM.

FOR MORE LOBBYISTS

JOSEPH R. GRUNDY

STOCKS CRASH IN RUSH TO SELL; BILLIONS LOST

Morgan, Mitchell Buying Stocks in Effort to Check Rush to Unload.

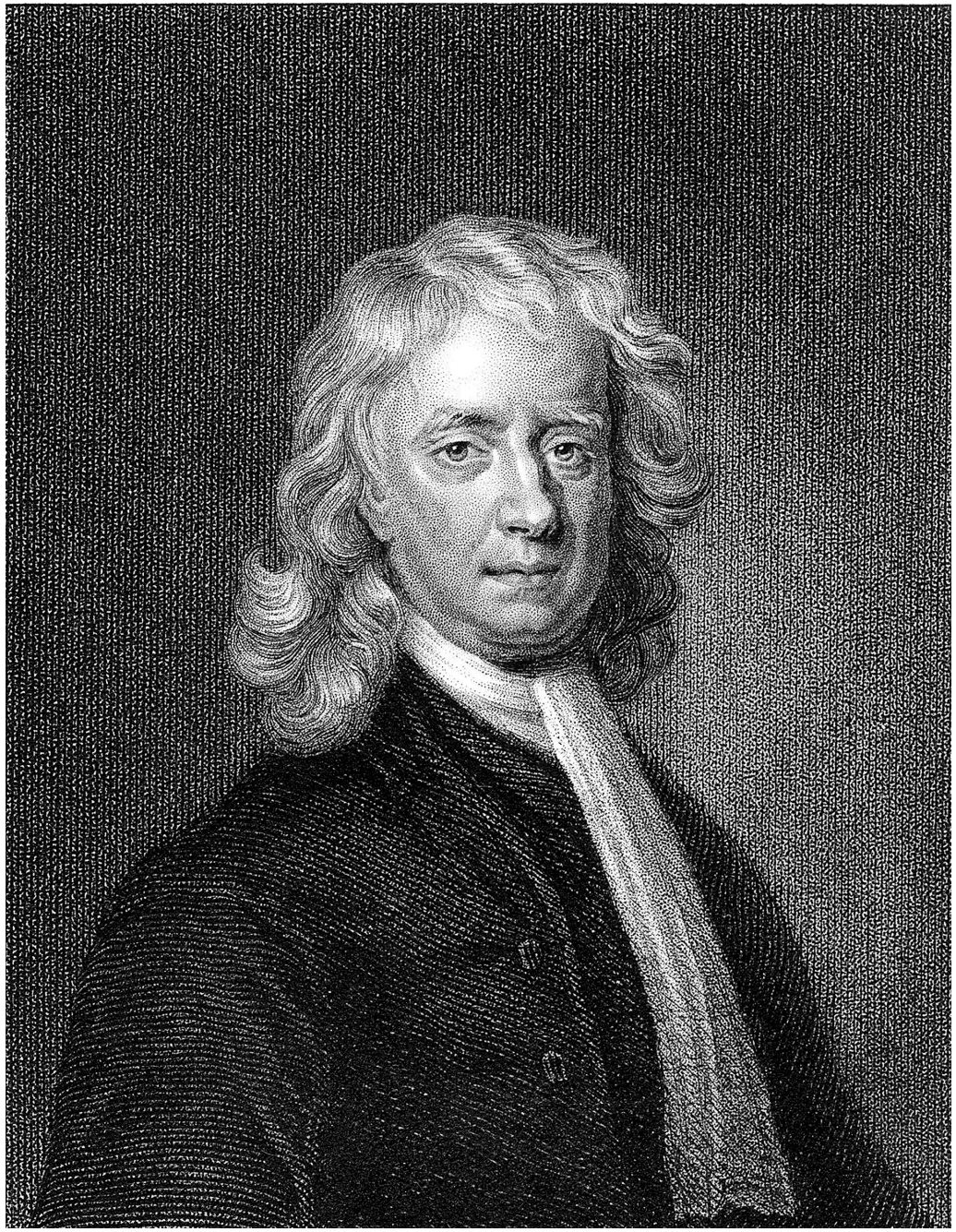
CARNEGIE CHARGE OF PAID ATHLETES ROUSES COLLEGES

HOOVER'S TRAIN HALTED BY AUTO PLACED ON RAILS

WARDER SOUGHT TO KEEP SEA TRIP

SOMERS NAMED AS HEAD OF NEW

Black Thursday October 24, 1929 - start of the Great Depression.



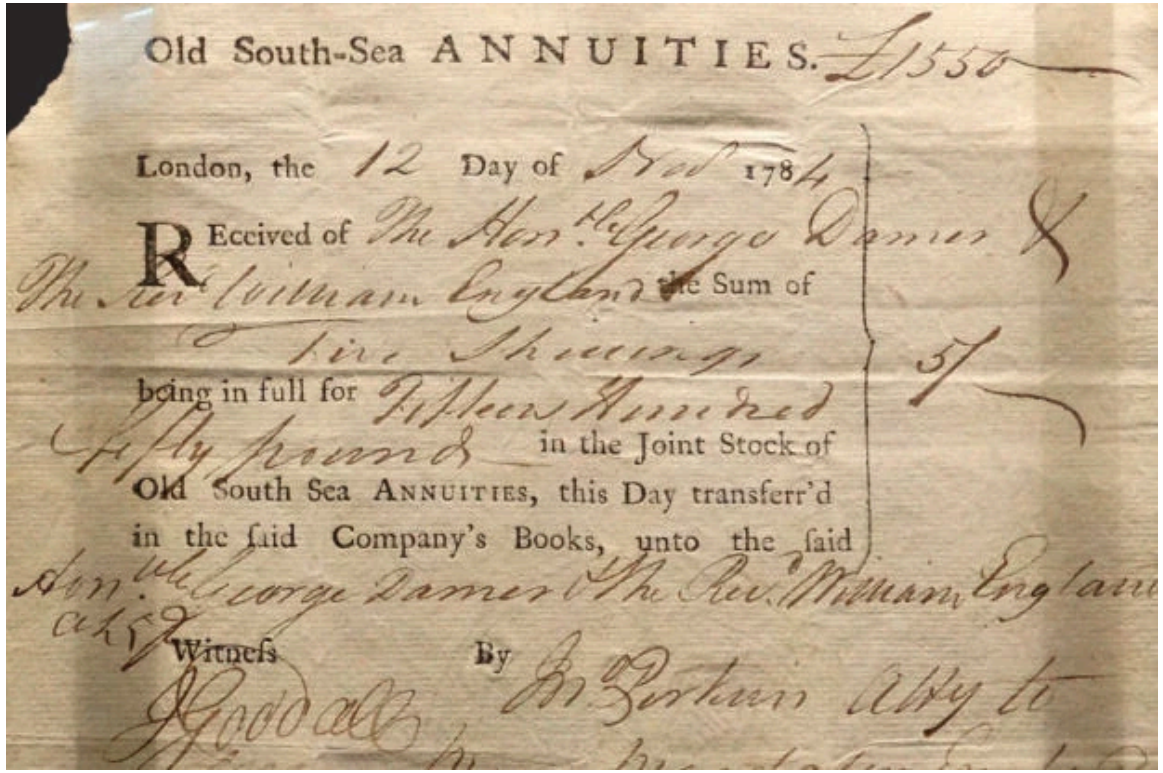
Sir Isaac Newton

Modelling the madness of people is more difficult than modelling the motion of planets.

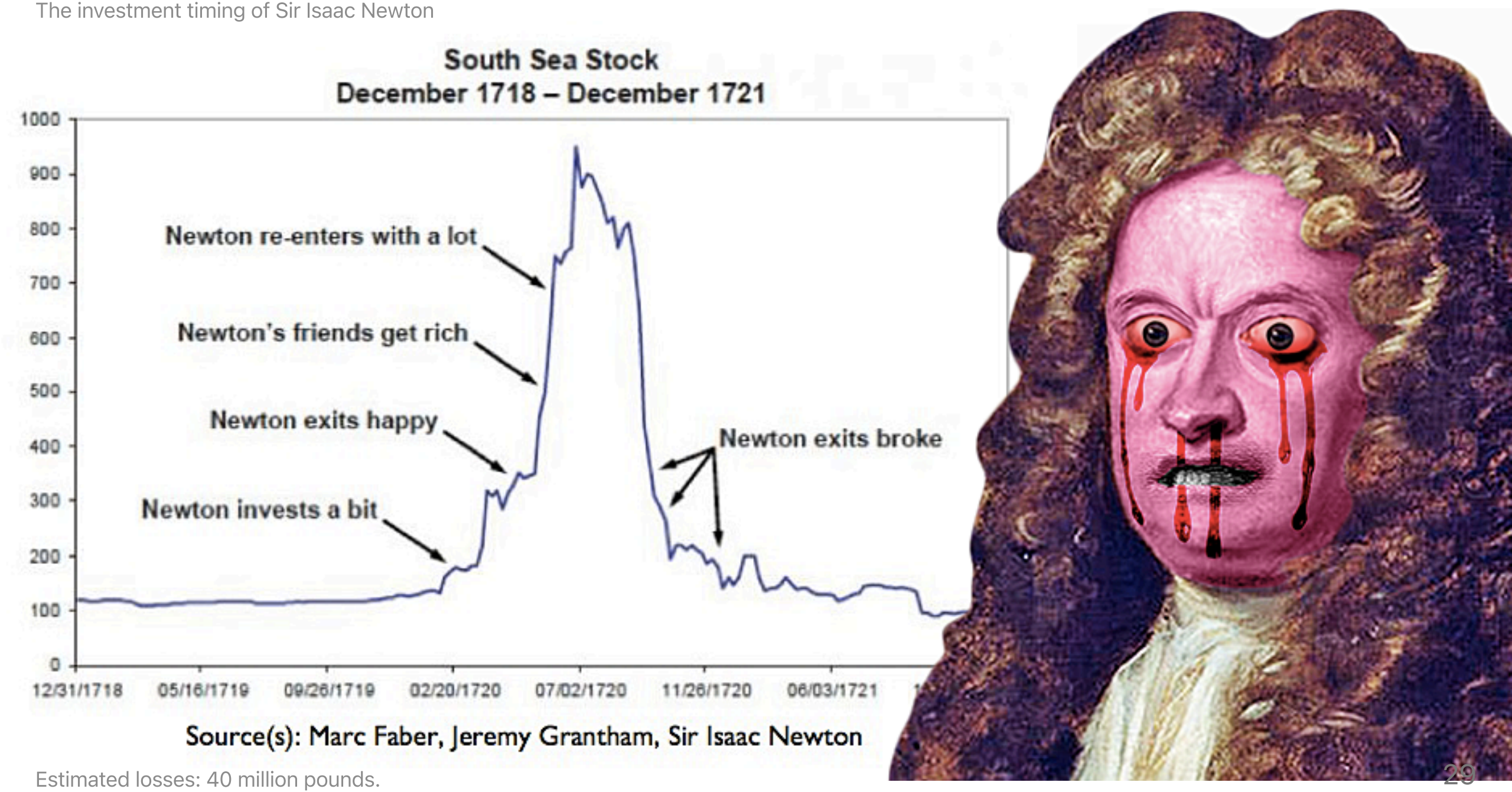
- Warden and then Master of the Royal Mint 1696-1727
- The calculus, optics, gravitation, motion.
- Investor

The South Sea Investment Opportunity

- formed in 1711 to consolidate government debt
- granted a monopoly on trade between Britain and Spain
- including the slave trade
- scheme to swap instruments of debt (government debt) for shares in the company



The investment timing of Sir Isaac Newton



Estimated losses: 40 million pounds.

Global Financial Crisis (GFC) of 2008

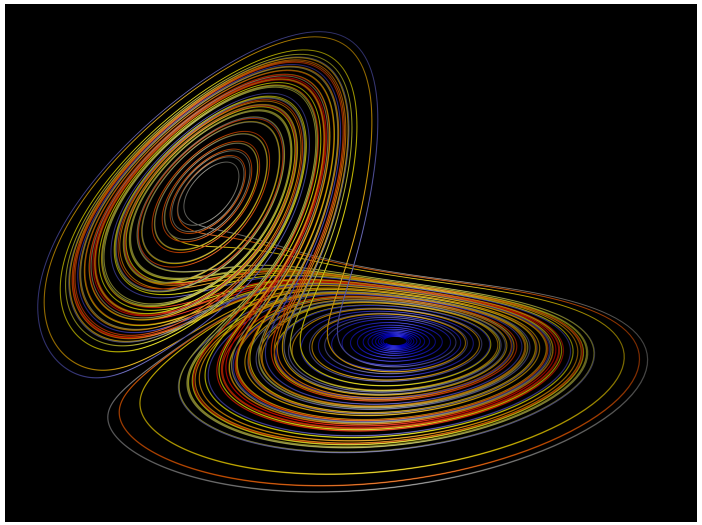
- structured financial products packaged subprime risk into seemingly respectable high-yield (HY) investments
- flawed models underestimated probability that multiple borrowers would default on their loans simultaneously.
- models neglected the very possibility of a global crisis, even as they contributed to triggering one.

Did we mentioned that free markets are wild markets?

Amazing movie.

What does physics have to offer?

- Models that explain how small perturbations can lead to wild effects.
- While a system may have an optimum state, it can be difficult to identify that the system never settles there.
- An optimum state can be hyper-fragile to small changes in the environment, and therefore often irrelevant to understanding what is going on.



The Lorenz attractor kind of looks like a butterfly.

The Call to Action

- Break away from classical economics.
- Develop completely different tools.
- Get behavioural economists and econo-physicists in same room and talking.

- Improve regulation.
- Scrutinize new financial products.
- Crash-test models against extreme scenarios outside the realm of current models
- Require approval by independent agencies

Treat finance like regulated industries: pharmacy, transport, etc

The response to this provocative 2008 article

Nature lists 4 citations:

- 2019: Physician Emigration: Should they Stay or Should they Go? A Policy Analysis
- 2020: SABCEMM: A Simulator for Agent-Based Computational Economic Market Models
- 2022: Impact of Twitter sentiment on stock price returns
- 2023: Microfounding GARCH models and beyond: a Kyle-inspired model with adaptive agents

But my favorite response is ...

Lucas Economics Needs A Scientific Revolution

- by Jean-Philippe Bouchaud
- head of research, Capital Fund Management
- physics professor, École Polytechnique, France.
- May 25, 2023 [1]

1. Professor Lucas passed away on May 15, 2023.

Professor Lucas

- American economist
- 1995 Nobel Prize winner in economics
- 10th most cited economist in the world [1]

What are rational expectations?

- A stable monetary policy that is consistent with individuals' rational expectations will be more effective in promoting economic stability than attempts to manipulate the money supply.

1. As of 2020

Counterpoints to Lucas-ian economics

Based on years of observing market participants:

- the fraction of truly rational people is *extremely* small
- crowd phenomenon are the main determinant of economic crises and financial crashes
- crowds can be rational even when invidividuals are not
- crowds can be *irrational* even when individuals are not

Counter to: people may not be rational, but they learn to be

Our world is no simple game:

- its dynamics is complex, non-ergodic, non-stationary.
- Patterns do not necessarily repeat.
- We learn the past … but how useful is it in the face of radical uncertainty?
- Things are changing faster than ever.

> *The problem with exponentials is that the greatest changes are in the most recent past.*
>
> Ray Kurzweil

CTA: we can (and should) do better

Goal

- if timing of crises can not be predicted, *at least* map out the mechanisms leading to economic instabilities.

Recommended Methods

- Make realistic models!
- Measure more things!
- Use numerical simulations!

Make Realistic Models!

A short review of theory and models.

- Markowitz and Modern Portfolio Theory
- Sharpe and Capital Asset Pricing Model
- Fama French

Photo by Niklas Hamann on Unsplash

Modern Portfolio Theory

- Performance of an individual stock is not as important as the performance and composition of an investor's entire portfolio.
- Assets can be mixed to maximize return for a given level of risk.
- The volatility of the portfolio depends not only on the volatility of the constituents but to what extent they go up and down together.

- Harry Markowitz
- The Journal of Finance in 1952
- 1990 Nobel Prize for Economics

Expected Return of a Portfolio

An investor holds a portfolio with

- $4,000 invested in Asset Z and
- $1,000 invested in Asset Y.

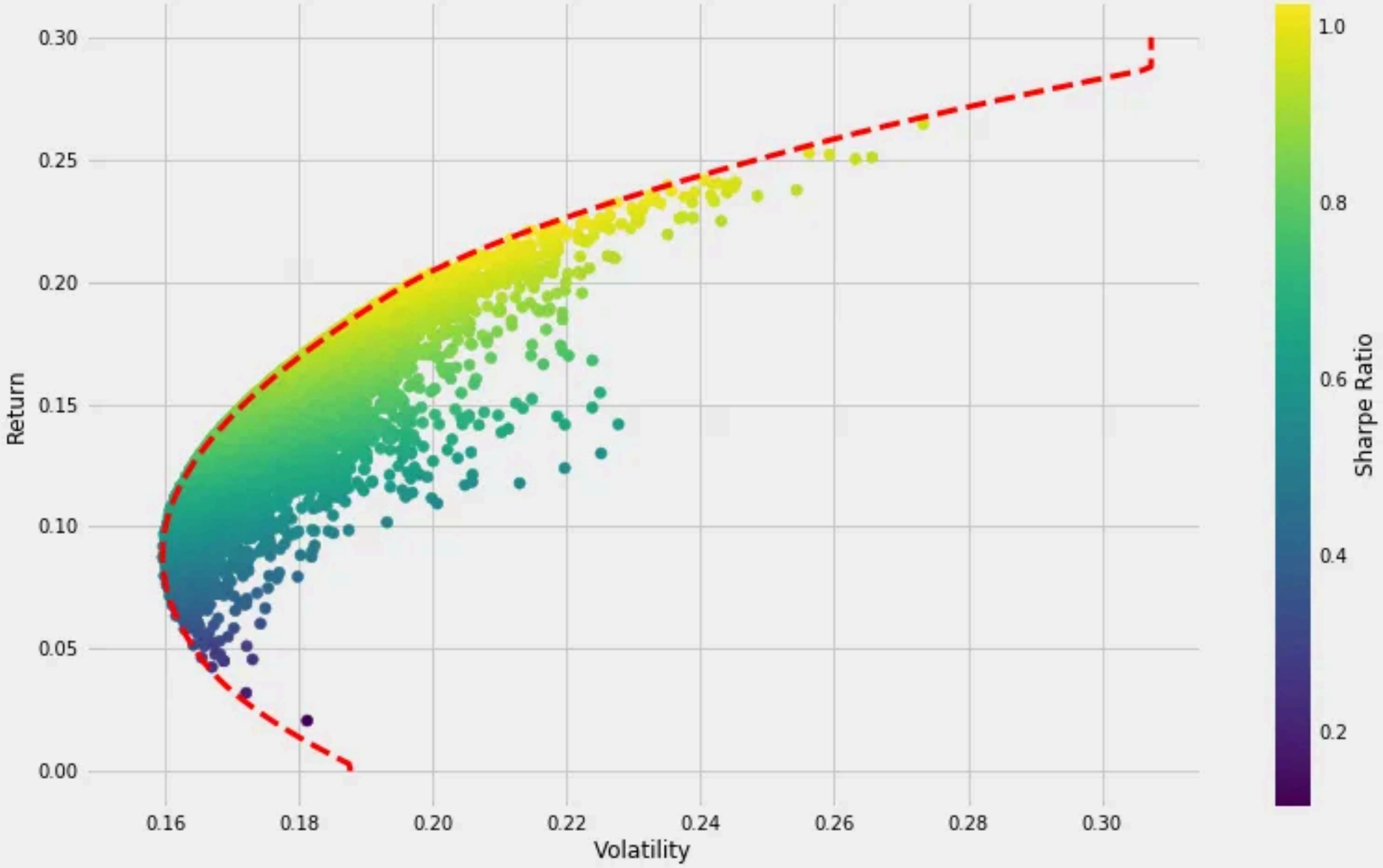
The expected returns are:

- 10% on Z
- 3% on Y.

The expected return of the portfolio is 8.6%.

$$[(4000/5000) \cdot 0.10] + [(1000/5000) \cdot 0.03] = [0.8 \cdot 0.10] + [0.2 \cdot 0.03] = 0.086$$

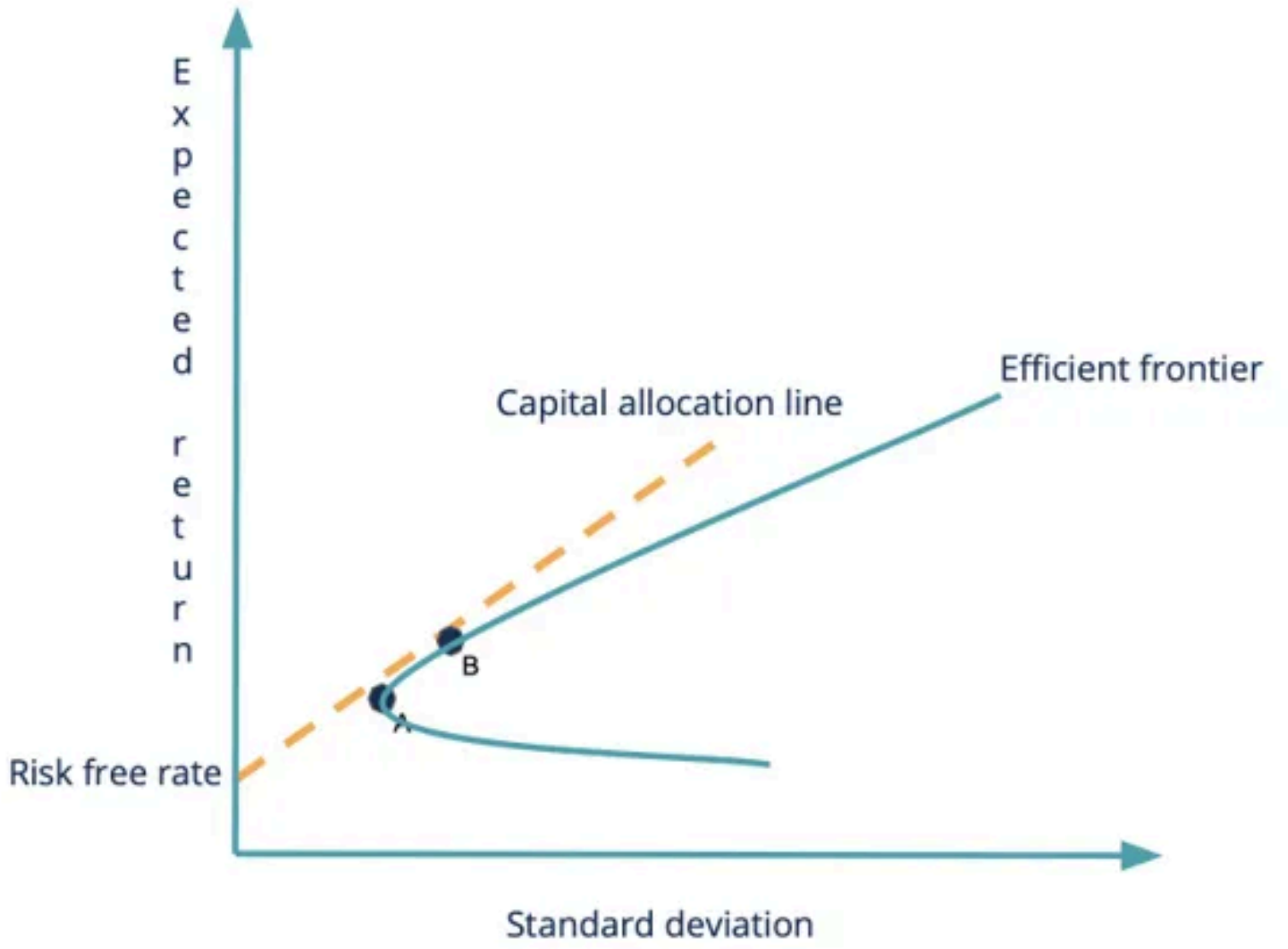
Make Realistic Models! Modern Portfolio Theory.



Consider all possible portfolios - which ones give higest expected return for a given level of risk?

Efficient frontier

A set of portfolios that maximizes expected returns for each level of standard deviation (risk).



Consider all possible portfolios - which ones give higest expected return for a given level of risk?

Capital Asset Pricing Model (CAPM)

Investors expect to be compensated for risk and the time value of money.

To determine fair value of a stock, compare its risk and the time value of money to its expected return.

$$ER_i = R_f + \beta_i(ER_m - R_f)$$

where:
ER_i is expected return of investment
R_f is risk-free rate (investing in Treasury bills)
β_i is beta of the investment and $ER_m - R_f$ is market risk premium

CAPM example

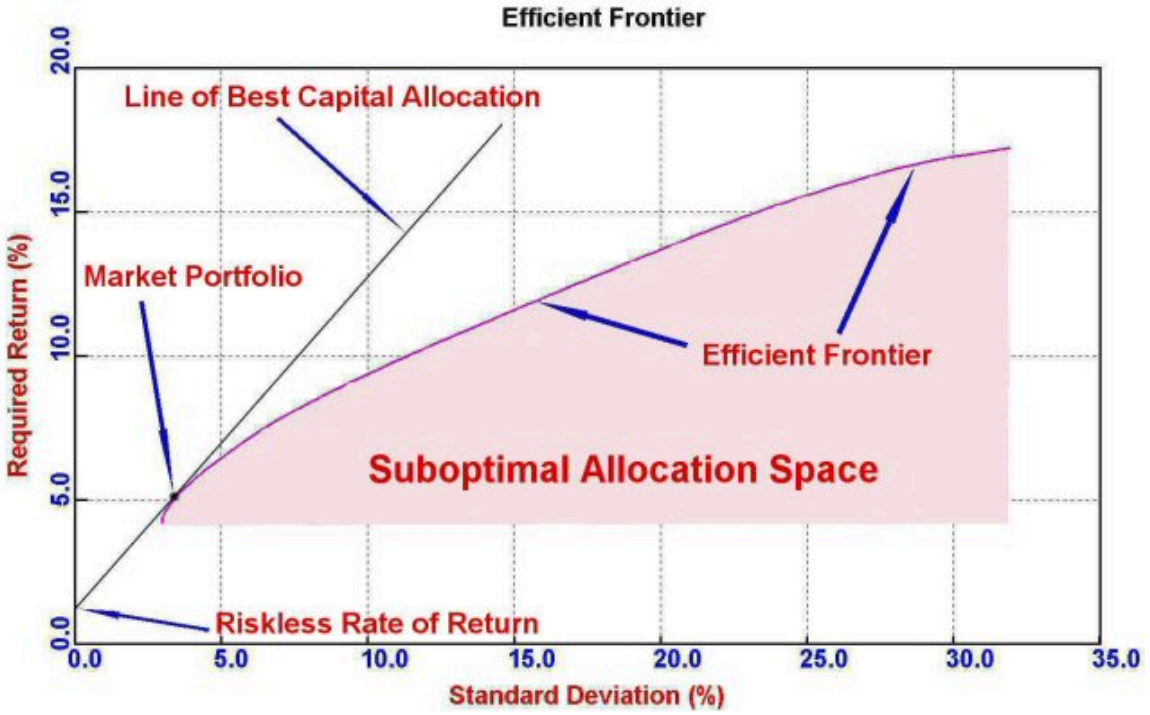
An investor considers a stock valued at $100 per share today that pays a 3% annual dividend.
The stock has a beta of 1.3 compared with the market.
Assume the risk-free rate is 3% (Treasury bills).
Last, the investor expects the market to rise in value by 8% per year.

The expected return of the stock based on the CAPM formula is 9.5%:

$$ER_i = 3\% + 1.3 \cdot (8\% - 3\%) = 9.5\%$$

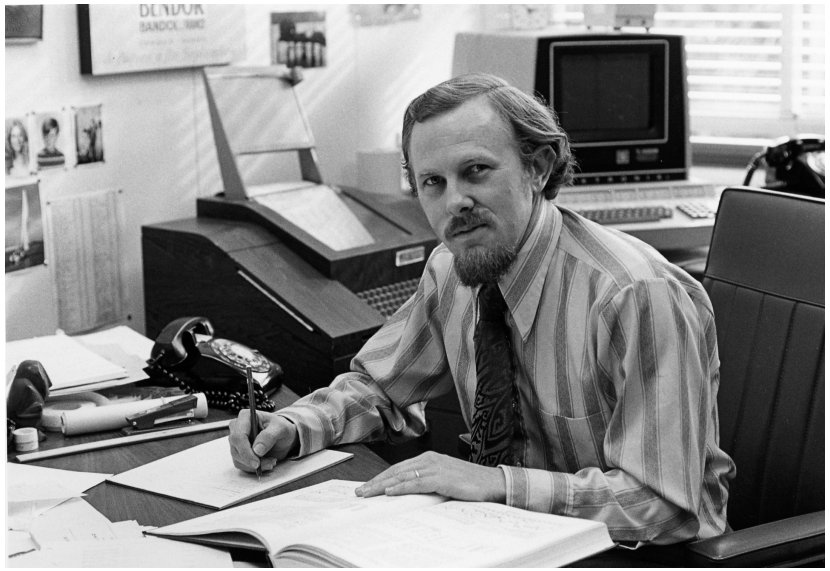
Sharpe Ratio

- start at the risk free rate
- draw a line of maximum slope that touches the efficient frontier
- the slope of this line is the Sharpe ratio



The slope of the CAL is called the Sharpe ratio, which is the increase in expected return per additional unit of standard deviation (reward-to-risk ratio).

Sharpe Ratio Thresholds	Rating
Less than 1	Bad
1 to 1.99	Adequate/good
2 to 2.99	Very good
Greater than 3	Excellent



Volatility, or beta

- Beta measures the volatility of a stock compared to the market as a whole.
- It needs to be compared to a benchmark.
- Often the S&P 500 is used as a benchmark ($\beta = 1.0$).
- If a stock i is riskier than the market, $\beta_i > 1$
- If a stock i is less risky than the market, $\beta_i < 1$

However, Fama and French found that differences in betas over a lengthy period did not explain the performance of different stocks.

The Fama French Three-Factor Model

expands on CAPM by adding:

- size risk
- value risk factors

Small Minus Big (SMB) - smaller companies outperform larger ones over the long-term.

High Minus Low (HML) - value stocks tend to outperform growth stocks.

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it}$$

The Fama and French Three-Factor Model was developed in 1992. Fama won the Nobel in 2013.

But this still wasn't good enough.

In 2014, Fama and French adapted their model to include five factors!

Take aways

There are some well known models. They continue to evolve.

Measure more things!

- Ratios
- Trends and Moving averages
- Oscillators
- Flows

- Red-winged Blackbird
- Agelaius phoeniceus
- Possibly North America's most abundant bird
- Photo by Joseph Corl on Unsplash

Metric	Formula
Sharpe Ratio	$\frac{R_p - R_f}{\sigma_p}$
Treynor Ratio	$\frac{R_p - R_f}{\beta_p}$
Jensen's alpha	$\alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$

R_p portfolio return

R_f risk-free rate

σ_p standard deviation of portfolio excess return

β_p portfolio beta

Trends can be identified with moving averages.

- Simple moving average
- Exponential moving average (EMA)
  - which puts more weight on recent prices

Changes in trends can be identified with:

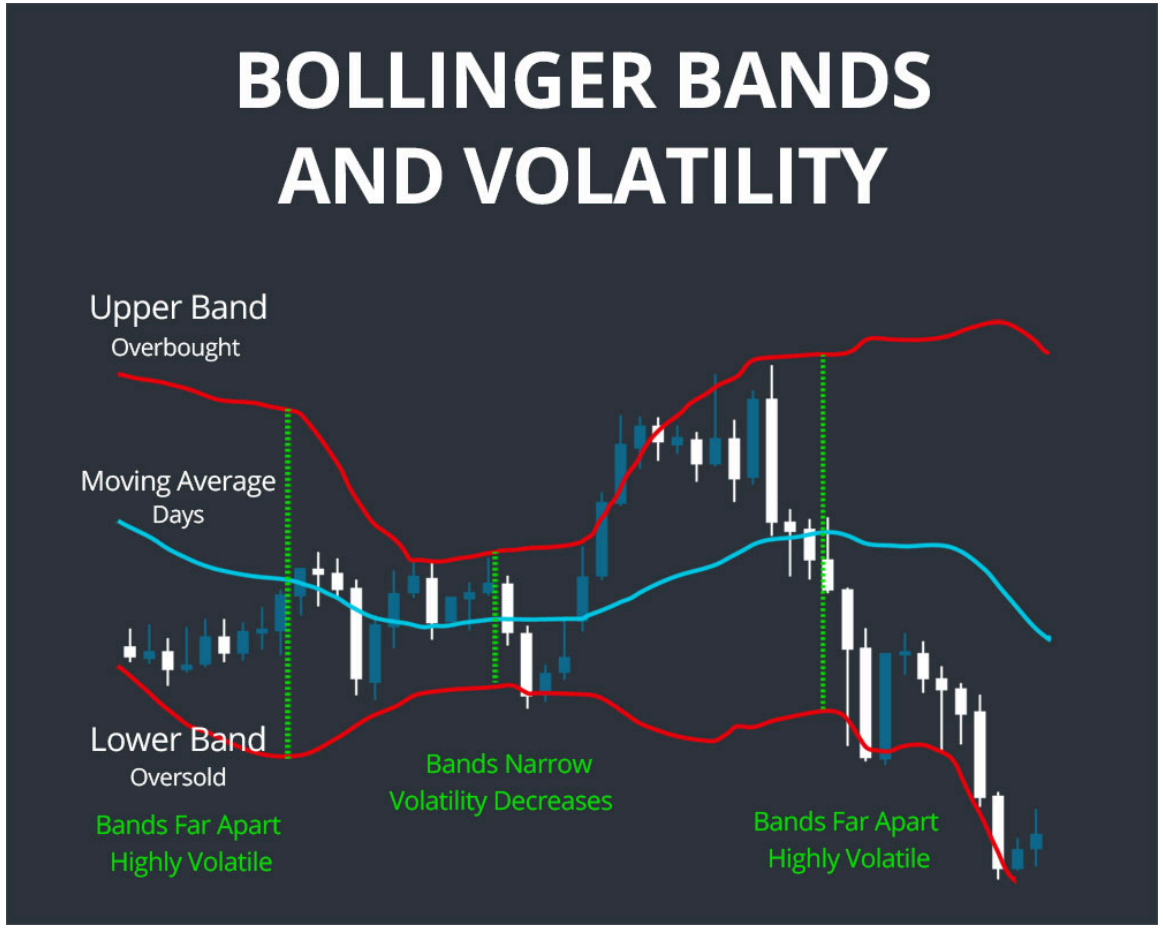
- Envelopes.
- Moving average convergence/divergence (MACD)

$$MACD = EMA_{short}(P) - EMA_{long}(P)$$

Example: Short: 12 days Long: 26 days

Bollinger Bands

- An envelope a standard deviation above and below a simple moving average of the price.
- Is an indicator to help determine whether prices are high or low on a relative basis.

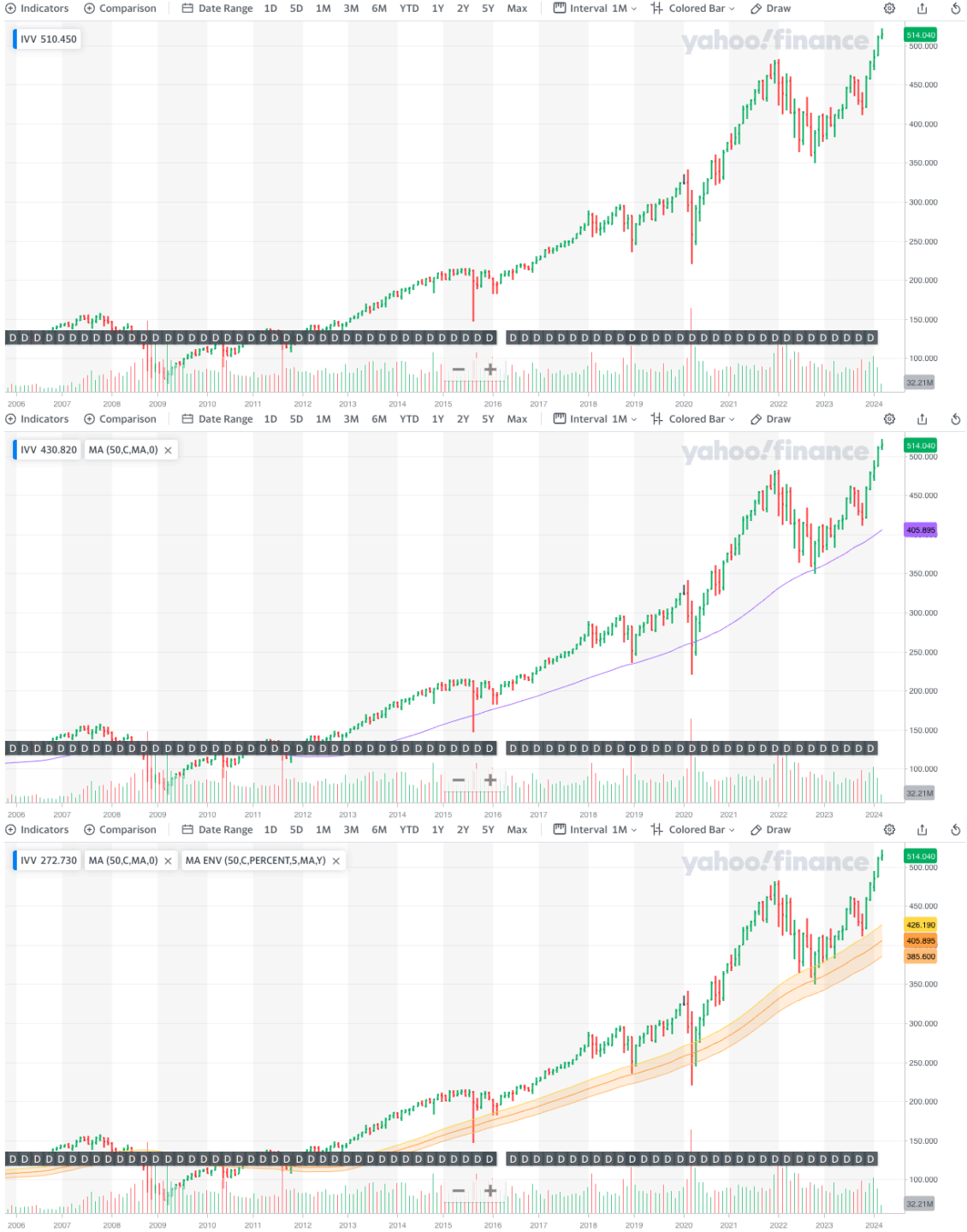


Example: Short: 12 days Long: 26 days

top: price vs time

mid: add moving average

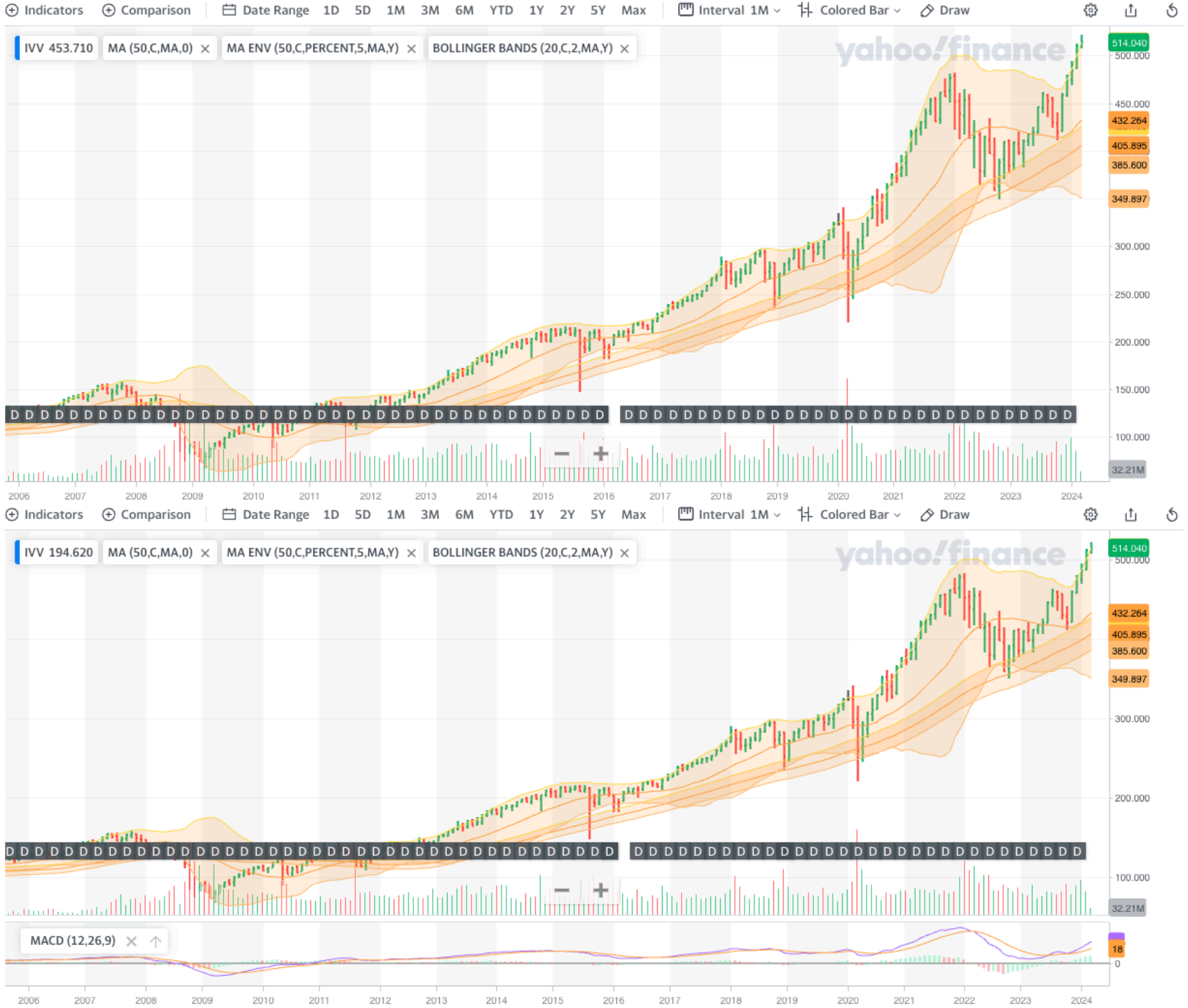
bottom: add moving average envelope



Source: Yahoo finance

top: add Bollinger Bands

bottom: add MACD



Source: Yahoo finance

Use Numerical Simulations!

Experiments

- Backtesting

Optimization

- Tax Loss Harvesting

Photo by Valdemaras D. on Unsplash

Source: Yahoo finance

Backtesting

- companies develop model portfolios to different investment styles
  - conservative, income with growth, growth with income, aggressive
- can be represented in equity and fixed income
- each model portfolio has its own benchmark
- themed approaches exist as well
  - alternative investments
    - REITs, commodities, infrastructure, private equity, hedge funds
    - crypto

Source: Yahoo finance

+Crypto Models

Experiments

Select from 3 portfolios

Benchmark Models

4 of 4 selected

Lookback Period

1 Year | 3 Year | 5 Year | MAX

Chart | Analytics | Holdings | About

Portfolio Performance

Moderate
Global Equity
Longevity
Select Moderate (including PIMCO ETFs)
Select Conservative (including PIMCO ETFs)
Select Aggressive (including PIMCO ETFs)
US Moderate Growth
US Growth
US Conservative Growth
US Aggressive Growth
MSCI ACWI ETF - Core US Aggregate Bond ETF 80 20
MSCI ACWI ETF - Core US Aggregate Bond ETF 60 40
MSCI ACWI ETF - Core US Aggregate Bond ETF 50 50
MSCI ACWI ETF - Core US Aggregate Bond ETF 30 70

Interactive Tool

Comparing the return on a $10,000 investment in different model portfolios.

Source: Yahoo finance

Tax Loss Harvesting Optionality

Ben Davis, Ph.D.

- Consortium for Data Analytics in Risk (CDAR)
- April 25, 2023

- CDAR is at Berkeley
- The Berkely mascot is Oski, a golden bear
- Golden bears (ursus arctos horribilis, previously californicus) are an extinct subspecies of brown bear
- Photo by John Thomas on Unsplash

Let's invest in ...

SPDR S&P 500 ETF Trust (SPY)

- SPY was created on January 22, 1993.
- Is the first US exchange traded fund (ETF) to be listed on a national stock exchange
- Very widely traded today.
- It tracks the Standard & Poor's 500 Index (S&P 500 Index)
- The S&P 500 Index is a market-capitalization-weighted index of the 500 leading publicly traded companies in the US

Name	Symbol	% Assets
Microsoft Corp	MSFT	7.18%
Apple Inc	AAPL	6.17%
NVIDIA Corp	NVDA	4.56%
Amazon.com Inc	AMZN	3.75%
Meta Platforms Inc Class A	META	2.54%
Berkshire Hathaway Inc Class B	BRK-B	1.74%
Alphabet Inc Class C	GOOG	1.63%
Eli Lilly and Co	LLY	1.40%
Broadcom Inc	AVGO	1.33%

Step	Description	Calculation
1.	Invest in SPY (an S&P 500 ETF)	245 shares at $600/share
2.	Portfolio cost basis	$147,000 = 245 x $600
3.	Suppose SPY falls 8.33%	$550/share
4.	Portfolio market value is now	$134,750 = 245 x $550
5.	Unrealized loss	-$12,250 = market value – cost basis
6.	Sell all SPY shares, realizing short-term capital loss	-$12,250

ChatGPT mostly formatted this table

- A loss offsets capital gains elsewhere in a portfolio
- This reduces the tax bill.

	Realized Gain or Loss	**Capital Gains Tax (40.8%)**
SPY	-$12,250	-$4,998
Elsewhere	$20,000	$8,160
Total Tax Due		$3,162

Investors have the right, but not the obligation, to sell assets held at a loss, resulting in valuable tax credits.
The *right but not the obligation* phrase reminds us of options.

Optimal Option Exercise

For a **perpetual American put** on a non-dividend paying equity, the **optimal policy** is to exercise when the underlying stock price falls below

$$P^* = \frac{r}{r + \sigma^2/2}\kappa$$

where r and σ are the continuous time risk-free rate and volatility of the underlying stock price κ .

A put option gives the owner the right to sell a specified amount of an underlying security at a specified price within a specified time.

Key Points about Option Problem

- the optimal policy is loss-depth trigger-based
- a simple analytical formula exists for the trigger price
- the trigger price depends on the stock volatility
  - as volatility goes up, the trigger price moves down (deeper trigger)
  - as volatility goes down, the trigger price moves up (shallower trigger)

Guess if government regulations make this simpler or not.

Wash Sales

When you sell or trade securities at a loss and within 30 days before or after the sale you:

1. Buy substantially identical securities,
2. Acquire substantially identical securities in a fully taxable trade...

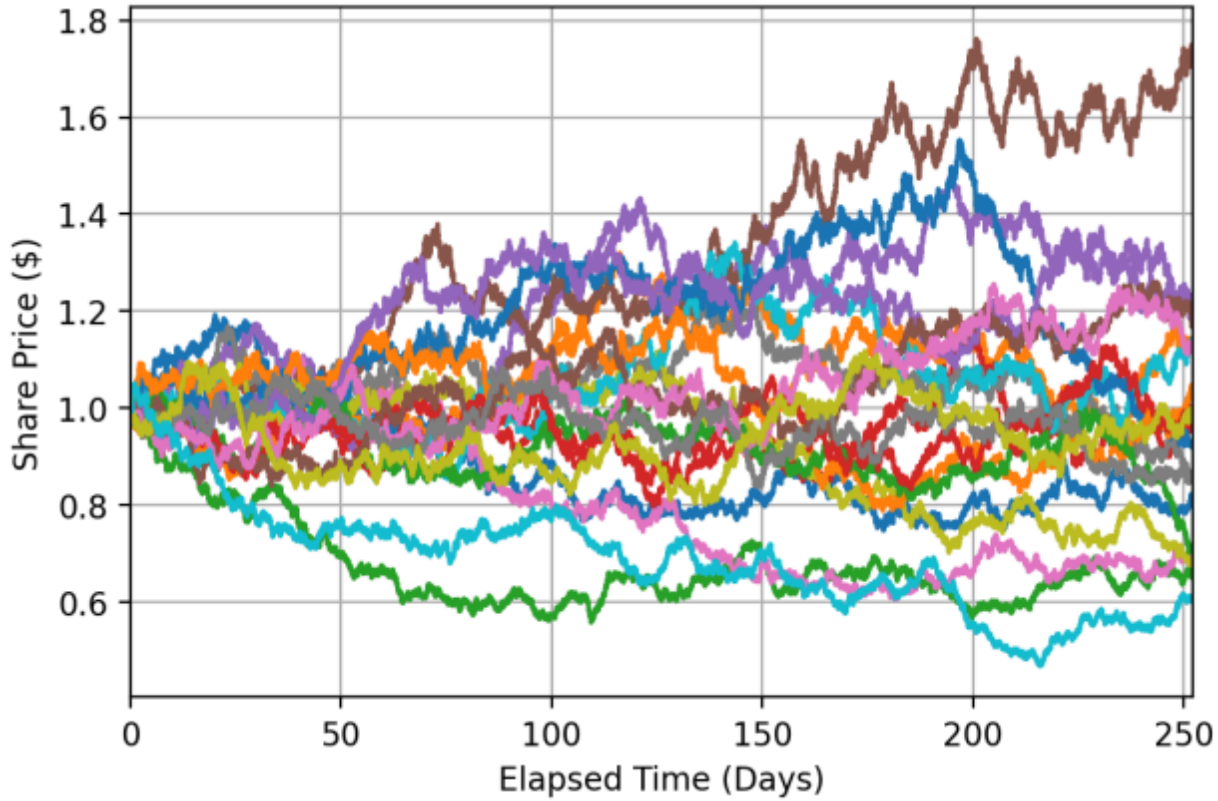
A taxable investor must adhere to the Internal Revenue Service (IRS) to claim deductions for realized capital losses.

The Stylized Model

- The ETF price exhibits geometric Brownian motion;
- Consider the expected losses at each time T
- Work with log losses for convenience
- Different triggers have different rates of expected loss accumulation

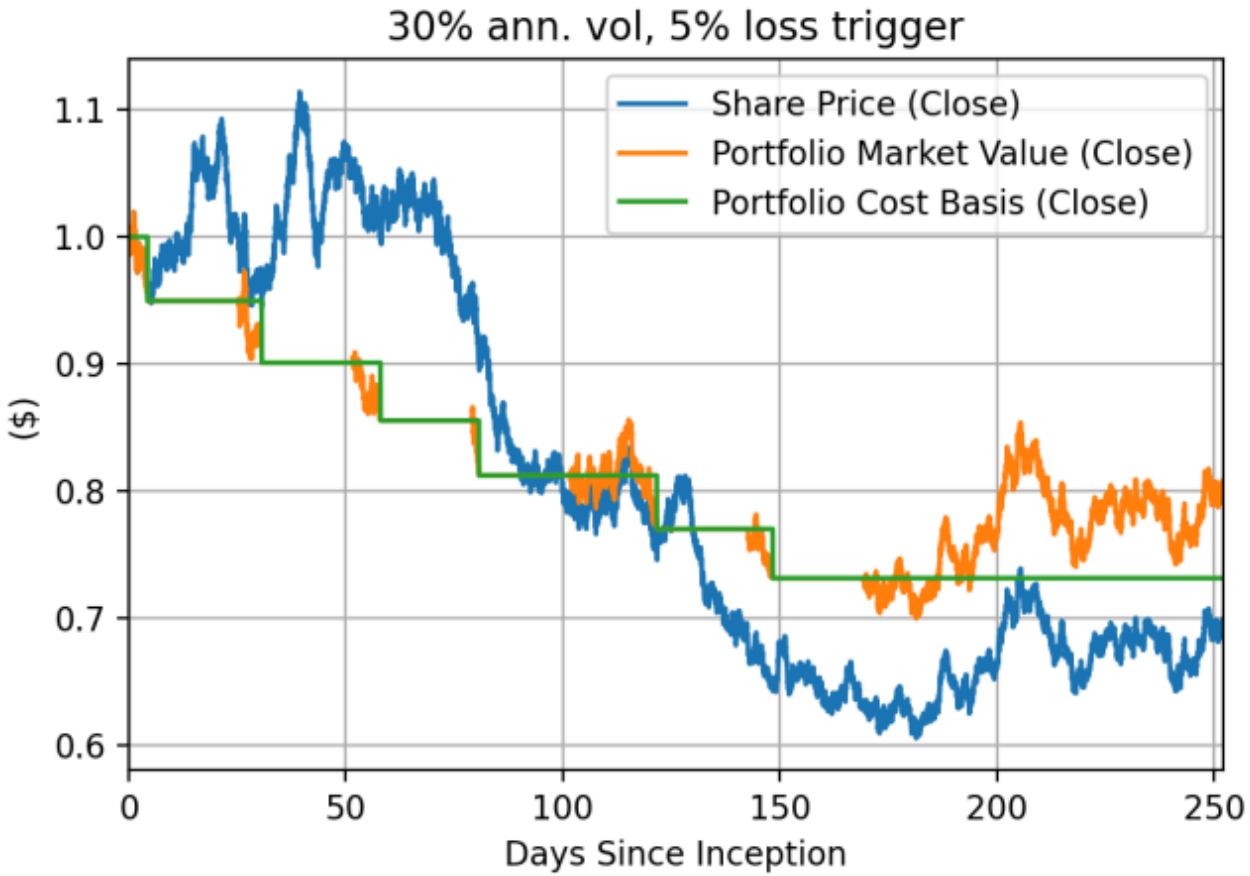
What the prices can look like

- geometric Brownian motion
- 30% annualized volatility
- 100 ticks per day over 1 year

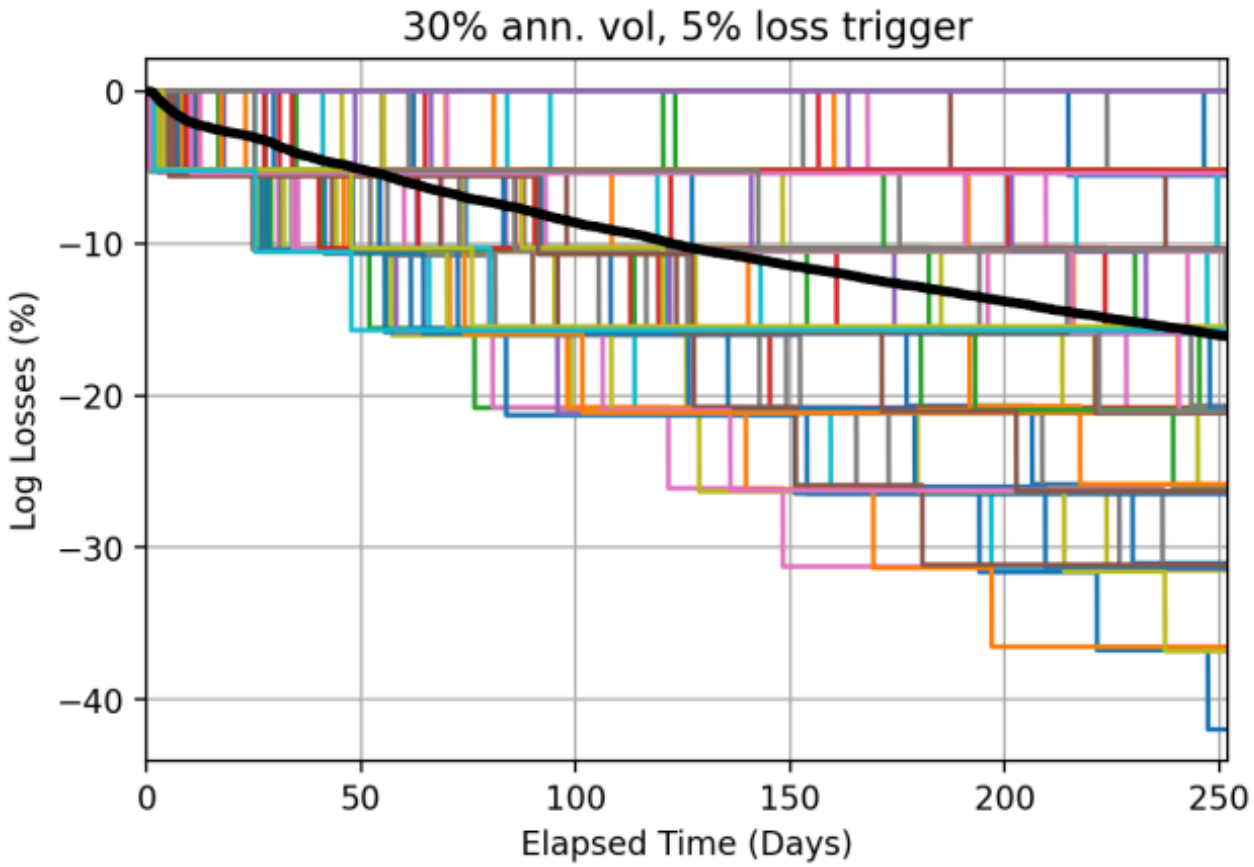


The investment process

- set a trigger price
- sell when the trigger price is reached
- hold cash during the wash sale period
- reinvest 100% back in the ETF

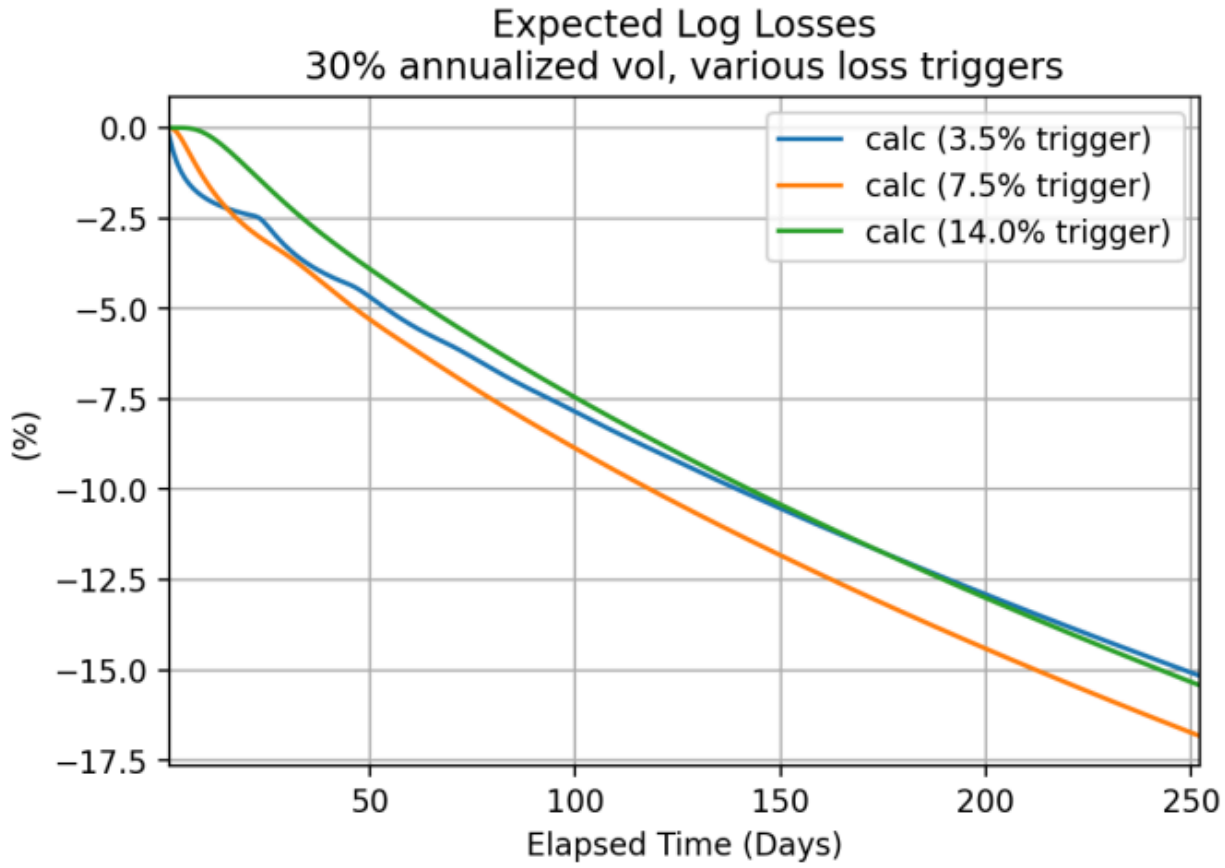


- Consider the distribution of simulated portfolio realized loss history (colorful stairs)
- Compute the expected value of losses at each time T (heavy black line)

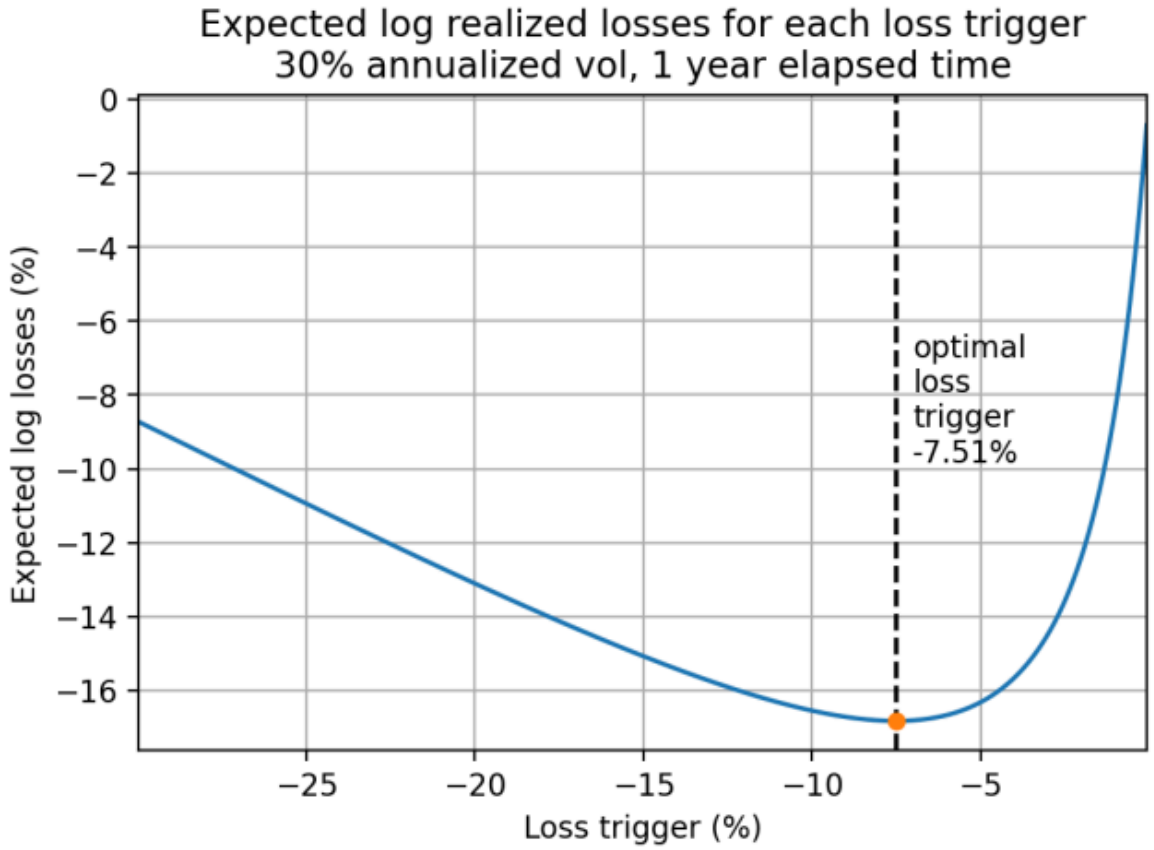


But wait! There is an analytical formula for the expected log losses.

- Different triggers have different rates of expected loss accumulation
- side effect of wash sale rule noticable at beginning



- the trigger depth with the largest expected log losses can be found using the analytical formula for expected log losses
- this provides more precision than Monte Carlo



The analytical formula

- Let σ be the volatility of the ETF log daily returns ($stdev(\log(1+r))$) where r represents the distribution of daily ETF price returns
- Let $\lambda = \| \log(1+L) \|$ where the trigger level L is expressed on a decimalized basis as a percentage of the cost basis; e.g. L = -0.05 represents a 5% loss trigger
- Let $z = \frac{\lambda}{\sigma}$ represent the trigger level expressed as standard deviations of ETF log daily returns
- The expected log loss at time T is given by

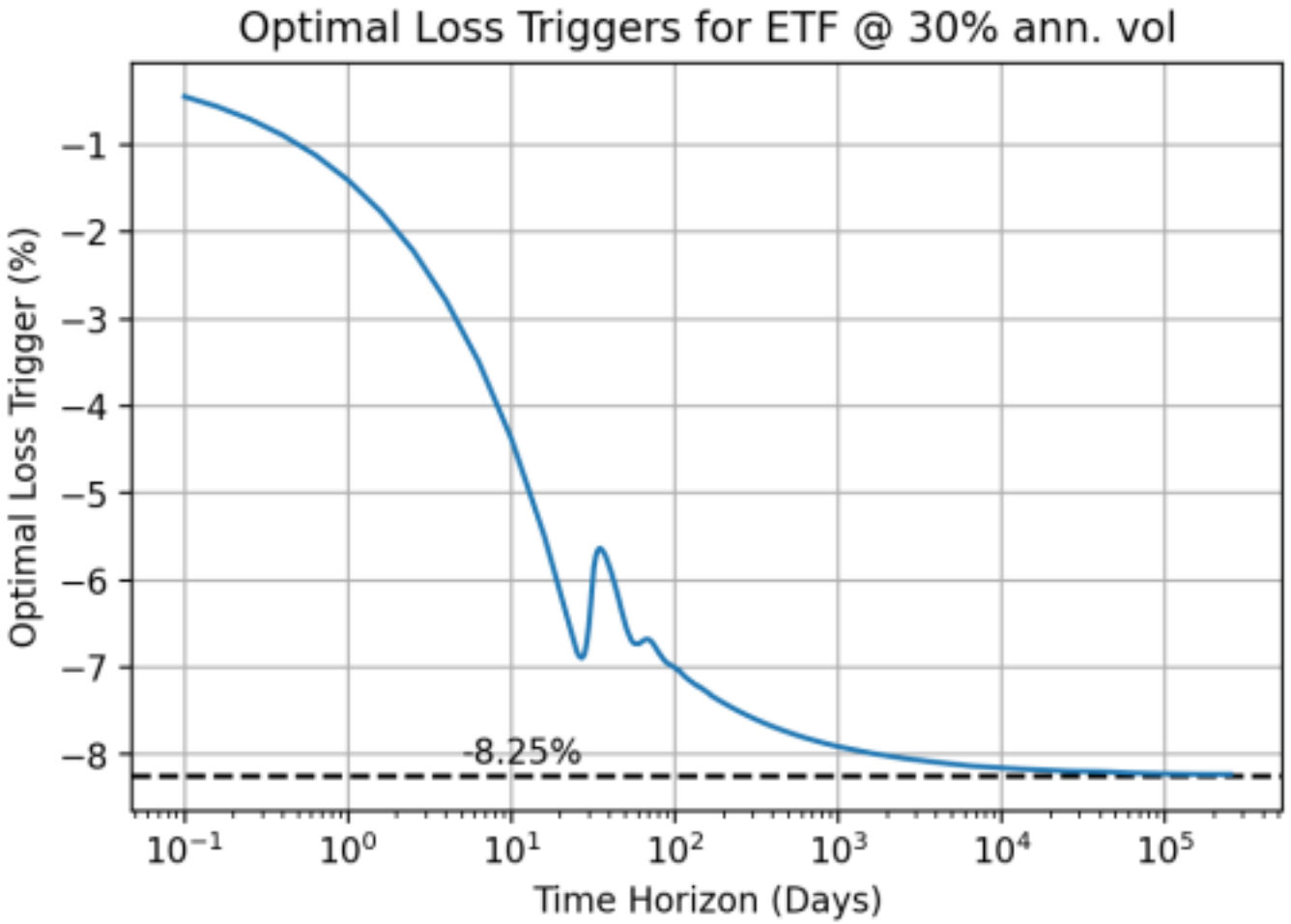
$$E[LogLoss(T)] = \lambda \cdot \sum_{m=0}^{M} \tilde{h}_m(T) \cdot m$$

where $M = ceil(T/21)$

$$\tilde{h}_m(T) = \begin{cases} erf\left(\frac{(m+1)z}{\sqrt{2(T-21m)}}\right) - erf\left(\frac{mz}{\sqrt{2(T-21(m-1))}}\right) & 21m < T \\ 1 - erf\left(\frac{mz}{\sqrt{2(T-21(m-1))}}\right) & 21(m-1) < T \leq 21m \\ 0 \qquad T \leq 21(m-1) \end{cases}$$

Optimal Trigger at Various Time Horizons

- from less than a day to 1000 years
- scaling laws guarantee plot is same, regardless of time unit



Long-Run Optimal Conjecture

The long-run optimal trigger of -8.25% corresponds to a log loss trigger of

$$\lambda_{opt,LR} = \ln\left(1 - \frac{8.25}{100}\right) = -8.61$$

relating volatility back to σ

$$vol \to \sigma = \sqrt{\ln\left(\frac{1 + \sqrt{1 + 4\left(\frac{vol}{\sqrt{12}}\right)}}{2}\right)}$$

and the conjecture is

$$\lambda_{opt,LR} = -\sigma\sqrt{\tau}$$

where τ is the duration of the wash sale period in multiples of 21 days.

Checklist

- ☑ We have models and they are evolving.
- ☑ We can measure so many things.
- ☑ We have lots of computing power and data to throw at simulations and analytics.

The Remaining Problem

Everything we reviewed existed *before* Professor Bouchard's 2023 call to action[1].

The Solution

Needs someone who understands matter and energy, systems and human nature.

It could be you.

- Magpie
- Photo by Daniil Komov on Unsplash

1. and most of it existed before the 2008 article

Questions?

About This Presentation

- Made with marp: https://marp.app/
- Animal images from unsplash.com
- Icons created by Freepik - Flaticon
- Slides are on github at the QR code[1].

1. As pdf or markdown.

Black-Scholes Equation

- invented in 1973[1]
- it assumes the probability of extreme price changes is negligible.

But: in real life stock prices are jerky.

Example

- Twenty years ago, unwarranted use of the model spiralled into the worldwide October 1987 crash;
- the Dow Jones index dropped 23% in a single day

The use of a crash-free model helped trigger a crash.

---

1 Taylor Swift's mother was 5 years old in 1973.

1. As pdf or markdown.

The evolution of an asset price S over time

$$dS = \sigma S dX + \mu S dt$$

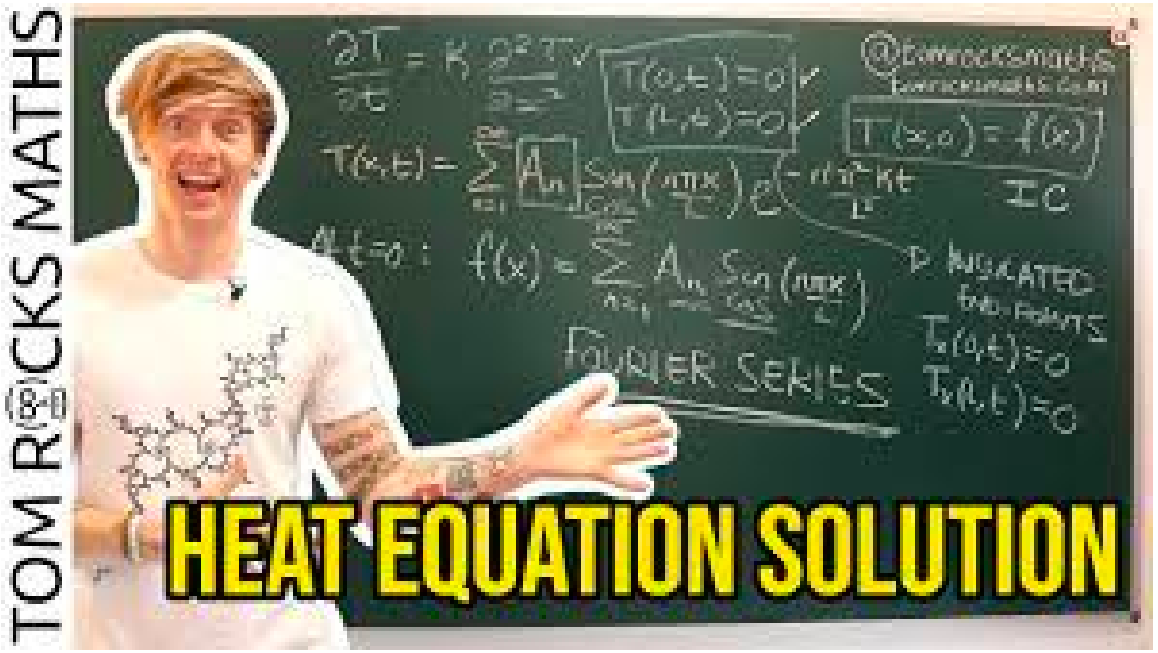
σ is the voltaility of the asset price.

dX is a time continuous stochastic process (Wiener process) where

- dX is a random variable from a normal distribution
- mean of dX = 0
- variance of dX is dt

The Heat Equation and its Solution

Credit: Tom Rocks Maths



- see Tom's youtube *Oxford Calculus* series
- Heat Equation Derivation
- How to Solve the Heat Equation

Deriving the Heat Equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

- ρ = density
- c = specific heat capacity
- heat in at start of rod: $q(a, t)$
- heat out at end of rod: $q(a + h, t)$

$$\lim_{x \to 0} \frac{1}{h} \int_a^{a+h} \rho c T_t dx = \lim_{h \to 0} \frac{-q(a,t) + q(a+h,t)}{h}$$

$$\rho c \frac{\partial T}{\partial t}|_{x=a} = -\frac{\partial q}{\partial x}|_{x=a}$$

Fouriers law

heat flux q

thermal conductivity k

$$q = -k\frac{\partial T}{\partial x}$$

leads to

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2}$$

κ thermal diffusivity

Solving the heat equation

Initial Boundary Value Problem (IBVP)

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

Boundary conditions at x = 0, L

$T(0, t) = 0$

$T(L, t) = 0$

$T(x, 0) = f(x)$ at $t = 0$

A Separable Solutions to the PDE

$$T(x,t) = F(x)G(t)$$

$$F(x)G'(t) = \kappa F''(x)G(t)$$

Group terms

$$\underbrace{\frac{1}{\kappa}\frac{G'(t)}{Gt}}_{\text{function of t only}} = \underbrace{\frac{F''(x)}{F(x)}}_{\text{function of x only}}$$

Solve for boundary conditions $F(0) = 0 = F(L)$