

Differential Equations Homework 2

Problem 1

Let $y(t)$ represent the intensity of an emotional state at time t .

- (a) Consider the differential equation

$$\frac{dy}{dt} = -ky, \quad k > 0.$$

- (a) Solve the differential equation.
(b) Explain whether this model is sufficient to represent the evolution of an emotional state over time. Justify your answer briefly.
(b) Explain how external factors such as visual or auditory stimuli may affect the rate of change $\frac{dy}{dt}$, even if these factors do not explicitly appear in the equation.

Problem 2

Suppose the emotional intensity $y(t)$ is influenced by a constant external input:

$$\frac{dy}{dt} = -ky + c, \quad k > 0, c > 0.$$

- (a) Find the equilibrium (steady-state) solution.
(b) Interpret the equilibrium value in terms of emotional balance.
(c) Give examples of visual or auditory characteristics (e.g., calm scenery, slow tempo sounds) that could be associated with such a constant input.

Problem 3

Consider the nonlinear model

$$\frac{dy}{dt} = -ky + ay(1 - y), \quad k, a > 0.$$

- (a) Find all equilibrium points.
(b) Determine the stability of each equilibrium.
(c) Explain how nonlinear feedback may represent emotional regulation rather than simple decay.

Problem 4

Different emotional states may require different balancing mechanisms.

Match each emotional category below with a suitable type of input that could support emotional balance, and briefly justify your choices.

- Happiness
- Sadness
- Anger
- Fear
- Loneliness
- Guilt/Shame
- Energy Level (Energetic / Fatigued)

Examples of inputs may include visual environments (e.g., open spaces, soft colors) or auditory characteristics (e.g., slow tempo, rhythmic patterns).

Problem 5

Assume two emotional components interact over time and are represented by $x(t)$ and $y(t)$:

$$\begin{aligned}\frac{dx}{dt} &= -ax + by, \\ \frac{dy}{dt} &= -cy,\end{aligned}\quad a, b, c > 0.$$

- (a) Solve the system.
- (b) Explain how interaction between emotional components may influence overall emotional balance.

Problem 6

Explain how the idea of emotional balance can be interpreted mathematically as the solution of a differential equation remaining bounded or returning toward equilibrium over time.

Problem 7

Discuss the limitations of modeling emotional processes using simple differential equations.

Problem 8

In first-order models, the rate parameter controls how fast solutions approach equilibrium. Discuss how choosing larger or smaller rate parameters affects the speed of convergence for different types of emotional responses such as anger, fear, or sadness.

Submission Details

- **Assignment Deadline:** 2 January, 2025 at 23:00
- **Assignment Submission Method:** online.yildiz.edu.tr
- **Assignment Report (1 document in PDF format) Content:**
 - Answers of all the questions with clear explanations by using L^AT_EX, then its compiled version into PDF format.
 - OR answers of all the questions with clear explanations written on paper, then its scanned version as PDF format.
- Each question has equal marks.