
1

Circuit Variables

Assessment Problems

- AP 1.1 Use a product of ratios to convert two-thirds the speed of light from meters per second to miles per second:

$$\left(\frac{2}{3}\right) \frac{3 \times 10^8 \text{ m}}{1 \text{ s}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = \frac{124,274.24 \text{ miles}}{1 \text{ s}}$$

Now set up a proportion to determine how long it takes this signal to travel 1100 miles:

$$\frac{124,274.24 \text{ miles}}{1 \text{ s}} = \frac{1100 \text{ miles}}{x \text{ s}}$$

Therefore,

$$x = \frac{1100}{124,274.24} = 0.00885 = 8.85 \times 10^{-3} \text{ s} = 8.85 \text{ ms}$$

- AP 1.2 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100 \text{ billion} = \$100 \times 10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = \$3.17/\text{ms}$$

- AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x) dx$$

We are given the expression for current, i , which can be substituted into the above expression. To find the total charge, we let $t \rightarrow \infty$ in the integral. Thus we have

$$\begin{aligned} q_{\text{total}} &= \int_0^\infty 20e^{-5000x} dx = \frac{20}{-5000} e^{-5000x} \Big|_0^\infty = \frac{20}{-5000} (e^{-\infty} - e^0) \\ &= \frac{20}{-5000} (0 - 1) = \frac{20}{5000} = 0.004 \text{ C} = 4000 \mu\text{C} \end{aligned}$$

- AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d}{dt} \left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2} \right) e^{-\alpha t} \right] \\ &= \frac{d}{dt} \left(\frac{1}{\alpha^2} \right) - \frac{d}{dt} \left(\frac{t}{\alpha} e^{-\alpha t} \right) - \frac{d}{dt} \left(\frac{1}{\alpha^2} e^{-\alpha t} \right) \\ &= 0 - \left(\frac{1}{\alpha} e^{-\alpha t} - \alpha \frac{t}{\alpha} e^{-\alpha t} \right) - \left(-\alpha \frac{1}{\alpha^2} e^{-\alpha t} \right) \\ &= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha} \right) e^{-\alpha t} \\ &= te^{-\alpha t} \end{aligned}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t :

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1 - \alpha t)e^{-\alpha t} = 0$$

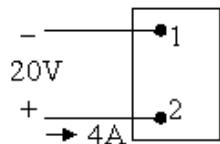
Since $e^{-\alpha t}$ never equals 0 for a finite value of t , the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t , the current is

$$i = \frac{1}{\alpha} e^{-\alpha/\alpha} = \frac{1}{\alpha} e^{-1}$$

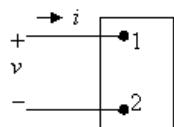
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α ,

$$i = \frac{1}{0.03679} e^{-1} \cong 10 \text{ A}$$

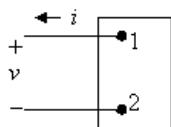
AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



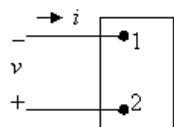
Also sketch the four figures from Fig. 1.6:



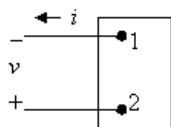
(a)



(b)



(c)



(d)

- [a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

- (a) $v = -20 \text{ V}$, $i = -4 \text{ A}$; (b) $v = -20 \text{ V}$, $i = 4 \text{ A}$
 (c) $v = 20 \text{ V}$, $i = -4 \text{ A}$; (d) $v = 20 \text{ V}$, $i = 4 \text{ A}$

- [b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80 \text{ W}$. Since the power is greater than 0, the box is absorbing power.

- [c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 [a] Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. To find the time at which the power is maximum, find the first derivative of the power with respect to time, set the resulting expression equal to zero, and solve for time:

$$p = (80,000te^{-500t})(15te^{-500t}) = 120 \times 10^4 t^2 e^{-1000t}$$

$$\frac{dp}{dt} = 240 \times 10^4 te^{-1000t} - 120 \times 10^4 t^2 e^{-1000t} = 0$$

Therefore,

$$240 \times 10^4 - 120 \times 10^7 t = 0$$

Solving,

$$t = \frac{240 \times 10^4}{120 \times 10^7} = 2 \times 10^{-3} = 2 \text{ ms}$$

- [b] The maximum power occurs at 2 ms, so find the value of the power at 2 ms:

$$p(0.002) = 120 \times 10^4 (0.002)^2 e^{-2} = 649.6 \text{ mW}$$

- [c] From Eq. (1.3), we know that power is the time rate of change of energy, or $p = dw/dt$. If we know the power, we can find the energy by integrating Eq. (1.3). To find the total energy, the upper limit of the integral is infinity:

$$\begin{aligned} w_{\text{total}} &= \int_0^\infty 120 \times 10^4 x^2 e^{-1000x} dx \\ &= \frac{120 \times 10^4}{(-1000)^3} e^{-1000x} [(-1000)^2 x^2 - 2(-1000)x + 2] \Big|_0^\infty \\ &= 0 - \frac{120 \times 10^4}{(-1000)^3} e^0 (0 - 0 + 2) = 2.4 \text{ mJ} \end{aligned}$$

- AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 $\frac{(260 \times 10^6)(540)}{10^9} = 104.4$ gigawatt-hours

P 1.2 $\frac{(480)(320) \text{ pixels}}{1 \text{ frame}} \cdot \frac{2 \text{ bytes}}{1 \text{ pixel}} \cdot \frac{30 \text{ frames}}{1 \text{ sec}} = 9.216 \times 10^6 \text{ bytes/sec}$

$$(9.216 \times 10^6 \text{ bytes/sec})(x \text{ secs}) = 32 \times 2^{30} \text{ bytes}$$

$$x = \frac{32 \times 2^{30}}{9.216 \times 10^6} = 3728 \text{ sec} = 62 \text{ min} \approx 1 \text{ hour of video}$$

P 1.3 [a] $\frac{20,000 \text{ photos}}{(11)(15)(1) \text{ mm}^3} = \frac{x \text{ photos}}{1 \text{ mm}^3}$

$$x = \frac{(20,000)(1)}{(11)(15)(1)} = 121 \text{ photos}$$

[b] $\frac{16 \times 2^{30} \text{ bytes}}{(11)(15)(1) \text{ mm}^3} = \frac{x \text{ bytes}}{(0.2)^3 \text{ mm}^3}$

$$x = \frac{(16 \times 2^{30})(0.008)}{(11)(15)(1)} = 832,963 \text{ bytes}$$

P 1.4 $(4 \text{ cond.}) \cdot (845 \text{ mi}) \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{2526 \text{ lb}}{1000 \text{ ft}} \cdot \frac{1 \text{ kg}}{2.2 \text{ lb}} = 20.5 \times 10^6 \text{ kg}$

P 1.5 Volume = area \times thickness

Convert values to millimeters, noting that $10 \text{ m}^2 = 10^6 \text{ mm}^2$

$$10^6 = (10 \times 10^6)(\text{thickness})$$

$$\Rightarrow \text{thickness} = \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm}$$

P 1.6 [a] We can set up a ratio to determine how long it takes the bamboo to grow $10 \mu\text{m}$. First, recall that $1 \text{ mm} = 10^3 \mu\text{m}$. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10 \mu\text{m}$:

$$\frac{10/3456 \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \quad \text{so} \quad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s}$$

$$[\text{b}] \frac{1 \text{ cell length}}{3.456 \text{ s}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} \cdot \frac{(24)(7) \text{ hr}}{1 \text{ week}} = 175,000 \text{ cell lengths/week}$$

P 1.7 [a] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 0.125e^{-2500t}$$

$$\text{Therefore, } dq = 0.125e^{-2500t} dt$$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 0.125 \int_0^t e^{-2500y} dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 0.125 \frac{e^{-2500y}}{-2500} \Big|_0^t = 50 \times 10^{-6} (1 - e^{-2500t})$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 50(1 - e^{-2500t}) \mu\text{C}$$

[b] As $t \rightarrow \infty$, $q_T = 50 \mu\text{C}$.

$$[\text{c}] q(0.5 \times 10^{-3}) = (50 \times 10^{-6})(1 - e^{(-2500)(0.0005)}) = 35.675 \mu\text{C}.$$

P 1.8 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20 \cos 5000t$$

$$\text{Therefore, } dq = 20 \cos 5000t dt$$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20 \int_0^t \cos 5000y dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20 \frac{\sin 5000y}{5000} \Big|_0^t = \frac{20}{5000} \sin 5000t - \frac{20}{5000} \sin 5000(0) = \frac{20}{5000} \sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4 \times 10^{-3} \sin 5000t \text{ C} = 4 \sin 5000t \text{ mC}$

P 1.9 [a] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 40te^{-500t}$$

Therefore, $dq = 40te^{-500t} dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 40 \int_0^t ye^{-500y} dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$\begin{aligned} q(t) - q(0) &= 40 \frac{e^{-500y}}{(-500)^2} (-500y - 1) \Big|_0^t = 160 \times 10^{-6} e^{-500t} (-500t - 1) + 160 \times 10^{-6} \\ &= 160 \times 10^{-6} (1 - 500te^{-500t} - e^{-500t}) \end{aligned}$$

But $q(0) = 0$ by hypothesis, so

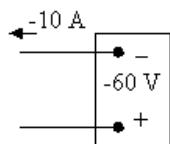
$$q(t) = 160(1 - 500te^{-500t} - e^{-500t}) \mu\text{C}$$

[b] $q(0.001) = (160)[1 - 500(0.001)e^{-500(0.001)} - e^{-500(0.001)}] = 14.4 \mu\text{C}$.

P 1.10 $n = \frac{35 \times 10^{-6} \text{ C/s}}{1.6022 \times 10^{-19} \text{ C/elec}} = 2.18 \times 10^{14} \text{ elec/s}$

P 1.11 $w = qV = (1.6022 \times 10^{-19})(6) = 9.61 \times 10^{-19} = 0.961 \text{ aJ}$

P 1.12 [a]



$$p = vi = (40)(-10) = -400 \text{ W}$$

Power is being delivered by the box.

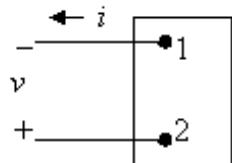
[b] Entering

[c] Gaining

P 1.13 [a] $p = vi = (-60)(-10) = 600 \text{ W}$, so power is being absorbed by the box.

[b] Entering

[c] Losing



- P 1.14 Assume we are standing at box A looking toward box B. Use the passive sign convention to get $p = vi$, since the current i is flowing into the + terminal of the voltage v . Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

- [a] $p = (30)(6) = 180 \text{ W}$ 180 W from A to B
- [b] $p = (-20)(-8) = 160 \text{ W}$ 160 W from A to B
- [c] $p = (-60)(4) = -240 \text{ W}$ 240 W from B to A
- [d] $p = (40)(-9) = -360 \text{ W}$ 360 W from B to A

- P 1.15 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery (the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,
 $p = vi = (30)(12) = 360 \text{ W}$.
 Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

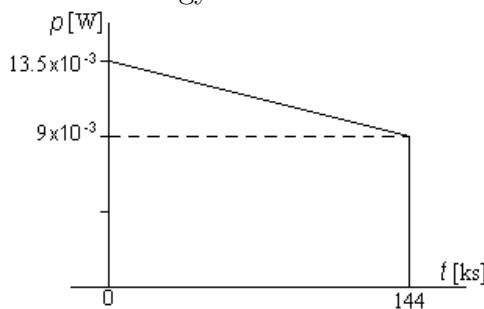
$$[b] w(t) = \int_0^t p dx; \quad 1 \text{ min} = 60 \text{ s}$$

$$w(60) = \int_0^{60} 360 dx$$

$$w = 360(60 - 0) = 360(60) = 21,600 \text{ J} = 21.6 \text{ kJ}$$

$$P 1.16 \quad p = vi; \quad w = \int_0^t p dx$$

Since the energy is the area under the power vs. time plot, let us plot p vs. t .



Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$$p(0) = (1.5)(9 \times 10^{-3}) = 13.5 \times 10^{-3} \text{ W}$$

$$p(144 \text{ ks}) = (1)(9 \times 10^{-3}) = 9 \times 10^{-3} \text{ W}$$

$$w = (9 \times 10^{-3})(144 \times 10^3) + \frac{1}{2}(13.5 \times 10^{-3} - 9 \times 10^{-3})(144 \times 10^3) = 1620 \text{ J}$$

$$\text{P 1.17 } p = (12)(100 \times 10^{-3}) = 1.2 \text{ W}; \quad 4 \text{ hr} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 14,400 \text{ s}$$

$$w(t) = \int_0^t p dt \quad w(14,400) = \int_0^{14,400} 1.2 dt = 1.2(14,400) = 17.28 \text{ kJ}$$

$$\text{P 1.18 [a]} \quad p = vi = (15e^{-250t})(0.04e^{-250t}) = 0.6e^{-500t} \text{ W}$$

$$p(0.01) = 0.6e^{-500(0.01)} = 0.6e^{-5} = 0.00404 = 4.04 \text{ mW}$$

$$\begin{aligned} \text{[b]} \quad w_{\text{total}} &= \int_0^\infty p(x) dx = \int_0^\infty 0.6e^{-500x} dx = \frac{0.6}{-500} e^{-500x} \Big|_0^\infty \\ &= -0.0012(e^{-\infty} - e^0) = 0.0012 = 1.2 \text{ mJ} \end{aligned}$$

$$\text{P 1.19 [a]} \quad p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t}) \text{ W}$$

$$\begin{aligned} \frac{dp}{dt} &= -3750e^{-1000t} + 7500e^{-2000t} = 0 \quad \text{so} \quad 2e^{-2000t} = e^{-1000t} \\ 2 &= e^{1000t} \quad \text{so} \quad \ln 2 = 1000t \quad \text{thus} \quad p \text{ is maximum at } t = 693.15 \mu\text{s} \end{aligned}$$

$$p_{\max} = p(693.15 \mu\text{s}) = 937.5 \text{ mW}$$

$$\begin{aligned} \text{[b]} \quad w &= \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}] dt = \left[\frac{3.75}{-1000} e^{-1000t} - \frac{3.75}{-2000} e^{-2000t} \right]_0^\infty \\ &= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ} \end{aligned}$$

$$\text{P 1.20 [a]} \quad p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$$

$$p(625 \mu\text{s}) = 42.2 \text{ mW}$$

$$\begin{aligned} \text{[b]} \quad w(t) &= \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}) dt \\ &= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t} \mu\text{J} \\ w(625 \mu\text{s}) &= 12.14 \mu\text{J} \end{aligned}$$

$$\text{[c]} \quad w_{\text{total}} = 140.625 \mu\text{J}$$

$$\begin{aligned}
 P 1.21 \quad [a] \quad p &= vi \\
 &= [(1500t + 1)e^{-750t}](0.04e^{-750t}) \\
 &= (60t + 0.04)e^{-1500t} \\
 \frac{dp}{dt} &= 60e^{-1500t} - 1500e^{-1500t}(60t + 0.04) \\
 &= -90,000te^{-1500t}
 \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so p_{\max} occurs at $t = 0$.

$$\begin{aligned}
 [b] \quad p_{\max} &= [(60)(0) + 0.04]e^0 = 0.04 \\
 &= 40 \text{ mW}
 \end{aligned}$$

$$\begin{aligned}
 [c] \quad w &= \int_0^t pdx \\
 w &= \int_0^t 60xe^{-1500x} dx + \int_0^t 0.04e^{-1500x} dx \\
 &= \frac{60e^{-1500x}}{(-1500)^2}(-1500x - 1) \Big|_0^t + 0.04 \frac{e^{-1500x}}{-1500} \Big|_0^t
 \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence
 $w = \frac{60}{225 \times 10^4} + \frac{0.04}{1500} = 53.33 \mu\text{J}$.

$$\begin{aligned}
 P 1.22 \quad [a] \quad p &= vi \\
 &= [(3200t + 3.2)e^{-1000t}][(160t + 0.16)e^{-1000t}] \\
 &= e^{-2000t}[512,000t^2 + 1024t + 0.512] \\
 \frac{dp}{dt} &= e^{-2000t}[1,024,000t + 1024] - 2000e^{-2000t}[512,000t^2 + 1024t + 0.512] \\
 &= -e^{-2000t}[1024 \times 10^6 t^2 + 1,024,000t]
 \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so p_{\max} occurs at $t = 0$.

$$\begin{aligned}
 [b] \quad p_{\max} &= e^{-0}[0 + 0 + 0.512] \\
 &= 512 \text{ mW}
 \end{aligned}$$

$$\begin{aligned}
 [c] \quad w &= \int_0^t pdx \\
 w &= \int_0^t 512,000x^2 e^{-2000x} dx + \int_0^t 1024x e^{-2000x} dx + \int_0^t 0.512 e^{-2000x} dx \\
 &= \frac{512,000e^{-2000x}}{-8 \times 10^9}[4 \times 10^6 x^2 + 4000x + 2] \Big|_0^t + \\
 &\quad \frac{1024e^{-2000x}}{4 \times 10^6}(-2000x - 1) \Big|_0^t + \frac{0.512e^{-2000x}}{-2000} \Big|_0^t
 \end{aligned}$$

When $t \rightarrow \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(512,000)(2)}{8 \times 10^9} + \frac{1024}{4 \times 10^6} + \frac{0.512}{2000}$$

$$w = 128 \times 10^{-6} + 256 \times 10^{-6} + 256 \times 10^{-6} = 640 \mu\text{J}.$$

- P 1.23 [a] We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t . The calculations are shown below:

$$p = 0 \quad t < 0, \quad p = 0 \quad t > 40 \text{ s}$$

$$p = vi = t(1 - 0.025t)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3 \text{ W} \quad 0 \leq t \leq 40 \text{ s}$$

$$\frac{dp}{dt} = 4 - 0.6t + 0.015t^2 = 0.015(t^2 - 40t + 266.67)$$

$$\frac{dp}{dt} = 0 \quad \text{when } t^2 - 40t + 266.67 = 0$$

$$t_1 = 8.453 \text{ s}; \quad t_2 = 31.547 \text{ s}$$

(using the polynomial solver on your calculator)

$$p(t_1) = 4(8.453) - 0.3(8.453)^2 + 0.005(8.453)^3 = 15.396 \text{ W}$$

$$p(t_2) = 4(31.547) - 0.3(31.547)^2 + 0.005(31.547)^3 = -15.396 \text{ W}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

- [b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\max} = 15.396 \text{ W}$ (delivered)
- [c] As we saw in part (a), the other “maximum” power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.
- [d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\max} = 15.396 \text{ W}$ (extracted)

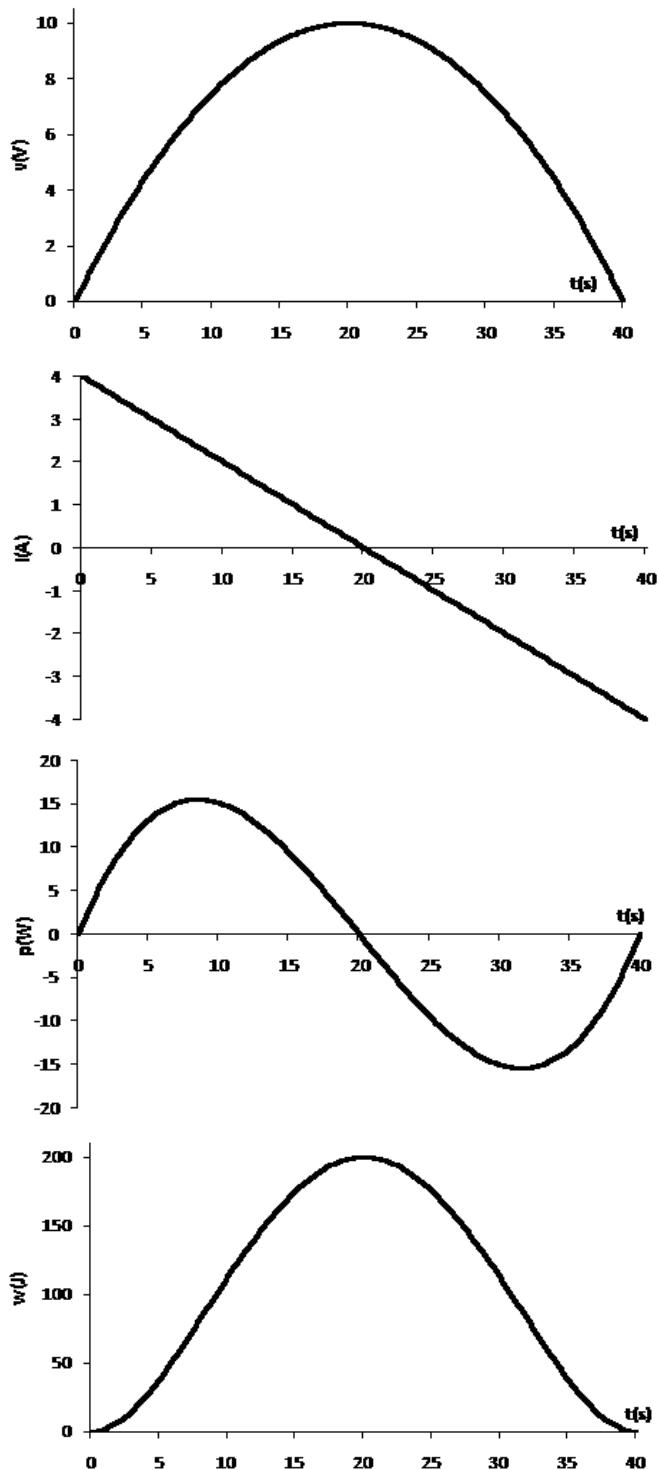
[e] $w = \int_0^t pdx = \int_0^t (4x - 0.3x^2 + 0.005x^3)dx = 2t^2 - 0.1t^3 + 0.00125t^4$

$$w(0) = 0 \text{ J} \quad w(30) = 112.5 \text{ J}$$

$$w(10) = 112.5 \text{ J} \quad w(40) = 0 \text{ J}$$

$$w(20) = 200 \text{ J}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:



P 1.24 [a] $v(10 \text{ ms}) = 400e^{-1} \sin 2 = 133.8 \text{ V}$
 $i(10 \text{ ms}) = 5e^{-1} \sin 2 = 1.67 \text{ A}$
 $p(10 \text{ ms}) = vi = 223.80 \text{ W}$

$$\begin{aligned}
 [\mathbf{b}] \quad p &= vi = 2000e^{-200t} \sin^2 200t \\
 &= 2000e^{-200t} \left[\frac{1}{2} - \frac{1}{2} \cos 400t \right] \\
 &= 1000e^{-200t} - 1000e^{-200t} \cos 400t \\
 w &= \int_0^\infty 1000e^{-200t} dt - \int_0^\infty 1000e^{-200t} \cos 400t dt \\
 &= 1000 \left. \frac{e^{-200t}}{-200} \right|_0^\infty \\
 &\quad - 1000 \left. \left\{ \frac{e^{-200t}}{(200)^2 + (400)^2} [-200 \cos 400t + 400 \sin 400t] \right\} \right|_0^\infty \\
 &= 5 - 1000 \left[\frac{200}{4 \times 10^4 + 16 \times 10^4} \right] = 5 - 1 \\
 w &= 4 \text{ J}
 \end{aligned}$$

P 1.25 [a] $p = vi = 2000 \cos(800\pi t) \sin(800\pi t) = 1000 \sin(1600\pi t)$ W

Therefore, $p_{\max} = 1000$ W

[b] $p_{\max}(\text{extracting}) = 1000$ W

$$\begin{aligned}
 [\mathbf{c}] \quad p_{\text{avg}} &= \frac{1}{2.5 \times 10^{-3}} \int_0^{2.5 \times 10^{-3}} 1000 \sin(1600\pi t) dt \\
 &= 4 \times 10^5 \left[\frac{-\cos 1600\pi t}{1600\pi} \right]_0^{2.5 \times 10^{-3}} = \frac{250}{\pi} [1 - \cos 4\pi] = 0
 \end{aligned}$$

[d]

$$\begin{aligned}
 p_{\text{avg}} &= \frac{1}{15.625 \times 10^{-3}} \int_0^{15.625 \times 10^{-3}} 1000 \sin(1600\pi t) dt \\
 &= 64 \times 10^3 \left[\frac{-\cos 1600\pi t}{1600\pi} \right]_0^{15.625 \times 10^{-3}} = \frac{40}{\pi} [1 - \cos 25\pi] = 25.46 \text{ W}
 \end{aligned}$$

P 1.26 [a] $q = \text{area under } i \text{ vs. } t \text{ plot}$

$$\begin{aligned}
 &= \frac{1}{2}(8)(12,000) + (16)(12,000) + \frac{1}{2}(16)(4000) \\
 &= 48,000 + 192,000 + 32,000 = 272,000 \text{ C}
 \end{aligned}$$

[b] $w = \int p dt = \int vi dt$

$$v = 250 \times 10^{-6}t + 8 \quad 0 \leq t \leq 16 \text{ ks}$$

$$0 \leq t \leq 12,000 \text{ s:}$$

$$i = 24 - 666.67 \times 10^{-6}t$$

$$p = 192 + 666.67 \times 10^{-6}t - 166.67 \times 10^{-9}t^2$$

$$\begin{aligned}
 w_1 &= \int_0^{12,000} (192 + 666.67 \times 10^{-6}t - 166.67 \times 10^{-9}t^2) dt \\
 &= (2304 + 48 - 96)10^3 = 2256 \text{ kJ}
 \end{aligned}$$

$12,000 \text{ s} \leq t \leq 16,000 \text{ s}$:

$$i = 64 - 4 \times 10^{-3}t$$

$$p = 512 - 16 \times 10^{-3}t - 10^{-6}t^2$$

$$w_2 = \int_{12,000}^{16,000} (512 - 16 \times 10^{-3}t - 10^{-6}t^2) dt$$

$$= (2048 - 896 - 789.33)10^3 = 362.667 \text{ kJ}$$

$$w_T = w_1 + w_2 = 2256 + 362.667 = 2618.667 \text{ kJ}$$

P 1.27 [a] $0 \text{ s} \leq t < 10 \text{ ms}$:

$$v = 8 \text{ V}; \quad i = 25t \text{ A}; \quad p = 200t \text{ W}$$

$10 \text{ ms} < t \leq 30 \text{ ms}$:

$$v = -8 \text{ V}; \quad i = 0.5 - 25t \text{ A}; \quad p = 200t - 4 \text{ W}$$

$30 \text{ ms} \leq t < 40 \text{ ms}$:

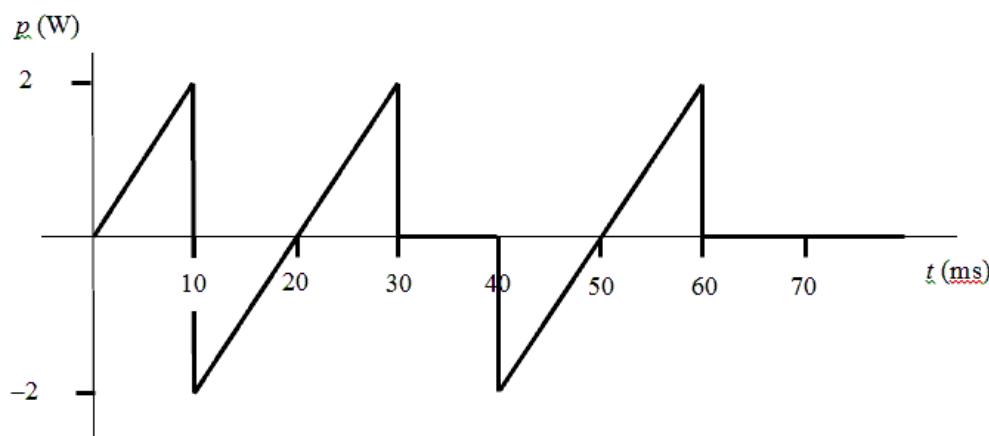
$$v = 0 \text{ V}; \quad i = -250 \text{ mA}; \quad p = 0 \text{ W}$$

$40 \text{ ms} < t \leq 60 \text{ ms}$:

$$v = 8 \text{ V}; \quad i = 25t - 1.25 \text{ A}; \quad p = 200t - 10 \text{ W}$$

$t > 60 \text{ ms}$:

$$v = 0 \text{ V}; \quad i = 250 \text{ mA}; \quad p = 0 \text{ W}$$



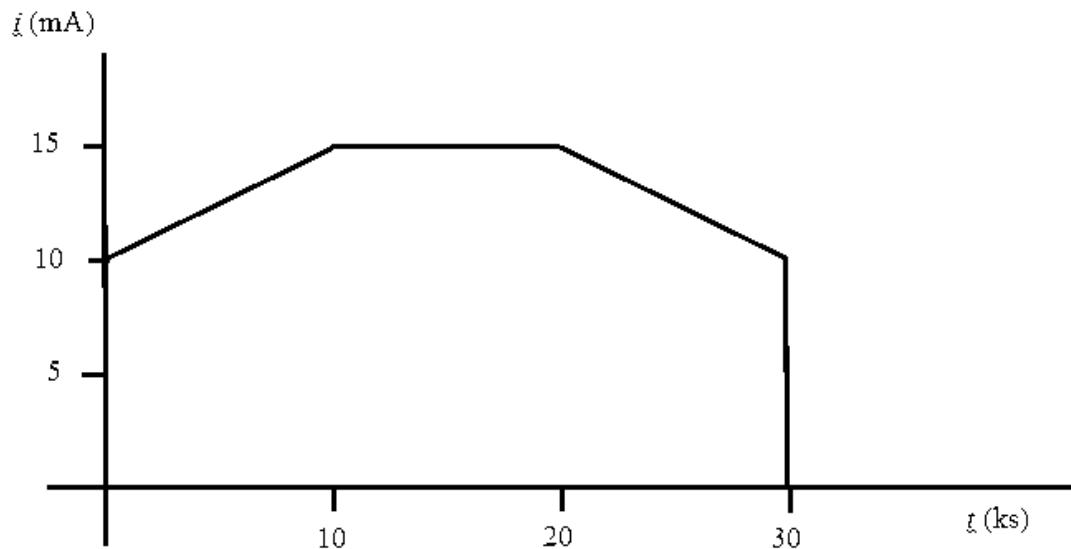
[b] Calculate the area under the curve from zero up to the desired time:

$$w(0.01) = \frac{1}{2}(2)(0.01) = 10 \text{ mJ}$$

$$w(0.03) = w(0.01) - \frac{1}{2}(2)(0.01) + \frac{1}{2}(2)(0.01) = 10 \text{ mJ}$$

$$w(0.08) = w(0.03) - \frac{1}{2}(2)(0.01) + \frac{1}{2}(2)(0.01) = 10 \text{ mJ}$$

P 1.28 [a]



[b] $i(t) = 10 + 0.5 \times 10^{-3}t$ mA, $0 \leq t \leq 10$ ks

$$i(t) = 15 \text{ mA}, \quad 10 \text{ ks} \leq t \leq 20 \text{ ks}$$

$$i(t) = 25 - 0.5 \times 10^{-3}t \text{ mA}, \quad 20 \text{ ks} \leq t \leq 30 \text{ ks}$$

$$i(t) = 0, \quad t > 30 \text{ ks}$$

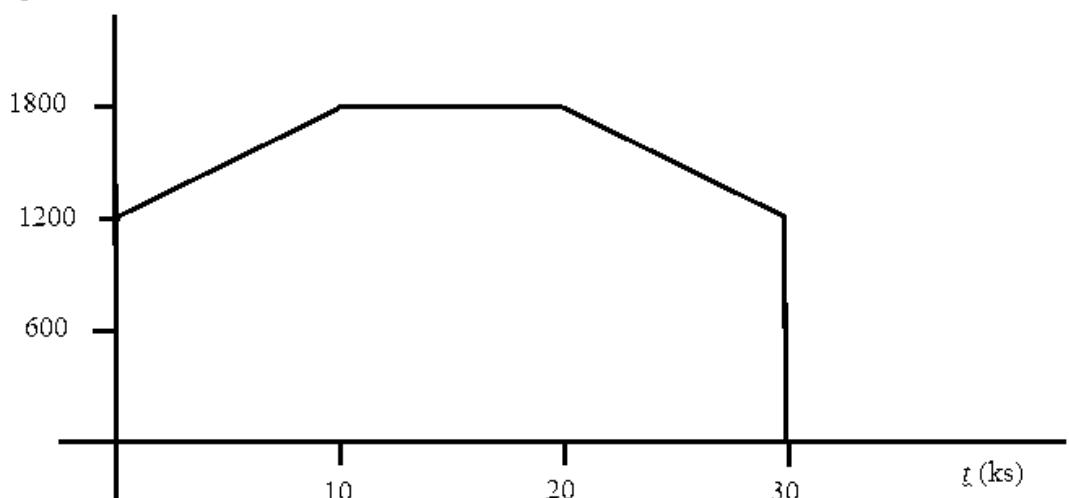
$$p = vi = 120i \text{ so}$$

$$p(t) = 1200 + 0.06t \text{ mW}, \quad 0 \leq t \leq 10 \text{ ks}$$

$$p(t) = 1800 \text{ mW}, \quad 10 \text{ ks} \leq t \leq 20 \text{ ks}$$

$$p(t) = 3000 - 0.06t \text{ mW}, \quad 20 \text{ ks} \leq t \leq 30 \text{ ks}$$

$$p(t) = 0, \quad t > 30 \text{ ks}$$

 p (mA)

[c] To find the energy, calculate the area under the plot of the power:

$$w(10 \text{ ks}) = \frac{1}{2}(0.6)(10,000) + (1.2)(10,000) = 15 \text{ kJ}$$

$$w(20 \text{ ks}) = w(10 \text{ ks}) + (1.8)(10,000) = 33 \text{ kJ}$$

$$w(10 \text{ ks}) = w(20 \text{ ks}) + \frac{1}{2}(0.6)(10,000) + (1.2)(10,000) = 48 \text{ kJ}$$

- P 1.29 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i , as shown below:

$$p_a = -v_a i_a = -(40)(-4 \times 10^{-3}) = 160 \text{ mW}$$

$$p_b = v_b i_b = (-24)(-4 \times 10^{-3}) = 96 \text{ mW}$$

$$p_c = -v_c i_c = -(-16)(4 \times 10^{-3}) = 64 \text{ mW}$$

$$p_d = -v_d i_d = -(-80)(-1.5 \times 10^{-3}) = -120 \text{ mW}$$

$$p_e = v_e i_e = (40)(2.5 \times 10^{-3}) = 100 \text{ mW}$$

$$p_f = v_f i_f = (120)(-2.5 \times 10^{-3}) = -300 \text{ mW}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas if the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$$\sum P_{\text{dev}} = 120 + 300 = 420 \text{ mW};$$

$$\sum P_{\text{abs}} = 160 + 96 + 64 + 100 = 420 \text{ mW}$$

Thus, the power balances and the total power absorbed in the circuit is 420 mW.

- P 1.30 $p_a = -v_a i_a = -(-3000)(-250 \times 10^{-6}) = -0.75 \text{ W}$

$$p_b = -v_b i_b = -(4000)(-400 \times 10^{-6}) = 1.6 \text{ W}$$

$$p_c = -v_c i_c = -(1000)(400 \times 10^{-6}) = -0.4 \text{ W}$$

$$p_d = v_d i_d = (1000)(150 \times 10^{-6}) = 0.15 \text{ W}$$

$$p_e = v_e i_e = (-4000)(200 \times 10^{-6}) = -0.8 \text{ W}$$

$$p_f = v_f i_f = (4000)(50 \times 10^{-6}) = 0.2 \text{ W}$$

Therefore,

$$\sum P_{\text{abs}} = 1.6 + 0.15 + 0.2 = 1.95 \text{ W}$$

$$\sum P_{\text{del}} = 0.75 + 0.4 + 0.8 = 1.95 \text{ W} = \sum P_{\text{abs}}$$

Thus, the interconnection does satisfy the power check.

P 1.31 [a] From the diagram and the table we have

$$p_a = -v_a i_a = -(46.16)(6) = -276.96 \text{ W}$$

$$p_b = v_b i_b = (14.16)(4.72) = 66.8352 \text{ W}$$

$$p_c = v_c i_c = (-32)(-6.4) = 204.8 \text{ W}$$

$$p_d = -v_d i_d = -(22)(1.28) = -28.16 \text{ W}$$

$$p_e = v_e i_e = (-33.6)(-1.68) = 56.448 \text{ W}$$

$$p_f = v_f i_f = (66)(0.4) = 26.4 \text{ W}$$

$$p_g = v_g i_g = (2.56)(1.28) = 3.2768 \text{ W}$$

$$p_h = -v_h i_h = -(-0.4)(0.4) = 0.16 \text{ W}$$

$$\sum P_{\text{del}} = 276.96 + 28.16 = 305.12 \text{ W}$$

$$\sum P_{\text{abs}} = 66.8352 + 204.8 + 56.448 + 26.4 + 3.2768 + 0.16 = 357.92 \text{ W}$$

Therefore, $\sum P_{\text{del}} \neq \sum P_{\text{abs}}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$-305.12 + 357.92 = 52.8 \text{ W}$$

One-half of this difference is 26.4 W, so it is likely that p_f is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the node connecting components f and h, the current i_f should be -0.4 A, not 0.4 A!) If the sign of p_f is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\sum P_{\text{del}} = 276.96 + 28.16 + 26.4 = 331.52 \text{ W}$$

$$\sum P_{\text{abs}} = 66.8352 + 204.8 + 56.448 + 3.2768 + 0.16 = 331.52 \text{ W}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.32 [a] Remember that if the circuit element is absorbing power, the power is positive, whereas if the circuit element is supplying power, the power is negative. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal: $\sum P_{\text{sup}} = 600 + 50 + 600 + 1250 = 2500 \text{ W}$;

$$\sum P_{\text{abs}} = 400 + 100 + 2000 = 2500 \text{ W}$$

Thus, the power balances.

[b] The current can be calculated using $i = p/v$ or $i = -p/v$, with proper application of the passive sign convention:

$$i_a = -p_a/v_a = -(-600)/(400) = 1.5 \text{ A}$$

$$i_b = p_b/v_b = (-50)/(-100) = 0.5 \text{ A}$$

$$i_c = p_c/v_c = (400)/(200) = 2.0 \text{ A}$$

$$i_d = p_d/v_d = (-600)/(300) = -2.0 \text{ A}$$

$$i_e = p_e/v_e = (100)/(-200) = -0.5 \text{ A}$$

$$i_f = -p_f/v_f = -(2000)/(500) = -4.0 \text{ A}$$

$$i_g = p_g/v_g = (-1250)/(-500) = 2.5 \text{ A}$$

P 1.33 [a] If the power balances, the sum of the power values should be zero:

$$p_{\text{total}} = 0.175 + 0.375 + 0.150 - 0.320 + 0.160 + 0.120 - 0.660 = 0$$

Thus, the power balances.

[b] When the power is positive, the element is absorbing power. Since elements a, b, c, e, and f have positive power, these elements are absorbing power.

[c] The voltage can be calculated using $v = p/i$ or $v = -p/i$, with proper application of the passive sign convention:

$$v_a = p_a/i_a = (0.175)/(0.025) = 7 \text{ V}$$

$$v_b = p_b/i_b = (0.375)/(0.075) = 5 \text{ V}$$

$$v_c = -p_c/i_c = -(0.150)/(-0.05) = 3 \text{ V}$$

$$v_d = p_d/i_d = (-0.320)/(0.04) = -8 \text{ V}$$

$$v_e = -p_e/i_e = -(0.160)/(0.02) = -8 \text{ V}$$

$$v_f = p_f/i_f = (0.120)/(-0.03) = -4 \text{ V}$$

$$v_g = p_g/i_g = (-0.66)/(0.055) = -12 \text{ V}$$

P 1.34 $p_a = v_a i_a = (120)(-10) = -1200 \text{ W}$

$$p_b = -v_b i_b = -(120)(9) = -1080 \text{ W}$$

$$p_c = v_c i_c = (10)(10) = 100 \text{ W}$$

$$p_d = -v_d i_d = -(10)(-1) = 10 \text{ W}$$

$$p_e = v_e i_e = (-10)(-9) = 90 \text{ W}$$

$$p_f = -v_f i_f = -(-100)(5) = 500 \text{ W}$$

$$p_g = v_g i_g = (120)(4) = 480 \text{ W}$$

$$p_h = v_h i_h = (-220)(-5) = 1100 \text{ W}$$

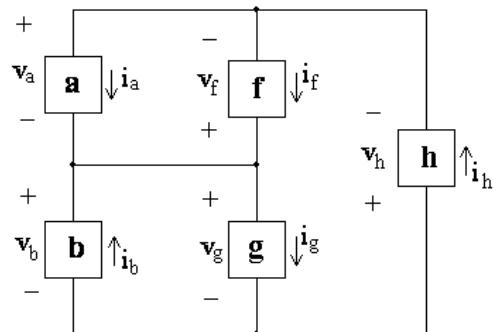
$$\sum P_{\text{del}} = 1200 + 1080 = 2280 \text{ W}$$

$$\sum P_{\text{abs}} = 100 + 10 + 90 + 500 + 480 + 1100 = 2280 \text{ W}$$

$$\text{Therefore, } \sum P_{\text{del}} = \sum P_{\text{abs}} = 2280 \text{ W}$$

Thus, the interconnection now satisfies the power check.

- P 1.35 [a] The revised circuit model is shown below:



- [b] The expression for the total power in this circuit is

$$\begin{aligned} &v_a i_a - v_b i_b - v_f i_f + v_g i_g + v_h i_h \\ &= (120)(-10) - (120)(10) - (-120)(3) + 120i_g + (-240)(-7) = 0 \end{aligned}$$

Therefore,

$$120i_g = 1200 + 1200 - 360 - 1680 = 360$$

so

$$i_g = \frac{360}{120} = 3 \text{ A}$$

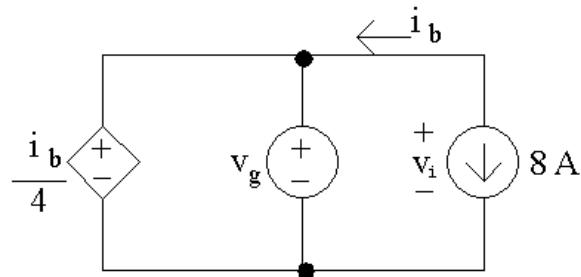
Thus, if the power in the modified circuit is balanced the current in component g is 3 A.

2

Circuit Elements

Assessment Problems

AP 2.1



- [a] Note that the current i_b is in the same circuit branch as the 8 A current source; however, i_b is defined in the opposite direction of the current source. Therefore,

$$i_b = -8 \text{ A}$$

Next, note that the dependent voltage source and the independent voltage source are in parallel with the same polarity. Therefore, their voltages are equal, and

$$v_g = \frac{i_b}{4} = \frac{-8}{4} = -2 \text{ V}$$

- [b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v_i . Note that the two independent sources are in parallel, and that the voltages v_g and v_i have the same polarities, so these voltages are equal:

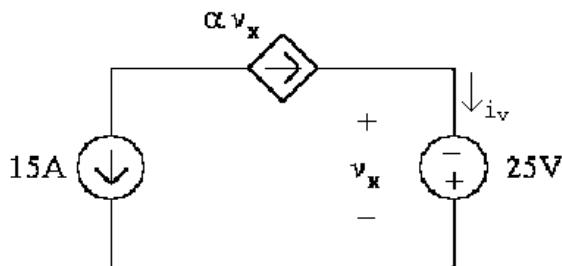
$$v_i = v_g = -2 \text{ V}$$

Using the passive sign convention,

$$p_s = (8 \text{ A})(v_i) = (8 \text{ A})(-2 \text{ V}) = -16 \text{ W}$$

Thus the current source generated 16 W of power.

AP 2.2



- [a] Note from the circuit that $v_x = -25$ V. To find α note that the two current sources are in the same branch of the circuit but their currents flow in opposite directions. Therefore

$$\alpha v_x = -15 \text{ A}$$

Solve the above equation for α and substitute for v_x ,

$$\alpha = \frac{-15 \text{ A}}{v_x} = \frac{-15 \text{ A}}{-25 \text{ V}} = 0.6 \text{ A/V}$$

- [b] To find the power associated with the voltage source we need to know the current, i_v . Note that this current is in the same branch of the circuit as the dependent current source and these two currents flow in the same direction. Therefore, the current i_v is the same as the current of the dependent source:

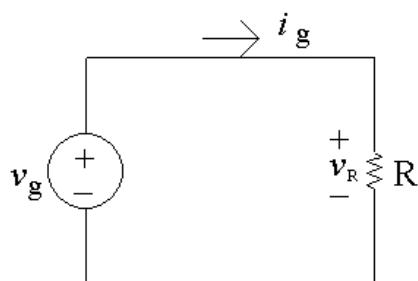
$$i_v = \alpha v_x = (0.6)(-25) = -15 \text{ A}$$

Using the passive sign convention,

$$p_s = -(i_v)(25 \text{ V}) = -(-15 \text{ A})(25 \text{ V}) = 375 \text{ W}.$$

Thus the voltage source dissipates 375 W.

AP 2.3



- [a] The resistor and the voltage source are in parallel and the resistor voltage and the voltage source have the same polarities. Therefore these two voltages are the same:

$$v_R = v_g = 1 \text{ kV}$$

Note from the circuit that the current through the resistor is $i_g = 5 \text{ mA}$. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v_R}{i_g} = \frac{1 \text{ kV}}{5 \text{ mA}} = 200 \text{ k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p_R = (v_R)(i_g) = (1 \text{ kV})(5 \text{ mA}) = 5 \text{ W}$$

The resistor is dissipating 5 W of power.

- [b] Note from part (a) the $v_R = v_g$ and $i_R = i_g$. The power delivered by the source is thus

$$p_{\text{source}} = -v_g i_g \quad \text{so} \quad v_g = -\frac{p_{\text{source}}}{i_g} = -\frac{-3 \text{ W}}{75 \text{ mA}} = 40 \text{ V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v_g}{i_g} = \frac{40 \text{ V}}{75 \text{ mA}} = 533.33 \Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p_R = -p_{\text{source}} = -(-3 \text{ W}) = 3 \text{ W}$$

- [c] Again, note the $i_R = i_g$. The power dissipated by the resistor can be determined from the resistor's current:

$$p_R = R(i_R)^2 = R(i_g)^2$$

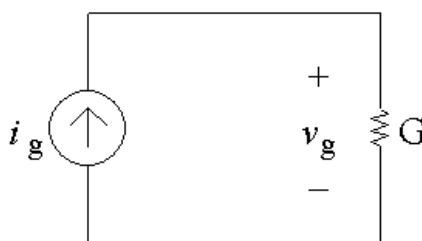
Solving for i_g ,

$$i_g^2 = \frac{p_R}{R} = \frac{480 \text{ mW}}{300 \Omega} = 0.0016 \quad \text{so} \quad i_g = \sqrt{0.0016} = 0.04 \text{ A} = 40 \text{ mA}$$

Then, since $v_R = v_g$

$$v_R = Ri_R = Ri_g = (300 \Omega)(40 \text{ mA}) = 12 \text{ V} \quad \text{so} \quad v_g = 12 \text{ V}$$

AP 2.4



- [a] Note from the circuit that the current through the conductance G is i_g , flowing from top to bottom, because the current source and the

conductance are in the same branch of the circuit so must have the same current. The voltage drop across the current source is v_g , positive at the top, because the current source and the conductance are also in parallel so must have the same voltage. From a version of Ohm's law,

$$v_g = \frac{i_g}{G} = \frac{0.5 \text{ A}}{50 \text{ mS}} = 10 \text{ V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v_g i_g = -(10)(0.5) = -5 \text{ W}$$

Thus the current source delivers 5 W to the circuit.

- [b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p_g = G v_g^2 \quad \text{so} \quad G = \frac{p_g}{v_g^2} = \frac{9}{15^2} = 0.04 \text{ S} = 40 \text{ mS}$$

$$i_g = G v_g = (40 \text{ mS})(15 \text{ V}) = 0.6 \text{ A}$$

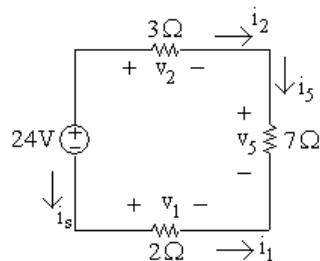
- [c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p_g = G v_g^2 \quad \text{so} \quad v_g^2 = \frac{p_g}{G} = \frac{8 \text{ W}}{200 \mu\text{S}} = 40,000$$

$$\text{Thus } v_g = \sqrt{40,000} = 200 \text{ V}$$

$$i_g = G v_g = (200 \mu\text{S})(200 \text{ V}) = 0.04 \text{ A} = 40 \text{ mA}$$

- AP 2.5 [a] Redraw the circuit with all of the voltages and currents labeled for every circuit element.



Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24 \text{ V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \quad v_5 = 7i_5; \quad v_1 = 2i_1$$

A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i_s = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5 :

$$v_2 = 3i_2 = 3i_5; \quad v_5 = 7i_5; \quad v_1 = 2i_1 = -2i_5$$

Now substitute these expressions for the three voltages into the first equation:

$$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4$ V

[c] $v_2 = 3i_5 = 3(2) = 6$ V

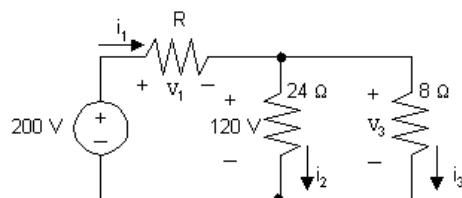
[d] $v_5 = 7i_5 = 7(2) = 14$ V

[e] A KCL equation at the lower left node gives $i_s = i_1$. Since $i_1 = -i_5$, $i_s = -2$ A. We can now compute the power associated with the voltage source:

$$p_{24} = (24)i_s = (24)(-2) = -48 \text{ W}$$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:



We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the 24 Ω resistor:

$$-120 \text{ V} + v_3 = 0$$

Use Ohm's law to calculate the voltage across the 8 Ω resistor in terms of its current:

$$v_3 = 8i_3$$

Substitute the expression for v_3 into the first equation:

$$-120 \text{ V} + 8i_3 = 0 \quad \text{so} \quad i_3 = \frac{120}{8} = 15 \text{ A}$$

Also use Ohm's law to calculate the value of the current through the 24Ω resistor:

$$i_2 = \frac{120\text{ V}}{24\Omega} = 5\text{ A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \quad \text{so} \quad i_1 = i_2 + i_3 = 5 + 15 = 20\text{ A}$$

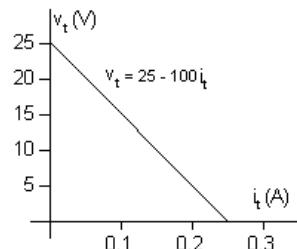
Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\text{ V} + v_1 + 120\text{ V} = 0 \quad \text{so} \quad v_1 = 200 - 120 = 80\text{ V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$R = \frac{v_1}{i_1} = \frac{80}{20} = 4\Omega$$

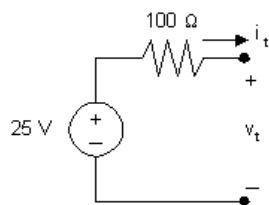
AP 2.7 [a] Plotting a graph of v_t versus i_t gives



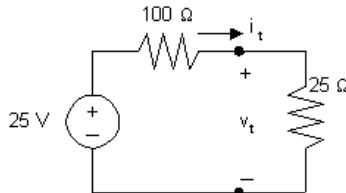
Note that when $i_t = 0$, $v_t = 25\text{ V}$; therefore the voltage source must be 25 V . Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor, as shown below:



- [b] Draw the circuit model from part (a) and attach a 25Ω resistor:



To find the power delivered to the 25Ω resistor we must calculate the current through the 25Ω resistor. Do this by first using KCL to recognize that the current in each of the components is i_t , flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i_t flowing through the resistors:

$$-25V + 100i_t + 25i_t = 0 \quad \text{so} \quad 125i_t = 25 \quad \text{so} \quad i_t = \frac{25}{125} = 0.2 \text{ A}$$

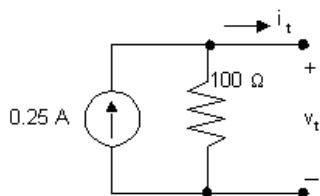
Thus, the power delivered to the 25Ω resistor is

$$p_{25} = (25)i_t^2 = (25)(0.2)^2 = 1 \text{ W.}$$

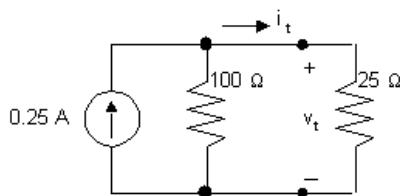
- AP 2.8 [a] From the graph in Assessment Problem 2.7(a), we see that when $v_t = 0$, $i_t = 0.25 \text{ A}$. Therefore the current source must be 0.25 A . Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100 \Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:



- [b] Draw the circuit model from part (a) and attach a 25Ω resistor:



Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v_t . Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law

to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

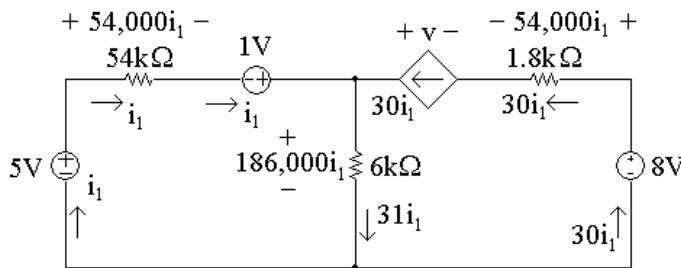
$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \quad \text{so} \quad 5v_t = 25, \quad \text{thus} \quad v_t = 5 \text{ V}$$

$$p_{25} = \frac{v_t^2}{25} = 1 \text{ W}.$$

- AP 2.9 First note that we know the current through all elements in the circuit except the $6 \text{ k}\Omega$ resistor (the current in the three elements to the left of the $6 \text{ k}\Omega$ resistor is i_1 ; the current in the three elements to the right of the $6 \text{ k}\Omega$ resistor is $30i_1$). To find the current in the $6 \text{ k}\Omega$ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\text{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1 . The results are shown in the figure below:



- [a] To find i_1 , write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5 \text{ V} + 54,000i_1 - 1 \text{ V} + 186,000i_1 = 0$$

Solving for i_1

$$54,000i_1 + 186,000i_1 = 6 \text{ V} \quad \text{so} \quad 240,000i_1 = 6 \text{ V}$$

Thus,

$$i_1 = \frac{6}{240,000} = 25 \mu\text{A}$$

- [b] Now that we have the value of i_1 , we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54,000i_1 + 8 \text{ V} - 186,000i_1 = 0$$

Thus,

$$v = 240,000i_1 - 8 \text{ V} = 240,000(25 \times 10^{-6}) - 8 \text{ V} = 6 \text{ V} - 8 \text{ V} = -2 \text{ V}$$

We now know the values of voltage and current for every circuit element.

Let's construct a power table:

Element	Current (μA)	Voltage (V)	Power Equation	Power (μW)
5 V	25	5	$p = -vi$	-125
54 k Ω	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	-25
6 k Ω	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	-2	$p = -vi$	1500
1.8 k Ω	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	-6000

- [c] The total power generated in the circuit is the sum of the negative power values in the power table:

$$-125 \mu\text{W} + -25 \mu\text{W} + -6000 \mu\text{W} = -6150 \mu\text{W}$$

Thus, the total power generated in the circuit is $6150 \mu\text{W}$.

- [d] The total power absorbed in the circuit is the sum of the positive power values in the power table:

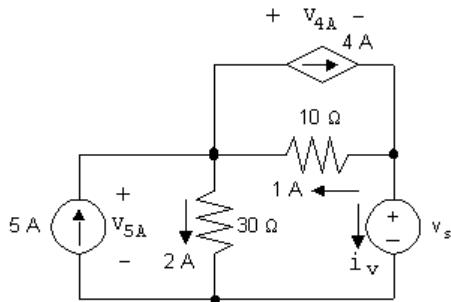
$$33.75 \mu\text{W} + 3603.75 \mu\text{W} + 1500 \mu\text{W} + 1012.5 \mu\text{W} = 6150 \mu\text{W}$$

Thus, the total power absorbed in the circuit is $6150 \mu\text{W}$.

- AP 2.10 Given that $i_\phi = 2 \text{ A}$, we know the current in the dependent source is $2i_\phi = 4 \text{ A}$. We can write a KCL equation at the left node to find the current in the 10Ω resistor. Summing the currents leaving the node,

$$-5 \text{ A} + 2 \text{ A} + 4 \text{ A} + i_{10\Omega} = 0 \quad \text{so} \quad i_{10\Omega} = 5 \text{ A} - 2 \text{ A} - 4 \text{ A} = -1 \text{ A}$$

Thus, the current in the 10Ω resistor is 1 A , flowing right to left, as seen in the circuit below.



- [a] To find v_s , write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v_s + (1 \text{ A})(10 \Omega) + (2 \text{ A})(30 \Omega) = 0 \quad \text{so} \quad v_s = 10 \text{ V} + 60 \text{ V} = 70 \text{ V}$$

- [b] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4 \text{ A} + 1 \text{ A} + i_v = 0 \quad \text{so} \quad i_v = 4 \text{ A} - 1 \text{ A} = 3 \text{ A}$$

The current in the voltage source is 3 A, flowing top to bottom. The power associated with this source is

$$p = vi = (70 \text{ V})(3 \text{ A}) = 210 \text{ W}$$

Thus, 210 W are absorbed by the voltage source.

- [c] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_{5A} + (2 \text{ A})(30 \Omega) = 0 \quad \text{so} \quad v_{5A} = 60 \text{ V}$$

The power associated with this source is

$$p = -v_{5A}i = -(60 \text{ V})(5 \text{ A}) = -300 \text{ W}$$

This source thus delivers 300 W of power to the circuit.

- [d] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_{4A} + (10 \Omega)(1 \text{ A}) = 0 \quad \text{so} \quad v_{4A} = -10 \text{ V}$$

The power associated with this source is

$$p = v_{4A}i = (-10 \text{ V})(4 \text{ A}) = -40 \text{ W}$$

This source thus delivers 40 W of power to the circuit.

- [e] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30 \Omega) + (i_{10\Omega})^2(10 \Omega) = (2)^2(30 \Omega) + (1)^2(10 \Omega) = 120 + 10 = 130 \text{ W}$$

Problems

P 2.1 [a] Yes, independent voltage sources can carry the 5 A current required by the connection; independent current source can support any voltage required by the connection, in this case 5 V, positive at the bottom.

[b] 20 V source: absorbing

15 V source: developing (delivering)

5 A source: developing (delivering)

[c] $P_{20V} = (20)(5) = 100 \text{ W}$ (abs)

$$P_{15V} = -(15)(5) = -75 \text{ W}$$
 (dev/del)

$$P_{5A} = -(5)(5) = -25 \text{ W}$$
 (dev/del)

$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 100 \text{ W}$$

[d] The interconnection is valid, but in this circuit the voltage drop across the 5 A current source is 35 V, positive at the top; 20 V source is developing (delivering), the 15 V source is developing (delivering), and the 5 A source is absorbing:

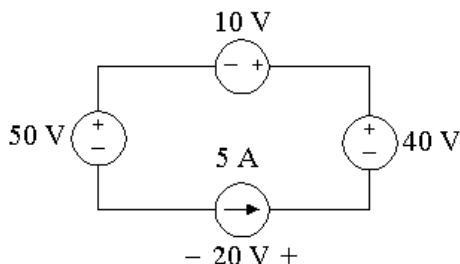
$$P_{20V} = -(20)(5) = -100 \text{ W}$$
 (dev/del)

$$P_{15V} = -(15)(5) = -75 \text{ W}$$
 (dev/del)

$$P_{5A} = (35)(5) = 175 \text{ W}$$
 (abs)

$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 175 \text{ W}$$

P 2.2 The interconnect is valid since the voltage sources can all carry 5 A of current supplied by the current source, and the current source can carry the voltage drop required by the interconnection. Note that the branch containing the 10 V, 40 V, and 5 A sources must have the same voltage drop as the branch containing the 50 V source, so the 5 A current source must have a voltage drop of 20 V, positive at the right. The voltages and currents are summarized in the circuit below:



$$P_{50V} = (50)(5) = 250 \text{ W (abs)}$$

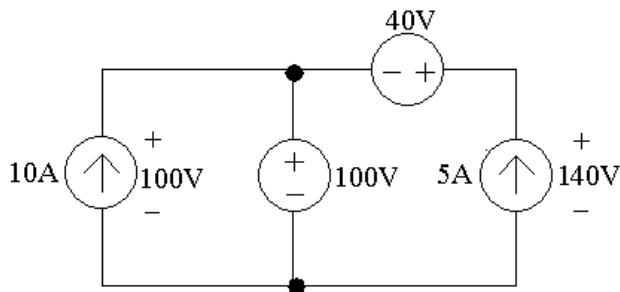
$$P_{10V} = (10)(5) = 50 \text{ W (abs)}$$

$$P_{40V} = -(40)(5) = -200 \text{ W (dev)}$$

$$P_{5A} = -(20)(5) = -100 \text{ W (dev)}$$

$$\sum P_{\text{dev}} = 300 \text{ W}$$

- P 2.3 The interconnection is valid. The 10 A current source has a voltage drop of 100 V, positive at the top, because the 100 V source supplies its voltage drop across a pair of terminals shared by the 10 A current source. The right hand branch of the circuit must also have a voltage drop of 100 V from the left terminal of the 40 V source to the bottom terminal of the 5 A current source, because this branch shares the same terminals as the 100 V source. This means that the voltage drop across the 5 A current source is 140 V, positive at the top. Also, the two voltage sources can carry the current required of the interconnection. This is summarized in the figure below:



From the values of voltage and current in the figure, the power supplied by the current sources is calculated as follows:

$$P_{10A} = -(100)(10) = -1000 \text{ W (1000 W supplied)}$$

$$P_{5A} = -(140)(5) = -700 \text{ W (700 W supplied)}$$

$$\sum P_{\text{dev}} = 1700 \text{ W}$$

- P 2.4 The interconnection is not valid. Note that the 3 A and 4 A sources are both connected in the same branch of the circuit. A valid interconnection would require these two current sources to supply the same current in the same direction, which they do not.

- P 2.5 The interconnection is valid, since the voltage sources can carry the currents supplied by the 2 A and 3 A current sources, and the current sources can carry whatever voltage drop from the top node to the bottom node is required by the interconnection. In particular, note the the voltage drop between the top and bottom nodes in the right hand branch must be the same as the voltage drop between the top and bottom nodes in the left hand branch. In particular, this means that

$$-v_1 + 8 \text{ V} = 12 \text{ V} + v_2$$

Hence any combination of v_1 and v_2 such that $v_1 + v_2 = -4\text{ V}$ is a valid solution.

- P 2.6 [a] Because both current sources are in the same branch of the circuit, their values must be the same. Therefore,

$$\frac{v_1}{50} = 0.4 \rightarrow v_1 = 0.4(50) = 20\text{ V}$$

[b] $p = v_1(0.4) = (20)(0.4) = 8\text{ W}$ (absorbed)

- P 2.7 [a] The voltage drop from the top node to the bottom node in this circuit must be the same for every path from the top to the bottom. Therefore, the voltages of the two voltage sources are equal:

$$-\alpha i_\Delta = 6$$

Also, the current i_Δ is in the same branch as the 15 mA current source, but in the opposite direction, so

$$i_\Delta = -0.015$$

Substituting,

$$-\alpha(-0.015) = 6 \rightarrow \alpha = \frac{6}{0.015} = 400$$

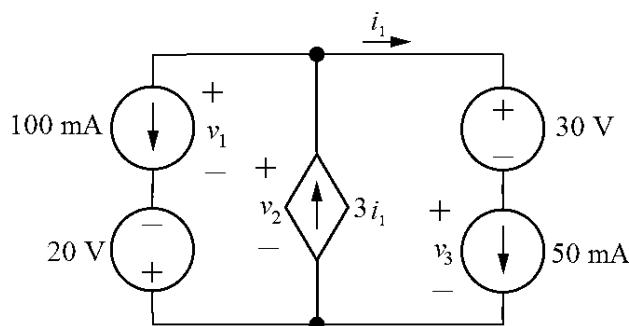
The interconnection is valid if $\alpha = 400\text{ V/A}$.

- [b] The voltage across the current source must equal the voltage across the 6 V source, since both are connected between the top and bottom nodes. Using the passive sign convention,

$$p = vi = (6)(0.015) = 0.09 = 90\text{ mW}$$

- [c] Since the power is positive, the current source is absorbing power.

- P 2.8 [a] Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_1 = 50\text{ mA}$.)
- [b] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1 , v_2 , and v_3 as shown:



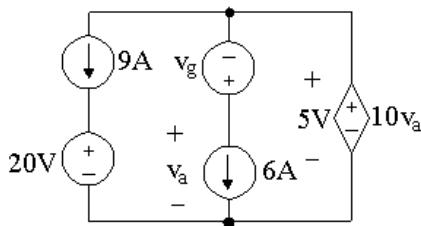
The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$$v_1 - 20 = v_2 = v_3 + 30$$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

- P 2.9 The interconnection is invalid. In the middle branch, the value of the current i_x must be 50 mA, since the 50 mA current source supplies current in this branch in the same direction as the current i_x . Therefore, the voltage supplied by the dependent voltage source in the right hand branch is $1800(0.05) = 90$ V. This gives a voltage drop from the top terminal to the bottom terminal in the right hand branch of $90 + 60 = 150$ V. But the voltage drop between these same terminals in the left hand branch is 30 V, due to the voltage source in that branch. Therefore, the interconnection is invalid.

- P 2.10



First, $10v_a = 5$ V, so $v_a = 0.5$ V. Then recognize that each of the three branches is connected between the same two nodes, so each of these branches must have the same voltage drop. The voltage drop across the middle branch is 5 V, and since $v_a = 0.5$ V, $v_g = 0.5 - 5 = -4.5$ V. Also, the voltage drop across the left branch is 5 V, so $20 + v_{9A} = 5$ V, and $v_{9A} = -15$ V, where v_{9A} is positive at the top. Note that the current through the 20 V source must be 9 A, flowing from top to bottom, and the current through the v_g is 6 A flowing from top to bottom. Let's find the power associated with the left and middle branches:

$$p_{9A} = (9)(-15) = -135 \text{ W}$$

$$p_{20V} = (9)(20) = 180 \text{ W}$$

$$p_{v_g} = -(6)(-4.5) = 27 \text{ W}$$

$$p_{6A} = (6)(0.5) = 3 \text{ W}$$

Since there is only one component left, we can find the total power:

$$p_{\text{total}} = -135 + 180 + 27 + 3 + p_{ds} = 75 + p_{ds} = 0$$

so p_{ds} must equal -75 W.

Therefore,

$$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 210 \text{ W}$$

P 2.11 [a] Using the passive sign convention and Ohm's law,

$$v = Ri = (3000)(0.015) = 45 \text{ V}$$

$$[b] P_R = \frac{v^2}{R} = \frac{45^2}{3000} = 0.675 = 675 \text{ mW}$$

[c] Using the passive sign convention with the current direction reversed,

$$v = -Ri = -(3000)(0.015) = -45 \text{ V}$$

$$P_R = \frac{v^2}{R} = \frac{-45^2}{3000} = 0.675 = 675 \text{ mW}$$

P 2.12 [a] Using the passive sign convention and Ohm's law,

$$i = -\frac{v}{R} = -\frac{40}{2500} = -0.016 = -16 \text{ mA}$$

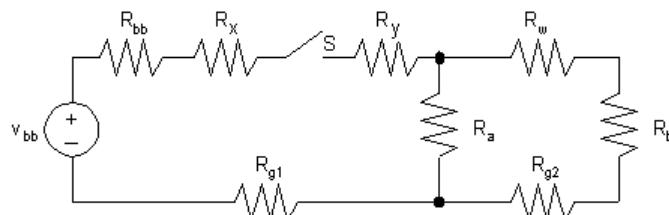
$$[b] P_R = Ri^2 = (2500)(-0.016)^2 = 0.64 = 640 \text{ mW}$$

[c] Using the passive sign convention with the voltage polarity reversed,

$$i = \frac{v}{R} = \frac{40}{2500} = 0.016 = 16 \text{ mA}$$

$$P_R = Ri^2 = (2500)(0.016)^2 = 0.64 = 640 \text{ mW}$$

P 2.13 [a]



[b] V_{bb} = no-load voltage of battery

R_{bb} = internal resistance of battery

R_x = resistance of wire between battery and switch

R_y = resistance of wire between switch and lamp A

R_a = resistance of lamp A

R_b = resistance of lamp B

R_w = resistance of wire between lamp A and lamp B

R_{g1} = resistance of frame between battery and lamp A

R_{g2} = resistance of frame between lamp A and lamp B

S = switch

- P 2.14 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.14(a),

$$v = Ri \quad \text{so} \quad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.14(b),

$$R = \frac{-7200}{-6} = \frac{-3600}{-3} = \frac{3600}{3} = \frac{7200}{6} = \frac{10,800}{9} = 1.2 \text{ k}\Omega$$

Note that this value is found in Appendix H.

- P 2.15 Since we know the device is a resistor, we can use the power equation. From Fig. P2.15(a),

$$p = vi = \frac{v^2}{R} \quad \text{so} \quad R = \frac{v^2}{p}$$

Using the values in the table of Fig. P2.13(b)

$$\begin{aligned} R &= \frac{(-8)^2}{640 \times 10^{-3}} = \frac{(-4)^2}{160 \times 10^{-3}} = \frac{(4)^2}{160 \times 10^{-3}} = \frac{(8)^2}{640 \times 10^{-3}} \\ &= \frac{(12)^2}{1440 \times 10^{-3}} = \frac{(16)^2}{2560 \times 10^{-3}} = 100 \Omega \end{aligned}$$

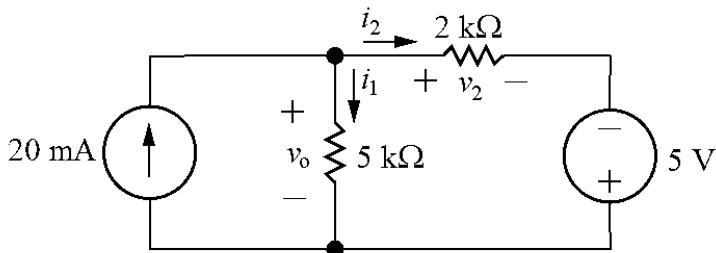
Note that this value is found in Appendix H.

- P 2.16 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.16(b),

$$\begin{aligned} \frac{8.25 \times 10^{-3}}{(0.5 \times 10^{-3})^2} &= \frac{33 \times 10^{-3}}{(1 \times 10^{-3})^2} = \frac{74.25 \times 10^{-3}}{(1.5 \times 10^{-3})^2} = \frac{132 \times 10^{-3}}{(2 \times 10^{-3})^2} \\ &= \frac{206.25 \times 10^{-3}}{(2.5 \times 10^{-3})^2} = \frac{297 \times 10^{-3}}{(3 \times 10^{-3})^2} = 33 \text{ k}\Omega \end{aligned}$$

Note that this is a value from Appendix H.

- P 2.17 Label the unknown resistor currents and voltages:



[a] KCL at the top node: $0.02 = i_1 + i_2$

KVL around the right loop: $-v_o + v_2 - 5 = 0$

Use Ohm's law to write the resistor voltages in the previous equation in terms of the resistor currents:

$$-5000i_1 + 2000i_2 - 5 = 0 \rightarrow -5000i_1 + 2000i_2 = 5$$

Multiply the KCL equation by -2000 and add it to the KVL equation to eliminate i_2 :

$$-2000(i_1 + i_2) + (-5000i_1 + 2000i_2) = -2000(0.02) + 5 \rightarrow -7000i_1 = -35$$

Solving,

$$i_1 = \frac{-35}{-7000} = 0.005 = 5 \text{ mA}$$

Therefore,

$$v_o = Ri_1 = (5000)(0.005) = 25 \text{ V}$$

[b] $p_{20\text{mA}} = -(0.02)v_o = -(0.02)(25) = -0.5 \text{ W}$

$$i_2 = 0.02 - i_1 = 0.02 - 0.005 = 0.015 \text{ A}$$

$$p_{5\text{V}} = -(5)i_2 = -(5)(0.015) = -0.075 \text{ W}$$

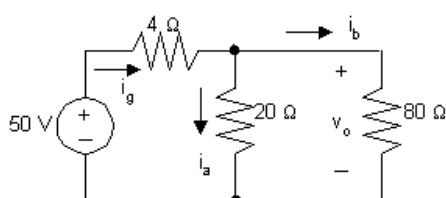
$$p_{5\text{k}} = 5000i_1^2 = 5000(0.005)^2 = 0.125 \text{ W}$$

$$p_{2\text{k}} = 2000i_2^2 = 2000(0.015)^2 = 0.45 \text{ W}$$

$$p_{\text{total}} = p_{20\text{mA}} + p_{5\text{V}} + p_{5\text{k}} + p_{2\text{k}} = -0.5 - 0.075 + 0.125 + 0.45 = 0$$

Thus the power in the circuit balances.

P 2.18 [a]



$$20i_a = 80i_b \quad i_g = i_a + i_b = 5i_b$$

$$i_a = 4i_b$$

$$50 = 4i_g + 80i_b = 20i_b + 80i_b = 100i_b$$

$$i_b = 0.5 \text{ A}, \text{ therefore, } i_a = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A}$$

[b] $i_b = 0.5 \text{ A}$

[c] $v_o = 80i_b = 40 \text{ V}$

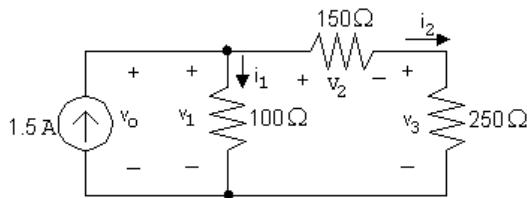
$$\begin{aligned}
 [\mathbf{d}] \quad p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25 \text{ W} \\
 p_{20\Omega} &= i_a^2(20) = (4)(20) = 80 \text{ W} \\
 p_{80\Omega} &= i_b^2(80) = 0.25(80) = 20 \text{ W}
 \end{aligned}$$

[e] p_{50V} (delivered) = $50i_g = 125 \text{ W}$
Check:

$$\sum P_{\text{dis}} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{\text{del}} = 125 \text{ W}$$

P 2.19



[a] Write a KCL equation at the top node:

$$-1.5 + i_1 + i_2 = 0 \quad \text{so} \quad i_1 + i_2 = 1.5$$

Write a KVL equation around the right loop:

$$-v_1 + v_2 + v_3 = 0$$

From Ohm's law,

$$v_1 = 100i_1, \quad v_2 = 150i_2, \quad v_3 = 250i_2$$

Substituting,

$$-100i_1 + 150i_2 + 250i_2 = 0 \quad \text{so} \quad -100i_1 + 400i_2 = 0$$

Solving the two equations for i_1 and i_2 simultaneously,

$$i_1 = 1.2 \text{ A} \quad \text{and} \quad i_2 = 0.3 \text{ A}$$

[b] Write a KVL equation clockwise around the left loop:

$$-v_o + v_1 = 0 \quad \text{but} \quad v_1 = 100i_1 = 100(1.2) = 120 \text{ V}$$

$$\text{So} \quad v_o = v_1 = 120 \text{ V}$$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$$p_{\text{source}} = -v_o(1.5) = -(120)(1.5) = -180 \text{ W}$$

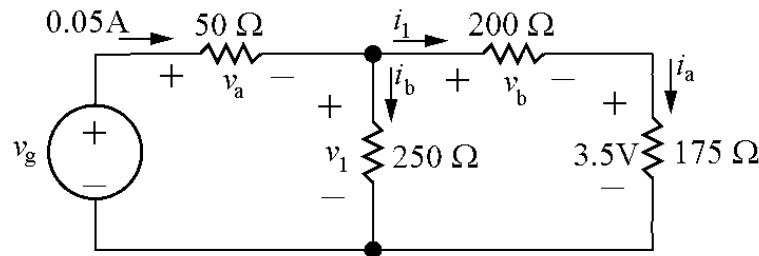
$$p_{100\Omega} = 1.2^2(100) = 144 \text{ W}$$

$$p_{150\Omega} = 0.3^2(150) = 13.5 \text{ W}$$

$$p_{250\Omega} = 0.3^2(250) = 22.5 \text{ W}$$

$$\sum P_{\text{dev}} = 180 \text{ W} \quad \sum P_{\text{abs}} = 144 + 13.5 + 22.5 = 180 \text{ W}$$

P 2.20 Label the unknown resistor voltages and currents:



[a] $i_a = \frac{3.5}{175} = 0.02 \text{ A}$ (Ohm's law)

$$i_1 = i_a = 0.02 \text{ A} \quad (\text{KCL})$$

[b] $v_b = 200i_1 = 200(0.02) = 4 \text{ V}$ (Ohm's law)

$$-v_1 + v_b + 3.5 = 0 \quad \text{so} \quad v_1 = 3.5 + v_b = 3.5 + 4 = 7.5 \text{ V} \quad (\text{KVL})$$

[c] $v_a = 0.05(50) = 2.5 \text{ V}$ (Ohm's law)

$$-v_g + v_a + v_1 = 0 \quad \text{so} \quad v_g = v_a + v_1 = 2.5 + 7.5 = 10 \text{ V} \quad (\text{KVL})$$

[d] $p_g = v_g(0.05) = 10(0.05) = 0.5 \text{ W}$

P 2.21 [a] Use KVL for the right loop to calculate the voltage drop across the right-hand branch v_o . This is also the voltage drop across the middle branch, so once v_o is known, use Ohm's law to calculate i_o :

$$v_o = 1000i_a + 4000i_a + 3000i_a = 8000i_a = 8000(0.002) = 16 \text{ V}$$

$$16 = 2000i_o$$

$$i_o = \frac{16}{2000} = 8 \text{ mA}$$

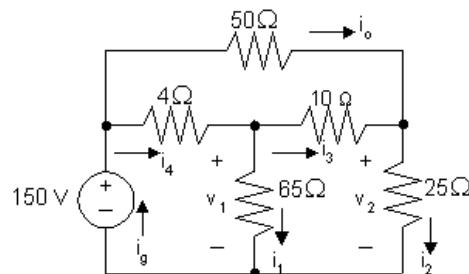
[b] KCL at the top node: $i_g = i_a + i_o = 0.002 + 0.008 = 0.010 \text{ A} = 10 \text{ mA}$.

[c] The voltage drop across the source is v_0 , seen by writing a KVL equation for the left loop. Thus,

$$p_g = -v_o i_g = -(16)(0.01) = -0.160 \text{ W} = -160 \text{ mW}.$$

Thus the source delivers 160 mW.

P 2.22 [a]



$$v_2 = 150 - 50(1) = 100 \text{ V}$$

$$i_2 = \frac{v_2}{25} = 4\text{A}$$

$$i_3 + 1 = i_2, \quad i_3 = 4 - 1 = 3\text{A}$$

$$v_1 = 10i_3 + 25i_2 = 10(3) + 25(4) = 130\text{V}$$

$$i_1 = \frac{v_1}{65} = \frac{130}{65} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 3 = 5\text{ A}$$

$$i_g = i_4 + i_o = 5 + 1 = 6\text{ A}$$

[b] $p_{4\Omega} = 5^2(4) = 100\text{ W}$

$$p_{50\Omega} = 1^2(50) = 50\text{ W}$$

$$p_{65\Omega} = 2^2(65) = 260\text{ W}$$

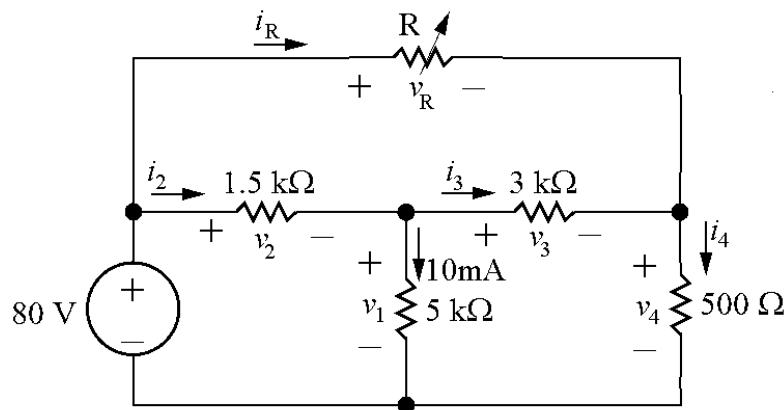
$$p_{10\Omega} = 3^2(10) = 90\text{ W}$$

$$p_{25\Omega} = 4^2(25) = 400\text{ W}$$

[c] $\sum P_{\text{dis}} = 100 + 50 + 260 + 90 + 400 = 900\text{ W}$

$$P_{\text{dev}} = 150i_g = 150(6) = 900\text{ W}$$

P 2.23 Label all unknown resistor voltages and currents:



Ohms' law for 5 kΩ resistor: $v_1 = (0.01)(5000) = 50\text{ V}$

KVL for lower left loop: $-80 + v_2 + 50 = 0 \rightarrow v_2 = 80 - 50 = 30\text{ V}$

Ohm's law for 1.5 kΩ resistor: $i_2 = v_2/1500 = 30/1500 = 20\text{ mA}$

KCL at center node:

$$i_2 = i_3 + 0.01 \rightarrow i_3 = i_2 - 0.01 = 0.02 - 0.01 = 0.01 = 10\text{ mA}$$

Ohm's law for 3 kΩ resistor $v_3 = 3000i_3 = 3000(0.01) = 30\text{ V}$

KVL for lower right loop:

$$-v_1 + v_3 + v_4 = 0 \rightarrow v_4 = v_1 - v_3 = 50 - 30 = 20\text{ V}$$

Ohm's law for 500Ω resistor: $i_4 = v_4/500 = 20/500 = 0.04 = 40 \text{ mA}$

KCL for right node:

$$i_3 + i_R = i_4 \rightarrow i_R = i_4 - i_3 = 0.04 - 0.01 = 0.03 = 30 \text{ mA}$$

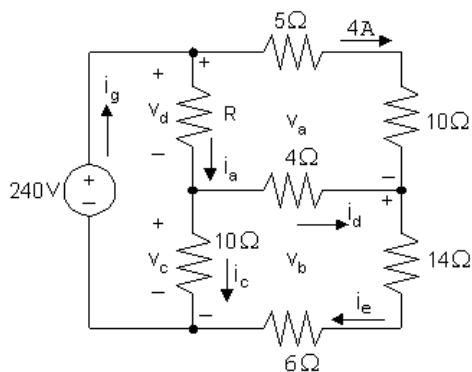
KVL for outer loop:

$$-80 + v_R + v_4 = 0 \rightarrow v_R = 80 - v_4 = 80 - 20 = 60 \text{ V}$$

Therefore,

$$R = \frac{v_R}{i_R} = \frac{60}{0.03} = 2000 = 2 \text{ k}\Omega$$

P 2.24 [a]



$$v_a = (5 + 10)(4) = 60 \text{ V}$$

$$-240 + v_a + v_b = 0 \quad \text{so} \quad v_b = 240 - v_a = 240 - 60 = 180 \text{ V}$$

$$i_e = v_b/(14 + 6) = 180/20 = 9 \text{ A}$$

$$i_d = i_e - 4 = 9 - 4 = 5 \text{ A}$$

$$v_c = 4i_d + v_b = 4(5) + 180 = 200 \text{ V}$$

$$i_c = v_c/10 = 200/10 = 20 \text{ A}$$

$$v_d = 240 - v_c = 240 - 200 = 40 \text{ V}$$

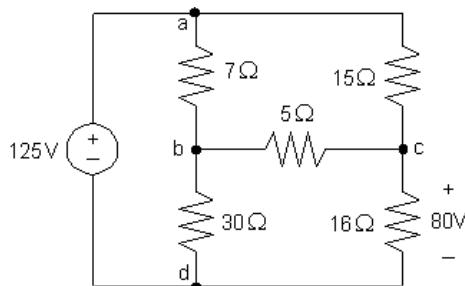
$$i_a = i_d + i_c = 5 + 20 = 25 \text{ A}$$

$$R = v_d/i_a = 40/25 = 1.6 \Omega$$

[b] $i_g = i_a + 4 = 25 + 4 = 29 \text{ A}$

$$p_g \text{ (supplied)} = (240)(29) = 6960 \text{ W}$$

P 2.25 [a]



$$i_{cd} = 80/16 = 5 \text{ A}$$

$$v_{ac} = 125 - 80 = 45 \quad \text{so} \quad i_{ac} = 45/15 = 3 \text{ A}$$

$$i_{ac} + i_{bc} = i_{cd} \quad \text{so} \quad i_{bc} = 5 - 3 = 2 \text{ A}$$

$$v_{ab} = 15i_{ac} - 5i_{bc} = 15(3) - 5(2) = 35 \text{ V} \quad \text{so} \quad i_{ab} = 35/7 = 5 \text{ A}$$

$$i_{bd} = i_{ab} - i_{bc} = 5 - 2 = 3 \text{ A}$$

Calculate the power dissipated by the resistors using the equation $p_R = R i_R^2$:

$$p_{7\Omega} = (7)(5)^2 = 175 \text{ W} \quad p_{30\Omega} = (30)(3)^2 = 270 \text{ W}$$

$$p_{15\Omega} = (15)(3)^2 = 135 \text{ W} \quad p_{16\Omega} = (16)(5)^2 = 400 \text{ W}$$

$$p_{5\Omega} = (5)(2)^2 = 20 \text{ W}$$

- [b] Calculate the current through the voltage source:

$$i_{ad} = -i_{ab} - i_{ac} = -5 - 3 = -8 \text{ A}$$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

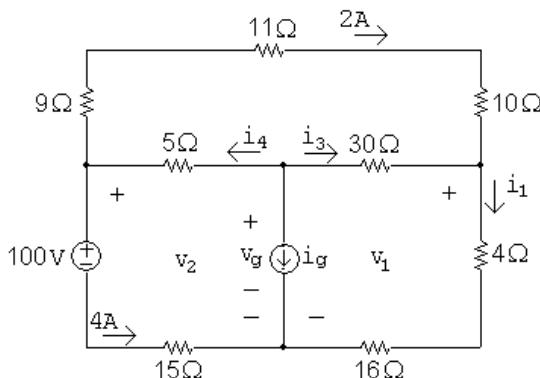
$$p_g = 125(-8) = -1000 \text{ W} \quad \text{thus} \quad p_g \text{ (supplied)} = 1000 \text{ W}$$

[c] $\sum P_{\text{dis}} = 175 + 270 + 135 + 400 + 20 = 1000 \text{ W}$

Therefore,

$$\sum P_{\text{supp}} = \sum P_{\text{dis}}$$

P 2.26 [a]



$$v_2 = 100 + 4(15) = 160 \text{ V}; \quad v_1 = 160 - (9 + 11 + 10)(2) = 100 \text{ V}$$

$$i_1 = \frac{v_1}{4 + 16} = \frac{100}{20} = 5 \text{ A}; \quad i_3 = i_1 - 2 = 5 - 2 = 3 \text{ A}$$

$$v_g = v_1 + 30i_3 = 100 + 30(3) = 190 \text{ V}$$

$$i_4 = 2 + 4 = 6 \text{ A}$$

$$i_g = -i_4 - i_3 = -6 - 3 = -9 \text{ A}$$

[b] Calculate power using the formula $p = Ri^2$:

$$\begin{array}{ll} p_{9\Omega} = (9)(2)^2 = 36 \text{ W}; & p_{11\Omega} = (11)(2)^2 = 44 \text{ W} \\ p_{10\Omega} = (10)(2)^2 = 40 \text{ W}; & p_{5\Omega} = (5)(6)^2 = 180 \text{ W} \\ p_{30\Omega} = (30)(3)^2 = 270 \text{ W}; & p_{4\Omega} = (4)(5)^2 = 100 \text{ W} \\ p_{16\Omega} = (16)(5)^2 = 400 \text{ W}; & p_{15\Omega} = (15)(4)^2 = 240 \text{ W} \end{array}$$

[c] $v_g = 190 \text{ V}$

[d] Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 180 + 270 + 100 + 400 + 240 = 1310 \text{ W}$$

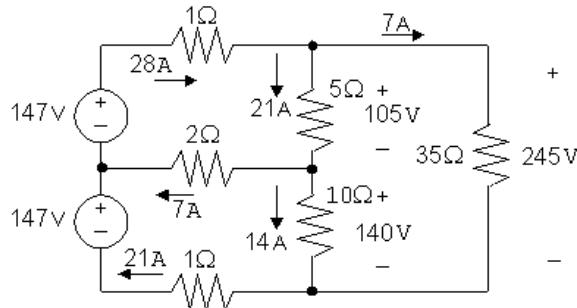
The power associated with the sources is

$$p_{\text{volt-source}} = (100)(4) = 400 \text{ W}$$

$$p_{\text{curr-source}} = v_g i_g = (190)(-9) = -1710 \text{ W}$$

Thus the total power dissipated is $1310 + 400 = 1710 \text{ W}$ and the total power developed is 1710 W , so the power balances.

- P 2.27 [a] Start by calculating the voltage drops due to the currents i_1 and i_2 . Then use KVL to calculate the voltage drop across and 35Ω resistor, and Ohm's law to find the current in the 35Ω resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle 2Ω resistor. These calculations are summarized in the figure below:



$$p_{147(\text{top})} = -(147)(28) = -4116 \text{ W}$$

$$p_{147(\text{bottom})} = -(147)(21) = -3087 \text{ W}$$

Therefore the top source supplies 4116 W of power and the bottom source supplies 3087 W of power.

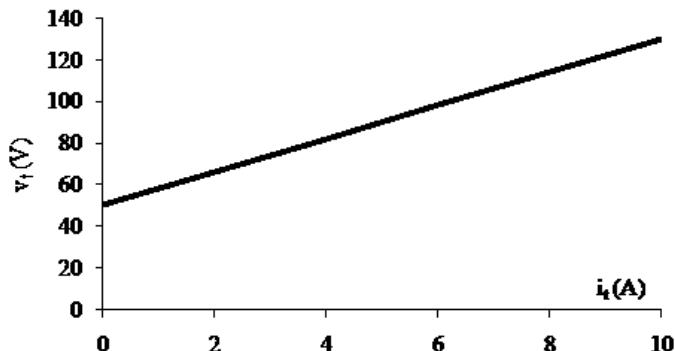
[b]

$$\begin{aligned} \sum P_{\text{dis}} &= (28)^2(1) + (7)^2(2) + (21)^2(1) + (21)^2(5) + (14)^2(10) + (7)^2(35) \\ &= 784 + 98 + 441 + 2205 + 1960 + 1715 = 7203 \text{ W} \end{aligned}$$

$$\sum P_{\text{sup}} = 4116 + 3087 = 7203 \text{ W}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 7203 \text{ W}$

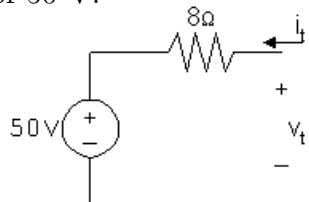
P 2.28 [a] Plot the $v - i$ characteristic



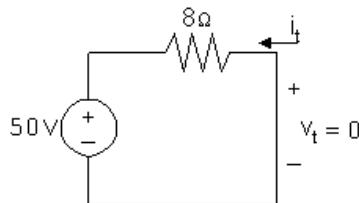
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(130 - 50)}{(10 - 0)} = 8 \Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal voltage source has a voltage of 50 V.



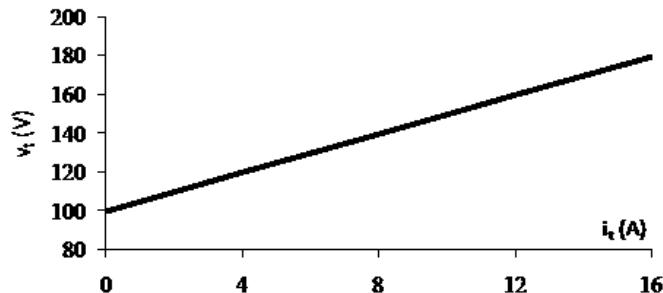
[b]



$$\text{When } v_t = 0, \quad i_t = \frac{-50}{8} = -6.25 \text{ A}$$

Note that this result can also be obtained by extrapolating the $v - i$ characteristic to $v_t = 0$.

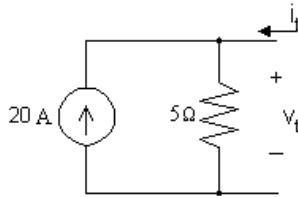
P 2.29 [a] Plot the $v - i$ characteristic:



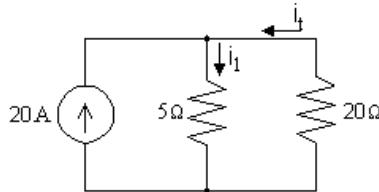
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(180 - 100)}{(16 - 0)} = 5 \Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal current source must have a current of $100/5 = 20$ A



[b] We attach a 20Ω resistor to the device model developed in part (a):



Write a KCL equation at the top node:

$$20 + i_t = i_1$$

Write a KVL equation for the right loop, in the direction of the two currents, using Ohm's law:

$$5i_1 + 20i_t = 0$$

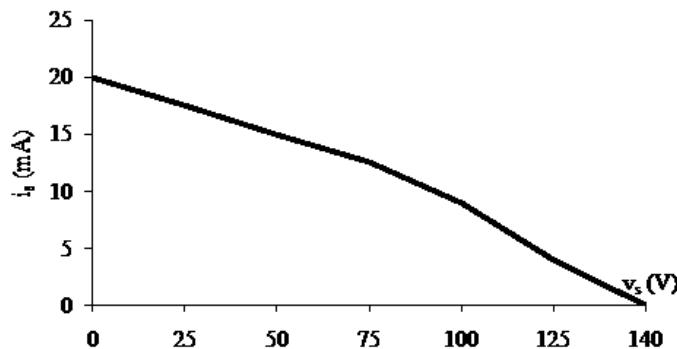
Combining the two equations and solving,

$$5(20 + i_t) + 20i_t = 0 \quad \text{so} \quad 25i_t = -100; \quad \text{thus} \quad i_t = -4 \text{ A}$$

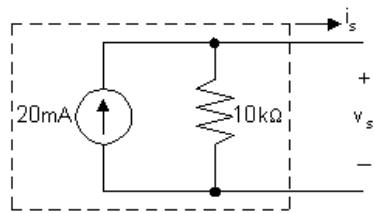
Now calculate the power dissipated by the resistor:

$$p_{20\Omega} = 20i_t^2 = 20(-4)^2 = 320 \text{ W}$$

P 2.30 [a]

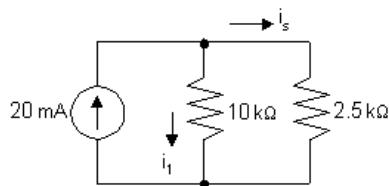


[b] $\Delta v = 25\text{V}; \quad \Delta i = 2.5 \text{ mA}; \quad R = \frac{\Delta v}{\Delta i} = 10 \text{ k}\Omega$



[c] $10,000i_1 = 2500i_s, \quad i_1 = 0.25i_s$

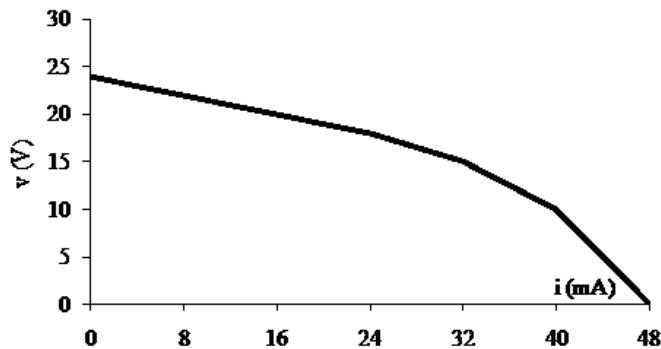
$$0.02 = i_1 + i_s = 1.25i_s, \quad i_s = 16 \text{ mA}$$



[d] $v_s(\text{open circuit}) = (20 \times 10^{-3})(10 \times 10^3) = 200 \text{ V}$

- [e] The open circuit voltage can be found in the table of values (or from the plot) as the value of the voltage v_s when the current $i_s = 0$. Thus, $v_s(\text{open circuit}) = 140 \text{ V}$ (from the table)
- [f] Linear model cannot predict the nonlinear behavior of the practical current source.

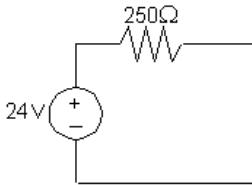
P 2.31 [a] Begin by constructing a plot of voltage versus current:



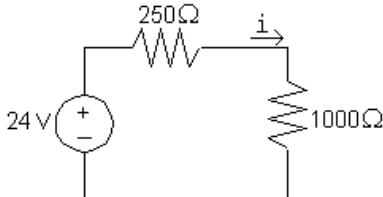
- [b] Since the plot is linear for $0 \leq i_s \leq 24 \text{ mA}$ and since $R = \Delta v / \Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{24 - 18}{0.024 - 0} = \frac{6}{0.024} = 250 \Omega$$

We can determine the value of the ideal voltage source by considering the value of v_s when $i_s = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 24 V. The model, valid for $0 \leq i_s \leq 24 \text{ mA}$, is shown below:



[c] The circuit is shown below:

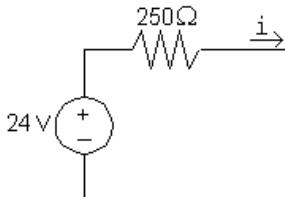


Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i :

$$-24 \text{ V} + 250i + 1000i = 0 \quad \text{so} \quad 1250i = 24 \text{ V}$$

$$\text{Thus, } i = \frac{24 \text{ V}}{1250 \Omega} = 19.2 \text{ mA}$$

[d] The circuit is shown below:



Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i :

$$-24 \text{ V} + 250i = 0 \quad \text{so} \quad 250i = 24 \text{ V}$$

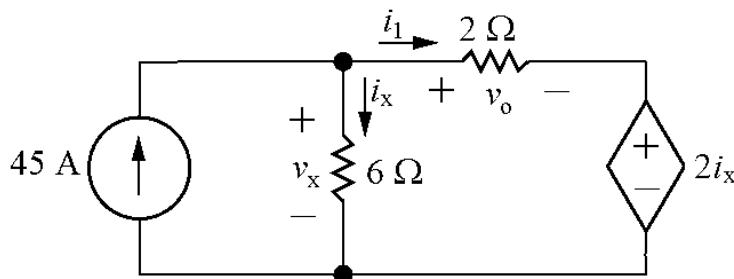
$$\text{Thus, } i = \frac{24 \text{ V}}{250 \Omega} = 96 \text{ mA}$$

[e] The short circuit current can be found in the table of values (or from the plot) as the value of the current i_s when the voltage $v_s = 0$. Thus,

$$i_{sc} = 48 \text{ mA} \quad (\text{from table})$$

[f] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i_s). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

P 2.32 Label unknown voltage and current:



$$-v_x + v_o + 2i_x = 0 \quad (\text{KVL})$$

$$v_x = 6i_x \quad (\text{Ohm's law})$$

Therefore

$$-6i_x + v_o + 2i_x = 0 \quad \text{so} \quad v_o = 4i_x$$

Thus

$$i_x = \frac{v_o}{4}$$

Also,

$$i_1 = \frac{v_o}{2} \quad (\text{Ohm's law})$$

$$45 = i_x + i_1 \quad (\text{KCL})$$

Substituting for the currents i_x and i_1 :

$$45 = \frac{v_o}{4} + \frac{v_o}{2} = \frac{3v_o}{4}$$

Thus

$$v_o = 45 \left(\frac{4}{3} \right) = 60 \text{ V}$$

The only two circuit elements that could supply power are the two sources, so calculate the power for each source:

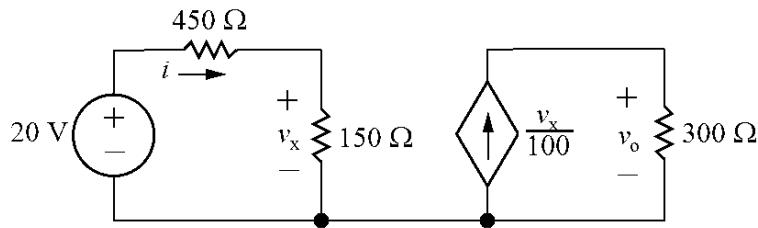
$$v_x = 6i_x = 6 \frac{v_o}{4} = 6(60/4) = 90 \text{ V}$$

$$p_{45V} = -45v_x = -45(90) = -4050 \text{ W}$$

$$p_{\text{d.s.}} = (2i_x)i_1 = 2(v_o/4)(v_o/2) = 2(60/4)(60/2) = 900 \text{ W}$$

Only the independent voltage source is supplying power, so the total power supplied is 4050 W.

P 2.33 Label unknown current:



$$-20 + 450i + 150i = 0 \quad (\text{KVL and Ohm's law})$$

$$\text{so } 600i = 20 \rightarrow i = 33.33 \text{ mA}$$

$$v_x = 150i = 150(0.0333) = 5 \text{ V} \quad (\text{Ohm's law})$$

$$v_o = 300 \left(\frac{v_x}{100} \right) = 300(5/100) = 15 \text{ V} \quad (\text{Ohm's law})$$

Calculate the power for all components:

$$p_{20V} = -20i = -20(0.0333) = -0.667 \text{ W}$$

$$p_{\text{d.s.}} = -v_o \left(\frac{v_x}{100} \right) = -(15)(5/100) = -0.75 \text{ W}$$

$$p_{450} = 450i^2 = 450(0.033)^2 = 0.5 \text{ W}$$

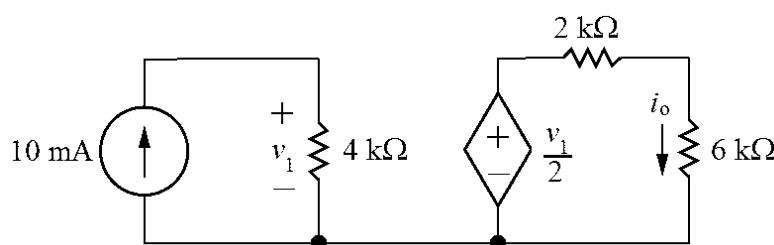
$$p_{150} = 150i^2 = 150(0.033)^2 = 0.1667 \text{ W}$$

$$p_{300} = \frac{v_o^2}{300} = \frac{15^2}{300} = 0.75 \text{ W}$$

Thus the total power absorbed is

$$p_{\text{abs}} = 0.5 + 0.1667 + 0.75 = 1.4167 \text{ W}$$

P 2.34 The circuit:



$$v_1 = (4000)(0.01) = 40 \text{ V} \quad (\text{Ohm's law})$$

$$\frac{v_1}{2} = 2000i_o + 6000i_o = 8000i_o \quad (\text{KVL})$$

Thus,

$$i_o = \frac{v_1/2}{8000} = \frac{40/2}{8000} = 0.0025 = 2.5 \text{ mA}$$

Calculate the power for all components:

$$p_{10\text{mA}} = -(0.01)v_1 = -(0.01)(40) = -0.4 \text{ W}$$

$$p_{\text{d.s.}} = -(v_1/2)i_o = -(40/2)(2.5 \times 10^{-3}) = -0.05 \text{ W}$$

$$p_{4\text{k}} = \frac{v_1^2}{4000} = \frac{40^2}{4000} = 0.4 \text{ W}$$

$$p_{2\text{k}} = 2000i_o^2 = 2000(2.5 \times 10^{-3})^2 = 0.0125 \text{ W}$$

$$p_{6\text{k}} = 6000i_o^2 = 6000(2.5 \times 10^{-3})^2 = 0.0375 \text{ W}$$

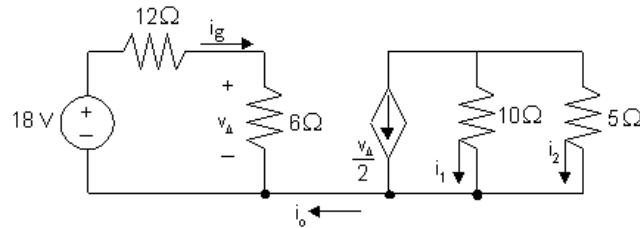
Therefore,

$$p_{\text{total}} = -0.4 - 0.05 + 0.4 + 0.0125 + 0.0375 = 0$$

Thus the power in the circuit balances.

- P 2.35 [a] $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



$$18 = (12 + 6)i_g \quad i_g = 1 \text{ A}$$

$$v_\Delta = 6i_g = 6V \quad v_\Delta/2 = 3 \text{ A}$$

$$10i_1 = 5i_2, \text{ so } i_1 + 2i_1 = -3 \text{ A; therefore, } i_1 = -1 \text{ A}$$

[c] $i_2 = 2i_1 = -2 \text{ A.}$

P 2.36 [a] $-50 - 20i_\sigma + 18i_\Delta = 0$

$$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0 \quad \text{so} \quad 18i_\Delta = 45i_\sigma$$

$$\text{Therefore, } -50 - 20i_\sigma + 45i_\sigma = 0, \quad \text{so} \quad i_\sigma = 2 \text{ A}$$

$$18i_\Delta = 45i_\sigma = 90; \quad \text{so} \quad i_\Delta = 5 \text{ A}$$

$$v_o = 40i_\sigma = 80 \text{ V}$$

[b] i_g = current out of the positive terminal of the 50 V source

v_d = voltage drop across the $8i_\Delta$ source

$$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47 \text{ A}$$

$$v_d = 80 - 20 = 60 \text{ V}$$

$$\sum P_{\text{gen}} = 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230 \text{ W}$$

$$\sum P_{\text{diss}} = 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20)$$

$$= (18)(25) + 10(47 - 5) + 4(40) + 40(60) + 40(20)$$

$$= 4230 \text{ W; Therefore,}$$

$$\sum P_{\text{gen}} = \sum P_{\text{diss}} = 4230 \text{ W}$$

P 2.37 $40i_2 + \frac{5}{40} + \frac{5}{10} = 0; \quad i_2 = -15.625 \text{ mA}$

$$v_1 = 80i_2 = -1.25 \text{ V}$$

$$25i_1 + \frac{(-1.25)}{20} + (-0.015625) = 0; \quad i_1 = 3.125 \text{ mA}$$

$$v_g = 60i_1 + 260i_1 = 320i_1$$

Therefore, $v_g = 1 \text{ V.}$

P 2.38 $i_E - i_B - i_C = 0$

$$i_C = \beta i_B \quad \text{therefore} \quad i_E = (1 + \beta)i_B$$

$$i_2 = -i_B + i_1$$

$$V_o + i_E R_E - (i_1 - i_B) R_2 = 0$$

$$-i_1 R_1 + V_{CC} - (i_1 - i_B) R_2 = 0 \quad \text{or} \quad i_1 = \frac{V_{CC} + i_B R_2}{R_1 + R_2}$$

$$V_o + i_E R_E + i_B R_2 - \frac{V_{CC} + i_B R_2}{R_1 + R_2} R_2 = 0$$

Now replace i_E by $(1 + \beta)i_B$ and solve for i_B . Thus

$$i_B = \frac{[V_{CC} R_2 / (R_1 + R_2)] - V_o}{(1 + \beta) R_E + R_1 R_2 / (R_1 + R_2)}$$

P 2.39 Here is Equation 2.25:

$$i_B = \frac{(V_{CC} R_2) / (R_1 + R_2) - V_o}{(R_1 R_2) / (R_1 + R_2) + (1 + \beta) R_E}$$

$$\frac{V_{CC} R_2}{R_1 + R_2} = \frac{(10)(60,000)}{100,000} = 6V$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{(40,000)(60,000)}{100,000} = 24 \text{ k}\Omega$$

$$i_B = \frac{6 - 0.6}{24,000 + 50(120)} = \frac{5.4}{30,000} = 0.18 \text{ mA}$$

$$i_C = \beta i_B = (49)(0.18) = 8.82 \text{ mA}$$

$$i_E = i_C + i_B = 8.82 + 0.18 = 9 \text{ mA}$$

$$v_{3d} = (0.009)(120) = 1.08V$$

$$v_{bd} = V_o + v_{3d} = 1.68V$$

$$i_2 = \frac{v_{bd}}{R_2} = \frac{1.68}{60,000} = 28 \mu\text{A}$$

$$i_1 = i_2 + i_B = 28 + 180 = 208 \mu\text{A}$$

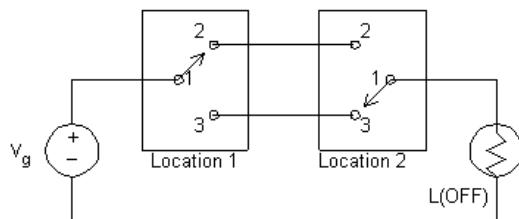
$$v_{ab} = 40,000(208 \times 10^{-6}) = 8.32V$$

$$i_{CC} = i_C + i_1 = 8.82 + 0.208 = 9.028 \text{ mA}$$

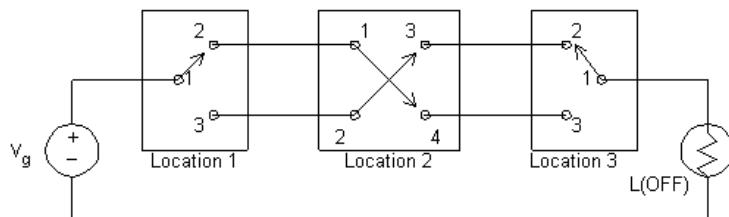
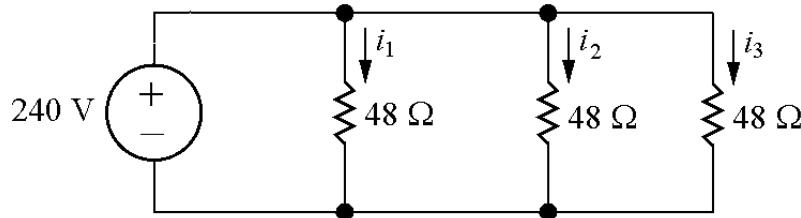
$$v_{13} + (8.82 \times 10^{-3})(750) + 1.08 = 10 \text{ V}$$

$$v_{13} = 2.305V$$

P 2.40 [a]



[b]

P 2.41 Each radiator is modeled as a 48Ω resistor:

Write a KVL equation for each of the three loops:

$$-240 + 48i_1 = 0 \rightarrow i_1 = \frac{240}{48} = 5 \text{ A}$$

$$-48i_1 + 48i_2 = 0 \rightarrow i_2 = i_1 = 5 \text{ A}$$

$$-48i_2 + 48i_3 = 0 \rightarrow i_3 = i_2 = 5 \text{ A}$$

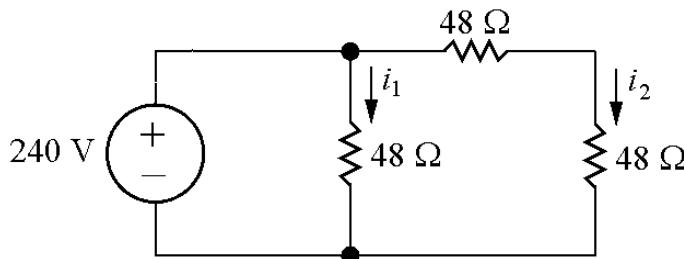
Therefore, the current through each radiator is 5 A and the power for each radiator is

$$p_{\text{rad}} = Ri^2 = 48(5)^2 = 1200 \text{ W}$$

There are three radiators, so the total power for this heating system is

$$p_{\text{total}} = 3p_{\text{rad}} = 3(1200) = 3600 \text{ W}$$

P 2.42 Each radiator is modeled as a 48Ω resistor:



Write a KVL equation for the left and right loops:

$$-240 + 48i_1 = 0 \rightarrow i_1 = \frac{240}{48} = 5 \text{ A}$$

$$-48i_1 + 48i_2 + 48i_2 = 0 \rightarrow i_2 = \frac{i_1}{2} = \frac{5}{2} = 2.5 \text{ A}$$

The power for the center radiator is

$$p_{\text{cen}} = 48i_1^2 = 48(5)^2 = 1200 \text{ W}$$

The power for each of the radiators on the right is

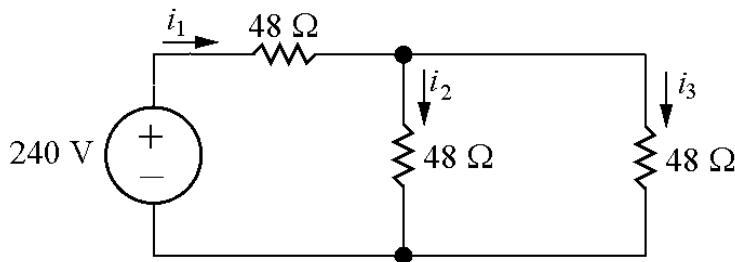
$$p_{\text{right}} = 48i_2^2 = 48(2.5)^2 = 300 \text{ W}$$

Thus the total power for this heating system is

$$p_{\text{total}} = p_{\text{cen}} + 2p_{\text{right}} = 1200 + 2(300) = 1800 \text{ W}$$

The center radiator produces 1200 W, just like the three radiators in Problem 2.41. But the other two radiators produce only 300 W each, which is 1/4th of the power of the radiators in Problem 2.41. The total power of this configuration is 1/2 of the total power in Fig. 2.41.

P 2.43 Each radiator is modeled as a 48Ω resistor:



Write a KVL equation for the left and right loops:

$$-240 + 48i_1 + 48i_2 = 0$$

$$-48i_2 + 48i_3 = 0 \rightarrow i_2 = i_3$$

Write a KCL equation at the top node:

$$i_1 = i_2 + i_3 \rightarrow i_1 = i_2 + i_2 = 2i_2$$

Substituting into the first KVL equation gives

$$-240 + 48(2i_2) + 48i_2 = 0 \rightarrow i_2 = \frac{240}{3(48)} = 1.67 \text{ A}$$

Solve for the currents i_1 and i_3 :

$$i_3 = i_2 = 1.67 \text{ A}; \quad i_1 = 2i_2 = 2(1.67) = 3.33 \text{ A}$$

Calculate the power for each radiator using the current for each radiator:

$$p_{\text{left}} = 48i_1^2 = 48(3.33)^2 = 533.33 \text{ W}$$

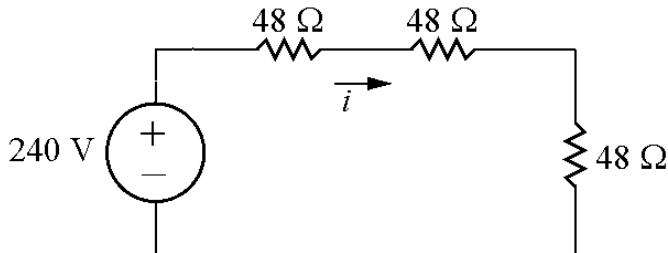
$$p_{\text{middle}} = p_{\text{right}} = 48i_2^2 = 48(1.67)^2 = 133.33 \text{ W}$$

Thus the total power for this heating system is

$$p_{\text{total}} = p_{\text{left}} + p_{\text{middle}} + p_{\text{right}} = 533.33 + 133.33 + 133.33 = 800 \text{ W}$$

All radiators in this configuration have much less power than their counterparts in Fig. 2.41. The total power for this configuration is only 22.2% of the total power for the heating system in Fig. 2.41.

- P 2.44 Each radiator is modeled as a 48Ω resistor:



Write a KVL equation for this loop:

$$-240 + 48i + 48i + 48i = 0 \rightarrow i = \frac{240}{3(48)} = 1.67 \text{ A}$$

Calculate the power for each radiator:

$$p_{\text{rad}} = 48i^2 = 48(1.67)^2 = 133.33 \text{ W}$$

Calculate the total power for this heating system:

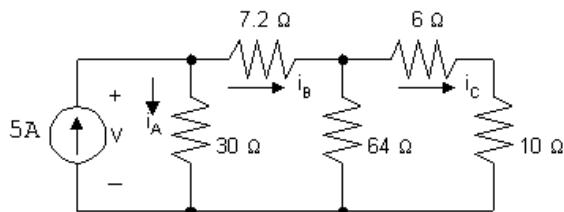
$$p_{\text{total}} = 3p_{\text{rad}} = 3(133.33) = 400 \text{ W}$$

Each radiator has much less power than the radiators in Fig. 2.41, and the total power of this configuration is just 1/9th of the total power in Fig. 2.41.

Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the 6Ω resistor and the 10Ω resistor in series:

$$6\Omega + 10\Omega = 16\Omega$$

Now combine this 16Ω resistor in parallel with the 64Ω resistor:

$$16\Omega \parallel 64\Omega = \frac{(16)(64)}{16 + 64} = \frac{1024}{80} = 12.8\Omega$$

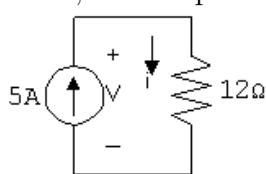
This equivalent 12.8Ω resistor is in series with the 7.2Ω resistor:

$$12.8\Omega + 7.2\Omega = 20\Omega$$

Finally, this equivalent 20Ω resistor is in parallel with the 30Ω resistor:

$$20\Omega \parallel 30\Omega = \frac{(20)(30)}{20 + 30} = \frac{600}{50} = 12\Omega$$

Thus, the simplified circuit is as shown:



- [a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the 12Ω equivalent resistor:

$$v = (12\Omega)(5 \text{ A}) = 60 \text{ V}$$

- [b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60 \text{ V})(5 \text{ A}) = -300 \text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

- [c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60 \text{ V}$, as calculated in part (a). This is also the voltage drop across the 30Ω resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60 \text{ V}}{30\Omega} = 2 \text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B :

$$-5 \text{ A} + i_A + i_B = 0 \quad \text{so} \quad i_B = 5 \text{ A} - i_A = 5 \text{ A} - 2 \text{ A} = 3 \text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

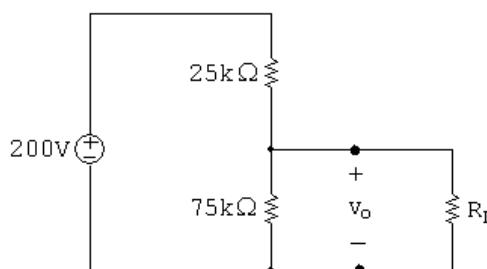
$$\text{So} \quad 16i_C = v - 7.2i_B = 60 \text{ V} - (7.2)(3) = 38.4 \text{ V}$$

$$\text{Thus} \quad i_C = \frac{38.4}{16} = 2.4 \text{ A}$$

Now that we have the current through the 10Ω resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\Omega} = (10)(2.4)^2 = 57.6 \text{ W}$$

AP 3.2



- [a] We can use voltage division to calculate the voltage v_o across the $75 \text{ k}\Omega$ resistor:

$$v_o(\text{no load}) = \frac{75,000}{75,000 + 25,000}(200 \text{ V}) = 150 \text{ V}$$

- [b] When we have a load resistance of $150 \text{ k}\Omega$ then the voltage v_o is across the parallel combination of the $75 \text{ k}\Omega$ resistor and the $150 \text{ k}\Omega$ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75 \text{ k}\Omega \parallel 150 \text{ k}\Omega = \frac{(75,000)(150,000)}{75,000 + 150,000} = 50,000 \Omega = 50 \text{ k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50,000}{50,000 + 25,000}(200 \text{ V}) = 133.3 \text{ V}$$

- [c] If the load terminals are short-circuited, the $75 \text{ k}\Omega$ resistor is effectively removed from the circuit, leaving only the voltage source and the $25 \text{ k}\Omega$ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200 \text{ V}}{25 \text{ k}\Omega} = 8 \text{ mA}$$

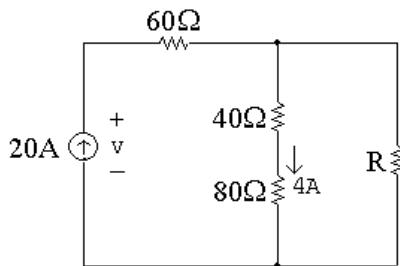
Now we can use the formula $p = Ri^2$ to find the power dissipated in the $25 \text{ k}\Omega$ resistor:

$$p_{25k} = (25,000)(0.008)^2 = 1.6 \text{ W}$$

- [d] The power dissipated in the $75 \text{ k}\Omega$ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V , so we can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75,000} = 0.3 \text{ W}$$

AP 3.3



- [a] We will write a current division equation for the current through the 80Ω resistor and use this equation to solve for R :

$$i_{80\Omega} = \frac{R}{R + 40\Omega + 80\Omega}(20 \text{ A}) = 4 \text{ A} \quad \text{so} \quad 20R = 4(R + 120)$$

$$\text{Thus} \quad 16R = 480 \quad \text{and} \quad R = \frac{480}{16} = 30 \Omega$$

- [b] With $R = 30\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40 + 80}{40 + 80 + 30}(20 \text{ A}) = 16 \text{ A} \quad \text{so} \quad p_R = (30)(16)^2 = 7680 \text{ W}$$

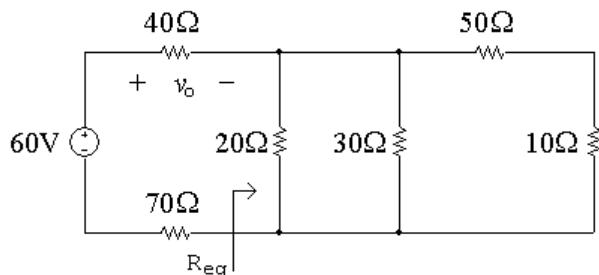
- [c] Write a KVL equation around the outer loop to solve for the voltage v , and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\Omega)(20 \text{ A}) + (30\Omega)(16 \text{ A}) = 0 \quad \text{so} \quad v = 1200 + 480 = 1680 \text{ V}$$

$$\text{Thus, } p_{\text{source}} = -(1680 \text{ V})(20 \text{ A}) = -33,600 \text{ W}$$

Thus, the current source generates 33,600 W of power.

AP 3.4



- [a] First we need to determine the equivalent resistance to the right of the 40Ω and 70Ω resistors:

$$R_{\text{eq}} = 20\Omega \parallel 30\Omega \parallel (50\Omega + 10\Omega) \quad \text{so} \quad \frac{1}{R_{\text{eq}}} = \frac{1}{20\Omega} + \frac{1}{30\Omega} + \frac{1}{60\Omega} = \frac{1}{10\Omega}$$

$$\text{Thus, } R_{\text{eq}} = 10\Omega$$

Now we can use voltage division to find the voltage v_o :

$$v_o = \frac{40}{40 + 10 + 70}(60 \text{ V}) = 20 \text{ V}$$

- [b] The current through the 40Ω resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20 \text{ V}}{40\Omega} = 0.5 \text{ A}$$

This current flows from left to right through the 40Ω resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the 20Ω resistor and the 50Ω and 10Ω resistors:

$$20\Omega \parallel (50\Omega + 10\Omega) = \frac{(20)(60)}{20 + 60} = 15\Omega$$

Now we use current division to find the current in the 30Ω branch:

$$i_{30\Omega} = \frac{15}{15 + 30}(0.5 \text{ A}) = 0.16667 \text{ A} = 166.67 \text{ mA}$$

[c] We can find the power dissipated by the 50Ω resistor if we can find the current in this resistor. We can use current division to find this current from the current in the 40Ω resistor, but first we need to calculate the equivalent resistance of the 20Ω branch and the 30Ω branch:

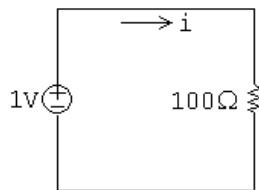
$$20\Omega \parallel 30\Omega = \frac{(20)(30)}{20 + 30} = 12\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12 + 50 + 10}(0.5\text{ A}) = 0.08333\text{ A}$$

$$\text{Thus, } p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$$

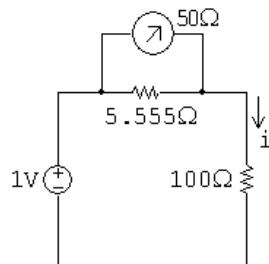
AP 3.5 [a]



We can find the current i using Ohm's law:

$$i = \frac{1\text{ V}}{100\Omega} = 0.01\text{ A} = 10\text{ mA}$$

[b]

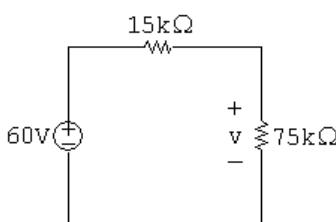


$$R_m = 50\Omega \parallel 5.555\Omega = 5\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\text{ V}}{100\Omega + 5\Omega} = 0.009524 = 9.524\text{ mA}$$

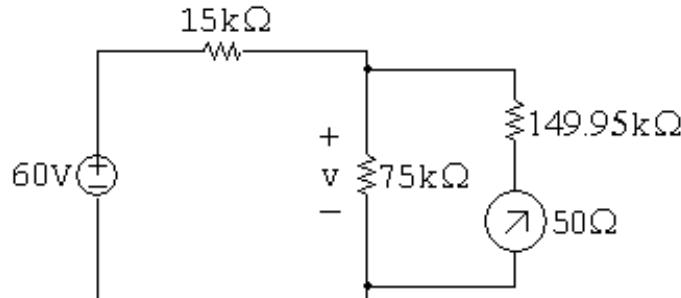
AP 3.6 [a]



Use voltage division to find the voltage v :

$$v = \frac{75,000}{75,000 + 15,000} (60 \text{ V}) = 50 \text{ V}$$

[b]



The meter resistance is a series combination of resistances:

$$R_m = 149,950 + 50 = 150,000 \Omega$$

We can use voltage division to find v , but first we must calculate the equivalent resistance of the parallel combination of the $75 \text{ k}\Omega$ resistor and the voltmeter:

$$75,000 \Omega \parallel 150,000 \Omega = \frac{(75,000)(150,000)}{75,000 + 150,000} = 50 \text{ k}\Omega$$

$$\text{Thus, } v_{\text{meas}} = \frac{50,000}{50,000 + 15,000} (60 \text{ V}) = 46.15 \text{ V}$$

AP 3.7 [a] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \quad \text{so} \quad R_x = \frac{(1000)(150)}{100} = 1500 \Omega = 1.5 \text{ k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x . We can find the current in the latter two branches using Ohm's law:

$$i_{R_1, R_3} = \frac{5 \text{ V}}{100 \Omega + 150 \Omega} = 20 \text{ mA}; \quad i_{R_2, R_x} = \frac{5 \text{ V}}{1000 + 1500} = 2 \text{ mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100 \Omega)(0.02 \text{ A})^2 = 40 \text{ mW}$$

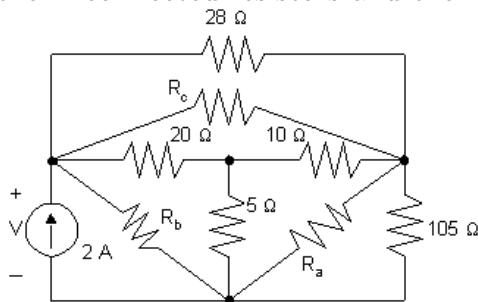
$$p_{150\Omega} = (150 \Omega)(0.02 \text{ A})^2 = 60 \text{ mW}$$

$$p_{1000\Omega} = (1000 \Omega)(0.002 \text{ A})^2 = 4 \text{ mW}$$

$$p_{1500\Omega} = (1500 \Omega)(0.002 \text{ A})^2 = 6 \text{ mW}$$

Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

- AP 3.8 Convert the three Y-connected resistors, 20Ω , 10Ω , and 5Ω to three Δ -connected resistors R_a , R_b , and R_c . To assist you the figure below has both the Y-connected resistors and the Δ -connected resistors

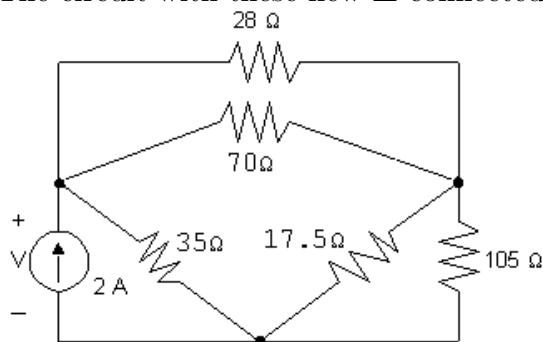


$$R_a = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5 \Omega$$

$$R_b = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35 \Omega$$

$$R_c = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70 \Omega$$

The circuit with these new Δ -connected resistors is shown below:



From this circuit we see that the 70Ω resistor is parallel to the 28Ω resistor:

$$70 \Omega \parallel 28 \Omega = \frac{(70)(28)}{70 + 28} = 20 \Omega$$

Also, the 17.5Ω resistor is parallel to the 105Ω resistor:

$$17.5 \Omega \parallel 105 \Omega = \frac{(17.5)(105)}{17.5 + 105} = 15 \Omega$$

Once the parallel combinations are made, we can see that the equivalent 20Ω resistor is in series with the equivalent 15Ω resistor, giving an equivalent resistance of $20\Omega + 15\Omega = 35\Omega$. Finally, this equivalent 35Ω resistor is in parallel with the other 35Ω resistor:

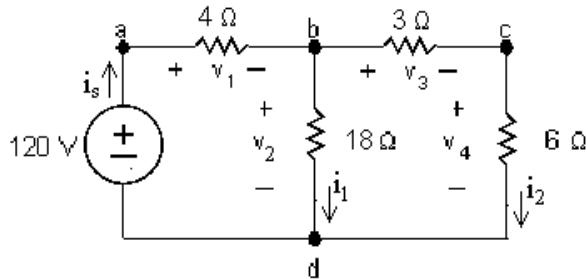
$$35\Omega \parallel 35\Omega = \frac{(35)(35)}{35 + 35} = 17.5\Omega$$

Thus, the resistance seen by the 2 A source is 17.5Ω , and the voltage can be calculated using Ohm's law:

$$v = (17.5\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

- P 3.1 [a] From Ex. 3-1: $i_1 = 4 \text{ A}$, $i_2 = 8 \text{ A}$, $i_s = 12 \text{ A}$
 at node b: $-12 + 4 + 8 = 0$, at node d: $12 - 4 - 8 = 0$

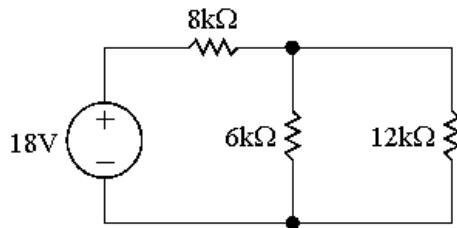


[b] $v_1 = 4i_s = 48 \text{ V}$ $v_3 = 3i_2 = 24 \text{ V}$
 $v_2 = 18i_1 = 72 \text{ V}$ $v_4 = 6i_2 = 48 \text{ V}$
 loop abda: $-120 + 48 + 72 = 0$,
 loop bcdb: $-72 + 24 + 48 = 0$,
 loop abcda: $-120 + 48 + 24 + 48 = 0$

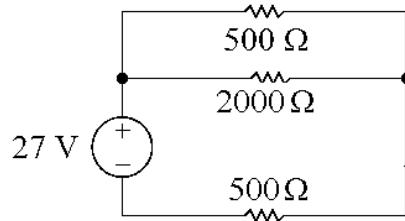
- P 3.2 [a] $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576 \text{ W}$ $p_{18\Omega} = (4)^2 18 = 288 \text{ W}$
 $p_{3\Omega} = (8)^2 3 = 192 \text{ W}$ $p_{6\Omega} = (8)^2 6 = 384 \text{ W}$

[b] $p_{120V}(\text{delivered}) = 120i_s = 120(12) = 1440 \text{ W}$
 [c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440 \text{ W}$

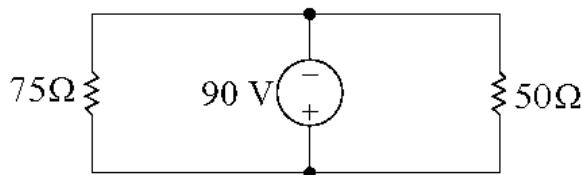
- P 3.3 [a] The 5 kΩ and 7 kΩ resistors are in series. The simplified circuit is shown below:



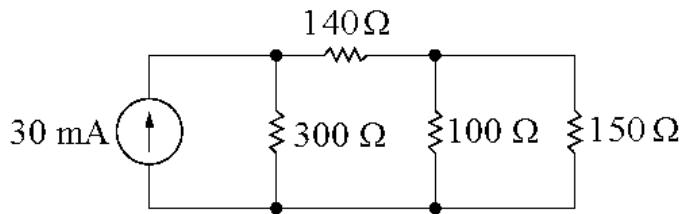
- [b] The 800Ω and 1200Ω resistors are in series, as are the 300Ω and 200Ω resistors. The simplified circuit is shown below:



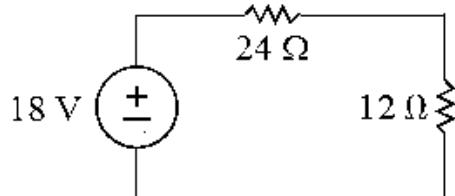
- [c] The 35Ω , 15Ω , and 25Ω resistors are in series, as are the 10Ω and 40Ω resistors. The simplified circuit is shown below:



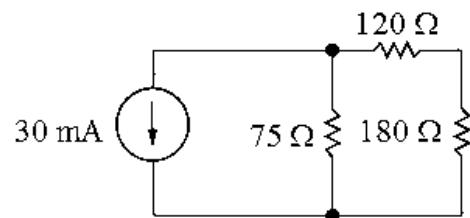
- [d] The 50Ω and 90Ω resistors are in series, as are the 80Ω and 70Ω resistors. The simplified circuit is shown below:



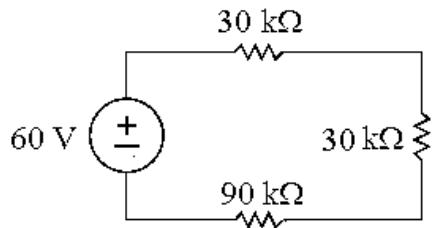
- P 3.4 [a] The 36Ω and 18Ω resistors are in parallel. The simplified circuit is shown below:



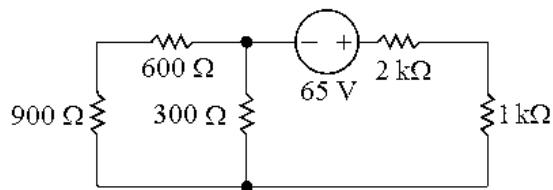
- [b] The 200Ω and 120Ω resistors are in parallel, as are the 210Ω and 280Ω resistors. The simplified circuit is shown below:



- [c] The $100\text{ k}\Omega$, $150\text{ k}\Omega$, and $60\text{ k}\Omega$ resistors are in parallel, as are the $75\text{ k}\Omega$ and $50\text{ k}\Omega$ resistors. The simplified circuit is shown below:



- [d] The $750\ \Omega$ and $500\ \Omega$ resistors are in parallel, as are the $1.5\ k\Omega$ and $3\ k\Omega$ resistors. The simplified circuit is shown below:



P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

- [a] Circuit in Fig. P3.3(a):

$$\begin{aligned} R_{eq} &= [(7000 + 5000)\parallel 6000] + 8000 = 12,000\parallel 6000 + 8000 \\ &= 4000 + 8000 = 12\ k\Omega \end{aligned}$$

Circuit in Fig. P3.3(b):

$$\begin{aligned} R_{eq} &= [500\parallel(800 + 1200)] + 300 + 200 = (500\parallel 2000) + 300 + 200 \\ &= 400 + 300 + 200 = 900\ \Omega \end{aligned}$$

Circuit in Fig. P3.3(c):

$$R_{eq} = (35 + 15 + 25)\parallel(10 + 40) = 75\parallel 50 = 30\ \Omega$$

Circuit in Fig. P3.3(d):

$$\begin{aligned} R_{eq} &= ((70 + 80)\parallel 100 + 50 + 90)\parallel 300 = [(150\parallel 100) + 50 + 90]\parallel 300 \\ &= (60 + 50 + 90)\parallel 300 = 200\parallel 300 = 120\ \Omega \end{aligned}$$

- [b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.3(a):

$$P = \frac{V_s^2}{R_{eq}} = \frac{18^2}{12,000} = 0.027 = 27\text{ mW}$$

For the circuit in Fig. P3.3(b):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{27^2}{900} = 0.81 = 810 \text{ mW}$$

For the circuit in Fig. P3.3(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{90^2}{30} = 270 \text{ W}$$

For the circuit in Fig. P3.3(d):

$$P = I_s^2(R_{\text{eq}}) = (0.03)^2(120) = 0.108 = 108 \text{ mW}$$

- P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] Circuit in Fig. P3.4(a):

$$R_{\text{eq}} = (36\parallel 18) + 24 = 12 + 24 = 36 \Omega$$

Circuit in Fig. P3.4(b):

$$R_{\text{eq}} = 200\parallel 120\parallel [(210\parallel 280) + 180] = 200\parallel 120\parallel (120 + 180) = 200\parallel 120\parallel 300 = 60 \Omega$$

Circuit in Fig. P3.4(c):

$$R_{\text{eq}} = (75 \text{ k}\parallel 50 \text{ k}) + (100 \text{ k}\parallel 150 \text{ k}\parallel 60 \text{ k}) + 90 \text{ k} = 30 \text{ k} + 30 \text{ k} + 90 \text{ k} = 150 \text{ k}\Omega$$

Circuit in Fig. P3.4(d):

$$\begin{aligned} R_{\text{eq}} &= [(600 + 900)\parallel 750\parallel 500] + (1500\parallel 3000) + 2000 = (1500\parallel 750\parallel 500) + 1000 + 2000 \\ &= 250 + 1000 + 2000 = 3250 = 3.25 \text{ k}\Omega \end{aligned}$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.4(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{18^2}{36} = 9 \text{ W}$$

For the circuit in Fig. P3.4(b):

$$P = I_s^2(R_{\text{eq}}) = (0.03)^2(60) = 0.054 = 54 \text{ mW}$$

For the circuit in Fig. P3.4(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{60^2}{150,000} = 0.024 = 24 \text{ mW}$$

For the circuit in Fig. P3.4(d):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{65^2}{3250} = 1.3 \text{ W}$$

P 3.7 [a] Circuit in Fig. P3.7(a):

$$\begin{aligned} R_{\text{eq}} &= ((15\parallel 60) + (30\parallel 45) + 20)\parallel 50 + 25 + 10 = [(12 + 18 + 20)\parallel 50] + 25 + 10 \\ &= (50\parallel 50) + 25 + 10 = 25 + 25 + 10 = 60 \Omega \end{aligned}$$

Circuit in Fig. P3.7(b) – begin by simplifying the 75Ω resistor and all resistors to its right:

$$[(18 + 12)\parallel 60 + 30]\parallel 75 = (30\parallel 60 + 30)\parallel 75 = (20 + 30)\parallel 75 = 50\parallel 75 = 30 \Omega$$

Now simplify the remainder of the circuit:

$$\begin{aligned} R_{\text{eq}} &= ((30 + 20)\parallel 50 + (20\parallel 60))\parallel 40 = [(50\parallel 50) + 15]\parallel 40 = (25 + 15)\parallel 40 \\ &= 40\parallel 40 = 20 \Omega \end{aligned}$$

Circuit in Fig. P3.7(c) – begin by simplifying the left and right sides of the circuit:

$$R_{\text{left}} = [(1800 + 1200)\parallel 2000] + 300 = (3000\parallel 2000) + 300 = 1200 + 300 = 1500 \Omega$$

$$R_{\text{right}} = [(500 + 2500)\parallel 1000] + 750 = (3000\parallel 1000) + 750 = 750 + 750 = 1500 \Omega$$

Now find the equivalent resistance seen by the source:

$$\begin{aligned} R_{\text{eq}} &= (R_{\text{left}}\parallel R_{\text{right}}) + 250 + 3000 = (1500\parallel 1500) + 250 + 3000 \\ &= 750 + 250 + 3000 = 4000 = 4 \text{ k}\Omega \end{aligned}$$

Circuit in Fig. P3.7(d):

$$\begin{aligned} R_{\text{eq}} &= ((750 + 250)\parallel 1000) + 100\parallel ((150 + 600)\parallel 500) + 300 \\ &= [(1000\parallel 1000) + 100]\parallel [(750\parallel 500) + 300] = (500 + 100)\parallel (300 + 300) \\ &= 600\parallel 600 = 300 \Omega \end{aligned}$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.7(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{30^2}{60} = 15 \text{ W}$$

For the circuit in Fig. P3.7(b):

$$P = I_s^2(R_{\text{eq}}) = (0.08)^2(20) = 0.128 = 128 \text{ mW}$$

For the circuit in Fig. P3.7(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{20^2}{4000} = 0.1 = 100 \text{ mW}$$

For the circuit in Fig. P3.7(d):

$$P = I_s^2(R_{\text{eq}}) = (0.05)^2(300) = 0.75 = 750 \text{ mW}$$

- P 3.8 [a] $R_{ab} = 24 + (90\parallel 60) + 12 = 24 + 36 + 12 = 72 \Omega$
 [b] $R_{ab} = [(4 \text{ k} + 6 \text{ k} + 2 \text{ k})\parallel 8 \text{ k}] + 5.2 \text{ k} = (12 \text{ k}\parallel 8 \text{ k}) + 5.2 \text{ k} = 4.8 \text{ k} + 5.2 \text{ k} = 10 \text{ k}\Omega$
 [c] $R_{ab} = 1200\parallel 720\parallel (320 + 480) = 1200\parallel 720\parallel 800 = 288 \Omega$

- P 3.9 Write an expression for the resistors in series and parallel from the right side of the circuit to the left. Then simplify the resulting expression from left to right to find the equivalent resistance.
- [a] $R_{ab} = [(26 + 10)\parallel 18 + 6]\parallel 36 = (36\parallel 18 + 6)\parallel 36 = (12 + 6)\parallel 36 = 18\parallel 36 = 12 \Omega$
 [b] $R_{ab} = [(12 + 18)\parallel 10\parallel 15\parallel 20 + 16]\parallel 30 + 4 + 14 = (30\parallel 10\parallel 15\parallel 20 + 16)\parallel 30 + 4 + 14$
 $= (4 + 16)\parallel 30 + 4 + 14 = 20\parallel 30 + 4 + 14 = 12 + 4 + 14 = 30 \Omega$
 [c] $R_{ab} = (500\parallel 1500\parallel 750 + 250)\parallel 2000 + 1000 = (250 + 250)\parallel 2000 + 1000$
 $= 500\parallel 2000 + 1000 = 400 + 1000 = 1400 \Omega$

[d] Note that the wire on the far right of the circuit effectively removes the 60Ω resistor!

$$\begin{aligned} R_{ab} &=([(30 + 18)\parallel 16 + 28]\parallel 40 + 20)\parallel 24 + 25 + 10]\parallel 50 \\ &=([(48\parallel 16 + 28)\parallel 40 + 20]\parallel 24 + 25 + 10)\parallel 50 \\ &=([(12 + 28)\parallel 40 + 20]\parallel 24 + 25 + 10)\parallel 50 = [(40\parallel 40 + 20)\parallel 24 + 25 + 10]\parallel 50 \\ &= [(20 + 20)\parallel 24 + 25 + 10]\parallel 50 = (40\parallel 24 + 25 + 10)\parallel 50 = (15 + 25 + 10)\parallel 50 \\ &= 50\parallel 50 = 25 \Omega \end{aligned}$$

- P 3.10 [a] $R + R = 2R$
 [b] $R + R + R + \dots + R = nR$
 [c] $R + R = 2R = 3000 \text{ so } R = 1500 = 1.5 \text{ k}\Omega$
 This is a resistor from Appendix H.
 [d] $nR = 4000; \text{ so if } n = 4, R = 1 \text{ k}\Omega$
 This is a resistor from Appendix H.

- P 3.11 [a] $R_{eq} = R\parallel R = \frac{R^2}{2R} = \frac{R}{2}$
 [b] $\begin{aligned} R_{eq} &= R\parallel R\parallel R\parallel \dots \parallel R \quad (n \text{ R's}) \\ &= R\parallel \frac{R}{n-1} \\ &= \frac{R^2/(n-1)}{R+R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n} \end{aligned}$

[c] $\frac{R}{2} = 5000 \text{ so } R = 10 \text{ k}\Omega$
 This is a resistor from Appendix H.

[d] $\frac{R}{n} = 4000 \quad \text{so} \quad R = 4000n$

If $n = 3 \quad r = 4000(3) = 12 \text{ k}\Omega$

This is a resistor from Appendix H. So put three 12k resistors in parallel to get 4k Ω .

P 3.12 [a] $v_o = \frac{160(3300)}{(4700 + 3300)} = 66 \text{ V}$

[b] $i = 160/8000 = 20 \text{ mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88 \text{ W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32 \text{ W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \quad \text{Therefore, } \left(\frac{94}{R_1}\right)^2 R_1 \leq 0.5$$

$$\text{Thus, } R_1 \geq \frac{94^2}{0.5} \quad \text{or} \quad R_1 \geq 17,672 \Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17,672}(160) = 66$$

$$\text{Thus, } R_2 = 12,408 \Omega$$

P 3.13 $4 = \frac{20R_2}{R_2 + 40} \quad \text{so} \quad R_2 = 10 \Omega$

$$3 = \frac{20R_e}{40 + R_e} \quad \text{so} \quad R_e = \frac{120}{17} \Omega$$

$$\text{Thus, } \frac{120}{17} = \frac{10R_L}{10 + R_L} \quad \text{so} \quad R_L = 24 \Omega$$

P 3.14 [a] $v_o = \frac{40R_2}{R_1 + R_2} = 8 \quad \text{so} \quad R_1 = 4R_2$

$$\text{Let } R_e = R_2 \| R_L = \frac{R_2 R_L}{R_2 + R_L}$$

$$v_o = \frac{40R_e}{R_1 + R_e} = 7.5 \quad \text{so} \quad R_1 = 4.33R_e$$

$$\text{Then, } 4R_2 = 4.33R_e = \frac{4.33(3600R_2)}{3600 + R_2}$$

$$\text{Thus, } R_2 = 300 \Omega \quad \text{and} \quad R_1 = 4(300) = 1200 \Omega$$

- [b] The resistor that must dissipate the most power is R_1 , as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 40 - 7.5 = 32.5 \text{ V}$$

$$p_{R_1} = \frac{32.5^2}{1200} = 880.2 \text{ m W}$$

Thus the minimum power rating for all resistors should be 1 W.

- P 3.15 Refer to the solution to Problem 3.16. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 1 W. Thus,

$$\frac{v_{R_1}^2}{1200} = 1 \quad \text{so} \quad v_{R_1} = 34.64 \text{ V}$$

$$v_o = 40 - 34.64 = 5.36 \text{ V}$$

$$\text{So, } \frac{40R_e}{1200 + R_e} = 5.36 \quad \text{and} \quad R_e = 185.68 \Omega$$

$$\text{Thus, } \frac{(300)R_L}{300 + R_L} = 185.68 \quad \text{and} \quad R_L = 487.26 \Omega$$

The minimum value for R_L from Appendix H is 560Ω .

- P 3.16 $R_{eq} = 10\parallel[6 + 5\|(8 + 12)] = 10\parallel(6 + 5\|20) = 10\parallel(6 + 4) = 5 \Omega$

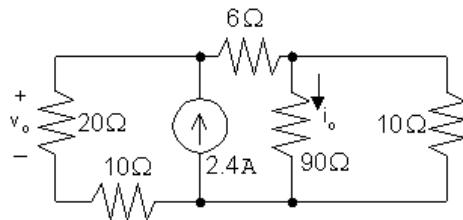
$$v_{10A} = v_{10\Omega} = (10 \text{ A})(5 \Omega) = 50 \text{ V}$$

Using voltage division:

$$v_{5\Omega} = \frac{5\|(8 + 12)}{6 + 5\|(8 + 12)}(50) = \frac{4}{6 + 4}(50) = 20 \text{ V}$$

$$\text{Thus, } p_{5\Omega} = \frac{v_{5\Omega}^2}{5} = \frac{20^2}{5} = 80 \text{ W}$$

P 3.17 [a]



$$R_{\text{eq}} = (10 + 20) \parallel [12 + (90 \parallel 10)] = 30 \parallel 15 = 10 \Omega$$

$$v_{2.4A} = 10(2.4) = 24 \text{ V}$$

$$v_o = v_{20\Omega} = \frac{20}{10 + 20}(24) = 16 \text{ V}$$

$$v_{90\Omega} = \frac{90 \parallel 10}{6 + (90 \parallel 10)}(24) = \frac{9}{15}(24) = 14.4 \text{ V}$$

$$i_o = \frac{14.4}{90} = 0.16 \text{ A}$$

$$[\text{b}] \quad p_{6\Omega} = \frac{(v_{2.4A} - v_{90\Omega})^2}{6} = \frac{(24 - 14.4)^2}{6} = 15.36 \text{ W}$$

$$[\text{c}] \quad p_{2.4A} = -(2.4)(24) = -57.6 \text{ W}$$

Thus the power developed by the current source is 57.6 W.

P 3.18 Begin by using KCL at the top node to relate the branch currents to the current supplied by the source. Then use the relationships among the branch currents to express every term in the KCL equation using just i_2 :

$$0.05 = i_1 + i_2 + i_3 + i_4 = 0.6i_2 + i_2 + 2i_2 + 4i_1 = 0.6i_2 + i_2 + 2i_2 + 4(0.6i_2) = 6i_2$$

Therefore,

$$i_2 = 0.05/6 = 0.00833 = 8.33 \text{ mA}$$

Find the remaining currents using the value of i_2 :

$$i_1 = 0.6i_2 = 0.6(0.00833) = 0.005 = 5 \text{ mA}$$

$$i_3 = 2i_2 = 2(0.00833) = 0.01667 = 16.67 \text{ mA}$$

$$i_4 = 4i_1 = 4(0.005) = 0.02 = 20 \text{ mA}$$

Since the resistors are in parallel, the same voltage, 25 V, appears across each of them. We know the current and the voltage for every resistor so we can use Ohm's law to calculate the values of the resistors:

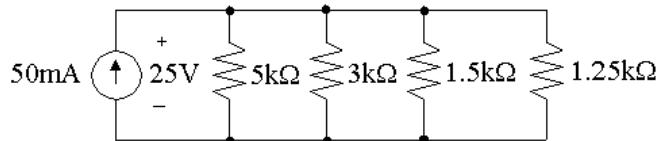
$$R_1 = 25/i_1 = 25/0.005 = 5000 = 5 \text{ k}\Omega$$

$$R_2 = 25/i_2 = 25/0.00833 = 3000 = 3 \text{ k}\Omega$$

$$R_3 = 25/i_3 = 25/0.01667 = 1500 = 1.5 \text{ k}\Omega$$

$$R_4 = 25/i_4 = 25/0.02 = 1250 = 1.25 \text{ k}\Omega$$

The resulting circuit is shown below:



$$\text{P 3.19} \quad \frac{(24)^2}{R_1 + R_2 + R_3} = 80, \quad \text{Therefore, } R_1 + R_2 + R_3 = 7.2 \Omega$$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

$$\text{Therefore, } 2(R_1 + R_2) = R_1 + R_2 + R_3$$

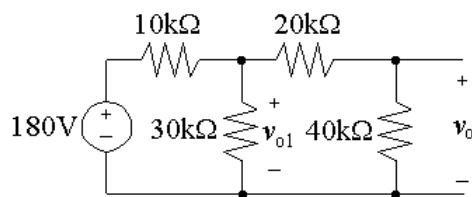
$$\text{Thus, } R_1 + R_2 = R_3; \quad 2R_3 = 7.2; \quad R_3 = 3.6 \Omega$$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$$

$$\text{Thus, } R_2 = 1.5 \Omega; \quad R_1 = 7.2 - R_2 - R_3 = 2.1 \Omega$$

P 3.20 [a]



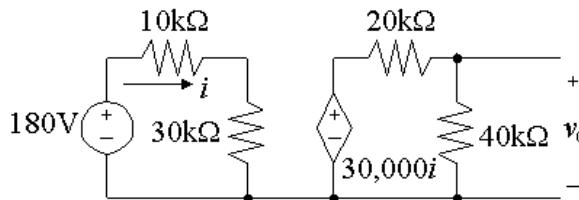
$$20 \text{ k}\Omega + 40 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$30 \text{ k}\Omega \parallel 60 \text{ k}\Omega = 20 \text{ k}\Omega$$

$$v_{o1} = \frac{20,000}{(10,000 + 20,000)}(180) = 120 \text{ V}$$

$$v_o = \frac{40,000}{60,000}(v_{o1}) = 80 \text{ V}$$

[b]



$$i = \frac{180}{40,000} = 4.5 \text{ mA}$$

$$30,000i = 135 \text{ V}$$

$$v_o = \frac{40,000}{60,000}(135) = 90 \text{ V}$$

- [c] It removes the loading effect of the second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{30,000}{40,000}(180) = 135 \text{ V}$$

Now note this is the input voltage to the second voltage divider when the current-controlled voltage source is used.

P 3.21 [a] At no load: $v_o = kv_s = \frac{R_2}{R_1 + R_2}v_s$.

$$\text{At full load: } v_o = \alpha v_s = \frac{R_e}{R_1 + R_e}v_s, \quad \text{where } R_e = \frac{R_o R_2}{R_o + R_2}$$

$$\begin{aligned} \text{Therefore } k &= \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_1 = \frac{(1-k)}{k}R_2 \\ \alpha &= \frac{R_e}{R_1 + R_e} \quad \text{and} \quad R_1 = \frac{(1-\alpha)}{\alpha}R_e \end{aligned}$$

$$\text{Thus } \left(\frac{1-\alpha}{\alpha}\right) \left[\frac{R_2 R_o}{R_o + R_2}\right] = \frac{(1-k)}{k} R_2$$

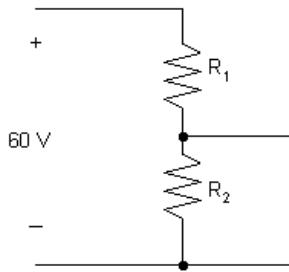
$$\text{Solving for } R_2 \text{ yields } R_2 = \frac{(k-\alpha)}{\alpha(1-k)} R_o$$

$$\text{Also, } R_1 = \frac{(1-k)}{k} R_2 \quad \therefore \quad R_1 = \frac{(k-\alpha)}{\alpha k} R_o$$

[b] $R_1 = \left(\frac{0.05}{0.68}\right) R_o = 2.5 \text{ k}\Omega$

$$R_2 = \left(\frac{0.05}{0.12}\right) R_o = 14.167 \text{ k}\Omega$$

[c]



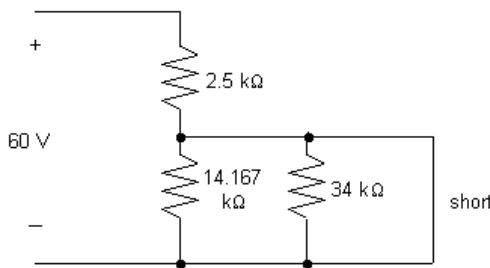
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\max)} = \frac{[(60)(0.85)]^2}{14,167} = 183.6 \text{ mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\max)} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60 \text{ mW}$$

[d]



$$P_{R_1} = \frac{(60)^2}{2500} = 1.44 \text{ W} = 1440 \text{ mW}$$

$$P_{R_2} = \frac{(0)^2}{14,167} = 0 \text{ W}$$

- P 3.22 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_o G_1 + v_o G_2 + \cdots + v_o G_N = v_o (G_1 + G_2 + \cdots + G_N)$$

$$\text{It follows that } v_o = \frac{i_g}{(G_1 + G_2 + \cdots + G_N)}$$

The current in the k^{th} branch is $i_k = v_o G_k$; Thus,

$$i_k = \frac{i_g G_k}{[G_1 + G_2 + \cdots + G_N]}$$

$$[b] i_5 = \frac{40(0.2)}{2 + 0.2 + 0.125 + 0.1 + 0.05 + 0.025} = 3.2 \text{ A}$$

- P 3.23 [a] The equivalent resistance of the $6 \text{ k}\Omega$ resistor and the resistors to its right is

$$6 \text{ k} \parallel (5 \text{ k} + 7 \text{ k}) = 6 \text{ k} \parallel 12 \text{ k} = 4 \text{ k}\Omega$$

Using voltage division,

$$v_{6k} = \frac{4000}{8000 + 4000}(18) = 6 \text{ V}$$

$$[b] v_{5k} = \frac{5000}{5000 + 7000}(6) = 2.5 \text{ V}$$

- P 3.24 [a] The equivalent resistance of the 100Ω resistor and the resistors to its right is

$$100\|(80 + 70) = 100\|150 = 60\Omega$$

Using current division,

$$i_{50} = \frac{(50 + 90 + 60)\|300}{50 + 90 + 60}(0.03) = \frac{120}{200}(0.03) = 0.018 = 18 \text{ mA}$$

$$[b] v_{70} = \frac{(80 + 70)\|100}{80 + 70}(0.018) = \frac{60}{150}(0.018) = 0.0072 = 7.2 \text{ mA}$$

- P 3.25 [a] The equivalent resistance of the circuit to the right of, and including, the 50Ω resistor is

$$[(60\|15) + (45\|30) + 20]\|50 = 25\Omega$$

Thus by voltage division,

$$v_{25} = \frac{25}{25 + 25 + 10}(30) = 12.5 \text{ V}$$

- [b] The current in the 25Ω resistor can be found from its voltage using Ohm's law:

$$i_{25} = \frac{12.5}{25} = 0.5 \text{ A}$$

- [c] The current in the 25Ω resistor divides between two branches – one containing 50Ω and one containing $(45\|30) + (15\|60) + 20 = 50\Omega$. Using current division,

$$i_{50} = \frac{50\|50}{50}(i_{25}) = \frac{25}{50}(0.5) = 0.25 \text{ A}$$

- [d] The voltage drop across the 50Ω resistor can be found using Ohm's law:

$$v_{50} = 50i_{50} = 50(0.25) = 12.5 \text{ V}$$

- [e] The voltage v_{50} divides across the equivalent resistance $(45\|30)\Omega$, the equivalent resistance $(15\|60)\Omega$, and the 20Ω resistor. Using voltage division,

$$v_{60} = v_{15\|60} = \frac{15\|60}{(15\|60) + (30\|45) + 20}(12.5) = \frac{12}{12 + 18 + 20}(12.5) = 3 \text{ V}$$

- P 3.26 [a] The equivalent resistance to the right of the 36Ω resistor is

$$6 + [18\parallel(26 + 10)] = 18\Omega$$

By current division,

$$i_{36} = \frac{36\parallel 18}{36}(0.45) = 0.15 = 150 \text{ mA}$$

- [b] Using Ohm's law,

$$v_{36} = 36i_{36} = 36(0.15) = 5.4 \text{ V}$$

- [c] Before using voltage division, find the equivalent resistance of the 18Ω resistor and the resistors to its right:

$$18\parallel(26 + 10) = 12\Omega$$

Now use voltage division:

$$v_{18} = \frac{12}{12 + 6}(5.4) = 3.6 \text{ V}$$

$$[d] v_{10} = \frac{10}{10 + 26}(3.6) = 1 \text{ V}$$

- P 3.27 [a] Begin by finding the equivalent resistance of the 30Ω resistor and all resistors to its right:

$$([(12 + 18)\parallel 10\parallel 15\parallel 20] + 16)\parallel 30 = 12\Omega$$

Now use voltage division to find the voltage across the 4Ω resistor:

$$v_4 = \frac{4}{4 + 12 + 14}(6) = 0.8 \text{ V}$$

- [b] Use Ohm's law to find the current in the 4Ω resistor:

$$i_4 = v_4/4 = 0.8/4 = 0.2 \text{ A}$$

- [c] Begin by finding the equivalent resistance of all resistors to the right of the 30Ω resistor:

$$([(12 + 18)\parallel 10\parallel 15\parallel 20] + 16) = 20\Omega$$

Now use current division:

$$i_{16} = \frac{30\parallel 20}{20}(0.2) = 0.12 = 120 \text{ mA}$$

- [d] Note that the current in the 16Ω resistor divides among four branches – 20Ω , 15Ω , 10Ω , and $(12 + 18)\Omega$:

$$i_{10} = \frac{20\parallel 15\parallel 10\parallel (12 + 18)}{10}(0.12) = 0.048 = 48 \text{ mA}$$

- [e] Use Ohm's law to find the voltage across the 10Ω resistor:

$$v_{10} = 10i_{10} = 10(0.048) = 0.48 \text{ V}$$

$$[\mathbf{f}] \quad v_{18} = \frac{18}{12+18}(0.48) = 0.288 = 288 \text{ mV}$$

$$\text{P 3.28} \quad [\mathbf{a}] \quad v_{6\text{k}} = \frac{6}{6+2}(18) = 13.5 \text{ V}$$

$$v_{3\text{k}} = \frac{3}{3+9}(18) = 4.5 \text{ V}$$

$$v_x = v_{6\text{k}} - v_{3\text{k}} = 13.5 - 4.5 = 9 \text{ V}$$

$$[\mathbf{b}] \quad v_{6\text{k}} = \frac{6}{8}(V_s) = 0.75V_s$$

$$v_{3\text{k}} = \frac{3}{12}(V_s) = 0.25V_s$$

$$v_x = (0.75V_s) - (0.25V_s) = 0.5V_s$$

- P 3.29 Use current division to find the current in the branch containing the 10 k and 15 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10 \text{ k} + 15 \text{ k})\|(3 \text{ k} + 12 \text{ k})}{10 \text{ k} + 15 \text{ k}}(18) = 6.75 \text{ mA}$$

Use Ohm's law to find the voltage drop across the 15 k resistor, positive at the top:

$$v_{15\text{k}} = -(6.75 \text{ m})(15 \text{ k}) = -101.25 \text{ V}$$

Find the current in the branch containing the 3 k and 12 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10 \text{ k} + 15 \text{ k})\|(3 \text{ k} + 12 \text{ k})}{3 \text{ k} + 12 \text{ k}}(18) = 11.25 \text{ mA}$$

Use Ohm's law to find the voltage drop across the 12 k resistor, positive at the top:

$$v_{12\text{k}} = -(12 \text{ k})(11.25 \text{ m}) = -135 \text{ V}$$

$$v_o = v_{15\text{k}} - v_{12\text{k}} = -101.25 - (-135) = 33.75 \text{ V}$$

- P 3.30 The equivalent resistance of the circuit to the right of the 90 Ω resistor is

$$R_{\text{eq}} = [(150\|75) + 40]\|(30 + 60) = 90\|90 = 45 \Omega$$

Use voltage division to find the voltage drop between the top and bottom nodes:

$$v_{\text{Req}} = \frac{45}{45 + 90}(3) = 1 \text{ V}$$

Use voltage division again to find v_1 from v_{Req} :

$$v_1 = \frac{150 \parallel 75}{150 \parallel 75 + 40}(1) = \frac{50}{90}(1) = \frac{5}{9} \text{ V}$$

Use voltage division one more time to find v_2 from v_{Req} :

$$v_2 = \frac{30}{30 + 60}(1) = \frac{1}{3} \text{ V}$$

- P 3.31 Find the equivalent resistance of all the resistors except the 2Ω :

$$5\Omega \parallel 20\Omega = 4\Omega; \quad 4\Omega + 6\Omega = 10\Omega; \quad 10\parallel(15+12+13) = 8\Omega = R_{\text{eq}}$$

Use Ohm's law to find the current i_g :

$$i_g = \frac{125}{2 + R_{\text{eq}}} = \frac{125}{2 + 8} = 12.5 \text{ A}$$

Use current division to find the current in the 6Ω resistor:

$$i_{6\Omega} = \frac{8}{6 + 4}(12.5) = 10 \text{ A}$$

Use current division again to find i_o :

$$i_o = \frac{5 \parallel 20}{20} i_{6\Omega} = \frac{5 \parallel 20}{20}(10) = 2 \text{ A}$$

- P 3.32 Use current division to find the current in the 8Ω resistor. Begin by finding the equivalent resistance of the 8Ω resistor and all resistors to its right:

$$R_{\text{eq}} = ((20 \parallel 80) + 4) \parallel 30 + 8 = 20\Omega$$

$$i_8 = \frac{60 \parallel R_{\text{eq}}}{R_{\text{eq}}}(0.25) = \frac{60 \parallel 20}{20}(0.25) = 0.1875 = 187.5 \text{ mA}$$

Use current division to find i_1 from i_8 :

$$i_1 = \frac{30 \parallel [4 + (80 \parallel 20)]}{30}(i_8) = \frac{30 \parallel 20}{30}(0.1875) = 0.075 = 75 \text{ mA}$$

Use current division to find $i_{4\Omega}$ from i_8 :

$$i_{4\Omega} = \frac{30\parallel[4 + (80\parallel 20)]}{4 + (80\parallel 20)}(i_8) = \frac{30\parallel 20}{20}(0.1875) = 0.1125 = 112.5 \text{ mA}$$

Finally, use current division to find i_2 from $i_{4\Omega}$:

$$i_2 = \frac{80\parallel 20}{20}(i_{4\Omega}) = \frac{80\parallel 20}{20}(0.1125) = 0.09 = 90 \text{ mA}$$

- P 3.33 The current in the shunt resistor at full-scale deflection is $i_A = i_{fullscale} - 3 \times 10^{-3}$ A. The voltage across R_A at full-scale deflection is always 150 mV; therefore,

$$R_A = \frac{150 \times 10^{-3}}{i_{fullscale} - 3 \times 10^{-3}} = \frac{150}{1000i_{fullscale} - 3}$$

[a] $R_A = \frac{150}{5000 - 3} = 30.018 \text{ m}\Omega$

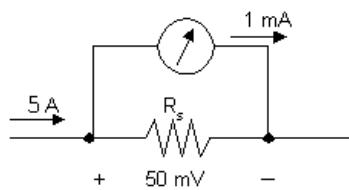
[b] Let R_m be the equivalent ammeter resistance:

$$R_m = \frac{0.15}{5} = 0.03 = 30 \text{ m}\Omega$$

[c] $R_A = \frac{150}{100 - 3} = 1.546 \Omega$

[d] $R_m = \frac{0.15}{0.1} = 1.5 \Omega$

- P 3.34



Original meter: $R_e = \frac{50 \times 10^{-3}}{5} = 0.01 \Omega$

Modified meter: $R_e = \frac{(0.02)(0.01)}{0.03} = 0.00667 \Omega$

$$\therefore (I_{fs})(0.00667) = 50 \times 10^{-3}$$

$$\therefore I_{fs} = 7.5 \text{ A}$$

- P 3.35 At full scale the voltage across the shunt resistor will be 200 mV; therefore the power dissipated will be

$$P_A = \frac{(200 \times 10^{-3})^2}{R_A}$$

$$\text{Therefore } R_A \geq \frac{(200 \times 10^{-3})^2}{1.0} = 40 \text{ m}\Omega$$

Otherwise the power dissipated in R_A will exceed its power rating of 1 W
When $R_A = 40 \text{ m}\Omega$, the shunt current will be

$$i_A = \frac{200 \times 10^{-3}}{40 \times 10^{-3}} = 5 \text{ A}$$

The measured current will be $i_{\text{meas}} = 5 + 0.002 = 5.002 \text{ A}$
 \therefore Full-scale reading for practical purposes is 5 A.

- P 3.36 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100 \text{ mV}}{2 \text{ mA}} = 50 \Omega$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50 + (25/12)}(i_{\text{meas}}) = \frac{25}{625}(i_{\text{meas}}) = \frac{1}{25}i_{\text{meas}}$$

- [b] At full scale, $i_{\text{meas}} = 5 \text{ A}$ and $i_m = 2 \text{ mA}$ so $5 - 0.002 = 4998 \text{ mA}$ flows through the resistor R_A :

$$R_A = \frac{100 \text{ mV}}{4998 \text{ mA}} = \frac{100}{4998} \Omega$$

$$i_m = \frac{(100/4998)}{50 + (100/4998)}(i_{\text{meas}}) = \frac{1}{2500}(i_{\text{meas}})$$

- [c] Yes

- P 3.37 For all full-scale readings the total resistance is

$$R_V + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\text{movement}} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20 \Omega$$

Therefore, $R_V = 1000$ (full-scale reading) – 20

[a] $R_V = 1000(50) - 20 = 49,980 \Omega$

[b] $R_V = 1000(5) - 20 = 4980 \Omega$

[c] $R_V = 1000(0.25) - 20 = 230 \Omega$

[d] $R_V = 1000(0.025) - 20 = 5 \Omega$

P 3.38 [a] $v_{\text{meas}} = (50 \times 10^{-3})[15\parallel 45\parallel(4980 + 20)] = 0.5612 \text{ V}$

[b] $v_{\text{true}} = (50 \times 10^{-3})(15\parallel 45) = 0.5625 \text{ V}$

$$\% \text{ error} = \left(\frac{0.5612}{0.5625} - 1 \right) \times 100 = -0.224\%$$

P 3.39 The measured value is $60\parallel 20.1 = 15.05618 \Omega$.

$$i_g = \frac{50}{(15.05618 + 10)} = 1.995526 \text{ A}; \quad i_{\text{meas}} = \frac{60}{80.1}(1.996) = 1.494768 \text{ A}$$

The true value is $60\parallel 20 = 15 \Omega$.

$$i_g = \frac{50}{(15 + 10)} = 2 \text{ A}; \quad i_{\text{true}} = \frac{60}{80}(2) = 1.5 \text{ A}$$

$$\% \text{ error} = \left[\frac{1.494768}{1.5} - 1 \right] \times 100 = -0.34878\% \approx -0.35\%$$

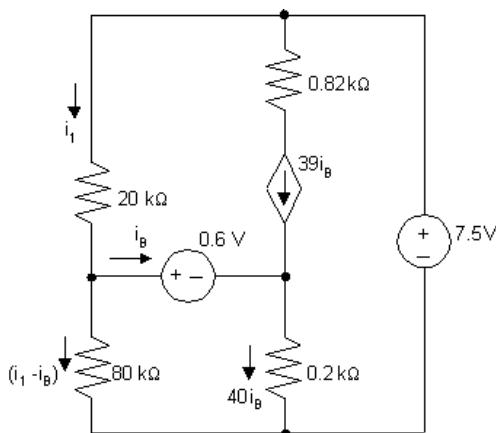
P 3.40 Begin by using current division to find the actual value of the current i_o :

$$i_{\text{true}} = \frac{15}{15 + 45}(50 \text{ mA}) = 12.5 \text{ mA}$$

$$i_{\text{meas}} = \frac{15}{15 + 45 + 0.1}(50 \text{ mA}) = 12.4792 \text{ mA}$$

$$\% \text{ error} = \left[\frac{12.4792}{12.5} - 1 \right] 100 = -0.166389\% \approx -0.17\%$$

P 3.41 [a]



$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_B) = 7.5$$

$$80 \times 10^3(i_1 - i_B) = 0.6 + 40i_B(0.2 \times 10^3)$$

$$\therefore 100i_1 - 80i_B = 7.5 \times 10^{-3}$$

$$80i_1 - 88i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225 \mu\text{A}$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad (\text{no change})$$

$$80 \times 10^3(i_1 - i_B) = 10^3i_B + 0.6 + 40i_B(200)$$

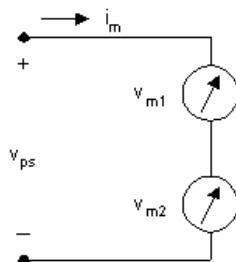
$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216 \mu\text{A}$

$$[\mathbf{c}] \% \text{ error} = \left(\frac{216}{225} - 1 \right) 100 = -4\%$$

P 3.42 [a] Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]



$$R_{m1} = (300)(900) = 270 \text{ k}\Omega; \quad R_{m2} = (150)(1200) = 180 \text{ k}\Omega$$

$$\therefore R_{m1} + R_{m2} = 450 \text{ k}\Omega$$

$$i_{1 \text{ max}} = \frac{300}{270} \times 10^{-3} = 1.11 \text{ mA}; \quad i_{2 \text{ max}} = \frac{150}{180} \times 10^{-3} = 0.833 \text{ mA}$$

$\therefore i_{\text{max}} = 0.833 \text{ mA}$ since meters are in series

$$v_{\text{max}} = (0.833 \times 10^{-3})(270 + 180)10^3 = 375 \text{ V}$$

Thus the meters can be used to measure the voltage.

$$[\mathbf{c}] i_m = \frac{320}{450 \times 10^3} = 0.711 \text{ mA}$$

$$v_{m1} = (0.711)(270) = 192 \text{ V}; \quad v_{m2} = (0.711)(180) = 128 \text{ V}$$

P 3.43 The current in the series-connected voltmeters is

$$i_m = \frac{205.2}{270,000} = \frac{136.8}{180,000} = 0.76 \text{ mA}$$

$$v_{50 \text{ k}\Omega} = (0.76 \times 10^{-3})(50,000) = 38 \text{ V}$$

$$V_{\text{power supply}} = 205.2 + 136.8 + 38 = 380 \text{ V}$$

P 3.44 $R_{\text{meter}} = R_m + R_{\text{movement}} = \frac{500 \text{ V}}{1 \text{ mA}} = 1000 \text{ k}\Omega$

$$v_{\text{meas}} = (50 \text{ k}\Omega \parallel 250 \text{ k}\Omega \parallel 1000 \text{ k}\Omega)(10 \text{ mA}) = (40 \text{ k}\Omega)(10 \text{ mA}) = 400 \text{ V}$$

$$v_{\text{true}} = (50 \text{ k}\Omega \parallel 250 \text{ k}\Omega)(10 \text{ mA}) = (41.67 \text{ k}\Omega)(10 \text{ mA}) = 416.67 \text{ V}$$

$$\% \text{ error} = \left(\frac{400}{416.67} - 1 \right) 100 = -4\%$$

P 3.45 [a] $v_{\text{meter}} = 180 \text{ V}$

[b] $R_{\text{meter}} = (100)(200) = 20 \text{ k}\Omega$

$$20 \parallel 70 = 15.555556 \text{ k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.555556} \times 15.555556 = 78.75 \text{ V}$$

[c] $20 \parallel 20 = 10 \text{ k}\Omega$

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5 \text{ V}$$

[d] $v_{\text{meter a}} = 180 \text{ V}$

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26 \text{ V}$$

No, because of the loading effect.

P 3.46 [a] $R_1 = (100/2)10^3 = 50 \text{ k}\Omega$

$$R_2 = (10/2)10^3 = 5 \text{ k}\Omega$$

$$R_3 = (1/2)10^3 = 500 \Omega$$

- [b] Let i_a = actual current in the movement
 i_d = design current in the movement

$$\text{Then \% error} = \left(\frac{i_a}{i_d} - 1 \right) 100$$

For the 100 V scale:

$$i_a = \frac{100}{50,000 + 25} = \frac{100}{50,025}, \quad i_d = \frac{100}{50,000}$$

$$\frac{i_a}{i_d} = \frac{50,000}{50,025} = 0.9995 \quad \% \text{ error} = (0.9995 - 1)100 = -0.05\%$$

For the 10 V scale:

$$\frac{i_a}{i_d} = \frac{5000}{5025} = 0.995 \quad \% \text{ error} = (0.995 - 1.0)100 = -0.4975\%$$

For the 1 V scale:

$$\frac{i_a}{i_d} = \frac{500}{525} = 0.9524 \quad \% \text{ error} = (0.9524 - 1.0)100 = -4.76\%$$

P 3.47 From the problem statement we have

$$50 = \frac{V_s(10)}{10 + R_s} \quad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$48.75 = \frac{V_s(6)}{6 + R_s} \quad (2)$$

- [a] From Eq (1) $10 + R_s = 0.2V_s$

$$\therefore R_s = 0.2V_s - 10$$

Substituting into Eq (2) yields

$$48.75 = \frac{6V_s}{0.2V_s - 4} \quad \text{or} \quad V_s = 52 \text{ mV}$$

- [b] From Eq (1)

$$50 = \frac{520}{10 + R_s} \quad \text{or} \quad 50R_s = 20$$

$$\text{So } R_s = 400 \text{ k}\Omega$$

P 3.48 [a] $R_{\text{movement}} = 50 \Omega$

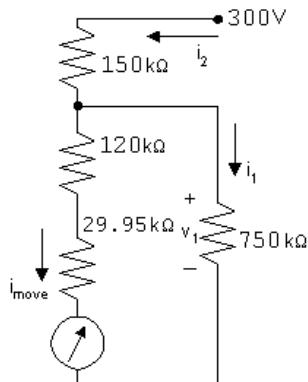
$$R_1 + R_{\text{movement}} = \frac{30}{1 \times 10^{-3}} = 30 \text{ k}\Omega \quad \therefore R_1 = 29,950 \Omega$$

$$R_2 + R_1 + R_{\text{movement}} = \frac{150}{1 \times 10^{-3}} = 150 \text{ k}\Omega \quad \therefore R_2 = 120 \text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{300}{1 \times 10^{-3}} = 300 \text{ k}\Omega$$

$$\therefore R_3 = 150 \text{ k}\Omega$$

[b]



$$v_1 = (0.96 \text{ m})(150 \text{ k}) = 144 \text{ V}$$

$$i_{\text{move}} = \frac{144}{120 + 29.95 + 0.05} = 0.96 \text{ mA}$$

$$i_1 = \frac{144}{750 \text{ k}} = 0.192 \text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 0.96 \text{ mA} + 0.192 \text{ mA} = 1.152 \text{ mA}$$

$$v_{\text{meas}} = v_x = 144 + 150i_2 = 316.8 \text{ V}$$

[c] $v_1 = 150 \text{ V}; \quad i_2 = 1 \text{ m} + 0.20 \text{ m} = 1.20 \text{ mA}$

$$i_1 = 150/750,000 = 0.20 \text{ mA}$$

$$\therefore v_{\text{meas}} = v_x = 150 + (150 \text{ k})(1.20 \text{ m}) = 330 \text{ V}$$

P 3.49 [a] $R_{\text{meter}} = 300 \text{ k}\Omega + 600 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 450 \text{ k}\Omega$

$$450 \parallel 360 = 200 \text{ k}\Omega$$

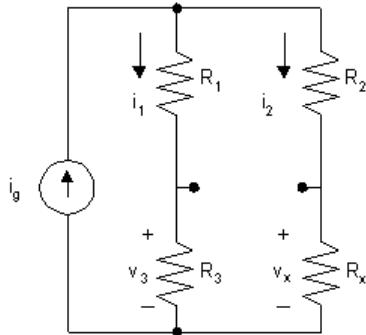
$$V_{\text{meter}} = \frac{200}{240}(600) = 500 \text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value} = \frac{360}{400}(600) = 540 \text{ V}$$

$$\% \text{ error} = \left(\frac{500}{540} - 1 \right) 100 = -7.41\%$$

- P 3.50 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

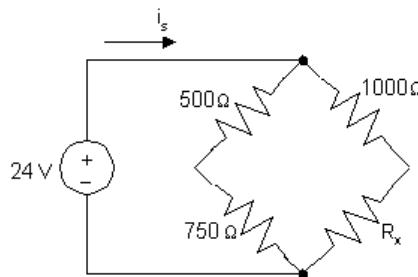
$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

$$\text{From which } R_x = \frac{R_2 R_3}{R_1}$$

- P 3.51 [a]



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(500)(R_x) = (1000)(750) \quad \text{so} \quad R_x = \frac{(1000)(750)}{500} = 1500 \Omega$$

- [b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 24 V:

$$i_s = \frac{24 \text{ V}}{500 \Omega + 750 \Omega} + \frac{24 \text{ V}}{1000 \Omega + 1500 \Omega} = 28.8 \text{ mA}$$

- [c] We can use Ohm's law to find the current in each branch:

$$i_{\text{left}} = \frac{24}{500 + 750} = 19.2 \text{ mA}$$

$$i_{\text{right}} = \frac{24}{1000 + 1500} = 9.6 \text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

$$p_{500} = (500)(0.0192)^2 = 184.32 \text{ mW} \quad p_{750} = (750)(0.0192)^2 = 276.18 \text{ mW}$$

$$p_{1000} = (1000)(0.0096)^2 = 92.16 \text{ mW} \quad p_{1500} = (1500)(0.0096)^2 = 138.24 \text{ mW}$$

Thus, the 750Ω resistor absorbs the most power; it absorbs 276.48 mW of power.

- [d] From the analysis in part (c), the 1000Ω resistor absorbs the least power; it absorbs 92.16 mW of power.

- P 3.52 Note the bridge structure is balanced, that is $15 \times 5 = 3 \times 25$, hence there is no current in the $5 \text{ k}\Omega$ resistor. It follows that the equivalent resistance of the circuit is

$$R_{\text{eq}} = 750 + (15,000 + 3000) \parallel (25,000 + 5000) = 750 + 11,250 = 12 \text{ k}\Omega$$

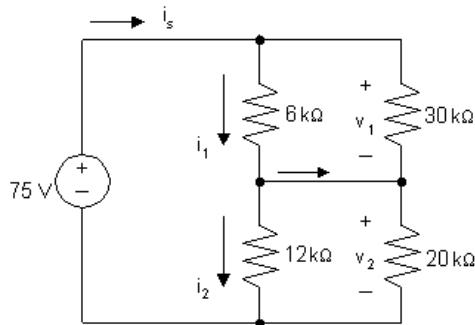
The source current is $192/12,000 = 16 \text{ mA}$.

The current down through the branch containing the $15 \text{ k}\Omega$ and $3 \text{ k}\Omega$ resistors is

$$i_{3k} = \frac{11,250}{18,000}(0.016) = 10 \text{ mA}$$

$$\therefore p_{3k} = 3000(0.01)^2 = 0.3 \text{ W}$$

- P 3.53 Redraw the circuit, replacing the detector branch with a short circuit.



$$6 \text{ k}\Omega \parallel 30 \text{ k}\Omega = 5 \text{ k}\Omega$$

$$12 \text{ k}\Omega \parallel 20 \text{ k}\Omega = 7.5 \text{ k}\Omega$$

$$i_s = \frac{75}{12,500} = 6 \text{ mA}$$

$$v_1 = 0.006(5000) = 30 \text{ V}$$

$$v_2 = 0.006(7500) = 45 \text{ V}$$

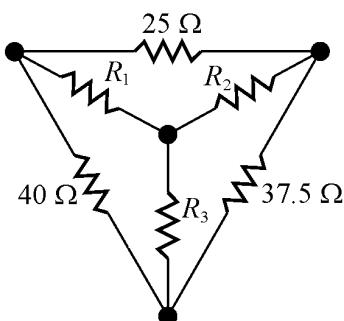
$$i_1 = \frac{30}{6000} = 5 \text{ mA}$$

$$i_2 = \frac{45}{12,000} = 3.75 \text{ mA}$$

$$i_d = i_1 - i_2 = 1.25 \text{ mA}$$

- P 3.54 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

- P 3.55 Use the figure below to transform the Δ to an equivalent Y:

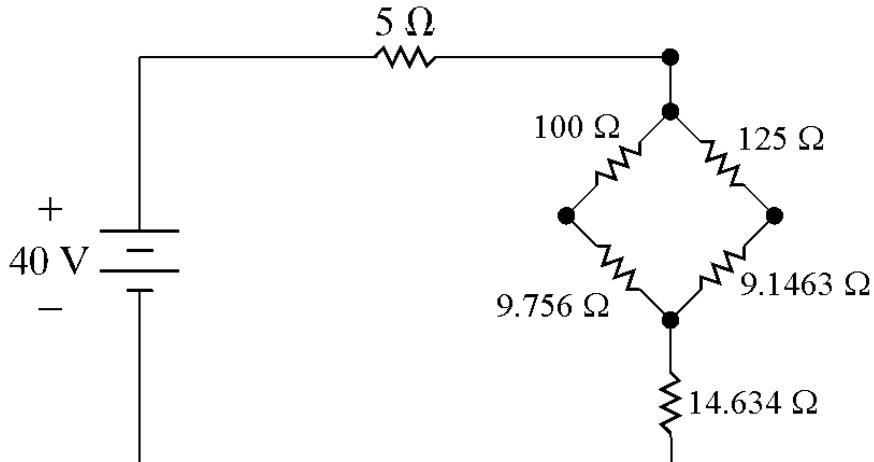


$$R_1 = \frac{(40)(25)}{40 + 25 + 37.5} = 9.756 \Omega$$

$$R_2 = \frac{(25)(37.5)}{40 + 25 + 37.5} = 9.1463 \Omega$$

$$R_3 = \frac{(40)(37.5)}{40 + 25 + 37.5} = 14.634 \Omega$$

Replace the Δ with its equivalent Y in the circuit to get the figure below:



Find the equivalent resistance to the right of the 5Ω resistor:

$$(100 + 9.756) \parallel (125 + 9.1463) + 14.634 = 75 \Omega$$

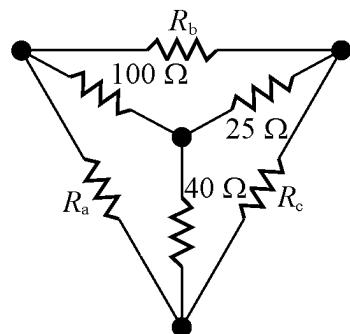
The equivalent resistance seen by the source is thus $5 + 75 = 80 \Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.56 Use the figure below to transform the Y to an equivalent Δ :

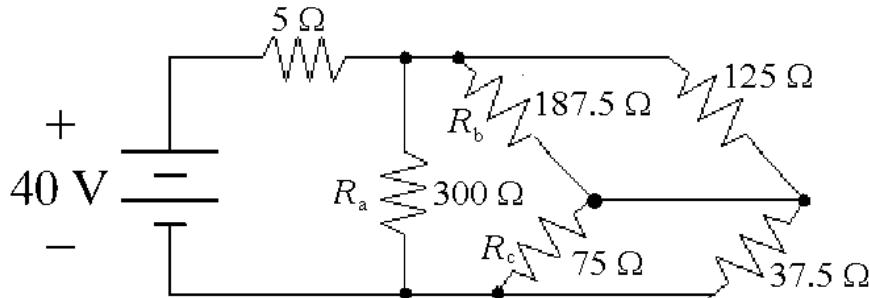


$$R_a = \frac{(25)(100) + (25)(40) + (40)(100)}{25} = \frac{7500}{25} = 300 \Omega$$

$$R_b = \frac{(25)(100) + (25)(40) + (40)(100)}{40} = \frac{7500}{40} = 187.5 \Omega$$

$$R_c = \frac{(25)(100) + (25)(40) + (40)(100)}{100} = \frac{7500}{100} = 75 \Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



Find the equivalent resistance to the right of the 5Ω resistor:

$$300\|[(125\|187.5) + (37.5\|75)] = 75 \Omega$$

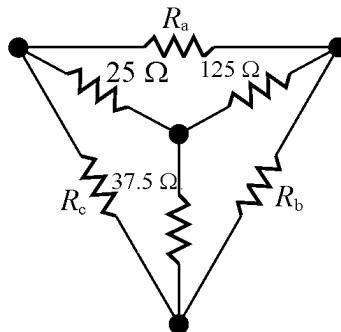
The equivalent resistance seen by the source is thus $5 + 75 = 80 \Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.57 Use the figure below to transform the Y to an equivalent Δ :

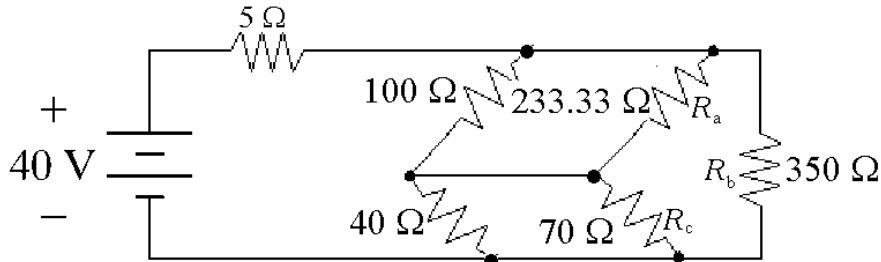


$$R_a = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{37.5} = \frac{8750}{37.5} = 233.33 \Omega$$

$$R_b = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{25} = \frac{8750}{25} = 350 \Omega$$

$$R_c = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{125} = \frac{8750}{125} = 70 \Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



Find the equivalent resistance to the right of the 5Ω resistor:

$$350 \parallel [(100 \parallel 233.33) + (40 \parallel 70)] = 75 \Omega$$

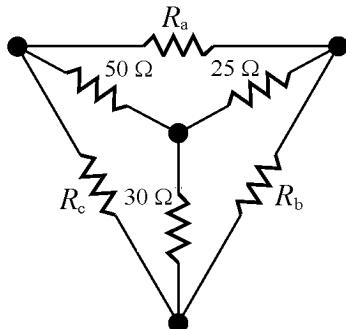
The equivalent resistance seen by the source is thus $5 + 75 = 80 \Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

- P 3.58 [a] Use the figure below to transform the Y to an equivalent Δ :

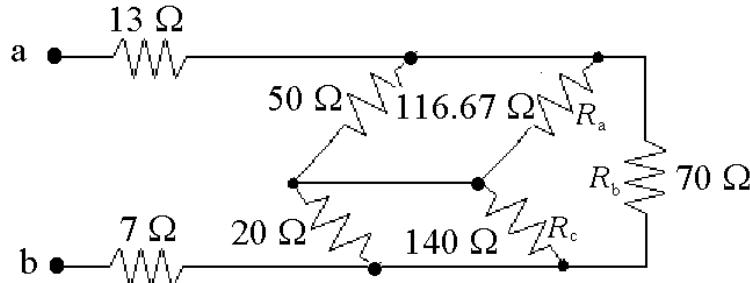


$$R_a = \frac{(25)(30) + (25)(50) + (30)(50)}{30} = \frac{3500}{30} = 116.67 \Omega$$

$$R_b = \frac{(25)(30) + (25)(50) + (30)(50)}{50} = \frac{3500}{50} = 70 \Omega$$

$$R_c = \frac{(25)(30) + (25)(50) + (30)(50)}{25} = \frac{3500}{25} = 140 \Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



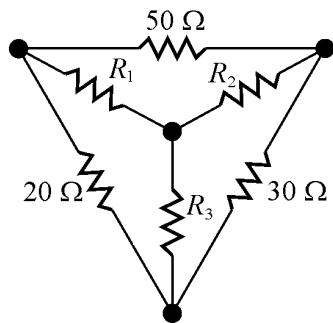
Find the equivalent resistance to the right of the 13Ω and 7Ω resistors:

$$70\parallel[(50\parallel 116.67) + (20\parallel 140)] = 30\Omega$$

Thus, the equivalent resistance seen from the terminals a-b is:

$$R_{ab} = 13 + 30 + 7 = 50\Omega$$

[b] Use the figure below to transform the Δ to an equivalent Y:

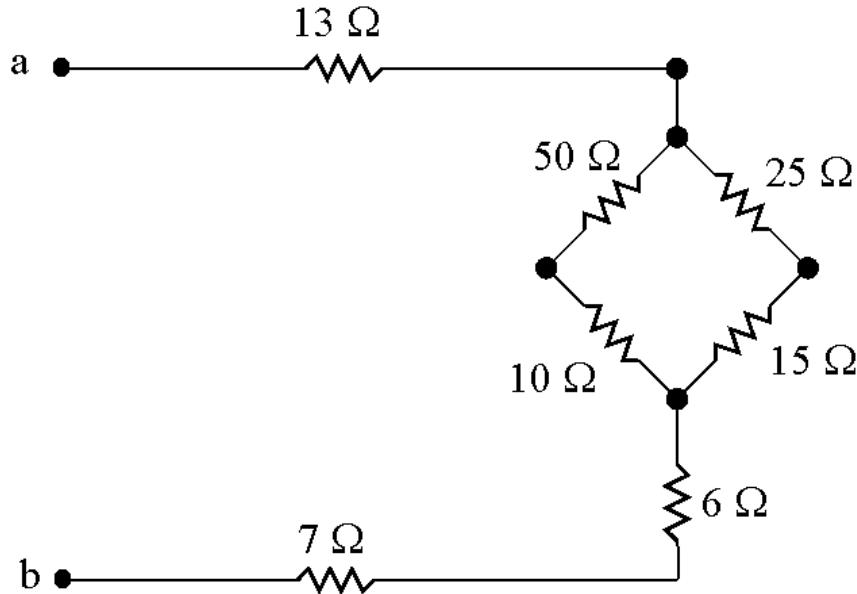


$$R_1 = \frac{(50)(20)}{50 + 20 + 30} = 10\Omega$$

$$R_2 = \frac{(50)(30)}{50 + 20 + 30} = 15\Omega$$

$$R_3 = \frac{(20)(30)}{50 + 20 + 30} = 6\Omega$$

Replace the Δ with its equivalent Y in the circuit to get the figure below:



Find the equivalent resistance to the right of the 13Ω and 7Ω resistors:

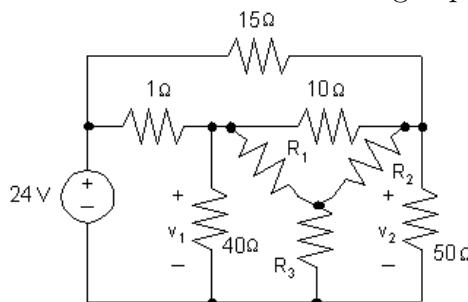
$$(50 + 10)\parallel(25 + 15) + 6 = 30\Omega$$

Thus, the equivalent resistance seen from the terminals a-b is:

$$R_{ab} = 13 + 30 + 7 = 50\Omega$$

- [c] Convert the delta connection $R_1—R_2—R_3$ to its equivalent wye.
Convert the wye connection $R_1—R_3—R_4$ to its equivalent delta.

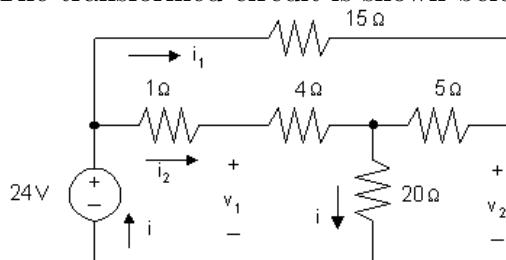
- P 3.59 Begin by transforming the Δ -connected resistors (10Ω , 40Ω , 50Ω) to Y-connected resistors. Both the Y-connected and Δ -connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(40)(10)}{10 + 40 + 50} = 4\Omega; \quad R_2 = \frac{(10)(50)}{10 + 40 + 50} = 5\Omega; \quad R_3 = \frac{(40)(50)}{10 + 40 + 50} = 20\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\text{eq}} = (15 + 5) \parallel (1 + 4) + 20 = 20 \parallel 5 + 20 = 4 + 20 = 24 \Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A}$$

Use current division to calculate the currents i_1 and i_2 . Note that the current i_1 flows in the branch containing the 15Ω and 5Ω series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the 1Ω and 4Ω resistors:

$$i_1 = \frac{4}{15 + 5}(i) = \frac{4}{20}(1 \text{ A}) = 0.2 \text{ A}, \quad \text{and} \quad i_2 = 1 \text{ A} - 0.2 \text{ A} = 0.8 \text{ A}$$

Now use KVL and Ohm's law to calculate v_1 . Note that v_1 is the sum of the voltage drop across the 4Ω resistor, $4i_2$, and the voltage drop across the 20Ω resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8 \text{ A}) + 20(1 \text{ A}) = 3.2 + 20 = 23.2 \text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2 . Note that v_2 is the sum of the voltage drop across the 5Ω resistor, $5i_1$, and the voltage drop across the 20Ω resistor, $20i$:

$$v_2 = 5i_1 + 20i = 5(0.2 \text{ A}) + 20(1 \text{ A}) = 1 + 20 = 21 \text{ V}$$

P 3.60 [a] Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5 \Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25 \Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25 \Omega$$

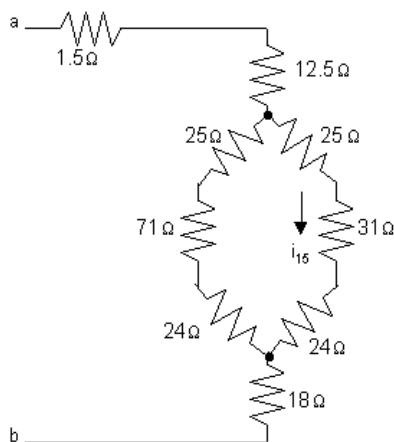
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\Omega$$

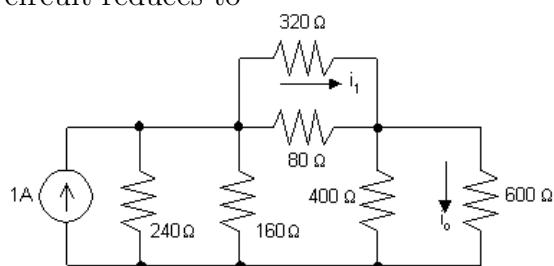
[b] When $v_{ab} = 400$ V

$$i_g = \frac{400}{80} = 5\text{ A}$$

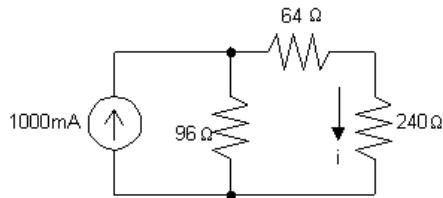
$$i_{31} = \frac{48}{80}(5) = 3\text{ A}$$

$$p_{31\Omega} = (31)(3)^2 = 279\text{ W}$$

P 3.61 [a] After the 20Ω — 100Ω — 50Ω wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to

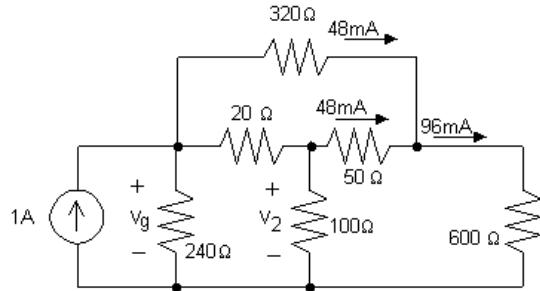


$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

$$[b] \quad i_1 = \frac{80}{400}(240) = 48 \text{ mA}$$

[c] Now that i_o and i_1 are known return to the original circuit



$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

$$[d] \quad v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96 \text{ V}$$

$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

$$\text{P 3.62} \quad 8 + 12 = 20 \Omega$$

$$20\parallel 60 = 15 \Omega$$

$$15 + 20 = 35 \Omega$$

$$35\parallel 140 = 28 \Omega$$

$$28 + 22 = 50 \Omega$$

$$50\parallel 75 = 30 \Omega$$

$$30 + 10 = 40 \Omega$$

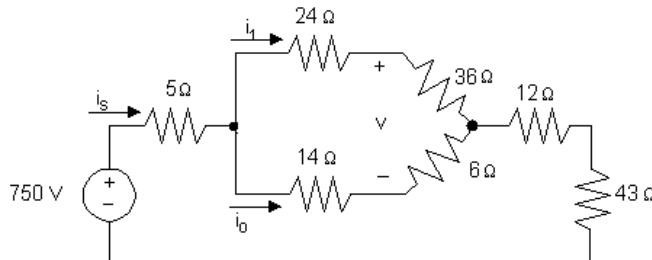
$$i_g = 240/40 = 6 \text{ A}$$

$$i_o = (6)(50)/125 = 2.4 \text{ A}$$

$$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72 \text{ A}$$

$$p_{140\Omega} = (0.72)^2(140) = 72.576 \text{ W}$$

- P 3.63 [a] Replace the 60—120—20 Ω delta with a wye equivalent to get



$$i_s = \frac{750}{5 + (24 + 36)\|(14 + 6) + 12 + 43} = \frac{750}{75} = 10 \text{ A}$$

$$i_1 = \frac{(24 + 36)\|(14 + 6)}{24 + 36}(10) = \frac{15}{60}(10) = 2.5 \text{ A}$$

[b] $i_o = 10 - 2.5 = 7.5 \text{ A}$

$$v = 36i_1 - 6i_o = 36(2.5) - 6(7.5) = 45 \text{ V}$$

[c] $i_2 = i_o + \frac{v}{60} = 7.5 + \frac{45}{60} = 8.25 \text{ A}$

[d] $P_{\text{supplied}} = (750)(10) = 7500 \text{ W}$

$$\begin{aligned} P 3.64 \quad G_a &= \frac{1}{R_a} = \frac{R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1 G_2 G_3)}{G_1 + G_2 + G_3} = \frac{G_2 G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_b and G_c .

- P 3.65 [a] Subtracting Eq. 3.42 from Eq. 3.43 gives

$$R_1 - R_2 = (R_c R_b - R_c R_a)/(R_a + R_b + R_c).$$

Adding this expression to Eq. 3.41 and solving for R_1 gives

$$R_1 = R_c R_b / (R_a + R_b + R_c).$$

To find R_2 , subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3 , subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43.

[b] Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_b = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a : First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or $R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c , use Eqs. 3.44–3.46 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.66 [a] $R_{ab} = 2R_1 + \frac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $2R_1 - R_L + \frac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Thus $R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be v_i/R_L .

Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $v_o = \frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L} R_L$

and $\frac{v_o}{v_i} = \frac{R_2}{2R_1 + R_2 + R_L}$

[b] $(300)^2 = 4(R_1 + R_2)R_1$

$$22,500 = R_1^2 + R_1R_2$$

$$\frac{v_o}{v_i} = 0.5 = \frac{R_2}{2R_1 + R_2 + 300}$$

$$\therefore R_1 + 0.5R_2 + 150 = R_2$$

$$0.5R_2 = R_1 + 150$$

$$R_2 = 2R_1 + 300$$

$$\therefore 22,500 = R_1^2 + R_1(2R_1 + 300) = 3R_1^2 + 300R_1$$

$$\therefore R_1^2 + 100R_1 - 7500 = 0$$

Solving,

$$R_1 = 50 \Omega$$

$$R_2 = 2(50) + 300 = 400 \Omega$$

- [c] From Appendix H, choose $R_1 = 47 \Omega$ and $R_2 = 390 \Omega$. For these values, $R_{ab} \neq R_L$, so the equations given in part (a) cannot be used. Instead

$$\begin{aligned} R_{ab} &= 2R_1 + [R_2\|(2R_1 + R_L)] = 2(47) + 390\|(2(47) + 300) \\ &= 94 + 390\|394 = 290 \Omega \end{aligned}$$

$$\% \text{ error} = \left(\frac{290}{300} - 1 \right) 100 = -3.33\%$$

Now calculate the ratio of the output voltage to the input voltage. Begin by finding the current through the top left R_1 resistor, called i_a :

$$i_a = \frac{v_i}{R_{ab}}$$

Now use current division to find the current through the R_L resistor, called i_L :

$$i_L = \frac{R_2}{R_2 + 2R_1 + R_L} i_a$$

Therefore, the output voltage, v_o , is equal to $R_L i_L$:

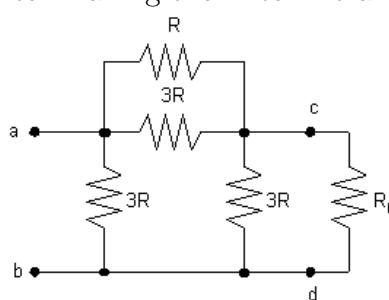
$$v_o = \frac{R_2 R_L v_i}{R_{ab}(R_2 + 2R_1 + R_L)}$$

Thus,

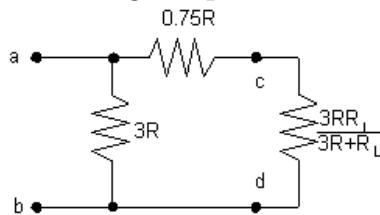
$$\frac{v_o}{v_i} = \frac{R_2 R_L}{R_{ab}(R_2 + 2R_1 + R_L)} = \frac{390(300)}{290(390 + 2(47) + 300)} = 0.5146$$

$$\% \text{ error} = \left(\frac{0.5146}{0.5} - 1 \right) 100 = 2.92\%$$

- P 3.67 [a] After making the Y-to- Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



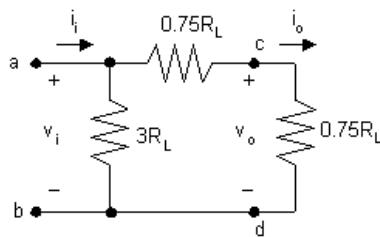
$$\text{Now note: } 0.75R + \frac{3RR_L}{3R + R_L} = \frac{2.25R^2 + 3.75RR_L}{3R + R_L}$$

$$\text{Therefore } R_{ab} = \frac{3R \left(\frac{2.25R^2 + 3.75RR_L}{3R + R_L} \right)}{3R + \left(\frac{2.25R^2 + 3.75RR_L}{3R + R_L} \right)} = \frac{3R(3R + 5R_L)}{15R + 9R_L}$$

$$\text{If } R = R_L, \text{ we have } R_{ab} = \frac{3R_L(8R_L)}{24R_L} = R_L$$

$$\text{Therefore } R_{ab} = R_L$$

[b] When $R = R_L$, the circuit reduces to



$$i_o = \frac{i_i(3R_L)}{4.5R_L} = \frac{1}{1.5}i_i = \frac{1}{1.5}\frac{v_i}{R_L}, \quad v_o = 0.75R_L i_o = \frac{1}{2}v_i,$$

$$\text{Therefore } \frac{v_o}{v_i} = 0.5$$

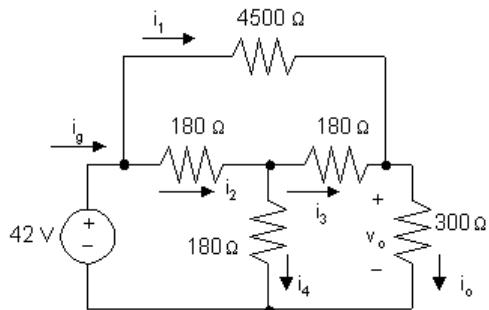
$$\text{P 3.68 [a]} \quad 3.5(3R - R_L) = 3R + R_L$$

$$10.5R - 1050 = 3R + 300$$

$$7.5R = 1350, \quad R = 180 \Omega$$

$$R_2 = \frac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500 \Omega$$

[b]



$$v_o = \frac{v_i}{3.5} = \frac{42}{3.5} = 12 \text{ V}$$

$$i_o = \frac{12}{300} = 40 \text{ mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67 \text{ mA}$$

$$i_g = \frac{42}{300} = 140 \text{ mA}$$

$$i_2 = 140 - 6.67 = 133.33 \text{ mA}$$

$$i_3 = 40 - 6.67 = 33.33 \text{ mA}$$

$$i_4 = 133.33 - 33.33 = 100 \text{ mA}$$

$$p_{4500 \text{ top}} = (6.67 \times 10^{-3})^2(4500) = 0.2 \text{ W}$$

$$p_{180 \text{ left}} = (133.33 \times 10^{-3})^2(180) = 3.2 \text{ W}$$

$$p_{180 \text{ right}} = (33.33 \times 10^{-3})^2(180) = 0.2 \text{ W}$$

$$p_{180 \text{ vertical}} = (100 \times 10^{-3})^2(180) = 0.48 \text{ W}$$

$$p_{300 \text{ load}} = (40 \times 10^{-3})^2(300) = 0.48 \text{ W}$$

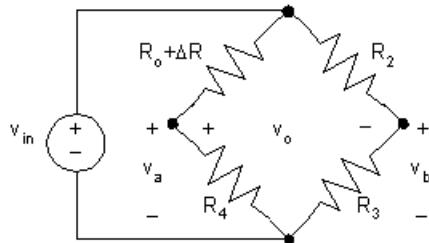
The 180 Ω resistor carrying i_2

[c] $p_{180 \text{ left}} = 3.2 \text{ W}$

[d] Two resistors dissipate minimum power – the 4500 Ω resistor and the 180 Ω resistor carrying i_3 .

[e] They both dissipate 0.2 W.

P 3.69 [a]



$$v_a = \frac{v_{\text{in}} R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3} v_{\text{in}}$$

$$v_o = v_a - v_b = \frac{R_4 v_{\text{in}}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3} v_{\text{in}}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4} v_{\text{in}} = \frac{R_3}{R_2 + R_3} v_{\text{in}}$$

$$\therefore \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

$$\begin{aligned} \text{Thus, } v_o &= \frac{R_4 v_{\text{in}}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{\text{in}}}{R_o + R_4} \\ &= R_4 v_{\text{in}} \left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4} \right] \\ &= \frac{R_4 v_{\text{in}} (-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R) R_4 v_{\text{in}}}{(R_o + R_4)^2}, \quad \text{since } \Delta R \ll R_4 \end{aligned}$$

[b] $\Delta R = 0.03 R_o$

$$R_o = \frac{R_2 R_4}{R_3} = \frac{(1000)(5000)}{500} = 10,000 \Omega$$

$$\Delta R = (0.03)(10^4) = 300 \Omega$$

$$\therefore v_o \approx \frac{-300(5000)(6)}{(15,000)^2} = -40 \text{ mV}$$

$$\begin{aligned} [\text{c}] \quad v_o &= \frac{-(\Delta R) R_4 v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15,300)(15,000)} \\ &= -39.2157 \text{ mV} \end{aligned}$$

P 3.70 [a] approx value = $\frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4)^2}$

$$\text{true value} = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \% \text{ error} = \left[\frac{R_o + R_4}{R_o + R_4 + \Delta R} - 1 \right] \times 100 = \frac{-\Delta R}{R_o + R_4} \times 100$$

Note that in the above expression, we take the ratio of the true value to the approximate value because both values are negative.

$$\text{But } R_o = \frac{R_2 R_4}{R_3}$$

$$\therefore \% \text{ error} = \frac{-R_3 \Delta R}{R_4(R_2 + R_3)}$$

[b] % error = $\frac{-(500)(300)}{(5000)(1500)} \times 100 = -2\%$

P 3.71 $\frac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \Delta R = 75 \Omega$$

$$\% \text{ change} = \frac{75}{10,000} \times 100 = 0.75\%$$

P 3.72 [a] Using the equation for voltage division,

$$V_y = \frac{\beta R_y}{\beta R_y + (1 - \beta)R_s} V_s = \frac{\beta R_y}{R_y} V_s = \beta V_s$$

[b] Since β represents the touch point with respect to the bottom of the screen, $(1 - \beta)$ represents the location of the touch point with respect to the top of the screen. Therefore, the y -coordinate of the pixel corresponding to the touch point is

$$y = (1 - \beta)p_y$$

Remember that the value of y is capped at $(p_y - 1)$.

- P 3.73 [a] Use the equations developed in the Practical Perspective and in Problem 3.72:

$$V_x = \alpha V_S \quad \text{so} \quad \alpha = \frac{V_x}{V_S} = \frac{1}{5} = 0.2$$

$$V_y = \beta V_S \quad \text{so} \quad \beta = \frac{V_y}{V_S} = \frac{3.75}{5} = 0.75$$

- [b] Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1 - \alpha)p_x = (1 - 0.2)(480) = 384$$

$$y = (1 - \beta)p_y = (1 - 0.75)(800) = 200$$

Therefore, the touch occurred in the upper right corner of the screen.

- P 3.74 Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1 - \alpha)p_x \quad \text{so} \quad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{480}{640} = 0.25$$

$$V_x = \alpha V_S = (0.25)(8) = 2 \text{ V}$$

$$y = (1 - \beta)p_y \quad \text{so} \quad \beta = 1 - \frac{y}{p_y} = 1 - \frac{192}{1024} = 0.8125$$

$$V_y = \beta V_S = (0.8125)(8) = 6.5 \text{ V}$$

- P 3.75 From the results of Problem 3.74, the voltages corresponding to the touch point (480, 192) are

$$V_{x1} = 2 \text{ V}; \quad V_{y1} = 6.5 \text{ V}$$

Now calculate the voltages corresponding to the touch point (240, 384):

$$x = (1 - \alpha)p_x \quad \text{so} \quad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{240}{640} = 0.625$$

$$V_{x2} = \alpha V_S = (0.625)(8) = 5 \text{ V}$$

$$y = (1 - \beta)p_y \quad \text{so} \quad \beta = 1 - \frac{y}{p_y} = 1 - \frac{384}{1024} = 0.625$$

$$V_{y2} = \beta V_S = (0.625)(8) = 5 \text{ V}$$

When the screen is touched at two points simultaneously, only the smaller of the two voltages in the x direction is sensed. The same is true in the y direction.

Therefore, the voltages actually sensed are

$$V_x = 2 \text{ V}; \quad V_y = 5 \text{ V}$$

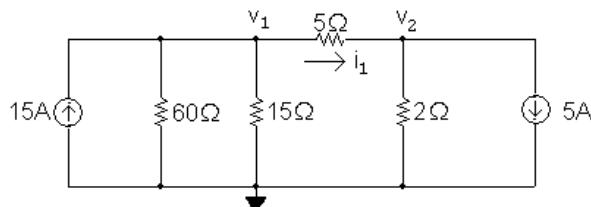
These two voltages identify the touch point as (480, 384), which does not correspond to either of the points actually touched! Therefore, the resistive touch screen is appropriate only for single point touches.

4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 [a] Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1 \left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5} \right) + v_2 \left(-\frac{1}{5} \right) = 15$$

$$v_1 \left(-\frac{1}{5} \right) + v_2 \left(\frac{1}{2} + \frac{1}{5} \right) = -5$$

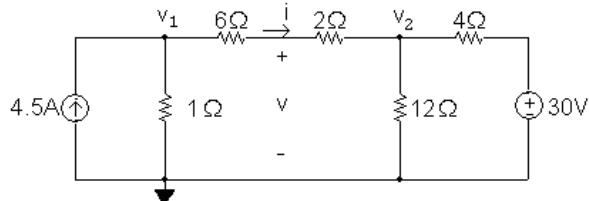
Solving, $v_1 = 60$ V and $v_2 = 10$ V;

Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15A} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900\text{ W}(\text{delivered})$

[c] $p_{5A} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50\text{ W} = -50\text{ W}(\text{delivered})$

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1 \left(1 + \frac{1}{8}\right) + v_2 \left(-\frac{1}{8}\right) = 4.5$$

$$v_1 \left(-\frac{1}{8}\right) + v_2 \left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6 \text{ V}$ $v_2 = 18 \text{ V}$

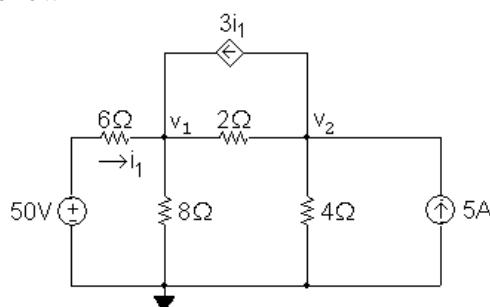
To find the voltage v , first find the current i through the series-connected 6Ω and 2Ω resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6 - 18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v :

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 [a] Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{aligned} v_1 \left(\frac{1}{6} + \frac{1}{8} + \frac{1}{2} \right) + v_2 \left(-\frac{1}{2} \right) + i_1(-3) &= \frac{50}{6} \\ v_1 \left(-\frac{1}{2} \right) + v_2 \left(\frac{1}{4} + \frac{1}{2} \right) + i_1(3) &= 5 \\ v_1 \left(\frac{1}{6} \right) + v_2(0) + i_1(1) &= \frac{50}{6} \end{aligned}$$

Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

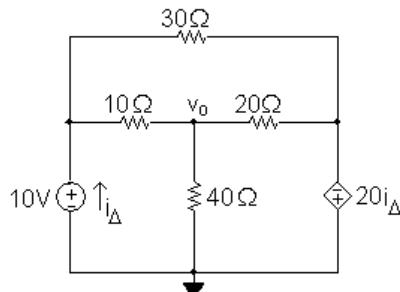
$$p_{50V} = -50i_1 = -150 \text{ W}$$

$$p_{5A} = -5(v_2) = -80 \text{ W}$$

$$p_{3i_1} = 3i_1(v_2 - v_1) = -144 \text{ W}$$

- [b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o - 10}{40} + \frac{v_o + 20i_\Delta}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

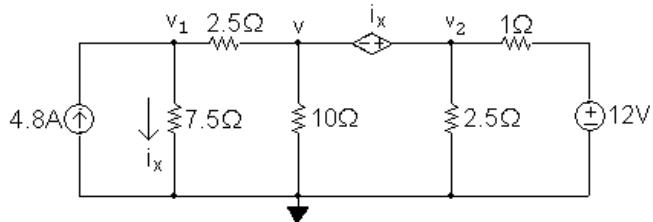
$$i_\Delta = i_{10\Omega} + i_{30\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$\begin{aligned} v_o \left(\frac{1}{40} + \frac{1}{10} + \frac{1}{20} \right) + i_\Delta(1) &= 1 \\ v_o \left(\frac{1}{10} \right) + i_\Delta \left(1 - \frac{20}{30} \right) &= 1 + \frac{10}{30} \end{aligned}$$

Solving, $i_\Delta = -3.2$ A and $v_o = 24$ V

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0$$

The supernode equation is

$$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

The constraint equation due to the dependent source is

$$i_x = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

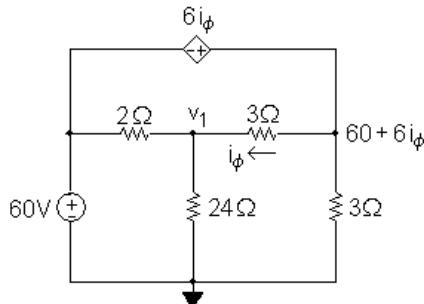
$$v + i_x = v_2$$

Place this set of equations in standard form:

$$\begin{aligned} v_1 \left(\frac{1}{7.5} + \frac{1}{2.5} \right) + v \left(-\frac{1}{2.5} \right) + v_2(0) + i_x(0) &= 4.8 \\ v_1 \left(-\frac{1}{2.5} \right) + v \left(\frac{1}{2.5} + \frac{1}{10} \right) + v_2 \left(\frac{1}{2.5} + 1 \right) + i_x(0) &= 12 \\ v_1 \left(-\frac{1}{7.5} \right) + v(0) + v_2(0) + i_x(1) &= 0 \\ v_1(0) + v(1) + v_2(-1) + i_x(1) &= 0 \end{aligned}$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

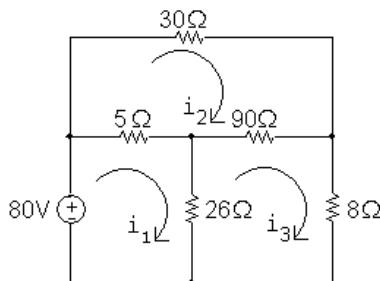
$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1 \left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3} \right) + i_\phi(-2) &= 30 + 20 \\ v_1 \left(\frac{1}{3} \right) + i_\phi(1 - 2) &= 20 \end{aligned}$$

Solving, $i_\phi = -4$ A and $v_1 = 48$ V

AP 4.7 [a] Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$-80 + 5(i_1 - i_2) + 26(i_1 - i_3) = 0$$

$$30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) = 0$$

$$8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) = 0$$

Place these equations in standard form:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$$i_1 = 5 \text{ A}; \quad i_2 = 2 \text{ A}; \quad i_3 = 2.5 \text{ A}$$

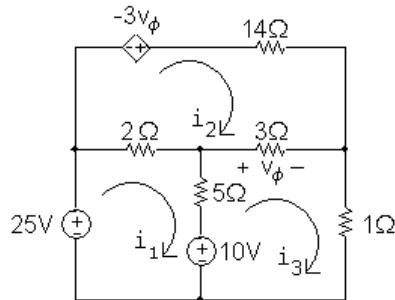
$$p_{80V} = -(80)i_1 = -(80)(5) = -400 \text{ W}$$

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50 \text{ W}$, so the 8Ω resistor dissipates 50 W.

AP 4.8 [a] $b = 8$, $n = 6$, $b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 3(i_3 - i_2)$$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0 \end{aligned}$$

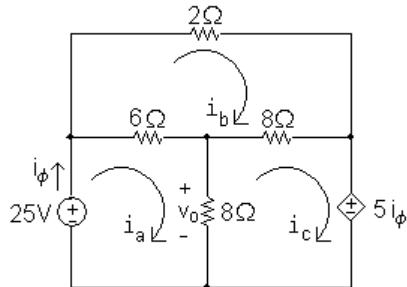
Solving

$$i_1 = 4 \text{ A}; \quad i_2 = -1 \text{ A}; \quad i_3 = 3 \text{ A}; \quad v_\phi = 12 \text{ V}$$

$$p_{ds} = -(-3v_\phi)i_2 = 3(12)(-1) = -36 \text{ W}$$

Thus, the dependent source is delivering 36 W, or absorbing -36 W .

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$-25 + 6(i_a - i_b) + 8(i_a - i_c) = 0$$

$$2i_b + 8(i_b - i_c) + 6(i_b - i_a) = 0$$

$$5i_\phi + 8(i_c - i_a) + 8(i_c - i_b) = 0$$

The dependent source constraint equation is $i_\phi = i_a$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$14i_a - 6i_b - 8i_c = 25$$

$$-6i_a + 16i_b - 8i_c = 0$$

$$-3i_a - 8i_b + 16i_c = 0$$

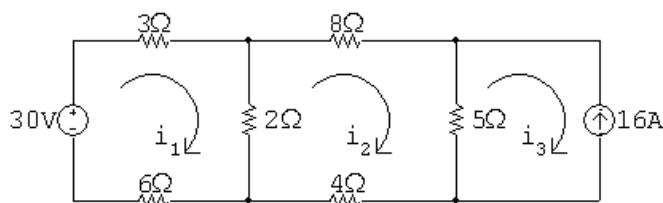
Solving,

$$i_a = 4 \text{ A}; \quad i_b = 2.5 \text{ A}; \quad i_c = 2 \text{ A}$$

Thus,

$$v_o = 8(i_a - i_c) = 8(4 - 2) = 16 \text{ V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

$$-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 = 0$$

$$8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) = 0$$

Place these equations in standard form:

$$11i_1 - 2i_2 = 30$$

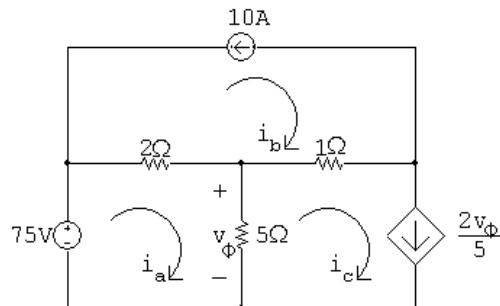
$$-2i_1 + 19i_2 = -80$$

Solving: $i_1 = 2 \text{ A}$, $i_2 = -4 \text{ A}$, $i_3 = -16 \text{ A}$

The current in the 2Ω resistor is $i_1 - i_2 = 6 \text{ A} \therefore p_{2\Omega} = (6)^2(2) = 72 \text{ W}$

Thus, the 2Ω resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_b = -10 \text{ A}; \quad i_c = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_a + 10) + 5(i_a - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

$$v_\phi = 5(i_a - i_c) = 5(i_a - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation in standard form:

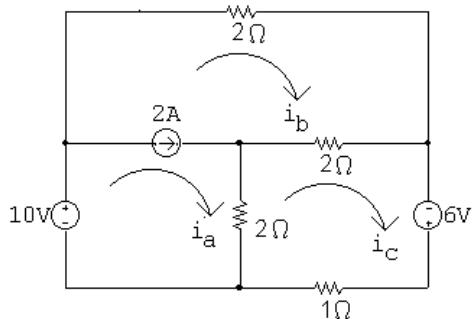
$$7i_a - 2v_\phi = 55$$

$$5i_a - 3v_\phi = 0$$

Solving: $i_a = 15 \text{ A}$; $i_b = -10 \text{ A}$; $i_c = 10 \text{ A}$; $v_\phi = 25 \text{ V}$

Thus, $i_a = 15 \text{ A}$.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b . Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

$$i_a - i_b = 2$$

Place these three equations in standard form:

$$2i_a + 4i_b - 4i_c = 10$$

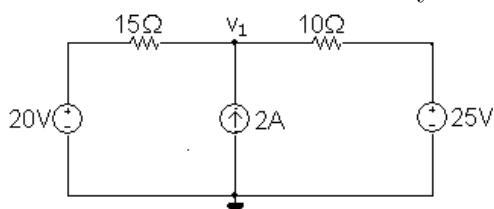
$$-2i_a - 2i_b + 5i_c = 6$$

$$i_a - i_b + 0i_c = 2$$

Solving, $i_a = 7 \text{ A}$; $i_b = 5 \text{ A}$; $i_c = 6 \text{ A}$

Thus, $p_{1\Omega} = i_c^2(1) = (6)^2(1) = 36 \text{ W}$

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1 :



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

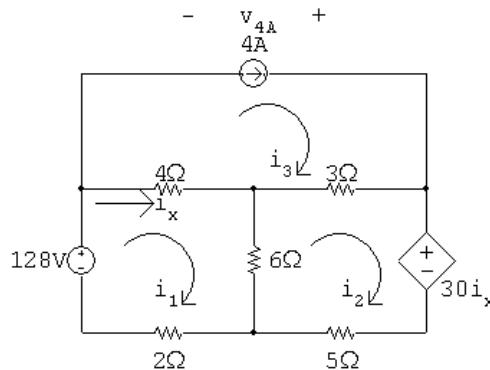
Rearranging and solving,

$$v_1 \left(\frac{1}{15} + \frac{1}{10} \right) = 2 + \frac{20}{15} + \frac{25}{10} \quad \therefore v_1 = 35 \text{ V}$$

$$p_{2A} = -35(2) = -70 \text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$-128 + 4(i_1 - 4) + 6(i_1 - i_2) + 2i_1 = 0$$

$$30i_x + 5i_2 + 6(i_2 - i_1) + 3(i_2 - 4) = 0$$

The constraint equation due to the dependent source is

$$i_x = i_1 - i_3 = i_1 - 4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$12i_1 - 6i_2 = 144$$

$$24i_1 + 14i_2 = 132$$

Solving,

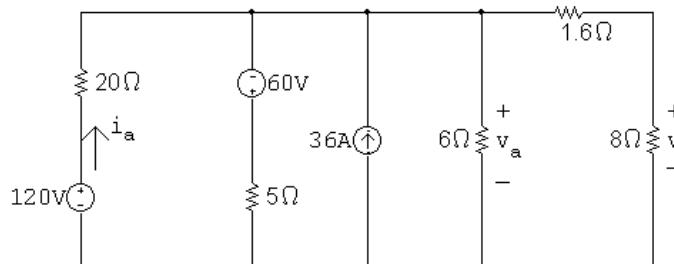
$$i_1 = 9 \text{ A}; \quad i_2 = -6 \text{ A}; \quad i_3 = 4 \text{ A}; \quad i_x = 9 - 4 = 5 \text{ A}$$

$$\therefore v_{4A} = 3(i_3 - i_2) - 4i_x = 10 \text{ V}$$

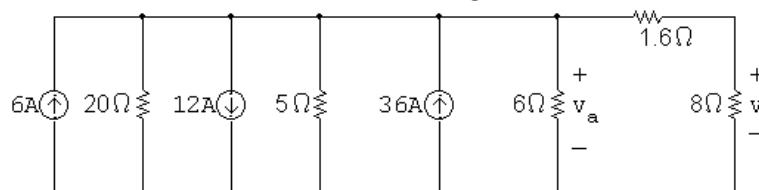
$$p_{4A} = -v_{4A}(4) = -(10)(4) = -40 \text{ W}$$

Thus, the 2 A current source delivers 40 W.

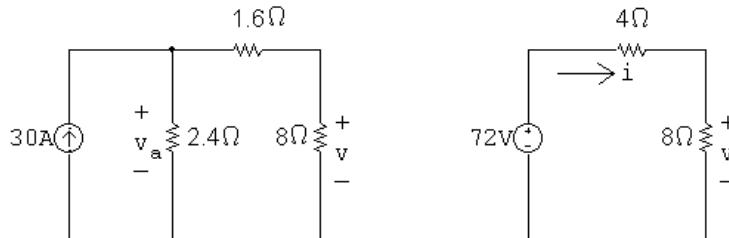
AP 4.15 [a] Redraw the circuit with a helpful voltage and current labeled:



Transform the 120 V source in series with the 20Ω resistor into a 6 A source in parallel with the 20Ω resistor. Also transform the -60 V source in series with the 5Ω resistor into a -12 A source in parallel with the 5Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20Ω, 5Ω, and 6Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4Ω resistor into a 72 V source in series with the 2.4Ω resistor. Combine the 2.4Ω resistor in series with the 1.6Ω resistor to get a very simple circuit that still maintains the voltage v . The resulting circuit is on the right.



Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{R} = \frac{48}{4} = 6 \text{ A}$$

Now use i to calculate v_a in the circuit on the left:

$$v_a = 6(1.6 + 8) = 57.6 \text{ V}$$

Returning back to the original circuit, note that the voltage v_a is also the voltage drop across the series combination of the 120 V source and 20Ω

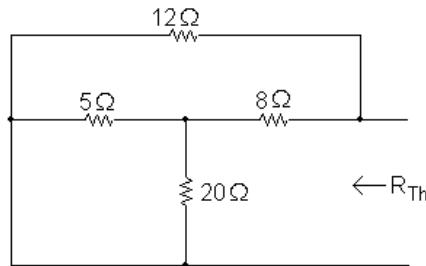
resistor. Use this fact to calculate the current in the 120 V source, i_a :

$$i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120V} = -(120)i_a = -(120)(3.12) = -374.40 \text{ W}$$

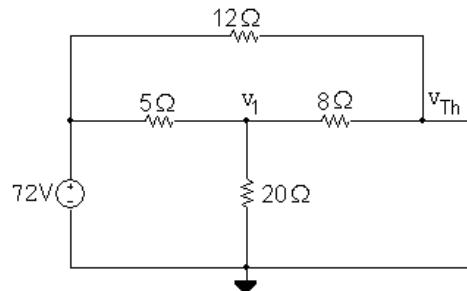
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th} , replace the 72 V source with a short circuit:



Note that the 5Ω and 20Ω resistors are in parallel, with an equivalent resistance of $5\parallel 20 = 4\Omega$. The equivalent 4Ω resistance is in series with the 8Ω resistor for an equivalent resistance of $4 + 8 = 12\Omega$. Finally, the 12Ω equivalent resistance is in parallel with the 12Ω resistor, so $R_{Th} = 12\parallel 12 = 6\Omega$.

Use node voltage analysis to find v_{Th} . Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

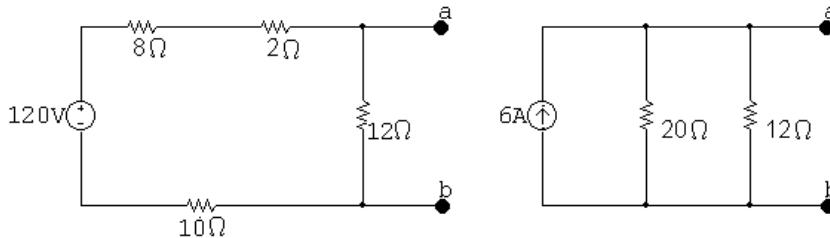
$$\begin{aligned}\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{Th}}{8} &= 0 \\ \frac{v_{Th} - v_1}{8} + \frac{v_{Th} - 72}{12} &= 0\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}v_1 \left(\frac{1}{5} + \frac{1}{20} + \frac{1}{8} \right) + v_{Th} \left(-\frac{1}{8} \right) &= \frac{72}{5} \\ v_1 \left(-\frac{1}{8} \right) + v_{Th} \left(\frac{1}{8} + \frac{1}{12} \right) &= 6\end{aligned}$$

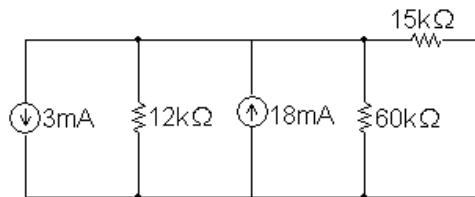
Solving, $v_1 = 60 \text{ V}$ and $v_{Th} = 64.8 \text{ V}$. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a 6Ω resistor.

AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and 8Ω resistor into a series combination of a 120 V source and an 8Ω resistor, as shown in the figure on the left. Next, combine the 2Ω , 8Ω and 10Ω resistors in series to give an equivalent 20Ω resistance. Then transform the series combination of the 120 V source and the 20Ω equivalent resistance into a parallel combination of a 6 A source and a 20Ω resistor, as shown in the figure on the right.

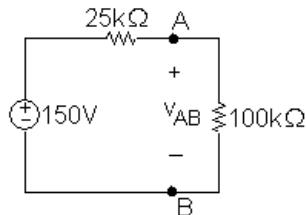


Finally, combine the 20Ω and 12Ω parallel resistors to give $R_N = 20\parallel 12 = 7.5\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a 7.5Ω resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and $12\text{ k}\Omega$ resistor into a parallel combination of a -3 mA source and $12\text{ k}\Omega$ resistor. The resulting circuit is shown below:



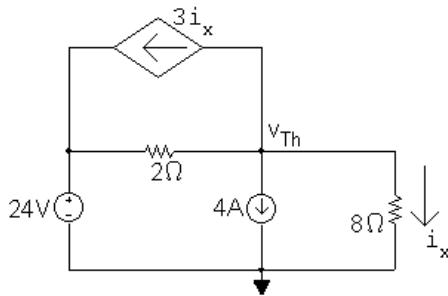
Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a $12\text{ k}\parallel 60\text{ k} = 10\text{ k}\Omega$ resistor. Then transform the 15 mA source in parallel with the $10\text{ k}\Omega$ resistor into a 150 V source in series with a $10\text{ k}\Omega$ resistor, and combine this $10\text{ k}\Omega$ resistor in series with the $15\text{ k}\Omega$ resistor. The Thévenin equivalent is thus a 150 V source in series with a $25\text{ k}\Omega$ resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a $100\text{ k}\Omega$ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB} :

$$v_{AB} = \frac{100,000}{125,000}(150) = 120\text{ V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th} , from the circuit below:



Summing the currents away from the node labeled v_{Th} We have

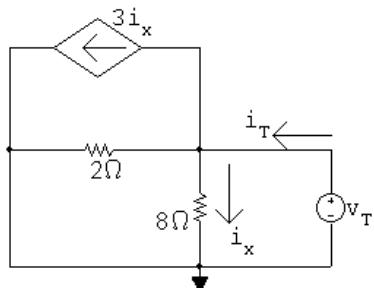
$$\frac{v_{Th}}{8} + 4 + 3i_x + \frac{v_{Th} - 24}{2} = 0$$

Also, using Ohm's law for the 8Ω resistor,

$$i_x = \frac{v_{Th}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{Th} = 8\text{ V}$.

Now calculate R_{Th} . To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_T , as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_T = i_x + 3i_x + v_T/2 = 4i_x + v_T/2$$

Use Ohm's law to determine i_x as a function of v_T :

$$i_x = v_T/8$$

Substitute the second equation into the first equation:

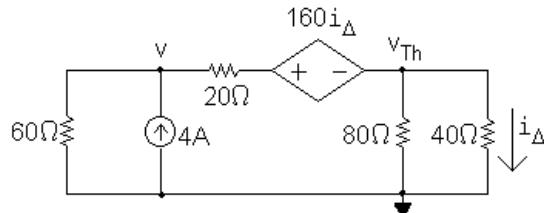
$$i_T = 4(v_T/8) + v_T/2 = v_T$$

Thus,

$$R_{\text{Th}} = v_{\text{Th}}/i_{\text{T}} = 1 \Omega$$

The Thévenin equivalent is an 8 V source in series with a 1Ω resistor.

- AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th} , using the node voltage method in the circuit below:



The node voltage equations are

$$\frac{v}{60} + \frac{v - (v_{\text{Th}} + 160i_{\Delta})}{20} - 4 = 0,$$

$$\frac{v_{\text{Th}}}{40} + \frac{v_{\text{Th}}}{80} + \frac{v_{\text{Th}} + 160i_{\Delta} - v}{20} = 0$$

The dependent source constraint equation is

$$i_{\Delta} = \frac{v_{\text{Th}}}{40}$$

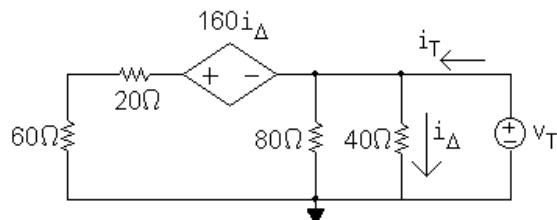
Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$v \left(\frac{1}{60} + \frac{1}{20} \right) + v_{\text{Th}} \left(-\frac{5}{20} \right) = 4$$

$$v \left(-\frac{1}{20} \right) + v_{\text{Th}} \left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20} \right) = 0$$

Solving, $v = 172.5$ V and $v_{\text{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th} . Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\text{T}} = \frac{v_{\text{T}}}{80} + \frac{v_{\text{T}}}{40} + \frac{v_{\text{T}} + 160i_{\Delta}}{80}$$

The dependent source constraint equation is

$$i_{\Delta} = \frac{v_T}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

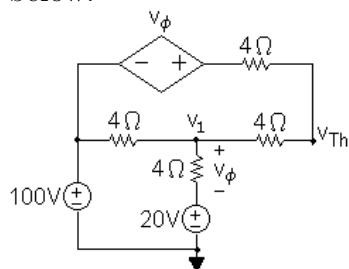
$$i_T = \frac{v_T}{10}$$

Therefore,

$$R_{Th} = \frac{v_T}{i_T} = 10 \Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a 10Ω resistor.

- AP 4.21 First find the Thévenin equivalent circuit. To find v_{Th} , create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\begin{aligned} \frac{v_{Th} - (100 + v_{\phi})}{4} + \frac{v_{Th} - v_1}{4} &= 0 \\ \frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{Th}}{4} &= 0 \end{aligned}$$

The dependent source constraint equation is

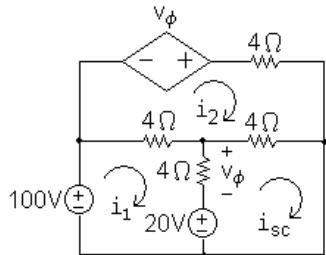
$$v_{\phi} = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned} v_{Th} \left(\frac{1}{4} + \frac{1}{4} \right) + v_1 \left(-\frac{1}{4} \right) + v_{\phi} \left(-\frac{1}{4} \right) &= 25 \\ v_{Th} \left(-\frac{1}{4} \right) + v_1 \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) + v_{\phi} (0) &= 30 \\ v_{Th} (0) + v_1 (1) + v_{\phi} (-1) &= 20 \end{aligned}$$

Solving, $v_{Th} = 120$ V, $v_1 = 80$ V, and $v_{\phi} = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_{sc}) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_{sc} - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{sc})$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_{sc} + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_{sc} - v_\phi &= 0 \\ 0i_1 - 4i_2 + 4i_{sc} - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_{sc} - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{120}{40} = 3\Omega$$

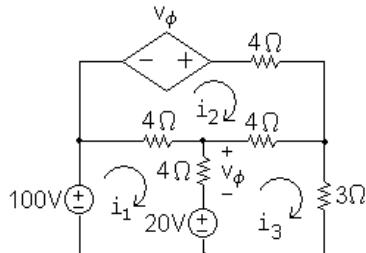
- [a] For maximum power transfer, $R = R_{Th} = 3\Omega$
- [b] The Thévenin voltage, $v_{Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{load} = \frac{120}{2} = 60\text{ V}$$

Therefore,

$$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200\text{ W}$$

AP 4.22 Substituting the value $R = 3\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$-100 + 4(i_1 - i_2) + v_\phi + 20 = 0$$

$$-v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$-20 - v_\phi + 4(i_3 - i_2) + 3i_3 = 0$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$4i_1 - 4i_2 + 0i_3 + v_\phi = 80$$

$$-4i_1 + 12i_2 - 4i_3 - v_\phi = 0$$

$$0i_1 - 4i_2 + 7i_3 - v_\phi = 20$$

$$4i_1 + 0i_2 - 4i_3 - v_\phi = 0$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

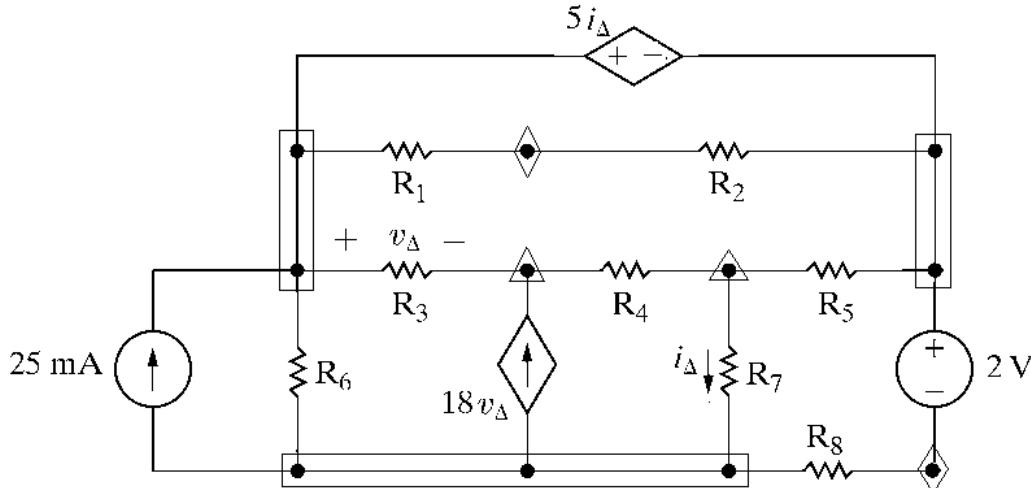
[a] $p_{100V} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{dep\ source} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1



- [a] 12 branches, 8 branches with resistors, 2 branches with independent sources, 2 branches with dependent sources.
- [b] The current is unknown in every branch except the one containing the 25 mA current source, so the current is unknown in 11 branches.
- [c] 10 essential branches – $R_1 - R_2$ forms an essential branch as does $R_8 - 2$ V. The remaining eight branches are essential branches that contain a single element.
- [d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 9 essential branches
- [e] From the figure there are 7 nodes – three identified by rectangular boxes, two identified by triangles, and two identified by diamonds.
- [f] There are 5 essential nodes, three identified with rectangular boxes and two identified with triangles.
- [g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

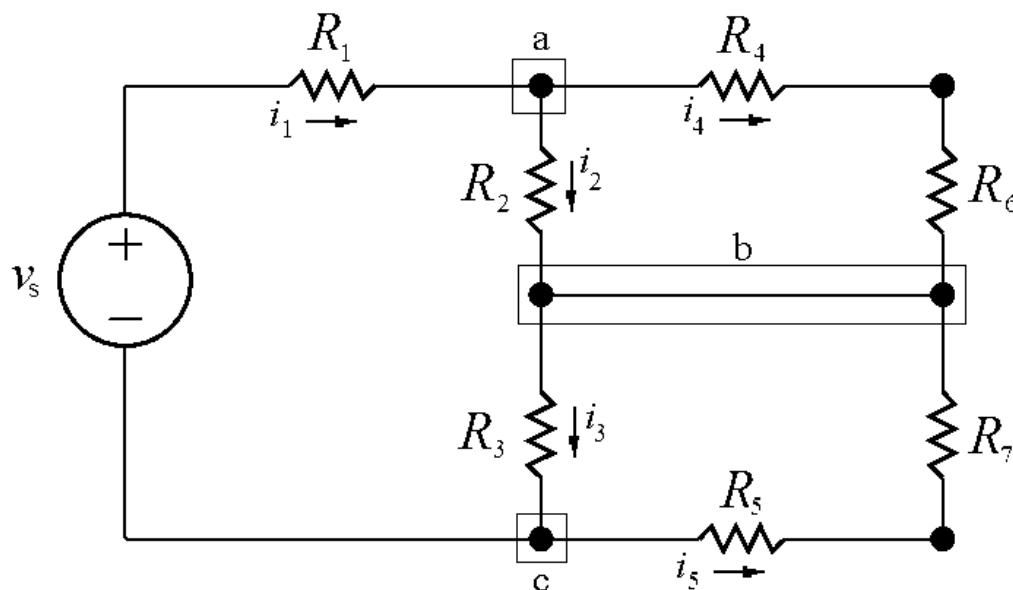
P 4.2

- [a] From Problem 4.1(d) there are 9 essential branches where the current is unknown, so we need 9 simultaneous equations to describe the circuit.
- [b] From Problem 4.1(f), there are 5 essential nodes, so we can apply KCL at $(5 - 1) = 4$ of these essential nodes. There would also be a dependent source constraint equation.
- [c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

- [d] We must avoid using the bottom left-most mesh, since it contains a current source, and we have no way of determining the voltage drop across a current source. The two meshes on the bottom that share the dependent source must be handled in a special way.

P 4.3 [a] There are eight circuit components, seven resistors and the voltage source. Therefore there are **eight** unknown currents. However, the voltage source and the R_1 resistor are in series, so have the same current. The R_4 and R_6 resistors are also in series, so have the same current. The R_5 and R_7 resistors are in series, so have the same current. Therefore, we only need 5 equations to find the 5 distinct currents in this circuit.

[b]



There are three essential nodes in this circuit, identified by the boxes. At two of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the third node would be dependent on the first two. Therefore there are **two** independent KCL equations.

- [c] Sum the currents at any two of the three essential nodes a, b, and c. Using nodes a and c we get

$$-i_1 + i_2 + i_4 = 0$$

$$i_1 - i_3 + i_5 = 0$$

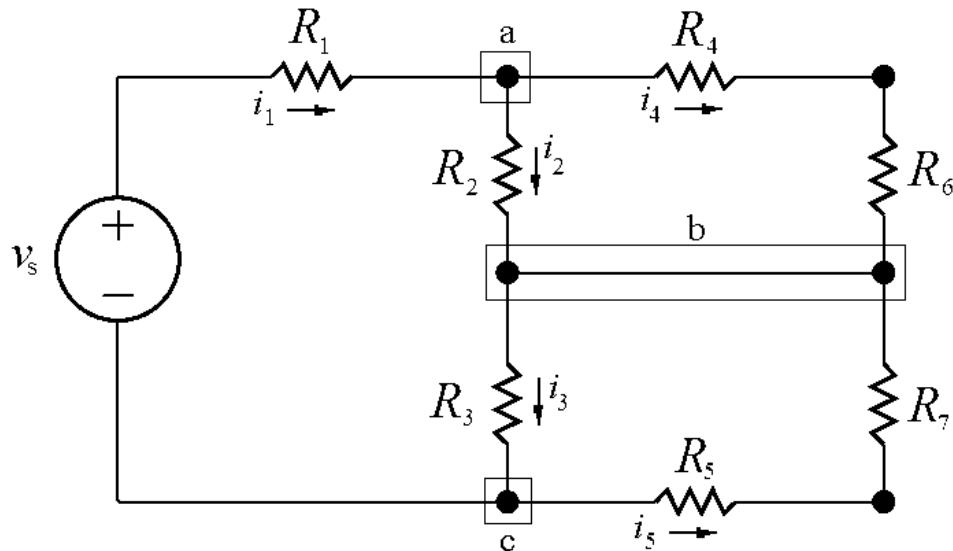
- [d] There are three meshes in this circuit: one on the left with the components v_s , R_1 , R_2 and R_3 ; one on the top right with components R_2 , R_4 , and R_6 ; and one on the bottom right with components R_3 , R_5 , and R_7 . We can write KVL equations for all three meshes, giving a total of **three** independent KVL equations.

[e] $-v_s + R_1 i_1 + R_2 i_2 + R_3 i_3 = 0$

$$R_4 i_4 + R_6 i_4 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 + R_7 i_5 = 0$$

P 4.4



[a] At node a: $-i_1 + i_2 + i_4 = 0$

At node b: $-i_2 + i_3 - i_4 - i_5 = 0$

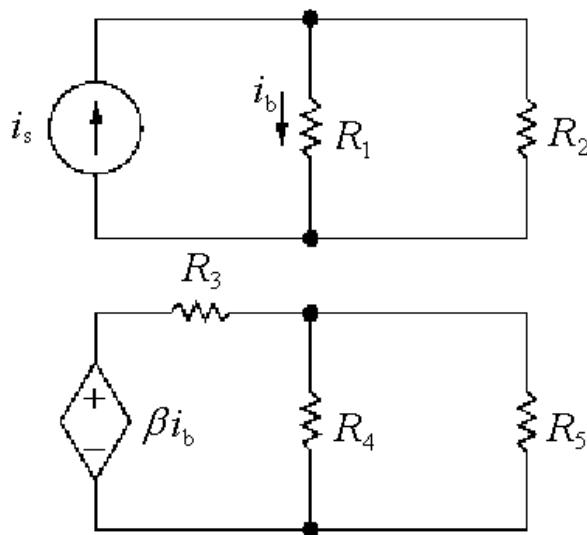
At node c: $i_1 - i_3 + i_5 = 0$

[b] There are many possible solutions. For example, adding the equations at nodes a and c gives the equation at node b:

$$(-i_1 + i_2 + i_4) + (i_1 - i_3 + i_5) = 0 \quad \text{so} \quad i_2 - i_3 + i_4 + i_5 = 0$$

This is the equation at node b with both sides multiplied by -1 .

P 4.5



- [a] As can be seen from the figure, the circuit has 2 separate parts.
- [b] There are 5 nodes – the four black dots and the node between the voltage source and the resistor R_3 .
- [c] There are 7 branches, each containing one of the seven circuit components.
- [d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.6 Use the lower terminal of the 25Ω resistor as the reference node.

$$\frac{v_o - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0$$

Solving, $v_o = 4 \text{ V}$

P 4.7 [a] From the solution to Problem 4.6 we know $v_o = 4 \text{ V}$, therefore

$$p_{40\text{mA}} = 0.04v_o = 0.16 \text{ W}$$

$$\therefore p_{40\text{mA}} (\text{developed}) = -160 \text{ mW}$$

- [b] The current into the negative terminal of the 24 V source is

$$i_g = \frac{24 - 4}{20 + 80} = 0.2 \text{ A}$$

$$p_{24\text{V}} = -24(0.2) = -4.8 \text{ W}$$

$$\therefore p_{24\text{V}} (\text{developed}) = 4800 \text{ mW}$$

[c] $p_{20\Omega} = (0.2)^2(20) = 800 \text{ mW}$

$$p_{80\Omega} = (0.2)^2(80) = 3200 \text{ mW}$$

$$p_{25\Omega} = (4)^2/25 = 640 \text{ mW}$$

$$\sum p_{\text{dev}} = 4800 \text{ mW}$$

$$\sum p_{\text{dis}} = 160 + 800 + 3200 + 640 = 4800 \text{ mW}$$

P 4.8 [a] $\frac{v_0 - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0; \quad v_o = 4 \text{ V}$

[b] Let v_x = voltage drop across 40 mA source

$$v_x = v_o - (50)(0.04) = 2 \text{ V}$$

$$p_{40\text{mA}} = (2)(0.04) = 80 \text{ mW} \quad \text{so} \quad p_{40\text{mA}} \text{ (developed)} = -80 \text{ mW}$$

[c] Let i_g = be the current into the positive terminal of the 24 V source

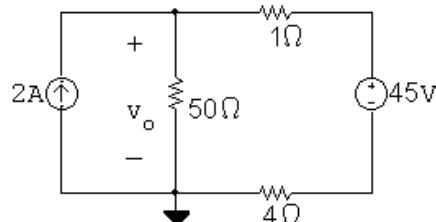
$$i_g = (4 - 24)/100 = -0.2 \text{ A}$$

$$p_{24\text{V}} = (-0.2)(24) = -4800 \text{ mW} \quad \text{so} \quad p_{24\text{V}} \text{ (developed)} = 4800 \text{ mW}$$

[d] $\sum p_{\text{dis}} = (0.2)^2(20) + (0.2)^2(80) + (4)^2/25 + (0.04)^2(50) + 0.08$
 $= 4800 \text{ mW}$

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.9



$$-2 + \frac{v_o}{50} + \frac{v_o - 45}{1 + 4} = 0$$

$$v_o = 50 \text{ V}$$

$$p_{2\text{A}} = -(50)(2) = -100 \text{ W} \quad (\text{delivering})$$

The 2 A source extracts -100 W from the circuit, because it delivers 100 W to the circuit.

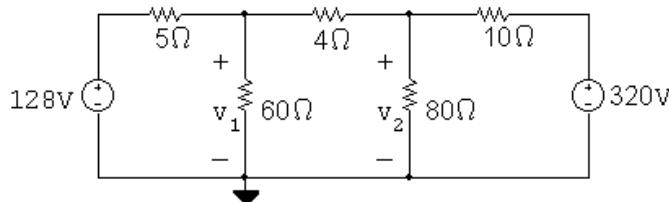
P 4.10 [a] $\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$

$$\therefore nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n} \sum_{k=1}^n v_k$$

[b] $v_o = \frac{1}{3}(100 + 80 - 60) = 40 \text{ V}$

P 4.11 [a]



$$\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} = 0$$

$$\frac{v_2 - v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} = 0$$

In standard form,

$$v_1 \left(\frac{1}{5} + \frac{1}{60} + \frac{1}{4} \right) + v_2 \left(-\frac{1}{4} \right) = \frac{128}{5}$$

$$v_1 \left(-\frac{1}{4} \right) + v_2 \left(\frac{1}{4} + \frac{1}{80} + \frac{1}{10} \right) = \frac{320}{10}$$

Solving, $v_1 = 162 \text{ V}$; $v_2 = 200 \text{ V}$

$$i_a = \frac{128 - 162}{5} = -6.8 \text{ A}$$

$$i_b = \frac{162}{60} = 2.7 \text{ A}$$

$$i_c = \frac{162 - 200}{4} = -9.5 \text{ A}$$

$$i_d = \frac{200}{80} = 2.5 \text{ A}$$

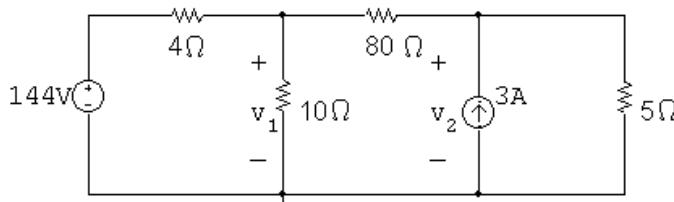
$$i_e = \frac{200 - 320}{10} = -12 \text{ A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4 \text{ W (abs)}$

$$p_{320V} = (320)(-12) = -3840 \text{ W (dev)}$$

Therefore, the total power developed is 3840 W.

P 4.12



$$\begin{aligned} \frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} &= 0 & \text{so} & \quad 29v_1 - v_2 = 2880 \\ -3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} &= 0 & \text{so} & \quad -v_1 + 17v_2 = 240 \end{aligned}$$

Solving, $v_1 = 100 \text{ V}; \quad v_2 = 20 \text{ V}$

P 4.13 $-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120 \text{ V}; \quad v_2 = 96 \text{ V}$

CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360 \text{ W}$$

$$p_{8\Omega} = \frac{(120 - 96)^2}{8} = 72 \text{ W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2 \text{ W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8 \text{ W}$$

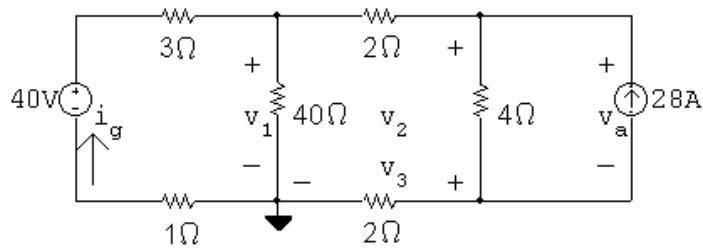
$$p_{6A} = -(6)(120) = -720 \text{ W}$$

$$p_{1A} = (1)(96) = 96 \text{ W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720 \text{ W}$$

$$\sum p_{\text{dev}} = 720 \text{ W} \quad (\text{CHECKS})$$

P 4.14 [a]



$$\frac{v_1}{40} + \frac{v_1 - 40}{4} + \frac{v_1 - v_2}{2} = 0 \quad \text{so} \quad 31v_1 - 20v_2 + 0v_3 = 400$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0 \quad \text{so} \quad -2v_1 + 3v_2 - v_3 = 112$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0 \quad \text{so} \quad 0v_1 - v_2 + 3v_3 = -112$$

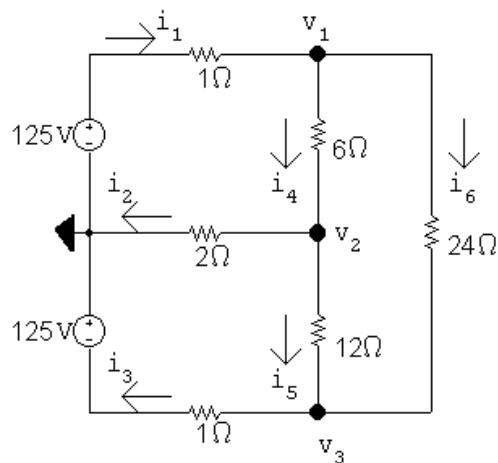
Solving, $v_1 = 60\text{ V}$; $v_2 = 73\text{ V}$; $v_3 = -13\text{ V}$,

[b] $i_g = \frac{40 - 60}{4} = -5\text{ A}$

$$p_g = (40)(-5) = -200\text{ W}$$

Thus the 40 V source delivers 200 W of power.

P 4.15 [a]



$$\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} = 0$$

$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

In standard form:

$$\begin{aligned} v_1 \left(\frac{1}{1} + \frac{1}{6} + \frac{1}{24} \right) + v_2 \left(-\frac{1}{6} \right) + v_3 \left(-\frac{1}{24} \right) &= 125 \\ v_1 \left(-\frac{1}{6} \right) + v_2 \left(\frac{1}{6} + \frac{1}{2} + \frac{1}{12} \right) + v_3 \left(-\frac{1}{12} \right) &= 0 \\ v_1 \left(-\frac{1}{24} \right) + v_2 \left(-\frac{1}{12} \right) + v_3 \left(\frac{1}{1} + \frac{1}{12} + \frac{1}{24} \right) &= -125 \end{aligned}$$

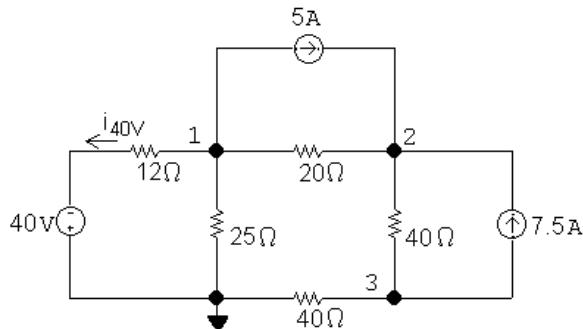
Solving, $v_1 = 101.24 \text{ V}$; $v_2 = 10.66 \text{ V}$; $v_3 = -106.57 \text{ V}$

$$\begin{aligned} \text{Thus, } i_1 &= \frac{125 - v_1}{1} = 23.76 \text{ A} & i_4 &= \frac{v_1 - v_2}{6} = 15.10 \text{ A} \\ i_2 &= \frac{v_2}{2} = 5.33 \text{ A} & i_5 &= \frac{v_2 - v_3}{12} = 9.77 \text{ A} \\ i_3 &= \frac{v_3 + 125}{1} = 18.43 \text{ A} & i_6 &= \frac{v_1 - v_3}{24} = 8.66 \text{ A} \end{aligned}$$

[b] $\sum P_{\text{dev}} = 125i_1 + 125i_3 = 5273.09 \text{ W}$

$$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09 \text{ W}$$

P 4.16



$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20} \right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

$$\text{Solving, } v_1 = -10 \text{ V}; \quad v_2 = 132 \text{ V}; \quad v_3 = -84 \text{ V}; \quad i_{40V} = \frac{-10 + 40}{12} = 2.5 \text{ A}$$

$$p_{5A} = 5(v_1 - v_2) = 5(-10 - 132) = -710 \text{ W} \quad (\text{del})$$

$$p_{7.5\Omega} = (-84 - 132)(7.5) = -1620 \text{ W} \quad (\text{del})$$

$$p_{40\Omega} = -(40)(2.5) = -100 \text{ W} \quad (\text{del})$$

$$p_{12\Omega} = (2.5)^2(12) = 75 \text{ W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4 \text{ W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2 \text{ W}$$

$$p_{40\Omega}(\text{lower}) = \frac{(v_3)^2}{40} = \frac{84^2}{40} = 176.4 \text{ W}$$

$$p_{40\Omega}(\text{right}) = \frac{(v_2 - v_3)^2}{40} = \frac{216^2}{40} = 1166.4 \text{ W}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 176.4 + 1166.4 = 2430 \text{ W}$$

$$\sum p_{\text{dev}} = 710 + 1620 + 100 = 2430 \text{ W} \quad (\text{CHECKS})$$

The total power dissipated in the circuit is 2430 W.

$$\text{P 4.17 } -3 + \frac{v_o}{200} + \frac{v_o + 5i_\Delta}{10} + \frac{v_o - 80}{20} = 0; \quad i_\Delta = \frac{v_o - 80}{20}$$

[a] Solving, $v_o = 50 \text{ V}$

$$[b] i_{\text{ds}} = \frac{v_o + 5i_\Delta}{10}$$

$$i_\Delta = (50 - 80)/20 = -1.5 \text{ A}$$

$$\therefore i_{\text{ds}} = 4.25 \text{ A}; \quad 5i_\Delta = -7.5 \text{ V} : \quad p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875 \text{ W}$$

$$[c] p_{3\Omega} = -3v_o = -3(50) = -150 \text{ W} \quad (\text{del})$$

$$p_{80\Omega} = 80i_\Delta = 80(-1.5) = -120 \text{ W} \quad (\text{del})$$

$$\sum p_{\text{del}} = 150 + 120 = 270 \text{ W}$$

CHECK:

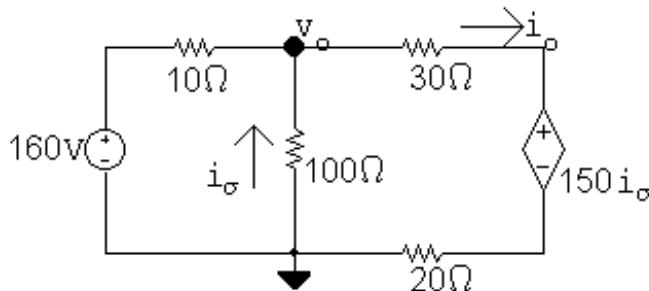
$$p_{200\Omega} = 2500/200 = 12.5 \text{ W}$$

$$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45 \text{ W}$$

$$p_{10\Omega} = (4.25)^2(10) = 180.625 \text{ W}$$

$$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270 \text{ W}$$

P 4.18



$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{50} = 0; \quad i_\sigma = -\frac{v_o}{100}$$

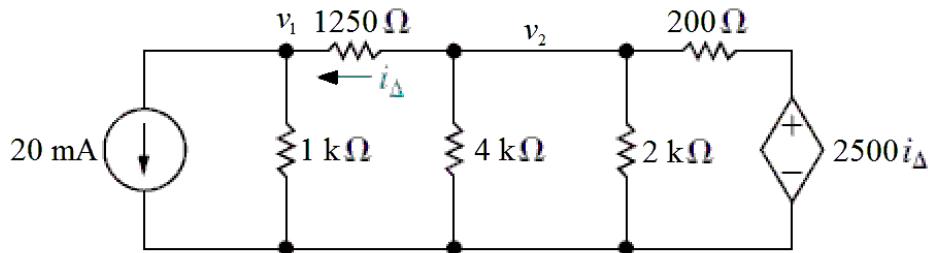
$$\text{Solving, } v_o = 100 \text{ V}; \quad i_\sigma = -1 \text{ A}$$

$$i_o = \frac{100 - (150)(-1)}{50} = 5 \text{ A}$$

$$p_{150i_\sigma} = 150i_\sigma i_o = -750 \text{ W}$$

. . . The dependent voltage source delivers 750 W to the circuit.

P 4.19



$$[\text{a}] \quad 0.02 + \frac{v_1}{1000} + \frac{v_1 - v_2}{1250} = 0$$

$$\frac{v_2 - v_1}{1250} + \frac{v_2}{4000} + \frac{v_2}{2000} + \frac{v_2 - 2500i_\Delta}{200} = 0$$

$$i_\Delta = \frac{v_2 - v_1}{1250}$$

Solving,

$$v_1 = 60 \text{ V}; \quad v_2 = 160 \text{ V}; \quad i_\Delta = 80 \text{ mA}$$

$$P_{20\text{mA}} = (0.02)v_1 = (0.02)(60) = 1.2 \text{ W} \text{ (absorbed)}$$

$$i_{ds} = \frac{v_2 - 2500i_\Delta}{200} = \frac{160 - (2500)(0.08)}{200} = -0.2 \text{ A}$$

$$P_{ds} = (2500i_\Delta)i_{ds} = 2500(0.08)(-0.2) = -40 \text{ W} \text{ (40 W developed)}$$

$$P_{\text{developed}} = 40 \text{ W}$$

$$[b] P_{1k} = \frac{v_1^2}{1000} = \frac{60^2}{1000} = 3.6 \text{ W}$$

$$P_{1250} = 1250i_\Delta^2 = 1250(0.08)^2 = 8 \text{ W}$$

$$P_{4k} = \frac{v_2^2}{4000} = \frac{160^2}{4000} = 6.4 \text{ W}$$

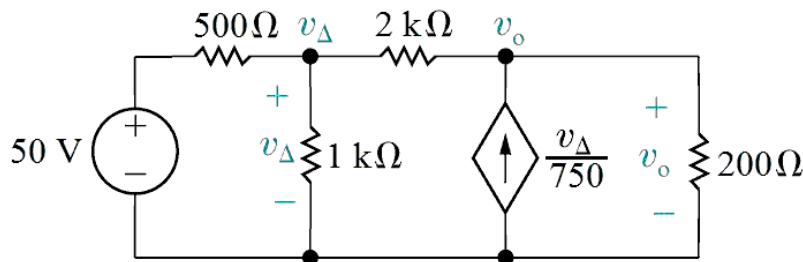
$$P_{2k} = \frac{v_2^2}{2000} = \frac{160^2}{2000} = 12.8 \text{ W}$$

$$P_{200} = 200i_{ds}^2 = 200(-0.2)^2 = 8 \text{ W}$$

$$P_{\text{absorbed}} = P_{20mA} + P_{1k} + P_{1250} + P_{4k} + P_{2k} + P_{200}$$

$$= 1.2 + 3.6 + 8 + 6.4 + 12.8 + 8 = 40 \text{ W} \text{ (check)}$$

P 4.20



$$[a] \frac{v_\Delta - 50}{500} + \frac{v_\Delta}{1000} + \frac{v_\Delta - v_o}{2000} = 0$$

$$\frac{v_o - v_\Delta}{2000} - \frac{v_\Delta}{750} + \frac{v_o}{200} = 0$$

Solving,

$$v_\Delta = 30 \text{ V}; \quad v_o = 10 \text{ V}$$

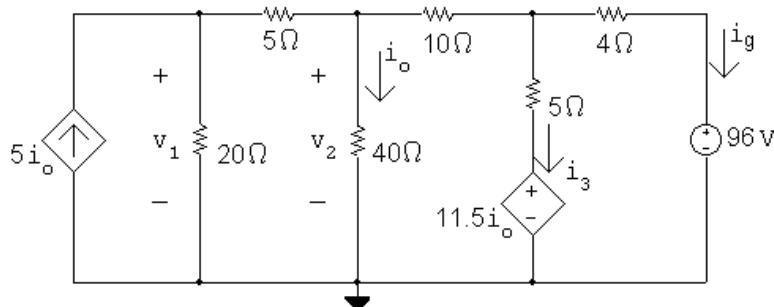
$$[b] i_{50V} = \frac{v_\Delta - 50}{500} = \frac{30 - 50}{500} = -0.04 \text{ A}$$

$$P_{50V} = 50i_{50V} = 50(-0.04) = -2 \text{ W} \quad (2 \text{ W supplied})$$

$$P_{ds} = -v_o \left(\frac{v_\Delta}{750} \right) = -(10)(30/750) = -0.4 \text{ W} \quad (0.4 \text{ W supplied})$$

$$P_{\text{total}} = 2 + 0.4 = 2.4 \text{ W supplied}$$

P 4.21 [a]



$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \quad \text{so} \quad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \quad \text{so} \quad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \quad \text{so} \quad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156 \text{ V}$; $v_2 = 120 \text{ V}$; $v_3 = 78 \text{ V}$

$$[b] \quad i_o = \frac{v_2}{40} = \frac{120}{40} = 3 \text{ A}$$

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7 \text{ A}$$

$$i_g = \frac{78 - 96}{4} = -4.5 \text{ A}$$

$$p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340 \text{ W (dev)}$$

$$p_{11.5i_o} = 11.5i_o i_3 = 11.5(3)(8.7) = 300.15 \text{ W (abs)}$$

$$p_{96V} = 96(-4.5) = -432 \text{ W (dev)}$$

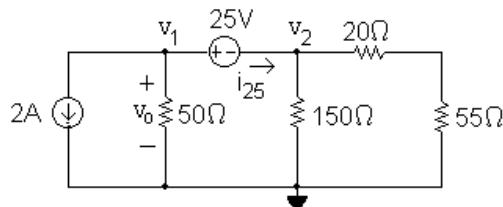
$$\sum p_{\text{dev}} = 2340 + 432 = 2772 \text{ W}$$

CHECK

$$\begin{aligned} \sum p_{\text{dis}} &= \frac{156^2}{20} + \frac{(156 - 120)^2}{5} + \frac{120^2}{40} + \frac{(120 - 78)^2}{50} \\ &\quad + (8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772 \text{ W} \end{aligned}$$

$$\therefore \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772 \text{ W}$$

P 4.22 [a]



This circuit has a supernode includes the nodes v_1 , v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$$

The supernode constraint equation is

$$v_1 - v_2 = 25$$

Place these two equations in standard form:

$$v_1 \left(\frac{1}{50} \right) + v_2 \left(\frac{1}{150} + \frac{1}{75} \right) = -2$$

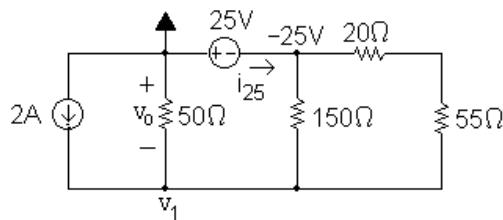
$$v_1(1) + v_2(-1) = 25$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V, so $v_o = v_1 = -37.5$ V.

$$p_{2A} = (2)v_o = (2)(-37.5) = -75 \text{ W}$$

The 2 A source delivers 75 W.

[b]



This circuit now has only one non-reference essential node where the voltage is not known – note that it is not a supernode. The KCL equation at v_1 is

$$-2 + \frac{v_1}{50} + \frac{v_1 + 25}{150} + \frac{v_1 + 25}{75} = 0$$

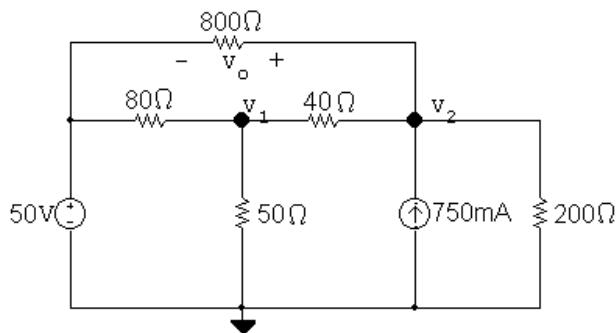
Solving, $v_1 = 37.5$ V so $v_0 = -v_1 = -37.5$ V.

$$p_{2A} = (2)v_o = (2)(-37.5) = -75 \text{ W}$$

The 2 A source delivers 75 W.

- [c] The choice of a reference node in part (b) resulted in one simple KCL equation, while the choice of a reference node in part (a) resulted in a supernode KCL equation and a second supernode constraint equation. Both methods give the same result but the choice of reference node in part (b) yielded fewer equations to solve, so is the preferred method.

P 4.23



The two node voltage equations are:

$$\begin{aligned}\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} &= 0 \\ \frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} &= 0\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}v_1 \left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40} \right) + v_2 \left(-\frac{1}{40} \right) &= \frac{50}{80} \\ v_1 \left(-\frac{1}{40} \right) + v_2 \left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800} \right) &= 0.75 + \frac{50}{800}\end{aligned}$$

Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.

Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.

POWER CHECK:

$$i_g = (50 - 34)/80 + (50 - 53.2)/800 = 196 \text{ mA}$$

$$p_{50V} = -(50)(0.196) = -9.8 \text{ W}$$

$$p_{80\Omega} = (50 - 34)^2/80 = 3.2 \text{ W}$$

$$p_{800\Omega} = (50 - 53.2)^2/800 = 12.8 \text{ mW}$$

$$p_{40\Omega} = (53.2 - 34)^2/40 = 9.216 \text{ W}$$

$$p_{50\Omega} = 34^2/50 = 23.12 \text{ W}$$

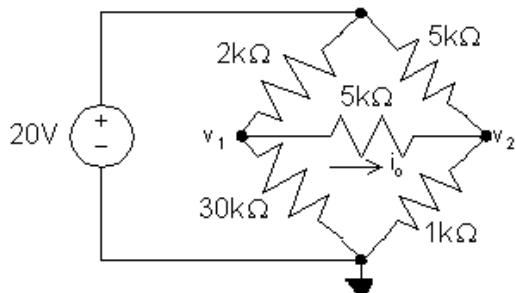
$$p_{200\Omega} = 53.2^2/200 = 14.1512 \text{ W}$$

$$p_{0.75A} = -(53.2)(0.75) = -39.9 \text{ W}$$

$$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7 \text{ W}$$

$$\sum p_{\text{del}} = 9.8 + 39.9 = 49.7 \text{ (check)}$$

P 4.24



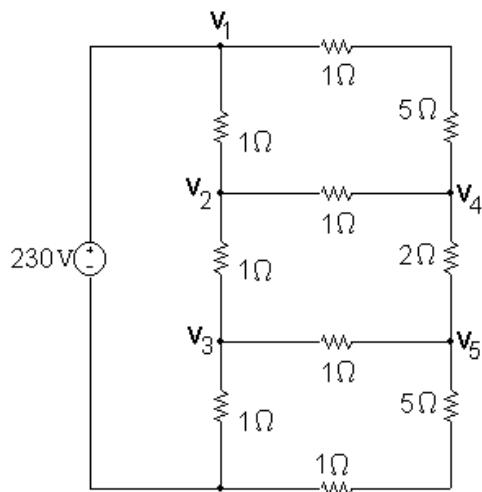
$$\frac{v_1}{30,000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \quad \text{so} \quad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \quad \text{so} \quad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15 \text{ V}$; $v_2 = 5 \text{ V}$

$$\text{Thus, } i_o = \frac{v_1 - v_2}{5000} = 2 \text{ mA}$$

P 4.25 [a]



$$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0 \quad \text{so} \quad 3v_2 - 1v_3 - 1v_4 + 0v_5 = 230$$

$$\frac{v_3 - v_2}{1} + \frac{v_3}{1} + \frac{v_3 - v_5}{1} = 0 \quad \text{so} \quad -1v_2 + 3v_3 + 0v_4 - 1v_5 = 0$$

$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0 \quad \text{so} \quad -12v_2 + 0v_3 + 20v_4 - 6v_5 = 460$$

$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0 \quad \text{so} \quad 0v_2 - 12v_3 - 6v_4 + 20v_5 = 0$$

Solving, $v_2 = 150 \text{ V}$; $v_3 = 80 \text{ V}$; $v_4 = 140 \text{ V}$; $v_5 = 90 \text{ V}$

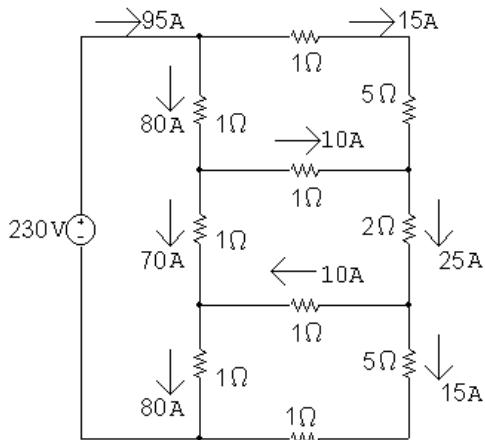
$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25 \text{ A}$$

$$P_{2\Omega} = (25)^2(2) = 1250 \text{ W}$$

$$\begin{aligned} [\mathbf{b}] \quad i_{230V} &= \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6} \\ &= \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95 \text{ A} \end{aligned}$$

$$P_{230V} = (230)(95) = 21,850 \text{ W}$$

Check:



$$\begin{aligned} \sum P_{\text{dis}} &= (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1) \\ &\quad + (10)^2(1) + (25)^2(2) + (15)^2(6) = 21,850 \text{ W} \end{aligned}$$

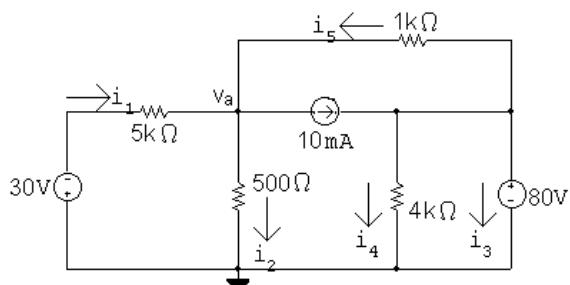
P 4.26 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta - 15}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 5v_\Delta}{20} + \frac{v_\Delta - 5v_\Delta}{40} = 0$$

$$12v_\Delta = 60; \quad v_\Delta = 5 \text{ V}$$

$$v_o = v_\Delta - 5v_\Delta = -4(5) = -20 \text{ V}$$

P 4.27 [a]



There is only one node voltage equation:

$$\frac{v_a + 30}{5000} + \frac{v_a}{500} + \frac{v_a - 80}{1000} + 0.01 = 0$$

Solving,

$$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0 \quad \text{so} \quad 16v_a = 320$$

$$\therefore v_a = 20 \text{ V}$$

Calculate the currents:

$$i_1 = (-30 - 20)/5000 = -10 \text{ mA}$$

$$i_2 = 20/500 = 40 \text{ mA}$$

$$i_4 = 80/4000 = 20 \text{ mA}$$

$$i_5 = (80 - 20)/1000 = 60 \text{ mA}$$

$$i_3 + i_4 + i_5 - 10 \text{ mA} = 0 \quad \text{so} \quad i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70 \text{ mA}$$

[b] $p_{30V} = (30)(-0.01) = -0.3 \text{ W}$

$$p_{10mA} = (20 - 80)(0.01) = -0.6 \text{ W}$$

$$p_{80V} = (80)(-0.07) = -5.6 \text{ W}$$

$$p_{5k} = (-0.01)^2(5000) = 0.5 \text{ W}$$

$$p_{500\Omega} = (0.04)^2(500) = 0.8 \text{ W}$$

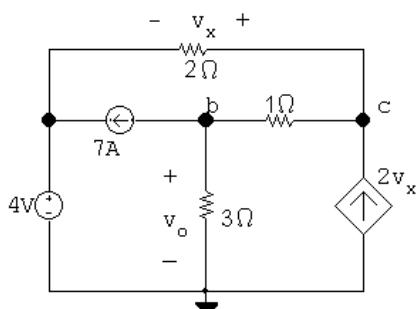
$$p_{1k} = (80 - 20)^2/(1000) = 3.6 \text{ W}$$

$$p_{4k} = (80)^2/(4000) = 1.6 \text{ W}$$

$$\sum p_{\text{abs}} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5 \text{ W}$$

$$\sum p_{\text{del}} = 0.3 + 0.6 + 5.6 = 6.5 \text{ W} \text{ (checks!)}$$

P 4.28



The two node voltage equations are:

$$7 + \frac{v_b}{3} + \frac{v_b - v_c}{1} = 0$$

$$-2v_x + \frac{v_c - v_b}{1} + \frac{v_c - 4}{2} = 0$$

The constraint equation for the dependent source is:

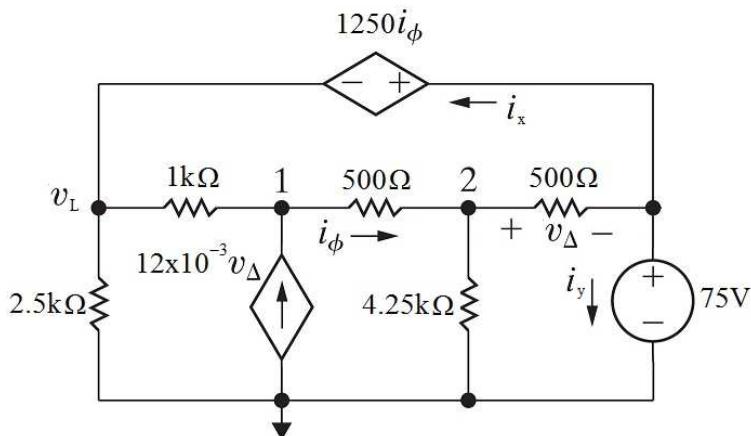
$$v_x = v_c - 4$$

Place these equations in standard form:

$$\begin{aligned} v_b \left(\frac{1}{3} + 1 \right) + v_c(-1) + v_x(0) &= -7 \\ v_b(-1) + v_c \left(1 + \frac{1}{2} \right) + v_x(-2) &= \frac{4}{2} \\ v_b(0) + v_c(1) + v_x(-1) &= 4 \end{aligned}$$

Solving, $v_c = 9\text{ V}$, $v_x = 5\text{ V}$, and $v_o = v_b = 1.5\text{ V}$

P 4.29



- [a] The left-most node voltage is $75 - 1250i_\phi$. The right-most node voltage is 75 V. Write KCL equations at the essential nodes labeled 1 and 2.
- [b] From the values given,

$$i_\phi = \frac{v_1 - v_2}{500} = \frac{105 - 85}{500} = 0.04 \text{ A}$$

$$v_\Delta = v_2 - 75 = 85 - 75 = 10 \text{ V}$$

$$v_L = 75 - (1250)(0.04) = 25 \text{ V}$$

$$i_x = \frac{v_L - v_1}{1000} + \frac{v_L}{2500} = \frac{25 - 105}{1000} + \frac{25}{2500} = -0.07 \text{ A}$$

$$i_y = \frac{v_2 - 75}{500} - i_x = \frac{85 - 75}{500} + 0.07 = 0.09 \text{ A}$$

Calculate the total power:

$$P_{\text{dstop}} = 1250i_\phi(i_x) = 1250(0.04)(-0.07) = -3.5 \text{ W}$$

$$P_{\text{dsbot}} = -v_1(12 \times 10^{-3}v_\Delta) = -(105)(12 \times 10^{-3})(10) = -12.6 \text{ W}$$

$$P_{75\text{V}} = 75i_y = 75(0.09) = 6.75 \text{ W}$$

$$P_{1k} = \frac{(v_L - v_1)^2}{1000} = \frac{(25 - 105)^2}{1000} = 6.4 \text{ W}$$

$$P_{2.5k} = \frac{v_L^2}{2500} = \frac{25^2}{2500} = 0.25 \text{ W}$$

$$P_{500\text{mid}} = 500i_\phi^2 = 500(0.04)^2 = 0.8 \text{ W}$$

$$P_{500\text{right}} = \frac{v_\Delta^2}{500} = \frac{10^2}{500} = 0.2 \text{ W}$$

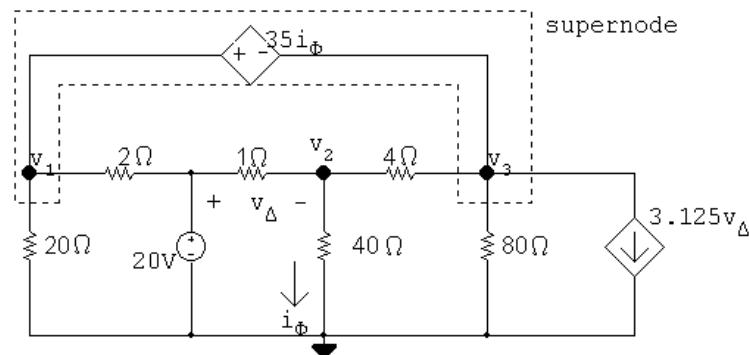
$$P_{4.25k} = \frac{v_2^2}{4250} = \frac{85^2}{4250} = 1.7 \text{ W}$$

[c] $P_{\text{supplied}} = 3.5 + 12.6 = 16.1 \text{ W}$

$$P_{\text{absorbed}} = 6.75 + 6.4 + 0.25 + 0.8 + 0.2 + 1.7 = 16.1 \text{ W} = P_{\text{supplied}}$$

Therefore the analyst is correct.

P 4.30



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35i_\phi = v_3$$

$$i_\phi = v_2/40$$

Solving, $v_1 = -20.25 \text{ V}$; $v_2 = 10 \text{ V}$; $v_3 = -29 \text{ V}$

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

$$p_g \text{ (delivered)} = 20(30.125) = 602.5 \text{ W}$$

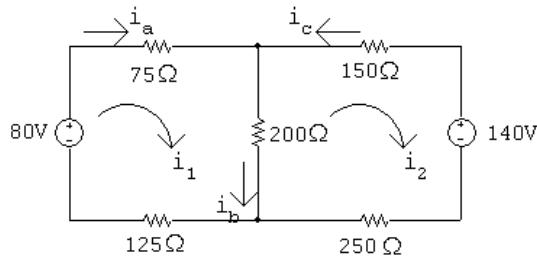
P 4.31 From Eq. 4.16, $i_B = v_c/(1 + \beta)R_E$

From Eq. 4.17, $i_B = (v_b - V_o)/(1 + \beta)R_E$

From Eq. 4.19,

$$\begin{aligned} i_B &= \frac{1}{(1 + \beta)R_E} \left[\frac{V_{CC}(1 + \beta)R_E R_2 + V_o R_1 R_2}{R_1 R_2 + (1 + \beta)R_E(R_1 + R_2)} - V_o \right] \\ &= \frac{V_{CC} R_2 - V_o (R_1 + R_2)}{R_1 R_2 + (1 + \beta)R_E(R_1 + R_2)} = \frac{[V_{CC} R_2 / (R_1 + R_2)] - V_o}{[R_1 R_2 / (R_1 + R_2)] + (1 + \beta)R_E} \end{aligned}$$

P 4.32 [a]



$$80 = 400i_1 - 200i_2$$

$$-140 = -200i_1 + 600i_2$$

$$\text{Solving, } i_1 = 0.1 \text{ A}; \quad i_2 = -0.2 \text{ A}$$

$$i_a = i_1 = 0.1 \text{ A}; \quad i_b = i_1 - i_2 = 0.3 \text{ A}; \quad i_c = -i_2 = 0.2 \text{ A}$$

[b] If the polarity of the 140 V source is reversed, we have

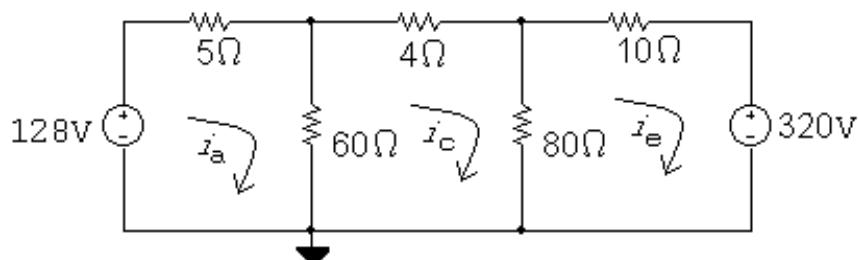
$$80 = 400i_1 - 200i_2$$

$$140 = -200i_1 + 600i_2$$

$$i_1 = 0.38 \text{ A} \quad \text{and} \quad i_2 = 0.36 \text{ A}$$

$$i_a = i_1 = 0.38 \text{ A}; \quad i_b = i_1 - i_2 = 0.02 \text{ A}; \quad i_c = -i_2 = -0.36 \text{ A}$$

P 4.33 [a]



The three mesh current equations are:

$$-128 + 5i_a + 60(i_a - i_c) = 0$$

$$4i_c + 80(i_c - i_e) + 60(i_c - i_a) = 0$$

$$320 + 80(i_e - i_c) + 10i_e = 0$$

Place these equations in standard form:

$$i_a(5 + 60) + i_c(-60) + i_e(0) = 128$$

$$i_a(-60) + i_c(4 + 80 + 60) + i_e(-80) = 0$$

$$i_a(0) + i_c(-80) + i_e(80 + 10) = -320$$

Solving, $i_a = -6.8 \text{ A}$; $i_c = -9.5 \text{ A}$; $i_e = -12 \text{ A}$

Now calculate the remaining branch currents:

$$i_b = i_a - i_c = 2.7 \text{ A}$$

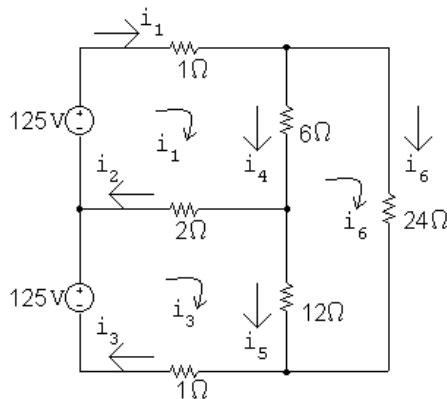
$$i_d = i_v - i_e = 2.5 \text{ A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4 \text{ W (abs)}$

$$p_{320V} = (320)(-12) = -3840 \text{ W (dev)}$$

Therefore, the total power developed is 3840 W.

P 4.34 [a]



The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$i_1(1 + 6 + 2) + i_3(-2) + i_6(-6) = 125$$

$$i_1(-6) + i_3(-12) + i_6(24 + 12 + 6) = 0$$

$$i_1(-2) + i_3(2 + 12 + 1) + i_6(-12) = 125$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A

Now calculate the remaining branch currents:

$$i_2 = i_1 - i_3 = 5.33 \text{ A}$$

$$i_4 = i_1 - i_6 = 15.10 \text{ A}$$

$$i_5 = i_3 - i_6 = 9.77 \text{ A}$$

$$[b] p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$$

$$= -2969.58 - 2303.51 = -5273 \text{ W}$$

Thus, the power developed in the circuit is 5273 W.

Now calculate the power absorbed by the resistors:

$$p_{1\text{top}} = (23.76)^2(1) = 564.39 \text{ W}$$

$$p_2 = (5.33)^2(2) = 56.79 \text{ W}$$

$$p_{1\text{bot}} = (18.43)^2(1) = 339.59 \text{ W}$$

$$p_6 = (15.10)^2(6) = 1367.64 \text{ W}$$

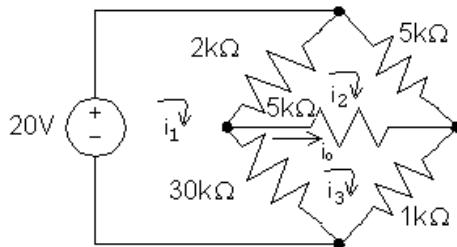
$$p_{12} = (9.77)^2(12) = 1145.22 \text{ W}$$

$$p_{24} = (8.66)^2(24) = 1799.47 \text{ W}$$

The power absorbed by the resistors is

$564.39 + 56.79 + 339.59 + 1367.64 + 1145.22 + 1799.47 = 5273 \text{ W}$ so the power balances.

P 4.35



The three mesh current equations are:

$$-20 + 2000(i_1 - i_2) + 30,000(i_1 - i_3) = 0$$

$$5000i_2 + 5000(i_2 - i_3) + 2000(i_2 - i_1) = 0$$

$$1000i_3 + 30,000(i_3 - i_1) + 5000(i_3 - i_2) = 0$$

Place these equations in standard form:

$$i_1(32,000) + i_2(-2000) + i_3(-30,000) = 20$$

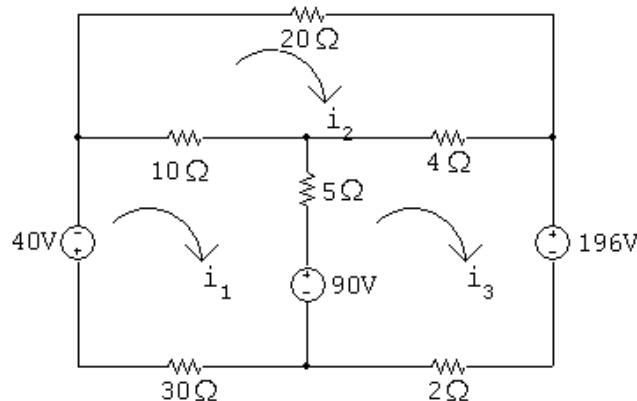
$$i_1(-2000) + i_2(12,000) + i_3(-5000) = 0$$

$$i_1(-30,000) + i_2(-5000) + i_3(36,000) = 0$$

Solving, $i_1 = 5.5 \text{ mA}$; $i_2 = 3 \text{ mA}$; $i_3 = 5 \text{ mA}$

Thus, $i_o = i_3 - i_2 = 2 \text{ mA}$.

P 4.36 [a]



$$40 + 10(i_1 - i_2) + 5(i_1 - i_3) + 90 + 30i_1 = 0$$

$$20i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$196 + 2i_3 - 90 + 5(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Solving, $i_1 = -5 \text{ A}$; $i_2 = -3 \text{ A}$; $i_3 = -13 \text{ A}$

$$p_{40} = 40i_1 = -200 \text{ W (del)}$$

$$p_{90} = 90(i_1 - i_3) = 720 \text{ W (abs)}$$

$$p_{196} = 196i_3 = -2548 \text{ W (del)}$$

$$\therefore \sum p_{\text{dev}} = 2748 \text{ W}$$

[b] $p_{20\Omega} = (-3)^2(20) = 180 \text{ W}$

$$p_{10\Omega} = (2)^2(10) = 40 \text{ W}$$

$$p_{4\Omega} = (10)^2(4) = 400 \text{ W}$$

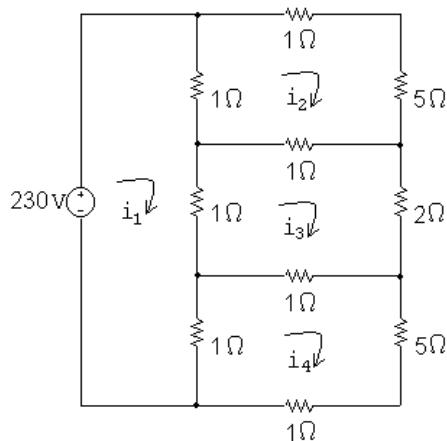
$$p_{5\Omega} = (8)^2(5) = 320 \text{ W}$$

$$p_{30\Omega} = (-5)^2(30) = 750 \text{ W}$$

$$p_{2\Omega} = (-13)^2(2) = 338 \text{ W}$$

$$\therefore \sum p_{\text{abs}} = 720 + 180 + 40 + 400 + 320 + 750 + 338 = 2748 \text{ W}$$

P 4.37 [a]



The four mesh current equations are:

$$-230 + 1(i_1 - i_2) + 1(i_1 - i_3) + 1(i_1 - i_4) = 0$$

$$6i_2 + 1(i_2 - i_3) + 1(i_2 - i_1) = 0$$

$$2i_3 + 1(i_3 - i_4) + 1(i_3 - i_1) + 1(i_3 - i_2) = 0$$

$$6i_4 + 1(i_4 - i_1) + 1(i_4 - i_3) = 0$$

Place these equations in standard form:

$$i_1(3) + i_2(-1) + i_3(-1) + i_4(-1) = 230$$

$$i_1(-1) + i_2(8) + i_3(-1) + i_4(0) = 0$$

$$i_1(-1) + i_2(-1) + i_3(5) + i_4(-1) = 0$$

$$i_1(-1) + i_2(0) + i_3(-1) + i_4(8) = 0$$

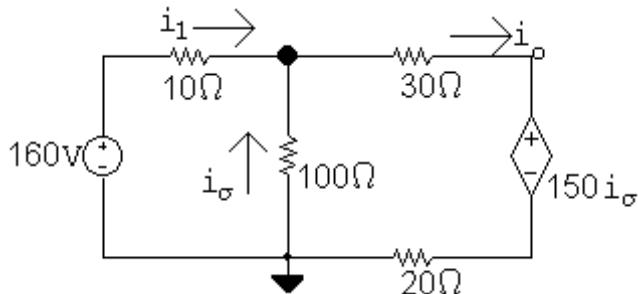
Solving, $i_1 = 95 \text{ A}$; $i_2 = 15 \text{ A}$; $i_3 = 25 \text{ A}$; $i_4 = 15 \text{ A}$

The power absorbed by the 5Ω resistor is

$$p_5 = i_3^2(2) = (25)^2(2) = 1250 \text{ W}$$

[b] $p_{230} = -(230)i_1 = -(230)(95) = -21,850 \text{ W}$

P 4.38



$$-160 + 10i_1 + 100(i_1 - i_o) = 0$$

$$30i_o + 150i_\sigma + 20i_o + 100i_\sigma = 0$$

$$i_\sigma = i_o - i_1$$

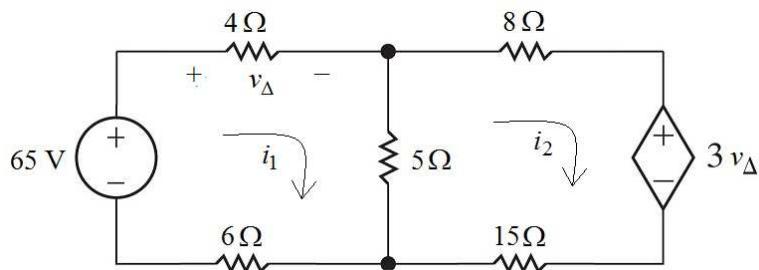
Solving,

$$i_1 = 6 \text{ A}; \quad i_o = 5 \text{ A}; \quad i_\sigma = -1 \text{ A}$$

$$P_{ds} = (150i_\sigma)i_o = 150(-1)(5) = -750 \text{ W}$$

Thus, 750 W is delivered by the dependent source.

P 4.39



$$-65 + 4i_1 + 5(i_1 - i_2) + 6i_1 = 0$$

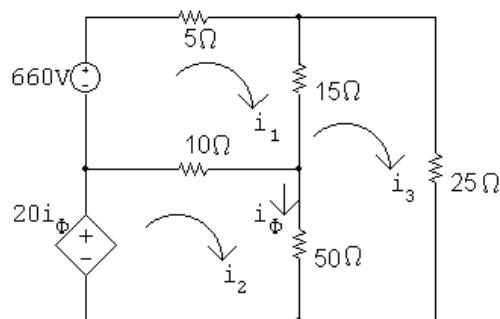
$$8i_2 + 3v_\Delta + 15i_2 + 5(i_2 - i_1) = 0$$

$$v_\Delta = 4i_1$$

$$\text{Solving, } i_1 = 4 \text{ A} \quad i_2 = -1 \text{ A} \quad v_\Delta = 16 \text{ V}$$

$$p_{15\Omega} = (-1)^2(15) = 15 \text{ W}$$

P 4.40



$$660 = 30i_1 - 10i_2 - 15i_3$$

$$20i_\Phi = -10i_1 + 60i_2 - 50i_3$$

$$0 = -15i_1 - 50i_2 + 90i_3$$

$$i_\Phi = i_2 - i_3$$

$$\text{Solving, } i_1 = 42 \text{ A}; \quad i_2 = 27 \text{ A}; \quad i_3 = 22 \text{ A}; \quad i_\Phi = 5 \text{ A}$$

$$20i_\Phi = 100 \text{ V}$$

$$p_{20i_\Phi} = -100i_2 = -100(27) = -2700 \text{ W}$$

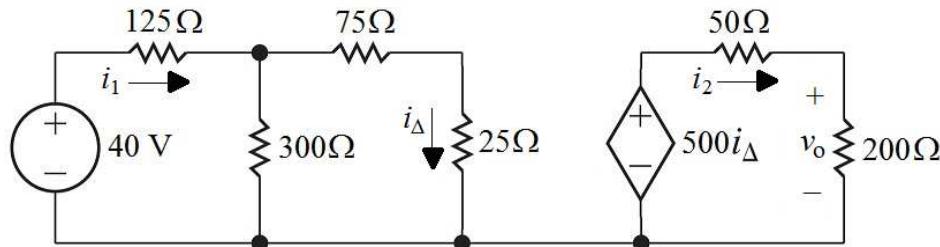
$$\therefore p_{20i_\Phi} \text{ (developed)} = 2700 \text{ W}$$

CHECK:

$$p_{660V} = -660(42) = -27,720 \text{ W (dev)}$$

$$\begin{aligned} \therefore \sum P_{\text{dev}} &= 27,720 + 2700 = 30,420 \text{ W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30,420 \text{ W} \end{aligned}$$

P 4.41 [a]



$$40 = 125i_1 + 300(i_1 - i_\Delta)$$

$$0 = 75i_\Delta + 25i_\Delta + 300(i_\Delta - i_1)$$

$$0 = 50i_2 + 200i_2 - 500i_\Delta$$

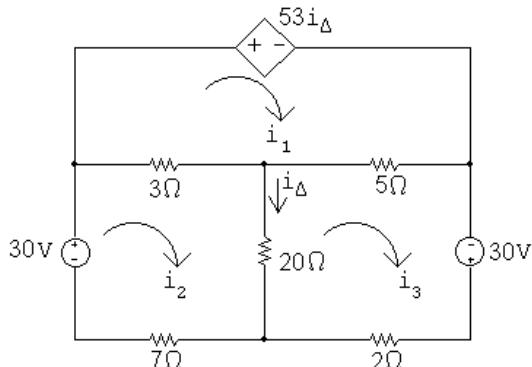
$$\text{Solving, } i_1 = 0.2 \text{ A; } i_2 = 0.3 \text{ A; } i_\Delta = 0.15 \text{ A}$$

$$v_o = 200i_2 = 200(0.3) = 60 \text{ V}$$

$$[\mathbf{b}] \quad p_{ds} = -(500i_\Delta)i_2 = -500(0.15)(0.3) = -22.5 \text{ W}$$

$$\therefore p_{ds} \text{ (delivered)} = 22.5 \text{ W}$$

P 4.42



Mesh equations:

$$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$$

$$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$$

$$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$$

Constraint equations:

$$i_\Delta = i_2 - i_3$$

Solving, $i_1 = 110 \text{ A}$; $i_2 = 52 \text{ A}$; $i_3 = 60 \text{ A}$; $i_\Delta = -8 \text{ A}$

$$p_{\text{depsource}} = 53i_\Delta i_1 = (53)(-8)(110) = -46,640 \text{ W}$$

Therefore, the dependent source is developing 46,640 W.

CHECK:

$$p_{30V} = -30i_2 = -1560 \text{ W} \text{ (left source)}$$

$$p_{30V} = -30i_3 = -1800 \text{ W} \text{ (right source)}$$

$$\sum p_{\text{dev}} = 46,640 + 1560 + 1800 = 50 \text{ kW}$$

$$p_{3\Omega} = (110 - 52)^2(3) = 10,092 \text{ W}$$

$$p_{5\Omega} = (110 - 60)^2(5) = 12,500 \text{ W}$$

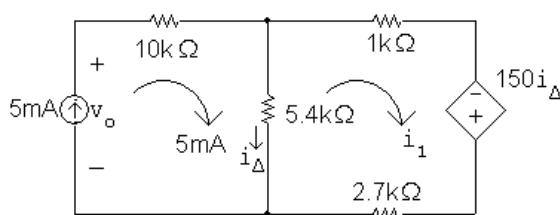
$$p_{20\Omega} = (-8)^2(20) = 1280 \text{ W}$$

$$p_{7\Omega} = (52)^2(7) = 18,928 \text{ W}$$

$$p_{2\Omega} = (60)^2(2) = 7200 \text{ W}$$

$$\sum p_{\text{diss}} = 10,092 + 12,500 + 1280 + 18,928 + 7200 = 50 \text{ kW}$$

P 4.43 [a]



The mesh current equation for the right mesh is:

$$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$$

$$\text{Solving, } 9250i_1 = 27.75 \quad \therefore i_1 = 3 \text{ mA}$$

$$\text{Then, } i_\Delta = 5 - 3 = 2 \text{ mA}$$

$$[b] \quad v_o = (0.005)(10,000) + (5400)(0.002) = 60.8 \text{ V}$$

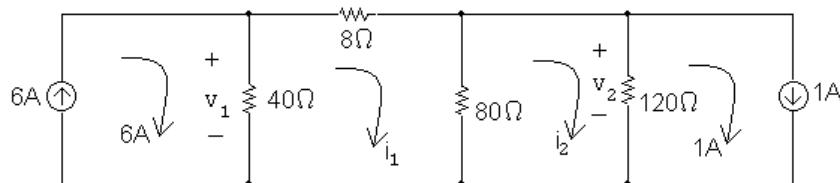
$$p_{5\text{mA}} = -(60.8)(0.005) = -304 \text{ mW}$$

Thus, the 5 mA source delivers 304 mW

$$[c] \quad p_{\text{dep source}} = -150i_\Delta i_1 = (-150)(0.002)(0.003) = -0.9 \text{ mW}$$

The dependent source delivers 0.9 mW.

P 4.44



Mesh equations:

$$128i_1 - 80i_2 = 240$$

$$-80i_1 + 200i_2 = 120$$

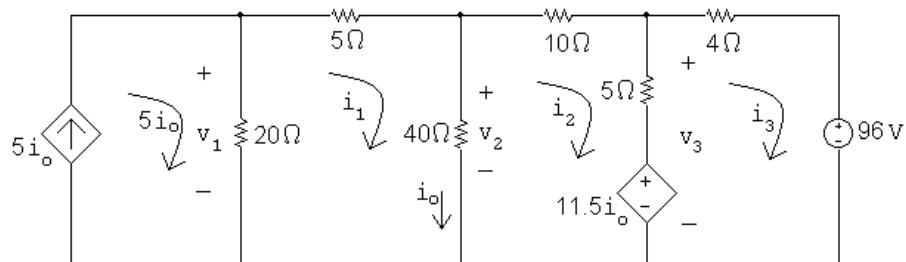
Solving,

$$i_1 = 3 \text{ A}; \quad i_2 = 1.8 \text{ A}$$

Therefore,

$$v_1 = 40(6 - 3) = 120 \text{ V}; \quad v_2 = 120(1.8 - 1) = 96 \text{ V}$$

P 4.45 [a]



Mesh equations:

$$65i_1 - 40i_2 + 0i_3 - 100i_o = 0$$

$$-40i_1 + 55i_2 - 5i_3 + 11.5i_o = 0$$

$$0i_1 - 5i_2 + 9i_3 - 11.5i_o = 0$$

$$-1i_1 + 1i_2 + 0i_3 + 1i_o = 0$$

Solving,

$$i_1 = 7.2 \text{ A}; \quad i_2 = 4.2 \text{ A}; \quad i_3 = -4.5 \text{ A}; \quad i_o = 3 \text{ A}$$

Therefore,

$$v_1 = 20[5(3) - 7.2] = 156 \text{ V}; \quad v_2 = 40(7.2 - 4.2) = 120 \text{ V}$$

$$v_3 = 5(4.2 + 4.5) + 11.5(3) = 78 \text{ V}$$

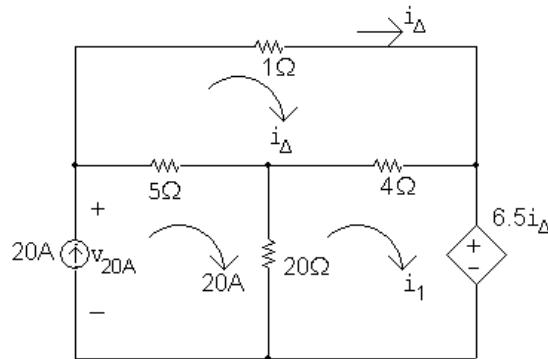
$$[b] \quad p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340 \text{ W}$$

$$p_{11.5i_o} = 11.5i_o(i_2 - i_3) = 11.5(3)(4.2 + 4.5) = 300.15 \text{ W}$$

$$p_{96V} = 96i_3 = 96(-4.5) = -432 \text{ W}$$

Thus, the total power dissipated in the circuit, which equals the total power developed in the circuit is $2340 + 432 = 2772 \text{ W}$.

P 4.46



Mesh equations:

$$10i_\Delta - 4i_1 = 0$$

$$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$$

$$\text{Solving, } i_1 = 15 \text{ A}; \quad i_\Delta = 16 \text{ A}$$

$$v_{20A} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120 \text{ V}$$

$$p_{20A} = -20v_{20A} = -(20)(120) = -2400 \text{ W (del)}$$

$$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560 \text{ W (abs)}$$

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.

CHECK:

$$p_{1\Omega} = (16)^2(1) = 256 \text{ W}$$

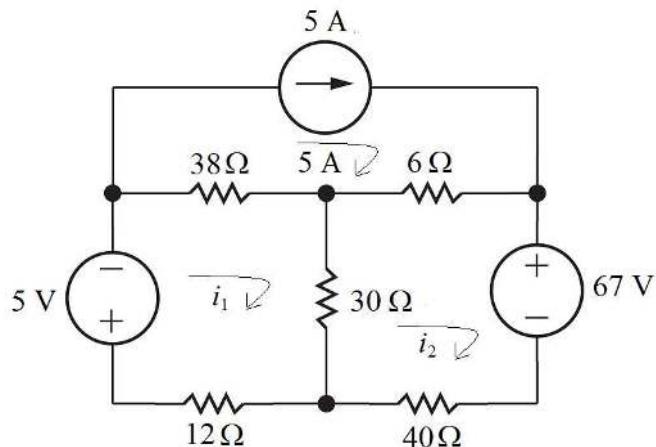
$$p_{5\Omega} = (20 - 16)^2(5) = 80 \text{ W}$$

$$p_{4\Omega} = (1)^2(4) = 4 \text{ W}$$

$$p_{20\Omega} = (20 - 15)^2(20) = 500 \text{ W}$$

$$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400 \text{ W (CHECKS)}$$

P 4.47



$$5 + 38(i_1 - 5) + 30(i_1 - i_2) + 12i_1 = 0$$

$$67 + 40i_2 + 30(i_2 - i_1) + 6(i_2 - 5) = 0$$

Solving, $i_1 = 2.5 \text{ A}$; $i_2 = 0.5 \text{ A}$

[a] $v_{5A} = 38(2.5 - 5) + 6(0.5 - 5)$

$$= -122 \text{ V}$$

$$p_{5A} = 5v_{5A} = 5(-122) = -610 \text{ W}$$

Therefore, the 5 A source delivers 610 W.

[b] $p_{5V} = 5(2.5) = 12.5 \text{ W}$

$$p_{67V} = 67(0.5) = 33.5 \text{ W}$$

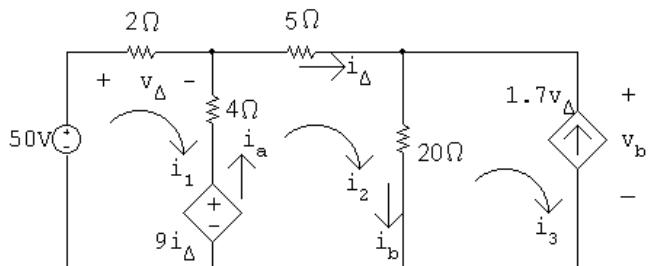
Therefore, only the current source delivers power and the total power delivered is 610 W.

[c] $\sum p_{\text{resistors}} = (2.5)^2(38) + (4.5)^2(6) + (2)^2(30) + (2.5)^2(12) + (0.5)^2(40)$

$$= 564 \text{ W}$$

$$\sum p_{\text{abs}} = 564 + 12.5 + 33.5 = 610 \text{ W} = \sum p_{\text{del}} \text{ (CHECKS)}$$

P 4.48 [a]



Mesh equations:

$$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$$

$$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$$

Constraint equations:

$$i_\Delta = i_2; \quad i_3 = -1.7v_\Delta; \quad v_\Delta = 2i_1$$

$$\text{Solving, } i_1 = -5 \text{ A}; \quad i_2 = 16 \text{ A}; \quad i_3 = 17 \text{ A}; \quad v_\Delta = -10 \text{ V}$$

$$9i_\Delta = 9(16) = 144 \text{ V}$$

$$i_a = i_2 - i_1 = 21 \text{ A}$$

$$i_b = i_2 - i_3 = -1 \text{ A}$$

$$v_b = 20i_b = -20 \text{ V}$$

$$p_{50V} = -50i_1 = 250 \text{ W (absorbing)}$$

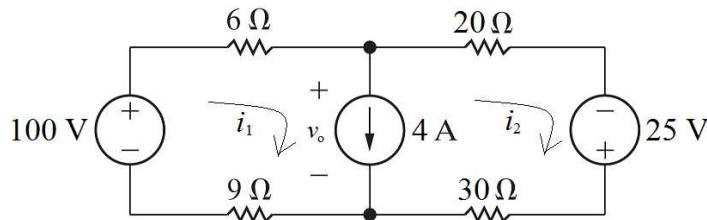
$$p_{9i_\Delta} = -i_a(9i_\Delta) = -(21)(144) = -3024 \text{ W (delivering)}$$

$$p_{1.7V} = -1.7v_\Delta v_b = i_3 v_b = (17)(-20) = -340 \text{ W (delivering)}$$

$$[b] \quad \sum P_{\text{dev}} = 3024 + 340 = 3364 \text{ W}$$

$$\begin{aligned} \sum P_{\text{dis}} &= 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20) \\ &= 3364 \text{ W} \end{aligned}$$

P 4.49



$$-100 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \quad i_1 - i_2 = 4$$

$$\text{Solving, } i_1 = 5 \text{ A}; \quad i_2 = 1 \text{ A}$$

$$p_{100V} = -100i_1 = -500 \text{ W (delivered)}$$

$$p_{6\Omega} = (5)^2(6) = 150 \text{ W}$$

$$p_{9\Omega} = (5)^2(9) = 225 \text{ W}$$

$$p_{20\Omega} = (1)^2(20) = 20 \text{ W}$$

$$p_{30\Omega} = (1)^2(30) = 30 \text{ W}$$

$$v_o = 20(1) - 25 + 30(1) = 25 \text{ V}$$

$$p_{4A} = 4v_o = 100 \text{ W}$$

$$p_{25V} = -25i_2 = -25 \text{ W (delivered)}$$

$$\sum p_{\text{dev}} = 150 + 225 + 20 + 30 + 100 = 525 \text{ W}$$

$$\sum p_{\text{diss}} = -500 - 25 = -525 \text{ W}$$

Thus the total power dissipated is 525 W.

P 4.50 [a] Summing around the supermesh used in the solution to Problem 4.49 gives

$$-67.5 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \quad i_1 - i_2 = 4$$

$$\therefore i_1 = 4.5 \text{ A}; \quad i_2 = 0.5 \text{ A}$$

$$p_{67.5V} = -67.5(4.5) = -303.75 \text{ W (del)}$$

$$v_o = 20(0.5) - 25 + 30(0.5) = 0 \text{ V}$$

$$p_{4A} = 4v_o = 0 \text{ W}$$

$$p_{25V} = -25i_2 = -12.5 \text{ W (del)}$$

$$\sum p_{\text{diss}} = (4.5)^2(6 + 9) + (0.5)^2(20 + 30) = 316.25 \text{ W}$$

$$\sum p_{\text{dev}} = 303.75 + 0 + 12.5 = 316.25 \text{ W} = \sum p_{\text{diss}}$$

- [b] With 4 A current source replaced with a short circuit

$$15i_1 = 67.5; \quad 50i_2 = 25$$

Solving,

$$i_1 = 4.5 \text{ A}, \quad i_2 = 0.5 \text{ A}$$

$$\therefore \sum P_{\text{sources}} = -(67.5)(4.5) - (25)(0.5) = -316.25 \text{ W}$$

- [c] A 4 A source with zero terminal voltage is equivalent to a short circuit carrying 4 A.

- [d] With the new value of the right-hand source, we want $v_o = 0$ but the current in the middle branch must still equal 4 A.

KVL left:

$$-100 + 6i_1 + 0 + 9i_1 = 0 \quad \text{so} \quad i_1 = 6.667 \text{ A}$$

$$i_1 - i_2 = 4 \quad \text{so} \quad i_2 = i_1 - 4 = 2.667 \text{ A}$$

KVL right:

$$20i_2 - V_2 + 30i_2 + 0 = 0 \quad \text{so} \quad V_2 = 50i_2 = 133.333 \text{ V}$$

To check these results, sum around the supermesh with the value of the source on the right as 133.333 V,

$$-100 + 6i_1 + 20i_2 - 133.333 + 30i_2 + 9i_1 = 0; \quad i_1 - i_2 = 4$$

Solving,

$$i_1 = 6.667 \text{ A}; \quad i_2 = 2.667 \text{ A}$$

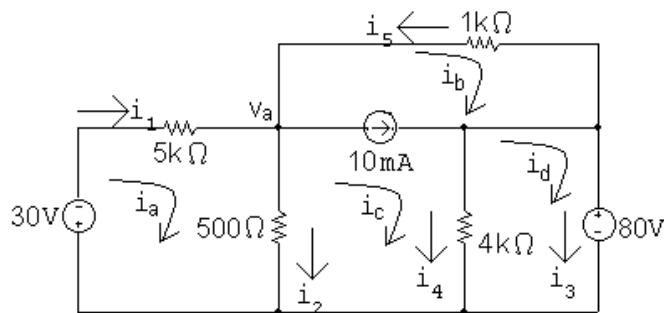
Therefore,

$$v_o = 20i_2 - 133.333 + 30i_2 = 0$$

Thus,

$$P_{4\text{A}} = 4v_o = 0 \text{ (checks)}$$

P 4.51 [a]



Supermesh equations:

$$1000i_b + 4000(i_c - i_d) + 500(i_c - i_a) = 0$$

$$i_c - i_b = 0.01$$

Two remaining mesh equations:

$$5500i_a - 500i_c = -30$$

$$4000i_d - 4000i_c = -80$$

In standard form,

$$-500i_a + 1000i_b + 4500i_c - 4000i_d = 0$$

$$0i_a - 1i_b + 1i_c + 0i_d = 0.01$$

$$5500i_a + 0i_b - 500i_c + 0i_d = -30$$

$$0i_a + 0i_b - 4000i_c + 4000i_d = -80$$

Solving:

$$i_a = -10 \text{ mA}; \quad i_b = -60 \text{ mA}; \quad i_c = -50 \text{ mA}; \quad i_d = -70 \text{ mA}$$

Then,

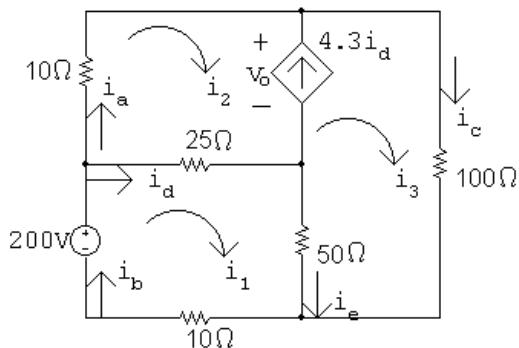
$$i_1 = i_a = -10 \text{ mA}; \quad i_2 = i_a - i_c = 40 \text{ mA}; \quad i_3 = i_d = -70 \text{ mA}$$

[b] $p_{\text{sources}} = 30(-0.01) + [1000(-0.06)](0.01) + 80(-0.07) = -6.5 \text{ W}$

$$p_{\text{resistors}} = 1000(0.06)^2 + 5000(0.01)^2 + 500(0.04)^2$$

$$+4000(-0.05 + 0.07)^2 = 6.5 \text{ W}$$

P 4.52 [a]



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \quad (\text{supermesh})$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

$$\text{Solving, } i_1 = 4.6 \text{ A}; \quad i_2 = 5.7 \text{ A}; \quad i_3 = 0.97 \text{ A}$$

$$i_a = i_2 = 5.7 \text{ A}; \quad i_b = i_1 = 4.6 \text{ A}$$

$$i_c = i_3 = 0.97 \text{ A}; \quad i_d = i_1 - i_2 = -1.1 \text{ A}$$

$$i_e = i_1 - i_3 = 3.63 \text{ A}$$

$$[b] \quad 10i_2 + v_o + 25(i_2 - i_1) = 0$$

$$\therefore v_o = -57 - 27.5 = -84.5 \text{ V}$$

$$p_{4.3i_d} = -v_o(4.3i_d) = -(-84.5)(4.3)(-1.1) = -399.685 \text{ W (dev)}$$

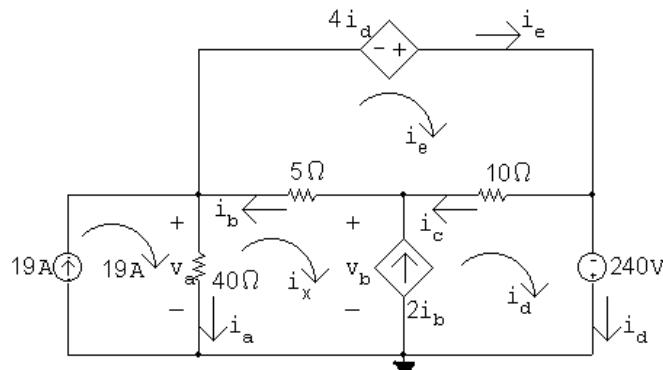
$$p_{200V} = -200(4.6) = -920 \text{ W (dev)}$$

$$\sum P_{\text{dev}} = 1319.685 \text{ W}$$

$$\begin{aligned} \sum P_{\text{dis}} &= (5.7)^2 10 + (1.1)^2 (25) + (0.97)^2 100 + (4.6)^2 (10) + \\ &\quad (3.63)^2 (50) \\ &= 1319.685 \text{ W} \end{aligned}$$

$$\therefore \sum P_{\text{dev}} = \sum P_{\text{dis}} = 1319.685 \text{ W}$$

P 4.53 [a]



$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

$$\text{Solving, } i_d = 10 \text{ A; } i_e = 18 \text{ A; } i_x = 26 \text{ A}$$

$$i_a = 19 - i_x = -7 \text{ A; } i_b = i_e - i_x = -8 \text{ A; } i_c = i_e - i_d = 8 \text{ A;}$$

[b] $v_a = 40i_a = -280 \text{ V; } v_b = 5i_b + 40i_a = -320 \text{ V}$

$$p_{19A} = -19v_a = 5320 \text{ W}$$

$$p_{4i_d} = -4i_d i_e = -720 \text{ W}$$

$$p_{2i_a} = -2i_b v_b = -5120 \text{ W}$$

$$p_{240V} = -240i_d = -2400 \text{ W}$$

$$p_{40\Omega} = (7)^2(40) = 1960 \text{ W} =$$

$$p_{5\Omega} = (8)^2(5) = 320 \text{ W}$$

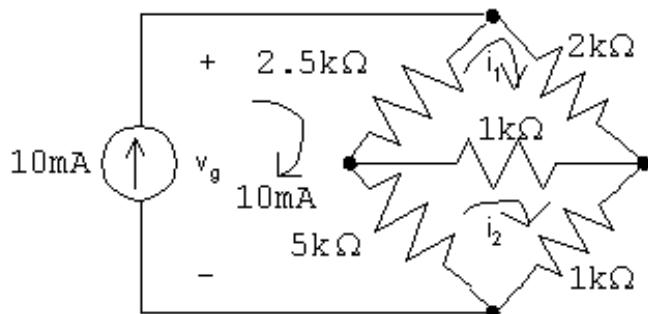
$$p_{10\Omega} = (8)^2(10) = 640 \text{ W}$$

$$\sum P_{\text{gen}} = 720 + 5120 + 2400 = 8240 \text{ W}$$

$$\sum P_{\text{diss}} = 5320 + 1960 + 320 + 640 = 8240 \text{ W}$$

P 4.54 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6 \text{ mA}$; $i_2 = 8 \text{ mA}$

Find the power in the $1 \text{ k}\Omega$ resistor:

$$i_{1k} = i_1 - i_2 = -2 \text{ mA}$$

$$p_{1k} = (-0.002)^2(1000) = 4 \text{ mW}$$

- [c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

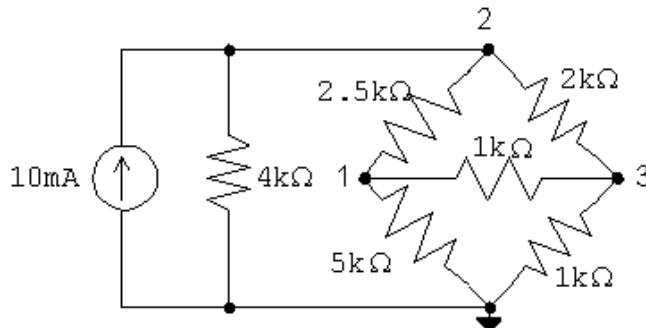
[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20 \text{ V}$

$$p_{10\text{mA}} = -(20)(0.01) = -200 \text{ mW}$$

Thus the 10 mA source develops 200 mW.

- P 4.55 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\frac{v_1}{5000} + \frac{v_1 - v_2}{2500} + \frac{v_1 - v_3}{1000} = 0$$

$$-0.01 + \frac{v_2}{4000} + \frac{v_2 - v_1}{2500} + \frac{v_2 - v_3}{2000} = 0$$

$$\frac{v_3 - v_1}{1000} + \frac{v_3 - v_2}{2000} + \frac{v_3}{1000} = 0$$

Put the equations in standard form:

$$v_1 \left(\frac{1}{5000} + \frac{1}{2500} + \frac{1}{1000} \right) + v_2 \left(-\frac{1}{2500} \right) + v_3 \left(-\frac{1}{1000} \right) = 0$$

$$v_1 \left(-\frac{1}{2500} \right) + v_2 \left(\frac{1}{4000} + \frac{1}{2500} + \frac{1}{2000} \right) + v_3 \left(-\frac{1}{2000} \right) = 0.01$$

$$v_1 \left(-\frac{1}{1000} \right) + v_2 \left(-\frac{1}{2000} \right) + v_3 \left(\frac{1}{2000} + \frac{1}{1000} + \frac{1}{1000} \right) = 0$$

Solving, $v_1 = 6.67 \text{ V}$; $v_2 = 13.33 \text{ V}$; $v_3 = 5.33 \text{ V}$

$$p_{10m} = -(13.33)(0.01) = -133.33 \text{ mW}$$

Therefore, the 10 mA source is developing 133.33 mW

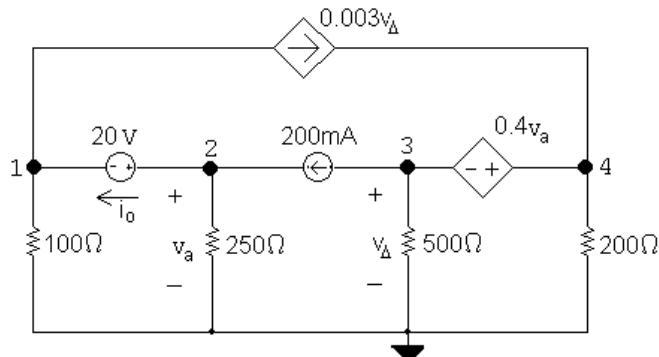
- P 4.56 [a] The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node

voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + 0.003v_{\Delta} + \frac{v_2}{250} - 0.2 = 0$$

$$0.2 + \frac{v_3}{100} + \frac{v_4}{200} - 0.003v_{\Delta} = 0$$

Constraints:

$$v_2 = v_a; \quad v_3 = v_{\Delta}; \quad v_4 - v_3 = 0.4v_a; \quad v_2 - v_1 = 20$$

$$\text{Solving, } v_1 = 24 \text{ V}; \quad v_2 = 44 \text{ V}; \quad v_3 = -72 \text{ V}; \quad v_4 = -54 \text{ V.}$$

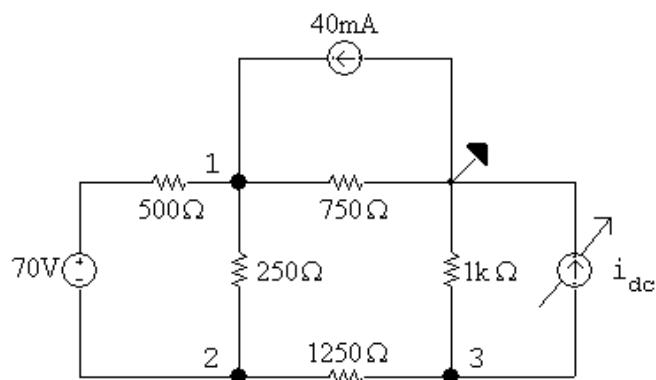
$$i_o = 0.2 - \frac{v_2}{250} = 24 \text{ mA}$$

$$p_{20V} = 20(0.024) = 480 \text{ mW}$$

Thus, the 20 V source absorbs 480 mW.

- P 4.57 [a] The mesh-current method does not directly involve the voltage drop across the 40 mA source. Instead, use the node-voltage method and choose the reference node so that a node voltage is identical to the voltage across the 40 mA source.

[b]



Since the 40 mA source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2 ; thus:

$$\frac{0 - (70 + v_2)}{500} + \frac{0 - v_2}{250} + \frac{0}{750} - 0.04 = 0$$

$$\therefore v_2 = -30 \text{ V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3 ; thus:

$$\frac{v_2 + 70 - 0}{500} + \frac{v_2 - 0}{250} + \frac{v_2 - v_3}{1250} = 0$$

$$\therefore v_3 = -80 \text{ V}$$

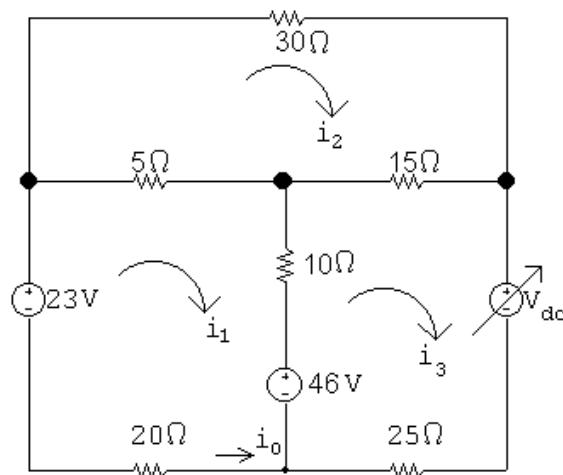
Now that we know v_3 we sum the currents away from node 3 to find i_{dc} ; thus:

$$\frac{v_3}{1000} + \frac{v_3 - v_2}{1250} + i_{dc} = 0$$

$$\therefore i_{dc} = 0.12 = 120 \text{ mA}$$

- P 4.58 [a] If the mesh-current method is used, then the value of the lower left mesh current is $i_o = 0$. This shortcut will simplify the set of KVL equations. The node-voltage method has no equivalent simplifying shortcut, so the mesh-current method is preferred.

[b]



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$-23 + 5(-i_2) + 10(-i_3) + 46 = 0$$

$$30i_2 + 15(i_2 - i_3) + 5i_2 = 0$$

$$V_{dc} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) = 0$$

Place the equations in standard form:

$$i_2(-5) + i_3(-10) + V_{dc}(0) = -23$$

$$i_2(30 + 15 + 5) + i_3(-15) + V_{dc}(0) = 0$$

$$i_2(-15) + i_3(25 + 10 + 15) + V_{dc}(1) = 46$$

Solving, $i_2 = 0.6 \text{ A}$; $i_3 = 2 \text{ A}$; $V_{dc} = -45 \text{ V}$

Thus, the value of V_{dc} required to make $i_o = 0$ is -45 V .

[c] Calculate the power:

$$p_{23V} = -(23)(0) = 0 \text{ W}$$

$$p_{46V} = -(46)(2) = -92 \text{ W}$$

$$p_{Vdc} = (-45)(2) = -90 \text{ W}$$

$$p_{30\Omega} = (30)(0.6)^2 = 10.8 \text{ W}$$

$$p_{5\Omega} = (5)(0.6)^2 = 1.8 \text{ W}$$

$$p_{15\Omega} = (15)(2 - 0.6)^2 = 29.4 \text{ W}$$

$$p_{10\Omega} = (10)(2)^2 = 40 \text{ W}$$

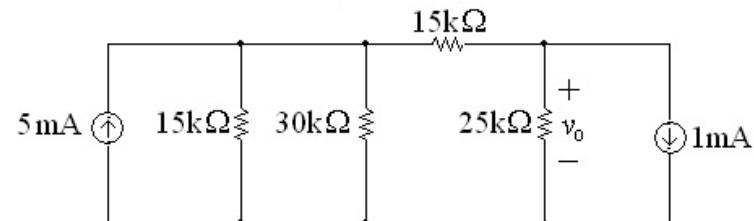
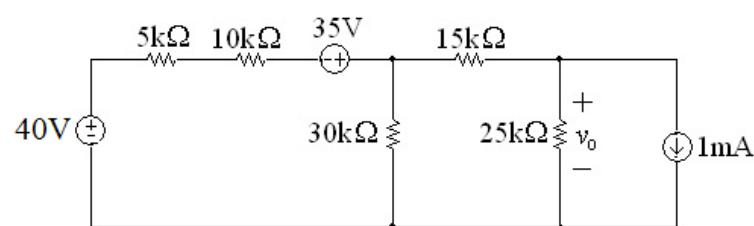
$$p_{20\Omega} = (20)(0)^2 = 0 \text{ W}$$

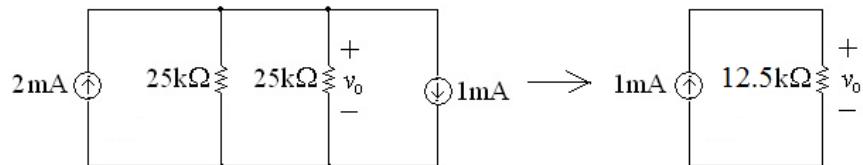
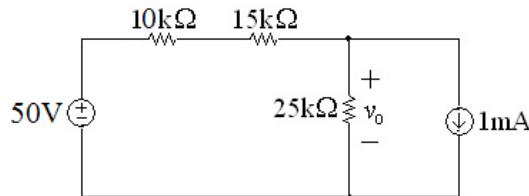
$$p_{25\Omega} = (25)(2)^2 = 100 \text{ W}$$

$$\sum p_{\text{dev}} = 92 + 90 = 182 \text{ W}$$

$$\sum p_{\text{dis}} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W} (\text{checks})$$

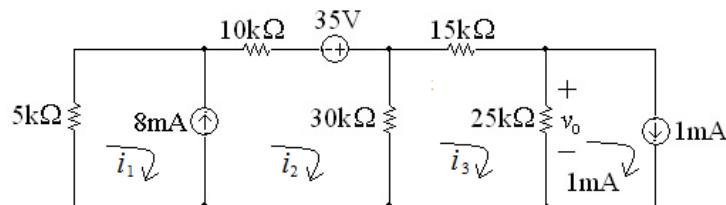
P 4.59 [a]





$$v_o = (12,500)(0.001) = 12.5 \text{ V}$$

[b]



$$5000i_1 + 40,000i_2 - 30,000i_3 = 35$$

$$i_2 - i_1 = 0.008$$

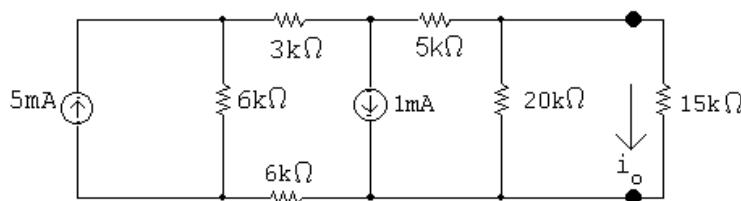
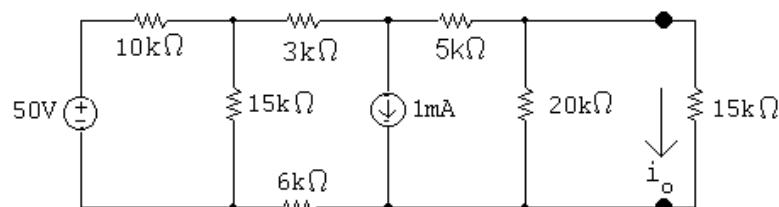
$$-30,000i_2 + 70,000i_3 = 25$$

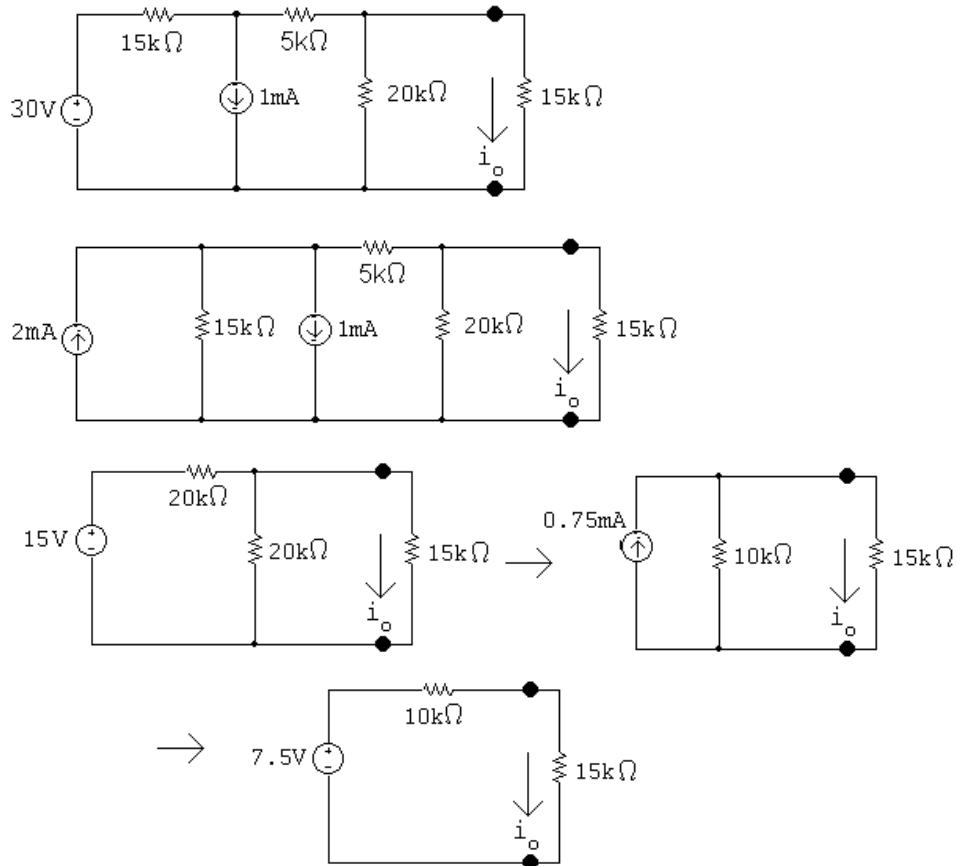
Solving,

$$i_1 = -5.33 \text{ mA}; \quad i_2 = 2.667 \text{ mA}; \quad i_3 = 1.5 \text{ mA}$$

$$v_o = (25,000)(i_3 - 0.001) = (25,000)(0.0005) = 12.5 \text{ V}$$

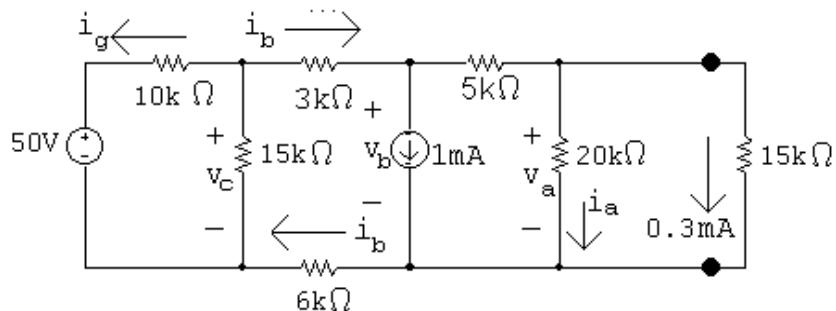
P 4.60 [a]





$$i_o = \frac{7.5}{25,000} = 0.3 \text{ mA}$$

[b]



$$v_a = (15,000)(0.0003) = 4.5 \text{ V}$$

$$i_a = \frac{v_a}{20,000} = 225 \mu\text{A}$$

$$i_b = 1 + 0.225 + 0.3 = 1.525 \text{ mA}$$

$$v_b = 5000(0.525 \times 10^{-3}) + 4.5 = 7.125 \text{ V}$$

$$v_c = 3000(1.525 \times 10^{-3}) + 7.125 + 6000(1.525 \times 10^{-3}) = 20.85 \text{ V}$$

$$i_g = \frac{20.85 - 50}{10,000} = -2.915 \text{ mA}$$

$$p_{50V} = (50)(-2.915 \times 10^{-3}) = -145.75 \text{ mW}$$

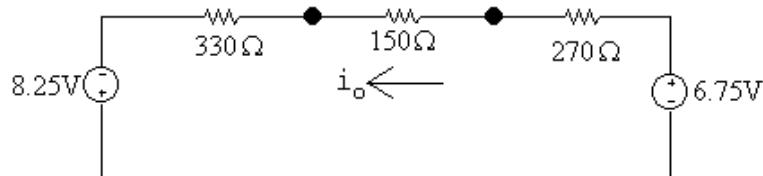
Check:

$$p_{1\text{mA}} = (7.125)(10^{-3}) = 7.125 \text{ mW}$$

$$\sum P_{\text{dev}} = 145.75 \text{ mW}$$

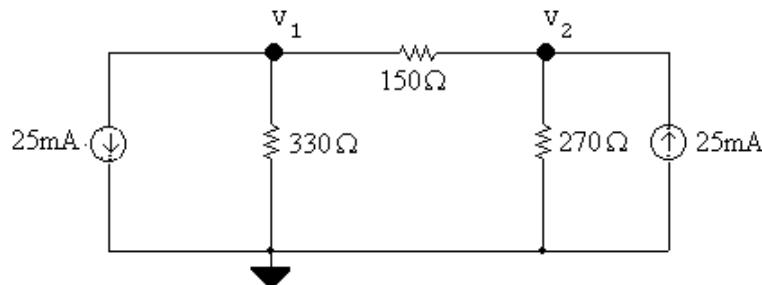
$$\begin{aligned} \sum P_{\text{dis}} &= (10,000)(2.915 \times 10^{-3})^2 + (20.85)^2/15,000 + (9000)(1.525 \times 10^{-3})^2 \\ &\quad + (5000)(0.525 \times 10^{-3})^2 + (20,000)(0.225 \times 10^{-3})^2 \\ &\quad + (15,000)(0.3 \times 10^{-3})^2 + 7.125 \times 10^{-3} \\ &= 145.75 \text{ mW} \end{aligned}$$

P 4.61 [a] Apply source transformations to both current sources to get



$$i_o = \frac{(6.75 + 8.25)}{330 + 150 + 270} = 20 \text{ mA}$$

[b]



The node voltage equations:

$$0.025 + \frac{v_1}{330} + \frac{v_1 - v_2}{150} = 0$$

$$\frac{v_2}{270} + \frac{v_2 - v_1}{150} - 0.025 = 0$$

Place these equations in standard form:

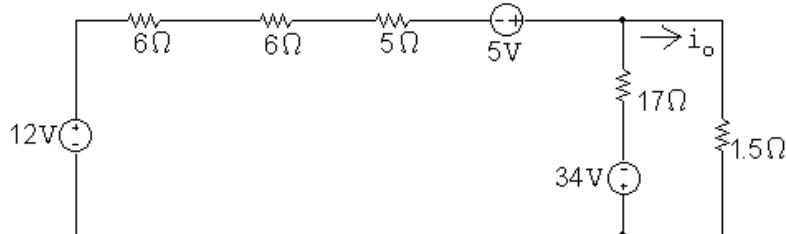
$$v_1 \left(\frac{1}{330} + \frac{1}{150} \right) + v_2 \left(-\frac{1}{150} \right) = -0.025$$

$$v_1 \left(-\frac{1}{150} \right) + v_2 \left(\frac{1}{270} + \frac{1}{150} \right) = 0.025$$

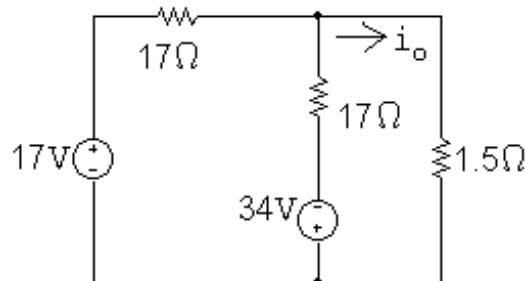
Solving, $v_1 = -1.65 \text{ V}$; $v_2 = 1.35 \text{ V}$

$$\therefore i_o = \frac{v_2 - v_1}{150} = 20 \text{ mA}$$

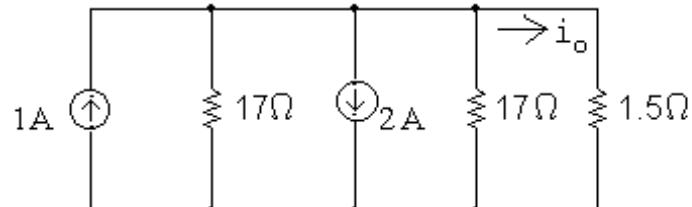
P 4.62 [a] Applying a source transformation to each current source yields



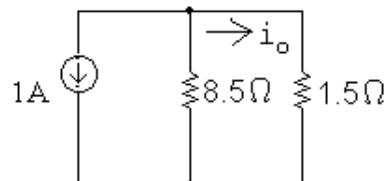
Now combine the 12 V and 5 V sources into a single voltage source and the 6Ω , 6Ω and 5Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

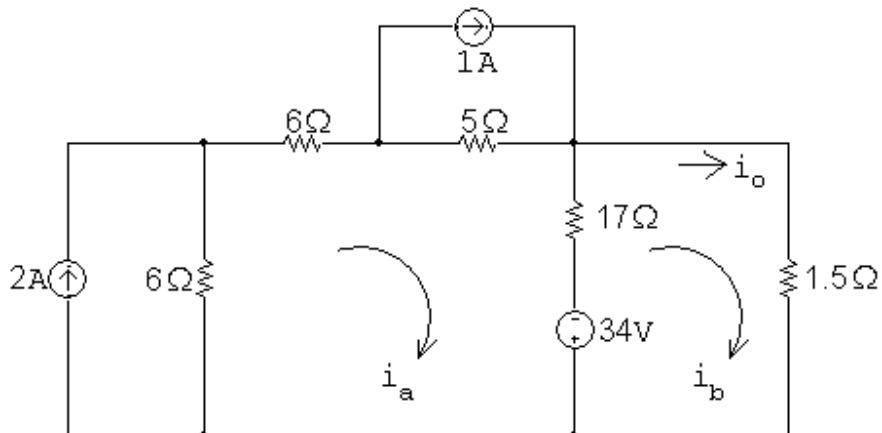


which can be reduced to



$$\therefore i_o = -\frac{8.5}{10}(1) = -0.85 \text{ A}$$

[b]

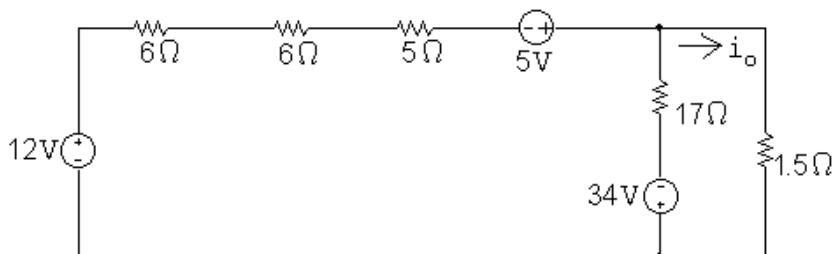


$$34i_a - 17i_b = 12 + 5 + 34 = 51$$

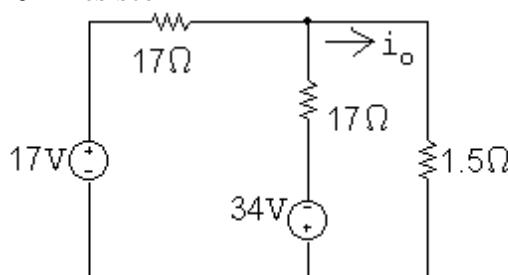
$$-17i_a + 18.5i_b = -34$$

$$\text{Solving, } i_b = -0.85 \text{ A} = i_o$$

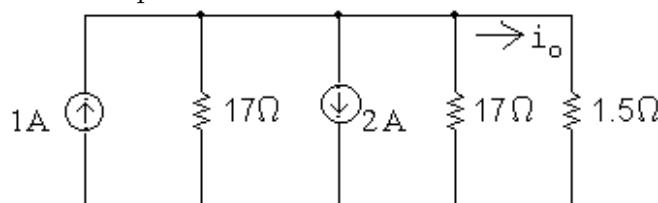
P 4.63 [a] First remove the 16Ω and 260Ω resistors:



Next use a source transformation to convert the 1 A current source and 40Ω resistor:

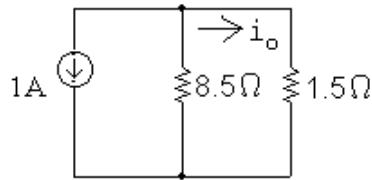


which simplifies to



$$\therefore v_o = \frac{250}{300}(480) = 400 \text{ V}$$

[b] Return to the original circuit with $v_o = 400$ V:



$$i_g = \frac{520}{260} + 1.6 = 3.6 \text{ A}$$

$$p_{520V} = -(520)(3.6) = -1872 \text{ W}$$

Therefore, the 520 V source is developing 1872 W.

[c] $v_1 = -520 + 1.6(4 + 250 + 6) = -104$ V

$$v_g = v_1 - 1(16) = -104 - 16 = -120 \text{ V}$$

$$p_{1A} = (1)(-120) = -120 \text{ W}$$

Therefore the 1 A source is developing 120 W.

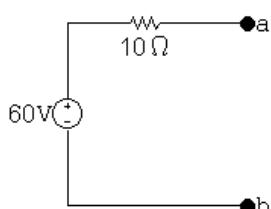
[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992 \text{ W}$

$$\sum p_{\text{diss}} = (1)^2(16) + \frac{(104)^2}{40} + \frac{(520)^2}{260} + (1.6)^2(260) = 1992 \text{ W}$$

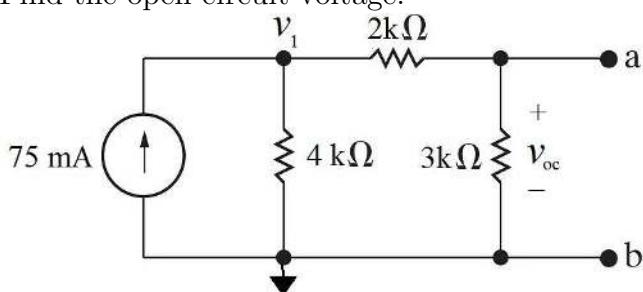
$$\therefore \sum p_{\text{diss}} = \sum p_{\text{dev}}$$

$$\text{P 4.64 } v_{\text{Th}} = \frac{30}{40}(80) = 60 \text{ V}$$

$$R_{\text{Th}} = 2.5 + \frac{(30)(10)}{40} = 10 \Omega$$



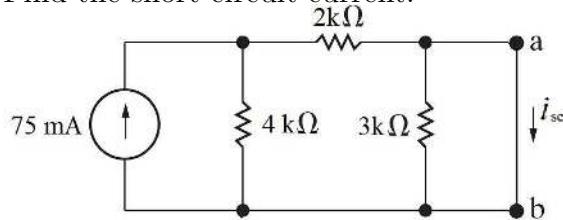
P 4.65 Find the open-circuit voltage:



$$-0.075 + \frac{v_1}{4000} + \frac{v_1}{5000} = 0$$

$$v_1 = 166.67 \text{ V}; \quad \text{so} \quad v_{oc} = \frac{3000}{5000} v_1 = 100 \text{ V}$$

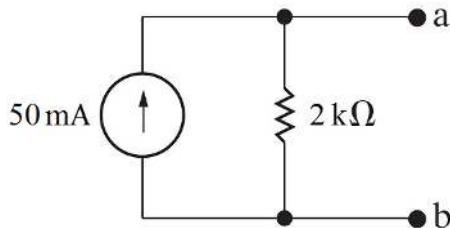
Find the short-circuit current:



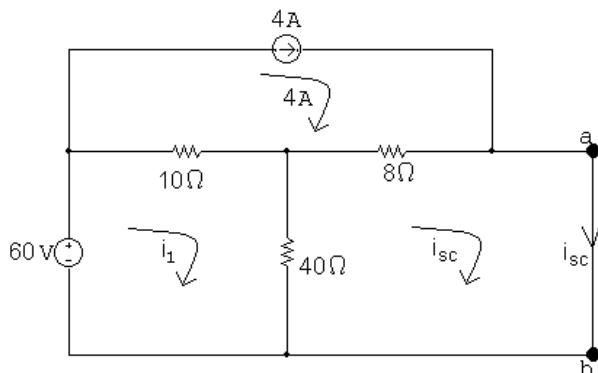
$$i_{sc} = \frac{4000 \parallel 2000}{2000} (0.075) = 50 \text{ mA}$$

Thus,

$$I_N = i_{sc} = 50 \text{ mA}; \quad R_N = \frac{v_{oc}}{i_{sc}} = \frac{100}{0.05} = 2 \text{ k}\Omega$$



P 4.66

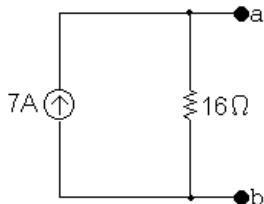


$$50i_1 - 40i_{sc} = 60 + 40$$

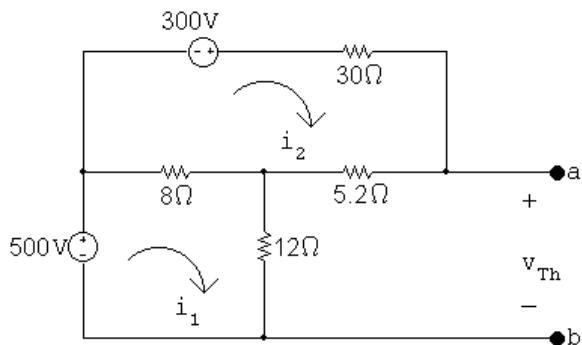
$$-40i_1 + 48i_{sc} = 32$$

$$\text{Solving, } i_{sc} = 7 \text{ A}$$

$$R_{Th} = 8 + \frac{(10)(40)}{50} = 16 \Omega$$



P 4.67 After making a source transformation the circuit becomes



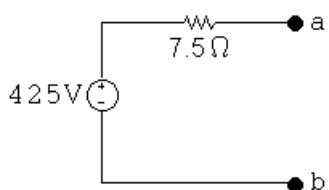
$$500 = 20i_1 - 8i_2$$

$$300 = -8i_1 + 43.2i_2$$

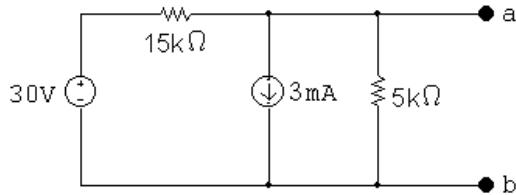
$$\therefore i_1 = 30 \text{ A} \text{ and } i_2 = 12.5 \text{ A}$$

$$v_{Th} = 12i_1 + 5.2i_2 = 425 \text{ V}$$

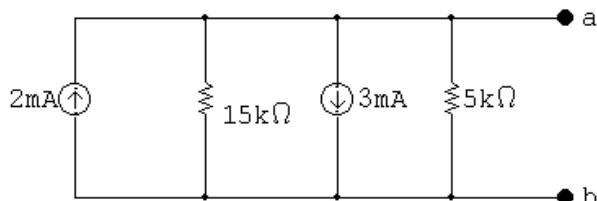
$$R_{Th} = (8\parallel 12 + 5.2)\parallel 30 = 7.5 \Omega$$



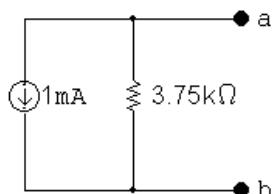
P 4.68 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



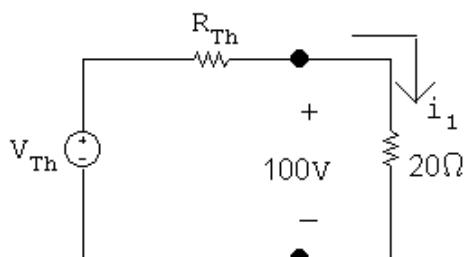
or



Therefore the Norton equivalent is

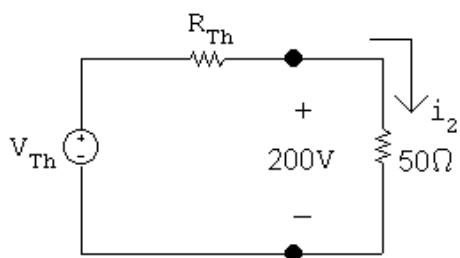


P 4.69



$$i_1 = 100/20 = 5 \text{ A}$$

$$100 = v_{\text{Th}} - 5R_{\text{Th}}, \quad v_{\text{Th}} = 100 + 5R_{\text{Th}}$$

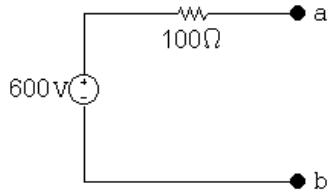


$$i_2 = 200/50 = 4 \text{ A}$$

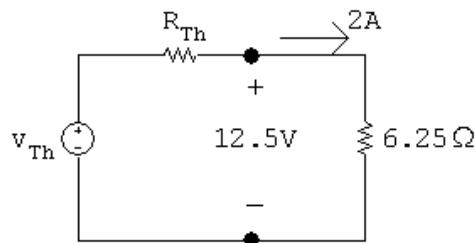
$$200 = v_{\text{Th}} - 4R_{\text{Th}}, \quad v_{\text{Th}} = 200 + 4R_{\text{Th}}$$

$$\therefore 100 + 5R_{Th} = 200 + 4R_{Th} \quad \text{so} \quad R_{Th} = 100\Omega$$

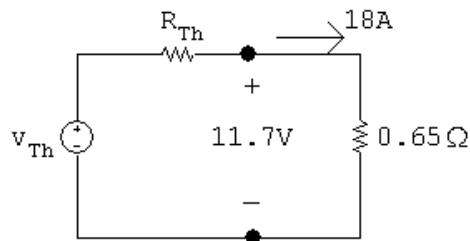
$$v_{Th} = 100 + 500 = 600 \text{ V}$$



P 4.70



$$12.5 = v_{Th} - 2R_{Th}$$



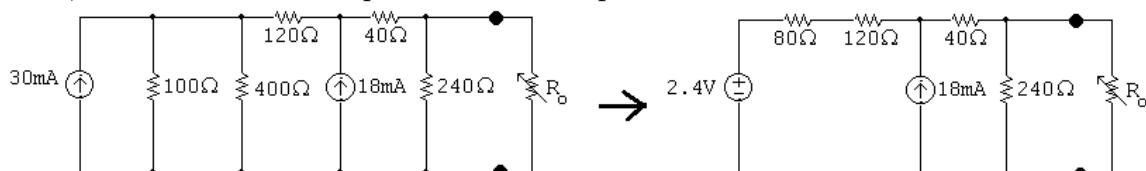
$$11.7 = v_{Th} - 18R_{Th}$$

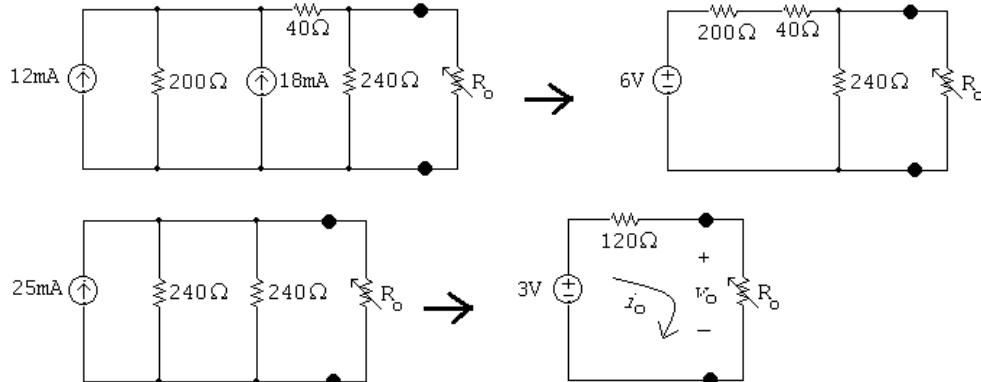
Solving the above equations for V_{Th} and R_{Th} yields

$$v_{Th} = 12.6 \text{ V}, \quad R_{Th} = 50 \text{ m}\Omega$$

$$\therefore I_N = 252 \text{ A}, \quad R_N = 50 \text{ m}\Omega$$

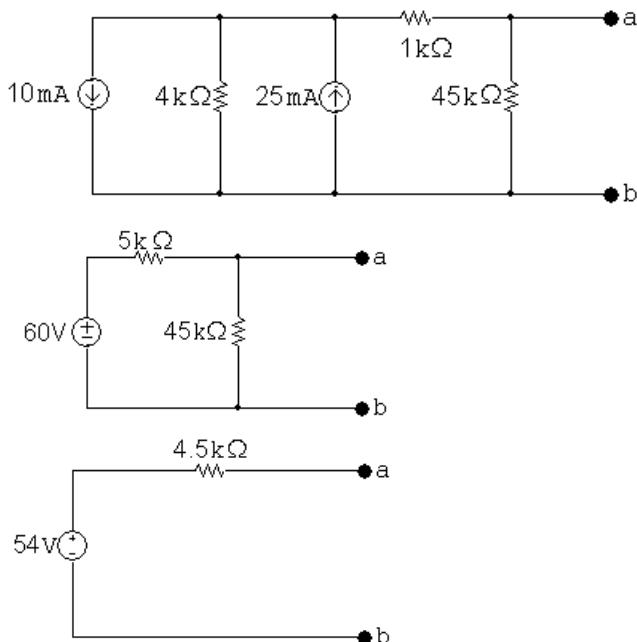
P 4.71 First, find the Thévenin equivalent with respect to R_o .



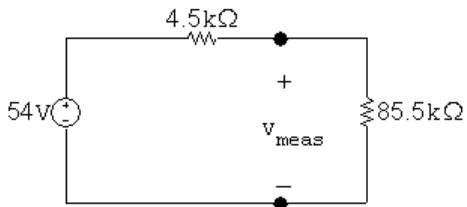


$R_o(\Omega)$	$i_o(\text{mA})$	$v_o(\text{V})$
100	13.64	1.364
120	12.5	1.5
150	11.11	1.667
180	10	1.8

- P 4.72 [a] First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.



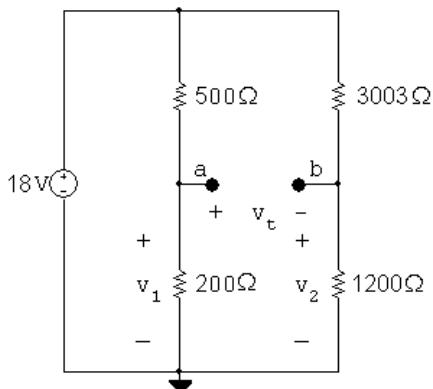
$$\therefore v_{\text{Th}} = 54 \text{ V} \quad R_{\text{Th}} = 4.5 \text{ k}\Omega$$



$$v_{\text{meas}} = \frac{54}{90}(85.5) = 51.3 \text{ V}$$

$$[\text{b}] \quad \% \text{error} = \left(\frac{51.3 - 54}{54} \right) \times 100 = -5\%$$

P 4.73

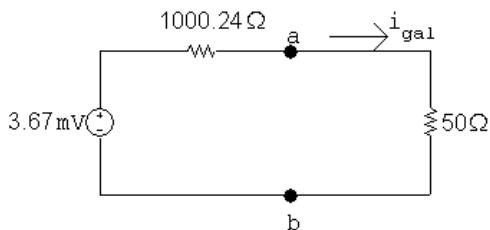


$$v_1 = \frac{200}{700}(18) = 5.143 \text{ V}$$

$$v_2 = \frac{1200}{4203}(18) = 5.139 \text{ V}$$

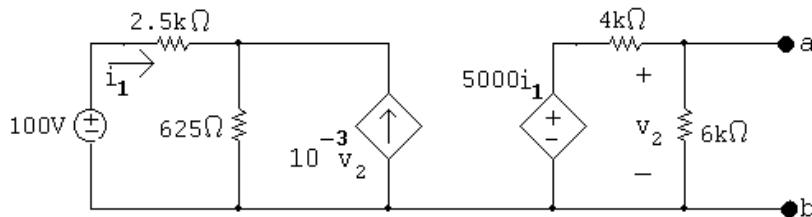
$$v_{\text{Th}} = v_1 - v_2 = 5.143 - 5.139 = 3.67 \text{ mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = 1000.24 \Omega$$



$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1050.24} = 3.5 \mu\text{A}$$

P 4.74



OPEN CIRCUIT

$$100 = 2500i_1 + 625(i_1 + 10^{-3}v_2)$$

$$v_2 = \frac{6000}{10,000}(5000i_1)$$

Solving,

$$i_1 = 0.02 \text{ A}; \quad v_2 = v_{oc} = 60 \text{ V}$$

SHORT CIRCUIT

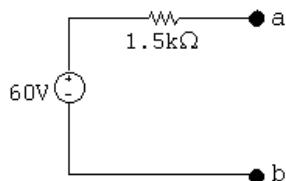
$$v_2 = 0; \quad i_{sc} = \frac{5000}{4000}i_1$$

$$i_1 = \frac{100}{2500 + 625} = 0.032 \text{ A}$$

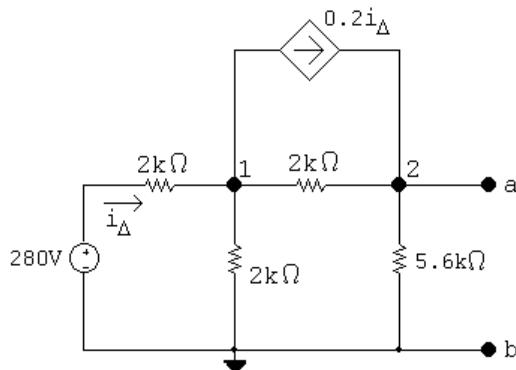
Thus,

$$i_{sc} = \frac{5}{4}i_1 = 0.04 \text{ A}$$

$$R_{Th} = \frac{60}{0.04} = 1.5 \text{ k}\Omega$$



P 4.75



The node voltage equations and dependant source equation are:

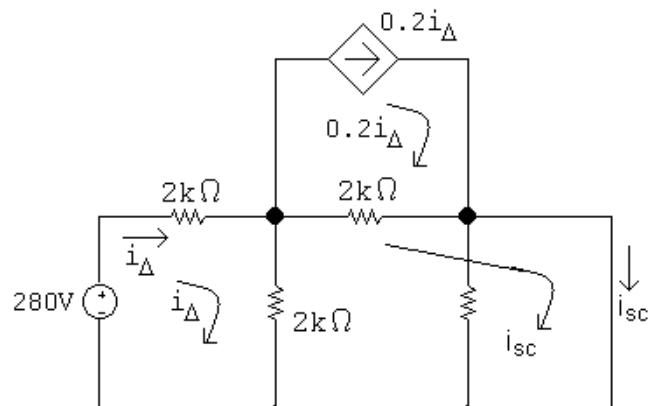
$$\begin{aligned}\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta &= 0 \\ \frac{v_2 - v_1}{2000} + \frac{v_2}{5600} - 0.2i_\Delta &= 0 \\ i_\Delta &= \frac{280 - v_1}{2000}\end{aligned}$$

In standard form:

$$\begin{aligned}v_1 \left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000} \right) + v_2 \left(-\frac{1}{2000} \right) + i_\Delta(0.2) &= \frac{280}{2000} \\ v_1 \left(-\frac{1}{2000} \right) + v_2 \left(\frac{1}{2000} + \frac{1}{5600} \right) + i_\Delta(-0.2) &= 0 \\ v_1 \left(\frac{1}{2000} \right) + v_2(0) + i_\Delta(1) &= \frac{280}{2000}\end{aligned}$$

Solving, $v_1 = 120 \text{ V}$; $v_2 = 112 \text{ V}$; $i_\Delta = 0.08 \text{ A}$

$V_{Th} = v_2 = 112 \text{ V}$



The mesh current equations are:

$$-280 + 2000i_{\Delta} + 2000(i_{\Delta} - i_{sc}) = 0$$

$$2000(i_{sc} - 0.2i_{\Delta}) + 2000(i_{sc} - i_{\Delta}) = 0$$

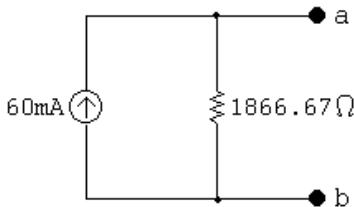
Put these equations in standard form:

$$i_{\Delta}(4000) + i_{sc}(-2000) = 280$$

$$i_{\Delta}(-2400) + i_{sc}(4000) = 0$$

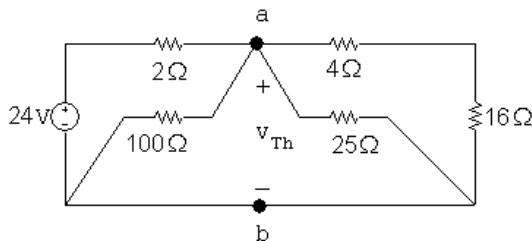
Solving, $i_{\Delta} = 0.1 \text{ A}$; $i_{sc} = 0.06 \text{ A}$

$$R_{Th} = \frac{112}{0.06} = 1866.67 \Omega$$



- P 4.76 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the 4.8Ω resistor.

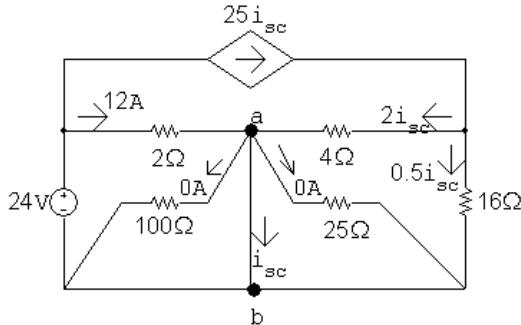
Thévenin voltage: note i_{ϕ} is zero.



$$\frac{v_{Th}}{100} + \frac{v_{Th}}{25} + \frac{v_{Th}}{20} + \frac{v_{Th} - 16}{2} = 0$$

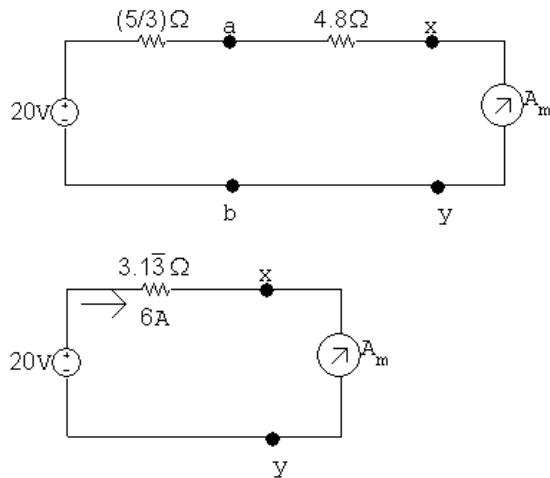
Solving, $v_{Th} = 20 \text{ V}$.

Short-circuit current:



$$i_{sc} = 12 + 2i_{sc}, \quad \therefore i_{sc} = -12 \text{ A}$$

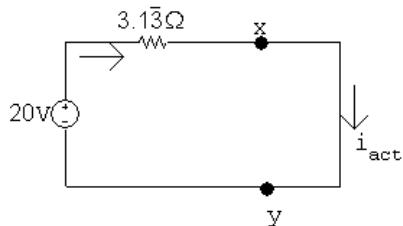
$$R_{Th} = \frac{20}{-12} = -(5/3) \Omega$$



$$R_{\text{total}} = \frac{20}{6} = 3.33 \Omega$$

$$R_{\text{meter}} = 3.33 - 3.13 = 0.2 \Omega$$

[b] Actual current:



$$i_{\text{actual}} = \frac{20}{3.13} = 6.38 \text{ A}$$

$$\% \text{ error} = \frac{6 - 6.38}{6.38} \times 100 = -6\%$$

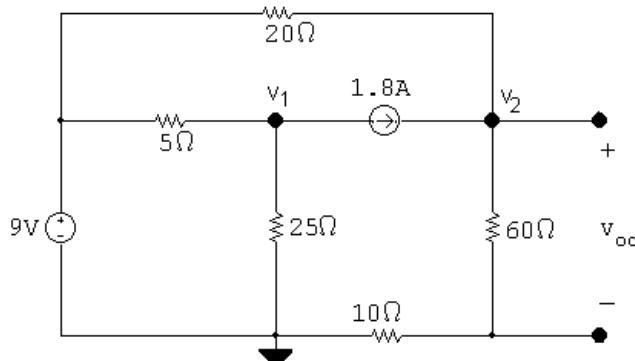
P 4.77 [a] Replace the voltage source with a short circuit and find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10 \parallel 30 + 2.5 = 10 \Omega$$

[b] Replace the current source with an open circuit and the voltage source with a short circuit. Find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10 \parallel 40 + 8 = 16 \Omega$$

P 4.78 [a] Open circuit:

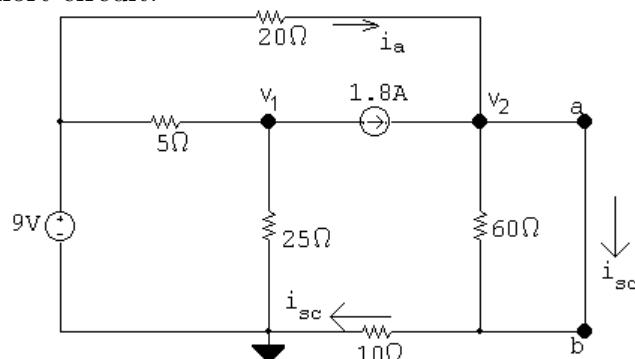


$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$$v_2 = 35 \text{ V}$$

$$v_{\text{Th}} = \frac{60}{70}v_2 = 30 \text{ V}$$

Short circuit:



$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

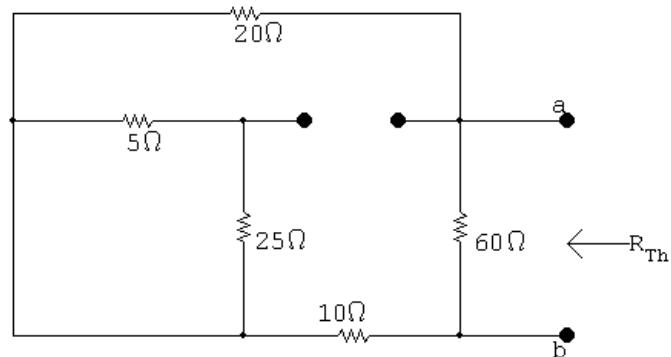
$$\therefore v_2 = 15 \text{ V}$$

$$i_a = \frac{9 - 15}{20} = -0.3 \text{ A}$$

$$i_{\text{sc}} = 1.8 - 0.3 = 1.5 \text{ A}$$

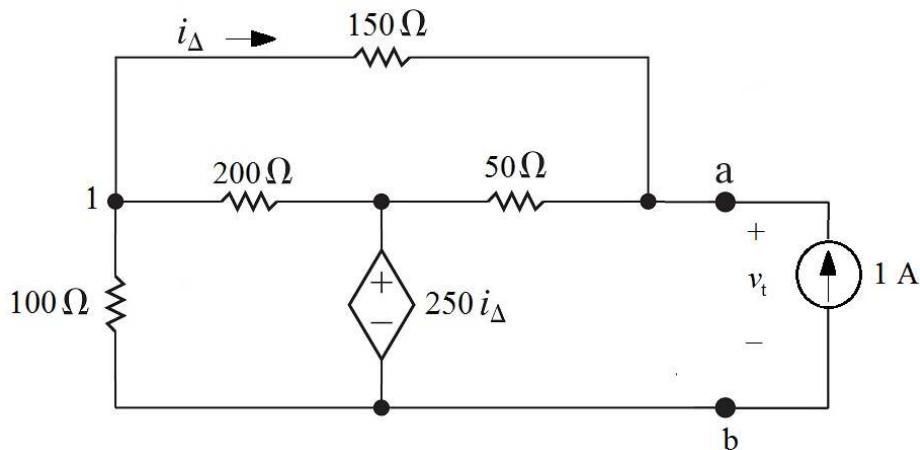
$$R_{\text{Th}} = \frac{30}{1.5} = 20 \Omega$$

[b]



$$R_{Th} = (20 + 10\parallel 60 = 20\Omega \text{ (CHECKS)})$$

P 4.79 $V_{Th} = 0$, since circuit contains no independent sources.



$$\frac{v_1}{100} + \frac{v_1 - 250i_\Delta}{200} + \frac{v_1 - v_t}{150} = 0$$

$$\frac{v_t - v_1}{150} + \frac{v_t - 250i_\Delta}{50} - 1 = 0$$

$$i_\Delta = \frac{v_t - v_1}{150}$$

In standard form:

$$v_1 \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{150} \right) + v_t \left(-\frac{1}{150} \right) + i_\Delta \left(-\frac{250}{200} \right) = 0$$

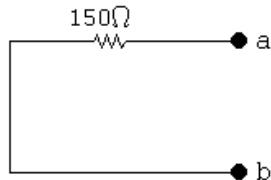
$$v_1 \left(-\frac{1}{150} \right) + v_t \left(\frac{1}{150} + \frac{1}{50} \right) + i_\Delta \left(-\frac{250}{50} \right) = 1$$

$$v_1 \left(-\frac{1}{150} \right) + v_t \left(\frac{1}{150} \right) + i_\Delta (-1) = 0$$

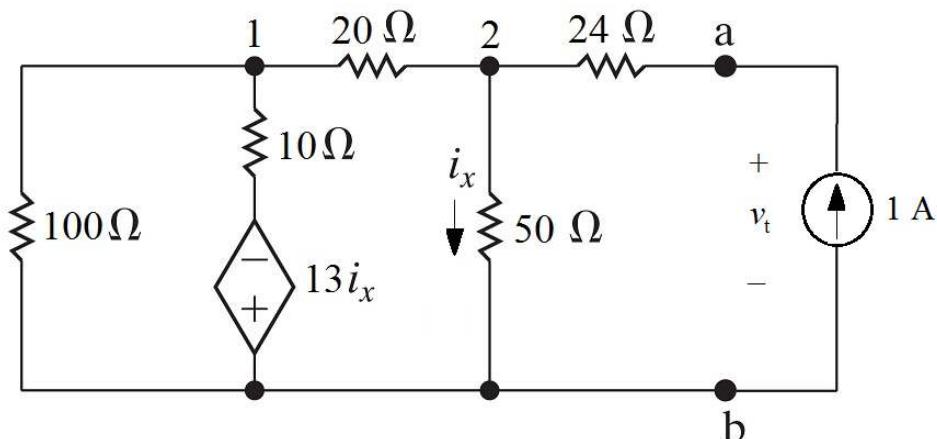
Solving,

$$v_1 = 75 \text{ V}; \quad v_t = 150 \text{ V}; \quad i_\Delta = 0.5 \text{ A}$$

$$\therefore R_{\text{Th}} = \frac{v_t}{1 \text{ A}} = 150 \Omega$$



- P 4.80 Since there is no independent source, $V_{\text{Th}} = 0$. Now apply a test source at the terminals a,b to find the Thévenin equivalent resistance:



$$\frac{v_1}{100} + \frac{v_1 + 13i_x}{10} + \frac{v_1 - v_2}{20} = 0$$

$$\frac{v_2 - v_1}{20} + \frac{v_2}{50} - 1 = 0$$

$$i_x = \frac{v_2}{50}$$

Solving,

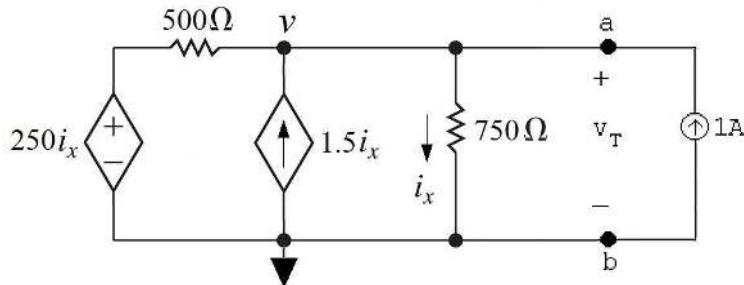
$$v_1 = 2.4 \text{ V}; \quad v_2 = 16 \text{ V}; \quad i_x = 0.32 \text{ A}$$

$$v_t - 24(1) = v_2; \quad \text{so} \quad v_t = 16 + 24 = 40 \text{ V}$$

$$R_{\text{Th}} = \frac{v_t}{1 \text{ A}} = 40 \Omega$$

The Thévenin equivalent is simply a 40Ω resistor.

P 4.81 $V_{Th} = 0$ since there are no independent sources in the circuit. Thus we need only find R_{Th} .



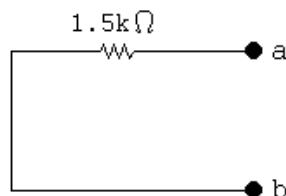
$$\frac{v - 250i_x}{500} - 1.5i_x + \frac{v}{750} - 1 = 0$$

$$i_x = \frac{v}{750}$$

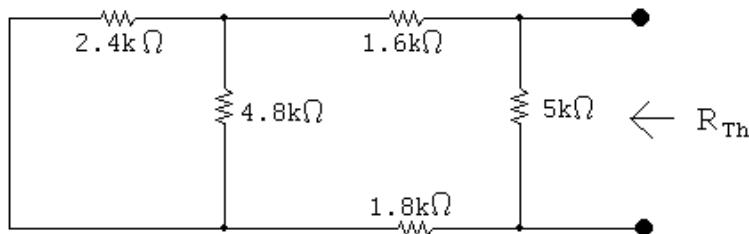
Solving,

$$v = 1500 \text{ V}; \quad i_x = 2 \text{ A}$$

$$R_{Th} = \frac{v}{1 \text{ A}} = 1500 = 1.5 \text{ k}\Omega$$



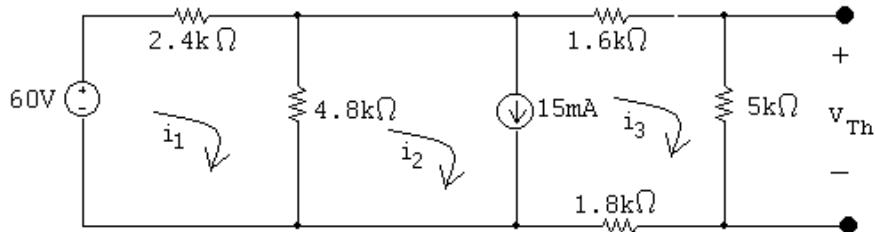
P 4.82 [a]



$$R_{Th} = 5000 \parallel (1600 + 2400 \parallel 4800 + 1800) = 2.5 \text{ k}\Omega$$

$$R_o = R_{Th} = 2.5 \text{ k}\Omega$$

[b]



$$7200i_1 - 4800i_2 = 60$$

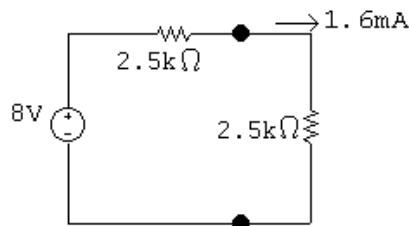
$$-4800i_1 + 4800i_2 + 8400i_3 = 0$$

$$i_2 - i_3 = 0.015$$

Solving,

$$i_1 = 19.4 \text{ mA}; \quad i_2 = 16.6 \text{ mA}; \quad i_3 = 1.6 \text{ mA}$$

$$v_{oc} = 5000i_3 = 8 \text{ V}$$



$$p_{\max} = (1.6 \times 10^{-3})^2(2500) = 6.4 \text{ mW}$$

- [c] The resistor closest to $2.5 \text{ k}\Omega$ from Appendix H has a value of $2.7 \text{ k}\Omega$. Use voltage division to find the voltage drop across this load resistor, and use the voltage to find the power delivered to it:

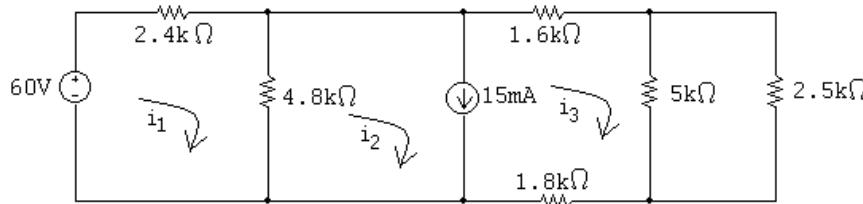
$$v_{2.7k} = \frac{2700}{2700 + 2500}(8) = 4.154 \text{ V}$$

$$p_{2.7k} = \frac{(4.154)^2}{2700} = 6.391 \text{ mW}$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\% \text{ error} = \left(\frac{6.391}{6.4} - 1 \right) (100) = -0.1\%$$

P 4.83 Write KVL equations for the left mesh and the supermesh, place them in standard form, and solve:



$$\text{At } i_1: -60 + 2400i_1 + 4800(i_1 - i_2) = 0$$

$$\text{Supermesh: } 4800(i_2 - i_1) + 1600i_3 + (5000\parallel 2500)i_3 + 1800i_3 = 0$$

$$\text{Constraint: } i_2 - i_3 = 0.015 = 0$$

Standard form:

$$i_1(7200) + i_2(-4800) + i_3(0) = 60$$

$$i_1(-4800) + i_2(4800) + i_3(5066.67) = 0$$

$$i_1(0) + i_2(1) + i_3(-1) = 0.015$$

Calculator solution:

$$i_1 = 19.933 \text{ mA}; \quad i_2 = 17.4 \text{ mA}; \quad i_3 = 2.4 \text{ mA}$$

Calculate voltage across the current source:

$$v_{15\text{mA}} = 4800(i_1 - i_2) = 12.16 \text{ V}$$

Calculate power delivered by the sources:

$$p_{15\text{mA}} = (0.015)(12.16) = 182.4 \text{ mW (abs)}$$

$$p_{60\text{V}} = -60i_1 = -60(0.019933) = -1.196 \text{ W (del)}$$

$$p_{\text{delivered}} = 1.196 \text{ W}$$

Calculate power absorbed by the $2.5 \text{ k}\Omega$ resistor and the percentage power:

$$i_{2.5\text{k}} = \frac{5000\parallel 2500}{2500}i_3 = 1.6 \text{ mA}$$

$$p_{2.5\text{k}} = (0.0016)^2(2500) = 6.4 \text{ mW}$$

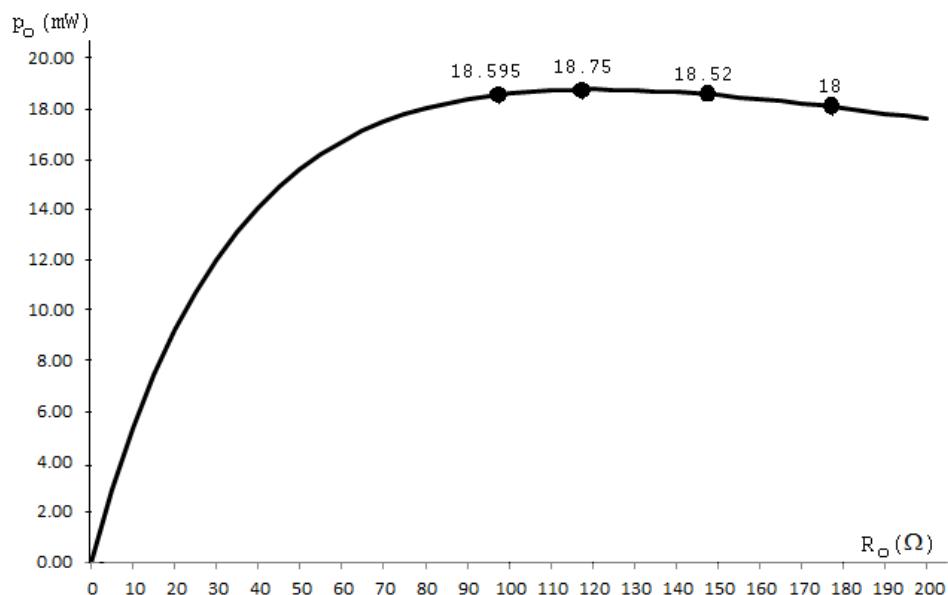
$$\% \text{ delivered to } R_o: \quad \frac{0.0064}{1.196}(100) = 0.535\%$$

P 4.84 [a] From the solution to Problem 4.71 we have

$R_o(\Omega)$	$p_o(\text{mW})$
100	18.595
120	18.75
150	18.52
180	18

The 120Ω resistor dissipates the most power, because its value is equal to the Thévenin equivalent resistance of the circuit.

[b]



[c] $R_o = 120\Omega$, $p_o = 18.75 \text{ mW}$, which is the maximum power that can be delivered to a load resistor.

P 4.85 [a] Since $0 \leq R_o \leq \infty$ maximum power will be delivered to the 6Ω resistor when $R_o = 0$.

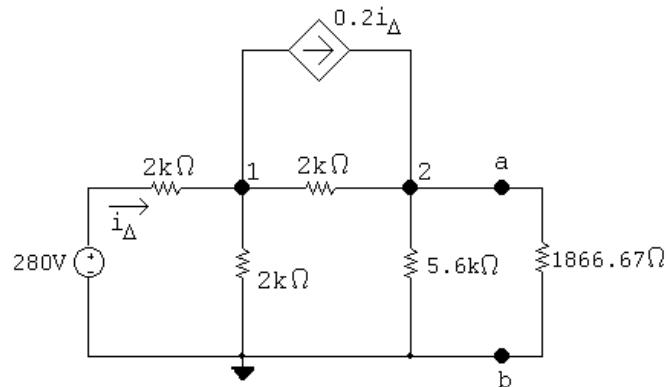
$$[b] P = \frac{30^2}{6} = 150 \text{ W}$$

P 4.86 [a] From the solution of Problem 4.75 we have $R_{\text{Th}} = 1866.67\Omega$ and $V_{\text{Th}} = 112 \text{ V}$. Therefore

$$R_o = R_{\text{Th}} = 1866.67\Omega$$

$$[b] p = \frac{(56)^2}{1866.67} = 1.68 \text{ W}$$

[c]



The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1866.67} - 0.2i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1 \left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000} \right) + v_2 \left(-\frac{1}{2000} \right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1 \left(-\frac{1}{2000} \right) + v_2 \left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1866.67} \right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

$$\text{Solving, } v_1 = 100 \text{ V; } v_2 = 56 \text{ V; } i_\Delta = 90 \text{ mA}$$

Calculate the power:

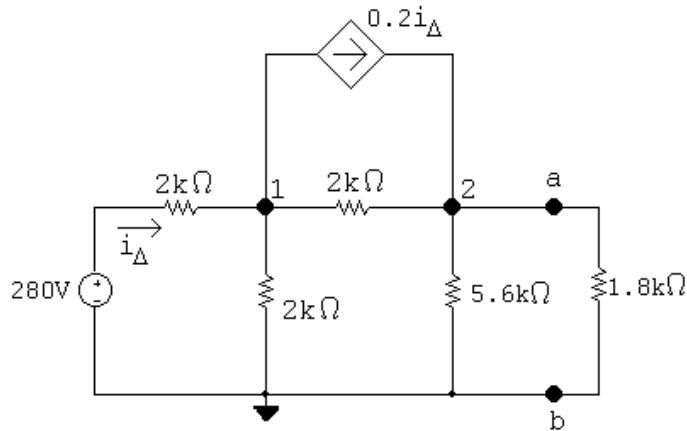
$$p_{280V} = -(280)(0.09) = -25.2 \text{ W}$$

$$p_{\text{dep source}} = (v_1 - v_2)(0.2i_\Delta) = 0.792 \text{ W}$$

$$\sum p_{\text{dev}} = 25.2 \text{ W}$$

$$\% \text{ delivered} = \frac{1.68}{25.2} \times 100 = 6.67\%$$

- [d] The 1.8 kΩ resistor in Appendix H is closest to the Thévenin equivalent resistance.
- [e] Substitute the 1.8 kΩ resistor into the original circuit and calculate the power developed by the sources in this circuit:



The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1800} - 0.2i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1 \left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000} \right) + v_2 \left(-\frac{1}{2000} \right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1 \left(-\frac{1}{2000} \right) + v_2 \left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1800} \right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

$$\text{Solving, } v_1 = 99.64 \text{ V}; \quad v_2 = 54.98 \text{ V}; \quad i_\Delta = 90.18 \text{ mA}$$

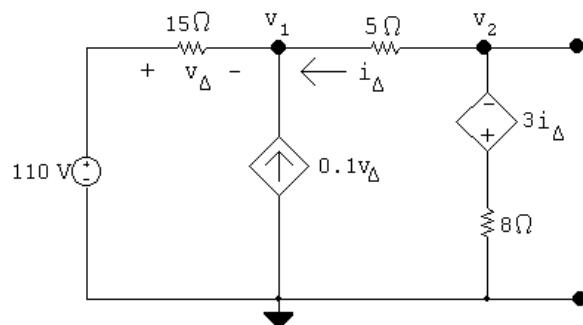
Calculate the power:

$$p_{280V} = -(280)(0.09018) = -25.25 \text{ W}$$

$$\sum p_{\text{dev}} = 25.25 \text{ mW}$$

$$p_L = (54.98)^2 / 1800 = 1.68 \text{ W} \quad \% \text{ delivered} = \frac{1.68}{25.25} \times 100 = 6.65\%$$

P 4.87 [a] Open circuit voltage



Node voltage equations:

$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - v_2}{5} = 0$$

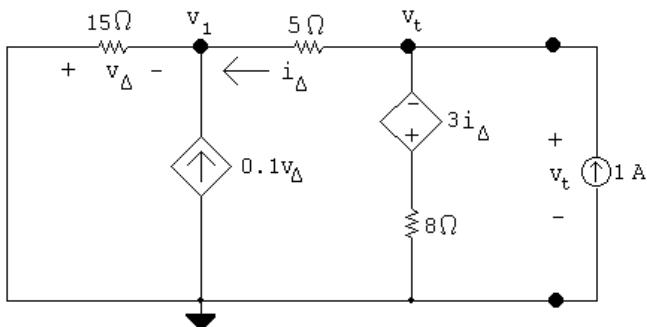
$$\frac{v_2 - v_1}{5} + \frac{v_2 + 3i_\Delta}{8} = 0$$

Constraint equations:

$$i_\Delta = \frac{v_2 - v_1}{5}; \quad v_\Delta = 110 - v_1$$

Solving, $v_2 = 55$ V = v_{Th}

Thévenin resistance using a test source:



$$\frac{v_1}{15} - 0.1v_\Delta + \frac{v_1 - v_t}{5} = 0$$

$$\frac{v_t - v_1}{5} + \frac{v_t + 3i_\Delta}{8} - 1 = 0$$

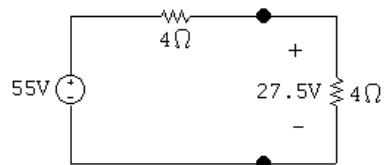
$$i_\Delta = \frac{v_t - v_1}{5}; \quad v_\Delta = -v_1$$

Solving, $v_t = 4$ V.

$$R_{\text{Th}} = \frac{v_t}{1} = 4 \Omega$$

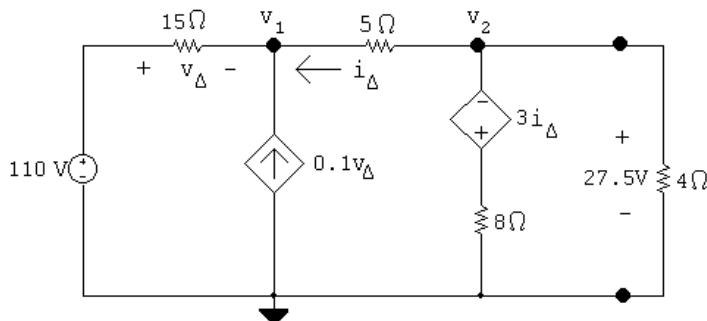
$$\therefore R_o = R_{\text{Th}} = 4 \Omega$$

[b]



$$p_{\text{max}} = \frac{(27.5)^2}{4} = 189.0625 \text{ W}$$

[c]



$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - 27.5}{5} = 0$$

$$v_\Delta = 110 - v_1$$

$$\text{Solving, } v_1 = 65 \text{ V,}$$

$$i_{110V} = \frac{65 - 110}{15} = -3 \text{ A}$$

$$p_{110V} = 110(-3) = -330 \text{ W}$$

$$i_\Delta = \frac{27.5 - 65}{5} = -7.5 \text{ A}$$

$$i_{CCVS} = \frac{27.5 + 3i_\Delta}{8} = 0.625 \text{ A}$$

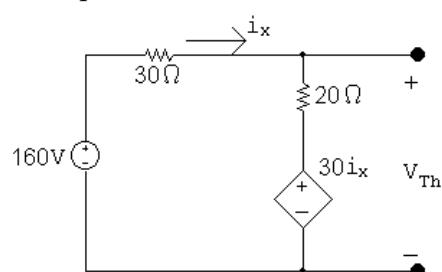
$$p_{CCVS} = -[3(-7.5)](0.625) = 14.0625 \text{ W}$$

$$p_{VCCS} = -[0.1(45)](65) = -292.5 \text{ W}$$

$$\sum p_{\text{dev}} = 330 + 292.5 = 622.5 \text{ W}$$

$$\% \text{ delivered} = \frac{189.0625}{622.5} \times 100 = 30.37\%$$

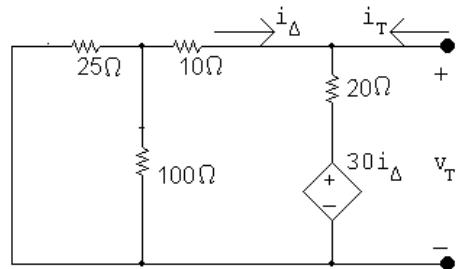
P 4.88 We begin by finding the Thévenin equivalent with respect to R_o . After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \quad i_\Delta = 2 \text{ A}$$

$$V_{Th} = 20i_{\Delta} + 30i_{\Delta} = 50i_{\Delta} = 100 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives

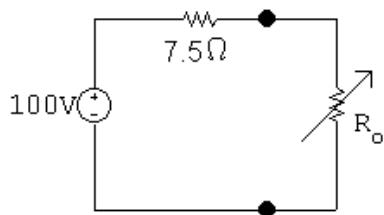


$$i_T = \frac{v_T}{30} + \frac{v_T - 30(-v_T/30)}{20}$$

$$\frac{i_T}{v_T} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{Th} = \frac{v_T}{i_T} = \frac{15}{2} = 7.5 \Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o} \right)^2 R_o = 250$$

$$\frac{10^4 R_o}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

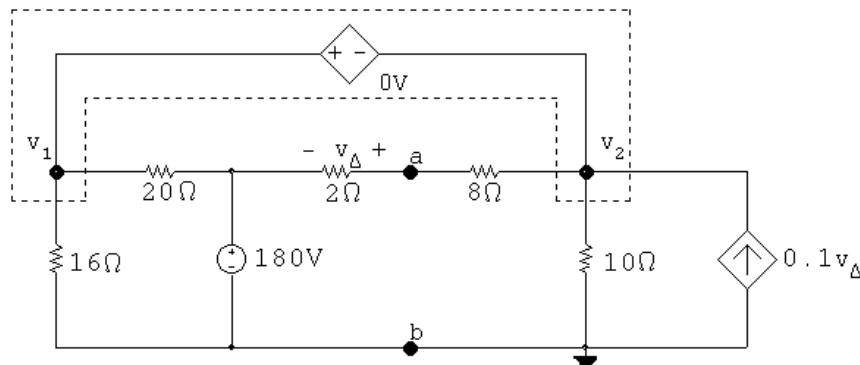
$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5 \Omega$$

$$R_o = 2.5 \Omega$$

P 4.89 [a] First find the Thévenin equivalent with respect to R_o .

Open circuit voltage: $i_\phi = 0; 184\phi = 0$



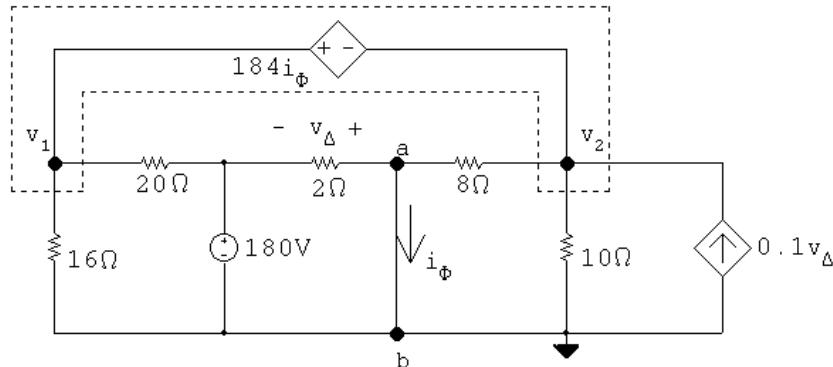
$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_1 - 180}{10} + \frac{v_1}{10} - 0.1v_\Delta = 0$$

$$v_\Delta = \frac{v_1 - 180}{10}(2) = 0.2v_1 - 36$$

$$v_1 = 80 \text{ V}; \quad v_\Delta = -20 \text{ V}$$

$$V_{Th} = 180 + v_\Delta = 180 - 20 = 160 \text{ V}$$

Short circuit current



$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2}{8} + \frac{v_2}{10} - 0.1(-180) = 0$$

$$v_2 + 184i_\phi = v_1$$

$$i_\phi = \frac{180}{2} + \frac{v_2}{8} = 90 + 0.125v_2$$

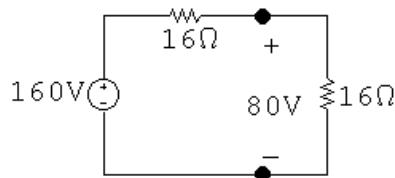
$$v_2 = -640 \text{ V}; \quad v_1 = 1200 \text{ V}$$

$$i_\phi = i_{sc} = 10 \text{ A}$$

$$R_{Th} = V_{Th}/i_{sc} = 160/10 = 16 \Omega$$

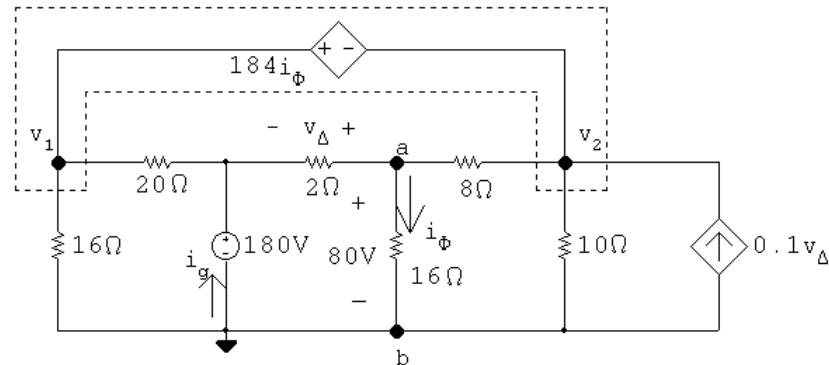
$$\therefore R_o = 16 \Omega$$

[b]



$$p_{\max} = (80)^2 / 16 = 400 \text{ W}$$

[c]



$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2 - 80}{8} + \frac{v_2}{10} - 0.1(80 - 180) = 0$$

$$v_2 + 184i_\phi = v_1; \quad i_\phi = 80/16 = 5 \text{ A}$$

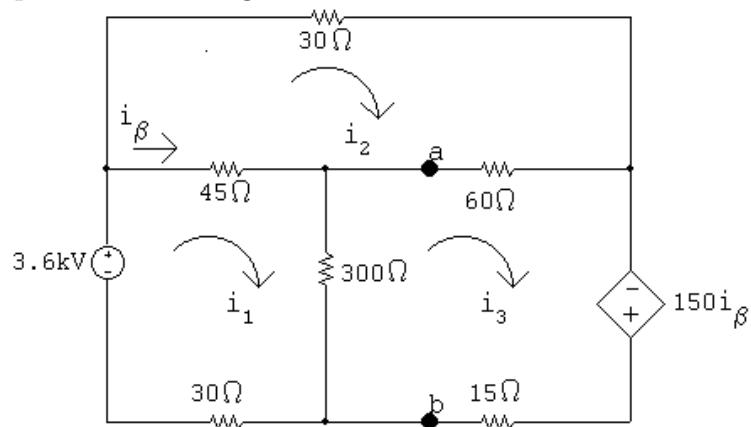
Therefore, $v_1 = 640 \text{ V}$ and $v_2 = -280 \text{ V}$; thus,

$$i_g = \frac{180 - 80}{2} + \frac{180 - 640}{20} = 27 \text{ A}$$

$$p_{180V} (\text{dev}) = (180)(27) = 4860 \text{ W}$$

P 4.90 [a] Find the Thévenin equivalent with respect to the terminals of R_L .

Open circuit voltage:



The mesh current equations are:

$$-3600 + 45(i_1 - i_2) + 300(i_1 - i_3) + 30i_1 = 0$$

$$30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) = 0$$

$$-150i_\beta + 15i_3 + 300(i_3 - i_1) + 60(i_3 - i_2) = 0$$

The dependent source constraint equation is:

$$i_\beta = i_1 - i_2$$

Place these equations in standard form:

$$i_1(45 + 300 + 30) + i_2(-45) + i_3(-300) + i_\beta(0) = 3600$$

$$i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) = 0$$

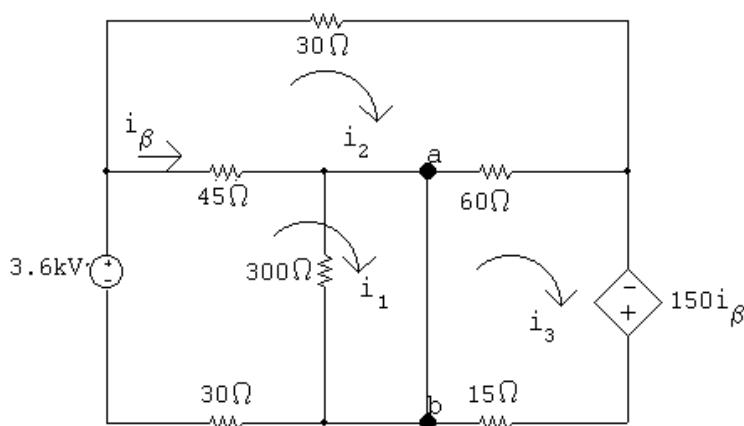
$$i_1(-300) + i_2(-60) + i_3(15 + 300 + 60) + i_\beta(-150) = 0$$

$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) = 0$$

Solving, $i_1 = 99.6 \text{ A}$; $i_2 = 78 \text{ A}$; $i_3 = 100.8 \text{ A}$; $i_\beta = 21.6 \text{ A}$

$$V_{\text{Th}} = 300(i_1 - i_3) = -360 \text{ V}$$

Short-circuit current:



The mesh current equations are:

$$-3600 + 45(i_1 - i_2) + 30i_1 = 0$$

$$30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) = 0$$

$$-150i_\beta + 15i_3 + 60(i_3 - i_2) = 0$$

The dependent source constraint equation is:

$$i_\beta = i_1 - i_2$$

Place these equations in standard form:

$$i_1(45 + 30) + i_2(-45) + i_3(0) + i_\beta(0) = 3600$$

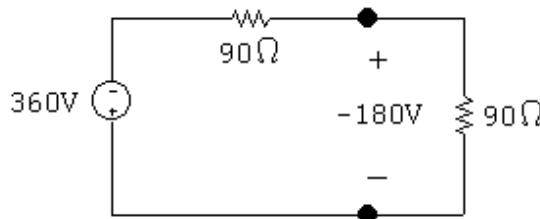
$$i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) = 0$$

$$i_1(0) + i_2(-60) + i_3(60 + 15) + i_\beta(-150) = 0$$

$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) = 0$$

Solving, $i_1 = 92 \text{ A}$; $i_2 = 73.33 \text{ A}$; $i_3 = 96 \text{ A}$; $i_\beta = 18.67 \text{ A}$

$$i_{sc} = i_1 - i_3 = -4 \text{ A}; \quad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-360}{-4} = 90 \Omega$$

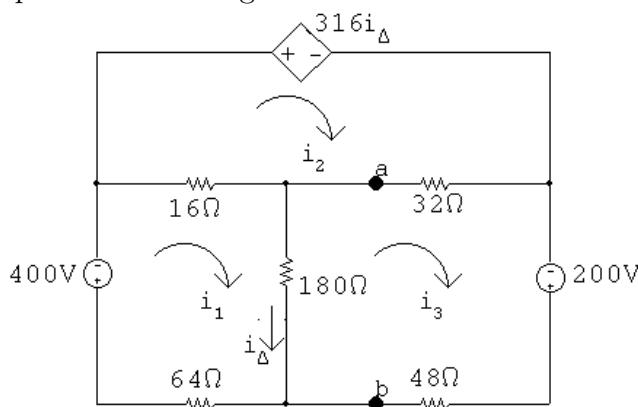


$$R_L = R_{Th} = 90 \Omega$$

$$[\text{b}] \quad p_{\max} = \frac{180^2}{90} = 360 \text{ W}$$

P 4.91 [a] We begin by finding the Thévenin equivalent with respect to the terminals of R_o .

Open circuit voltage



The mesh current equations are:

$$260i_1 - 16i_2 - 180i_3 = -400$$

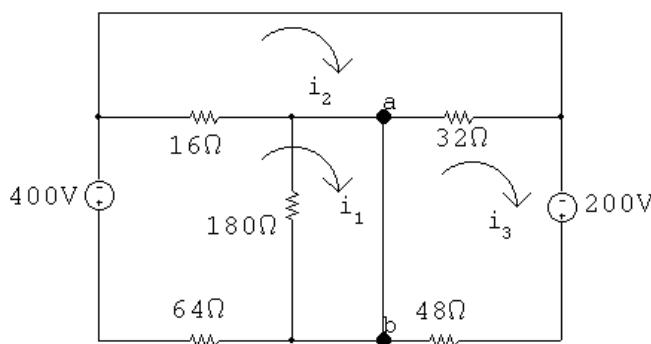
$$-16i_1 + 48i_2 - 32i_3 + 316(i_1 - i_3) = 0$$

$$-180i_1 - 32i_2 + 260i_3 = 200$$

Solving, $i_1 = 3 \text{ A}$; $i_2 = 17.5 \text{ A}$; $i_3 = 5 \text{ A}$; $i_\Delta = i_1 - i_3 = -2 \text{ A}$

Also, $V_{Th} = v_{oc} = 180i_\Delta = -360 \text{ V}$

Now find the short-circuit current.



Note with the short circuit from a to b that i_{Δ} is zero, hence $316i_{\Delta}$ is also zero.

The mesh currents are:

$$80i_1 - 16i_2 + 0i_3 = -400$$

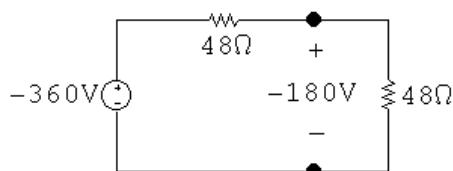
$$-16i_1 + 48i_2 - 32i_3 = 0$$

$$0i_1 - 32i_2 + 80i_3 = 200$$

Solving, $i_1 = -5 \text{ A}$; $i_2 = 0 \text{ A}$; $i_3 = 2.5 \text{ A}$

Then, $i_{sc} = i_1 - i_3 = -7.5 \text{ A}$

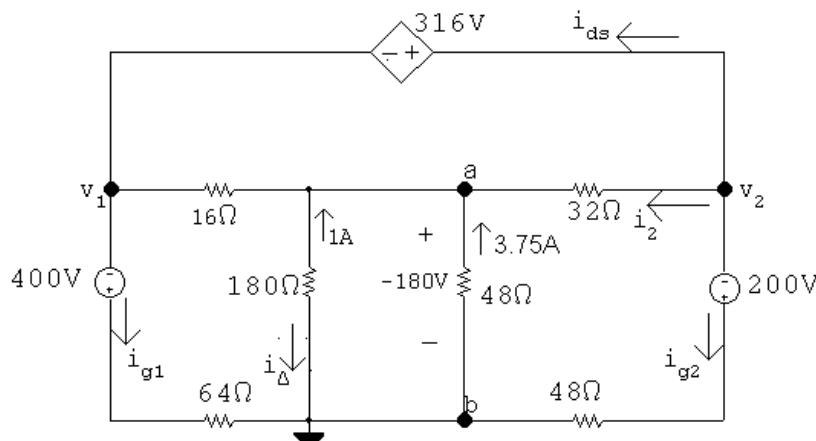
$$R_{Th} = \frac{-360}{-7.5} = 48 \Omega$$



For maximum power transfer $R_o = R_{Th} = 48 \Omega$

$$[\text{b}] \quad p_{\max} = \frac{180^2}{48} = 675 \text{ W}$$

- [c] The problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_{Δ} is -1 A , and hence $316i_{\Delta}$ is -316 V .



Using the node voltage method to find v_1 and v_2 yields

$$\frac{v_1 + 400}{64} + \frac{v_1 + 180}{16} + \frac{v_2 + 180}{32} + \frac{v_2 + 200}{48} = 0$$

$$v_2 - v_1 = 316$$

Solving, $v_1 = -336 \text{ V}$; $v_2 = -20 \text{ V}$.

It follows that

$$\begin{aligned} i_{g_1} &= \frac{-336 + 400}{64} = 1 \text{ A} \\ i_{g_2} &= \frac{-20 + 200}{48} = 3.75 \text{ A} \\ i_2 &= \frac{-20 + 180}{32} = 5 \text{ A} \\ i_{ds} &= -5 - 3.75 = -8.75 \text{ A} \\ p_{400V} &= -400i_{g_1} = -400 \text{ W} \\ p_{200V} &= -200i_{g_2} = -750 \text{ W} \\ p_{ds} &= 316i_{ds} = -2765 \text{ W} \end{aligned}$$

$$\therefore \sum p_{\text{dev}} = 400 + 750 + 2765 = 3915 \text{ W}$$

$$\therefore \% \text{ delivered} = \frac{675}{3915}(100) = 17.24\%$$

$\therefore 17.24\%$ of developed power is delivered to load

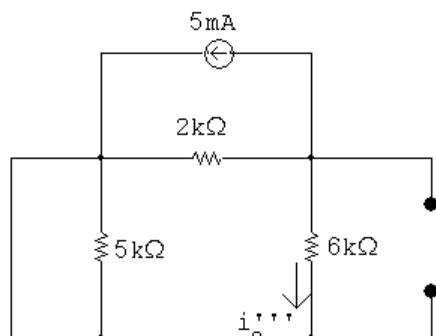
- [d] The resistor from Appendix H that is closest to the Thévenin resistance is 47Ω . To calculate the power delivered to a 47Ω load resistor, calculate the current using the Thévenin circuit and use it to find the power delivered to the load resistor:

$$i_{47} = \frac{-360}{47 + 48} = 3.7895 \text{ A}$$

$$p_{47} = 47(3.7895)^2 = 674.9 \text{ W}$$

Thus, using a 47Ω resistor selected from Appendix H will cause 674.9 W of power to be delivered to the load, compared to the maximum power of 675 W that will be delivered if a 48Ω resistor is used.

- P 4.92 [a] By hypothesis $i'_o + i''_o = 3 \text{ mA}$.



$$i'''_o = -5 \frac{(2)}{(8)} = -1.25 \text{ mA}; \quad \therefore i_o = 3.5 - 1.25 = 2.25 \text{ mA}$$

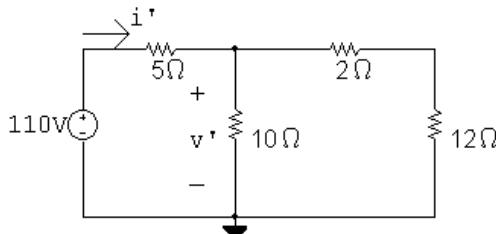
[b] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$\therefore v_b = 13.5 \text{ V}$$

$$i_o = \frac{v_b}{6} = 2.25 \text{ mA}$$

P 4.93 [a] 110 V source acting alone:

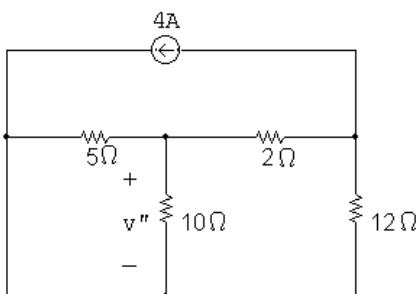


$$R_e = \frac{10(14)}{24} = \frac{35}{6} \Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13} \text{ A}$$

$$v' = \left(\frac{35}{6}\right) \left(\frac{132}{13}\right) = \frac{770}{13} \text{ V} = 59.231 \text{ V}$$

4 A source acting alone:

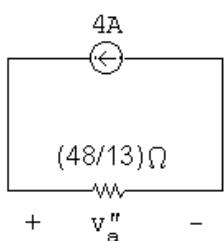


$$5\Omega \parallel 10\Omega = 50/15 = 10/3 \Omega$$

$$10/3 + 2 = 16/3 \Omega$$

$$16/3 \parallel 12 = 48/13 \Omega$$

Hence our circuit reduces to:



It follows that

$$v_a'' = 4(48/13) = (192/13) \text{ V}$$

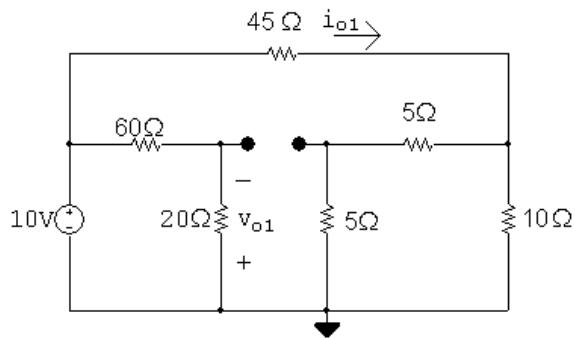
and

$$v'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13) \text{ V} = -9.231 \text{ V}$$

$$\therefore v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50 \text{ V}$$

$$[\text{b}] p = \frac{v^2}{10} = 250 \text{ W}$$

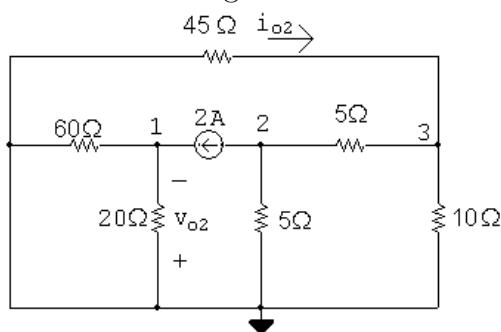
P 4.94 Voltage source acting alone:



$$i_{o1} = \frac{10}{45 + (5+5)\parallel 10} = \frac{10}{45+5} = 0.2 \text{ A}$$

$$v_{o1} = \frac{20}{20+60}(-10) = -2.5 \text{ V}$$

Current source acting alone:



$$\frac{v_2}{5} + 2 + \frac{v_2 - v_3}{5} = 0$$

$$\frac{v_3}{10} + \frac{v_3 - v_2}{5} + \frac{v_3}{45} = 0$$

Solving, $v_2 = -7.25 \text{ V} = v_{o2}$; $v_3 = -4.5 \text{ V}$

$$i_{o2} = -\frac{v_3}{45} = -0.1 \text{ A}$$

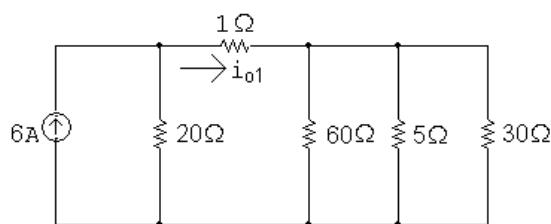
$$i_{20} = \frac{60 \parallel 20}{20}(2) = 1.5 \text{ A}$$

$$v_{o2} = -20i_{20} = -20(1.5) = -30 \text{ V}$$

$$\therefore v_o = v_{o1} + v_{o2} = -2.5 - 30 = -32.5 \text{ V}$$

$$i_o = i_{o1} + i_{o2} = 0.2 + 0.1 = 0.3 \text{ A}$$

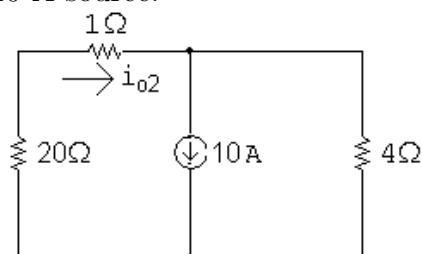
P 4.95 6 A source:



$$30 \Omega \parallel 5 \Omega \parallel 60 \Omega = 4 \Omega$$

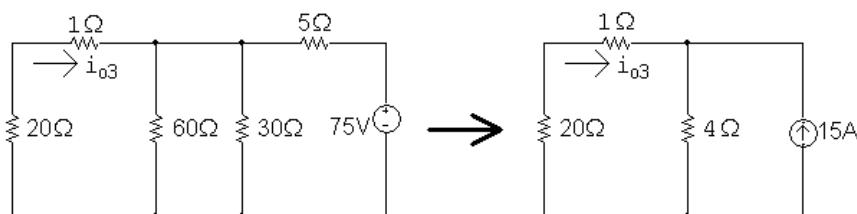
$$\therefore i_{o1} = \frac{20}{20+5}(6) = 4.8 \text{ A}$$

10 A source:



$$i_{o2} = \frac{4}{25}(10) = 1.6 \text{ A}$$

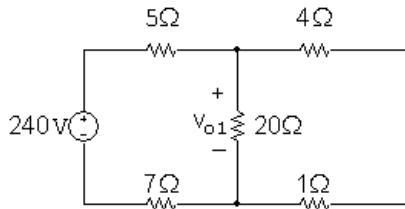
75 V source:



$$i_{o3} = -\frac{4}{25}(15) = -2.4 \text{ A}$$

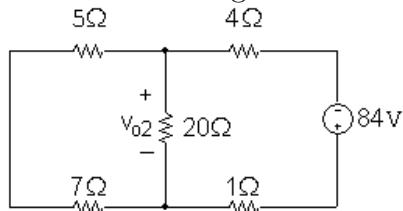
$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4 \text{ A}$$

P 4.96 240 V source acting alone:



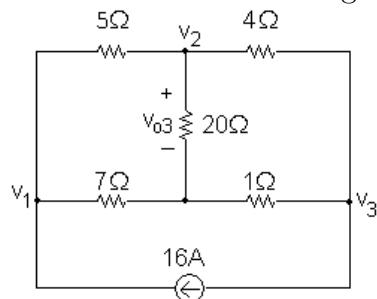
$$v_{o1} = \frac{20\parallel 5}{5 + 7 + 20\parallel 5}(240) = 60 \text{ V}$$

84 V source acting alone:



$$v_{o2} = \frac{20\parallel 12}{1 + 4 + 20\parallel 12}(-84) = -50.4 \text{ V}$$

16 A current source acting alone:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

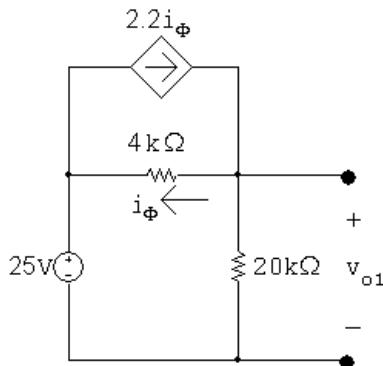
$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4 \text{ V} = v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28 \text{ V}$$

P 4.97 Voltage source acting alone:

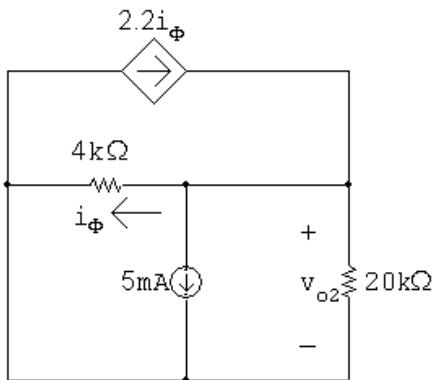


$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20,000} - 2.2 \left(\frac{v_{o1} - 25}{4000} \right) = 0$$

$$\text{Simplifying } 5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$$

$$\therefore v_{o1} = 30 \text{ V}$$

Current source acting alone:



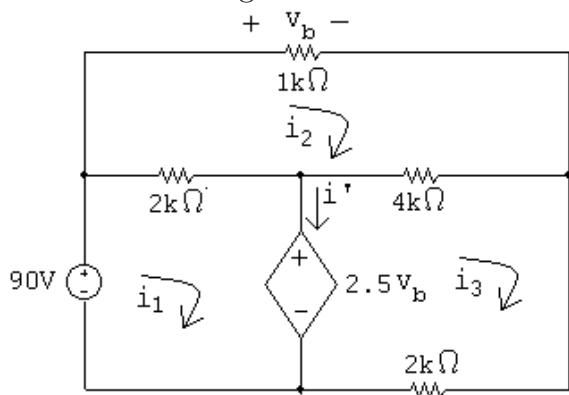
$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20,000} + 0.005 - 2.2 \left(\frac{v_{o2}}{4000} \right) = 0$$

$$\text{Simplifying } 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$$

$$\therefore v_{o2} = 20 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 30 + 20 = 50 \text{ V}$$

P 4.98 90-V source acting alone:



$$2000(i_1 - i_2) + 2.5v_b = 90$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

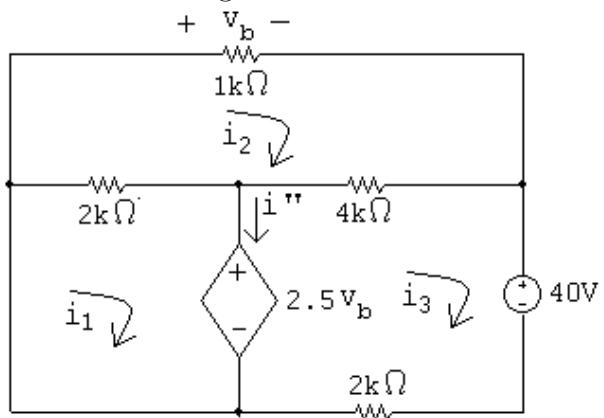
$$-4000i_2 + 6000i_3 - 2.5v_b = 0$$

$$v_b = 1000i_2$$

Solving,

$$i_1 = 37.895 \text{ mA}; \quad i_3 = 30.789 \text{ mA}; \quad i' = i_1 - i_3 = 7.105 \text{ mA}$$

40-V source acting alone:



$$2000(i_1 - i_2) + 2.5v_b = 0$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

$$-4000i_2 + 6000i_3 - 2.5v_b = -40$$

$$v_b = 1000i_2$$

Solving,

$$i_1 = 2.105 \text{ mA}; \quad i_3 = -15.789 \text{ mA}; \quad i'' = i_1 - i_3 = 17.895 \text{ mA}$$

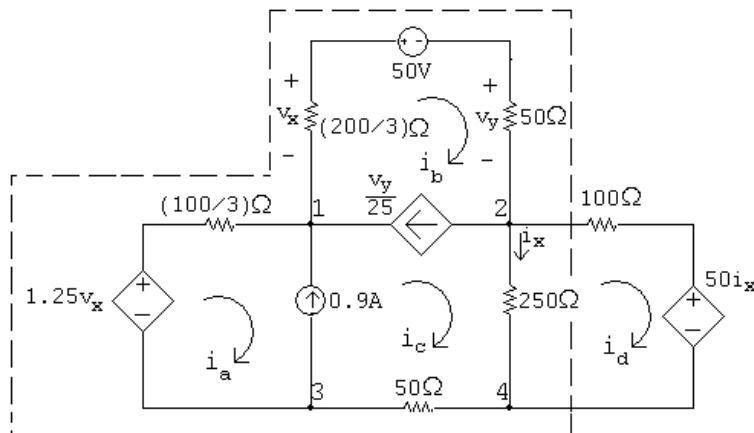
$$\text{Hence, } i = i' + i'' = 7.105 + 17.895 = 25 \text{ mA}$$

P 4.99 [a] In studying the circuit in Fig. P4.99 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node-voltage approach will require solving three node voltage equations along with equations involving v_x , v_y , and i_x .

The mesh-current approach will require writing one mesh equation and one supermesh equation plus five constraint equations involving the five sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 50 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a , i_c and i_d .

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-1.25v_x + (100/3)i_a + (200/3)i_b + 50 + 50i_b + 250(i_c - i_d) + 50i_c = 0$$

The remaining mesh equation is

$$50i_x + 350i_d - 250i_c = 0$$

The constraint equations are

$$\frac{v_y}{25} = i_b - i_c; \quad 0.9 = i_c - i_a; \quad v_x = -(200/3)i_b$$

$$v_y = 50i_b; \quad i_x = i_c - i_d$$

Solving,

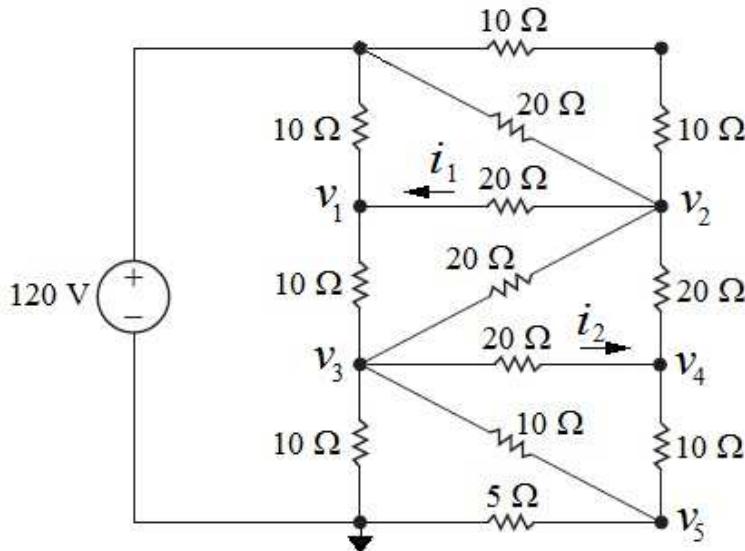
$$i_a = -0.3 \text{ A}; \quad i_b = -0.6 \text{ A}; \quad i_c = 0.6 \text{ A}; \quad i_d = 0.4 \text{ A}$$

Finally,

$$p_{50V} = 50i_b = -30 \text{ W}$$

The 50 V source delivers 30 W of power.

P 4.100



$$\text{At } v_1: \frac{v_1 - 120}{10} + \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{10} = 0$$

$$\text{At } v_2: \frac{v_2 - 120}{20} + \frac{v_2 - 120}{20} + \frac{v_2 - v_1}{20} + \frac{v_1 - v_3}{20} + \frac{v_2 - v_4}{20} = 0$$

$$\text{At } v_3: \frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{20} + \frac{v_3 - v_4}{20} + \frac{v_3}{10} + \frac{v_3 - v_5}{10} = 0$$

$$\text{At } v_4: \frac{v_4 - v_2}{20} + \frac{v_4 - v_3}{20} + \frac{v_4 - v_5}{10} = 0$$

$$\text{At } v_5: \frac{v_5 - v_4}{10} + \frac{v_5 - v_3}{10} + \frac{v_5}{5} = 0$$

A calculator solution yields

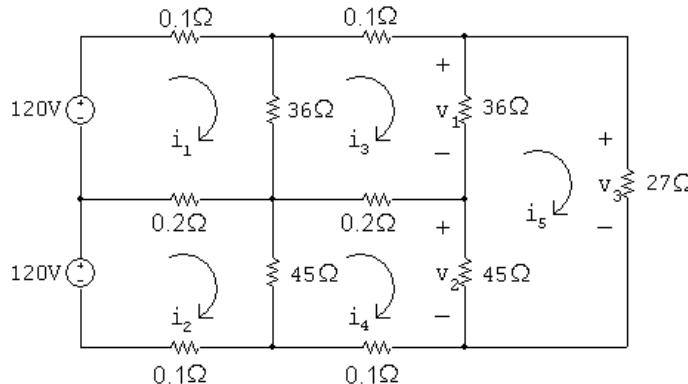
$$v_1 = 80 \text{ A}; \quad v_2 = 80 \text{ A};$$

$$v_3 = 40 \text{ A}; \quad v_4 = 40 \text{ A};$$

$$v_5 = 20 \text{ A}.$$

$$\therefore i_1 = \frac{v_2 - v_1}{20} = 0 \text{ A}; \quad i_1 = \frac{v_3 - v_4}{20} = 0 \text{ A}$$

P 4.101



The mesh equations are:

$$\begin{aligned} i_1(36.3) + i_2(-0.2) + i_3(-36) + i_4(0) + i_5(0) &= 120 \\ i_1(-0.2) + i_2(45.3) + i_3(0) + i_4(-45) + i_5(0) &= 120 \\ i_1(-36) + i_2(0) + i_3(72.3) + i_4(-0.2) + i_5(-36) &= 0 \\ i_1(0) + i_2(-45) + i_3(-0.2) + i_4(90.3) + i_5(-45) &= 0 \\ i_1(0) + i_2(0) + i_3(-36) + i_4(-45) + i_5(108) &= 0 \end{aligned}$$

Solving,

$$i_1 = 15.226 \text{ A}; \quad i_2 = 13.953 \text{ A}; \quad i_3 = 11.942 \text{ A}; \quad i_4 = 11.314 \text{ A}; \quad i_5 = 8.695 \text{ A}$$

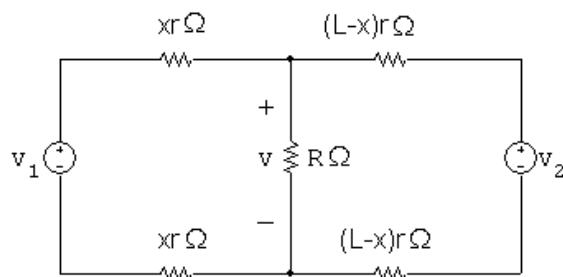
Find the requested voltages:

$$v_1 = 36(i_3 - i_5) = 118.6 \text{ V}$$

$$v_2 = 45(i_4 - i_5) = 117.8 \text{ V}$$

$$v_3 = 27i_5 = 234.8 \text{ V}$$

P 4.102 [a]



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L - x)} = 0$$

$$v \left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L - x)} \right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L - x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$$\frac{dv}{dx} = 0 \quad \text{when numerator is zero.}$$

The numerator simplifies to

$$x^2 + \frac{2Lv_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1} \left\{ -v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_2 - v_1)^2} \right\}$$

$$[c] \quad x = \frac{L}{v_2 - v_1} \left\{ -v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_1 - v_2)^2} \right\}$$

$$v_2 = 1200 \text{ V}, \quad v_1 = 1000 \text{ V}, \quad L = 16 \text{ km}$$

$$r = 5 \times 10^{-5} \Omega/\text{m}; \quad R = 3.9 \Omega$$

$$\frac{L}{v_2 - v_1} = \frac{16,000}{1200 - 1000} = 80; \quad v_1v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

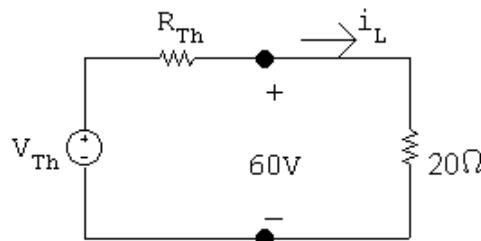
$$x = 80 \{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80 \{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$\begin{aligned} v_{\min} &= \frac{v_1RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2} \\ &= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16,000) + 10 \times 10^{-5}(16,000)(4000) - 10 \times 10^{-5}(16 \times 10^6)} \\ &= 975 \text{ V} \end{aligned}$$

P 4.103 [a]



$$v_{oc} = V_{Th} = 75 \text{ V}; \quad i_L = \frac{60}{20} = 3 \text{ A}; \quad i_L = \frac{75 - 60}{R_{Th}} = \frac{15}{R_{Th}}$$

Therefore $R_{\text{Th}} = \frac{15}{3} = 5 \Omega$

$$[\mathbf{b}] \quad i_L = \frac{v_o}{R_L} = \frac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$$

$$\text{Therefore } R_{\text{Th}} = \frac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\frac{V_{\text{Th}}}{v_o} - 1 \right) R_L$$

$$\text{P 4.104} \quad \frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 4.105 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A} = -14.5833 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1 \text{ A}$

$$\therefore \Delta v_1 = \left(-\frac{175}{12}\right)(-1) = \frac{175}{12} = 14.583 \text{ V}$$

Thus, $v_1 = 25 + 14.583 = 39.583 \text{ V}$

Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5 \text{ V}$

The PSpice solution is

$$v_1 = 39.583 \text{ V}$$

and

$$v_2 = 102.5 \text{ V}$$

These values are in agreement with our predicted values.

P 4.106 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1 \text{ A}$

$$\therefore \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5 \text{ V}$

Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105 \text{ V}$

The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 4.107 From the solutions to Problems 4.104 — 4.106 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \quad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \quad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1 \text{ A}$$

$$\Delta I_{g2} = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$$\Delta v_2 = 12.5 + 15 = 27.5 \text{ V}$$

Hence

$$v_1 = 25 + 27.0833 = 52.0833 \text{ V}$$

$$v_2 = 90 + 27.5 = 117.5 \text{ V}$$

The PSpice solution is

$$v_1 = 52.0830 \text{ V}$$

and

$$v_2 = 117.5 \text{ V}$$

These values are in agreement with our predicted values.

P 4.108 By hypothesis,

$$\Delta R_1 = 27.5 - 25 = 2.5 \Omega$$

$$\Delta R_2 = 4.5 - 5 = -0.5 \Omega$$

$$\Delta R_3 = 55 - 50 = 5 \Omega$$

$$\Delta R_4 = 67.5 - 75 = -7.5 \Omega$$

So

$$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168 \text{ V}$$

$$\therefore v_1 = 25 + 4.9168 = 29.9168 \text{ V}$$

$$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1 \text{ V}$$

$$\therefore v_2 = 90 - 1.1 = 88.9 \text{ V}$$

The PSpice solution is

$$v_1 = 29.6710 \text{ V}$$

and

$$v_2 = 88.5260 \text{ V}$$

Note our predicted values are within a fraction of a volt of the actual values.

5

The Operational Amplifier

Assessment Problems

AP 5.1 [a] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \quad \text{so} \quad v_o = -5v_s$$

$$v_s(\text{ V}) \quad 0.4 \quad 2.0 \quad 3.5 \quad -0.6 \quad -1.6 \quad -2.4$$

$$v_o(\text{ V}) \quad -2.0 \quad -10.0 \quad -15.0 \quad 3.0 \quad 8.0 \quad 10.0$$

Two of the values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \quad v_s = -2 \text{ V}$$

$$\text{Therefore } -2 \leq v_s \leq 3 \text{ V}$$

AP 5.2 From Assessment Problem 5.1

$$\begin{aligned} v_o &= (-R_f/R_i)v_s = (-R_x/16,000)v_s = (-R_x/16,000)(-0.640) \\ &= 0.64R_x/16,000 = 4 \times 10^{-5}R_x \end{aligned}$$

Use the negative power supply value to determine one limit on the value of R_x :

$$4 \times 10^{-5}R_x = -15 \quad \text{so} \quad R_x = -15/4 \times 10^{-5} = -375 \text{ k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x :

$$4 \times 10^{-5} R_x = 10 \quad \text{so} \quad R_x = 10/4 \times 10^{-5} = 250 \text{ k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250 \text{ k}\Omega$$

AP 5.3 [a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_a)v_a + (-R_f/R_b)v_b = -(250/5)v_a - (250/25)v_b = -50v_a - 10v_b$$

Substituting the values for v_a and v_b :

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_a - 10(0.25) = -50v_a - 2.5 = -10 \text{ V}$$

Therefore $50v_a = 7.5$, so $v_a = 0.15 \text{ V}$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_b = -5 - 10v_b = -10 \text{ V};$$

Therefore $10v_b = 5$, so $v_b = 0.5 \text{ V}$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.

Repeat part (a):

$$v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_a + 2.5 = -10 \text{ V}; \quad 50v_a = 12.5, \quad v_a = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_b = 15 \text{ V}; \quad 10v_b = 20; \quad v_b = 2.0 \text{ V}$$

AP 5.4 [a] Write a node voltage equation at v_n ; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63,000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \quad \text{so} \quad v_o = 15v_n$$

Now use voltage division to calculate v_p . We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15,000 + R_x} (0.400)$$

Now substitute the value $R_x = 60$ kΩ:

$$v_p = \frac{60,000}{15,000 + 60,000} (0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_o

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

- [b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15 \left(\frac{0.4R_x}{15,000 + R_x} \right) = 5$$

$$15(0.4R_x) = 5(15,000 + R_x) \quad \text{so} \quad R_x = 75 \text{ k}\Omega$$

- AP 5.5 [a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_b - \frac{50}{10}v_a$$

Simplify this expression and substitute in the value for v_b :

$$v_o = 5(v_b - v_a) = 20 - 5v_a$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 6 \text{ V}$$

Therefore $2 \leq v_a \leq 6 \text{ V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)} v_b - 5v_a = 4v_b - 5v_a$$

Now substitute the value for v_b :

$$v_o = 4(4) - 5v_a = 16 - 5v_a$$

Set this expression for v_o to the positive power supply value:

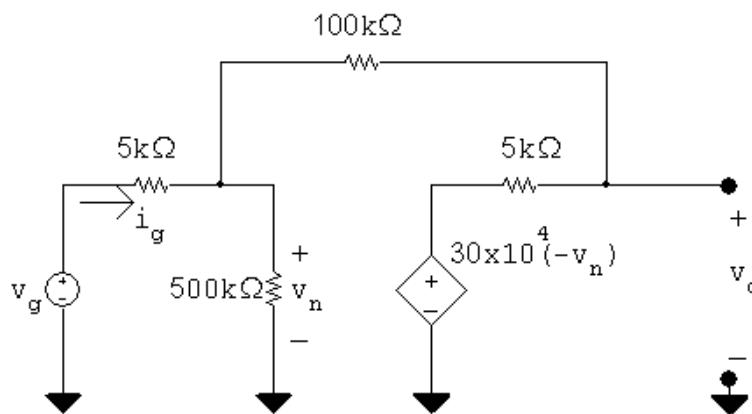
$$16 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 5.2 \text{ V}$$

$$\text{Therefore } 1.2 \leq v_a \leq 5.2 \text{ V}$$

AP 5.6 [a] Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500,000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300,000(-v_n)}{5000} + \frac{v_o - v_n}{100,000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o :

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6,000,445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985 v_g; \quad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n :

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \quad v_n = 69.995 \mu \text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35 \Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

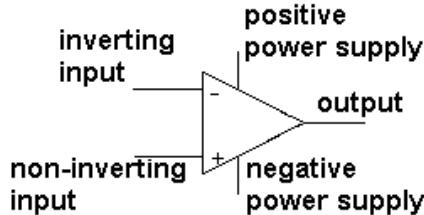
Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$

Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000 \Omega$$

Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



- [b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0 \text{ A}$.
- [c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.
- [d] Write a node voltage equation at v_n :

$$\frac{v_n - 2}{4000} + \frac{v_n - v_o}{12,000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2}{4000} - \frac{v_o}{12,000} = 0 \quad \text{so} \quad v_o = -6 \text{ V}$$

P 5.2 [a] Let the value of the voltage source be v_s :

$$\frac{v_n - v_s}{4000} + \frac{v_n - v_o}{12,000} = 0$$

But $v_n = v_p = 0$. Therefore,

$$v_o = -\frac{12,000}{4000}v_s = -3v_s$$

When $v_s = -6 \text{ V}$, $v_o = -3(-6) = 18 \text{ V}$; saturates at $v_o = 15 \text{ V}$.

When $v_s = -3.5 \text{ V}$, $v_o = -3(-3.5) = 10.5 \text{ V}$.

When $v_s = -1.25 \text{ V}$, $v_o = -3(-1.25) = 3.75 \text{ V}$.

When $v_s = 2.4 \text{ V}$, $v_o = -3(2.4) = -7.2 \text{ V}$.

When $v_s = 4.5 \text{ V}$, $v_o = -3(4.5) = -13.5 \text{ V}$.

When $v_s = 5.4 \text{ V}$, $v_o = -3(5.4) = -16.2 \text{ V}$; saturates at $v_o = -15 \text{ V}$.

$$[b] -3v_s = 15 \quad \text{so} \quad v_s = \frac{15}{-3} = -5 \text{ V}$$

$$-3v_s = -15 \quad \text{so} \quad v_s = \frac{-15}{-3} = 5 \text{ V}$$

The range of source voltages that avoids saturation is $-5 \text{ V} \leq v_s \leq 5 \text{ V}$.

$$\text{P 5.3} \quad v_p = \frac{5000}{5000 + 10,000}(6) = 2 \text{ V} = v_n$$

$$\frac{v_n + 5}{3000} + \frac{v_n - v_o}{6000} = 0$$

$$2(2 + 5) + (2 - v_o) = 0$$

$$v_o = 16 \text{ V}$$

$$i_L = \frac{v_o}{8000} = \frac{16}{8000} = 2000 \times 10^{-6}$$

$$i_L = 2 \text{ mA}$$

$$\text{P 5.4} \quad \frac{v_b - v_a}{5000} + \frac{v_b - v_o}{40,000} = 0, \quad \text{therefore } v_o = 9v_b - 8v_a$$

[a] $v_a = 1.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = -12 \text{ V}$

[b] $v_a = -0.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = 4 \text{ V}$

[c] $v_a = 1 \text{ V}, \quad v_b = 2.5 \text{ V}, \quad v_o = 14.5 \text{ V}$

[d] $v_a = 2.5 \text{ V}, \quad v_b = 1 \text{ V}, \quad v_o = -11 \text{ V}$

[e] $v_a = 2.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = -20 \text{ V} \quad (\text{saturates at } -16 \text{ V})$

[f] If $v_b = 2 \text{ V}, \quad v_o = 18 - 8v_a = \pm 16$

$$\therefore 0.25 \text{ V} \leq v_a \leq 4.25 \text{ V}$$

$$\text{P 5.5} \quad v_o = -(0.5 \times 10^{-3})(10,000) = -5 \text{ V}$$

$$\therefore i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1 \text{ mA}$$

$$\text{P 5.6} \quad \text{[a]} \quad i_a = \frac{240 \times 10^{-3}}{8000} = 30 \mu\text{A}$$

$$v_1 = -40 \times 10^3 i_2 = -3 \text{ V}$$

[b] $\frac{0 - v_a}{60,000} = i_a \quad \text{so} \quad v_a = -60,000i_a = -1.8 \text{ V}$

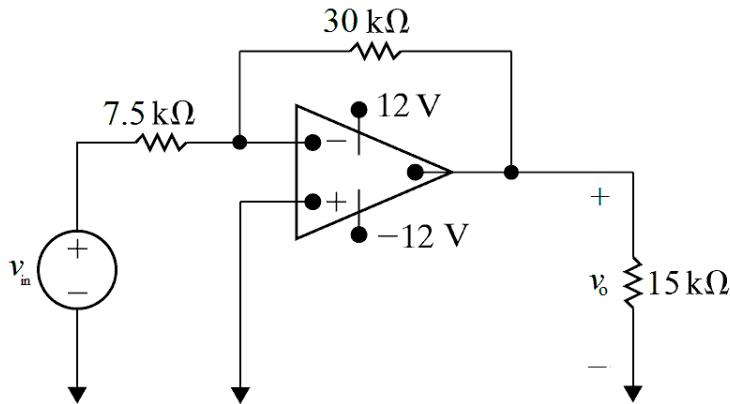
[c] $\frac{v_a}{60,000} + \frac{v_a}{40,000} + \frac{v_a - v_o}{30,000} = 0$

$$\therefore v_o = 2.25v_a = -4.05 \text{ V}$$

[d] $i_o = \frac{-v_o}{20,000} + \frac{v_a - v_o}{30,000} = 277.5 \mu\text{A}$

- P 5.7 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\text{ M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.8 [a] $\frac{30,000}{R_{\text{in}}} = 4$ so $R_{\text{in}} = \frac{30,000}{4} = 7500 = 7.5\text{ k}\Omega$



[b] $-4v_{\text{in}} = 12$ so $v_{\text{in}} = \frac{12}{-4} = -3\text{ V}$

$$-4v_{\text{in}} = -12 \quad \text{so} \quad v_{\text{in}} = \frac{12}{4} = 3\text{ V}$$

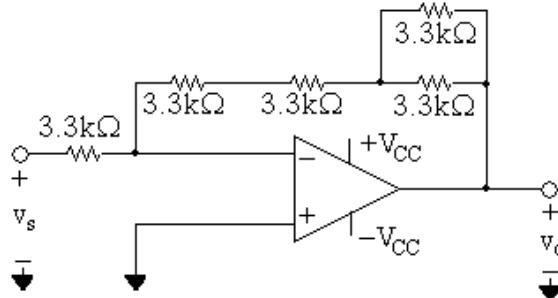
$$\therefore -3\text{ V} \leq v_{\text{in}} \leq 3\text{ V}$$

[c] $-\frac{R_f}{7500}(2) = -12 \quad \text{so} \quad R_f = 45\text{ k}\Omega$

$$\left| \frac{v_o}{v_{\text{in}}} \right| = \frac{R_f}{R_{\text{in}}} = \frac{45,000}{7500} = 6$$

The amplifier has a gain of 6.

- P 5.9 [a] The gain of an inverting amplifier is the negative of the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 2.5, the feedback resistor must be 2.5 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present one here that uses $3.3\text{ k}\Omega$ resistors. Use a single $3.3\text{ k}\Omega$ resistor as the input resistor. Then construct a network of $3.3\text{ k}\Omega$ resistors whose equivalent resistance is $2.5(3.3) = 8.25\text{ k}\Omega$ by connecting two resistors in parallel and connecting the parallel resistors in series with two other resistors. The resulting circuit is shown here:



- [b] To amplify signals in the range -2 V to 3 V without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain.

$$-2.5(-2) = 5 \text{ V} \quad \text{and} \quad -2.5(3) = -7.5 \text{ V}$$

Thus, the power supplies should have values of -7.5 V and 5 V .

P 5.10 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50,000} = 0$$

$$-25v_g - v_\Delta = 0, \quad \therefore v_\Delta = -25(40 \times 10^{-3}) = -1 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50,000} + \frac{v_\Delta - v_o}{(1 - \alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1 - \alpha} = 0$$

$$v_\Delta \left(\frac{1}{\alpha} + 2 + \frac{1}{1 - \alpha} \right) = \frac{v_o}{1 - \alpha}$$

$$\therefore v_o = -1 \left[1 + 2(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right]$$

$$\text{When } \alpha = 0.2, \quad v_o = -1(1 + 1.6 + 4) = -6.6 \text{ V}$$

$$\text{When } \alpha = 1, \quad v_o = -1(1 + 0 + 0) = -1 \text{ V}$$

$$\therefore -6.6 \text{ V} \leq v_o \leq -1 \text{ V}$$

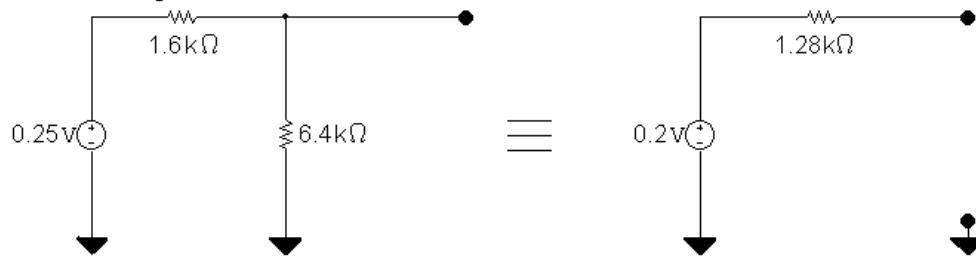
[b] $-1 \left[1 + 2(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right] = -7$

$$\alpha + 2\alpha(1 - \alpha) + (1 - \alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

- P 5.11 [a] Replace the combination of v_g , $1.6\text{ k}\Omega$, and the $6.4\text{ k}\Omega$ resistors with its Thévenin equivalent.



$$\text{Then } v_o = \frac{-[12 + \sigma 50]}{1.28} (0.20)$$

At saturation $v_o = -5\text{ V}$; therefore

$$-\left(\frac{12 + \sigma 50}{1.28}\right) (0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

Thus for $0 \leq \sigma \leq 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272$, $v_o = \frac{-(12 + 13.6)}{1.28} (0.20) = -4\text{ V}$

$$\text{Also } \frac{v_o}{10} + \frac{v_o}{25.6} + i_o = 0$$

$$\therefore i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6} \text{ mA} = 556.25 \mu\text{A}$$

- P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{44}v_a - \frac{220}{27.5}v_b - \frac{220}{80}v_c = -5 - 12 + 11 = -6\text{ V}$

[c] $v_o = -6 - 8v_b = \pm 10$

$$\therefore v_b = -0.5\text{ V} \quad \text{when } v_o = 10\text{ V};$$

$$v_b = 2\text{ V} \quad \text{when } v_o = -10\text{ V}$$

$$\therefore -0.5\text{ V} \leq v_b \leq 2\text{ V}$$

P 5.13 $v_o = -\left[\frac{R_f}{4000}(0.2) + \frac{R_f}{5000}(0.15) + \frac{R_f}{20,000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_f; \quad R_f = 60\text{ k}\Omega; \quad \therefore 0 \leq R_f \leq 60\text{ k}\Omega$$

- P 5.14 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_d - v_a}{60,000} + \frac{v_d - v_b}{20,000} + \frac{v_d - v_c}{36,000} + \frac{v_d}{270,000} + \frac{v_d - v_o}{180,000} = 0$$

Multiply through by 180,000, plug in the values of input voltages, and rearrange to solve for v_o :

$$v_o = 180,000 \left(\frac{3}{60,000} + \frac{-3}{20,000} + \frac{1}{36,000} + \frac{6}{270,000} + \frac{6}{180,000} \right)$$

$$= -3 \text{ V}$$

- [b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{6-3}{60,000} + \frac{6-9}{20,000} + \frac{6-v_c}{36,000} + \frac{6}{270,000} + \frac{6-v_o}{180,000} = 0$$

Simplify and solve for v_o :

$$9 - 27 + 30 - 5v_c + 4 + 6 - v_o = 0 \quad \text{so} \quad v_o = 22 - 5v_c$$

Set v_o to the positive power supply voltage and solve for v_c :

$$22 - 5v_c = 10 \quad \therefore \quad v_c = 2.4 \text{ V}$$

Set v_o to the negative power supply voltage and solve for v_c :

$$22 - 5v_c = -10 \quad \therefore \quad v_c = 6.4 \text{ V}$$

Therefore,

$$2.4 \text{ V} \leq v_c \leq 6.4 \text{ V}$$

P 5.15 [a] $\frac{6-3}{60,000} + \frac{6-9}{20,000} + \frac{6-5}{36,000} + \frac{6}{270,000} + \frac{6-v_o}{R_f} = 0$

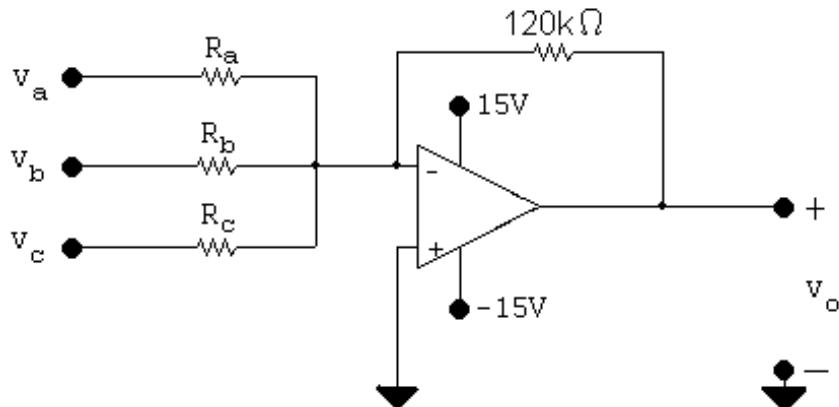
$$\frac{6-v_o}{R_f} = -5 \times 10^{-5} \quad \text{so} \quad R_f = \frac{6-v_o}{-5 \times 10^{-5}}$$

$$\text{For } v_o = 10 \text{ V}, \quad R_f = 80 \text{ k}\Omega$$

$$\text{For } v_o = -10 \text{ V}, \quad R_f < 0 \quad \text{so this solution is not possible.}$$

[b] $i_o = -(i_f + i_{16k}) = -\left[\frac{10-6}{80,000} + \frac{10}{16,000}\right] = -675 \mu\text{A}$

P 5.16 [a]



$$\frac{120,000}{R_a} = 8 \quad \text{so} \quad R_a = \frac{120,000}{8} = 15 \text{ k}\Omega$$

$$\frac{120,000}{R_b} = 5 \quad \text{so} \quad R_b = \frac{120,000}{5} = 24 \text{ k}\Omega$$

$$\frac{120,000}{R_c} = 12 \quad \text{so} \quad R_c = \frac{120,000}{12} = 10 \text{ k}\Omega$$

[b] $-[8(2) + 5v_b + 12(-1)] = -4 - 5v_b$

$$-4 - 5v_b = -15 \quad \text{so} \quad 5v_b = 11 \quad \text{thus} \quad v_b = \frac{11}{5} = 2.2 \text{ V}$$

$$-4 - 5v_b = 15 \quad \text{so} \quad -5v_b = 19 \quad \text{thus} \quad v_b = \frac{-19}{5} = -3.8 \text{ V}$$

Thus,

$$-3.8 \text{ V} \leq v_b \leq 2.2 \text{ V}$$

P 5.17 We want the following expression for the output voltage:

$$v_o = -(8v_a + 4v_b + 10v_c + 6v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 8, 4, 10, and 6 – say 120 kΩ:

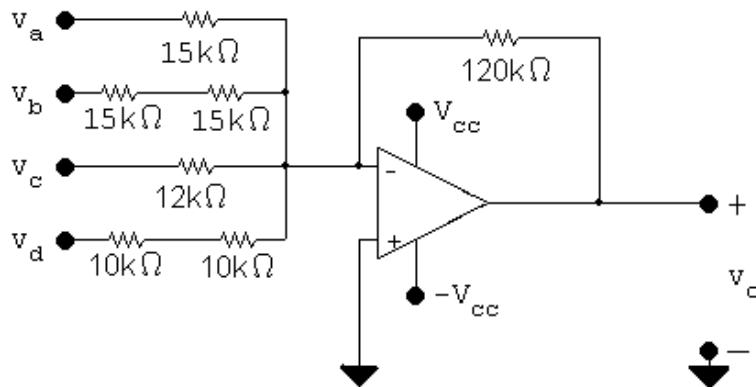
$$v_o = - \left[\frac{120\text{k}}{R_a} v_a + \frac{120\text{k}}{R_b} v_b + \frac{120\text{k}}{R_c} v_c + \frac{120\text{k}}{R_d} v_d \right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore R_a = 120,000/8 = 15 \text{ k}\Omega \quad R_c = 120,000/10 = 12 \text{ k}\Omega$$

$$R_b = 120,000/4 = 30 \text{ k}\Omega \quad R_d = 120,000/6 = 20 \text{ k}\Omega$$

Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_f = 120\text{ k}\Omega$, $R_a = 15\text{ k}\Omega$, and $R_c = 12\text{ k}\Omega$ are already values from Appendix H. Create $R_b = 30\text{ k}\Omega$ by combining two $15\text{ k}\Omega$ resistors in series. Create $R_d = 20\text{ k}\Omega$ by combining two $10\text{ k}\Omega$ resistors in series. Of course there are many other acceptable possibilities. The final circuit is shown here:



- P 5.18 [a] The circuit shown is a non-inverting amplifier.
 [b] We assume the op amp to be ideal, so $v_n = v_p = 2\text{ V}$. Write a KCL equation at v_n :

$$\frac{2}{25,000} + \frac{2 - v_o}{150,000} = 0$$

Solving,

$$v_o = 14\text{ V}.$$

- P 5.19 [a] This circuit is an example of the non-inverting amplifier.
 [b] Use voltage division to calculate v_p :

$$v_p = \frac{8000}{8000 + 32,000} v_s = \frac{v_s}{5}$$

Write a KCL equation at $v_n = v_p = v_s/5$:

$$\frac{v_s/5}{7000} + \frac{v_s/5 - v_o}{56,000} = 0$$

Solving,

$$v_o = 8v_s/5 + v_s/5 = 1.8v_s$$

[c] $1.8v_s = 12$ so $v_s = 6.67\text{ V}$

$$1.8v_s = -15 \quad \text{so} \quad v_s = -8.33\text{ V}$$

Thus, $-8.33\text{ V} \leq v_s \leq 6.67\text{ V}$.

P 5.20 [a] $v_p = v_n = \frac{68}{80}v_g = 0.85v_g$

$$\therefore \frac{0.85v_g}{30,000} + \frac{0.85v_g - v_o}{63,000} = 0;$$

$$\therefore v_o = 2.635v_g = 2.635(4), \quad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55 \text{ V}, \quad -4.55 \leq v_g \leq 4.55 \text{ V}$$

[c] $\frac{0.85v_g}{30,000} + \frac{0.85v_g - v_o}{R_f} = 0$

$$\left(\frac{0.85R_f}{30,000} + 0.85 \right) v_g = v_o = \pm 12$$

$$\therefore 1.7R_f + 51 = \pm 360; \quad 1.7R_f = 360 - 51; \quad R_f = 181.76 \text{ k}\Omega$$

P 5.21 [a] From Eq. 5.18,

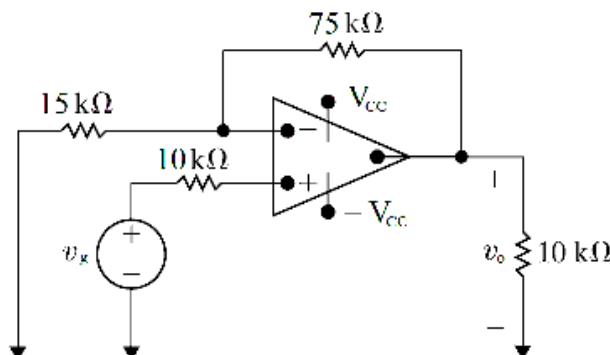
$$v_o = \frac{R_s + R_f}{R_s} v_g \quad \text{so} \quad \frac{v_o}{v_g} = 1 + \frac{R_f}{R_s} = 6$$

So,

$$\frac{R_f}{R_s} = 5$$

Thus,

$$R_s = \frac{R_f}{5} = \frac{75,000}{5} = 15 \text{ k}\Omega$$



[b] $v_o = 6v_g$

When $v_g = -2.5 \text{ V}$, $v_o = 6(-2.5) = -15 \text{ V}$.

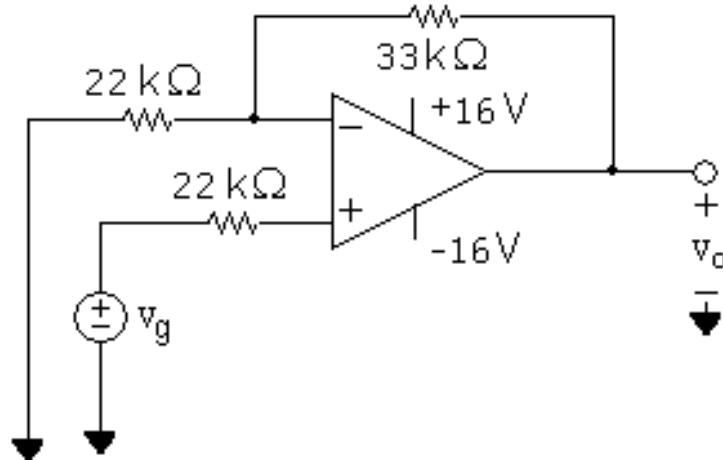
When $v_g = 1.5 \text{ V}$, $v_o = 6(1.5) = 9 \text{ V}$.

The power supplies can be set at 9 V and -15 V.

P 5.22 [a] From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 2.5 \quad \text{so} \quad R_s + R_f = 2.5R_s \quad \text{and therefore} \quad R_f = 1.5R_s$$

Choose $R_s = 22\text{k}\Omega$, which is a component in Appendix H. Then $R_f = (1.5)(22) = 33\text{k}\Omega$, which is also a resistor value in Appendix H. The resulting non-inverting amplifier circuit is shown here:



$$[b] \quad v_o = 2.5v_g = 16 \quad \text{so} \quad v_g = 6.4 \text{ V}$$

$$v_o = 2.5v_g = -16 \quad \text{so} \quad v_g = -6.4 \text{ V}$$

Therefore,

$$-6.4 \text{ V} \leq v_g \leq 6.4 \text{ V}$$

P 5.23 [a] This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s :

$$\frac{v_p - 5}{16,000} + \frac{v_p - v_s}{24,000} = 0$$

$$3v_p - 15 + 2v_p - 2v_s = 0 \quad \text{so} \quad v_p = 2v_s/5 + 3$$

Now write a KCL equation at v_n and solve for v_o :

$$\frac{v_n}{24,000} + \frac{v_n - v_o}{96,000} = 0 \quad \text{so} \quad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(2v_s/5 + 3) = 2v_s + 15$$

$$[c] \quad 2v_s + 15 = 10 \quad \text{so} \quad v_s = -2.5 \text{ V}$$

$$2v_s + 15 = -10 \quad \text{so} \quad v_s = -12.5 \text{ V}$$

Thus, $-12.5 \text{ V} \leq v_s \leq -2.5 \text{ V}$.

$$\text{P 5.24 [a]} \quad \frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$$

$$\therefore v_p = \frac{R_b R_c}{D} v_a + \frac{R_a R_c}{D} v_b + \frac{R_a R_b}{D} v_c$$

where $D = R_b R_c + R_a R_c + R_a R_b$

$$\frac{v_n}{20,000} + \frac{v_n - v_o}{100,000} = 0$$

$$\left(\frac{100,000}{20,000} + 1 \right) v_n = 6v_n = v_o$$

$$\therefore v_o = \frac{6R_b R_c}{D} v_a + \frac{6R_a R_c}{D} v_b + \frac{6R_a R_b}{D} v_c$$

By hypothesis,

$$\frac{6R_b R_c}{D} = 1; \quad \frac{6R_a R_c}{D} = 2; \quad \frac{6R_a R_b}{D} = 3$$

Then

$$\frac{6R_a R_b / D}{6R_a R_c / D} = \frac{3}{2} \quad \text{so} \quad R_b = 1.5R_c$$

But from the circuit

$$R_b = 15 \text{ k}\Omega \quad \text{so} \quad R_c = 10 \text{ k}\Omega$$

Similarly,

$$\frac{6R_b R_c / D}{6R_a R_b / D} = \frac{1}{3} \quad \text{so} \quad 3R_c = R_a$$

Thus,

$$R_a = 30 \text{ k}\Omega$$

$$[\text{b}] \quad v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8 \text{ V}$$

$$v_n = v_o / 6 = 0.8 \text{ V} = v_p$$

$$i_a = \frac{v_a - v_p}{30,000} = \frac{0.7 - 0.8}{30,000} = -3.33 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{15,000} = \frac{0.4 - 0.8}{15,000} = -26.67 \mu\text{A}$$

$$i_c = \frac{v_c - v_p}{10,000} = \frac{1.1 - 0.8}{10,000} = 30 \mu\text{A}$$

Check:

$$i_a + i_b + i_c = 0? \quad -3.33 - 26.67 + 30 = 0 \text{ (checks)}$$

P 5.25 [a] Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v'_o}{R_b} + i_n = 0, \quad i_n = 0$$

Therefore

$$\frac{v'_o}{R_b} = -\frac{v_a}{R_a}, \quad v'_o = -\frac{R_b}{R_a}v_a$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_n}{R_a} + \frac{v_n - v''_o}{R_b} + i_n = 0, \quad i_n = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b}\right) \left(\frac{R_d}{R_c + R_d}\right) v_b - \frac{v''_o}{R_b} = 0$$

$$v''_o = \left(\frac{R_b}{R_a} + 1\right) \left(\frac{R_d}{R_c + R_d}\right) v_b = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b$$

$$v_o = v'_o + v''_o = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b - \frac{R_b}{R_a} v_a$$

[b] $\frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}$, therefore $R_d(R_a + R_b) = R_b(R_c + R_d)$

$$R_d R_a = R_b R_c, \quad \text{therefore} \quad \frac{R_a}{R_b} = \frac{R_c}{R_d}$$

$$\text{When } \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}$$

$$\text{Eq. (5.22) reduces to } v_o = \frac{R_b}{R_a} v_b - \frac{R_b}{R_a} v_a = \frac{R_b}{R_a} (v_b - v_a).$$

P 5.26 [a] This is a difference amplifier circuit.

[b] Use Eq. 5.22 with $R_a = 5 \text{ k}\Omega$, $R_b = 20 \text{ k}\Omega$, $R_c = 8 \text{ k}\Omega$, $R_d = 2 \text{ k}\Omega$, and $v_b = 5 \text{ V}$:

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a = \frac{2(5 + 20)}{5(8 + 2)} (5) - \frac{20}{5} v_a = 5 - 4v_a$$

[c] $\frac{2000(5000 + R_f)}{5000(8000 + 2000)} (5) - \frac{R_f}{5000} (2) = \frac{5000 + R_f}{5000} - \frac{2R_f}{5000} = 1 - \frac{R_f}{5000}$

$$1 - \frac{R_f}{5000} = 10 \quad \text{so} \quad R_f < 0 \text{ which is not a possible solution.}$$

$$1 - \frac{R_f}{5000} = -10 \quad \text{so} \quad R_f = 5000(11) = 55 \text{ k}\Omega$$

$$\text{P 5.27 [a]} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{120(24 + 75)}{24(130 + 120)}(5) - \frac{75}{24}(8)$$

$$v_o = 9.9 - 25 = -15.1 \text{ V}$$

$$\text{[b]} \quad \frac{v_1 - 8}{24,000} + \frac{v_1 + 15.1}{75,000} = 0 \quad \text{so} \quad v_1 = 2.4 \text{ V}$$

$$i_a = \frac{8 - 2.4}{24,000} = 233 \mu \text{ A}$$

$$R_{\text{ina}} = \frac{v_a}{i_a} = \frac{8}{233 \times 10^{-6}} = 34.3 \text{ k}\Omega$$

$$\text{[c]} \quad R_{\text{in b}} = R_c + R_d = 250 \text{ k}\Omega$$

$$\text{P 5.28} \quad v_p = \frac{1500}{9000}(-18) = -3 \text{ V} = v_n$$

$$\frac{-3 + 18}{1600} + \frac{-3 - v_o}{R_f} = 0$$

$$\therefore v_o = 0.009375R_f - 3$$

$$v_o = 9 \text{ V}; \quad R_f = 1280 \Omega$$

$$v_o = -9 \text{ V}; \quad R_f = -640 \Omega$$

$$\text{But } R_f \geq 0, \quad \therefore R_f = 1.28 \text{ k}\Omega$$

$$\text{P 5.29} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

$$\text{By hypothesis: } R_b/R_a = 4; \quad R_c + R_d = 470 \text{ k}\Omega; \quad \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 3$$

$$\therefore \frac{R_d}{R_a} \frac{(R_a + 4R_a)}{470,000} = 3 \quad \text{so} \quad R_d = 282 \text{ k}\Omega; \quad R_c = 188 \text{ k}\Omega$$

Create $R_d = 282 \text{ k}\Omega$ by combining a $270 \text{ k}\Omega$ resistor and a $12 \text{ k}\Omega$ resistor in series. Create $R_c = 188 \text{ k}\Omega$ by combining a $120 \text{ k}\Omega$ resistor and a $68 \text{ k}\Omega$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

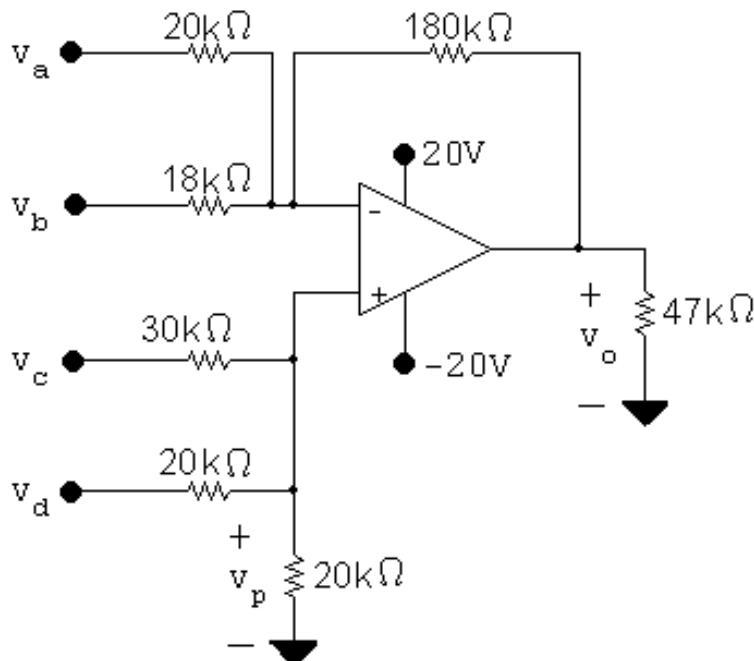
$$\therefore v_n \left(1 + \frac{R_a}{R_b}\right) = v_a; \quad v_n = 0.8v_a$$

$$i_a = \frac{v_a - 0.8v_a}{R_a} = 0.2 \frac{v_a}{R_a}; \quad R_{in} = \frac{v_a}{i_a} = 5R_a = 22 \text{ k}\Omega$$

$$\therefore R_a = 4.4 \text{ k}\Omega; \quad R_b = 17.6 \text{ k}\Omega$$

Create $R_a = 4.4 \text{ k}\Omega$ by combining two $2.2 \text{ k}\Omega$ resistors in series. Create $R_b = 17.6 \text{ k}\Omega$ by combining a $12 \text{ k}\Omega$ resistor and a $5.6 \text{ k}\Omega$ resistor in series.

P 5.30 [a]



$$\frac{v_p}{20,000} + \frac{v_p - v_c}{30,000} + \frac{v_p - v_d}{20,000} = 0$$

$$\therefore 8v_p = 2v_c + 3v_d = 8v_n$$

$$\frac{v_n - v_a}{20,000} + \frac{v_n - v_b}{18,000} + \frac{v_n - v_o}{180,000} = 0$$

$$\begin{aligned} \therefore v_o &= 20v_n - 9v_a - 10v_b \\ &= 20[(1/4)v_c + (3/8)v_d] - 9v_a - 10v_b \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16 \text{ V} \end{aligned}$$

$$[b] \quad v_o = 5v_c + 30 - 9 - 20 = 5v_c + 1$$

$$\pm 20 = 5v_c + 1$$

$$\therefore v_b = -4.2 \text{ V} \quad \text{and} \quad v_b = 3.8 \text{ V}$$

$$\therefore -4.2 \text{ V} \leq v_b \leq 3.8 \text{ V}$$

$$\text{P 5.31 } v_p = R_b i_b = v_n$$

$$\frac{R_b i_b}{2000} + \frac{R_b i_b - v_o}{R_f} - i_a = 0$$

$$\therefore R_b i_b \left(\frac{1}{2000} + \frac{1}{R_f} \right) - i_a = \frac{v_o}{R_f}$$

$$\therefore R_b i_b \left(1 + \frac{R_f}{2000} \right) - R_f i_a = v_o$$

By hypothesis, $v_o = 8000(i_b - i_a)$. Therefore,

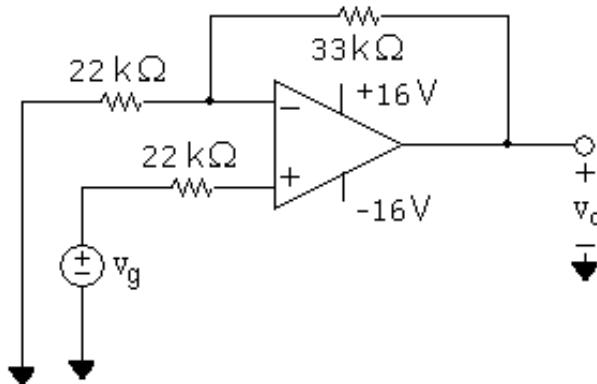
$$R_f = 8 \text{ k}\Omega \quad (\text{use } 3.3 \text{ k}\Omega \text{ and } 4.7 \text{ k}\Omega \text{ resistors in series})$$

$$R_b \left(1 + \frac{8000}{2000} \right) = 8000 \quad \text{so} \quad R_b = 1.6 \text{ k}\Omega$$

To construct the 1.6 kΩ resistor, combine 270 Ω, 330 Ω, and 1 kΩ resistors in series.

$$\begin{aligned} \text{P 5.32 [a]} \quad v_p &= \frac{\alpha R_g}{\alpha R_g + (R_g - \alpha R_g)} v_g & v_o &= \left(1 + \frac{R_f}{R_1} \right) \alpha v_g - \frac{R_f}{R_1} v_g \\ v_n &= v_p = \alpha v_g & &= (\alpha v_g - v_g)4 + \alpha v_g \\ \frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} &= 0 & &= [(\alpha - 1)4 + \alpha]v_g \\ (v_n - v_g) \frac{R_f}{R_1} + v_n - v_o &= 0 & &= (5\alpha - 4)v_g \\ & & &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1 \right) v_g - \left(\frac{R_f}{R_1} \right) v_g$$

Therefore,

$$\text{slope} = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad \text{intercept} = - \left(\frac{R_f}{R_1} \right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad 4 = - \left(\frac{R_f}{R_1} \right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \quad \frac{R_f}{R_1} = 2$$

P 5.33 [a] $A_{dm} = \frac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{cm} = \frac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] CMRR = $\left| \frac{24.98}{0.04} \right| = 624.50$

P 5.34 $A_{cm} = \frac{(3000)(6000) - (6000)R_x}{3000(6000 + R_x)}$

$$A_{dm} = \frac{6000(3000 + 6000) + 6000(6000 + R_x)}{2(3000)(6000 + R_x)}$$

$$\frac{A_{dm}}{A_{cm}} = \frac{R_x + 15,000}{2(3000 - R_x)}$$

$$\therefore \frac{R_x + 15,000}{2(3000 - R_x)} = \pm 1500 \quad \text{for the limits on the value of } R_x$$

If we use +1500 $R_x = 2994 \Omega$

If we use -1500 $R_x = 3006 \Omega$

$$2994 \Omega \leq R_x \leq 3006 \Omega$$

- P 5.35 It follows directly from the circuit that $v_o = -(75/15)v_g = -5v_g$
 From the plot of v_g we have $v_g = 0, t < 0$

$$v_g = t \quad 0 \leq t \leq 2$$

$$v_g = 4 - t \quad 2 \leq t \leq 6$$

$$v_g = t - 8 \quad 6 \leq t \leq 10$$

$$v_g = 12 - t \quad 10 \leq t \leq 14$$

$$v_g = t - 16 \quad 14 \leq t \leq 18, \text{ etc.}$$

Therefore

$$v_o = -5t \quad 0 \leq t \leq 2$$

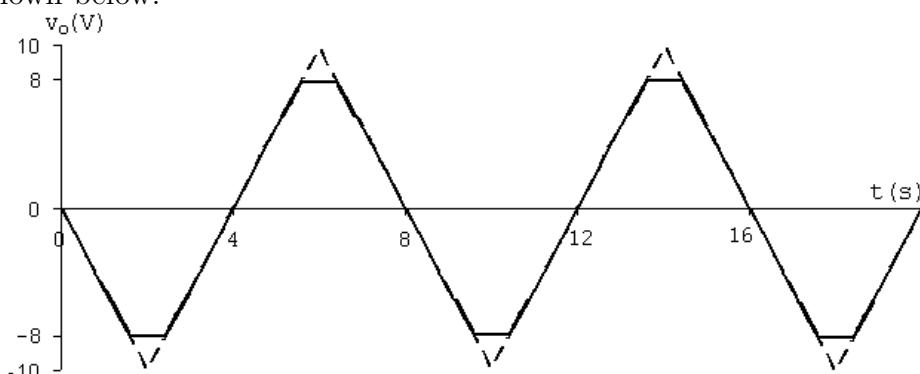
$$v_o = 5t - 20 \quad 2 \leq t \leq 6$$

$$v_o = 40 - 5t \quad 6 \leq t \leq 10$$

$$v_o = 5t - 60 \quad 10 \leq t \leq 14$$

$$v_o = 80 - 5t \quad 14 \leq t \leq 18, \text{ etc.}$$

These expressions for v_o are valid as long as the op amp is not saturated.
 Since the peak values of v_o are ± 9 , the output is clipped at ± 9 . The plot is shown below.



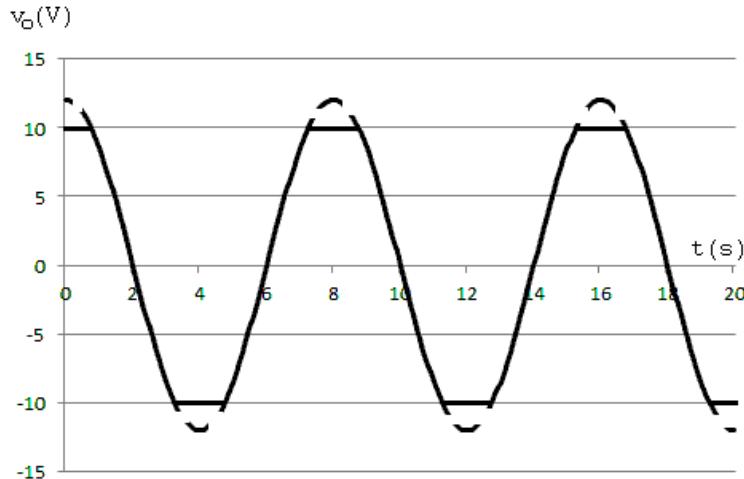
- P 5.36 $v_p = \frac{5.4}{7.2}v_g = 0.75v_g = 3\cos(\pi/4)t \text{ V}$

$$\frac{v_n}{20,000} + \frac{v_n - v_o}{60,000} = 0$$

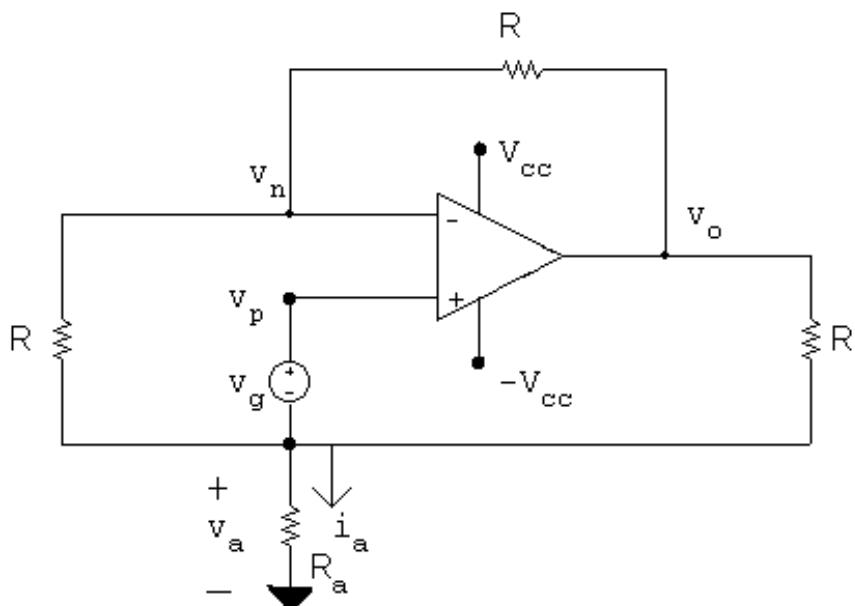
$$4v_n = v_o; \quad v_n = v_p$$

$$\therefore v_o = 12 \cos(\pi/4)t \text{ V} \quad 0 \leq t \leq \infty$$

but saturation occurs at $v_o = \pm 10 \text{ V}$



P 5.37 [a]



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a \left[\frac{1}{R_a} + \frac{2}{R} \right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a \left(2 + \frac{R}{R_a} \right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore v_a - v_o = -2v_g \quad (1)$$

$$2v_a + v_a \left(\frac{R}{R_a} \right) - v_a - v_g = v_o$$

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) - v_o = v_g \quad (2)$$

Now combining equations (1) and (2) yields

$$-v_a \frac{R}{R_a} = -3v_g$$

$$\text{or } v_a = 3v_g \frac{R_a}{R}$$

$$\text{Hence } i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \quad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm V_{cc}$

$$\therefore v_a = \pm V_{cc} - 2v_g \quad (3)$$

and

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) = \pm V_{cc} + v_g \quad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g}$$

$$\therefore \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm V_{cc} - 2v_g}$$

$$\text{or } R_a = \frac{(\pm V_{cc} - 2v_g)}{3v_g} R \quad \text{Q.E.D.}$$

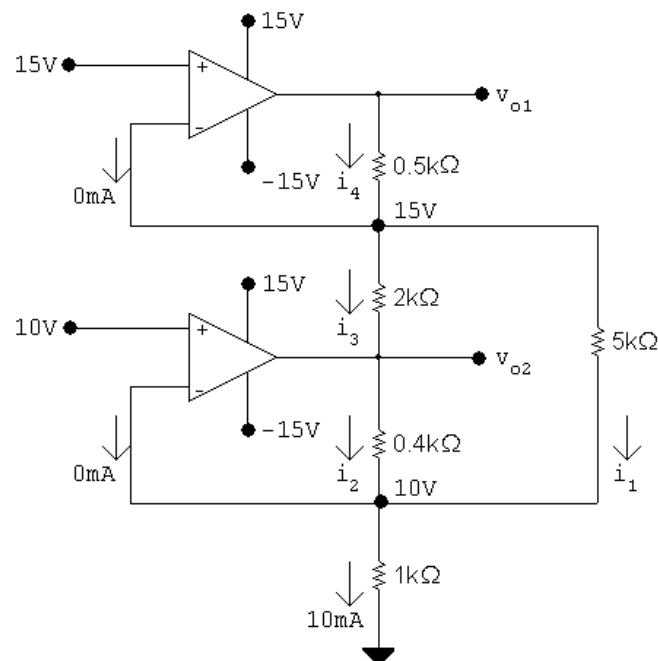
$$\text{P 5.38 [a]} \quad v_p = v_s, \quad v_n = \frac{R_1 v_o}{R_1 + R_2}, \quad v_n = v_p$$

$$\text{Therefore } v_o = \left(\frac{R_1 + R_2}{R_1} \right) v_s = \left(1 + \frac{R_2}{R_1} \right) v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.39



$$i_1 = \frac{15 - 10}{5000} = 1 \text{ mA}$$

$$i_2 + i_1 + 0 = 10 \text{ mA}; \quad i_2 = 9 \text{ mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7 \text{ mA}$$

$$i_4 = i_3 + i_1 = 1.7 \text{ mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

P 5.40 [a] $p_{16\text{k}\Omega} = \frac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4 \mu\text{W}$

[b] $v_{16\text{k}\Omega} = \left(\frac{16}{64}\right)(320) = 80 \text{ mV}$

$$p_{16\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4 \mu\text{W}$$

[c] $\frac{p_a}{p_b} = \frac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.41 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \quad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = 5.7 \text{ mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1 \text{ V}$

Thus

$$\frac{-47}{10}(v_L) = 1$$

$$v_L = -\frac{10}{47} = -212.77 \text{ mV}$$

P 5.42 [a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2 \text{ mA}$$

$$\text{For } R_L = 4 \text{ k}\Omega \quad v_o = (4 + 4)(2) = 16 \text{ V}$$

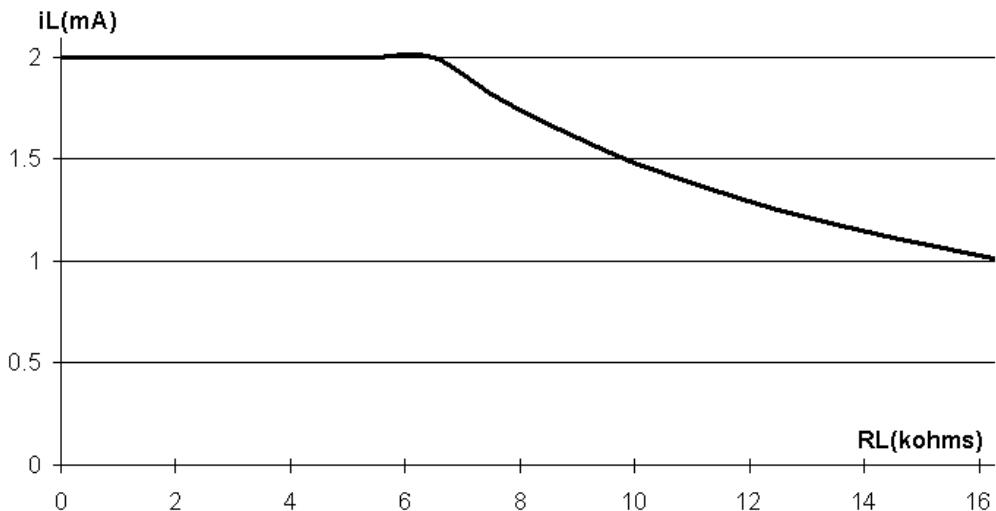
Now since $v_o < 20 \text{ V}$ our assumption of linear operation is correct, therefore

$$i_L = 2 \text{ mA}$$

[b] $20 = 2(4 + R_L); \quad R_L = 6 \text{ k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L . From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6 \text{ k}\Omega$. Therefore when $R_L = 6 \text{ k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16,000) = 1 \text{ mA}$. To justify neglecting the current into the op-amp assume the drop across the $50 \text{ k}\Omega$ resistor is negligible, since the input resistance to the op-amp is at least $500 \text{ k}\Omega$. Then $i_p = i_n = (8 - 4)/(500 \times 10^3) = 8 \mu\text{A}$. But $8 \mu\text{A} \ll 1 \text{ mA}$, hence our assumption is reasonable.

[d]



P 5.43 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}} \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)}{(R - \Delta R) \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)} - \frac{v_o}{R_f}$$

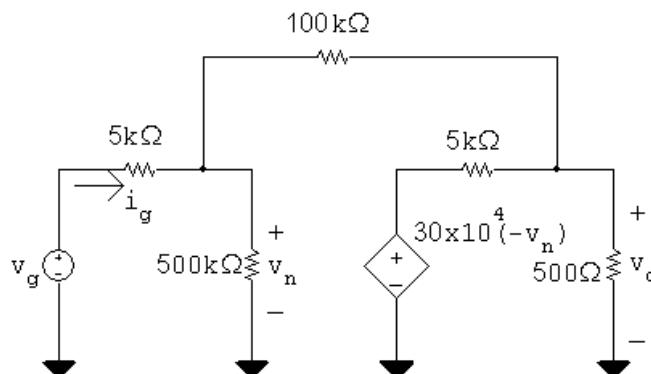
Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}} \left(\frac{1}{R - \Delta R} - \frac{1}{R + \Delta R} \right)$$

Thus,

$$v_o = v_{\text{ref}} \left(\frac{2\Delta R}{R^2 - (\Delta R)^2} \right) R_f$$

P 5.44 [a] Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} + \frac{v_n}{500,000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100,000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844 \text{ V}; \quad v_n = 736.1 \mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

- [b] From the solution in part (a), $v_n = 736.1 \mu\text{V}$.

$$[c] i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 736.1 \times 10^{-6} v_g}{5000}$$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68 \Omega$$

- [d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

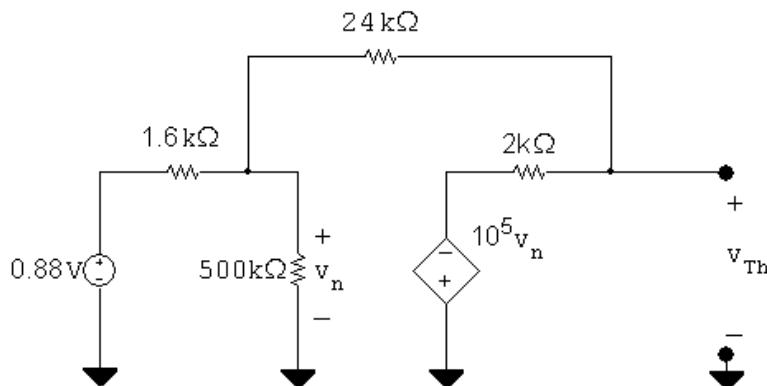
$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite.

Therefore,

$$v_n = v_p = 0 \text{ V}; \quad R_g = 5000 \Omega$$

P 5.45 [a]

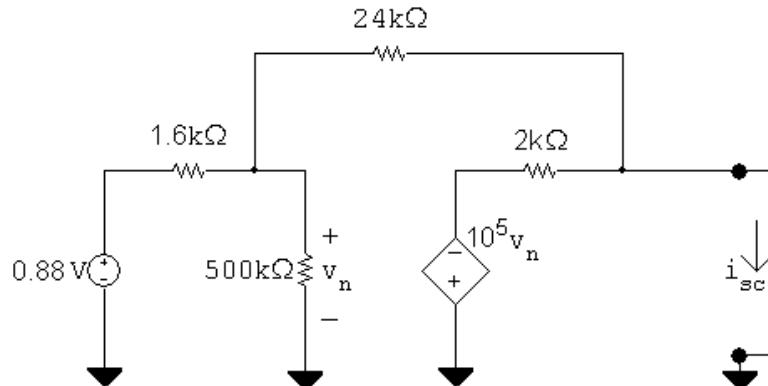


$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500,000} + \frac{v_n - v_{Th}}{24,000} = 0$$

$$\frac{v_{\text{Th}} + 10^5 v_n}{2000} + \frac{v_{\text{Th}} - v_n}{24,000} = 0$$

Solving, $v_{\text{Th}} = -13.198$ V

Short-circuit current calculation:

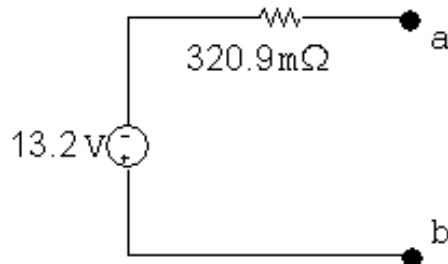


$$\frac{v_n}{500,000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24,000} = 0$$

$$\therefore v_n = 0.8225 \text{ V}$$

$$i_{\text{sc}} = \frac{v_n}{24,000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

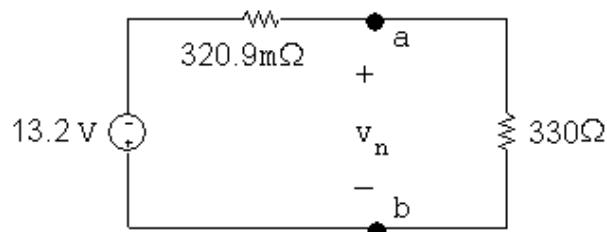
$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 320.9 \text{ m}\Omega$$



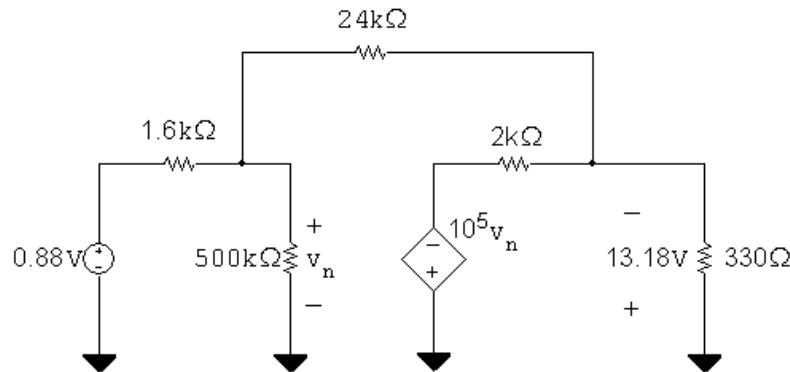
- [b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\text{Th}} = 320.9 \text{ m}\Omega$$

- [c]



$$v_o = \left(\frac{330}{330.3209} \right) (-13.2) = -13.18 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500,000} + \frac{v_n + 13.18}{24,000} = 0$$

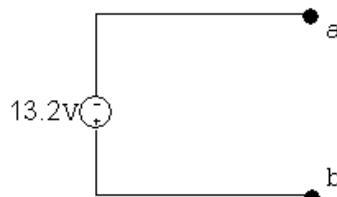
$$\therefore v_n = 942 \mu V$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41 \mu A$$

$$R_g = \frac{0.88}{i_g} = 1601.71 \Omega$$

P 5.46 [a] $v_{Th} = -\frac{24,000}{1600}(0.88) = -13.2 \text{ V}$

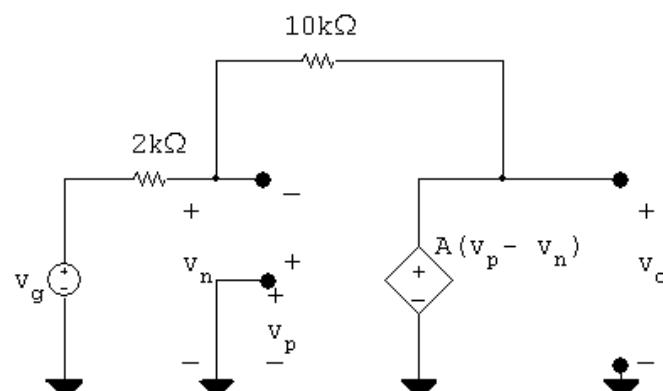
$R_{Th} = 0$, since op-amp is ideal



[b] $R_o = R_{Th} = 0 \Omega$

[c] $R_g = 1.6 \text{ k}\Omega$ since $v_n = 0$

P 5.47 [a]



$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10,000} = 0$$

$$\therefore v_o = 6v_n - 5v_g$$

$$\text{Also } v_o = A(v_p - v_n) = -Av_n$$

$$\therefore v_n = \frac{-v_o}{A}$$

$$\therefore v_o \left(1 + \frac{6}{A}\right) = -5v_g$$

$$v_o = \frac{-5A}{(6+A)}v_g$$

$$[\mathbf{b}] v_o = \frac{-5(150)(1)}{156} = -4.81 \text{ V}$$

$$[\mathbf{c}] v_o = \frac{-5}{1 + (6/A)}(1) = -5 \text{ V}$$

$$[\mathbf{d}] \frac{-5A}{A+6}(1) = -0.99(5) \quad \text{so} \quad -5A = -4.95(A+6)$$

$$\therefore -0.05A = -29.7 \quad \text{so} \quad A = 594$$

$$\text{P 5.48 [a]} \frac{v_n}{16,000} + \frac{v_n - v_g}{800,000} + \frac{v_n - v_o}{200,000} = 0 \quad \text{or} \quad 55v_n - 4v_o = v_g \quad \text{Eq (1)}$$

$$\frac{v_o}{20,000} + \frac{v_o - v_n}{200,000} + \frac{v_o - 50,000(v_p - v_n)}{8000} = 0$$

$$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$$

$$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$$

$$36v_o + 874,999v_n = 875,000v_g \quad \text{Eq (2)}$$

Let $v_g = 1 \text{ V}$ and solve Eqs. (1) and (2) simultaneously:

$$v_n = 999.446 \text{ mV} \quad \text{and} \quad v_o = 13.49 \text{ V}$$

$$\therefore \frac{v_o}{v_g} = 13.49$$

[b] From part (a), $v_n = 999.446 \text{ mV}$.

$$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834 \text{ mV}$$

[c] $v_p - v_n = 387.78 \mu\text{V}$

$$[\mathbf{d}] i_g = \frac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47 \text{ pA}$$

$$[\text{e}] \quad \frac{v_g}{16,000} + \frac{v_g - v_o}{200,000} = 0, \quad \text{since } v_n = v_p = v_g$$

$$\therefore v_o = 13.5v_g, \quad \frac{v_o}{v_g} = 13.5$$

$$v_n = v_p = 1 \text{ V}; \quad v_p - v_n = 0 \text{ V}; \quad i_g = 0 \text{ A}$$

P 5.49 [a] Use Eq. 5.61 to solve for R_f ; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2 \text{ k}\Omega$$

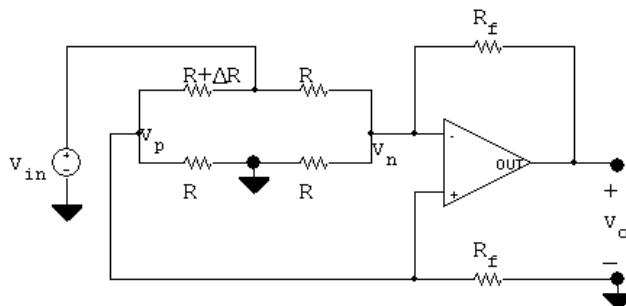
[b] Now solve for Δ given $v_o = 50 \text{ mV}$:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12 \text{ m}\Omega$$

P 5.50 [a]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore v_p \left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1} \right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f} \right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

$$\begin{aligned}
 \therefore v_n \left[\frac{R + 2R_f}{RR_f} \right] - \frac{v_{\text{in}}}{R} &= \frac{v_o}{R_f} \\
 \therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_f} \right] \left[\frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} \right] - \frac{v_{\text{in}}}{R} \\
 \therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R} \right] v_{\text{in}} \\
 \therefore v_o &= \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f}{R[R_1(R + R_f) + RR_f]} v_{\text{in}}
 \end{aligned}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R + R_f)R_f v_{\text{in}}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R + 2R_f)}$$

$$[\mathbf{b}] \quad v_o \approx \frac{47 \times 10^4 (48 \times 10^4) (-95) 15}{10^8 (95 \times 10^4)} \approx -3.384 \text{ V}$$

$$[\mathbf{c}] \quad v_o = \frac{-95(48 \times 10^4)(47 \times 10^4)15}{10^4[(1.0095)10^4(48 \times 10^4) + 47 \times 10^8]} = -3.368 \text{ V}$$

$$\begin{aligned}
 \text{P 5.51} \quad [\mathbf{a}] \quad v_o &\approx \frac{(R + R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R + 2R_f)} \\
 v_o &= \frac{(R + R_f)(-\Delta R)R_f v_{\text{in}}}{R[(R + \Delta R)(R + R_f) + RR_f]}
 \end{aligned}$$

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R + \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{R[(R + \Delta R)(R + R_f) + RR_f] - R^2(R + 2R_f)}{R^2(R + 2R_f)}$$

$$= \frac{\Delta R}{R} \frac{(R + R_f)}{(R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

$$[\mathbf{b}] \quad \% \text{ error} = \frac{95(48 \times 10^4) \times 100}{10^4(95 \times 10^4)} = 0.48\%$$

$$\text{P 5.52} \quad 1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \Delta R = \frac{9500}{48} = 197.91667 \Omega$$

$$\therefore \% \text{ change in } R = \frac{197.91667}{10^4} \times 100 \approx 1.98\%$$

P 5.53 [a] It follows directly from the solution to Problem 5.50 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{in}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{in}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{in}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.50 that

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810 \Omega \quad \therefore \Delta R = 10,000 - 9810 = 190 \Omega$

$$\therefore v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

$$[\text{d}] \% \text{ error} = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$

6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 [a] $i_g = 8e^{-300t} - 8e^{-1200t}$ A

$$v = L \frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t} \text{ V}, \quad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8 \text{ V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54 \text{ ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}$ W

[d] $\frac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \quad t = \frac{\ln 1.45}{900} = 411.05 \mu\text{s}$$

$$x = 11.0523, \quad t = \frac{\ln 11.05}{900} = 2.67 \text{ ms}$$

p is maximum at $t = 411.05 \mu\text{s}$

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72 \text{ W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54 \text{ ms}$.

[g] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78 \text{ A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6 \text{ mJ}$$

$$\text{AP 6.2 [a]} \quad i = C \frac{dv}{dt} = 24 \times 10^{-6} \frac{d}{dt} [e^{-15,000t} \sin 30,000t] \\ = [0.72 \cos 30,000t - 0.36 \sin 30,000t] e^{-15,000t} \text{ A}, \quad i(0^+) = 0.72 \text{ A}$$

$$\text{[b]} \quad i\left(\frac{\pi}{80} \text{ ms}\right) = -31.66 \text{ mA}, \quad v\left(\frac{\pi}{80} \text{ ms}\right) = 20.505 \text{ V}, \\ p = vi = -649.23 \text{ mW}$$

$$\text{[c]} \quad w = \left(\frac{1}{2}\right) Cv^2 = 126.13 \mu\text{J}$$

$$\text{AP 6.3 [a]} \quad v = \left(\frac{1}{C}\right) \int_{0^-}^t i \, dx + v(0^-) \\ = \frac{1}{0.6 \times 10^{-6}} \int_{0^-}^t 3 \cos 50,000x \, dx = 100 \sin 50,000t \text{ V}$$

$$\text{[b]} \quad p(t) = vi = [300 \cos 50,000t] \sin 50,000t \\ = 150 \sin 100,000t \text{ W}, \quad p_{(\max)} = 150 \text{ W}$$

$$\text{[c]} \quad w_{(\max)} = \left(\frac{1}{2}\right) Cv_{\max}^2 = 0.30(100)^2 = 3000 \mu\text{J} = 3 \text{ mJ}$$

$$\text{AP 6.4 [a]} \quad L_{\text{eq}} = \frac{60(240)}{300} = 48 \text{ mH}$$

$$\text{[b]} \quad i(0^+) = 3 + -5 = -2 \text{ A}$$

$$\text{[c]} \quad i = \frac{125}{6} \int_{0^+}^t (-0.03e^{-5x}) \, dx - 2 = 0.125e^{-5t} - 2.125 \text{ A}$$

$$\text{[d]} \quad i_1 = \frac{50}{3} \int_{0^+}^t (-0.03e^{-5x}) \, dx + 3 = 0.1e^{-5t} + 2.9 \text{ A}$$

$$i_2 = \frac{25}{6} \int_{0^+}^t (-0.03e^{-5x}) \, dx - 5 = 0.025e^{-5t} - 5.025 \text{ A}$$

$$i_1 + i_2 = i$$

$$\text{AP 6.5} \quad v_1 = 0.5 \times 10^6 \int_{0^+}^t 240 \times 10^{-6} e^{-10x} \, dx - 10 = -12e^{-10t} + 2 \text{ V}$$

$$v_2 = 0.125 \times 10^6 \int_{0^+}^t 240 \times 10^{-6} e^{-10x} \, dx - 5 = -3e^{-10t} - 2 \text{ V}$$

$$v_1(\infty) = 2 \text{ V}, \quad v_2(\infty) = -2 \text{ V}$$

$$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20 \mu\text{J}$$

AP 6.6 [a] Summing the voltages around mesh 1 yields

$$4 \frac{di_1}{dt} + 8 \frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4 \frac{di_1}{dt} + 25i_1 + 8 \frac{di_2}{dt} - 20i_2 = - \left(5i_g + 8 \frac{di_g}{dt} \right)$$

Summing the voltages around mesh 2 yields

$$16 \frac{d(i_2 + i_g)}{dt} + 8 \frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8 \frac{di_1}{dt} - 20i_1 + 16 \frac{di_2}{dt} + 800i_2 = -16 \frac{di_g}{dt}$$

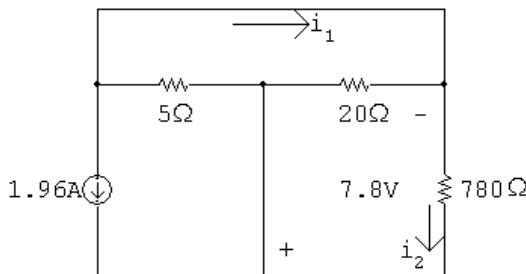
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \quad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4A; \quad i_2(\infty) = -0.01A$$

When $t = \infty$ the circuit reduces to



$$\therefore i_1(\infty) = - \left(\frac{7.8}{20} + \frac{7.8}{780} \right) = -0.4A; \quad i_2(\infty) = - \frac{7.8}{780} = -0.01A$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

$$\text{Also, } \frac{di_g}{dt} = 7.84e^{-4t}$$

Thus

$$4 \frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8 \frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8 \frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$\begin{aligned} & 185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t} \\ & + 0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}] \\ & - 9.8 + (300 - 240 - 40 - 20)e^{-5t} \\ & + (185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t}) \\ & - 9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t} \\ & - 9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad (\text{OK}) \end{aligned}$$

Also,

$$8 \frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16 \frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16 \frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$\begin{aligned} & 371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t} \\ & - 8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t} \\ & (8 - 8) + (800 - 480 - 240 - 80)e^{-5t} \\ & + (371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t} \\ & (800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t} \\ & - 125.44e^{-4t} = -125.44e^{-4t} \quad (\text{OK}) \end{aligned}$$

Problems

P 6.1 [a] $v = L \frac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t) \text{ mV}$$

[b] $i(5 \text{ ms}) = 25(0.005)(e^{-2.5}) = 10.26 \text{ mA}$

$$v(5 \text{ ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73 \mu\text{V}$$

$$p(5 \text{ ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74 \mu\text{W}$$

[c] delivering $4.74 \mu\text{W}$

[d] $i(5 \text{ ms}) = 10.26 \text{ mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9 \text{ nJ}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 500t = 0 \quad \text{or} \quad t = 2 \text{ ms}$$

$$i_{\max} = 25(0.002)e^{-1} = 18.39 \text{ mA}$$

$$w_{\max} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38 \text{ nJ}$$

P 6.2 [a]

$i = 0$	$t < 0$
$i = 4t \text{ A}$	$0 \leq t \leq 25 \text{ ms}$
$i = 0.2 - 4t \text{ A}$	$25 \leq t \leq 50 \text{ ms}$
$i = 0$	$50 \text{ ms} < t$

[b] $v = L \frac{di}{dt} = 500 \times 10^{-3}(4) = 2 \text{ V}$ $0 \leq t \leq 25 \text{ ms}$

$$v = 500 \times 10^{-3}(-4) = -2 \text{ V} \quad 25 \leq t \leq 50 \text{ ms}$$

$v = 0$	$t < 0$
$v = 2 \text{ V}$	$0 < t < 25 \text{ ms}$
$v = -2 \text{ V}$	$25 < t < 50 \text{ ms}$
$v = 0$	$50 \text{ ms} < t$

$$p = vi$$

$$\begin{aligned}
p &= 0 & t < 0 \\
p &= (4t)(2) = 8t \text{ W} & 0 < t < 25 \text{ ms} \\
p &= (0.2 - 4t)(-2) = 8t - 0.4 \text{ W} & 25 < t < 50 \text{ ms} \\
p &= 0 & 50 \text{ ms} < t \\
w &= 0 & t < 0 \\
w &= \int_0^t (8x) dx = 8 \frac{x^2}{2} \Big|_0^t = 4t^2 \text{ J} & 0 < t < 25 \text{ ms} \\
w &= \int_{0.025}^t (8x - 0.4) dx + 2.5 \times 10^{-3} & \\
&= 4x^2 - 0.4x \Big|_{0.025}^t + 2.5 \times 10^{-3} & \\
&= 4t^2 - 0.4t + 10 \times 10^{-3} \text{ J} & 25 < t < 50 \text{ ms} \\
w &= 0 & 10 \text{ ms} < t
\end{aligned}$$

P 6.3 [a] $i(0) = A_1 + A_2 = 0.12$

$$\frac{di}{dt} = -500A_1e^{-500t} - 2000A_2e^{-2000t}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t} \text{ V}$$

$$v(0) = -25A_1 - 100A_2 = 3$$

Solving, $A_1 = 0.2$ and $A_2 = -0.08$

Thus,

$$i = 200e^{-500t} - 80e^{-2000t} \text{ mA} \quad t \geq 0$$

$$v = -5e^{-500t} + 8e^{-2000t} \text{ V}, \quad t \geq 0$$

[b] $i = 0$ when $200e^{-500t} = 80e^{-2000t}$

Therefore

$$e^{1500t} = 0.4 \quad \text{so} \quad t = -610.86 \mu\text{s} \quad \text{which is not possible!}$$

$$v = 0 \quad \text{when} \quad 5e^{-500t} = 8e^{-2000t}$$

Therefore

$$e^{1500t} = 1.6 \quad \text{so} \quad t = 313.34 \mu\text{s}$$

Thus the power is zero at $t = 313.34 \mu\text{s}$.

P 6.4 [a] From Problem 6.3 we have

$$i = A_1 e^{-500t} + A_2 e^{-2000t} \text{ A}$$

$$v = -25A_1 e^{-500t} - 100A_2 e^{-2000t} \text{ V}$$

$$i(0) = A_1 + A_2 = 0.12$$

$$v(0) = -25A_1 - 100A_2 = -18$$

$$\text{Solving, } A_1 = -0.08; \quad A_2 = 0.2$$

Thus,

$$i = -80e^{-500t} + 200e^{-2000t} \text{ mA} \quad t \geq 0$$

$$v = 2e^{-500t} - 20e^{-2000t} \text{ V} \quad t \geq 0$$

[b] $i = 0$ when $80e^{-500t} = 200e^{-2000t}$

$$\therefore e^{1500t} = 2.5 \quad \text{so} \quad t = 610.86 \mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t < 610.86 \mu\text{s} \quad \text{and} \quad i < 0 \quad \text{for} \quad 610.86 \mu\text{s} < t < \infty$$

$$v = 0 \quad \text{when} \quad 2e^{-500t} = 20e^{-2000t}$$

$$\therefore e^{1500t} = 10 \quad \text{so} \quad t = 1535.06 \mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t < 1535.06 \mu\text{s} \quad \text{and} \quad v > 0 \quad \text{for} \quad 1535.06 \mu\text{s} < t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t < 610.86 \mu\text{s} \quad \text{and} \quad 1535.06 \mu\text{s} < t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 610.86 \mu\text{s} < t < 1535.06 \mu\text{s} \quad (\text{inductor stores energy})$$

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.05)(0.12)^2 = 360 \mu\text{J}$$

$$p = vi = -0.16e^{-1000t} + 2e^{-2500t} - 4e^{-4000t} \text{ W}$$

For $t > 0$:

$$\begin{aligned} w &= \int_0^\infty -0.16e^{-1000t} dt + \int_0^\infty 2e^{-2500t} dt - \int_0^\infty 4e^{-4000t} dt \\ &= \frac{-0.16e^{-1000t}}{-1000} \Big|_0^\infty + \frac{2e^{-2500t}}{-2500} \Big|_0^\infty - \frac{4e^{-4000t}}{-4000} \Big|_0^\infty \end{aligned}$$

$$\begin{aligned}
 &= (-1.6 + 8 - 10) \times 10^{-4} \\
 &= -360 \mu\text{J}
 \end{aligned}$$

Thus, the energy stored equals the energy extracted.

P 6.5 $i = (B_1 \cos 200t + B_2 \sin 200t)e^{-50t}$

$$i(0) = B_1 = 75 \text{ mA}$$

$$\frac{di}{dt} = (B_1 \cos 200t + B_2 \sin 200t)(-50e^{-50t}) + e^{-50t}(-200B_1 \sin 200t + 200B_2 \cos 200t)$$

$$= [(200B_2 - 50B_1) \cos 200t - (200B_1 + 50B_2) \sin 200t]e^{-50t}$$

$$v = 0.2 \frac{di}{dt} = [(40B_2 - 10B_1) \cos 200t - (40B_1 + 10B_2) \sin 200t]e^{-50t}$$

$$v(0) = 4.25 = 40B_2 - 10B_1 = 40B_2 - 0.75 \quad \therefore B_2 = 125 \text{ mA}$$

Thus,

$$i = (75 \cos 200t + 125 \sin 200t)e^{-50t} \text{ mA}, \quad t \geq 0$$

$$v = (4.25 \cos 200t - 4.25 \sin 200t)e^{-50t} \text{ V}, \quad t \geq 0$$

$$i(0.025) = -28.25 \text{ mA}; \quad v(0.025) = 1.513 \text{ V}$$

$$p(0.025) = (-28.25)(1.513) = -42.7 \text{ mW} \text{ delivering}$$

P 6.6 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$W = \int_0^\infty p dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}] dx = 0.2 \text{ J}$$

This is energy stored in the inductor at $t = \infty$.

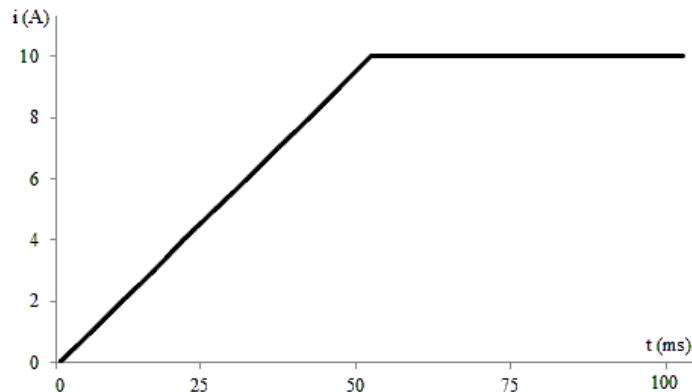
P 6.7 [a] $0 \leq t \leq 50 \text{ ms}$:

$$\begin{aligned}
 i &= \frac{1}{L} \int_0^t v_s dx + i(0) = \frac{10^6}{750} \int_0^t 0.15 dx + 0 \\
 &= 200x \Big|_0^t = 200t \text{ A}
 \end{aligned}$$

$$i(0.05) = 200(0.05) = 10 \text{ A}$$

$$t \geq 50 \text{ ms} : \quad i = \frac{10^6}{750} \int_{50 \times 10^{-3}}^t (0) dx + 10 = 10 \text{ A}$$

[b] $i = 200t \text{ A}, \quad 0 \leq t \leq 50 \text{ ms}; \quad i = 10 \text{ A}, \quad t \geq 50 \text{ ms}$



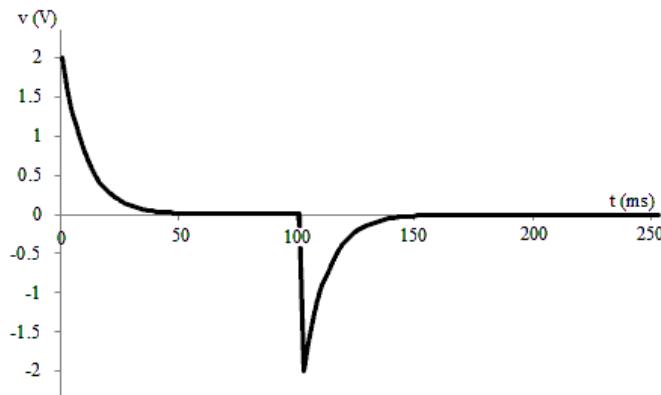
P 6.8 $0 \leq t \leq 100 \text{ ms} :$

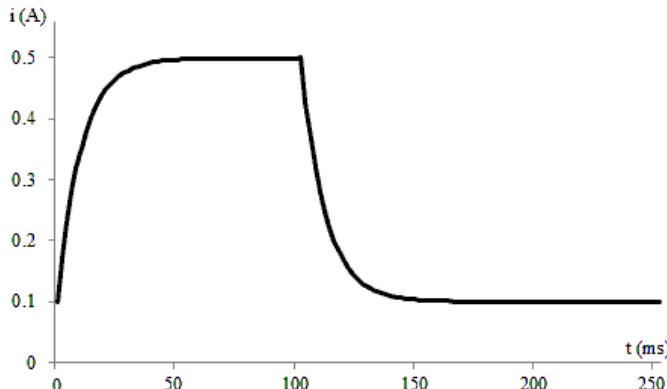
$$\begin{aligned} i_L &= \frac{10^3}{50} \int_0^t 2e^{-100x} dx + 0.1 = 40 \frac{e^{-100x}}{-100} \Big|_0^t + 0.1 \\ &= -0.4e^{-100t} + 0.5 \text{ A} \end{aligned}$$

$$i_L(0.1) = -0.4e^{-10} + 0.5 = 0.5 \text{ A}$$

$t \geq 100 \text{ ms} :$

$$\begin{aligned} i_L &= \frac{10^3}{50} \int_{0.1}^t -2e^{-100(x-0.1)} dx + 0.5 = -40 \frac{e^{-100(x-0.1)}}{-100} \Big|_{0.1}^t + 0.5 \\ &= 0.4e^{-100(t-0.1)} + 0.1 \text{ A} \end{aligned}$$





P 6.9 [a] $0 \leq t \leq 25$ ms :

$$v = 800t$$

$$i = \frac{1}{10} \int_0^t 800x \, dx + 0 = 80 \frac{x^2}{2} \Big|_0^t$$

$$i = 40t^2 \text{ A}$$

$25 \text{ ms} \leq t \leq 75 \text{ ms} :$

$$v = 20$$

$$i(0.025) = 25 \text{ mA}$$

$$\therefore i = \frac{1}{10} \int_{0.025}^t 20 \, dx + 0.025$$

$$= 2x \Big|_{0.025}^t + 0.025$$

$$= 2t - 0.025 \text{ A}$$

$75 \text{ ms} \leq t \leq 125 \text{ ms} :$

$$v = 80 - 800t \text{ V}$$

$$i(0.075) = 2(0.075) - 0.025 = 0.125 \text{ A}$$

$$i = \frac{1}{10} \int_{0.075}^t (80 - 800x) \, dx + 0.125$$

$$= \left(8x - \frac{80x^2}{2} \right) \Big|_{0.075}^t + 0.125$$

$$= 8t - 40t^2 - 0.25 \text{ A}$$

$125 \text{ ms} \leq t \leq 150 \text{ ms} :$

$$v = 800t - 120$$

$$i(0.125) = 8(0.125) - 40(0.125)^2 - 0.25 = 0.125 \text{ A}$$

$$\begin{aligned} i &= \frac{1}{10} \int_{0.125}^t (800x - 120) dx + 0.125 \\ &= \left(\frac{80x^2}{2} - 12x \right) \Big|_{0.125}^t + 0.125 \\ &= 40t^2 - 12t + 1 \text{ A} \end{aligned}$$

$t \geq 150 \text{ ms} :$

$$v = 0$$

$$i(0.150) = 40(0.15)^2 - 12(0.15) + 1 = 0.1 \text{ A}$$

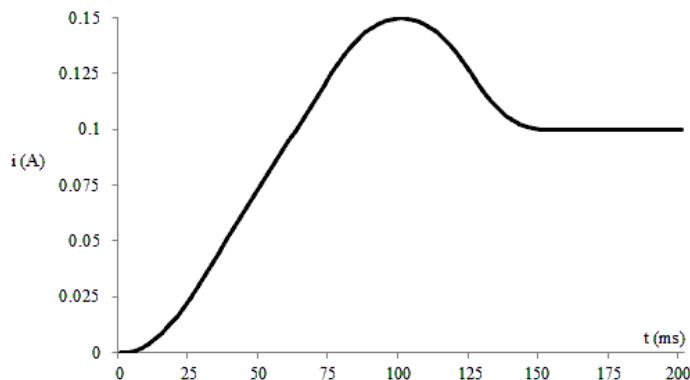
$$\begin{aligned} i &= \frac{1}{10} \int_{0.15}^t 0 dx + 0.1 \\ &= 0.1 \text{ A} \end{aligned}$$

[b] $v = 0$ at $t = 100 \text{ ms}$ and $t = 150 \text{ ms}$

$$i(0.1) = 8(0.1) - 40(0.1)^2 - 0.25 = 0.15 \text{ A}$$

$$i(0.15) = 0.1 \text{ A}$$

[c]



$$\begin{aligned} \text{P 6.10 [a]} \quad i &= \frac{1}{0.1} \int_0^t 20 \cos 80x dx \\ &= 200 \frac{\sin 80x}{80} \Big|_0^t \\ &= 2.5 \sin 80t \text{ A} \end{aligned}$$

$$[b] \quad p = vi = (20 \cos 80t)(2.5 \sin 80t)$$

$$= 50 \cos 80t \sin 80t$$

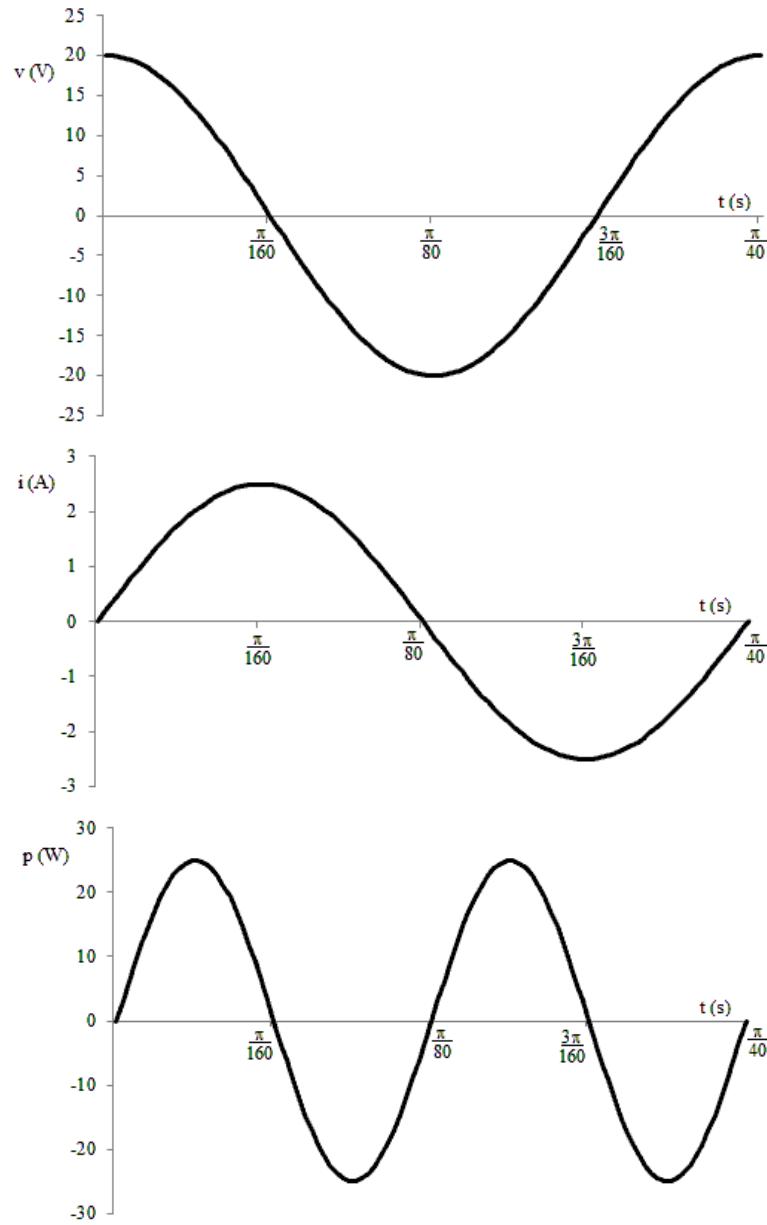
$$p = 25 \sin 160t \text{ W}$$

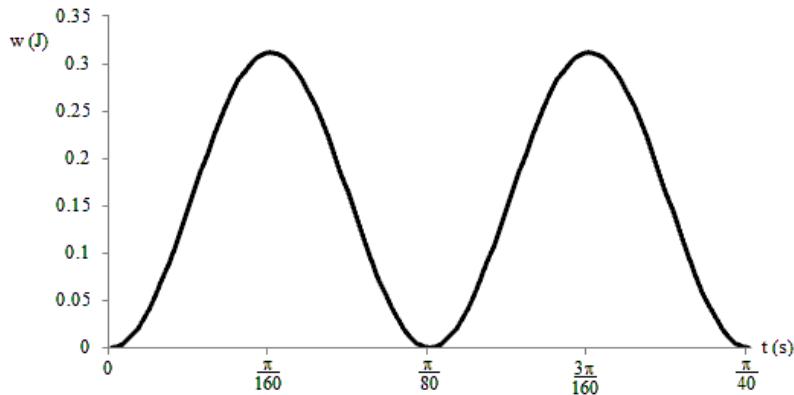
$$w = \frac{1}{2} Li^2$$

$$= \frac{1}{2}(0.1)(2.5 \sin 80t)^2$$

$$= 312.5 \sin^2 80t \text{ mJ}$$

$$w = (156.25 - 156.25 \cos 160t) \text{ mJ}$$





- [c] Absorbing power: Delivering power:
- $$0 \leq t \leq \pi/160 \text{ s} \quad \pi/160 \leq t \leq \pi/80 \text{ s}$$
- $$\pi/80 \leq t \leq 3\pi/160 \text{ s} \quad 3\pi/160 \leq t \leq \pi/40 \text{ s}$$

P 6.11 [a] $v = L \frac{di}{dt}$

$$\begin{aligned}
 v &= -25 \times 10^{-3} \frac{d}{dt} [10 \cos 400t + 5 \sin 400t] e^{-200t} \\
 &= -25 \times 10^{-3} (-200e^{-200t} [10 \cos 400t + 5 \sin 400t] \\
 &\quad + e^{-200t} [-4000 \sin 400t + 2000 \cos 400t]) \\
 v &= -25 \times 10^{-3} e^{-200t} (-1000 \sin 400t - 4000 \sin 400t) \\
 &= -25 \times 10^{-3} e^{-200t} (-5000 \sin 400t) \\
 &= 125e^{-200t} \sin 400t \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \frac{dv}{dt} &= 125(e^{-200t}(400) \cos 400t - 200e^{-200t} \sin 400t) \\
 &= 25,000e^{-200t}(2 \cos 400t - \sin 400t) \text{ V/s}
 \end{aligned}$$

$$\begin{aligned}
 \frac{dv}{dt} &= 0 \quad \text{when} \quad 2 \cos 400t = \sin 400t \\
 \therefore \tan 400t &= 2, \quad 400t = 1.11; \quad t = 2.77 \text{ ms}
 \end{aligned}$$

[b] $v(2.77 \text{ ms}) = 125e^{-0.55} \sin 1.11 = 64.27 \text{ V}$

P 6.12 For $0 \leq t \leq 1.6 \text{ s}$:

$$i_L = \frac{1}{5} \int_0^t 3 \times 10^{-3} dx + 0 = 0.6 \times 10^{-3} t$$

$$i_L(1.6 \text{ s}) = (0.6 \times 10^{-3})(1.6) = 0.96 \text{ mA}$$

$$R_m = (20)(1000) = 20 \text{ k}\Omega$$

$$v_m(1.6 \text{ s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2 \text{ V}$$

P 6.13 [a] $i = C \frac{dv}{dt} = (5 \times 10^{-6})[500t(-2500)e^{-2500t} + 500e^{-2500t}]$
 $= 2.5 \times 10^{-3}e^{-2500t}(1 - 2500t) \text{ A}$

[b] $v(100 \mu) = 500(100 \times 10^{-6})e^{-0.25} = 38.94 \text{ mV}$
 $i(100 \mu) = (2.5 \times 10^{-3})e^{-0.25}(1 - 0.25) = 1.46 \text{ mA}$
 $p(100 \mu) = vi = (38.94 \times 10^{-3})(1.46 \times 10^{-3}) = 56.86 \mu\text{W}$

[c] $p > 0$, so the capacitor is absorbing power.

[d] $v(100 \mu) = 38.94 \text{ mV}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(5 \times 10^{-6})(38.94 \times 10^{-3})^2 = 3.79 \text{ nJ}$$

[e] The energy is maximum when the voltage is maximum:

$$\frac{dv}{dt} = 0 \text{ when } (1 - 2500t) = 0 \text{ or } t = 0.4 \text{ ms}$$

$$v_{\max} = 500(0.4 \times 10^{-3})^2 e^{-1} = 73.58 \text{ mV}$$

$$p_{\max} = \frac{1}{2}Cv_{\max}^2 = 13.53 \text{ nJ}$$

P 6.14 [a] $v = 0 \quad t < 0$
 $v = 10t \text{ A} \quad 0 \leq t \leq 2 \text{ s}$
 $v = 40 - 10t \text{ A} \quad 2 \leq t \leq 6 \text{ s}$
 $v = 10t - 80 \text{ A} \quad 6 \leq t \leq 8 \text{ s}$
 $v = 0 \quad 8 \text{ s} < t$

[b] $i = C \frac{dv}{dt}$
 $i = 0 \quad t < 0$
 $i = 2 \text{ mA} \quad 0 < t < 2 \text{ s}$
 $i = -2 \text{ mA} \quad 2 < t < 6 \text{ s}$
 $i = 2 \text{ mA} \quad 6 < t < 8 \text{ s}$
 $i = 0 \quad 8 \text{ s} < t$

$$p = vi$$

$$p = 0 \quad t < 0$$

$$p = (10t)(0.002) = 0.02t \text{ W} \quad 0 < t < 2 \text{ s}$$

$$p = (40 - 10t)(-0.002) = 0.02t - 0.08 \text{ W} \quad 2 < t < 6 \text{ s}$$

$$p = (10t - 80)(0.002) = 0.02t - 0.16 \text{ W} \quad 6 < t < 8 \text{ s}$$

$$p = 0 \quad 8 \text{ s} < t$$

$$w = \int p dx$$

$$w = 0 \quad t < 0$$

$$w = \int_0^t (0.02x) dx = 0.01x^2 \Big|_0^t = 0.01t^2 \text{ J} \quad 0 < t < 2 \text{ s}$$

$$w = \int_2^t (0.02x - 0.08) dx + 0.04$$

$$= (0.01x^2 - 0.08x) \Big|_2^t + 0.04$$

$$= 0.01t^2 - 0.08t + 0.16 \text{ J} \quad 2 < t < 6 \text{ s}$$

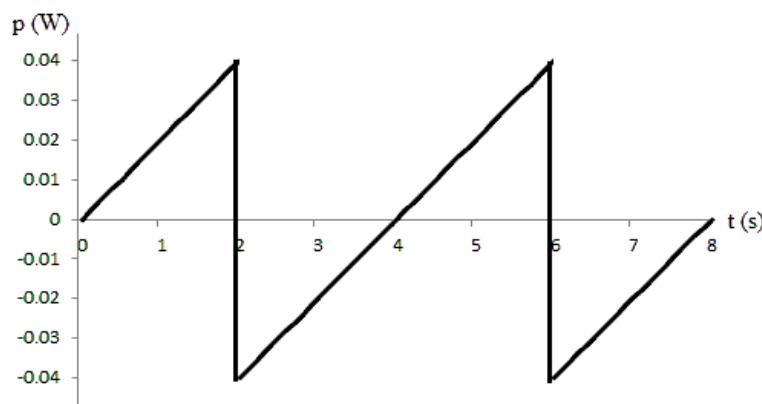
$$w = \int_6^t (0.02x - 0.16) dx + 0.04$$

$$= (0.01x^2 - 0.16x) \Big|_6^t + 0.04$$

$$= 0.01t^2 - 0.16t + 0.64 \text{ J} \quad 6 < t < 8 \text{ s}$$

$$w = 0 \quad 8 \text{ s} < t$$

[c]



From the plot of power above, it is clear that power is being absorbed for $0 < t < 2$ s and for $4 < t < 6$ s, because $p > 0$. Likewise, power is being delivered for $2 < t < 4$ s and $6 < t < 8$ s, because $p < 0$.

$$\text{P 6.15 [a]} \quad w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(5 \times 10^{-6})(60)^2 = 9 \text{ mJ}$$

$$\text{[b]} \quad v = (A_1 + A_2t)e^{-1500t}$$

$$v(0) = A_1 = 60 \text{ V}$$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1 + A_2t) + e^{-1500t}(A_2)$$

$$= (-1500A_2t - 1500A_1 + A_2)e^{-1500t}$$

$$\frac{dv(0)}{dt} = A_2 - 1500A_1$$

$$i = C \frac{dv}{dt}, \quad i(0) = C \frac{dv(0)}{dt}$$

$$\therefore \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{100 \times 10^{-3}}{5 \times 10^{-6}} = 20 \times 10^3$$

$$\therefore 20 \times 10^3 = A_2 - 1500(60)$$

$$\text{Thus, } A_2 = 20 \times 10^3 + 90 \times 10^3 = 110 \times 10^3 \frac{\text{V}}{\text{s}}$$

$$\text{[c]} \quad v = (60 + 110 \times 10^3 t)e^{-1500t}$$

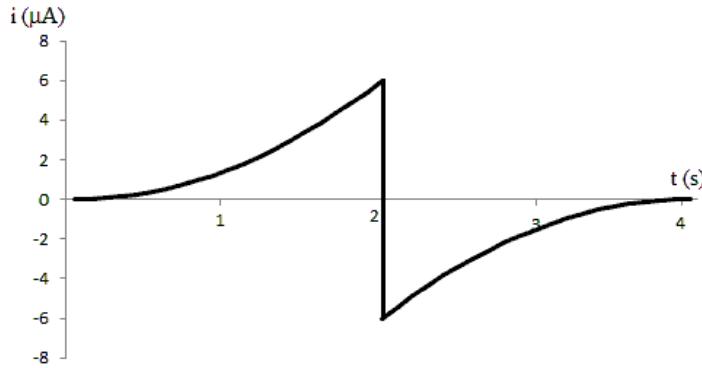
$$i = C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt} (60 + 110 \times 10^3 t) e^{-1500t}$$

$$\begin{aligned} i &= (5 \times 10^{-6})[110,000e^{-1500t} - 1500(60 + 110,000t)e^{-1500t}] \\ &= (0.1 - 825t)e^{-1500t} \text{ A}, \quad t \geq 0 \end{aligned}$$

$$\text{P 6.16} \quad i_C = C(dv/dt)$$

$$0 < t < 2 \text{ s} : \quad i_C = 100 \times 10^{-9}(15)t^2 = 1.5 \times 10^{-6}t^2 \text{ A}$$

$$2 < t < 4 \text{ s} : \quad i_C = 100 \times 10^{-9}(-15)(t-4)^2 = -1.5 \times 10^{-6}(t-4)^2 \text{ A}$$



P 6.17 [a] $i = C \frac{dv}{dt} = 0, \quad t < 0$

[b] $i = C \frac{dv}{dt} = 120 \times 10^{-6} \frac{d}{dt} [30 + 5e^{-500t} (6 \cos 2000t + \sin 2000t)]$

$$= 120 \times 10^{-6} [5(-500)e^{-500t} (6 \cos 2000t + \sin 2000t)]$$

$$+ 5(2000)e^{-500t} (-6 \sin 2000t + \cos 2000t)]$$

$$= -0.6e^{-500t} [\cos 2000t + 12.5 \sin 2000t] \text{ A}, \quad t \geq 0$$

[c] no, $v(0^-) = 60 \text{ V}$
 $v(0^+) = 30 + 5(6) = 60 \text{ V}$

[d] yes, $i(0^-) = 0 \text{ A}$
 $i(0^+) = -0.6 \text{ A}$

[e] $v(\infty) = 30 \text{ V}$

$$w = \frac{1}{2} Cv^2 = \frac{1}{2} (120 \times 10^{-6})(30)^2 = 54 \text{ mJ}$$

P 6.18 [a] $v(20 \mu\text{s}) = 12.5 \times 10^9 (20 \times 10^{-6})^2 = 5 \text{ V}$ (end of first interval)

$$v(20 \mu\text{s}) = 10^6 (20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10$$

$$= 5 \text{ V} \text{ (start of second interval)}$$

$$v(40 \mu\text{s}) = 10^6 (40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10$$

$$= 10 \text{ V} \text{ (end of second interval)}$$

[b] $p(10 \mu\text{s}) = 62.5 \times 10^{12} (10^{-5})^3 = 62.5 \text{ mW}, \quad v(10 \mu\text{s}) = 1.25 \text{ V},$

$$i(10 \mu\text{s}) = 50 \text{ mA}, \quad p(10 \mu\text{s}) = vi = (1.25)(50 \text{ m}) = 62.5 \text{ mW} \text{ (checks)}$$

$$p(30 \mu\text{s}) = 437.50 \text{ mW}, \quad v(30 \mu\text{s}) = 8.75 \text{ V}, \quad i(30 \mu\text{s}) = 0.05 \text{ A}$$

$$p(30 \mu\text{s}) = vi = (8.75)(0.05) = 62.5 \text{ mW} \text{ (checks)}$$

$$[c] \quad w(10\mu s) = 15.625 \times 10^{12} (10 \times 10^{-6})^4 = 0.15625 \mu J$$

$$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625 \mu J$$

$$w(30\mu s) = 7.65625 \mu J$$

$$w(30\mu s) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625 \mu J$$

$$\begin{aligned} P 6.19 \quad [a] \quad v &= \frac{1}{0.5 \times 10^{-6}} \int_0^{500 \times 10^{-6}} 50 \times 10^{-3} e^{-2000t} dt - 20 \\ &= 100 \times 10^3 \frac{e^{-2000t}}{-2000} \Big|_0^{500 \times 10^{-6}} - 20 \\ &= 50(1 - e^{-1}) - 20 = 11.61 V \\ w &= \frac{1}{2} Cv^2 = \frac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7 \mu J \end{aligned}$$

$$[b] \quad v(\infty) = 50 - 20 = 30 V$$

$$w(\infty) = \frac{1}{2}(0.5 \times 10^{-6})(30)^2 = 225 \mu J$$

$$\begin{aligned} P 6.20 \quad [a] \quad i &= \frac{5}{2 \times 10^{-3}} t = 2500t \quad 0 \leq t \leq 2 \text{ ms} \\ i &= \frac{-10}{4 \times 10^{-3}} t + 10 = 10 - 2500t \quad 2 \leq t \leq 6 \text{ ms} \\ i &= \frac{10}{4 \times 10^{-3}} t - 20 = 2500t - 20 \quad 6 \leq t \leq 10 \text{ ms} \\ i &= \frac{-5}{2 \times 10^{-3}} t + 30 = 30 - 2500t \quad 10 \leq t \leq 12 \text{ ms} \\ q &= \int_0^{0.002} 2500t dt + \int_{0.002}^{0.006} (10 - 2500t) dt \\ &= \frac{2500t^2}{2} \Big|_0^{0.002} + \left(10t - \frac{2500t^2}{2} \right) \Big|_{0.002}^{0.006} \\ &= 0.005 - 0 + (0.06 - 0.045) - (0.02 - 0.005) \\ &= 5 \text{ mC} \end{aligned}$$

$$\begin{aligned} [b] \quad v &= 0.5 \times 10^6 \int_0^{0.002} 2500x dx + 0.5 \times 10^6 \int_{0.002}^{0.006} (10 - 2500x) dx \\ &\quad + 0.5 \times 10^6 \int_{0.006}^{0.01} (2500x - 20) dx \\ &= 0.5 \times 10^6 \left[\frac{2500x^2}{2} \Big|_0^{0.002} + 10x \Big|_{0.002}^{0.006} - \frac{2500x^2}{2} \Big|_{0.002}^{0.006} + \frac{2500x^2}{2} \Big|_{0.006}^{0.01} - 20x \Big|_{0.006}^{0.01} \right] \end{aligned}$$

$$\begin{aligned}
&= 0.5 \times 10^6 [(0.005 - 0) + (0.06 - 0.02) - (0.045 - 0.005) \\
&\quad + (0.125 - 0.045) - (0.2 - 0.12)] \\
&= 2500 \text{ V}
\end{aligned}$$

$$v(10 \text{ ms}) = 2500 \text{ V}$$

$$\begin{aligned}
[\mathbf{c}] \quad v(12 \text{ ms}) &= v(10 \text{ ms}) + 0.5 \times 10^6 \int_{0.01}^{0.012} (30 - 2500x) dx \\
&= 2500 + 0.5 \times 10^6 \left(30x - \frac{2500x^2}{2} \right) \Big|_{0.01}^{0.012} \\
&= 2500 + 0.5 \times 10^6 (0.36 - 0.18 - 0.3 + 0.125) \\
&= 2500 + 2500 = 5000 \text{ V} \\
w &= \frac{1}{2} C v^2 = \frac{1}{2} (2 \times 10^{-6}) (5000)^2 = 25 \text{ J}
\end{aligned}$$

P 6.21 [a] $0 \leq t \leq 10 \mu\text{s}$

$$C = 0.1 \mu\text{F} \quad \frac{1}{C} = 10 \times 10^6$$

$$v = 10 \times 10^6 \int_0^t -0.05 dx + 15$$

$$v = -50 \times 10^4 t + 15 \text{ V} \quad 0 \leq t \leq 10 \mu\text{s}$$

$$v(10 \mu\text{s}) = -5 + 15 = 10 \text{ V}$$

[b] $10 \mu\text{s} \leq t \leq 20 \mu\text{s}$

$$v = 10 \times 10^6 \int_{10 \times 10^{-6}}^t 0.1 dx + 10 = 10^6 t - 10 + 10$$

$$v = 10^6 t \text{ V} \quad 10 \leq t \leq 20 \mu\text{s}$$

$$v(20 \mu\text{s}) = 10^6 (20 \times 10^{-6}) = 20 \text{ V}$$

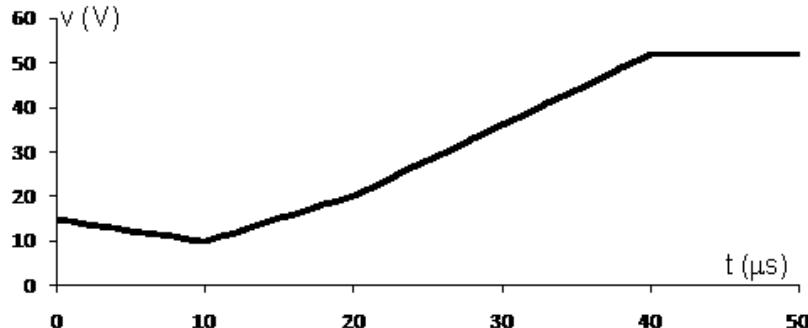
[c] $20 \mu\text{s} \leq t \leq 40 \mu\text{s}$

$$v = 10 \times 10^6 \int_{20 \times 10^{-6}}^t 1.6 dx + 20 = 1.6 \times 10^6 t - 32 + 20$$

$$v = 1.6 \times 10^6 t - 12 \text{ V}, \quad 20 \mu\text{s} \leq t \leq 40 \mu\text{s}$$

[d] $40 \mu\text{s} \leq t < \infty$

$$v(40 \mu\text{s}) = 64 - 12 = 52 \text{ V} \quad 40 \mu\text{s} \leq t < \infty$$



P 6.22 [a] $15 \parallel 30 = 10 \text{ mH}$

$$10 + 10 = 20 \text{ mH}$$

$$20 \parallel 20 = 10 \text{ mH}$$

$$12 \parallel 24 = 8 \text{ mH}$$

$$10 + 8 = 18 \text{ mH}$$

$$18 \parallel 9 = 6 \text{ mH}$$

$$L_{ab} = 6 + 8 = 14 \text{ mH}$$

[b] $12 + 18 = 30 \mu\text{H}$

$$30 \parallel 20 = 12 \mu\text{H}$$

$$12 + 38 = 50 \mu\text{H}$$

$$30 \parallel 75 \parallel 50 = 15 \mu\text{H}$$

$$15 + 15 = 30 \mu\text{H}$$

$$30 \parallel 60 = 20 \mu\text{H}$$

$$L_{ab} = 20 + 25 = 45 \mu\text{H}$$

P 6.23 [a] Combine two 10 mH inductors in parallel to get a 5 mH equivalent inductor. Then combine this parallel pair in series with three 1 mH inductors:

$$10 \text{ mH} \parallel 10 \text{ mH} + 1 \text{ mH} + 1 \text{ mH} + 1 \text{ mH} = 8 \text{ mH}$$

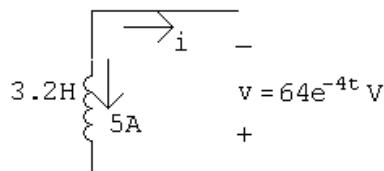
[b] Combine two $10 \mu\text{H}$ inductors in parallel to get a $5 \mu\text{H}$ inductor. Then combine this parallel pair in series with four more $10 \mu\text{H}$ inductors:

$$10 \mu\text{H} \parallel 10 \mu\text{H} + 10 \mu\text{H} + 10 \mu\text{H} + 10 \mu\text{H} + 10 \mu\text{H} = 45 \mu\text{H}$$

- [c] Combine two $100\ \mu\text{H}$ inductors in parallel to get a $50\ \mu\text{H}$ inductor. Then combine this parallel pair with a $100\ \mu\text{H}$ inductor and three $10\ \mu\text{H}$ inductors in series:

$$100\ \mu\text{H} \parallel 100\ \mu\text{H} + 100\ \mu\text{H} + 10\ \mu\text{H} + 10\ \mu\text{H} + 10\ \mu\text{H} = 180\ \mu\text{H}$$

P 6.24 [a]



$$3.2 \frac{di}{dt} = 64e^{-4t} \quad \text{so} \quad \frac{di}{dt} = 20e^{-4t}$$

$$i(t) = 20 \int_0^t e^{-4x} dx - 5$$

$$= 20 \frac{e^{-4x}}{-4} \Big|_0^t - 5$$

$$i(t) = -5e^{-4t} \text{ A}$$

$$[b] 4 \frac{di_1}{dt} = 64e^{-4t}$$

$$i_1(t) = 16 \int_0^t e^{-4x} dx - 10$$

$$= 16 \frac{e^{-4x}}{-4} \Big|_0^t - 10$$

$$i_1(t) = -4e^{-4t} - 6 \text{ A}$$

$$[c] 16 \frac{di_2}{dt} = 64e^{-4t} \quad \text{so} \quad \frac{di_2}{dt} = 4e^{-4t}$$

$$i_2(t) = 4 \int_0^t e^{-4x} dx + 5$$

$$= 4 \frac{e^{-4x}}{-4} \Big|_0^t + 5$$

$$i_2(t) = -e^{-4t} + 6 \text{ A}$$

$$[d] p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t} \text{ W}$$

$$w = \int_0^\infty p dt = \int_0^\infty 320e^{-8t} dt$$

$$= 320 \frac{e^{-8t}}{-8} \Big|_0^\infty$$

$$= 40 \text{ J}$$

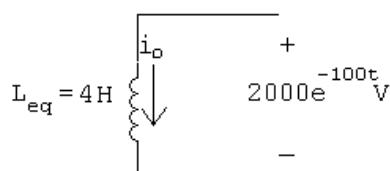
[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400 \text{ J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360 \text{ J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360 \text{ J} \quad \text{checks}$

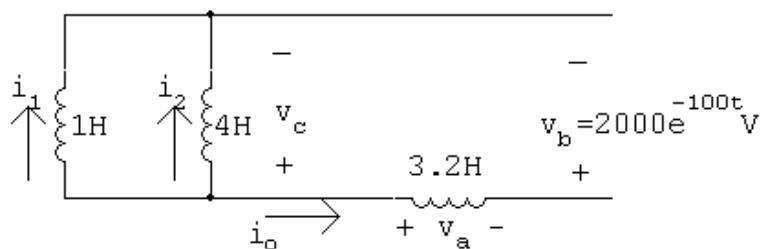
P 6.25 [a] $i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5 \text{ A}$

[b]



$$\begin{aligned} i_o &= -\frac{1}{4} \int_0^t 2000e^{-100x} dx + 5 = -500 \frac{e^{-100x}}{-100} \Big|_0^t + 5 \\ &= 5(e^{-100t} - 1) + 5 = 5e^{-100t} \text{ A}, \quad t \geq 0 \end{aligned}$$

[c]



$$v_a = 3.2(-500e^{-100t}) = -1600e^{-100t} \text{ V}$$

$$v_c = v_a + v_b = -1600e^{-100t} + 2000e^{-100t}$$

$$= 400e^{-100t} \text{ V}$$

$$i_1 = \frac{1}{1} \int_0^t 400e^{-100x} dx - 6$$

$$= -4e^{-100t} + 4 - 6$$

$$i_1 = -4e^{-100t} - 2 \text{ A} \quad t \geq 0$$

$$\begin{aligned}
 [\mathbf{d}] \quad i_2 &= \frac{1}{4} \int_0^t 400e^{-100x} dx + 1 \\
 &= -e^{-100t} + 2 \text{ A}, \quad t \geq 0 \\
 [\mathbf{e}] \quad w(0) &= \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60 \text{ J} \\
 [\mathbf{f}] \quad w_{\text{del}} &= \frac{1}{2}(4)(5)^2 = 50 \text{ J} \\
 [\mathbf{g}] \quad w_{\text{trapped}} &= 60 - 50 = 10 \text{ J} \\
 \text{or} \quad w_{\text{trapped}} &= \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 + 10 \text{ J} \text{ (check)}
 \end{aligned}$$

$$\text{P 6.26} \quad v_b = 2000e^{-100t} \text{ V}$$

$$i_o = 5e^{-100t} \text{ A}$$

$$p = 10,000e^{-200t} \text{ W}$$

$$w = \int_0^t 10^4 e^{-200x} dx = 10,000 \frac{e^{-200x}}{-200} \Big|_0^t = 50(1 - e^{-200t}) \text{ J}$$

$$w_{\text{total}} = 50 \text{ J}$$

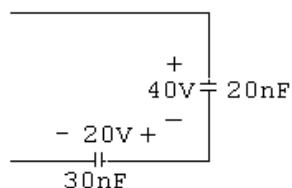
$$80\%w_{\text{total}} = 40 \text{ J}$$

Thus,

$$50 - 50e^{-200t} = 40; \quad e^{200t} = 5; \quad \therefore t = 8.05 \text{ ms}$$

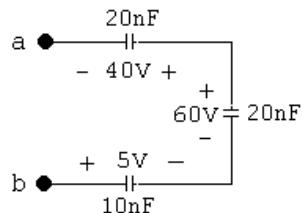
$$\text{P 6.27} \quad [\mathbf{a}] \quad \frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}; \quad C_1 = 16 \text{ nF}$$

$$C_2 = 4 + 16 = 20 \text{ nF}$$



$$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}; \quad C_3 = 12 \text{ nF}$$

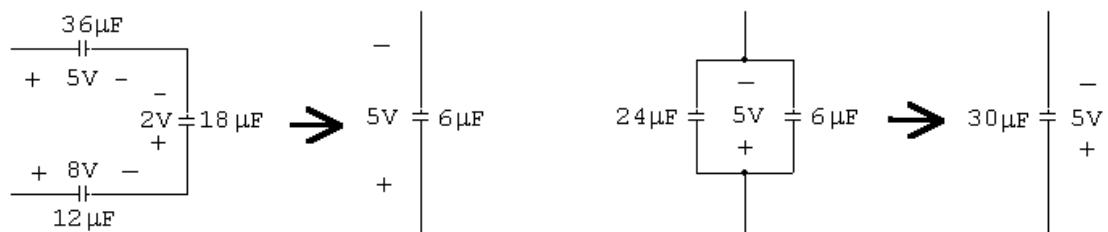
$$C_4 = 12 + 8 = 20 \text{ nF}$$



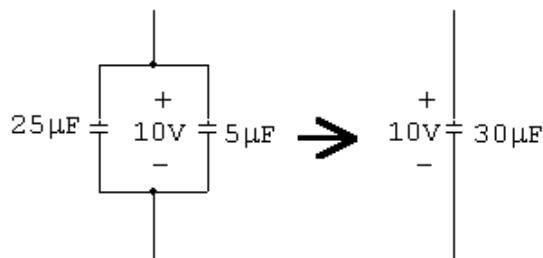
$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}; \quad C_5 = 5 \text{nF}$$

Equivalent capacitance is 5 nF with an initial voltage drop of +15 V.

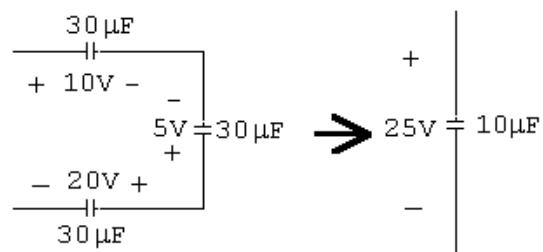
[b] $\frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \quad \therefore C_{\text{eq}} = 6 \mu\text{F} \quad 24 + 6 = 30 \mu\text{F}$



$$25 + 5 = 30 \mu\text{F}$$



$$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30} \quad \therefore C_{\text{eq}} = 10 \mu\text{F}$$



Equivalent capacitance is 10 μF with an initial voltage drop of +25 V.

- P 6.28 [a] Combine a 470 pF capacitor and a 10 pF capacitor in parallel to get a 480 pF capacitor:

$$(470 \text{ p}) \text{ in parallel with } (10 \text{ p}) = 470 \text{ p} + 10 \text{ p} = 480 \text{ pF}$$

[b] Create a 1200 nF capacitor as follows:

(1 μ) in parallel with (0.1 μ) in parallel with (0.1 μ)

$$= 1000 \text{ n} + 100 \text{ n} + 100 \text{ n} = 1200 \text{ nF}$$

Create a second 1200 nF capacitor using the same three resistors. Place these two 1200 nF in series:

$$(1200 \text{ n}) \text{ in series with } (1200 \text{ n}) = \frac{(1200 \text{ n})(1200 \text{ n})}{1200 \text{ n} + 1200 \text{ n}} = 600 \text{ nF}$$

[a] Combine two 220 μ F capacitors in series to get a 110 μ F capacitor. Then combine the series pair in parallel with a 10 μ F capacitor to get 120 μ F:

[(220 μ) in series with (220 μ)] in parallel with (10 μ)

$$= \frac{(220 \mu)(220 \mu)}{220 \mu + 220 \mu} + 10 \mu = 120 \mu\text{F}$$

P 6.29 From Figure 6.17(a) we have

$$v = \frac{1}{C_1} \int_0^t i \, dx + v_1(0) + \frac{1}{C_2} \int_0^t i \, dx + v_2(0) + \dots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \dots \right] \int_0^t i \, dx + v_1(0) + v_2(0) + \dots$$

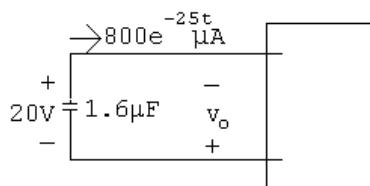
$$\text{Therefore } \frac{1}{C_{\text{eq}}} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \dots \right], \quad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \dots$$

P 6.30 From Fig. 6.18(a)

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots = [C_1 + C_2 + \dots] \frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \dots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq} .

P 6.31 [a]



$$v_o = \frac{10^6}{1.6} \int_0^t 800 \times 10^{-6} e^{-25x} dx - 20$$

$$= 500 \frac{e^{-25x}}{-25} \Big|_0^t - 20$$

$$= -20e^{-25t} \text{ V}, \quad t \geq 0$$

$$[\mathbf{b}] \quad v_1 = \frac{10^6}{2} (800 \times 10^{-6}) \frac{e^{-25x}}{-25} \Big|_0^t + 5$$

$$= -16e^{-25t} + 21 \text{ V}, \quad t \geq 0$$

$$[\mathbf{c}] \quad v_2 = \frac{10^6}{8} (800 \times 10^{-6}) \frac{e^{-25x}}{-25} \Big|_0^t - 25$$

$$= -4e^{-25t} - 21 \text{ V}, \quad t \geq 0$$

$$[\mathbf{d}] \quad p = -vi = -(-20e^{-25t})(800 \times 10^{-6})e^{-25t}$$

$$= 16 \times 10^{-3} e^{-50t}$$

$$w = \int_0^\infty 16 \times 10^{-3} e^{-50t} dt$$

$$= 16 \times 10^{-3} \frac{e^{-50t}}{-50} \Big|_0^\infty$$

$$= -0.32 \times 10^{-3} (0 - 1) = 320 \mu\text{J}$$

$$[\mathbf{e}] \quad w = \frac{1}{2}(2 \times 10^{-6})(5)^2 + \frac{1}{2}(8 \times 10^{-6})(25)^2 \\ = 2525 \mu\text{J}$$

$$[\mathbf{f}] \quad w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525 - 320 = 2205 \mu\text{J}$$

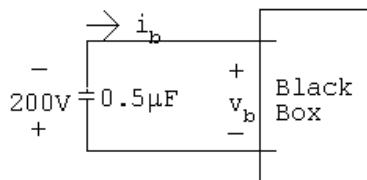
$$[\mathbf{g}] \quad w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(21)^2 + \frac{1}{2}(8 \times 10^{-6})(-21)^2 \\ = 2205 \mu\text{J}$$

$$\text{P 6.32} \quad \frac{1}{C_e} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25} = \frac{10}{5} = 2$$

$$\therefore C_2 = 0.5 \mu\text{F}$$

$$v_b = 20 - 250 + 30 = -200 \text{ V}$$

[a]



$$v_b = -\frac{10^6}{0.5} \int_0^t -5 \times 10^{-3} e^{-50x} dx - 200$$

$$= 10,000 \frac{e^{-50x}}{-50} \Big|_0^t - 200$$

$$= -200e^{-50t} V$$

$$[b] \quad v_a = -\frac{10^6}{0.5} \int_0^t -5 \times 10^{-3} e^{-50x} dx - 20$$

$$= 20(e^{-50t} - 1) - 20$$

$$= 20e^{-50t} - 40 V$$

$$[c] \quad v_c = \frac{10^6}{1.25} \int_0^t -5 \times 10^{-3} e^{-50x} dx - 30$$

$$= 80(e^{-50t} - 1) - 30$$

$$= 80e^{-50t} - 110 V$$

$$[d] \quad v_d = 10^6 \int_0^t -5 \times 10^{-3} e^{-50x} dx + 250$$

$$= 100(e^{-50t} - 1) + 250$$

$$= 100e^{-50t} + 150 V$$

$$\text{CHECK: } v_b = -v_c - v_d - v_a$$

$$= -200e^{-50t} V \quad (\text{checks})$$

$$[e] \quad i_1 = 0.2 \times 10^{-6} \frac{d}{dt} [100e^{-50t} + 150]$$

$$= 0.2 \times 10^{-6} (-5000e^{-50t}) = -e^{-50t} mA$$

$$[f] \quad i_2 = 0.8 \times 10^{-6} \frac{d}{dt} [100e^{-50t} + 150] = -4e^{-50t} mA$$

$$\text{CHECK: } i_b = i_1 + i_2 = -5e^{-50t} mA \quad (\text{OK})$$

$$\begin{aligned} \text{P 6.33 [a]} \quad w(0) &= \frac{1}{2}(0.2 \times 10^{-6})(250)^2 + \frac{1}{2}(0.8 \times 10^{-6})(250)^2 + \frac{1}{2}(5 \times 10^{-6})(20)^2 \\ &\quad + \frac{1}{2}(1.25 \times 10^{-6})(30)^2 \\ &= 32,812.5 \mu\text{J} \end{aligned}$$

$$\begin{aligned} \text{[b]} \quad w(\infty) &= \frac{1}{2}(5 \times 10^{-6})(40)^2 + \frac{1}{2}(1.25 \times 10^{-6})(110)^2 + \frac{1}{2}(0.2 \times 10^{-6})(150)^2 \\ &\quad + \frac{1}{2}(0.8 \times 10^{-6})(150)^2 \\ &= 22,812.5 \mu\text{J} \end{aligned}$$

$$\text{[c]} \quad w = \frac{1}{2}(0.5 \times 10^{-6})(200)^2 = 10,000 \mu\text{J}$$

CHECK: $32,812.5 - 22,812.5 = 10,000 \mu\text{J}$

$$\text{[d]} \quad \% \text{ delivered} = \frac{10,000}{32,812.5} \times 100 = 30.48\%$$

$$\text{[e]} \quad w = \int_0^t (-0.005e^{-50x})(-200e^{-50x}) dx = \int_0^t e^{-100x} dx$$

$$= 10(1 - e^{-100t}) \text{ mJ}$$

$$\therefore 10^{-2}(1 - e^{-100t}) = 7.5 \times 10^{-3}; \quad e^{-100t} = 0.25$$

$$\text{Thus, } t = \frac{\ln 4}{100} = 13.86 \text{ ms.}$$

$$\text{P 6.34} \quad v_c = -\frac{1}{10 \times 10^{-6}} \left(\int_0^t 0.2e^{-800x} dx - \int_0^t 0.04e^{-200x} dx \right) + 5$$

$$= 25(e^{-800t} - 1) - 20(e^{-200t} - 1) + 5$$

$$= 25e^{-800t} - 20e^{-200t} \text{ V}$$

$$v_L = 150 \times 10^{-3} \frac{di_o}{dt}$$

$$= 150 \times 10^{-3}(-160e^{-800t} + 8e^{-200t})$$

$$= -24e^{-800t} + 1.2e^{-200t} \text{ V}$$

$$v_o = v_c - v_L$$

$$= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t})$$

$$= 49e^{-800t} - 21.2e^{-200t} \text{ V}, t > 0$$

$$\begin{aligned} \text{P 6.35} \quad \frac{di_o}{dt} &= (2)\{e^{-5000t}[-1000 \sin 1000t + 5000 \cos 1000t] \\ &\quad + (-5000e^{-5000t})[\cos 1000t + 5 \sin 1000t]\} \\ &= e^{-5000t}\{-52,000 \sin 1000t\} \text{ V} \end{aligned}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

$$\therefore 50 \times 10^{-3} \frac{di_o}{dt}(0^+) = 0 \quad \text{so} \quad v_2(0^+) = 0$$

$$v_1(0^+) = 25i_o(0^+) + v_2(0^+) = 25(2) + 0 = 50 \text{ V}$$

- P 6.36 [a] Rearrange by organizing the equations by di_1/dt , i_1 , di_2/dt , i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4 \frac{di_1}{dt} + 25i_1 - 8 \frac{di_2}{dt} - 20i_2 = 5i_g - 8 \frac{di_g}{dt}$$

$$-8 \frac{di_1}{dt} - 20i_1 + 16 \frac{di_2}{dt} + 80i_2 = 16 \frac{di_g}{dt}$$

- [b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4 \frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8 \frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8 \frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad (\text{OK})$$

$$8 \frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16 \frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16 \frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+ 80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad (\text{OK})$$

P 6.37 [a] Yes, using KVL around the lower right loop

$$v_o = v_{20\Omega} + v_{60\Omega} = 20(i_2 - i_1) + 60i_2$$

$$\begin{aligned} [\text{b}] \quad v_o &= 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 20(-3 - 116e^{-5t} + 119e^{-4t}) + 60 - 3120e^{-5t} + 3060e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t} \text{ V} \end{aligned}$$

$$\begin{aligned} [\text{c}] \quad v_o &= L_2 \frac{d}{dt}(i_g - i_2) + M \frac{di_1}{dt} \\ &= 16 \frac{d}{dt}(15 + 36e^{-5t} - 51e^{-4t}) + 8 \frac{d}{dt}(4 + 64e^{-5t} - 68e^{-4t}) \\ &= -2880e^{-5t} + 3264e^{-4t} - 2560e^{-5t} + 2176e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t} \text{ V} \end{aligned}$$

$$\begin{aligned} \text{P 6.38} \quad [\text{a}] \quad v_g &= 5(i_g - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 60 + 5780e^{-4t} - 5840e^{-5t} \text{ V} \end{aligned}$$

$$[\text{b}] \quad v_g(0) = 60 + 5780 - 5840 = 0 \text{ V}$$

$$\begin{aligned}
 [\mathbf{c}] \quad p_{\text{dev}} &= v_g i_g \\
 &= 960 + 92,480e^{-4t} - 94,400e^{-5t} - 92,480e^{-9t} + \\
 &\quad 93,440e^{-10t} \text{W}
 \end{aligned}$$

$$[\mathbf{d}] \quad p_{\text{dev}}(\infty) = 960 \text{ W}$$

$$[\mathbf{e}] \quad i_1(\infty) = 4 \text{ A}; \quad i_2(\infty) = 1 \text{ A}; \quad i_g(\infty) = 16 \text{ A};$$

$$p_{5\Omega} = (16 - 4)^2(5) = 720 \text{ W}$$

$$p_{20\Omega} = 3^2(20) = 180 \text{ W}$$

$$p_{60\Omega} = 1^2(60) = 60 \text{ W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960 \text{ W}$$

$$\therefore \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960 \text{ W}$$

$$\text{P 6.39} \quad [\mathbf{a}] \quad 0.5 \frac{di_g}{dt} + 0.2 \frac{di_2}{dt} + 10i_2 = 0$$

$$0.2 \frac{di_2}{dt} + 10i_2 = -0.5 \frac{di_g}{dt}$$

$$[\mathbf{b}] \quad i_2 = 625e^{-10t} - 250e^{-50t} \text{ mA}$$

$$\frac{di_2}{dt} = -6.25e^{-10t} + 12.5e^{-50t} \text{ A/s}$$

$$i_g = e^{-10t} - 10 \text{ A}$$

$$\frac{di_g}{dt} = -10e^{-10t} \text{ A/s}$$

$$0.2 \frac{di_2}{dt} + 10i_2 = 5e^{-10t} \quad \text{and} \quad -0.5 \frac{di_g}{dt} = 5e^{-10t}$$

$$[\mathbf{c}] \quad v_1 = 5 \frac{di_g}{dt} + 0.5 \frac{di_2}{dt}$$

$$\begin{aligned}
 &= 5(-10e^{-10t}) + 0.5(-6.25e^{-10t} + 12.5e^{-50t}) \\
 &= -53.125e^{-10t} + 6.25e^{-50t} \text{ V}, \quad t > 0
 \end{aligned}$$

$$[\mathbf{d}] \quad v_1(0) = -53.125 + 6.25 = -46.875 \text{ V}; \quad \text{Also}$$

$$\begin{aligned}
 v_1(0) &= 5 \frac{di_g}{dt}(0) + 0.5 \frac{di_2}{dt}(0) \\
 &= 5(-10) + 0.5(-6.25 + 12.5) = -46.875 \text{ V}
 \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.40 [a] $v_{ab} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt} = (L_1 + L_2 + 2M) \frac{di}{dt}$

It follows that $L_{ab} = (L_1 + L_2 + 2M)$

[b] $v_{ab} = L_1 \frac{di}{dt} - M \frac{di}{dt} + L_2 \frac{di}{dt} - M \frac{di}{dt} = (L_1 + L_2 - 2M) \frac{di}{dt}$

Therefore $L_{ab} = (L_1 + L_2 - 2M)$

P 6.41 [a] $v_{ab} = L_1 \frac{d(i_1 - i_2)}{dt} + M \frac{di_2}{dt}$
 $0 = L_1 \frac{d(i_2 - i_1)}{dt} - M \frac{di_2}{dt} + M \frac{d(i_1 - i_2)}{dt} + L_2 \frac{di_2}{dt}$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{ab} = L_1 \frac{di_1}{dt} + (M - L_1) \frac{di_2}{dt}$$

and

$$0 = (M - L_1) \frac{di_1}{dt} + (L_1 + L_2 - 2M) \frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1 L_2 - M^2} v_{ab}$$

from which we have

$$v_{ab} = \left(\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \right) \left(\frac{di_1}{dt} \right)$$

$$\therefore L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

- [b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

- P 6.42 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks downscale, the induced voltage across the voltmeter must be negative at its positive terminal. Therefore, the voltage is negative at the positive terminal of the voltmeter.

Thus, the upper terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the upper terminal of the unmarked coil.

P 6.43 [a] Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.44 [a] $\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right) \left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right) \left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right) \left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.45 [a] $k = \frac{M}{\sqrt{L_1L_2}} = \frac{22.8}{\sqrt{576}} = 0.95$

[b] $M_{\max} = \sqrt{576} = 24 \text{ mH}$

[c] $\frac{L_1}{L_2} = \frac{N_1^2 \mathcal{P}_1}{N_2^2 \mathcal{P}_2} = \left(\frac{N_1}{N_2}\right)^2$

$$\therefore \left(\frac{N_1}{N_2}\right)^2 = \frac{60}{9.6} = 6.25$$

$$\frac{N_1}{N_2} = \sqrt{6.25} = 2.5$$

P 6.46 [a] $L_2 = \left(\frac{M^2}{k^2 L_1}\right) = \frac{(0.09)^2}{(0.75)^2(0.288)} = 50 \text{ mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{0.288}{(1200)^2} = 0.2 \times 10^{-6} \text{ Wb/A}$

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6} \text{ Wb/A}$$

P 6.47 [a] $W = (0.5)L_1 i_1^2 + (0.5)L_2 i_2^2 + M i_1 i_2$

$$M = 0.85 \sqrt{(18)(32)} = 20.4 \text{ mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6 \text{ mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6 \text{ mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4 \text{ mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4 \text{ mJ}$

P 6.48 [a] $M = 1.0 \sqrt{(18)(32)} = 24 \text{ mH}, \quad i_1 = 6 \text{ A}$

$$\text{Therefore } 16i_2^2 + 144i_2 + 324 = 0, \quad i_2^2 + 9i_2 + 20.25 = 0$$

$$\text{Therefore } i_2 = -\left(\frac{9}{2}\right) \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$$

$$\text{Therefore } i_2 = -4.5 \text{ A}$$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 [a] $L_1 = N_1^2 \mathcal{P}_1; \quad \mathcal{P}_1 = \frac{72 \times 10^{-3}}{6.25 \times 10^4} = 1152 \text{ nWb/A}$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.2; \quad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore 1152 \times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 192 \text{ nWb/A}; \quad \mathcal{P}_{21} = 960 \text{ nWb/A}$$

$$M = k\sqrt{L_1 L_2} = (2/3)\sqrt{(0.072)(0.0405)} = 36 \text{ mH}$$

$$N_2 = \frac{M}{N_1 \mathcal{P}_{21}} = \frac{36 \times 10^{-3}}{(250)(960 \times 10^{-9})} = 150 \text{ turns}$$

[b] $\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{40.5 \times 10^{-3}}{(150)^2} = 1800 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 192 \text{ nWb/A}$ [see part (a)]

[d] $\frac{\phi_{22}}{\phi_{12}} = \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \frac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \frac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 960 \text{ nWb/A}; \quad \mathcal{P}_2 = 1800 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1800}{960} - 1 = 0.875$$

P 6.50 $\mathcal{P}_1 = \frac{L_1}{N_1^2} = 2 \text{ nWb/A}; \quad \mathcal{P}_2 = \frac{L_2}{N_2^2} = 2 \text{ nWb/A}; \quad M = k\sqrt{L_1 L_2} = 180 \mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

- P 6.51 When the touchscreen in the mutual-capacitance design is touched at the point x, y , the touch capacitance C_t is present in series with the mutual capacitance at the touch point, C_{mxy} . Remember that capacitances combine in series the way that resistances combine in parallel. The resulting mutual capacitance is

$$C'_{mxy} = \frac{C_{mxy} C_t}{C_{mxy} + C_t}$$

- P 6.52 [a] The self-capacitance and the touch capacitance are effectively connected in parallel. Therefore, the capacitance at the x-grid electrode closest to the touch point with respect to ground is

$$C_x = C_p + C_t = 30 \text{ pF} + 15 \text{ pF} = 45 \text{ pF}.$$

The same capacitance exists at the y-grid electrode closest to the touch point with respect to ground.

- [b] The mutual-capacitance and the touch capacitance are effectively connected in series. Therefore, the mutual capacitance between the x-grid and y-grid electrodes closest to the touch point is

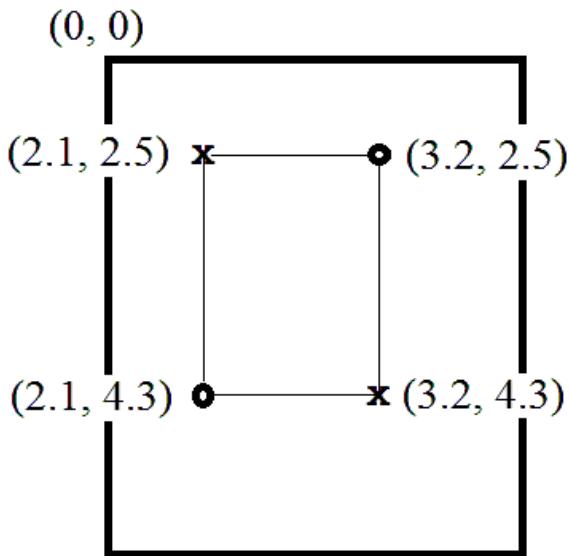
$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t} = \frac{(30)(15)}{30 + 15} = 10 \text{ pF.}$$

- [c] In the self-capacitance design, touching the screen increases the capacitance being measured at the point of touch. For example, in part (a) the measured capacitance before the touch is 30 pF and after the touch is 45 pF. In the mutual-capacitance design, touching the screen decreases the capacitance being measured at the point of touch. For example, in part (b) the measured capacitance before the touch is 30 pF and after the touch is 10 pF.

- P 6.53 [a] The four touch points identified are the two actual touch points and two ghost touch points. Their coordinates, in inches from the upper left corner of the screen, are

$$(2.1, 4.3); \quad (3.2, 2.5); \quad (2.1, 2.5); \quad \text{and} \quad (3.2, 4.3)$$

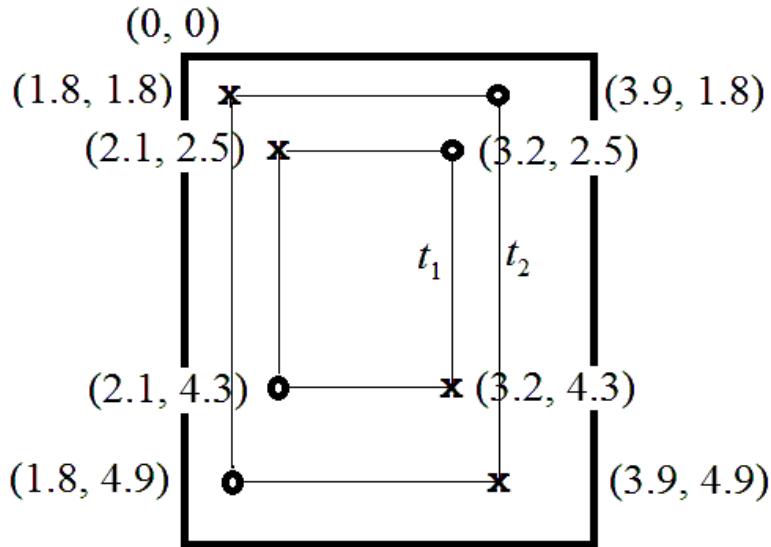
These four coordinates identify a rectangle within the screen, shown below.



- [b] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$$(1.8, 4.9); \quad (3.9, 1.8); \quad (1.8, 1.8); \quad \text{and} \quad (3.9, 4.9)$$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2 , as shown here:

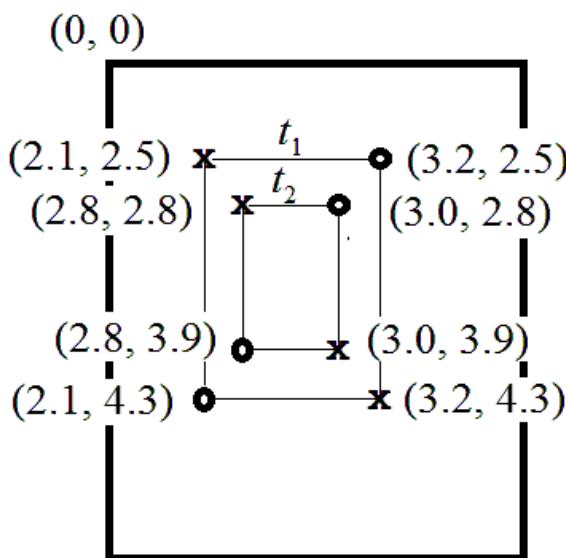


Note that the rectangle at time t_2 is larger than the rectangle at time t_1 , so the software would recognize the two fingers are moving toward the edges of the screen. This pinch gesture thus specifies a zoom-in for the screen.

- [c] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$$(2.8, 3.9); \quad (3.0, 2.8); \quad (2.8, 2.8); \quad \text{and} \quad (3.0, 3.9)$$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2 , as shown here:



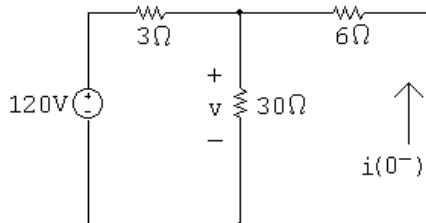
Here, the rectangle at time t_2 is smaller than the rectangle at time t_1 , so the software would recognize the two fingers are moving toward the middle of the screen. This pinch gesture thus specifies a zoom-out for the screen.

7

Response of First-Order RL and RC Circuits

Assessment Problems

- AP 7.1 [a] The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the 2Ω resistor from the circuit.



First combine the 30Ω and 6Ω resistors in parallel:

$$30\parallel 6 = 5\Omega$$

Use voltage division to find the voltage drop across the parallel resistors:

$$v = \frac{5}{5+3}(120) = 75\text{ V}$$

Now find the current using Ohm's law:

$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\text{ A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\text{ mJ}$

- [c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the 2Ω resistor remains connected to the inductor. Thus,

$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\text{ ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\text{ A}, \quad t \geq 0$

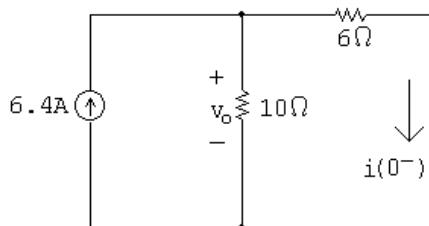
[e] $i(5\text{ ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\text{ A}$

So $w(5\text{ ms}) = \frac{1}{2}Li^2(5\text{ ms}) = \frac{1}{2}(8) \times 10^{-3}(3.58)^2 = 51.3\text{ mJ}$

$$w(\text{dis}) = 625 - 51.3 = 573.7 \text{ mJ}$$

$$\% \text{ dissipated} = \left(\frac{573.7}{625} \right) 100 = 91.8\%$$

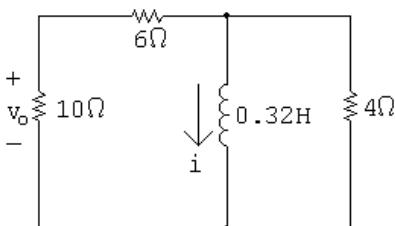
AP 7.2 [a] First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,

$$i(0^-) = \frac{10}{10 + 6}(6.4) = 4 \text{ A}$$

Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$$R_{\text{eq}} = 4\|(6 + 10) = 3.2 \Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1 \text{ s}$$

Use the initial inductor current and the time constant to find the current in the inductor:

$$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t} \text{ A}, \quad t \geq 0$$

Use current division to find the current in the 10Ω resistor:

$$i_o(t) = \frac{4}{4 + 10 + 6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t} \text{ A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the 10Ω resistor:

$$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t} \text{ V}, \quad t \geq 0^+$$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56 \text{ J}$$

Find the energy dissipated in the 4Ω resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L \frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t} \text{ V}, \quad t \geq 0^+$$

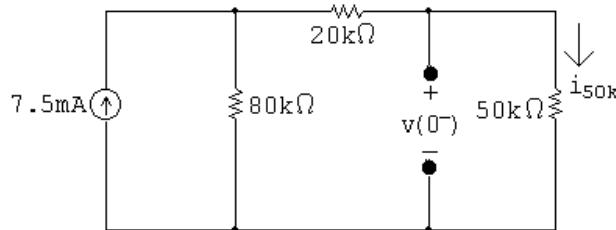
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t} \text{ W}, \quad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t} dt = 2.048 \text{ J}$$

Find the percentage of the initial energy in the inductor dissipated in the 4Ω resistor:

$$\% \text{ dissipated} = \left(\frac{2.048}{2.56} \right) 100 = 80\%$$

- AP 7.3 [a] The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\text{k}\Omega$ resistor. First use current division to find the current through the $50\text{k}\Omega$ resistor:

$$i_{50k} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3} (7.5 \times 10^{-3}) = 4 \text{ mA}$$

Use Ohm's law to find the voltage drop:

$$v(0^-) = (50 \times 10^3) i_{50k} = (50 \times 10^3)(0.004) = 200 \text{ V}$$

- [b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\text{k}\Omega$ resistor remains connected to the capacitor. Thus,

$$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20 \text{ ms}$$

[c] $v(t) = v(0^-) e^{-t/\tau} = 200 e^{-t/0.02} = 200 e^{-50t} \text{ V}, \quad t \geq 0$

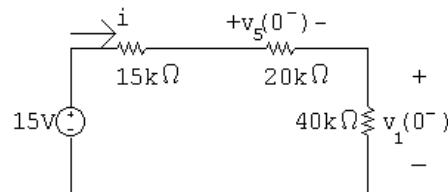
[d] $w(0) = \frac{1}{2} Cv^2 = \frac{1}{2} (0.4 \times 10^{-6})(200)^2 = 8 \text{ mJ}$

[e] $w(t) = \frac{1}{2} Cv^2(t) = \frac{1}{2} (0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t} \text{ mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3} e^{-100t} = 2 \times 10^{-3}, \quad e^{100t} = 4, \quad t = (\ln 4)/100 = 13.86 \text{ ms}$$

- AP 7.4 [a] This circuit is actually two RC circuits in series, and the requested voltage, v_o , is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75,000} = 0.2 \text{ mA}, \quad v_5(0^-) = 4 \text{ V}, \quad v_1(0^-) = 8 \text{ V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5 \mu\text{F} - 20 \text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1 \mu\text{F} - 40 \text{k}\Omega$ subcircuit:

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100 \text{ ms}; \quad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40 \text{ ms}$$

Therefore,

$$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t} \text{ V}, \quad t \geq 0$$

$$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t} \text{ V}, \quad t \geq 0$$

Finally,

$$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}] \text{ V}, \quad t \geq 0$$

- [b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$$v_1(60 \text{ ms}) = 8e^{-25(0.06)} \cong 1.79 \text{ V}, \quad v_5(60 \text{ ms}) = 4e^{-10(0.06)} \cong 2.20 \text{ V}$$

$$w_1(60 \text{ ms}) = \frac{1}{2}Cv_1^2(60 \text{ ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59 \mu\text{J}$$

$$w_5(60 \text{ ms}) = \frac{1}{2}Cv_5^2(60 \text{ ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05 \mu\text{J}$$

$$w(60 \text{ ms}) = 1.59 + 12.05 = 13.64 \mu\text{J}$$

Find the initial energy from the initial voltage:

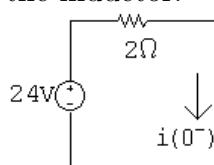
$$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72 \mu\text{J}$$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$$w_{\text{diss}} = w(0) - w(60 \text{ ms}) = 72 - 13.64 = 58.36 \mu\text{J}$$

$$\% \text{ dissipated} = (58.36 \times 10^{-6} / 72 \times 10^{-6})(100) = 81.05 \%$$

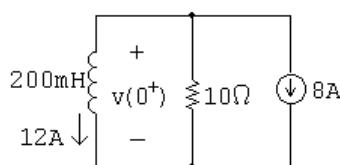
- AP 7.5 [a] Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$$i(0^-) = 24/2 = 12 \text{ A} = i(0^+)$$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

- [b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+ . Note that this is the same as the voltage drop across the 10Ω resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

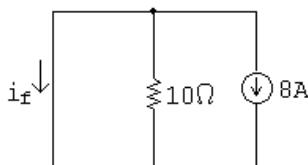


$$v(0^+) = -10(8 + 12) = -200 \text{ V}$$

- [c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the 10Ω resistor is connected to the inductor for $t > 0$. Thus,

$$\tau = L/R = (200 \times 10^{-3}/10) = 20 \text{ ms}$$

- [d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$$i_f = -8 \text{ A}$$

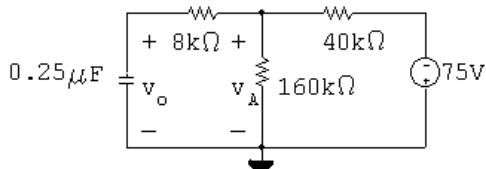
Now,

$$\begin{aligned} i(t) &= i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02} \\ &= -8 + 20e^{-50t} \text{ A}, \quad t \geq 0 \end{aligned}$$

- [e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L \frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t} \text{ V}, \quad t \geq 0^+$$

AP 7.6 [a]



From Example 7.6,

$$v_o(t) = -60 + 90e^{-100t} \text{ V}$$

Write a KCL equation at the top node and use it to find the relationship between v_o and v_A :

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160,000} + \frac{v_A + 75}{40,000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

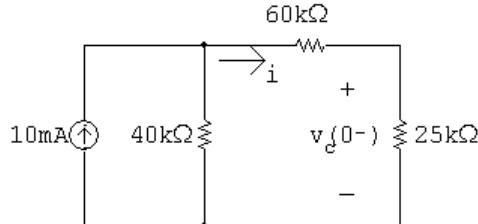
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A :

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t} \text{ V}, \quad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

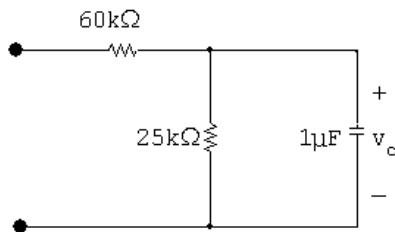
AP 7.7 [a] Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3} \right) (10 \times 10^{-3}) = 3.2 \text{ mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80 \text{ V} \quad \text{so} \quad v_c(0^+) = 80 \text{ V}$$

Now use the next circuit, valid for $0 \leq t \leq 10 \text{ ms}$, to calculate $v_c(t)$ for that interval:



For $0 \leq t \leq 100 \text{ ms}$:

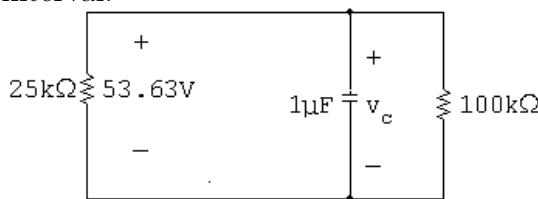
$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25 \text{ ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t} \text{ V} \quad 0 \leq t \leq 10 \text{ ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10 \text{ ms}$, using the capacitor voltage from the previous interval:

$$v_c(0.01) = 80e^{-40(0.01)} = 53.63 \text{ V}$$

Now use the next circuit, valid for $t \geq 10 \text{ ms}$, to calculate $v_c(t)$ for that interval:



For $t \geq 10 \text{ ms}$:

$$R_{\text{eq}} = 25 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 20 \text{ k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02 \text{ s}$$

$$\text{Therefore } v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)} \text{ V}, \quad t \geq 0.01 \text{ s}$$

- [c] To calculate the energy dissipated in the $25\text{ k}\Omega$ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\text{ k}\Omega} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25,000} dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25,000} dt = 2.91\text{ mJ}$$

- [d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

$$w_{100\text{ k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100,000} dt = 0.29\text{ mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the $25\text{ k}\Omega$ resistor and the $100\text{ k}\Omega$ resistor.

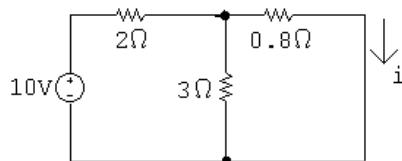
$$\text{Check: } w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\text{ mJ}$$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\text{ mJ}$$

- AP 7.8 [a] Prior to switch A closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.

At the instant A is closed, $i(0^+) = 0$.

For $0 \leq t \leq 1\text{ s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\parallel 0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\parallel 0.8)} = 3.8\text{ A}$$

The final current in the inductor, which is equal to the current in the 0.8Ω resistor is

$$I_F = \frac{3}{3 + 0.8}(3.8) = 3\text{ A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\parallel 3) + 0.8]\parallel 3\parallel 6 = 1\Omega \quad \tau = \frac{L}{R} = \frac{2}{1} = 2\text{ s}$$

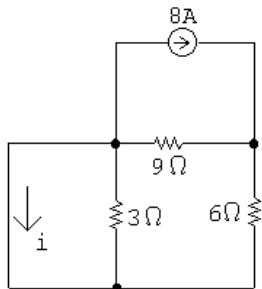
Therefore,

$$i = i_F + [i(0^+) - i_F]e^{-t/\tau} = 3 - 3e^{-0.5t}\text{ A}, \quad 0 \leq t \leq 1\text{ s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-0.5} = 1.18 \text{ A}$$

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9+6}(-8) = -4.8 \text{ A}$$

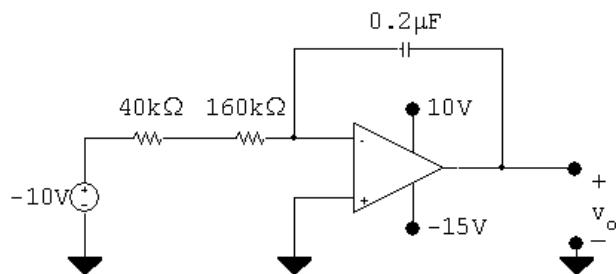
The equivalent resistance seen by the inductor is used to calculate the time constant:

$$3\parallel(9+6) = 2.5 \Omega \quad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8 \text{ s}$$

Therefore,

$$\begin{aligned} i &= i_F + [i(1^+) - i_F]e^{-(t-1)/\tau} \\ &= -4.8 + 5.98e^{-1.25(t-1)} \text{ A}, \quad t \geq 1 \text{ s} \end{aligned}$$

AP 7.9 $0 \leq t \leq 32 \text{ ms}$:

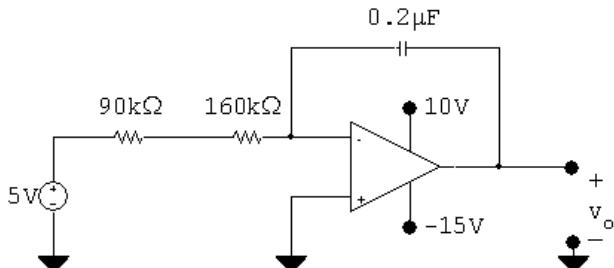


$$v_o = -\frac{1}{RC_f} \int_0^{32 \times 10^{-3}} -10 dt + 0 = -\frac{1}{RC_f} (-10t) \Big|_0^{32 \times 10^{-3}} = -\frac{1}{RC_f} (-320 \times 10^{-3})$$

$$RC_f = (200 \times 10^3)(0.2 \times 10^{-6}) = 40 \times 10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320 \times 10^{-3}) = 8 \text{ V}$$

$t \geq 32 \text{ ms}$:



$$v_o = -\frac{1}{RC_f} \int_{32 \times 10^{-3}}^t 5 dy + 8 = -\frac{1}{RC_f}(5y) \Big|_{32 \times 10^{-3}}^t + 8 = -\frac{1}{RC_f}5(t - 32 \times 10^{-3}) + 8$$

$$RC_f = (250 \times 10^3)(0.2 \times 10^{-6}) = 50 \times 10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32 \times 10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262 \text{ ms}$$

- AP 7.10 [a] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$$

$$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \quad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t} \text{ V}; \quad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o :

$$\frac{v_n}{10,000} + \frac{v_n - v_o}{40,000} = 0$$

$$\therefore v_o = 5v_n = 5v_p = -10 + 10e^{-625t} \text{ V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11 \text{ ms}$$

- [b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t} \text{ V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t} \text{ V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \quad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76 \text{ ms}$$

Problems

P 7.1 [a] $i_o(0) = \frac{20}{16 + 12 + 4 + 8} = \frac{20}{40} = 0.5 \text{ A}$

$$i_o(\infty) = 0 \text{ A}$$

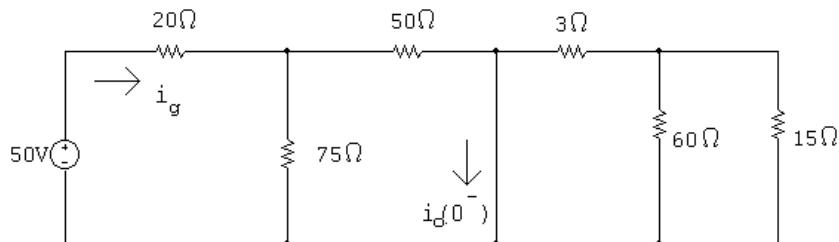
[b] $i_o = 0.5e^{-t/\tau}; \quad \tau = \frac{L}{R} = \frac{80 \times 10^{-3}}{12 + 8} = 4 \text{ ms}$

$$i_o = 0.5e^{-250t} \text{ A}, \quad t \geq 0$$

[c] $0.5e^{-250t} = 0.1$

$$e^{250t} = 5 \quad \therefore t = 6.44 \text{ ms}$$

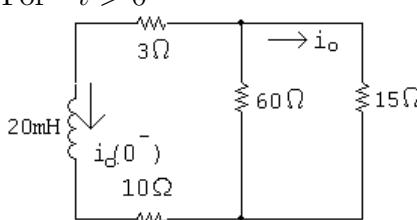
P 7.2 [a] For $t < 0$



$$i_g = \frac{50}{20 + (75\parallel 50)} = \frac{50}{50} = 1 \text{ A}$$

$$i_o(0^-) = \frac{50}{75 + 50}(1) = 0.4 \text{ A} = i_o(0^+)$$

For $t > 0$



$$i_o(t) = i_o(0^+)e^{-t/\tau} \text{ A}, \quad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.02}{3 + 60\parallel 15} = 1.33 \text{ ms}; \quad \frac{1}{\tau} = 750$$

$$i_o(t) = 0.4e^{-750t} \text{ A}, \quad t \geq 0$$

[b] $v_L = L \frac{di_o}{dt} = 0.02(-750)(0.4e^{-750t}) = -6e^{-750t} \text{ V}$

$$v_o = \frac{60\parallel 15}{3 + 60\parallel 15} v_L = \frac{12}{15}(-6e^{-750t}) = -4.8e^{-750t} \text{ V} \quad t \geq 0^+$$

P 7.3 [a] $i(0) = \frac{60}{120} = 0.5 \text{ A}$

[b] $\tau = \frac{L}{R} = \frac{0.32}{160} = 2 \text{ ms}$

[c] $i = 0.5e^{-500t} \text{ A}, \quad t \geq 0$

$$v_1 = L \frac{di}{dt} (0.5e^{-500t}) = -80e^{-500t} \text{ V} \quad t \geq 0^+$$

$$v_2 = -70i = -35e^{-500t} \text{ V} \quad t \geq 0^+$$

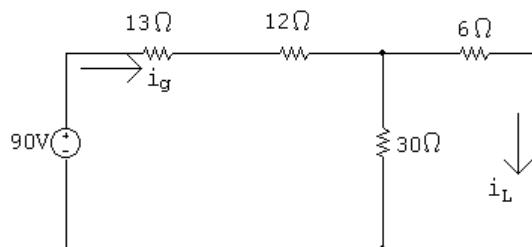
[d] $w(0) = \frac{1}{2}(0.32)(0.5)^2 = 40 \text{ mJ}$

$$w_{90\Omega} = \int_0^t 90(0.25e^{-1000x}) dx = 22.5 \frac{e^{-1000x}}{-1000} \Big|_0^t = 22.5(1 - e^{-1000t}) \text{ mJ}$$

$$w_{90\Omega}(1 \text{ ms}) = 0.0225(1 - e^{-1}) = 14.22 \text{ mJ}$$

$$\% \text{ dissipated} = \frac{14.22}{40}(100) = 35.6\%$$

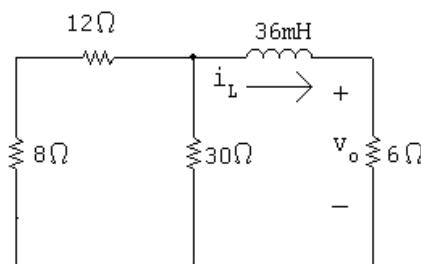
P 7.4 $t < 0$:



$$i_g = \frac{90}{13 + 12 + 6\parallel 30} = 3 \text{ A}$$

$$i_L(0^-) = \frac{30}{36}(3) = 2.5 \text{ A}$$

$t > 0$:



$$R_e = 6 + 30\|(8 + 12) = 6 + 12 = 18 \Omega$$

$$\tau = \frac{L}{R_e} = \frac{36 \times 10^{-3}}{18} = 2 \text{ ms}; \quad \frac{1}{\tau} = 500$$

$$\therefore i_L = 2.5e^{-500t} \text{ A}$$

$$v_o = 6i_o = 15e^{-500t} \text{ V}, \quad t \geq 0^+$$

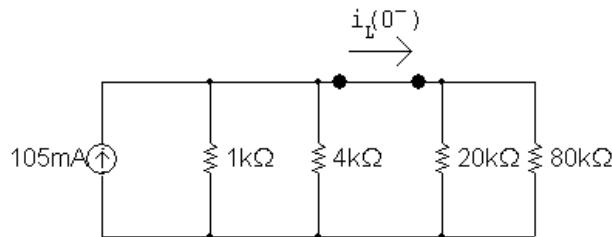
$$\text{P 7.5} \quad p_{6\Omega} = \frac{v_o^2}{6} = \frac{(15)^2}{6} e^{-1000t} = 37.5e^{-1000t} \text{ W}$$

$$w_{6\Omega} = \int_0^\infty 37.5e^{-1000t} dt = 37.5 \frac{e^{-1000t}}{-1000} \Big|_0^\infty = 37.5 \text{ mJ}$$

$$w(0) = \frac{1}{2}(36 \times 10^{-3})(2.5)^2 = 112.5 \text{ mJ}$$

$$\% \text{ diss} = \frac{37.5}{112.5}(100) = 33.33\%$$

P 7.6 [a] $t < 0$

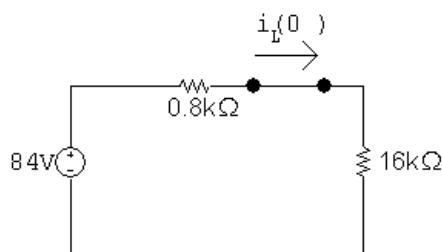


Simplify this circuit by creating a Thévenin equivalent to the left of the inductor and an equivalent resistance to the right of the inductor:

$$1\text{k}\Omega \parallel 4\text{k}\Omega = 0.8\text{k}\Omega$$

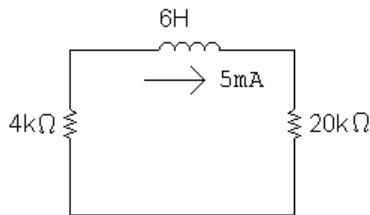
$$20\text{k}\Omega \parallel 80\text{k}\Omega = 16\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^3) = 84 \text{ V}$$



$$i_L(0+) = \frac{84}{16,800} = 5 \text{ mA}$$

$$t > 0$$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250 \mu\text{s}; \quad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t} \text{ mA}, \quad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6} e^{-8000t} (4000) = 0.10 e^{-8000t} \text{ W}$$

$$w_{\text{diss}} = \int_0^t 0.10 e^{-8000x} dx = 12.5 \times 10^{-6} [1 - e^{-8000t}] \text{ J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75 \mu\text{J}$$

$$0.10w(0) = 7.5 \mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \quad \therefore e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54 \mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t}) \mu\text{J}$

$$w_{\text{diss}}(114.54 \mu\text{s}) = 45 \mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.7 [a] $v_o(t) = v_o(0^+) e^{-t/\tau}$

$$\therefore v_o(0^+) e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore e^{10^{-3}/\tau} = 2$$

$$\therefore \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43 \text{ mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30 \text{ mV}$

$$v_o(t) = -0.03e^{-t/\tau} \text{ V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5} e^{-2t/\tau}$$

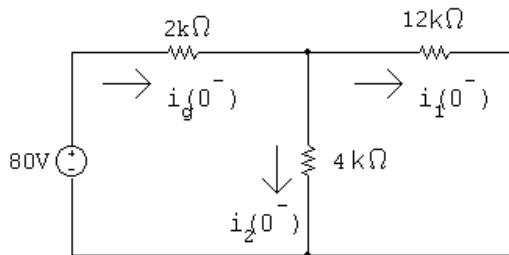
$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5} e^{-2t/\tau} dt = 4.5\tau \times 10^{-5} (1 - e^{-2 \times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000 \ln 2} \quad \therefore \quad w_{10\Omega} = 48.69 \text{ nJ}$$

$$w_L(0) = \frac{1}{2} L i_L^2(0) = \frac{1}{2} (14.43 \times 10^{-3}) (3 \times 10^{-3})^2 = 64.92 \text{ nJ}$$

$$\% \text{ diss in 1 ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 7.8 [a] $t < 0$



$$4 \text{ k}\Omega \parallel 12 \text{ k}\Omega = 3 \text{ k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{80}{(2000 + 3000)} = 16 \text{ mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{3000}{12,000} (0.016) = 4 \text{ mA}$$

$$i_2(0^-) = \frac{3000}{4000} (0.016) = 12 \text{ mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 4 \text{ mA}$$

$$i_2(0^+) = -i_1(0^+) = -4 \text{ mA} \quad (\text{when switch is open})$$

$$[c] \quad \tau = \frac{L}{R} = \frac{0.64 \times 10^{-3}}{16 \times 10^3} = 4 \times 10^{-5} \text{ s}; \quad \frac{1}{\tau} = 25,000$$

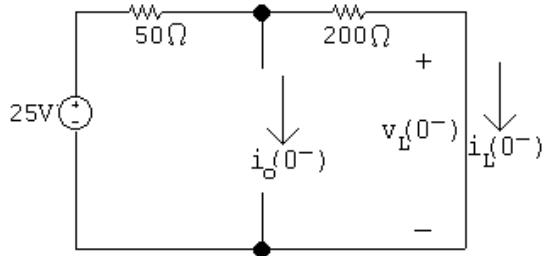
$$i_1(t) = i_1(0^+) e^{-t/\tau} = 4e^{-25,000t} \text{ mA}, \quad t \geq 0$$

$$[d] \quad i_2(t) = -i_1(t) \quad \text{when } t \geq 0^+$$

$$\therefore i_2(t) = -4e^{-25,000t} \text{ mA}, \quad t \geq 0^+$$

- [e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 12 mA and $i_2(0^+) = -4$ mA.

P 7.9 [a] For $t = 0^-$ the circuit is:

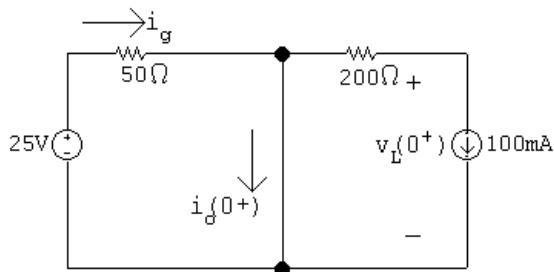


$$i_o(0^-) = 0 \text{ since the switch is open}$$

$$i_L(0^-) = \frac{25}{250} = 0.1 = 100 \text{ mA}$$

$$v_L(0^-) = 0 \text{ since the inductor behaves like a short circuit}$$

[b] For $t = 0^+$ the circuit is:



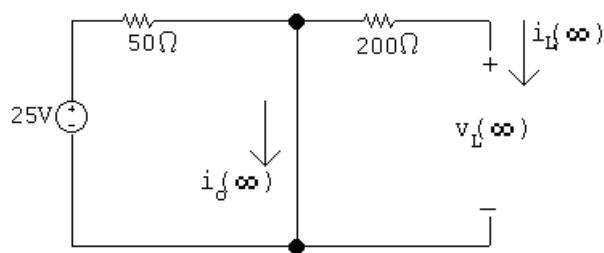
$$i_L(0^+) = i_L(0^-) = 100 \text{ mA}$$

$$i_g = \frac{25}{50} = 0.5 = 500 \text{ mA}$$

$$i_o(0^+) = i_g - i_L(0^+) = 500 - 100 = 400 \text{ mA}$$

$$200i_L(0^+) + v_L(0^+) = 0 \quad \therefore \quad v_L(0^+) = -200i_L(0^+) = -20 \text{ V}$$

[c] As $t \rightarrow \infty$ the circuit is:



$$i_L(\infty) = 0; \quad v_L(\infty) = 0$$

$$i_o(\infty) = \frac{25}{50} = 500 \text{ mA}$$

$$[\mathbf{d}] \quad \tau = \frac{L}{R} = \frac{0.05}{200} = 0.25 \text{ ms}$$

$$i_L(t) = 0 + (0.1 - 0)e^{-4000t} = 0.1e^{-4000t} \text{ A}$$

$$[\mathbf{e}] \quad i_o(t) = i_g - i_L = 0.5 - 0.1e^{-4000t} \text{ A}$$

$$[\mathbf{f}] \quad v_L(t) = L \frac{di_L}{dt} = 0.05(-4000)(0.1)e^{-4000t} = -20e^{-4000t} \text{ V}$$

$$\text{P 7.10} \quad w(0) = \frac{1}{2}(10 \times 10^{-3})(5)^2 = 125 \text{ mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \quad i(t) = 5e^{-t/\tau} \text{ A}$$

$$\therefore w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau} \text{ mJ}$$

$$w(10 \mu\text{s}) = 125e^{-20 \times 10^{-6}/\tau} \text{ mJ}$$

$$\therefore 125e^{-20 \times 10^{-6}/\tau} = 112.5 \quad \text{so} \quad e^{20 \times 10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3} \ln(10/9)}{20 \times 10^{-6}} = 52.68 \Omega$$

$$\text{P 7.11} \quad [\mathbf{a}] \quad w(0) = \frac{1}{2}LI_g^2$$

$$\begin{aligned} w_{\text{diss}} &= \int_0^{t_o} I_g^2 R e^{-2t/\tau} dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)} \Big|_0^{t_o} \\ &= \frac{1}{2} I_g^2 R \tau (1 - e^{-2t_o/\tau}) = \frac{1}{2} I_g^2 L (1 - e^{-2t_o/\tau}) \end{aligned}$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma \left(\frac{1}{2}LI_g^2 \right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \quad e^{2t_o/\tau} = \frac{1}{(1 - \sigma)}$$

$$\frac{2t_o}{\tau} = \ln \left[\frac{1}{(1-\sigma)} \right]; \quad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L \ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \frac{(10 \times 10^{-3}) \ln[1/0.9]}{20 \times 10^{-6}}$

$$R = 52.68 \Omega$$

P 7.12 [a] $R = \frac{v}{i} = 25 \Omega$

[b] $\tau = \frac{1}{10} = 100 \text{ ms}$

[c] $\tau = \frac{L}{R} = 0.1$

$$L = (0.1)(25) = 2.5 \text{ H}$$

[d] $w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(2.5)(6.4)^2 = 51.2 \text{ J}$

[e] $w_{\text{diss}} = \int_0^t 1024e^{-20x} dx = 1024 \frac{e^{-20x}}{-20} \Big|_0^t = 51.2(1 - e^{-20t}) \text{ J}$

$$\% \text{ dissipated} = \frac{51.2(1 - e^{-20t})}{51.2}(100) = 100(1 - e^{-20t})$$

$$\therefore 100(1 - e^{-20t}) = 60 \quad \text{so} \quad e^{-20t} = 0.4$$

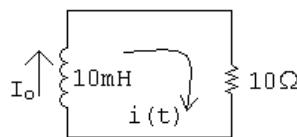
Therefore $t = \frac{1}{20} \ln 2.5 = 45.81 \text{ ms}$

P 7.13 [a] Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$$R = \frac{L}{\tau}$$

Choose a 10 mH inductor from Appendix H. Then,

$$R = \frac{0.01}{0.001} = 10 \Omega \quad \text{which is a resistor value from Appendix H.}$$



[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t} \text{ mA}, \quad t \geq 0$

$$[c] \quad w(0) = \frac{1}{2}LI_o^2 = \frac{1}{2}(0.01)(0.01)^2 = 0.5 \mu\text{J}$$

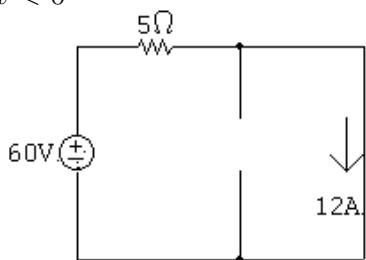
$$w(t) = \frac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5 \times 10^{-6}e^{-2000t}$$

$$\text{So } 0.5 \times 10^{-6}e^{-2000t} = \frac{1}{2}w(0) = 0.25 \times 10^{-6}$$

$$e^{-2000t} = 0.5 \quad \text{then} \quad e^{2000t} = 2$$

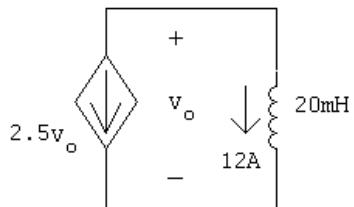
$$\therefore t = \frac{\ln 2}{2000} = 346.57 \mu\text{s} \quad (\text{for a } 10 \text{ mH inductor})$$

P 7.14 $t < 0$

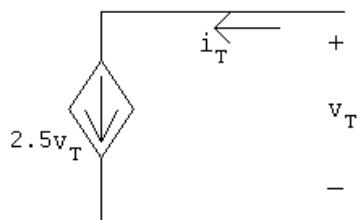


$$i_L(0^-) = i_L(0^+) = 12 \text{ A}$$

$t > 0$

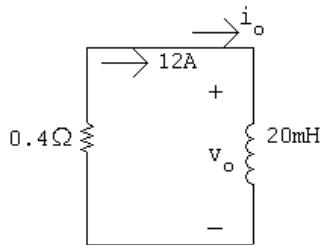


Find Thévenin resistance seen by inductor:



$$i_T = 2.5v_T; \quad \frac{v_T}{i_T} = R_{Th} = \frac{1}{2.5} = 0.4 \Omega$$

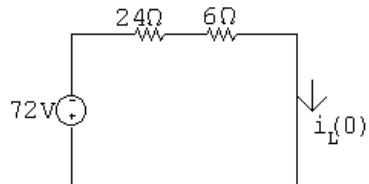
$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{0.4} = 50 \text{ ms}; \quad 1/\tau = 20$$



$$i_o = 12e^{-20t} \text{ A}, \quad t \geq 0$$

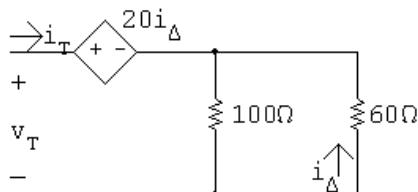
$$v_o = L \frac{di_o}{dt} = (20 \times 10^{-3})(-240e^{-20t}) = -4.8e^{-20t} \text{ V}, \quad t \geq 0^+$$

P 7.15 [a] $t < 0$:



$$i_L(0) = -\frac{72}{24 + 6} = -2.4 \text{ A}$$

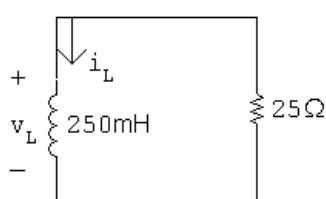
$t > 0$:



$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T \frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{Th} = -12.5 + 37.5 = 25 \Omega$$



$$\tau = \frac{L}{R} = \frac{250 \times 10^{-3}}{25} \quad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t} \text{ A}, \quad t \geq 0$$

[b] $v_L = 250 \times 10^{-3} (240e^{-100t}) = 60e^{-100t}$ V, $t \geq 0^+$

[c] $i_\Delta = 0.625i_L = -1.5e^{-100t}$ A $t \geq 0^+$

P 7.16 $w(0) = \frac{1}{2}(250 \times 10^{-3})(-2.4)^2 = 720$ mJ

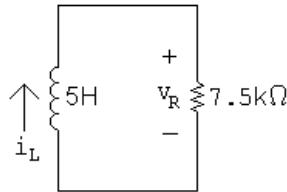
$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}$$
 W

$$w_{60\Omega} = \int_0^\infty 135e^{-200t} dt = 135 \frac{e^{-200t}}{-200} \Big|_0^\infty = 675$$
 mJ

$$\% \text{ dissipated} = \frac{675}{720}(100) = 93.75\%$$

P 7.17 [a] $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5$$
 H



$$i_L(t) = i_L(0)e^{-t/\tau} \text{ mA}; \quad i_L(0) = 2 \text{ A}; \quad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t} \text{ A}, \quad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15,000e^{-1500t} \text{ V}, \quad t \geq 0^+$$

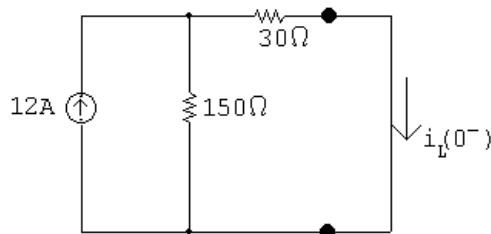
$$v_o = -3.75 \frac{di_L}{dt} = 11,250e^{-1500t} \text{ V}, \quad t \geq 0^+$$

[b] $i_o = \frac{-1}{6} \int_0^t 11,250e^{-1500x} dx + 0 = 1.25e^{-1500t} - 1.25$ A

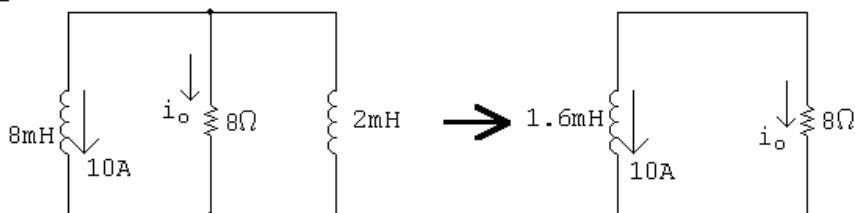
P 7.18 [a] From the solution to Problem 7.17,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10$$
 J

[b] $w_{\text{trapped}} = \frac{1}{2}(10)(1.25)^2 + \frac{1}{2}(6)(1.25)^2 = 12.5$ J

P 7.19 [a] $t < 0$ 

$$i_L(0^-) = \frac{150}{180}(12) = 10 \text{ A}$$

 $t \geq 0$ 

$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \quad 1/\tau = 5000$$

$$i_o = -10e^{-5000t} \text{ A} \quad t \geq 0$$

$$[\mathbf{b}] \quad w_{\text{del}} = \frac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80 \text{ mJ}$$

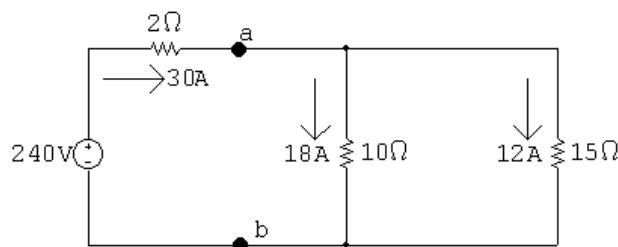
$$[\mathbf{c}] \quad 0.95w_{\text{del}} = 76 \text{ mJ}$$

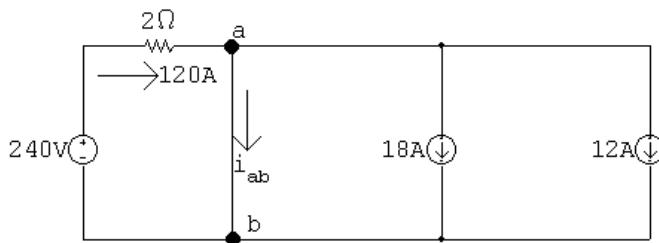
$$\therefore 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10,000t}) dt$$

$$\therefore 76 \times 10^{-3} = -80 \times 10^{-3} e^{-10,000t} \Big|_0^{t_o} = 80 \times 10^{-3} (1 - e^{-10,000t_o})$$

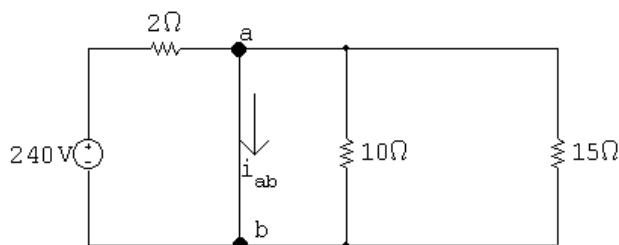
$$\therefore e^{-10,000t_o} = 0.05 \quad \text{so} \quad t_o = 299.57 \mu\text{s}$$

$$\therefore \frac{t_o}{\tau} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{so} \quad t_o \approx 1.498\tau$$

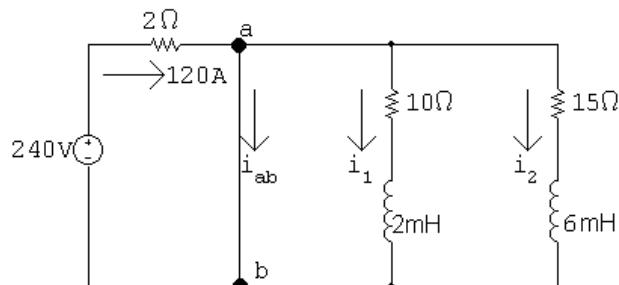
P 7.20 [a] $t < 0$:

$t = 0^+$:

$$120 = i_{ab} + 18 + 12, \quad i_{ab} = 90 \text{ A}, \quad t = 0^+$$

[b] At $t = \infty$:

$$i_{ab} = 240/2 = 120 \text{ A}, \quad t = \infty$$



$$[c] \quad i_1(0) = 18, \quad \tau_1 = \frac{2 \times 10^{-3}}{10} = 0.2 \text{ ms}$$

$$i_2(0) = 12, \quad \tau_2 = \frac{6 \times 10^{-3}}{15} = 0.4 \text{ ms}$$

$$i_1(t) = 18e^{-5000t} \text{ A}, \quad t \geq 0$$

$$i_2(t) = 12e^{-2500t} \text{ A}, \quad t \geq 0$$

$$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t} \text{ A}, \quad t \geq 0$$

$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$

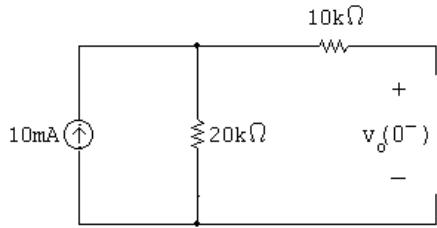
$$6 = 18e^{-5000t} + 12e^{-2500t}$$

$$\text{Let } x = e^{-2500t} \quad \text{so} \quad 6 = 18x^2 + 12x$$

$$\text{Solving } x = \frac{1}{3} = e^{-2500t}$$

$$\therefore e^{2500t} = 3 \quad \text{and} \quad t = \frac{\ln 3}{2500} = 439.44 \mu\text{s}$$

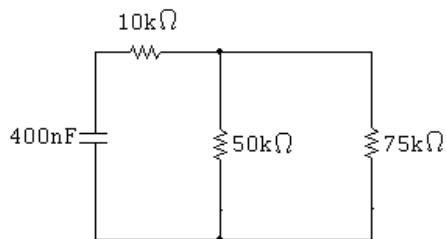
P 7.21 [a] For $t < 0$:



$$v(0) = 20,000(0.01) = 200 \text{ V}$$

$$[\mathbf{b}] \quad w(0) = \frac{1}{2} Cv(0)^2 = \frac{1}{2}(400 \times 10^{-9})(200)^2 = 8 \text{ mJ}$$

[c] For $t > 0$:

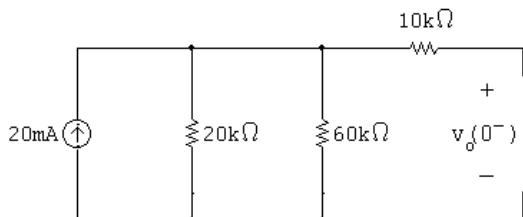


$$R_{\text{eq}} = 10,000 + 50,000\parallel 75,000 = 40 \text{ k}\Omega$$

$$\tau = R_{\text{eq}}C = (40,000)(400 \times 10^{-9}) = 16 \text{ ms}$$

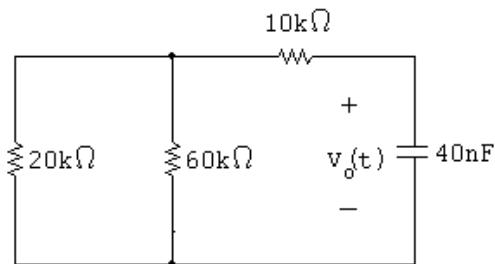
$$[\mathbf{d}] \quad v(t) = v(0)e^{-t/\tau} = 200e^{-62.5t} \text{ V} \quad t \geq 0$$

P 7.22 For $t < 0$:



$$V_o = (20,000\parallel 60,000)(20 \times 10^{-3}) = 300 \text{ V}$$

For $t \geq 0$:

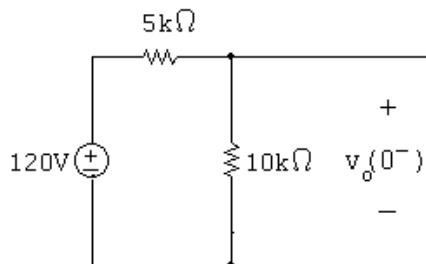


$$R_{\text{eq}} = 10,000 + (20,000\parallel 60,000) = 25\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (25,000)(40 \times 10^{-9}) = 1\text{ ms}$$

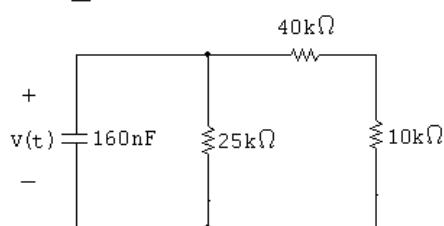
$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\text{ V} \quad t \geq 0$$

P 7.23 [a] For $t < 0$:



$$V_o = \frac{10,000}{15,000}(120) = 80\text{ V}$$

For $t \geq 0$:

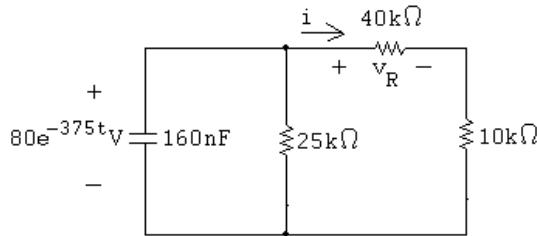


$$R_{\text{eq}} = 25,000\parallel(40,000 + 10,000) = 16.67\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (16,666/67)(160 \times 10^{-9}) = 2.67\text{ ms}$$

$$v(t) = V_o e^{-t/\tau} = 80e^{-375t}\text{ V} \quad t \geq 0$$

[b] For $t \geq 0$:



$$v_R(t) = \frac{40,000}{50,000}(80e^{-375t}) = 64e^{-375t} \text{ V}$$

$$i(t) = \frac{v_R}{40,000} = 1.6e^{-375t} \text{ mA}, \quad t \geq 0^+$$

P 7.24 Using the results of Problem 7.23:

$$w(0) = \frac{1}{2}CV_o^2 = \frac{1}{2}(160 \times 10^{-9})(80)^2 = 512 \mu\text{J}$$

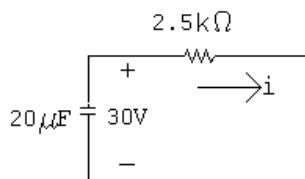
$$p_{40k} = Ri^2 = (40,000)(1.6 \times 10^{-3}e^{-375t})^2 = 0.1024e^{-750t}$$

$$w_{40k} = \int_0^\infty p_{40k} dt = \int_0^\infty 0.1024e^{-750t} dt = \frac{0.1024e^{-750t}}{-750} \Big|_0^\infty = 136.53 \mu\text{J}$$

$$\text{percent} = \frac{136.53}{512}(100) = 26.67\%$$

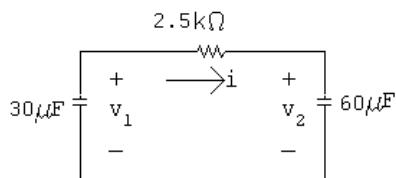
P 7.25 [a] $v_1(0^-) = v_1(0^+) = (0.006)(5000) = 30 \text{ V}$ $v_2(0^+) = 0$

$$C_{\text{eq}} = (30)(40)/90 = 20 \mu\text{F}$$



$$\tau = (2.5 \times 10^3)(20 \times 10^{-6}) = 50 \text{ ms}; \quad \frac{1}{\tau} = 20$$

$$i = \frac{30}{2500}e^{-20t} = 12e^{-20t} \text{ mA}, \quad t \geq 0^+$$



$$v_1 = \frac{-1}{30^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x} dx + 30 = 20e^{-20t} + 10 \text{ V}, \quad t \geq 0$$

$$v_2 = \frac{1}{60 \times 10^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x} dx + 0 = -10e^{-20t} + 10 \text{ V}, \quad t \geq 0$$

[b] $w(0) = \frac{1}{2}(30 \times 10^{-6})(30)^2 = 13.5 \text{ mJ}$

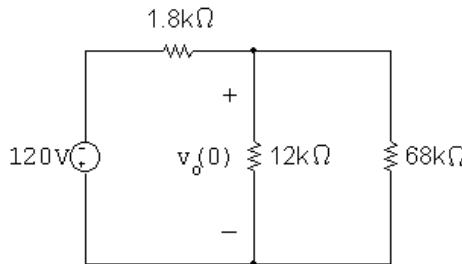
[c] $w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-6})(10)^2 + \frac{1}{2}(60 \times 10^{-6})(10)^2 = 4.5 \text{ mJ}.$

The energy dissipated by the $2.5 \text{ k}\Omega$ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(20 \times 10^{-6})(30)^2 = 9 \text{ mJ}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 4.5 + 9 = 13.5 \text{ mJ}; \quad w(0) = 13.5 \text{ mJ}.$

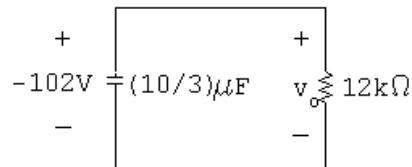
P 7.26 [a] $t < 0$:



$$R_{\text{eq}} = 12 \text{ k} \parallel 8 \text{ k} = 10.2 \text{ k}\Omega$$

$$v_o(0) = \frac{10,200}{10,200 + 1800}(-120) = -102 \text{ V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6}](12,000) = 40 \text{ ms}; \quad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t} \text{ V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12,000} = 867 \times 10^{-3} e^{-50t} \text{ W}$$

$$\begin{aligned} w_{\text{diss}} &= \int_0^{12 \times 10^{-3}} 867 \times 10^{-3} e^{-50t} dt \\ &= 17.34 \times 10^{-3} (1 - e^{-50(12 \times 10^{-3})}) = 7824 \mu\text{J} \end{aligned}$$

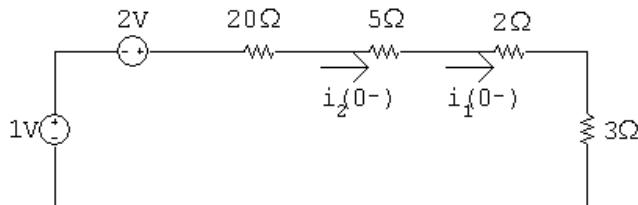
$$[\mathbf{b}] \quad w(0) = \left(\frac{1}{2}\right) \left(\frac{10}{3}\right) (102)^2 \times 10^{-6} = 17.34 \text{ mJ}$$

$$0.75w(0) = 13 \text{ mJ}$$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x} dx = 13 \times 10^{-3}$$

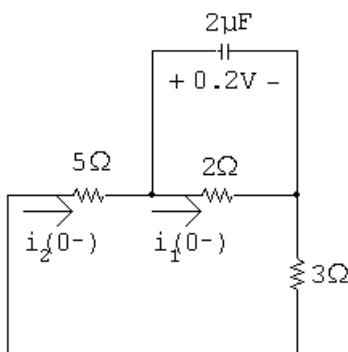
$$\therefore 1 - e^{-50t_o} = 0.75; \quad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73 \text{ ms}$$

P 7.27 [a] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100 \text{ mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100 \text{ mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25 \text{ mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100 \text{ mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100 \text{ mA} \quad \text{and} \quad i_2(0^+) = 25 \text{ mA}$$

$$[\mathbf{e}] \quad v_c = 0.2e^{-t/\tau} \text{ V}, \quad t \geq 0$$

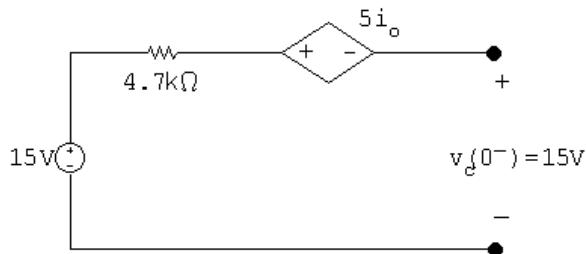
$$\tau = R_e C = 1.6(2 \times 10^{-6}) = 3.2 \mu\text{s}; \quad \frac{1}{\tau} = 312,500$$

$$v_c = 0.2e^{-312,000t} \text{ V}, \quad t \geq 0$$

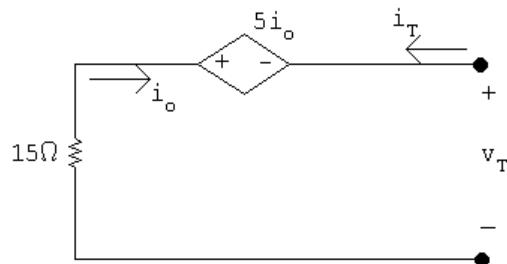
$$i_1 = \frac{v_c}{2} = 0.1e^{-312,000t} \text{ A}, \quad t \geq 0$$

$$[\text{f}] \quad i_2 = \frac{-v_c}{8} = -25e^{-312,000t} \text{ mA}, \quad t \geq 0^+$$

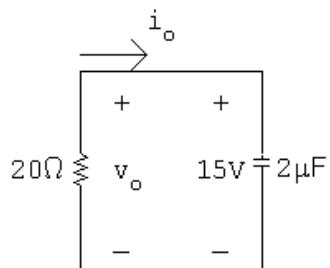
P 7.28 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20 \Omega$$



$$\tau = RC = 40 \mu\text{s}; \quad \frac{1}{\tau} = 25,000$$

$$v_o = 15e^{-25,000t} \text{ V}, \quad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25,000t} \text{ A}, \quad t \geq 0^+$$

P 7.29 [a] $R = \frac{v}{i} = 8 \text{ k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 500; \quad C = \frac{1}{(500)(8000)} = 0.25 \mu\text{F}$

[c] $\tau = \frac{1}{500} = 2 \text{ ms}$

[d] $w(0) = \frac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648 \mu\text{J}$

[e] $w_{\text{diss}} = \int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)} dt$

$$= 0.648 \frac{e^{-1000t}}{-1000} \Big|_0^{t_o} = 648(1 - e^{-1000t_o}) \mu\text{J}$$

$$\%_{\text{diss}} = 100(1 - e^{-1000t_o}) = 68 \quad \text{so} \quad e^{1000t_o} = 3.125$$

$$\therefore t = \frac{\ln 3.125}{1000} = 1139 \mu\text{s}$$

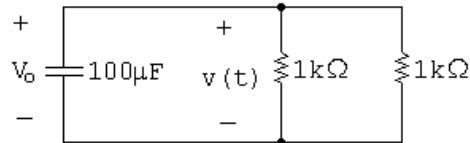
- P 7.30 [a] Note that there are many different possible correct solutions to this problem.

$$R = \frac{\tau}{C}$$

Choose a $100 \mu\text{F}$ capacitor from Appendix H. Then,

$$R = \frac{0.05}{100 \times 10^{-6}} = 500 \Omega$$

Construct a 500Ω resistor by combining two $1 \text{k}\Omega$ resistors in parallel:

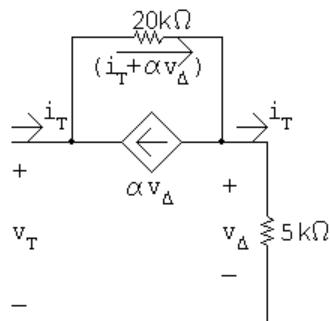


[b] $v(t) = V_o e^{-t/\tau} = 50 e^{-20t} \text{ V}, \quad t \geq 0$

[c] $50 e^{-20t} = 10 \quad \text{so} \quad e^{20t} = 5$

$$\therefore t = \frac{\ln 5}{20} = 80.47 \text{ ms}$$

- P 7.31 [a]



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_{\Delta} = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha (5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25,000 + 100 \times 10^6 \alpha$$

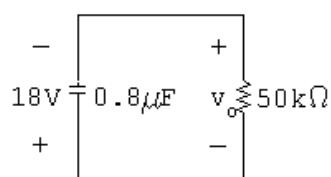
$$\tau = R_{\text{Th}} C = 40 \times 10^{-3} = R_{\text{Th}} (0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50 \text{ k}\Omega = 25,000 + 100 \times 10^6 \alpha$$

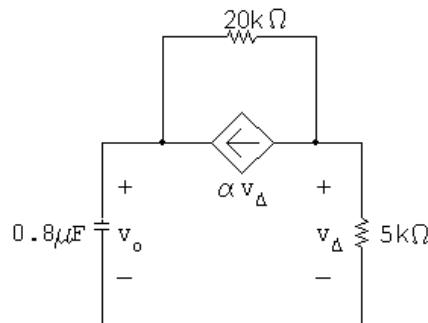
$$\alpha = \frac{25,000}{100 \times 10^6} = 2.5 \times 10^{-4} \text{ A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18 \text{ V} \quad t < 0$

$t > 0$:



$$v_o = -18e^{-25t} \text{ V}, \quad t \geq 0$$

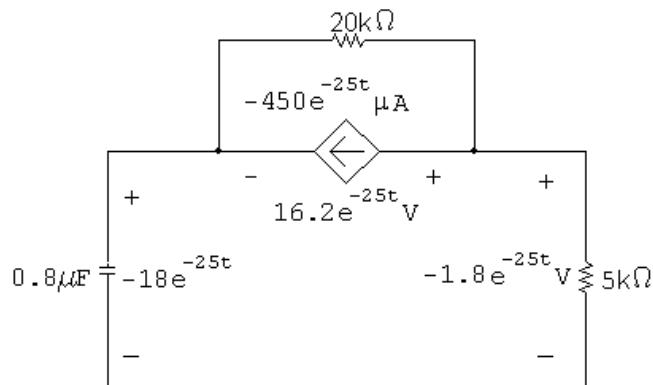


$$\frac{v_{\Delta}}{5000} + \frac{v_{\Delta} - v_o}{20,000} + 2.5 \times 10^{-4} v_{\Delta} = 0$$

$$4v_{\Delta} + v_{\Delta} - v_o + 5v_{\Delta} = 0$$

$$\therefore v_{\Delta} = \frac{v_o}{10} = -1.8e^{-25t} \text{ V}, \quad t \geq 0^+$$

P 7.32 [a]



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6} e^{-25t}) = -7290 \times 10^{-6} e^{-50t} \text{ W}$$

$$w_{ds} = \int_0^\infty p_{ds} dt = -145.8 \mu\text{J}.$$

\therefore dependent source is delivering $145.8 \mu\text{J}$.

$$[\mathbf{b}] w_{5k} = \int_0^\infty (5000)(0.36 \times 10^{-3} e^{-25t})^2 dt = 648 \times 10^{-6} \int_0^\infty e^{-50t} dt = 12.96 \mu\text{J}$$

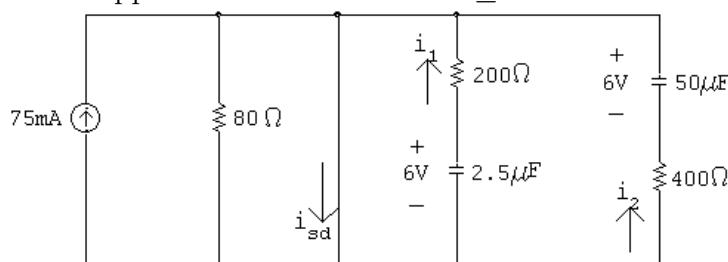
$$w_{20k} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20,000} dt = 13,122 \times 10^{-6} \int_0^\infty e^{-50t} dt = 262.44 \mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6 \mu\text{J}$$

$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4 \mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4 \mu\text{J}.$$

P 7.33 [a] At $t = 0^-$ the voltage on each capacitor will be 6 V (0.075×80), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore i_{sd}(0^+) = 0.075 + \frac{6}{200} + \frac{6}{400} = 120 \text{ mA}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore i_{sd}(\infty) = 75 \text{ mA}$$

[b] $i_{sd}(t) = 0.075 + i_1(t) + i_2(t)$

$$\tau_1 = 200(25 \times 10^{-6}) = 5 \text{ ms}$$

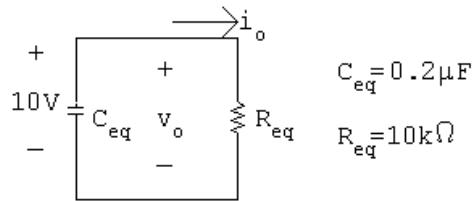
$$\tau_2 = 400(50 \times 10^{-6}) = 20 \text{ ms}$$

$$\therefore i_1(t) = 30e^{-200t} \text{ mA}, \quad t \geq 0^+$$

$$i_2(t) = 15e^{-50t} \text{ mA}, \quad t \geq 0$$

$$\therefore i_{sd} = 75 + 30e^{-200t} + 15e^{-50t} \text{ mA}, \quad t \geq 0^+$$

P 7.34 [a] The equivalent circuit for $t > 0$:



$$\tau = 2 \text{ ms}; \quad 1/\tau = 500$$

$$v_o = 10e^{-500t} \text{ V}, \quad t \geq 0$$

$$i_o = e^{-500t} \text{ mA}, \quad t \geq 0^+$$

$$i_{24k\Omega} = e^{-500t} \left(\frac{16}{40} \right) = 0.4e^{-500t} \text{ mA}, \quad t \geq 0^+$$

$$p_{24k\Omega} = (0.16 \times 10^{-6} e^{-1000t})(24,000) = 3.84e^{-1000t} \text{ mW}$$

$$w_{24k\Omega} = \int_0^\infty 3.84 \times 10^{-3} e^{-1000t} dt = -3.84 \times 10^{-6}(0 - 1) = 3.84 \mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45 \text{ mJ}$$

$$\% \text{ diss } (24 \text{ k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3} e^{-500t})^2 = 0.4 \times 10^{-3} e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400} dt = 0.40 \mu\text{J}$$

$$\% \text{ diss } (400 \text{ }\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t} \left(\frac{24}{40} \right) = 0.6e^{-500t} \text{ mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3} e^{-500t})^2 (16,000) = 5.76 \times 10^{-3} e^{-1000t} \text{ W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3} e^{-1000t} dt = 5.76 \mu\text{J}$$

$$\% \text{ diss } (16 \text{ k}\Omega) = 0.4\%$$

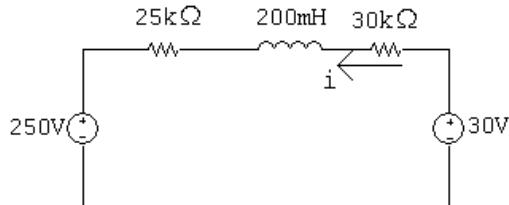
[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10 \mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44 \text{ mJ}$$

$$\% \text{ trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

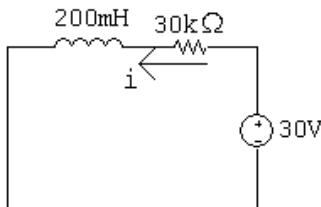
- P 7.35 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 200 mH inductor. We get



$$i(0^-) = \frac{30 - 250}{25 \text{ k} + 30 \text{ k}} = -4 \text{ mA}$$

$$i(0^-) = i(0^+) = -4 \text{ mA}$$

- [b] For $t > 0$, the circuit reduces to



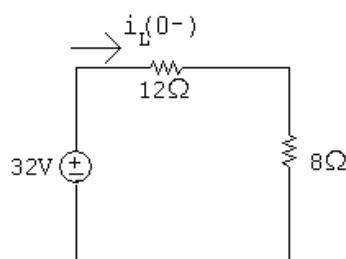
Therefore $i(\infty) = 30/30,000 = 1 \text{ mA}$

[c] $\tau = \frac{L}{R} = \frac{200 \times 10^{-3}}{30,000} = 6.67 \mu\text{s}$

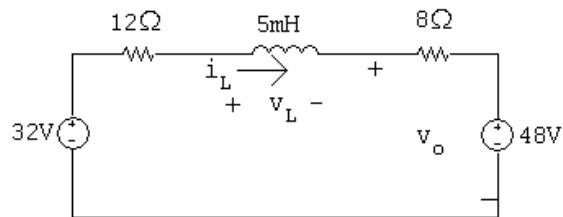
[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.001 + [-0.004 - 0.001]e^{-150,000t} = 1 - 5e^{-150,000t} \text{ mA}, \quad t \geq 0$$

- P 7.36 [a] $t < 0$



$$i_L(0^-) = \frac{32}{20} = 1.6 \text{ A}$$

$t > 0$ 

$$i_L(\infty) = \frac{32 - 48}{12 + 8} = -0.8 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{12 + 8} = 250 \mu\text{s}; \quad \frac{1}{\tau} = 4000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -0.8 + (1.6 + 0.8)e^{-4000t} = -0.8 + 2.4e^{-4000t} \text{ A}, \quad t \geq 0$$

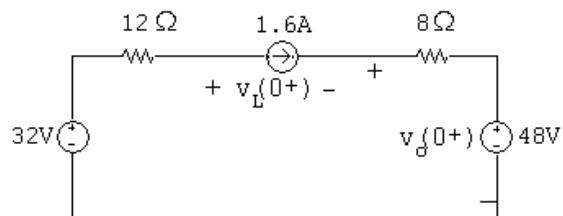
$$v_o = 8i_L + 48 = 8(-0.8 + 2.4e^{-4000t}) + 48 = 41.6 + 19.2e^{-4000t} \text{ V}, \quad t \geq 0$$

[b] $v_L = L \frac{di_L}{dt} = 5 \times 10^{-3}(-4000)[2.4e^{-4000t}] = -48e^{-4000t} \text{ V}, \quad t \geq 0^+$

$$v_L(0^+) = -48 \text{ V}$$

From part (a) $v_o(0^+) = 0 \text{ V}$

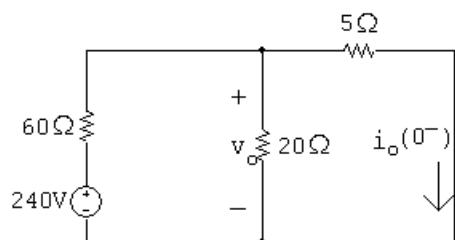
Check: at $t = 0^+$ the circuit is:



$$v_o(0^+) = 48 + (8 \Omega)(1.6 \text{ A}) = 60.8 \text{ V}; \quad v_L(0^+) + v_o(0^+) = 12(-1.6) + 32$$

$$\therefore v_L(0^+) = -19.2 + 32 - 60.8 = -48 \text{ V}$$

P 7.37 [a] $t < 0$



KVL equation at the top node:

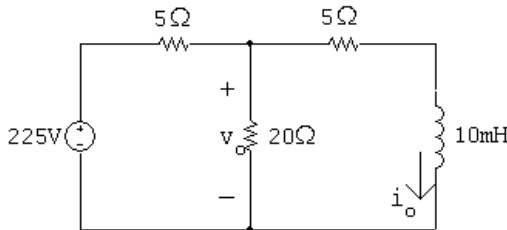
$$\frac{v_o - 240}{60} + \frac{v_o}{20} + \frac{v_o}{5} = 0$$

Multiply by 60 and solve:

$$240 = (3 + 1 + 12)v_o; \quad v_o = 15 \text{ V}$$

$$\therefore i_o(0^-) = \frac{v_o}{5} = 15/5 = 3 \text{ A}$$

$t > 0$



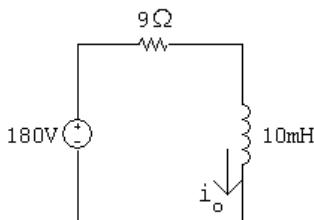
Use voltage division to find the Thévenin voltage:

$$V_{Th} = v_o = \frac{20}{20+5}(225) = 180 \text{ V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{Th} = 5 + 20\parallel 5 = 5 + 4 = 9 \Omega$$

The simplified circuit is:

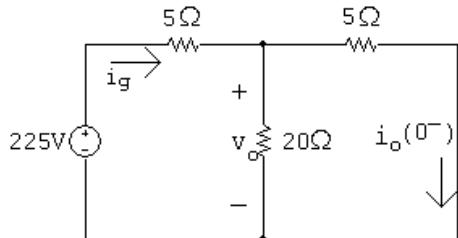


$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{9} = 1.11 \text{ ms}; \quad \frac{1}{\tau} = 900$$

$$i_o(\infty) = \frac{190}{9} = 20 \text{ A}$$

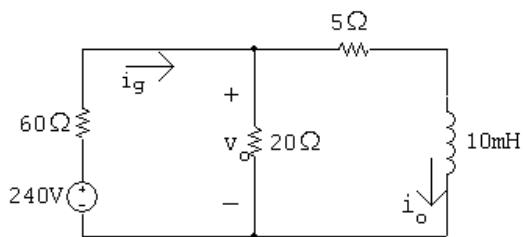
$$\begin{aligned} \therefore i_o &= i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau} \\ &= 20 + (3 - 20)e^{-900t} = 20 - 17e^{-900t} \text{ A}, \quad t \geq 0 \end{aligned}$$

$$\begin{aligned} [\mathbf{b}] \quad v_o &= 5i_o + L \frac{di_o}{dt} \\ &= 5(20 - 17e^{-900t}) + 0.01(-900)(17e^{-900t}) \\ &= 100 - 85e^{-900t} + 153e^{-900t} \\ v_o &= 100 + 68e^{-900t} \text{ V}, \quad t \geq 0^+ \end{aligned}$$

P 7.38 [a] $t < 0$ 

$$i_g = \frac{225}{5 + 20\parallel 5} = \frac{225}{9} = 25 \text{ A}$$

$$\therefore i_o(0^-) = \frac{20\parallel 5}{5}(25) = 20 \text{ A}$$

 $t > 0$ 

$$i_g(\infty) = \frac{240}{60 + 20\parallel 5} = \frac{240}{64} = 3.75 \text{ A}$$

$$i_o(\infty) = \frac{20\parallel 5}{5} i_g(\infty) = 3 \text{ A}$$

$$R_{\text{eq}} = 5 + 20\parallel 60 = 3 + 15 = 20 \Omega$$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{10 \times 10^{-3}}{20} = 0.5 \text{ ms}; \quad \frac{1}{\tau} = 2000$$

$$\therefore i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (20 - 3)e^{-2000t} = 3 + 17e^{-2000t} \text{ A}, \quad t \geq 0$$

$$\begin{aligned} [\text{b}] \quad v_o &= 5i_o + L \frac{di_o}{dt} \\ &= 5(3 + 17e^{-2000t}) + 0.01(-2000)(17e^{-2000t}) \\ &= 15 + 65e^{-2000t} - 340e^{-2000t} \\ v_o &= 15 - 255e^{-2000t} \text{ V}, \quad t \geq 0^+ \end{aligned}$$

P 7.39 [a] From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R} \right) e^{-(R/L)t}$$

$$v = (V_s - I_o R) e^{-(R/L)t}$$

$$\therefore \frac{V_s}{R} = 4; \quad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_o R = -80; \quad \frac{R}{L} = 40$$

$$\therefore I_o = 4 + \frac{V_s}{R} = 8 \text{ A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \quad R = 20 \Omega$$

$$V_s = 80 \text{ V}; \quad L = \frac{R}{40} = 0.5 \text{ H}$$

[b] $i = 4 + 4e^{-40t}; \quad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$.

$$x^2 + 2x - 1.25 = 0; \quad \text{Solving, } x = 0.5; \quad x = -2.5$$

But $x \geq 0$ for all t . Thus,

$$e^{-40t} = 0.5; \quad e^{40t} = 2; \quad t = 25 \ln 2 = 17.33 \text{ ms}$$

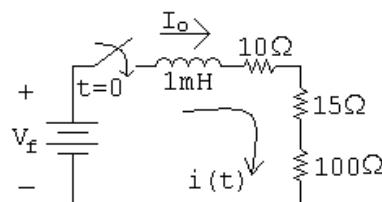
P 7.40 [a] Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125 \Omega$$

Construct the resistance needed by combining 100Ω , 10Ω , and 15Ω resistors in series:



[b] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0 \text{ A}; \quad I_f = \frac{V_f}{R} = \frac{25}{125} = 200 \text{ mA}$$

$$\therefore i(t) = 200 + (0 - 200)e^{-125,000t} \text{ mA} = 200 - 200e^{-125,000t} \text{ mA}, \quad t \geq 0$$

[c] $i(t) = 0.2 - 0.2e^{-125,000t} = (0.75)(0.2) = 0.15$

$$e^{-125,000t} = 0.25 \quad \text{so} \quad e^{125,000t} = 4$$

$$\therefore t = \frac{\ln 4}{125,000} = 11.09 \mu\text{s}$$

P 7.41 [a] $v_o(0^+) = -I_g R_2; \quad \tau = \frac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1 + R_2)/L]t} \text{ V}, \quad t \geq 0^+$$

[b] $v_o(0^+) \rightarrow \infty$, and the duration of $v_o(t) \rightarrow \text{zero}$

[c] $v_{sw} = R_2 i_o; \quad \tau = \frac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \quad i_o(\infty) = I_g \frac{R_1}{R_1 + R_2}$$

Therefore $i_o(t) = \frac{I_g R_1}{R_1 + R_2} + \left[I_g - \frac{I_g R_1}{R_1 + R_2} \right] e^{-[(R_1 + R_2)/L]t}$

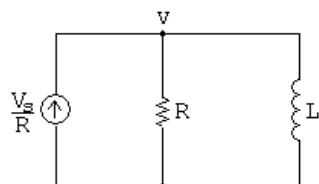
$$i_o(t) = \frac{R_1 I_g}{(R_1 + R_2)} + \frac{R_2 I_g}{(R_1 + R_2)} e^{-[(R_1 + R_2)/L]t}$$

Therefore $v_{sw} = \frac{R_1 I_g}{(1 + R_1/R_2)} + \frac{R_2 I_g}{(1 + R_1/R_2)} e^{-[(R_1 + R_2)/L]t}, \quad t \geq 0^+$

[d] $|v_{sw}(0^+)| \rightarrow \infty$; duration $\rightarrow 0$

P 7.42 Opening the inductive circuit causes a very large voltage to be induced across the inductor L . This voltage also appears across the switch (part [d] of Problem 7.41), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.43 [a]



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L} \int_0^t v dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = 0$$

$$\therefore \frac{dv}{dt} + \frac{R}{L} v = 0$$

$$[b] \frac{dv}{dt} = -\frac{R}{L} v$$

$$\frac{dv}{dt} dt = -\frac{R}{L} v dt \quad \text{so} \quad dv = -\frac{R}{L} v dt$$

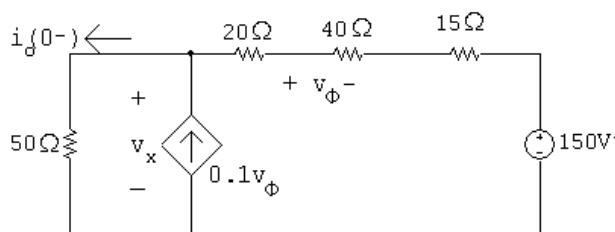
$$\frac{dv}{v} = -\frac{R}{L} dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L} \int_0^t dy$$

$$\ln \frac{v(t)}{V_o} = -\frac{R}{L} t$$

$$\therefore v(t) = V_o e^{-(R/L)t} = (V_s - RI_o) e^{-(R/L)t}$$

P 7.44 For $t < 0$

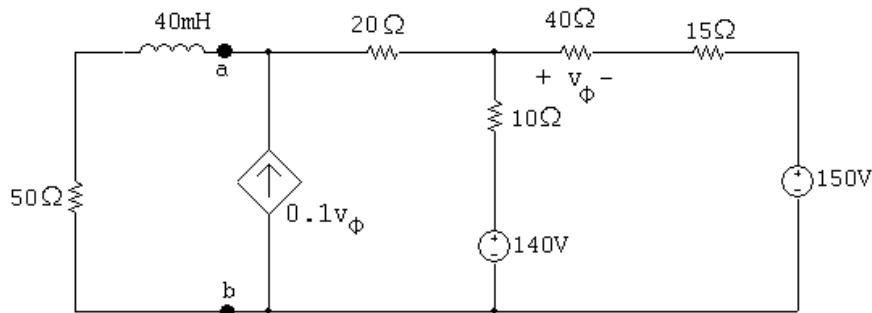


$$\frac{v_x}{50} - 0.1v_\phi + \frac{v_x - 150}{75} = 0$$

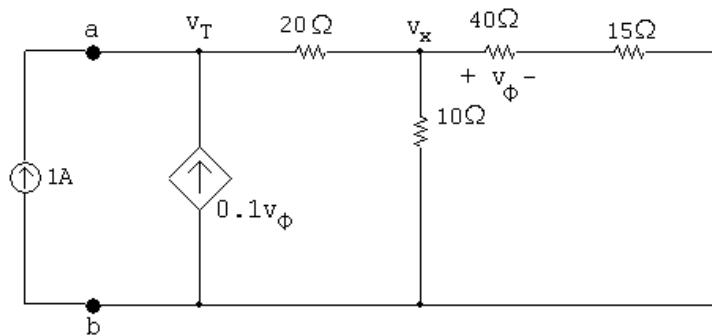
$$v_\phi = \frac{40}{75}(v_x - 150)$$

Solving,

$$v_x = 300 \text{ V}; \quad i_o(0^-) = \frac{v_x}{50} = 6 \text{ A}$$

$t > 0$ 

Find Thévenin equivalent with respect to a, b. Use a test source to find the Thévenin equivalent resistance:



$$-1 - 0.1v_\phi + \frac{v_T - v_x}{20} = 0$$

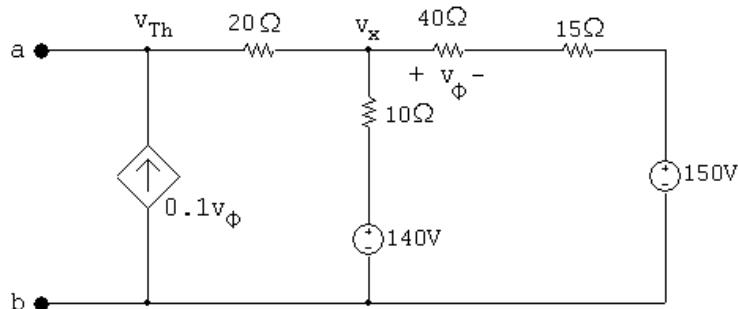
$$\frac{v_x - v_T}{20} + \frac{v_x}{10} + \frac{v_x}{55} = 0$$

$$v_\phi = \frac{40}{55}v_x$$

Solving,

$$v_T = 74\text{ V} \quad \text{so} \quad R_{\text{Th}} = \frac{v_T}{1\text{ A}} = 74\Omega$$

Find the open circuit voltage with respect to a, b:



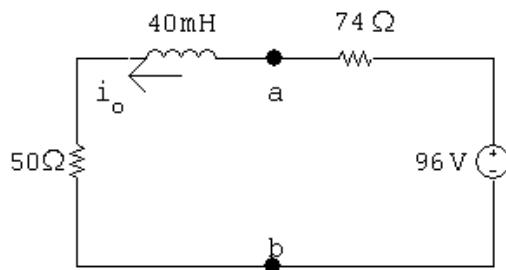
$$-0.1v_\phi + \frac{v_{\text{Th}} - v_x}{20} = 0$$

$$\frac{v_x - v_{\text{Th}}}{20} + \frac{v_x - 140}{10} + \frac{v_x - 150}{55} = 0$$

$$v_\phi = \frac{40}{55}(v_x - 150)$$

Solving,

$$v_{\text{Th}} = 96 \text{ V}$$

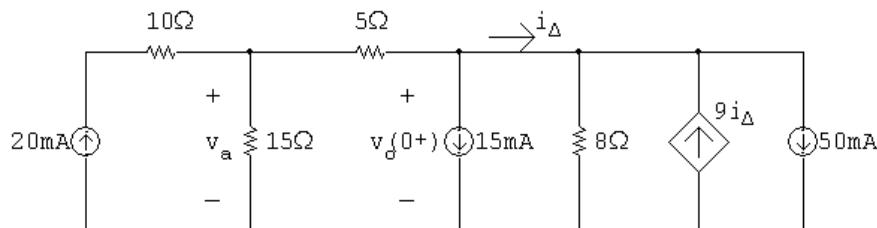


$$i_o(\infty) = 96/124 = 0.774 \text{ A}$$

$$\tau = \frac{40 \times 10^{-3}}{124} = 0.3226 \text{ ms}; \quad 1/\tau = 3100$$

$$i_o = 0.774 + (6 - 0.774)e^{-3100t} = 0.774 + 5.226e^{-3100t} \text{ A}, \quad t \geq 0$$

P 7.45 $t > 0$; calculate $v_o(0^+)$



$$\frac{v_a}{15} + \frac{v_a - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore v_a = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_a}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_a - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_{\Delta} = \frac{v_o(0^+)}{8} - 9i_{\Delta} + 50 \times 10^{-3}$$

$$\therefore i_{\Delta} = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore 360i_{\Delta} = 4.5v_o(0^+) + 1800 \times 10^{-3}$$

$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

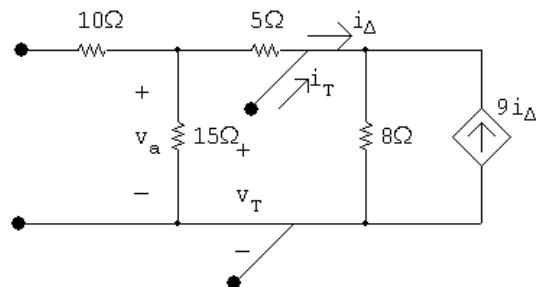
$$\therefore 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$

$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \quad v_o(0^+) = -80 \text{ mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_{\Delta}$$

$$i_{\Delta} = \frac{v_T}{8} - 9i_{\Delta} \quad \therefore 10i_{\Delta} = \frac{v_T}{8}; \quad i_{\Delta} = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16} \text{ S}$$

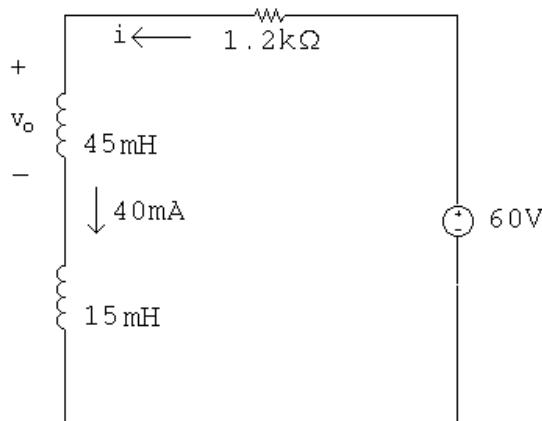
$$\therefore R_{Th} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25 \text{ ms}; \quad 1/\tau = 4000$$

$$\therefore v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t} \text{ mV}, \quad t \geq 0^+$$

P 7.46 For $t < 0$, $i_{45\text{mH}}(0) = 80\text{V}/2000\Omega = 40\text{mA}$

For $t > 0$, after making a Thévenin equivalent of the circuit to the right of the inductors we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R} \right) e^{-t/\tau}$$

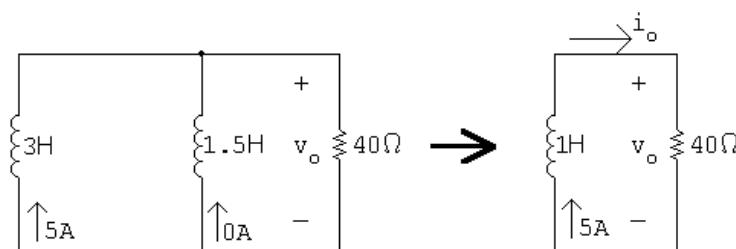
$$\frac{1}{\tau} = \frac{R}{L} = \frac{1200}{60 \times 10^{-3}} = 20,000$$

$$I_o = 40\text{mA}; \quad I_f = \frac{V_s}{R} = \frac{60}{1200} = 50\text{mA}$$

$$i = 0.05 + (0.04 - 0.05)e^{-20,000t} = 50 - 10e^{-20,000t} \text{mA}, \quad t \geq 0$$

$$v_o = 0.045 \frac{di}{dt} = 0.045(-0.01)(-20,000e^{-20,000t}) = 9e^{-20,000t} \text{V}, \quad t \geq 0^+$$

P 7.47 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t} \text{A}, \quad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t} \text{V}, \quad t > 0^+$$

$$200e^{-40t} = 100; \quad e^{40t} = 2$$

$$\therefore t = \frac{1}{40} \ln 2 = 17.33 \text{ ms}$$

P 7.48 [a] $w_{\text{diss}} = \frac{1}{2} L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5 \text{ J}$

[b] $i_{3H} = \frac{1}{3} \int_0^t (200)e^{-40x} dx - 5$
 $= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33 \text{ A}$

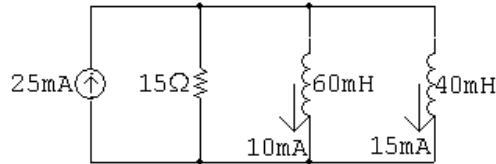
$$i_{1.5H} = \frac{1}{1.5} \int_0^t (200)e^{-40x} dx + 0$$

 $= -3.33e^{-40t} + 3.33 \text{ A}$

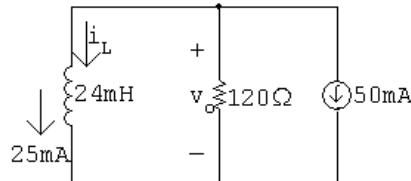
$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25 \text{ J}$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5 \text{ J}$

P 7.49 [a] $t < 0$



$t > 0$



$i_L(0^-) = i_L(0^+) = 25 \text{ mA}; \quad \tau = \frac{24 \times 10^{-3}}{120} = 0.2 \text{ ms}; \quad \frac{1}{\tau} = 5000$

$i_L(\infty) = -50 \text{ mA}$

$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t} \text{ mA}, \quad t \geq 0$

$v_o = -120[75 \times 10^{-3} e^{-5000t}] = -9e^{-5000t} \text{ V}, \quad t \geq 0^+$

[b] $i_1 = \frac{1}{60 \times 10^{-3}} \int_0^t -9e^{-5000x} dx + 10 \times 10^{-3} = (30e^{-5000t} - 20) \text{ mA}, \quad t \geq 0$

[c] $i_2 = \frac{1}{40 \times 10^{-3}} \int_0^t -9e^{-5000x} dx + 15 \times 10^{-3} = (45e^{-5000t} - 30) \text{ mA}, \quad t \geq 0$

P 7.50 [a] Let v be the voltage drop across the parallel branches, positive at the top node, then

$-I_g + \frac{v}{R_g} + \frac{1}{L_1} \int_0^t v dx + \frac{1}{L_2} \int_0^t v dx = 0$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \int_0^t v \, dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e} \int_0^t v \, dx = I_g$$

$$\frac{1}{R_g} \frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e} v = 0$$

$$\text{Therefore } v = I_g R_g e^{-t/\tau}; \quad \tau = L_e / R_g$$

Thus

$$i_1 = \frac{1}{L_1} \int_0^t I_g R_g e^{-x/\tau} \, dx = \frac{I_g R_g}{L_1} \frac{e^{-x/\tau}}{(-1/\tau)} \Big|_0^t = \frac{I_g L_e}{L_1} (1 - e^{-t/\tau})$$

$$i_1 = \frac{I_g L_2}{L_1 + L_2} (1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2} (1 - e^{-t/\tau})$$

$$[\mathbf{b}] \quad i_1(\infty) = \frac{L_2}{L_1 + L_2} I_g; \quad i_2(\infty) = \frac{L_1}{L_1 + L_2} I_g$$

P 7.51 [a] $v_c(0^+) = -120 \text{ V}$

[b] Use voltage division to find the final value of voltage:

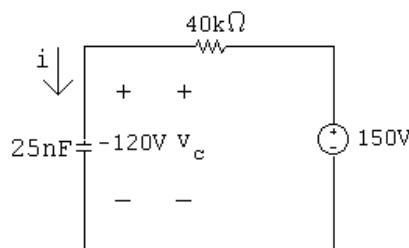
$$v_c(\infty) = \frac{150,000}{200,000} (200) = 150 \text{ V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = 150 \text{ V}, \quad R_{\text{Th}} = 2500 + 150 \text{ k} \parallel 50 \text{ k} = 40 \text{ k}\Omega,$$

$$\text{Therefore } \tau = R_{\text{eq}} C = (40,000)(25 \times 10^{-9}) = 1 \text{ ms}$$

The simplified circuit for $t > 0$ is:



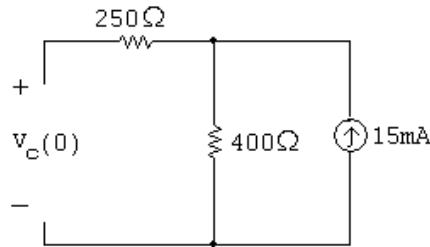
$$[\mathbf{d}] \quad i(0^+) = \frac{150 - (-120)}{40,000} = 6.75 \text{ mA}$$

$$[\mathbf{e}] \quad v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)] e^{-t/\tau}$$

$$= 150 + (-120 - 150) e^{-t/\tau} = 150 - 270 e^{-1000t} \text{ V}, \quad t \geq 0$$

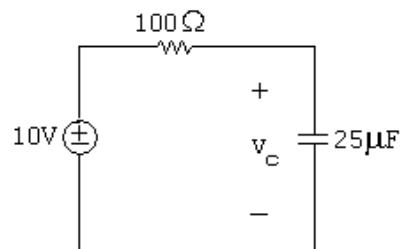
$$[\text{f}] \quad i = C \frac{dv_c}{dt} = (25 \times 10^{-9})(-1000)(-270e^{-1000t}) = 6.75e^{-1000t} \text{ mA}, \quad t \geq 0^+$$

P 7.52 [a] for $t < 0$:



$$v_c(0) = 400(0.015) = 6 \text{ V}$$

For $t \geq 0$:

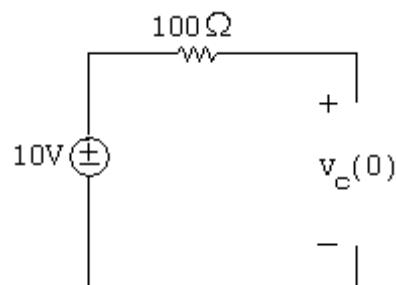


$$v_c(\infty) = 10 \text{ V}$$

$$R_{\text{eq}} = 20 \Omega \quad \text{so} \quad \tau = R_{\text{eq}}C = 250(25 \times 10^{-6}) = 6.25 \text{ ms}$$

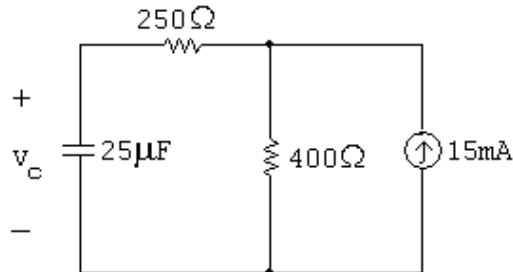
$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 10 + (6 - 10)e^{-160t} = 10 - 4e^{-160t} \text{ V}$$

[b] For $t < 0$:



$$v_c(0) = 10 \text{ V}$$

For $t \geq 0$:



$$v_c(\infty) = 400(0.015) = 6 \text{ V}$$

$$R_{\text{eq}} = 100 + 400 = 500 \Omega \quad \text{so} \quad \tau = R_{\text{eq}}C = 500(25 \times 10^{-6}) = 12.5 \text{ ms}$$

$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 6 + (10 - 6)e^{-80t} = 6 + 4e^{-80t} \text{ V}$$

P 7.53 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9 \text{ k}}{9 \text{ k} + 3 \text{ k}}(120) = 90 \text{ V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60 \text{ V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60 \text{ V}, \quad R_{\text{Th}} = 10 \text{ k} + 40 \text{ k} = 50 \text{ k}\Omega$$

$$\tau = R_{\text{Th}}C = 1 \text{ ms} = 1000 \mu\text{s}$$

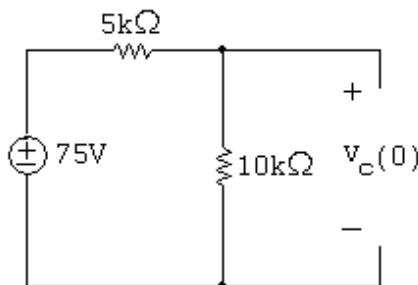
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t} \text{ V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

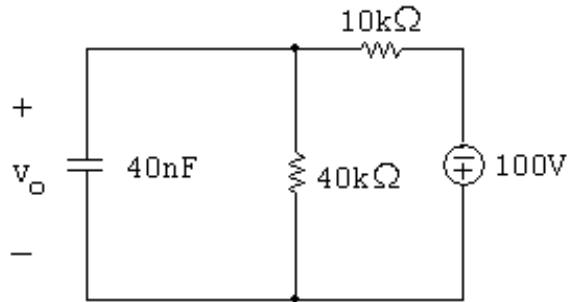
$$\text{Therefore } t = \frac{\ln(150/60)}{1000} = 916.3 \mu\text{s}$$

P 7.54 [a] For $t < 0$:



$$v_o(0) = \frac{10,000}{15,000}(75) = 50 \text{ V}$$

For $t \geq 0$:



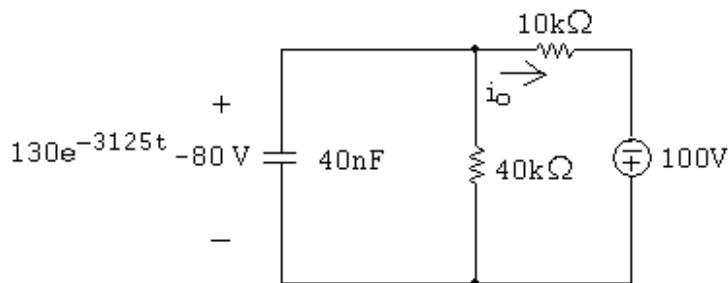
$$v_o(\infty) = \frac{40,000}{50,000}(-100) = -80 \text{ V}$$

$$R_{\text{eq}} = 40 \text{ k} \parallel 10 \text{ k} = 8 \text{ k}\Omega$$

$$\tau = R_{\text{eq}}C = (8000)(40 \times 10^{-9}) = 0.32 \text{ ms}$$

$$\begin{aligned} v_o(t) &= v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau} = -80 + (50 + 80)e^{-3125t} \\ &= -80 + 130e^{-3125t} \text{ V} \end{aligned}$$

[b] For $t \geq 0$:

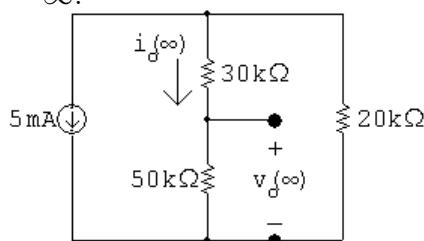


$$i_o = \frac{130e^{-3125t} - 80 + 100}{10,000} = 13e^{-3125t} + 2 \text{ mA}$$

P 7.55 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2 \text{ mA}; \quad v_o(0^-) = (2 \times 10^{-3})(50,000) = 100 \text{ V}$$

$t = \infty$:

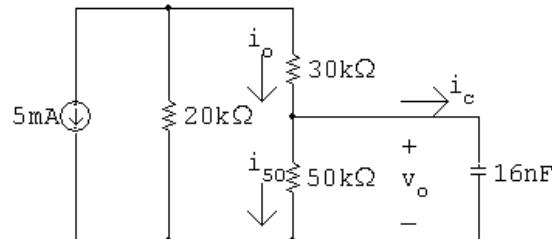


$$i_o(\infty) = -5 \times 10^{-3} \left(\frac{20}{100} \right) = -1 \text{ mA}; \quad v_o(\infty) = i_o(\infty)(50,000) = -50 \text{ V}$$

$$R_{Th} = 50\text{ k}\Omega \parallel 50\text{ k}\Omega = 25\text{ k}\Omega; \quad C = 16\text{ nF}$$

$$\tau = (25,000)(16 \times 10^{-9}) = 0.4\text{ ms}; \quad \frac{1}{\tau} = 2500$$

$$\therefore v_o(t) = -50 + 150e^{-2500t}\text{ V}, \quad t \geq 0$$

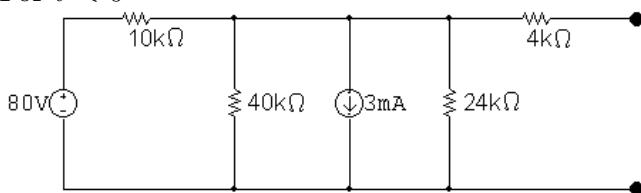


$$i_c = C \frac{dv_o}{dt} = -6e^{-2500t} \text{ mA}, \quad t \geq 0^+$$

$$i_{50k} = \frac{v_o}{50,000} = -1 + 3e^{-2500t} \text{ mA}, \quad t \geq 0^+$$

$$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t}) \text{ mA}, \quad t \geq 0^+$$

P 7.56 For $t < 0$



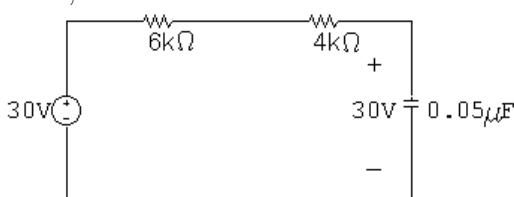
Simplify the circuit:

$$80/10,000 = 8 \text{ mA}, \quad 10\text{ k}\Omega \parallel 40\text{ k}\Omega \parallel 24\text{ k}\Omega = 6\text{ k}\Omega$$

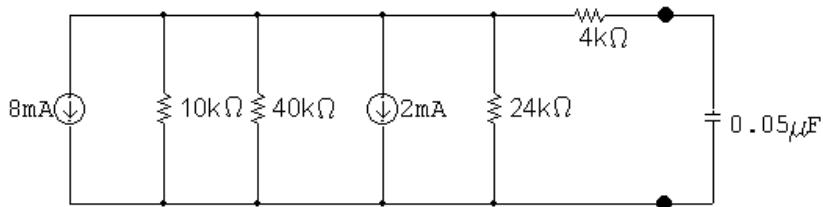
$$8 \text{ mA} - 3 \text{ mA} = 5 \text{ mA}$$

$$5 \text{ mA} \times 6 \text{ k}\Omega = 30 \text{ V}$$

Thus, for $t < 0$



$$\therefore v_o(0^-) = v_o(0^+) = 30 \text{ V}$$

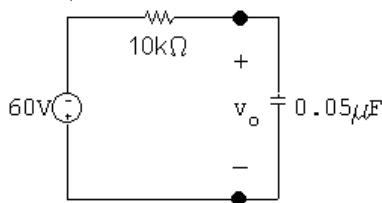
$t > 0$ 

Simplify the circuit:

$$8\text{ mA} + 2\text{ mA} = 10\text{ mA}$$

$$10\text{ k}\parallel 40\text{ k}\parallel 24\text{ k} = 6\text{ k}\Omega$$

$$(10\text{ mA})(6\text{ k}\Omega) = 60\text{ V}$$

Thus, for $t > 0$ 

$$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\text{ V}$$

$$\tau = RC = (10\text{ k})(0.05\mu) = 0.5\text{ ms}; \quad \frac{1}{\tau} = 2000$$

$$\begin{aligned} v_o &= v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t} \\ &= -60 + 90e^{-2000t}\text{ V} \quad t \geq 0 \end{aligned}$$

P 7.57 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40 + 60}(50) = 30\text{ V}$$

Use Ohm's law to find the final value of voltage:

$$v_o(\infty) = (-5\text{ mA})(20\text{ k}\Omega) = -100\text{ V}$$

$$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\text{ ms}; \quad \frac{1}{\tau} = 200$$

$$\begin{aligned} v_o &= v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} \\ &= -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\text{ V}, \quad t \geq 0 \end{aligned}$$

P 7.58 [a] $v = I_s R + (V_o - I_s R) e^{-t/RC}$ $i = \left(I_s - \frac{V_o}{R} \right) e^{-t/RC}$

$$\therefore I_s R = 40, \quad V_o - I_s R = -24$$

$$\therefore V_o = 16 \text{ V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \quad I_s - \frac{16}{R} = 3 \times 10^{-3}; \quad R = \frac{40}{I_s}$$

$$\therefore I_s - 0.4I_s = 3 \times 10^{-3}; \quad I_s = 5 \text{ mA}$$

$$R = \frac{40}{5} \times 10^3 = 8 \text{ k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50 \text{ nF}; \quad \tau = RC = \frac{1}{2500} = 400 \mu\text{s}$$

[b] $v(\infty) = 40 \text{ V}$

$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40 \mu\text{J}$$

$$0.81w(\infty) = 32.4 \mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \quad v(t_o) = 36 \text{ V}$$

$$40 - 24e^{-2500t_o} = 36; \quad e^{2500t_o} = 6; \quad \therefore t_o = 716.70 \mu\text{s}$$

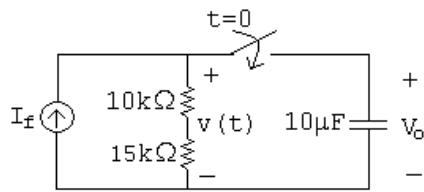
P 7.59 [a] Note that there are many different possible solutions to this problem.

$$R = \frac{\tau}{C}$$

Choose a $10 \mu\text{H}$ capacitor from Appendix H. Then,

$$R = \frac{0.25}{10 \times 10^{-6}} = 25 \text{ k}\Omega$$

Construct the resistance needed by combining $10 \text{ k}\Omega$ and $15 \text{ k}\Omega$ resistors in series:



[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

$$V_o = 100 \text{ V}; \quad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25 \text{ V}$$

$$\therefore v(t) = 25 + (100 - 25)e^{-4t} \text{ V} = 25 + 75e^{-4t} \text{ V}, \quad t \geq 0$$

$$[c] \quad v(t) = 25 + 75e^{-4t} = 50 \quad \text{so} \quad e^{-4t} = \frac{1}{3}$$

$$\therefore t = \frac{\ln 3}{4} = 274.65 \text{ ms}$$

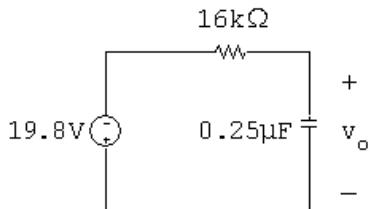
P 7.60 For $t > 0$

$$V_{\text{Th}} = (-25)(16,000)i_b = -400 \times 10^3 i_b$$

$$i_b = \frac{33,000}{80,000}(120 \times 10^{-6}) = 49.5 \mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8 \text{ V}$$

$$R_{\text{Th}} = 16 \text{ k}\Omega$$



$$v_o(\infty) = -19.8 \text{ V}; \quad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4 \text{ ms}; \quad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t} \text{ V}, \quad t \geq 0$$

$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2 \text{ J}$$

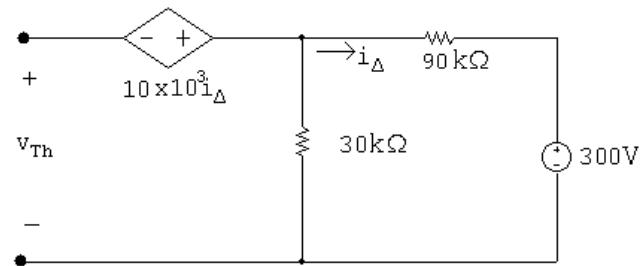
$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

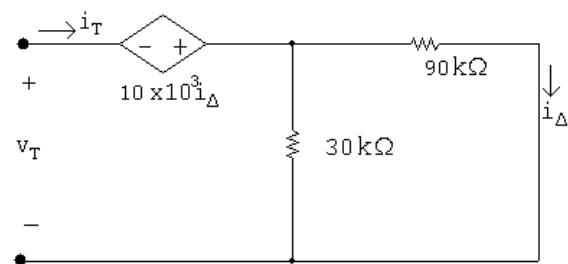
$$e^{-250t} = 0.4 \quad \therefore t = 3.67 \text{ ms}$$

P 7.61 For $t < 0$, $v_o(0) = 90 \text{ V}$

$t > 0$:



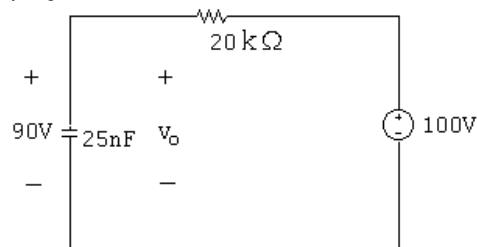
$$v_{\text{Th}} = -10 \times 10^3 i_{\Delta} + (30/120)(300) = -10 \times 10^3 \left(\frac{-300}{120 \times 10^3} \right) + 75 = 100 \text{ V}$$



$$\begin{aligned} v_T &= -10 \times 10^3 i_{\Delta} + 22.5 \times 10^3 i_T = -10 \times 10^3 (30/120) i_T + 22.5 \times 10^3 i_T \\ &= 20 \times 10^3 i_T \end{aligned}$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 20 \text{ k}\Omega$$

$t > 0$



$$v_o = 100 + (90 - 100)e^{-t/\tau}$$

$$\tau = RC = (20 \times 10^3)(25 \times 10^{-9}) = 500 \times 10^{-6}; \quad \frac{1}{\tau} = 2000$$

$$v_o = 100 - 10e^{-2000t} \text{ V}, \quad t \geq 0$$

P 7.62 From Problem 7.61,

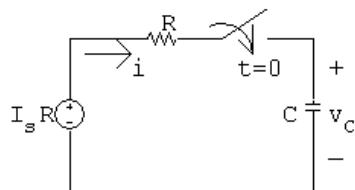
$$v_o(0) = 100 \text{ V}; \quad v_o(\infty) = 90 \text{ V}$$

$$R_{\text{Th}} = 40 \text{ k}\Omega$$

$$\tau = (40)(25 \times 10^{-6}) = 10^{-3}; \quad \frac{1}{\tau} = 1000$$

$$v = 90 + (100 - 90)e^{-1000t} = 90 + 10e^{-1000t} \text{ V}, \quad t \geq 0$$

P 7.63 [a]



$$I_s R = Ri + \frac{1}{C} \int_{0^+}^t i \, dx + V_o$$

$$0 = R \frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \frac{di}{dt} + \frac{i}{RC} = 0$$

$$[b] \quad \frac{di}{dt} = -\frac{i}{RC}; \quad \frac{di}{i} = -\frac{dt}{RC}$$

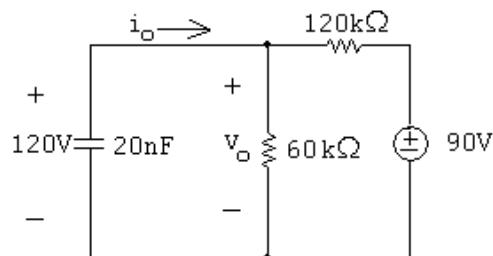
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC} \int_{0^+}^t dx$$

$$\ln \frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+) e^{-t/RC}; \quad i(0^+) = \frac{I_s R - V_o}{R} = \left(I_s - \frac{V_o}{R} \right)$$

$$\therefore i(t) = \left(I_s - \frac{V_o}{R} \right) e^{-t/RC}$$

P 7.64 [a] For $t > 0$:



$$v(\infty) = \frac{60}{180}(90) = 30 \text{ V}$$

$$R_{\text{eq}} = 60 \text{k} \parallel 120 \text{k} = 40 \text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40 \times 10^3)(20 \times 10^{-9}) = 0.8 \text{ ms}; \quad \frac{1}{\tau} = 1250$$

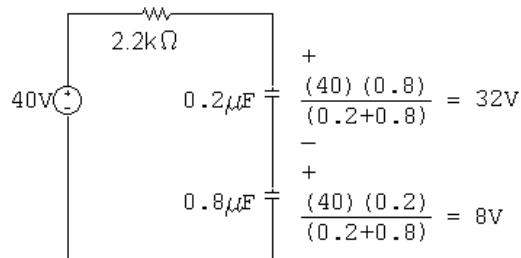
$$v_o = 30 + (120 - 30)e^{-1250t} = 30 + 90e^{-1250t} \text{ V}, \quad t \geq 0^+$$

$$[\mathbf{b}] \quad i_o = \frac{v_o}{60,000} \frac{90}{120,000} = \frac{30 + 90e^{-1250t}}{60,000} + \frac{30 + 90e^{-1250t} - 90}{120,000}$$

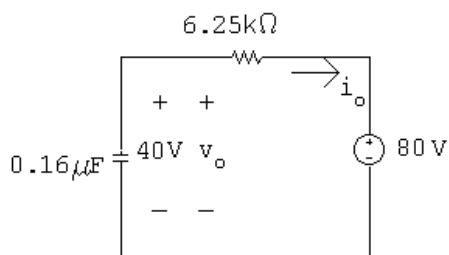
$$= 2.25e^{-1250t} \text{ mA}$$

$$v_1 = \frac{1}{60 \times 10^{-9}} \times 2.25 \times 10^{-3} \int_0^t e^{-1250x} dx = -30e^{-1250t} + 30 \text{ V}, \quad t \geq 0$$

P 7.65 [a] $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40 \text{ V}$$

$$v_o(\infty) = 80 \text{ V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1 \text{ ms}; \quad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t} \text{ V}, \quad t \geq 0$$

$$[\mathbf{b}] \quad i_o = -C \frac{dv_o}{dt} = -0.16 \times 10^{-6} [40,000e^{-1000t}]$$

$$= -6.4e^{-1000t} \text{ mA}; \quad t \geq 0^+$$

$$[\mathbf{c}] \quad v_1 = \frac{-1}{0.2 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x} dx + 32$$

$$= 64 - 32e^{-1000t} \text{ V}, \quad t \geq 0$$

$$[\mathbf{d}] \quad v_2 = \frac{-1}{0.8 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x} dx + 8 \\ = 16 - 8e^{-1000t} \text{ V}, \quad t \geq 0$$

$$[\mathbf{e}] \quad w_{\text{trapped}} = \frac{1}{2}(0.2 \times 10^{-6})(64)^2 + \frac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512 \mu\text{J}.$$

P 7.66 [a] Let i be the current in the clockwise direction around the circuit. Then

$$\begin{aligned} V_g &= iR_g + \frac{1}{C_1} \int_0^t i dx + \frac{1}{C_2} \int_0^t i dx \\ &= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \int_0^t i dx = iR_g + \frac{1}{C_e} \int_0^t i dx \end{aligned}$$

Now differentiate the equation

$$0 = R_g \frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_g C_e} i = 0$$

$$\text{Therefore } i = \frac{V_g}{R_g} e^{-t/R_g C_e} = \frac{V_g}{R_g} e^{-t/\tau}; \quad \tau = R_g C_e$$

$$v_1(t) = \frac{1}{C_1} \int_0^t \frac{V_g}{R_g} e^{-x/\tau} dx = \frac{V_g}{R_g C_1} \frac{e^{-x/\tau}}{-1/\tau} \Big|_0^t = -\frac{V_g C_e}{C_1} (e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_g C_2}{C_1 + C_2} (1 - e^{-t/\tau}); \quad \tau = R_g C_e$$

$$v_2(t) = \frac{V_g C_1}{C_1 + C_2} (1 - e^{-t/\tau}); \quad \tau = R_g C_e$$

$$[\mathbf{b}] \quad v_1(\infty) = \frac{C_2}{C_1 + C_2} V_g; \quad v_2(\infty) = \frac{C_1}{C_1 + C_2} V_g$$

P 7.67 [a] $L_{\text{eq}} = \frac{(3)(15)}{3+15} = 2.5 \text{ H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3} \text{ s}$$

$$i_o(0) = 0; \quad i_o(\infty) = \frac{120}{7.5} = 16 \text{ A}$$

$$\therefore i_o = 16 - 16e^{-3t} \text{ A}, \quad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t} \text{ V}, \quad t \geq 0^+$$

$$i_1 = \frac{1}{3} \int_0^t 120e^{-3x} dx = \frac{40}{3} - \frac{40}{3}e^{-3t} \text{ A}, \quad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t} \text{ A}, \quad t \geq 0$$

- [b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
 $v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3 \frac{di_1}{dt} = 120e^{-3t} \text{ V}, \quad t \geq 0^+$$

or

$$v_o = 15 \frac{di_2}{dt} = 120e^{-3t} \text{ V}, \quad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.68 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50 \text{ mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \quad \frac{1}{\tau} = 5000$$

$$\therefore i_o(t) = 40 - 40e^{-5000t} \text{ mA}, \quad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t}) = 10e^{-5000t} \text{ V}, \quad t \geq 0^+$

[c] $v_o = 0.5 \frac{di_1}{dt} - 0.25 \frac{di_2}{dt} = 10e^{-5000t} \text{ V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t} \text{ A/s}$$

$$\therefore \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore 10e^{-5000t} = 0.5 \frac{di_1}{dt} - 50e^{-5000t} + 0.25 \frac{di_1}{dt}$$

$$\therefore 0.75 \frac{di_1}{dt} = 60e^{-5000t}; \quad di_1 = 80e^{-5000t} dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y} dy$$

$$i_1 = \frac{80}{-5000} e^{-5000y} \Big|_0^t = 16 - 16e^{-5000t} \text{ mA}, \quad t \geq 0$$

$$\begin{aligned} [\mathbf{d}] \quad i_2 &= i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t} \\ &= 24 - 24e^{-5000t} \text{ mA}, \quad t \geq 0 \end{aligned}$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}} \frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5 \frac{di_1}{dt} - 0.25 \frac{di_2}{dt} = 10e^{-5000t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25 \frac{di_2}{dt} - 0.25 \frac{di_1}{dt} = 10e^{-5000t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

$$v_o(0^+) = 10 \text{ V, which agrees with } i_o(0^+) = 0 \text{ A}$$

$$i_o(\infty) = 40 \text{ mA}; \quad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2 \text{ mWb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = (16 \text{ m})(500) + (24 \text{ m})(-250) = 2 \text{ mWb-turns (ok)}$$

$$i_2(\infty)L_2 + i_1(\infty)M = (24 \text{ m})(250) + (16 \text{ m})(-250) = 2 \text{ mWb-turns (ok)}$$

Therefore, the final values of i_o , i_1 , and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

$$\text{P 7.69 [a]} \quad L_{\text{eq}} = 0.02 + 0.04 + 2(0.015) = 0.09 = 90 \text{ mH}$$

$$\tau = \frac{L}{R} = \frac{0.09}{4500} = 20 \mu\text{s}; \quad \frac{1}{\tau} = 50,000$$

$$i = 20 - 20e^{-50,000t} \text{ mA, } t \geq 0$$

$$[\mathbf{b}] \quad v_1(t) = 0.02 \frac{di}{dt} + 0.015 \frac{di}{dt} = 0.035 \frac{di}{dt} = 0.035(1000e^{-50,000t}) = 35e^{-50,000t} \text{ V, } t \geq 0^+$$

$$[\mathbf{c}] \quad v_2(t) = 0.04 \frac{di}{dt} + 0.015 \frac{di}{dt} = 0.055 \frac{di}{dt} = 0.055(1000e^{-50,000t}) = 55e^{-50,000t} \text{ V, } t \geq 0^+$$

$$[\mathbf{d}] \quad i(0) = 0.02 - 0.02 = 0, \text{ which agrees with initial conditions.}$$

$$90 = 4500i + v_1 + v_2 = 4500(0.02 - 0.02e^{-50,000t}) + 35e^{-50,000t} + 55e^{-50,000t} = 90 \text{ V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$.

Thus, the answers make sense in terms of known circuit behavior.

P 7.70 [a] $L_{\text{eq}} = 0.02 + 0.04 - 2(0.015) = 0.03 = 30 \text{ mH}$

$$\tau = \frac{L}{R} = \frac{0.03}{4500} = 6.67 \mu\text{s}; \quad \frac{1}{\tau} = 150,000$$

$$i = 0.02 - 0.02e^{-150,000t} \text{ A}, \quad t \geq 0$$

$$[b] v_1(t) = 0.02 \frac{di}{dt} - 0.015 \frac{di}{dt} = 0.005 \frac{di}{dt} = 0.005(3000e^{-150,000t})$$

$$= 15e^{-150,000t} \text{ V}, \quad t \geq 0^+$$

$$[c] v_2(t) = 0.04 \frac{di}{dt} - 0.015 \frac{di}{dt} = 0.025 \frac{di}{dt} = 0.025(3000e^{-150,000t})$$

$$= 75e^{-150,000t} \text{ V}, \quad t \geq 0^+$$

[d] $i(0) = 0$, which agrees with initial conditions.

$$90 = 4500i_1 + v_1 + v_2 = 4500(0.02 - 0.02e^{-150,000t}) + 15e^{-150,000t}$$

$$+ 75e^{-150,000t} = 90 \text{ V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.71 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1 \text{ H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \quad \frac{1}{\tau} = 20$$

$$\therefore i_o(t) = 4 - 4e^{-20t} \text{ A}, \quad t \geq 0$$

$$[b] v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t} \text{ V}, \quad t \geq 0^+$$

$$[c] v_o = 5 \frac{di_1}{dt} - 5 \frac{di_2}{dt} = 80e^{-20t} \text{ V}$$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t} \text{ A/s}$$

$$\therefore \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore 80e^{-20t} = 5 \frac{di_1}{dt} - 400e^{-20t} + 5 \frac{di_1}{dt}$$

$$\therefore 10 \frac{di_1}{dt} = 480e^{-20t}; \quad di_1 = 48e^{-20t} dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y} dy$$

$$i_1 = \frac{48}{-20} e^{-20y} \Big|_0^t = 2.4 - 2.4e^{-20t} \text{ A}, \quad t \geq 0$$

$$[\mathbf{d}] \quad i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$$

$$= 1.6 - 1.6e^{-20t} \text{ A}, \quad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}} \frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5 \frac{di_1}{dt} - 5 \frac{di_2}{dt} = 80e^{-20t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10 \frac{di_2}{dt} - 5 \frac{di_1}{dt} = 80e^{-20t} \text{ V}, \quad t \geq 0^+ \text{ (checks)}$$

$$v_o(0^+) = 80 \text{ V, which agrees with } i_o(0^+) = 0 \text{ A}$$

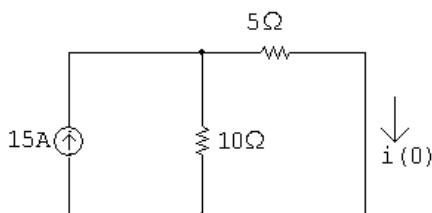
$$i_o(\infty) = 4 \text{ A}; \quad i_o(\infty)L_{\text{eq}} = (4)(1) = 4 \text{ Wb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4 \text{ Wb-turns (ok)}$$

$$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4 \text{ Wb-turns (ok)}$$

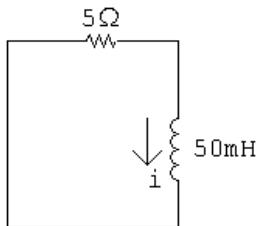
Therefore, the final values of i_o , i_1 , and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.72 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10 \text{ A}$$

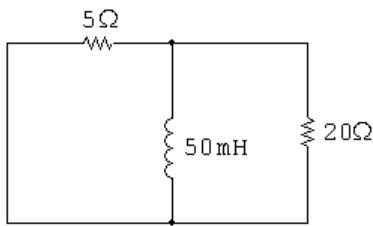
$0 \leq t \leq 10 \text{ ms}:$



$$i = 10e^{-100t} \text{ A}$$

$$i(10 \text{ ms}) = 10e^{-1} = 3.68 \text{ A}$$

$10 \text{ ms} \leq t \leq 20 \text{ ms}:$



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4 \Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)} \text{ A}$$

$20 \text{ ms} \leq t < \infty:$

$$i(20 \text{ ms}) = 3.68e^{-80(0.02-0.01)} = 1.65 \text{ A}$$

$$i = 1.65e^{-100(t-0.02)} \text{ A}$$

$$v_o = L \frac{di}{dt}; \quad L = 50 \text{ mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)} \text{ V}, \quad t > 20^+ \text{ ms}$$

$$v_o(25 \text{ ms}) = -8.26e^{-100(0.025-0.02)} = -5.013 \text{ V}$$

P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(50 \text{ mH})(10 \text{ A})^2 = 2.5 \text{ J}$$

$$0.04w(0) = 0.1 \text{ J}$$

$$\therefore \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2 \text{ A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

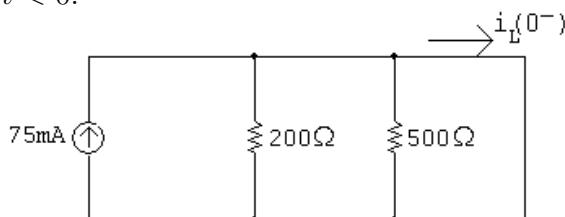
$$i(10 \text{ ms}) = 3.68 \text{ A} \quad \text{and} \quad i(20 \text{ ms}) = 1.65 \text{ A}$$

For $10 \text{ ms} \leq t \leq 20 \text{ ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

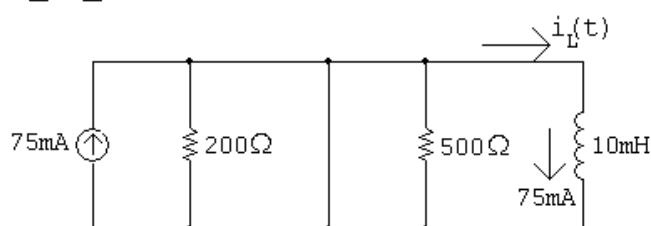
$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6 \text{ ms}$$

P 7.74 $t < 0$:



$$i_L(0^-) = 75 \text{ mA} = i_L(0^+)$$

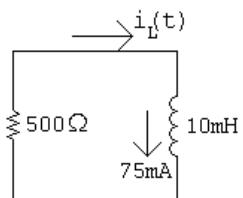
$0 \leq t \leq 25 \text{ ms}$:



$$\tau = 0.01/0 = \infty$$

$$i_L(t) = 0.075e^{-t/\infty} = 0.075e^{-0} = 75 \text{ mA}$$

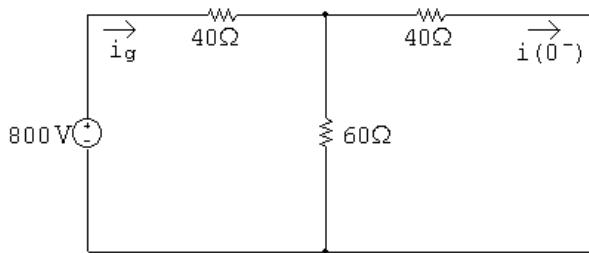
$25 \text{ ms} \leq t :$



$$\tau = \frac{0.01}{500} = 20 \mu\text{s}; \quad 1/\tau = 50,000$$

$$i_L(t) = 75e^{-50,000(t-0.025)} \text{ mA}, \quad t \geq 25 \text{ ms}$$

P 7.75 [a] $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60 \| 40} = 12.5 \text{ A}$$

Using current division,

$$i(0^-) = \frac{60}{60 + 40}(12.5) = 7.5 \text{ A} = i(0^+)$$

[b] $0 \leq t \leq 1 \text{ ms}$:

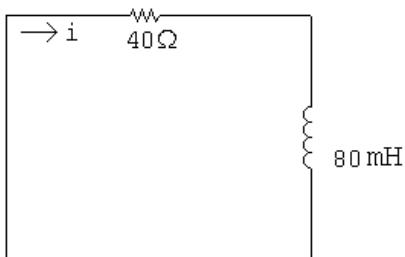
$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120 \| 60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200 \times 10^{-6})} = 7.5e^{-0.2} = 6.14 \text{ A}$$

[c] $i(1 \text{ ms}) = 7.5e^{-1} = 2.7591 \text{ A}$

$1 \text{ ms} \leq t:$ 

$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1 \text{ ms}) e^{-(t-1 \text{ ms})/\tau} = 2.7591 e^{-500(t-0.001)} \text{ A}$$

$$i(6 \text{ ms}) = 2.7591 e^{-500(0.005)} = 2.7591 e^{-2.5} = 226.48 \text{ mA}$$

[d] $0 \leq t \leq 1 \text{ ms}:$

$$i = 7.5 e^{-1000t}$$

$$v = L \frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5 e^{-1000t}) = -600 e^{-1000t} \text{ V}$$

$$v(1^- \text{ ms}) = -600 e^{-1} = -220.73 \text{ V}$$

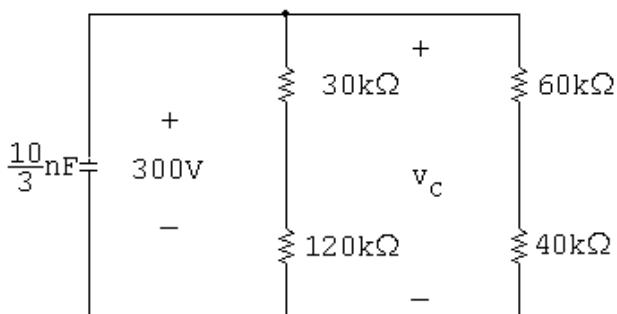
[e] $1 \text{ ms} \leq t \leq \infty:$

$$i = 2.759 e^{-500(t-0.001)}$$

$$v = L \frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759 e^{-500(t-0.001)})$$

$$= -110.4 e^{-500(t-0.001)} \text{ V}$$

$$v(1^+ \text{ ms}) = -110.4 \text{ V}$$

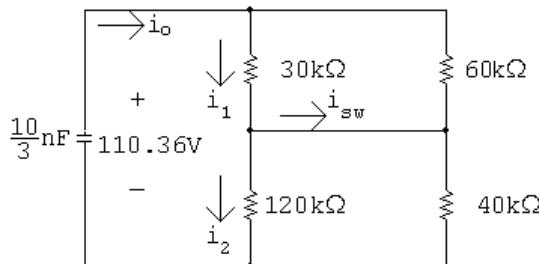
P 7.76 $0 \leq t \leq 200 \mu\text{s};$ 

$$R_e = 150 \parallel 100 = 60 \text{ k}\Omega; \quad \tau = \left(\frac{10}{3} \times 10^{-9}\right) (60,000) = 200 \mu\text{s}$$

$$v_c = 300e^{-5000t} \text{ V}$$

$$v_c(200 \mu\text{s}) = 300e^{-1} = 110.36 \text{ V}$$

$200 \mu\text{s} \leq t < \infty$:



$$R_e = 30\parallel 60 + 120\parallel 40 = 20 + 30 = 50 \text{ k}\Omega$$

$$\tau = \left(\frac{10}{3} \times 10^{-9}\right) (50,000) = 166.67 \mu\text{s}; \quad \frac{1}{\tau} = 6000$$

$$v_c = 110.36e^{-6000(t - 200 \mu\text{s})} \text{ V}$$

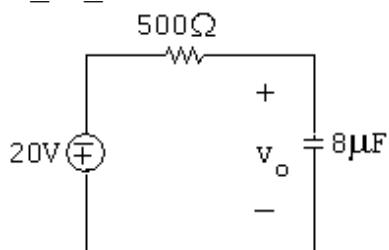
$$v_c(300 \mu\text{s}) = 110.36e^{-6000(100 \mu\text{s})} = 60.57 \text{ V}$$

$$i_o(300 \mu\text{s}) = \frac{60.57}{50,000} = 1.21 \text{ mA}$$

$$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \quad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$$

$$i_{sw} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50 \text{ mA}$$

P 7.77 $0 \leq t \leq 2.5 \text{ ms}$:

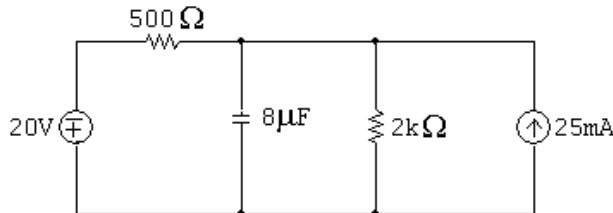


$$\tau = RC = (500)(8 \times 10^{-6}) = 4 \text{ ms}; \quad 1/\tau = 250$$

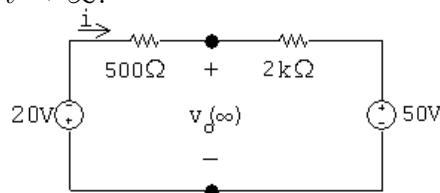
$$v_o(0) = 0 \text{ V}; \quad v_o(\infty) = -20 \text{ V}$$

$$v_o = -20 + 20e^{-250t} \text{ V} \quad 0 \leq t \leq 2.5 \text{ ms}$$

$2.5 \text{ ms} \leq t$:



$t \rightarrow \infty$:



$$i = \frac{-70 \text{ V}}{2.5 \text{ k}\Omega} = -28 \text{ mA}$$

$$v_o(\infty) = (-28 \times 10^{-3})(2000) + 50 = -6 \text{ V}$$

$$v_o(0.0025) = -20 + 20e^{-0.625} = -9.29 \text{ V}$$

$$v_o = -6 + (-9.29 + 6)e^{-(t-0.0025)/\tau}$$

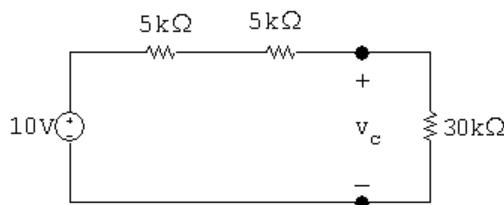
$$R_{\text{Th}} = 2000 \parallel 500 = 400 \Omega$$

$$\tau = (400)(8 \times 10^{-6}) = 3.2 \text{ ms}; \quad 1/\tau = 312.5$$

$$v_o = -6 - 3.29e^{-312.5(t-0.0025)} \quad 2.5 \text{ ms} \leq t$$

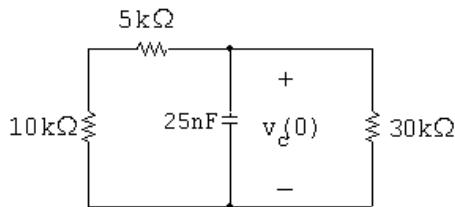
- P 7.78 Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5 \text{ V}$$

$0 \leq t \leq 0.2 \text{ ms}:$



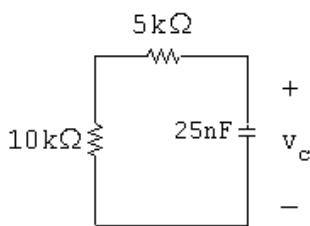
$$\tau = R_e C, \quad R_e = 15,000 \parallel 30,000 = 10 \text{ k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25 \text{ ms}, \quad \frac{1}{\tau} = 4000$$

$$v_c = 7.5e^{-4000t} \text{ V}, \quad t \geq 0$$

$$v_c(0.2 \text{ ms}) = 7.5e^{-0.8} = 3.37 \text{ V}$$

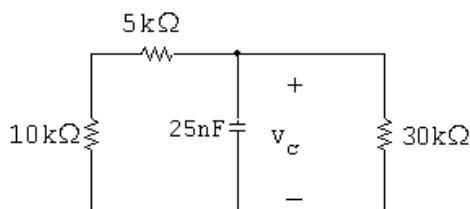
$0.2 \text{ ms} \leq t \leq 0.8 \text{ ms}:$



$$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375 \mu\text{s}, \quad \frac{1}{\tau} = 2666.67$$

$$v_c = 3.37e^{-2666.67(t-200 \times 10^{-6})} \text{ V}$$

$0.8 \text{ ms} \leq t <:$



$$\tau = 0.25 \text{ ms}, \quad \frac{1}{\tau} = 4000$$

$$v_c(0.8 \text{ ms}) = 3.37e^{-2666.67(800 - 200) \times 10^{-6}} = 3.37e^{-1.6} = 0.68 \text{ V}$$

$$v_c = 0.68e^{-4000(t-0.8 \times 10^{-3})} \text{ V}$$

$$v_c(1 \text{ ms}) = 0.68e^{-4000(1 - 0.8) \times 10^{-3}} = 0.68e^{-0.8} = 0.306 \text{ V}$$

$$v_o = (10/15)(0.306) = 0.204 \text{ V}$$

P 7.79 $w(0) = \frac{1}{2}(25 \times 10^{-9})(7.5)^2 = 703.125 \text{ nJ}$
 $0 \leq t \leq 200 \mu\text{s}:$

$$v_c = 7.5e^{-4000t}; \quad v_c^2 = 56.25e^{-8000t}$$

$$p_{30k} = 1.875e^{-8000t} \text{ mW}$$

$$\begin{aligned} w_{30k} &= \int_0^{200 \times 10^{-6}} 1.875 \times 10^{-3} e^{-8000t} dt \\ &= 1.875 \times 10^{-3} \frac{e^{-8000t}}{-8000} \Big|_0^{200 \times 10^{-6}} \\ &= -234.375 \times 10^{-9} (e^{-1.6} - 1) = 187.1 \text{ nJ} \end{aligned}$$

$0.8 \text{ ms} \leq t:$

$$v_c = 0.68e^{-4000(t-0.8 \times 10^{-3})} \text{ V}; \quad v_c^2 = 0.46e^{-8000(t-0.8 \times 10^{-3})}$$

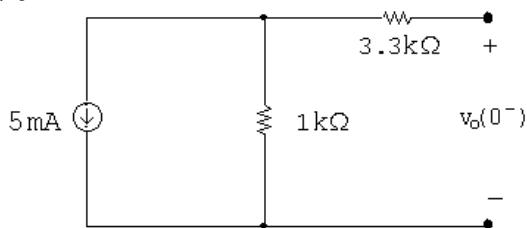
$$p_{30k} = 15.33e^{-8000(t-0.8 \times 10^{-3})} \mu\text{W}$$

$$\begin{aligned} w_{30k} &= \int_{0.8 \times 10^{-3}}^{\infty} 15.33 \times 10^{-6} e^{-8000(t-0.8 \times 10^{-3})} dt \\ &= 15.33 \times 10^{-6} \frac{e^{-8000(t-0.8 \times 10^{-3})}}{-8000} \Big|_{0.8 \times 10^{-3}}^{\infty} \\ &= -1.9 \times 10^{-9} (0 - 1) = 1.9 \text{ nJ} \end{aligned}$$

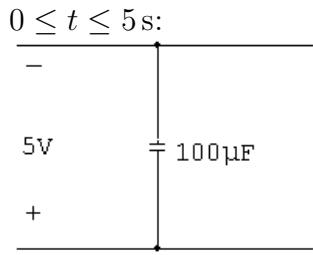
$$w_{30k} = 187.1 + 1.9 = 189 \text{ nJ}$$

$$\% = \frac{189}{703.125} (100) = 26.88\%$$

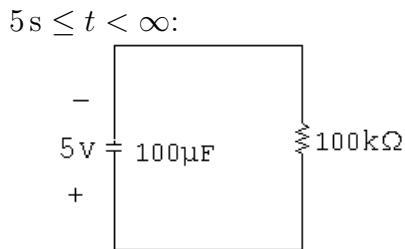
P 7.80 $t < 0:$



$$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5 \text{ V} = v_c(0^+)$$



$$\tau = \infty; \quad 1/\tau = 0; \quad v_o = -5e^{-0} = -5 \text{ V}$$



$$\tau = (100)(0.1) = 10 \text{ s}; \quad 1/\tau = 0.1; \quad v_o = -5e^{-0.1(t-5)} \text{ V}$$

Summary:

$$v_o = -5 \text{ V}, \quad 0 \leq t \leq 5 \text{ s}$$

$$v_o = -5e^{-0.1(t-5)} \text{ V}, \quad 5 \text{ s} \leq t < \infty$$

P 7.81 [a] $i_o(0) = 0; \quad i_o(\infty) = 50 \text{ mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{3000}{75} \times 10^3 = 40,000$$

$$i_o = (50 - 50e^{-40,000t}) \text{ mA}, \quad 0 \leq t \leq 25 \mu\text{s}$$

$$v_o = 0.075 \frac{di_o}{dt} = 150e^{-40,000t} \text{ V}, \quad 0 \leq t \leq 25 \mu\text{s}$$

$25 \mu\text{s} \leq t:$

$$i_o(25 \mu\text{s}) = 50 - 50e^{-1} = 31.6 \text{ mA}; \quad i_o(\infty) = 0$$

$$i_o = 31.6e^{-40,000(t-25 \times 10^{-6})} \text{ mA}$$

$$v_o = 0.075 \frac{di_o}{dt} = -94.82e^{-40,000(t-25 \mu\text{s})} \text{ V}$$

$$\therefore t < 0 : \quad v_o = 0$$

$$0 \leq t \leq 25 \mu\text{s} : \quad v_o = 150e^{-40,000t} \text{ V}$$

$$25 \mu\text{s} \leq t : \quad v_o = -94.82e^{-40,000(t-25 \mu\text{s})} \text{ V}$$

[b] $v_o(25^- \mu\text{s}) = 150e^{-1} = 55.18 \text{ V}$

$$v_o(25^+ \mu\text{s}) = -94.82 \text{ V}$$

[c] $i_o(25^- \mu\text{s}) = i_o(25^+ \mu\text{s}) = 31.6 \text{ mA}$

P 7.82 [a] $0 \leq t \leq 2.5 \text{ ms}$

$$v_o(0^+) = 80 \text{ V}; \quad v_o(\infty) = 0$$

$$\tau = \frac{L}{R} = 2 \text{ ms}; \quad 1/\tau = 500$$

$$v_o(t) = 80e^{-500t} \text{ V}, \quad 0^+ \leq t \leq 2.5^- \text{ ms}$$

$$v_o(2.5^- \text{ ms}) = 80e^{-1.25} = 22.92 \text{ V}$$

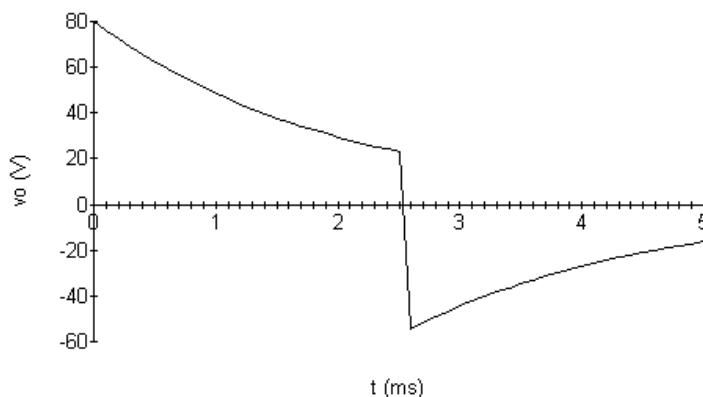
$$i_o(2.5^- \text{ ms}) = \frac{(80 - 22.92)}{20} = 2.85 \text{ A}$$

$$v_o(2.5^+ \text{ ms}) = -20(2.85) = -57.08 \text{ V}$$

$$v_o(\infty) = 0; \quad \tau = 2 \text{ ms}; \quad 1/\tau = 500$$

$$v_o = -57.08e^{-500(t - 0.0025)} \text{ V} \quad t \geq 2.5^+ \text{ ms}$$

[b]



[c] $v_o(5 \text{ ms}) = -16.35 \text{ V}$

$$i_o = \frac{+16.35}{20} = 817.68 \text{ mA}$$

P 7.83 [a] $t < 0; \quad v_o = 0$

$0 \leq t \leq 25 \mu\text{s};$

$$\tau = (4000)(50 \times 10^{-9}) = 0.2 \text{ ms}; \quad 1/\tau = 5000$$

$$v_o = 10 - 10e^{-5000t} \text{ V}, \quad 0 \leq t \leq 25 \mu\text{s}$$

$$v_o(25 \mu\text{s}) = 10(1 - e^{-0.125}) = 1.175 \text{ V}$$

$25 \mu\text{s} \leq t \leq 50 \mu\text{s}$:

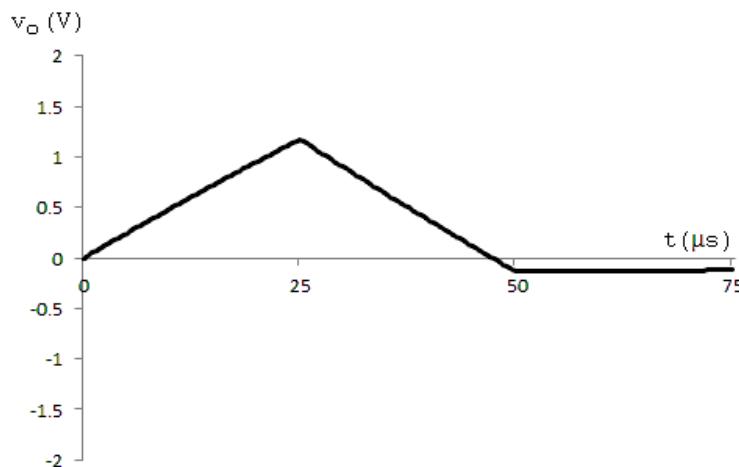
$$v_o = -10 + 11.175e^{-5000(t-25 \times 10^{-6})} \text{ V}, \quad 25 \mu\text{s} \leq t \leq 50 \mu\text{s}$$

$$v_o(50 \mu\text{s}) = -10 + 11.175e^{-0.125} = -0.138 \text{ V}$$

$t \geq 50 \mu\text{s}$:

$$v_o = -0.138e^{-5000(t-50 \times 10^{-6})} \text{ V}, \quad t \geq 50 \mu\text{s}$$

[b]



[c] $t \leq 0 : v_o = 0$

$0 \leq t \leq 25 \mu\text{s}$:

$$\tau = (800)(50 \times 10^{-9}) = 40 \mu\text{s} \quad 1/\tau = 25,000$$

$$v_o = 10 - 10e^{-25,000t} \text{ V}, \quad 0 \leq t \leq 25 \mu\text{s}$$

$$v_o(25 \mu\text{s}) = 10 - 10e^{-0.625} = 4.65 \text{ V}$$

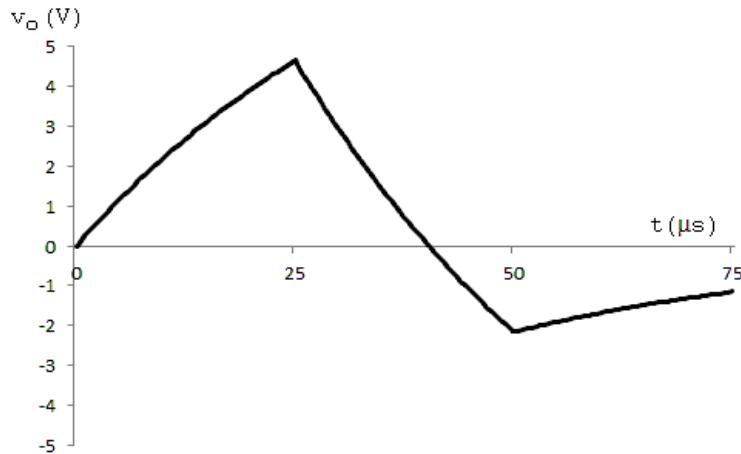
$25 \mu\text{s} \leq t \leq 50 \mu\text{s}$:

$$v_o = -10 + 14.65e^{-25,000(t-25 \times 10^{-6})} \text{ V}, \quad 25 \mu\text{s} \leq t \leq 50 \mu\text{s}$$

$$v_o(50 \mu\text{s}) = -10 + 14.65e^{-0.625} = -2.16 \text{ V}$$

$t \geq 50 \mu\text{s}$:

$$v_o = -2.16e^{-25,000(t-50 \times 10^{-6})} \text{ V}, \quad t \geq 50 \mu\text{s}$$



P 7.84 [a] $0 \leq t \leq 1 \text{ ms}$:

$$v_c(0^+) = 0; \quad v_c(\infty) = 50 \text{ V};$$

$$RC = 400 \times 10^3 (0.01 \times 10^{-6}) = 4 \text{ ms}; \quad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t} \text{ V}, \quad 0 \leq t \leq 1 \text{ ms}$$

$1 \text{ ms} \leq t < \infty$:

$$v_c(1 \text{ ms}) = 50 - 50e^{-0.25} = 11.06 \text{ V}$$

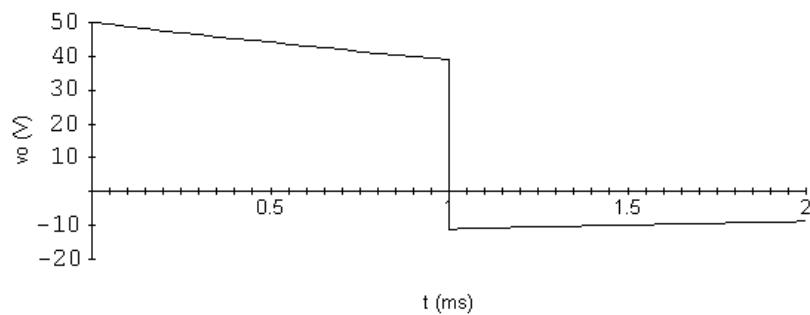
$$v_c(\infty) = 0 \text{ V}$$

$$\tau = 4 \text{ ms}; \quad 1/\tau = 250$$

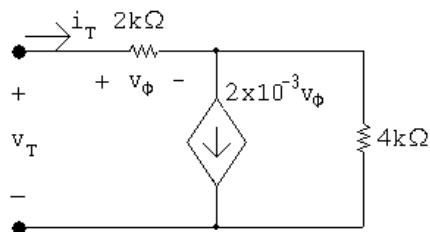
$$v_c = 11.06e^{-250(t - 0.001)} \text{ V}$$

$$v_o = -v_c = -11.06e^{-250(t - 0.001)} \text{ V}, \quad t \geq 1 \text{ ms}$$

[b]



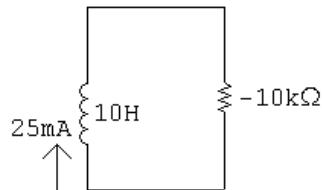
P 7.85



$$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10,000$$

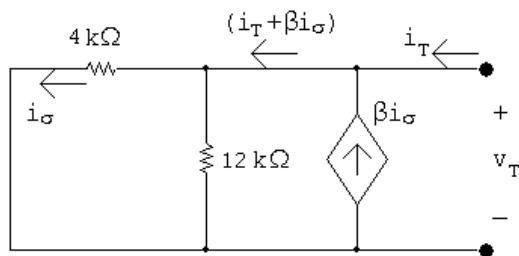


$$\tau = \frac{10}{-10,000} = -1 \text{ ms}; \quad 1/\tau = -1000$$

$$i = 25e^{1000t} \text{ mA}$$

$$\therefore 25e^{1000t} \times 10^{-3} = 5; \quad t = \frac{\ln 200}{1000} = 5.3 \text{ ms}$$

P 7.86 [a]



Using Ohm's law,

$$v_T = 4000i_\sigma$$

Using current division,

$$i_\sigma = \frac{12,000}{12,000 + 4000}(i_T + \beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

Solve for i_σ :

$$i_\sigma(1 - 0.75\beta) = 0.75i_T$$

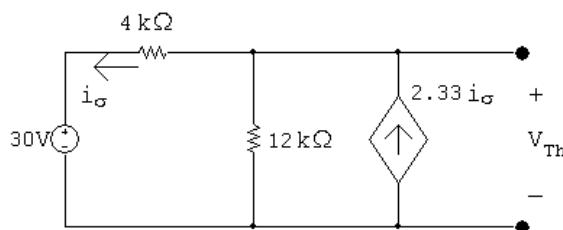
$$i_\sigma = \frac{0.75i_T}{1 - 0.75\beta}; \quad v_T = 4000i_\sigma = \frac{3000i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{Th} = -4\text{k}\Omega$:

$$R_{Th} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$$1 - 0.75\beta = -0.75 \quad \therefore \quad \beta = 2.33$$

[b] Find V_{Th} ;



Write a KCL equation at the top node:

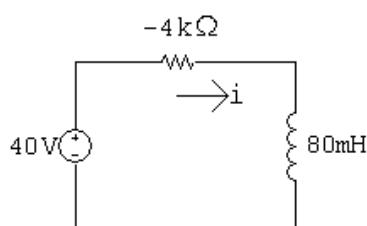
$$\frac{V_{Th} - 30}{4000} + \frac{V_{Th}}{12,000} - 2.33i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{Th} - 30)}{4000} = 0$$

Solving,

$$V_{Th} = 40\text{ V}$$



Write a KVL equation around the loop:

$$40 = -4000i + 0.08 \frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 500 + 50,000i = 25,000(i + 0.01)$$

Separate the variables and integrate to find i :

$$\frac{di}{i + 0.01} = 50,000 dt$$

$$\int_0^i \frac{dx}{x + 0.01} = \int_0^t 50,000 \, dx$$

$$\therefore i = -10 + 10e^{50,000t} \text{ mA}$$

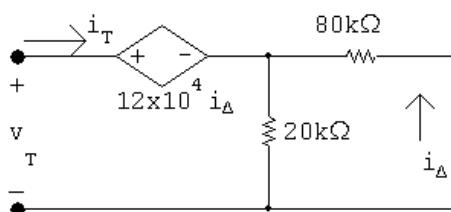
$$\frac{di}{dt} = (10 \times 10^{-3})(50,000)e^{50,000t} = 500e^{50,000t}$$

Solve for the arc time:

$$v = 0.08 \frac{di}{dt} = 40e^{50,000t} = 30,000; \quad e^{50,000t} = 750$$

$$\therefore t = \frac{\ln 750}{50,000} = 132.4 \mu\text{s}$$

P 7.87 $t > 0$:



$$v_T = 12 \times 10^4 i_{\Delta} + 16 \times 10^3 i_T$$

$$i_{\Delta} = -\frac{20}{100}i_T = -0.2i_T$$

$$\therefore v_T = -24 \times 10^3 i_T + 16 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = -8 \text{ k}\Omega$$

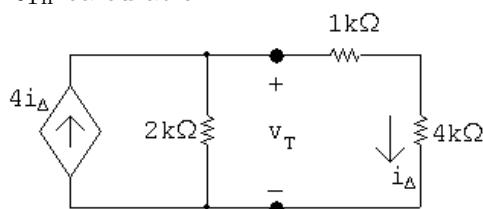
$$\tau = RC = (-8 \times 10^3)(2.5 \times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t} \text{ V}; \quad 20e^{50t} = 20,000$$

$$50t \equiv \ln 1000 \quad \therefore \quad t = 138.16 \text{ ms}$$

P 7.88 Find the Thévenin equivalent with respect to the terminals of the capacitor.

R_{Th} calculation:

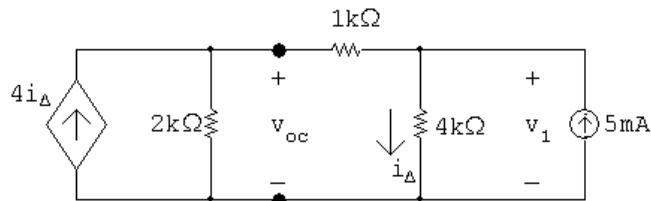


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \frac{i_T}{v_T} = \frac{5 + 2 - 8}{10,000} = -\frac{1}{10,000}$$

$$\frac{v_T}{i_T} = -\frac{10,000}{1} = -10 \text{ k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

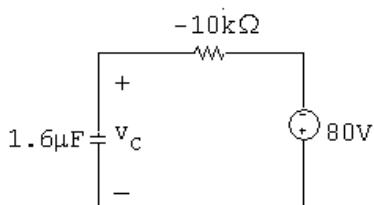
$$\frac{v_{oc}}{2000} + \frac{v_{oc} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{oc}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{oc} = -80 \text{ V}$, $v_1 = -60 \text{ V}$



$$v_c(0) = 0; \quad v_c(\infty) = -80 \text{ V}$$

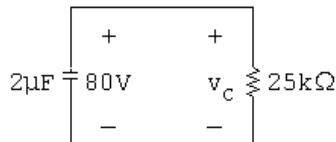
$$\tau = RC = (-10,000)(1.6 \times 10^{-6}) = -16 \text{ ms}; \quad \frac{1}{\tau} = -62.5$$

$$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14,400$$

Solve for the time of the maximum voltage rating:

$$e^{62.5t} = 181; \quad 62.5t = \ln 181; \quad t = 83.09 \text{ ms}$$

P 7.89 [a]



$$\tau = (25)(2) \times 10^{-3} = 50 \text{ ms}; \quad 1/\tau = 20$$

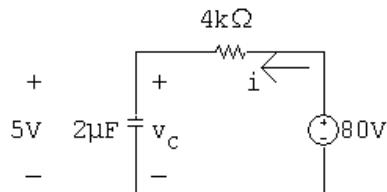
$$v_c(0^+) = 80 \text{ V}; \quad v_c(\infty) = 0$$

$$v_c = 80e^{-20t} \text{ V}$$

$$\therefore 80e^{-20t} = 5; \quad e^{20t} = 16; \quad t = \frac{\ln 16}{20} = 138.63 \text{ ms}$$

[b] $0^+ \leq t \leq 138.63^- \text{ ms}$:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t} \text{ mA}$$

 $t \geq 138.63^+ \text{ ms}$:

$$\tau = (2)(4) \times 10^{-3} = 8 \text{ ms}; \quad 1/\tau = 125$$

$$v_c(138.63^+ \text{ ms}) = 5 \text{ V}; \quad v_c(\infty) = 80 \text{ V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)} \text{ V}, \quad t \geq 138.63 \text{ ms}$$

$$\begin{aligned} i &= 2 \times 10^{-6}(9375)e^{-125(t-0.13863)} \\ &= 18.75e^{-125(t-0.13863)} \text{ mA}, \quad t \geq 138.63^+ \text{ ms} \end{aligned}$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \quad \Delta t = \frac{\ln 6.25}{125} \cong 14.66 \text{ ms}$$

$$\text{P 7.90} \quad v_o = -\frac{1}{R(0.5 \times 10^{-6})} \int_0^t 4 \, dx + 0 = \frac{-4t}{R(0.5 \times 10^{-6})}$$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12 \text{ k}\Omega$$

$$\text{P 7.91} \quad v_o = \frac{-4t}{R(0.5 \times 10^{-6})} + 6 = \frac{-4(40 \times 10^{-3})}{R(0.5 \times 10^{-6})} + 6 = -10$$

$$\therefore R = \frac{-4(40 \times 10^{-3})}{-16(0.5 \times 10^{-6})} = 20 \text{ k}\Omega$$

$$\text{P 7.92} \quad RC = (80 \times 10^3)(250 \times 10^{-9}) = 20 \text{ ms}; \quad \frac{1}{RC} = 50$$

[a] $t < 0 : \quad v_o = 0$

[b] $0 \leq t \leq 2 \text{ s} :$

$$v_o = -50 \int_0^t 0.075 dx = -3.75t \text{ V}$$

[c] $2 \text{ s} \leq t \leq 4 \text{ s} ;$

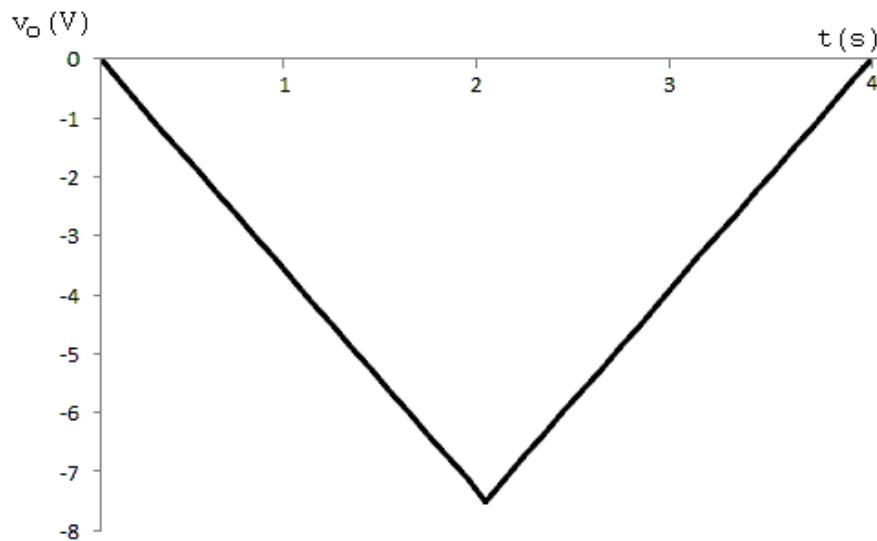
$$v_o(2) = -3.75(2) = -7.5 \text{ V}$$

$$v_o(t) = -50 \int_2^t -0.075 dx - 7.5 = 3.75(t - 2) - 7.5 = 3.75t - 15 \text{ V}$$

[d] $t \geq 4 \text{ s} :$

$$v_o(4) = 15 - 15 = 0 \text{ V}$$

$$v_o(t) = 0 \text{ V}$$



P 7.93 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0 - v_g}{R_i} + \frac{0 - v_o}{R_f} + C_f \frac{d}{dt}(0 - v_o) = 0$$

$$\therefore \frac{dv_o}{dt} + \frac{1}{R_f C_f} v_o = -\frac{v_g}{R_i C_f}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s = -v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o = \frac{-v_g R_f}{R_i} + \left(V_o - \frac{-v_g R_f}{R_i} \right) e^{-t/R_f C_f}$$

[a] $v_o = 0, \quad t < 0$

[b] $R_f C_f = (4 \times 10^6)(250 \times 10^{-9}) = 1; \quad \frac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = \frac{-(0.075)(4 \times 10^6)}{80,000} = -3.75$$

$$V_o = v_o(0) = 0$$

$$\therefore v_o = -3.75 + (0 + 3.75)e^{-t} = -3.75(1 - e^{-t}) \text{ V}, \quad 0 \leq t \leq 2 \text{ s}$$

[c] $\frac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = \frac{-(-0.075)(4 \times 10^6)}{80,000} = 3.75$$

$$V_o = v_o(2) = -3.75(1 - e^{-2}) = -3.24 \text{ V}$$

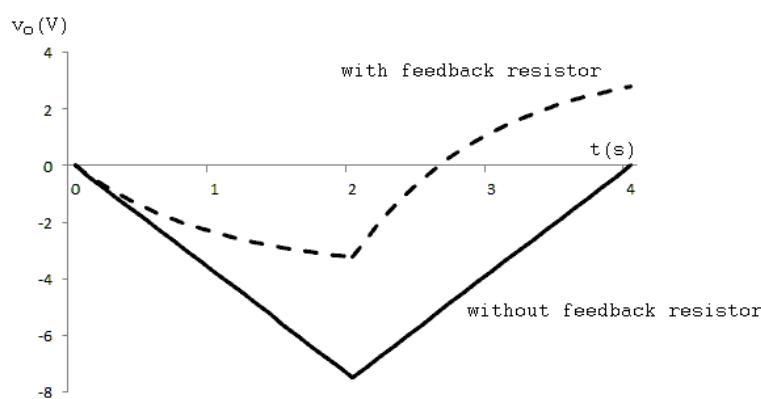
$$\begin{aligned} \therefore v_o &= 3.75 + [-3.24 - 3.75]e^{-(t-2)} \\ &= 3.75 - 6.99e^{-(t-2)} \text{ V}, \quad 2 \text{ s} \leq t \leq 4 \text{ s} \end{aligned}$$

[d] $\frac{1}{R_f C_f} = 1$

$$\frac{-v_g R_f}{R_i} = 0$$

$$V_o = v_o(4) = 3.75 - 6.99e^{-2} = 2.8 \text{ V}$$

$$v_o = 0 + (2.8 - 0)e^{-(t-4)} = 2.8e^{-(t-4)} \text{ V}, \quad 4 \text{ s} \leq t$$



$$\text{P 7.94 [a]} \quad \frac{Cdv_p}{dt} + \frac{v_p - v_b}{R} = 0; \quad \text{therefore} \quad \frac{dv_p}{dt} + \frac{1}{RC}v_p = \frac{v_b}{RC}$$

$$\frac{v_n - v_a}{R} + C \frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_a}{RC}$$

But $v_n = v_p$

$$\text{Therefore} \quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_b}{RC}$$

$$\text{Therefore} \quad \frac{dv_o}{dt} = \frac{1}{RC}(v_b - v_a); \quad v_o = \frac{1}{RC} \int_0^t (v_b - v_a) dy$$

- [b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

$$\text{[c]} \quad v_o = \frac{1}{RC} \int_0^t (v_b - v_a) dx$$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5 \text{ ms}$$

$$v_b - v_a = -25 \text{ mV}$$

$$v_o = \frac{1}{0.0005} \int_0^t -25 \times 10^{-3} dx = -50t$$

$$-50t_{\text{sat}} = -6; \quad t_{\text{sat}} = 120 \text{ ms}$$

- P 7.95 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36 \text{ V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80,000} + 2.5 \times 10^{-6} \frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore 2.5 \times 10^{-6} \frac{dv_o}{dt} = \frac{-50}{80,000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore dv_o = -250 dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250 \int_0^t dy \quad \therefore v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20 \text{ V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \quad t = \frac{20}{250} = 80 \text{ ms}$$

P 7.96 [a] $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \quad \frac{1}{RC} = 1,250,000$
 $0 \leq t \leq 1 \mu\text{s}:$

$$v_g = 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -1.25 \times 10^6 \int_0^t 2 \times 10^6 x \, dx + 0 \\ &= -2.5 \times 10^{12} \frac{x^2}{2} \Big|_0^t = -125 \times 10^{10} t^2 \text{ V}, \quad 0 \leq t \leq 1 \mu\text{s} \end{aligned}$$

$$v_o(1 \mu\text{s}) = -125 \times 10^{10} (1 \times 10^{-6})^2 = -1.25 \text{ V}$$

$1 \mu\text{s} \leq t \leq 3 \mu\text{s}:$

$$v_g = 4 - 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -125 \times 10^4 \int_{1 \times 10^{-6}}^t (4 - 2 \times 10^6 x) \, dx - 1.25 \\ &= -125 \times 10^4 \left[4x \Big|_{1 \times 10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2} \Big|_{1 \times 10^{-6}}^t \right] - 1.25 \\ &= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25 \\ &= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5 \text{ V}, \quad 1 \mu\text{s} \leq t \leq 3 \mu\text{s} \end{aligned}$$

$$v_o(3 \mu\text{s}) = 125 \times 10^{10} (3 \times 10^{-6})^2 - 5 \times 10^6 (3 \times 10^{-6}) + 2.5$$

$$= -1.25$$

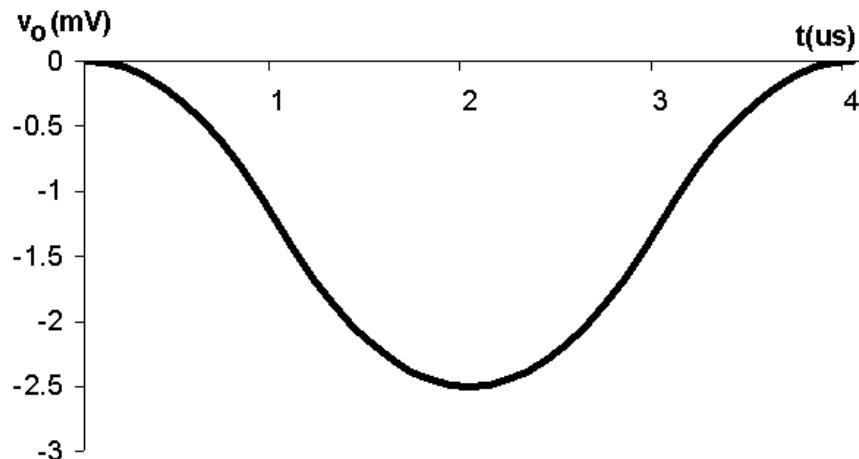
$3 \mu\text{s} \leq t \leq 4 \mu\text{s}:$

$$v_g = -8 + 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -125 \times 10^4 \int_{3 \times 10^{-6}}^t (-8 + 2 \times 10^6 x) \, dx - 1.25 \\ &= -125 \times 10^4 \left[-8x \Big|_{3 \times 10^{-6}}^t + 2 \times 10^6 \frac{x^2}{2} \Big|_{3 \times 10^{-6}}^t \right] - 1.25 \\ &= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25 \\ &= -125 \times 10^{10} t^2 + 10^7 t - 20 \text{ V}, \quad 3 \mu\text{s} \leq t \leq 4 \mu\text{s} \end{aligned}$$

$$v_o(4 \mu\text{s}) = -125 \times 10^{10} (4 \times 10^{-6})^2 + 10^7 (4 \times 10^{-6}) - 20 = 0$$

[b]



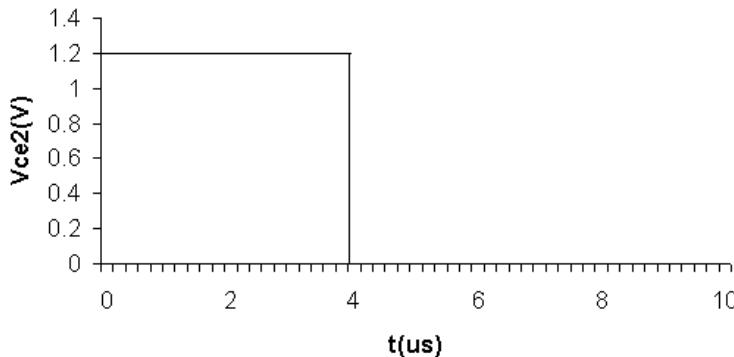
- [c] The output voltage will also repeat. This follows from the observation that at $t = 4 \mu s$ the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4 \mu s$ as it was at $t = 0$, thus as v_g repeats itself, so will v_o .

- P 7.97 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC} , positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.
- [b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.
- [c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C , it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

- P 7.98 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L} \right) R_1 = \frac{6(5)}{25} = 1.2 \text{ V}$$

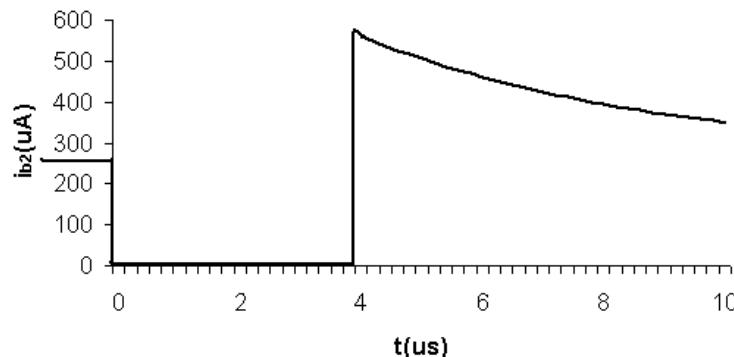
T_2 remains open for $(23,083)(250) \times 10^{-12} \ln 2 \cong 4 \mu s$.



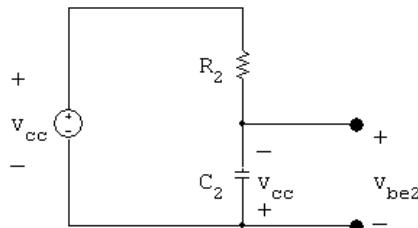
[b] $i_{b2} = \frac{V_{CC}}{R} = 259.93 \mu A, \quad -5 \leq t \leq 0 \mu s$

$$i_{b2} = 0, \quad 0 < t < RC \ln 2$$

$$\begin{aligned} i_{b2} &= \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L} e^{-(t-RC \ln 2)/R_L C} \\ &= 259.93 + 300e^{-0.2 \times 10^6 (t-4 \times 10^{-6})} \mu A, \quad RC \ln 2 < t \end{aligned}$$

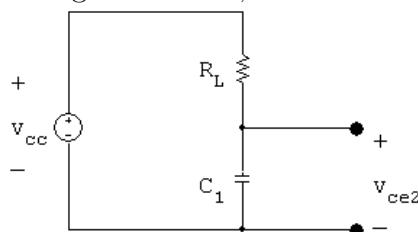


- P 7.99 [a] While T_2 has been ON, C_2 is charged to V_{CC} , positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC} , positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

- [b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1 , thus



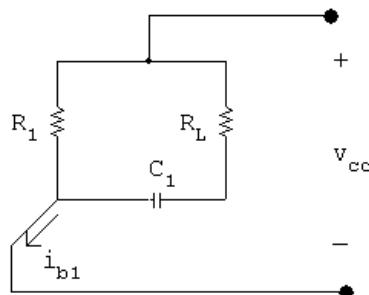
It follows that $v_{ce2} = V_{CC} - V_{CC}e^{-t/R_L C_1}$.

- [c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

- [d] When $t = R_2C_2 \ln 2$, we have

$$v_{ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_L C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

- [e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have

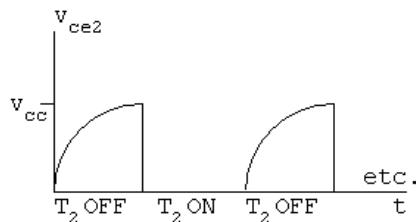


$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-t/R_L C_1}$$

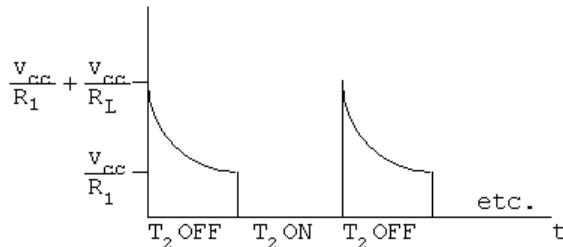
- [f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

[g]



[h]



P 7.100 [a] $t_{OFF2} = R_2 C_2 \ln 2 = 18 \times 10^3 (2 \times 10^{-9}) \ln 2 \cong 25 \mu\text{s}$

[b] $t_{ON2} = R_1 C_1 \ln 2 \cong 25 \mu\text{s}$

[c] $t_{OFF1} = R_1 C_1 \ln 2 \cong 25 \mu\text{s}$

[d] $t_{ON1} = R_2 C_2 \ln 2 \cong 25 \mu\text{s}$

[e] $i_{b1} = \frac{9}{3} + \frac{9}{18} = 3.5 \text{ mA}$

[f] $i_{b1} = \frac{9}{18} + \frac{9}{3} e^{-6 \ln 2} \cong 0.5469 \text{ mA}$

[g] $v_{ce2} = 9 - 9 e^{-6 \ln 2} \cong 8.86 \text{ V}$

P 7.101 [a] $t_{OFF2} = R_2 C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35 \mu\text{s}$

[b] $t_{ON2} = R_1 C_1 \ln 2 \cong 37.4 \mu\text{s}$

[c] $t_{OFF1} = R_1 C_1 \ln 2 \cong 37.4 \mu\text{s}$

[d] $t_{ON1} = R_2 C_2 \ln 2 = 35 \mu\text{s}$

[e] $i_{b1} = 3.5 \text{ mA}$

[f] $i_{b1} = \frac{9}{18} + 3 e^{-5.6 \ln 2} \cong 0.562 \text{ mA}$

[g] $v_{ce2} = 9 - 9 e^{-5.6 \ln 2} \cong 8.81 \text{ V}$

Note in this circuit T_2 is OFF $35 \mu\text{s}$ and ON $37.4 \mu\text{s}$ of every cycle, whereas T_1 is ON $35 \mu\text{s}$ and OFF $37.4 \mu\text{s}$ every cycle.

P 7.102 If $R_1 = R_2 = 50 R_L = 100 \text{ k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49 \text{ pF}; \quad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37 \text{ pF}$$

If $R_1 = R_2 = 6R_L = 12\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\text{nF}; \quad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\text{nF}$$

Therefore $692.49\text{pF} \leq C_1 \leq 5.77\text{nF}$ and $519.37\text{pF} \leq C_2 \leq 4.33\text{nF}$

P 7.103 [a] $0 \leq t \leq 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60} \right) e^{-t/\tau} \quad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore e^{30/L} = 3; \quad L = \frac{30}{\ln 3} = 27.31\text{H}$$

[b] $0 \leq t \leq t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0 \right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore 0.4 = 0.5e^{-60t_r/L}; \quad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\text{s}$$

P 7.104 From the Practical Perspective,

$$v_C(t) = 0.75V_S = V_S(1 - e^{-t/RC})$$

$$0.25 = e^{-t/RC} \quad \text{so} \quad t = -RC \ln 0.25$$

In the above equation, t is the number of seconds it takes to charge the capacitor to $0.75V_S$, so it is a period. We want to calculate the heart rate, which is a frequency in beats per minute, so $H = 60/t$. Thus,

$$H = \frac{60}{-RC \ln 0.25}$$

P 7.105 In this problem, $V_{max} = 0.6V_S$, so the equation for heart rate in beats per minute is

$$H = \frac{60}{-RC \ln 0.4}$$

Given $R = 150\text{k}\Omega$ and $C = 6\mu\text{F}$,

$$H = \frac{60}{-(150,000)(6 \times 10^{-6}) \ln 0.4} = 72.76$$

Therefore, the heart rate is about 73 beats per minute.

P 7.106 From the Practical Perspective,

$$v_C(t) = V_{max} = V_S(1 - e^{-t/RC})$$

Solve this equation for the resistance R :

$$\frac{V_{max}}{V_S} = 1 - e^{-t/RC} \quad \text{so} \quad e^{-t/RC} = 1 - \frac{V_{max}}{V_S}$$

$$\text{Then, } \frac{-t}{RC} = \ln\left(1 - \frac{V_{max}}{V_S}\right)$$

$$\therefore R = \frac{-t}{C \ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

In the above equation, t is the time it takes to charge the capacitor to a voltage of V_{max} . But t and the heart rate H are related as follows:

$$H = \frac{60}{t}$$

Therefore,

$$R = \frac{-60}{HC \ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

P 7.107 From Problem 7.106,

$$R = \frac{-60}{HC \ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

Note that from the problem statement,

$$\frac{V_{max}}{V_S} = 0.68$$

Therefore,

$$R = \frac{-60}{(70)(2.5 \times 10^{-6}) \ln(1 - 0.68)} = 301 \text{ k}\Omega$$

8

Natural and Step Responses of *RLC* Circuits

Assessment Problems

$$\text{AP 8.1 [a]} \frac{1}{(2RC)^2} = \frac{1}{LC}, \quad \text{therefore } C = 500 \text{ nF}$$

$$[\text{b}] \quad \alpha = 5000 = \frac{1}{2RC}, \quad \text{therefore } C = 1 \mu\text{F}$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000) \text{ rad/s}$$

$$[\text{c}] \quad \frac{1}{\sqrt{LC}} = 20,000, \quad \text{therefore } C = 125 \text{ nF}$$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2} \right] 10^3,$$

$$s_1 = -5.36 \text{ krad/s}, \quad s_2 = -74.64 \text{ krad/s}$$

$$\begin{aligned} \text{AP 8.2 } i_L &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20,000x}] dx + 30 \times 10^{-3} \\ &= 20 \left\{ \frac{-14e^{-5000x}}{-5000} \Big|_0^t + \frac{26e^{-20,000t}}{-20,000} \Big|_0^t \right\} + 30 \times 10^{-3} \\ &= 56 \times 10^{-3} (e^{-5000t} - 1) - 26 \times 10^{-3} (e^{-20,000t} - 1) + 30 \times 10^{-3} \\ &= [56e^{-5000t} - 56 - 26e^{-20,000t} + 26 + 30] \text{ mA} \\ &= 56e^{-5000t} - 26e^{-20,000t} \text{ mA}, \quad t \geq 0 \end{aligned}$$

AP 8.3 From the given values of R , L , and C , $s_1 = -10 \text{ krad/s}$ and $s_2 = -40 \text{ krad/s}$.

$$[\text{a}] \quad v(0^-) = v(0^+) = 0, \quad \text{therefore } i_R(0^+) = 0$$

[b] $i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4 + 0) = 4 \text{ A}$

[c] $C \frac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \quad \text{therefore} \quad \frac{dv_c(0^+)}{dt} = \frac{4}{C} = 4 \times 10^8 \text{ V/s}$

[d] $v = [A_1 e^{-10,000t} + A_2 e^{-40,000t}] \text{ V}, \quad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \quad \frac{dv(0^+)}{dt} = -10,000A_1 - 40,000A_2$$

$$\text{Therefore } A_1 + A_2 = 0, \quad -A_1 - 4A_2 = 40,000; \quad A_1 = 40,000/3 \text{ V}$$

[e] $A_2 = -40,000/3 \text{ V}$

[f] $v = [40,000/3][e^{-10,000t} - e^{-40,000t}] \text{ V}, \quad t \geq 0$

AP 8.4 [a] $\frac{1}{2RC} = 8000, \quad \text{therefore} \quad R = 62.5 \Omega$

[b] $i_R(0^+) = \frac{10 \text{ V}}{62.5 \Omega} = 160 \text{ mA}$

$$i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240 \text{ mA} = C \frac{dv(0^+)}{dt}$$

$$\text{Therefore } \frac{dv(0^+)}{dt} = \frac{-240 \text{ m}}{C} = -240 \text{ kV/s}$$

[c] $B_1 = v(0^+) = 10 \text{ V}, \quad \frac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

$$\text{Therefore } 6000B_2 - 8000B_1 = -240,000, \quad B_2 = (-80/3) \text{ V}$$

[d] $i_L = -(i_R + i_C); \quad i_R = v/R; \quad i_C = C \frac{dv}{dt}$

$$v = e^{-8000t}[10 \cos 6000t - \frac{80}{3} \sin 6000t] \text{ V}$$

$$\text{Therefore } i_R = e^{-8000t}[160 \cos 6000t - \frac{1280}{3} \sin 6000t] \text{ mA}$$

$$i_C = e^{-8000t}[-240 \cos 6000t + \frac{460}{3} \sin 6000t] \text{ mA}$$

$$i_L = 10e^{-8000t}[8 \cos 6000t + \frac{82}{3} \sin 6000t] \text{ mA}, \quad t \geq 0$$

AP 8.5 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = \frac{10^6}{4}, \quad \text{therefore} \quad \frac{1}{2RC} = 500, \quad R = 100 \Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3}, \quad \text{therefore} \quad V_0 = 50 \text{ V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3}, \quad I_0 = 250 \text{ mA}$

[d] $D_2 = v(0^+) = 50, \quad \frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$$i_R(0^+) = \frac{50}{100} = 500 \text{ mA}$$

Therefore $i_C(0^+) = -(500 + 250) = -750 \text{ mA}$

Therefore $\frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75,000 \text{ V/s}$

Therefore $D_1 - \alpha D_2 = -75,000; \quad \alpha = \frac{1}{2RC} = 500, \quad D_1 = -50,000 \text{ V/s}$

[e] $v = [50e^{-500t} - 50,000te^{-500t}] \text{ V}$

$$i_R = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}] \text{ A}, \quad t \geq 0^+$$

AP 8.6 [a] $i_R(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08 \text{ A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58 \text{ A}$

[c] $\frac{di_L(0^+)}{dt} = \frac{V_o}{L} = \frac{40}{0.64} = 62.5 \text{ A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000; \quad \frac{1}{LC} = 1,562,500; \quad s_{1,2} = -1000 \pm j750 \text{ rad/s}$

[e] $i_L = i_f + B'_1 e^{-\alpha t} \cos \omega_d t + B'_2 e^{-\alpha t} \sin \omega_d t, \quad i_f = I = -1 \text{ A}$

$$i_L(0^+) = 0.5 = i_f + B'_1, \quad \text{therefore } B'_1 = 1.5 \text{ A}$$

$$\frac{di_L(0^+)}{dt} = 62.5 = -\alpha B'_1 + \omega_d B'_2, \quad \text{therefore } B'_2 = (25/12) \text{ A}$$

Therefore $i_L(t) = -1 + e^{-1000t}[1.5 \cos 750t + (25/12) \sin 750t] \text{ A}, \quad t \geq 0$

[f] $v(t) = \frac{L di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3) \sin 750t] \text{ V} \quad t \geq 0$

AP 8.7 [a] $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\frac{15 \text{ k}}{15 \text{ k} + 9 \text{ k}} \right) (80) = 50 \text{ V}$

[c] $50 + 80i(0^+) + L \frac{di(0^+)}{dt} = 100, \quad \frac{di(0^+)}{dt} = 10,000 \text{ A/s}$

[d] $\alpha = 8000; \quad \frac{1}{LC} = 100 \times 10^6; \quad s_{1,2} = -8000 \pm j6000 \text{ rad/s}$

[e] $i = i_f + e^{-\alpha t}[B'_1 \cos \omega_d t + B'_2 \sin \omega_d t]; \quad i_f = 0, \quad i(0^+) = 0$

$$\text{Therefore } B'_1 = 0; \quad \frac{di(0^+)}{dt} = 10,000 = -\alpha B'_1 + \omega_d B'_2$$

Therefore $B'_2 = 1.67 \text{ A}; \quad i = 1.67e^{-8000t} \sin 6000t \text{ A}, \quad t \geq 0$

$$\text{AP 8.8 } v_c(t) = v_f + e^{-\alpha t} [B'_1 \cos \omega_d t + B'_2 \sin \omega_d t], \quad v_f = 100 \text{ V}$$

$$v_c(0^+) = 50 \text{ V}; \quad \frac{dv_c(0^+)}{dt} = 0; \quad \text{therefore } 50 = 100 + B'_1$$

$$B'_1 = -50 \text{ V}; \quad 0 = -\alpha B'_1 + \omega_d B'_2$$

$$\text{Therefore } B'_2 = \frac{\alpha}{\omega_d} B'_1 = \left(\frac{8000}{6000} \right) (-50) = -66.67 \text{ V}$$

$$\text{Therefore } v_c(t) = 100 - e^{-8000t} [50 \cos 6000t + 66.67 \sin 6000t] \text{ V}, \quad t \geq 0$$

Problems

$$\text{P 8.1 [a]} \quad i_R(0) = \frac{25}{125} = 200 \text{ mA}$$

$$i_L(0) = -300 \text{ mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 300 - 200 = 100 \text{ mA}$$

$$\text{[b]} \quad \alpha = \frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(8 \times 10^{-6})}} = 1000$$

$\alpha^2 < \omega_0^2$ The response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1000^2 - 800^2} = 600$$

$$v = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t$$

$$v(0) = B_1 = 25$$

$$\frac{dv}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} i_C(0)$$

$$\text{So, } -800(25) + 600B_2 = \frac{1}{5 \times 10^{-6}}(0.1) = 20,000$$

$$\therefore B_2 = \frac{20,000 + 800(25)}{600} = 66.67$$

$$v = 25e^{-800t} \cos 600t + 66.67e^{-800t} \sin 600t \text{ V}, \quad t \geq 0$$

$$\begin{aligned}
 [\mathbf{c}] \quad i_C &= C \frac{dv}{dt} \\
 &= 5 \times 10^{-6} [20,000e^{-800t} \cos 600t - 68,333.33e^{-800t} \sin 600t] \\
 &= 100e^{-800t} \cos 600t - 341.67e^{-800t} \sin 600t \text{ mA} \\
 i_R &= \frac{v}{R} = 200e^{-800t} \cos 600t + 533.36e^{-800t} \sin 600t \text{ mA} \\
 i_L &= -i_C - i_R = -300e^{-800t} \cos 600t - 191.7e^{-800t} \sin 600t \text{ mA}, \quad t \geq 0
 \end{aligned}$$

$$\text{P 8.2} \quad \frac{1}{2RC} = \frac{1}{2(100)(5 \times 10^{-6})} = 1000$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 = \omega_0^2$ So the response is critically damped

$$v(t) = D_1 te^{-1000t} + D_2 e^{-1000t}$$

$$v(0^+) = 25 \text{ V} = D_2$$

$$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\text{So, } D_1 - 1000(25) = \frac{1}{5 \times 10^{-6}} \left(0.3 - \frac{25}{100} \right)$$

$$\therefore D_1 = 35,000$$

$$v(t) = 35,000e^{-1000t} + 25e^{-1000t} \text{ V}, \quad t \geq 0$$

$$\text{P 8.3} \quad \alpha = \frac{1}{2RC} = \frac{1}{2(80)(5 \times 10^{-6})} = 1250$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 > \omega_0^2$ So the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v(t) = A_1 e^{-500t} + A_2 e^{-2000t}$$

$$v(0) = A_1 + A_2 = 25$$

$$\frac{dv}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right) = \frac{1}{5 \times 10^{-6}} \left(0.3 - \frac{25}{80} \right) = -2500$$

$$\text{Solving, } A_1 = 31.67, \quad A_2 = -6.67$$

$$v(t) = 31.67e^{-500t} - 6.67e^{-2000t} \text{ V}, \quad t \geq 0$$

P 8.4 [a] $\alpha = \frac{1}{2RC} = \frac{10^{12}}{(4000)(10)} = 25,000$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25,000 \pm \sqrt{625 \times 10^6 - 400 \times 10^6} = -25,000 \pm 15,000$$

$$s_1 = -10,000 \text{ rad/s}$$

$$s_2 = -40,000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \alpha^2 = \omega_o^2 - \omega_d^2 = 4 \times 10^8 - 144 \times 10^6 = 256 \times 10^6$$

$$\alpha = 16 \times 10^3 = 16,000$$

$$\frac{1}{2RC} = 16,000; \quad \therefore R = \frac{10^9}{(32,000)(10)} = 3125 \Omega$$

[d] $s_1 = -16,000 + j12,000 \text{ rad/s}; \quad s_2 = -16,000 - j12,000 \text{ rad/s}$

[e] $\alpha = 4 \times 10^4 = \frac{1}{2RC}; \quad \therefore R = \frac{1}{2C(4 \times 10^4)} = 2500 \Omega$

P 8.5 [a] $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -1000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -4000$$

$$\text{Adding the above equations, } -2\alpha = -5000$$

$$\alpha = 2500 \text{ rad/s}$$

$$-2500 \pm \sqrt{2500^2 - \omega_0^2} = -1000 \quad \text{so} \quad \sqrt{2500^2 - \omega_0^2} = 1500$$

$$\therefore -\omega_0^2 = 1500^2 - 2500^2 \quad \text{thus} \quad \omega_0 = 2000$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.01)C} = 2000^2 \quad \text{so} \quad C = \frac{1}{(0.01)2000^2} = 25 \mu\text{F}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2R(25 \times 10^{-6})} = 2500 \quad \text{so} \quad R = \frac{1}{2(25 \times 10^{-6})(2500)} = 8 \Omega$$

$$[\mathbf{b}] \quad i_R = \frac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t} \text{ A}, \quad t \geq 0^+$$

$$i_C = C \frac{dv(t)}{dt} = 9e^{-4000t} - e^{-1000t} \text{ A}, \quad t \geq 0^+$$

$$i_L = -(i_R + i_C) = 2.25e^{-4000t} - 4e^{-1000t} \text{ A}, \quad t \geq 0$$

P 8.6 [a] $\alpha = 400$; $\omega_d = 300$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^4 + 16 \times 10^4 = 25 \times 10^4$$

$$\frac{1}{LC} = 25 \times 10^4$$

$$L = \frac{1}{(25 \times 10^4)(250 \times 10^{-6})} = 16 \text{ mH}$$

$$[\mathbf{b}] \quad \alpha = \frac{1}{2RC}$$

$$\therefore R = \frac{1}{2\alpha C} = \frac{1}{(800)(250 \times 10^{-6})} = 5 \Omega$$

$$[\mathbf{c}] \quad V_o = v(0) = 120 \text{ V}$$

$$[\mathbf{d}] \quad I_o = i_L(0) = -i_R(0) - i_C(0)$$

$$i_R(0) = \frac{120}{5} = 24 \text{ A}$$

$$i_C(0) = C \frac{dv}{dt}(0) = 250 \times 10^{-6} [-400(120) + 300(80)] = -6 \text{ A}$$

$$\therefore I_o = -24 + 6 = -18 \text{ A}$$

$$[\mathbf{e}] \quad i_C(t) = 250 \times 10^{-6} \frac{dv(t)}{dt} = e^{-400t} (-17 \sin 300t - 6 \cos 300t) \text{ A}$$

$$i_R(t) = \frac{v(t)}{5} = e^{-400t} (24 \cos 300t + 16 \sin 300t) \text{ A}$$

$$i_L(t) = -i_R(t) - i_C(t)$$

$$= e^{-400t} (-18 \cos 300t + \sin 300t) \text{ A}, \quad t \geq 0$$

Check:

$$L \frac{di_L}{dt} = 16 \times 10^{-3} e^{-400t} [7500 \cos 300t + 5000 \sin 300t]$$

$$v(t) = e^{-400t} [120 \cos 300t + 80 \sin 300t] \text{ V}$$

$$\text{P 8.7} \quad [\text{a}] \quad \left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (80)^2$$

$$\therefore C = \frac{1}{2(50)(80)} = 125 \mu\text{F}$$

$$\frac{1}{LC} = 80^2$$

$$\therefore L = \frac{1}{80^2(125 \times 10^{-6})} = 1.25 \text{ H}$$

$$v(0) = D_2 = 5 \text{ V}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore D_1 - 80(5) = \frac{1}{125 \times 10^{-6}} \left(0.025 - \frac{5}{50} \right) \quad \text{so} \quad D_1 = -200$$

$$[\text{b}] \quad v = -200te^{-80t} + 5e^{-80t} \text{ V}, \quad t \geq 0$$

$$\frac{dv}{dt} = [16,000t - 600]e^{-80t}$$

$$i_C = C \frac{dv}{dt} = (2000t - 75)e^{-80t} \text{ mA}, \quad t \geq 0^+$$

$$\text{P 8.8} \quad [\text{a}] \quad \omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20 \times 10^{-3})(500 \times 10^{-9})}} = 10,000$$

$$\alpha = \frac{1}{2RC} = 10,000; \quad R = \frac{1}{2(10,000)(500 \times 10^{-9})} = 100 \Omega$$

$$[\text{b}] \quad v(t) = D_1 te^{-10,000t} + D_2 e^{-10,000t}$$

$$v(0) = 40 \text{ V} = D_2$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore D_1 - 10,000D_2 = \frac{1}{500 \times 10^{-9}} \left(-0.12 - \frac{40}{100} \right) \quad \text{so} \quad D_1 = 640,000$$

$$\therefore v(t) = (40 - 640,000t)e^{-10,000t} \text{ V}, \quad t \geq 0$$

$$[\text{c}] \quad i_C(t) = 0 \text{ when } \frac{dv}{dt}(t) = 0$$

$$\frac{dv}{dt} = (64 \times 10^8 t - 1040 \times 10^3)e^{-10,000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 640 \times 10^8 t_1 = 1040 \times 10^3; \quad \therefore t_1 = 162.5 \mu\text{s}$$

$$v(162.5 \mu\text{s}) = (-64)e^{-1.625} = -12.6 \text{ V}$$

$$[\mathbf{d}] \quad w(0) = \frac{1}{2}(500 \times 10^{-9})(40)^2 + \frac{1}{2}(0.02)(0.012)^2 = 544 \mu\text{J}$$

$$w(162.5 \mu\text{s}) = \frac{1}{2}(500 \times 10^{-9})(12.6)^2 + \frac{1}{2}(0.02) \left(\frac{12.6}{100}\right)^2 = 198.5 \mu\text{J}$$

$$\% \text{ remaining} = \frac{198.5}{544}(100) = 36.5\%$$

$$\text{P 8.9} \quad \alpha = 500/2 = 250$$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(500)(18)} = 1000 \Omega$$

$$v(0^+) = -11 + 20 = 9 \text{ V}$$

$$i_R(0^+) = \frac{9}{1000} = 9 \text{ mA}$$

$$\frac{dv}{dt} = 1100e^{-100t} - 8000e^{-400t}$$

$$\frac{dv(0^+)}{dt} = 1100 - 8000 = -6900 \text{ V/s}$$

$$i_C(0^+) = 2 \times 10^{-6}(-6900) = -13.8 \text{ mA}$$

$$i_L(0^+) = -[i_R(0^+) + i_C(0^+)] = -[9 - 13.8] = 4.8 \text{ mA}$$

$$\text{P 8.10} \quad [\mathbf{a}] \quad \alpha = \frac{1}{2RC} = 1250, \quad \omega_o = 10^3, \quad \text{therefore overdamped}$$

$$s_1 = -500, \quad s_2 = -2000$$

$$\text{therefore } v = A_1 e^{-500t} + A_2 e^{-2000t}$$

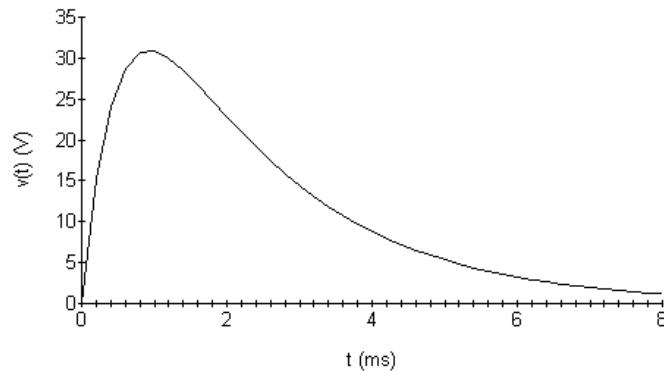
$$v(0^+) = 0 = A_1 + A_2; \quad \left[\frac{dv(0^+)}{dt} \right] = \frac{i_C(0^+)}{C} = 98,000 \text{ V/s}$$

$$\text{Therefore } -500A_1 - 2000A_2 = 98,000$$

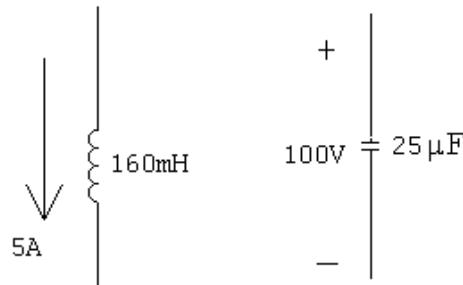
$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15} \right] [e^{-500t} - e^{-2000t}] \text{ V}, \quad t \geq 0$$

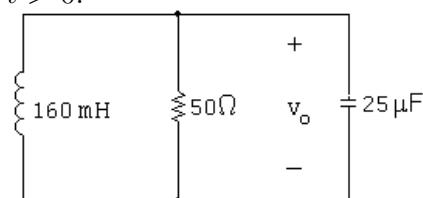
[b]

Example 8.4: $v_{\max} \cong 74.1 \text{ V}$ at 1.4 msExample 8.5: $v_{\max} \cong 36.1 \text{ V}$ at 1.0 msProblem 8.17: $v_{\max} \cong 30.9 \text{ V}$ at 0.92 ms

P 8.11 $t < 0 :$ $V_o = 100 \text{ V}, I_o = 5 \text{ A}$



$t > 0:$



$$\alpha = \frac{1}{2RC} = \frac{1}{2(50)(25 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$$

$\alpha^2 < \omega_0^2$ Response is underdamped

$$\omega_d = \sqrt{500^2 - 400^2} = 300$$

$$\therefore v_o = B_1 e^{-400t} \cos 300t + B_2 e^{-400t} \sin 300t$$

$$v_o(0) = B_1 = 100$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore -(400)(100) + 300B_2 = \frac{1}{25 \times 10^{-6}} \left(-5 - \frac{100}{50} \right) \quad \text{so} \quad B_2 = -800$$

$$\therefore v_o = 100e^{-400t} \cos 300t - 800e^{-400t} \sin 300t \text{ V}, \quad t \geq 0$$

P 8.12 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25 \times 10^{-6})} = 500$$

$$\alpha^2 = \omega_0^2 \quad \text{Response is critically damped}$$

$$v_o(t) = D_1 te^{-500t} = D_2 e^{-500t}$$

$$v_o(0) = D_2 = 100 \text{ V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore D_1 - 500(100) = \frac{1}{25 \times 10^{-6}} \left(-5 - \frac{100}{40} \right) \quad \text{so} \quad D_1 = -250,000$$

$$v_o(t) = -250,000te^{-500t} + 100e^{-500t} \text{ V}, \quad t \geq 0$$

P 8.13 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(32)(25 \times 10^{-6})} = 625$$

$$\therefore \alpha^2 > \omega_o^2 \quad (\text{overdamped})$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000$$

$$v_o(t) = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$v_o(0) = A_1 + A_2 = 100 \text{ V}$$

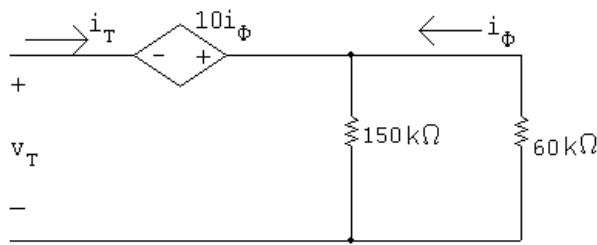
$$\frac{dv_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore -250A_1 - 1000A_2 = \frac{1}{25 \times 10^{-6}} \left(-5 - \frac{100}{32} \right) = -325,000$$

$$\text{Solving, } A_1 = -300, \quad A_2 = 400$$

$$v_o(t) = -300e^{-250t} + 400e^{-1000t} \text{ V}, \quad t \geq 0$$

P 8.14



$$v_T = -10i_\phi + i_T \left(\frac{(150)(60)}{210} \right) = -10 \frac{-i_T(150)}{210} + i_t \frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50 \Omega$$

$$V_o = \frac{4000}{10,000}(50) = 20 \text{ V}; \quad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{20}{50} = -0.4 \text{ A}$$

$$\frac{i_C(0)}{C} = \frac{-0.4}{8 \times 10^{-6}} = -50,000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2 \times 10^{-3})(8 \times 10^{-6})}} = 1562.5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

$$v_o = B_1 e^{-1250t} \cos 937.5t + B_2 e^{-1250t} \sin 937.5t$$

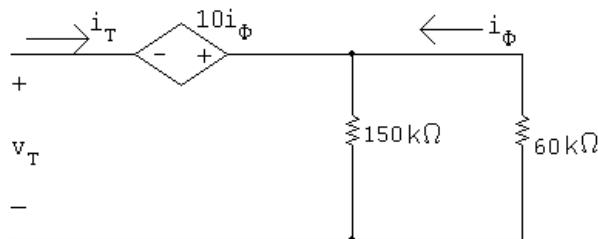
$$v_o(0) = B_1 20 \text{ V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C}$$

$$\therefore -1250(20) + 937.5 B_2 = -50,000 \quad \text{so} \quad B_2 = -26.67$$

$$v_o = 20e^{-1250t} \cos 937.5t - 26.67e^{-1250t} \sin 937.5t \text{ V}, \quad t \geq 0$$

P 8.15



$$v_T = -10i_\phi + i_T \left(\frac{(150)(60)}{210} \right) = -10 \frac{-i_T(150)}{210} + i_t \frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50 \Omega$$

$$V_o = \frac{4000}{10,000}(50) = 20 \text{ V}; \quad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80 \times 10^{-3})(8 \times 10^{-6})}} = 1250 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ so the response is critically damped

$$v_o = D_1 t e^{-1250t} + D_2 e^{-1250t}$$

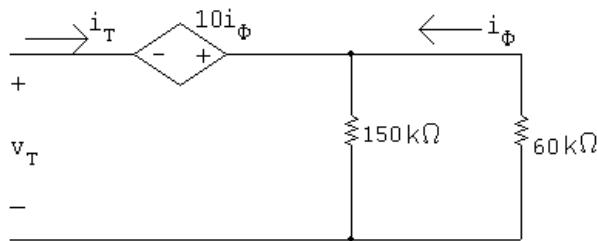
$$v_o(0) = D_2 = 20 \text{ V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore D_1 - 1250(20) = \frac{1}{8 \times 10^{-6}} \left(0 - \frac{20}{50} \right) \quad \text{so} \quad D_1 = -25,000$$

$$v_o = -25,000 t e^{-1250t} + 20 e^{-1250t} \text{ V}, \quad t \geq 0$$

P 8.16



$$v_T = -10i_\phi + i_T \left(\frac{(150)(60)}{210} \right) = -10 \frac{-i_T(150)}{210} + i_t \frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50 \Omega$$

$$V_o = \frac{4000}{10,000}(50) = 20 \text{ V}; \quad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(125 \times 10^{-3})(8 \times 10^{-6})}} = 1000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v_o = A_1 e^{-500t} + A_2 e^{-1000t}$$

$$v_o(0) = A_1 + A_2 = 20 \text{ V}$$

$$\frac{dv_o}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C} \left(-I_0 - \frac{V_0}{R} \right)$$

$$\therefore -500A_1 - 2000A_2 = \frac{1}{8 \times 10^{-6}} \left(0 - \frac{20}{50} \right) = -50,000$$

Solving, $A_1 = -6.67$, $A_2 = 26.67$

$$v_o = -6.67e^{-500t} + 26.67e^{-2000t} \text{ V}, \quad t \geq 0$$

P 8.17 [a] $\frac{1}{LC} = 5000^2$

There are many possible solutions. This one begins by choosing $L = 10 \text{ mH}$. Then,

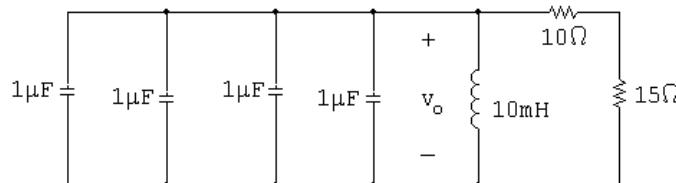
$$C = \frac{1}{(10 \times 10^{-3})(5000)^2} = 4 \mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $1 \mu\text{F}$ capacitors in parallel.

Critically damped: $\alpha = \omega_0 = 5000$ so $\frac{1}{2RC} = 5000$

$$\therefore R = \frac{1}{2(4 \times 10^{-6})(5000)} = 25 \Omega$$

We can create this resistor value using components from Appendix H by combining a 10Ω resistor and a 15Ω resistor in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000 \pm 0$

Therefore there are two repeated real roots at -5000 rad/s .

P 8.18 [a] Underdamped response:

$$\alpha < \omega_0 \quad \text{so} \quad \alpha < 5000$$

Therefore we choose a larger resistor value than the one used in Problem 8.17. Choose $R = 100 \Omega$:

$$\alpha = \frac{1}{2(100)(4 \times 10^{-6})} = 1250$$

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 5000^2} = -1250 \pm j4841.23 \text{ rad/s}$$

[b] Overdamped response:

$$\alpha > \omega_0 \quad \text{so} \quad \alpha > 5000$$

Therefore we choose a smaller resistor value than the one used in Problem 8.17. Choose $R = 20 \Omega$:

$$\alpha = \frac{1}{2(20)(4 \times 10^{-6})} = 6250$$

$$s_{1,2} = -1250 \pm \sqrt{6250^2 - 5000^2} = -1250 \pm 3750$$

$$= -2500 \text{ rad/s; and } -10,000 \text{ rad/s}$$

$$\text{P 8.19 [a]} \quad \alpha = \frac{1}{2RC} = 800 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = 10^6$$

$$\omega_d = \sqrt{10^6 - 800^2} = 600 \text{ rad/s}$$

$$\therefore v = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t$$

$$v(0) = B_1 = 30$$

$$i_R(0^+) = \frac{30}{5000} = 6 \text{ mA}; \quad i_C(0^+) = -12 \text{ mA}$$

$$\therefore \frac{dv}{dt}(0^+) = \frac{-0.012}{125 \times 10^{-9}} = -96,000 \text{ V/s}$$

$$-96,000 = -\alpha B_1 + \omega_d B_2 = -(800)(30) + 600B_2$$

$$\therefore B_2 = -120$$

$$\therefore v = 30e^{-800t} \cos 600t - 120e^{-800t} \sin 600t \text{ V}, \quad t \geq 0$$

$$\text{[b]} \quad \frac{dv}{dt} = 6000e^{-800t}(13 \sin 600t - 16 \cos 600t)$$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 16 \cos 600t = 13 \sin 600t \quad \text{or} \quad \tan 600t = \frac{16}{13}$$

$$\therefore 600t_1 = 0.8885, \quad t_1 = 1.48 \text{ ms}$$

$$600t_2 = 0.8885 + \pi, \quad t_2 = 6.72 \text{ ms}$$

$$600t_3 = 0.8885 + 2\pi, \quad t_3 = 11.95 \text{ ms}$$

$$\text{[c]} \quad t_3 - t_1 = 10.47 \text{ ms}; \quad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{600} = 10.47 \text{ ms}$$

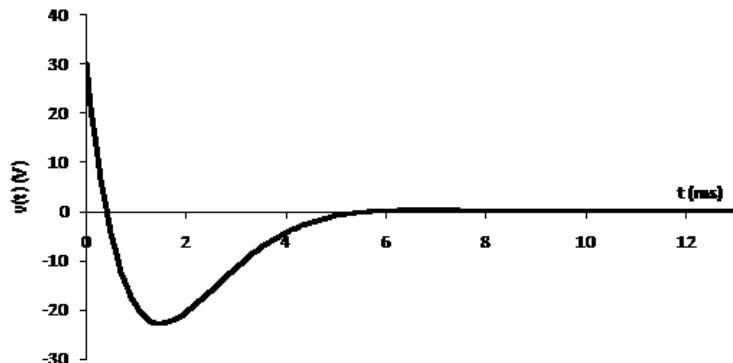
$$\text{[d]} \quad t_2 - t_1 = 5.24 \text{ ms}; \quad \frac{T_d}{2} = \frac{10.48}{2} = 5.24 \text{ ms}$$

$$\text{[e]} \quad v(t_1) = 30e^{-(1.184)}(\cos 0.8885 - 4 \sin 0.8885) = -22.7 \text{ V}$$

$$v(t_2) = 30e^{-(5.376)}(\cos 4.032 - 4 \sin 4.032) = 0.334 \text{ V}$$

$$v(t_3) = 30e^{-(9.56)}(\cos 7.17 - 4 \sin 7.17) = -5.22 \text{ mV}$$

[f]



P 8.20 [a] $\alpha = 0$; $\omega_d = \omega_o = \sqrt{10^6} = 1000 \text{ rad/s}$

$$v = B_1 \cos \omega_o t + B_2 \sin \omega_o t; \quad v(0) = B_1 = 30$$

$$C \frac{dv}{dt}(0) = -i_L(0) = -0.006$$

$$-48,000 = -\alpha B_1 + \omega_d B_2 = -0 + 1000 B_2$$

$$\therefore B_2 = \frac{-48,000}{1000} = -48 \text{ V}$$

$$v = 30 \cos 1000t - 48 \sin 1000t \text{ V}, \quad t \geq 0$$

[b] $2\pi f = 1000; \quad f = \frac{1000}{2\pi} \cong 159.15 \text{ Hz}$

[c] $\sqrt{30^2 + 48^2} = 56.6 \text{ V}$

P 8.21 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2 , respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinant is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinants are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

$$\text{and } N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$$

$$\text{It follows that } A_1 = \frac{N_1}{\Delta} = \frac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$$

$$\text{and } A_2 = \frac{N_2}{\Delta} = \frac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$$

We see from these expressions that $A_1 = A_2^*$.

- P 8.22 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.21 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.12 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but } K_2 = \frac{dv(0^+)}{dt} \quad \text{and } K_1 = B_1.$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

- P 8.23 [a] $2\alpha = 1000$; $\alpha = 500 \text{ rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \quad \omega_o = 400 \text{ rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4 \mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2 (4 \times 10^{-6})} = 1.5625 \text{ H}$$

$$i_C(0^+) = A_1 + A_2 = 45 \text{ mA}$$

$$\frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt} = 0$$

$$\frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{1.5625} = 0 \text{ A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R} \frac{dv(0)}{dt} = \frac{1}{R} \frac{i_C(0)}{C} = \frac{45 \times 10^{-3}}{(250)(4 \times 10^{-6})} = 45 \text{ A/s}$$

$$\therefore \frac{di_C(0)}{dt} = 0 - 45 = -45 \text{ A/s}$$

$$\therefore 200A_1 + 800A_2 = 45; \quad A_1 + A_2 = 0.045$$

$$\text{Solving, } A_1 = -15 \text{ mA}; \quad A_2 = 60 \text{ mA}$$

$$\therefore i_C = -15e^{-200t} + 60e^{-800t} \text{ mA}, \quad t \geq 0^+$$

[b] By hypothesis

$$v = A_3 e^{-200t} + A_4 e^{-800t}, \quad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11,250 \text{ V/s}$$

$$-200A_3 - 800A_4 = 11,250; \quad \therefore A_3 = 18.75 \text{ V}; \quad A_4 = -18.75 \text{ V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t} \text{ V}, \quad t \geq 0$$

$$[c] i_R(t) = \frac{v}{250} = 75e^{-200t} - 75e^{-800t} \text{ mA}, \quad t \geq 0^+$$

[d] $i_L = -i_R - i_C$

$$i_L = -60e^{-200t} + 15e^{-800t} \text{ mA}, \quad t \geq 0$$

$$\text{P 8.24 [a]} v = L \left(\frac{di_L}{dt} \right) = 16[e^{-20,000t} - e^{-80,000t}] \text{ V}, \quad t \geq 0$$

$$[b] i_R = \frac{v}{R} = 40[e^{-20,000t} - e^{-80,000t}] \text{ mA}, \quad t \geq 0^+$$

$$[c] i_C = I - i_L - i_R = [-8e^{-20,000t} + 32e^{-80,000t}] \text{ mA}, \quad t \geq 0^+$$

$$\text{P 8.25 [a]} \quad v = L \left(\frac{di_L}{dt} \right) = 40e^{-32,000t} \sin 24,000t \text{ V}, \quad t \geq 0$$

$$\begin{aligned} \text{[b]} \quad i_C(t) &= I - i_R - i_L = 24 \times 10^{-3} - \frac{v}{625} - i_L \\ &= [24e^{-32,000t} \cos 24,000t - 32e^{-32,000t} \sin 24,000t] \text{ mA}, \quad t \geq 0^+ \end{aligned}$$

$$\text{P 8.26} \quad v = L \left(\frac{di_L}{dt} \right) = 960,000t e^{-40,000t} \text{ V}, \quad t \geq 0$$

$$\text{P 8.27} \quad \omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(12.5)(62.5 \times 10^{-6})} = 640 \text{ rad/s} \quad \therefore \text{ underdamped}$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$I_f = 2 \text{ A}$$

$$i_L = 2 + B'_1 e^{-640t} \cos 480t + B'_2 e^{-640t} \sin 480t$$

$$i_L(0) = 2 + B'_1 = 1 \quad \text{so} \quad B'_1 = -1$$

$$\frac{di_L}{dt}(0) = -\alpha B'_1 + \omega_d B'_2 = \frac{V_0}{L}$$

$$\therefore -640(-1) + 480B'_2 = \frac{50}{25 \times 10^{-3}} \quad \text{so} \quad B'_2 = 2.83$$

$$i_L(t) = 2 - e^{-640t} \cos 480t + 2.83e^{-640t} \sin 480t \text{ A}, \quad t \geq 0$$

$$\text{P 8.28} \quad \alpha = \frac{1}{2RC} = \frac{1}{2(8)(62.5 \times 10^{-6})} = 1000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

$$\text{Overdamped: } s_{1,2} = -1000 \pm \sqrt{1000^2 - 800^2} = -400, -1600 \text{ rad/s}$$

$$I_f = 2 \text{ A}$$

$$i_L = 2 + A'_1 e^{-400t} + A'_2 e^{-1600t}$$

$$i_L(0) = 2 + A'_1 + A'_2 = 1 \quad \text{so} \quad A'_1 + A'_2 = -1$$

$$\frac{di_L}{dt}(0) = -400A'_1 - 1600A'_2 = \frac{V_0}{L} = \frac{50}{25 \times 10^{-3}} = 2000$$

$$\text{Solving, } A'_1 = \frac{1}{3}, \quad A'_2 = -\frac{4}{3}$$

$$i_L(t) = 2 + \frac{1}{3}e^{-400t} - \frac{4}{3}e^{-1600t} \text{ A}, \quad t \geq 0$$

$$\text{P 8.29} \quad \alpha = \frac{1}{2RC} = \frac{1}{2(10)(62.5 \times 10^{-6})} = 800$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

$$\alpha^2 = \omega_0^2 \quad \text{Critically damped}$$

$$I_f = 2 \text{ A}$$

$$i_L = 2 + D'_1 te^{-800t} + D'_2 e^{-800t}$$

$$i_L(0) = 2 + D'_2 = 1; \quad \therefore D'_2 = -1 \text{ A}$$

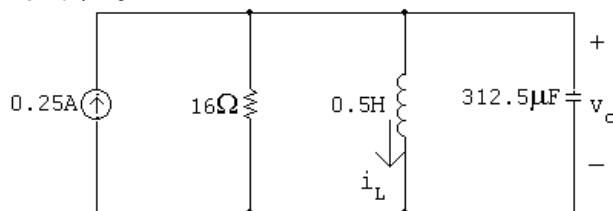
$$\frac{di_L}{dt}(0) = D'_1 - \alpha D'_2 = \frac{V_0}{L}$$

$$\therefore D'_1 - 800(-1) = \frac{50}{25 \times 10^{-3}} \quad \text{so} \quad D'_1 = 1200$$

$$i_L = 2 + 1200te^{-800t} - e^{-800t} \text{ A}, \quad t \geq 0$$

$$\text{P 8.30} \quad i_L(0^-) = i_L(0^+) = \frac{4}{16} = 0.25 \text{ A}$$

For $t > 0$



$$\alpha = \frac{1}{2RC} = 100 \text{ rad/s}; \quad \omega_o^2 = \frac{1}{LC} = 80^2 \quad \text{so} \quad \omega_o = 80 \text{ rad/s}$$

$$s_{1,2} = -100 \pm \sqrt{100^2 - 80^2} = -40, -160 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f; \quad A'_1 + A'_2 = v(0) = 0$$

$$v_o = A'_1 e^{-40t} + A'_2 e^{-160t}$$

$$i_C(0^+) = -0.25 + 0.25 + 0 = 0$$

$$\therefore \frac{dv_o}{dt} = 0 = -40A'_1 - 160A'_2$$

$$\text{Solving, } A'_1 = 0; \quad A'_2 = 0$$

$$\therefore v_o = 0 \text{ for } t \geq 0$$

$$\text{Note: } v_o(0) = 0; \quad v_o(\infty) = 0; \quad \frac{dv_o(0)}{dt} = 0$$

Hence, the 0.25 A current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit, the 4 V source sustains a current of 0.25 A in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

$$\text{P 8.31 } \alpha = \frac{1}{2RC} = \frac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$$\alpha^2 = \omega_0^2 \quad \text{Critically damped}$$

$$V_0 = v_C(0) = 60 \text{ V}; \quad I_0 = i_o(0) = 0; \quad I_f = i_o(\infty) = \frac{60}{20} = 3 \text{ A}$$

$$i_o = 3 + D'_1 te^{-800t} + D'_2 e^{-800t}$$

$$i_o(0) = 3 + D'_2 = 0 \quad \text{so} \quad D'_2 = -3$$

$$\frac{di_o}{dt}(0) = D'_1 - \alpha D'_2 = \frac{V_0}{L} \quad \text{so} \quad D'_1 - 800(-3) = \frac{60}{50 \times 10^{-3}}$$

$$\text{Solving, } D'_1 = -1200$$

$$i_o(t) = 3 - 1200te^{-800t} - 3e^{-800t} \text{ A}, \quad t \geq 0$$

P 8.32 [a] $\alpha = \frac{1}{2RC} = \frac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ Critically damped

$$V_f = v_C(\infty) = 0$$

$$v_o = D'_1 te^{-800t} + D'_2 e^{-800t}$$

$$v_o(0) = D'_2 = 60$$

$$\frac{v_o}{dt}(0) = D'_1 - \alpha D'_2 = 0 \quad \text{so} \quad D'_1 = (800)(60) = 48,000$$

$$\therefore v_o(t) = 48,000te^{-800t} + 60e^{-800t} \text{ V}, \quad t \geq 0$$

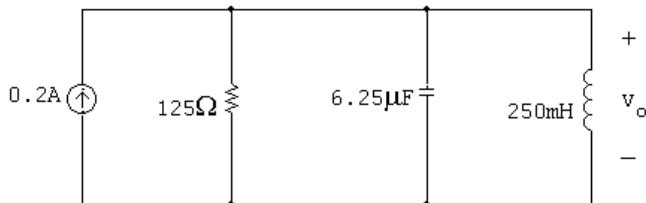
[b] $v_o = L \frac{di_o}{dt} = (50 \times 10^{-3}) \frac{d}{dt} (3 - 1200te^{-800t} - 3e^{-800t})$

$$v_o(t) = (50 \times 10^{-3})(960,000te^{-800t} + 1200e^{-800t})$$

$$= 48,000te^{-800t} + 60e^{-800t} \text{ V}, \quad t \geq 0$$

Thus the solutions for $i_o(t)$ and $v_o(t)$ agree.

P 8.33 For $t > 0$



$$\alpha = \frac{1}{2RC} = 640; \quad \frac{1}{LC} = 64 \times 10^4$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$i_o = I_f + B'_1 e^{-640t} \cos 480t + B'_2 e^{-640t} \sin 480t$$

$$I_f = \frac{25}{125} = 0.2 \text{ A}$$

$$i_o(0) = 0.2 + B'_1 = 0 \quad \text{so} \quad B'_1 = -0.2$$

$$\frac{di_o}{dt}(0) = -\alpha B'_1 + \omega_d B'_2 = \frac{V_0}{L} \quad \text{so} \quad -640(-0.2) + 480B'_2 = 0$$

$$\text{Solving, } B'_2 = -0.267$$

$$i_o(t) = 0.2 - 0.2e^{-640t} \cos 480t - 0.267e^{-640t} \sin 480t \text{ A}, \quad t \geq 0$$

$$P\ 8.34 \quad [a] \quad \alpha = \frac{1}{2RC} = 640; \quad \frac{1}{LC} = 64 \times 10^4$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$v_o = V_f + B'_1 e^{-640t} \cos 480t + B'_2 e^{-640t} \sin 480t$$

$$V_f = 0; \quad V_o(0^+) = 0; \quad i_C(0^+) = 0.2 \text{ A}$$

$$v_o(0) = 0 + B'_1 = 0 \quad \text{so} \quad B'_1 = 0$$

$$\frac{dv_o}{dt}(0) = -640(0) + 480B'_2 = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 32,000 \quad \text{so} \quad B'_2 = 66.67$$

$$\therefore v_o(t) = 66.67e^{-640t} \sin 480t \text{ V}, \quad t \geq 0$$

[b] From the solution to Problem 8.33,

$$i_o(t) = 0.2 - 0.2e^{-640t} \cos 480t - 0.267e^{-640t} \sin 480t \text{ A}$$

$$v_o = L \frac{di_o}{dt} = (0.25)(266.67e^{-640t} \sin 480t) = 66.67e^{-640t} \sin 480t \text{ V}, \quad t \geq 0$$

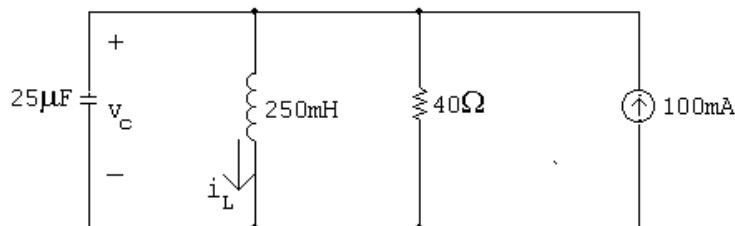
Thus the solutions to Problems 8.33 and 8.34 are consistent.

P 8.35 $t < 0$:

$$V_0 = v_o(0^-) = v_o(0^+) = \frac{3000}{4000}(100) = 75 \text{ V}$$

$$I_0 = i_L(0^-) = i_L(0^+) = 100 \text{ mA}$$

$t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25 \times 10^{-6})} = 500 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(25 \times 10^{-6})}} = 400$$

$$\therefore \alpha^2 > \omega_o^2 \quad \text{overdamped}$$

$$s_{1,2} = -500 \pm \sqrt{500^2 - 400^2} = -200, -800$$

[a] $i_L = I_f + A_1 e^{-200t} + A_2 e^{-800t}$

$$I_f = 100 \text{ mA}$$

$$i_L(0) = 0.1 + A_1 + A_2 = 0.1 \quad \text{so} \quad A_1 + A_2 = 0$$

$$\frac{di_L}{dt}(0) = -200A_1 - 800A_2 = \frac{V_0}{L} = \frac{75}{0.25} = 300$$

$$\text{Solving, } A_1 = 0.5, \quad A_2 = -0.5$$

$$\therefore i_L(t) = 0.1 + 0.5e^{-200t} - 0.5e^{-800t} \text{ A}$$

[b] $v_C(t) = v_L(t) = L \frac{di_L}{dt} = (0.25)(-100e^{-200t} + 400e^{-800t})$

$$= -25e^{-200t} + 100e^{-800t} \text{ V}, \quad t \geq 0$$

P 8.36 [a] $w_L = \int_0^\infty pdt = \int_0^\infty v_o i_L dt$

$$v_o = -25e^{-200t} + 100e^{-800t} \text{ V}$$

$$i_L = 0.1 + 0.5e^{-200t} - 0.5e^{-800t} \text{ A}$$

$$p = -2.5e^{-200t} - 12.5e^{-400t} + 10e^{-800t} + 62.5e^{-1000t} - 50e^{-1600t} \text{ W}$$

$$w_L = -2.5 \int_0^\infty e^{-200t} dt - 12.5 \int_0^\infty e^{-400t} dt + 10 \int_0^\infty e^{-800t} dt$$

$$+ 62.5 \int_0^\infty e^{-1000t} dt - 50 \int_0^\infty e^{-1600t} dt$$

$$= -2.5 \frac{e^{-200t}}{-200} \Big|_0^\infty + -12.5 \frac{e^{-400t}}{-400} \Big|_0^\infty$$

$$= 10 \frac{e^{-800t}}{-800} \Big|_0^\infty + 62.5 \frac{e^{-1000t}}{-1000} \Big|_0^\infty$$

$$- 50 \frac{e^{-1600t}}{-1600} \Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_L = \frac{-2.5}{200} - \frac{-12.5}{400} + \frac{10}{800} + \frac{62.5}{1000} - \frac{50}{1600} = 0 \text{ J}$$

Since the initial and final values of the current in the inductor are the same, the initial and final values of the energy in the inductor are the same. Thus, there is no net energy delivered to the inductor.

[b] $v_o = -25e^{-200t} + 100e^{-800t}$ V

$$i_R = \frac{v_o}{40} = -0.625e^{-200t} + 2.5e^{-800t}$$
 A

$$p_R = v_o i_R = 15.625e^{-400t} - 125e^{-1000t} + 250e^{-1600t}$$
 W

$$w_R = \int_0^\infty p_R dt$$

$$= 15.625 \int_0^\infty e^{-400t} dt - 125 \int_0^\infty e^{-1000t} dt + 250 \int_0^\infty e^{-1600t} dt$$

$$= 15.625 \frac{e^{-400t}}{-400} \Big|_0^\infty - 125 \frac{e^{-1000t}}{-1000} \Big|_0^\infty + 250 \frac{e^{-1600t}}{-1600} \Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_R = \frac{15.625}{400} - \frac{125}{400} + \frac{250}{1600} = 70.3125$$
 mJ

[c] $0.1 = i_R + i_C + i_L$ (mA)

$$\begin{aligned} i_C &= 0.1 - (-0.625e^{-200t} + 2.5e^{-800t}) - (0.1 + 0.5e^{-200t} - 0.5e^{-800t}) \\ &= 0.125e^{-200t} - 2e^{-800t}$$
 A

$$\begin{aligned} p_C &= v_o i_C = [-25e^{-200t} + 100e^{-800t}][0.125e^{-200t} - 2e^{-800t}] \\ &= -3.125e^{-400t} + 62.5e^{-1000t} - 200e^{-1600t}$$
 W

$$\begin{aligned} w_C &= -3.125 \int_0^\infty e^{-400t} dt + 62.5 \int_0^\infty e^{-1000t} dt - 200 \int_0^\infty e^{-1600t} dt \\ &= -3.125 \frac{e^{-400t}}{-400} \Big|_0^\infty + 62.5 \frac{e^{-1000t}}{-1000} \Big|_0^\infty - 200 \frac{e^{-1600t}}{-1600} \Big|_0^\infty \end{aligned}$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-3.125}{400} + \frac{62.5}{1000} - \frac{200}{1600} = -70.3125$$
 mJ

[d] $i_s = 100$ mA

$$p_s(\text{del}) = 0.1v = -2.5e^{-200t} + 10e^{-800t}$$
 W

$$\begin{aligned} w_s &= -2.5 \int_0^\infty e^{-200t} dt + 10 \int_0^\infty e^{-800t} dt \\ &= -2.5 \frac{e^{-200t}}{-200} \Big|_0^\infty + 10 \frac{e^{-800t}}{-800} \Big|_0^\infty = \frac{-2.5}{200} + \frac{10}{800} = 0 \end{aligned}$$

[e] $w_L = 0 \text{ J}$

$$w_R = 70.3125 \text{ mJ} \quad (\text{absorbed})$$

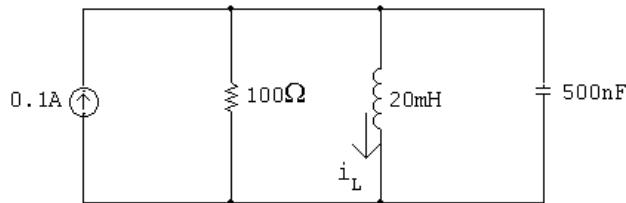
$$w_C = 70.3125 \text{ mJ} \quad (\text{delivered})$$

$$w_S = 0 \text{ mJ}$$

$$\sum w_{\text{del}} = w_{\text{abs}} = 70.3125 \text{ mJ.}$$

P 8.37 $t < 0 : i_L(0^-) = \frac{36}{300} = 0.12 \text{ A}; v_C(0^-) = 0 \text{ V}$

The circuit reduces to:



$$i_L(\infty) = 0.1 \text{ A}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20 \times 10^{-3})(500 \times 10^{-9})}} = 10,000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(100)(500 \times 10^{-9})} = 10,000 \text{ rad/s}$$

Critically damped:

$$i_L = 0.1 + D'_1 t e^{-10,000t} + D'_2 e^{-10,000t}$$

$$i_L(0) = 0.1 + D'_2 = 0.12 \quad \text{so} \quad D'_2 = 0.02$$

$$\frac{di_L}{dt}(0) = D'_1 - \alpha D'_2 = \frac{V_0}{L} \quad \text{so} \quad D'_1 - (10,000)(0.02) = 0$$

$$\text{Solving, } D'_1 = 200$$

$$i_L(t) = 0.1 + 200t e^{-10,000t} + 0.02e^{-10,000t} \text{ A}, \quad t \geq 0$$

$$\text{P 8.38 } v_C(0^+) = \frac{1}{2}(80) = 40 \text{ V}$$

$$i_L(0^+) = 10 \text{ A}; \quad i_L(\infty) = \frac{80}{10} = 8 \text{ A}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)(250 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(16 \times 10^{-3})(250 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2; \quad \therefore \text{ underdamped}$$

$$\omega_d = \sqrt{500^2 - 400^2} = 300 \text{ rad/s}$$

$$i_L = 8 + B'_1 e^{-400t} \cos 300t + B'_2 e^{-400t} \sin 300t$$

$$i_L(0) = 8 + B'_1 = 10; \quad B'_1 = 10 - 8 = 2 \text{ A}$$

$$\frac{di_L}{dt}(0) = -\alpha B'_1 + \omega_d B'_2 = \frac{V_0}{L}$$

$$\therefore -400(2) + 300B'_2 = \frac{40}{0.016} \quad \text{so} \quad B'_2 = 11 \text{ A}$$

$$\therefore i_L = 8 + 2e^{-400t} \cos 300t + 11e^{-400t} \sin 300t \text{ A}, \quad t \geq 0$$

$$\text{P 8.39 } \alpha = 2000 \text{ rad/s}; \quad \omega_d = 1500 \text{ rad/s}$$

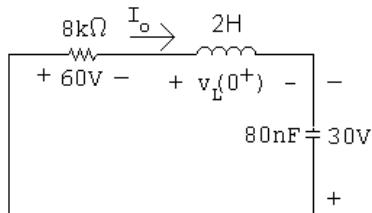
$$\omega_o^2 - \alpha^2 = 225 \times 10^4; \quad \omega_o^2 = 625 \times 10^4; \quad w_o = 25,000 \text{ rad/s}$$

$$\alpha = \frac{R}{2L} = 2000; \quad R = 4000L$$

$$\frac{1}{LC} = 625 \times 10^4; \quad L = \frac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2 \text{ H}$$

$$\therefore R = 8 \text{ k}\Omega$$

$$i(0^+) = B_1 = 7.5 \text{ mA}; \quad \text{at } t = 0^+$$



$$60 + v_L(0^+) - 30 = 0; \quad \therefore v_L(0^+) = -30\text{ V}$$

$$\frac{di(0^+)}{dt} = \frac{-30}{2} = -15\text{ A/s}$$

$$\therefore \frac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$$

$$\therefore 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \quad \therefore B_2 = 0\text{ A}$$

$$\therefore i = 7.5e^{-2000t} \cos 1500t \text{ mA}, \quad t \geq 0$$

P 8.40 From Prob. 8.39 we know v_c will be of the form

$$v_c = B_3 e^{-2000t} \cos 1500t + B_4 e^{-2000t} \sin 1500t$$

From Prob. 8.39 we have

$$v_c(0) = -30\text{ V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = \frac{7.5 \times 10^{-3}}{80 \times 10^{-9}} = 93.75 \times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93,750$$

$$\therefore 1500B_4 = 2000(-30) + 93,750; \quad B_4 = 22.5\text{ V}$$

$$v_c(t) = -30e^{-2000t} \cos 1500t + 22.5e^{-2000t} \sin 1500t \text{ V} \quad t \geq 0$$

P 8.41 [a] $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \quad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16,000$

$$\therefore \alpha = 10,000 \text{ rad/s}, \quad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10,000; \quad R = 20,000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \quad L = \frac{10^9}{64 \times 10^6 (31.25)} = 0.5\text{ H}$$

$$R = 10,000 \Omega$$

[b] $i(0) = 0$

$$L \frac{di(0)}{dt} = v_c(0); \quad \frac{1}{2}(31.25) \times 10^{-9} v_c^2(0) = 9 \times 10^{-6}$$

$$\therefore v_c^2(0) = 576; \quad v_c(0) = 24 \text{ V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48 \text{ A/s}$$

[c] $i(t) = A_1 e^{-4000t} + A_2 e^{-16,000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16,000A_2 = 48$$

Solving,

$$\therefore A_1 = 4 \text{ mA}; \quad A_2 = -4 \text{ mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16,000t} \text{ mA}, \quad t \geq 0$$

[d] $\frac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16,000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16,000t} = 16e^{-4000t}$$

$$\text{or } e^{12,000t} = 4$$

$$\therefore t = \frac{\ln 4}{12,000} = 115.52 \mu\text{s}$$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89 \text{ mA}$

[f] $v_L(t) = 0.5 \frac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}] \text{ V}, \quad t \geq 0^+$

P 8.42 [a] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(125)(0.32)} = 25 \times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000 \text{ rad/s}$$

$$\therefore R = (5000)(2)L = 1250 \Omega$$

[b] $i(0) = i_L(0) = 6 \text{ mA}$

$$v_L(0) = 15 - (0.006)(1250) = 7.5 \text{ V}$$

$$\frac{di}{dt}(0) = \frac{7.5}{0.125} = 60 \text{ A/s}$$

[c] $v_C = D_1 e^{-5000t} + D_2 e^{-5000t}$

$$v_C(0) = D_2 = 15 \text{ V}$$

$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C} = -18,750$$

$$\therefore D_1 = 56,250 \text{ V/s}$$

$$v_C = 56,250t e^{-5000t} + 15e^{-5000t} \text{ V}, \quad t \geq 0$$

P 8.43 [a] $\frac{1}{LC} = 20,000^2$

There are many possible solutions. This one begins by choosing $L = 1 \text{ mH}$. Then,

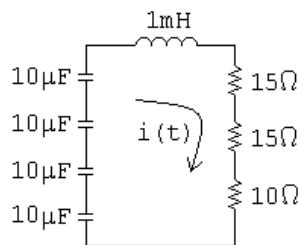
$$C = \frac{1}{(1 \times 10^{-3})(20,000)^2} = 2.5 \mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $10 \mu\text{F}$ capacitors in series.

$$\text{Critically damped: } \alpha = \omega_0 = 20,000 \quad \text{so} \quad \frac{R}{2L} = 20,000$$

$$\therefore R = 2(10^{-3})(20,000) = 40 \Omega$$

We can create this resistor value using components from Appendix H by combining a 10Ω resistor and two 15Ω resistors in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -20,000 \pm 0$

Therefore there are two repeated real roots at $-20,000 \text{ rad/s}$.

P 8.44 [a] Underdamped response:

$$\alpha < \omega_0 \quad \text{so} \quad \alpha < 20,000$$

Therefore we choose a larger resistor value than the one used in Problem 8.40 to give a smaller value of α . For convenience, pick $\alpha = 16,000 \text{ rad/s}$:

$$\alpha = \frac{R}{2L} = 16,000 \quad \text{so} \quad R = 2(16,000)(10^{-3}) = 32 \Omega$$

We can create a 32Ω resistance by combining a 10Ω resistor and a 22Ω resistor in series.

$$s_{1,2} = -16,000 \pm \sqrt{16,000^2 - 20,000^2} = -16,000 \pm j12,000 \text{ rad/s}$$

[b] Overdamped response:

$$\alpha > \omega_0 \quad \text{so} \quad \alpha > 20,000$$

Therefore we choose a smaller resistor value than the one used in Problem 8.40. Choose $R = 50\Omega$, which can be created by combining two 100Ω resistors in parallel:

$$\alpha = \frac{R}{2L} = 25,000$$

$$s_{1,2} = -25,000 \pm \sqrt{25,000^2 - 20,000^2} = -25,000 \pm 15,000$$

$$= -10,000 \text{ rad/s; and } -40,000 \text{ rad/s}$$

P 8.45 [a] $t < 0$:

$$i_o = \frac{100}{50} = 2 \text{ A}; \quad v_o = -4(100) = -400 \text{ V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(0.4)} = 625 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.4)(10 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000 \text{ rad/s}$$

$$i_o = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$i_o(0) = A_1 + A_2 = 2$$

$$\frac{di_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{L}(-V_0 - RI_0) = -1500$$

$$\text{Solving, } A_1 = \frac{2}{3}; \quad A_2 = \frac{4}{3}$$

$$\therefore i_o(t) = \frac{2}{3}e^{-250t} + \frac{4}{3}e^{-1000t} \text{ A}, \quad t \geq 0$$

$$\begin{aligned}
 [\mathbf{b}] \quad v_o(t) &= \frac{1}{10 \times 10^{-6}} \int_0^t i_o(x) dx - 400 \\
 &= 10^5 \left(\int_0^t \frac{2}{3} e^{-250x} dx + \int_0^t \frac{4}{3} e^{-1000x} dx \right) - 400 \\
 &= 10^5 \left(\frac{(2/3)e^{-250x}}{-250} \Big|_0^t + \frac{(4/3)e^{-1000x}}{-1000} \Big|_0^t \right) - 400 \\
 &= -266.67e^{-250t} - 133.33e^{-1000t} \text{ V}, \quad t \geq 0
 \end{aligned}$$

P 8.46 $t < 0$: $I_0 = -75 \text{ mA}$; $V_0 = 0$

$t > 0$:

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80 \times 10^{-3})(200 \times 10^{-6})}} = 250$$

$$\alpha = \frac{R}{2L} = \frac{40}{2(80 \times 10^{-3})} = 250$$

Critically damped:

$$i = D_1 t e^{-250t} + D_2 e^{-250t}$$

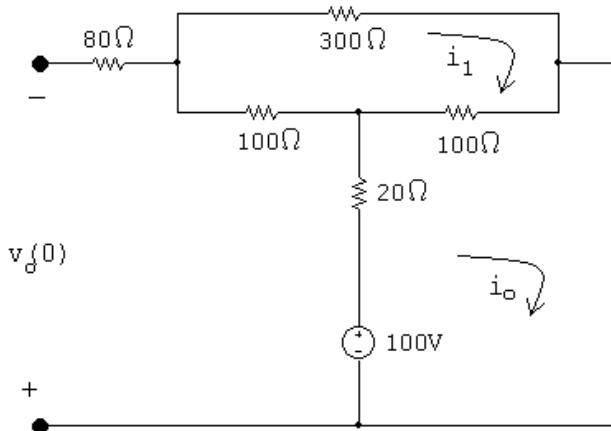
$$i(0) = D_2 = I_0 = -0.075$$

$$\frac{di}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{L}(-V_0 - RI_0)$$

$$\text{So } D_1 - (250)(-0.075) = \frac{1}{80 \times 10^{-3}}(0 - (40)(-0.075))$$

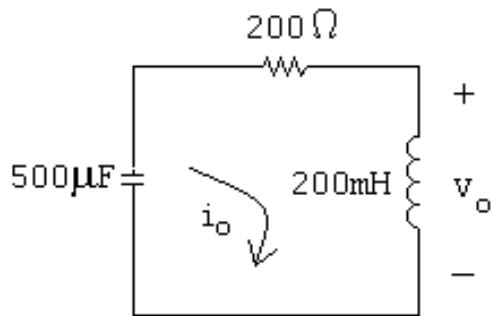
$$\text{Solving, } D_1 = 18.75$$

$$\therefore i(t) = 18.750t e^{-250t} - 75e^{-250t} \text{ mA}, \quad t \geq 0$$

P 8.47 $t < 0$:

$$500i_1 - 100i_o = 0; \quad -100i_1 + 120i_o = 100$$

$$\text{Solving, } i_1 = 0.2 \text{ A}; \quad i_o = 1 \text{ A}; \quad V_o = -100 + 20i_o + 100i_1 = -60 \text{ V}$$

 $t > 0$:

$$\alpha = \frac{R}{2L} = \frac{200}{2(0.2)} = 500$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.2)(36.25 \times 10^{-6})}} = 400$$

$$\alpha^2 > \omega_o^2 \quad \text{overdamped}$$

$$s_{1,2} = -100 \pm \sqrt{500^2 - 400^2} = -200, -800 \text{ rad/s}$$

$$i_o = A_1 e^{-200t} + A_2 e^{-800t}$$

$$i_o(0) = A_1 + A_2 = 1$$

$$\frac{di_o}{dt}(0) = -200A_1 - 800A_2 = \frac{1}{L}(-V_0 - RI_0) = -700$$

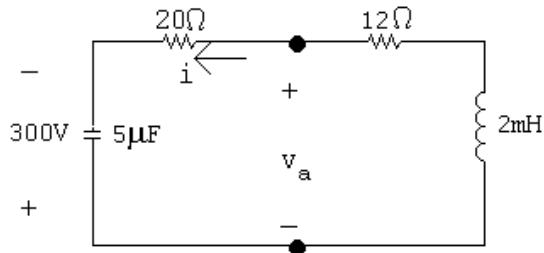
Solving, $A_1 = 166.67 \text{ mA}$; $A_2 = 833.33 \text{ mA}$

$$\therefore i_o(t) = 166.67e^{-200t} + 833.33e^{-800t} \text{ mA}, \quad t \geq 0$$

$$v_o(t) = L \frac{di_o}{dt} = (0.2)[(-200)0.16667e^{-200t} + (-800)0.83333e^{-800t}]$$

$$= -6.67e^{-200t} - 133.33e^{-800t} \text{ V}, \quad t \geq 0$$

P 8.48 [a] For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$$v_a(0^+) = -300 \text{ V}$$

$$[b] \quad v_a = 20i + \frac{1}{5 \times 10^{-6}} \int_0^t i \, dx - 300$$

$$\frac{dv_a}{dt} = 20 \frac{di}{dt} + 200,000i$$

$$\frac{dv_a(0^+)}{dt} = 20 \frac{di(0^+)}{dt} + 200,000i(0^+) = 20 \frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{1}{L}(-V_0 - RI_0) = \frac{1}{2 \times 10^{-3}}(300) = 150,000$$

$$\therefore \frac{dv_a(0^+)}{dt} = 3 \times 10^6 \text{ V/s}$$

$$[c] \quad \alpha = \frac{R}{2L} = \frac{32}{2(2 \times 10^{-3})} = 8000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(2 \times 10^{-3})(5 \times 10^{-6})}} = 10,000$$

Underdamped:

$$\omega_d = \sqrt{10,000^2 - 8000^2} = 6000$$

$$v_a = B_1 e^{-8000t} \cos 6000t + B_2 e^{-8000t} \sin 6000t$$

$$v_a(0) = B_1 = -300 \text{ V}$$

$$\frac{dv_a(0)}{dt} = -8000B_1 + 6000B_2 = 3 \times 10^6; \quad \therefore B_2 = 100 \text{ V}$$

$$v_a(t) = -300e^{-8000t} \cos 6000t + 100e^{-8000t} \sin 6000t \text{ V}, \quad t \geq 0^+$$

$$\text{P 8.49} \quad \alpha = \frac{R}{2L} = \frac{200}{2(0.025)} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(16 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_0^2 : \quad \text{underdamped}$$

$$\omega_d = \sqrt{500^2 - 400^2} = 300 \text{ rad/s}$$

$$v_o = V_f + B'_1 e^{-400t} \cos 300t + B'_2 e^{-400t} \sin 300t$$

$$v_o(\infty) = 200(0.08) = 16 \text{ V}$$

$$v_o(0) = 0 = V_f + B'_1 = 0 \quad \text{so} \quad B'_1 = -16 \text{ V}$$

$$\frac{dv_o}{dt}(0) = 0 = -400B'_1 + 300B'_2 \quad \text{so} \quad B'_2 = -21.33 \text{ V}$$

$$\therefore v_o(t) = 16 - 16e^{-400t} \cos 300t - 21.33e^{-400t} \sin 300t \text{ V}, \quad t \geq 0$$

$$\text{P 8.50} \quad \alpha = \frac{R}{2L} = \frac{250}{2(0.025)} = 500 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(16 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 = \omega_0^2 : \quad \text{critically damped}$$

$$v_o = V_f + D'_1 t e^{-500t} + D'_2 e^{-500t}$$

$$v_o(0) = 0 = V_f + D'_2$$

$$v_o(\infty) = (250)(0.08) = 20 \text{ V}; \quad \therefore D'_2 = -20 \text{ V}$$

$$\frac{dv_o}{dt}(0) = 0 = D'_1 - \alpha D'_2 \quad \text{so} \quad D'_1 = (500)(-20) = -10,000 \text{ V/s}$$

$$\therefore v_o(t) = 20 - 10,000t e^{-500t} - 20e^{-500t} \text{ V}, \quad t \geq 0$$

$$P\ 8.51 \quad \alpha = \frac{R}{2L} = \frac{312.5}{2(0.025)} = 625 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(16 \times 10^{-6})}} = 500 \text{ rad/s}$$

$\alpha^2 > \omega_0^2$: overdamped

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000 \text{ rad/s}$$

$$v_o = V_f + A'_1 e^{-250t} + A'_2 e^{-1000t}$$

$$v_o(0) = 0 = V_f + A'_1 + A'_2$$

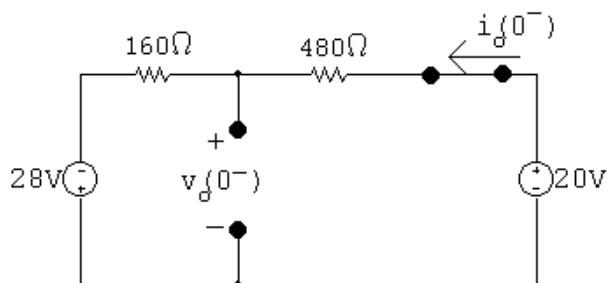
$$v_o(\infty) = (312.5)(08) = 25 \text{ V}; \quad \therefore A'_1 + A'_2 = -25 \text{ V}$$

$$\frac{dv_o}{dt}(0) = 0 = -250A'_1 - 1000A'_2$$

$$\text{Solving, } A'_1 = -33.33 \text{ V}; \quad A'_2 = 8.33 \text{ V}$$

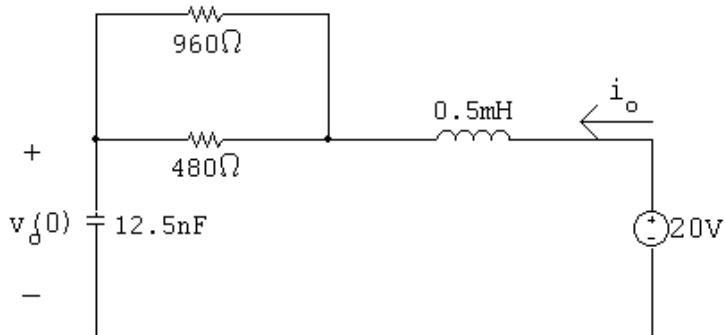
$$v_o(t) = 25 - 33.33e^{-250t} + 8.33e^{-1000t} \text{ V}, \quad t \geq 0$$

P 8.52 $t < 0$:



$$i_o(0^-) = \frac{20 + 28}{160 + 480} = 75 \text{ mA}$$

$$v_o(0^-) = 20 - 480(0.075) = -16 \text{ V}$$



As $t \rightarrow \infty$, $V_f = 20$ V.

$$R_{\text{eq}} = 960\parallel 480 = 320\Omega$$

$$\alpha = \frac{R_{\text{eq}}}{2L} = \frac{320}{2(0.5 \times 10^{-3})} = 320,000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.5 \times 10^{-3})(12.5 \times 10^{-9})}} = 400,000 \text{ rad/s}$$

$$\alpha^2 < \omega_0^2 : \quad \text{underdamped}$$

$$\omega_d = \sqrt{400,000^2 - 320,000^2} = 240,000 \text{ rad/s}$$

$$v_o = 20 + B'_1 e^{-320,000t} \cos 240,000t + B'_2 e^{-320,000t} \sin 240,000t$$

$$v_o(0) = 20 + B'_1 = -16 \quad \text{so} \quad B'_1 = -36 \text{ V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B'_1 + \omega_d B'_2 = \frac{I_0}{C} \quad \text{so} \quad -320,000(-36) + 240,000B'_2 = \frac{75 \times 10^{-3}}{12.5 \times 10^{-9}}$$

$$\text{solving,} \quad B'_2 = -23$$

$$\therefore v_o(t) = 20 - 36e^{-320,000t} \cos 240,000t - 23e^{-320,000t} \sin 240,000t \text{ V} \quad t \geq 0$$

$$\text{P 8.53} \quad i_C(0) = 0; \quad v_o(0) = 200 \text{ V}$$

$$\alpha = \frac{R}{2L} = \frac{25}{2(0.25)} = 50 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(2.5 \times 10^{-3})}} = 40 \text{ rad/s}$$

$\therefore \alpha^2 > \omega_o^2$; overdamped

$$s_{1,2} = -50 \pm \sqrt{50^2 - 40^2} = -20, -80 \text{ rad/s}$$

$$v_o = V_f + A'_1 e^{-20t} + A'_2 e^{-80t}$$

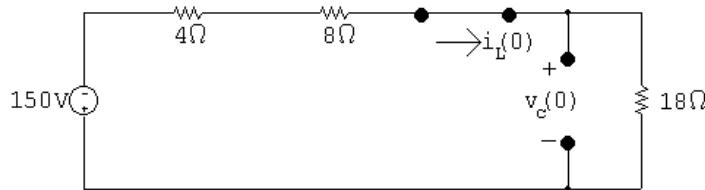
$$V_f = 150 \text{ V}; \quad v_o(0) = 150 + A'_1 + A'_2 = 200 \quad \text{so} \quad A'_1 + A'_2 = 50$$

$$\frac{v_o}{dt}(0) = -20A'_1 - 80A'_2 = \frac{I_0}{C} = 0$$

$$\text{Solving, } A'_1 = 66.67; \quad A'_2 = -16.67$$

$$\therefore v_o(t) = 150 + 66.67e^{-20t} - 16.67e^{-80t} \text{ V}, \quad t \geq 0$$

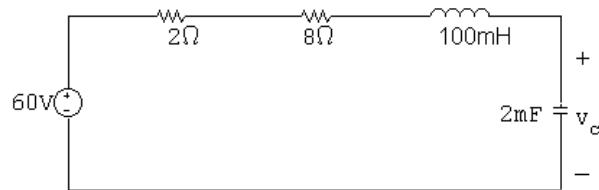
P 8.54 $t < 0$:



$$i_L(0) = \frac{-150}{30} = -5 \text{ A}$$

$$v_C(0) = 18i_L(0) = -90 \text{ V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2 \times 10^{-3})} = 5000$$

$\omega_o > \alpha^2 \quad \therefore \text{underdamped}$

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B'_1 e^{-50t} \cos 50t + B'_2 e^{-50t} \sin 50t$$

$$v_c(0) = -90 = 60 + B'_1 \quad \therefore \quad B'_1 = -150$$

$$C \frac{dv_c}{dt}(0) = -5; \quad \frac{dv_c}{dt}(0) = \frac{-5}{2 \times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B'_1 + 50B_2 = -2500 \quad \therefore \quad B'_2 = -200$$

$$v_c = 60 - 150e^{-50t} \cos 50t - 200e^{-50t} \sin 50t \text{ V}, \quad t \geq 0$$

P 8.55 [a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B'_1 e^{-\alpha t} \cos \omega_d t + B'_2 e^{-\alpha t} \sin \omega_d t$$

$$i_f = i(\infty) = 0, \quad i(0) = \frac{V_g}{R} = B'_1$$

$$\text{Therefore } i = B'_1 e^{-\alpha t} \cos \omega_d t + B'_2 e^{-\alpha t} \sin \omega_d t$$

$$L \frac{di(0)}{dt} = 0, \quad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B'_2 - \alpha B'_1) \cos \omega_d t - (\alpha B'_2 + \omega_d B'_1) \sin \omega_d t] e^{-\alpha t}$$

$$\text{Therefore } \omega_d B'_2 - \alpha B'_1 = 0; \quad B'_2 = \frac{\alpha}{\omega_d} B'_1 = \frac{\alpha}{\omega_d} \frac{V_g}{R}$$

Therefore

$$\begin{aligned} v_o &= L \frac{di}{dt} = - \left\{ L \left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R} \right) \sin \omega_d t \right\} e^{-\alpha t} \\ &= - \left\{ \frac{LV_g}{R} \left(\frac{\alpha^2}{\omega_d} + \omega_d \right) \sin \omega_d t \right\} e^{-\alpha t} \\ &= - \frac{V_g L}{R} \left(\frac{\alpha^2 + \omega_d^2}{\omega_d} \right) e^{-\alpha t} \sin \omega_d t \\ &= - \frac{V_g L}{R} \left(\frac{\omega_o^2}{\omega_d} \right) e^{-\alpha t} \sin \omega_d t \\ &= - \frac{V_g L}{R \omega_d} \left(\frac{1}{LC} \right) e^{-\alpha t} \sin \omega_d t \end{aligned}$$

$$v_o = - \frac{V_g}{RC \omega_d} e^{-\alpha t} \sin \omega_d t \text{ V}, \quad t \geq 0$$

$$[\mathbf{b}] \quad \frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC} \{\omega_d \cos \omega_d t - \alpha \sin \omega_d t\} e^{-\alpha t}$$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan \omega_d t = \frac{\omega_d}{\alpha}$$

$$\text{Therefore } \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad (\text{smallest } t)$$

$$t = \frac{1}{\omega_d} \tan^{-1} \left(\frac{\omega_d}{\alpha} \right)$$

P 8.56 [a] From Problem 8.55 we have

$$v_o = \frac{-V_g}{RC\omega_d} e^{-\alpha t} \sin \omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{480}{2(8 \times 10^{-3})} = 30,000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(8 \times 10^{-3})(50 \times 10^{-9})}} = 50,000 \text{ rad/s}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 40,000 \text{ rad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-24)}{(480)(50 \times 10^{-9})(40 \times 10^3)} = 25$$

$$\therefore v_o = 25e^{-30,000t} \sin 40,000t \text{ V}$$

[b] From Problem 8.55

$$t_d = \frac{1}{\omega_d} \tan^{-1} \left(\frac{\omega_d}{\alpha} \right) = \frac{1}{40,000} \tan^{-1} \left(\frac{40,000}{30,000} \right)$$

$$t_d = 23.18 \mu\text{s}$$

$$[\mathbf{c}] \quad v_{\max} = 25e^{-0.03(23.18)} \sin[(0.04)(23.18)] = 9.98 \text{ V}$$

$$[\mathbf{d}] \quad R = 96 \Omega; \quad \alpha = 6000 \text{ rad/s}$$

$$\omega_d = 49,638.7 \text{ rad/s}$$

$$v_o = 100.73e^{-6000t} \sin 49,638.7t \text{ V}, \quad t \geq 0$$

$$t_d = 29.22 \mu\text{s}$$

$$v_{\max} = 83.92 \text{ V}$$

P 8.57 [a] $v_c = V_f + [B'_1 \cos \omega_d t + B'_2 \sin \omega_d t] e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B'_2 - \alpha B'_1) \cos \omega_d t - (\alpha B'_2 + \omega_d B'_1) \sin \omega_d t] e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

$$\text{It follows that } B'_1 = -V_f \quad \text{and} \quad B'_2 = \frac{\alpha B'_1}{\omega_d}$$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d \right) V_f e^{-\alpha t} \sin \omega_d t$$

$$\text{But } V_f = V \quad \text{and} \quad \frac{\alpha^2}{\omega_d} + \omega_d = \frac{\alpha^2 + \omega_d^2}{\omega_d} = \frac{\omega_o^2}{\omega_d}$$

$$\text{Therefore } \frac{dv_c}{dt} = \left(\frac{\omega_o^2}{\omega_d} \right) V e^{-\alpha t} \sin \omega_d t$$

[b] $\frac{dv_c}{dt} = 0 \quad \text{when} \quad \sin \omega_d t = 0, \quad \text{or} \quad \omega_d t = n\pi$

$$\text{where } n = 0, 1, 2, 3, \dots$$

$$\text{Therefore } t = \frac{n\pi}{\omega_d}$$

[c] When $t_n = \frac{n\pi}{\omega_d}$, $\cos \omega_d t_n = \cos n\pi = (-1)^n$

$$\text{and} \quad \sin \omega_d t_n = \sin n\pi = 0$$

$$\text{Therefore } v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$$

[d] It follows from [c] that

$$v(t_1) = V + V e^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + V e^{-(3\alpha\pi/\omega_d)}$$

$$\text{Therefore } \frac{v_c(t_1) - V}{v_c(t_3) - V} = \frac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$$

$$\text{But } \frac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad \text{thus} \quad \alpha = \frac{1}{T_d} \ln \frac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$$

P 8.58 $\frac{1}{T_d} \ln \left\{ \frac{v_c(t_1) - V}{v_c(t_3) - V} \right\}; \quad T_d = t_3 - t_1 = \frac{3\pi}{7} - \frac{\pi}{7} = \frac{2\pi}{7} \text{ ms}$

$$\alpha = \frac{7000}{2\pi} \ln \left[\frac{63.84}{26.02} \right] = 1000; \quad \omega_d = \frac{2\pi}{T_d} = 7000 \text{ rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49 \times 10^6 + 10^6 = 50 \times 10^6$$

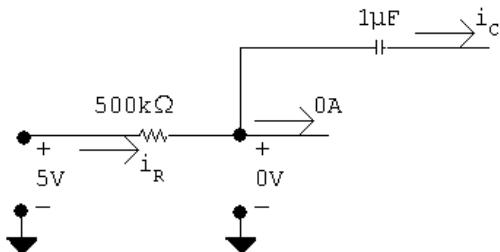
$$L = \frac{1}{(50 \times 10^6)(0.1 \times 10^{-6})} = 200 \text{ mH}; \quad R = 2\alpha L = 400 \Omega$$

P 8.59 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500 \text{ k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1 \mu\text{F}$ capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.60 [a] From Example 8.13 $\frac{d^2v_o}{dt^2} = 2$

therefore $\frac{dg(t)}{dt} = 2, \quad g(t) = \frac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \quad g(t) = 2t + g(0); \quad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500} \times 10^{-3} = 10 \mu\text{A} = i_C = -C \frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10 \times 10^{-6}}{1 \times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t dt - 10 dt$$

$$v_o - v_o(0) = t^2 - 10t; \quad v_o(0) = 8 \text{ V}$$

$$v_o = t^2 - 10t + 8, \quad 0 \leq t \leq t_{\text{sat}}$$

$$[b] \quad t^2 - 10t + 8 = -9$$

$$t^2 - 10t + 17 = 0$$

$$t \cong 2.17 \text{ s}$$

P 8.61 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_a = 100 \text{ k}\Omega; \quad C_1 = 0.1 \mu\text{F}; \quad R_b = 25 \text{ k}\Omega; \quad C_2 = 1 \mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_a C_1}\right) \left(\frac{1}{R_b C_2}\right) v_g; \quad \frac{1}{R_a C_1} = 100 \quad \frac{1}{R_b C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \quad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \frac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6 \text{ V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095 \text{ s}$$

[c] $\frac{dv_{o1}}{dt} = -\frac{1}{R_a C_1} v_g = -25$

[d] Since $v_{o1}(0) = 0$, $v_{o1} = -25t \text{ V}$

At $t = 0.1095 \text{ s}$, $v_{o1} \cong -2.74 \text{ V}$

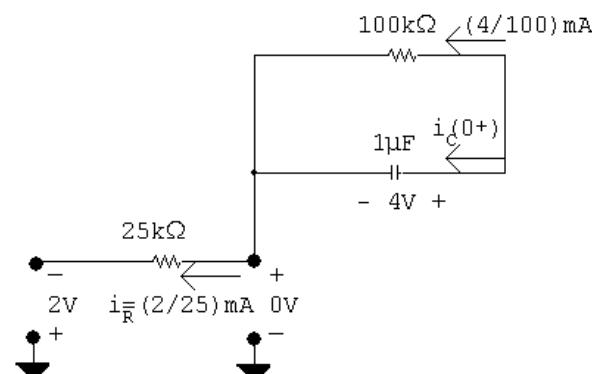
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \leq t \leq 0.1095 \text{ s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2 \text{ V}$ and $v_o(0) = 4 \text{ V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A'_1 e^{-10t} + A'_2 e^{-20t}$ (from Example 8.14)

$$v_o(0) = 4 = 5 + A'_1 + A'_2$$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100} \text{ mA} = C \frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40 \text{ V/s}$$

$$\frac{dv_o}{dt} = -10A'_1 e^{-10t} - 20A'_2 e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A'_1 - 20A'_2 = 40$$

Therefore $-A'_1 - 2A'_2 = 4$ and $A'_1 + A'_2 = -1$

Thus, $A'_1 = 2$ and $A'_2 = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t} \text{ V}$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

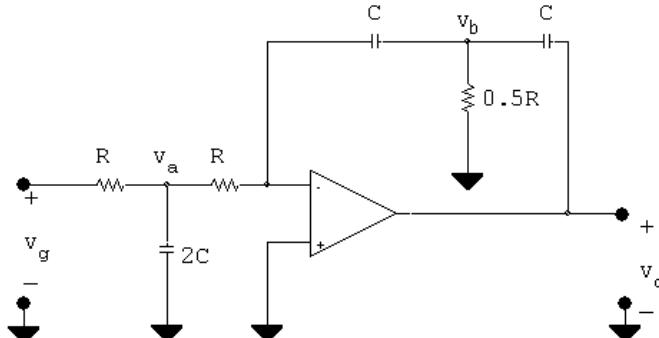
[d] From Example 8.14:

$$v_{o1}(\infty) = -1.25 \text{ V}; \quad v_1(0) = -2 \text{ V} \quad (\text{given})$$

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t} \text{ V}$$

P 8.62 [a]



$$2C \frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

$$(1) \text{ Therefore } \frac{dv_a}{dt} + \frac{v_a}{RC} = \frac{v_g}{2RC}$$

$$\frac{0 - v_a}{R} + C \frac{d(0 - v_b)}{dt} = 0$$

$$(2) \text{ Therefore } \frac{dv_b}{dt} + \frac{v_a}{RC} = 0, \quad v_a = -RC \frac{dv_b}{dt}$$

$$\frac{2v_b}{R} + C \frac{dv_b}{dt} + C \frac{d(v_b - v_o)}{dt} = 0$$

$$(3) \text{ Therefore } \frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2} \frac{dv_o}{dt}$$

$$\text{From (2) we have } \frac{dv_a}{dt} = -RC \frac{d^2v_b}{dt^2} \quad \text{and} \quad v_a = -RC \frac{dv_b}{dt}$$

When these are substituted into (1) we get

$$(4) \quad -RC \frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$$

Now differentiate (3) to get

$$(5) \quad \frac{d^2v_b}{dt^2} + \frac{1}{RC} \frac{dv_b}{dt} = \frac{1}{2} \frac{d^2v_o}{dt^2}$$

But from (4) we have

$$(6) \quad \frac{d^2v_b}{dt^2} + \frac{1}{RC} \frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

$$[b] \text{ When } R_1C_1 = R_2C_2 = RC : \quad \frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

$$\text{P 8.63} \quad [a] \quad \frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1R_2C_2}v_g$$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2) \times 10^{-6} \times 10^{-6}} = 250$$

$$\therefore \frac{d^2v_o}{dt^2} = 250v_g$$

$$0 \leq t \leq 0.5^- :$$

$$v_g = 80 \text{ mV}$$

$$\frac{d^2v_o}{dt^2} = 20$$

$$\text{Let } g(t) = \frac{dv_o}{dt}, \quad \text{then } \frac{dg}{dt} = 20 \quad \text{or} \quad dg = 20 dt$$

$$\int_{g(0)}^{g(t)} dx = 20 \int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20 \int_0^t x dx; \quad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2 \text{ V}, \quad 0 \leq t \leq 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1 C_1} v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6 dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6 \int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \quad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t \text{ V}, \quad 0 \leq t \leq 0.5^-$$

$0.5^+ \leq t \leq t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \quad \text{let } g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \quad dg(t) = -10 dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10 \int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C \frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80 \text{ V}$$

$$\therefore C \frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^3} = 2 \mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10 \text{ V/s}$$

$$\therefore g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore dv_o = -10t dt + 15 dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^t -10y dy + \int_{0.5^+}^t 15 dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^t + 15y \Big|_{0.5}^t$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5 \text{ V}$$

$$\therefore v_o(t) = -5t^2 + 15t - 3.75 \text{ V}, \quad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \quad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$dv_{o1} = 0.8 dt; \quad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8 \int_{0.5^+}^t dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \quad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8 \text{ V}$$

$$\therefore v_{o1}(t) = 0.8t - 1.2 \text{ V}, \quad 0.5^+ \leq t \leq t_{\text{sat}}$$

Summary:

$$0 \leq t \leq 0.5^- \text{ s} : \quad v_{o1} = -1.6t \text{ V}, \quad v_o = 10t^2 \text{ V}$$

$$0.5^+ \text{ s} \leq t \leq t_{\text{sat}} : \quad v_{o1} = 0.8t - 1.2 \text{ V}, \quad v_o = -5t^2 + 15t - 3.75 \text{ V}$$

$$[b] -12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$$

$$\therefore 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

$$\text{Solving, } t_{\text{sat}} = 3.5 \text{ sec}$$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6 \text{ V}$$

$$P 8.64 \quad \tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50 \text{ s}$$

$$\frac{1}{\tau_1} = 2; \quad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1 \text{ s}; \quad \therefore \frac{1}{\tau_2} = 1$$

$$\therefore \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \quad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A'_1 e^{-t} + A'_2 e^{-2t}; \quad V_f = \frac{20}{2} = 10 \text{ V}$$

$$v_o = 10 + A'_1 e^{-t} + A'_2 e^{-2t}$$

$$v_o(0) = 0 = 10 + A'_1 + A'_2; \quad \frac{dv_o}{dt}(0) = 0 = -A'_1 - 2A'_2$$

$$\therefore A'_1 = -20, \quad A'_2 = 10 \text{ V}$$

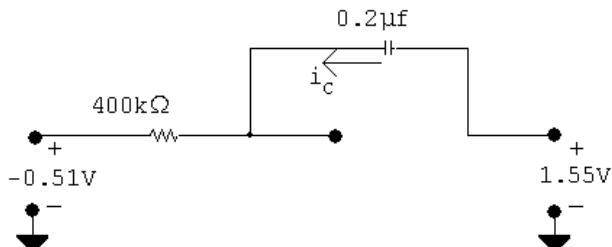
$$v_o(t) = 10 - 20e^{-t} + 10e^{-2t} \text{ V}, \quad 0 \leq t \leq 0.5 \text{ s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \quad \therefore v_{o1} = -0.8 + 0.8e^{-2t} \text{ V}, \quad 0 \leq t \leq 0.5 \text{ s}$$

$$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55 \text{ V}$$

$$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51 \text{ V}$$

At $t = 0.5 \text{ s}$



$$i_C = \frac{0 + 0.51}{400 \times 10^3} = 1.26 \mu\text{A}$$

$$C \frac{dv_o}{dt} = 1.26 \mu\text{A}; \quad \frac{dv_o}{dt} = \frac{1.26}{0.2} = 6.32 \text{ V/s}$$

$0.5 \text{ s} \leq t \leq \infty$:

$$\frac{d^2v_o}{dt^2} + 3 \frac{dv_o}{dt} + 2 = -10$$

$$v_o(\infty) = -5$$

$$\therefore v_o = -5 + A'_1 e^{-(t-0.5)} + A'_2 e^{-2(t-0.5)}$$

$$1.55 = -5 + A'_1 + A'_2$$

$$\frac{dv_o}{dt}(0.5) = 6.32 = -A'_1 - 2A'_2$$

$$\therefore A'_1 + A'_2 = 6.55; \quad -A'_1 - 2A'_2 = 6.32$$

Solving,

$$A'_1 = 19.42; \quad A'_2 = -12.87$$

$$\therefore v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)} \text{ V}, \quad 0.5 \leq t \leq \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

$$\therefore v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)} \text{ V}, \quad 0.5 \leq t \leq \infty$$

P 8.65 [a] $f(t) = \text{inertial force} + \text{frictional force} + \text{spring force}$

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

$$[b] \frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right) \left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right) x$$

Given $v_A = \frac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1 C_1} \int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1 C_1} \frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2 C_2} \int_0^t v_B dy = \frac{1}{R_1 R_2 C_1 C_2} x$$

$$v_D = -\frac{R_3}{R_4} \cdot v_B = \frac{R_3}{R_4 R_1 C_1} \frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right] v_C = \left[\frac{R_5 + R_6}{R_6}\right] \cdot \frac{1}{R_1 R_2 C_1 C_2} \cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right] f(t), \quad v_A = -(v_D + v_E + v_F)$$

$$\text{Therefore } \frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right] f(t) - \left[\frac{R_3}{R_4 R_1 C_1}\right] \frac{dx}{dt} - \left[\frac{R_5 + R_6}{R_6 R_1 R_2 C_1 C_2}\right] x$$

$$\text{Therefore } M = \frac{R_7}{R_8}, \quad D = \frac{R_3 R_7}{R_8 R_4 R_1 C_1} \quad \text{and} \quad K = \frac{R_7 (R_5 + R_6)}{R_8 R_6 R_1 R_2 C_1 C_2}$$

Box Number	Function
1	inverting and scaling
2	summing and inverting
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.66 [a] $\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(5 \times 10^{-9})(2 \times 10^{-12})}} = 10^{10} \text{ rad/sec}$

$$\therefore f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59 \times 10^9 \text{ Hz} = 1.59 \text{ GHz}$$

[b] $I_0 = \frac{10}{25} = 0.4 \text{ A}$

$$w(0) = \frac{1}{2}LI_0^2 = \frac{1}{2}(5 \times 10^{-9})(0.4)^2 = 4 \times 10^{-10} \text{ J} = 0.4 \text{ nJ}$$

[c] Because the inductor and capacitor are assumed to be ideal, none of the initial energy will ever be dissipated, so for all $t \geq 0$ the 0.4 nJ will be continually exchanged between the inductor and capacitor.

P 8.67 [a] $\omega_0 = 2\pi f_0 = 2\pi(2 \times 10^9) = 4\pi \times 10^9 \text{ rad/s}$

$$\omega_0^2 = \frac{1}{LC}$$

$$\therefore C = \frac{1}{L\omega_0^2} = \frac{1}{(10^{-9})(4\pi \times 10^9)^2} = 6.33 \times 10^{-12} = 6.33 \text{ pF}$$

[b] $v_o(t) = \frac{V}{\omega_0 RC} \sin \omega_0 t = \frac{4}{(4\pi \times 10^9)(10)(6.33 \times 10^{-12})} \sin 4\pi \times 10^9 t$
 $= 5.03 \sin 4\pi \times 10^9 t \text{ V}, \quad t \geq 0$

P 8.68 [a] $\alpha = \frac{R}{2L} = \frac{0.01}{2(5 \times 10^{-9})} = 10^6 \text{ rad/s}$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(5 \times 10^{-9})(2 \times 10^{-12})}} = 10^{10} \text{ rad/s}$$

[b] $\omega_0^2 > \alpha^2$ so the response is underdamped

$$[\text{c}] \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(10^{10})^2 - (10^6)^2} \approx 10^{10} \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59 \times 10^9 \text{ Hz} = 1.59 \text{ GHz}$$

Therefore the addition of the $10 \text{ m}\Omega$ resistance does not change the frequency of oscillation.

- [d] Because of the added resistance, the oscillation now occurs within a decaying exponential envelope (see Fig. 8.9). The form of the exponential envelope is $e^{-\alpha t} = e^{-10^6 t}$. Let's assume that the oscillation is described by the function $Ke^{-10^6 t} \cos 10^{10} t$, where the maximum magnitude, K , exists at $t = 0$. How long does it take before the magnitude of the oscillation has decayed to $0.01K$?

$$Ke^{-10^6 t} = 0.01K \quad \text{so} \quad e^{-10^6 t} = 0.01$$

$$\therefore t = \frac{\ln 0.01}{-10^6} = 4.6 \times 10^{-6} \text{ s} = 4.6 \mu\text{s}$$

Therefore, the oscillations will persist for only $4.6 \mu\text{s}$, due to the presence of a small amount of resistance in the circuit. This is why an LC oscillator is not used in the clock generator circuit.

9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 [a] $\mathbf{V} = 170/\underline{-40^\circ}$ V

[b] $10 \sin(1000t + 20^\circ) = 10 \cos(1000t - 70^\circ)$

$$\therefore \mathbf{I} = 10/\underline{-70^\circ}$$
 A

[c] $\mathbf{I} = 5/\underline{36.87^\circ} + 10/\underline{-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18/\underline{-26.57^\circ}$$
 A

[d] $\sin(20,000\pi t + 30^\circ) = \cos(20,000\pi t - 60^\circ)$

Thus,

$$\begin{aligned}\mathbf{V} &= 300/\underline{45^\circ} - 100/\underline{-60^\circ} = 212.13 + j212.13 - (50 - j86.60) \\ &= 162.13 + j298.73 = 339.90/\underline{61.51^\circ}$$
 mV

AP 9.2 [a] $v = 18.6 \cos(\omega t - 54^\circ)$ V

[b] $\mathbf{I} = 20/\underline{45^\circ} - 50/\underline{-30^\circ} = 14.14 + j14.14 - 43.3 + j25$
 $= -29.16 + j39.14 = 48.81/\underline{126.68^\circ}$

Therefore $i = 48.81 \cos(\omega t + 126.68^\circ)$ mA

[c] $\mathbf{V} = 20 + j80 - 30/\underline{15^\circ} = 20 + j80 - 28.98 - j7.76$
 $= -8.98 + j72.24 = 72.79/\underline{97.08^\circ}$
 $v = 72.79 \cos(\omega t + 97.08^\circ)$ V

AP 9.3 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200 \Omega$

[b] $Z_L = j\omega L = j200 \Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10/30^\circ)(200/90^\circ) \times 10^{-3} = 2/120^\circ \text{ V}$

[d] $v_L = 2 \cos(10,000t + 120^\circ) \text{ V}$

AP 9.4 [a] $X_C = \frac{-1}{\omega C} = \frac{-1}{4000(5 \times 10^{-6})} = -50 \Omega$

[b] $Z_C = jX_C = -j50 \Omega$

[c] $\mathbf{I} = \frac{\mathbf{V}}{Z_C} = \frac{30/25^\circ}{50/-90^\circ} = 0.6/115^\circ \text{ A}$

[d] $i = 0.6 \cos(4000t + 115^\circ) \text{ A}$

AP 9.5 $\mathbf{I}_1 = 100/25^\circ = 90.63 + j42.26$

$\mathbf{I}_2 = 100/145^\circ = -81.92 + j57.36$

$\mathbf{I}_3 = 100/-95^\circ = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0) \text{ A}, \quad \text{therefore } i_4 = 0 \text{ A}$

AP 9.6 [a] $\mathbf{I} = \frac{125/-60^\circ}{|Z|/\theta_z} = \frac{125}{|Z|} / (-60 - \theta_Z)^\circ$

But $-60 - \theta_Z = -105^\circ \quad \therefore \theta_Z = 45^\circ$

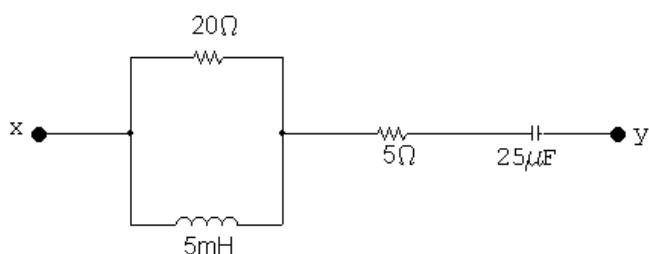
$Z = 90 + j160 + jX_C$

$\therefore X_C = -70 \Omega; \quad X_C = -\frac{1}{\omega C} = -70$

$\therefore C = \frac{1}{(70)(5000)} = 2.86 \mu\text{F}$

[b] $\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{125/-60^\circ}{(90 + j90)} = 0.982/-105^\circ \text{ A}; \quad \therefore |\mathbf{I}| = 0.982 \text{ A}$

AP 9.7 [a]



$\omega = 2000 \text{ rad/s}$

$\omega L = 10 \Omega, \quad \frac{-1}{\omega C} = -20 \Omega$

$$\begin{aligned} Z_{xy} &= 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20 \\ &= 4 + j8 + 5 - j20 = (9 - j12) \Omega \end{aligned}$$

[b] $\omega L = 40 \Omega, \quad \frac{-1}{\omega C} = -5 \Omega$

$$\begin{aligned} Z_{xy} &= 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40} \right] \\ &= 5 - j5 + 16 + j8 = (21 + j3) \Omega \end{aligned}$$

[c] $Z_{xy} = \left[\frac{20(j\omega L)}{20 + j\omega L} \right] + \left(5 - \frac{j10^6}{25\omega} \right)$
 $= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000 \text{ rad/s}$.

[d] $Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15 \Omega$

- AP 9.8 The frequency 4000 rad/s was found to give $Z_{xy} = 15 \Omega$ in Assessment Problem 9.7. Thus,

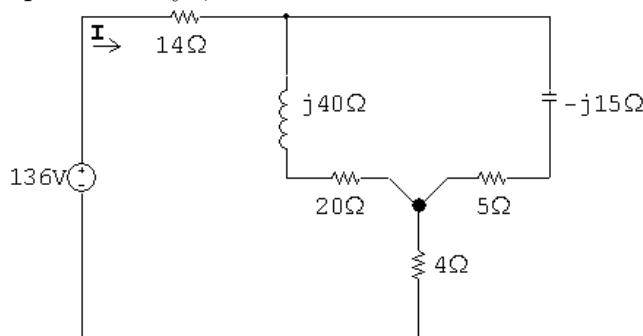
$$\mathbf{V} = 150/\underline{0^\circ}, \quad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{xy}} = \frac{150/\underline{0^\circ}}{15} = 10/\underline{0^\circ} \text{ A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07/\underline{-45^\circ} \text{ A}$$

$$i_L = 7.07 \cos(4000t - 45^\circ) \text{ A}, \quad I_m = 7.07 \text{ A}$$

- AP 9.9 After replacing the delta made up of the 50Ω , 40Ω , and 10Ω resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40) \parallel (5 - j15) = (12 - j16) \Omega$$

Therefore $\mathbf{I} = \frac{136/0^\circ}{14 + 12 - j16 + 4} = 4/28.07^\circ \text{ A}$

AP 9.10

$$\mathbf{V}_1 = 240/53.13^\circ = 144 + j192 \text{ V}$$

$$\mathbf{V}_2 = 96/-90^\circ = -j96 \text{ V}$$

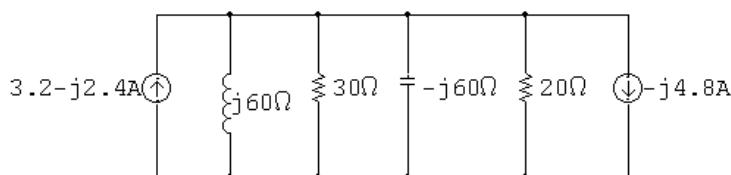
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60 \Omega$$

$$\frac{1}{j\omega C} = -j \frac{6 \times 10^6}{(4000)(25)} = -j60 \Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4 \text{ A}$$

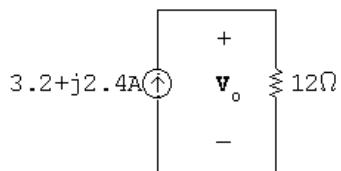
$$\frac{\mathbf{V}_2}{20} = -j \frac{96}{20} = -j4.8 \text{ A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12 \Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8 \text{ V} = 48/36.87^\circ \text{ V}$$

$$v_o = 48 \cos(4000t + 36.87^\circ) \text{ V}$$

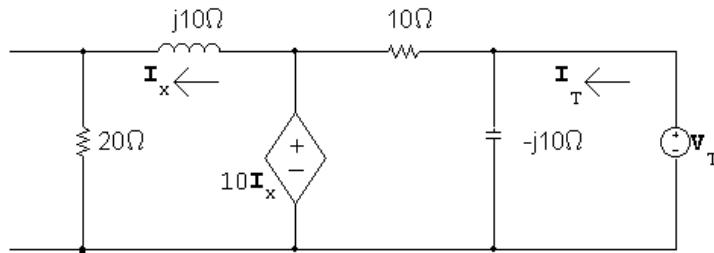
AP 9.11 Use the lower node as the reference node. Let \mathbf{V}_1 = node voltage across the 20Ω resistor and \mathbf{V}_{Th} = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2/\underline{45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{Th} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for \mathbf{V}_{Th} gives $\mathbf{V}_{Th} = 10/\underline{45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

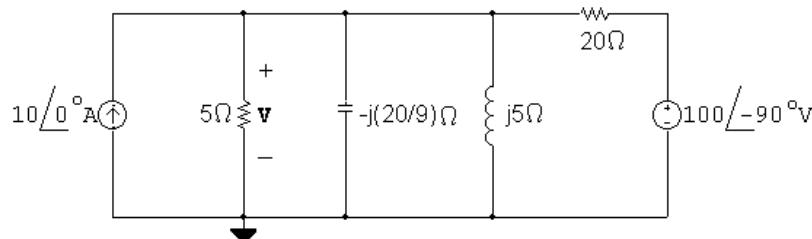
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{Th} = (5 - j5) \Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100/\underline{-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62/-71.57^\circ$

Therefore $v = 31.62 \cos(50,000t - 71.57^\circ)$ V

- AP 9.13 Let \mathbf{I}_a , \mathbf{I}_b , and \mathbf{I}_c be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$$

and

$$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b),$$

therefore

$$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)].$$

Solving for $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07/3.95^\circ$ A.

- AP 9.14 [a] $M = 0.4\sqrt{0.0625} = 0.1$ H, $\omega M = 80$ Ω

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300) \Omega$$

$$\text{Therefore } |Z_{22}| = 500 \Omega, \quad Z_{22}^* = (400 - j300) \Omega$$

$$Z_\tau = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68) \Omega$$

$$[b] \quad \mathbf{I}_1 = \frac{245.20}{184 + 100 + j400 + Z_\tau} = 0.50/-53.13^\circ \text{ A}$$

$$i_1 = 0.5 \cos(800t - 53.13^\circ) \text{ A}$$

$$[c] \quad \mathbf{I}_2 = \left(\frac{j\omega M}{Z_{22}}\right) \mathbf{I}_1 = \frac{j80}{500/36.87^\circ} (0.5/-53.13^\circ) = 0.08/0^\circ \text{ A}$$

$$i_2 = 80 \cos 800t \text{ mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 / 0^\circ}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5 / 36.87^\circ \text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1 \mathbf{I}_1 = 25,000 / 0^\circ - (4 + j3)(1500 + j6000)$$

$$= 37,000 - j28,500$$

$$\mathbf{V}_2 = -\frac{1}{25} \mathbf{V}_1 = -1480 + j1140 = 1868.15 / 142.39^\circ \text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15 / 142.39^\circ}{4 - j14.4} = 125 / 216.87^\circ \text{ A}$$

Problems

P 9.1 [a] $\omega = 2\pi f = 800 \text{ rad/s}$, $f = \frac{\omega}{2\pi} = 127.32 \text{ Hz}$

[b] $T = 1/f = 7.85 \text{ ms}$

[c] $I_m = 125 \text{ mA}$

[d] $i(0) = 125 \cos(36.87^\circ) = 100 \text{ mA}$

[e] $\phi = 36.87^\circ$; $\phi = \frac{36.87^\circ (2\pi)}{360^\circ} = 0.6435 \text{ rad}$

[f] $i = 0$ when $800t + 36.87^\circ = 90^\circ$. Now resolve the units:

$$(800 \text{ rad/s})t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.927 \text{ rad}, \quad t = 1.16 \text{ ms}$$

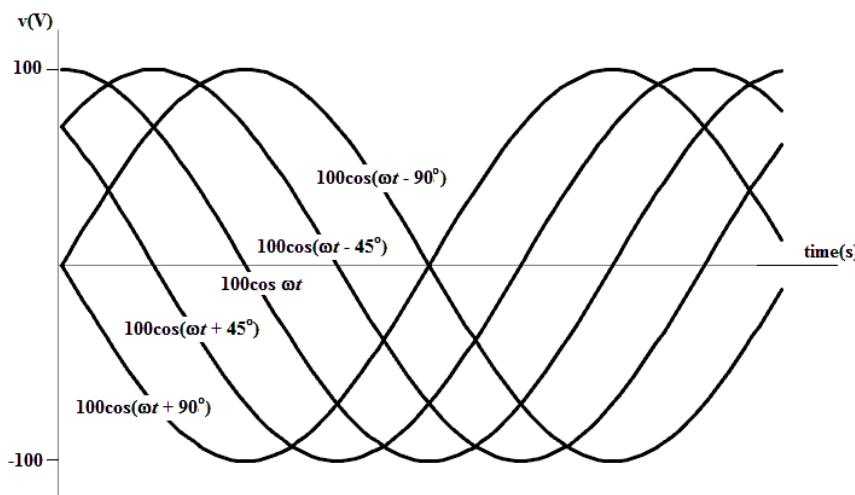
[g] $(di/dt) = (-0.125)800 \sin(800t + 36.87^\circ)$

$$(di/dt) = 0 \quad \text{when} \quad 800t + 36.87^\circ = 180^\circ$$

$$\text{or} \quad 800t = \frac{143.13^\circ}{57.3^\circ/\text{rad}} = 2.498 \text{ rad}$$

Therefore $t = 3.12 \text{ ms}$

P 9.2



[a] Right as ϕ becomes more negative

[b] Left

P 9.3 [a] 25 V

[b] $2\pi f = 400\pi$; $f = 200 \text{ Hz}$

[c] $\omega = 400\pi = 1256.64 \text{ rad/s}$

[d] $\theta(\text{rad}) = 60^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{\pi}{3} = 1.0472 \text{ rad}$

[e] $\theta = 60^\circ$

[f] $T = \frac{1}{f} = \frac{1}{200} = 5 \text{ ms}$

[g] $400\pi t + \frac{\pi}{3} = \frac{\pi}{2}; \quad \therefore 400\pi t = \frac{\pi}{6}$

$$\therefore t = \frac{1}{2400} = 416.67 \mu\text{s}$$

[h] $v = 25 \cos \left[400\pi \left(t - \frac{0.005}{6} \right) + \frac{\pi}{3} \right]$

$$= 25 \cos [400\pi t - (\pi/3) + (\pi/3)]$$

$$= 25 \cos 400\pi t \text{ V}$$

[i] $400\pi(t + t_o) + (\pi/3) = 400\pi t + (3\pi/2)$

$$\therefore 400\pi t_o = \frac{7\pi}{6}; \quad t_o = \frac{7}{2400} = 2.92 \text{ ms}$$

P 9.4 [a] By hypothesis

$$v = 50 \cos(\omega t + \theta)$$

$$\frac{dv}{dt} = -50\omega \sin(\omega t + \theta)$$

$$\therefore 50\omega = 750\pi; \quad \omega = 15\pi \text{ rad/s}$$

[b] At $t = (40/3)$ ms, the argument of the cosine function must equal 90° .

Remember that we must convert ωt to degrees:

$$15\pi \left(\frac{0.04}{3} \right) \frac{360^\circ}{2\pi} + \theta = 90^\circ$$

Solving,

$$\theta = 90^\circ - 36^\circ = 54^\circ$$

The problem description says that the voltage is increasing, but the derivative calculated in part (a) is negative. Therefore we need to shift the phase angle by 180° to $54 - 180 = -126^\circ$, effectively multiplying the expression for the voltage by -1 , so that the expression for the voltage is increasing.

$$\therefore v = 50 \cos(15\pi t - 126^\circ) \text{ V}$$

P 9.5 [a] $\frac{T}{2} = 25 - 5 = 20 \text{ ms}; \quad T = 40 \text{ ms}$

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25 \text{ Hz}$$

[b] $i = I_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 50\pi \text{ rad/s}$$

$$50\pi(5 \times 10^{-3}) + \theta = 0; \quad \therefore \theta = \frac{-\pi}{4} \text{ rad} = -45^\circ$$

$$i = I_m \sin[50\pi t - 45^\circ]$$

$$0.5 = I_m \sin -45^\circ; \quad I_m = -70.71 \text{ mA}$$

$$i = -70.71 \sin[50\pi t - 45^\circ] = 70.71 \cos[50\pi t + 45^\circ] \text{ mA}$$

P 9.6 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(240) = 339.41 \text{ V}$

P 9.7 $V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t dt}$

$$\int_0^{T/2} V_m^2 \sin^2 \left(\frac{2\pi}{T} t \right) dt = \frac{V_m^2}{2} \int_0^{T/2} \left(1 - \cos \frac{4\pi}{T} t \right) dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T} \frac{V_m^2 T}{4}} = \frac{V_m}{2}$

P 9.8 $\int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi) dt = V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2} \cos(2\omega t + 2\phi) dt$
 $= \frac{V_m^2}{2} \left\{ \int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi) dt \right\}$
 $= \frac{V_m^2}{2} \left\{ T + \frac{1}{2\omega} [\sin(2\omega t + 2\phi)] \Big|_{t_o}^{t_o+T} \right\}$
 $= \frac{V_m^2}{2} \left\{ T + \frac{1}{2\omega} [\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)] \right\}$
 $= V_m^2 \left(\frac{T}{2} \right) + \frac{1}{2\omega} (0) = V_m^2 \left(\frac{T}{2} \right)$

P 9.9 [a] The numerical values of the terms in Eq. 9.8 are

$$V_m = 75, \quad R/L = 5333.33, \quad \omega L = 300$$

$$\sqrt{R^2 + \omega^2 L^2} = 500$$

$$\phi = -60^\circ, \quad \theta = \tan^{-1} 300/400, \quad \theta = 36.87^\circ$$

Substitute these values into Equation 9.9:

$$i = [-17.94e^{-5333.33t} + 150 \cos(4000t - 96.87^\circ)] \text{ mA}, \quad t \geq 0$$

[b] Transient component $= -17.94e^{-5333.33t} \text{ mA}$

Steady-state component $= 150 \cos(4000t - 96.87^\circ) \text{ mA}$

[c] By direct substitution into Eq 9.9 in part (a), $i(750 \mu\text{s}) = 38.44 \text{ mA}$

[d] 150 mA , 4000 rad/s , -96.87°

[e] The current lags the voltage by 36.87° .

P 9.10 [a] From Eq. 9.9 we have

$$L \frac{di}{dt} = \frac{V_m R \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$Ri = \frac{-V_m R \cos(\phi - \theta) e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R \cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$L \frac{di}{dt} + Ri = V_m \left[\frac{R \cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \right]$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos \theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin \theta$$

Therefore the right-hand side reduces to

$$V_m \cos(\omega t + \phi)$$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

[b] $i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

Therefore

$$L \frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

$$L \frac{di_{ss}}{dt} + Ri_{ss} = V_m \left[\frac{R \cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \right]$$

$$= V_m \cos(\omega t + \phi)$$

P 9.11 [a] $\mathbf{Y} = 30/\underline{-160^\circ} + 15/\underline{70^\circ} = 29.38/\underline{170.56^\circ}$

$$y = 28.38 \cos(200t + 170.56^\circ)$$

[b] $\mathbf{Y} = 90/\underline{-110^\circ} + 60/\underline{-70^\circ} = 141.33/\underline{-94.16^\circ}$

$$y = 141.33 \cos(50t - 94.16^\circ)$$

$$[\mathbf{c}] \quad \mathbf{Y} = 50/\underline{-60^\circ} + 25/\underline{20^\circ} - 75/\underline{-30^\circ} = 16.7/\underline{170.52^\circ}$$

$$y = 16.7 \cos(5000t + 170.52^\circ)$$

$$[\mathbf{d}] \quad \mathbf{Y} = 10/\underline{30^\circ} + 10/\underline{-90^\circ} + 10/\underline{150^\circ} = 0$$

$$y = 0$$

P 9.12 [a] 400 Hz

$$[\mathbf{b}] \quad \theta_v = 0^\circ$$

$$\mathbf{I} = \frac{100/\underline{0^\circ}}{j\omega L} = \frac{100}{\omega L}/\underline{-90^\circ}; \quad \theta_i = -90^\circ$$

$$[\mathbf{c}] \quad \frac{100}{\omega L} = 20; \quad \omega L = 5 \Omega$$

$$[\mathbf{d}] \quad L = \frac{5}{800\pi} = 1.99 \text{ mH}$$

$$[\mathbf{e}] \quad Z_L = j\omega L = j5 \Omega$$

P 9.13 [a] $\omega = 2\pi f = 160\pi \times 10^3 = 502.65 \text{ krad/s} = 502,654.82 \text{ rad/s}$

$$[\mathbf{b}] \quad \mathbf{I} = \frac{25 \times 10^{-3}/\underline{0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})/\underline{0^\circ} = 25 \times 10^{-3}\omega C/\underline{90^\circ}$$

$$\therefore \theta_i = 90^\circ$$

$$[\mathbf{c}] \quad 628.32 \times 10^{-6} = 25 \times 10^{-3} \omega C$$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79 \Omega, \quad \therefore X_C = -39.79 \Omega$$

$$[\mathbf{d}] \quad C = \frac{1}{39.79(\omega)} = \frac{1}{(39.79)(160\pi \times 10^3)}$$

$$C = 0.05 \times 10^{-6} = 0.05 \mu\text{F}$$

$$[\mathbf{e}] \quad Z_c = j \left(\frac{-1}{\omega C} \right) = -j39.79 \Omega$$

P 9.14 [a] $\mathbf{V}_g = 300/\underline{78^\circ}; \quad \mathbf{I}_g = 6/\underline{33^\circ}$

$$\therefore Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300/\underline{78^\circ}}{6/\underline{33^\circ}} = 50/\underline{45^\circ} \Omega$$

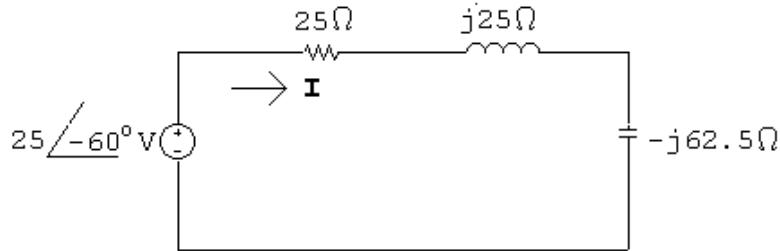
[b] i_g lags v_g by 45° :

$$2\pi f = 5000\pi; \quad f = 2500 \text{ Hz}; \quad T = 1/f = 400 \mu\text{s}$$

$$\therefore i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400 \mu\text{s}) = 50 \mu\text{s}$$

P 9.15 [a] $Z_L = j(500)(50 \times 10^{-3}) = j25 \Omega$

$$Z_C = \frac{-j}{(500)(32 \times 10^{-6})} = -j62.5 \Omega$$

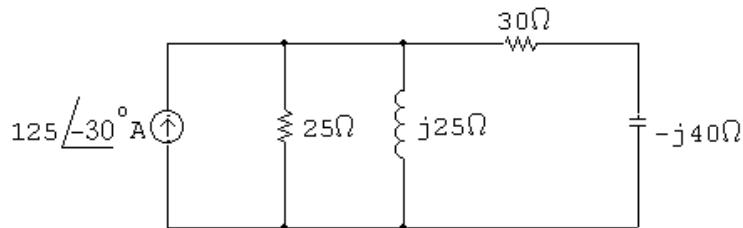


[b] $\mathbf{I} = \frac{25/-60^\circ}{25 + j25 - j62.5} = 554.7/-3.69^\circ \text{ mA}$

[c] $i = 554.7 \cos(500t - 3.69^\circ) \text{ mA}$

P 9.16 [a] $j\omega L = j(2500)(0.01) = j25 \Omega$

$$\frac{1}{j\omega C} = -j \frac{1}{(2500)(10 \times 10^{-6})} = -j40 \Omega; \quad \mathbf{I}_g = 125/-30^\circ \text{ A}$$



[b] $\mathbf{V}_o = 125/-30^\circ Z_e$

$$Z_e = \frac{1}{Y_e}; \quad Y_e = \frac{1}{25} + \frac{1}{j25} + \frac{1}{30 - j40}$$

$$Y_e = 52 - j24 \text{ mS}$$

$$Z_e = \frac{1}{0.052 - j0.024} = 17.46/24.78^\circ \Omega$$

$$\mathbf{V}_o = (125/-30^\circ)(17.46/24.78^\circ) = 2182.6/-5.22^\circ \text{ V}$$

[c] $v_o = 2182.6 \cos(2500t - 5.22^\circ) \text{ V}$

P 9.17 [a]
$$\begin{aligned} Y &= \frac{1}{3+j4} + \frac{1}{16-j12} + \frac{1}{-j4} \\ &= 0.12 - j0.16 + 0.04 + j0.03 + j0.25 \\ &= 160 + j120 = 200/36.87^\circ \text{ mS} \end{aligned}$$

[b] $G = 160 \text{ mS}$

[c] $B = 120 \text{ mS}$

[d] $\mathbf{I} = 8/\underline{0^\circ} \text{ A}, \quad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2/\underline{36.87^\circ}} = 40/\underline{-36.87^\circ} \text{ V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40/\underline{-36.87^\circ}}{4/\underline{-90^\circ}} = 10/\underline{53.13^\circ} \text{ A}$$

$$i_C = 10 \cos(\omega t + 53.13^\circ) \text{ A}, \quad I_m = 10 \text{ A}$$

P 9.18 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \frac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500 \Omega$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625 \text{ mH}$$

P 9.19 [a] $Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $Y_2 = Y_1$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \frac{8000^2 + 1000^2(4)^2}{8000} = 10 \text{ k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20 \text{ H}$$

P 9.20 [a] $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

$$[\mathbf{b}] \quad R_1 = \frac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-4})^2} = 200 \Omega$$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5 \text{nF}$$

P 9.21 [a] $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $Y_1 = Y_2$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

$$[\mathbf{b}] \quad R_2 = \frac{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2}{(50 \times 10^3)^2(1000)(40 \times 10^{-9})^2} = 1250 \Omega$$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2} = 8 \text{nF}$$

P 9.22 $Z_{ab} = 5 + j8 + 10\| - j20 + (8 + j16)\|(40 - j80)$

$$= 5 + j8 + 8 - j4 + 12 + j16 = 25 + j20 \Omega = 32.02/\underline{38.66^\circ} \Omega$$

P 9.23 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125 \text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8 \Omega$$

$$Z_{ab} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12 \Omega$$

$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{16 - j12} = 0.04 + j0.03 \text{S}$$

$$= 40 + j30 \text{mS} = 50/\underline{36.87^\circ} \text{mS}$$

P 9.24 [a] $\frac{1}{j\omega C} + R\|j\omega L = \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R}$

$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$

$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω :

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250,000$$

$$\therefore \omega = 500 \text{ rad/s}$$

$$[\mathbf{b}] Z_{ab}(500) = -j100 + \frac{(200)(j200)}{200 + j200} = 100 \Omega$$

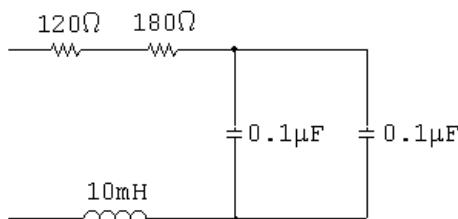
P 9.25 [a] $R = 300 \Omega = 120 \Omega + 180 \Omega$

$$\omega L - \frac{1}{\omega C} = -400 \quad \text{so} \quad 10,000L - \frac{1}{10,000C} = -400$$

Choose $L = 10 \text{ mH}$. Then,

$$\frac{1}{10,000C} = 100 + 400 \quad \text{so} \quad C = \frac{1}{10,000(500)} = 0.2 \mu\text{F}$$

We can achieve the desired capacitance by combining two $0.1 \mu\text{F}$ capacitors in parallel. The final circuit is shown here:



$$[\mathbf{b}] 0.01\omega = \frac{1}{\omega(0.2 \times 10^{-6})} \quad \text{so} \quad \omega^2 = \frac{1}{0.01(0.2 \times 10^{-6})} = 5 \times 10^8$$

$$\therefore \omega = 22,360.7 \text{ rad/s}$$

P 9.26 [a] Using the notation and results from Problem 9.19:

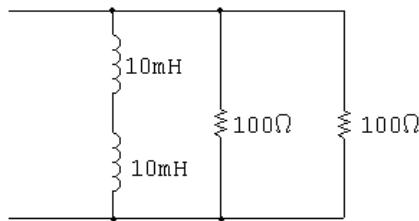
$$R \parallel L = 40 + j20 \quad \text{so} \quad R_1 = 40, \quad L_1 = \frac{20}{5000} = 4 \text{ mH}$$

$$R_2 = \frac{40^2 + 5000^2(0.004)^2}{40} = 50 \Omega$$

$$L_2 = \frac{40^2 + 5000^2(0.004)^2}{5000^2(0.004)} = 20 \text{ mH}$$

$$R_2 \parallel j\omega L_2 = 50 \parallel j100 = 40 + j20 \Omega \quad (\text{checks})$$

The circuit, using combinations of components from Appendix H, is shown here:



- [b] Using the notation and results from Problem 9.21:

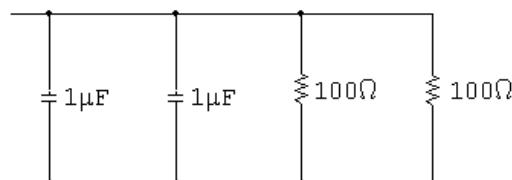
$$R \parallel C = 40 - j20 \quad \text{so} \quad R_1 = 40, \quad C_1 = 10 \mu F$$

$$R_2 = \frac{1 + 5000^2(40)^2(10 \mu)^2}{5000^2(40)(10 \mu)^2} = 50 \Omega$$

$$C_2 = \frac{10 \mu}{1 + 5000^2(40)^2(10 \mu)^2} = 2 \mu F$$

$$R_2 \parallel (-j/\omega C_2) = 50 \parallel (-j100) = 40 - j20 \Omega \quad (\text{checks})$$

The circuit, using combinations of components from Appendix H, is shown here:



P 9.27 [a] $(40 + j20) \parallel (-j/\omega C) = 50 \parallel j100 \parallel (-j/\omega C)$

To cancel out the $j100 \Omega$ impedance, the capacitive impedance must be $-j100 \Omega$:

$$\frac{-j}{5000C} = -j100 \quad \text{so} \quad C = \frac{1}{(100)(5000)} = 2 \mu F$$

Check:

$$R \parallel j\omega L \parallel (-j/\omega C) = 50 \parallel j100 \parallel (-j100) = 50 \Omega$$

Create the equivalent of a $2 \mu F$ capacitor from components in Appendix H by combining two $1 \mu F$ capacitors in parallel.

[b] $(40 - j20) \parallel (j\omega L) = 50 \parallel (-j100) \parallel (j\omega L)$

To cancel out the $-j100 \Omega$ impedance, the inductive impedance must be $j100 \Omega$:

$$j5000L = j100 \quad \text{so} \quad L = \frac{100}{5000} = 20 \text{ mH}$$

Check:

$$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50 \Omega$$

Create the equivalent of a 20 mH inductor from components in Appendix H by combining two 10 mH inductors in series.

$$\text{P 9.28 } Z = 4000 + j(2000)(0.5) - j\frac{1}{(2000)(100 \times 10^{-9})}$$

$$= 3000 + j1000 - j5000 = 3000 - j4000 \Omega$$

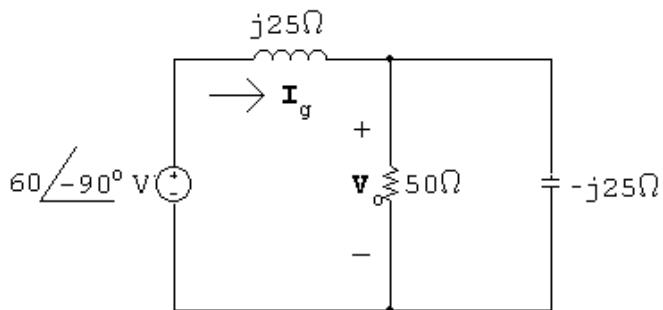
$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{80/0^\circ}{3000 - j4000} = 9.6 + j12.8 = 16/53.13^\circ \text{ mA}$$

$$i_o(t) = 16 \cos(2000t + 53.13^\circ) \text{ mA}$$

$$\text{P 9.29 } \frac{1}{j\omega C} = \frac{1}{j(5 \times 10^{-6})(8000)} = -j25 \Omega$$

$$j\omega L = j8000(3.125 \times 10^{-3}) = j25 \Omega$$

$$\mathbf{V}_g = 60/-90^\circ \text{ V}$$



$$Z_e = j25 + (50\| -j25) = 10 + j5 \Omega$$

$$\mathbf{I}_g = \frac{60/-90^\circ}{10 + j5} = -j2.4 - j4.8 \text{ A}$$

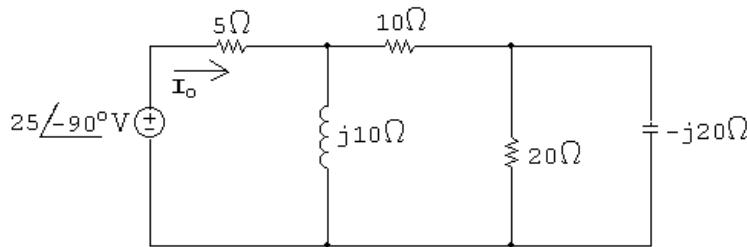
$$\mathbf{V}_o = (50\| -j25)\mathbf{I}_g = (10 - j20)(-j2.4 - j4.8) = -120 \text{ V}$$

$$v_o = -120 \cos 8000t \text{ V}$$

P 9.30 $\mathbf{V}_s = 25/\underline{-90^\circ} \text{ V}$

$$\frac{1}{j\omega C} = -j20 \Omega$$

$$j\omega L = j10 \Omega$$



$$Z_{\text{eq}} = 5 + j10 \parallel (10 + 20 \parallel -j20) = 10 + j10 \Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_{\text{eq}}} = \frac{25/\underline{-90^\circ}}{10 + j10} = -1.25 - j1.25 = 1.77/\underline{-135^\circ} \text{ A}$$

$$i_o = 1.77 \cos(4000t - 135^\circ) \text{ A}$$

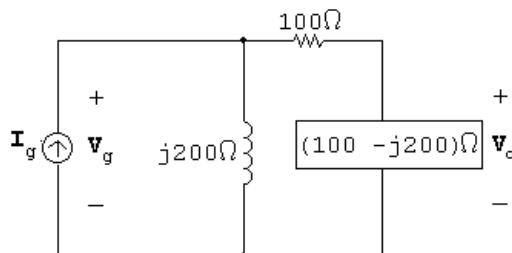
P 9.31 [a] $\frac{1}{j\omega C} = -j250 \Omega$

$$j\omega L = j200 \Omega$$

$$Z_e = j200 \parallel (100 + 500 \parallel -j250) = 200 + j200 \Omega$$

$$\mathbf{I}_g = 0.025/\underline{0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 0.025(200 + j200) = 5 + j5 \text{ V}$$



$$\mathbf{V}_o = \frac{100 - j200}{200 - j200}(5 + j5) = 5 + j2.5 = 5.59/\underline{26.57^\circ} \text{ V}$$

$$v_o = 5.59 \cos(50,000t + 26.57^\circ) \text{ V}$$

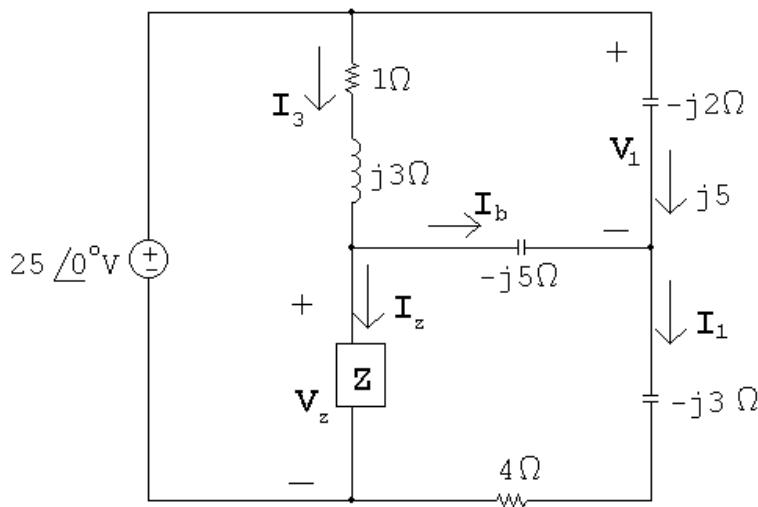
$$[\text{b}] \quad \omega = 2\pi f = 50,000; \quad f = \frac{25,000}{\pi}$$

$$T = \frac{1}{f} = \frac{\pi}{25,000} = 40\pi \mu\text{s}$$

$$\therefore \frac{26.57}{360}(40\pi \mu\text{s}) = 9.27 \mu\text{s}$$

$\therefore v_o$ leads i_g by $9.27 \mu\text{s}$.

P 9.32



$$\mathbf{V}_1 = j5(-j2) = 10 \text{ V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8 \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2 \text{ A}$$

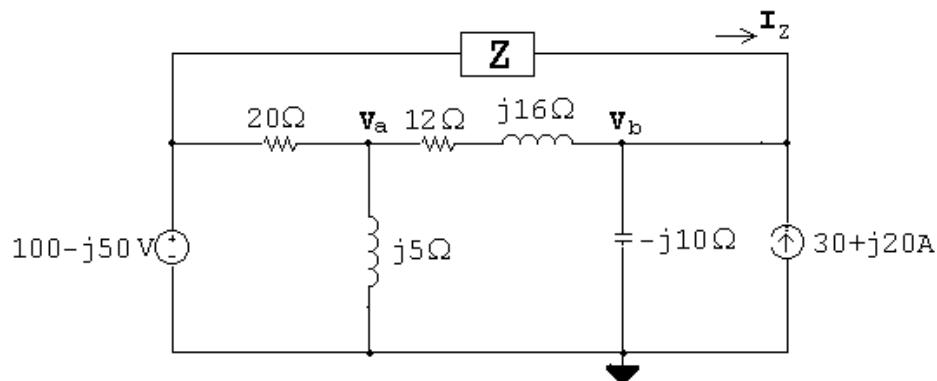
$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12 \text{ V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6 \text{ A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4 \text{ A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88 \Omega$$

P 9.33



$$\frac{\mathbf{V}_a - (100 - j50)}{20} + \frac{\mathbf{V}_a}{j5} + \frac{\mathbf{V}_a - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_a = 40 + j30 \text{ V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

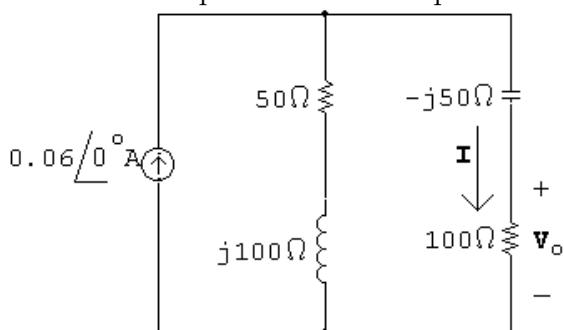
$$\mathbf{I}_Z = -30 - j10 \text{ A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2 \Omega$$

P 9.34 $Z_L = j(10,000)(10 \times 10^{-3}) = j100 \Omega$

$$Z_C = \frac{-j}{(10,000)(2 \times 10^{-6})} = -j50 \Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(50 + j100)}{50 + j100 + 100 - j50} (0.06) = 30 + j30 \text{ mA}$$

$$\mathbf{V}_o = 100\mathbf{I} = 3 + j3 = 4.24/45^\circ$$

$$v_o = 4.24 \cos(10,000t + 45^\circ) \text{ V}$$

P 9.35 $\mathbf{V}_g = 40/-15^\circ \text{ V}; \quad \mathbf{I}_g = 40/-68.13^\circ \text{ mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000/53.13^\circ \Omega = 600 + j800 \Omega$$

$$Z = 600 + j \left(3.2\omega - \frac{0.4 \times 10^6}{\omega} \right)$$

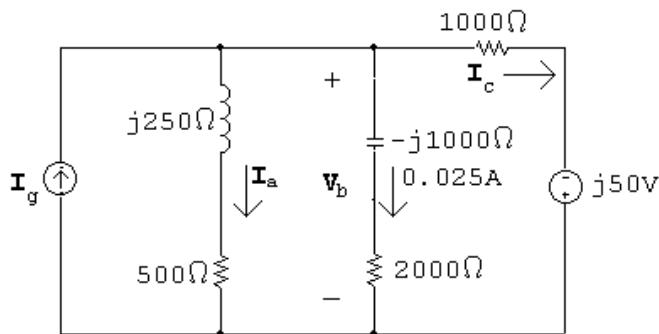
$$\therefore 3.2\omega - \frac{0.4 \times 10^6}{\omega} = 800$$

$$\therefore \omega^2 - 250\omega - 125,000 = 0$$

Solving,

$$\omega = 500 \text{ rad/s}$$

P 9.36 [a]



$$\mathbf{V}_b = (2000 - j1000)(0.025) = 50 - j25 \text{ V}$$

$$\mathbf{I}_a = \frac{50 - j25}{500 + j250} = 60 - j80 \text{ mA} = 100/-53.13^\circ \text{ mA}$$

$$\mathbf{I}_c = \frac{50 - j25 + j50}{1000} = 50 + j25 \text{ mA} = 55.9/26.57^\circ \text{ mA}$$

$$\mathbf{I}_g = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 135 - j55 \text{ mA} = 145.77/-22.17^\circ \text{ mA}$$

[b] $i_a = 100 \cos(1500t - 53.13^\circ)$ mA

$$i_c = 55.9 \cos(1500t + 26.57^\circ)$$
 mA

$$i_g = 145.77 \cos(1500t - 22.17^\circ)$$
 mA

- P 9.37 [a] In order for v_g and i_g to be in phase, the impedance to the right of the 500Ω resistor must be purely real:

$$\begin{aligned} Z_{eq} &= j\omega L \parallel (R + 1/\omega C) = \frac{j\omega L(R + 1/j\omega C)}{j\omega L + R + 1/j\omega C} \\ &= \frac{j\omega L(j\omega RC + 1)}{j\omega RC - \omega^2 LC + 1} \\ &= \frac{(-\omega^2 RLC + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC + j\omega RC)(1 - \omega^2 LC - j\omega RC)} \end{aligned}$$

The denominator of the above expression is purely real. Now set the imaginary part of the numerator in that expression to zero and solve for ω :

$$\omega L(1 - \omega^2 LC) + \omega^3 R^2 lC^2 = 0$$

$$\text{So } \omega^2 = \frac{1}{LC - R^2 C^2} = \frac{1}{(0.2)(10^{-6}) - 200^2 (10^{-6})^2} = 6,250,000$$

$$\therefore \omega = 2500 \text{ rad/s} \quad \text{and} \quad f = 397.9 \text{ Hz}$$

[b] $Z_{eq} = 500 + j500 \parallel (200 - j400) = 1500\Omega$

$$\mathbf{I}_g = \frac{90/0^\circ}{1500} = 60/0^\circ$$
 mA

$$i_g(t) = 60 \cos 2500t$$
 mA

- P 9.38 [a] For i_g and v_g to be in phase, the impedance to the right of the 480Ω resistor must be purely real.

$$\begin{aligned} j\omega L + \frac{(1/j\omega C)(200)}{(1/j\omega C) + 200} &= j\omega L + \frac{200}{1 + 200j\omega C} \\ &= j\omega L + \frac{200(1 - 200j\omega C)}{1 + 200^2 \omega^2 C^2} \\ &\underline{\underline{j\omega L(1 + 200^2 \omega^2 C^2) + 200(1 - 200j\omega C)}} \\ &\qquad\qquad\qquad 1 + 200^2 \omega^2 C^2 \end{aligned}$$

In the above expression the denominator is purely real. So set the imaginary part of the numerator to zero and solve for ω :

$$\omega L(1 + 200^2 \omega^2 C^2) - 200^2 \omega C = 0$$

$$\omega^2 = \frac{200^2 C - L}{200^2 C} = \frac{200^2 (3.125 \times 10^{-6}) - 0.1}{200^2 (3.125 \times 10^{-6})^2} = 64,000$$

$$\therefore \omega = 800 \text{ rad/s}$$

[b] When $\omega = 800 \text{ rad/s}$

$$Z_g = 480 \| [j80 + (200\| - j400)] = 120 \Omega$$

$$\therefore \mathbf{V}_g = Z_g \mathbf{I}_g = 120(0.06) = 7.2 \text{ V}$$

$$v_o = 7.2 \cos 800t \text{ V}$$

- P 9.39 [a] The voltage and current are in phase when the impedance to the left of the $1 \text{ k}\omega$ resistor is purely real:

$$\begin{aligned} Z_{\text{eq}} &= \frac{1}{j\omega C} \| (R + j\omega L) = \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC} \\ &= \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 + \omega^2 LC)^2 + \omega^2 R^2 C^2} \end{aligned}$$

The denominator in the above expression is purely real, so set the imaginary part of the expression's numerator to zero and solve for ω :

$$-\omega R^2 C + \omega L - \omega^3 L^2 C = 0$$

$$\omega^2 = \frac{L - R^2 C}{L^2 C} = \frac{(0.01) - 240^2 (62.5 \times 10^{-9})}{(0.01)^2 (62.5 \times 10^{-9})^2} = 1024 \times 10^6$$

$$\therefore \omega = 32,000 \text{ rad/s}$$

[b] $Z_t = 1000 + (-j500) \| (240 + j320) = 1666.67 \Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_T} = \frac{15/\underline{0^\circ}}{1666.67} = 9/\underline{0^\circ} \text{ mA}$$

$$i_o = 9 \cos 32,000t \text{ mA}$$

- P 9.40 [a] $Z_C = \frac{-j}{(1000)(10^{-6})} = -j1000 \Omega$

$$Z_1 = 2500 \| j\omega L = \frac{2500 j\omega L}{2500 + j\omega L} = \frac{j2500\omega L(2500 - j\omega L)}{2500^2 + \omega^2 L^2}$$

$$Z_T = 500 + Z_C + Z_1 = 500 - j1000 + \frac{2500\omega^2 L^2 + 2500^2 j\omega L}{2500^2 + \omega^2 L^2}$$

Z_T is resistive when

$$\frac{2500^2 \omega L}{2500^2 + \omega^2 L^2} = 1000 \quad \text{or}$$

$$L^2(1000\omega^2) - L(2500^2\omega) + 1000(2500^2) = 0$$

For $\omega = 1000$:

$$L^2 - 6.25L + 6.25 = 0$$

Solving, $L_1 = 5$ H and $L_2 = 1.25$ H.

[b] When $L = 5$ H:

$$Z_T = 500 - j1000 + 2500||j5000 = 2500 \Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{2500} = 16/0^\circ \text{ mA}$$

$$i_g = 16 \cos 1000t \text{ mA}$$

When $L = 1.25$ H:

$$Z_T = 500 - j1000 + 2500||j1250 = 1000 \Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{1000} = 40/0^\circ \text{ mA}$$

$$i_g = 40 \cos 1000t \text{ mA}$$

$$\text{P 9.41 [a]} \quad Z_p = \frac{\frac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC}$$

$$\begin{aligned} &= \frac{10,000}{1 + j(5000)(10,000)C} = \frac{10,000}{1 + j50 \times 10^6 C} \\ &= \frac{10,000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2} \end{aligned}$$

$$= \frac{10,000}{1 + 25 \times 10^{14} C^2} - j \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$j\omega L = j5000(0.8) = j4000$$

$$\therefore 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$\therefore 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$$

$$\therefore C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$$

Solving,

$$C_1 = 40 \text{ nF} \quad C_2 = 10 \text{ nF}$$

$$[b] R_e = \frac{10,000}{1 + 25 \times 10^{14} C^2}$$

When $C = 40 \text{ nF}$ $R_e = 2000 \Omega$;

$$\mathbf{I}_g = \frac{80/0^\circ}{2000} = 40/0^\circ \text{ mA}; \quad i_g = 40 \cos 5000t \text{ mA}$$

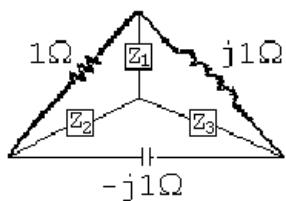
When $C = 10 \text{ nF}$ $R_e = 8000 \Omega$;

$$\mathbf{I}_g = \frac{80/0^\circ}{8000} = 10/0^\circ \text{ mA}; \quad i_g = 10 \cos 5000t \text{ mA}$$

P 9.42 Simplify the top triangle using series and parallel combinations:

$$(1 + j1) \parallel (1 - j1) = 1 \Omega$$

Convert the lower left delta to a wye:

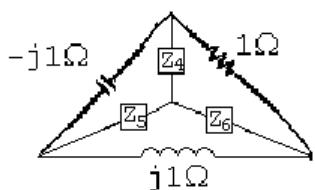


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1 \Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1 \Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1 \Omega$$

Convert the lower right delta to a wye:

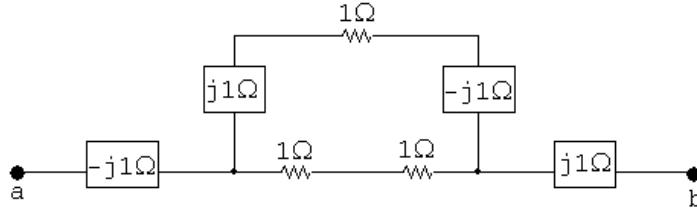


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1 \Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1 \Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1 \Omega$$

The resulting circuit is shown below:



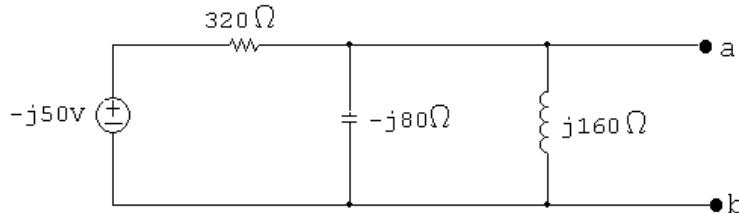
Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1) \parallel (1 + 1) = 1 \parallel 2 = 2/3 \Omega$$

$$Z_{ab} = -j1 + 2/3 + j1 = 2/3 \Omega$$

P 9.43 [a] $j\omega L = j(400)(400) \times 10^{-3} = j160 \Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(31.25 \times 10^{-6})} = -j80 \Omega$$



Using voltage division,

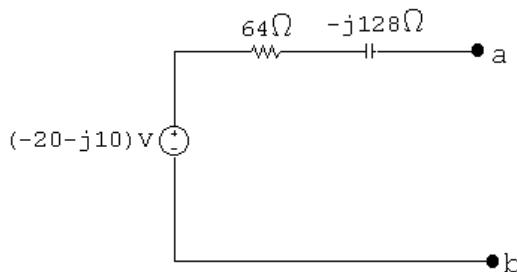
$$V_{ab} = \frac{-j80 \parallel j160}{320 + (-j80 \parallel j160)} (-j50) = -20 - j10 \text{ V}$$

$$V_{Th} = V_{ab} = -20 - j10 \text{ V}$$

- [b] Remove the voltage source and combine impedances in parallel to find $Z_{Th} = Z_{ab}$:

$$Z_{Th} = Z_{ab} = 320 \parallel -j80 \parallel j160 = 64 - j128 \Omega$$

[c]



P 9.44 Step 1 to Step 2:

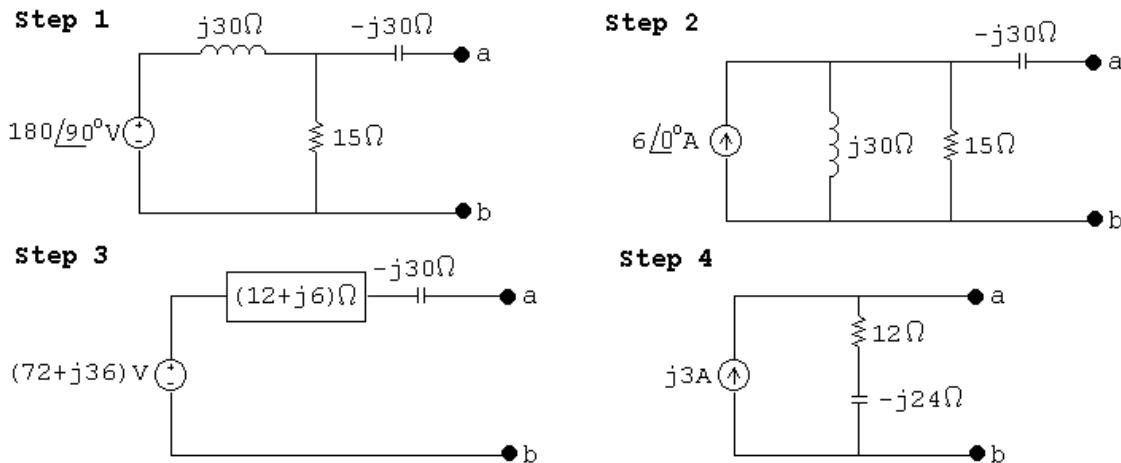
$$\frac{180/90^\circ}{j30} = 6/0^\circ \text{ A}$$

Step 2 to Step 3:

$$(j30)\parallel 15 = 12 + j6 \Omega; \quad (6/0^\circ)(12 + j6) = 72 + j36 \text{ V}$$

Step 3 to Step 4:

$$Z_N = 12 + j6 - j30 = 12 - j24 \Omega; \quad I_N = \frac{72 + j36}{12 - j24} = j3 \text{ A}$$



P 9.45 Step 1 to Step 2:

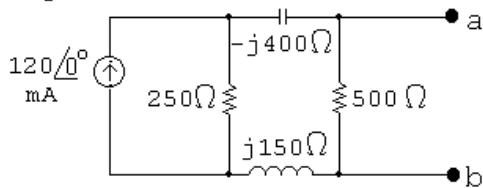
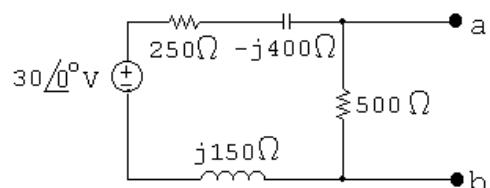
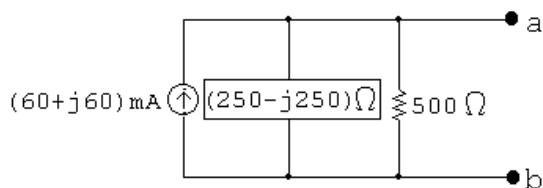
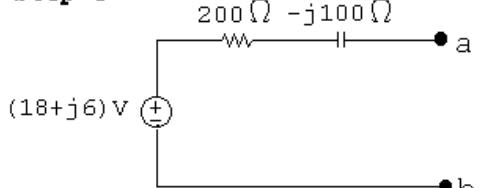
$$(0.12/0^\circ)(250) = 30/0^\circ \text{ V}$$

Step 2 to Step 3:

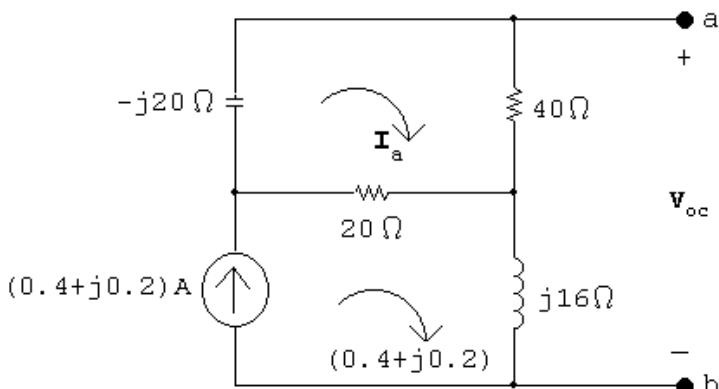
$$250 - j400 + j150 = 250 - j250 \Omega; \quad \frac{30/0^\circ}{250 - j250} = 60 - j60 \text{ mA}$$

Step 3 to Step 4:

$$(250 - j250)\parallel 500 = 200 - j100 \Omega; \quad (200 - j100)(0.06 - j0.06) = 18 - j6 \text{ V}$$

Step 1**Step 2****Step 3****Step 4**

P 9.46 Open circuit voltage:



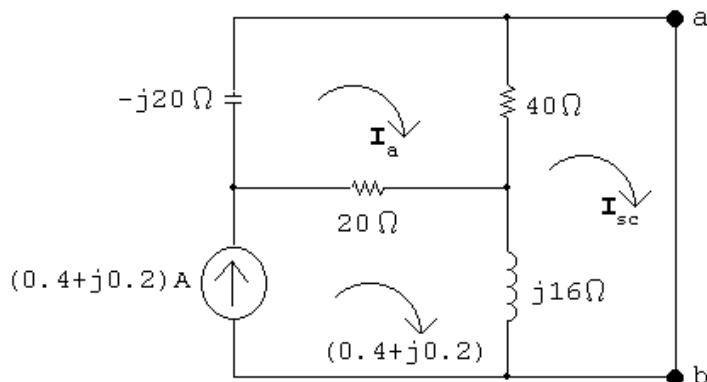
$$-j20I_a + 40I_a + 20(I_a - 0.4 - j0.2) = 0$$

Solving,

$$I_a = \frac{20(0.4 + j0.2)}{60 - j20} = 0.1 + j0.1 \text{ A}$$

$$V_{oc} = 40I_a + j16(0.4 + j0.2) = 0.8 + j10.4 \text{ V}$$

Short circuit current:



$$-j20I_a + 40(I_a - I_{sc}) + 20(I_a - 0.4 - j0.2) = 0$$

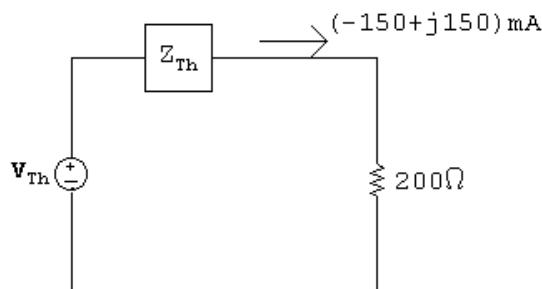
$$40(I_{sc} - I_a) + j16(I_{sc} - 0.4 - j0.2) = 0$$

Solving,

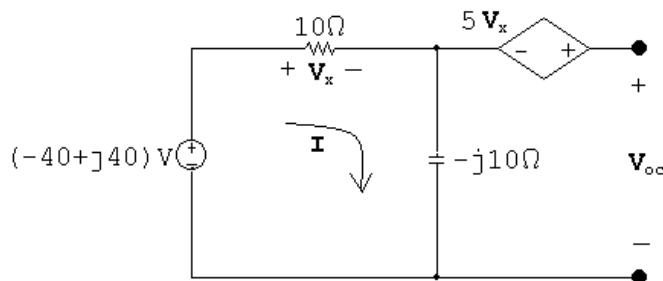
$$\mathbf{I}_{sc} = 0.3 + j0.5 \text{ A}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{0.8 + j10.4}{0.3 + j0.5} = 16 + j8 \Omega$$

P 9.47



$$\mathbf{V}_{Th} = (-0.15 + j0.15)Z_{Th} + (-30 + j30)$$



$$\frac{j200}{Z_{Th} + j200} \mathbf{V}_{Th} = -40 - j40 \quad \text{so} \quad -j200\mathbf{V}_{Th} + (-40 - j40)(Z_{Th} + j200)$$

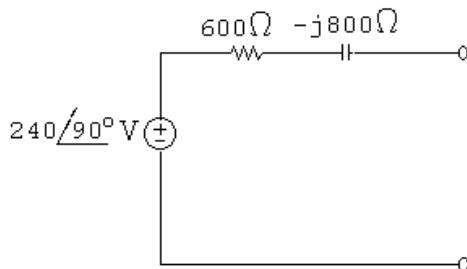
Place the two equations in standard form:

$$\mathbf{V}_{\text{Th}} + (0.15 - j0.15)Z_{\text{Th}} = -30 - j30$$

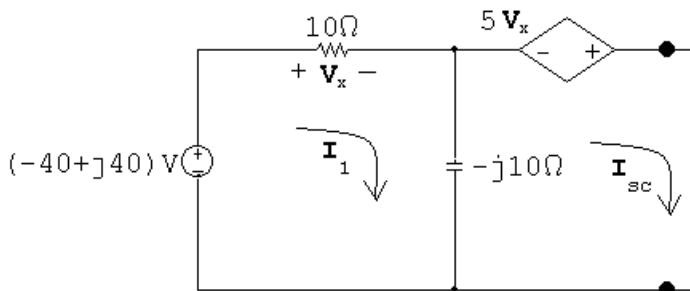
$$j200\mathbf{V}_{\text{Th}} + (40 + j40)C_{\text{Th}} = (-40 - j40)(j200)$$

Solving,

$$\mathbf{V}_{\text{Th}} = j240 \text{ V}; \quad Z_{\text{Th}} = 600 - j800 \Omega$$



P 9.48 Short circuit current



$$\mathbf{V}_x - j10(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) = -40 + j40$$

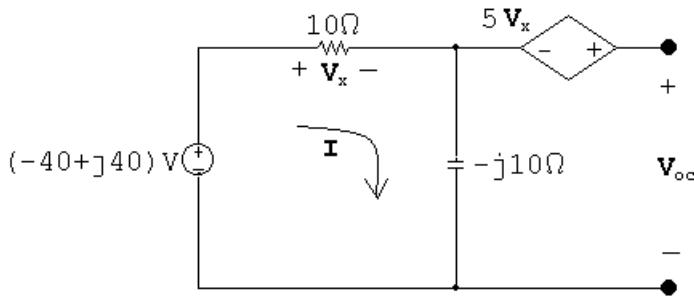
$$-5\mathbf{V}_x - j10(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) = 0$$

$$\mathbf{V}_x = 10\mathbf{I}_1$$

Solving,

$$\mathbf{I}_N = \mathbf{I}_{\text{sc}} = 6 + j4 \text{ A}$$

Open circuit voltage



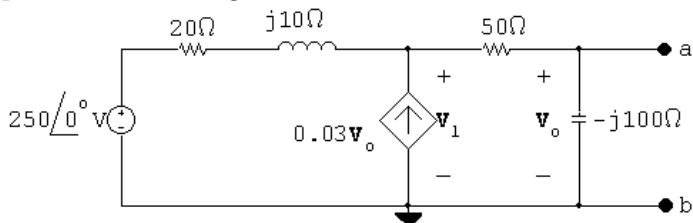
$$I = \frac{-40 + j40}{10 - j10} = -4 \text{ A}$$

$$\mathbf{V}_x = 10\mathbf{I} = -40 \text{ V}$$

$$\mathbf{V}_{oc} = 5\mathbf{V}_x - j10\mathbf{I} = -200 + j40 \text{ V}$$

$$Z_N = \frac{-200 + j40}{6 + j4} = -20 + j20 \Omega$$

P 9.49 Open circuit voltage:



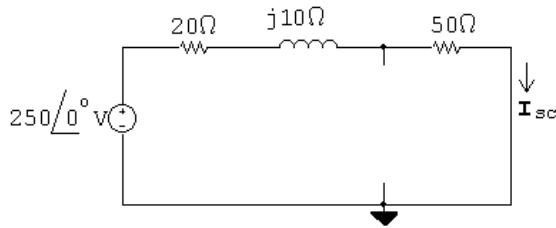
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \mathbf{V}_o = \frac{-j100}{50 - j100} \mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250 \text{ V}; \quad \mathbf{V}_o = 300 - j400 \text{ V} = \mathbf{V}_{Th}$$

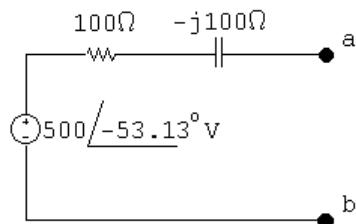
Short circuit current:



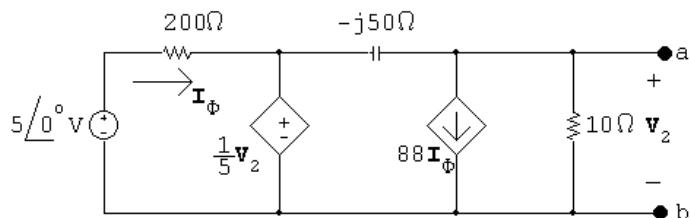
$$\mathbf{I}_{sc} = \frac{250\angle 0^\circ}{70 + j10} = 3.5 - j0.5 \text{ A}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100 \Omega$$

The Thévenin equivalent circuit:



P 9.50 Open circuit voltage:



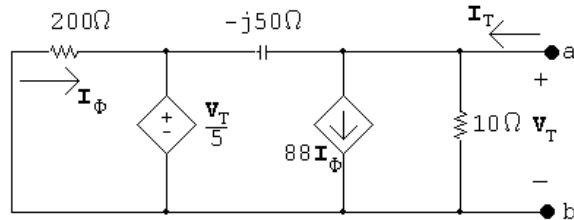
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\angle 126.87^\circ \text{ V} = \mathbf{V}_{Th}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

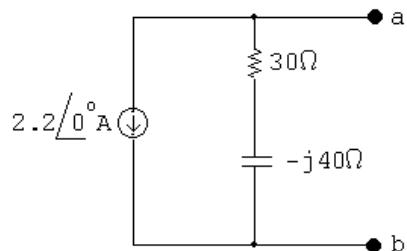
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T \left(\frac{1}{10} - 88 \frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50} \right)$$

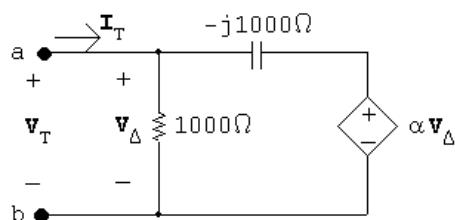
$$\therefore \quad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{Th}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{Th}}{Z_{Th}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0 \text{ A}$$

The Norton equivalent circuit:



P 9.51 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j-1+\alpha}{j1000}$$

$$\therefore Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000 \Omega$

$$[c] Z_{\text{Th}} = 500 - j500 = \frac{j1000}{\alpha - 1 + j}$$

$$= \frac{1000}{(\alpha - 1)^2 + 1} + j \frac{1000(\alpha - 1)}{(\alpha - 1)^2 + 1}$$

Equate the real parts:

$$\frac{1000}{(\alpha - 1)^2 + 1} = 500 \quad \therefore (\alpha - 1)^2 + 1 = 2$$

$$\therefore (\alpha - 1)^2 = 1 \quad \text{so } \alpha = 0$$

Check the imaginary parts:

$$\left. \frac{(\alpha - 1)1000}{(\alpha - 1)^2 + 1} \right|_{\alpha=1} = -500$$

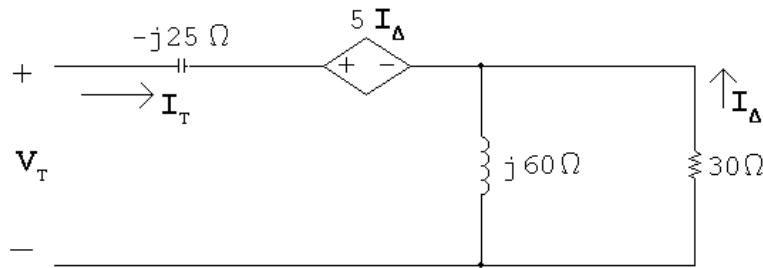
Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \frac{1000}{(\alpha - 1)^2 + 1} + j \frac{1000(\alpha - 1)}{(\alpha - 1)^2 + 1}$

For $\text{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \leq 10$.

P 9.52 $j\omega L = j100 \times 10^3 (0.6 \times 10^{-3}) = j60 \Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25 \Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_{\Delta} - 30\mathbf{I}_{\Delta}$$

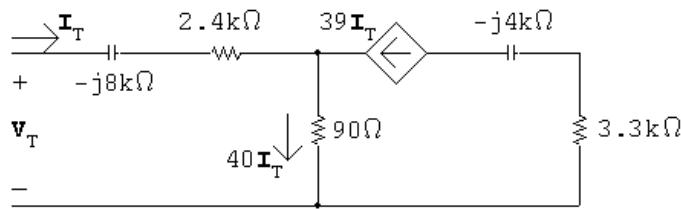
$$\mathbf{I}_{\Delta} = \frac{-j60}{30 + j60} \mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25 \frac{j60}{30 + j60} \mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{ab} = 20 - j15 = 25/\underline{-36.87^\circ} \Omega$$

P 9.53 $\frac{1}{\omega C_1} = \frac{10^9}{50,000(2.5)} = 8 \text{ k}\Omega$

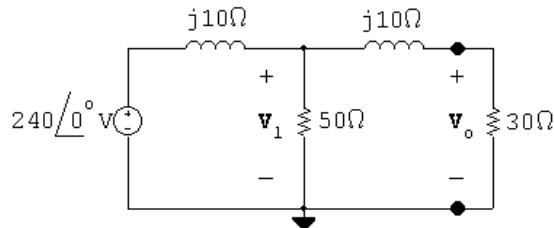
$$\frac{1}{\omega C_2} = \frac{10^9}{50,000(5)} = 4 \text{ k}\Omega$$



$$\mathbf{V}_T = (2400 - j8000)\mathbf{I}_T + 40\mathbf{I}_T(90)$$

$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 6000 - j8000 \Omega$$

P 9.54



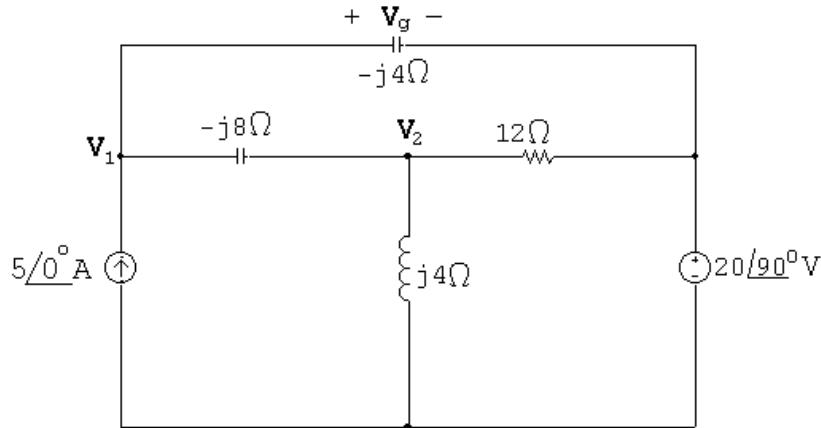
$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for \mathbf{V}_1 yields

$$\mathbf{V}_1 = 198.63/\underline{-24.44^\circ} \text{ V}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43/\underline{-42.88^\circ} \text{ V}$$

P 9.55 Set up the frequency domain circuit to use the node voltage method:



$$\text{At } \mathbf{V}_1: \quad -5\angle 0^\circ + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\angle 90^\circ}{-j4} = 0$$

$$\text{At } \mathbf{V}_2: \quad \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\angle 90^\circ}{12} = 0$$

In standard form:

$$\mathbf{V}_1 \left(\frac{1}{-j8} + \frac{1}{-j4} \right) + \mathbf{V}_2 \left(-\frac{1}{-j8} \right) = 5\angle 0^\circ + \frac{20\angle 90^\circ}{-j4}$$

$$\mathbf{V}_1 \left(-\frac{1}{-j8} \right) + \mathbf{V}_2 \left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12} \right) = \frac{20\angle 90^\circ}{12}$$

Solving on a calculator:

$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \quad \mathbf{V}_2 = -8 + j4$$

Thus

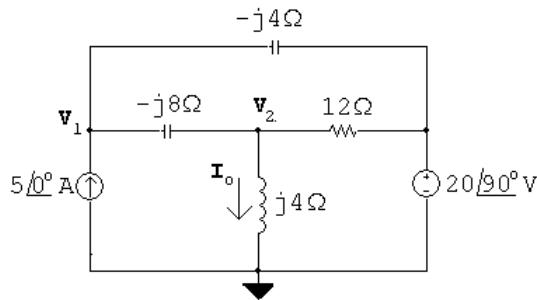
$$\mathbf{V}_g = \mathbf{V}_1 - 20\angle 90^\circ = -\frac{8}{3} - j\frac{56}{3} \text{ V}$$

$$\text{P 9.56} \quad j\omega L = j(2500)(1.6 \times 10^{-3}) = j4 \Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(2500)(100 \times 10^{-6})} = -j4 \Omega$$

$$\mathbf{I}_g = 5\angle 0^\circ \text{ A}$$

$$\mathbf{V}_g = 20/90^\circ \text{ V}$$



$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - j20}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - j20}{12} = 0$$

Solving,

$$\mathbf{V}_2 = -8 + j4 \text{ V}; \quad \mathbf{I}_o = \frac{\mathbf{V}_2}{j4} = 1 + j2 = 2.24/63.43^\circ \text{ A}$$

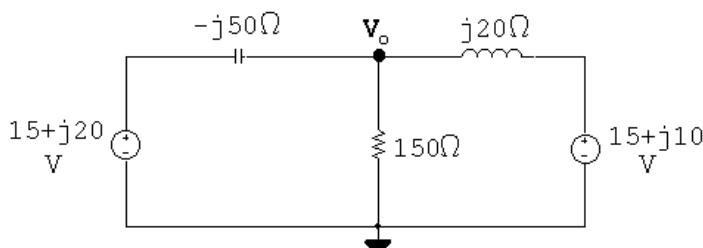
$$i_o = 2.24 \cos(2500t + 63.43^\circ) \text{ A}$$

$$\text{P 9.57} \quad j\omega L = j(400)(50 \times 10^{-3}) = j20 \Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(50 \times 10^{-6})} = -j50 \Omega$$

$$\mathbf{V}_{g1} = 25/53.13^\circ = 15 + j20 \text{ V}$$

$$\mathbf{V}_{g2} = 18.03/33.69^\circ = 15 + j10 \text{ V}$$



$$\frac{\mathbf{V}_o - (15 + j20)}{-j50} + \frac{\mathbf{V}_o}{150} + \frac{\mathbf{V}_o - (15 + j10)}{j20} = 0$$

Solving,

$$\mathbf{V}_o = 15/0^\circ$$

$$v_o(t) = 15 \cos 400t \text{ V}$$

P 9.58 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

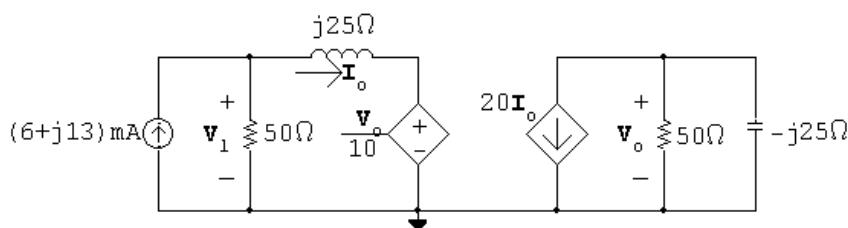
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80/\underline{90^\circ} \text{ V}$$

P 9.59



$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

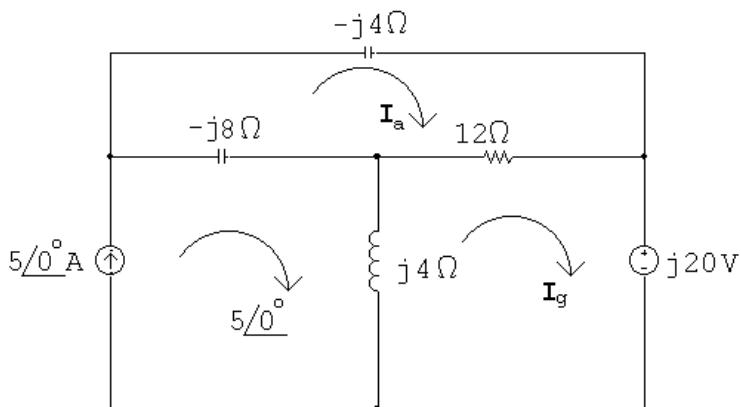
$$\therefore \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10/\underline{0^\circ} \text{ mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47/\underline{116.57^\circ} \text{ V}$$

P 9.60



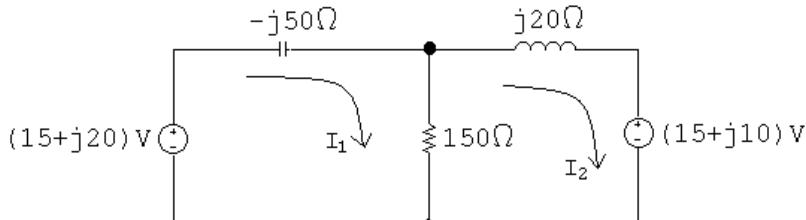
$$(12 - j12)\mathbf{I}_a - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47/\underline{-26.57^\circ} \text{ A}$$

P 9.61 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-(15 + j20) - j50\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$15 + j10 + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(150 - j50) + \mathbf{I}_2(-150) = 15 + j20$$

$$\mathbf{I}_1(-150) + \mathbf{I}_2(150 + j20) = -(15 + j10)$$

Solving on a calculator yields:

$$\mathbf{I}_1 = -0.4 \text{ A}; \quad \mathbf{I}_2 = -0.5 \text{ A}$$

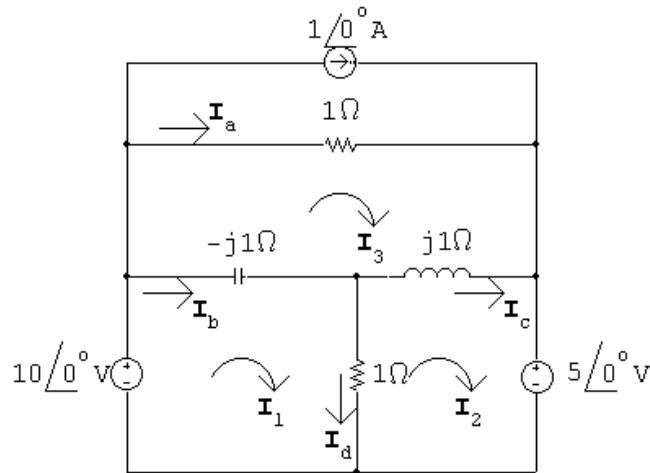
Thus,

$$\mathbf{V}_o = 150(\mathbf{I}_1 - \mathbf{I}_2) = 15 \text{ V}$$

and

$$v_o(t) = 15 \cos 400t \text{ V}$$

P 9.62



$$10\angle 0^\circ = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\angle 0^\circ = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 11 + j10 \text{ A}; \quad \mathbf{I}_2 = 11 + j5 \text{ A}; \quad \mathbf{I}_3 = 6 \text{ A}$$

$$\mathbf{I}_a = \mathbf{I}_3 - 1 = 5 \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10 \text{ A}$$

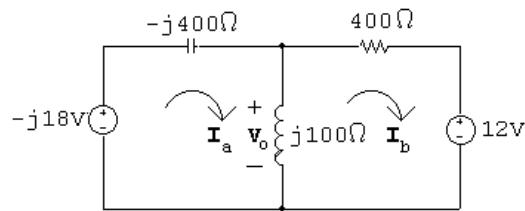
$$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5 \text{ A}$$

$$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = j5 \text{ A}$$

$$P\ 9.63 \quad \mathbf{V}_a = -j18\text{ V}; \quad \mathbf{V}_b = 12\text{ V}$$

$$j\omega L = j(4000)(25 \times 10^{-3}) = j100\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{4000(625 \times 10^{-6})} = -j400\Omega$$



$$-j18 = -j300\mathbf{I}_a - j100\mathbf{I}_b$$

$$-12 = -j100\mathbf{I}_a + (400 + j100)\mathbf{I}_b$$

Solving,

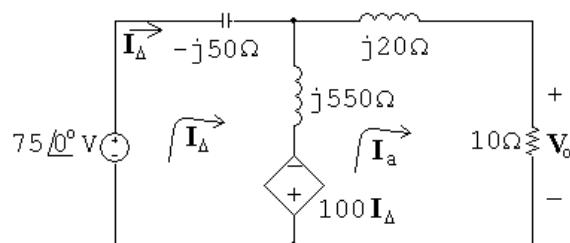
$$\mathbf{I}_a = 67.5 - j7.5\text{ mA}; \quad \mathbf{I}_b = -22.5 + j22.5\text{ mA}$$

$$\mathbf{V}_o = j100(\mathbf{I}_a - \mathbf{I}_b) = 3 + j9 = 9.49\angle 71.57^\circ \text{ A}$$

$$v_o(t) = 9.49 \cos(4000t + 71.57^\circ) \text{ A}$$

$$P\ 9.64 \quad j\omega L_1 = j5000(4 \times 10^{-3}) = j20\Omega; \quad j\omega L_2 = j5000(110 \times 10^{-3}) = j550\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(4 \times 10^{-6})} = -j50\Omega$$



$$75\angle 0^\circ = j500\mathbf{I}_\Delta - 100\mathbf{I}_\Delta - j550\mathbf{I}_a$$

$$0 = (10 + j20)\mathbf{I}_a + 100\mathbf{I}_\Delta + j550(\mathbf{I}_a - \mathbf{I}_\Delta)$$

Solving,

$$\mathbf{I}_a = j2.5 \text{ A}$$

$$\mathbf{V}_o = 10\mathbf{I}_a = j25 = 25/\underline{90^\circ}$$

$$v_o = 25 \cos(5000t - 90^\circ) = 25 \sin 5000t \text{ V}$$

$$\text{P 9.65} \quad \frac{1}{j\omega c} = \frac{-j}{(100,000)(3.125 \times 10^{-9})} = -j3200 \Omega$$

$$j\omega L = j(100,000)(80 \times 10^{-3}) - j8000 \Omega$$

Let

$$Z_1 = 1200 - j3200 \Omega; \quad Z_2 = 2400 + j8000 \Omega$$

$$\mathbf{V}_o = \frac{Z_2}{Z_1 + Z_2} \mathbf{V}_g = \frac{2400 + j8000}{(3600 + j4800)} (120) = 156.8 + j57.6 = 167.045/\underline{20.17^\circ}$$

$$\therefore v_o = 167.045 \cos(100,000t + 20.17^\circ) \text{ V}$$

$$\text{P 9.66} \quad \frac{1}{j\omega C} = \frac{-j}{(250)(100 \times 10^{-6})} = -j40 \Omega$$

$$j\omega L = j(250)(0.8) = j200 \Omega$$

$$\text{Let } Z_1 = 20 - j40 \Omega; \quad Z_2 = 100 + j200 \Omega$$

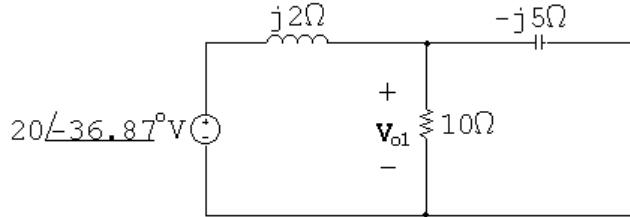
$$\mathbf{I}_g = 60/\underline{0^\circ} \text{ mA}$$

$$\begin{aligned} \mathbf{I}_o &= \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{(0.06)(20 - j40)}{(120 + j160)} \\ &= -6 - j12 \text{ mA} = 13.42/\underline{-116.57^\circ} \text{ mA} \end{aligned}$$

$$i_o = 13.42 \cos(250t - 116.57^\circ) \text{ mA}$$

- P 9.67 [a] Superposition must be used because the frequencies of the two sources are different.

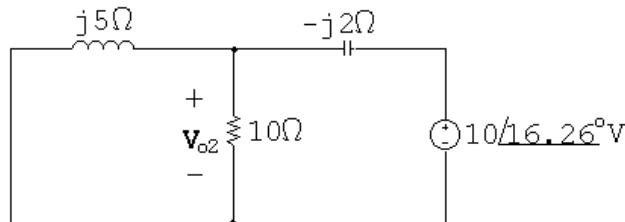
[b] For $\omega = 2000$ rad/s:



$$10\parallel -j5 = 2 - j4 \Omega$$

$$\text{so } \mathbf{V}_{o1} = \frac{2 - j4}{2 - j4 + j2} (20 \angle -36.87^\circ) = 31.62 \angle -55.3^\circ \text{ V}$$

For $\omega = 5000$ rad/s:



$$j5\parallel 10 = 2 + j4 \Omega$$

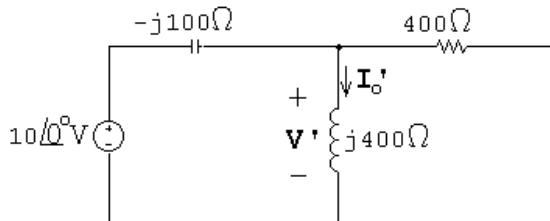
$$\mathbf{V}_{o2} = \frac{2 + j4}{2 + j4 - j2} (10 \angle 16.26^\circ) = 15.81 \angle 34.69^\circ \text{ V}$$

Thus,

$$v_o(t) = [31.62 \cos(2000t - 55.3^\circ) + 15.81 \cos(5000t + 34.69^\circ)] \text{ V}$$

P 9.68 [a] Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 16,000$ rad/s:



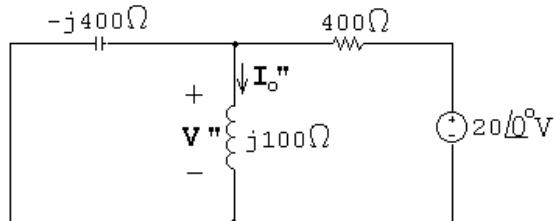
$$\frac{\mathbf{V}'_o - 10}{-j100} + \frac{\mathbf{V}'_o}{j400} + \frac{\mathbf{V}'_o}{400} = 0$$

$$\mathbf{V}'_o \left(\frac{1}{-j100} + \frac{1}{j400} + \frac{1}{400} \right) = \frac{10}{-j100}$$

$$\therefore \mathbf{V}'_o = 12 + j4 \text{ V}$$

$$\mathbf{I}'_o = \frac{\mathbf{V}'_o}{400} = 10 - j30 \text{ mA} = 31.62 \angle -71.57^\circ \text{ mA}$$

For $\omega = 4000 \text{ rad/s}$:



$$\frac{\mathbf{V}_o''}{-j400} + \frac{\mathbf{V}_o''}{j100} + \frac{\mathbf{V}_o'' - 20}{400} = 0$$

$$\mathbf{V}_o''(j - j4 + 1) = 20 \quad \text{so} \quad \mathbf{V}_o'' = \frac{20}{1 - j3} = 2 + j6 \text{ V}$$

$$\therefore \mathbf{I}_o'' = \frac{\mathbf{V}''}{j100} = \frac{2 + j6}{-j100} = 60 - j20 \text{ mA} = 63.25 \angle -18.43^\circ \text{ mA}$$

Thus,

$$i_o(t) = [31.26 \cos(16,000t - 71.57^\circ) + 63.25 \cos(4000t - 18.43^\circ)] \text{ mA}$$

$$\text{P 9.69 } \mathbf{V}_g = 20 \angle 0^\circ \text{ V}; \quad \frac{1}{j\omega C} = -j400 \Omega$$

Let \mathbf{V}_a = voltage across the capacitor, positive at upper terminal
Then:

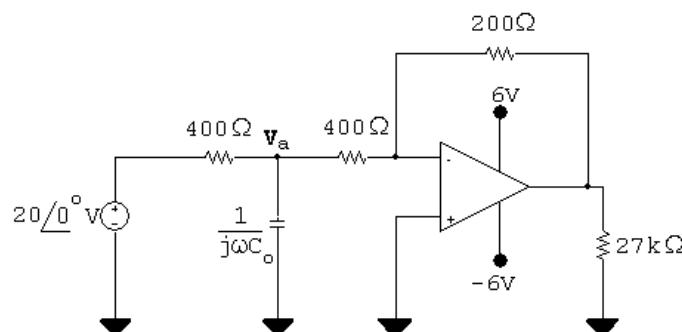
$$\frac{\mathbf{V}_a - 20 \angle 0^\circ}{400} + \frac{\mathbf{V}_a}{-j400} + \frac{\mathbf{V}_a}{400} = 0; \quad \therefore \mathbf{V}_a = (8 - j4) \text{ V}$$

$$\frac{0 - \mathbf{V}_a}{400} + \frac{0 - \mathbf{V}_o}{200} = 0; \quad \mathbf{V}_o = -\frac{\mathbf{V}_a}{2}$$

$$\therefore \mathbf{V}_o = -4 + j2 = 4.47 \angle 153.43^\circ \text{ V}$$

$$v_o = 4.47 \cos(5000t + 153.43^\circ) \text{ V}$$

P 9.70 [a]



$$\frac{\mathbf{V}_a - 20 \angle 0^\circ}{400} + j\omega C_o \mathbf{V}_a + \frac{\mathbf{V}_a}{400} = 0$$

$$\mathbf{V}_a = \frac{20}{2 + j400\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_a}{2}$$

$$\mathbf{V}_o = \frac{-10}{2 + j2 \times 10^6 C_o} = \frac{10/\underline{180^\circ}}{2 + j2 \times 10^6 C_o}$$

\therefore denominator angle = 45°

$$\text{so } 2 \times 10^6 C_o = 2 \quad \therefore \quad C = 1 \mu\text{F}$$

$$[\text{b}] \quad \mathbf{V}_o = \frac{10/\underline{180^\circ}}{2 + j2} = 3.54/\underline{135^\circ} \text{ V}$$

$$v_o = 3.54 \cos(5000t + 135^\circ) \text{ V}$$

$$\text{P 9.71} \quad [\text{a}] \quad \mathbf{V}_g = 25/\underline{0^\circ} \text{ V}$$

$$\mathbf{V}_p = \frac{20}{100} \mathbf{V}_g = 5/\underline{0^\circ}; \quad \mathbf{V}_n = \mathbf{V}_p = 5/\underline{0^\circ} \text{ V}$$

$$\frac{5}{80,000} + \frac{5 - \mathbf{V}_o}{Z_p} = 0$$

$$Z_p = -j80,000 \| 40,000 = 32,000 - j16,000 \Omega$$

$$\mathbf{V}_o = \frac{5Z_p}{80,000} + 5 = 7 - j = 7.07/\underline{-8.13^\circ}$$

$$v_o = 7.07 \cos(50,000t - 8.13^\circ) \text{ V}$$

$$[\text{b}] \quad \mathbf{V}_p = 0.2V_m/\underline{0^\circ}; \quad \mathbf{V}_n = \mathbf{V}_p = 0.2V_m/\underline{0^\circ}$$

$$\frac{0.2V_m}{80,000} + \frac{0.2V_m - \mathbf{V}_o}{32,000 - j16,000} = 0$$

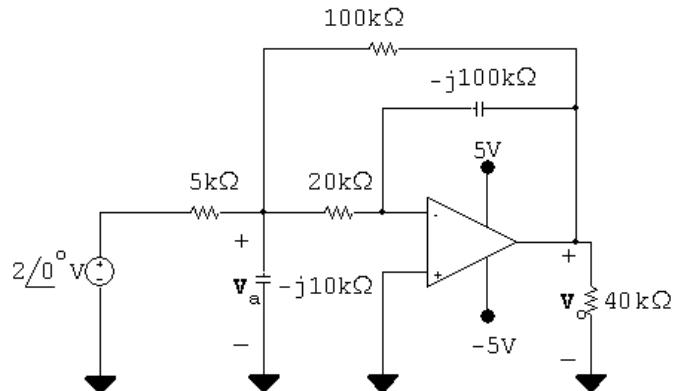
$$\therefore \mathbf{V}_o = 0.2V_m + \frac{32,000 - j16,000}{80,000} V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore |V_m(0.28 - j0.04)| \leq 10$$

$$\therefore V_m \leq 35.36 \text{ V}$$

$$\text{P 9.72} \quad \frac{1}{j\omega C_1} = -j10 \text{ k}\Omega$$

$$\frac{1}{j\omega C_2} = -j100 \text{ k}\Omega$$



$$\frac{\mathbf{V}_a - 2}{5000} + \frac{\mathbf{V}_a}{-j10,000} + \frac{\mathbf{V}_a}{20,000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{100,000} = 0$$

$$20\mathbf{V}_a - 40 + j10\mathbf{V}_a + 5\mathbf{V}_a + \mathbf{V}_a - \mathbf{V}_o = 0$$

$$\therefore (26 + j10)\mathbf{V}_a - \mathbf{V}_o = 40$$

$$\frac{0 - \mathbf{V}_a}{20,000} + \frac{0 - \mathbf{V}_o}{-j100,000} = 0$$

$$j5\mathbf{V}_a - \mathbf{V}_o = 0$$

Solving,

$$\mathbf{V}_o = 1.43 + j7.42 = 7.56/\underline{79.09^\circ} \text{ V}$$

$$v_o(t) = 7.56 \cos(10^6 t + 79.09^\circ) \text{ V}$$

$$\text{P 9.73 [a]} \quad \frac{1}{j\omega C} = \frac{-j10^9}{(10^6)(10)} = -j100 \Omega$$

$$\mathbf{V}_g = 30/\underline{0^\circ} \text{ V}$$

$$\mathbf{V}_p = \frac{\mathbf{V}_g(1/j\omega C_o)}{25 + (1/j\omega C_o)} = \frac{30/\underline{0^\circ}}{1 + j25\omega C_o} = \mathbf{V}_n$$

$$\frac{\mathbf{V}_n}{100} + \frac{\mathbf{V}_n - \mathbf{V}_o}{-j100} = 0$$

$$\mathbf{V}_o = \frac{1 + j1}{j} \mathbf{V}_n = (1 - j1) \mathbf{V}_n = \frac{30(1 - j1)}{1 + j25\omega C_o}$$

$$|\mathbf{V}_o| = \frac{30\sqrt{2}}{\sqrt{1 + 625\omega^2 C_o^2}} = 6$$

Solving,

$$C_o = 280 \text{ nF}$$

$$[\mathbf{b}] \quad \mathbf{V}_o = \frac{30(1 - j1)}{1 + j7} = 6/-126.87^\circ$$

$$v_o = 6 \cos(10^6 t - 126.87^\circ) \text{ V}$$

$$\text{P 9.74} \quad j\omega L_1 = j50 \Omega$$

$$j\omega L_2 = j32 \Omega$$

$$\frac{1}{j\omega C} = -j20 \Omega$$

$$j\omega M = j(4 \times 10^3)k\sqrt{(12.5)(8)} \times 10^{-3} = j40k \Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12 \Omega$$

$$Z_{22}^* = 5 - j12 \Omega$$

$$Z_r = \left[\frac{40k}{|5 + j12|} \right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$$Z_{ab} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$$

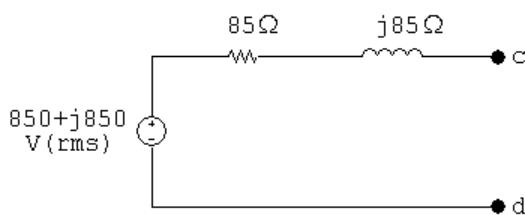
$$\therefore Z_{ab} = 20 + (47.337)(0.44) = 40.83 \Omega$$

P 9.75 Remove the voltage source to find the equivalent impedance:

$$Z_{Th} = 45 + j125 + \left(\frac{20}{|5 + j5|} \right)^2 (5 - j5) = 85 + j85 \Omega$$

Using voltage division:

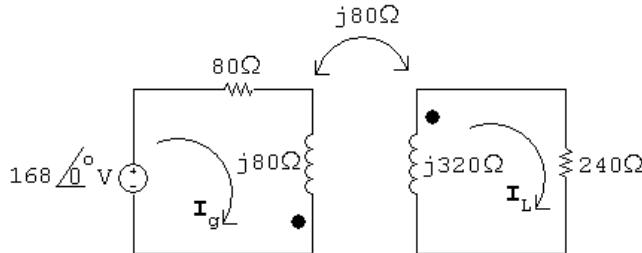
$$\mathbf{V}_{Th} = \mathbf{V}_{cd} = j20I_1 = j20 \left(\frac{425}{5 + j5} \right) = 850 + j850 \text{ V}$$



P 9.76 [a] $j\omega L_1 = j(800)(100 \times 10^{-3}) = j80 \Omega$

$$j\omega L_2 = j(800)(400 \times 10^{-3}) = j320 \Omega$$

$$j\omega M = j80 \Omega$$



$$168 = (80 + j80)\mathbf{I}_g + j80\mathbf{I}_L$$

$$0 = j80\mathbf{I}_g + (240 + j320)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 1.2 - j0.9 \text{ A}; \quad \mathbf{I}_L = -0.3 \text{ A}$$

$$i_g = 1.5 \cos(800t - 36.87^\circ) \text{ A}$$

$$i_L = 0.3 \cos(5000t - 180^\circ) \text{ A}$$

[b] $k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.1}{\sqrt{(0.1)(0.4)}} = 0.5$

[c] When $t = 625\pi \mu s$,

$$800t = (800)(625\pi) \times 10^{-6} = 0.5\pi = \pi/2 \text{ rad} = 90^\circ$$

$$i_g(625\pi \mu s) = 1.5 \cos(53.13^\circ) = 0.9 \text{ A}$$

$$i_L(625\pi \mu s) = 0.3 \cos(-90^\circ) = 0 \text{ A}$$

$$w = \frac{1}{2}L_1 i_g^2 + \frac{1}{2}L_2 i_L^2 + Mi_g i_L = \frac{1}{2}(100 \times 10^{-3})(0.81) + 0 + 0 = 40.5 \text{ mJ}$$

When $t = 1250\pi \mu s$,

$$800t = \pi \text{ rad} = 180^\circ$$

$$i_g(1250\pi \mu s) = 1.5 \cos(180 - 36.87) = -1.2 \text{ A}$$

$$i_L(1250\pi \mu s) = 0.3 \cos(180 - 180) = 0.3 \text{ A}$$

$$w = \frac{1}{2}(100 \times 10^{-3})(1.44) + \frac{1}{2}(400 \times 10^{-3})(0.09)$$

$$+ 100 \times 10^{-3}(-1.2)(0.3) = 54 \text{ mJ}$$

$$\text{P 9.77 [a]} \quad j\omega L_1 = j(200 \times 10^3)(10^{-3}) = j200 \Omega$$

$$j\omega L_2 = j(200 \times 10^3)(4 \times 10^{-3}) = j800 \Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200 \times 10^3)(12.5 \times 10^{-9})} = -j400 \Omega$$

$$\therefore Z_{22} = 100 + 200 + j800 - j400 = 300 + j400 \Omega$$

$$\therefore Z_{22}^* = 300 - j400 \Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500} \right]^2 (300 - j400) = k^2(192 - j256) \Omega$$

$$Z_{\text{in}} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\text{in}}| = [(200 + 192k)^2 + (200 - 256k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(200 + 192k)^2 + (200 - 256k)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore k^2 = 0.125; \quad \therefore k = \sqrt{0.125} = 0.3536$$

$$\begin{aligned} \text{[b]} \quad Z_{\text{in}} (\text{min}) &= 200 + 192(0.125) + j[200 - 0.125(256)] \\ &= 224 + j168 = 280/\underline{-36.87^\circ} \Omega \end{aligned}$$

$$\mathbf{I}_1 (\text{max}) = \frac{560/\underline{0^\circ}}{224 + j168} = 2/\underline{-36.87^\circ} \text{ A}$$

$$\therefore i_1 (\text{peak}) = 2 \text{ A}$$

Note — You can test that the k value obtained from setting $d|Z_{\text{in}}|/dt = 0$ leads to a minimum by noting $0 \leq k \leq 1$. If $k = 1$,

$$Z_{\text{in}} = 392 - j56 = 395.98/\underline{-8.13^\circ} \Omega$$

Thus,

$$|Z_{\text{in}}|_{k=1} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$$Z_{\text{in}} = 200 + j200 = 282.84/\underline{45^\circ} \Omega$$

Thus,

$$|Z_{\text{in}}|_{k=0} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

P 9.78 [a] $j\omega L_L = j20 \Omega$

$$j\omega L_2 = j100 \Omega$$

$$Z_{22} = 100 + 60 + j20 + j100 = 160 + j120 \Omega$$

$$Z_{22}^* = 160 - j120 \Omega$$

$$\omega M = 54 \Omega$$

$$Z_r = \left(\frac{54}{200} \right)^2 [160 - j120] = 11.66 - j8.75 \Omega$$

[b] $Z_{ab} = R_1 + j\omega L_1 + Z_r = 8.34 + j36 + 11.66 - j8.75 = 20 + j27.25 \Omega$

P 9.79 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2 \omega^2 L_1 L_2$ and then write X_{ab} as

$$\begin{aligned} X_{ab} &= \omega L_1 - \frac{k^2 \omega^2 L_1 L_2 (\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2} \\ &= \omega L_1 \left\{ 1 - \frac{k^2 \omega L_2 (\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2} \right\} \end{aligned}$$

For X_{ab} to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2 \omega L_2 (\omega L_2 + \omega L_L)$$

or

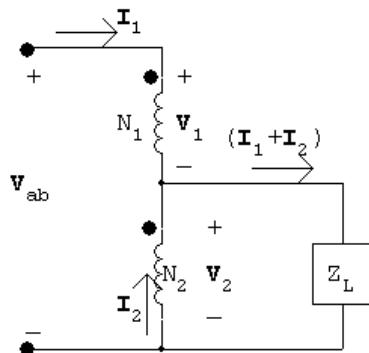
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2 \omega L_2 (\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2 (1 - k^2) + \omega L_2 \omega L_L (2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$, so it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

P 9.80 [a]



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \quad \mathbf{V}_2 = \frac{N_2}{N_1} \mathbf{V}_1$$

$$N_1 \mathbf{I}_1 = N_2 \mathbf{I}_2, \quad \mathbf{I}_2 = \frac{N_1}{N_2} \mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2) Z_L = \mathbf{I}_1 \left(1 + \frac{N_1}{N_2} \right) Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1 \right) \mathbf{V}_2 = \left(1 + \frac{N_1}{N_2} \right)^2 Z_L \mathbf{I}_1$$

$$\therefore Z_{ab} = \frac{(1 + N_1/N_2)^2 Z_L \mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = \left(1 + \frac{N_1}{N_2} \right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \quad \mathbf{V}_1 = \frac{-N_1}{N_2} \mathbf{V}_2$$

$$N_1 \mathbf{I}_1 = -N_2 \mathbf{I}_2, \quad \mathbf{I}_2 = \frac{-N_1}{N_2} \mathbf{I}_1$$

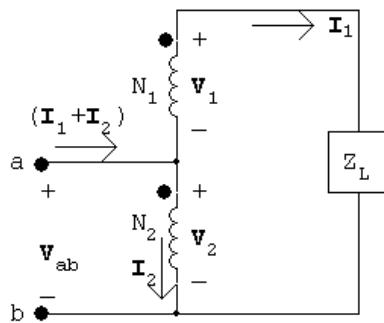
As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1) Z_L \quad \text{and} \quad Z_{ab} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{ab} = \frac{(1 - N_1/N_2) \mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2) Z_L \mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.81 [a]



$$N_1 \mathbf{I}_1 = N_2 \mathbf{I}_2, \quad \mathbf{I}_2 = \frac{N_1}{N_2} \mathbf{I}_1$$

$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \quad \mathbf{V}_1 = \frac{N_1}{N_2} \mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L \mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1 \right) \mathbf{V}_2$$

$$Z_{ab} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore Z_{ab} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2} \mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2} \mathbf{I}_1$$

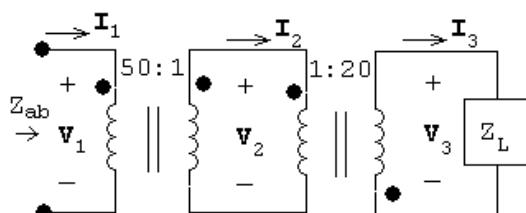
As before

$$Z_{ab} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L \mathbf{I}_1$$

$$\therefore Z_{ab} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L \mathbf{I}_1}{[1 - (N_1/N_2)]^2 \mathbf{I}_1}$$

$$Z_{ab} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.82



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$\frac{\mathbf{V}_2}{1} = \frac{-\mathbf{V}_3}{20}; \quad 1\mathbf{I}_2 = -20\mathbf{I}_3$$

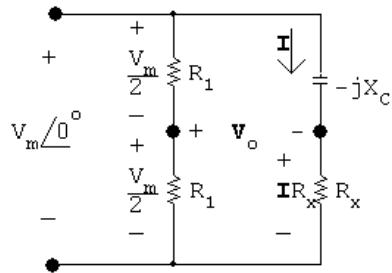
$$\frac{\mathbf{V}_1}{50} = \frac{\mathbf{V}_2}{1}; \quad 50\mathbf{I}_1 = 1\mathbf{I}_2$$

$$Z_{ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

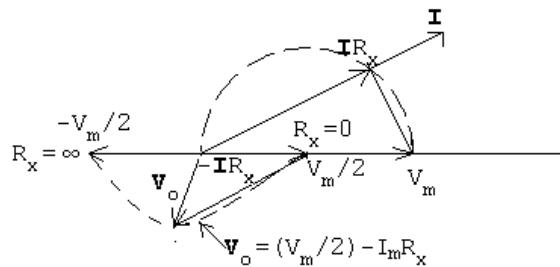
$$\begin{aligned} Z_{ab} &= \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{50\mathbf{V}_2}{\mathbf{I}_2/50} = \frac{50^2\mathbf{V}_2}{\mathbf{I}_2} \\ &= \frac{50^2(-\mathbf{V}_3/20)}{-20\mathbf{I}_3} = \frac{(50)^2\mathbf{V}_3}{(20)^2\mathbf{I}_3} = 6.25Z_L = 6.25(200/\underline{-45^\circ}) = 1250/\underline{-45^\circ} \Omega \end{aligned}$$

P 9.83 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \quad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞ , the amplitude of v_o remains constant and its phase angle increases from 0° to -180° , as shown in the following phasor diagram:



$$\text{P 9.84 [a]} \quad \mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5) \text{ A}$$

$$\mathbf{V}_s = 240/\underline{0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11/\underline{1.68^\circ} \text{ V}$$

[b] Use the capacitor to eliminate the j component of \mathbf{I} , therefore

$$\mathbf{I}_c = j7.5 \text{ A}, \quad Z_c = \frac{240}{j7.5} = -j32 \Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13/\underline{1.90^\circ} \text{ V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5) \text{ A}$$

$$\begin{aligned}\mathbf{V}_s &= 240/\underline{\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)] \\ &= (247 - 0.8I_c) + j(7.25 + 0.1I_c)\end{aligned}$$

It follows that

$$240 \cos \alpha = (247 - 0.8I_c) \quad \text{and} \quad 240 \sin \alpha = (7.25 + 0.1I_c)$$

Now square each term and then add to generate the quadratic equation

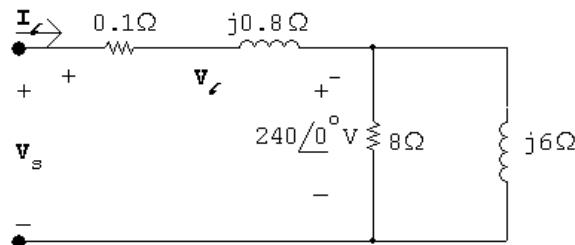
$$I_c^2 - 605.77I_c + 5325.48 = 0; \quad I_c = 302.88 \pm 293.96$$

Therefore

$$I_c = 8.92 \text{ A} \quad (\text{smallest value}) \quad \text{and} \quad Z_c = 240/j8.92 = -j26.90 \Omega.$$

Therefore, the capacitive reactance is -26.90Ω .

P 9.85 [a]

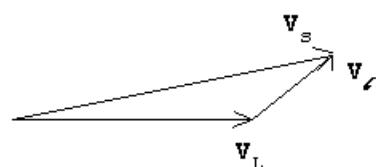


$$\mathbf{I}_\ell = \frac{240}{8} + \frac{240}{j6} = 30 - j40 \text{ A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 35 + j20 = 40.31/\underline{29.74^\circ} \text{ V}$$

$$\mathbf{V}_s = 240/\underline{0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73/\underline{4.16^\circ} \text{ V}$$

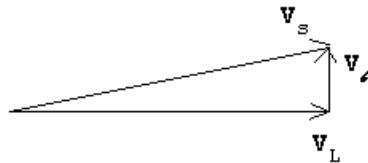
[b]



$$[c] \quad \mathbf{I}_\ell = 30 - j40 + \frac{240}{-j5} = 30 + j8 \text{ A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = -3.4 + j24.8 = 25.03/\underline{97.81^\circ}$$

$$\mathbf{V}_s = 240/\underline{0^\circ} + \mathbf{V}_\ell = 236.6 + j24.8 = 237.9/\underline{5.98^\circ}$$



P 9.86 [a] $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02/\underline{-30.5^\circ} \text{ A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44/\underline{-25.87^\circ} \text{ A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5/\underline{0^\circ} \text{ A}; \quad \mathbf{I}_5 = \frac{120}{12} = 10/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86/\underline{-36.87^\circ} \text{ A}$$

[b] When fuse A is interrupted,

$$\mathbf{I}_1 = 0 \quad \mathbf{I}_3 = 15 \text{ A} \quad \mathbf{I}_5 = 10 \text{ A}$$

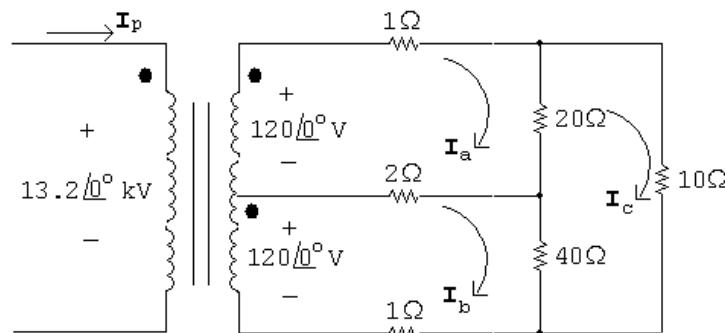
$$\mathbf{I}_2 = 10 + 5 = 15 \text{ A} \quad \mathbf{I}_4 = -5 \text{ A} \quad \mathbf{I}_6 = 5 \text{ A}$$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the 12Ω load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.87 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120/\underline{0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120/\underline{0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24/\underline{0^\circ} \text{ A} \quad \mathbf{I}_b = 21.96/\underline{0^\circ} \text{ A} \quad \mathbf{I}_c = 19.40/\underline{0^\circ} \text{ A}$$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_4 = \mathbf{I}_c = 19.40/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6/\underline{0^\circ} \text{ A}$$

$$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55/\underline{0^\circ} \text{ A}$$

- [b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13,200}{N_1} = \frac{240}{2N_2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{1}{110}$$

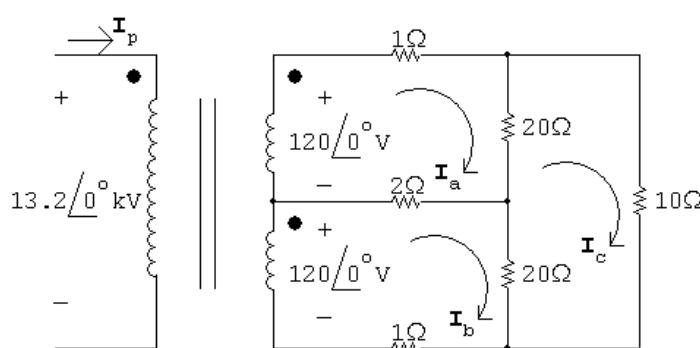
The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2(\mathbf{I}_1 + \mathbf{I}_3)$$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42/\underline{0^\circ} \text{ A}$$

P 9.88 [a]



The three mesh current equations are

$$120/\underline{0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120/\underline{0^\circ} = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24/\underline{0^\circ} \text{ A}; \quad \mathbf{I}_b = 24/\underline{0^\circ} \text{ A}; \quad \mathbf{I}_c = 19.2/\underline{0^\circ} \text{ A}$$

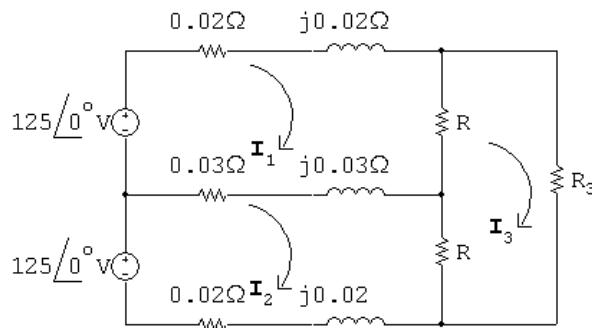
$$\therefore \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0 \text{ A}$$

$$[\text{b}] \quad \mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b)$$

$$= \frac{1}{110}(24 + 24) = 0.436/\underline{0^\circ} \text{ A}$$

- [c] Yes; when the two 120 V loads are equal, there is no current in the “neutral” line, so no power is lost to this line. Since you pay for power, the cost is lower when the loads are equal.

P 9.89 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

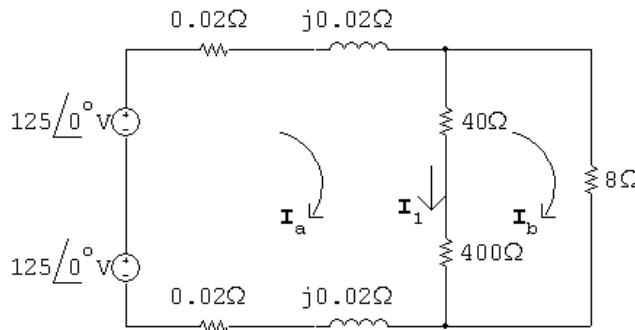
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 0 \text{ A}$$

$$[\text{b}] \quad \mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3); \quad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$$250 = (440.04 + j0.04)\mathbf{I}_a - 440\mathbf{I}_b$$

$$0 = -440\mathbf{I}_a + 448\mathbf{I}_b$$

Solving,

$$\mathbf{I}_a = 31.656207 - j0.160343 \text{ A}$$

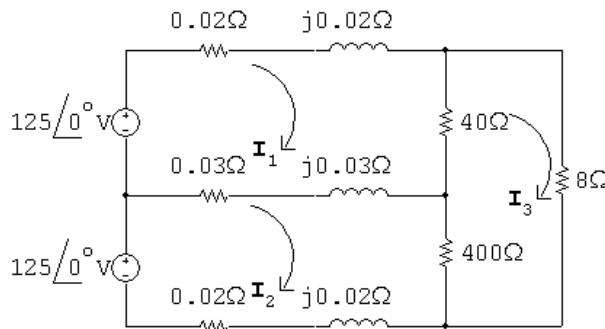
$$\mathbf{I}_b = 31.090917 - j0.157479 \text{ A}$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.56529 - j0.002864 \text{ A}$$

$$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.612 - j0.11456 = 22.612/-0.290282^\circ \text{ V}$$

$$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.116 - j1.1456 = 226.1189/-0.290282^\circ \text{ V}$$

[d]



$$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$$

$$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 34.19 - j0.182 \text{ A}$$

$$\mathbf{I}_2 = 31.396 - j0.164 \text{ A}$$

$$\mathbf{I}_3 = 31.085 - j0.163 \text{ A}$$

$$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.2/-0.35^\circ \text{ V}$$

$$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.4/-0.18^\circ \text{ V}$$

- [e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.90 [a] Let N_1 = primary winding turns and $2N_2$ = secondary winding turns.
Then

$$\frac{14,000}{N_1} = \frac{250}{2N_2}; \quad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$$\mathbf{I}_p = 2a\mathbf{I}_a$$

$$\begin{aligned} \therefore \mathbf{I}_p &= \frac{2N_2\mathbf{I}_a}{N_1} = \frac{1}{56}\mathbf{I}_a \\ &= \frac{1}{56}(31.656 - j0.16) \end{aligned}$$

$$\mathbf{I}_p = 565.3 - j2.9 \text{ mA}$$

In part d),

$$\mathbf{I}_p N_1 = \mathbf{I}_1 N_2 + \mathbf{I}_2 N_2$$

$$\begin{aligned} \therefore \mathbf{I}_p &= \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2) \\ &= \frac{1}{112}(34.19 - j0.182 + 31.396 - j0.164) \\ &= \frac{1}{112}(65.586 - j0.346) \end{aligned}$$

$$\mathbf{I}_p = 585.6 - j3.1 \text{ mA}$$

- [b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100/\underline{-45^\circ}$ V, $\mathbf{I} = 20/\underline{15^\circ}$ A

Therefore

$$P = \frac{1}{2}(100)(20) \cos[-45 - (15)] = 500 \text{ W}, \quad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin -60^\circ = -866.03 \text{ VAR}, \quad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100/\underline{-45^\circ}$, $\mathbf{I} = 20/\underline{165^\circ}$

$$P = 1000 \cos(-210^\circ) = -866.03 \text{ W}, \quad \text{B} \rightarrow \text{A}$$

$$Q = 1000 \sin(-210^\circ) = 500 \text{ VAR}, \quad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100/\underline{-45^\circ}$, $\mathbf{I} = 20/\underline{-105^\circ}$

$$P = 1000 \cos(60^\circ) = 500 \text{ W}, \quad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin(60^\circ) = 866.03 \text{ VAR}, \quad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100/\underline{0^\circ}$, $\mathbf{I} = 20/\underline{120^\circ}$

$$P = 1000 \cos(-120^\circ) = -500 \text{ W}, \quad \text{B} \rightarrow \text{A}$$

$$Q = 1000 \sin(-120^\circ) = -866.03 \text{ VAR}, \quad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

$$\text{From Ex. 9.4 } I_{\text{eff}} = \frac{I_\rho}{\sqrt{3}} = \frac{0.18}{\sqrt{3}} \text{ A}$$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3} \right) (5000) = 54 \text{ W}$$

$$\text{AP 10.4 [a]} \quad Z = (39 + j26) \parallel (-j52) = 48 - j20 = 52/-22.62^\circ \Omega$$

$$\text{Therefore } \mathbf{I}_\ell = \frac{250/0^\circ}{48 - j20 + 1 + j4} = 4.85/18.08^\circ \text{ A (rms)}$$

$$\mathbf{V}_L = Z \mathbf{I}_\ell = (52/-22.62^\circ)(4.85/18.08^\circ) = 252.20/-4.54^\circ \text{ V (rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{39 + j26} = 5.38/-38.23^\circ \text{ A (rms)}$$

$$\begin{aligned} \text{[b]} \quad S_L &= \mathbf{V}_L \mathbf{I}_L^* = (252.20/-4.54^\circ)(5.38/+38.23^\circ) = 1357/33.69^\circ \\ &= (1129.09 + j752.73) \text{ VA} \end{aligned}$$

$$P_L = 1129.09 \text{ W}; \quad Q_L = 752.73 \text{ VAR}$$

$$\text{[c]} \quad P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52 \text{ W}; \quad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09 \text{ VAR}$$

$$\text{[d]} \quad S_g(\text{delivering}) = 250 \mathbf{I}_\ell^* = (1152.62 - j376.36) \text{ VA}$$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

$$\text{[e]} \quad Q_{\text{cap}} = \frac{|\mathbf{V}_L|^2}{-52} = \frac{(252.20)^2}{-52} = -1223.18 \text{ VAR}$$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

$$\text{Check: } 94.09 + 752.73 + 376.36 = 1223.18 \text{ VAR and}$$

$$1129.09 + 23.52 = 1152.62 \text{ W}$$

AP 10.5 Series circuit derivation:

$$S = 250 \mathbf{I}^* = (40,000 - j30,000)$$

$$\text{Therefore } \mathbf{I}^* = 160 - j120 = 200/-36.87^\circ \text{ A (rms)}$$

$$\mathbf{I} = 200/36.87^\circ \text{ A (rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200/36.87^\circ} = 1.25/-36.87^\circ = (1 - j0.75) \Omega$$

$$\text{Therefore } R = 1 \Omega, \quad X_C = -0.75 \Omega$$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \quad \text{therefore} \quad R = \frac{(250)^2}{40,000} = 1.5625 \Omega$$

$$Q = \frac{(250)^2}{X_C}; \quad \text{therefore} \quad X_C = \frac{(250)^2}{-30,000} = -2.083 \Omega$$

AP 10.6

$$S_1 = 15,000(0.6) + j15,000(0.8) = 9000 + j12,000 \text{ VA}$$

$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600 \text{ VA}$$

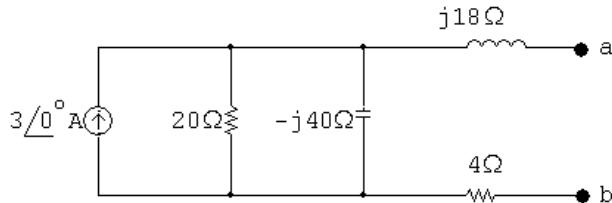
$$S_T = S_1 + S_2 = 13,800 + j8400 \text{ VA}$$

$$S_T = 200\mathbf{I}^*; \quad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \quad \mathbf{I} = 69 - j42 \text{ A}$$

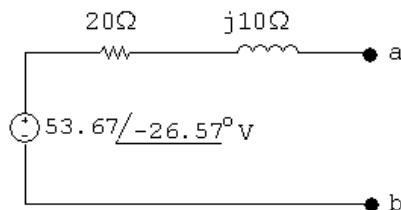
$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64/\underline{15.91^\circ} \text{ V (rms)}$$

AP 10.7 [a] The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{Th} = 3 \frac{-j800}{20 - j40} = 48 - j24 = 53.67/\underline{-26.57^\circ} \text{ V}$$

$$Z_{Th} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36/\underline{26.57^\circ} \Omega$$

For maximum power transfer, $Z_L = (20 - j10) \Omega$

$$[b] \quad I = \frac{53.67/-26.57^\circ}{40} = 1.34/-26.57^\circ \text{ A}$$

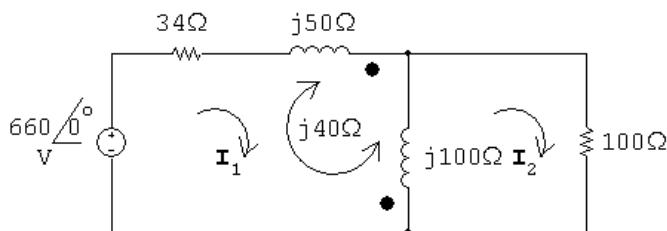
$$\text{Therefore } P = \left(\frac{1.34}{\sqrt{2}}\right)^2 20 = 17.96 \text{ W}$$

$$[c] \quad R_L = |Z_{\text{Th}}| = 22.36 \Omega$$

$$[d] \quad I = \frac{53.67/-26.57^\circ}{42.36 + j10} = 1.23/-39.85^\circ \text{ A}$$

$$\text{Therefore } P = \left(\frac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17 \text{ W}$$

AP 10.8



Mesh current equations:

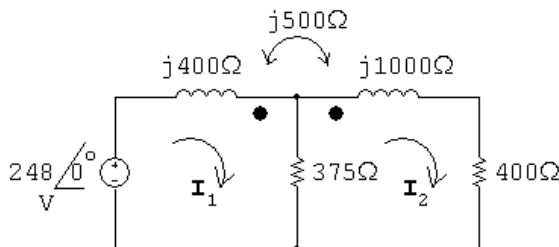
$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5/\underline{0^\circ} \text{ A}; \quad \therefore P = \frac{1}{2}(3.5)^2(100) = 612.50 \text{ W}$$

AP 10.9 [a]



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62 \text{ A}; \quad \mathbf{I}_2 = 0.4 - j0.3 = 0.5/\underline{-36.87^\circ}$$

$$\therefore P = \frac{1}{2}(0.25)(400) = 50 \text{ W}$$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32 \text{ A}$

$$P_{375} = \frac{1}{2} |\mathbf{I}_1 - \mathbf{I}_2|^2 (375) = 49.20 \text{ W}$$

[c] $P_g = \frac{1}{2} (248)(0.8) = 99.20 \text{ W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20 \text{ W} \quad (\text{checks})$$

AP 10.10 [a] $V_{\text{Th}} = 210 \text{ V}; \quad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \quad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$
Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \mathbf{I}_2 = 14 \text{ A}; \quad R_{\text{Th}} = \frac{210}{14} = 15 \Omega$$

[b] $P_{\text{max}} = \left(\frac{210}{30}\right)^2 15 = 735 \text{ W}$

AP 10.11 [a] $\mathbf{V}_{\text{Th}} = -4(146/\underline{0^\circ}) = -584/\underline{0^\circ} \text{ V (rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \quad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146/\underline{0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \mathbf{I}_2 = -146/365 = -0.40 \text{ A}; \quad R_{\text{Th}} = \frac{-584}{-0.4} = 1460 \Omega$$

[b] $P = \left(\frac{-584}{2920}\right)^2 1460 = 58.40 \text{ W}$

Problems

P 10.1 [a] $P = \frac{1}{2}(250)(4) \cos(45 + 30) = 500 \cos 75^\circ = 129.41 \text{ W}$ (abs)

$$Q = 500 \sin 75^\circ = 482.96 \text{ VAR}$$
 (abs)

[b] $P = \frac{1}{2}(18)(5) \cos(30 + 75) = 45 \cos(105^\circ) = -11.65 \text{ W}$ (del)

$$Q = 45 \sin(105^\circ) = 43.47 \text{ VAR}$$
 (abs)

[c] $P = \frac{1}{2}(150)(2) \cos(-65 - 50) = 150 \cos(-115^\circ) = -63.39 \text{ W}$ (del)

$$Q = 150 \sin(-115^\circ) = -135.95 \text{ VAR}$$
 (del)

[d] $P = \frac{1}{2}(80)(10) \cos(120 - 170) = 400 \cos(-50^\circ) = 257.12 \text{ W}$ (abs)

$$Q = 400 \sin(-50^\circ) = -306.42 \text{ VAR}$$
 (del)

P 10.2 [a] coffee maker = 1200 W frying pan = 1196 W

$$\text{microwave oven} = 1450 \text{ W} \quad \text{toaster} = 1146 \text{ W}$$

$$\sum P = 4992 \text{ W}$$

Therefore $I_{\text{eff}} = \frac{4992}{120} = 41.6 \text{ A}$

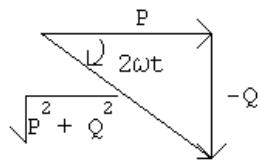
The breaker will not trip.

[b] $\sum P = 4992 + 860 + 630 = 6482 \text{ W}; \quad I_{\text{eff}} = \frac{6482}{120} = 54.02 \text{ A}$

The breaker will trip because the current is greater than 20 A.

P 10.3 $p = P + P \cos 2\omega t - Q \sin 2\omega t; \quad \frac{dp}{dt} = -2\omega P \sin 2\omega t - 2\omega Q \cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P \sin 2\omega t = 2\omega Q \cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2 + Q^2}}; \quad \sin 2\omega t = -\frac{Q}{\sqrt{P^2 + Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

$$\text{Therefore } p_{\max} = P + P \cdot \frac{P}{\sqrt{P^2 + Q^2}} - \frac{Q(-Q)}{\sqrt{P^2 + Q^2}} = P + \sqrt{P^2 + Q^2}$$

$$\text{and } p_{\min} = P - P \cdot \frac{P}{\sqrt{P^2 + Q^2}} - Q \cdot \frac{Q}{\sqrt{P^2 + Q^2}} = P - \sqrt{P^2 + Q^2}$$

$$\text{P 10.4 [a]} \quad P = \frac{1}{2} \frac{(240)^2}{480} = 60 \text{ W}$$

$$-\frac{1}{\omega C} = \frac{-9 \times 10^6}{(5000)(5)} = -360 \Omega$$

$$Q = \frac{1}{2} \frac{(240)^2}{(-360)} = -80 \text{ VAR}$$

$$p_{\max} = P + \sqrt{P^2 + Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160 \text{ W (del)}$$

$$[\text{b}] \quad p_{\min} = 60 - \sqrt{60^2 + 80^2} = -40 \text{ W (abs)}$$

$$[\text{c}] \quad P = 60 \text{ W from (a)}$$

$$[\text{d}] \quad Q = -80 \text{ VAR from (a)}$$

$$[\text{e}] \quad \text{generates, because } Q < 0$$

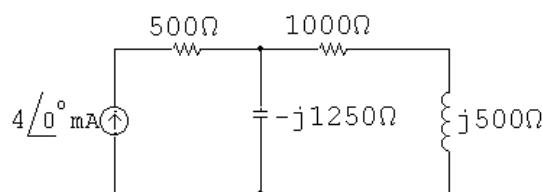
$$[\text{f}] \quad \text{pf} = \cos(\theta_v - \theta_i)$$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83/\underline{53.13^\circ} \text{ A}$$

$$\therefore \text{pf} = \cos(0 - 53.13^\circ) = 0.6 \text{ leading}$$

$$[\text{g}] \quad \text{rf} = \sin(-53.13^\circ) = -0.8$$

$$\text{P 10.5} \quad \mathbf{I}_g = 4/\underline{0^\circ} \text{ mA}; \quad \frac{1}{j\omega C} = -j1250 \Omega; \quad j\omega L = j500 \Omega$$

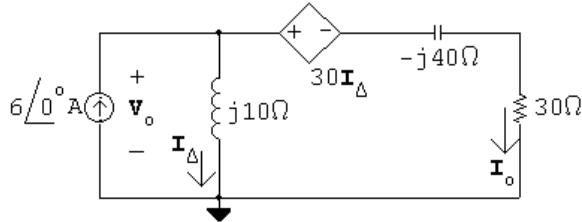


$$Z_{\text{eq}} = 500 + [-j1250||(1000 + j500)] = 1500 - j500 \Omega$$

$$P_g = -\frac{1}{2}|I|^2 \text{Re}\{Z_{\text{eq}}\} = -\frac{1}{2}(0.004)^2(1500) = -12 \text{ mW}$$

The source delivers 12 mW of power to the circuit.

$$\text{P 10.6} \quad j\omega L = j20,000(0.5 \times 10^{-3}) = j10 \Omega; \quad \frac{1}{j\omega C} = \frac{10^6}{j20,000(1.25)} = -j40 \Omega$$



$$-6 + \frac{\mathbf{V}_o}{j10} + \frac{\mathbf{V}_o - 30(\mathbf{V}_o/j10)}{30 - j40} = 0$$

$$\therefore \mathbf{V}_o \left[\frac{1}{j10} + \frac{1 + j3}{30 - j40} \right] = 6$$

$$\therefore \mathbf{V}_o = 100/\underline{126.87^\circ} \text{ V}$$

$$\therefore \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j10} = 10/\underline{36.87^\circ} \text{ A}$$

$$\mathbf{I}_o = 6/\underline{0^\circ} - \mathbf{I}_\Delta = 6 - 8 - j6 = -2 - j6 = 6.32/\underline{-108.43^\circ} \text{ A}$$

$$P_{30\Omega} = \frac{1}{2} |\mathbf{I}_o|^2 30 = 600 \text{ W}$$

$$\text{P 10.7} \quad Z_f = -j10,000 \| 20,000 = 4000 - j8000 \Omega$$

$$Z_i = 2000 - j2000 \Omega$$

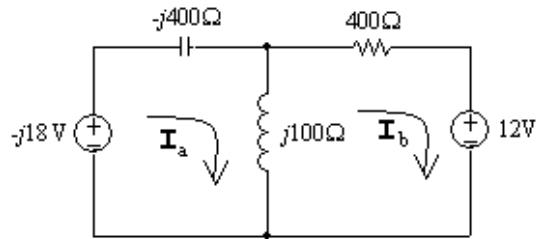
$$\therefore \frac{Z_f}{Z_i} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_f}{Z_i} \mathbf{V}_g; \quad \mathbf{V}_g = 1/\underline{0^\circ} \text{ V}$$

$$\mathbf{V}_o = -(3 - j1)(1) = -3 + j1 = 3.16/\underline{161.57^\circ} \text{ V}$$

$$P = \frac{1}{2} \frac{V_m^2}{R} = \frac{1}{2} \frac{(10)^2}{1000} = 5 \times 10^{-3} = 5 \text{ mW}$$

P 10.8 [a] From the solution to Problem 9.63 we have:



$$\mathbf{I}_a = 67.5 - j7.5 \text{ mA} \quad \text{and} \quad \mathbf{I}_b = -22.5 + j22.5 \text{ mA}$$

$$S_{-j18V} = -\frac{1}{2}(-j18)(0.0675 + j0.0075) = -67.5 + j607.5 \text{ mVA}$$

$$P_{-j18V} = -67.5 \text{ mW}; \quad \text{and} \quad Q_{-j18V} = 607.5 \text{ mVAR}$$

$$S_{12V} = \frac{1}{2}(12)(-0.0225 - j0.0225) = -135 - j135 \text{ mVA}$$

$$P_{12V} = -135 \text{ mW}; \quad \text{and} \quad Q_{12V} = -135 \text{ mVAR}$$

$$P_{-j400\Omega} = 0 \text{ W}; \quad \text{and} \quad Q_{-j400\Omega} = -\frac{1}{2}(400)|\mathbf{I}_a|^2 = -922.5 \text{ mVAR}$$

$$P_{400\Omega} = \frac{1}{2}(400)|\mathbf{I}_b|^2 = 202.5 \text{ mW}; \quad \text{and} \quad Q_{400\Omega} = 0 \text{ mVAR}$$

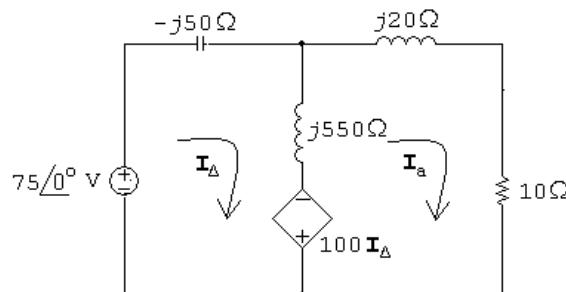
$$P_{j100\Omega} = 0 \text{ W}; \quad \text{and} \quad Q_{j100\Omega} = \frac{1}{2}(100)|\mathbf{I}_a - \mathbf{I}_b|^2 = 450 \text{ mVAR}$$

[b] $\sum P_{\text{gen}} = 67.5 + 135 = 202.5 \text{ mW} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 135 + 922.5 = 1057.5 \text{ mVAR}$

$$\sum Q_{\text{abs}} = 607.5 + 450 = 1057.5 \text{ mVAR} \text{ (check)}$$

P 10.9 [a] From the solution to Problem 9.64 we have



$$\mathbf{I}_\Delta = 0.5 + j2.5 \text{ A}; \quad \text{and} \quad \mathbf{I}_a = j2.5 \text{ A}$$

$$S_{75V} = -\frac{1}{2}(75)(0.5 - j2.5) = -18.75 + j93.75 \text{ VA}$$

$$P_{75V} = -18.75 \text{ W}; \quad \text{and} \quad Q_{75V} = 93.75 \text{ VAR}$$

$$S_{\text{dep.source}} = \frac{1}{2}(100\mathbf{I}_\Delta)(\mathbf{I}_a - \mathbf{I}_\Delta)^* = -12.5 - j62.5 \text{ VA}$$

$$P_{\text{dep.source}} = -12.5 \text{ W}; \quad \text{and} \quad Q_{\text{dep.source}} = -62.5 \text{ VAR}$$

$$P_{-j50\Omega} = 0 \text{ W}; \quad \text{and} \quad Q_{-j50\Omega} = -\frac{1}{2}(50)|\mathbf{I}_\Delta|^2 = -162.5 \text{ VAR}$$

$$P_{j20\Omega} = 0 \text{ W}; \quad \text{and} \quad Q_{j20\Omega} = \frac{1}{2}(20)|\mathbf{I}_a|^2 = 62.5 \text{ VAR}$$

$$P_{10\Omega} = \frac{1}{2}(10)|\mathbf{I}_a|^2 = 31.25 \text{ W}; \quad \text{and} \quad Q_{10\Omega} = 0 \text{ VAR}$$

$$P_{j550\Omega} = 0, \text{W}; \quad \text{and} \quad Q_{j550\Omega} = \frac{1}{2}(550)|\mathbf{I}_\Delta - \mathbf{I}_a|^2 = 68.75 \text{ VAR}$$

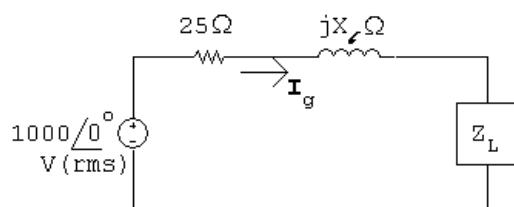
[b] $\sum P_{\text{dev}} = 18.75 + 12.5 = 31.25 \text{ W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 162.5 + 62.5 = 225 \text{ VAR}$

$$\sum Q_{\text{abs}} = 93.75 + 62.5 + 68.5 = 225 \text{ VAR} = \sum Q_{\text{dev}}$$

P 10.10 [a] line loss = $8000 - 6000 = 2000 \text{ W}$

$$\text{line loss} = |\mathbf{I}_g|^2 25 \quad \therefore |\mathbf{I}_g|^2 = 80$$

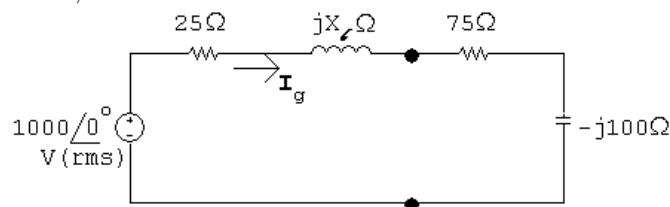


$$|\mathbf{I}_g| = \sqrt{80} \text{ A}$$

$$|\mathbf{I}_g|^2 R_L = 6000 \quad \therefore R_L = 75 \Omega$$

$$|\mathbf{I}_g|^2 X_L = -8000 \quad \therefore X_L = -100 \Omega$$

Thus,



$$|Z| = \sqrt{(100)^2 + (X_\ell - 100)^2} \quad |\mathbf{I}_g| = \frac{1000}{\sqrt{10,000 + (X_\ell - 100)^2}}$$

$$\therefore 10,000 + (X_\ell - 100)^2 = \frac{10^6}{80} = 12,500$$

Solving, $(X_\ell - 100) = \pm 50$.

Thus, $X_\ell = 150 \Omega$ or $X_\ell = 50 \Omega$

[b] If $X_\ell = 150 \Omega$:

$$\mathbf{I}_g = \frac{1000}{100 + j50} = 8 - j4 \text{ A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 - j4000 \text{ VA}$$

Thus, the voltage source is delivering 8 kW and 4 kvars.

$$Q_{j150} = |\mathbf{I}_g|^2 X_\ell = 80(150) = 12 \text{ kvars}$$

Therefore the line reactance is absorbing 12 kvars.

$$Q_{-j100} = |\mathbf{I}_g|^2 X_L = 80(-100) = -8 \text{ kvars}$$

Therefore the load reactance is generating 8 kvars.

$$\sum Q_{\text{gen}} = 12 \text{ kvars} = \sum Q_{\text{abs}}$$

If $X_\ell = 50 \Omega$:

$$\mathbf{I}_g = \frac{1000}{100 - j50} = 8 + j4 \text{ A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 + j4000 \text{ VA}$$

Thus, the voltage source is delivering 8 kW and absorbing 4 kvars.

$$Q_{j50} = |\mathbf{I}_g|^2 (50) = 80(50) = 4 \text{ kvars}$$

Therefore the line reactance is absorbing 4 kvars. The load continues to deliver 8 kvars.

$$\sum Q_{\text{gen}} = 8 \text{ kvars} = \sum Q_{\text{abs}}$$

P 10.11 [a] $I_{\text{eff}} = 40/115 \cong 0.35 \text{ A}$

[b] $I_{\text{eff}} = 130/115 \cong 1.13 \text{ A}$

P 10.12 $i(t) = 250t \quad 0 \leq t \leq 80 \text{ ms}$

$$i(t) = 100 - 1000t \quad 80 \text{ ms} \leq t \leq 100 \text{ ms}$$

$$\therefore I_{\text{rms}} = \sqrt{\frac{1}{0.1} \left\{ \int_0^{0.08} (250)^2 t^2 dt + \int_{0.08}^{0.1} (100 - 1000t)^2 dt \right\}}$$

$$\int_0^{0.08} (250)^2 t^2 dt = (250)^2 \frac{t^3}{3} \Big|_0^{0.08} = \frac{32}{3}$$

$$(100 - 1000t)^2 = 10^4 - 2 \times 10^5 t + 10^6 t^2$$

$$\int_{0.08}^{0.1} 10^4 dt = 200$$

$$\int_{0.08}^{0.1} 2 \times 10^5 t dt = 10^5 t^2 \Big|_{0.08}^{0.1} = 360$$

$$10^6 \int_{0.08}^{0.1} t^2 dt = \frac{10^6}{3} t^3 \Big|_{0.08}^{0.1} = \frac{488}{3}$$

$$\therefore I_{\text{rms}} = \sqrt{10\{(32/3) + 225 - 360 + (488/3)\}} = 11.55 \text{ A}$$

$$\text{P 10.13 } P = I_{\text{rms}}^2 R \quad \therefore R = \frac{1280}{(11.55)^2} = 9.6 \Omega$$

$$\text{P 10.14 [a]} \quad A = 40^2(5) + (-40)^2(5) = 16,000$$

$$\text{mean value} = \frac{A}{20} = 800$$

$$V_{\text{rms}} = \sqrt{800} = 28.28 \text{ V(rms)}$$

$$\text{[b]} \quad P = \frac{V_{\text{rms}}^2}{R} = \frac{800}{40} = 20 \text{ W}$$

$$\text{[c]} \quad R = \frac{V_{\text{rms}}^2}{P} = \frac{800}{0.01} = 80 \text{ k}\Omega$$

$$\text{P 10.15 [a]} \quad \text{Area under one cycle of } v_g^2:$$

$$\begin{aligned} A &= (100)(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 100(25 \times 10^{-6}) \\ &= 1000(25 \times 10^{-6}) \end{aligned}$$

Mean value of v_g^2 :

$$\text{M.V.} = \frac{A}{100 \times 10^{-6}} = \frac{1000(25 \times 10^{-6})}{100 \times 10^{-6}} = 250$$

$$\therefore V_{\text{rms}} = \sqrt{250} = 15.81 \text{ V (rms)}$$

$$\text{[b]} \quad P = \frac{V_{\text{rms}}^2}{R} = \frac{250}{4} = 62.5 \text{ W}$$

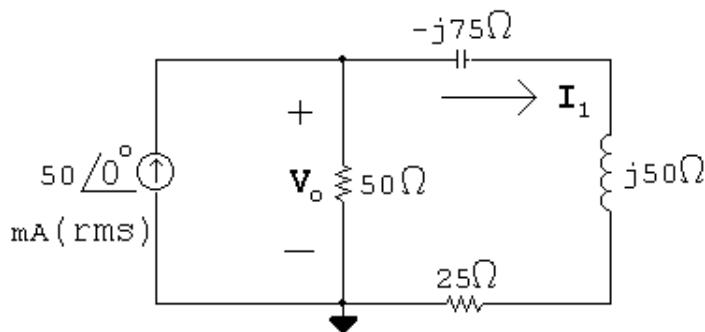
$$\text{P 10.16 } W_{\text{dc}} = \frac{V_{\text{dc}}^2}{R} T; \quad W_s = \int_{t_o}^{t_o+T} \frac{v_s^2}{R} dt$$

$$\therefore \frac{V_{\text{dc}}^2}{R} T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R} dt$$

$$V_{\text{dc}}^2 = \frac{1}{T} \int_{t_o}^{t_o+T} v_s^2 dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T} \int_{t_o}^{t_o+T} v_s^2 dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.17 [a]



$$Z_{\text{eq}} = 50 \parallel (25 - j25) = 20 - j10 \Omega$$

$$\therefore \mathbf{V}_o = 0.05 Z_{\text{eq}} = 1 - j0.5 \text{ V(rms)}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{25 - j25} = 30 + j10 \text{ mA(rms)}$$

$$\begin{aligned} S_g &= -\mathbf{V}_g \mathbf{I}_g^* = -(1 - j0.5)(0.05) \\ &= -50 + j25 \text{ mVA} \end{aligned}$$

[b] Source is delivering 50 mW.

[c] Source is absorbing 25 mVAR.

[d] $Q_{\text{cap}} = -|\mathbf{I}_1|^2(75) = -75 \text{ mVAR}$

$$P_{50\Omega} = \frac{|\mathbf{V}_o|^2}{50} = 25 \text{ mW}$$

$$P_{25\Omega} = |\mathbf{I}_1|^2(25) = 25 \text{ mW}$$

$$Q_{\text{ind}} = |\mathbf{I}_1|^2(50) = 50 \text{ mVAR}$$

$$P_{\text{middle branch}} = 25 \text{ mW}; \quad Q_{\text{middle branch}} = 0$$

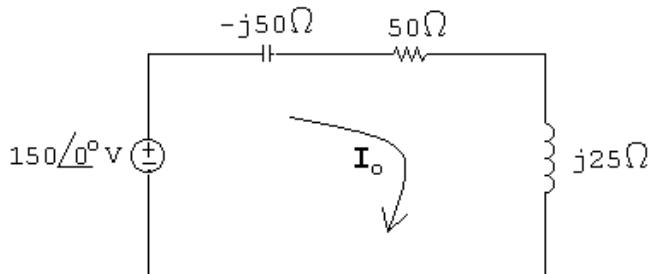
$$P_{\text{right branch}} = 25 \text{ mW}; \quad Q_{\text{right branch}} = -75 + 50 = -25 \text{ mVAR}$$

$$[\mathbf{e}] \sum P_{\text{del}} = 50 \text{ mW}$$

$$\sum P_{\text{diss}} = 25 + 25 = 50 \text{ mW} = \sum P_{\text{del}}$$

$$[\mathbf{f}] \sum Q_{\text{abs}} = 25 + 50 = 75 \text{ mVAR} = \sum Q_{\text{dev}}$$

P 10.18 $j\omega L = j25 \Omega$; $\frac{1}{j\omega C} = -j75 \Omega$



$$\mathbf{I}_o = \frac{j150}{50 - j25} = 2.4 + j1.2 \text{ A}$$

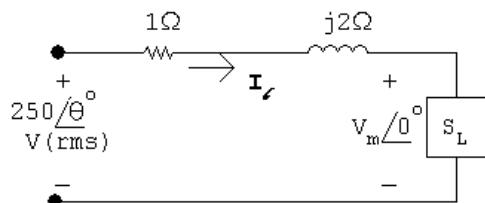
$$P = \frac{1}{2} |\mathbf{I}_o|^2 (50) = \frac{1}{2} (7.2)(50) = 180 \text{ W}$$

$$Q = \frac{1}{2} |\mathbf{I}_o|^2 (25) = 90 \text{ VAR}$$

$$S = P + jQ = 180 + j90 \text{ VA}$$

$$|S| = 201.25 \text{ VA}$$

P 10.19 [a] Let $\mathbf{V}_L = V_m \angle 0^\circ$:



$$S_L = 2500(0.8 + j0.6) = 2000 + j1500 \text{ VA}$$

$$\mathbf{I}_\ell^* = \frac{2000}{V_m} + j\frac{1500}{V_m}; \quad \mathbf{I}_\ell = \frac{2000}{V_m} - j\frac{1500}{V_m}$$

$$250 \angle \theta^\circ = V_m + \left(\frac{2000}{V_m} - j\frac{1500}{V_m} \right) (1 + j2)$$

$$250V_m \angle \theta^\circ = V_m^2 + (2000 - j1500)(1 + j2) = V_m^2 + 5000 + j2500$$

$$250V_m \cos \theta = V_m^2 + 5000; \quad 250V_m \sin \theta = 2500$$

$$(250)^2 V_m^2 = (V_m^2 + 5000)^2 + 2500^2$$

$$62,500V_m^2 = V_m^4 + 10,000V_m^2 + 31.25 \times 10^6$$

or

$$V_m^4 - 52,500V_m^2 + 31.25 \times 10^6 = 0$$

Solving,

$$V_m^2 = 26,250 \pm 25,647.86; \quad V_m = 227.81 \text{ V and } V_m = 24.54 \text{ V}$$

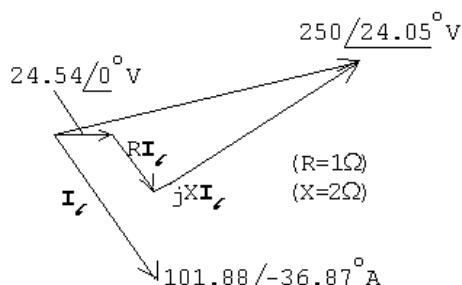
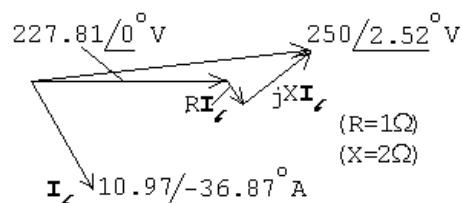
If $V_m = 227.81$ V:

$$\sin \theta = \frac{2500}{(227.81)(250)} = 0.044; \quad \therefore \theta = 2.52^\circ$$

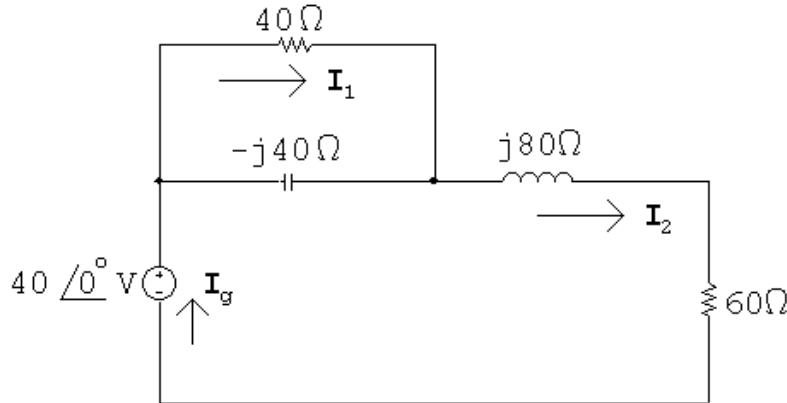
If $V_m = 24.54$ V:

$$\sin \theta = \frac{2500}{(24.54)(250)} = 0.4075; \quad \therefore \theta = 24.05^\circ$$

[b]



P 10.20 [a] $\frac{1}{j\omega C} = -j40 \Omega; \quad j\omega L = j80 \Omega$



$$Z_{eq} = 40 \parallel -j40 + j80 + 60 = 80 + j60 \Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{80 + j60} = 0.32 - j0.24 \text{ A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g \mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8 \text{ VA}$$

$$P = 6.4 \text{ W (del)}; \quad Q = 4.8 \text{ VAR (del)}$$

$$|S| = |S_g| = 8 \text{ VA}$$

[b] $\mathbf{I}_1 = \frac{-j40}{40 - j40} \mathbf{I}_g = 0.04 - j0.28 \text{ A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6 \text{ W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8 \text{ W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4 \text{ W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04 \text{ A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6 \text{ VAR (del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4 \text{ VAR (abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8 \text{ VAR} = \sum Q_{\text{dev}}$$

P 10.21 $S_T = 40,800 + j30,600 \text{ VA}$

$$S_1 = 20,000(0.96 - j0.28) = 19,200 - j5600 \text{ VA}$$

$$S_2 = S_T - S_1 = 21,600 + j36,200 = 42,154.48/\underline{59.176^\circ} \text{ VA}$$

$$\text{rf} = \sin(59.176^\circ) = 0.8587$$

$$\text{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

P 10.22 [a] $S_1 = 10,000 - j4000 \text{ VA}$

$$S_2 = \frac{|\mathbf{V}_L|^2}{Z_2^*} = \frac{(1000)^2}{60 - j80} = 6 + j8 \text{ kVA}$$

$$S_1 + S_2 = 16 + j4 \text{ kVA}$$

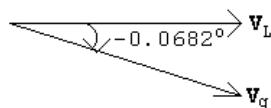
$$1000\mathbf{I}_L^* = 16,000 + j4000; \quad \therefore \mathbf{I}_L = 16 - j4 \text{ A(rms)}$$

$$\begin{aligned} \mathbf{V}_g &= \mathbf{V}_L + \mathbf{I}_L(0.5 + j0.05) = 1000 + (16 - j4)(0.5 + j0.05) \\ &= 1008.2 - j1.2 = 1008.2/\underline{-0.0682^\circ} \text{ Vrms} \end{aligned}$$

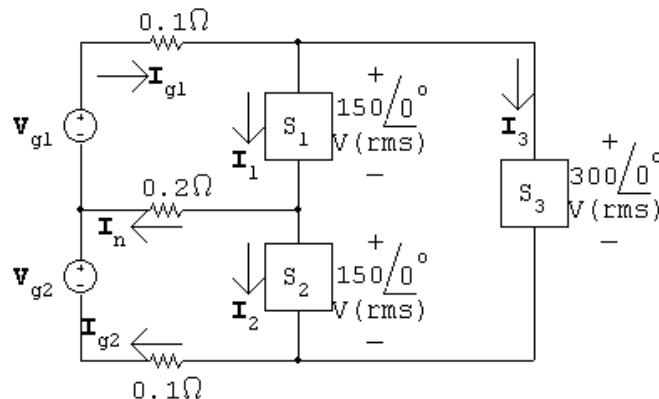
[b] $T = \frac{1}{f} = \frac{1}{50} = 20 \text{ ms}$

$$\frac{-0.0682^\circ}{360^\circ} = \frac{t}{20 \text{ ms}}; \quad \therefore t = -3.79 \mu\text{s}$$

[c] \mathbf{V}_L leads \mathbf{V}_g by 0.0682° or $3.79 \mu\text{s}$



P 10.23 [a]



$$\mathbf{I}_1 = \frac{6000 - j3000}{150} = 40 - j20 \text{ A (rms)}$$

$$\mathbf{I}_2 = \frac{7500 + j4500}{150} = 50 + j30 \text{ A (rms)}$$

$$\mathbf{I}_3 = \frac{12,000 - j9000}{300} = 40 - j30 \text{ A (rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 80 - j50 \text{ A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -10 - j50 \text{ A (rms)}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 90 + j0 \text{ A}$$

$$\mathbf{V}_{g1} = 0.1\mathbf{I}_{g1} + 150 + 0.2\mathbf{I}_n = 156 - j15 \text{ V (rms)}$$

$$\mathbf{V}_{g2} = -0.2\mathbf{I}_n + 150 + 0.1\mathbf{I}_{g2} = 161 + j10 \text{ V (rms)}$$

$$S_{g1} = -[(156 - j15)(80 + j50)] = -[13,230 + j6600] \text{ VA}$$

$$S_{g2} = -[(161 + j10)(90 + j0)] = -[14,490 + j900] \text{ VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

$$[\mathbf{b}] \quad P_{0.1} = |\mathbf{I}_{g1}|^2(0.1) = 890 \text{ W}$$

$$P_{0.2} = |\mathbf{I}_n|^2(0.2) = 520 \text{ W}$$

$$P_{0.1} = |\mathbf{I}_{g2}|^2(0.1) = 810 \text{ W}$$

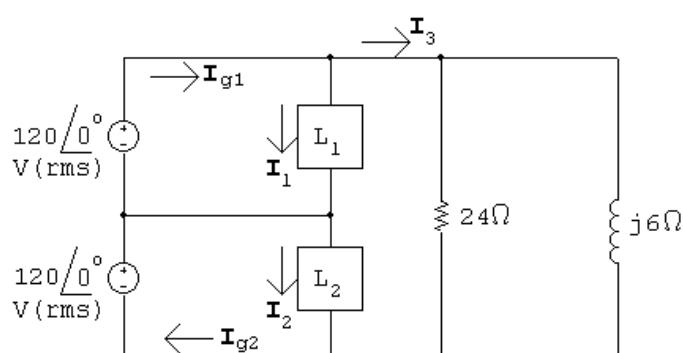
$$\sum P_{\text{dis}} = 890 + 520 + 810 + 6000 + 7500 + 12,000 = 27,720 \text{ W}$$

$$\sum P_{\text{dev}} = 13,230 + 14,490 = 27,720 \text{ W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 3000 + 9000 = 12,000 \text{ VAR}$$

$$\sum Q_{\text{del}} = 4500 + 6600 + 900 = 12,000 \text{ VAR} = \sum Q_{\text{abs}}$$

P 10.24 [a]



$$120\mathbf{I}_1^* = 4800 - j2400; \quad \therefore \quad \mathbf{I}_1 = 40 + j20 \text{ A (rms)}$$

$$120\mathbf{I}_2^* = 4800 + j3600; \quad \therefore \mathbf{I}_2 = 40 - j30 \text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{24} + \frac{240}{j6} = 10 - j40 \text{ A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 50 - j20 \text{ A}$$

$$S_{g1} = -120(50 + j20) = -6000 - j2400 \text{ VA}$$

Thus the \mathbf{V}_{g1} source is delivering 6 kW and 2.4 kvars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 50 - j70 \text{ A(rms)}$$

$$S_{g2} = -120(50 + j70) = -6000 - j8400 \text{ VA}$$

Thus the \mathbf{V}_{g2} source is delivering 6 kW and 8.4 kvars.

[b] $\sum P_{\text{gen}} = 6 + 6 = 12 \text{ kW}$

$$\sum P_{\text{abs}} = 4800 + 4800 + \frac{(240)^2}{24} = 12 \text{ kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 2400 + 8400 + 2400 = 13.2 \text{ kVAR}$$

$$\sum Q_{\text{abs}} = 3600 + \frac{(240)^2}{6} = 13.2 \text{ kVAR} = \sum Q_{\text{del}}$$

P 10.25 $S_1 = 1146 + 1200 + 1450 = 3796 + j0 \text{ VA}$

$$\therefore \mathbf{I}_1 = \frac{3796}{110} = 34.51 \text{ A}$$

$$S_2 = 145 + 630 + 1322 = 2097 + j0 \text{ VA}$$

$$\therefore \mathbf{I}_2 = \frac{2097}{110} = 19.064 \text{ A}$$

$$S_3 = 512 + 4856 = 5368 + j0 \text{ VA}$$

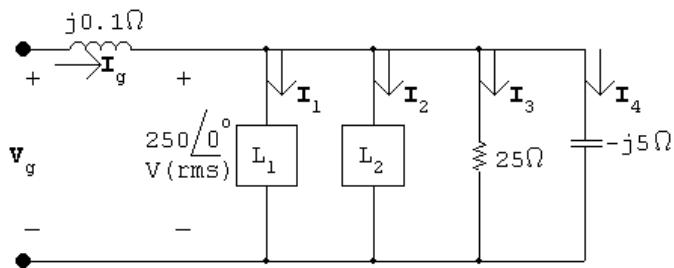
$$\therefore \mathbf{I}_3 = \frac{5368}{220} = 24.4 \text{ A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 58.91 \text{ A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 43.464 \text{ A}$$

The breaker protecting the upper service conductor will trip because its feeder current exceeds 50 A. The breaker protecting the lower service conductor will not trip because its feeder current is less than 50 A. Thus, service to Loads 1 and 3 will be interrupted.

P 10.26



$$250\mathbf{I}_1^* = 6000 - j8000$$

$$\mathbf{I}_1^* = 24 - j32; \quad \therefore \quad \mathbf{I}_1 = 24 + j32 \text{ A (rms)}$$

$$250\mathbf{I}_2^* = 9000 + j3000$$

$$\mathbf{I}_2^* = 36 + j12; \quad \therefore \quad \mathbf{I}_2 = 36 - j12 \text{ A (rms)}$$

$$\mathbf{I}_3 = \frac{250/0^\circ}{25} = 10 + j0 \text{ A}; \quad \mathbf{I}_4 = \frac{250/0^\circ}{-j5} = 0 + j50 \text{ A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70 + j70 \text{ A}$$

$$\mathbf{V}_g = 250 + (70 + j70)(j0.1) = 243 + j7 = 243.1/1.65^\circ \text{ V (rms)}$$

P 10.27 [a] From Problem 9.78,

$$Z_{ab} = 20 + j27.25 \quad \text{so}$$

$$\mathbf{I}_1 = \frac{75}{10 + j12.75 + 20 + 27.25} = \frac{75}{30 + j40} = 0.9 - j1.2 \text{ A}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}} \mathbf{I}_1 = \frac{j54}{160 + j120} (0.9 - j1.2) = 405 + j0 \text{ mA}$$

$$\mathbf{V}_L = (60 + j20)(0.405 + j0) = 24.3 + j8.1$$

$$|\mathbf{V}_L| = 25.61 \text{ V}$$

$$[\mathbf{b}] \quad P_g(\text{ideal}) = 75(0.9) = 67.5 \text{ W}$$

$$P_g(\text{practical}) = 67.5 - |\mathbf{I}_1|^2(10) = 45 \text{ W}$$

$$P_L = |\mathbf{I}_2|^2(60) = 9.8415 \text{ W}$$

$$\% \text{ delivered} = \frac{9.8415}{45}(100) = 21.87\%$$

P 10.28 [a] $S_1 = 3 + j0 \text{ kVA}$; $S_2 = 4 - j3 \text{ kVA}$; $S_3 = 5 - j6 \text{ kVA}$

$$S_T = S_1 + S_2 + S_3 = 12 - j9 \text{ kVA}$$

$$300\mathbf{I}^* = (12 - j9) \times 10^3; \quad \therefore \mathbf{I} = 40 + j30 \text{ A}$$

$$Z = \frac{300}{40 + j30} = 4.8 - j3.6 \Omega = 6 \angle -36.87^\circ \Omega$$

[b] pf = $\cos(-36.87^\circ) = 0.8$ leading

P 10.29 [a] From the solution to Problem 10.28 we have

$$\mathbf{I}_L = 40 + j30 \text{ A (rms)}$$

$$\begin{aligned} \therefore \mathbf{V}_s &= 300 \angle 0^\circ + (40 + j30)(0.2 + j0.05) = 306.5 + j8 \\ &= 306.6 \angle 1.5^\circ \text{ V (rms)} \end{aligned}$$

[b] $|\mathbf{I}_L| = \sqrt{16,000}$

$$P_\ell = (2500)(0.2) = 500 \text{ W} \quad Q_\ell = (2500)(0.05) = 125 \text{ VAR}$$

[c] $P_s = 12,000 + 500 = 12.5 \text{ kW} \quad Q_s = -9000 + 125 = -8.875 \text{ kVAR}$

[d] $\eta = \frac{12}{12.5}(100) = 96\%$

P 10.30 [a] $Z_1 = 240 + j70 = 250 \angle 16.26^\circ \Omega$

$$\text{pf} = \cos(16.26^\circ) = 0.96 \text{ lagging}$$

$$\text{rf} = \sin(16.26^\circ) = 0.28$$

$$Z_2 = 160 - j120 = 200 \angle -36.87^\circ \Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.8 \text{ leading}$$

$$\text{rf} = \sin(-36.87^\circ) = -0.6$$

$$Z_3 = 30 - j40 = 50 \angle -53.13^\circ \Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

$$\text{rf} = \sin(-53.13^\circ) = -0.8$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250 \angle 16.26^\circ}; \quad Y_2 = \frac{1}{200 \angle -36.87^\circ}; \quad Y_3 = \frac{1}{50 \angle -53.13^\circ}$$

$$Y = 19.84 + j17.88 \text{ mS}$$

$$Z = \frac{1}{Y} = 37.44 / -42.03^\circ \Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.31 [a] $\mathbf{I} = \frac{270/0^\circ}{36 + j48} = 2.7 - j3.6 = 4.5 / -53.13^\circ \text{ A(rms)}$

$$P = (4.5)^2(6) = 121.5 \text{ W}$$

[b] $Y_L = \frac{1}{30 + j40} = 12 - j16 \text{ mS}$

$$\therefore X_C = \frac{1}{-16 \times 10^{-3}} = -62.5 \Omega$$

[c] $Z_L = \frac{1}{12 \times 10^{-3}} = 83.33 \Omega$

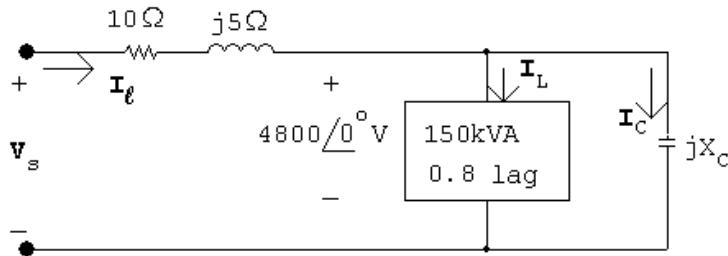
[d] $\mathbf{I} = \frac{270/0^\circ}{89.33 + j8} = 3.01 / -5.12^\circ \text{ A}$

$$P = (3.01)^2(6) = 54.37 \text{ W}$$

[e] $\% = \frac{54.37}{121.5}(100) = 44.75\%$

Thus the power loss after the capacitor is added is 44.75% of the power loss before the capacitor is added.

P 10.32



$$\mathbf{I}_L = \frac{120,000 - j90,000}{4800} = 25 - j18.75 \text{ A (rms)}$$

$$\mathbf{I}_C = \frac{4800}{jX_C} = -j \frac{4800}{X_C} = jI_C$$

$$\mathbf{I}_e = 25 - j18.75 + jI_C = 25 + j(I_C - 18.75)$$

$$\begin{aligned}\mathbf{V}_s &= 4800 + (10 + j5)[25 + j(I_C - 18.75)] \\ &= (5143.75 - 5I_C) + j(-62.5 + 10I_C)\end{aligned}$$

$$|\mathbf{V}_s|^2 = (5143.75 - 5I_C)^2 + (-62.5 + 10I_C)^2 = (4800)^2$$

$$\therefore 125I_C^2 - 52,687.5I_C + 3,422,070.313 = 0$$

$$I_C = 341.284 \text{ A(rms)} \quad \text{and} \quad I_C = 80.2165 \text{ A(rms)}^*$$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ .

$$\therefore X_C = -\frac{4800}{80.2165} = -59.838$$

$$\therefore C = \frac{1}{(59.838)(120\pi)} = 44.43 \mu\text{F}$$

P 10.33 [a] $S_L = 20,000(0.85 + j0.53) = 17,000 + j10,535.65 \text{ VA}$

$$125\mathbf{I}_L^* = (17,000 + j10,535.65); \quad \mathbf{I}_L^* = 136 + j84.29 \text{ A(rms)}$$

$$\therefore \mathbf{I}_L = 136 - j84.29 \text{ A(rms)}$$

$$\begin{aligned}\mathbf{V}_s &= 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04 \\ &= 133.48/\underline{4.31^\circ} \text{ V(rms)}\end{aligned}$$

$$|\mathbf{V}_s| = 133.48 \text{ V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256 \text{ W}$

[c] $\frac{(125)^2}{X_C} = -10,535.65; \quad X_C = -1.48306 \Omega$

$$-\frac{1}{\omega C} = -1.48306; \quad C = \frac{1}{(1.48306)(120\pi)} = 1788.59 \mu\text{F}$$

[d] $\mathbf{I}_\ell = 136 + j0 \text{ A(rms)}$

$$\begin{aligned}\mathbf{V}_s &= 125 + 136(0.01 + j0.08) = 126.36 + j10.88 \\ &= 126.83/\underline{4.92^\circ} \text{ V(rms)}\end{aligned}$$

$$|\mathbf{V}_s| = 126.83 \text{ V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96 \text{ W}$

P 10.34 [a] $S_o = \text{original load} = 1600 + j\frac{1600}{0.8}(0.6) = 1600 + j1200 \text{ kVA}$

$$S_f = \text{final load} = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560 \text{ kVA}$$

$$\therefore Q_{\text{added}} = 560 - 1200 = -640 \text{ kVAR}$$

[b] deliver

[c] $S_a = \text{added load} = 320 - j640 = 715.54/-63.43^\circ \text{ kVA}$

$$\text{pf} = \cos(-63.43) = 0.447 \text{ leading}$$

[d] $\mathbf{I}_L^* = \frac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500 \text{ A}$

$$\mathbf{I}_L = 666.67 - j500 = 833.33/-36.87^\circ \text{ A(rms)}$$

$$|\mathbf{I}_L| = 833.33 \text{ A(rms)}$$

[e] $\mathbf{I}_L^* = \frac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

$$\mathbf{I}_L = 800 - j233.33 = 833.33/-16.26^\circ \text{ A(rms)}$$

$$|\mathbf{I}_L| = 833.33 \text{ A(rms)}$$

P 10.35 [a] $P_{\text{before}} = P_{\text{after}} = (833.33)^2(0.25) = 173.611 \text{ kW}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.25 + j0.1)$

$$= 2616.67 - j58.33 = 2617.32/-1.28^\circ \text{ V(rms)}$$

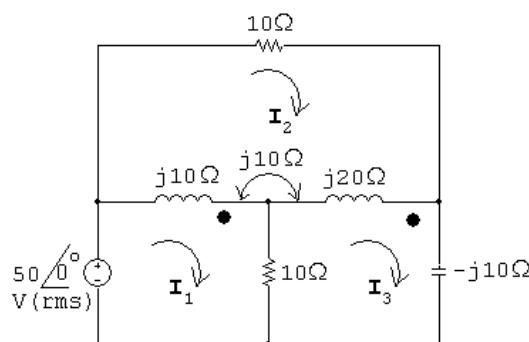
$$|\mathbf{V}_s(\text{before})| = 2617.32 \text{ V(rms)}$$

$$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.25 + j0.1)$$

$$= 2623.33 + j21.67 = 2623.42/0.47^\circ \text{ V(rms)}$$

$$|\mathbf{V}_s(\text{after})| = 2623.42 \text{ V(rms)}$$

P 10.36 [a]



$$50 = j10(\mathbf{I}_1 - \mathbf{I}_2) + j10(\mathbf{I}_3 - \mathbf{I}_2) + 10(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 10\mathbf{I}_2 + j20(\mathbf{I}_2 - \mathbf{I}_3) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = -j10\mathbf{I}_3 + 10(\mathbf{I}_3 - \mathbf{I}_1) + j20(\mathbf{I}_3 - \mathbf{I}_2) + j10(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

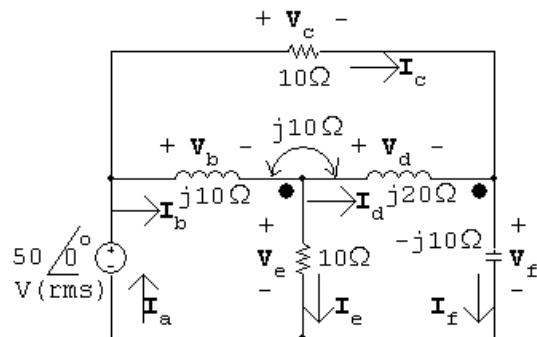
$$\mathbf{I}_1 = 5.5 + j0.5 \text{ A(rms)}; \quad \mathbf{I}_2 = 3 + j2 \text{ A(rms)}; \quad \mathbf{I}_3 = 2 + j2 \text{ A(rms)}$$

$$\mathbf{I}_a = \mathbf{I}_1 = 5.5 + j0.5 \text{ A} \quad \mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_2 = 2.5 - j1.5 \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_2 = 3 + j2 \text{ A} \quad \mathbf{I}_d = \mathbf{I}_3 - \mathbf{I}_2 = -1 \text{ A}$$

$$\mathbf{I}_e = \mathbf{I}_1 - \mathbf{I}_3 = 3.5 - j1.5 \text{ A} \quad \mathbf{I}_f = \mathbf{I}_3 = 2 + j2 \text{ A}$$

[b]



$$\mathbf{V}_a = 50 \text{ V}$$

$$\mathbf{V}_b = j10\mathbf{I}_b + j10\mathbf{I}_d = 15 + j15 \text{ V}$$

$$\mathbf{V}_c = 10\mathbf{I}_c = 30 + j20 \text{ V}$$

$$\mathbf{V}_d = j20\mathbf{I}_d + j10\mathbf{I}_b = 15 + j5 \text{ V}$$

$$\mathbf{V}_e = 10\mathbf{I}_e = 35 - j15 \text{ V}$$

$$\mathbf{V}_f = -j10\mathbf{I}_f = 20 - j20 \text{ V}$$

$$S_a = -50\mathbf{I}_a^* = -275 + j25 \text{ VA}$$

$$S_b = \mathbf{V}_b \mathbf{I}_b^* = 15 + j60 \text{ VA}$$

$$S_c = \mathbf{V}_c \mathbf{I}_c^* = 130 + j0 \text{ VA}$$

$$S_d = \mathbf{V}_d \mathbf{I}_d^* = -15 - j5 \text{ VA}$$

$$S_e = \mathbf{V}_e \mathbf{I}_e^* = 145 - j0 \text{ VA}$$

$$S_f = \mathbf{V}_f \mathbf{I}_f^* = 0 - j80 \text{ VA}$$

[c] $\sum P_{\text{dev}} = 275 + 15 = 290 \text{ W}$

$$\sum P_{\text{abs}} = 15 + 130 + 145 = 290 \text{ W}$$

Note that the total power absorbed by the coupled coils is zero:

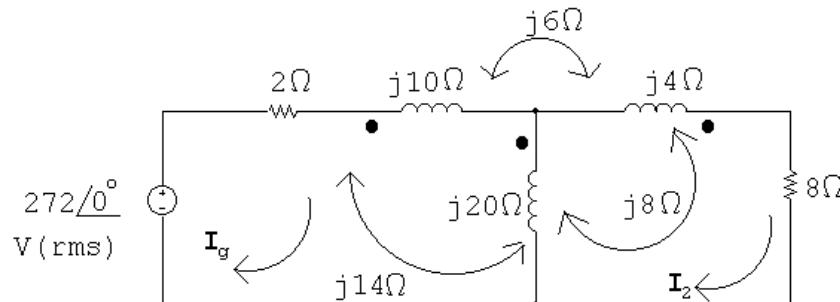
$$15 - 15 = 0 = P_b + P_d$$

[d] $\sum Q_{\text{dev}} = 5 + 80 = 85 \text{ VAR}$

$$\sum Q_{\text{abs}} = 25 + 60 = 85 \text{ VAR}$$

$\sum Q$ absorbed by the coupled coils is $Q_b + Q_d = 55$

P 10.37 [a]



$$272\angle 0^\circ = 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2$$

$$+j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)$$

$$0 = j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2$$

$$+j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2$$

Solving,

$$I_g = 20 - j4 \text{ A(rms)}; \quad \mathbf{I}_2 = 24\angle 0^\circ \text{ A(rms)}$$

$$P_{8\Omega} = (24)^2(8) = 4608 \text{ W}$$

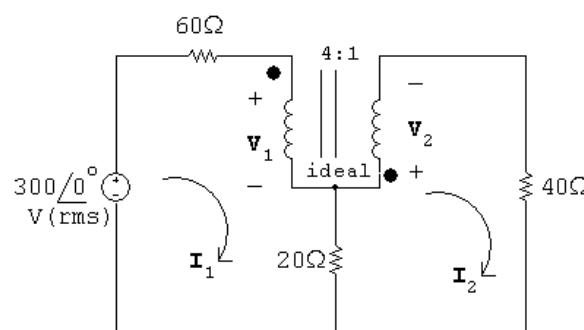
[b] $P_g(\text{developed}) = (272)(20) = 5440 \text{ W}$

[c] $Z_{ab} = \frac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \frac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\angle 13.28^\circ \Omega$

[d] $P_{2\Omega} = |I_g|^2(2) = 832 \text{ W}$

$$\sum P_{\text{diss}} = 832 + 4608 = 5440 \text{ W} = \sum P_{\text{dev}}$$

P 10.38 [a]



$$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$$

$$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \quad \mathbf{I}_2 = -4\mathbf{I}_1$$

Solving,

$$\mathbf{V}_1 = 260 \text{ V (rms)}; \quad \mathbf{V}_2 = 65 \text{ V (rms)}$$

$$\mathbf{I}_1 = 0.25 \text{ A (rms)}; \quad \mathbf{I}_2 = -1.0 \text{ A (rms)}$$

$$\mathbf{V}_{5A} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285 \text{ V (rms)}$$

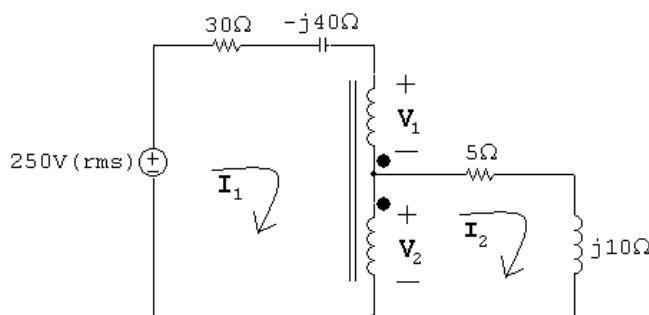
$$\therefore P = -(285)(5) = -1425 \text{ W}$$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25 \text{ A (rms)}$

$$\therefore P_{20\Omega} = (1.25)^2(20) = 31.25 \text{ W}$$

P 10.39 [a]



$$(30 - j40)\mathbf{I}_1 + \mathbf{V}_1 + \mathbf{V}_2 = 250$$

$$(5 + j10)\mathbf{I}_2 - \mathbf{V}_2 = 0$$

$$\frac{\mathbf{V}_1}{900} = \frac{-\mathbf{V}_2}{300}$$

$$900\mathbf{I}_1 = 300(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$$\mathbf{V}_1 = 150 + j300 \text{ V (rms)}; \quad \mathbf{V}_2 = -50 - j100 \text{ V (rms)}$$

$$\mathbf{I}_1 = 5 + j0 \text{ A (rms)}; \quad \mathbf{I}_2 = -10 + j0 \text{ A (rms)}$$

$$P_{30\Omega} = (5)^2(30) = 750 \text{ W}; \quad \text{and} \quad P_{5\Omega} = (10)^2(5) = 500 \text{ W}$$

[b] $P_g = -250(5/0^\circ) = -1250 \text{ W}$

$$\sum P_{\text{abs}} = 750 + 500 = 1250 \text{ W} = \sum P_{\text{dev}}$$

$$P \text{ 10.40 [a]} \quad 25a_1^2 + 4a_2^2 = 500$$

$$\mathbf{I}_{25} = a_1 \mathbf{I}; \quad P_{25} = a_1^2 \mathbf{I}^2 (25)$$

$$\mathbf{I}_4 = a_2 \mathbf{I}; \quad P_4 = a_2^2 \mathbf{I}^2 (4)$$

$$P_4 = 4P_{25}; \quad a_2^2 \mathbf{I}^2 4 = 100a_1^2 \mathbf{I}^2$$

$$\therefore 100a_1^2 = 4a_2^2$$

$$25a_1^2 + 100a_1^2 = 500; \quad a_1 = 2$$

$$25(4) + 4a_2^2 = 500; \quad a_2 = 10$$

$$[\text{b}] \quad \mathbf{I} = \frac{2000/0^\circ}{500 + 500} = 2/0^\circ \text{ A(rms)}$$

$$\mathbf{I}_{25} = a_1 \mathbf{I} = 4 \text{ A}$$

$$P_{25\Omega} = (16)(25) = 400 \text{ W}$$

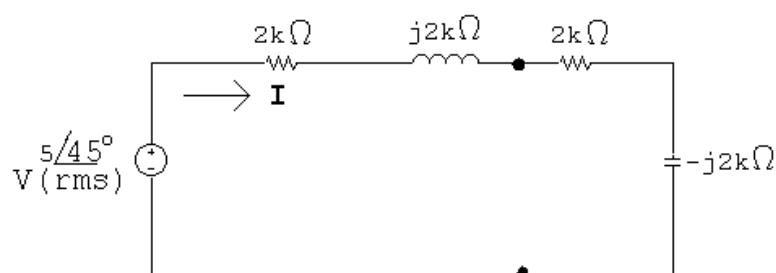
$$[\text{c}] \quad \mathbf{I}_4 = a_2 \mathbf{I} = 10(2) = 20 \text{ A(rms)}$$

$$\mathbf{V}_4 = (20)(4) = 80/0^\circ \text{ V(rms)}$$

$$P \text{ 10.41 [a]} \quad Z_{\text{Th}} = j4000 + \frac{(4000)(-j4000)}{4000 - j4000} = 2000 + j2000 \Omega$$

$$\therefore Z_L = Z_{\text{Th}}^* = 2000 - j2000 \Omega$$

$$[\text{b}] \quad \mathbf{V}_{\text{Th}} = \frac{10/0^\circ(4000)}{4000 - j4000} = 5 + j5 = 5\sqrt{2}/45^\circ \text{ V}$$



$$\mathbf{I} = \frac{5\sqrt{2}/45^\circ}{4000} = 1.25\sqrt{2}/45^\circ \text{ mA}$$

$$|\mathbf{I}_{\text{rms}}| = 1.25 \text{ mA}$$

$$P_{\text{load}} = (0.00125)^2 (2000) = 3.125 \text{ mW}$$

- [c] The closest resistor values from Appendix H are $1.8\text{ k}\Omega$ and $2.2\text{ k}\Omega$. Find the capacitor value:

$$\frac{1}{8000C} = 2000 \quad \text{so} \quad C = 62.5\text{ nF}$$

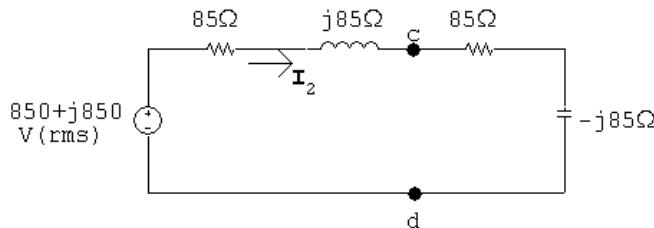
The closest capacitor value is 47 nF . Choose $R = 1.8\text{ k}\Omega$:

$$\begin{aligned} \mathbf{I} &= \frac{5/45^\circ}{2000 + j2000 + 1800 - j2659.57} = 0.7462 + j1.06\text{ mA(rms)} \\ &= 1.3/54.85^\circ\text{ mA(rms)} \end{aligned}$$

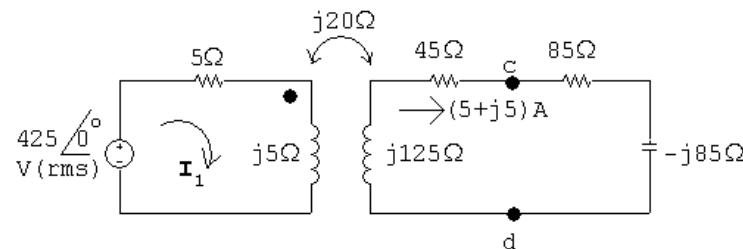
$$P_{\text{load}} = (0.0013)^2(1800) = 3.03\text{ mW} \quad (\text{instead of } 3.125\text{ mW})$$

Therefore, the load impedance is $Z_L = 1800 - j2659.57\Omega$.

- P 10.42 [a] From Problem 9.75, $Z_{\text{Th}} = 85 + j85\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\text{ V}$. Thus, for maximum power transfer, $Z_L = Z_{\text{Th}}^* = 85 - j85\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\text{ A}$$



$$425/0^\circ = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\text{ A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18,062.5 + j9562.5\text{ VA}$$

$$P_g = 18,062.5\text{ W}$$

[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11,562.5 + 2250 = 13,812.5\text{ W}$

$$\% \text{ loss in transformer} = \frac{13,812.5}{18,062.5}(100) = 76.47\%$$

P 10.43 [a] $\frac{156 - j42 - 300}{Z_{Th}} + \frac{156 - j42}{200 - j500} = 0$
 $\therefore Z_{Th} = \frac{300 - 156 + j42}{0.18 + j0.24} = 400 - j300 \Omega$

$$\therefore Z_L = 400 + j300 \Omega$$

[b] $\mathbf{I} = \frac{300/0^\circ}{800/0^\circ} = 0.375/0^\circ \text{ A(rms)}$

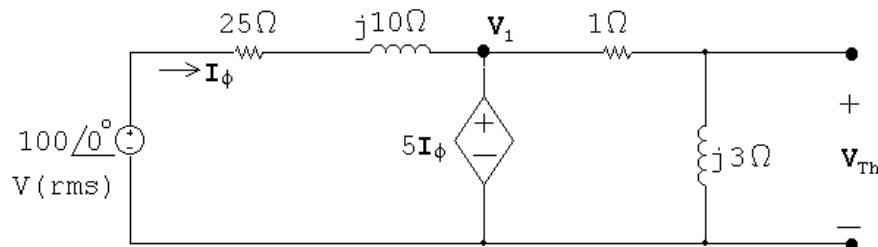
$$P = (0.375)^2(400) = 56.25 \text{ W}$$

[c] Let $R = 180 \Omega + 220 \Omega = 400 \Omega$

$$2\pi(50)L = 300 \quad \text{so} \quad L = \frac{300}{100\pi} = 95.5 \text{ mH}$$

Use 9 series-connected 10 mH inductors to get 90 mH. Use 2 parallel-connected 10 mH inductors to get 5 mH. Use 2 parallel-connected 1 mH inductors to get 0.5 mH. Combine the 90 mH, the 5 mH, and the 0.5 mH in series to get 95.5 mH.

P 10.44 [a] Open circuit voltage:



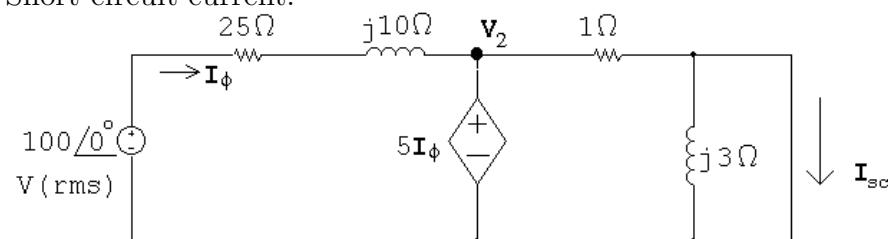
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5 \frac{100 - 5\mathbf{I}\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}\phi = \frac{100}{30 + j10} = 3 - j \text{ A}$$

$$\mathbf{V}_{Th} = \frac{j3}{1 + j3}(5\mathbf{I}\phi) = 15 \text{ V}$$

Short circuit current:



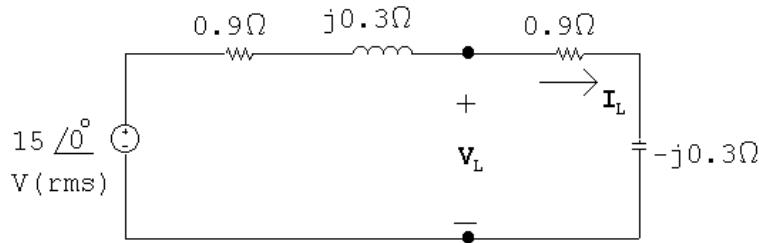
$$\mathbf{V}_2 = 5\mathbf{I}\phi = \frac{100 - 5\mathbf{I}\phi}{25 + j10}$$

$$\mathbf{I}_\phi = 3 - j1 \text{ A}$$

$$\mathbf{I}_{sc} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5 \text{ A}$$

$$Z_{Th} = \frac{15}{15 - j5} = 0.9 + j0.3 \Omega$$

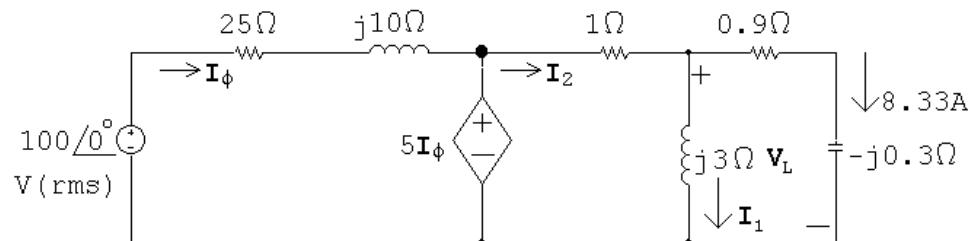
$$Z_L = Z_{Th}^* = 0.9 - j0.3 \Omega$$



$$\mathbf{I}_L = \frac{0.3}{1.8} = 8.33 \text{ A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5 \text{ W}$$

[b] $\mathbf{V}_L = (0.9 - j0.3)(8.33) = 7.5 - j2.5 \text{ V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_L}{j3} = -0.833 - j2.5 \text{ A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_L = 7.5 - j2.5 \text{ A(rms)}$$

$$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1 \text{ A}$$

$$\mathbf{I}_{d.s.} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5 \text{ A}$$

$$S_g = -100(3 + j1) = -300 - j100 \text{ VA}$$

$$S_{d.s.} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0 \text{ VA}$$

$$P_{dev} = 300 + 75 = 375 \text{ W}$$

$$\% \text{ developed} = \frac{62.5}{375}(100) = 16.67\%$$

Checks:

$$P_{25\Omega} = (10)(25) = 250 \text{ W}$$

$$P_{1\Omega} = (67.5)(1) = 67.5 \text{ W}$$

$$P_{0.9\Omega} = 62.5 \text{ W}$$

$$\sum P_{\text{abs}} = 230 + 62.5 + 67.5 = 375 = \sum P_{\text{dev}}$$

$$Q_{j10} = (10)(10) = 100 \text{ VAR}$$

$$Q_{j3} = (6.94)(3) = 20.82 \text{ VAR}$$

$$Q_{-j0.3} = (69.4)(-0.3) = -20.82 \text{ VAR}$$

$$Q_{\text{source}} = -100 \text{ VAR}$$

$$\sum Q = 100 + 20.82 - 20.82 - 100 = 0$$

$$\text{P 10.45 } Z_L = |Z_L| \angle \theta^\circ = |Z_L| \cos \theta^\circ + j |Z_L| \sin \theta^\circ$$

$$\text{Thus } |\mathbf{I}| = \frac{|\mathbf{V}_{\text{Th}}|}{\sqrt{(R_{\text{Th}} + |Z_L| \cos \theta)^2 + (X_{\text{Th}} + |Z_L| \sin \theta)^2}}$$

$$\text{Therefore } P = \frac{0.5 |\mathbf{V}_{\text{Th}}|^2 |Z_L| \cos \theta}{(R_{\text{Th}} + |Z_L| \cos \theta)^2 + (X_{\text{Th}} + |Z_L| \sin \theta)^2}$$

Let D = denominator in the expression for P , then

$$\frac{dP}{d|Z_L|} = \frac{(0.5 |\mathbf{V}_{\text{Th}}|^2 \cos \theta)(D \cdot 1 - |Z_L| dD/d|Z_L|)}{D^2}$$

$$\frac{dD}{d|Z_L|} = 2(R_{\text{Th}} + |Z_L| \cos \theta) \cos \theta + 2(X_{\text{Th}} + |Z_L| \sin \theta) \sin \theta$$

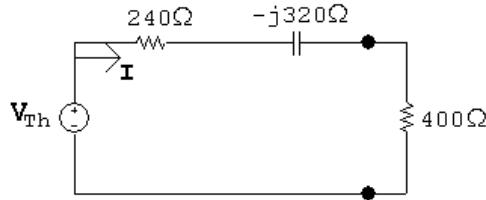
$$\frac{dP}{d|Z_L|} = 0 \quad \text{when } D = |Z_L| \left(\frac{dD}{d|Z_L|} \right)$$

Substituting the expressions for D and $(dD/d|Z_L|)$ into this equation gives us the relationship $R_{\text{Th}}^2 + X_{\text{Th}}^2 = |Z_L|^2$ or $|Z_{\text{Th}}| = |Z_L|$.

$$\text{P 10.46 [a]} \quad Z_{\text{Th}} = 200 - j480 + \frac{(j200)(500 + j300)}{500 + j500} = 240 - j320 = 400 \angle -53.13^\circ \Omega$$

$$\therefore R = |Z_{\text{Th}}| = 400 \Omega$$

[b] $\mathbf{V}_{Th} = \frac{j200}{500 + j300 + j200} (300/0^\circ) = 60 + j60 \text{ V(rms)}$



$$\mathbf{I} = \frac{60 + j60}{640 - j320} = 37.5 + j112.5 \text{ mA(rms)} = 118.59/71.57^\circ \text{ mA(rms)}$$

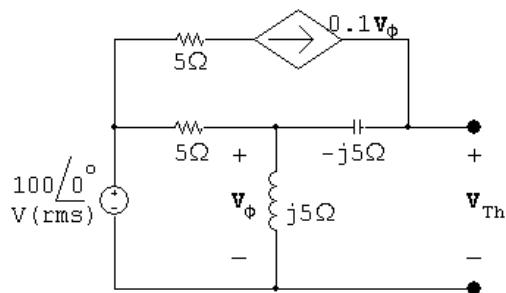
$$P = (0.11859)^2(400) = 5.625 \text{ W}$$

[c] Pick the 390Ω resistor from Appendix H for the closest match:

$$\mathbf{I} = \frac{60 + j60}{630 - j320} = 120.084/71.93^\circ \text{ mA(rms)}$$

$$P = (0.120084)^2(390) = 5.624 \text{ W}$$

P 10.47 [a] Open circuit voltage:

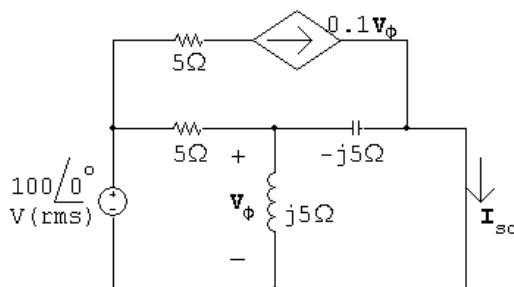


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \mathbf{V}_\phi = 40 + j80 \text{ V(rms)}$$

$$\mathbf{V}_{Th} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60 \text{ V(rms)}$$

Short circuit current:



$$\mathbf{I}_{sc} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

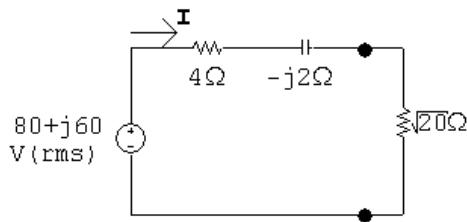
$$\therefore \mathbf{V}_\phi = 100 \text{ V(rms)}$$

$$\mathbf{I}_{sc} = (0.1 + j0.2)(100) = 10 + j20 \text{ A(rms)}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{80 + j60}{10 + j20} = 4 - j2 \Omega$$

$$\therefore R_o = |Z_{Th}| = 4.47 \Omega$$

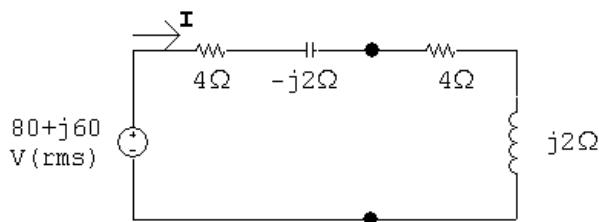
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82 \text{ A (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17 \text{ W}$$

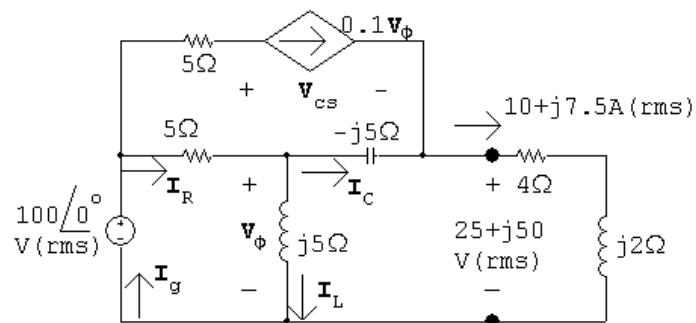
[c]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5 \text{ A (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625 \text{ W}$$

[d]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

$$\mathbf{V}_\phi = 50 + j25 \text{ V (rms)}$$

$$0.1\mathbf{V}_\phi = 5 + j2.5 \text{ V (rms)}$$

$$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$$

$$\mathbf{I}_C = 5 + j5 \text{ A (rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10 \text{ A (rms)}$$

$$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5 \text{ A (rms)}$$

$$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5 \text{ A (rms)}$$

$$S_g = -100\mathbf{I}_g^* = -1500 - j250 \text{ VA}$$

$$100 = 5(5 + j2.5) + \mathbf{V}_{cs} + 25 + j50 \quad \therefore \quad \mathbf{V}_{cs} = 50 - j62.5 \text{ V (rms)}$$

$$S_{cs} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5 \text{ VA}$$

Thus,

$$\sum P_{\text{dev}} = 1500$$

$$\% \text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

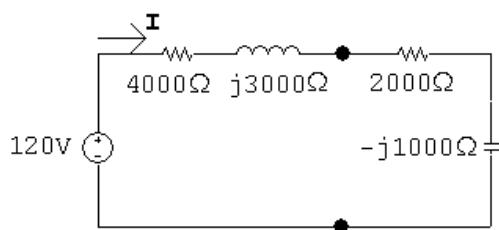
P 10.48 [a] First find the Thévenin equivalent:

$$j\omega L = j3000 \Omega$$

$$Z_{\text{Th}} = 6000 \| 12,000 + j3000 = 4000 + j3000 \Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12,000}{6000 + 12,000}(180) = 120 \text{ V}$$

$$\frac{-j}{\omega C} = -j1000 \Omega$$



$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6 \text{ mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360 \text{ mW}$$

[b] Set $C_o = 0.1 \mu\text{F}$ so $-j/\omega C = -j2000 \Omega$

Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1 \Omega$$

$$\therefore R_o = 4000 \Omega$$

$$[\text{c}] \quad \mathbf{I} = \frac{120}{8000 + j1000} = 14.77 - j1.85 \text{ mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1 \text{ mW}$$

Yes; $443.1 \text{ mW} > 360 \text{ mW}$

$$[\text{d}] \quad \mathbf{I} = \frac{120}{8000} = 15 \text{ mA}$$

$$P = \frac{1}{2}(0.015)^2(4000) = 450 \text{ mW}$$

[e] $R_o = 4000 \Omega$; $C_o = 66.67 \text{ nF}$

[f] Yes; $450 \text{ mW} > 443.1 \text{ mW}$

P 10.49 [a] Set $C_o = 0.1 \mu\text{F}$, so $-j/\omega C = -j2000 \Omega$; also set $R_o = 4123.1 \Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79 \text{ mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18 \text{ mW}$$

[b] Yes; $443.18 \text{ mW} > 360 \text{ mW}$

[c] Yes; $443.18 \text{ mW} < 450 \text{ mW}$

P 10.50 [a] $\frac{1}{\omega C} = 100 \Omega$; $C = \frac{1}{(60)(200\pi)} = 26.53 \mu\text{F}$

$$\begin{aligned} [\text{b}] \quad \mathbf{V}_{\text{swo}} &= 4000 + (40)(1.25 + j10) = 4050 + j400 \\ &= 4069.71/\underline{5.64^\circ} \text{ V(rms)} \end{aligned}$$

$$\mathbf{V}_{\text{sw}} = 4000 + (40 - j40)(1.25 + j10) = 4450 + j350 = 4463.73/\underline{4.50^\circ} \text{ V(rms)}$$

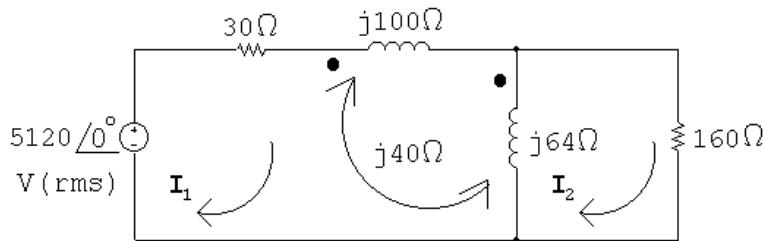
$$\% \text{ increase} = \left(\frac{4463.73}{4069.71} - 1 \right) (100) = 9.68\%$$

$$[\text{c}] \quad P_{\ell\text{wo}} = (40\sqrt{2})^2(1.25) = 4000 \text{ W}$$

$$P_{\ell\text{w}} = 40^2(1.25) = 2000 \text{ W}$$

$$\% \text{ increase} = \left(\frac{4000}{2000} - 1 \right) (100) = 100\%$$

P 10.51 [a]



$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) + j64(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1$$

$$0 = 160\mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1$$

Solving,

$$\mathbf{I}_1 = 8 - j20 \text{ A(rms)}; \quad \mathbf{I}_2 = 13\angle0^\circ \text{ A(rms)}$$

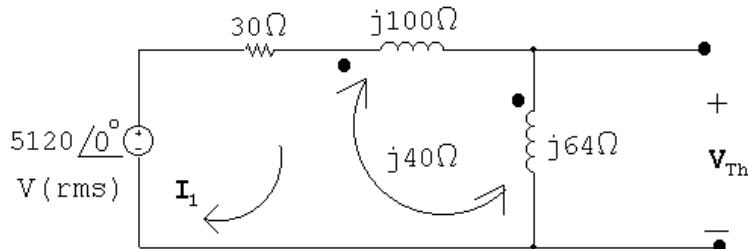
$$\therefore \mathbf{V}_o = (13)(160) = 2080 \text{ V(rms)}$$

[b] $P = (13)^2(160) = 27,040 \text{ W}$

[c] $S_g = -(5120)(8 + j20) = -40,960 - j102,400 \text{ VA} \quad \therefore P_g = -40,960 \text{ W}$

$$\% \text{ delivered} = \frac{27,040}{40,960}(100) = 66\%$$

P 10.52 [a] Open circuit voltage:

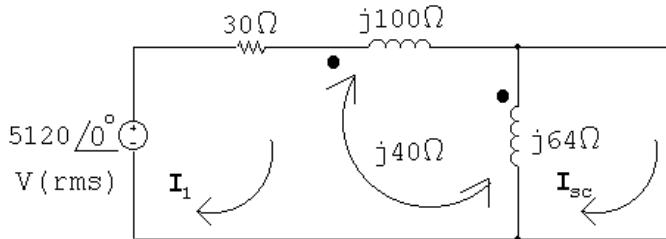


$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40\mathbf{I}_1 + j64\mathbf{I}_1 + j40\mathbf{I}_1$$

$$\therefore \mathbf{I}_1 = \frac{5120}{30 + j244} = 2.54 - j20.67 \text{ A(rms)}$$

$$\mathbf{V}_{Th} = j64\mathbf{I}_1 + j40\mathbf{I}_1 = j104\mathbf{I}_1 = 2149.8 + j264.32 \text{ V}$$

Short circuit current:



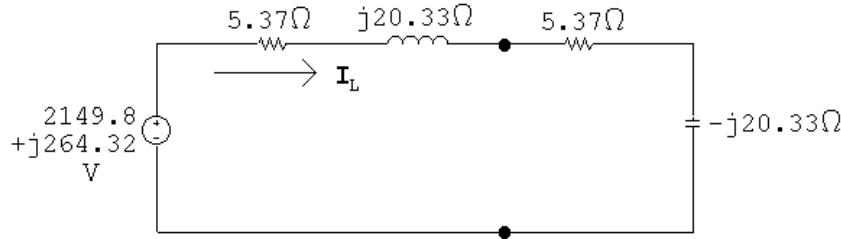
$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_{sc}) + j64(\mathbf{I}_1 - \mathbf{I}_{sc}) + j40\mathbf{I}_1$$

$$0 = j64(\mathbf{I}_{sc} - \mathbf{I}_1) - j40\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{sc} = 38.25 - j95.63 \text{ A}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{2149.8 + j264.32}{38.25 - j95.63} = 5.37 + j20.33 \Omega$$



$$\mathbf{I}_L = \frac{2149.8 + j264.32}{10.74} = 200.17 + j24.61 = 201.67/\underline{7.01^\circ} \text{ A}$$

$$P_L = (201.67)^2(5.37) = 218.4 \text{ kW}$$

$$[\text{b}] \quad Z_o \mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 = 0 \quad \text{so} \quad \mathbf{I}_1 = \frac{Z_o + j64}{j104} \mathbf{I}_2$$

$$\therefore \mathbf{I}_1 = \frac{5.37 - j20.33 + j64}{j104} (200.17 + j24.61) = 85.32/\underline{0^\circ} \text{ A (rms)}$$

$$P_{dev} = (5120)(85.32) = 436.8 \text{ kW}$$

- [c] Begin by choosing the capacitor value from Appendix H that is closest to the required reactive impedance, assuming the frequency of the source is 60 Hz:

$$20.33 = \frac{1}{2\pi(60)C} \quad \text{so} \quad C = \frac{1}{2\pi(60)(20.33)} = 130.48 \mu\text{F}$$

Choose the capacitor value closest to this capacitance from Appendix H, which is $100 \mu\text{F}$. Then,

$$X_L = -\frac{1}{2\pi(60)(100 \times 10^{-6})} = -26.5258 \Omega$$

Now set R_L as close as possible to $\sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$:

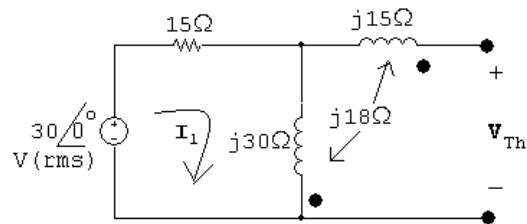
$$R_L = \sqrt{5.37^2 + (20.33 - 26.5258)^2} = 8.2 \Omega$$

The closest single resistor value from Appendix H is 10Ω . The resulting real power developed by the source is calculated below, using the Thévenin equivalent circuit:

$$\mathbf{I} = \frac{2149.8 + j264.32}{5.37 + j20.33 + 10 - j26.5258} = 130.7/\underline{28.96^\circ}$$

$$P = |2149.8 + j264.32|(130.7) = 283.1 \text{ kW} \quad (\text{instead of } 436.8 \text{ kW})$$

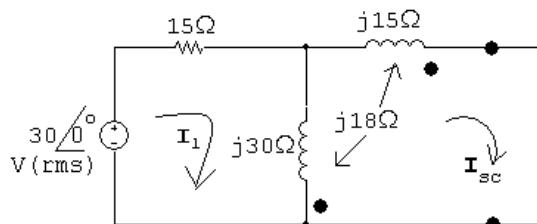
P 10.53 Open circuit voltage:



$$\mathbf{I}_1 = \frac{30\angle 0^\circ}{15 + j30} = 0.4 - j0.8 \text{ A}$$

$$\mathbf{V}_{\text{Th}} = j30\mathbf{I}_1 - j18\mathbf{I}_1 = j12\mathbf{I}_1 = 9.6 + j4.8 = 10.73\angle 26.57^\circ$$

Short circuit current:



$$30\angle 0^\circ = 15\mathbf{I}_1 + j30(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j18\mathbf{I}_{\text{sc}}$$

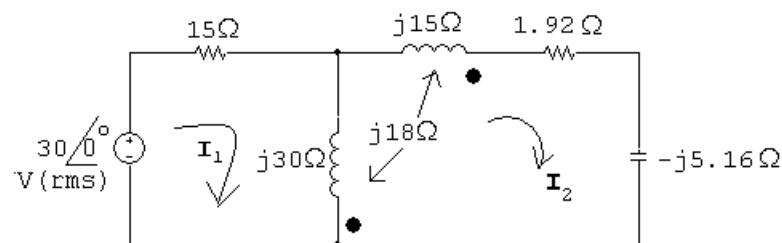
$$0 = j15\mathbf{I}_{\text{sc}} - j18(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j30(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j18\mathbf{I}_{\text{sc}}$$

Solving,

$$\mathbf{I}_{\text{sc}} = 1.95\angle -43.025^\circ \text{ A}$$

$$Z_{\text{Th}} = \frac{9.6 + j4.8}{1.95\angle -43.025^\circ} = 1.92 + j5.16 \Omega$$

$$\therefore \mathbf{I}_2 = \frac{9.6 + j4.8}{3.84} = 2.795\angle 26.57^\circ \text{ A}$$

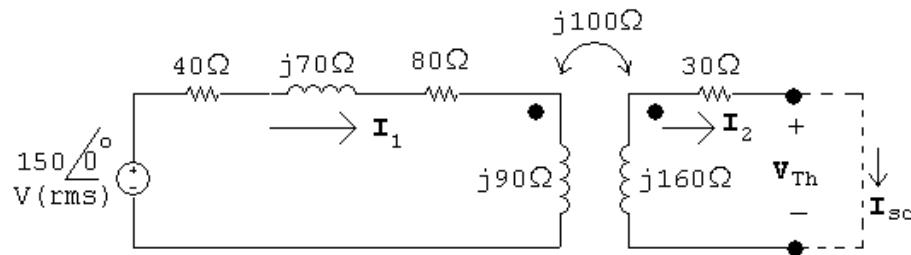


$$30\angle 0^\circ = 15\mathbf{I}_1 + j30(\mathbf{I}_1 i \mathbf{I}_2) + j18\mathbf{I}_2$$

$$\therefore \mathbf{I}_1 = \frac{30 + j12\mathbf{I}_2}{15 + j30} = \frac{30 + j12(2.795/\underline{26.57^\circ})}{15 + j30} = 1 \text{ A}$$

$$Z_g = \frac{30/0^\circ}{1} = 30 + j0 = 30/\underline{0^\circ} \Omega$$

P 10.54 [a]



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{150}{120 + j160}(j100) = 60 + j45 \text{ V}$$

Short circuit:

$$(120 + j160)\mathbf{I}_1 - j100\mathbf{I}_{\text{sc}} = 150$$

$$-j100\mathbf{I}_1 + (30 + j160)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 0.5 - j2.5 \text{ A}$$

$$Z_{\text{Th}} = \frac{60 + j45}{0.5 - j2.5} = 64 + j112 \Omega$$

$$\therefore Z_L = Z_{\text{Th}}^* = 64 - j112 \Omega$$

$$\mathbf{I}_L = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}} + Z_L} = \frac{60 + j45}{128} = 0.586/\underline{36.87^\circ} \text{ A (rms)}$$

$$P_L = |\mathbf{I}_L|^2 (64) = 21.98 \text{ W}$$

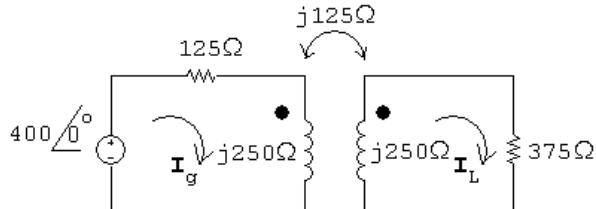
$$[\text{b}] \quad \mathbf{I}_1 = \frac{Z_{22}\mathbf{I}_2}{j\omega M} = \frac{94 + j48}{j100}(0.586/\underline{36.87^\circ}) = 0.62/\underline{-26.08^\circ} \text{ A (rms)}$$

$$P_{\text{transformer}} = (150)(0.62) \cos(-26.08^\circ) - (0.62)^2(40) = 68.15 \text{ W}$$

$$\% \text{ delivered} = \frac{21.98}{68.15}(100) = 32.25\%$$

P 10.55 [a] $j\omega L_1 = j\omega L_2 = j(400)(625 \times 10^{-3}) = j250 \Omega$

$$j\omega M = j(400)(312.5 \times 10^{-3}) = j125 \Omega$$



$$400 = (125 + j250)\mathbf{I}_g - j125\mathbf{I}_L$$

$$0 = -j125\mathbf{I}_g + (375 + j250)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 0.8 - j1.2 \text{ A}; \quad \mathbf{I}_L = 0.4 \text{ A}$$

Thus,

$$i_g = 1.44 \cos(400t - 56.31^\circ) \text{ A}$$

$$i_L = 0.4 \cos 400t \text{ A}$$

[b] $k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.3125}{0.625} = 0.5$

[c] When $t = 1.25\pi \text{ ms}$:

$$400t = (400)(1.25\pi) \times 10^{-3} = 0.5\pi \text{ rad} = 90^\circ$$

$$i_g(1.25\pi \text{ ms}) = 1.44 \cos(90^\circ - 56.31^\circ) = 1.2 \text{ A}$$

$$i_L(1.25\pi \text{ ms}) = 0.4 \cos(90^\circ) = 0 \text{ A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - Mi_1 i_2 = \frac{1}{2}(0.625)(1.2)^2 + 0 - 0 = 450 \text{ mJ}$$

When $t = 2.5\pi \text{ ms}$:

$$400t = (400)(2.5\pi) \times 10^{-3} = \pi = 180^\circ$$

$$i_g(2.5\pi \text{ ms}) = 1.44 \cos(180^\circ - 56.31^\circ) = -0.8 \text{ A}$$

$$i_L(2.5\pi \text{ ms}) = 0.4 \cos(180^\circ) = -0.4 \text{ A}$$

$$w = \frac{1}{2}(0.625)(0.8)^2 + \frac{1}{2}(0.625)(-0.4)^2 - (0.3125)(-0.8)(-0.4) = 150 \text{ mJ}$$

[d] From (a), $I_L = 0.4 \text{ A}$,

$$\therefore P = \frac{1}{2}(0.4)^2(375) = 30 \text{ W}$$

[e] Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{400}{125 + j250} (j125) = 160 + j80 \text{ V}$$

Short circuit:

$$400 = (125 + j250)\mathbf{I}_1 - j125\mathbf{I}_{\text{sc}}$$

$$0 = -j125\mathbf{I}_1 + j250\mathbf{I}_{\text{sc}}$$

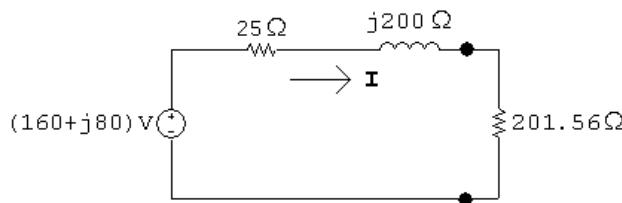
Solving,

$$\mathbf{I}_{\text{sc}} = 0.4923 - j0.7385$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{160 + j80}{0.4923 - j0.7385} = 25 + j200 \Omega$$

$$\therefore R_L = 201.56 \Omega$$

[f]



$$\mathbf{I} = \frac{160 + j80}{226.56 + j200} = 0.592 \angle -14.87^\circ \text{ A}$$

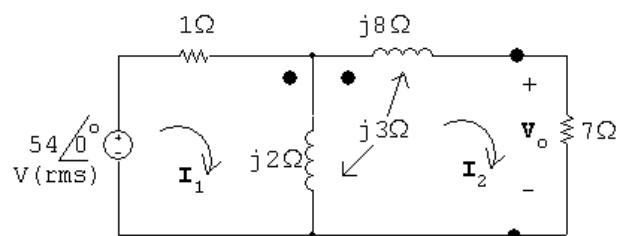
$$P = \frac{1}{2}(0.592)^2(201.56) = 35.3 \text{ W}$$

[g] $Z_L = Z_{\text{Th}}^* = 25 - j200 \Omega$

[h] $\mathbf{I} = \frac{160 + j80}{50} = 3.58 \angle 26.57^\circ$

$$P = \frac{1}{2}(3.58)^2(25) = 160 \text{ W}$$

P 10.56 [a]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$$\mathbf{I}_1 = 12 - j21 \text{ A (rms)}; \quad \mathbf{I}_2 = -3 \text{ A (rms)}$$

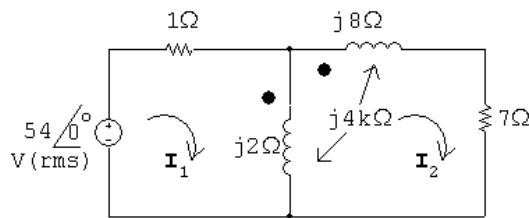
$$\mathbf{V}_o = 7\mathbf{I}_2 = -21/0^\circ \text{ V(rms)}$$

[b] $P = |\mathbf{I}_2|^2(7) = 63 \text{ W}$

[c] $P_g = (54)(12) = 648 \text{ W}$

$$\% \text{ delivered} = \frac{63}{648}(100) = 9.72\%$$

P 10.57 [a]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = \frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) - (4k - 2)}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[b] When $\mathbf{I}_2 = 0$

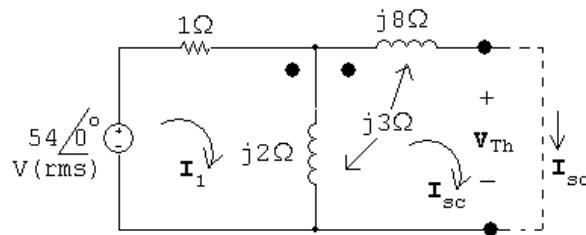
$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6 \text{ A(rms)}$$

$$P_g = (54)(10.8) = 583.2 \text{ W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2 \text{ W}$$

P 10.58 [a]



Open circuit:

$$\mathbf{V}_{\text{Th}} = -j3\mathbf{I}_1 + j2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6$$

$$\mathbf{V}_{\text{Th}} = -21.6 - j10.8 \text{ V}$$

Short circuit:

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j3\mathbf{I}_{\text{sc}}$$

$$0 = j2(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j3\mathbf{I}_{\text{sc}} + j8\mathbf{I}_{\text{sc}} + j3(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

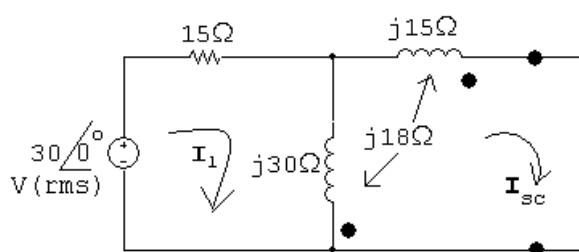
Solving,

$$\mathbf{I}_{\text{sc}} = -3.32 + j5.82$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + j3.6 = 3.6 \angle 86.86^\circ \Omega$$

$$\therefore R_L = |Z_{\text{Th}}| = 3.6 \Omega$$

[b]



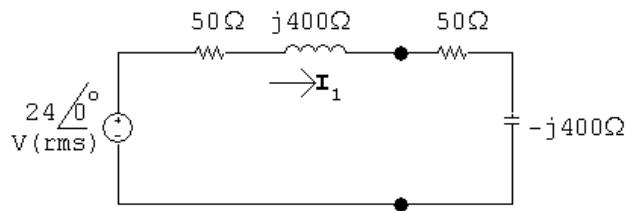
$$\mathbf{I} = \frac{-21.6 - j10.8}{3.8 + j3.6} = 4.614 \angle 163.1^\circ$$

$$P = |\mathbf{I}|^2(3.6) = 76.6 \text{ W, which is greater than when } R_L = 7 \Omega$$

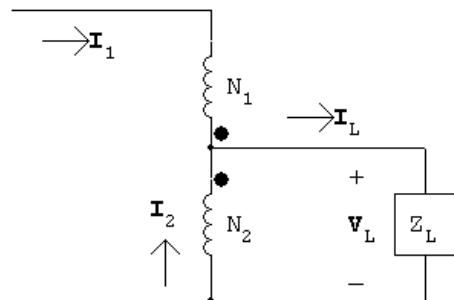
P 10.59 [a] $Z_{ab} = 50 - j400 = \left(1 - \frac{N_1}{N_2}\right)^2 Z_L$

$$\therefore Z_L = \frac{1}{(1-6)^2}(50 - j400) = 2 - j16 \Omega$$

[b]



$$\mathbf{I}_1 = \frac{24}{100} = 240/0^\circ \text{ mA}$$



$$N_1 \mathbf{I}_1 = -N_2 \mathbf{I}_2$$

$$\mathbf{I}_2 = -6\mathbf{I}_1 = -1.44/0^\circ \text{ A}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 = -1.68/0^\circ \text{ A}$$

$$\mathbf{V}_L = (2 - j16)\mathbf{I}_L = -3.36 + j26.88 = 27.1/97.13^\circ \text{ V(rms)}$$

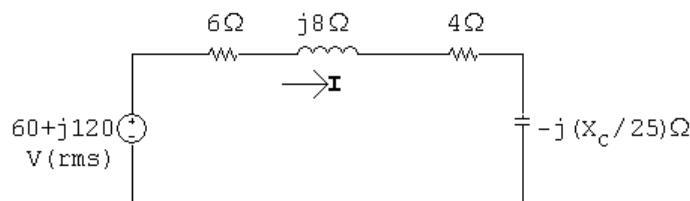
P 10.60 [a] Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{Th} = (15)(20\|j10) = 60 + j120 \text{ V}$$

$$Z_{Th} = 2 + 20\|j10 = 6 + j8 \Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 - jX_C) = 4 - j\frac{X_C}{25} \Omega$$



Now maximize \mathbf{I} by setting $(X_C/25) = 8 \Omega$:

$$\therefore C = \frac{1}{200(20 \times 10^3)} = 0.25 \mu\text{F}$$

$$[b] \quad \mathbf{I} = \frac{60 + j120}{10} = 6 + j12 \text{ A}$$

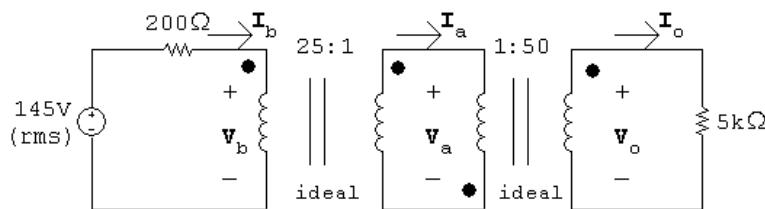
$$P = |\mathbf{I}|^2(4) = 720 \text{ W}$$

$$[c] \quad \frac{R_o}{25} = 6 \Omega; \quad \therefore R_o = 150 \Omega$$

$$[d] \quad \mathbf{I} = \frac{60 + j120}{12} = 5 + j10 \text{ A}$$

$$P = |\mathbf{I}|^2(6) = 750 \text{ W}$$

P 10.61



$$\frac{\mathbf{V}_a}{1} = \frac{-\mathbf{V}_o}{50}; \quad 1\mathbf{I}_a = -50\mathbf{I}_o$$

$$\therefore \frac{\mathbf{V}_a}{\mathbf{I}_a} = \frac{-\mathbf{V}_o/50}{-50\mathbf{I}_o} = \frac{\mathbf{V}_o/\mathbf{I}_o}{50^2} = \frac{5000}{50^2} = 2 \Omega$$

$$\frac{\mathbf{V}_b}{25} = \frac{\mathbf{V}_a}{1}; \quad 25\mathbf{I}_b = 1\mathbf{I}_a$$

$$\therefore \frac{\mathbf{V}_b/25}{25\mathbf{I}_b} = \frac{\mathbf{V}_b/\mathbf{I}_b}{25^2} = \frac{\mathbf{V}_a}{\mathbf{I}_a}$$

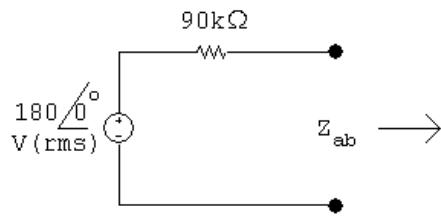
$$\therefore \frac{\mathbf{V}_b}{\mathbf{I}_b} = 25^2 \frac{\mathbf{V}_a}{\mathbf{I}_a} = 25^2(2) = 1250 \Omega$$

Thus $\mathbf{I}_b = [145/(200 + 1250)] = 100 \text{ mA (rms)}$; since the ideal transformers are lossless, $P_{5\text{k}\Omega} = P_{1250\Omega}$, and the power delivered to the 1250Ω resistor is $(0.1)^2(1250)$ or 12.5 W .

P 10.62 [a] $\frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{25^2(5000)}{a^2} = 200 \Omega; \quad \text{therefore } a^2 = 15,625, \quad a = 125$

[b] $\mathbf{I}_b = \frac{145}{400} = 362.5 \text{ mA}; \quad P = (0.3625)^2(200) = 26.28125 \text{ W}$

P 10.63 [a]

For maximum power transfer, Z_{ab} = 90 kΩ

$$Z_{ab} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_L$$

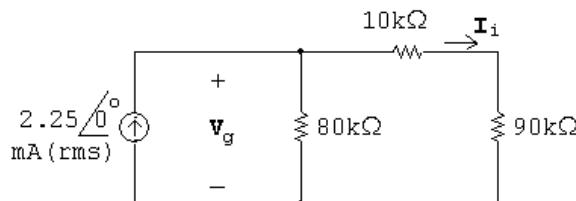
$$\therefore \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{90,000}{400} = 225$$

$$1 - \frac{N_1}{N_2} = \pm 15; \quad \frac{N_1}{N_2} = 15 + 1 = 16$$

$$[b] P = |\mathbf{I}_i|^2 (90,000) = \left(\frac{180}{180,000}\right)^2 (90,000) = 90 \text{ mW}$$

$$[c] \mathbf{V}_1 = R_i \mathbf{I}_i = (90,000) \left(\frac{180}{180,000}\right) = 90 \text{ V}$$

[d]



$$\mathbf{V}_g = (2.25 \times 10^{-3})(100,000 \| 80,000) = 100 \text{ V}$$

$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225 \text{ mW}$$

$$\% \text{ delivered} = \frac{90}{225}(100) = 40\%$$

$$P 10.64 [a] Z_{Th} = 720 + j1500 + \left(\frac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700/\underline{36.87^\circ} \Omega$$

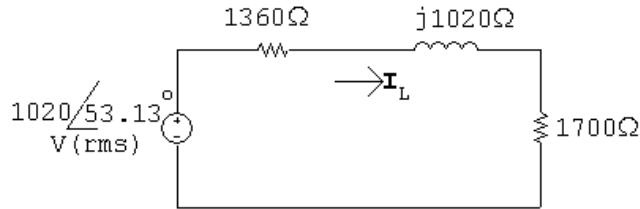
$$\therefore Z_{ab} = 1700 \Omega$$

$$Z_{ab} = \frac{Z_L}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

$$[b] \quad V_{Th} = \frac{255/0^\circ}{40 + j30} (j200) = 1020/53.13^\circ \text{ V}$$

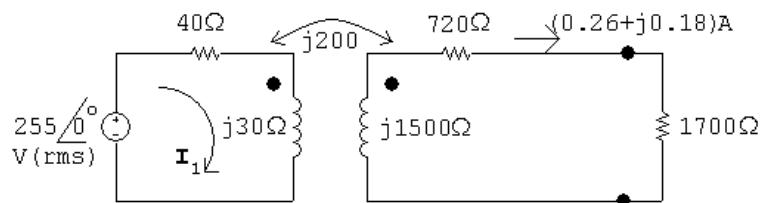


$$I_L = \frac{1020/53.13^\circ}{3060 + j1020} = 0.316/34.7^\circ \text{ A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$$P = |\mathbf{I}|^2(1700) = 170 \text{ W}$$

[c]



$$255/0^\circ = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

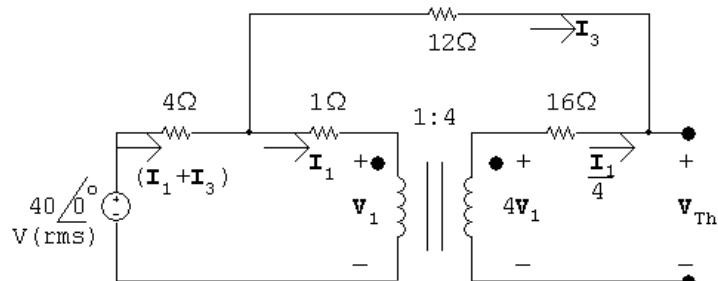
$$\therefore \mathbf{I}_1 = 4.13 - j1.80 \text{ A(rms)}$$

$$P_{\text{gen}} = (255)(4.13) = 1053 \text{ W}$$

$$P_{\text{diss}} = 1053 - 170 = 883 \text{ W}$$

$$\% \text{ dissipated} = \frac{883}{1053}(100) = 83.85\%$$

P 10.65 [a] Open circuit voltage:



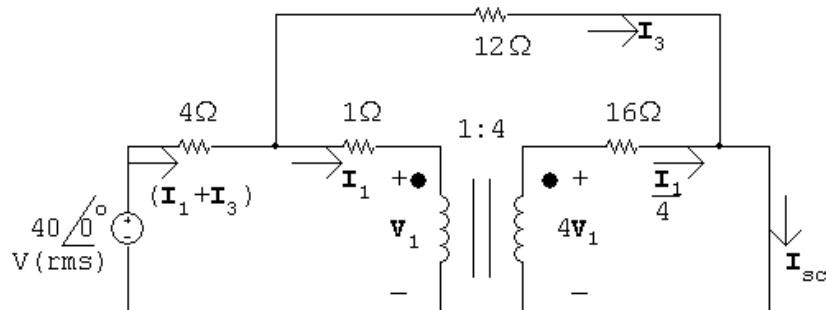
$$40/0^\circ = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + V_{Th}$$

$$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \quad \mathbf{I}_1 = -4\mathbf{I}_3$$

Solving,

$$\mathbf{V}_{\text{Th}} = 40/\underline{0^\circ} \text{ V}$$

Short circuit current:



$$40/\underline{0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \quad \therefore \quad \mathbf{V}_1 = \mathbf{I}_1$$

$$\therefore 40/\underline{0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$$

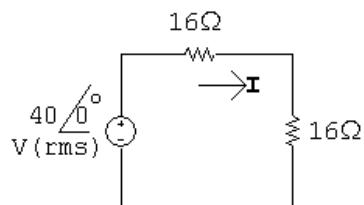
Also,

$$40/\underline{0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 6 \text{ A}; \quad \mathbf{I}_3 = 1 \text{ A}; \quad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5 \text{ A}$$

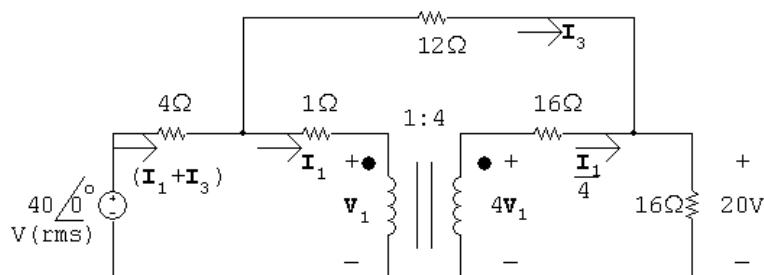
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16 \Omega$$



$$\mathbf{I} = \frac{40/\underline{0^\circ}}{32} = 1.25/\underline{0^\circ} \text{ A(rms)}$$

$$P = (1.25)^2(16) = 25 \text{ W}$$

[b]



$$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$$

$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \quad \therefore \quad \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore \quad \mathbf{I}_1 = 6 \text{ A}; \quad \mathbf{I}_3 = -0.25 \text{ A}; \quad \mathbf{I}_1 + \mathbf{I}_3 = 5.75/0^\circ \text{ A}$$

$$P_{40\text{V}}(\text{developed}) = 40(5.75) = 230 \text{ W}$$

$$\therefore \% \text{ delivered} = \frac{25}{230}(100) = 10.87\%$$

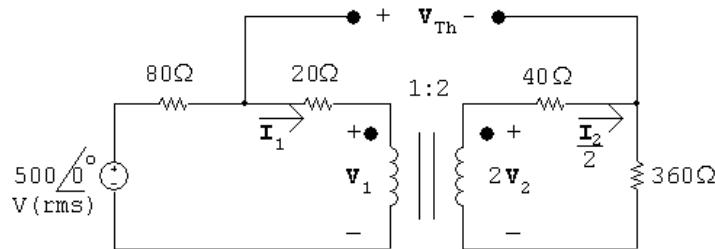
[c] $P_{R_L} = 25 \text{ W}; \quad P_{16\Omega} = (1.5)^2(16) = 36 \text{ W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25 \text{ W}; \quad P_{1\Omega} = (6)^2(1) = 36 \text{ W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75 \text{ W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230 \text{ W} = \sum P_{\text{dev}}$$

P 10.66 [a] Open circuit voltage:



$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \quad \therefore \quad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \quad \therefore \quad \mathbf{V}_1 = 100\mathbf{I}_1$$

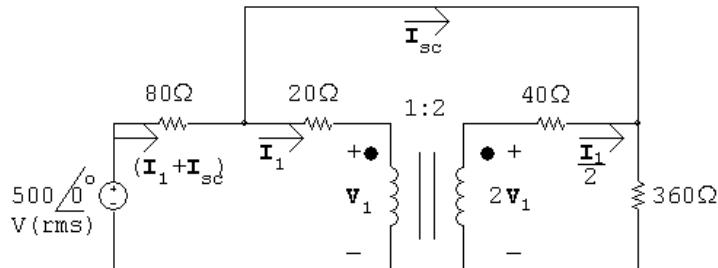
$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \quad \therefore \quad \mathbf{I}_1 = 500/200 = 2.5 \text{ A}$$

$$\therefore \quad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25 \text{ A}$$

$$\mathbf{V}_1 = 100(2.5) = 250 \text{ V}; \quad \mathbf{V}_2 = 2\mathbf{V}_1 = 500 \text{ V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150 \text{ V(rms)}$$

Short circuit current:



$$500 = 80(\mathbf{I}_{sc} + \mathbf{I}_1) + 360(\mathbf{I}_{sc} + 0.5\mathbf{I}_1)$$

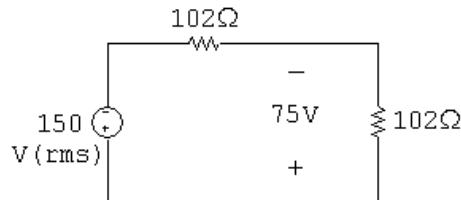
$$2\mathbf{V}_1 = 40 \frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{sc} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{sc}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

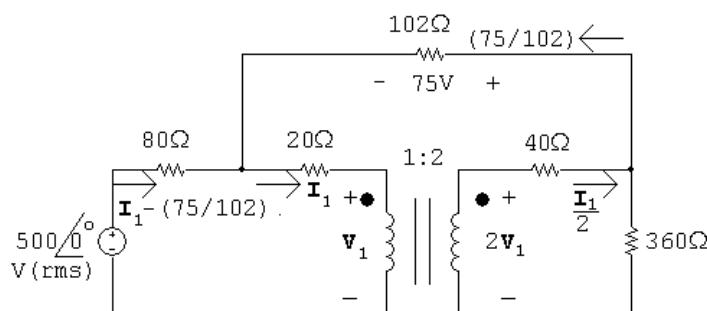
$$\mathbf{I}_{sc} = -1.47 \text{ A}$$

$$R_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{-150}{-1.47} = 102 \Omega$$



$$P = \frac{75^2}{102} = 55.15 \text{ W}$$

[b]



$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27,000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_2$$

$$\therefore \mathbf{I}_1 = 3.456 \text{ A}$$

$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.35 \text{ W}$$

$$\% \text{ delivered} = \frac{55.15}{1360.35}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80(\mathbf{I}_1 + \mathbf{I}_L)^2 = 592.13 \text{ W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86 \text{ W}$$

$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43 \text{ W}$$

$$P_{102\Omega} = 102\mathbf{I}_L^2 = 55.15 \text{ W}$$

$$P_{360\Omega} = 360(\mathbf{I}_2 + \mathbf{I}_L)^2 = 354.73 \text{ W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3 \text{ W} = \sum P_{\text{dev}}$$

P 10.67 [a] $\frac{30[5(44.28) + 19(15.77)]}{1000} = 15.63 \text{ kWh}$

[b] $\frac{30[5(44.28) + 19(8.9)]}{1000} = 11.72 \text{ kWh}$

[c] $\frac{30[5(44.28) + 19(4.42)]}{1000} = 9.16 \text{ kWh}$

[d] $\frac{30[5(44.28) + 19(0)]}{1000} = 6.64 \text{ kWh}$

Note that this is about 40 % of the amount of total power consumed in part (a).

P 10.68 [a] $\frac{30[0.2(1433) + 23.8(3.08)]}{1000} = 10.8 \text{ kWh}$

[b] The standby power consumed in one month by the microwave oven when in the ready state is

$$\frac{30[23.8(3.08)]}{1000} = 2.2 \text{ kWh}$$

This is $(2.2/10.8) * 100 = 20.4\%$ of the total power consumed by the microwave in one month. Since it is not practical to unplug the microwave when you are not using it, this is the cost associated with having a microwave oven.

P 10.69 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25 \Omega$

$$\mathbf{I} = \frac{120}{5 + j94.25} = 1.27/\underline{-86.96^\circ} \text{ A(rms)}$$

$$P = R_1|\mathbf{I}|^2 = 5(1.27)^2 = 8.06 \text{ W}$$

P 10.70 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25 \Omega$

$$\mathbf{I} = \frac{120}{0.05 + j94.25} = 1.27/\underline{-86.97^\circ} \text{ A(rms)}$$

$$P = R_1 |\mathbf{I}|^2 = 0.05(1.27)^2 = 81.1 \text{ mW}$$

Note that while the current supplied by the voltage source is virtually identical to that calculated in Problem 10.69, the much smaller value of transformer resistance results in a much smaller value of real power consumed by the transformer.

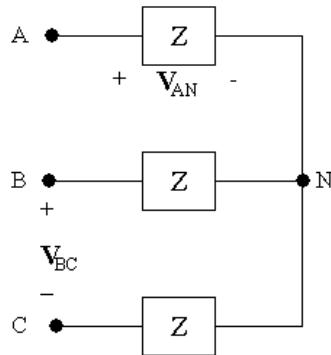
- P 10.71 An ideal transformer has no resistance, so consumes no real power. This is one of the important characteristics of ideal transformers.

11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know \mathbf{V}_{AN} and wish to find \mathbf{V}_{BC} . To do this, write a KVL equation to find \mathbf{V}_{AB} , and use the known phase angle relationship between \mathbf{V}_{AB} and \mathbf{V}_{BC} to find \mathbf{V}_{BC} .

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{BN} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since \mathbf{V}_{AN} , \mathbf{V}_{BN} , and \mathbf{V}_{CN} form a balanced set, and $\mathbf{V}_{AN} = 240/\underline{-30^\circ}\text{V}$, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|/\underline{|\mathbf{V}_{AN}| - 120^\circ} = 240/\underline{-30^\circ - 120^\circ} = 240/\underline{-150^\circ}\text{V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240/\underline{-30^\circ}) - (240/\underline{-150^\circ}) = 415.46/\underline{0^\circ}\text{V}$$

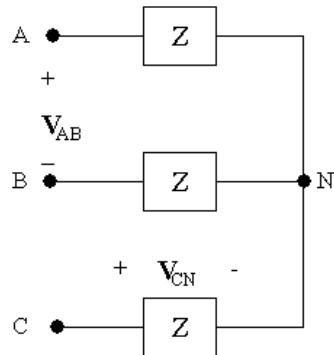
Since \mathbf{V}_{AB} , \mathbf{V}_{BC} , and \mathbf{V}_{CA} form a balanced set with a positive phase sequence, we can find \mathbf{V}_{BC} from \mathbf{V}_{AB} :

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|/\underline{(|\mathbf{V}_{AB}| - 120^\circ)} = 415.69/\underline{0^\circ - 120^\circ} = 415.69/\underline{-120^\circ}\text{V}$$

Thus,

$$\mathbf{V}_{BC} = 415.69/\underline{-120^\circ} \text{ V}$$

AP 11.2 Make a sketch:



We know \mathbf{V}_{CN} and wish to find \mathbf{V}_{AB} . To do this, write a KVL equation to find \mathbf{V}_{BC} , and use the known phase angle relationship between \mathbf{V}_{AB} and \mathbf{V}_{BC} to find \mathbf{V}_{AB} .

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} + \mathbf{V}_{NC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}$$

Since \mathbf{V}_{AN} , \mathbf{V}_{BN} , and \mathbf{V}_{CN} form a balanced set, and $\mathbf{V}_{CN} = 450/\underline{-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{BN} = |\mathbf{V}_{CN}|/\underline{|\mathbf{V}_{CN} - 120^\circ|} = 450/\underline{-23^\circ - 120^\circ} = 450/\underline{-145^\circ} \text{ V}$$

Then,

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = (450/\underline{-145^\circ}) - (450/\underline{-25^\circ}) = 779.42/\underline{-175^\circ} \text{ V}$$

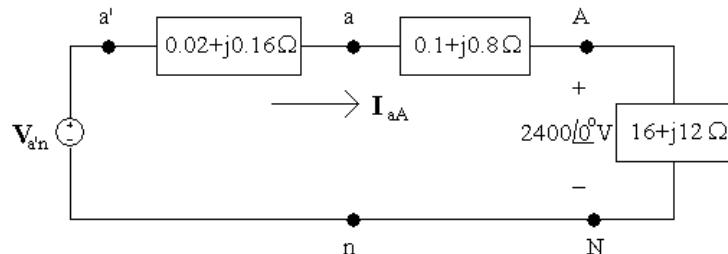
Since \mathbf{V}_{AB} , \mathbf{V}_{BC} , and \mathbf{V}_{CA} form a balanced set with a negative phase sequence, we can find \mathbf{V}_{AB} from \mathbf{V}_{BC} :

$$\mathbf{V}_{AB} = |\mathbf{V}_{BC}|/\underline{|\mathbf{V}_{BC} - 120^\circ|} = 779.42/\underline{-295^\circ} \text{ V}$$

But we normally want phase angle values between $+180^\circ$ and -180° . We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{AB} = 779.42/\underline{65^\circ} \text{ V}$$

AP 11.3 Sketch the a-phase circuit:



- [a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that \mathbf{I}_{aA} , \mathbf{I}_{bB} , and \mathbf{I}_{cC} form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, \mathbf{V}_{AN} , has a phase angle of 0° .

$$2400/0^\circ = \mathbf{I}_{aA}(16 + j12)$$

so

$$\mathbf{I}_{aA} = \frac{2400/0^\circ}{16 + j12} = 96 - j72 = 120/-36.87^\circ \text{ A}$$

With an acb phase sequence,

$$\underline{\mathbf{I}}_{bB} = \underline{\mathbf{I}}_{aA} + 120^\circ \quad \text{and} \quad \underline{\mathbf{I}}_{cC} = \underline{\mathbf{I}}_{aA} - 120^\circ$$

so

$$\mathbf{I}_{aA} = 120/-36.87^\circ \text{ A}$$

$$\mathbf{I}_{bB} = 120/83.13^\circ \text{ A}$$

$$\mathbf{I}_{cC} = 120/-156.87^\circ \text{ A}$$

- [b] The line voltages at the source are \mathbf{V}_{ab} , \mathbf{V}_{bc} , and \mathbf{V}_{ca} . They form a balanced set. To find \mathbf{V}_{ab} , use the a-phase circuit to find \mathbf{V}_{AN} , and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\begin{aligned} \mathbf{V}_{an} &= \mathbf{V}_{aA} + \mathbf{V}_{AN} = (0.1 + j0.8)\mathbf{I}_{aA} + 2400/0^\circ \\ &= (0.1 + j0.8)(96 - j72) + 2400/0^\circ = 2467.2 + j69.6 \\ &\quad 2468.18/1.62^\circ \text{ V} \end{aligned}$$

From Fig. 11.9(b),

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}/-30^\circ) = 4275.02/-28.38^\circ \text{ V}$$

With an acb phase sequence,

$$\underline{\mathbf{V}}_{bc} = \underline{\mathbf{V}}_{ab} + 120^\circ \quad \text{and} \quad \underline{\mathbf{V}}_{ca} = \underline{\mathbf{V}}_{ab} - 120^\circ$$

so

$$\mathbf{V}_{ab} = 4275.02/-28.38^\circ \text{ V}$$

$$\mathbf{V}_{bc} = 4275.02/91.62^\circ \text{ V}$$

$$\mathbf{V}_{ca} = 4275.02/-148.38^\circ \text{ V}$$

[c] Using KVL on the a-phase circuit

$$\begin{aligned}\mathbf{V}_{a'n} &= \mathbf{V}_{a'a} + \mathbf{V}_{an} = (0.2 + j0.16)\mathbf{I}_{aA} + \mathbf{V}_{an} \\ &= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9) \\ &= 2480.64 + j83.52 = 2482.05/\underline{1.93^\circ} \text{ V}\end{aligned}$$

With an acb phase sequence,

$$\underline{\mathbf{V}}_{b'n} = \underline{\mathbf{V}}_{a'n} + 120^\circ \quad \text{and} \quad \underline{\mathbf{V}}_{c'n} = \underline{\mathbf{V}}_{a'n} - 120^\circ$$

so

$$\mathbf{V}_{a'n} = 2482.05/\underline{1.93^\circ} \text{ V}$$

$$\mathbf{V}_{b'n} = 2482.05/\underline{121.93^\circ} \text{ V}$$

$$\mathbf{V}_{c'n} = 2482.05/\underline{-118.07^\circ} \text{ V}$$

AP 11.4

$$\mathbf{I}_{CC} = (\sqrt{3}/\underline{-30^\circ})\mathbf{I}_{CA} = (\sqrt{3}/\underline{-30^\circ}) \cdot 8/\underline{-15^\circ} = 13.86/\underline{-45^\circ} \text{ A}$$

AP 11.5

$$\begin{aligned}\mathbf{I}_{aA} &= 12/\underline{(65^\circ - 120^\circ)} = 12/\underline{-55^\circ} \\ \mathbf{I}_{AB} &= \left[\left(\frac{1}{\sqrt{3}} \right) \underline{-30^\circ} \right] \mathbf{I}_{aA} = \left(\frac{\underline{-30^\circ}}{\sqrt{3}} \right) \cdot 12/\underline{-55^\circ} \\ &= 6.93/\underline{-85^\circ} \text{ A}\end{aligned}$$

$$\text{AP 11.6 [a]} \quad \mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}} \right) \underline{30^\circ} \right] [69.28/\underline{-10^\circ}] = 40/\underline{20^\circ} \text{ A}$$

$$\text{Therefore } Z_\phi = \frac{4160/\underline{0^\circ}}{40/\underline{20^\circ}} = 104/\underline{-20^\circ} \Omega$$

$$\text{[b]} \quad \mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}} \right) \underline{-30^\circ} \right] [69.28/\underline{-10^\circ}] = 40/\underline{-40^\circ} \text{ A}$$

$$\text{Therefore } Z_\phi = 104/\underline{40^\circ} \Omega$$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50/\underline{-53.13^\circ} \text{ A}$$

$$\text{Therefore } |\mathbf{I}_{aA}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60 \text{ A}$$

$$\text{AP 11.8 [a]} \quad |S| = \sqrt{3}(208)(73.8) = 26,587.67 \text{ VA}$$

$$Q = \sqrt{(26,587.67)^2 - (22,659)^2} = 13,909.50 \text{ VAR}$$

$$[\mathbf{b}] \text{ pf} = \frac{22,659}{26,587.67} = 0.8522 \quad \text{lagging}$$

$$\text{AP 11.9 [a]} \quad \mathbf{V}_{\text{AN}} = \left(\frac{2450}{\sqrt{3}} \right) /0^\circ \text{ V}; \quad \mathbf{V}_{\text{AN}} \mathbf{I}_{\text{aA}}^* = S_\phi = 144 + j192 \text{ kVA}$$

Therefore

$$\mathbf{I}_{\text{aA}}^* = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7) \text{ A}$$

$$\mathbf{I}_{\text{aA}} = 101.8 - j135.7 = 169.67 / -53.13^\circ \text{ A}$$

$$|\mathbf{I}_{\text{aA}}| = 169.67 \text{ A}$$

$$[\mathbf{b}] \quad P = \frac{(2450)^2}{R}; \quad \text{therefore} \quad R = \frac{(2450)^2}{144,000} = 41.68 \Omega$$

$$Q = \frac{(2450)^2}{X}; \quad \text{therefore} \quad X = \frac{(2450)^2}{192,000} = 31.26 \Omega$$

$$[\mathbf{c}] \quad Z_\phi = \frac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \frac{2450/\sqrt{3}}{169.67 / -53.13^\circ} = 8.34 / 53.13^\circ = (5 + j6.67) \Omega$$

$$\therefore R = 5 \Omega, \quad X = 6.67 \Omega$$

Problems

P 11.1 [a] First, convert the cosine waveforms to phasors:

$$\mathbf{V}_a = 137/\underline{63^\circ}; \quad \mathbf{V}_b = 137/\underline{-57^\circ}; \quad \mathbf{V}_c = 137/\underline{183^\circ}$$

Subtract the phase angle of the a-phase from all phase angles:

$$\underline{\mathbf{V}'_a} = 63^\circ - 63^\circ = 0^\circ$$

$$\underline{\mathbf{V}'_b} = -57^\circ - 63^\circ = -120^\circ$$

$$\underline{\mathbf{V}'_c} = 183^\circ - 63^\circ = 120^\circ$$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors, making sure that all waveforms are represented as cosines:

$$\mathbf{V}_a = 820/\underline{-36^\circ}; \quad \mathbf{V}_b = 820/\underline{84^\circ}; \quad \mathbf{V}_c = 820/\underline{-156^\circ}$$

Subtract the phase angle of the a-phase from all phase angles:

$$\underline{\mathbf{V}'_a} = -36^\circ + 36^\circ = 0^\circ$$

$$\underline{\mathbf{V}'_b} = 84^\circ + 36^\circ = 120^\circ$$

$$\underline{\mathbf{V}'_c} = -156^\circ + 36^\circ = -120^\circ$$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.2 [a] $\mathbf{V}_a = 48/\underline{-45^\circ} \text{ V}$

$$\mathbf{V}_b = 48/\underline{-165^\circ} \text{ V}$$

$$\mathbf{V}_c = 48/\underline{75^\circ} \text{ V}$$

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 188/\underline{60^\circ} \text{ V}$

$$\mathbf{V}_b = -188/\underline{0^\circ} \text{ V} = 188/\underline{180^\circ} \text{ V}$$

$$\mathbf{V}_c = 188/\underline{-60^\circ} \text{ V}$$

Balanced, negative phase sequence

[c] $\mathbf{V}_a = 426/\underline{0^\circ} \text{ V}$

$$\mathbf{V}_b = 462/\underline{120^\circ} \text{ V}$$

$$\mathbf{V}_c = 426/\underline{-120^\circ} \text{ V}$$

Unbalanced due to unequal amplitudes

[d] $\mathbf{V}_a = 1121/\underline{-20^\circ} \text{ V}$

$$\mathbf{V}_b = 1121/\underline{-140^\circ} \text{ V}$$

$$\mathbf{V}_c = 1121/\underline{100^\circ} \text{ V}$$

Balanced, positive phase sequence

[e] $\mathbf{V}_a = 540/\underline{-90^\circ} \text{ V}$

$$\mathbf{V}_b = 540/\underline{-120^\circ} \text{ V}$$

$$\mathbf{V}_c = 540/\underline{120^\circ} \text{ V}$$

Unbalanced due to unequal phase separation

[f] $\mathbf{V}_a = 144/\underline{80^\circ} \text{ V}$

$$\mathbf{V}_b = 144/\underline{-160^\circ} \text{ V}$$

$$\mathbf{V}_c = 144/\underline{-40^\circ} \text{ V}$$

Balanced, negative phase sequence

P 11.3 $\mathbf{V}_a = V_m/\underline{0^\circ} = V_m + j0$

$$\mathbf{V}_b = V_m/\underline{-120^\circ} = -V_m(0.5 + j0.866)$$

$$\mathbf{V}_c = V_m/\underline{120^\circ} = V_m(-0.5 + j0.866)$$

$$\begin{aligned}\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866) \\ &= V_m(0) = 0\end{aligned}$$

P 11.4 $\mathbf{I} = \frac{188/\underline{60^\circ} + 188/\underline{180^\circ} + 188/\underline{-60^\circ}}{3(R_W + jX_W)} = 0$

P 11.5 $\mathbf{I} = \frac{426/\underline{0^\circ} + 462/\underline{120^\circ} + 426/\underline{-120^\circ}}{3(R_W + jX_W)} = \frac{36/\underline{120^\circ}}{3(R_W + jX_W)}$

- P 11.6 [a] The voltage sources form a balanced set, the source impedances are equal and the line impedances are equal. But the load impedances are not equal. Therefore, the circuit is unbalanced. Also,

$$\mathbf{I}_{aA} = \frac{110}{32 - j24} = 2.75/\underline{36.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{bB} = \frac{110/-120^\circ}{6+j8} = 11/-173.13^\circ \text{ A (rms)}$$

$$\mathbf{I}_{cC} = \frac{110/120^\circ}{40+j30} = 2.2/83.13^\circ \text{ A (rms)}$$

The magnitudes are unequal and the phase angles are not 120° apart, so the currents are not balanced and thus the circuit is not balanced.

b] $\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 11.79/67.58^\circ \text{ A (rms)}$

P 11.7 [a] $\mathbf{I}_{aA} = \frac{277/0^\circ}{80+j60} = 2.77/-36.87^\circ \text{ A (rms)}$

$$\mathbf{I}_{bB} = \frac{277/-120^\circ}{80+j60} = 2.77/-156.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{cC} = \frac{277/120^\circ}{80+j60} = 2.77/83.13^\circ \text{ A (rms)}$$

$$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

[b] $\mathbf{V}_{AN} = (78+j54)\mathbf{I}_{aA} = 262.79/-2.17^\circ \text{ V (rms)}$

[c] $\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$

$$\mathbf{V}_{BN} = (77+j56)\mathbf{I}_{bB} = 263.73/-120.84^\circ \text{ V (rms)}$$

$$\mathbf{V}_{AB} = 262.79/-2.17^\circ - 263.73/-120.84^\circ = 452.89/28.55^\circ \text{ V (rms)}$$

[d] Unbalanced — see conditions for a balanced circuit in the text

P 11.8 $Z_{ga} + Z_{la} + Z_{L_a} = 60 + j80 \Omega$

$$Z_{gb} + Z_{lb} + Z_{L_b} = 90 + j120 \Omega$$

$$Z_{gc} + Z_{lc} + Z_{L_c} = 30 + j40 \Omega$$

$$\frac{\mathbf{V}_N - 320}{60 + j80} + \frac{\mathbf{V}_N - 320/-120^\circ}{90 + j120} + \frac{\mathbf{V}_N - 320/120^\circ}{30 + j40} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for \mathbf{V}_N yields

$$\mathbf{V}_N = 49.47/75.14^\circ \text{ V (rms)}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{20} = 2.47/75.14^\circ \text{ A (rms)}$$

$$P 11.9 \quad \mathbf{V}_{AN} = 285/\underline{-45^\circ} V$$

$$\mathbf{V}_{BN} = 285/\underline{-165^\circ} V$$

$$\mathbf{V}_{CN} = 285/\underline{75^\circ} V$$

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 498.83/\underline{-15^\circ} V$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 498.83/\underline{-135^\circ} V$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = 498.83/\underline{105^\circ} V$$

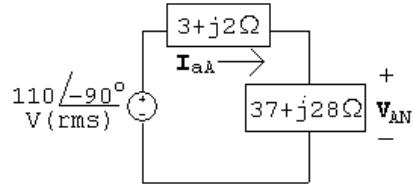
$$v_{AB} = 498.83 \cos(\omega t - 15^\circ) V$$

$$v_{BC} = 498.83 \cos(\omega t - 135^\circ) V$$

$$v_{CA} = 498.83 \cos(\omega t + 105^\circ) V$$

$$P 11.10 \quad [a] \quad \mathbf{V}_{an} = 1/\sqrt{3}/\underline{-30^\circ} \mathbf{V}_{ab} = 110/\underline{-90^\circ} V \text{ (rms)}$$

The a-phase circuit is

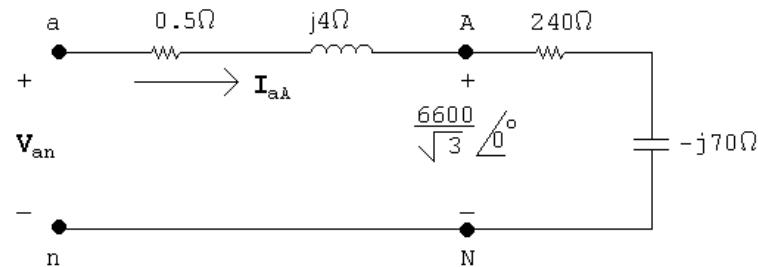


$$[b] \quad \mathbf{I}_{aA} = \frac{110/\underline{-90^\circ}}{40 + j30} = 2.2/\underline{-126.87^\circ} A \text{ (rms)}$$

$$[c] \quad \mathbf{V}_{AN} = (37 + j28)\mathbf{I}_{aA} = 102.08/\underline{-89.75^\circ} V \text{ (rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}/\underline{30^\circ} \mathbf{V}_{AN} = 176.81/\underline{-59.75^\circ} A \text{ (rms)}$$

$$P 11.11 \quad [a]$$



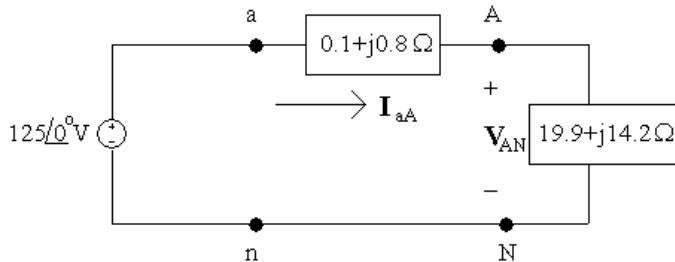
$$\mathbf{I}_{aA} = \frac{6600}{\sqrt{3}(240 - j70)} = 15.24/\underline{16.26^\circ} A \text{ (rms)}$$

$$|\mathbf{I}_{aA}| = |\mathbf{I}_L| = 15.24 A \text{ (rms)}$$

[b] $\mathbf{V}_{an} = (15.24/\underline{16.26^\circ})(240 - j66) = 3801.24/\underline{0.91^\circ}$

$$|\mathbf{V}_{ab}| = \sqrt{3}(3801.24) = 6583.94 \text{ V (rms)}$$

P 11.12 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\begin{aligned} \mathbf{I}_{aA} &= \frac{125/\underline{0^\circ}}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125/\underline{0^\circ}}{20 + j15} \\ &= 4 - j3 = 5/\underline{-36.87^\circ} \text{ A (rms)} \end{aligned}$$

Find the other line currents using the abc phase sequence:

$$\mathbf{I}_{bB} = 5/\underline{-36.87^\circ + 120^\circ} = 5/\underline{83.13^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{cC} = 5/\underline{-36.87^\circ - 120^\circ} = 5/\underline{-156.87^\circ} \text{ A (rms)}$$

[b] The phase voltage at the source is $\mathbf{V}_{an} = 125/\underline{0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, \mathbf{V}_{ab} , from the phase voltage:

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}/\underline{-30^\circ}) = 216.51/\underline{-30^\circ} \text{ V (rms)}$$

Find the other line voltages using the abc phase sequence:

$$\mathbf{V}_{bc} = 216.51/\underline{-30^\circ + 120^\circ} = 216.51/\underline{90^\circ} \text{ V (rms)}$$

$$\mathbf{V}_{ca} = 216.51/\underline{-30^\circ - 120^\circ} = 216.51/\underline{-150^\circ} \text{ V (rms)}$$

[c] The phase voltage at the load in the a-phase is \mathbf{V}_{AN} . Calculate its value using \mathbf{I}_{aA} and the load impedance:

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_L = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23/\underline{-1.36^\circ} \text{ V (rms)}$$

Find the phase voltage at the load for the b- and c-phases using the abc sequence:

$$\mathbf{V}_{BN} = 122.23/\underline{-1.36^\circ + 120^\circ} = 122.23/\underline{118.64^\circ} \text{ V (rms)}$$

$$\mathbf{V}_{CN} = 122.23/\underline{-1.36^\circ - 120^\circ} = 122.23/\underline{-121.36^\circ} \text{ V (rms)}$$

- [d] The line voltage at the load in the a-phase is \mathbf{V}_{AB} . Find this line voltage from the phase voltage at the load in the a-phase, \mathbf{V}_{AN} , using Fig, 11.9(b):

$$\mathbf{V}_{AB} = \mathbf{V}_{AN}(\sqrt{3}/-30^\circ) = 211.72/-31.36^\circ \text{ V (rms)}$$

Find the line voltage at the load for the b- and c-phases using the abc sequence:

$$\mathbf{V}_{BC} = 211.72/-31.36^\circ + 120^\circ = 211.72/88.64^\circ \text{ V (rms)}$$

$$\mathbf{V}_{CA} = 211.72/-31.36^\circ - 120^\circ = 211.72/-151.36^\circ \text{ V (rms)}$$

P 11.13 [a] $\mathbf{I}_{AB} = \frac{7200}{216 - j288} = 20/53.13^\circ \text{ A (rms)}$

$$\mathbf{I}_{BC} = 20/173.13^\circ \text{ A (rms)}$$

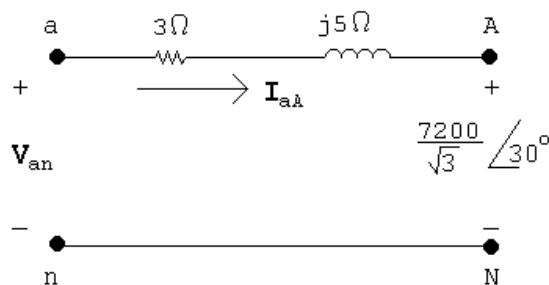
$$\mathbf{I}_{CA} = 20/-66.87^\circ \text{ A (rms)}$$

[b] $\mathbf{I}_{aA} = \sqrt{3}/30^\circ \mathbf{I}_{AB} = 34.64/83.13^\circ \text{ A (rms)}$

$$\mathbf{I}_{bB} = 34.64/-156.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{cC} = 34.64/-36.87^\circ \text{ A (rms)}$$

[c]



$$\begin{aligned}\mathbf{V}_{an} &= \frac{7200}{\sqrt{3}}/30^\circ + (3 + j5)(34.64/83.13^\circ) \\ &= 4085/32.62^\circ \text{ V (rms)}\end{aligned}$$

$$\mathbf{V}_{ab} = \sqrt{3}/-30^\circ \mathbf{V}_{an} = 7075.43/2.62^\circ \text{ V (rms)}$$

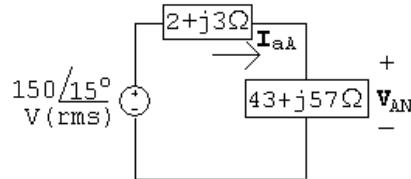
$$\mathbf{V}_{bc} = 7075.43/122.62^\circ \text{ V (rms)}$$

$$\mathbf{V}_{ca} = 7075.43/-117.38^\circ \text{ V (rms)}$$

P 11.14 [a] $\mathbf{V}_{an} = \mathbf{V}_{bn} - 1/120^\circ = 150/15^\circ \text{ V (rms)}$

$$Z_y = Z_\Delta/3 = 43 + j57 \Omega$$

The a-phase circuit is



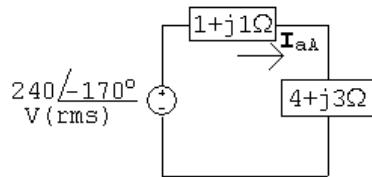
$$[b] \quad I_{aA} = \frac{150\angle 15^\circ}{45 + j60} = 2\angle -38.13^\circ \text{ A (rms)}$$

$$[c] \quad V_{AN} = (43 + j57)I_{aA} = 142.8\angle 14.84^\circ \text{ V (rms)}$$

$$V_{AB} = \sqrt{3}\angle -30^\circ V_{AN} = 247.34\angle -15.16^\circ \text{ A (rms)}$$

$$\text{P 11.15 } Z_y = Z_\Delta / 3 = 4 + j3 \Omega$$

The a-phase circuit is



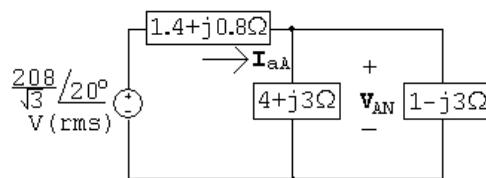
$$I_{aA} = \frac{240\angle -170^\circ}{(1 + j1) + (4 + j3)} = 37.48\angle 151.34^\circ \text{ A (rms)}$$

$$I_{AB} = \frac{1}{\sqrt{3}}\angle -30^\circ I_{aA} = 21.64\angle 121.34^\circ \text{ A (rms)}$$

$$\text{P 11.16 } V_{an} = 1/\sqrt{3}\angle -30^\circ V_{ab} = \frac{208}{\sqrt{3}}\angle 20^\circ \text{ V (rms)}$$

$$Z_y = Z_\Delta / 3 = 1 - j3 \Omega$$

The a-phase circuit is

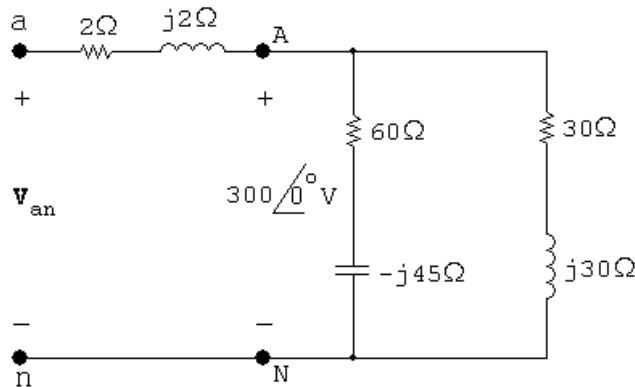


$$Z_{eq} = (4 + j3) \parallel (1 - j3) = 2.6 - j1.8 \Omega$$

$$V_{AN} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)} \left(\frac{208}{\sqrt{3}} \right) \angle 20^\circ = 92.1\angle -0.66^\circ \text{ V (rms)}$$

$$V_{AB} = \sqrt{3}\angle 30^\circ V_{AN} = 159.5\angle 29.34^\circ \text{ V (rms)}$$

P 11.17 [a]



$$\mathbf{I}_{aA} = \frac{300}{60 - j45} + \frac{300}{30 + j30} = 8.6 \angle -17.59^\circ \text{ A (rms)}$$

$$|\mathbf{I}_{aA}| = 8.6 \text{ A (rms)}$$

$$[\mathbf{b}] \quad \mathbf{I}_{AB} = \frac{300\sqrt{3}\angle 30^\circ}{90 + j90} = 4.08 \angle -15^\circ \text{ A (rms)}$$

$$|\mathbf{I}_{AB}| = 4.08 \text{ A (rms)}$$

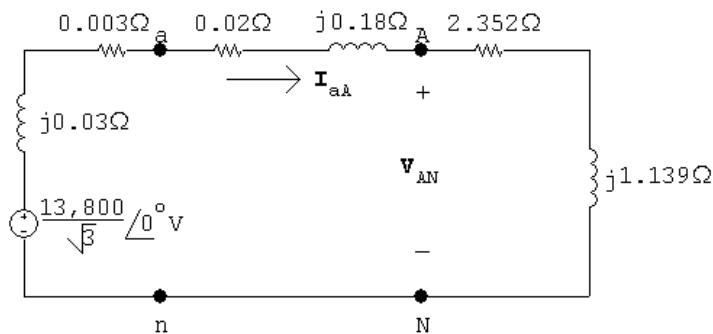
$$[\mathbf{c}] \quad \mathbf{I}_{AN} = \frac{300\angle 0^\circ}{60 - j45} = 4 \angle 36.87^\circ \text{ A (rms)}$$

$$|\mathbf{I}_{AN}| = 4 \text{ A (rms)}$$

$$[\mathbf{d}] \quad \mathbf{V}_{an} = (8.6 \angle -17.59^\circ)(2 + j2) + 300\angle 0^\circ = 321.79 \angle 1.99^\circ \text{ V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(321.79) = 557.37 \text{ V (rms)}$$

P 11.18 [a]



$$[\mathbf{b}] \quad \mathbf{I}_{aA} = \frac{13,800}{\sqrt{3}(2.375 + j1.349)} = 2917 \angle -29.6^\circ \text{ A (rms)}$$

$$|\mathbf{I}_{aA}| = 2917 \text{ A (rms)}$$

$$[\mathbf{c}] \quad \mathbf{V}_{AN} = (2.352 + j1.139)(2917 \angle -29.6^\circ) = 7622.93 \angle -3.76^\circ \text{ V (rms)}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}|\mathbf{V}_{AN}| = 13,203.31 \text{ V (rms)}$$

$$[\mathbf{d}] \quad \mathbf{V}_{\text{an}} = (2.372 + j1.319)(2917/\underline{-29.6^\circ}) = 7616.93/\underline{-0.52^\circ} \text{ V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 13,712.52 \text{ V (rms)}$$

$$[\mathbf{e}] \quad |\mathbf{I}_{\text{AB}}| = \frac{|\mathbf{I}_{\text{aA}}|}{\sqrt{3}} = 1684.13 \text{ A (rms)}$$

$$[\mathbf{f}] \quad |\mathbf{I}_{\text{ab}}| = |\mathbf{I}_{\text{AB}}| = 1684.13 \text{ A (rms)}$$

$$\text{P 11.19 } [\mathbf{a}] \quad \mathbf{I}_{\text{AB}} = \frac{13,200/\underline{0^\circ}}{100 - j75} = 105.6/\underline{36.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{BC}} = 105.6/\underline{156.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{CA}} = 105.6/\underline{-83.13^\circ} \text{ A (rms)}$$

$$[\mathbf{b}] \quad \mathbf{I}_{\text{aA}} = \sqrt{3}/\underline{-30^\circ} \mathbf{I}_{\text{AB}} = 182.9/\underline{66.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{bB}} = 182.9/\underline{-173.13^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 182.9/\underline{-53.13^\circ} \text{ A (rms)}$$

$$[\mathbf{c}] \quad \mathbf{I}_{\text{ba}} = \mathbf{I}_{\text{AB}} = 105.6/\underline{36.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{cb}} = \mathbf{I}_{\text{BC}} = 105.6/\underline{156.87^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{ac}} = \mathbf{I}_{\text{CA}} = 105.6/\underline{-83.13^\circ} \text{ A (rms)}$$

$$\text{P 11.20 } [\mathbf{a}] \quad \mathbf{I}_{\text{AB}} = \frac{480/\underline{0^\circ}}{2.4 - j0.7} = 192/\underline{16.26^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{BC}} = \frac{480/\underline{120^\circ}}{8 + j6} = 48/\underline{83.13^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\text{CA}} = \frac{480/\underline{-120^\circ}}{20} = 24/\underline{-120^\circ} \text{ A (rms)}$$

$$[\mathbf{b}] \quad \mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} - \mathbf{I}_{\text{CA}}$$

$$= 210/\underline{20.79^\circ}$$

$$\mathbf{I}_{\text{bB}} = \mathbf{I}_{\text{BC}} - \mathbf{I}_{\text{AB}}$$

$$= 178.68/\underline{-178.04^\circ}$$

$$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CA}} - \mathbf{I}_{\text{BC}}$$

$$= 70.7/\underline{-104.53^\circ}$$

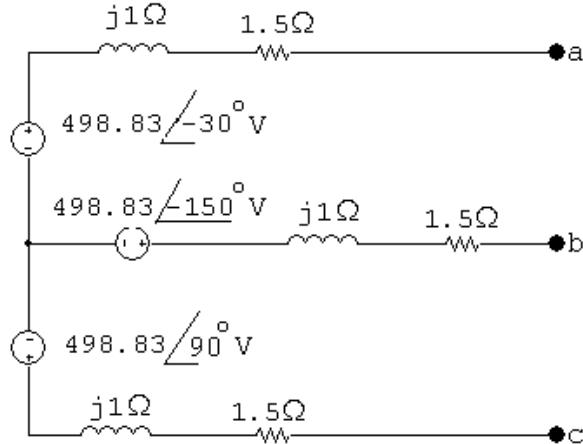
P 11.21 [a] Since the phase sequence is abc (positive) we have:

$$\mathbf{V}_{an} = 498.83/-30^\circ \text{ V (rms)}$$

$$\mathbf{V}_{bn} = 498.83/-150^\circ \text{ V (rms)}$$

$$\mathbf{V}_{cn} = 498.83/90^\circ \text{ V (rms)}$$

$$Z_Y = \frac{1}{3}Z_\Delta = 1.5 + j1\Omega/\phi$$



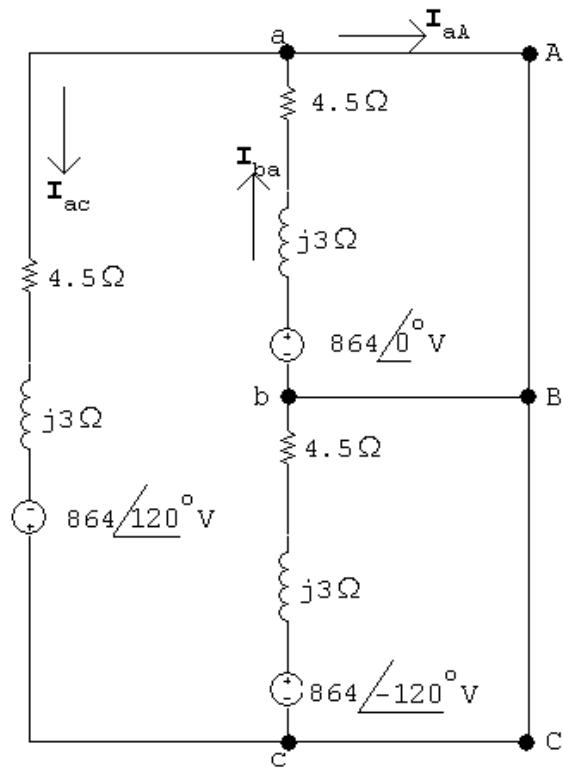
[b] $\mathbf{V}_{ab} = 498.83/-30^\circ - 498.83/-150^\circ = 498.83\sqrt{3}/0^\circ = 864/0^\circ \text{ V (rms)}$

Since the phase sequence is positive, it follows that

$$\mathbf{V}_{bc} = 864/-120^\circ \text{ V (rms)}$$

$$\mathbf{V}_{ca} = 864/120^\circ \text{ V (rms)}$$

[c]



$$I_{ba} = \frac{864}{4.5 + j3} = 159.75 \angle -33.69^\circ \text{ A (rms)}$$

$$I_{ac} = 159.75 \angle 86.31^\circ \text{ A (rms)}$$

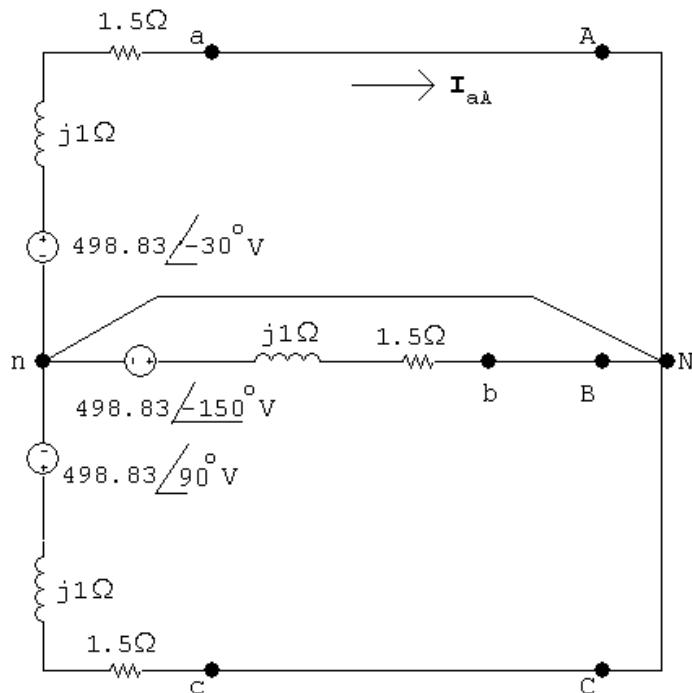
$$I_{aA} = I_{ba} - I_{ac} = 276.70 \angle -63.69^\circ \text{ A (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$I_{bB} = 276.70 \angle 176.31^\circ \text{ A (rms)}$$

$$I_{cC} = 276.70 \angle -56.31^\circ \text{ A (rms)}$$

[d]



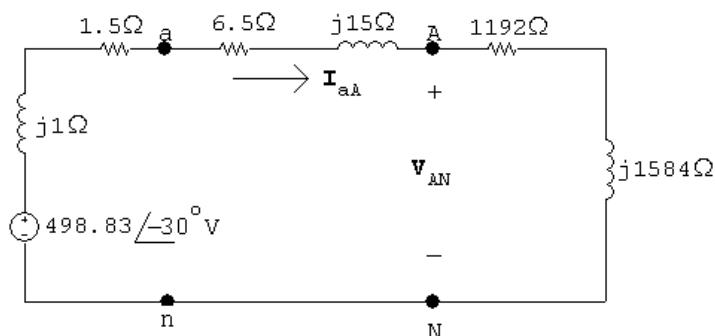
$$I_{aA} = \frac{498.83 \angle -30^\circ}{1.5 + j1} = 276.70 \angle -63.69^\circ \text{ A (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$I_{bB} = 276.70 \angle 176.31^\circ \text{ A (rms)}$$

$$I_{cC} = 276.70 \angle 56.31^\circ \text{ A (rms)}$$

P 11.22 [a]



$$[b] I_{aA} = \frac{498.83 \angle -30^\circ}{1200 + j1600} = 249.42 \angle -83.13^\circ \text{ mA (rms)}$$

$$V_{AN} = (1192 + j1584)(0.24942 \angle -83.13^\circ) = 494.45 \angle -30.09^\circ \text{ V (rms)}$$

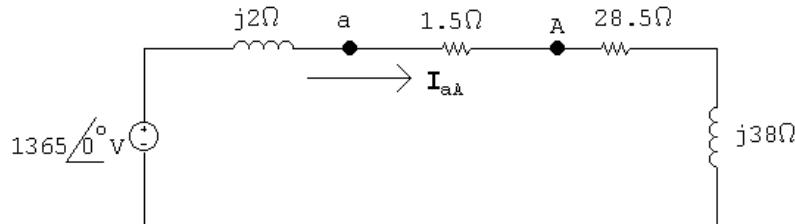
$$|V_{AB}| = \sqrt{3}(494.45) = 856.41 \text{ V (rms)}$$

$$[c] |I_{ab}| = \frac{0.24942}{\sqrt{3}} = 144 \text{ mA (rms)}$$

$$[d] V_{an} = (1198.5 + j1599)(0.24942 / -83.13^\circ) = 498.42 / -29.98^\circ \text{ V (rms)}$$

$$|V_{ab}| = \sqrt{3}(498.42) = 863.29 \text{ V (rms)}$$

P 11.23 [a]



$$I_{aA} = \frac{1365/0^\circ}{30 + j40} = 27.3 / -53.13^\circ \text{ A (rms)}$$

$$I_{CA} = \frac{I_{aA}}{\sqrt{3}} / 150^\circ = 15.76 / 96.87^\circ \text{ A (rms)}$$

$$[b] S_{g/\phi} = -1365 I_{aA}^* = -22,358.75 - j29,811.56 \text{ VA}$$

$$\therefore P_{\text{developed}/\text{phase}} = 22.359 \text{ kW}$$

$$P_{\text{absorbed}/\text{phase}} = |I_{aA}|^2 28.5 = 21.241 \text{ kW}$$

$$\% \text{ delivered} = \frac{21.241}{22.359} (100) = 95\%$$

P 11.24 The complex power of the source per phase is

$S_s = 20,000 / (\cos^{-1} 0.6) = 20,000 / 53.13^\circ = 12,000 + j16,000 \text{ kVA}$. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$$S_s = S_1 + S_2 \quad \text{so} \quad 12,000 + j16,000 = 10,000 + S_2$$

$$\therefore S_2 = 2000 + j16,000 \text{ VA}$$

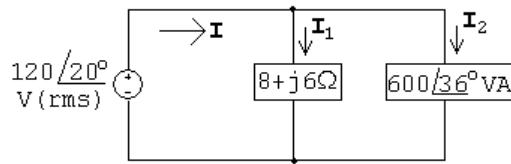
$$\text{Also, } S_2 = \frac{|V_{\text{rms}}|^2}{Z_2^*}$$

$$|V_{\text{rms}}| = \frac{|V_{\text{load}}|}{\sqrt{3}} = 120 \text{ V (rms)}$$

$$\text{Thus, } Z_2^* = \frac{|V_{\text{rms}}|^2}{S_2} = \frac{(120)^2}{2000 + j16,000} = 0.11 - j0.89 \Omega$$

$$\therefore Z_2 = 0.11 + j0.89 \Omega$$

P 11.25 The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120/20^\circ}{8+j6} = 12/-16.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_2^* = \frac{600/36^\circ}{120/20^\circ} = 5/16^\circ \text{ A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12/-16.87^\circ + 5/-16^\circ = 17/-16.61^\circ \text{ A (rms)}$$

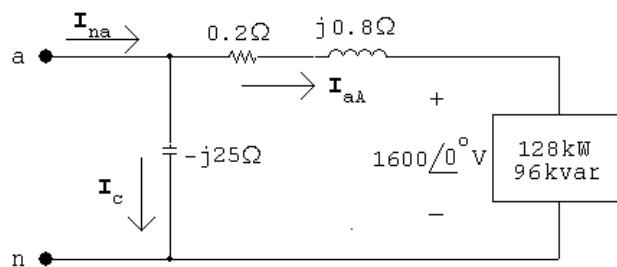
$$S_a = \mathbf{V}\mathbf{I}^* = (120/20^\circ)(17/16.61^\circ) = 2040/36.61^\circ \text{ VA}$$

$$S_T = 3S_a = 6120/36.61^\circ \text{ VA}$$

$$\text{P 11.26 [a]} \quad \mathbf{I}_{aA}^* = \frac{(128+j96)10^3}{1600} = 80+j60$$

$$\mathbf{I}_{aA} = 80-j60 \text{ A (rms)}$$

$$\mathbf{V}_{an} = 1600 + (80-j60)(0.2+j0.8) = 1664+j52 \text{ V (rms)}$$



$$\mathbf{I}_C = \frac{1664+j52}{-j25} = -2.08+j66.56 \text{ A (rms)}$$

$$\mathbf{I}_{na} = \mathbf{I}_{aA} + \mathbf{I}_C = 77.92+j6.56 = 78.2/4.81^\circ \text{ A (rms)}$$

$$\text{[b]} \quad S_{g/\phi} = -(1664+j52)(77.92-j6.56) = -130,000+j6864 \text{ VA}$$

$$S_{gT} = 3S_{g/\phi} = -390,000+j20,592 \text{ VA}$$

Therefore, the source is delivering 390 kW and absorbing 20.592 kvars.

[c] $P_{\text{del}} = 390 \text{ kW}$

$$\begin{aligned} P_{\text{abs}} &= 3(128,000) + 3|\mathbf{I}_{\text{aA}}|^2(0.2) \\ &= 390 \text{ kW} = P_{\text{del}} \end{aligned}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(25) = 332,592 \text{ VAR}$

$$\begin{aligned} Q_{\text{abs}} &= 3(96,000) + 3|\mathbf{I}_{\text{aA}}|^2(0.8) + 20,592 \\ &= 332,592 \text{ VAR} = Q_{\text{del}} \end{aligned}$$

P 11.27 [a] $S_{T\Delta} = 14,000/\underline{41.41^\circ} - 9000/\underline{53.13^\circ} = 5.5/\underline{22^\circ} \text{ kVA}$

$$S_{\Delta} = S_{T\Delta}/3 = 1833.46/\underline{22^\circ} \text{ VA}$$

[b] $|\mathbf{V}_{\text{an}}| = \left| \frac{3000/\underline{53.13^\circ}}{10/\underline{-30^\circ}} \right| = 300 \text{ V (rms)}$

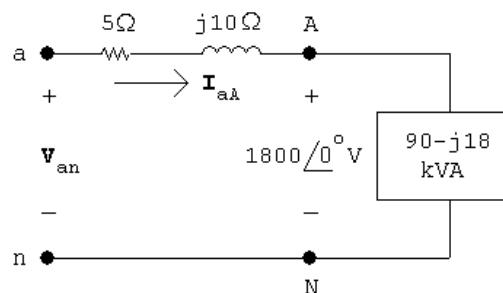
$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62 \text{ V (rms)}$$

P 11.28 [a] $S_{1/\phi} = 60,000(0.866) + j60,000(0.5) = 51,960 + j30,000 \text{ VA}$

$$S_{2/\phi} = 50,000(0.28) - j50,000(0.96) = 14,000 - j48,000 \text{ VA}$$

$$S_{3/\phi} = 24,040 \text{ VA}$$

$$S_{T/\phi} = S_1 + S_2 + S_3 = 90,000 - j18,000 \text{ VA}$$



$$\therefore \mathbf{I}_{\text{aA}}^* = \frac{90,000 - j18,000}{1800} = 50 - j10$$

$$\therefore \mathbf{I}_{\text{aA}} = 50 + j10 \text{ A}$$

$$\mathbf{V}_{\text{an}} = 1800 + (50 + j10)(5 + j10) = 1950 + j550 = 2026.08/\underline{15.75^\circ} \text{ V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2026.08) = 3509.27 \text{ V (rms)}$$

[b] $S_{g/\phi} = (1950 + j550)(50 - j10) = 103 + j0.8 \text{ kVA}$

$$\% \text{ efficiency} = \frac{90,000}{103,000}(100) = 87.38\%$$

P 11.29 [a] $S_1 = 10,200(0.87) + j10,200(0.493) = 8874 + j5029.13 \text{ VA}$

$$S_2 = 4200 + j1913.6 \text{ VA}$$

$$\sqrt{3}V_L I_L \sin \theta_3 = 7250; \quad \sin \theta_3 = \frac{7250}{\sqrt{3}(220)(36.8)} = 0.517$$

$$\text{Therefore } \cos \theta_3 = 0.856$$

Therefore

$$P_3 = \frac{7250}{0.517} \times 0.856 = 12,003.9 \text{ W}$$

$$S_3 = 12,003.9 + j7250 \text{ VA}$$

$$S_T = S_1 + S_2 + S_3 = 25.078 + j14.192 \text{ kVA}$$

$$S_{T/\phi} = \frac{1}{3}S_T = 8359.3 + j4730.7 \text{ VA}$$

$$\frac{220}{\sqrt{3}} \mathbf{I}_{aA}^* = (8359.3 + j4730.7); \quad \mathbf{I}_{aA}^* = 65.81 + j37.24 \text{ A}$$

$$\mathbf{I}_{aA} = 65.81 - j37.24 = 75.62 / -29.51^\circ \text{ A (rms)}$$

[b] pf = $\cos(0^\circ - 29.51^\circ) = 0.87$ lagging

P 11.30 From the solution to Problem 11.18 we have:

$$S_{AB} = (480/0^\circ)(192/-16.26^\circ) = 88,473.7 - j25,804.5 \text{ VA}$$

$$S_{BC} = (480/120^\circ)(48/-83.13^\circ) = 18,431.98 + j13,824.03 \text{ VA}$$

$$S_{CA} = (480/-120^\circ)(24/120^\circ) = 11,520 + j0 \text{ VA}$$

P 11.31 Let p_a , p_b , and p_c represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_a = v_{an} i_{aA} = [V_m \cos \omega t][I_m \cos(\omega t - \theta_\phi)]$$

$$p_b = v_{bn} i_{bB} = [V_m \cos(\omega t - 120^\circ)][I_m \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_c = v_{cn} i_{cC} = [V_m \cos(\omega t + 120^\circ)][I_m \cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_a + p_b + p_c$, so

$$p_T = V_m I_m [\cos \omega t \cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ) \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$+ \cos(\omega t + 120^\circ) \cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$\begin{aligned} p_T &= V_m I_m [\cos \omega t (1 + 2 \cos^2 120^\circ) \cos(\omega t - \theta_\phi) \\ &\quad + 2 \sin \omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m [\cos \omega t \cos(\omega t - \theta_\phi) + \sin \omega t \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m \cos \theta_\phi \end{aligned}$$

$$\text{P 11.32 } |I_{\text{line}}| = \frac{1600}{240/\sqrt{3}} = 11.547 \text{ A (rms)}$$

$$|Z_y| = \frac{|V|}{|I|} = \frac{240/\sqrt{3}}{11.547} = 12$$

$$Z_y = 12 / \underline{-50^\circ} \Omega$$

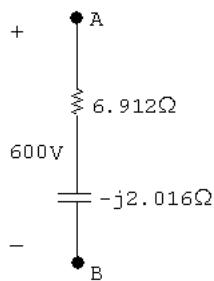
$$Z_\Delta = 3Z_y = 36 / \underline{-50^\circ} = 23.14 - j27.58 \Omega/\phi$$

P 11.33 Assume a Δ -connected load (series):

$$S_\phi = \frac{1}{3}(150 \times 10^3)(0.96 - j0.28) = 48,000 - j14,000 \text{ VA}$$

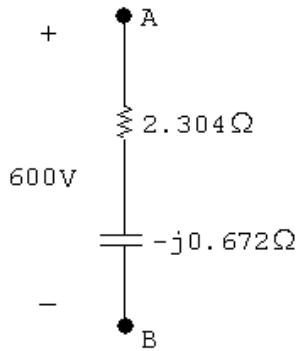
$$Z_{\Delta\phi}^* = \frac{|600|^2}{48,000 - j14,000} = 6.912 + j2.016 \Omega/\phi$$

$$Z_{\Delta\phi} = 6.912 - j2.016 \Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3} Z_{\Delta\phi} = 2.304 - j0.672 \Omega/\phi$$



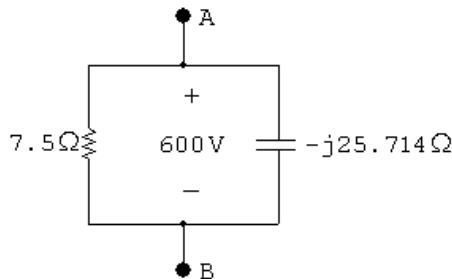
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|600|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|600|^2}{48,000} = 7.5 \Omega$$

$$Q_\phi = \frac{|600|^2}{X_\Delta}$$

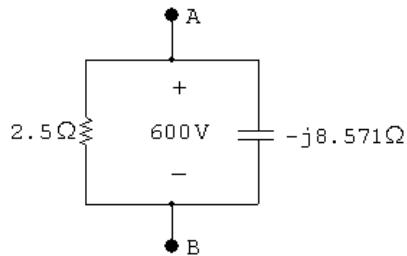
$$X_{\Delta\phi} = \frac{|600|^2}{-14,000} = -25.714 \Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 2.5 \Omega$$

$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = -8.571 \Omega$$



P 11.34 [a] $P_{\text{OUT}} = 746 \times 100 = 74,600 \text{ W}$

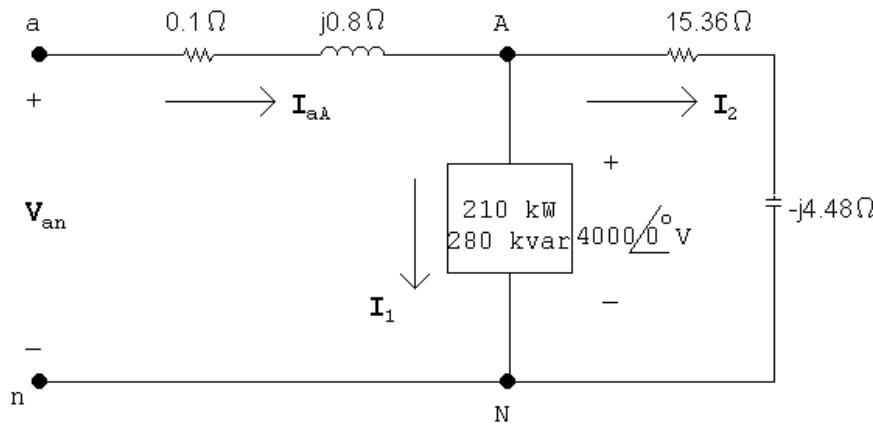
$$P_{\text{IN}} = 74,600 / (0.97) = 76,907.22 \text{ W}$$

$$\sqrt{3}V_L I_L \cos \theta = 76,907.22$$

$$I_L = \frac{76,907.22}{\sqrt{3}(208)(0.88)} = 242.58 \text{ A (rms)}$$

$$[b] Q = \sqrt{3}V_L I_L \sin \phi = \sqrt{3}(208)(242.58)(0.475) = 41,511.90 \text{ VAR}$$

P 11.35



$$4000\mathbf{I}_1^* = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70 \text{ A (rms)}$$

$$\mathbf{I}_1 = 52.5 - j70 \text{ A (rms)}$$

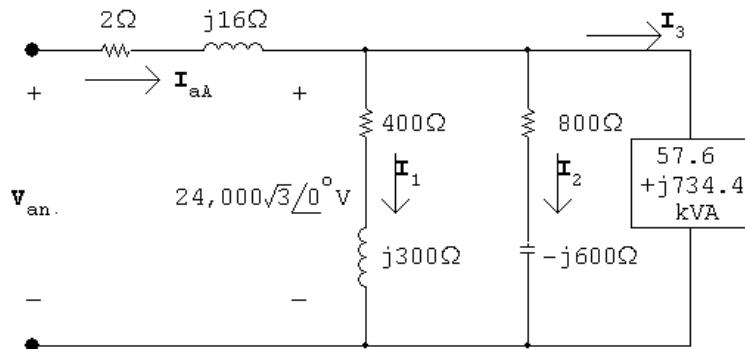
$$\mathbf{I}_2 = \frac{4000\angle 0^\circ}{15.36 - j4.48} = 240 + j70 \text{ A (rms)}$$

$$\therefore \mathbf{I}_{aA} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0 \text{ A (rms)}$$

$$\mathbf{V}_{an} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\angle 3.32^\circ \text{ V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 6990.62 \text{ V (rms)}$$

P 11.36 [a]



$$I_1 = \frac{24,000\sqrt{3}/0^\circ}{400 + j300} = 66.5 - j49.9 \text{ A (rms)}$$

$$I_2 = \frac{24,000\sqrt{3}/0^\circ}{800 - j600} = 33.3 + j24.9 \text{ A (rms)}$$

$$I_3^* = \frac{57,600 + j734,400}{24,000\sqrt{3}} = 1.4 + j17.7$$

$$I_3 = 1.4 - j17.7 \text{ A (rms)}$$

$$I_{aA} = I_1 + I_2 + I_3 = 101.2 - j42.7 \text{ A} = 109.8/-22.9^\circ \text{ A (rms)}$$

$$V_{an} = (2 + j16)(101.2 - j42.7) + 24,000\sqrt{3} = 42,454.8 + j1533.8 \text{ V (rms)}$$

$$S_\phi = V_{an} I_{aA}^* = (42,454.8 + j1533.8)(101.2 + j42.7)$$

$$= 4,230,932.5 + j1,968,040.5 \text{ VA}$$

$$S_T = 3S_\phi = 12,692.8 + j5904.1 \text{ kVA}$$

$$[b] S_{1/\phi} = 24,000\sqrt{3}(66.5 + j49.9) = 2764.4 + j2074.3 \text{ kVA}$$

$$S_{2/\phi} = 24,000\sqrt{3}(33.3 - j24.9) = 1384.3 - j1035.1 \text{ kVA}$$

$$S_{3/\phi} = 57.6 + j734.4 \text{ kVA}$$

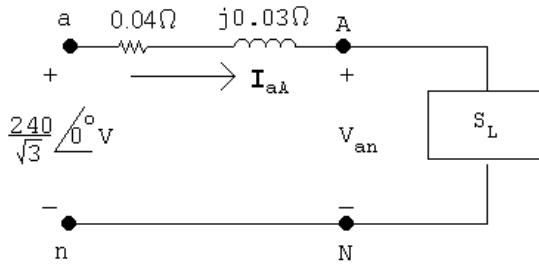
$$S_\phi(\text{load}) = 4206.3 + j1773.6 \text{ kVA}$$

$$\% \text{ delivered} = \left(\frac{4206.3}{4230.9} \right) (100) = 99.4\%$$

P 11.37 [a] $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9803.73 \text{ VA}$

$$I_{aA}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.76 + j70.76 \text{ A (rms)}$$

$$I_{aA} = 70.76 - j70.76 \text{ A (rms)}$$



$$\begin{aligned} \mathbf{V}_{AN} &= \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.76 - j70.76) \\ &= 133.61 + j0.71 = 133.61/0.30^\circ \text{ V (rms)} \end{aligned}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}(133.61) = 231.42 \text{ V (rms)}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9404 + j9504.5 \text{ VA}$

$$S_L = 3S_{L/\phi} = 28,212 + j28,513 \text{ VA}$$

Check:

$$S_g = 41,600(0.7071 + j0.7071) = 29,415 + j29,415 \text{ VA}$$

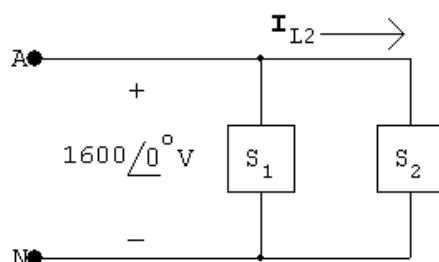
$$P_\ell = 3|\mathbf{I}_{aA}|^2(0.04) = 1202 \text{ W}$$

$$P_g = P_L + P_\ell = 28,212 + 1202 = 29,414 \text{ W (checks)}$$

$$Q_\ell = 3|\mathbf{I}_{aA}|^2(0.03) = 901 \text{ VAR}$$

$$Q_g = Q_L + Q_\ell = 28,513 + 901 = 29,414 \text{ VAR (checks)}$$

P 11.38 [a]



$$S_g = \frac{1}{3}(540)(0.96 + j0.28) = 172.8 + j50.4 \text{ kVA}$$

$$S_1 = \frac{1}{3}(28.4 - j208.8) = 12.8 - j69.6 \text{ kVA}$$

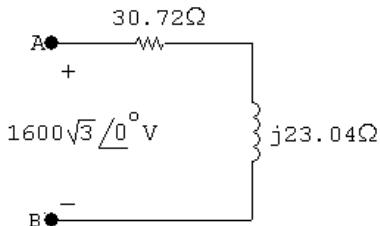
$$S_2 = S_g - S_1 = 160 + j120 \text{ kVA}$$

$$\therefore \mathbf{I}_{L2}^* = \frac{(160 + j120)10^3}{1600} = 100 + j75$$

$$\mathbf{I}_{L2} = 100 - j75 \text{ A (rms)}$$

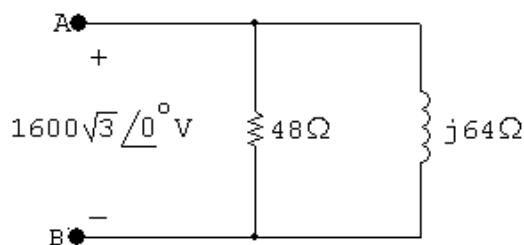
$$Z_Y = \frac{1600}{100 - j75} = 10.24 + j7.68 \Omega$$

$$Z_\Delta = 3Z_Y = 30.72 + j23.04 \Omega$$

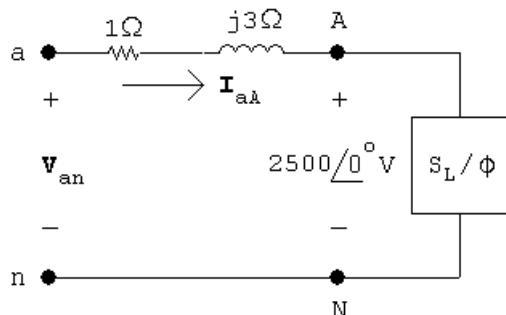


$$[b] R = \frac{(1600)^2}{160 \times 10^3} = 16 \Omega \rightarrow R_\Delta = 3R = 48 \Omega$$

$$X_L = \frac{(1600)^2}{120 \times 10^3} = 21.33 \Omega \rightarrow X_{L\Delta} = 3X_L = 64 \Omega$$



P 11.39 [a]



$$S_{L/\phi} = \frac{1}{3} \left[900 + j \frac{900}{0.6} (0.8) \right] 10^3 = 300,000 + j400,000 \text{ VA}$$

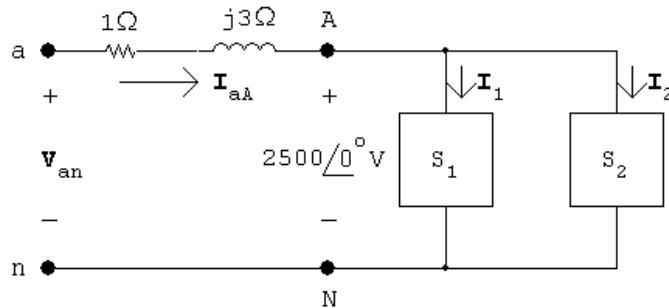
$$\mathbf{I}_{aA}^* = \frac{300,000 + j400,000}{2500} = 120 + j160 \text{ A (rms)}$$

$$\mathbf{I}_{aA} = 120 - j160 \text{ A (rms)}$$

$$\begin{aligned} \mathbf{V}_{an} &= 2500 + (1 + j3)(120 - j160) \\ &= 3100 + j200 = 3106.44/3.69^\circ \text{ V (rms)} \end{aligned}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(3106.44) = 5380.5 \text{ V (rms)}$$

[b]



$$\mathbf{I}_1 = 120 - j160 \text{ A} \quad (\text{from part [a]})$$

$$S_2 = 0 - j\frac{1}{3}(1125) \times 10^3 = -j375,000 \text{ VAR}$$

$$\mathbf{I}_2^* = \frac{-j375,000}{2500} = -j150 \text{ A (rms)}$$

$$\therefore \mathbf{I}_2 = j150 \text{ A (rms)}$$

$$\mathbf{I}_{aA} = 120 - j160 + j150 = 120 - j10 \text{ A (rms)}$$

$$\begin{aligned} \mathbf{V}_{an} &= 2500 + (120 - j10)(1 + j3) \\ &= 2650 + j350 = 2673.01/\underline{7.52^\circ} \text{ V (rms)} \end{aligned}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(2673.01) = 4629.8 \text{ V (rms)}$$

[c] $|\mathbf{I}_{aA}| = 200 \text{ A (rms)}$

$$P_{\text{loss}/\phi} = (200)^2(1) = 40 \text{ kW}$$

$$P_{g/\phi} = 300,000 + 40,000 = 340 \text{ kW}$$

$$\% \eta = \frac{300}{340}(100) = 88.2\%$$

[d] $|\mathbf{I}_{aA}| = 120.416 \text{ A (rms)}$

$$P_{\ell/\phi} = (120.416)^2(1) = 14,500 \text{ W}$$

$$\% \eta = \frac{300,000}{314,500}(100) = 95.4\%$$

$$[\mathbf{e}] \ Z_{\text{cap/Y}} = \frac{2500^2}{j375,000} = -j16.67 \Omega$$

$$\therefore \frac{1}{\omega C} = 16.67; \quad C = \frac{1}{(16.67)(120\pi)} = 159.155 \mu\text{F}$$

P 11.40 [a] From Assessment Problem 11.9, $\mathbf{I}_{\text{aA}} = (101.8 - j135.7) \text{ A}$ (rms)

$$\text{Therefore } \mathbf{I}_{\text{cap}} = j135.7 \text{ A (rms)}$$

$$\text{Therefore } Z_{CY} = \frac{2450/\sqrt{3}}{j135.7} = -j10.42 \Omega$$

$$\text{Therefore } C_Y = \frac{1}{(10.42)(2\pi)(60)} = 254.5 \mu\text{F}$$

$$Z_{C\Delta} = (-j10.42)(3) = -j31.26 \Omega$$

$$\text{Therefore } C_\Delta = \frac{254.5}{3} = 84.84 \mu\text{F}$$

[b] $C_Y = 254.5 \mu\text{F}$

[c] $|\mathbf{I}_{\text{aA}}| = 101.8 \text{ A}$ (rms)

P 11.41 $W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}| \cos(\underline{\mathbf{V}_{\text{AB}}} - \underline{\mathbf{I}_{\text{aA}}}) = (199.58)(2.4) \cos(65.68^\circ) = 197.26 \text{ W}$

$$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}| \cos(\underline{\mathbf{V}_{\text{CB}}} - \underline{\mathbf{I}_{\text{cC}}}) = (199.58)(2.4) \cos(5.68^\circ) = 476.64 \text{ W}$$

$$\text{CHECK: } W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9 \text{ W}$$

P 11.42 $\tan \phi = \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.75$

$$\therefore \phi = 36.87^\circ$$

$$\therefore 2400\sqrt{3}|\mathbf{I}_L| \cos 66.87^\circ = 40,823.09$$

$$|\mathbf{I}_L| = 25 \text{ A}$$

$$|Z| = \frac{2400}{25} = 96 \Omega \quad \therefore Z = 96/\underline{36.87^\circ} \Omega$$

P 11.43 $\mathbf{I}_{\text{aA}} = \frac{\mathbf{V}_{\text{AN}}}{Z_\phi} = |\mathbf{I}_L|/\underline{-\theta_\phi} \text{ A},$

$$Z_\phi = |Z|/\underline{\theta_\phi}, \quad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|/\underline{-90^\circ} \text{ V},$$

$$\begin{aligned} W_m &= |\mathbf{V}_L||\mathbf{I}_L| \cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L||\mathbf{I}_L| \cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L||\mathbf{I}_L| \sin \theta_\phi, \end{aligned}$$

$$\text{therefore } \sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L||\mathbf{I}_L| \sin \theta_\phi = Q_{\text{total}}$$

P 11.44 [a] $Z = 16 - j12 = 20/\underline{-36.87^\circ} \Omega$

$$\mathbf{V}_{AN} = 680/\underline{0^\circ} V; \quad \therefore \mathbf{I}_{aA} = 34/\underline{36.87^\circ} A$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 680\sqrt{3}/\underline{-90^\circ} V$$

$$W_m = (680\sqrt{3})(34) \cos(-90 - 36.87^\circ) = -24,027.07 W$$

$$\sqrt{3}W_m = -41,616.1 W$$

[b] $Q_\phi = (34^2)(-12) = -13,872 \text{ VAR}$

$$Q_T = 3Q_\phi = -41,616 \text{ VAR} = \sqrt{3}W_m$$

P 11.45 [a] $W_2 - W_1 = V_L I_L [\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)]$

$$= V_L I_L [\cos \theta \cos 30^\circ + \sin \theta \sin 30^\circ$$

$$- \cos \theta \cos 30^\circ + \sin \theta \sin 30^\circ]$$

$$= 2V_L I_L \sin \theta \sin 30^\circ = V_L I_L \sin \theta,$$

$$\text{therefore } \sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L \sin \theta = Q_T$$

[b] $Z_\phi = (8 + j6) \Omega$

$$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592 \text{ VAR},$$

$$Q_T = 3(12)^2(6) = 2592 \text{ VAR};$$

$$Z_\phi = (8 - j6) \Omega$$

$$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592 \text{ VAR},$$

$$Q_T = 3(12)^2(-6) = -2592 \text{ VAR};$$

$$Z_\phi = 5(1 + j\sqrt{3}) \Omega$$

$$Q_T = \sqrt{3}[2160 - 0] = 3741.23 \text{ VAR},$$

$$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23 \text{ VAR};$$

$$Z_\phi = 10/\underline{75^\circ} \Omega$$

$$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80 \text{ VAR},$$

$$Q_T = 3(12)^2[-10 \sin 75^\circ] = -4172.80 \text{ VAR}$$

$$\text{P 11.46 } Z_\phi = |Z| \underline{\theta} = \frac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}}$$

$$\theta = \underline{\mathbf{V}_{\text{AN}}} - \underline{\mathbf{I}_{\text{aA}}}$$

$$\theta_1 = \underline{\mathbf{V}_{\text{AB}}} - \underline{\mathbf{I}_{\text{aA}}}$$

For a positive phase sequence,

$$\underline{\mathbf{V}_{\text{AB}}} = \underline{\mathbf{V}_{\text{AN}}} + 30^\circ$$

Thus,

$$\theta_1 = \underline{\mathbf{V}_{\text{AN}}} + 30^\circ - \underline{\mathbf{I}_{\text{aA}}} = \theta + 30^\circ$$

Similarly,

$$Z_\phi = |Z| \underline{\theta} = \frac{\mathbf{V}_{\text{CN}}}{\mathbf{I}_{\text{cC}}}$$

$$\theta = \underline{\mathbf{V}_{\text{CN}}} - \underline{\mathbf{I}_{\text{cC}}}$$

$$\theta_2 = \underline{\mathbf{V}_{\text{CB}}} - \underline{\mathbf{I}_{\text{cC}}}$$

For a positive phase sequence,

$$\underline{\mathbf{V}_{\text{CB}}} = \underline{\mathbf{V}_{\text{BA}}} - 120^\circ = \underline{\mathbf{V}_{\text{AB}}} + 60^\circ$$

$$\underline{\mathbf{I}_{\text{cC}}} = \underline{\mathbf{I}_{\text{aA}}} + 120^\circ$$

Thus,

$$\begin{aligned}\theta_2 &= \underline{\mathbf{V}_{\text{AB}}} + 60^\circ - (\underline{\mathbf{I}_{\text{aA}}} + 120^\circ) = \theta_1 - 60^\circ \\ &= \theta + 30^\circ - 60^\circ = \theta - 30^\circ\end{aligned}$$

$$\text{P 11.47 [a]} \quad Z_\phi = 100 - j75 = 125 \angle -36.87^\circ \Omega$$

$$S_\phi = \frac{(13,200)^2}{125 \angle -36.87^\circ} = 1,115,136 + j836,352 \text{ VA}$$

$$[b] \frac{13,200}{\sqrt{3}} / 30^\circ \mathbf{I}_{aA}^* = S_\phi \quad \text{so} \quad \mathbf{I}_{aA} = 182.9 / 66.87^\circ$$

$$W_{m1} = (13,200)(182.9) \cos(0 - 66.87^\circ) = 948,401.92 \text{ W}$$

$$W_{m2} = (13,200)(182.9) \cos(-60^\circ + 53.13^\circ) = 2,397,006.08 \text{ W}$$

$$\text{Check: } P_T = 3(1,115,136) \text{ W} = W_{m1} + W_{m2}.$$

P 11.48 From the solution to Prob. 11.20 we have

$$\mathbf{I}_{aA} = 210 / 20.79^\circ \text{ A} \quad \text{and} \quad \mathbf{I}_{bB} = 178.68 / -178.04^\circ \text{ A}$$

$$\begin{aligned} [a] \quad W_1 &= |\mathbf{V}_{ac}| |\mathbf{I}_{aA}| \cos(\theta_{ac} - \theta_{aA}) \\ &= 480(210) \cos(60^\circ - 20.79^\circ) = 78,103.2 \text{ W} \end{aligned}$$

$$\begin{aligned} [b] \quad W_2 &= |\mathbf{V}_{bc}| |\mathbf{I}_{bB}| \cos(\theta_{bc} - \theta_{bB}) \\ &= 480(178.68) \cos(120^\circ + 178.04^\circ) = 40,317.7 \text{ W} \end{aligned}$$

$$[c] \quad W_1 + W_2 = 118,421 \text{ W}$$

$$P_{AB} = (192)^2(2.4) = 88,473.6 \text{ W}$$

$$P_{BC} = (48)^2(8) = 18,432 \text{ W}$$

$$P_{CA} = (24)^2(20) = 11,520 \text{ W}$$

$$P_{AB} + P_{BC} + P_{CA} = 118,425.7$$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

$$P 11.49 \quad [a] \quad \mathbf{I}_{aA}^* = \frac{144(0.96 - j0.28)10^3}{7200} = 20 / -16.26^\circ \text{ A}$$

$$\mathbf{V}_{BN} = 7200 / -120^\circ \text{ V}; \quad \mathbf{V}_{CN} = 7200 / 120^\circ \text{ V}$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 7200\sqrt{3} / -90^\circ \text{ V}$$

$$\mathbf{I}_{bB} = 20 / -103.74^\circ \text{ A}$$

$$W_{m1} = (7200\sqrt{3})(20) \cos(-90^\circ + 103.74^\circ) = 242,278.14 \text{ W}$$

[b] Current coil in line aA, measure \mathbf{I}_{aA} .

Voltage coil across AC, measure \mathbf{V}_{AC} .

$$[c] \quad I_{aA} = 20 / 16.76^\circ \text{ A}$$

$$\mathbf{V}_{CA} = \mathbf{V}_{AN} - \mathbf{V}_{CN} = 7200\sqrt{3} / -30^\circ \text{ V}$$

$$W_{m2} = (7200\sqrt{3})(20) \cos(-30^\circ - 16.26^\circ) = 172,441.86 \text{ W}$$

[d] $W_{m1} + W_{m2} = 414.72 \text{ kW}$

$$P_T = 432,000(0.96) = 414.72 \text{ kW} = W_{m1} + W_{m2}$$

P 11.50 [a] $W_1 = |\mathbf{V}_{BA}| |\mathbf{I}_{bB}| \cos \theta$

Negative phase sequence:

$$\mathbf{V}_{BA} = 240\sqrt{3}/150^\circ \text{ V}$$

$$\mathbf{I}_{aA} = \frac{240/0^\circ}{13.33/-30^\circ} = 18/30^\circ \text{ A}$$

$$\mathbf{I}_{bB} = 18/150^\circ \text{ A}$$

$$W_1 = (18)(240)\sqrt{3} \cos 0^\circ = 7482.46 \text{ W}$$

$$W_2 = |\mathbf{V}_{CA}| |\mathbf{I}_{cC}| \cos \theta$$

$$\mathbf{V}_{CA} = 240\sqrt{3}/-150^\circ \text{ V}$$

$$\mathbf{I}_{cC} = 18/-90^\circ \text{ A}$$

$$W_2 = (18)(240)\sqrt{3} \cos(-60^\circ) = 3741.23 \text{ W}$$

[b] $P_\phi = (18)^2(40/3) \cos(-30^\circ) = 3741.23 \text{ W}$

$$P_T = 3P_\phi = 11,223.69 \text{ W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11,223.69 \text{ W}$$

$$\therefore W_1 + W_2 = P_T \quad (\text{checks})$$

P 11.51 [a] $Z = \frac{1}{3}Z_\Delta = 4.48 + j15.36 = 16/73.74^\circ \Omega$

$$\mathbf{I}_{aA} = \frac{600/0^\circ}{16/73.74^\circ} = 37.5/-73.74^\circ \text{ A}$$

$$\mathbf{I}_{bB} = 37.5/-193.74^\circ \text{ A}$$

$$\mathbf{V}_{AC} = 600\sqrt{3}/-30^\circ \text{ V}$$

$$\mathbf{V}_{BC} = 600\sqrt{3}/-90^\circ \text{ V}$$

$$W_1 = (600\sqrt{3})(37.5) \cos(-30 + 73.74^\circ) = 28,156.15 \text{ W}$$

$$W_2 = (600\sqrt{3})(37.5) \cos(-90 + 193.74^\circ) = -9256.15 \text{ W}$$

[b] $W_1 + W_2 = 18,900 \text{ W}$

$$P_T = 3(37.5)^2(13.44/3) = 18,900 \text{ W}$$

$$[c] \sqrt{3}(W_1 - W_2) = 64,800 \text{ VAR}$$

$$Q_T = 3(37.5)^2(46.08/3) = 64,800 \text{ VAR}$$

P 11.52 [a] Negative phase sequence:

$$\mathbf{V}_{AB} = 240\sqrt{3}/-30^\circ \text{ V}$$

$$\mathbf{V}_{BC} = 240\sqrt{3}/90^\circ \text{ V}$$

$$\mathbf{V}_{CA} = 240\sqrt{3}/-150^\circ \text{ V}$$

$$\mathbf{I}_{AB} = \frac{240\sqrt{3}/-30^\circ}{20/30^\circ} = 20.78/-60^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \frac{240\sqrt{3}/90^\circ}{60/0^\circ} = 6.93/90^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \frac{240\sqrt{3}/-150^\circ}{40/-30^\circ} = 10.39/-120^\circ \text{ A}$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC} = 18/-30^\circ \text{ A}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CB} + \mathbf{I}_{CA} = \mathbf{I}_{CA} + \mathbf{I}_{BC} = 16.75/-108.06^\circ$$

$$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46 \text{ W}$$

$$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23 \text{ W}$$

[b] $W_{m1} + W_{m2} = 14,103.69 \text{ W}$

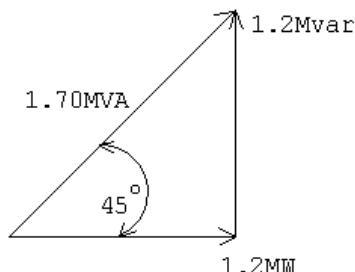
$$P_A = (12\sqrt{3})^2(20 \cos 30^\circ) = 7482.46 \text{ W}$$

$$P_B = (4\sqrt{3})^2(60) = 2880 \text{ W}$$

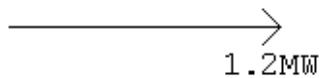
$$P_C = (6\sqrt{3})^2[40 \cos(-30^\circ)] = 3741.23 \text{ W}$$

$$P_A + P_B + P_C = 14,103.69 = W_{m1} + W_{m2}$$

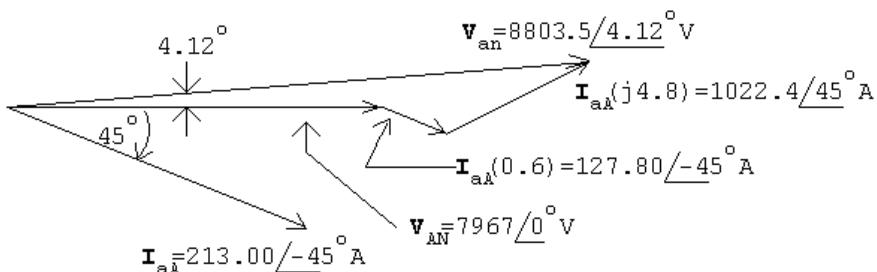
P 11.53 [a]



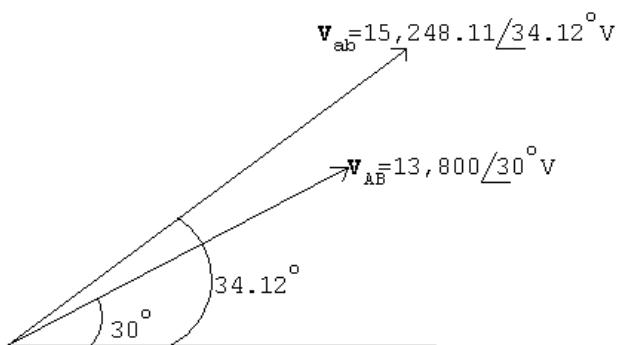
[b]



[c]



[d]



$$\text{P 11.54 [a]} \quad Q = \frac{|\mathbf{V}|^2}{X_C}$$

$$\therefore |X_C| = \frac{(13,800)^2}{1.2 \times 10^6} = 158.70 \Omega$$

$$\therefore \frac{1}{\omega C} = 158.70; \quad C = \frac{1}{2\pi(60)(158.70)} = 16.71 \mu\text{F}$$

$$\text{[b]} \quad |X_C| = \frac{(13,800/\sqrt{3})^2}{1.2 \times 10^6} = \frac{1}{3}(158.70)$$

$$\therefore C = 3(16.71) = 50.14 \mu\text{F}$$

P 11.55 [a] The capacitor from Appendix H whose value is closest to $50.14 \mu\text{F}$ is $47 \mu\text{F}$.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(47 \times 10^{-6})} = 56.4 \Omega$$

$$Q = \frac{|\mathbf{V}|^2}{3X_C} = \frac{(13,800)^2}{3(56.4)} = 1,124,775.6 \text{ VAR}$$

$$[b] \quad \mathbf{I}_{aA}^* = \frac{1,200,000 + j75,224}{13,800/\sqrt{3}} = 150.6 + j9.4 \text{ A}$$

$$\mathbf{V}_{an} = \frac{13,800}{\sqrt{3}} \underline{0^\circ} + (0.6 + j4.8)(150.6 - j9.4) = 8134.8 \underline{5.06^\circ}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(8134.8) = 14,089.9 \text{ V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

- P 11.56 [a] The capacitor from Appendix H whose value is closest to $16.71 \mu\text{F}$ is $22 \mu\text{F}$.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(22 \times 10^{-6})} = 120.57 \Omega$$

$$Q = \frac{|V|^2}{X_C} = \frac{(13,800)^2}{120.57} = 1,579,497 \text{ VAR}/\phi$$

$$[b] \quad \mathbf{I}_{aA}^* = \frac{1,200,000 - j379,497}{13,800/\sqrt{3}} = 50.2 - j15.9 \text{ A}$$

$$\mathbf{V}_{an} = \frac{13,800}{\sqrt{3}} \underline{0^\circ} + (0.6 + j4.8)(50.2 + j15.9) = 7897.8 \underline{1.76^\circ}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(7897.8) = 13,679.4 \text{ V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

- P 11.57 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13,800}{\sqrt{3}} \mathbf{I}_{aA}^* = -j1.2 \times 10^6$$

$$\text{or} \quad \mathbf{I}_{aA}^* = -j150.61 \text{ A}$$

$$\text{Hence} \quad \mathbf{I}_{aA} = j150.61 \text{ A}$$

Now,

$$\mathbf{V}_{an} = \frac{13,800}{\sqrt{3}} \underline{0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05 \underline{0.71^\circ} \text{ V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{ab}| = \sqrt{3}(7245.05) = 12,548.80 \text{ V.}$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.58 Before the capacitors are added the total line loss is

$$P_L = 3|150.61 + j150.61|^2(0.6) = 81.66 \text{ kW}$$

After the capacitors are added the total line loss is

$$P_L = 3|150.61|^2(0.6) = 40.83 \text{ kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.59 [a] $\frac{13,800}{\sqrt{3}} \mathbf{I}_{aA}^* = 60 \times 10^3 + j160 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{aA}^* = \frac{60\sqrt{3} - j1040\sqrt{3}}{13.8} = 7.53 - j130.53 \text{ A}$$

$$\therefore \mathbf{I}_{aA} = 7.53 + j130.53 \text{ A}$$

$$\begin{aligned} \mathbf{V}_{an} &= \frac{13,800}{\sqrt{3}} \underline{/0^\circ} + (0.6 + j4.8)(7.53 + j130.53) \\ &= 7345.41 + j114.46 = 7346.3 \underline{/0.89^\circ} \text{ V} \end{aligned}$$

$$\therefore |\mathbf{V}_{ab}| = \sqrt{3}(7346.3) = 12,724.16 \text{ V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.60 [a] $\frac{13,800}{\sqrt{3}} \mathbf{I}_{aA}^* = (60 + j160) \times 10^3$

$$\mathbf{I}_{aA}^* = \frac{60\sqrt{3} + j160\sqrt{3}}{13.8} = 7.53 + j20.08 \text{ A}$$

$$\therefore \mathbf{I}_{aA} = 7.53 - j20.08 \text{ A}$$

$$\begin{aligned} \mathbf{V}_{an} &= \frac{13,800}{\sqrt{3}} \underline{/0^\circ} + (0.6 + j4.8)(7.53 - j20.08) \\ &= 8068.34 + j24.10 = 8068.38 \underline{/0.17^\circ} \text{ V} \end{aligned}$$

$$\therefore |\mathbf{V}_{ab}| = \sqrt{3}(8068.38) = 13,974.77 \text{ V}$$

[b] Yes: $13 \text{ kV} < 13,974.77 < 14.6 \text{ kV}$

[c] $P_{loss} = 3|7.53 + j130.53|^2(0.6) = 30.77 \text{ kW}$

[d] $P_{loss} = 3|7.53 - j20.08|^2(0.6) = 0.83 \text{ kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.

12

Introduction to the Laplace Transform

Assessment Problems

$$\text{AP 12.1 [a]} \cosh \beta t = \frac{e^{\beta t} + e^{-\beta t}}{2}$$

Therefore,

$$\begin{aligned}\mathcal{L}\{\cosh \beta t\} &= \frac{1}{2} \int_{0^-}^{\infty} [e^{(s-\beta)t} + e^{-(s-\beta)t}] dt \\ &= \frac{1}{2} \left[\frac{e^{-(s-\beta)t}}{-(s-\beta)} \Big|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)} \Big|_{0^-}^{\infty} \right] \\ &= \frac{1}{2} \left(\frac{1}{s-\beta} + \frac{1}{s+\beta} \right) = \frac{s}{s^2 - \beta^2}\end{aligned}$$

$$[\text{b}] \sinh \beta t = \frac{e^{\beta t} - e^{-\beta t}}{2}$$

Therefore,

$$\begin{aligned}\mathcal{L}\{\sinh \beta t\} &= \frac{1}{2} \int_{0^-}^{\infty} [e^{-(s-\beta)t} - e^{-(s+\beta)t}] dt \\ &= \frac{1}{2} \left[\frac{e^{-(s-\beta)t}}{-(s-\beta)} \Big|_{0^-}^{\infty} - \frac{1}{2} \left[\frac{e^{-(s+\beta)t}}{-(s+\beta)} \Big|_{0^-}^{\infty} \right] \right] \\ &= \frac{1}{2} \left(\frac{1}{s-\beta} - \frac{1}{s+\beta} \right) = \frac{\beta}{(s^2 - \beta^2)}\end{aligned}$$

$$\text{AP 12.2 [a]} \text{ Let } f(t) = te^{-at}:$$

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

$$\text{Now, } \mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds}$$

$$\text{So, } \mathcal{L}\{t \cdot te^{-at}\} = -\frac{d}{ds} \left[\frac{1}{(s+a)^2} \right] = \frac{2}{(s+a)^3}$$

[b] Let $f(t) = e^{-at} \sinh \beta t$, then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2 - \beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2 - \beta^2} - 0 = \frac{\beta s}{(s+a)^2 - \beta^2}$$

[c] Let $f(t) = \cos \omega t$. Then

$$F(s) = \frac{s}{(s^2 + \omega^2)} \quad \text{and} \quad \frac{dF(s)}{ds} = \frac{-(s^2 - \omega^2)}{(s^2 + \omega^2)^2}$$

$$\text{Therefore } \mathcal{L}\{t \cos \omega t\} = -\frac{dF(s)}{ds} = \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$$

AP 12.3

$$F(s) = \frac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6 - 26 + 26}{(1)(2)} = 3; \quad K_2 = \frac{24 - 52 + 26}{(-1)(1)} = 2$$

$$K_3 = \frac{54 - 78 + 26}{(-2)(-1)} = 1$$

$$\text{Therefore } f(t) = [3e^{-t} + 2e^{-2t} + e^{-3t}] u(t)$$

AP 12.4

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)} = \frac{K_1}{s+3} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$$

$$K_1 = \frac{63 - 189 - 134}{1(2)} = 4; \quad K_2 = \frac{112 - 252 + 134}{(-1)(1)} = 6$$

$$K_3 = \frac{175 - 315 + 134}{(-2)(-1)} = -3$$

$$f(t) = [4e^{-3t} + 6e^{-4t} - 3e^{-5t}] u(t)$$

AP 12.5

$$F(s) = \frac{10(s^2 + 119)}{(s + 5)(s^2 + 10s + 169)}$$

$$s_{1,2} = -5 \pm \sqrt{25 - 169} = -5 \pm j12$$

$$F(s) = \frac{K_1}{s + 5} + \frac{K_2}{s + 5 - j12} + \frac{K_2^*}{s + 5 + j12}$$

$$K_1 = \frac{10(25 + 119)}{25 - 50 + 169} = 10$$

$$K_2 = \frac{10[(-5 + j12)^2 + 119]}{(j12)(j24)} = j4.17 = 4.17/\underline{90^\circ}$$

Therefore

$$\begin{aligned} f(t) &= [10e^{-5t} + 8.33e^{-5t} \cos(12t + 90^\circ)] u(t) \\ &= [10e^{-5t} - 8.33e^{-5t} \sin 12t] u(t) \end{aligned}$$

AP 12.6

$$F(s) = \frac{4s^2 + 7s + 1}{s(s+1)^2} = \frac{K_0}{s} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}$$

$$K_0 = \frac{1}{(1)^2} = 1; \quad K_1 = \frac{4 - 7 + 1}{-1} = 2$$

$$\begin{aligned} K_2 &= \frac{d}{ds} \left[\frac{4s^2 + 7s + 1}{s} \right]_{s=-1} = \frac{s(8s+7) - (4s^2 + 7s + 1)}{s^2} \Big|_{s=-1} \\ &= \frac{1+2}{1} = 3 \end{aligned}$$

$$\text{Therefore } f(t) = [1 + 2te^{-t} + 3e^{-t}] u(t)$$

AP 12.7

$$\begin{aligned} F(s) &= \frac{40}{(s^2 + 4s + 5)^2} = \frac{40}{(s + 2 - j1)^2(s + 2 + j1)^2} \\ &= \frac{K_1}{(s + 2 - j1)^2} + \frac{K_2}{(s + 2 - j1)} + \frac{K_1^*}{(s + 2 + j1)^2} \\ &\quad + \frac{K_2^*}{(s + 2 + j1)} \end{aligned}$$

$$K_1 = \frac{40}{(j2)^2} = -10 = 10/\underline{180^\circ} \quad \text{and} \quad K_1^* = -10$$

$$K_2 = \frac{d}{ds} \left[\frac{40}{(s+2+j1)^2} \right]_{s=-2+j1} = \frac{-80(j2)}{(j2)^4} = -j10 = 10/\underline{-90^\circ}$$

$$K_2^* = j10$$

Therefore

$$\begin{aligned} f(t) &= [20te^{-2t} \cos(t + 180^\circ) + 20e^{-2t} \cos(t - 90^\circ)] u(t) \\ &= 20e^{-2t} [\sin t - t \cos t] u(t) \end{aligned}$$

AP 12.8

$$F(s) = \frac{5s^2 + 29s + 32}{(s+2)(s+4)} = \frac{5s^2 + 29s + 32}{s^2 + 6s + 8} = 5 - \frac{s+8}{(s+2)(s+4)}$$

$$\frac{s+8}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{-2+8}{2} = 3; \quad K_2 = \frac{-4+8}{-2} = -2$$

Therefore,

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$f(t) = 5\delta(t) + [-3e^{-2t} + 2e^{-4t}]u(t)$$

AP 12.9

$$F(s) = \frac{2s^3 + 8s^2 + 2s - 4}{s^2 + 5s + 4} = 2s - 2 + \frac{4(s+1)}{(s+1)(s+4)} = 2s - 2 + \frac{4}{s+4}$$

$$f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}u(t)$$

AP 12.10

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \left[\frac{7s^3[1 + (9/s) + (134/(7s^2))]}{s^3[1 + (3/s)][1 + (4/s)][1 + (5/s)]} \right] = 7$$

$$\therefore f(0^+) = 7$$

$$\lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} \left[\frac{7s^3 + 63s^2 + 134s}{(s+3)(s+4)(s+5)} \right] = 0$$

$$\therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \left[\frac{s^3[4 + (7/s) + (1/s)^2]}{s^3[1 + (1/s)]^2} \right] = 4$$

$$\therefore f(0^+) = 4$$

$$\lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} \left[\frac{4s^2 + 7s + 1}{(s + 1)^2} \right] = 1$$

$$\therefore f(\infty) = 1$$

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \left[\frac{40s}{s^4[1 + (4/s) + (5/s^2)]^2} \right] = 0$$

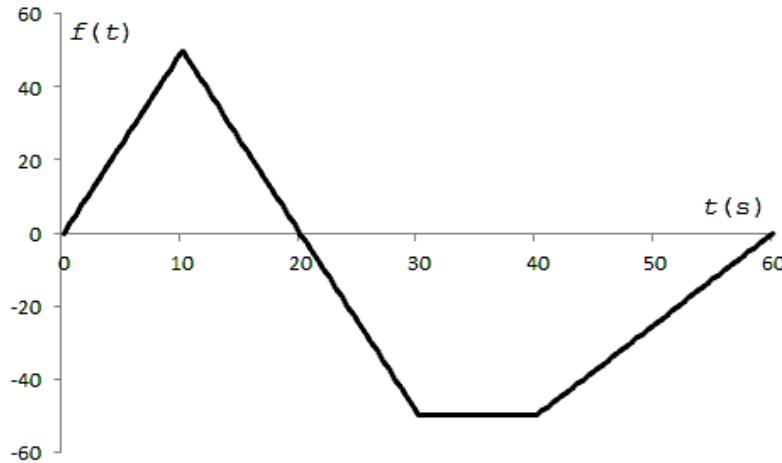
$$\therefore f(0^+) = 0$$

$$\lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} \left[\frac{40s}{(s^2 + 4s + 5)^2} \right] = 0$$

$$\therefore f(\infty) = 0$$

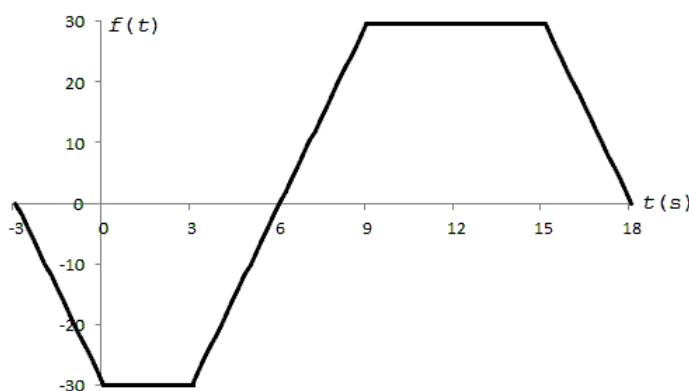
Problems

P 12.1 [a]



$$\begin{aligned}
 \text{[b]} \quad f(t) = & 5t[u(t) - u(t - 10)] + (100 - 5t)[u(t - 10) - u(t - 30)] \\
 & - 50[u(t - 30) - u(t - 40)] \\
 & + (2.5t - 150)[u(t - 40) - u(t - 60)]
 \end{aligned}$$

P 12.2



$$\text{P 12.3 [a]} \quad (-3t - 15)[u(t + 5) - u(t)] + (-3t + 15)[u(t) - u(t - 5)]$$

$$= -3(t + 5)u(t + 5) + 30u(t) + 3(t - 5)u(t - 5)$$

$$\text{[b]} \quad (5t + 20)[u(t + 4) - u(t + 2)] - 5t[u(t + 2) - u(t - 2)]$$

$$+ (5t - 20)[u(t - 2) - u(t - 4)]$$

$$= 5(t + 4)u(t + 4) - 10(t + 2)u(t + 2) + 10(t - 2)u(t - 2)$$

$$- 5(t - 4)u(t - 4)$$

P 12.4 [a] $6.25t[u(t) - u(t-8)] + 50[u(t-8) - u(t-12)]$
 $+ (125 - 6.25t)[u(t-12) - u(t-20)]$

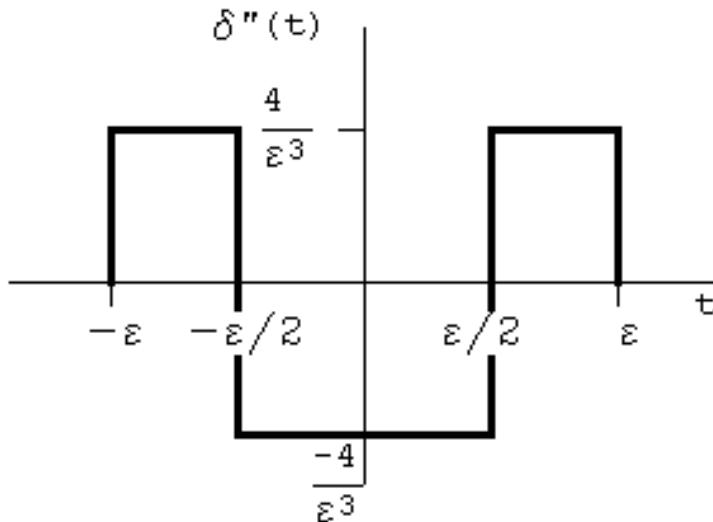
[b] $25e^{-t}[u(t) - u(t-2)]$
[c] $(30 - 2t)[u(t) - u(t-5)] + 20[u(t-5) - u(t-15)]$
 $+ (50 - 2t)[u(t-15) - u(t-25)]$

P 12.5 As $\varepsilon \rightarrow 0$ the amplitude $\rightarrow \infty$; the duration $\rightarrow 0$; and the area is independent of ε , i.e.,

$$A = \int_{-\infty}^{\infty} \frac{1}{\pi \varepsilon^2 + t^2} dt = 1$$

P 12.6 [a] $A = \left(\frac{1}{2}\right)bh = \left(\frac{1}{2}\right)(2\varepsilon)\left(\frac{1}{\varepsilon}\right) = 1$
[b] 0; [c] ∞

P 12.7



$$F(s) = \int_{-\varepsilon}^{-\varepsilon/2} \frac{4}{\varepsilon^3} e^{-st} dt + \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{-4}{\varepsilon^3}\right) e^{-st} dt + \int_{\varepsilon/2}^{\varepsilon} \frac{4}{\varepsilon^3} e^{-st} dt$$

$$\text{Therefore } F(s) = \frac{4}{s\varepsilon^3} [e^{s\varepsilon} - 2e^{s\varepsilon/2} + 2e^{-s\varepsilon/2} - e^{-s\varepsilon}]$$

$$\mathcal{L}\{\delta''(t)\} = \lim_{\varepsilon \rightarrow 0} F(s)$$

After applying L'Hopital's rule three times, we have

$$\lim_{\varepsilon \rightarrow 0} \frac{2s}{3} \left[se^{s\varepsilon} - \frac{s}{4} e^{s\varepsilon/2} - \frac{s}{4} e^{-s\varepsilon/2} + se^{-s\varepsilon} \right] = \frac{2s}{3} \left(\frac{3s}{2} \right)$$

$$\text{Therefore } \mathcal{L}\{\delta''(t)\} = s^2$$

$$\text{P 12.8 [a]} \quad I = \int_{-1}^3 (t^3 + 2)\delta(t) dt + \int_{-1}^3 8(t^3 + 2)\delta(t - 1) dt \\ = (0^3 + 2) + 8(1^3 + 2) = 2 + 8(3) = 26$$

$$\text{[b]} \quad I = \int_{-2}^2 t^2\delta(t) dt + \int_{-2}^2 t^2\delta(t + 1.5) dt + \int_{-2}^2 \delta(t - 3) dt \\ = 0^2 + (-1.5)^2 + 0 = 2.25$$

$$\text{P 12.9} \quad F(s) = \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} e^{-st} dt = \frac{e^{s\varepsilon} - e^{-s\varepsilon}}{2\varepsilon s}$$

$$F(s) = \frac{1}{2s} \lim_{\varepsilon \rightarrow 0} \left[\frac{se^{s\varepsilon} + se^{-s\varepsilon}}{1} \right] = \frac{1}{2s} \cdot \frac{2s}{1} = 1$$

$$\text{P 12.10} \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(4 + j\omega)}{(9 + j\omega)} \cdot \pi\delta(\omega) \cdot e^{jt\omega} d\omega = \left(\frac{1}{2\pi} \right) \left(\frac{4 + j0}{9 + j0} \pi e^{-jt0} \right) = \frac{2}{9}$$

$$\text{P 12.11} \quad \mathcal{L} \left\{ \frac{d^n f(t)}{dt^n} \right\} = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \dots,$$

Therefore

$$\mathcal{L}\{\delta^n(t)\} = s^n(1) - s^{n-1}\delta(0^-) - s^{n-2}\delta'(0^-) - s^{n-3}\delta''(0^-) - \dots = s^n$$

$$\text{P 12.12 [a]} \quad \text{Let } dv = \delta'(t - a) dt, \quad v = \delta(t - a)$$

$$u = f(t), \quad du = f'(t) dt$$

Therefore

$$\int_{-\infty}^{\infty} f(t)\delta'(t - a) dt = f(t)\delta(t - a) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t - a)f'(t) dt \\ = 0 - f'(a)$$

$$\text{[b]} \quad \mathcal{L}\{\delta'(t)\} = \int_{0^-}^{\infty} \delta'(t)e^{-st} dt = - \left[\frac{d(e^{-st})}{dt} \right]_{t=0} = - \left[-se^{-st} \right]_{t=0} = s$$

$$\text{P 12.13 [a]} \quad \mathcal{L}\{20e^{-500(t-10)}u(t-10)\} = \frac{20e^{-10s}}{(s+500)}$$

[b] First rewrite $f(t)$ as

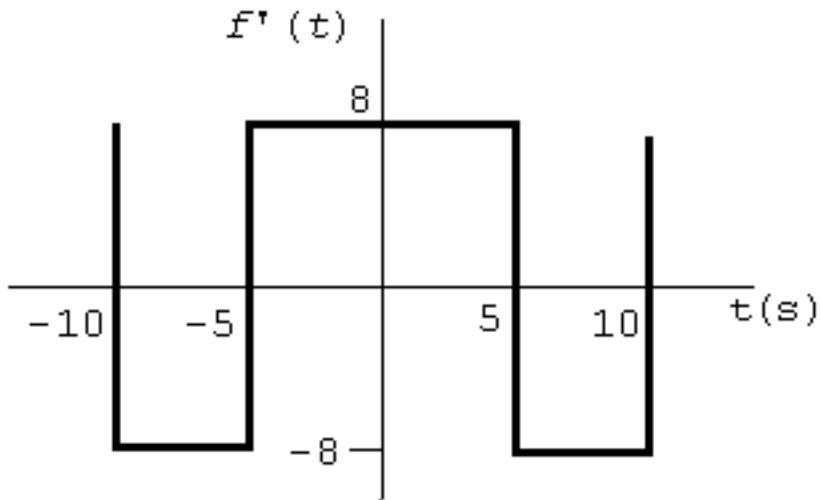
$$\begin{aligned} f(t) &= (5t + 20)u(t + 4) - (10t + 20)u(t + 2) \\ &\quad + (10t - 20)u(t - 2) - (5t - 20)u(t - 4) \\ &= 5(t + 4)u(t + 4) - 10(t + 2)u(t + 2) \\ &\quad + 10(t - 2)u(t - 2) - 5(t - 4)u(t - 4) \end{aligned}$$

$$\therefore F(s) = \frac{5[e^{4s} - 2e^{2s} + 2e^{-2s} - e^{-4s}]}{s^2}$$

P 12.14 [a] $f(t) = (-8t - 80)[u(t + 10) - u(t + 5)]$
 $+ 8t[u(t + 5) - u(t - 5)]$
 $+ (-8t + 80)[u(t - 5) - u(t - 10)]$
 $= -8(t + 10)u(t + 10) + 16(t + 5)u(t + 5)$
 $- 16(t - 5)u(t - 5) + 8(t - 10)u(t - 10)$

$$\therefore F(s) = \frac{8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]}{s^2}$$

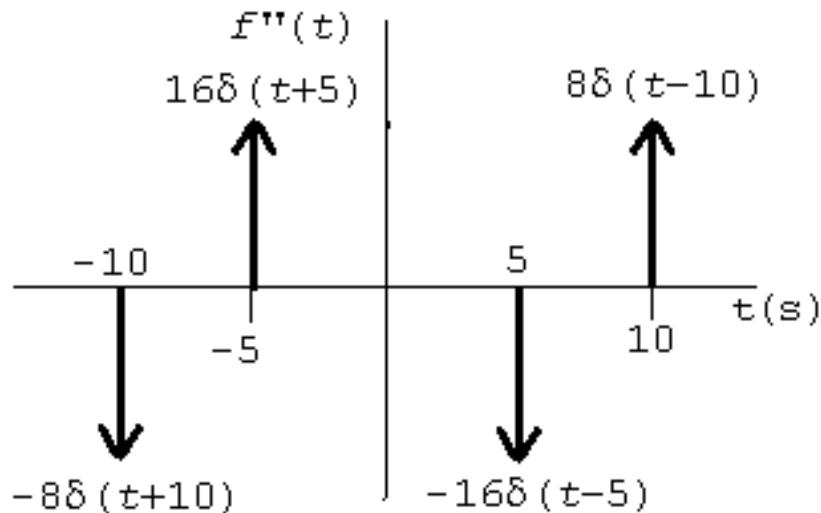
[b]



$$f'(t) = -8[u(t + 10) - u(t + 5)] + 8[u(t + 5) - u(t - 5)]
+ (-8)[u(t - 5) - u(t - 10)]
= -8u(t + 10) + 16u(t + 5) - 16u(t - 5) + 8u(t - 10)$$

$$\mathcal{L}\{f'(t)\} = \frac{8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]}{s}$$

[c]



$$f''(t) = -8\delta(t+10) + 16\delta(t+5) - 16\delta(t-5) + 8\delta(t-10)$$

$$\mathcal{L}\{f''(t)\} = 8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]$$

$$\text{P 12.15 } \mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^{\infty} [e^{-at}f(t)]e^{-st} dt = \int_{0^-}^{\infty} f(t)e^{-(s+a)t} dt = F(s+a)$$

$$\text{P 12.16 } \mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st} dt$$

$$\text{Let } u = at, \quad du = a dt, \quad u = 0^- \quad \text{when } t = 0^-$$

$$\text{and } u = \infty \quad \text{when } t = \infty$$

$$\text{Therefore } \mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s} \frac{du}{a} = \frac{1}{a} F(s/a)$$

$$\text{P 12.17 [a]} \quad \mathcal{L}\{te^{-at}\} = \int_{0^-}^{\infty} te^{-(s+a)t} dt$$

$$= \frac{e^{-(s+a)t}}{(s+a)^2} \left[-(s+a)t - 1 \right]_{0^-}^{\infty}$$

$$= 0 + \frac{1}{(s+a)^2}$$

$$\therefore \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

$$[\mathbf{b}] \quad \mathcal{L} \left\{ \frac{d}{dt} (te^{-at}) u(t) \right\} = \frac{s}{(s+a)^2} - 0$$

$$\mathcal{L} \left\{ \frac{d}{dt} (te^{-at}) u(t) \right\} = \frac{s}{(s+a)^2}$$

$$[\mathbf{c}] \quad \frac{d}{dt} (te^{-at}) = -ate^{-at} + e^{-at}$$

$$\mathcal{L} \{-ate^{-at} + e^{-at}\} = \frac{-a}{(s+a)^2} + \frac{1}{(s+a)} = \frac{-a}{(s+a)^2} + \frac{s+a}{(s+a)^2}$$

$$\therefore \quad \mathcal{L} \left\{ \frac{d}{dt} (te^{-at}) \right\} = \frac{s}{(s+a)^2} \quad \text{CHECKS}$$

$$\text{P 12.18} \quad [\mathbf{a}] \quad \mathcal{L} \left\{ \int_{0^-}^t e^{-ax} dx \right\} = \frac{F(s)}{s} = \frac{1}{s(s+a)}$$

$$[\mathbf{b}] \quad \int_{0^-}^t e^{-ax} dx = \frac{1}{a} - \frac{e^{-at}}{a}$$

$$\mathcal{L} \left\{ \frac{1}{a} - \frac{e^{-at}}{a} \right\} = \frac{1}{a} \left[\frac{1}{s} - \frac{1}{s+a} \right] = \frac{1}{s(s+a)}$$

$$\text{P 12.19} \quad [\mathbf{a}] \quad \int_{0^-}^t x dx = \frac{t^2}{2}$$

$$\begin{aligned} \mathcal{L} \left\{ \frac{t^2}{2} \right\} &= \frac{1}{2} \int_{0^-}^{\infty} t^2 e^{-st} dt \\ &= \frac{1}{2} \left[\frac{e^{-st}}{-s^3} (s^2 t^2 + 2st + 2) \Big|_{0^-}^{\infty} \right] \\ &= \frac{1}{2s^3} (2) = \frac{1}{s^3} \end{aligned}$$

$$\therefore \quad \mathcal{L} \left\{ \int_{0^-}^t x dx \right\} = \frac{1}{s^3}$$

$$[\mathbf{b}] \quad \mathcal{L} \left\{ \int_{0^-}^t x dx \right\} = \frac{\mathcal{L}\{t\}}{s} = \frac{1/s^2}{s} = \frac{1}{s^3}$$

$$\therefore \quad \mathcal{L} \left\{ \int_{0^-}^t x dx \right\} = \frac{1}{s^3} \quad \text{CHECKS}$$

$$\text{P 12.20} \quad [\mathbf{a}] \quad \mathcal{L}\{t\} = \frac{1}{s^2}; \quad \text{therefore} \quad \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

$$[\mathbf{b}] \sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{j2}$$

Therefore

$$\begin{aligned}\mathcal{L}\{\sin \omega t\} &= \left(\frac{1}{j2}\right) \left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right) = \left(\frac{1}{j2}\right) \left(\frac{2j\omega}{s^2 + \omega^2} \right) \\ &= \frac{\omega}{s^2 + \omega^2}\end{aligned}$$

$$[\mathbf{c}] \sin(\omega t + \theta) = (\sin \omega t \cos \theta + \cos \omega t \sin \theta)$$

Therefore

$$\begin{aligned}\mathcal{L}\{\sin(\omega t + \theta)\} &= \cos \theta \mathcal{L}\{\sin \omega t\} + \sin \theta \mathcal{L}\{\cos \omega t\} \\ &= \frac{\omega \cos \theta + s \sin \theta}{s^2 + \omega^2}\end{aligned}$$

$$[\mathbf{d}] \mathcal{L}\{t\} = \int_0^\infty te^{-st} dt = \frac{e^{-st}}{s^2}(-st - 1) \Big|_0^\infty = 0 - \frac{1}{s^2}(0 - 1) = \frac{1}{s^2}$$

$$[\mathbf{e}] f(t) = \cosh t \cosh \theta + \sinh t \sinh \theta$$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2 - 1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2 - 1}$$

$$\begin{aligned}\therefore \mathcal{L}\{\cosh(t + \theta)\} &= \cosh \theta \left[\frac{s}{(s^2 - 1)} \right] + \sinh \theta \left[\frac{1}{s^2 - 1} \right] \\ &= \frac{\sinh \theta + s[\cosh \theta]}{(s^2 - 1)}\end{aligned}$$

$$\text{P 12.21 } [\mathbf{a}] \frac{dF(s)}{ds} = \frac{d}{ds} \left[\int_{0^-}^\infty f(t)e^{-st} dt \right] = - \int_{0^-}^\infty tf(t)e^{-st} dt$$

$$\text{Therefore } \mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds}$$

$$[\mathbf{b}] \frac{d^2F(s)}{ds^2} = \int_{0^-}^\infty t^2 f(t)e^{-st} dt; \quad \frac{d^3F(s)}{ds^3} = \int_{0^-}^\infty -t^3 f(t)e^{-st} dt$$

$$\text{Therefore } \frac{d^n F(s)}{ds^n} = (-1)^n \int_{0^-}^\infty t^n f(t)e^{-st} dt = (-1)^n \mathcal{L}\{t^n f(t)\}$$

$$[\mathbf{c}] \mathcal{L}\{t^5\} = \mathcal{L}\{t^4 t\} = (-1)^4 \frac{d^4}{ds^4} \left(\frac{1}{s^2} \right) = \frac{120}{s^6}$$

$$\mathcal{L}\{t \sin \beta t\} = (-1)^1 \frac{d}{ds} \left(\frac{\beta}{s^2 + \beta^2} \right) = \frac{2\beta s}{(s^2 + \beta^2)^2}$$

$$\mathcal{L}\{te^{-t} \cosh t\}:$$

From Assessment Problem 12.1(a),

$$F(s) = \mathcal{L}\{\cosh t\} = \frac{s}{s^2 - 1}$$

$$\frac{dF}{ds} = \frac{(s^2 - 1)1 - s(2s)}{(s^2 - 1)^2} = -\frac{s^2 + 1}{(s^2 - 1)^2}$$

$$\text{Therefore } -\frac{dF}{ds} = \frac{s^2 + 1}{(s^2 - 1)^2}$$

Thus

$$\mathcal{L}\{t \cosh t\} = \frac{s^2 + 1}{(s^2 - 1)^2}$$

$$\mathcal{L}\{e^{-t}t \cosh t\} = \frac{(s+1)^2 + 1}{[(s+1)^2 - 1]^2} = \frac{s^2 + 2s + 2}{s^2(s+2)^2}$$

P 12.22 [a] $\mathcal{L}\left\{\frac{d \sin \omega t}{dt} u(t)\right\} = \frac{s\omega}{s^2 + \omega^2} - \sin(0) = \frac{s\omega}{s^2 + \omega^2}$

[b] $\mathcal{L}\left\{\frac{d \cos \omega t}{dt} u(t)\right\} = \frac{s^2}{s^2 + \omega^2} - \cos(0) = \frac{s^2}{s^2 + \omega^2} - 1 = \frac{-\omega^2}{s^2 + \omega^2}$

[c] $\mathcal{L}\left\{\frac{d^3(t^2)}{dt^3} u(t)\right\} = s^3 \left(\frac{2}{s^3}\right) - s^2(0) - s(0) - 2(0) = 2$

[d] $\frac{d \sin \omega t}{dt} = (\cos \omega t) \cdot \omega, \quad \mathcal{L}\{\omega \cos \omega t\} = \frac{\omega s}{s^2 + \omega^2}$

$$\frac{d \cos \omega t}{dt} = -\omega \sin \omega t$$

$$\mathcal{L}\{-\omega \sin \omega t\} = -\frac{\omega^2}{s^2 + \omega^2}$$

$$\frac{d^3(t^2 u(t))}{dt^3} = 2\delta(t); \quad \mathcal{L}\{2\delta(t)\} = 2$$

P 12.23 [a] $\mathcal{L}\{f'(t)\} = \int_{-\varepsilon}^{\varepsilon} \frac{e^{-st}}{2\varepsilon} dt + \int_{\varepsilon}^{\infty} -ae^{-a(t-\varepsilon)} e^{-st} dt$

$$= \frac{1}{2s\varepsilon}(e^{s\varepsilon} - e^{-s\varepsilon}) - \left(\frac{a}{s+a}\right) e^{-s\varepsilon} = F(s)$$

$$\lim_{\varepsilon \rightarrow 0} F(s) = 1 - \frac{a}{s+a} = \frac{s}{s+a}$$

$$[\text{b}] \quad \mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$

$$\text{Therefore } \mathcal{L}\{f'(t)\} = sF(s) - f(0^-) = \frac{s}{s+a} - 0 = \frac{s}{s+a}$$

$$\text{P 12.24 [a]} \quad f_1(t) = e^{-at} \cos \omega t; \quad F_1(s) = \frac{s+a}{(s+a)^2 + \omega^2}$$

$$F(s) = sF_1(s) - f_1(0^-) = \frac{s(s+a)}{(s+a)^2 + \omega^2} - 1 = \frac{-a^2 - sa - \omega^2}{(s+a)^2 + \omega^2}$$

$$[\text{b}] \quad f_1(t) = e^{-at} \sin \omega t; \quad F_1(s) = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$F(s) = \frac{F_1(s)}{s} = \frac{\omega}{s[(s+a)^2 + \omega^2]}$$

$$[\text{c}] \quad \frac{d}{dt}[e^{-at} \cos \omega t] = -\omega e^{-at} \sin \omega t - ae^{-at} \cos \omega t$$

$$\text{Therefore } F(s) = \frac{-\omega^2 - a(s+a)}{(s+a)^2 + \omega^2} = \frac{-a^2 - sa - \omega^2}{(s+a)^2 + \omega^2}$$

$$\int_{0^-}^t e^{-ax} \sin \omega x \, dx = \frac{-ae^{-at} \sin \omega t - \omega e^{-at} \cos \omega t + \omega}{a^2 + \omega^2}$$

Therefore

$$\begin{aligned} F(s) &= \frac{1}{a^2 + \omega^2} \left[\frac{-a\omega}{(s+a)^2 + \omega^2} - \frac{\omega(s+a)}{(s+a)^2 + \omega^2} + \frac{\omega}{s} \right] \\ &= \frac{\omega}{s[(s+a)^2 + \omega^2]} \end{aligned}$$

$$\text{P 12.25 [a]} \quad \int_s^\infty F(u) \, du = \int_s^\infty \left[\int_{0^-}^\infty f(t) e^{-ut} \, dt \right] \, du = \int_{0^-}^\infty \left[\int_s^\infty f(t) e^{-ut} \, du \right] \, dt$$

$$= \int_{0^-}^\infty f(t) \int_s^\infty e^{-ut} \, du \, dt = \int_{0^-}^\infty f(t) \left[\frac{e^{-tu}}{-t} \Big|_s^\infty \right] \, dt$$

$$= \int_{0^-}^\infty f(t) \left[\frac{-e^{-st}}{-t} \right] \, dt = \mathcal{L} \left\{ \frac{f(t)}{t} \right\}$$

$$[\text{b}] \quad \mathcal{L}\{t \sin \beta t\} = \frac{2\beta s}{(s^2 + \beta^2)^2}$$

$$\text{therefore } \mathcal{L} \left\{ \frac{t \sin \beta t}{t} \right\} = \int_s^\infty \left[\frac{2\beta u}{(u^2 + \beta^2)^2} \right] \, du$$

Let $\omega = u^2 + \beta^2$, then $\omega = s^2 + \beta^2$ when $u = s$, and $\omega = \infty$ when $u = \infty$; also $d\omega = 2u \, du$, thus

$$\mathcal{L} \left\{ \frac{t \sin \beta t}{t} \right\} = \beta \int_{s^2 + \beta^2}^\infty \left[\frac{d\omega}{\omega^2} \right] = \beta \left(\frac{-1}{\omega} \right) \Big|_{s^2 + \beta^2}^\infty = \frac{\beta}{s^2 + \beta^2}$$

P 12.26 $I_g(s) = \frac{5s}{s^2 + 400}; \quad \frac{1}{RC} = 16; \quad \frac{1}{LC} = 100; \quad \frac{1}{C} = 20$

$$\frac{V(s)}{R} + \frac{1}{L} \frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_g(s)$$

$$V(s) \left[\frac{1}{R} + \frac{1}{Ls} + sC \right] = I_g(s)$$

$$\begin{aligned} V(s) &= \frac{I_g(s)}{\frac{1}{R} + \frac{1}{Ls} + sC} = \frac{LsI_g(s)}{\frac{sL}{R} + 1 + s^2LC} = \frac{\frac{1}{C}sI_g(s)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \\ &= \frac{(20)(5)s^2}{(s^2 + 16s + 100)(s^2 + 400)} = \frac{100s^2}{(s^2 + 16s + 100)(s^2 + 400)} \end{aligned}$$

P 12.27 [a] $I_{dc} = \frac{1}{L} \int_0^t v_o dx + \frac{v_o}{R} + C \frac{dv_o}{dt}$

[b] $\frac{I_{dc}}{s} = \frac{V_o(s)}{sL} + \frac{V_o(s)}{R} + sCV_o(s)$

$$\therefore V_o(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = C \frac{dv_o}{dt}$

$$\therefore I_o(s) = sCV_o(s) = \frac{sI_{dc}}{s^2 + (1/RC)s + (1/LC)}$$

P 12.28 [a] For $t \geq 0^+$:

$$\frac{v_o}{R} + C \frac{dv_o}{dt} + i_o = 0$$

$$v_o = L \frac{di_o}{dt}; \quad \frac{dv_o}{dt} = L \frac{d^2i_o}{dt^2}$$

$$\therefore \frac{L}{R} \frac{di_o}{dt} + LC \frac{d^2i_o}{dt^2} + i_o = 0$$

$$\text{or } \frac{d^2i_o}{dt^2} + \frac{1}{RC} \frac{di_o}{dt} + \frac{1}{LC} i_o = 0$$

[b] $s^2 I_o(s) - sI_{dc} - 0 + \frac{1}{RC}[sI_o(s) - I_{dc}] + \frac{1}{LC} I_o(s) = 0$

$$I_o(s) \left[s^2 + \frac{1}{RC}s + \frac{1}{LC} \right] = I_{dc}(s + 1/RC)$$

$$I_o(s) = \frac{I_{dc}[s + (1/RC)]}{[s^2 + (1/RC)s + (1/LC)]}$$

P 12.29 [a] For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \quad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

[b] $s^2V_o(s) - sV_{dc} - 0 + \frac{R}{L}[sV_o(s) - V_{dc}] + \frac{1}{LC}V_o(s) = 0$

$$V_o(s) \left[s^2 + \frac{R}{L}s + \frac{1}{LC} \right] = V_{dc}(s + R/L)$$

$$V_o(s) = \frac{V_{dc}[s + (R/L)]}{[s^2 + (R/L)s + (1/LC)]}$$

P 12.30 [a] $\frac{v_o - V_{dc}}{R} + \frac{1}{L} \int_0^t v_o dx + C\frac{dv_o}{dt} = 0$

$$\therefore v_o + \frac{R}{L} \int_0^t v_o dx + RC\frac{dv_o}{dt} = V_{dc}$$

[b] $V_o + \frac{R}{L}\frac{V_o}{s} + RCsV_o = \frac{V_{dc}}{s}$

$$\therefore sLV_o + RV_o + RCLs^2V_o = LV_{dc}$$

$$\therefore V_o(s) = \frac{(1/RC)V_{dc}}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = \frac{1}{L} \int_0^t v_o dx$

$$I_o(s) = \frac{V_o}{sL} = \frac{V_{dc}/RLC}{s[s^2 + (1/RC)s + (1/LC)]}$$

P 12.31 [a] $\frac{1}{L} \int_0^t v_1 d\tau + \frac{v_1 - v_2}{R} = i_g u(t)$

and

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0$$

$$[\mathbf{b}] \quad \frac{V_1}{sL} + \frac{V_1 - V_2}{R} = I_g$$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0$$

or

$$(R + sL)V_1(s) - sLV_2(s) = RLsI_g(s)$$

$$-V_1(s) + (RCs + 1)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

$$\begin{aligned} \text{P 12.32 [a]} \quad & 625 = 150i_1 + 62.5\frac{di_1}{dt} + 25\frac{d}{dt}(i_2 - i_1) + 12.5\frac{d}{dt}(i_1 - i_2) - 25\frac{di_1}{dt} \\ & 0 = 12.5\frac{d}{dt}(i_2 - i_1) + 25\frac{di_1}{dt} + 100i_2 \end{aligned}$$

Simplifying the above equations gives:

$$625 = 150i_1 + 25\frac{di_1}{dt} + 12.5\frac{di_2}{dt}$$

$$0 = 100i_2 + 12.5\frac{di_1}{dt} + 12.5\frac{di_2}{dt}$$

$$[\mathbf{b}] \quad \frac{625}{s} = (25s + 150)I_1(s) + 12.5sI_2(s)$$

$$0 = 12.5sI_1(s) + (12.5s + 100)I_2(s)$$

[c] Solving the equations in (b),

$$I_1(s) = \frac{50(s+8)}{s(s+4)(s+24)}$$

$$I_2(s) = \frac{-50}{(s+4)(s+24)}$$

P 12.33 From Problem 12.26:

$$V(s) = \frac{100s^2}{(s^2 + 16s + 100)(s^2 + 400)}$$

$$s^2 + 16s + 100 = (s + 8 + j6)(s + 8 - j6); \quad s^2 + 400 = (s - j20)(s + j20)$$

Therefore

$$\begin{aligned} V(s) &= \frac{100s^2}{(s+8+j6)(s+8-j6)(s-j20)(s+j20)} \\ &= \frac{K_1}{s+8-j6} + \frac{K_1^*}{s+8+j6} + \frac{K_2}{s-j20} + \frac{K_2^*}{s+j20} \end{aligned}$$

$$K_1 = \left. \frac{100s^2}{(s+8+j6)(s^2+400)} \right|_{s=-8+j6} = 1.9 / -151.1^\circ$$

$$K_2 = \left. \frac{100s^2}{(s+j20)(s^2+16s+100)} \right|_{s=j20} = 2.28 / -43.15^\circ$$

Therefore

$$v(t) = [3.8e^{-8t} \cos(6t - 151.1^\circ) + 4.56 \cos(20t - 43.15^\circ)]u(t) \text{ V}$$

$$\text{P 12.34 [a]} \quad \frac{1}{RC} = \frac{1}{(20)(20 \times 10^{-6})} = 2500$$

$$\frac{1}{LC} = \frac{1}{(0.05)(20 \times 10^{-6})} = 10^6; \quad \frac{1}{C} = 50,000$$

$$V_o(s) = \frac{50,000I_{\text{dc}}}{s + 2500s + 10^6}$$

$$= \frac{50,000I_{\text{dc}}}{(s+500)(s+2000)}$$

$$= \frac{3750}{(s+500)(s+2000)}$$

$$= \frac{K_1}{s+500} + \frac{K_2}{s+2000}$$

$$K_1 = \frac{3750}{1500} = 2.5; \quad K_2 = \frac{3750}{-1500} = -2.5$$

$$V_o(s) = \frac{2.5}{s+500} - \frac{2.5}{s+2000}$$

$$v_o(t) = [2.5e^{-500t} - 2.5e^{-2000t}]u(t) \text{ V}$$

$$\text{[b]} \quad I_o(s) = \frac{0.075s}{(s+500)(s+2000)}$$

$$= \frac{K_1}{s+500} + \frac{K_2}{s+2000}$$

$$K_1 = \frac{0.075(-500)}{1500} = -0.025$$

$$K_2 = \frac{0.075(-2000)}{-1500} = 0.1$$

$$I_o(s) = \frac{-0.025}{s + 500} + \frac{0.1}{s + 2000}$$

$$i_o(t) = (-25e^{-500t} + 100e^{-2000t})u(t) \text{ mA}$$

[c] $i_o(0) = 100 - 25 = 75 \text{ mA}$

Yes. The initial inductor current is zero by hypothesis, the initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at $t = 0$ the source current appears in the capacitor.

P 12.35 $\frac{1}{RC} = 200,000$; $\frac{1}{LC} = 10^{10}$

$$I_o(s) = \frac{0.04(s + 200,000)}{s^2 + 200,000s + 10^{10}}$$

$$s_{1,2} = -100,000$$

$$I_o(s) = \frac{0.04(s + 200,000)}{(s + 100,000)^2} = \frac{K_1}{(s + 100,000)^2} + \frac{K_2}{s + 100,000}$$

$$K_1 = 0.04(s + 200,000) \Big|_{s=-100,000} = 4000$$

$$K_2 = \frac{d}{ds} [0.04(s + 200,000)]_{s=-100,000} = 0.04$$

$$I_o(s) = \frac{4000}{(s + 100,000)^2} + \frac{0.04}{s + 100,000}$$

$$i_o(t) = [4000te^{-100,000t} + 0.04e^{-100,000t}]u(t) \text{ V}$$

P 12.36 $\frac{R}{L} = 10,000$; $\frac{1}{LC} = 16 \times 10^6$

$$V_o(s) = \frac{120(s + 10,000)}{s^2 + 10,000s + 16 \times 10^6}$$

$$= \frac{120(s + 10,000)}{(s + 2000)(s + 8000)} = \frac{K_1}{s + 2000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{120(8000)}{6000} = 160 \text{ V}; \quad K_2 = \frac{120(2000)}{-6000} = -40 \text{ V}$$

$$V_o(s) = \frac{160}{s + 2000} - \frac{40}{s + 8000}$$

$$v_o(t) = [160e^{-2000t} - 40e^{-8000t}]u(t) \text{ V}$$

$$\text{P 12.37 [a]} \quad \frac{1}{LC} = \frac{1}{(1.6)(5 \times 10^{-6})} = 125 \times 10^3$$

$$\frac{1}{RC} = \frac{1}{(2000)(5 \times 10^{-6})} = 100$$

$$V_o(s) = \frac{5600}{s^2 + 100s + 125 \times 10^3}$$

$$s_{1,2} = -50 \pm j350 \text{ rad/s}$$

$$\begin{aligned} V_o(s) &= \frac{5600}{(s + 50 - j350)(s + 50 + j350)} \\ &= \frac{K_1}{s + 50 - j350} + \frac{K_1^*}{s + 50 + j350} \end{aligned}$$

$$K_1 = \frac{5600}{j700} = 8 \angle -90^\circ$$

$$\begin{aligned} v_o(t) &= 16e^{-50t} \cos(350t - 90^\circ) u(t) \text{ V} \\ &= [16e^{-50t} \sin 350t] u(t) \text{ V} \end{aligned}$$

$$\begin{aligned} \text{[b]} \quad I_o(s) &= \frac{56(62.5)}{s(s + 50 - j350)(s + 50 + j350)} \\ &= \frac{K_1}{s} + \frac{K_2}{s + 50 - j350} + \frac{K_2^*}{s + 50 + j350} \end{aligned}$$

$$K_1 = \frac{3500}{125 \times 10^3} = 28 \text{ mA}$$

$$K_2 = \frac{3500}{(-50 + j350)(j700)} = 14.14 \angle 171.87^\circ \text{ mA}$$

$$i_o(t) = [28 + 28.28e^{-50t} \cos(350t + 171.87^\circ)] u(t) \text{ mA}$$

$$\text{P 12.38} \quad \frac{1}{C} = 64 \times 10^6; \quad \frac{1}{LC} = 1600 \times 10^6; \quad \frac{R}{L} = 100,000; \quad I_g = \frac{0.15}{s}$$

$$V_2(s) = \frac{96 \times 10^5}{s^2 + 10^5 s + 1600 \times 10^6}$$

$$s_1 = -20,000; \quad s_2 = -80,000$$

$$V_2(s) = \frac{96 \times 10^5}{(s + 20,000)(s + 80,000)}$$

$$= \frac{160}{s + 20,000} - \frac{160}{s + 80,000}$$

$$v_2(t) = [160e^{-20,000t} - 160e^{-80,000t}] u(t) \text{ V}$$

P 12.39 [a] $I_1(s) = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+24}$

$$K_1 = \frac{(50)(8)}{(4)(24)} = 4.167; \quad K_2 = \frac{(50)(4)}{(-4)(20)} = -2.5$$

$$K_3 = \frac{(50)(-16)}{(-24)(-20)} = -1.667$$

$$I_1(s) = \left(\frac{4.167}{s} - \frac{2.5}{s+4} - \frac{1.667}{s+24} \right)$$

$$i_1(t) = (4.167 - 2.5e^{-4t} - 1.667e^{-24t})u(t) \text{ A}$$

$$I_2(s) = \frac{K_1}{s+4} + \frac{K_2}{s+24}$$

$$K_1 = \frac{-50}{20} = -2.5; \quad K_2 = \frac{-50}{-20} = 2.5$$

$$I_2(s) = \left(\frac{-2.5}{s+4} + \frac{2.5}{s+24} \right)$$

$$i_2(t) = (2.5e^{-24t} - 2.5e^{-4t})u(t) \text{ A}$$

[b] $i_1(\infty) = 4.167 \text{ A}; \quad i_2(\infty) = 0 \text{ A}$

[c] Yes, at $t = \infty$

$$i_1 = \frac{625}{150} = 4.167 \text{ A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 12.5 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

P 12.40 [a] $F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+8}$

$$K_1 = \frac{6(s+10)}{(s+8)} \Big|_{s=-5} = 10$$

$$K_2 = \frac{6(s+10)}{(s+5)} \Big|_{s=-8} = -4$$

$$f(t) = [10e^{-5t} - 4e^{-8t}]u(t)$$

[b] $F(s) = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{s+7}$

$$K_1 = \frac{20s^2 + 141s + 315}{(s+3)(s+7)} \Big|_{s=0} = 15$$

$$K_2 = \frac{20s^2 + 141s + 315}{s(s+7)} \Big|_{s=-3} = -6$$

$$K_3 = \frac{20s^2 + 141s + 315}{s(s+3)} \Big|_{s=-7} = 11$$

$$f(t) = [15 - 6e^{-3t} + 11e^{-7t}]u(t)$$

[c] $F(s) = \frac{K_1}{s+2} + \frac{K_2}{s+4} + \frac{K_3}{s+6}$

$$K_1 = \frac{15s^2 + 112s + 228}{(s+4)(s+6)} \Big|_{s=-2} = 8$$

$$K_2 = \frac{15s^2 + 112s + 228}{(s+2)(s+6)} \Big|_{s=-4} = -5$$

$$K_3 = \frac{15s^2 + 112s + 228}{(s+2)(s+4)} \Big|_{s=-6} = 12$$

$$f(t) = [8e^{-2t} - 5e^{-4t} + 12e^{-6t}]u(t)$$

[d] $F(s) = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2} + \frac{K_3}{s+3}$

$$K_1 = \frac{2s^3 + 33s^2 + 93s + 54}{(s+1)(s+2)(s+3)} \Big|_{s=0} = 9$$

$$K_2 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+2)(s+3)} \Big|_{s=-1} = 4$$

$$K_3 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+1)(s+3)} \Big|_{s=-2} = -8$$

$$K_4 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+1)(s+2)} \Big|_{s=-3} = -3$$

$$f(t) = [9 + 4e^{-t} - 8e^{-2t} - 3e^{-3t}]u(t)$$

P 12.41 [a] $F(s) = \frac{K_1}{s+7-j14} + \frac{K_1^*}{s+7+j14}$

$$K_1 = \frac{280}{s+7+j14} \Big|_{s=-7+j14} = -j10 = 10/\underline{-90^\circ}$$

$$f(t) = [20e^{-7t} \cos(14t - 90^\circ)]u(t) = [20e^{-7t} \sin 14t]u(t)$$

$$[\mathbf{b}] \quad F(s) = \frac{K_1}{s} + \frac{K_2}{s+5-j8} + \frac{K_2^*}{s+5+j8}$$

$$K_1 = \left. \frac{-s^2 + 52s + 445}{s^2 + 10s + 89} \right|_{s=0} = 5$$

$$K_2 = \left. \frac{-s^2 + 52s + 445}{s(s+5+j8)} \right|_{s=-5+j8} = -3 - j2 = 3.6 \angle -146.31^\circ$$

$$f(t) = [5 + 7.2e^{-5t} \cos(8t - 146.31^\circ)]u(t)$$

$$[\mathbf{c}] \quad F(s) = \frac{K_1}{s+6} + \frac{K_2}{s+2-j4} + \frac{K_2^*}{s+2+j4}$$

$$K_1 = \left. \frac{14s^2 + 56s + 152}{s^2 + 4s + 20} \right|_{s=-6} = 10$$

$$K_2 = \left. \frac{14s^2 + 56s + 152}{(s+6)(s+2+j4)} \right|_{s=-2+j4} = 2 + j2 = 2.83 \angle 45^\circ$$

$$f(t) = [10e^{-6t} + 5.66e^{-2t} \cos(4t + 45^\circ)]u(t)$$

$$[\mathbf{d}] \quad F(s) = \frac{K_1}{s+5-j3} + \frac{K_1^*}{s+5+j3} + \frac{K_2}{s+4-j2} + \frac{K_2^*}{s+4+j2}$$

$$K_1 = \left. \frac{8(s+1)^2}{(s+5+j3)(s^2 + 8s + 20)} \right|_{s=-5+j3} = 4.62 \angle -40.04^\circ$$

$$K_2 = \left. \frac{8(s+1)^2}{(s+4+j2)(s^2 + 10s + 34)} \right|_{s=-4+j2} = 3.61 \angle 168.93^\circ$$

$$f(t) = [9.25e^{-5t} \cos(3t - 40.05^\circ) + 7.21e^{-4t} \cos(2t + 168.93^\circ)]u(t)$$

$$\text{P 12.42 } [\mathbf{a}] \quad F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+8}$$

$$K_1 = \left. \frac{320}{s+8} \right|_{s=0} = 40$$

$$K_2 = \left. \frac{d}{ds} \left[\frac{320}{s+8} \right] = \left[\frac{-320}{(s+8)^2} \right] \right|_{s=0} = -5$$

$$K_3 = \left. \frac{320}{s^2} \right|_{s=-8} = 5$$

$$f(t) = [40t - 5 + 5e^{-8t}]u(t)$$

$$[\mathbf{b}] \quad F(s) = \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$$

$$K_1 = \frac{80(s+3)}{(s+2)^2} \Big|_{s=0} = 60$$

$$K_2 = \frac{80(s+3)}{s} \Big|_{s=-2} = -40$$

$$K_3 = \frac{d}{ds} \left[\frac{80(s+3)}{s} \right] = \left[\frac{80}{s} - \frac{80(s+3)}{s^2} \right]_{s=-2} = -60$$

$$f(t) = [60 - 40te^{-2t} - 60e^{-2t}]u(t)$$

$$[\mathbf{c}] \quad F(s) = \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1} + \frac{K_3}{s+3-j4} + \frac{K_3^*}{s+3+j4}$$

$$K_1 = \frac{60(s+5)}{s^2 + 6s + 25} \Big|_{s=-1} = 12$$

$$K_2 = \frac{d}{ds} \left[\frac{60(s+5)}{s^2 + 6s + 25} \right] = \left[\frac{60}{s^2 + 6s + 25} - \frac{60(s+5)(2s+6)}{(s^2 + 6s + 25)^2} \right]_{s=-1} = 0.6$$

$$K_3 = \frac{60(s+5)}{(s+1)^2(s+3+j4)} \Big|_{s=-3+j4} = 1.68/\underline{100.305^\circ}$$

$$f(t) = [12te^{-t} + 0.6e^{-t} + 3.35e^{-3t} \cos(4t + 100.305^\circ)]u(t)$$

$$[\mathbf{d}] \quad F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{(s+5)^2} + \frac{K_4}{s+5}$$

$$K_1 = \frac{25(s+4)^2}{(s+5)^2} \Big|_{s=0} = 16$$

$$K_2 = \frac{d}{ds} \left[\frac{25(s+4)^2}{(s+5)^2} \right] = \left[\frac{25(2)(s+4)}{(s+5)^2} - \frac{25(2)(s+4)^2}{(s+5)^3} \right]_{s=0} = 1.6$$

$$K_3 = \frac{25(s+4)^2}{s^2} \Big|_{s=-5} = 1$$

$$K_4 = \frac{d}{ds} \left[\frac{25(s+4)^2}{s^2} \right] = \left[\frac{25(2)(s+4)}{s^2} - \frac{25(2)(s+4)^2}{s^3} \right]_{s=-5} = -1.6$$

$$f(t) = [16t + 1.6 + te^{-5t} - 1.6e^{-5t}]u(t)$$

$$\text{P 12.43 [a]} \quad F(s) = \frac{K_1}{s} + \frac{K_2}{(s+3)^3} + \frac{K_3}{(s+3)^2} + \frac{K_4}{s+3}$$

$$K_1 = \frac{135}{(s+3)^3} \Big|_{s=0} = 5$$

$$K_2 = \frac{135}{s} \Big|_{s=-3} = -45$$

$$K_3 = \frac{d}{ds} \left[\frac{135}{s} \right] = \left[\frac{-135}{s^2} \right]_{s=-3} = -15$$

$$K_4 = \frac{1}{2} \frac{d}{ds} \left[\frac{-135}{s^2} \right] = \left[\frac{1}{2} (-2) \left(\frac{-135}{s^3} \right) \right]_{s=-3} = -5$$

$$f(t) = [5 - 22.5t^2 e^{-3t} - 15te^{-3t} - 5e^{-3t}]u(t)$$

$$\text{[b]} \quad F(s) = \frac{K_1}{(s+1-j1)^2} + \frac{K_1^*}{(s+1+j1)^2} + \frac{K_2}{s+1-j1} + \frac{K_2^*}{s+1+j1}$$

$$K_1 = \frac{10(s+2)^2}{(s+1+j1)^2} \Big|_{s=-1+j1} = -j5 = 5/\underline{-90^\circ}$$

$$K_2 = \frac{d}{ds} \left[\frac{10(s+2)^2}{(s+1+j1)^2} \right] = \left[\frac{10(2)(s+2)}{(s+1+j1)^2} - \frac{10(2)(s+2)^2}{(s+1+j1)^3} \right]_{s=0} \\ = -j5 = 5/\underline{-90^\circ}$$

$$f(t) = [10te^{-t} \cos(t-90^\circ) + 10e^{-t} \cos(t-90^\circ)]u(t)$$

[c]

$$F(s) = \frac{s^2 + 15s + 54}{20s + 144} = \frac{25s^2 + 395s + 1494}{25s^2 + 375s + 1350}$$

$$F(s) = 25 + \frac{20s + 144}{s^2 + 15s + 54} = 25 + \frac{K_1}{s+6} + \frac{K_2}{s+9}$$

$$K_1 = \frac{20s + 144}{s+9} \Big|_{s=-6} = 8$$

$$K_2 = \frac{20s + 144}{s+6} \Big|_{s=-9} = 12$$

$$f(t) = 25\delta(t) + [8e^{-6t} + 12e^{-9t}]u(t)$$

[d]

$$F(s) = \frac{5s - 15}{s^2 + 7s + 10} = \frac{5s - 15}{\overline{\begin{array}{r} 5s^3 + 20s^2 - 49s - 108 \\ 5s^3 + 35s^2 + 50s \\ \hline -15s^2 - 99s - 108 \\ \hline -15s^2 - 105s - 150 \\ \hline 6s + 42 \end{array}}}$$

$$F(s) = 5s - 15 + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$K_1 = \left. \frac{6s + 42}{s + 5} \right|_{s=-2} = 10$$

$$K_2 = \left. \frac{6s + 42}{s + 2} \right|_{s=-5} = -4$$

$$f(t) = 5\delta'(t) - 15\delta(t) + [10e^{-2t} - 4e^{-5t}]u(t)$$

$$\begin{aligned} \text{P 12.44 } f(t) &= \mathcal{L}^{-1} \left\{ \frac{K}{s + \alpha - j\beta} + \frac{K^*}{s + \alpha + j\beta} \right\} \\ &= Ke^{-\alpha t} e^{j\beta t} + K^* e^{-\alpha t} e^{-j\beta t} \\ &= |K| e^{-\alpha t} [e^{j\theta} e^{j\beta t} + e^{-j\theta} e^{-j\beta t}] \\ &= |K| e^{-\alpha t} [e^{j(\beta t + \theta)} + e^{-j(\beta t + \theta)}] \\ &= 2|K| e^{-\alpha t} \cos(\beta t + \theta) \end{aligned}$$

$$\text{P 12.45 [a]} \quad \mathcal{L}\{t^n f(t)\} = (-1)^n \left[\frac{d^n F(s)}{ds^n} \right]$$

$$\text{Let } f(t) = 1, \text{ then } F(s) = \frac{1}{s}, \text{ thus } \frac{d^n F(s)}{ds^n} = \frac{(-1)^n n!}{s^{(n+1)}}$$

$$\text{Therefore } \mathcal{L}\{t^n\} = (-1)^n \left[\frac{(-1)^n n!}{s^{(n+1)}} \right] = \frac{n!}{s^{(n+1)}}$$

$$\text{It follows that } \mathcal{L}\{t^{(r-1)}\} = \frac{(r-1)!}{s^r}$$

$$\text{and } \mathcal{L}\{t^{(r-1)} e^{-at}\} = \frac{(r-1)!}{(s+a)^r}$$

$$\text{Therefore } \frac{K}{(r-1)!} \mathcal{L}\{t^{r-1} e^{-at}\} = \frac{K}{(s+a)^r} = \mathcal{L} \left\{ \frac{K t^{r-1} e^{-at}}{(r-1)!} \right\}$$

$$[b] f(t) = \mathcal{L}^{-1} \left\{ \frac{K}{(s + \alpha - j\beta)^r} + \frac{K^*}{(s + \alpha + j\beta)^r} \right\}$$

Therefore

$$\begin{aligned} f(t) &= \frac{Kt^{r-1}}{(r-1)!} e^{-(\alpha-j\beta)t} + \frac{K^*t^{r-1}}{(r-1)!} e^{-(\alpha+j\beta)t} \\ &= \frac{|K|t^{r-1}e^{-\alpha t}}{(r-1)!} [e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}] \\ &= \left[\frac{2|K|t^{r-1}e^{-\alpha t}}{(r-1)!} \right] \cos(\beta t + \theta) \end{aligned}$$

$$P 12.46 [a] \lim_{s \rightarrow \infty} sV(s) = \lim_{s \rightarrow \infty} \left[\frac{1.92s^3}{s^4[1 + (1.6/s) + (1/s^2)][1 + (1/s^2)]} \right] = 0$$

Therefore $v(0^+) = 0$

[b] No, V has a pair of poles on the imaginary axis.

$$P 12.47 sV_o(s) = \frac{(I_{dc}/C)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s \rightarrow 0} sV_o(s) = 0, \quad \therefore v_o(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sV_o(s) = 0, \quad \therefore v_o(0^+) = 0$$

$$sI_o(s) = \frac{s^2 I_{dc}}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s \rightarrow 0} sI_o(s) = 0, \quad \therefore i_o(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sI_o(s) = I_{dc}, \quad \therefore i_o(0^+) = I_{dc}$$

$$P 12.48 sI_o(s) = \frac{I_{dc}s[s + (1/RC)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s \rightarrow 0} sI_o(s) = 0, \quad \therefore i_o(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sI_o(s) = I_{dc}, \quad \therefore i_o(0^+) = I_{dc}$$

$$\text{P 12.49 } sV_o(s) = \frac{sV_{\text{dc}}/RC}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s \rightarrow 0} sV_o(s) = 0, \quad \therefore v_o(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sV_o(s) = 0, \quad \therefore v_o(0^+) = 0$$

$$sI_o(s) = \frac{V_{\text{dc}}/RC}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s \rightarrow 0} sI_o(s) = \frac{V_{\text{dc}}/RLC}{1/LC} = \frac{V_{\text{dc}}}{R}, \quad \therefore i_o(\infty) = \frac{V_{\text{dc}}}{R}$$

$$\lim_{s \rightarrow \infty} sI_o(s) = 0, \quad \therefore i_o(0^+) = 0$$

$$\text{P 12.50 [a]} \quad sF(s) = \frac{6s^2 + 60s}{(s+5)(s+8)}$$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 6, \quad \therefore f(0^+) = 6$$

$$\text{[b]} \quad sF(s) = \frac{20s^2 + 141s + 315}{(s^2 + 10s + 21)}$$

$$\lim_{s \rightarrow 0} sF(s) = 15; \quad \therefore f(\infty) = 15$$

$$\lim_{s \rightarrow \infty} sF(s) = 20, \quad \therefore f(0^+) = 20$$

$$\text{[c]} \quad sF(s) = \frac{15s^3 + 112s^2 + 228s}{(s+2)(s+4)(s+6)}$$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 15, \quad \therefore f(0^+) = 15$$

$$\text{[d]} \quad sF(s) = \frac{2s^3 + 33s^2 + 93s + 54}{(s+1)(s^2 + 5s + 6)}$$

$$\lim_{s \rightarrow 0} sF(s) = 9, \quad \therefore f(\infty) = 9$$

$$\lim_{s \rightarrow \infty} sF(s) = 2, \quad \therefore f(0^+) = 2$$

P 12.51 [a] $sF(s) = \frac{280s}{s^2 + 14s + 245}$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[b] $sF(s) = \frac{-s^2 + 52s + 445}{s^2 + 10s + 89}$

$$\lim_{s \rightarrow 0} sF(s) = 5, \quad \therefore f(\infty) = 5$$

$$\lim_{s \rightarrow \infty} sF(s) = -1, \quad \therefore f(0^+) = -1$$

[c] $sF(s) = \frac{14s^3 + 56s^2 + 152s}{(s+6)(s^2 + 4s + 20)}$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 14, \quad \therefore f(0^+) = 14$$

[d] $sF(s) = \frac{8s(s+1)^2}{(s^2 + 10s + 34)(s^2 + 8s + 20)}$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

P 12.52 [a] $sF(s) = \frac{320}{s(s+8)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem here.

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[b] $sF(s) = \frac{80(s+3)}{(s+2)^2}$

$$\lim_{s \rightarrow 0} sF(s) = 60, \quad \therefore f(\infty) = 60$$

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[c] $sF(s) = \frac{60s(s+5)}{(s+1)^2(s^2 + 6s + 25)}$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[d] $sF(s) = \frac{25(s+4)^2}{s(s+5)^2}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem here.

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

P 12.53 [a] $sF(s) = \frac{135}{(s+3)^3}$

$$\lim_{s \rightarrow 0} sF(s) = 5, \quad \therefore f(\infty) = 5$$

$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[b] $sF(s) = \frac{10s(s+2)^2}{(s^2 + 2s + 2)^2}$

$$\lim_{s \rightarrow 0} sF(s) = 0, \quad \therefore f(\infty) = 0$$

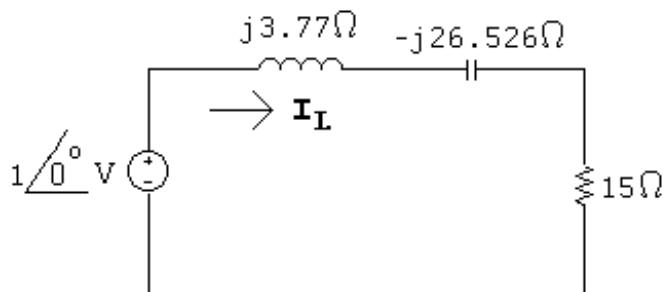
$$\lim_{s \rightarrow \infty} sF(s) = 0, \quad \therefore f(0^+) = 0$$

[c] This $F(s)$ function is an improper rational function, and thus the corresponding $f(t)$ function contains impulses ($\delta(t)$). Neither the initial value theorem nor the final value theorem may be applied to this $F(s)$ function!

[d] This $F(s)$ function is an improper rational function, and thus the corresponding $f(t)$ function contains impulses ($\delta(t)$). Neither the initial value theorem nor the final value theorem may be applied to this $F(s)$ function!

P 12.54 [a] $Z_L = j120\pi(0.01) = j3.77 \Omega; \quad Z_C = \frac{-j}{120\pi(100 \times 10^{-6})} = -j26.526 \Omega$

The phasor-transformed circuit is



$$I_L = \frac{1}{15 + j3.77 - j26.526} = 36.69 \angle 56.61^\circ \text{ mA}$$

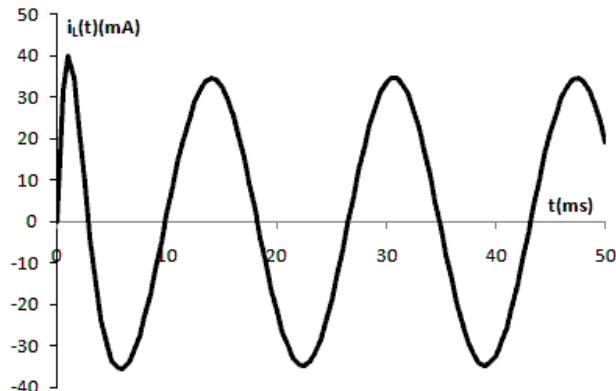
$$\therefore i_{L-ss}(t) = 36.69 \cos(120\pi t + 56.61^\circ) \text{ mA}$$

- [b] The steady-state response is the second term in Eq. 12.109, which matches the steady-state response just derived in part (a).

P 12.55 The transient and steady-state components are both proportional to the magnitude of the input voltage. Therefore,

$$K = \frac{40}{42.26} = 0.947$$

So if we make the amplitude of the sinusoidal source 0.947 instead of 1, the current will not exceed the 40 mA limit. A plot of the current through the inductor is shown below with the amplitude of the sinusoidal source set at 0.947.



P 12.56 We begin by using Eq. 12.105, and changing the right-hand side so it is the Laplace transform of Kte^{-100t} :

$$15I_L(s) + 0.01sI_L(s) + 10^4 \frac{I_L(s)}{s} = \frac{A}{(s + 100)^2}$$

Solving for $I_L(s)$,

$$I_L(s) = \frac{100Ks}{(s^2 + 1500s + 10^6)(s + 100)^2} = \frac{K_1}{s + 750 - j661.44} + \frac{K_1^*}{s + 750 + j661.44}$$

$$+ \frac{K_2}{(s + 100)^2} + \frac{K_3}{s + 100}$$

$$K_1 = \frac{100Ks}{(s + 750 + j661.44)(s + 100)^2} \Big|_{s=-750+j661.44} = 87.9K/139.59^\circ \mu\text{A}$$

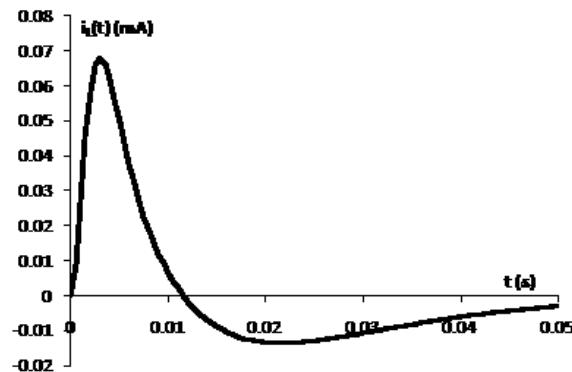
$$K_2 = \frac{100Ks}{(s^2 + 1500s + 10^6)} \Big|_{s=-100} = -11.63K \text{ mA}$$

$$K_3 = \frac{d}{ds} \left[\frac{100Ks}{(s^2 + 1500s + 10^6)} \right]_{s=-100} = 133.86K \mu\text{A}$$

Therefore,

$$i_L(t) = K[0.176e^{-750t} \cos(661.44t + 139.59^\circ) - 11.63te^{-100t} + 0.134e^{-100t}]u(t) \text{ mA}$$

Plot the expression above with $K = 1$:



The maximum value of the inductor current is $0.068K$ mA. Therefore,

$$K = \frac{40}{0.068} = 588$$

So the inductor current rating will not be exceeded if the input to the RLC circuit is $588te^{-100t}$ V.

13

The Laplace Transform in Circuit Analysis

Assessment Problems

$$\text{AP 13.1 [a]} \quad Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80,000; \quad \frac{1}{LC} = 25 \times 10^8$$

$$\text{Therefore } Y = \frac{25 \times 10^{-9}(s^2 + 80,000s + 25 \times 10^8)}{s}$$

$$[b] \quad z_{1,2} = -40,000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40,000 \pm j30,000 \text{ rad/s}$$

$$-z_1 = -40,000 - j30,000 \text{ rad/s}$$

$$-z_2 = -40,000 + j30,000 \text{ rad/s}$$

$$p_1 = 0 \text{ rad/s}$$

$$\text{AP 13.2 [a]} \quad Z = 2000 + \frac{1}{Y} = 2000 + \frac{4 \times 10^7 s}{s^2 + 80,000s + 25 \times 10^8}$$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80,000s + 25 \times 10^8} = \frac{2000(s + 50,000)^2}{s^2 + 80,000s + 25 \times 10^8}$$

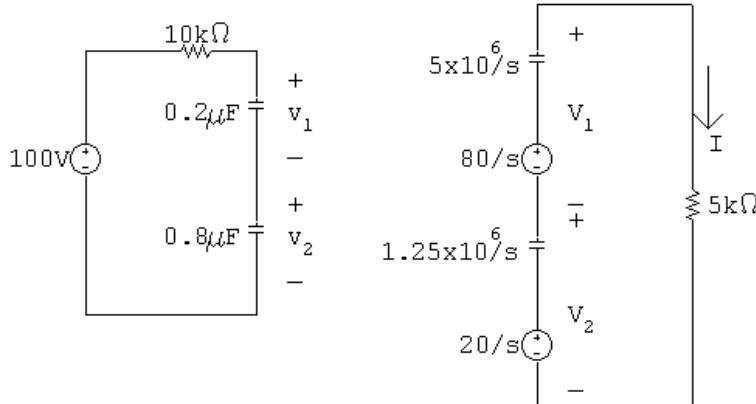
$$[b] \quad -z_1 = -z_2 = -50,000 \text{ rad/s}$$

$$-p_1 = -40,000 - j30,000 \text{ rad/s}$$

$$-p_2 = -40,000 + j30,000 \text{ rad/s}$$

AP 13.3 [a] At $t = 0^-$, $0.2v_1 = (0.8)v_2$; $v_1 = 4v_2$; $v_1 + v_2 = 100 \text{ V}$

Therefore $v_1(0^-) = 80V = v_1(0^+)$; $v_2(0^-) = 20V = v_2(0^+)$



$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

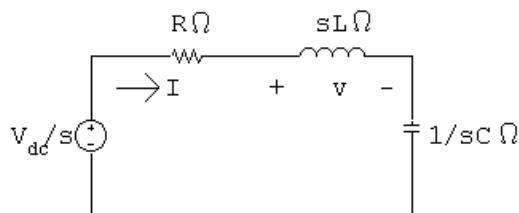
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s} \left(\frac{20 \times 10^{-3}}{s + 1250} \right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s} \left(\frac{20 \times 10^{-3}}{s + 1250} \right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t) \text{ mA}; \quad v_1 = 80e^{-1250t}u(t) \text{ V}$

$$v_2 = 20e^{-1250t}u(t) \text{ V}$$

AP 13.4 [a]



$$I = \frac{V_{dc}/s}{R + sL + (1/sC)} = \frac{V_{dc}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{dc}}{L} = 40; \quad \frac{R}{L} = 1.2; \quad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{40}{j1.6} = -j25 = 25/-90^\circ; \quad K_1^* = 25/90^\circ$$

[b] $i = 50e^{-0.6t} \cos(0.8t - 90^\circ) = [50e^{-0.6t} \sin 0.8t]u(t)$ A

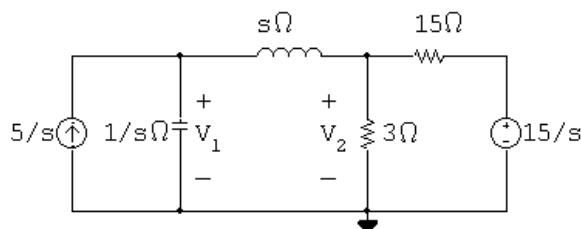
[c] $V = sLI = \frac{160s}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)}$

$$= \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{160(-0.6 + j0.8)}{j1.6} = 100/36.87^\circ$$

[d] $v(t) = [200e^{-0.6t} \cos(0.8t + 36.87^\circ)]u(t)$ V

AP 13.5 [a]



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2 + 2.5s + 1)}, \quad V_2 = \frac{2.5(s^2 + 6)}{s(s^2 + 2.5s + 1)}$$

[b] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s + 0.5} + \frac{5/3}{s + 2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s + 0.5} + \frac{25/3}{s + 2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t} \right] u(t) \text{ V} \quad \text{and}$$

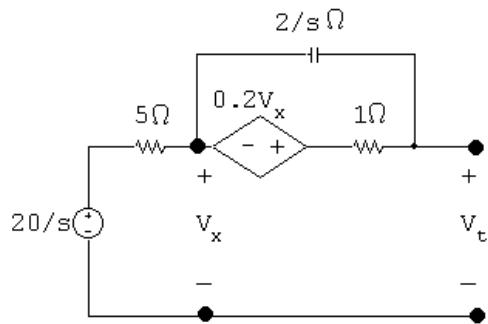
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t} \right] u(t) \text{ V}$$

[c] $v_1(0^+) = 15 - \frac{50}{3} + \frac{5}{3} = 0$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5 \text{ V}$$

[d] $v_1(\infty) = 15 \text{ V}; \quad v_2(\infty) = 15 \text{ V}$

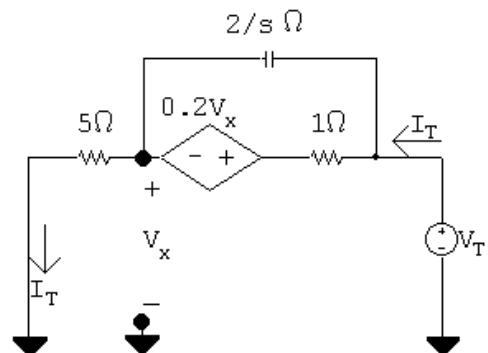
AP 13.6 [a]



With no load across terminals $a - b$ $V_x = 20/s$:

$$\frac{1}{2} \left[\frac{20}{s} - V_{Th} \right] s + \left[1.2 \left(\frac{20}{s} \right) - V_{Th} \right] = 0$$

therefore $V_{Th} = \frac{20(s + 2.4)}{s(s + 2)}$



$$V_x = 5I_T \quad \text{and} \quad Z_{Th} = \frac{V_T}{I_T}$$

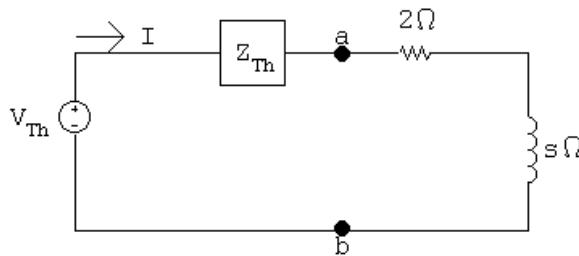
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$$14I_T = V_T s + 5sI_T + 2V_T; \quad \text{therefore} \quad Z_{Th} = \frac{5(s + 2.8)}{s + 2}$$

[b]



$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + 2 + s} = \frac{20(s + 2.4)}{s(s + 3)(s + 6)}$$

AP 13.7 [a] $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$$1.25e^{-t} = 3.75e^{-3t} \quad \text{or} \quad e^{2t} = 3, \quad t = 0.5(\ln 3) = 549.31 \text{ ms}$$

$$i_2(\max) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13 \text{ mA}$$

[b] From Eqs. 13.68 and 13.69, we have

$$\Delta = 12(s^2 + 4s + 3) = 12(s + 1)(s + 3) \quad \text{and} \quad N_1 = 60(s + 2)$$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s + 2)}{(s + 1)(s + 3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s + 1} + \frac{2.5}{s + 3}$$

Therefore we get

$$i_1 = 2.5[e^{-t} + e^{-3t}]u(t) \text{ A}$$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\frac{di_1(0.54931)}{dt} = -2.89 \text{ A/s}$

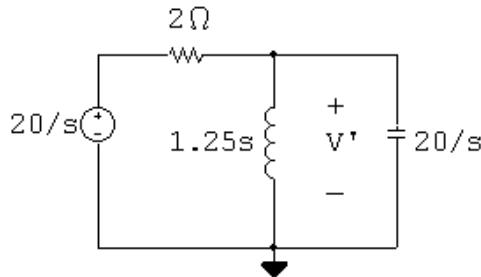
[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2 \left(\frac{di_2}{dt} \right) = 0 \quad \text{and} \quad i_2 = - \left(\frac{M}{12} \right) \left(\frac{di_1}{dt} \right)$

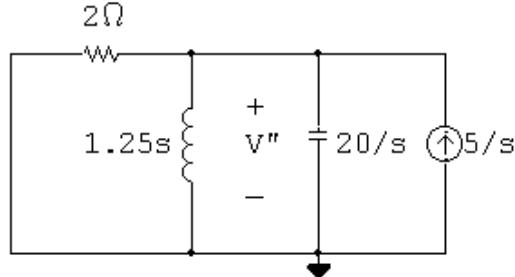
[e] $i_2(\max) = \frac{-2(-2.89)}{12} = 481.13 \text{ mA} \quad (\text{checks})$

AP 13.8 [a] The s -domain circuit with the voltage source acting alone is



$$\begin{aligned} \frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} &= 0 \\ V' &= \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8} \\ v' &= \frac{100}{3}[e^{-2t} - e^{-8t}]u(t) \text{ V} \end{aligned}$$

[b] With the current source acting alone,



$$\begin{aligned} \frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} &= \frac{5}{s} \\ V'' &= \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8} \\ v'' &= \frac{50}{3}[e^{-2t} - e^{-8t}]u(t) \text{ V} \end{aligned}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)$ V

AP 13.9 [a] $\frac{V_o}{s+2} + \frac{V_o s}{10} = I_g$; therefore $\frac{V_o}{I_g} = H(s) = \frac{10(s+2)}{s^2 + 2s + 10}$

[b] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 [a]

$$V_o = \frac{10(s+2)}{s^2 + 2s + 10} \cdot \frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$$K_o = 2; \quad K_1 = 5/3/-126.87^\circ; \quad K_1^* = 5/3/126.87^\circ$$

$$v_o = [2 + (10/3)e^{-t} \cos(3t - 126.87^\circ)]u(t) \text{ V}$$

$$[\mathbf{b}] \quad V_o = \frac{10(s+2)}{s^2 + 2s + 10} \cdot 1 = \frac{K_2}{s+1-j3} + \frac{K_2^*}{s+1+j3}$$

$$K_2 = 5.27/-18.43^\circ; \quad K_2^* = 5.27/18.43^\circ$$

$$v_o = [10.54e^{-t} \cos(3t - 18.43^\circ)]u(t) \text{ V}$$

AP 13.11 [a]

$$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$$

$$\begin{aligned} v_o(t) &= 10,000 \cos \theta e^{-70t} \cos 240t - 10,000 \sin \theta e^{-70t} \sin 240t \\ &= 9600e^{-70t} \cos 240t - 2800e^{-70t} \sin 240t \end{aligned}$$

$$\begin{aligned} \text{Therefore } H(s) &= \frac{9600(s+70)}{(s+70)^2 + (240)^2} - \frac{2800(240)}{(s+70)^2 + (240)^2} \\ &= \frac{9600s}{s^2 + 140s + 62,500} \end{aligned}$$

$$\begin{aligned} [\mathbf{b}] \quad V_o(s) &= H(s) \cdot \frac{1}{s} = \frac{9600}{s^2 + 140s + 62,500} \\ &= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240} \end{aligned}$$

$$K_1 = \frac{9600}{j480} = -j20 = 20/-90^\circ$$

Therefore

$$v_o(t) = [40e^{-70t} \cos(240t - 90^\circ)]u(t) \text{ V} = [40e^{-70t} \sin 240t]u(t) \text{ V}$$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2 + 2s + 10}$$

$$\text{Therefore } H(j4) = \frac{10(2+j4)}{10 - 16 + j8} = 4.47/-63.43^\circ$$

Thus,

$$v_o = (10)(4.47) \cos(4t - 63.43^\circ) = 44.7 \cos(4t - 63.43^\circ) \text{ V}$$

AP 13.13 [a]

$$\text{Let } R_1 = 10 \text{ k}\Omega, \quad R_2 = 50 \text{ k}\Omega, \quad C = 400 \text{ pF}, \quad R_2C = 2 \times 10^{-5}$$

$$\text{then } V_1 = V_2 = \frac{V_g R_2}{R_2 + (1/sC)}$$

$$\text{Also } \frac{V_1 - V_g}{R_1} + \frac{V_1 - V_o}{R_1} = 0$$

$$\text{therefore } V_o = 2V_1 - V_g$$

$$\text{Now solving for } V_o/V_g, \text{ we get } H(s) = \frac{R_2Cs - 1}{R_2Cs + 1}$$

$$\text{It follows that } H(j50,000) = \frac{j - 1}{j + 1} = j1 = 1/\underline{90^\circ}$$

$$\text{Therefore } v_o = 10 \cos(50,000t + 90^\circ) \text{ V}$$

$$[\text{b}] \text{ Replacing } R_2 \text{ by } R_x \text{ gives us } H(s) = \frac{R_xCs - 1}{R_xCs + 1}$$

Therefore

$$H(j50,000) = \frac{j20 \times 10^{-6}R_x - 1}{j20 \times 10^{-6}R_x + 1} = \frac{R_x + j50,000}{R_x - j50,000}$$

Thus,

$$\frac{50,000}{R_x} = \tan 60^\circ = 1.7321, \quad R_x = 28,867.51 \Omega$$

Problems

P 13.1 $i = \frac{1}{L} \int_{0^-}^t v d\tau + I_0;$ therefore $I = \left(\frac{1}{L}\right) \left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.2 $V_{\text{Th}} = V_{ab} = CV_0 \left(\frac{1}{sC}\right) = \frac{V_0}{s}; \quad Z_{\text{Th}} = \frac{1}{sC}$

P 13.3 $I_{sc_{ab}} = I_N = \frac{-LI_0}{sL} = \frac{-I_0}{s}; \quad Z_N = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.4 [a] $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{s^2 + 32,000s + 4 \times 10^8}{s}$$

[b] $s_{1,2} = -16,000 \pm j12,000 \text{ rad/s}$

Zeros at $-16,000 + j12,000 \text{ rad/s}$ and $-16,000 - j12,000 \text{ rad/s}$

Pole at 0.

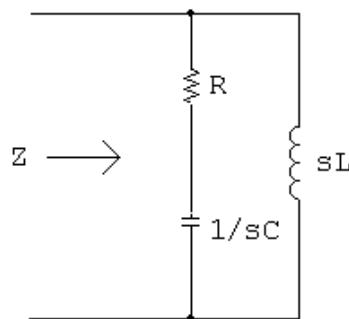
P 13.5 [a] $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{16 \times 10^9 s}{s^2 + 2 \times 10^6 s + 64 \times 10^{10}}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -400 \text{ krad/s}$ and $-p_2 = -1600 \text{ krad/s}$

P 13.6 [a]



$$Z = \frac{(R + 1/sC)(sL)}{R + sL + (1/sC)} = \frac{(Rs)(s + 1/RC)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = 500; \quad \frac{1}{RC} = 80; \quad \frac{1}{LC} = 40 \times 10^3$$

$$Z = \frac{200s(s + 80)}{s^2 + 500s + 40 \times 10^3}$$

$$[b] \quad Z = \frac{200s(s+80)}{(s+100)(s+400)}$$

$$z_1 = 0; \quad -z_2 = -80 \text{ rad/s}$$

$$-p_1 = -100 \text{ rad/s}; \quad -p_2 = -400 \text{ rad/s}$$

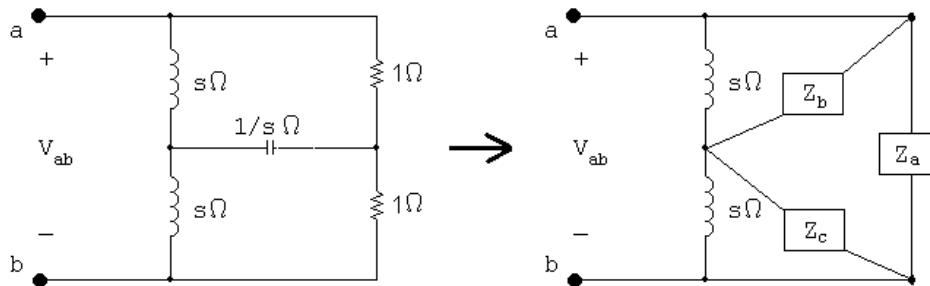
$$\text{P 13.7} \quad \left(1\|s + \frac{1}{s}\right)\|1 = \left(\frac{s}{s+1} + \frac{1}{s}\right)\|1 = \frac{s^2 + s + 1}{s(s+1)}\|1$$

$$= \frac{\frac{s^2 + s + 1}{s(s+1)}}{\frac{s^2 + s + 1}{s(s+1)} + 1} = \frac{s^2 + s + 1}{2s^2 + 2s + 1} = \frac{0.5(s^2 + s + 1)}{s^2 + s + 0.5}$$

$$-z_1 = -0.5 + j0.866 \text{ rad/s}; \quad -z_2 = -0.5 - j0.866 \text{ rad/s}$$

$$-p_1 = -0.5 + j0.5 \text{ rad/s}; \quad -p_2 = -0.5 - j0.5 \text{ rad/s}$$

- P 13.8 Transform the Y-connection of the two resistors and the capacitor into the equivalent delta-connection:



where

$$Z_a = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{(1/s)} = s + 2$$

$$Z_b = Z_c = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{1} = \frac{s+2}{s}$$

Then

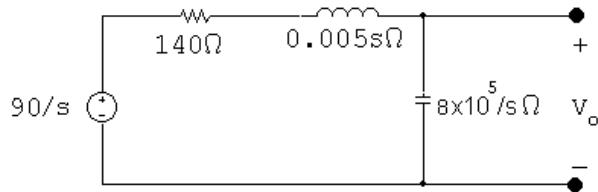
$$Z_{ab} = Z_a\|(s\|Z_c) + (s\|Z_b)] = Z_a\|2(s\|Z_b)$$

$$s\|Z_b = \frac{s[(s+2)/s]}{s + [(s+2)/s]} = \frac{s(s+2)}{s^2 + s + 2}$$

$$\begin{aligned}
 Z_{ab} &= (s+2) \left\| \frac{2s(s+2)}{s^2+s+2} \right. = \frac{\frac{2s(s+2)^2}{s^2+s+2}}{s+2 + \frac{2s(s+2)}{s^2+s+2}} \\
 &= \frac{2s(s+2)^2}{(s+2)(s^2+s+2) + 2s(s+2)} = \frac{2s(s+2)}{s^2+3s+2}
 \end{aligned}$$

Zeros at 0 and -2 rad/s; poles at -1 rad/s and -2 rad/s.

P 13.9



$$\begin{aligned}
 V_o &= \frac{(90/s)(8 \times 10^5/s)}{140 + 0.005s + (8 \times 10^5/s)} \\
 &= \frac{144 \times 10^8}{s(s^2 + 28,000s + 16 \times 10^7)} \\
 &= \frac{144 \times 10^8}{s(s + 8000)(s + 20,000)} \\
 &= \frac{K_1}{s} + \frac{K_2}{s + 8000} + \frac{K_3}{s + 20,000}
 \end{aligned}$$

$$K_1 = \frac{144 \times 10^8}{16 \times 10^7} = 90$$

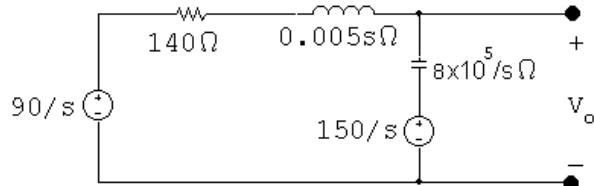
$$K_2 = \frac{144 \times 10^8}{(-8000)(12,000)} = -150$$

$$K_3 = \frac{144 \times 10^8}{(-12,000)(-20,000)} = 60$$

$$V_o = \frac{90}{s} - \frac{150}{s + 8000} + \frac{60}{s + 20,000}$$

$$v_o(t) = [90 - 150e^{-8000t} + 60e^{-20,000t}]u(t) \text{ V}$$

P 13.10 With a non-zero initial voltage on the capacitor, the s-domain circuit becomes:



$$\frac{V_o - 90/s}{0.005s + 140} + \frac{(V_o - 150/s)s}{8 \times 10^5} = 0$$

$$V_o \left[\frac{200}{s + 28,000} + \frac{s}{8 \times 10^5} \right] = \frac{150}{8 \times 10^5} + \frac{18,000}{s(s + 28,000)}$$

$$\begin{aligned} \therefore V_o &= \frac{150(s^2 + 28,000s + 96 \times 10^6)}{s(s + 8000)(s + 20,000)} \\ &= \frac{K_1}{s} + \frac{K_2}{s + 8000} + \frac{K_3}{s + 20,000} \end{aligned}$$

$$K_1 = \frac{144 \times 10^8}{160 \times 10^6} = 90$$

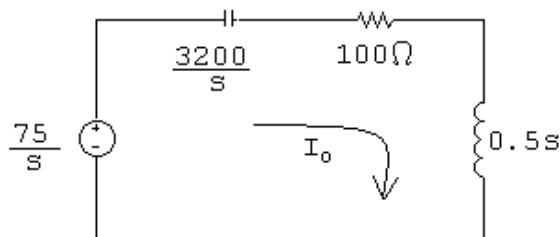
$$K_2 = \frac{150(s^2 + 28,000s + 96 \times 10^6)}{s(s + 20,000)} \Big|_{s=-8000} = 100$$

$$K_3 = \frac{150(s^2 + 28,000s + 96 \times 10^6)}{s(s + 8000)} \Big|_{s=-20,000} = -40$$

$$V_o = \frac{90}{s} + \frac{100}{s + 8000} - \frac{40}{s + 20,000}$$

$$v_o(t) = [90 + 100e^{-8000t} - 40e^{-20,000t}]u(t) \text{ V}$$

P 13.11 [a] For $t > 0$:



$$\begin{aligned}
 [\mathbf{b}] \quad I_o &= \frac{75/s}{(3200/s) + 100 + 0.5s} \\
 &= \frac{75}{0.5s^2 + 100s + 3200} \\
 &= \frac{150}{(s^2 + 200s + 6400)} = \frac{150}{(s + 40)(s + 160)}
 \end{aligned}$$

$$[\mathbf{c}] \quad I_o = \frac{K_1}{s + 40} + \frac{K_2}{s + 160}$$

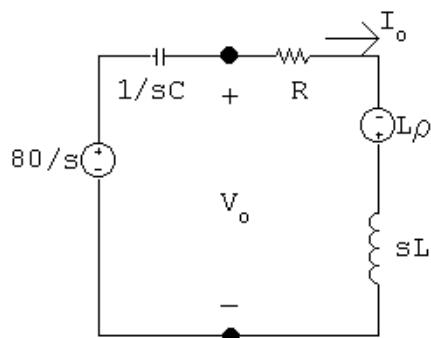
$$K_1 = \frac{150}{s + 160} \Big|_{s=-40} = 1.25$$

$$K_2 = \frac{150}{s + 40} \Big|_{s=-160} = -1.25$$

$$I_o = \frac{1.25}{s + 40} - \frac{1.25}{s + 160}$$

$$i_o(t) = (1.25e^{-40t} - 1.25e^{-160t})u(t) \text{ A}$$

$$\text{P 13.12 } [\mathbf{a}] \quad i_o(0^-) = \frac{20}{4000} = 5 \text{ mA}$$



$$\begin{aligned}
 I_o &= \frac{80/s + L\rho}{R + sL + 1/sC} = \frac{sC(80/s + L\rho)}{s^2LC + RsC + 1} \\
 &= \frac{80/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{20,000 + s(0.1)}{s^2 + 200,000s + 10^{10}} \\
 &= \frac{0.1(s + 200,000)}{s^2 + 200,000s + 10^{10}} = \frac{K_1}{(s + 100,000)^2} + \frac{K_2}{s + 100,000}
 \end{aligned}$$

$$K_1 = 10,000; \quad K_2 = 0.1$$

$$i_o(t) = [10,000te^{-100,000t} + 0.1e^{-100,000t}]u(t) \text{ A}$$

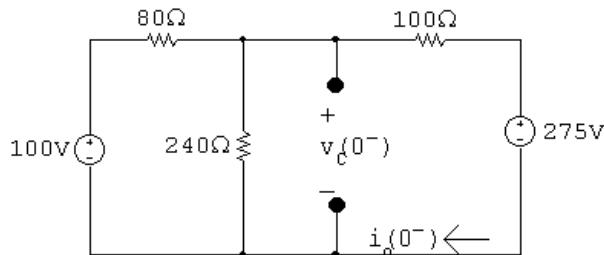
$$[b] V_o = (R + sL)I_o - L\rho = \frac{(800 + 0.004s)(0.1s + 20,000)}{s^2 + 200,000s + 10^{10}} - 4 \times 10^{-4}$$

$$= \frac{80(s + 150,000)}{(s + 100,000)^2} = \frac{K_1}{(s + 100,000)^2} + \frac{K_2}{s + 100,000}$$

$$K_1 = 4 \times 10^6 \quad K_2 = 80$$

$$v_o(t) = [4 \times 10^6 t e^{-100,000t} + 80e^{-100,000t}]u(t) \text{ A}$$

P 13.13 [a] For $t < 0$:



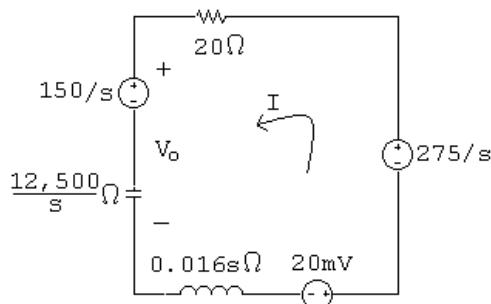
$$\frac{V_c - 100}{80} + \frac{V_c}{240} + \frac{V_c - 275}{100} = 0$$

$$V_c \left(\frac{1}{80} + \frac{1}{240} + \frac{1}{100} \right) = \frac{100}{80} + \frac{275}{100}$$

$$V_c = 150 \text{ V}$$

$$i_L(0^-) = \frac{150 - 275}{100} = -1.25 \text{ A}$$

For $t > 0$:



$$[b] V_o = \frac{12,500}{s} I + \frac{150}{s}$$

$$0 = -\frac{275}{s} + 20I + \frac{12,500}{s} I + \frac{150}{s} - 20 \times 10^{-3} + 0.016sI$$

$$I \left(20 + \frac{12,500}{s} + 0.016s \right) = \frac{125}{s} + 20 \times 10^{-3}$$

$$\therefore I = \frac{7812.5 + 1.25s}{s^2 + 1250s + 781,250}$$

$$V_o = \frac{12,500}{s} \left(\frac{7812.5 + 1.25s}{s^2 + 1250s + 781,250} \right) + \frac{150}{s}$$

$$= \frac{150s^2 + 203,125s + 214,843,750}{s(s^2 + 1250s + 781,250)}$$

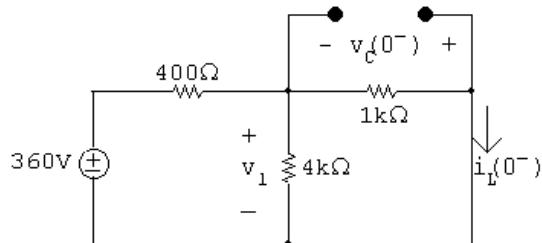
[c] $V_o = \frac{K_1}{s} + \frac{K_2}{s + 625 - j625} + \frac{K_2^*}{s + 625 + j625}$

$$K_1 = \frac{150s^2 + 203,125s + 214,843,750}{s^2 + 1250s + 781,250} \Big|_{s=0} = 275$$

$$K_2 = \frac{150s^2 + 203,125s + 214,843,750}{s(s + 625 + j625)} \Big|_{s=-625+j625} = 80.04/141.34^\circ$$

$$v_o(t) = [2755 + 160.08e^{-625t} \cos(625t + 141.34^\circ)]u(t) \text{ V}$$

P 13.14 [a] For $t < 0$:

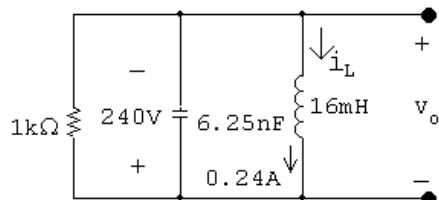


$$v_1 = \frac{4000\parallel 1000}{400 + 4000\parallel 1000}(360) = 240 \text{ V}$$

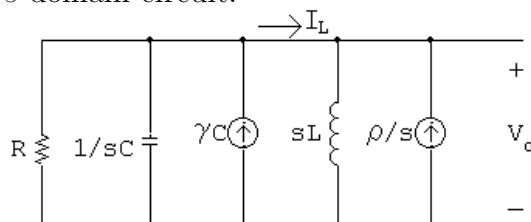
$$i_L(0^-) = \frac{240}{1000} = 0.24 \text{ A}$$

$$v_C(0^-) = -v_1 = -240 \text{ V}$$

For $t = 0^+$:



s -domain circuit:



where

$$R = 1 \text{ k}\Omega; \quad C = 6.25 \text{ nF}; \quad \gamma = -240 \text{ V};$$

$$L = 16 \text{ mH}; \quad \text{and} \quad \rho = -0.24 \text{ A}$$

$$[\mathbf{b}] \frac{V_o}{R} + V_o s C - \gamma C + \frac{V_o}{s L} - \frac{\rho}{s} = 0$$

$$\therefore V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-0.24}{(-240)(6.25 \times 10^{-9})} = 160,000$$

$$\frac{1}{RC} = \frac{1}{(1000)(6.25 \times 10^{-9})} = 160,000$$

$$\frac{1}{LC} = \frac{1}{(16 \times 10^{-3})(6.25 \times 10^{-9})} = 10^{10}$$

$$V_o = \frac{-240(s + 160,000)}{s^2 + 160,000s + 10^{10}}$$

$$[\mathbf{c}] I_L = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{0.016s} + \frac{0.24}{s}$$

$$= \frac{-15,000(s + 160,000)}{s(s^2 + 160,000s + 10^{10})} + \frac{0.24}{s} = \frac{0.24(s + 97,500)}{(s^2 + 160,000s + 10^{10})}$$

$$[\mathbf{d}] V_o = \frac{-240(s + 160,000)}{s^2 + 160,000s + 10^{10}}$$

$$= \frac{K_1}{s + 80,000 - j60,000} + \frac{K_1^*}{s + 80,000 + j60,000}$$

$$K_1 = \frac{-240(s + 160,000)}{s + 80,000 + j60,000} \Big|_{s=-80,000+j60,000} = 200/\underline{126.87^\circ}$$

$$v_o(t) = [400e^{-80,000t} \cos(60,000t + 126.87^\circ)]u(t) \text{ V}$$

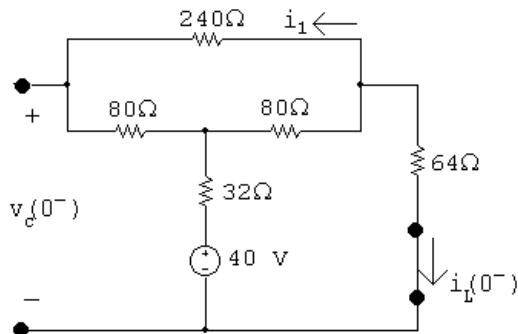
$$[\mathbf{e}] I_L = \frac{0.24(s + 97,500)}{s^2 + 160,000s + 10^{10}}$$

$$= \frac{K_1}{s + 80,000 - j60,000} + \frac{K_1^*}{s + 80,000 + j60,000}$$

$$K_1 = \frac{0.24(s + 97,500)}{s + 80,000 + j60,000} \Big|_{s=-80,000+j60,000} = 0.125/\underline{-16.26^\circ}$$

$$i_L(t) = [0.5e^{-80,000t} \cos(60,000t - 16.26^\circ)]u(t) \text{ A}$$

P 13.15 [a] For $t < 0$:

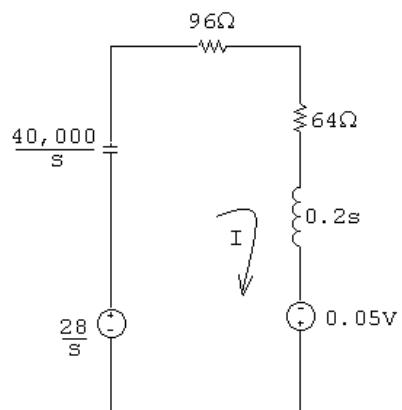


$$i_L(0^-) = \frac{40}{32 + 80\parallel 320 + 64} = \frac{40}{160} = 0.25 \text{ A}$$

$$i_1 = \frac{80}{400}(-0.25) = -0.05 \text{ A}$$

$$v_C(0^-) = 80(-0.05) + 32(-0.25) + 40 = 28 \text{ V}$$

For $t > 0$:



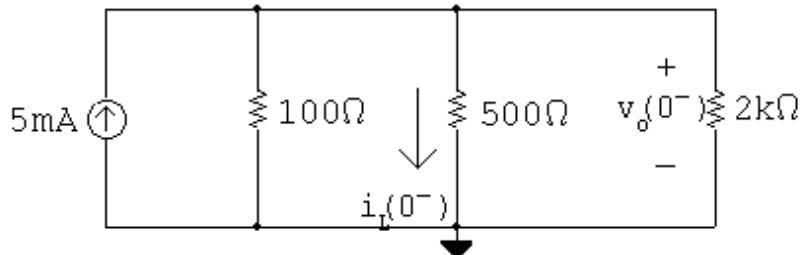
$$[b] (160 + 0.2s + 40,000/s)I = 0.05 + \frac{28}{s}$$

$$\therefore I = \frac{0.25(s + 560)}{s^2 + 800s + 200,000}$$

$$= \frac{K_1}{s + 400 - j200} + \frac{K_1^*}{s + 400 + j200}$$

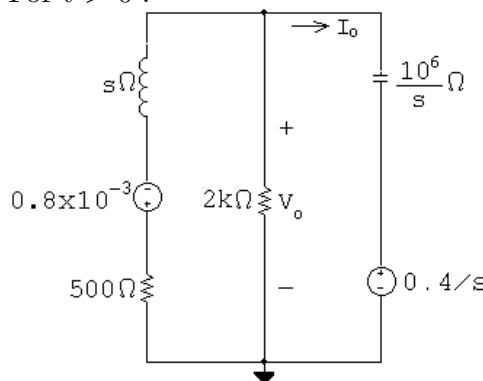
$$K_1 = \frac{0.25(s + 560)}{s + 400 + j200} \Big|_{s=-400+j200} = 0.16/-38.66^\circ$$

$$[c] i_o(t) = 0.32e^{-400t} \cos(200t - 38.66^\circ)u(t) \text{ A}$$

P 13.16 $v_o(0^-) < 0$.

$$v_o(0^-) = (100\parallel 500\parallel 2000)(0.005) = 0.4 \text{ V}$$

$$i_L(0^-) = \frac{v_o(0^-)}{500} = 0.8 \text{ mA}$$

For $t > 0$:

$$\frac{V_o + 0.8 \times 10^{-3}}{500 + s} + \frac{V_o}{2000} + \frac{V_o - (0.4/s)}{10^6/s} = 0$$

$$V_o \left(\frac{1}{500 + s} + \frac{1}{2000} + \frac{s}{10^6} \right) = 0.4 \times 10^{-6} - \frac{0.8 \times 10^{-3}}{500 + s}$$

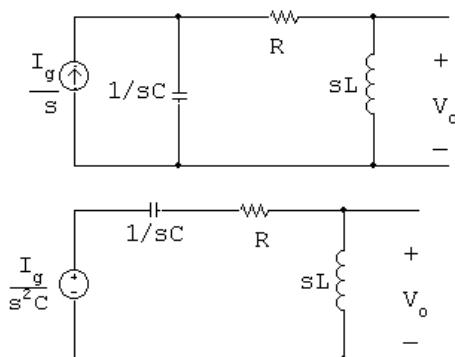
$$\therefore V_o = \frac{0.4(s - 1500)}{s^2 + 1000s + 125 \times 10^4}$$

$$= \frac{K_1}{s + 500 - j1000} + \frac{K_1^*}{s + 500 + j1000}$$

$$K_1 = \frac{0.4(s - 1500)}{s + 500 + j1000} \Big|_{s=-500+j1000} = 0.447/j63.43^\circ$$

$$v_o(t) = [0.894e^{-500t} \cos(1000t + 63.43^\circ)]u(t) \text{ V}$$

P 13.17 [a]



$$V_o = \frac{sL(I_g/sC)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{0.06}{0.001} = 60$$

$$\frac{R}{L} = 140; \quad \frac{1}{LC} = 4000$$

$$V_o = \frac{60}{s^2 + 140s + 4000}$$

$$[b] \quad sV_o = \frac{60s}{s^2 + 140s + 4000}$$

$$\lim_{s \rightarrow 0} sV_o = 0; \quad \therefore v_o(\infty) = 0$$

$$\lim_{s \rightarrow \infty} sV_o = 0; \quad \therefore v_o(0^+) = 0$$

$$[c] \quad V_o = \frac{60}{(s+40)(s+100)} = \frac{1}{s+40} + \frac{-1}{s+100}$$

$$v_o = [e^{-40t} - e^{-100t}]u(t) \text{ V}$$

$$\text{P 13.18} \quad I_C = \frac{I_g}{s} - \frac{V_o}{sL} = \frac{0.06}{s} - \frac{240}{s(s^2 + 140s + 4000)}$$

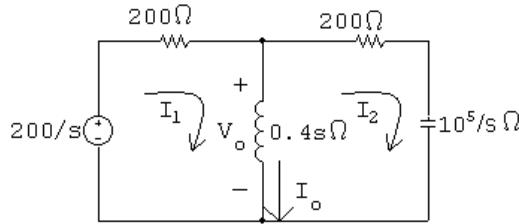
$$= \frac{0.06}{s} - \frac{0.06}{s} + \frac{0.1}{s+40} - \frac{0.04}{s+100}$$

$$\therefore i_C(t) = [100e^{-40t} - 40e^{-100t}]u(t) \text{ mA}$$

Check:

$$i_C(0^+) = 60 \text{ mA (ok)}; \quad i_C(\infty) = 0 \text{ (ok)}$$

P 13.19 [a]



$$200i_1 + 0.4s(I_1 - I_2) = \frac{200}{s}; \quad 200I_2 + \frac{10^5}{s}I_2 + 0.4s(I_2 - I_1) = 0$$

Solving the second equation for I_1 :

$$I_1 = \frac{s^2 + 500s + 25 \times 10^4}{s^2} I_2$$

Substituting into the first equation and solving for I_2 :

$$(0.4s + 200) \frac{s^2 + 500s + 25 \times 10^4}{s^2} - 0.4s = \frac{200}{s}$$

$$\therefore I_2 = \frac{0.5s}{s^2 + 500s + 125,000}$$

$$\begin{aligned} \therefore I_1 &= \frac{s^2 + 500s + 25 \times 10^4}{s^2} \cdot \frac{0.5s}{s^2 + 500s + 125,000} \\ &= \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125,000)} \end{aligned}$$

$$\begin{aligned} I_o &= I_1 - I_2 = \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125,000)} - \frac{0.5s}{s^2 + 500s + 125,000} \\ &= \frac{250(s + 500)}{s(s^2 + 500s + 125,000)} \end{aligned}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 250 - j250} + \frac{K_2^*}{s + 250 + j250}$$

$$K_1 = 1; \quad K_2 = 0.5/-180^\circ = -0.5$$

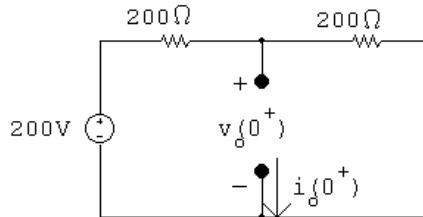
$$\therefore i_o(t) = [1 - 1e^{-250t} \cos 250t]u(t) \text{ A}$$

$$[b] \quad V_o = 0.4sI_o = \frac{100(s + 500)}{s^2 + 500s + 125,000} = \frac{K_1}{s + 250 - j250} + \frac{K_1^*}{s + 250 + j250}$$

$$K_1 = 70.71/-45^\circ$$

$$\therefore v_o(t) = 141.42e^{-250t} \cos(250t - 45^\circ)u(t) \text{ V}$$

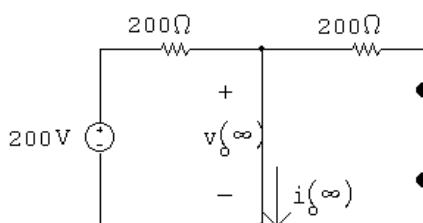
[c] At $t = 0^+$ the circuit is



$$\therefore v_o(0^+) = 100 \text{ V} = 141.42 \cos(-45^\circ); \quad i_o(0^+) = 0$$

Both values agree with our solutions for v_o and i_o .

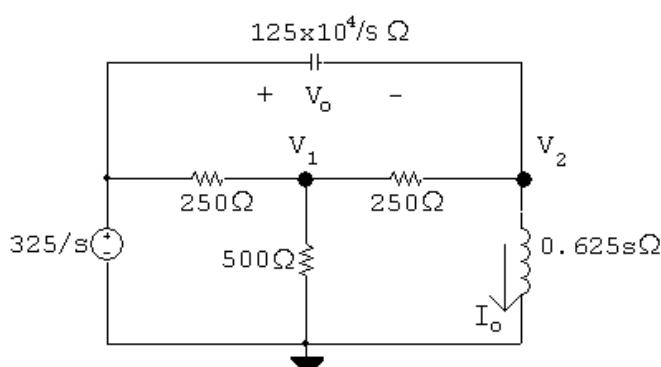
At $t = \infty$ the circuit is



$$\therefore v_o(\infty) = 0; \quad i_o(\infty) = 1 \text{ A}$$

Both values agree with our solutions for v_o and i_o .

P 13.20 [a]



$$\frac{V_1 - 325/s}{250} + \frac{V_1}{500} + \frac{V_1 - V_2}{250} = 0$$

$$\frac{V_2}{0.625s} + \frac{V_2 - V_1}{250} + \frac{(V_2 - 325/s)s}{125 \times 10^4} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{650}{s}$$

$$-5000sV_1 + (s^2 + 5000s + 2 \times 10^6)V_2 = 325s$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -5000s & s^2 + 5000s + 2 \times 10^6 \end{vmatrix} = 5(s+1000)(s+2000)$$

$$N_2 = \begin{vmatrix} 5 & 650/s \\ -5000s & 325s \end{vmatrix} = 1625(s+2000)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{1625(s+2000)}{5(s+1000)(s+2000)} = \frac{325}{s+1000}$$

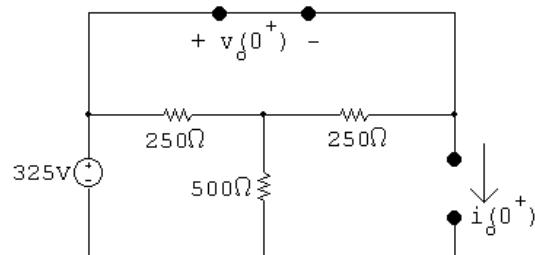
$$V_o = \frac{325}{s} - \frac{325}{s+1000} = \frac{325,000}{s(s+1000)}$$

$$I_o = \frac{V_2}{0.625s} = \frac{520}{s(s+1000)} = \frac{0.52}{s} - \frac{0.52}{s+1000}$$

[b] $v_o(t) = (325 - 325e^{-1000t})u(t)$ V

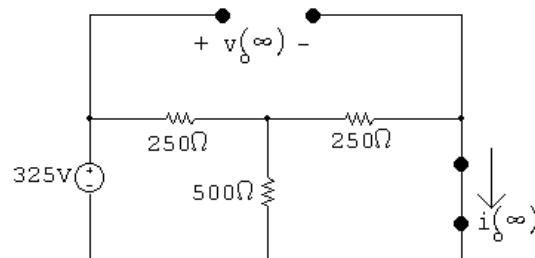
$$i_o(t) = (520 - 520e^{-1000t})u(t)$$
 mA

[c] At $t = 0^+$ the circuit is



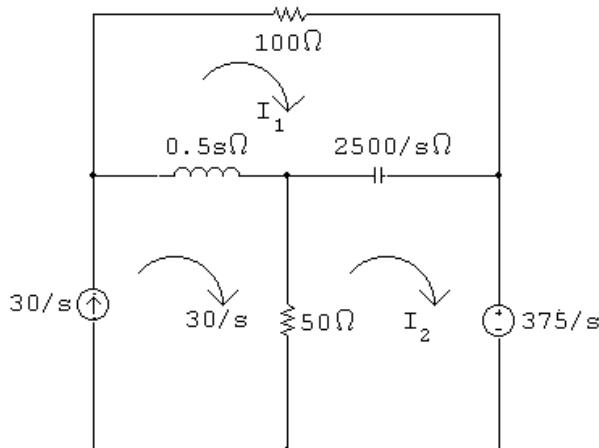
$$v_o(0^+) = 0; \quad i_o(0^+) = 0 \quad \text{Checks}$$

At $t = \infty$ the circuit is



$$v_o(\infty) = 325 \text{ V}; \quad i_o(\infty) = \frac{325}{250 + (500\parallel 250)} \cdot \frac{500}{750} = 0.52 \text{ A} \quad \text{Checks}$$

P 13.21 [a]



$$0 = 0.5s(I_1 - 30/s) + \frac{2500}{s}(I_1 - I_2) + 100I_1$$

$$\frac{-375}{s} = \frac{2500}{s}(I_2 - I_1) + 50(I_2 - 30/s)$$

or

$$(s^2 + 200s + 5000)I_1 - 5000I_2 = 30s$$

$$-50I_1 + (s + 50)I_2 = 22.5$$

$$\Delta = \begin{vmatrix} (s^2 + 200s + 5000) & -5000 \\ -50 & (s + 50) \end{vmatrix} = s(s + 100)(s + 150)$$

$$N_1 = \begin{vmatrix} 30s & -5000 \\ 22.5 & (s + 50) \end{vmatrix} = 30(s^2 + 50s + 3750)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{30(s^2 + 50s + 3750)}{s(s + 100)(s + 150)}$$

$$N_2 = \begin{vmatrix} (s^2 + 200s + 5000) & 30s \\ -50 & 22.5 \end{vmatrix} = 22.5s^2 + 6000s + 112,500$$

$$I_2 = \frac{N_2}{\Delta} = \frac{22.5s^2 + 6000s + 112,500}{s(s + 100)(s + 150)}$$

$$[b] \quad sI_1 = \frac{30(s^2 + 50s + 3750)}{(s + 100)(s + 150)}$$

$$\lim_{s \rightarrow \infty} sI_1 = i_1(0^+) = 30 \text{ A}; \quad \lim_{s \rightarrow 0} sI_1 = i_1(\infty) = 7.5 \text{ A}$$

$$sI_2 = \frac{22.5s^2 + 6000s + 112,500}{(s + 100)(s + 150)}$$

$$\lim_{s \rightarrow \infty} sI_2 = i_2(0^+) = 22.5 \text{ A}; \quad \lim_{s \rightarrow 0} sI_2 = i_2(\infty) = 7.5 \text{ A}$$

[c] $I_1 = \frac{30(s^2 + 50s + 3750)}{s(s + 100)(s + 150)} = \frac{K_1}{s} + \frac{K_2}{s + 100} + \frac{K_3}{s + 150}$

$$K_1 = 7.5; \quad K_2 = -52.5; \quad K_3 = 75$$

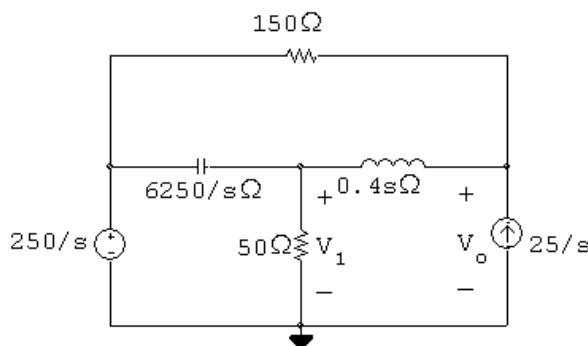
$$i_1(t) = [7.5 - 52.5e^{-100t} + 75e^{-150t}]u(t) \text{ A}$$

$$I_2 = \frac{22.5s^2 + 6000s + 112,500}{s(s + 100)(s + 150)} = \frac{K_1}{s} + \frac{K_2}{s + 100} + \frac{K_3}{s + 150}$$

$$K_1 = 7.5; \quad K_2 = 52.5; \quad K_3 = -37.5$$

$$i_2(t) = [7.5 + 52.5e^{-100t} - 37.5e^{-250t}]u(t) \text{ A}$$

P 13.22 [a]



$$\frac{V_1}{50} + \frac{V_1 - 250/s}{6250/s} + \frac{V_1 - V_o}{0.4s} = 0$$

$$\frac{-25}{s} + \frac{V_o - V_1}{0.4s} + \frac{V_o - 250/s}{150} = 0$$

Simplifying,

$$(s^2 + 125s + 15,625)V_1 - 15,625V_o = 250s$$

$$-375V_1 + (s + 375)V_o = 4000$$

$$\Delta = \begin{vmatrix} (s^2 + 125s + 15,625) & -15,625 \\ -375 & (s + 375) \end{vmatrix} = s(s + 250)^2$$

$$N_o = \begin{vmatrix} (s^2 + 125s + 15,625) & 250s \\ -375 & 4000 \end{vmatrix} = 4000s^2 + 593,750s + 625 \times 10^5$$

$$V_o = \frac{N_o}{\Delta} = \frac{4000s^2 + 593,750s + 625 \times 10^5}{s(s+250)^2} = \frac{K_1}{s} + \frac{K_2}{(s+250)^2} + \frac{K_3}{s+250}$$

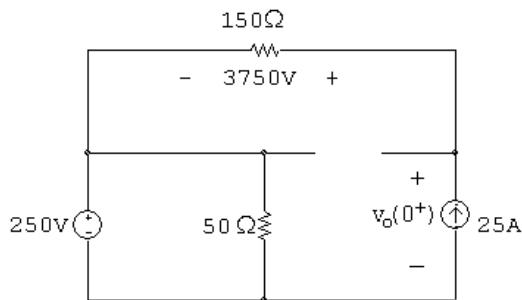
$$K_1 = 1000; \quad K_2 = -656,250$$

$$K_3 = 25 \frac{d}{ds} \left[\frac{4000s^2 + 593,750s + 625 \times 10^5}{s} \right]_{s=-250} = 3000$$

$$\therefore V_o = \frac{1000}{s} - \frac{656,250}{(s+250)^2} + \frac{3000}{s+250}$$

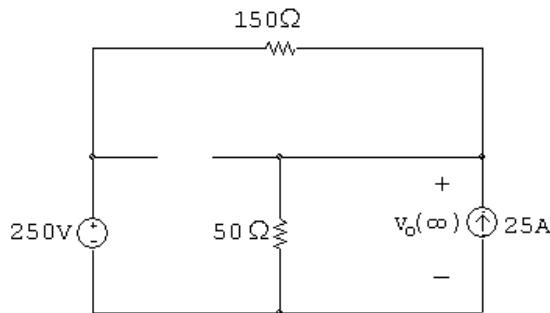
[b] $v_o(t) = [1 - 656.25t e^{-250t} + 3e^{-250t}]u(t)$ kV

[c] At $t = 0^+$:



$$v_o(0^+) = 250 + 3750 = 4 \text{ kV} \text{ (checks)}$$

At $t = \infty$:

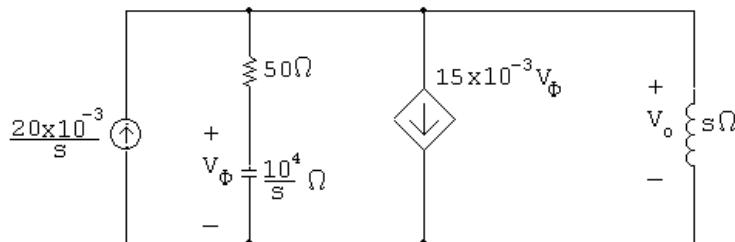


$$\frac{v_o(\infty)}{50} - 25 + \frac{v_o(\infty) - 250}{150} = 0$$

$$\therefore 3v_o(\infty) - 3750 + v_o(\infty) - 250 = 0; \quad \therefore 4v_o(\infty) = 4000$$

$$\therefore v_o(\infty) = 1 \text{ kV} \text{ (checks)}$$

P 13.23



$$\frac{20 \times 10^{-3}}{s} = \frac{V_o}{50 + 10^4/s} + 15 \times 10^{-3} V_\phi + \frac{V_o}{s}$$

$$V_\phi = \frac{10^4/s}{50 + 10^4/s} V_o = \frac{10^4 V_o}{50s + 10^4}$$

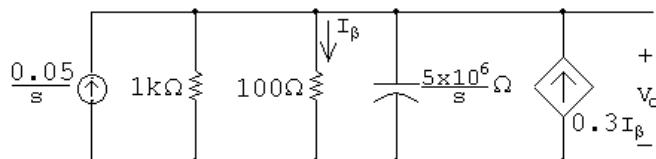
$$\therefore \frac{20 \times 10^{-3}}{s} = \frac{V_o s}{50s + 10^4} + \frac{150 V_o}{50s + 10^4} + \frac{V_o}{s}$$

$$\therefore V_o = \frac{s + 200}{s^2 + 200s + 10^4} = \frac{K_1}{(s + 100)^2} + \frac{K_2}{s + 100}$$

$$K_1 = 100; \quad K_2 = 1$$

$$V_o = \frac{100}{(s + 100)^2} + \frac{1}{s + 100}$$

$$v_o(t) = [100te^{-100t} + e^{-100t}]u(t) \text{ V}$$

P 13.24 $v_C(0^-) = v_C(0^+) = 0$ 

$$\frac{0.05}{s} = \frac{V_o}{1000} + \frac{V_o}{100} + \frac{V_o s}{5 \times 10^6} - \frac{0.3 V_o}{100}$$

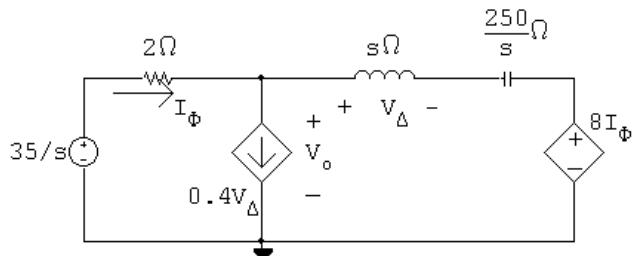
$$\frac{250 \times 10^3}{s} = (5000 + 50,000 + s - 15,000)V_o$$

$$V_o = \frac{250 \times 10^3}{s(s + 40,000)} = \frac{K_1}{s} + \frac{K_2}{s + 40,000}$$

$$= \frac{6.25}{s} - \frac{6.25}{s + 40,000}$$

$$v_o(t) = [6.25 - 6.25e^{-40,000t}]u(t) \text{ V}$$

P 13.25 [a]



$$\frac{V_o - 35/s}{2} + 0.4V_\Delta + \frac{V_o - 8I_\phi}{s + (250/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 8I_\phi}{s + (250/s)} \right] s; \quad I_\phi = \frac{(35/s) - V_o}{2}$$

Solving for V_o yields:

$$V_o = \frac{29.4s^2 + 56s + 1750}{s(s^2 + 2s + 50)} = \frac{29.4s^2 + 56s + 1750}{s(s+1-j7)(s+1+j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s+1-j7} + \frac{K_2^*}{s+1+j7}$$

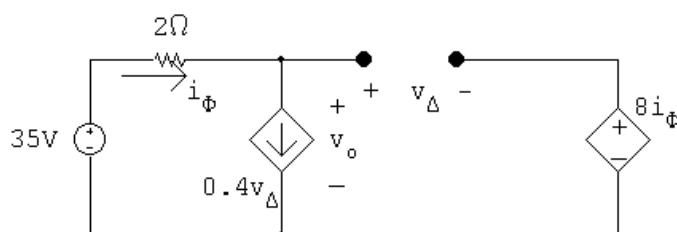
$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50} \Big|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s+1+j7)} \Big|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86/\underline{167.91^\circ}$$

$$\therefore v_o(t) = [35 + 5.73e^{-t} \cos(7t + 167.91^\circ)]u(t) \text{ V}$$

[b] At $t = 0^+$ $v_o = 35 + 5.73 \cos(167.91^\circ) = 29.4 \text{ V}$

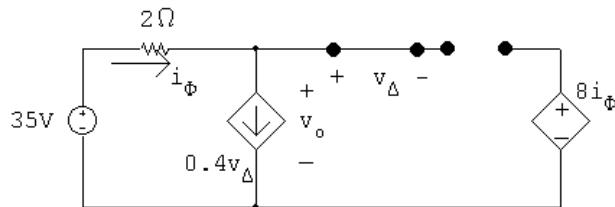


$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \quad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2 \text{ V}$$

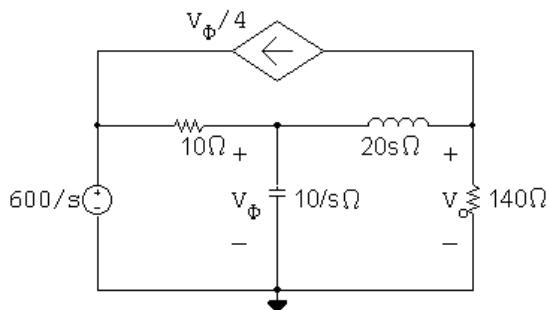
$$v_o + (0.8)\frac{v_o}{4.2} = 35; \quad \therefore v_o(0^+) = 29.4 \text{ V (checks)}$$

At $t = \infty$, the circuit is



$$v_D = 0, \quad i_\phi = 0 \quad \therefore v_o = 35 \text{ V} (\text{checks})$$

P 13.26 [a]



$$\frac{V_\phi}{10/s} + \frac{V_\phi - (600/s)}{10} + \frac{V_\phi - V_o}{20s} = 0$$

$$\frac{V_o}{140} + \frac{V_o - V_\phi}{20s} + \frac{V_\phi}{4} = 0$$

Simplifying,

$$(2s^2 + 2s + 1)V_\phi - V_o = 1200$$

$$(35s - 7)V_\phi + (s + 7)V_o = 0$$

$$\Delta = \begin{vmatrix} 2s^2 + 2s + 1 & -1 \\ 35s - 7 & s + 7 \end{vmatrix} = 2s(s^2 + 8s + 25)$$

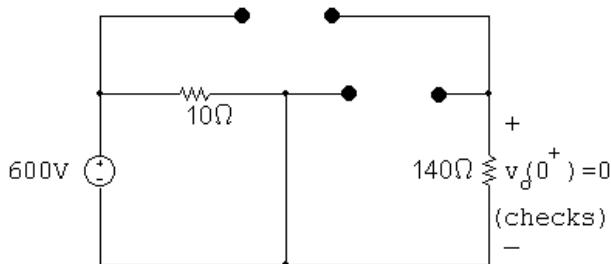
$$N_2 = \begin{vmatrix} 2s^2 + 2s + 1 & 1200 \\ 35s - 7 & 0 \end{vmatrix} = -42,000s + 8400$$

$$V_o = \frac{N_2}{\Delta} = \frac{-21,000s + 4200}{s(s^2 + 8s + 25)} = \frac{-4200(5s - 1)}{s(s^2 + 8s + 25)}$$

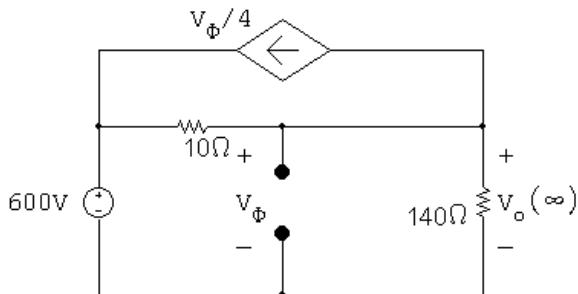
$$[b] \quad v_o(0^+) = \lim_{s \rightarrow \infty} sV_o = 0$$

$$v_o(\infty) = \lim_{s \rightarrow 0} sV_o = \frac{4200}{25} = 168$$

[c] At $t = 0^+$ the circuit is



At $t = \infty$ the circuit is



$$\frac{V_\phi - 600}{10} + \frac{V_\phi}{140} + \frac{V_\phi}{4} = 0$$

$$\therefore V_\phi = 168 \text{ V} = V_o(\infty) \quad (\text{checks})$$

$$[d] V_o = \frac{N_2}{\Delta} = \frac{-21,000s + 4200}{s(s^2 + 8s + 25)} = \frac{K_1}{s} + \frac{K_2}{s + 4 - j3} + \frac{K_2^*}{s + 4 + j3}$$

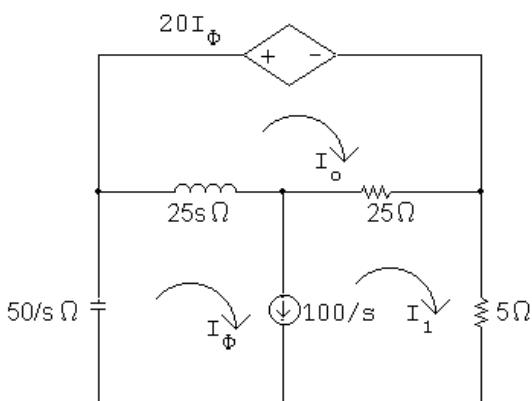
$$K_1 = \frac{4200}{25} = 168$$

$$K_2 = \frac{-21,000(-4 + j3) + 4200}{(-4 + j3)(j6)} = -84 + j3612 = 3612.98 \angle 91.33^\circ$$

$$v_o(t) = [168 + 7225.95e^{-4t} \cos(3t + 91.33^\circ)]u(t) \text{ V}$$

$$\text{Check: } v_o(0^+) = 0 \text{ V}; \quad v_o(\infty) = 168 \text{ V}$$

P 13.27 [a]



$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

$$25s(I_\phi - I_o) + \frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s}; \quad \therefore I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-5s - 1)I_\phi + (5s + 5)I_o = -500/s$$

$$(5s^2 + 6s + 10)I_\phi + (-5s^2 - 5s)I_o = 600$$

$$\Delta = \begin{vmatrix} -5s - 1 & 5s + 5 \\ 5s^2 + 6s + 10 & -5s^2 - 5s \end{vmatrix} = -25(s^2 + 3s + 2)$$

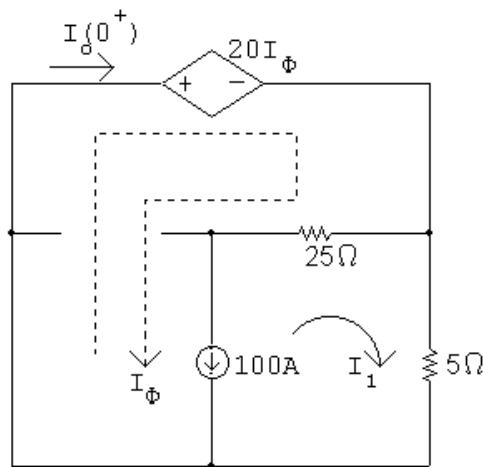
$$N_2 = \begin{vmatrix} -5s - 1 & -500/s \\ 5s^2 + 6s + 10 & 600 \end{vmatrix} = -\frac{500}{s}(s^2 - 4.8s - 10)$$

$$I_o = \frac{N_2}{\Delta} = \frac{20s^2 - 96s - 200}{s(s+1)(s+2)}$$

[b] $i_o(0^+) = \lim_{s \rightarrow \infty} sI_o = 20 \text{ A}$

$$i_o(\infty) = \lim_{s \rightarrow 0} sI_o = \frac{-200}{2} = -100 \text{ A}$$

[c] At $t = 0^+$ the circuit is

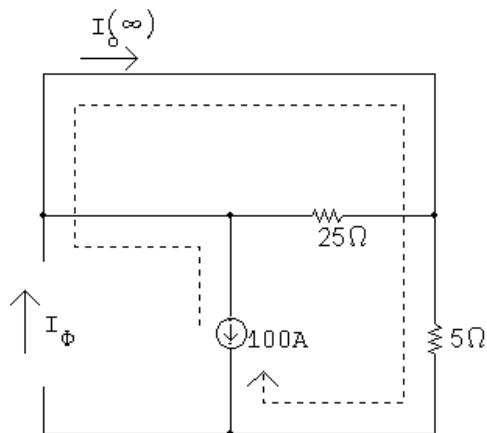


$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore 20I_\phi + 5(I_\phi - 100) = 0; \quad 25I_\phi = 500$$

$$\therefore I_\phi = I_o(0^+) = 20 \text{ A} \text{ (checks)}$$

At $t + \infty$ the circuit is



$$I_o(\infty) = -100 \text{ A} \text{ (checks)}$$

$$[\text{d}] \quad I_o = \frac{20s^2 - 96s - 200}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = \frac{-200}{(1)(2)} = -100; \quad K_2 = \frac{20 + 96 - 200}{(-1)(1)} = 84$$

$$K_3 = \frac{80 + 192 - 200}{(-2)(-1)} = 36$$

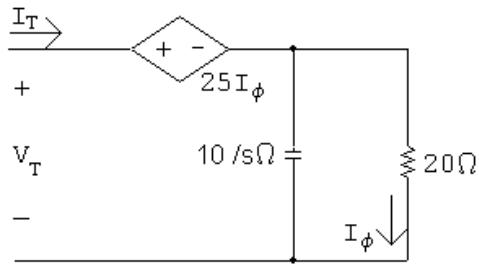
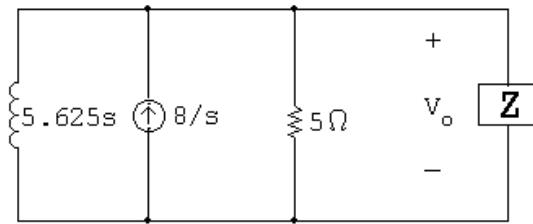
$$I_o = \frac{-100}{s} + \frac{84}{s+1} + \frac{36}{s+2}$$

$$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t) \text{ A}$$

$$i_o(\infty) = -100 \text{ A} \text{ (checks)}$$

$$i_o(0^+) = -100 + 84 + 36 = 20 \text{ A} \text{ (checks)}$$

P 13.28 [a] $i_L(0^-) = i_L(0^+) = \frac{24}{3} = 8 \text{ A}$ directed upward



$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20 + (10/s)} \right] I_T = \frac{25I_T(10/s)}{20 + (10/s)} + \left(\frac{200}{10 + 20s} \right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250 + 200}{20s + 10} = \frac{45}{2s + 1}$$

$$\frac{V_o}{5} + \frac{V_o(2s + 1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

$$\frac{[9s + (2s + 1)s + 8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2 + 10s + 8] = 360$$

$$V_o = \frac{360}{2s^2 + 10s + 8} = \frac{180}{s^2 + 5s + 4}$$

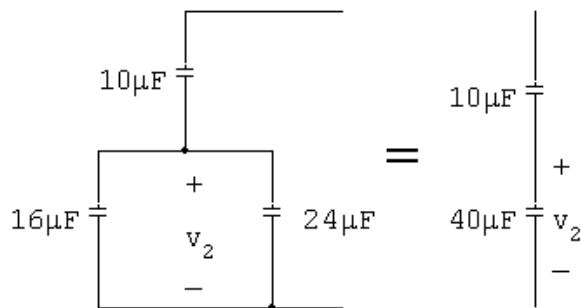
[b] $V_o = \frac{180}{(s + 1)(s + 4)} = \frac{K_1}{s + 1} + \frac{K_2}{s + 4}$

$$K_1 = \frac{180}{3} = 60; \quad K_2 = \frac{180}{-3} = -60$$

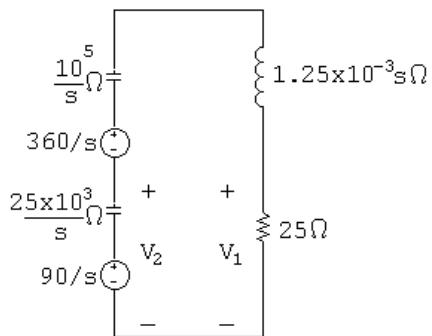
$$V_o = \frac{60}{s + 1} - \frac{60}{s + 4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t) \text{ V}$$

P 13.29 [a] For $t < 0$:



$$V_2 = \frac{10}{10 + 40} (450) = 90 \text{ V}$$



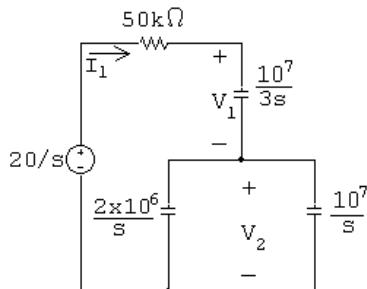
$$\begin{aligned} [\mathbf{b}] \quad V_1 &= \frac{25(450/s)}{(125,000/s) + 25 + 1.25 \times 10^{-3}s} \\ &= \frac{9 \times 10^6}{s^2 + 20,000s + 10^8} = \frac{9 \times 10^6}{(s + 10,000)^2} \end{aligned}$$

$$v_1(t) = (9 \times 10^6 t e^{-10,000t}) u(t) \text{ V}$$

$$\begin{aligned} [\mathbf{c}] \quad V_2 &= \frac{90}{s} - \frac{(25,000/s)(450/s)}{(125,000/s) + 1.25 \times 10^{-3}s + 25} \\ &= \frac{90(s + 20,000)}{s^2 + 20,000s + 10^8} \\ &= \frac{900,000}{(s + 10,000)^2} + \frac{90}{s + 10,000} \end{aligned}$$

$$v_2(t) = [9 \times 10^5 t e^{-10,000t} + 90 e^{-10,000t}] u(t) \text{ V}$$

P 13.30 [a]



$$[b] \quad Z_{\text{eq}} = 50,000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}/s^2}{12 \times 10^6/s}$$

$$= 50,000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}}{12 \times 10^6 s}$$

$$= \frac{100,000s + 10^7}{2s}$$

$$I_1 = \frac{20/s}{Z_{\text{eq}}} = \frac{0.4 \times 10^{-3}}{s + 100}$$

$$V_1 = \frac{10^7}{3s} I_1 = \frac{4000/3}{s(s + 100)}$$

$$V_2 = \frac{10^7}{6s} \cdot \frac{0.4 \times 10^{-4}}{s + 100} = \frac{2000/3}{s(s + 100)}$$

$$[c] \quad i_1(t) = 0.4e^{-100t}u(t) \text{ mA}$$

$$V_1 = \frac{40/3}{s} - \frac{40/3}{s + 100}; \quad v_1(t) = (40/3)(1 - 1e^{-100t})u(t) \text{ V}$$

$$V_2 = \frac{20/3}{s} - \frac{20/3}{s + 100}; \quad v_2(t) = (20/3)(1 - 1e^{-100t})u(t) \text{ V}$$

$$[d] \quad i_1(0^+) = 0.4 \text{ mA}$$

$$i_1(0^+) = \frac{20}{50} \times 10^{-3} = 0.44 \text{ mA (checks)}$$

$$v_1(0^+) = 0; \quad v_2(0^+) = 0 \text{ (checks)}$$

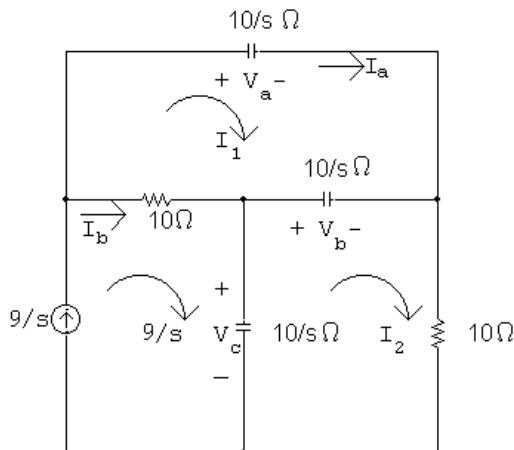
$$v_1(\infty) = 40/3 \text{ V}; \quad v_2(\infty) = 20/3 \text{ V (checks)}$$

$$v_1(\infty) + v_2(\infty) = 20 \text{ V (checks)}$$

$$(0.3 \times 10^{-6})v_1(\infty) = 4 \mu\text{C}$$

$$(0.6 \times 10^{-6})v_2(\infty) = 4 \mu\text{C (checks)}$$

P 13.31 [a]



$$\frac{10}{s}I_1 + \frac{10}{s}(I_1 - I_2) + 10(I_1 - 9/s) = 0$$

$$\frac{10}{s}(I_2 - 9/s) + \frac{10}{s}(I_2 - I_1) + 10I_2 = 0$$

Simplifying,

$$(s+2)I_1 - I_2 = 9$$

$$-I_1 + (s+2)I_2 = \frac{9}{s}$$

$$\Delta = \begin{vmatrix} (s+2) & -1 \\ -1 & (s+2) \end{vmatrix} = s^2 + 4s + 3 = (s+1)(s+3)$$

$$N_1 = \begin{vmatrix} 9 & -1 \\ 9/s & (s+2) \end{vmatrix} = \frac{9s^2 + 18s + 9}{s} = \frac{9}{s}(s+1)^2$$

$$I_1 = \frac{N_1}{\Delta} = \frac{9}{s} \left[\frac{(s+1)^2}{(s+1)(s+3)} \right] = \frac{9(s+1)}{s(s+3)}$$

$$N_2 = \begin{vmatrix} (s+2) & 9 \\ -1 & 9/s \end{vmatrix} = \frac{18}{s}(s+1)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{18(s+1)}{s(s+1)(s+3)} = \frac{18}{s(s+3)}$$

$$I_a = I_1 = \frac{9(s+1)}{s(s+3)} = \frac{3}{s} + \frac{6}{s+3}$$

$$I_b = \frac{9}{s} - I_1 = \frac{9}{s} - \frac{9(s+1)}{s(s+3)} = \frac{6}{s} - \frac{6}{s+3}$$

[b] $i_a(t) = 3(1 + 2e^{-3t})u(t)$ A

$$i_b(t) = 6(1 - e^{-3t})u(t)$$
 A

[c] $V_a = \frac{10}{s}I_b = \frac{10}{s}\left(\frac{3}{s} + \frac{6}{s+3}\right)$

$$= \frac{30}{s^2} + \frac{60}{s(s+3)} = \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

$$V_b = \frac{10}{s}(I_2 - I_1) = \frac{10}{s}\left[\left(\frac{6}{s} - \frac{6}{s+3}\right) - \left(\frac{3}{s} + \frac{6}{s+3}\right)\right]$$

$$= \frac{10}{s}\left[\frac{3}{s} - \frac{12}{s+3}\right] = \frac{30}{s^2} - \frac{40}{s} + \frac{40}{s+3}$$

$$V_c = \frac{10}{s}(9/s - I_2) = \frac{10}{s}\left(\frac{9}{s} - \frac{6}{s} + \frac{6}{s+3}\right)$$

$$= \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

[d] $v_a(t) = [30t + 20 - 20e^{-3t}]u(t)$ V

$$v_b(t) = [30t - 40 + 40e^{-3t}]u(t)$$
 V

$$v_c(t) = [30t + 20 - 20e^{-3t}]u(t)$$
 V

[e] Calculating the time when the capacitor voltage drop first reaches 1000 V:

$$30t + 20 - 20e^{-3t} = 1000 \quad \text{or} \quad 30t - 40 + 40e^{-3t} = 1000$$

Note that in either of these expressions the exponential term is negligible when compared to the other terms. Thus,

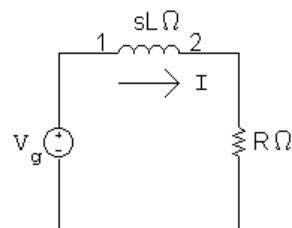
$$30t + 20 = 1000 \quad \text{or} \quad 30t - 40 = 1000$$

Thus,

$$t = \frac{980}{30} = 32.67 \text{ s} \quad \text{or} \quad t = \frac{1040}{30} = 34.67 \text{ s}$$

Therefore, the breakdown will occur at $t = 32.67$ s.

P 13.32 [a] The s -domain equivalent circuit is



$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \quad V_g = \frac{V_m(\omega \cos \phi + s \sin \phi)}{s^2 + \omega^2}$$

$$I = \frac{K_0}{s + R/L} + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_0 = \frac{V_m(\omega L \cos \phi - R \sin \phi)}{R^2 + \omega^2 L^2}, \quad K_1 = \frac{V_m/\phi - 90^\circ - \theta(\omega)}{2\sqrt{R^2 + \omega^2 L^2}}$$

where $\tan \theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L \cos \phi - R \sin \phi)}{R^2 + \omega^2 L^2} e^{-(R/L)t} + \frac{V_m \sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2 + \omega^2 L^2}}$$

[b] $i_{ss}(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin[\omega t + \phi - \theta(\omega)]$

[c] $i_{tr} = \frac{V_m(\omega L \cos \phi - R \sin \phi)}{R^2 + \omega^2 L^2} e^{-(R/L)t}$

[d] $\mathbf{I} = \frac{\mathbf{V}_g}{R + j\omega L}, \quad \mathbf{V}_g = V_m/\phi - 90^\circ$

$$\text{Therefore } \mathbf{I} = \frac{V_m/\phi - 90^\circ}{\sqrt{R^2 + \omega^2 L^2}/\theta(\omega)} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}/\phi - \theta(\omega) - 90^\circ$$

$$\text{Therefore } i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin[\omega t + \phi - \theta(\omega)]$$

[e] The transient component vanishes when

$$\omega L \cos \phi = R \sin \phi \quad \text{or} \quad \tan \phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.33 $v_C = 12 \times 10^5 t e^{-5000t} \text{ V}$, $C = 5 \mu\text{F}$; therefore

$$i_C = C \left(\frac{dv_C}{dt} \right) = 6e^{-5000t}(1 - 5000t) \text{ A}$$

$i_C > 0$ when $1 > 5000t$ or $i_C < 0$ when $0 < t < 200 \mu\text{s}$

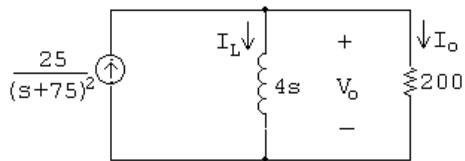
and $i_C < 0$ when $t > 200 \mu\text{s}$

$i_C = 0$ when $1 - 5000t = 0$, or $t = 200 \mu\text{s}$

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34 [a]



$$200\parallel 4s = \frac{800s}{4s + 200} = \frac{200s}{s + 50}$$

$$V_o = \frac{200s}{s + 50} \left[\frac{25}{(s + 75)^2} \right] = \frac{5000s}{(s + 50)(s + 75)^2}$$

$$I_o = \frac{V_o}{200} = \frac{25s}{(s + 50)(s + 75)^2}$$

$$I_L = \frac{V_o}{4s} = \frac{1250}{(s + 50)(s + 75)^2}$$

[b] $V_o = \frac{K_1}{s + 50} + \frac{K_2}{(s + 75)^2} + \frac{K_3}{s + 75}$

$$K_1 = \frac{5000s}{(s + 75)^2} \Big|_{s=-50} = -400$$

$$K_2 = \frac{5000s}{(s + 50)} \Big|_{s=-75} = 15,000$$

$$K_3 = \frac{d}{ds} \left[\frac{5000s}{s + 50} \right]_{s=-75} = \left[\frac{5000}{s + 50} - \frac{5000s}{(s + 50)^2} \right]_{s=-75} = 400$$

$$v_o(t) = [-400e^{-50t} + 15,000te^{-75t} + 400e^{-75t}]u(t) \text{ V}$$

$$I_o = \frac{K_1}{s + 50} + \frac{K_2}{(s + 75)^2} + \frac{K_3}{s + 75}$$

$$K_1 = \frac{25s}{(s + 75)^2} \Big|_{s=-50} = -2$$

$$K_2 = \frac{25s}{(s + 50)} \Big|_{s=-75} = 75$$

$$K_3 = \frac{d}{ds} \left[\frac{25s}{s + 50} \right]_{s=-75} = \left[\frac{25}{s + 50} - \frac{25s}{(s + 50)^2} \right]_{s=-75} = 2$$

$$i_o(t) = [-2e^{-50t} + 75te^{-75t} + 2e^{-75t}]u(t) \text{ V}$$

$$I_L = \frac{K_1}{s + 50} + \frac{K_2}{(s + 75)^2} + \frac{K_3}{s + 75}$$

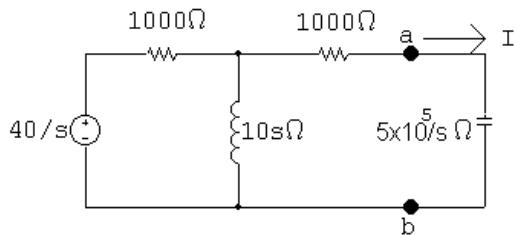
$$K_1 = \frac{1250}{(s+75)^2} \Big|_{s=-50} = 2$$

$$K_2 = \frac{1250}{(s+50)} \Big|_{s=-75} = -50$$

$$K_3 = \frac{d}{ds} \left[\frac{1250}{s+50} \right]_{s=-75} = \left[-\frac{1250}{(s+50)^2} \right]_{s=-75} = -2$$

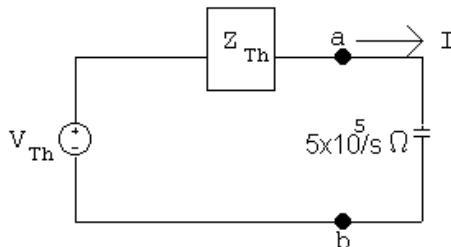
$$i_L(t) = [2e^{-50t} - 50te^{-75t} - 2e^{-75t}]u(t) \text{ V}$$

P 13.35



$$V_{Th} = \frac{10s}{10s + 1000} \cdot \frac{40}{s} = \frac{400}{10s + 1000} = \frac{40}{s + 100}$$

$$Z_{Th} = 1000 + 1000\parallel 10s = 1000 + \frac{10,000s}{10s + 1000} = \frac{2000(s + 50)}{s + 100}$$



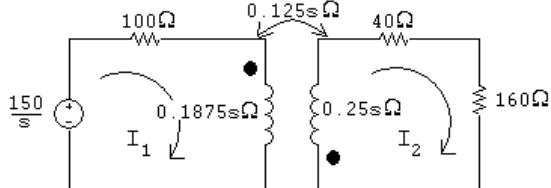
$$I = \frac{40/(s + 100)}{(5 \times 10^5)/s + 2000(s + 50)/(s + 100)} = \frac{40s}{2000s^2 + 600,000s + 5 \times 10^7}$$

$$= \frac{0.02s}{s^2 + 300s + 25,000} = \frac{K_1}{s + 150 - j50} + \frac{K_1^*}{s + 150 + j50}$$

$$K_1 = \frac{0.02s}{s + 150 + j50} \Big|_{s=-150+j50} = 31.62 \times 10^{-3} / 71.57^\circ$$

$$i(t) = 63.25e^{-150t} \cos(50t + 71.57^\circ)u(t) \text{ mA}$$

P 13.36 [a]



$$\frac{150}{s} = (100 + 0.1875s)I_1 + 0.125sI_2$$

$$0 = 0.125sI_1 + (200 + 0.25s)I_2$$

$$\Delta = \begin{vmatrix} 0.1875s + 100 & 0.125s \\ 0.125s & 0.25s + 200 \end{vmatrix} = 0.03125(s + 400)(s + 1600)$$

$$N_1 = \begin{vmatrix} 150/s & 0.125s \\ 0 & 0.25s + 200 \end{vmatrix} = \frac{37.5(s + 800)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1200(s + 800)}{s(s + 400)(s + 1600)}$$

$$[b] sI_1 = \frac{1200(s + 800)}{(s + 400)(s + 1600)}$$

$$\lim_{s \rightarrow 0} sI_1 = i_1(\infty) = 1.5 \text{ A}$$

$$\lim_{s \rightarrow \infty} sI_1 = i_1(0) = 0$$

$$[c] I_1 = \frac{K_1}{s} + \frac{K_2}{s + 400} + \frac{K_3}{s + 1600}$$

$$K_1 = 1.5; \quad K_2 = -1; \quad K_3 = -0.5$$

$$i_1(t) = (1.5 - e^{-400t} - 0.5e^{-1600t})u(t) \text{ A}$$

P 13.37 [a] From the solution to Problem 13.36 we have

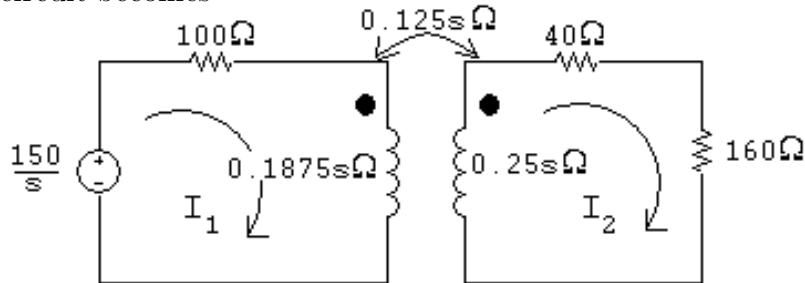
$$N_2 = \begin{vmatrix} 0.1875s + 100 & 150/s \\ 0.125s & 0 \end{vmatrix} = -18.75$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-600}{(s + 400)(s + 1600)} = \frac{K_1}{s + 400} + \frac{K_2}{s + 1600}$$

$$K_1 = \frac{-600}{1200} = -0.5; \quad K_2 = \frac{-600}{-1200} = 0.5$$

$$i_2(t) = (-0.5e^{-40t} + 0.5e^{-1600t})u(t) \text{ A}$$

- [b] Reversing the dot on the 250 mH coil will reverse the sign of M , thus the circuit becomes



The two simultaneous equations are

$$\frac{150}{s} = (100 + 0.1875s)I_1 - 0.125sI_2$$

$$0 = -0.125sI_1 + (0.25s + 200)I_2$$

When these equations are compared to those derived in Problem 13.36 we see the only difference is the algebraic sign of the $0.125s$ term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2 . Hence,

$$i_2(t) = (0.5e^{-400t} - 0.5e^{-1600t})u(t) \text{ A}$$

P 13.38 [a] $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2$

$$W = 0.16(300)^2 + 0.36(200)^2 + 0.24(200)(300) = 43.2 \text{ kJ}$$

[b] $240i_1 + 0.32\frac{di_1}{dt} - 0.24\frac{di_2}{dt} = 0$

$$540i_2 + 0.72\frac{di_2}{dt} - 0.24\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$$240I_1 + 0.32(sI_1 - 300) - 0.24(sI_2 + 200) = 0$$

$$540I_2 + 0.72(sI_2 + 200) - 0.24(sI_1 - 300) = 0$$

In standard form,

$$(0.32s + 240)I_1 - 0.24sI_2 = 144$$

$$-0.24sI_1 + (0.72s + 540)I_2 = -216$$

$$\Delta = \begin{vmatrix} 0.32s + 240 & -0.24s \\ -0.24s & 0.72s + 540 \end{vmatrix} = 0.1728(s + 500)(s + 1500)$$

$$N_1 = \begin{vmatrix} 144 & -0.24s \\ -216 & 0.72s + 540 \end{vmatrix} = 51.84(s + 1500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 144 \\ -0.24s & -216 \end{vmatrix} = -103.68(s + 1500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{51.84(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{300}{s + 500}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-103.68(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{-200}{s + 500}$$

[c] $i_1(t) = 300e^{-500t}u(t)$ A; $i_2(t) = -200e^{-500t}u(t)$ A

[d] $W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-1000t})(240) dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000} \Big|_0^\infty = 21,600$ J

$$W_{540\Omega} = \int_0^\infty (4 \times 10^4 e^{-1000t})(540) dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000} \Big|_0^\infty = 21,600$$
 J

$$W_{240\Omega} + W_{540\Omega} = 43.2$$
 kJ

[e] $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2 = 14,400 + 14,400 - 14,400 = 14.4$ kJ

With the dot reversed the s -domain equations are

$$(0.32s + 240)I_1 + 0.24sI_2 = 48$$

$$0.24sI_1 + (0.72s + 540)I_2 = -72$$

As before, $\Delta = 0.1728(s + 500)(s + 1500)$. Now,

$$N_1 = \begin{vmatrix} 48 & -0.24s \\ -72 & 0.72s + 540 \end{vmatrix} = 51.84(s + 500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 48 \\ 0.24s & -72 \end{vmatrix} = -34.56(s + 500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{300}{s + 1500}; \quad I_2 = \frac{N_2}{\Delta} = \frac{-200}{s + 1500}$$

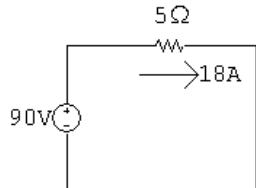
$i_1(t) = 300e^{-1500t}u(t)$ A; $i_2(t) = -200e^{-1500t}u(t)$ A

$$W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-3000t})(240) dt = 7200$$
 J

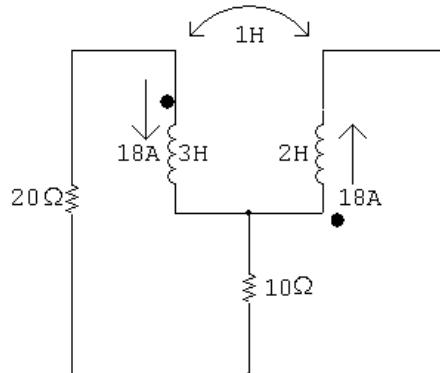
$$W_{540\Omega} = \int_0^\infty (4 \times 10^4 e^{-3000t})(540) dt = 7200 \text{ J}$$

$$W_{120\Omega} + W_{270\Omega} = 14.4 \text{ kJ}$$

P 13.39 For $t < 0$:



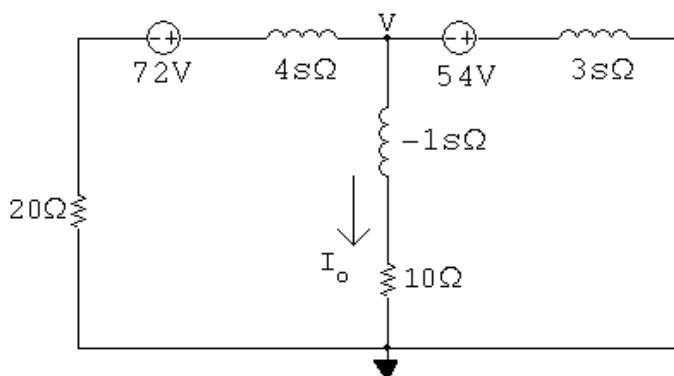
For $t > 0^+$:



Note that because of the dot locations on the coils, the sign of the mutual inductance is negative! (See Example C.1 in Appendix C.)

$$L_1 - M = 3 + 1 = 4 \text{ H}; \quad L_2 - M = 2 + 1 = 3 \text{ H}$$

$$18 \times 4 = 72; \quad 18 \times 3 = 54$$



$$\frac{V - 72}{4s + 20} + \frac{V}{-s + 10} + \frac{V + 54}{3s} = 0$$

$$V \left(\frac{1}{4s + 20} + \frac{1}{-s + 10} + \frac{1}{3s} \right) = \frac{72}{4s + 20} - \frac{54}{3s}$$

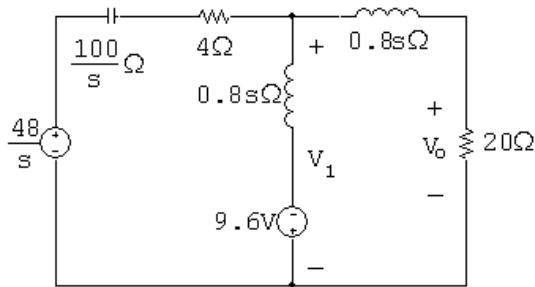
$$V \left[\frac{3s(-s+10) + 3s(4s+20) + (4s+20)(-s+10)}{3s(-s+10)(4s+20)} \right] = \frac{72(3s) - 54(4s+20)}{3s(4s+20)}$$

$$V = \frac{[72(3s) - 54(4s+20)](-s+10)}{5s^2 + 110s + 200}$$

$$I_o = \frac{V}{-s+10} = \frac{-108}{(s+2)(s+20)} = \frac{-1.2}{s+2} + \frac{1.2}{s+20}$$

$$i_o(t) = 1.2[e^{-20t} - e^{-2t}]u(t) \text{ A}$$

P 13.40 The s -domain equivalent circuit is



$$\frac{V_1 - 48/s}{4 + (100/s)} + \frac{V_1 + 9.6}{0.8s} + \frac{V_1}{0.8s + 20} = 0$$

$$V_1 = \frac{-1200}{s^2 + 10s + 125}$$

$$V_o = \frac{20}{0.8s + 20} V_1 = \frac{-30,000}{(s+25)(s+5-j10)(s+5+j10)}$$

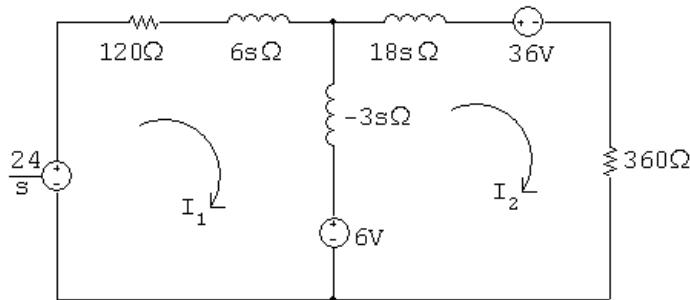
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \frac{-30,000}{s^2 + 10s + 125} \Big|_{s=-25} = -60$$

$$K_2 = \frac{-30,000}{(s+25)(s+5+j10)} \Big|_{s=-5+j10} = 67.08 \angle 63.43^\circ$$

$$v_o(t) = [-60e^{-25t} + 134.16e^{-5t} \cos(10t + 63.43^\circ)]u(t) \text{ V}$$

P 13.41 [a] *s*-domain equivalent circuit is



$$\text{Note: } i_2(0^+) = -\frac{20}{10} = -2 \text{ A}$$

$$[b] \frac{24}{s} = (120 + 3s)I_1 + 3sI_2 + 6$$

$$0 = -6 + 3sI_1 + (360 + 15s)I_2 + 36$$

In standard form,

$$(s + 40)I_1 + sI_2 = (8/s) - 2$$

$$sI_1 + (5s + 120)I_2 = -10$$

$$\Delta = \begin{vmatrix} s + 40 & s \\ s & 5s + 120 \end{vmatrix} = 4(s + 20)(s + 60)$$

$$N_1 = \begin{vmatrix} (8/s) - 2 & s \\ -10 & 5s + 120 \end{vmatrix} = \frac{-200(s - 4.8)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{-50(s - 4.8)}{s(s + 20)(s + 60)}$$

$$[c] sI_1 = \frac{-50(s - 4.8)}{(s + 20)(s + 60)}$$

$$\lim_{s \rightarrow \infty} sI_1 = i_1(0^+) = 0 \text{ A}$$

$$\lim_{s \rightarrow 0} sI_1 = i_1(\infty) = \frac{(-50)(-4.8)}{(20)(60)} = 0.2 \text{ A}$$

$$[d] I_1 = \frac{K_1}{s} + \frac{K_2}{s + 20} + \frac{K_3}{s + 60}$$

$$K_1 = \frac{240}{1200} = 0.2; \quad K_2 = \frac{-50(-20) + 240}{(-20)(40)} = -1.55$$

$$K_3 = \frac{-50(-60) + 240}{(-60)(-40)} = 1.35$$

$$i_1(t) = [0.2 - 1.55e^{-20t} + 1.35e^{-60t}]u(t) \text{ A}$$

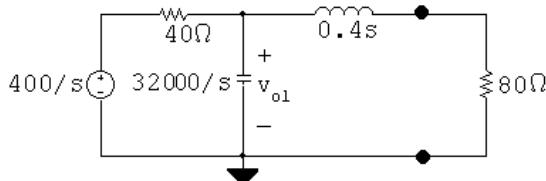
$$P\ 13.42 \quad \Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$$

$$N_2 = \begin{vmatrix} Y_{11} [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} \quad (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

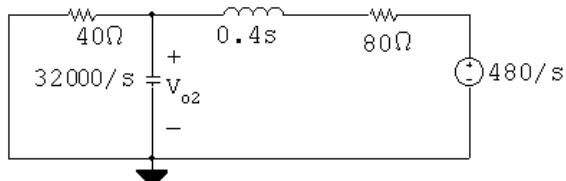
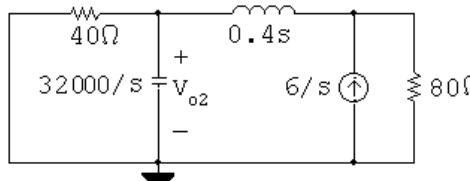
Substitution and simplification lead directly to Eq. 13.90.

P 13.43 [a] Voltage source acting alone:



$$\frac{V_{o1} - 400/s}{40} + \frac{V_{o1}s}{32,000} + \frac{V_{o1}}{80 + 0.4s} = 0$$

$$\therefore V_{o1} = \frac{32 \times 10^4(s + 200)}{s(s + 400)(s + 600)}$$



$$\frac{V_{o2}}{40} + \frac{V_{o2}s}{32,000} + \frac{V_{o2} - 480/s}{0.4s + 80} = 0$$

$$\therefore V_{o2} = \frac{38.4 \times 10^6}{s(s + 400)(s + 600)}$$

$$V_o = V_{o1} + V_{o2} = \frac{32 \times 10^4(s + 200) + 38.4 \times 10^6}{s(s + 400)(s + 600)} = \frac{32 \times 10^4(s + 320)}{s(s + 400)(s + 600)}$$

$$\begin{aligned}
 [\mathbf{b}] \quad V_o &= \frac{K_1}{s} + \frac{K_2}{s+400} + \frac{K_3}{s+600} \\
 K_1 &= \frac{(32 \times 10^4)(320)}{(400)(600)} = 426.67; \quad K_2 = \frac{32 \times 10^4(-80)}{(-400)(200)} = 320; \\
 K_3 &= \frac{32 \times 10^4(-280)}{(-600)(-200)} = -746.67 \\
 v_o(t) &= [426.67 + 320e^{-400t} - 746.67e^{-600t}]u(t) \text{ V}
 \end{aligned}$$

P 13.44 [a] $V_o = -\frac{Z_f}{Z_i}V_g$

$$Z_f = \frac{2000(1.6 \times 10^6/s)}{2000 + 1.6 \times 10^6/s} = \frac{1.6 \times 10^6}{s + 800}$$

$$Z_i = 800 + \frac{10^6}{3.125s} = \frac{800(s + 400)}{s}$$

$$V_g = \frac{5000}{s^2}$$

$$\therefore V_o = \frac{-10^7}{s(s + 400)(s + 800)}$$

$$[\mathbf{b}] \quad V_o = \frac{K_1}{s} + \frac{K_2}{s+400} + \frac{K_3}{s+800}$$

$$K_1 = \frac{-10^7}{(400)(800)} = -31.25$$

$$K_2 = \frac{-10^7}{(-400)(400)} = 62.5$$

$$K_3 = \frac{-10^7}{(-800)(-400)} = -31.25$$

$$\therefore v_o(t) = (-31.25 + 62.5e^{-400t} - 31.25e^{-800t})u(t) \text{ V}$$

$$[\mathbf{c}] \quad -31.25 + 62.5e^{-400t_s} - 31.25e^{-800t_s} = -10$$

$$\therefore 62.5e^{-400t_s} - 31.25e^{-800t_s} = 21.25$$

Let $x = e^{-400t_s}$. Then

$$62.5x - 31.25x^2 = 21.25; \quad \text{or } x^2 - 2x + 0.68 = 0$$

Solving,

$$x = 0.4343$$

$$\therefore e^{-400t_s} = 0.4343; \quad \therefore t_s = 2.1 \text{ ms}$$

$$[\text{d}] \quad v_g = m t u(t); \quad V_g = \frac{m}{s^2}$$

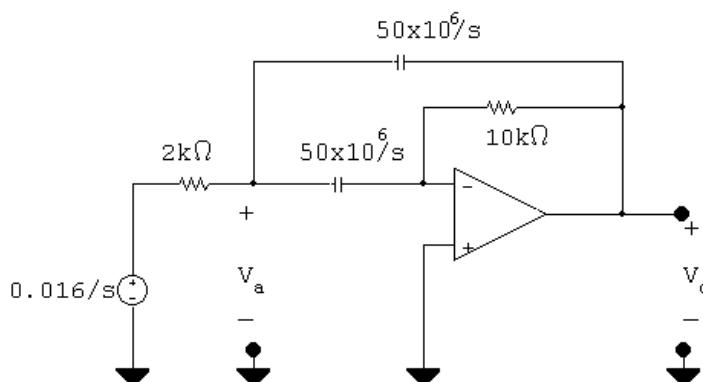
$$V_o = \frac{-2000m}{s(s+400)(s+800)}$$

$$K_1 = \frac{-2000m}{(400)(800)} = \frac{-2000m}{32 \times 10^4}$$

$$\therefore -10 = \frac{-2000m}{32 \times 10^4} \quad \therefore m = 1600 \text{ V/s}$$

Thus, m must be less than or equal to 1600 V/s to avoid saturation.

P 13.45



$$\frac{V_a - 0.016/s}{2000} + \frac{V_a s}{50 \times 10^6} + \frac{(V_a - V_o)s}{50 \times 10^6} = 0$$

$$\frac{(0 - V_a)s}{50 \times 10^6} + \frac{(0 - V_o)}{10,000} = 0$$

$$V_a = \frac{-5000V_o}{s}$$

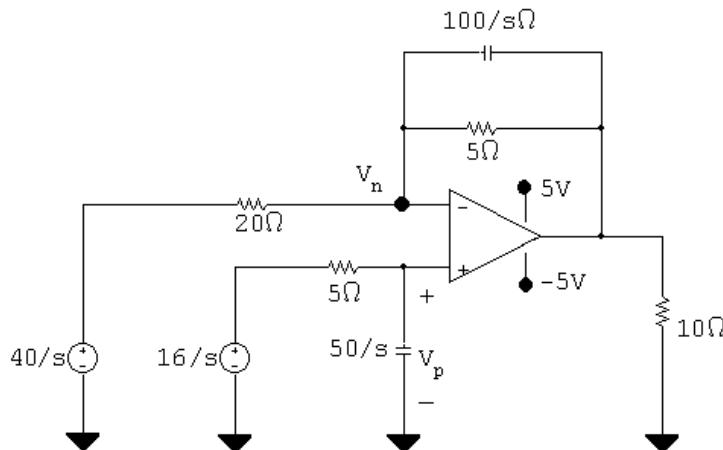
$$\therefore \frac{-5000V_o}{s}(2s + 25,000) - sV_o = 25,000 \left(\frac{0.016}{s} \right)$$

$$V_o = \frac{-4000}{(s + 5000 - j10,000)(s + 5000 + j10,000)}$$

$$K_1 = \frac{-400}{j10,000} = j0.02 = 0.02/\underline{90^\circ}$$

$$v_o(t) = 40e^{-5000t} \cos(10,000t + 90^\circ) = -40e^{-5000t} \sin(10,000t)u(t) \text{ mV}$$

P 13.46 [a]



$$V_p = \frac{50/s}{5 + 50/s} V_{g2} = \frac{50}{5s + 50} V_{g2}$$

$$\frac{V_p - 40/s}{20} + \frac{V_p - V_o}{5} + \frac{V_p - V_o}{100/s} = 0$$

$$V_p \left(\frac{1}{20} + \frac{1}{5} + \frac{s}{100} \right) - V_o \left(\frac{1}{5} + \frac{s}{100} \right) = \frac{2}{s}$$

$$\frac{s+25}{100} \left(\frac{50}{5s+50} \right) \frac{16}{s} - \frac{2}{s} = V_o \left(\frac{1}{5} + \frac{s}{100} \right) = V_o \left(\frac{s+20}{100} \right)$$

$$V_o = \frac{100}{s+20} \left[\frac{16(s+25)}{10(s+10)(s)} - \frac{2}{s} \right] = \frac{-40s+2000}{s(s+10)(s+20)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+20}$$

$$K_1 = 10; \quad K_2 = -24; \quad K_3 = 14$$

$$\therefore v_o(t) = [10 - 24e^{-10t} + 14e^{-20t}]u(t) \text{ V}$$

[b] $10 - 24x + 14x^2 = 5$

$$14x^2 - 24x + 5 = 0$$

$$x = 0 \quad \text{or} \quad 0.242691$$

$$e^{-10t} = 0.242691 \quad \therefore t = 141.60 \text{ ms}$$

P 13.47 [a] Let v_a be the voltage across the $0.5 \mu\text{F}$ capacitor, positive at the upper terminal.

Let v_b be the voltage across the $100 \text{ k}\Omega$ resistor, positive at the upper terminal.

Also note

$$\frac{10^6}{0.5s} = \frac{2 \times 10^6}{s} \quad \text{and} \quad \frac{10^6}{0.25s} = \frac{4 \times 10^6}{s}; \quad V_g = \frac{0.5}{s}$$

$$\frac{sV_a}{s \times 10^6} + \frac{V_a - (0.5/s)}{200,000} + \frac{V_a}{200,000} = 0$$

$$sV_a + 10V_a - \frac{5}{s} + 10V_a = 0$$

$$V_a = \frac{5}{s(s+20)}$$

$$\frac{0 - V_a}{200,000} + \frac{(0 - V_b)s}{4 \times 10^6} = 0$$

$$\therefore V_b = -\frac{20}{s}V_a = \frac{-100}{s^2(s+20)}$$

$$\frac{V_b}{100,000} + \frac{(V_b - 0)s}{4 \times 10^6} + \frac{(V_b - V_o)s}{4 \times 10^6} = 0$$

$$40V_b + sV_b + sV_b = sV_o$$

$$\therefore V_o = \frac{2(s+20)V_b}{s}; \quad V_o = 2 \left(\frac{-100}{s^3} \right) = \frac{-200}{s^3}$$

[b] $v_o(t) = -100t^2u(t)$ V

[c] $-100t^2 = -4; \quad t = 0.2$ s = 200 ms

P 13.48 [a] $\frac{1/sC}{R + 1/sC} = \frac{1}{RsC + 1} = \frac{1/RC}{s + 1/RC}$

There are no zeros, and a single pole at $-1/RC$ rad/sec.

[b] $\frac{R}{R + sL} = \frac{R/L}{s + R/L}$

There are no zeros, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 10 \Omega; \quad L = 10 \text{ mH}; \quad C = 100 \mu\text{F}$$

P 13.49 [a] $\frac{R}{R + 1/sC} = \frac{RsC}{RsC + 1} = \frac{s}{s + 1/RC}$

There is a single zero at 0 rad/sec, and a single pole at $-1/RC$ rad/sec.

[b] $\frac{sL}{R + sL} = \frac{s}{s + R/L}$

There is a single zero at 0 rad/sec, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 100 \Omega; \quad L = 10 \text{ mH}; \quad C = 1 \mu\text{F}$$

P 13.50 [a] $\frac{R}{1/sC + sL + R} = \frac{(R/L)s}{s^2 + (R/L)s + 1/LC}$

There is a single zero at 0 rad/sec, and two poles:

$$p_1 = -(R/2L) + \sqrt{(R/2L)^2 - (1/LC)}; \quad p_2 = -(R/2L) - \sqrt{(R/2L)^2 - (1/LC)}$$

[b] There are several possible solutions. One is

$$R = 250 \Omega; \quad L = 10 \text{ mH}; \quad C = 1 \mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -5000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -20,000 \text{ rad/sec}$$

[c] There are several possible solutions. One is

$$R = 200 \Omega; \quad L = 10 \text{ mH}; \quad C = 1 \mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -10,000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -10,000 \text{ rad/sec}$$

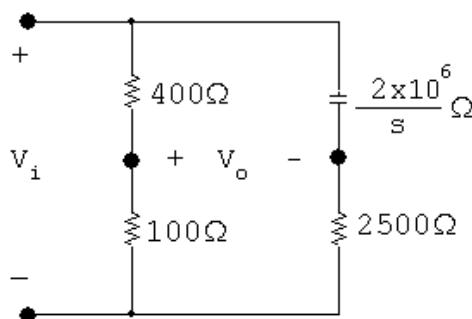
[d] There are several possible solutions. One is

$$R = 120 \Omega; \quad L = 10 \text{ mH}; \quad C = 1 \mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -6000 + j8000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -6000 - j8000 \text{ rad/sec}$$

P 13.51 [a]



$$\frac{100}{500}V_i = V_o + \frac{2500V_i}{2500 + (2 \times 10^6 / s)}$$

$$0.2V_i - \frac{sV_i}{s + 800} = V_o$$

$$\therefore \frac{V_o}{V_i} = H(s) = \frac{-0.8(s - 200)}{(s + 800)}$$

[b] $-z_1 = 200 \text{ rad/s}$

$$-p_1 = -800 \text{ rad/s}$$

$$\text{P 13.52 [a]} \frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1}$$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{25}{s + 25}; \quad -p_1 = -25 \text{ rad/s}$$

$$\text{[b]} \frac{V_o}{V_i} = \frac{R}{R + 1/sC} = \frac{RCs}{RCs + 1} = \frac{s}{s + (1/RC)}$$

$$= \frac{s}{s + 25}; \quad z_1 = 0, \quad -p_1 = -25 \text{ rad/s}$$

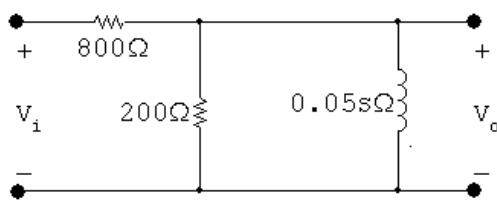
$$\text{[c]} \frac{V_o}{V_i} = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 2000}$$

$$z_1 = 0; \quad -p_1 = -2000 \text{ rad/s}$$

$$\text{[d]} \frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{R/L}{s + (R/L)} = \frac{2000}{s + 2000}$$

$$-p_1 = -2000 \text{ rad/s}$$

[e]



$$\frac{V_o}{0.05s} + \frac{V_o}{200} + \frac{V_o - V_i}{800} = 0$$

$$sV_o + 4sV_o + 16,000V_o = sV_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{0.2s}{s + 3200}$$

$$z_1 = 0; \quad -p_1 = -3200 \text{ rad/s}$$

P 13.53 [a] Let $R_1 = 250 \text{ k}\Omega$; $R_2 = 125 \text{ k}\Omega$; $C_2 = 1.6 \text{ nF}$; and $C_f = 0.4 \text{ nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s \left(s + \frac{C_2 + C_f}{C_2 C_f R_2}\right)}$$

$$\frac{1}{C_f} = 2.5 \times 10^9$$

$$\frac{1}{R_2 C_2} = \frac{62.5 \times 10^7}{125 \times 10^3} = 5000 \text{ rad/s}$$

$$\frac{C_2 + C_f}{C_2 C_f R_2} = \frac{2 \times 10^{-9}}{(0.64 \times 10^{-18})(125 \times 10^3)} = 25,000 \text{ rad/s}$$

$$\therefore Z_f = \frac{2.5 \times 10^9(s + 5000)}{s(s + 25,000)} \Omega$$

$$Z_i = R_1 = 250 \times 10^3 \Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-10^4(s + 5000)}{s(s + 25,000)}$$

[b] $-z_1 = -5000 \text{ rad/s}$

$$-p_1 = 0; \quad -p_2 = -25,000 \text{ rad/s}$$

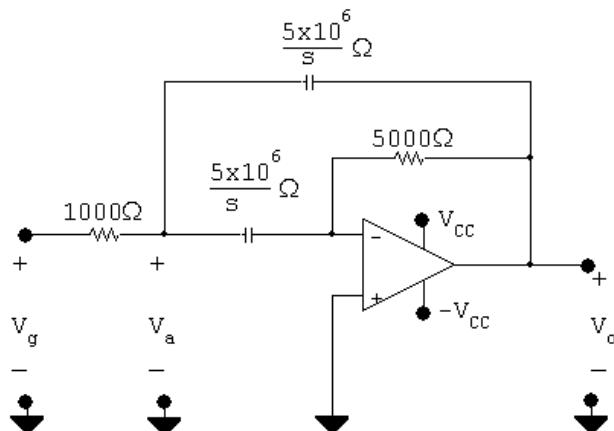
P 13.54 [a] $Z_i = 250 + \frac{31,250}{s} = \frac{250(s + 125)}{s}$

$$Z_f = \frac{25 \times 10^4}{s} \| 10,000 = \frac{25 \times 10^4}{s + 25}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-25 \times 10^4 / (s + 25)}{250(s + 125)/s} = \frac{-1000s}{(s + 25)(s + 125)}$$

[b] Zero at $z_1 = 0$; Poles at $-p_1 = -25 \text{ rad/s}$ and $-p_2 = -125 \text{ rad/s}$

P 13.55 [a]



$$\frac{V_a - V_g}{1000} + \frac{sV_a}{5 \times 10^6} + \frac{(V_a - V_o)s}{5 \times 10^6} = 0$$

$$5000V_a - 5000V_g + 2sV_a - sV_o = 0$$

$$(5000 + 2s)V_a - sV_o = 5000V_g$$

$$\frac{(0 - V_a)s}{5 \times 10^6} + \frac{0 - V_o}{5000} = 0$$

$$-sV_a - 1000V_o = 0; \quad \therefore V_a - \frac{-1000}{s}V_o$$

$$(2s + 5000) \left(\frac{-1000}{s} \right) V_o - sV_o = 5000V_g$$

$$1000V_o(2s + 5000) + s^2V_o = -5000sV_g$$

$$V_o(s^2 + 2000s + 5 \times 10^6) = -5000sV_g$$

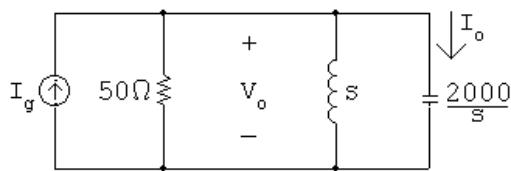
$$\frac{V_o}{V_g} = \frac{-5000s}{s^2 + 2000s + 5 \times 10^6}$$

$$s_{1,2} = -1000 \pm \sqrt{10^6 - 5 \times 10^6} = -1000 \pm j2000$$

$$\frac{V_o}{V_g} = \frac{-5000s}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

[b] $z_1 = 0; \quad -p_1 = -1000 + j2000; \quad -p_2 = -1000 - j2000$

P 13.56 [a]



$$\frac{V_o}{50} + \frac{V_o}{s} + \frac{V_o s}{2000} = I_g$$

$$V_o = \frac{2000s}{s^2 + 40s + 2000} \cdot I_g \quad \text{and} \quad I_o = 5 \times 10^{-4}sV_o$$

$$\therefore H(s) = \frac{I_o}{I_g} = \frac{s^2}{s^2 + 40s + 2000}$$

[b] $I_g = \frac{0.025s}{s^2 + 40,000} \quad \text{so} \quad I_o = \frac{(s^2)(0.025s)}{(s + 20 - j40)(s + 20 + j40)(s^2 + 40,000)}$

$$I_o = \frac{0.025s^3}{(s + 20 - j40)(s + 20 + j40)(s + j200)(s - j200)}$$

[c] Damped sinusoid of the form

$$Me^{-20t} \cos(40t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N \cos(200t + \theta_2)$$

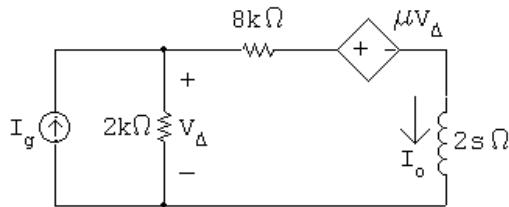
[e] $I_o = \frac{K_1}{s + 20 - j40} + \frac{K_1^*}{s + 20 + j40} + \frac{K_2}{s - j200} + \frac{K_2^*}{s + j200}$

$$K_1 = \frac{0.025(-20 + j40)^3}{(j80)(-20 - j160)(-20 + j240)} = 719.77 \times 10^{-6} / -97.94^\circ$$

$$K_2 = \frac{0.025(j200)^3}{(j400)(20 + j160)(20 + j240)} = 12.88 \times 10^{-3} / 11.89^\circ$$

$$i_o(t) = [1.44e^{-20t} \cos(40t - 97.94^\circ) + 25.76 \cos(200t + 11.89^\circ)] \text{ mA}$$

P 13.57 [a]



$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore I_o = \frac{1000(1-\mu)}{s+1000(5-\mu)} I_g$$

$$\therefore H(s) = \frac{1000(1-\mu)}{s+1000(5-\mu)}$$

[b] $\mu < 5$

[c]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20,000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15,000/s(s+1000)$
5	$-4000/s$	$-20,000/s^2$
6	$-5000/(s-1000)$	$-25,000/s(s-1000)$

 $\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \quad i_o = [2.5 - 2.5e^{-8000t}]u(t) \text{ A}$$

 $\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \quad i_o = [1 - e^{-5000t}]u(t) \text{ A}$$

 $\mu = 4$:

$$I_o = \frac{-15}{s} - \frac{15}{s+1000}; \quad i_o = [-15 + 15e^{-1000t}]u(t) \text{ A}$$

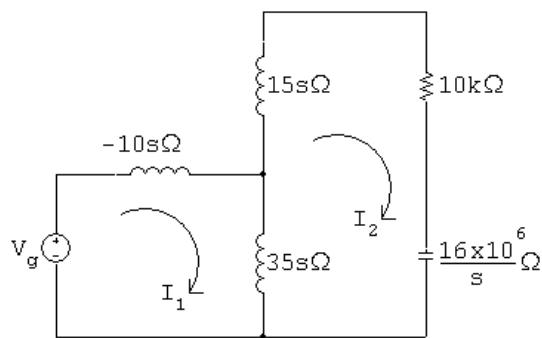
 $\mu = 5$:

$$I_o = \frac{-20,000}{s^2}; \quad i_o = -20,000t u(t) \text{ A}$$

 $\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s-1000}; \quad i_o = 25[1 - e^{1000t}]u(t) \text{ A}$$

P 13.58



$$V_g = 25sI_1 - 35sI_2$$

$$0 = -35sI_1 + \left(50s + 10,000 + \frac{16 \times 10^6}{s} \right) I_2$$

$$\Delta = \begin{vmatrix} 25s & -35s \\ -35s & 50s + 10,000 + 16 \times 10^6/s \end{vmatrix} = 25(s + 2000)(s + 8000)$$

$$N_2 = \begin{vmatrix} 25s & V_g \\ -35s & 0 \end{vmatrix} = 35sV_g$$

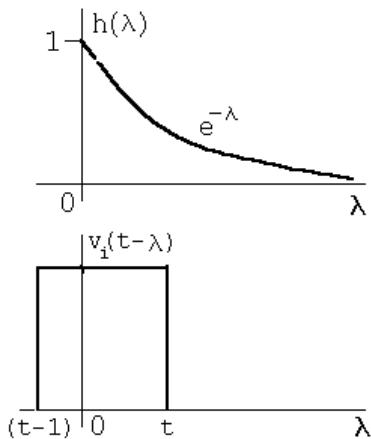
$$I_2 = \frac{N_2}{\Delta} = \frac{35sV_g}{25(s + 2000)(s + 8000)}$$

$$H(s) = \frac{I_2}{V_g} = \frac{1.4s}{(s + 2000)(s + 8000)}$$

$$\therefore z_1 = 0; \quad -p_1 = -2000 \text{ rad/s}; \quad -p_2 = -8000 \text{ rad/s}$$

P 13.59 $H(s) = \frac{V_o}{V_i} = \frac{1}{s+1}; \quad h(t) = e^{-t}$

For $0 \leq t \leq 1$:



$$v_o = \int_0^t e^{-\lambda} d\lambda = (1 - e^{-t}) V$$

For $1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda} d\lambda = (e - 1)e^{-t} V$$

P 13.60 $H(s) = \frac{V_o}{V_i} = \frac{s}{s+1} = 1 - \frac{1}{s+1}; \quad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

For $0 \leq t \leq 1$:

$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}] d\lambda = [1 + e^{-\lambda}] \Big|_0^t = e^{-t} V$$

For $1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda}) d\lambda = e^{-\lambda} \Big|_{t-1}^t = (1 - e)e^{-t} V$$

P 13.61 [a] From Problem 13.52(a)

$$H(s) = \frac{25}{s + 25}$$

$$h(\lambda) = 25e^{-25\lambda}$$

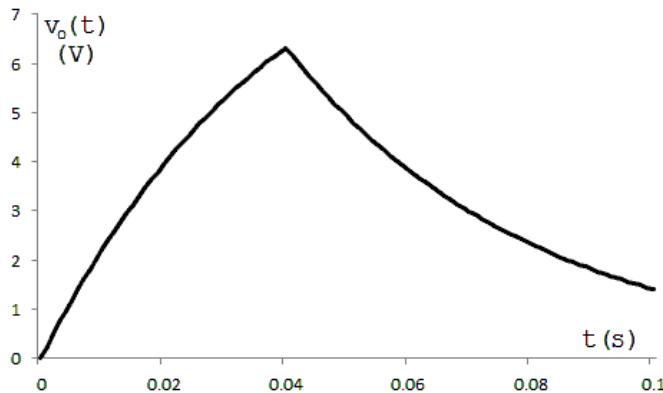
$0 \leq t \leq 40 \text{ ms}$:

$$v_o = \int_0^t 10(25)e^{-25\lambda} d\lambda = 10(1 - e^{-25t}) \text{ V}$$

$40 \text{ ms} \leq t \leq \infty$:

$$v_o = \int_{t=0.04}^t 10(25)e^{-25\lambda} d\lambda = 10(e - 1)e^{-25t} \text{ V}$$

[b]



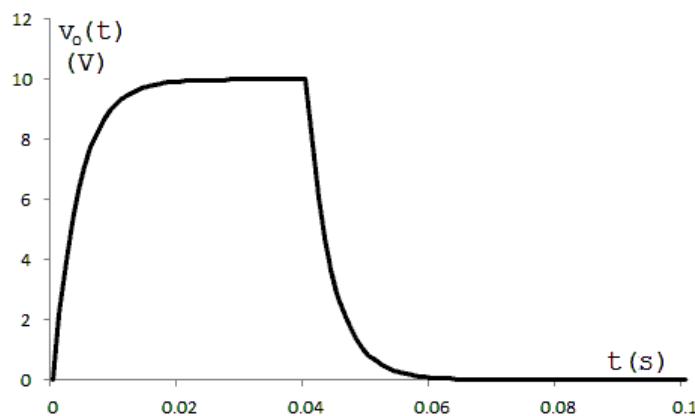
P 13.62 [a] $H(s) = \frac{250}{s + 250} \quad \therefore h(\lambda) = 250e^{-250\lambda}$

$0 \leq t \leq 40 \text{ ms}$:

$$v_o = \int_0^t 10(250)e^{-250\lambda} d\lambda = 10(1 - e^{-250t}) \text{ V}$$

$40 \text{ ms} \leq t \leq \infty$:

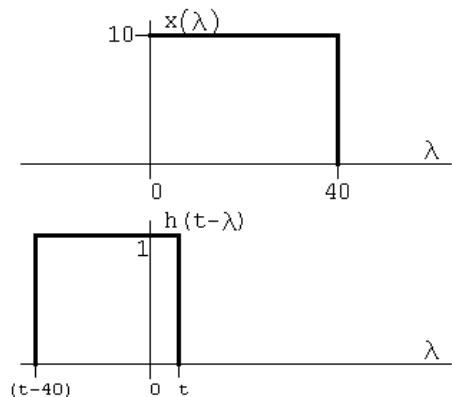
$$v_o = \int_{t=0.04}^t 10(250)e^{-250\lambda} d\lambda = 10(e^{10} - 1)e^{-250t} \text{ V}$$



[b] decrease

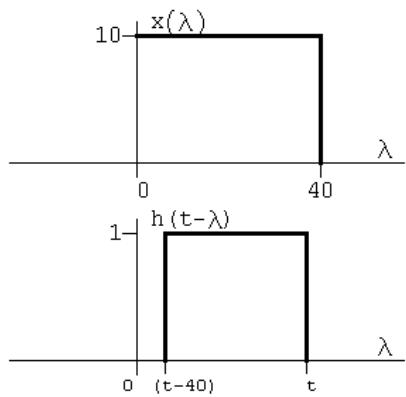
[c] The circuit with $R = 200 \Omega$.

P 13.63 [a] $0 \leq t \leq 40$:



$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

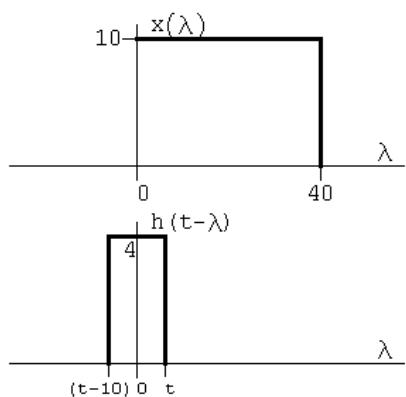
$40 \leq t \leq 80$:



$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

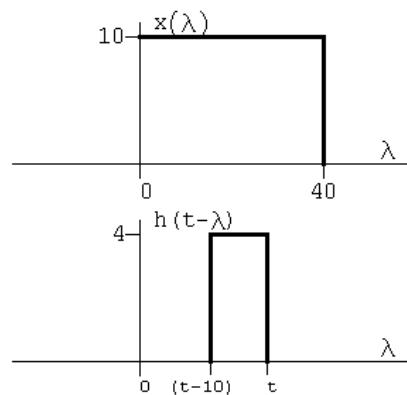
$$t \geq 80 : \quad y(t) = 0$$

[b] $0 \leq t \leq 10$:



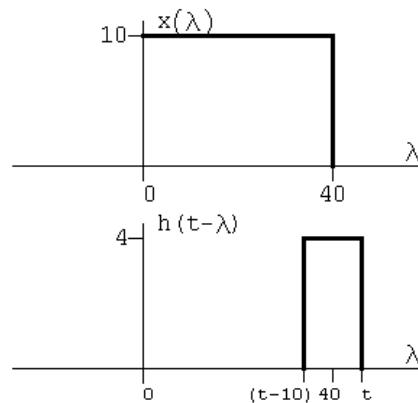
$$y(t) = \int_0^t 40 d\lambda = 40\lambda \Big|_0^t = 40t$$

$10 \leq t \leq 40$:



$$y(t) = \int_{t-10}^t 40 d\lambda = 40\lambda \Big|_{t-10}^t = 400$$

$40 \leq t \leq 50$:



$$y(t) = \int_{t-10}^{40} 40 d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50 - t)$$

$$t \geq 50 : \quad y(t) = 0$$

[c] The expressions are

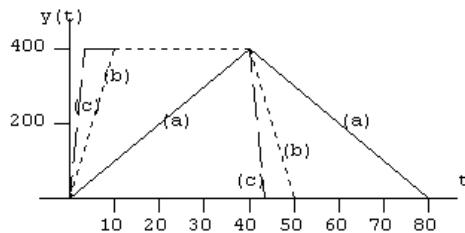
$$0 \leq t \leq 1 : \quad y(t) = \int_0^t 400 d\lambda = 400\lambda \Big|_0^t = 400t$$

$$1 \leq t \leq 40 : \quad y(t) = \int_{t-1}^t 400 d\lambda = 400\lambda \Big|_{t-1}^t = 400$$

$$40 \leq t \leq 41 : \quad y(t) = \int_{t-1}^{40} 400 d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41 - t)$$

$$41 \leq t < \infty : \quad y(t) = 0$$

[d]

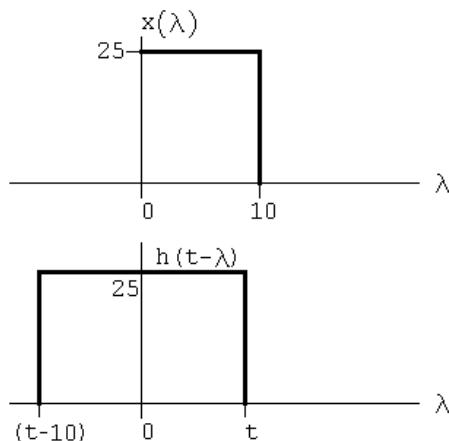


- [e] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$\begin{aligned} y(t) &= \int_0^t h(t-\lambda)x(\lambda) d\lambda \rightarrow \int_0^t 40\delta(t-\lambda)x(\lambda) d\lambda \\ &\rightarrow 40x(t) \end{aligned}$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.64 [a]

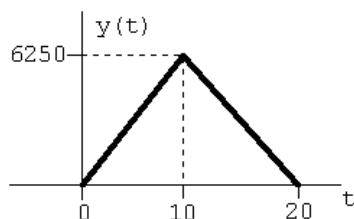


$$y(t) = 0 \quad t < 0$$

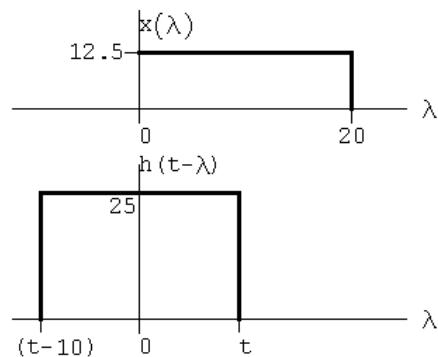
$$0 \leq t \leq 10 : \quad y(t) = \int_0^t 625 d\lambda = 625t$$

$$10 \leq t \leq 20 : \quad y(t) = \int_{t-10}^{10} 625 d\lambda = 625(10 - t + 10) = 625(20 - t)$$

$$20 \leq t < \infty : \quad y(t) = 0$$



[b]



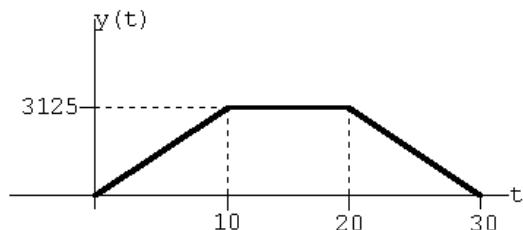
$$y(t) = 0 \quad t < 0$$

$$0 \leq t \leq 10 : \quad y(t) = \int_0^t 312.5 d\lambda = 312.5t$$

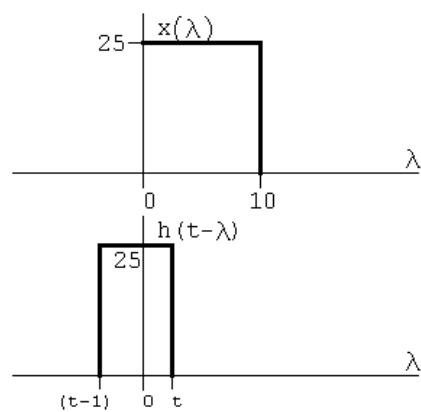
$$10 \leq t \leq 20 : \quad y(t) = \int_0^{10} 312.5 d\lambda = 3125$$

$$20 \leq t \leq 30 : \quad y(t) = \int_{t-20}^{10} 312.5 d\lambda = 312.5(30 - t)$$

$$30 \leq t < \infty : \quad y(t) = 0$$



[c]



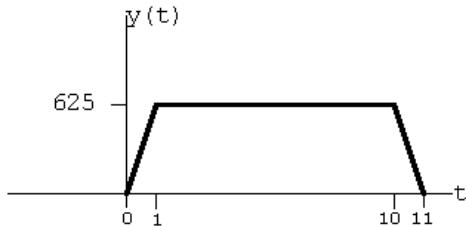
$$y(t) = 0 \quad t < 0$$

$$0 \leq t \leq 1 : \quad y(t) = \int_0^t 625 d\lambda = 625t$$

$$1 \leq t \leq 10 : \quad y(t) = \int_0^1 625 d\lambda = 625$$

$$10 \leq t \leq 11 : \quad y(t) = \int_{t-10}^1 625 d\lambda = 625(11-t)$$

$$11 \leq t < \infty : \quad y(t) = 0$$



P 13.65 [a] $-1 \leq t \leq 4$:

$$v_o = \int_0^{t+1} 10\lambda d\lambda = 5\lambda^2 \Big|_0^{t+1} = 5t^2 + 10t + 5 \text{ V}$$

$4 \leq t \leq 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda d\lambda = 5\lambda^2 \Big|_{t-4}^{t+1} = 50t - 75 \text{ V}$$

$9 \leq t \leq 14$:

$$\begin{aligned} v_o &= 10 \int_{t-4}^{10} \lambda d\lambda + 10 \int_{10}^{t+1} 10 d\lambda \\ &= 5\lambda^2 \Big|_{t-4}^{10} + 100\lambda \Big|_{10}^{t+1} = -5t^2 + 140t - 480 \text{ V} \end{aligned}$$

$14 \leq t \leq 19$:

$$v_o = 100 \int_{t-4}^{t+1} d\lambda = 500 \text{ V}$$

$19 \leq t \leq 24$:

$$\begin{aligned} v_o &= \int_{t-4}^{20} 100\lambda d\lambda + \int_{20}^{t+2} 10(30-\lambda) d\lambda \\ &= 100\lambda \Big|_{t-2}^{20} + 300\lambda \Big|_{20}^{t+1} - 5\lambda^2 \Big|_{20}^{t+2} \\ &= -5t^2 + 190t - 1305 \text{ V} \end{aligned}$$

$24 \leq t \leq 29$:

$$\begin{aligned} v_o &= 10 \int_{t-4}^{t+1} (30-\lambda) d\lambda = 300\lambda \Big|_{t-4}^{t+1} - 5\lambda^2 \Big|_{t-4}^{t+1} \\ &= 1575 - 50t \text{ V} \end{aligned}$$

$29 \leq t \leq 34$:

$$v_o = 10 \int_{t=4}^{30} (30 - \lambda) d\lambda = 300\lambda \Big|_{t=4}^{30} - 5\lambda^2 \Big|_{t=2}^{30}$$

$$= 5t^2 - 340t + 5780 \text{ V}$$

Summary:

$$v_o = 0 \quad -\infty \leq t \leq -1$$

$$v_o = 5t^2 + 10t + 5 \text{ V} \quad -1 \leq t \leq 4$$

$$v_o = 50t - 75 \text{ V} \quad 4 \leq t \leq 9$$

$$v_o = -5t^2 + 140t - 480 \text{ V} \quad 9 \leq t \leq 14$$

$$v_o = 500 \text{ V} \quad 14 \leq t \leq 19$$

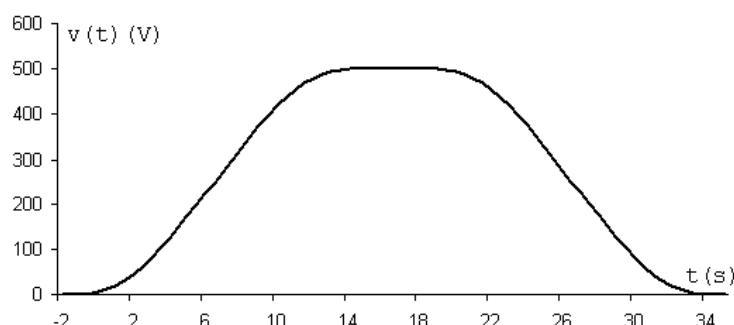
$$v_o = -5t^2 + 190t - 1305 \text{ V} \quad 19 \leq t \leq 24$$

$$v_o = 1575 - 50t \text{ V} \quad 24 \leq t \leq 29$$

$$v_o = 5t^2 - 340t + 5780 \text{ V} \quad 29 \leq t \leq 34$$

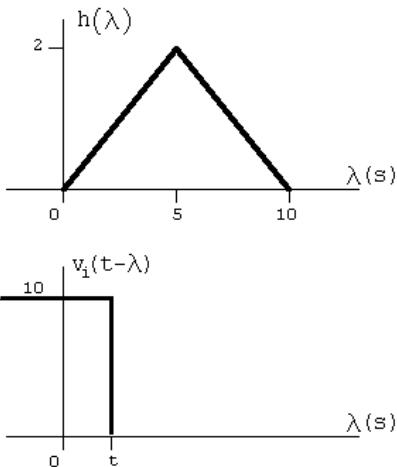
$$v_o = 0 \quad 34 \leq t \leq \infty$$

[b]



P 13.66 [a] $h(\lambda) = \frac{2}{5}\lambda \quad 0 \leq \lambda \leq 5$

$$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \quad 5 \leq \lambda \leq 10$$



$0 \leq t \leq 5$:

$$v_o = 10 \int_0^t \frac{2}{5} \lambda d\lambda = 2t^2$$

$5 \leq t \leq 10$:

$$\begin{aligned} v_o &= 10 \int_0^5 \frac{2}{5} \lambda d\lambda + 10 \int_5^t \left(4 - \frac{2}{5}\lambda\right) d\lambda \\ &= \frac{4\lambda^2}{2} \Big|_0^5 + 40\lambda \Big|_5^t - \frac{4\lambda^2}{2} \Big|_5^t \\ &= -100 + 40t - 2t^2 \end{aligned}$$

$10 \leq t \leq \infty$:

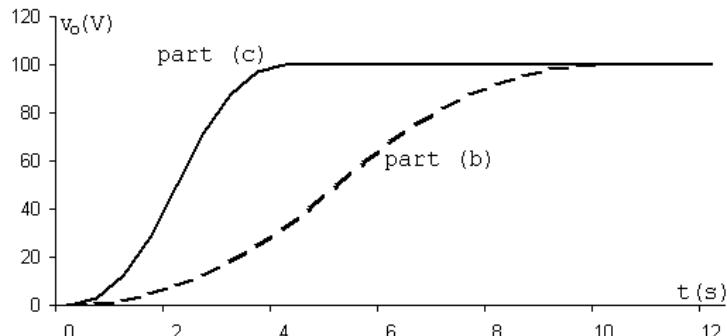
$$\begin{aligned} v_o &= 10 \int_0^5 \frac{2}{5} \lambda d\lambda + 10 \int_5^{10} \left(4 - \frac{2}{5}\lambda\right) d\lambda \\ &= \frac{4\lambda^2}{2} \Big|_0^5 + 40\lambda \Big|_5^{10} - \frac{4\lambda^2}{2} \Big|_5^{10} \\ &= 50 + 200 - 150 = 100 \end{aligned}$$

$$v_o = 2t^2 \text{ V} \quad 0 \leq t \leq 5$$

$$v_o = 40t - 100 - 2t^2 \text{ V} \quad 5 \leq t \leq 10$$

$$v_o = 100 \text{ V} \quad 10 \leq t \leq \infty$$

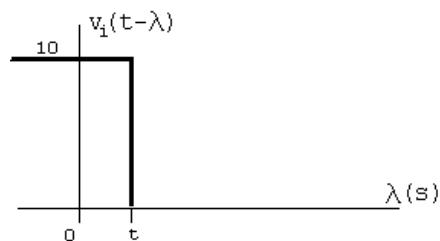
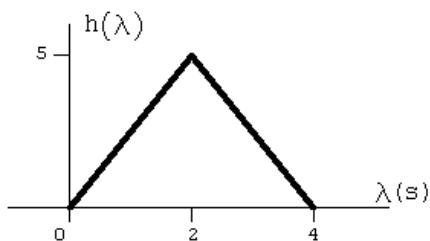
[b]



$$[c] \text{ Area} = \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10 \quad \text{so} \quad h = 5$$

$$h(\lambda) = \frac{5}{2}\lambda \quad 0 \leq \lambda \leq 2$$

$$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \quad 2 \leq \lambda \leq 4$$



$$0 \leq t \leq 2:$$

$$v_o = 10 \int_0^t \frac{5}{2}\lambda d\lambda = 12.5t^2$$

$$2 \leq t \leq 4:$$

$$\begin{aligned} v_o &= 10 \int_0^2 \frac{5}{2}\lambda d\lambda + 10 \int_2^t \left(10 - \frac{5}{2}\lambda\right) d\lambda \\ &= \frac{25\lambda^2}{2} \Big|_0^2 + 100\lambda \Big|_2^t - \frac{25\lambda^2}{2} \Big|_2^t \\ &= -100 + 100t - 12.5t^2 \end{aligned}$$

$4 \leq t \leq \infty$:

$$\begin{aligned} v_o &= 10 \int_0^2 \frac{5}{2} \lambda d\lambda + 10 \int_2^4 \left(10 - \frac{5}{2} \lambda\right) d\lambda \\ &= \frac{25\lambda^2}{2} \Big|_0^2 + 100\lambda \Big|_2^4 - \frac{25\lambda^2}{2} \Big|_2^4 \\ &= 50 + 200 - 150 = 100 \end{aligned}$$

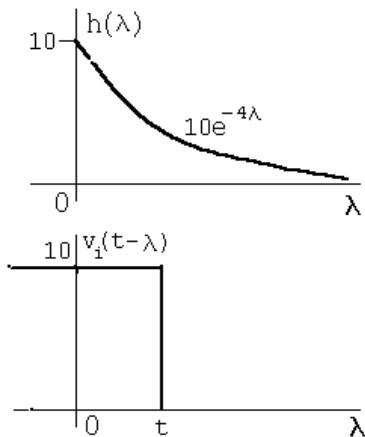
$$v_o = 12.5t^2 \text{ V} \quad 0 \leq t \leq 2$$

$$v_o = 100t - 100 - 12.5t^2 \text{ V} \quad 2 \leq t \leq 4$$

$$v_o = 100 \text{ V} \quad 4 \leq t \leq \infty$$

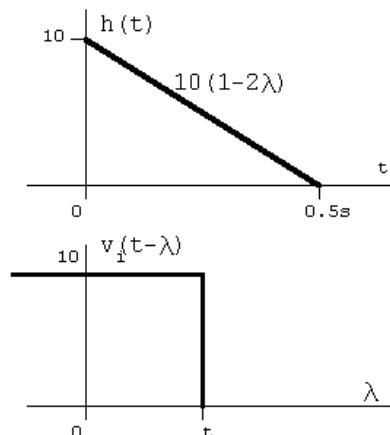
- [d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.67 [a]



$$\begin{aligned} v_o &= \int_0^t 10(10e^{-4\lambda}) d\lambda \\ &= 100 \frac{e^{-4\lambda}}{-4} \Big|_0^t = -25[e^{-4t} - 1] \\ &= 25(1 - e^{-4t}) \text{ V}, \quad 0 \leq t \leq \infty \end{aligned}$$

[b]



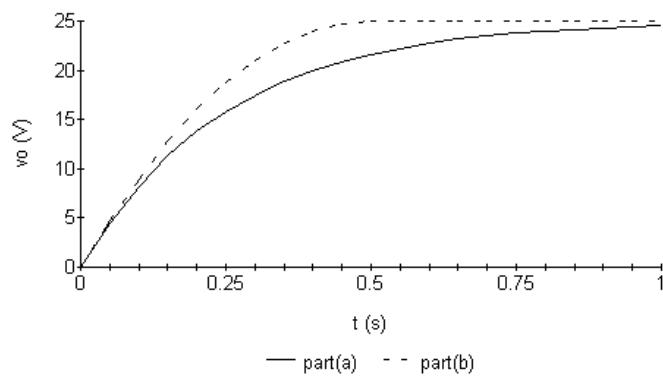
$$0 \leq t \leq 0.5:$$

$$v_o = \int_0^t 100(1 - 2\lambda) d\lambda = 100(\lambda - \lambda^2) \Big|_0^t = 100t(1 - t)$$

$$0.5 \leq t \leq \infty:$$

$$v_o = \int_0^{0.5} 100(1 - 2\lambda) d\lambda = 100(\lambda - \lambda^2) \Big|_0^{0.5} = 25$$

[c]



$$\text{P 13.68 } H(s) = \frac{16s}{40 + 4s + 16s} = \frac{0.8s}{s + 2} = 0.8 \left(1 - \frac{2}{s + 2}\right) = 0.8 - \frac{1.6}{s + 2}$$

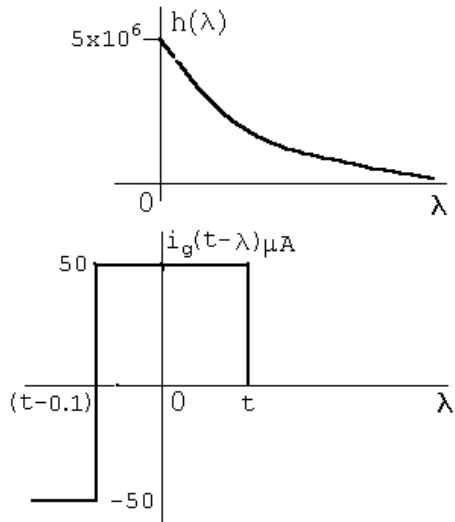
$$h(\lambda) = 0.8\delta(\lambda) - 1.6e^{-2\lambda}u(\lambda)$$

$$\begin{aligned} v_o &= \int_0^t 75[0.8\delta(\lambda) - 1.6e^{-2\lambda}] d\lambda = \int_0^t 60\delta(\lambda) d\lambda - 120 \int_0^t e^{-2\lambda} d\lambda \\ &= 60 - 120 \frac{e^{-2\lambda}}{-2} \Big|_0^t = 60 + 60(e^{-2t} - 1) \\ &= 60e^{-2t}u(t) \text{ V} \end{aligned}$$

$$\text{P 13.69 [a]} \quad I_o = \frac{V_o}{10^5} + \frac{V_o s}{5 \times 10^6} = \frac{V_o(s + 50)}{5 \times 10^6}$$

$$\frac{V_o}{I_g} = H(s) = \frac{5 \times 10^6}{s + 50}$$

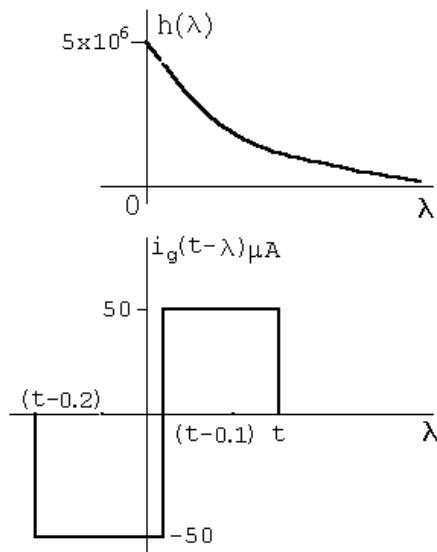
$$h(\lambda) = 5 \times 10^6 e^{-50\lambda} u(\lambda)$$



$0 \leq t \leq 0.1 \text{ s}$:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6) e^{-50\lambda} d\lambda = 250 \frac{e^{-50\lambda}}{-50} \Big|_0^t = 5(1 - e^{-50t}) \text{ V}$$

$0.1 \text{ s} \leq t \leq 0.2 \text{ s}$:



$$v_o = \int_0^{t-0.1} (-50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda} d\lambda)$$

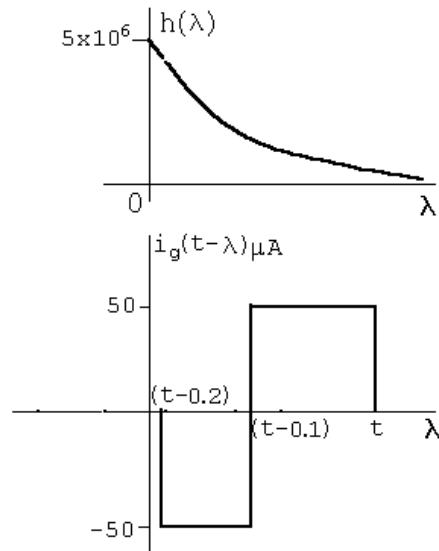
$$+ \int_{t-0.1}^t (50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda} d\lambda)$$

$$= -250 \frac{e^{-50\lambda}}{-50} \Big|_0^{t-0.1} + 250 \frac{e^{-50\lambda}}{-50} \Big|_{t-0.1}^t$$

$$= 5 \left[e^{-50(t-0.1)} - 1 \right] - 5 \left[e^{-50t} - e^{-50(t-0.1)} \right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5] \text{ V}$$

$0.2 \text{ s} \leq t \leq \infty$:



$$\begin{aligned} v_o &= \int_{t-0.2}^{t-0.1} -250e^{-50\lambda} d\lambda + \int_{t-0.1}^t 250e^{-50\lambda} d\lambda \\ &= \left[5e^{-50\lambda} \Big|_{t-0.2}^{t-0.1} -5e^{-50\lambda} \Big|_{t-0.1}^t \right] \\ v_o &= [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}] \text{ V} \end{aligned}$$

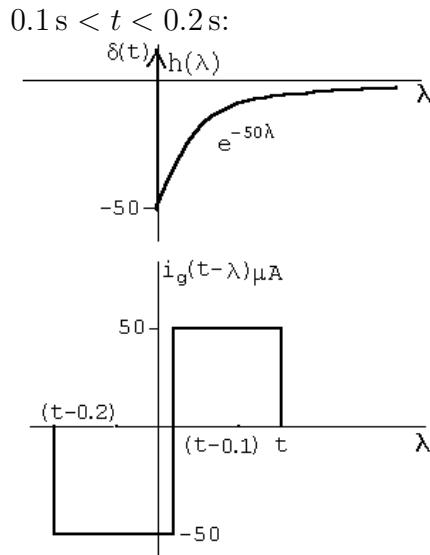
$$[\mathbf{b}] \quad I_o = \frac{V_o s}{5 \times 10^6} = \frac{s}{5 \times 10^6} \cdot \frac{5 \times 10^6 I_g}{s + 50}$$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s + 50} = 1 - \frac{50}{s + 50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

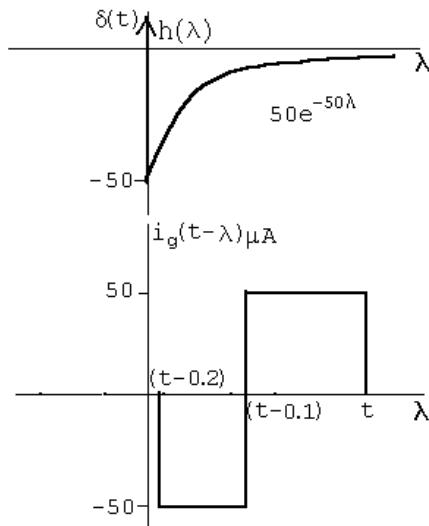
$0 < t < 0.1 \text{ s}$:

$$\begin{aligned} i_o &= \int_0^t (50 \times 10^{-6}) [\delta(\lambda) - 50e^{-50\lambda}] d\lambda \\ &= 50 \times 10^{-6} - 25 \times 10^{-3} \frac{e^{-50\lambda}}{-50} \Big|_0^t \\ &= 50 \times 10^{-6} + 50 \times 10^{-6} [e^{-50t} - 1] = 50e^{-50t} \mu\text{A} \end{aligned}$$



$$\begin{aligned}
 i_o &= \int_0^{t-0.1} (-50 \times 10^{-6}) [\delta(\lambda) - 50e^{-50\lambda}] d\lambda \\
 &\quad + \int_{t-0.1}^t (50 \times 10^{-6})(-50e^{-50\lambda}) d\lambda \\
 &= -50 \times 10^{-6} + 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50} \Big|_0^{t-0.1} - 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50} \Big|_{t-0.1}^t \\
 &= -50 \times 10^{-6} - 50 \times 10^{-6} e^{-50(t-0.1)} + 50 \times 10^{-6} \\
 &\quad + 50 \times 10^{-6} e^{-50t} - 50 \times 10^{-6} e^{-50(t-0.1)} \\
 &= 50e^{-50t} - 100e^{-50(t-0.1)} \mu A
 \end{aligned}$$

$0.2 \text{ s} < t < \infty$:



$$\begin{aligned} i_o &= \int_{t-0.2}^{t-0.1} (-50 \times 10^{-6}) (-50e^{-50\lambda}) d\lambda \\ &\quad + \int_{t-0.1}^t (50 \times 10^{-6}) (-50e^{-50\lambda}) d\lambda \\ &= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)} \mu\text{A} \end{aligned}$$

[c] At $t = 0.1^-$:

$$v_o = 5(1 - e^{-5}) = 4.97 \text{ V}; \quad i_{100\text{k}\Omega} = \frac{4.97}{0.1} = 49.66 \mu\text{A}$$

$$\therefore i_o = 50 - 49.66 = 0.34 \mu\text{A}$$

From the solution for i_o we have

$$i_o(0.1^-) = 50e^{-5} = 0.34 \mu\text{A} \quad (\text{checks})$$

At $t = 0.1^+$:

$$v_o(0.1^+) = v_o(0.1^-) = 4.97 \text{ V}$$

$$i_{100\text{k}\Omega} = 49.66 \mu\text{A}$$

$$\therefore i_o(0.1^+) = -(50 + 49.66) = -99.66 \mu\text{A}$$

From the solution for i_o we have

$$i_o(0.1^+) = 50e^{-5} - 100 = 99.66 \mu\text{A} \quad (\text{checks})$$

At $t = 0.2^-$:

$$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93 \text{ V}$$

$$i_{100k\Omega} = 49.33 \mu A$$

$$i_o = -50 + 49.33 = -0.67 \mu A$$

From the solution for i_o ,

$$v_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67 \mu A \quad (\text{checks})$$

At $t = 0.2^+$:

$$v_o(0.2^+) = v_o(0.2^-) = -4.93 V; \quad i_{100k\Omega} = -49.33 \mu A$$

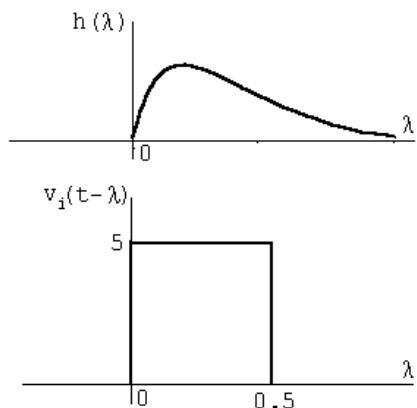
$$i_o = 0 + 49.33 = 49.33 \mu A$$

From the solution for i_o ,

$$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33 \mu A \quad (\text{checks})$$

$$\begin{aligned} \text{P 13.70 [a]} \quad H(s) &= \frac{V_o}{V_i} = \frac{1/LC}{s^2 + (R/L)s + (1/LC)} \\ &= \frac{100}{s^2 + 20s + 100} = \frac{100}{(s + 10)^2} \end{aligned}$$

$$h(\lambda) = 100\lambda e^{-10\lambda} u(\lambda)$$



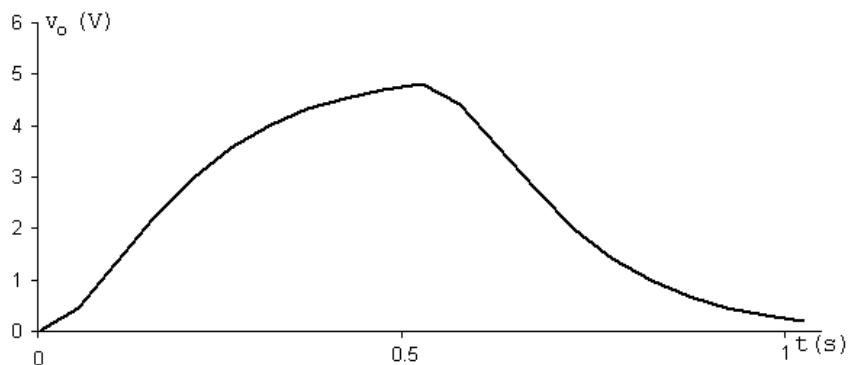
$0 \leq t \leq 0.5$:

$$\begin{aligned} v_o &= 500 \int_0^t \lambda e^{-10\lambda} d\lambda \\ &= 500 \left\{ \frac{e^{-10\lambda}}{100} (-10\lambda - 1) \Big|_0^t \right\} \\ &= 5[1 - e^{-10t}(10t + 1)] \end{aligned}$$

$0.5 \leq t \leq \infty$:

$$\begin{aligned} v_o &= 500 \int_{t-0.5}^t \lambda e^{-10\lambda} d\lambda \\ &= 500 \left\{ \frac{e^{-10\lambda}}{100} (-10\lambda - 1) \Big|_{t-0.5}^t \right\} \\ &= 5e^{-10t} [e^5(10t - 4) - 10t - 1] \end{aligned}$$

[b]

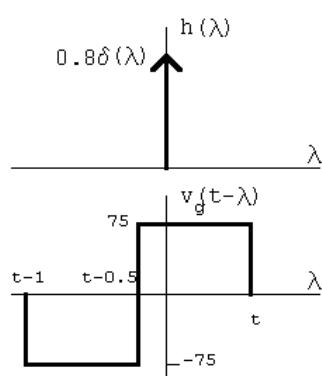


P 13.71 [a] $V_o = \frac{16}{20} V_g$

$$\therefore H(s) = \frac{V_o}{V_g} = \frac{4}{5}$$

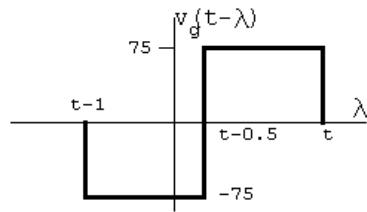
$$h(\lambda) = 0.8\delta(\lambda)$$

[b]



$$0 < t < 0.5 \text{ s} : \quad v_o = \int_0^t 75[0.8\delta(\lambda)] d\lambda = 60 \text{ V}$$

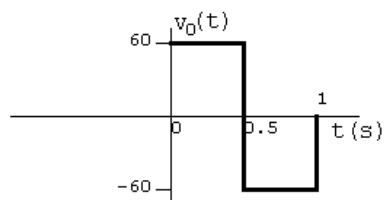
$0.5 \text{ s} \leq t \leq 1.0 \text{ s}$:



$$v_o = \int_0^{t-0.5} -75[0.8\delta(\lambda)] d\lambda = -60 \text{ V}$$

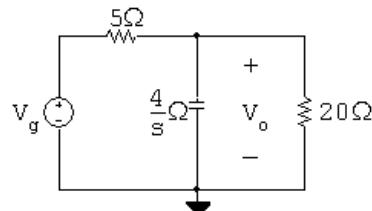
$$1 \text{ s} < t < \infty : \quad v_o = 0$$

[c]



Yes, because the circuit has no memory.

P 13.72 [a]

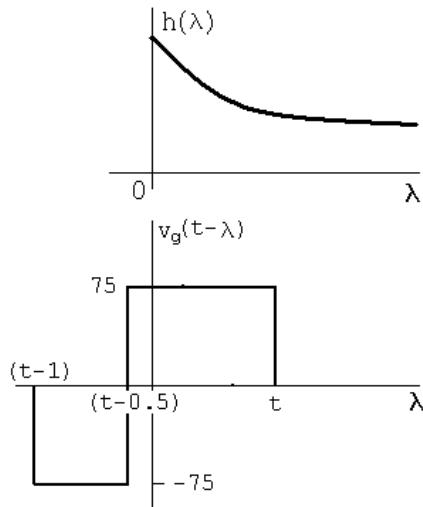


$$\frac{V_o - V_g}{5} + \frac{V_o s}{4} + \frac{V_o}{20} = 0$$

$$(5s + 5)V_o = 4V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.8}{s + 1}; \quad h(\lambda) = 0.8e^{-\lambda}u(\lambda)$$

[b]

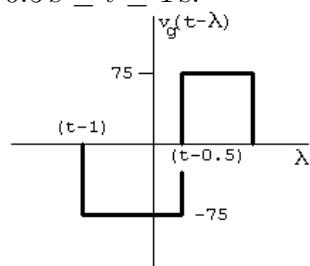


$$0 \leq t \leq 0.5 \text{ s};$$

$$v_o = \int_0^t 75(0.8e^{-\lambda}) d\lambda = 60 \frac{e^{-\lambda}}{-1} \Big|_0^t$$

$$v_o = 60 - 60e^{-t} \text{ V}, \quad 0 \leq t \leq 0.5 \text{ s}$$

$$0.5 \text{ s} \leq t \leq 1 \text{ s}: \quad$$



$$v_o = \int_0^{t-0.5} (-75)(0.8e^{-\lambda}) d\lambda + \int_{t-0.5}^t 75(0.8e^{-\lambda}) d\lambda$$

$$= -60 \frac{e^{-\lambda}}{-1} \Big|_0^{t-0.5} + 60 \frac{e^{-\lambda}}{-1} \Big|_{t-0.5}^t$$

$$= 120e^{-(t-0.5)} - 60e^{-t} - 60 \text{ V}, \quad 0.5 \text{ s} \leq t \leq 1 \text{ s}$$

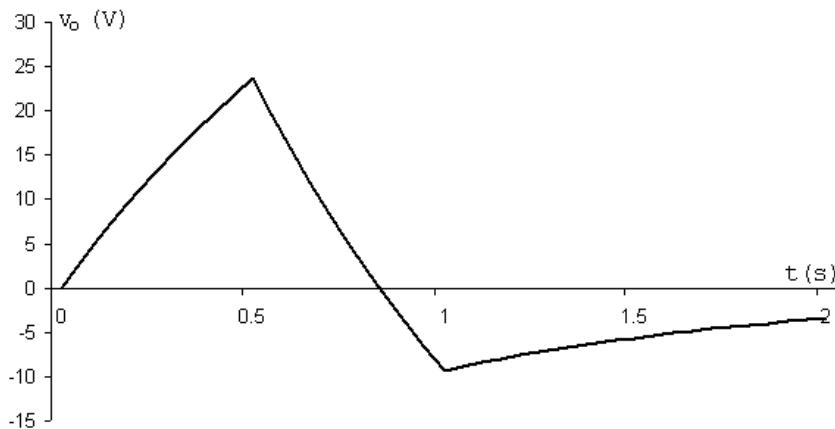
$$1 \text{ s} \leq t \leq \infty;$$

$$v_o = \int_{t-1}^{t-0.5} (-75)(0.8e^{-\lambda}) d\lambda + \int_{t-0.5}^t 75(0.8e^{-\lambda}) d\lambda$$

$$= -60 \frac{e^{-\lambda}}{-1} \Big|_{t-1}^{t-0.5} + 60 \frac{e^{-\lambda}}{-1} \Big|_{t-0.5}^t$$

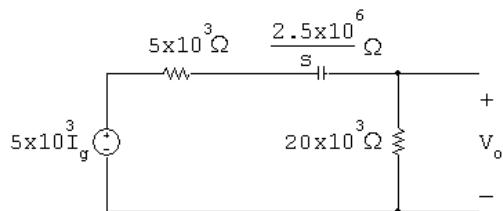
$$= 120e^{-(t-0.5)} - 60e^{-(t-1)} - 60e^{-t} \text{ V}, \quad 1 \text{ s} \leq t \leq \infty$$

[c]



[d] No, the circuit has memory because of the capacitive storage element.

P 13.73



$$V_o = \frac{5 \times 10^3 I_g}{25 \times 10^3 + 2.5 \times 10^6/s} (20 \times 10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s + 100}$$

$$H(s) = 4000 \left[1 - \frac{100}{s + 100} \right] = 4000 - \frac{4 \times 10^5}{s + 100}$$

$$h(t) = 4000\delta(t) - 4 \times 10^5 e^{-100t}$$

$$\begin{aligned}
 v_o &= \int_0^{10^{-3}} (-20 \times 10^{-3}) [4000\delta(\lambda) - 4 \times 10^5 e^{-100\lambda}] d\lambda \\
 &\quad + \int_{10^{-3}}^{5 \times 10^{-3}} (10 \times 10^{-3}) [-4 \times 10^5 e^{-100\lambda}] d\lambda \\
 &= -80 + 8000 \int_0^{10^{-3}} e^{-100\lambda} d\lambda - 4000 \int_{10^{-3}}^{5 \times 10^{-3}} e^{-100\lambda} d\lambda \\
 &= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1}) \\
 &= 40e^{-0.5} - 120e^{-0.1} = -84.32 \text{ V}
 \end{aligned}$$

Alternate:

$$I_g = \int_0^{4 \times 10^{-3}} (10 \times 10^{-3}) e^{-st} dt + \int_{4 \times 10^{-3}}^{6 \times 10^{-3}} (-20 \times 10^{-3}) e^{-st} dt$$

$$= \left[\frac{10}{s} - \frac{30}{s} e^{-4 \times 10^{-3}s} + \frac{20}{s} e^{-6 \times 10^{-3}s} \right] \times 10^{-3}$$

$$V_o = I_g H(s) = \frac{40}{s+100} [1 - 3e^{-4 \times 10^{-3}s} + 2e^{-6 \times 10^{-3}s}]$$

$$= \frac{40}{s+100} - \frac{120e^{-4 \times 10^{-3}s}}{s+100} + \frac{80e^{-6 \times 10^{-3}s}}{s+100}$$

$$\begin{aligned} v_o(t) &= 40e^{-100t} - 120e^{-100(t-4 \times 10^{-3})} u(t - 4 \times 10^{-3}) \\ &\quad + 80e^{-100(t-6 \times 10^{-3})} u(t - 6 \times 10^{-3}) \end{aligned}$$

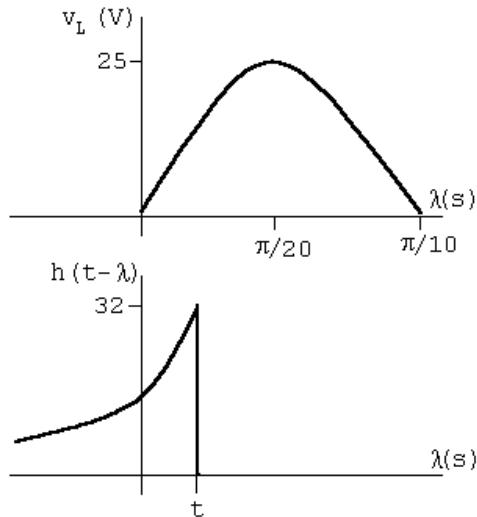
$$v_o(5 \times 10^{-3}) = 40e^{-0.5} - 120e^{-0.1} + 80(0) = -84.32 \text{ V} \quad (\text{checks})$$

$$\text{P 13.74 } v_i = 25 \sin 10\lambda [u(\lambda) - u(\lambda - \pi/10)]$$

$$H(s) = \frac{32}{s+32}$$

$$h(\lambda) = 32e^{-32\lambda}$$

$$h(t-\lambda) = 32e^{-32(t-\lambda)} = 32e^{-32t}e^{32\lambda}$$



$$\begin{aligned}
 v_o &= 800e^{-32t} \int_0^t e^{32\lambda} \sin 10\lambda d\lambda \\
 &= 800e^{-32t} \left[\frac{e^{32\lambda}}{32^2 + 10^2} (32 \sin 10\lambda - 10 \cos 10\lambda) \Big|_0^t \right] \\
 &= \frac{800e^{-32t}}{1124} [e^{32t} (32 \sin 10t - 10 \cos 10t) + 10] \\
 &= \frac{800}{1124} [32 \sin 10t - 10 \cos 10t + 10e^{-32t}]
 \end{aligned}$$

$$v_o(0.075) = 10.96 \text{ V}$$

$$\begin{aligned}
 \text{P 13.75 [a]} \quad Y(s) &= \int_0^\infty y(t)e^{-st} dt \\
 Y(s) &= \int_0^\infty e^{-st} \left[\int_0^\infty h(\lambda)x(t-\lambda) d\lambda \right] dt \\
 &= \int_0^\infty \int_0^\infty e^{-st} h(\lambda)x(t-\lambda) d\lambda dt \\
 &= \int_0^\infty h(\lambda) \int_0^\infty e^{-st} x(t-\lambda) dt d\lambda
 \end{aligned}$$

But $x(t-\lambda) = 0$ when $t < \lambda$.

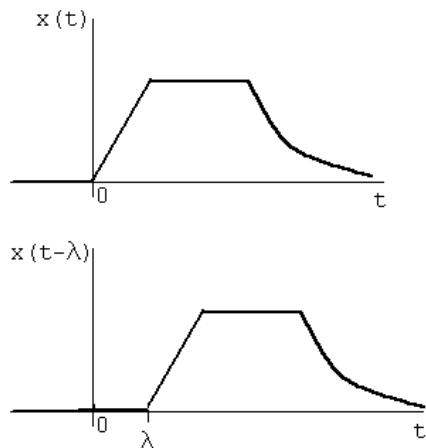
$$\text{Therefore } Y(s) = \int_0^\infty h(\lambda) \int_\lambda^\infty e^{-st} x(t-\lambda) dt d\lambda$$

Let $u = t - \lambda$; $du = dt$; $u = 0$ when $t = \lambda$; $u = \infty$ when $t = \infty$.

$$\begin{aligned}
 Y(s) &= \int_0^\infty h(\lambda) \int_0^\infty e^{-s(u+\lambda)} x(u) du d\lambda \\
 &= \int_0^\infty h(\lambda) e^{-s\lambda} \int_0^\infty e^{-su} x(u) du d\lambda \\
 &= \int_0^\infty h(\lambda) e^{-s\lambda} X(s) d\lambda = H(s) X(s)
 \end{aligned}$$

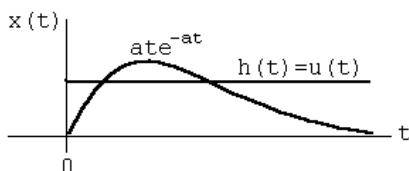
Note on $x(t-\lambda) = 0$, $t < \lambda$

We are using one-sided Laplace transforms; therefore $h(t)$ and $x(t)$ are assumed zero for $t < 0$.



$$[\mathbf{b}] \quad F(s) = \frac{a}{s(s+a)^2} = \frac{1}{s} \cdot \frac{a}{(s+a)^2} = H(s)X(s)$$

$$\therefore h(t) = u(t), \quad x(t) = at e^{-at} u(t)$$



$$\begin{aligned}
 \therefore f(t) &= \int_0^t (1) a \lambda e^{-a\lambda} d\lambda = a \left[\frac{e^{-a\lambda}}{a^2} (-a\lambda - 1) \right]_0^t \\
 &= \frac{1}{a} [e^{-at}(-at - 1) - 1(-1)] = \frac{1}{a} [1 - e^{-at} - ate^{-at}] \\
 &= \left[\frac{1}{a} - \frac{1}{a} e^{-at} - te^{-at} \right] u(t)
 \end{aligned}$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \quad K_1 = -1; \quad K_2 = \frac{d}{ds} \left(\frac{a}{s} \right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a} e^{-at} \right] u(t)$$

P 13.76 [a] $H(s) = \frac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_f C_f)} = \frac{10^7}{s + 5000}$$

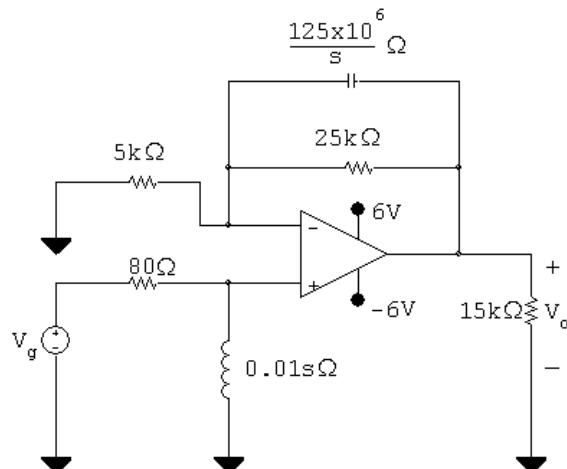
$$Z_i = \frac{R_i[s + (1/R_i C_i)]}{s} = \frac{200(s + 2000)}{s}$$

$$H(s) = \frac{-50,000s}{(s + 2000)(s + 5000)}$$

[b] $H(j3000) = \frac{-50,000(j3000)}{(2000 + j3000)(5000 + j3000)} = 7.135/-177.27^\circ$

$$v_o(t) = 7.135 \cos(3000t - 177.27^\circ) V$$

P 13.77 [a]



$$V_p = \frac{0.01s}{80 + 0.01s} V_g = \frac{s}{s + 8000} V_g$$

$$V_n = V_p$$

$$\frac{V_n}{5000} + \frac{V_n - V_o}{25,000} + (V_n - V_o)8 \times 10^{-9}s = 0$$

$$5V_n + V_n - V_o + (V_n - V_o)2 \times 10^{-4}s = 0$$

$$6V_n + 2 \times 10^{-4}sV_n = V_o + 2 \times 10^{-4}sV_o$$

$$2 \times 10^{-4}V_n(s + 30,000) = 2 \times 10^{-4}V_o(s + 5000)$$

$$V_o = \frac{s + 30,000}{s + 5000} V_i = \left(\frac{s + 30,000}{s + 5000} \right) \left(\frac{sV_g}{s + 8000} \right)$$

$$H(s) = \frac{V_o}{V_g} = \frac{s(s + 30,000)}{(s + 5000)(s + 8000)}$$

[b] $v_g = 0.6u(t); \quad V_g = \frac{0.6}{s}$

$$V_o = \frac{0.6(s + 30,000)}{(s + 5000)(s + 8000)} = \frac{K_1}{s + 5000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{0.6(25,000)}{3000} = 5; \quad K_2 = \frac{0.6(22,000)}{-3000} = -4.4$$

$$\therefore v_o(t) = (5e^{-5000t} - 4.4e^{-8000t})u(t) \text{ V}$$

[c] $V_g = 2 \cos 10,000t \text{ V}$

$$H(j\omega) = \frac{j10,000(30,000 + j10,000)}{(5000 + j10,000)(8000 + j10,000)} = 2.21/\underline{-6.34^\circ}$$

$$\therefore v_o = 4.42 \cos(10,000t - 6.34^\circ) \text{ V}$$

P 13.78 $H(j20) = \frac{25(8 + j20)}{-400 + j1200 + 150} = 0.44/\underline{-33.57^\circ}$

$$\therefore v_o(t) = 4.4 \cos(20t - 33.57^\circ) \text{ V}$$

P 13.79 $H(j500) = \frac{125(400 + j500)}{(j500)(10^4 - 500^2 + j10^5)} = 0.6157 \times 10^{-3}/\underline{163.96^\circ}$

$$\therefore i_o(t) = 49.3 \cos(500t + 163.96^\circ) \text{ mA}$$

P 13.80 $V_o = \frac{50}{s + 8000} - \frac{20}{s + 5000} = \frac{30(s + 3000)}{(s + 5000)(s + 8000)}$

$$V_o = H(s)V_g = H(s) \left(\frac{30}{s} \right)$$

$$\therefore H(s) = \frac{s(s + 3000)}{(s + 5000)(s + 8000)}$$

$$H(j6000) = \frac{(j6000)(3000 + j6000)}{(5000 + j6000)(8000 + j6000)} = 0.52/\underline{66.37^\circ}$$

$$\therefore v_o(t) = 61.84 \cos(6000t + 66.37^\circ) \text{ V}$$

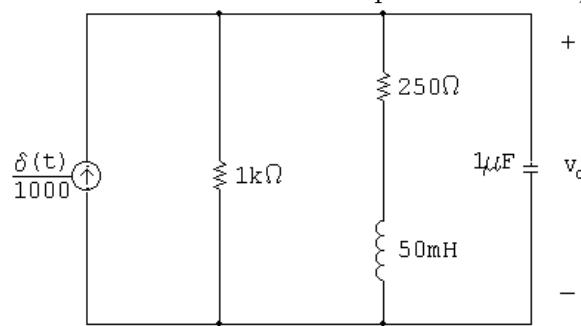
P 13.81 Original charge on C_1 ; $q_1 = V_0 C_1$

$$\text{The charge transferred to } C_2; \quad q_2 = V_0 C_e = \frac{V_0 C_1 C_2}{C_1 + C_2}$$

$$\text{The charge remaining on } C_1; \quad q'_1 = q_1 - q_2 = \frac{V_0 C_1^2}{C_1 + C_2}$$

$$\text{Therefore } V_2 = \frac{q_2}{C_2} = \frac{V_0 C_1}{C_1 + C_2} \quad \text{and} \quad V_1 = \frac{q'_1}{C_1} = \frac{V_0 C_1}{C_1 + C_2}$$

P 13.82 [a] After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



$$\text{Therefore } v_o(0^+) = 10^6 \int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000} \right] dt = 1000 \text{ V}$$

$$\text{Therefore } w_C = (0.5)Cv^2 = 0.5 \text{ J}$$

$$[\mathbf{b}] \quad i_L(0^+) = 0; \quad \text{therefore } w_L = 0 \text{ J}$$

$$[\mathbf{c}] \quad V_o(10^{-6})s + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Therefore

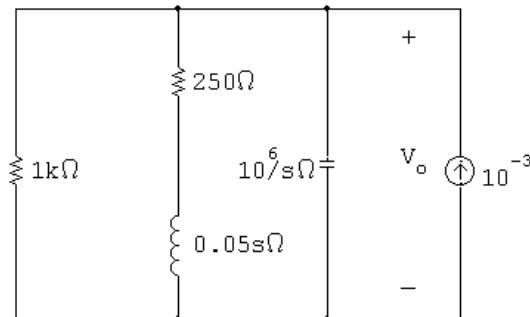
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$$K_1 = 559.02/-26.57^\circ; \quad K_1^* = 559.02/26.57^\circ$$

$$v_o = [1118.03e^{-3000t} \cos(4000t - 26.57^\circ)]u(t) \text{ V}$$

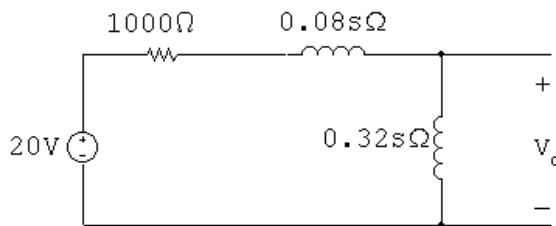
[d] The s -domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.83 [a]



$$\begin{aligned} V_o &= \frac{20}{1000 + 0.4s} \cdot 0.32s \\ &= \frac{16s}{s + 2500} = 16 - \frac{40,000}{s + 2500} \end{aligned}$$

$$v_o(t) = 16\delta(t) - 40,000e^{-2500t}u(t) \text{ V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

$$i_L(0) = \frac{10^3}{400} \int_{0^-}^{0^+} 20\delta(t) dt = 50 \text{ A}$$

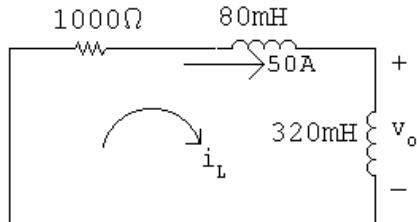
It follows that since $i_L(0^-) = 0$,

$$\frac{di_L}{dt}(0) = 50\delta(t)$$

$$\therefore v_o(0) = (0.32)(50\delta(t)) = 16\delta(t)$$

This agrees with our solution.

At $t = 0^+$ our circuit is



$$\therefore i_L(t) = 50e^{-t/\tau} \text{ A}, \quad t \geq 0^+$$

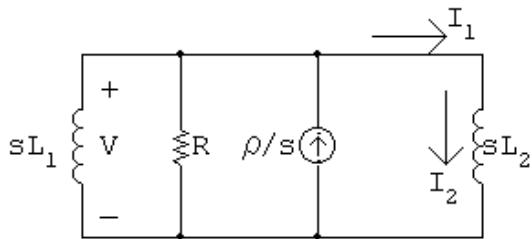
$$\tau = L/R = 0.4 \text{ ms}$$

$$\therefore i_L(t) = 50e^{-2500t} \text{ A}, \quad t \geq 0^+$$

$$v_o(t) = 0.32 \frac{di_L}{dt} = -40,000e^{-2500t} \text{ V}, \quad t \geq 0^+$$

which agrees with our solution.

P 13.84 [a] The s -domain circuit is



$$\text{The node-voltage equation is } \frac{V}{sL_1} + \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho}{s}$$

$$\text{Therefore } V = \frac{\rho R}{s + (R/L_e)} \quad \text{where } L_e = \frac{L_1 L_2}{L_1 + L_2}$$

$$\text{Therefore } v = \rho R e^{-(R/L_e)t} u(t) \text{ V}$$

$$[b] I_1 = \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho[s + (R/L_e)]}{s[s + (R/L_e)]} = \frac{K_0}{s} + \frac{K_1}{s + (R/L_e)}$$

$$K_0 = \frac{\rho L_1}{L_1 + L_2}; \quad K_1 = \frac{\rho L_2}{L_1 + L_2}$$

$$\text{Thus we have } i_1 = \frac{\rho}{L_1 + L_2} [L_1 + L_2 e^{-(R/L_e)t}] u(t) \text{ A}$$

$$[c] I_2 = \frac{V}{sL_2} = \frac{(\rho R/L_2)}{s[s + (R/L_e)]} = \frac{K_2}{s} + \frac{K_3}{s + (R/L_e)}$$

$$K_2 = \frac{\rho L_1}{L_1 + L_2}; \quad K_3 = \frac{-\rho L_1}{L_1 + L_2}$$

Therefore $i_2 = \frac{\rho L_1}{L_1 + L_2} [1 - e^{-(R/L_e)t}] u(t)$

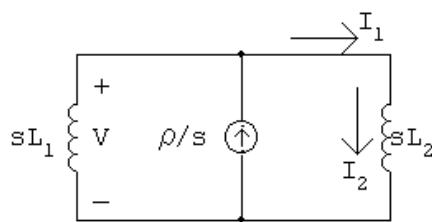
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.85 [a] As $R \rightarrow \infty$, $v(t) \rightarrow \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e .

$$i_1(t) \rightarrow \frac{\rho L_1}{L_1 + L_2} u(t) \text{ A as } R \rightarrow \infty$$

$$i_2(t) \rightarrow \frac{\rho L_1}{L_1 + L_2} u(t) \text{ A as } R \rightarrow \infty$$

[b] The s -domain circuit is



$$\frac{V}{sL_1} + \frac{V}{sL_2} = \frac{\rho}{s}; \quad \text{therefore } V = \frac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$$

$$\text{Therefore } v(t) = \rho L_e \delta(t)$$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2} \right) \left(\frac{1}{s} \right)$$

$$\text{Therefore } i_1 = i_2 = \frac{\rho L_1}{L_1 + L_2} u(t) \text{ A}$$

P 13.86 [a] $Z_1 = \frac{1/C_1}{s + 1/R_1 C_1} = \frac{25 \times 10^{10}}{s + 20 \times 10^4} \Omega$

$$Z_2 = \frac{1/C_2}{s + 1/R_2 C_2} = \frac{6.25 \times 10^{10}}{s + 12,500} \Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s + 12,500)}{6.25 \times 10^{10}} + \frac{V_0(s + 20 \times 10^4)}{25 \times 10^{10}} = \frac{10}{s} \frac{(s + 20 \times 10^4)}{25 \times 10^{10}}$$

$$V_0 = \frac{2(s + 200,000)}{s(s + 50,000)} = \frac{K_1}{s} + \frac{K_2}{s + 50,000}$$

$$K_1 = \frac{2(200,000)}{50,000} = 8$$

$$K_2 = \frac{2(150,000)}{-50,000} = -6$$

$$\therefore v_o = [8 - 6e^{-50,000t}]u(t) \text{ V}$$

$$\begin{aligned} [\mathbf{b}] \quad I_0 &= \frac{V_0}{Z_2} = \frac{2(s + 200,000)(s + 12,500)}{s(s + 50,000)6.25 \times 10^{10}} \\ &= 32 \times 10^{-12} \left[1 + \frac{162,500s + 25 \times 10^8}{s(s + 50,000)} \right] \\ &= 32 \times 10^{-12} \left[1 + \frac{K_1}{s} + \frac{K_2}{s + 50,000} \right] \end{aligned}$$

$$K_1 = 50,000; \quad K_2 = 112,500$$

$$i_o = 32\delta(t) + [1.6 \times 10^6 + 3.6 \times 10^6 e^{-50,000t}]u(t) \text{ pA}$$

[c] When $C_1 = 64 \text{ pF}$

$$Z_1 = \frac{156.25 \times 10^8}{s + 12,500} \Omega$$

$$\frac{V_0(s + 12,500)}{625 \times 10^8} + \frac{V_0(s + 12,500)}{156.25 \times 10^8} = \frac{10}{s} \frac{(s + 12,500)}{156.25 \times 10^8}$$

$$\therefore V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$$v_o = 8u(t) \text{ V}$$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s} \frac{(s + 12,500)}{6.25 \times 10^{10}} = 128 \times 10^{-12} \left[1 + \frac{12,500}{s} \right]$$

$$i_o(t) = 128\delta(t) + 1.6 \times 10^6 u(t) \text{ pA}$$

$$\text{P 13.87 Let } a = \frac{1}{R_1 C_1} = \frac{1}{R_2 C_2}$$

$$\text{Then } Z_1 = \frac{1}{C_1(s + a)} \quad \text{and} \quad Z_2 = \frac{1}{C_2(s + a)}$$

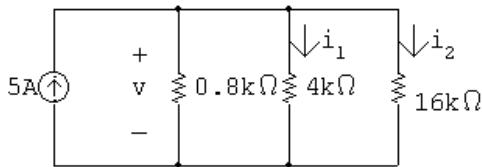
$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_o C_2(s + a) + V_0 C_1(s + a) = (10/s) C_1(s + a)$$

$$V_o = \frac{10}{s} \left(\frac{C_1}{C_1 + C_2} \right)$$

Thus, v_o is the input scaled by the factor $\frac{C_1}{C_1 + C_2}$.

P 13.88 [a] For $t < 0$:



$$R_{eq} = 0.8 \text{ k}\Omega \| 4 \text{ k}\Omega \| 16 \text{ k}\Omega = 0.64 \text{ k}\Omega; \quad v = 5(640) = 3200 \text{ V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8 \text{ A}; \quad i_2(0^-) = \frac{3200}{1600} = 0.2 \text{ A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

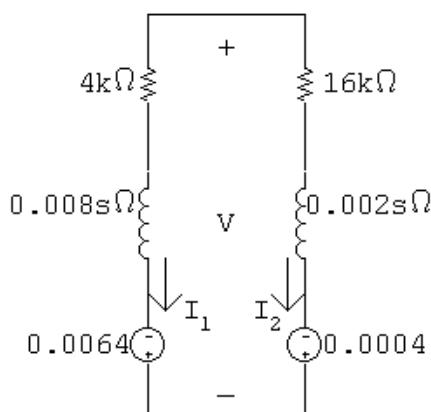
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \quad \text{therefore } \Delta i_1 = -0.2 \text{ A}$$

$$\Delta i_2 = -0.8 \text{ A}; \quad i_1(0^+) = 0.8 - 0.2 = 0.6 \text{ A}$$

[c] $i_2(0^-) = 0.2 \text{ A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6 \text{ A}$

[e] The s -domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20,000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2 \times 10^6 t} u(t) \text{ A}$$

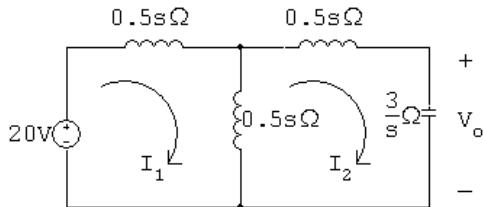
$$[\mathbf{f}] \quad i_2(t) = -i_1(t) = -0.6e^{-2 \times 10^6 t} u(t) \text{ A}$$

$$[\mathbf{g}] \quad V = -0.0064 + (0.008s + 4000)I_1 = \frac{-0.0016(s + 6.5 \times 10^6)}{s + 2 \times 10^6}$$

$$= -1.6 \times 10^{-3} - \frac{7200}{s + 2 \times 10^6}$$

$$v(t) = [-1.6 \times 10^{-3} \delta(t)] - [7200e^{-2 \times 10^6 t} u(t)] \text{ V}$$

P 13.89 [a]



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s + 3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2 + 4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s + 3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2 + 60}{s} = \frac{20(s^2 + 3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2 + 3)}{s(0.75)(s^2 + 4)} = \frac{80}{3} \cdot \frac{s^2 + 3}{s(s^2 + 4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

$$K_0 = \frac{80}{3} \left(\frac{3}{4}\right) = 20; \quad K_1 = \frac{80}{3} \left[\frac{-4 + 3}{(j2)(j4)} \right] = \frac{10}{3} / 0^\circ$$

$$\therefore i_1 = \left[20 + \frac{20}{3} \cos 2t \right] u(t) \text{ A}$$

$$[\mathbf{b}] \quad N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2 + 4)} = \frac{40}{3} \left(\frac{s}{s^2 + 4} \right) = \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

$$K_1 = \frac{40}{3} \left(\frac{j2}{j4} \right) = \frac{20}{3} / 0^\circ$$

$$i_2 = \left[\frac{40}{3} \cos 2t \right] u(t) \text{ A}$$

[c] $V_0 = \frac{3}{s} I_2 = \left(\frac{3}{s} \right) \frac{40}{3} \left(\frac{s}{s^2 + 4} \right) = \frac{40}{s^2 + 4} = \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$

$$K_1 = \frac{40}{j4} = -j10 = 10 / 90^\circ$$

$$v_o = 20 \cos(2t - 90^\circ) = 20 \sin 2t$$

$$v_o = [20 \sin 2t] u(t) \text{ V}$$

- [d] Let us begin by noting i_1 jumps from 0 to $(80/3)$ A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to $(40/3)$ A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3} \sin 2t \text{ A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3} \sin 2t \text{ A/s}$$

From the circuit diagram we have

$$\begin{aligned} 20\delta(t) &= 1 \frac{di_1}{dt} - 0.5 \frac{di_2}{dt} \\ &= \frac{80}{3}\delta(t) - \frac{40}{3} \sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3} \sin 2t \\ &= 20\delta(t) \end{aligned}$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.

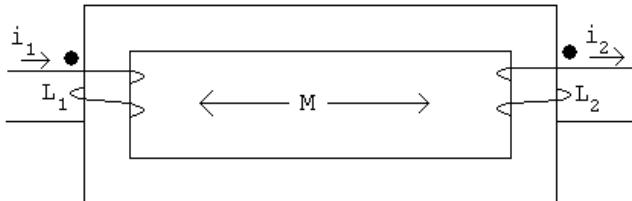
Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate. Thus the fact that i_1 , i_2 , and v_o exist for all time is consistent with known circuit behavior.

Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.

Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20. For the given polarity dots and reference directions of i_1 and i_2 we have

$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

$$\begin{aligned}\lambda(0^+) &= 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right) \\ &= \frac{120}{3} - \frac{60}{3} = 20 \quad (\text{checks})\end{aligned}$$



P 13.90 [a] For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$$v_1 + v_2 = 100; \quad \text{therefore } v_1(0^-) = 80 \text{ V}$$

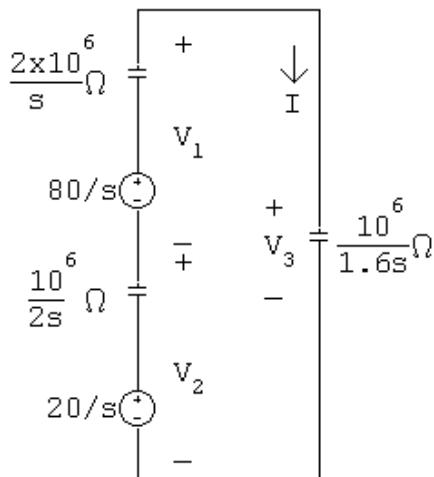
[b] $v_2(0^-) = 20 \text{ V}$

[c] $v_3(0^-) = 0 \text{ V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$$i(t) = 32\delta(t) \mu\text{A}$$



$$[e] v_1(0^+) = -\frac{10^6}{0.5} \int_{0^-}^{0^+} 32 \times 10^{-6} \delta(t) dt + 80 = -64 + 80 = 16 \text{ V}$$

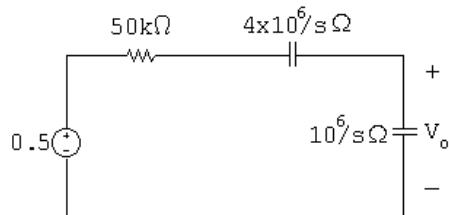
$$[f] v_2(0^+) = -\frac{10^6}{2} \int_{0^-}^{0^+} 32 \times 10^{-6} \delta(t) dt + 20 = -16 + 20 = 4 \text{ V}$$

$$[g] V_3 = \frac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \frac{20}{s}$$

$$v_3(t) = 20u(t) \text{ V}; \quad v_3(0^+) = 20 \text{ V}$$

$$\text{Check: } v_1(0^+) + v_2(0^+) = v_3(0^+)$$

P 13.91 [a]



$$V_o = \frac{0.5}{50,000 + 5 \times 10^6/s} \cdot \frac{10^6}{s}$$

$$\frac{500,000}{50,000s + 5 \times 10^6} = \frac{10}{s + 100}$$

$$v_o = 10e^{-100t}u(t) \text{ V}$$

[b] At $t = 0$ the current in the $1\mu\text{F}$ capacitor is $10\delta(t)\mu\text{A}$

$$\therefore v_o(0^+) = 10^6 \int_{0^-}^{0^+} 10 \times 10^{-6}\delta(t) dt = 10 \text{ V}$$

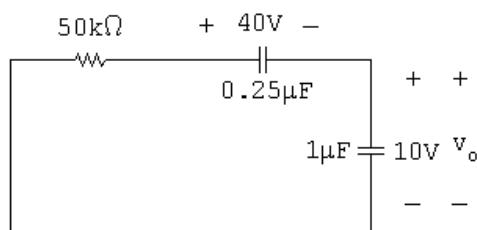
After the impulsive current has charged the $1\mu\text{F}$ capacitor to 10 V it discharges through the $50\text{ k}\Omega$ resistor.

$$C_e = \frac{C_1 C_2}{C_1 + C_2} = \frac{0.25}{1.25} = 0.2\mu\text{F}$$

$$\tau = (50,000)(0.2 \times 10^{-6}) = 10^{-2}$$

$$\frac{1}{\tau} = 100 \text{ (checks)}$$

Note – after the impulsive current passes the circuit becomes



The solution for v_o in this circuit is also

$$v_o = 10e^{-100t}u(t) \text{ V}$$

P 13.92 [a] The circuit parameters are

$$R_a = \frac{120^2}{1200} = 12\Omega \quad R_b = \frac{120^2}{1800} = 8\Omega \quad X_a = \frac{120^2}{350} = \frac{288}{7}\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120/0^\circ}{12} = 10/0^\circ \text{ A(rms)} \quad \mathbf{I}_2 = \frac{120/0^\circ}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}/-90^\circ \text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{120/\underline{0^\circ}}{8} = 15/\underline{0^\circ} \text{ A(rms)}$$

$$\therefore \mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17/\underline{-6.65^\circ} \text{ A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2} \cos(\omega t - 90^\circ) \text{ A} \quad \text{and} \quad i_o = 25.17\sqrt{2} \cos(\omega t - 6.65^\circ) \text{ A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0 \text{ A} \quad \text{and} \quad i_o(0^-) = i_o(0^+) = 25\sqrt{2} \text{ A}$$

- [b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically. Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore V_0 = \frac{(R_a/L_\ell)V_g + I_o R_a}{s + [R_a(L_a + L_\ell)]/L_a L_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12 \Omega$, and $I_0 R_a = 300\sqrt{2}$ V. Thus,

$$\begin{aligned} V_0 &= \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14,400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi} \\ &= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi} \end{aligned}$$

The coefficients are

$$K_1 = -121.18\sqrt{2} \text{ V} \quad K_2 = 61.03\sqrt{2}/\underline{6.85^\circ} \text{ V} \quad K_2^* = 61.03\sqrt{2}/\underline{-6.85^\circ}$$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2} \cos(120\pi t + 6.85^\circ) \text{ V}$$

Initially,

$$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2} \cos 6.85^\circ = 300\sqrt{2} \text{ V}$$

Note that at $t = 0^+$ the initial value of i_L , which is $25\sqrt{2}$ A, exists in the 12Ω resistor R_a . Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

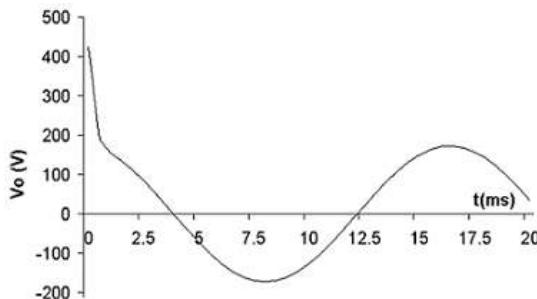
- [c] The phasor domain equivalent circuit has a $j1 \Omega$ inductive impedance in series with the parallel combination of a 12Ω resistive impedance and a $j1440/35 \Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120/\underline{0^\circ} + (25.17/\underline{-6.65^\circ})(j1) = 125.43/\underline{11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43/\underline{11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

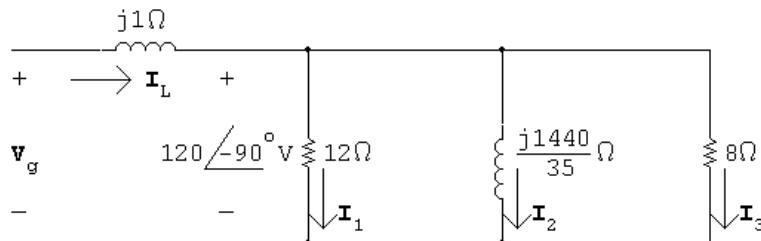
$$\therefore \mathbf{V}_o = 122.06 / 6.85^\circ \text{ V(rms)}$$

Therefore, $v_0 = 122.06\sqrt{2} \cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

- [d] A plot of v_0 , generated in Excel, is shown below.



- P 13.93 [a] At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10 / -90^\circ \text{ A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12} / 180^\circ \text{ A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15 / -90^\circ \text{ A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17 / -96.65^\circ \text{ A (rms)}$$

$$i_L = 25.17\sqrt{2} \cos(120\pi t - 96.65^\circ) \text{ A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2} \text{ A}$$

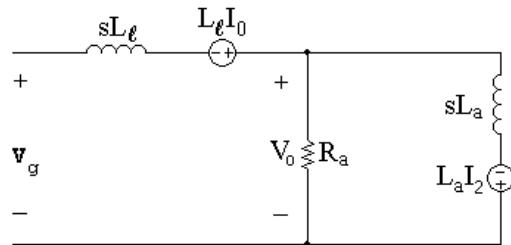
$$i_2 = \frac{35}{12}\sqrt{2} \cos(120\pi t + 180^\circ) \text{ A}$$

$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2} \text{ A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\begin{aligned}
 \mathbf{V}_g &= -j120 + 25 - j\frac{35}{12} \\
 &= 25 - j122.92 = 125.43 \angle -78.50^\circ \text{ V (rms)} \\
 v_g &= 125.43\sqrt{2} \cos(120\pi t - 78.50^\circ) \text{ V} \\
 &= 125.43\sqrt{2}[\cos 120\pi t \cos 78.50^\circ + \sin 120\pi t \sin 78.50^\circ] \\
 &= 25\sqrt{2} \cos 120\pi t + 122.92\sqrt{2} \sin 120\pi t \\
 \therefore V_g &= \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}
 \end{aligned}$$

s-domain circuit:



where

$$L_l = \frac{1}{120\pi} \text{ H}; \quad L_a = \frac{12}{35\pi} \text{ H}; \quad R_a = 12 \Omega$$

$$i_L(0) = -2.92\sqrt{2} \text{ A}; \quad i_2(0) = -2.92\sqrt{2} \text{ A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_g R_a / L_l}{[s + R_a(L_l + L_a)/L_a L_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_l L_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_l L_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\begin{aligned}
 \therefore V_o &= \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]} K_2^* \\
 &= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}
 \end{aligned}$$

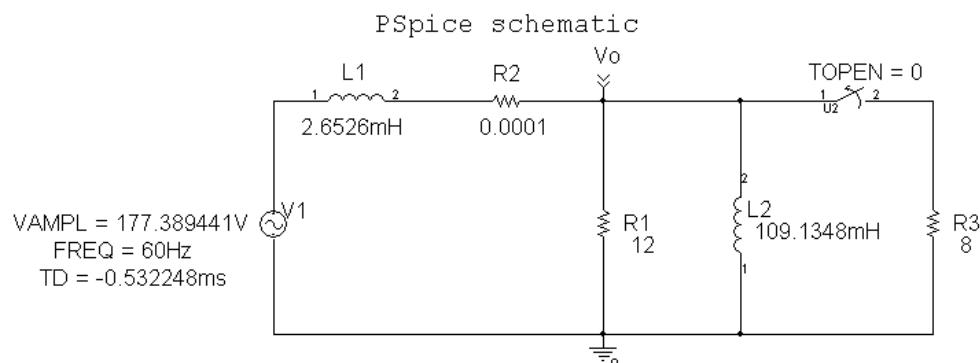
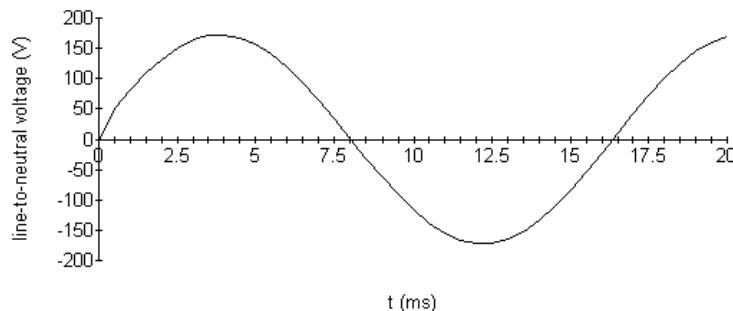
$$K_1 = -14.55\sqrt{2} \quad K_2 = 61.03\sqrt{2}/-83.15^\circ$$

$$\therefore v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$$

Check:

$$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$$

[b]



PSpice output file

```
***** 07/15/01 07:40:45 ***** PSpice Lite (Mar 2000) *****
** Profile: "SCHEMATIC1-tran" [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]
***** CIRCUIT DESCRIPTION
*****
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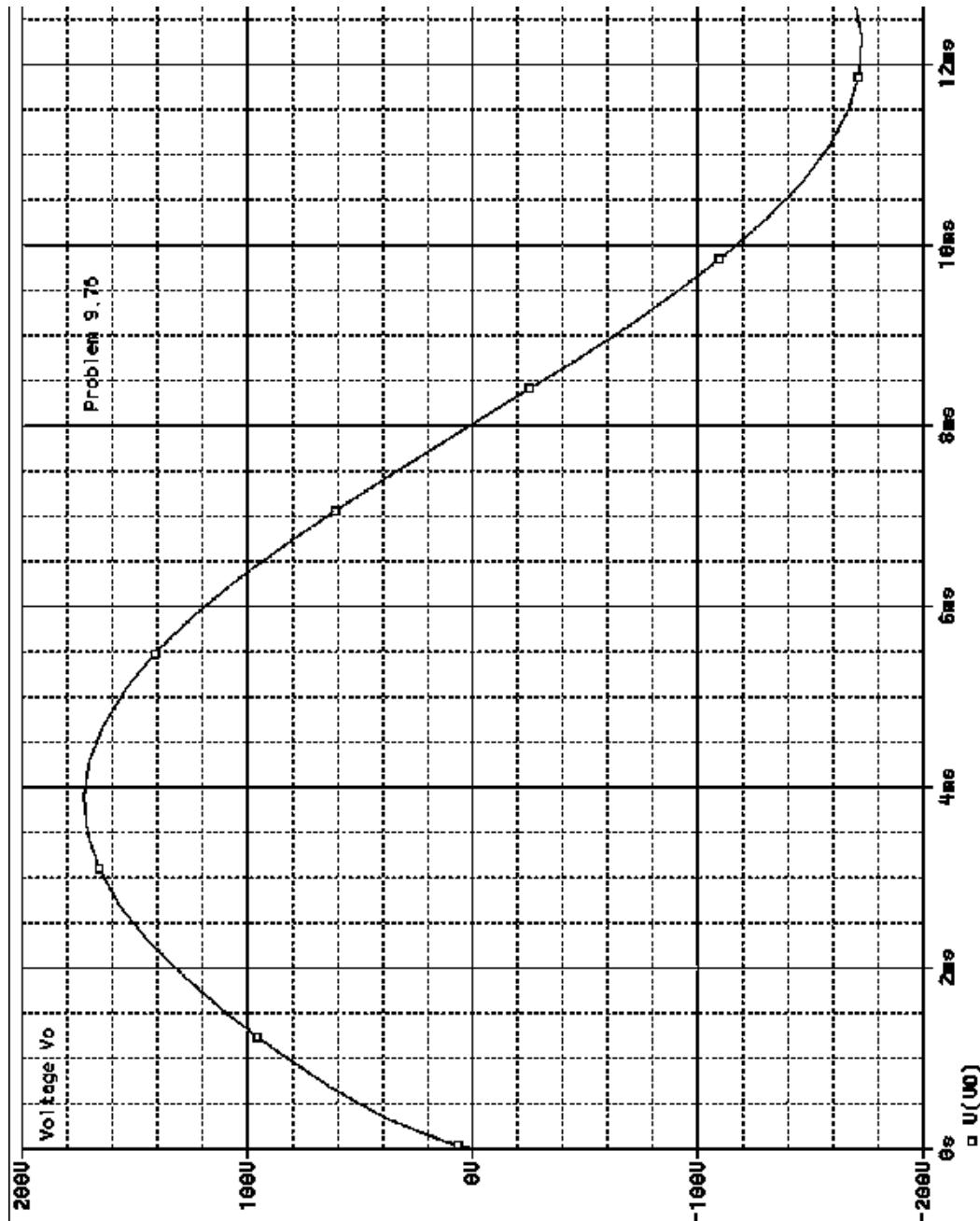
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** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN 0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"

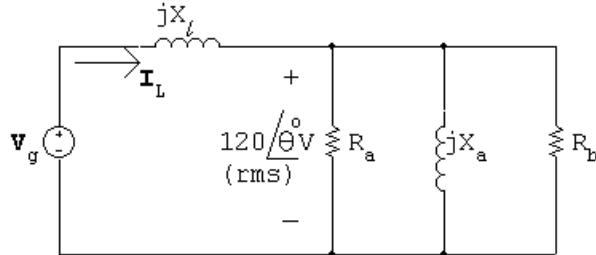
***** INCLUDING p9_76-SCHEMATIC1.net *****
* source P9_76
V_V1      N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1      N00637 N01311 2.6526mH IC=0
L_L2      0 VO 109.1348mH IC=0
R_R1      0 VO 12
R_R2      VO N01311 0.0001
R_R3      0 N01959 8
K_U2      VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+ Ropen=1Meg

***** RESUMING p9_76-SCHEMATIC1-tran.sim.cir *****
.END
```



- [c] In Problem 13.92, the line-to-neutral voltage spikes at $300\sqrt{2}$ V. Here the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.94 [a] First find V_g before R_b is disconnected. The phasor domain circuit is



$$\begin{aligned}\mathbf{I}_L &= \frac{120/\theta^\circ}{R_a} + \frac{120/\theta^\circ}{R_b} + \frac{120/\theta^\circ}{jX_a} \\ &= \frac{120/\theta^\circ}{R_a R_b X_a} [(R_a + R_b) X_a = j R_a R_b]\end{aligned}$$

Since $X_L = 1\Omega$ we have

$$\mathbf{V}_g = 120/\theta^\circ + \frac{120/\theta^\circ}{R_a R_b X_a} [R_a R_b + j(R_a + R_b) X_a]$$

$$R_a = 12\Omega; \quad R_b = 8\Omega; \quad X_a = \frac{1440}{35}\Omega$$

$$\begin{aligned}\mathbf{V}_g &= \frac{120/\theta^\circ}{1400} (1475 + j300) \\ &= \frac{25}{12}/\theta^\circ (59 + j12) = 125.43/\theta^\circ\end{aligned}$$

$$v_g = 125.43\sqrt{2} \cos(120\pi t + \theta + 11.50^\circ)\text{V}$$

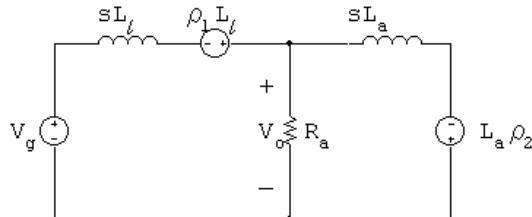
Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2} (\cos 120\pi t \cos \beta - \sin 120\pi t \sin \beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s \cos \beta - 120\pi \sin \beta)}{s^2 + (120\pi)^2}$$

The s -domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_a(0^+)$.

The s -domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{s L_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{s L_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a / L_l + (\rho_1 - \rho_2) R_a}{[s + \frac{(L_a + L_l) R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi} \text{ H}; \quad L_a = \frac{12}{35\pi} \text{ H}; \quad R_a = 12 \Omega; \quad R_b = 8 \Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2 .

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\begin{aligned} \mathbf{I}_L &= \frac{120/\theta^\circ}{R_a R_b X_a} [(R_a + R_b) X_a - j R_a R_b] \\ &= \frac{120/\theta^\circ}{96(1440/35)} \left[\frac{(20)(1440)}{35} - j96 \right] \\ &= 25.17/\underline{\theta - 6.65^\circ} \text{ A(rms)} \end{aligned}$$

$$\therefore i_L = 25.17\sqrt{2} \cos(120\pi t + \theta - 6.65^\circ) \text{ A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2} \cos(\theta - 6.65^\circ) \text{ A}$$

$$\therefore \rho_1 = 25\sqrt{2} \cos \theta + 2.92\sqrt{2} \sin \theta \text{ A}$$

$$\mathbf{I}_2 = \frac{120/\theta^\circ}{j(1440/35)} = \frac{35}{12} \underline{\theta - 90^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2} \cos(120\pi t + \theta - 90^\circ) \text{ A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2} \sin \theta = 2.92\sqrt{2} \sin \theta \text{ A}$$

$$\therefore \rho_1 = \rho_2 = 25\sqrt{2} \cos \theta$$

$$(\rho_1 - \rho_2) R_a = 300\sqrt{2} \cos \theta$$

$$\begin{aligned} \therefore V_o &= \frac{1440\pi}{s + 1475\pi} \cdot V_g + \frac{300\sqrt{2} \cos \theta}{s + 1475\pi} \\ &= \frac{1440\pi}{s + 1475\pi} \left[\frac{125.43\sqrt{2}(s \cos \beta - 120\pi \sin \beta)}{s^2 + 14,400\pi^2} \right] + \frac{300\sqrt{2} \cos \theta}{s + 1475\pi} \\ &= \frac{K_1 + 300\sqrt{2} \cos \theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} \end{aligned}$$

Now

$$\begin{aligned} K_1 &= \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi \cos \beta - 120\pi \sin \beta]}{1475^2\pi^2 + 14,400\pi^2} \\ &= \frac{-1440(125.43\sqrt{2})[1475 \cos \beta + 120 \sin \beta]}{1475^2 + 14,000} \end{aligned}$$

Since $\beta = \theta + 11.50^\circ$, K_1 reduces to

$$K_1 = -121.18\sqrt{2} \cos \theta + 14.55\sqrt{2} \sin \theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2} \cos \theta$. It follows that

$$14.55\sqrt{2} \sin \theta = -178.82\sqrt{2} \cos \theta$$

$$\text{or } \tan \theta = -12.29$$

$$\text{Therefore, } \theta = -85.35^\circ$$

[b] When $\theta = -85.35^\circ$, $\beta = -73.85^\circ$

$$\begin{aligned} \therefore K_2 &= \frac{1440\pi(125.43\sqrt{2})[-120\pi \sin(-73.85^\circ) + j120\pi \cos(-73.85^\circ)]}{(1475\pi + j120\pi)(j240\pi)} \\ &= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475} \\ &= 61.03\sqrt{2}/-78.50^\circ \end{aligned}$$

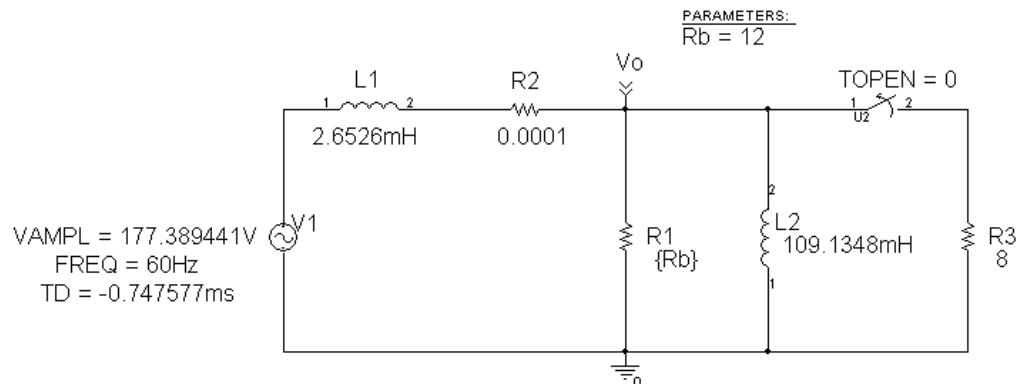
$$\therefore v_o = 122.06\sqrt{2} \cos(120\pi t - 78.50^\circ) \text{ V} \quad t > 0$$

$$= 172.61 \cos(120\pi t - 78.50^\circ) \text{ V} \quad t > 0$$

[c] $v_{o1} = 169.71 \cos(120\pi t - 85.35^\circ) \text{ V} \quad t < 0$

$$v_{o2} = 172.61 \cos(120\pi t - 78.50^\circ) \text{ V} \quad t > 0$$

PSpice schematic



PSPICE output file

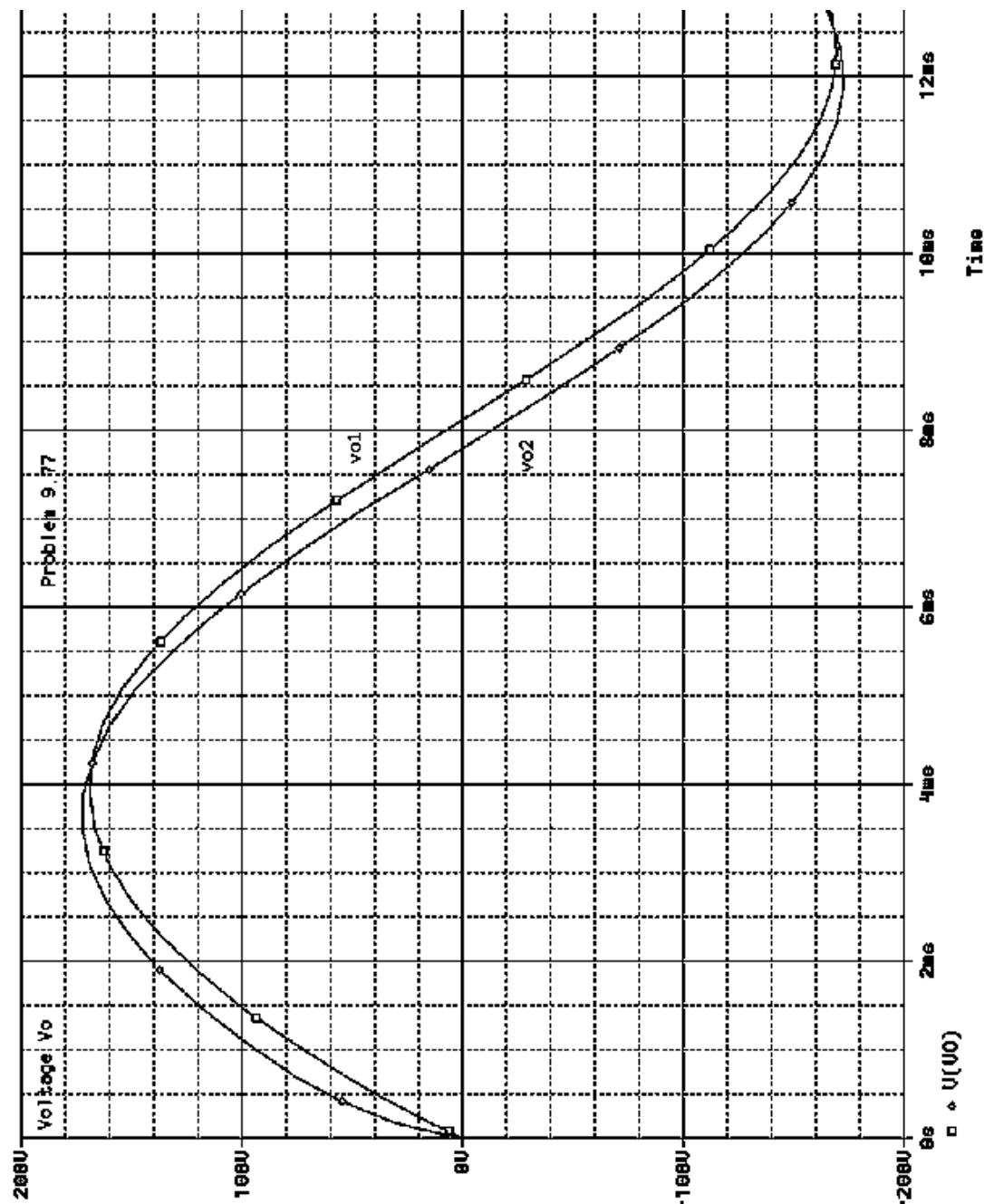
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** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

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* Local Libraries :
* From [PSpice NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
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*Analysis directives:
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.STEP PARAM Rb LIST 4.8 12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"

**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1 N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1 N00637 N01311 2.6526mH IC=0
L_L2 0 VO 109.1348mH IC=0
R_R1 0 VO {Rb}
R_R2 VO N01311 0.0001
R_R3 0 N01959 8
X_U2 VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+ Ropen=1Meg
.PARAM Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
```



14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8 \text{ kHz}, \quad \omega_c = 2\pi f_c = 16\pi \text{ krad/s}$$

$$\omega_c = \frac{1}{RC}; \quad R = 10 \text{ k}\Omega;$$

$$\therefore C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99 \text{ nF}$$

AP 14.2 [a] $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi \text{ krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40 \text{ H}$$

$$[\mathbf{b}] \quad H(j\omega) = \frac{\omega_c}{\omega_c + j\omega} = \frac{4000\pi}{4000\pi + j\omega}$$

When $\omega = 2\pi f = 2\pi(50,000) = 100,000\pi \text{ rad/s}$

$$H(j100,000\pi) = \frac{4000\pi}{4000\pi + j100,000\pi} = \frac{1}{1 + j25} = 0.04/\underline{87.71^\circ}$$

$$\therefore |H(j100,000\pi)| = 0.04$$

$$[\mathbf{c}] \quad \theta(100,000\pi) = -87.71^\circ$$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43 \text{ Mrad/s}$$

AP 14.4 [a] $\omega_c = \frac{1}{RC} = \frac{10^6}{R} = \frac{10^6}{100} = 10 \text{ krad/s}$

[b] $\omega_c = \frac{10^6}{5000} = 200 \text{ rad/s}$

[c] $\omega_c = \frac{10^6}{3 \times 10^4} = 33.33 \text{ rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L . Then

$$Z = \frac{R_L}{(R_L C s + 1)}$$

$$\begin{aligned} \text{Thus } H(s) &= \frac{Z}{R+Z} = \frac{R_L}{R(R_L C s + 1) + R_L} \\ &= \frac{(1/RC)}{s + \frac{R+R_L}{R_L} \left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K} \left(\frac{1}{RC}\right)} \end{aligned}$$

$$\text{where } K = \frac{R_L}{R + R_L}$$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2 (0.1 \times 10^{-6})} = 1.76 \text{ mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10 \Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi \text{ rad/s};$$

$$\beta = 2\pi(500) = 1000\pi \text{ rad/s}; \quad R = 250 \Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27 \mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2 (1.27)} = 4.97 \text{ mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2 (0.2 \times 10^{-6})} = 5.07 \text{ mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98 \text{ k}\Omega$$

AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2 \times 10^{-6})} = 31.66 \text{ mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o R C$$

$$\therefore R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2 \times 10^{-6})} = 9.95 \text{ k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi \text{ rad/s}$$

$$C = 500 \text{ nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17 \text{ mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$$

$$\therefore R = \frac{1}{\omega_o C Q} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92 \Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20,000) = 40\pi \text{ krad/s}; \quad R = 100 \Omega; \quad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi \times 10^3)} = 3.98 \text{ mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92 \text{ nF}$$

Problems

P 14.1 [a] $\omega_c = \frac{R}{L} = \frac{1200}{50 \times 10^{-3}} = 24 \text{ krad/s}$

$$\therefore f_c = \frac{\omega_c}{2\pi} = \frac{24,000}{2\pi} = 3819.72 \text{ Hz}$$

[b] $H(s) = \frac{\omega_c}{s + \omega_c} = \frac{24,000}{s + 24,000}$

$$H(j\omega) = \frac{24,000}{24,000 + j\omega}$$

$$H(j\omega_c) = \frac{24,000}{24,000 + j24,000} = 0.7071/\underline{-45^\circ}$$

$$H(j0.125\omega_c) = \frac{24,000}{24,000 + j3000} = 0.9923/\underline{-7.125^\circ}$$

$$H(j8\omega_c) = \frac{24,000}{24,000 + j192,000} = 0.124/\underline{-82.875^\circ}$$

[c] $v_o(t)|_{\omega_c} = 14.142 \cos(24,000t - 45^\circ) \text{ V}$

$$v_o(t)|_{0.125\omega_c} = 19.846 \cos(3000t - 7.125^\circ) \text{ V}$$

$$v_o(t)|_{8\omega_c} = 2.48 \cos(192,000t - 82.875^\circ) \text{ V}$$

P 14.2 [a] $\omega_c = \frac{1}{RC} = \frac{1}{(160)(5 \times 10^{-6})} = 1250 \text{ rad/s}$

$$f_c = \frac{\omega_c}{2\pi} = 198.94 \text{ Hz}$$

[b] $H(j\omega) = \frac{\omega_c}{s + \omega_c} = \frac{1250}{s + 1250}$

$$H(j\omega) = \frac{1250}{1250 + j\omega}$$

$$H(j\omega_c) = \frac{1250}{1250 + j1250} = 0.7071/\underline{-45^\circ}$$

$$H(j0.1\omega_c) = \frac{1250}{1250 + j125} = 0.9950/\underline{-5.71^\circ}$$

$$H(j10\omega_c) = \frac{1250}{1250 + j12,500} = 0.0995/\underline{-84.29^\circ}$$

$$\begin{aligned}
 [\mathbf{c}] \quad v_o(t)|_{\omega_c} &= 25(0.7071) \cos(1250t - 45^\circ) \\
 &= 17.68 \cos(1250t - 45^\circ) \text{ mV} \\
 v_o(t)|_{0.1\omega_c} &= 25(0.9950) \cos(125t - 5.71^\circ) \\
 &= 24.88 \cos(125t - 5.71^\circ) \text{ mV} \\
 v_o(t)|_{10\omega_c} &= 25(0.0995) \cos(12,500t - 84.29^\circ) \\
 &= 2.49 \cos(12,500t - 84.29^\circ) \text{ mV}
 \end{aligned}$$

P 14.3 [a] $H(s) = \frac{V_o}{V_i} = \frac{R}{sL + R + R_l} = \frac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \frac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\max}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \frac{R}{R + R_l}$

[d] $|H(j\omega_c)| = \frac{R}{\sqrt{2}(R + R_l)} = \frac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore \omega_c^2 = \left(\frac{R + R_l}{L}\right)^2; \quad \therefore \omega_c = (R + R_l)/L$$

[e] $\omega_c = \frac{1200 + 300}{0.05} = 30,000 \text{ rad/s}$

$$H(j\omega) = \frac{24,000}{30,000 + j\omega}$$

$$H(j0) = 0.8$$

$$H(j30,000) = \frac{0.8}{\sqrt{2}} / \underline{-45^\circ} = 0.5657 / \underline{-45^\circ}$$

$$H(j6000) = \frac{24,000}{30,000 + j6000} = 0.7845 / \underline{-11.31^\circ}$$

$$H(j150,000) = \frac{24,000}{30,000 + j150,000} = 0.1569 / \underline{-78.69^\circ}$$

P 14.4 [a] Let $Z = \frac{R_L(1/sC)}{R_L + 1/sC} = \frac{R_L}{R_LCs + 1}$

$$\begin{aligned}\text{Then } H(s) &= \frac{Z}{Z + R} \\ &= \frac{R_L}{RR_LCs + R + R_L} \\ &= \frac{(1/RC)}{s + \left(\frac{R + R_L}{RR_LC}\right)}\end{aligned}$$

[b] $|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + [(R + R_L)/RR_LC]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$.

[c] $|H(j\omega)|_{\max} = \frac{R_L}{R + R_L}$

[d] $|H(j\omega_c)| = \frac{R_L}{\sqrt{2}(R + R_L)} = \frac{(1/RC)}{\sqrt{\omega_C^2 + [(R + R_L)/RR_LC]^2}}$

$$\therefore \omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC} (1 + (R/R_L))$$

[e] $\omega_c = \frac{1}{(160)(5 \times 10^{-6})} [1 + (160/320)] = 1250(1 + 0.5) = 1875 \text{ rad/s}$

$$H(j0) = \frac{1250}{1875} = 0.667/\underline{0^\circ}$$

$$H(j\omega_c) = \frac{1250}{1875 + j1875} = 0.4714/\underline{-45^\circ}$$

$$H(j0.2\omega_c) = \frac{1250}{1875 + j375} = 0.6537/\underline{-11.31^\circ}$$

$$H(j5\omega_c) = \frac{1250}{1875 + j9375} = 0.1307/\underline{-78.69^\circ}$$

P 14.5 [a] $Z_L = j\omega L = j0L = 0$ so it is a short circuit.

At $\omega = 0$, $V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit.

At $\omega = \infty$, $V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

$$[d] \quad H(s) = \frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$[e] \quad \omega_c = \frac{R}{L} = \frac{330}{0.01} = 33 \text{ krad/s}$$

P 14.6 [a] $H(s) = \frac{V_o}{V_i} = \frac{R\|R_L}{R\|R_L + sL} = \frac{\frac{R}{L} \left(\frac{R_L}{R + R_L} \right)}{s + \frac{R}{L} \left(\frac{R_L}{R + R_L} \right)}$

[b] $\omega_{c(UL)} = \frac{R}{L}; \quad \omega_{c(L)} = \frac{R}{L} \left(\frac{R_L}{R + R_L} \right)$ so the cutoff frequencies are different.

$$H(0)_{(UL)} = 1; \quad H(0)_{(L)} = 1 \quad \text{so the passband gains are the same.}$$

$$[c] \quad \omega_{c(UL)} = 33,000 \text{ rad/s}$$

$$\omega_{c(L)} = 33,000 - 0.05(33,000) = 31,350 \text{ rad/s}$$

$$31,350 = \frac{330}{0.01} \left(\frac{R_L}{330 + R_L} \right) \quad \text{so} \quad \frac{R_L}{330 + R_L} = 0.95$$

$$\therefore 0.05R_L = 313.5 \quad \text{so} \quad R_L \geq 6270 \Omega$$

P 14.7 [a] $\frac{R}{L} = 10,000\pi \text{ rad/s}$

$$R = (0.001)(10,000)(\pi) = 31.42 \Omega$$

[b] $R_e = 31.42\|68 = 21.49 \Omega$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 21,488.34 \text{ rad/s}$$

$$\therefore f_{\text{loaded}} = 3419.98 \text{ Hz}$$

[c] The 33Ω resistor in Appendix H is closest to the desired value of 31.42Ω .
Therefore,

$$\omega_c = 33 \text{ krad/s} \quad \text{so} \quad f_c = 5252.11 \text{ Hz}$$

P 14.8 [a] $f_c = \frac{1600}{2\pi} = 254.65 \text{ Hz}$

[b] $1600 = \frac{R}{0.01} \quad \text{so} \quad R = 1600(0.01) = 16 \Omega$

[c] $\omega'_c = 1600 - 160 = 1440 \text{ rad/s}$

$$R' = 1440(0.01) = 14.4 \Omega$$

$$16\|R_L = 14.4 \quad \text{so} \quad \frac{16R_L}{16 + R_L} = 14.4$$

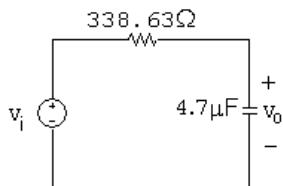
$$\text{Solving,} \quad R_L = 144 \Omega$$

- [d] The magnitude of $H(j\omega)$ when $\omega = 0$ is still 1. The resistive load affects the cutoff frequency but not the magnitude of this low-pass filter.

P 14.9 [a] $\omega_c = 2\pi(100) = 628.32 \text{ rad/s}$

$$[b] \omega_c = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\omega_c C} = \frac{1}{(628.32)(4.7 \times 10^{-6})} = 338.63 \Omega$$

[c]



$$[d] H(s) = \frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1/RC}{s + 1/RC} = \frac{628.32}{s + 628.32}$$

$$[e] H(s) = \frac{V_o}{V_i} = \frac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \frac{1/RC}{s + \left(\frac{R + R_L}{R_L}\right)1/RC} = \frac{628.32}{s + 2(628.32)}$$

$$[f] \omega_c = 2(628.32) = 1256.64 \text{ rad/s}$$

$$[g] H(0) = 1/2$$

P 14.10 [a] $f_c = \frac{\omega_c}{2\pi} = \frac{50,000}{2\pi} = \frac{50}{2\pi} \times 10^3 = 7957.75 \text{ Hz}$

$$[b] \frac{1}{RC} = 50 \times 10^3$$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40 \Omega$$

- [c] With a load resistor added in parallel with the capacitor the transfer function becomes

$$\begin{aligned} H(s) &= \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC} \\ &= \frac{R_L}{RR_LsC + R + R_L} = \frac{1/RC}{s + [(R + R_L)/RR_LC]} \end{aligned}$$

This transfer function is in the form of a low-pass filter, with a cutoff frequency equal to the quantity added to s in the denominator. Therefore,

$$\omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC} \left(1 + \frac{R}{R_L}\right)$$

$$\therefore \frac{R}{R_L} = 0.05 \quad \therefore R_L = 20R = 800 \Omega$$

$$[d] \ H(j0) = \frac{R_L}{R + R_L} = \frac{800}{840} = 0.9524$$

P 14.11 [a] $\frac{1}{RC} = \frac{1}{(20)(80 \times 10^{-6})} = 625 \text{ rad/s}$

$$f_c = \frac{625}{2\pi} = 99.47 \text{ Hz}$$

[b] $H(s) = \frac{s}{s + \omega_c} \quad \therefore \quad H(j\omega) = \frac{j\omega}{625 + j\omega}$

$$H(j\omega_c) = H(j6250) = \frac{j625}{625 + j625} = 0.7071/\underline{45^\circ}$$

$$H(j0.125\omega_c) = H(j78.125) = \frac{j78.125}{625 + j78.125} = 0.124/\underline{82.87^\circ}$$

$$H(j8\omega_c) = H(j5000\omega_c) = \frac{j5000}{625 + j5000} = 0.9923/\underline{7.125^\circ}$$

[c] $v_o(t)|_{\omega_c} = (0.7071)(75) \cos(625t + 45^\circ)$

$$= 53.03 \cos(625t + 45^\circ) \text{ mV}$$

$$v_o(t)|_{0.125\omega_c} = (0.124)(75) \cos(78.125t + 82.87^\circ)$$

$$= 9.3 \cos(78.125t + 82.87^\circ) \text{ mV}$$

$$v_o(t)|_{8\omega_c} = (0.9923)(75) \cos(5000t + 7.125^\circ)$$

$$= 74.42 \cos(5000t + 7.125^\circ) \text{ mV}$$

P 14.12 [a] $H(s) = \frac{V_o}{V_i} = \frac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R + R_c} \cdot \frac{s}{[s + (1/(R + R_c)C)]}$$

[b] $H(j\omega) = \frac{R}{R + R_c} \cdot \frac{j\omega}{j\omega + (1/(R + R_c)C)}$

$$|H(j\omega)| = \frac{R}{R + R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R + R_c)^2 C^2}}}$$

The magnitude will be maximum when $\omega = \infty$.

[c] $|H(j\omega)|_{\max} = \frac{R}{R + R_c}$

$$[\mathbf{d}] \quad |H(j\omega_c)| = \frac{R\omega_c}{(R + R_c)\sqrt{\omega_c^2 + [1/(R + R_c)C]^2}}$$

$$\therefore |H(j\omega)| = \frac{R}{\sqrt{2}(R + R_c)} \quad \text{when}$$

$$\therefore \omega_c^2 = \frac{1}{(R + R_c)^2 C^2}$$

$$\text{or } \omega_c = \frac{1}{(R + R_c)C}$$

$$[\mathbf{e}] \quad \omega_c = \frac{1}{(25)(80 \times 10^{-6})} = 500 \text{ rad/s}$$

$$\frac{R}{R + R_c} = \frac{20}{25} = 0.8$$

$$\therefore H(j\omega) = \frac{0.8j\omega}{500 + j\omega}$$

$$H(j\omega_c) = \frac{(0.8)j500}{500 + j500} = 0.5657/\underline{45^\circ}$$

$$H(j0.125\omega_c) = \frac{(0.8)j62.5}{500 + j62.5} = 0.099/\underline{82.87^\circ}$$

$$H(j8\omega_c) = \frac{(0.8)j4000}{500 + j4000} = 0.7938/\underline{7.125^\circ}$$

$$\text{P 14.13} \quad [\mathbf{a}] \quad \omega_c = \frac{1}{RC} = 2\pi(300) = 600\pi \text{ rad/s}$$

$$\therefore R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100 \times 10^{-9})} = 5305.16 \Omega = 5.305 \text{ k}\Omega$$

$$[\mathbf{b}] \quad R_e = 5305.16 \| 47,000 = 4767.08 \Omega$$

$$\omega_c = \frac{1}{R_e C} = \frac{1}{(4767.08)(100 \times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86 \text{ Hz}$$

P 14.14 [a] For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.

For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.

Thus, the circuit is a high-pass filter.

$$[\mathbf{b}] \quad H(s) = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 15,000}$$

[c] $\omega_c = \frac{R}{L} = 15,000 \text{ rad/s}$

[d] $|H(jR/L)| = \left| \frac{jR/L}{jR/L + R/L} \right| = \left| \frac{j}{j+1} \right| = \frac{1}{\sqrt{2}}$

P 14.15 [a] $H(s) = \frac{V_o}{V_i} = \frac{R_L \| sL}{R + R_L \| sL} = \frac{s \left(\frac{R_L}{R + R_L} \right)}{s + \frac{R}{L} \left(\frac{R_L}{R + R_L} \right)}$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(15,000)}$$

[b] $\omega_c = \frac{R}{L} \left(\frac{R_L}{R + R_L} \right) = \frac{1}{2}(15,000) = 7500 \text{ rad/s}$

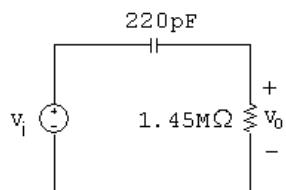
[c] $\omega_{c(L)} = \frac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.16 [a] $\omega_c = 2\pi(500) = 3141.59 \text{ rad/s}$

[b] $\omega_c = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\omega_c C} = \frac{1}{(3141.59)(220 \times 10^{-12})} = 1.45 \text{ M}\Omega$

[c]



[d] $H(s) = \frac{V_o}{V_i} = \frac{R}{R + 1/sC} = \frac{s}{s + 1/RC} = \frac{s}{s + 3141.59}$

[e] $H(s) = \frac{V_o}{V_i} = \frac{R \| R_L}{R \| R_L + (1/sC)} = \frac{s}{s + \left(\frac{R + R_L}{R_L} \right) 1/RC} = \frac{s}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19 \text{ rad/s}$

[g] $H(\infty) = 1$

P 14.17 [a] $R = \omega_c L = (1500 \times 10^3)(100 \times 10^{-6}) = 150 \Omega \quad (\text{a value from Appendix H})$

[b] With a load resistor in parallel with the inductor, the transfer function becomes

$$H(s) = \frac{sL \| R_L}{R + sL \| R_L} = \frac{sLR_L}{R(sL + R_L) + sLR_L} = \frac{s[R_L/(R + R_L)]}{s + [RR_L/(R + R_L)]}$$

This transfer function is in the form of a high-pass filter whose cutoff frequency is the quantity added to s in the denominator. Thus,

$$\omega_c = \frac{RR_L}{L(R + R_L)}$$

Substituting in the values of R and L from part (a), we can solve for the value of load resistance that gives a cutoff frequency of 1200 krad/s:

$$\frac{150R_L}{100 \times 10^{-6}(150 + R_L)} = 1200 \times 10^3 \quad \text{so} \quad R_L = 600 \Omega$$

The smallest resistor from Appendix H that is larger than 600Ω is 680Ω .

$$\text{P 14.18 [a]} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{(10 \times 10^{-3})(10 \times 10^{-9})} = 10^{10}$$

$$\omega_o = 10^5 \text{ rad/s} = 100 \text{ krad/s}$$

$$\text{[b]} \quad f_o = \frac{\omega_o}{2\pi} = \frac{10^5}{2\pi} = 15.9 \text{ kHz}$$

$$\text{[c]} \quad Q = \omega_o RC = (100 \times 10^3)(8000)(10 \times 10^{-9}) = 8$$

$$\text{[d]} \quad \omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 10^5 \left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}} \right] = 93.95 \text{ krad/s}$$

$$\text{[e]} \quad \therefore f_{c1} = \frac{\omega_{c1}}{2\pi} = 14.95 \text{ kHz}$$

$$\text{[f]} \quad \omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 10^5 \left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}} \right] = 106.45 \text{ krad/s}$$

$$\text{[g]} \quad \therefore f_{c2} = \frac{\omega_{c2}}{2\pi} = 16.94 \text{ kHz}$$

$$\text{[h]} \quad \beta = \frac{\omega_o}{Q} = \frac{10^5}{8} = 12.5 \text{ krad/s or } 1.99 \text{ kHz}$$

$$\text{P 14.19} \quad \omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110 \text{ krad/s}$$

$$f_o = \frac{\omega_o}{2\pi} = 17.51 \text{ kHz}$$

$$\beta = 121 - 100 = 21 \text{ krad/s or } 2.79 \text{ kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{110}{21} = 5.24$$

$$\text{P 14.20 } \beta = \frac{\omega_o}{Q} = \frac{50,000}{4} = 12.5 \text{ krad/s}; \quad \frac{12,500}{2\pi} = 1.99 \text{ kHz}$$

$$\omega_{c2} = 50,000 \left[\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8} \right)^2} \right] = 56.64 \text{ krad/s}$$

$$f_{c2} = \frac{56.64 \text{ k}}{2\pi} = 9.01 \text{ kHz}$$

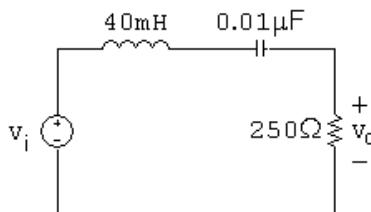
$$\omega_{c1} = 50,000 \left[-\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8} \right)^2} \right] = 44.14 \text{ krad/s}$$

$$f_{c1} = \frac{44.14 \text{ k}}{2\pi} = 7.02 \text{ kHz}$$

$$\text{P 14.21 [a]} \omega_o = \sqrt{1/LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(50,000)^2 (0.01 \times 10^{-6})} = 40 \text{ mH}$$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{50,000}{8} = 6250 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (40 \times 10^{-3})(6250) = 250 \Omega$$



[b] From part (a), \$\beta = 6250 \text{ rad/s}\$. Then,

$$\omega_{c1,2} = \pm \frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm \frac{6250}{2} + \sqrt{\left(\frac{6250}{2}\right)^2 + 50,000^2} = \pm 3125 + 50,097.56$$

$$\omega_{c1} = 46,972.56 \text{ rad/s} \quad \omega_{c2} = 53,222.56 \text{ rad/s}$$

$$\text{P 14.22 } H(j\omega) = \frac{j\omega(6250)}{50,000^2 - \omega^2 + j\omega(6250)}$$

$$\text{[a]} \quad H(j50,000) = \frac{j50,000(6250)}{50,000^2 - 50,000^2 + j(50,000)(6250)} = 1$$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 5 \cos 50,000t \text{ V}$$

$$[\mathbf{b}] \quad H(j46,972.56) = \frac{j46,972.56(6250)}{50,000^2 - 46,972.56^2 + j(46,972.56)(6250)} = \frac{1}{\sqrt{2}} \underline{/45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}} \underline{/45^\circ} V_i \quad \therefore \quad v_o(t) = 3.54 \cos(46,972.56t + 45^\circ) \text{ V}$$

$$[\mathbf{c}] \quad H(j53,222.56) = \frac{j53,222.56(6250)}{50,000^2 - 53,222.56^2 + j(53,222.56)(6250)} = \frac{1}{\sqrt{2}} \underline/{-45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}} \underline/{-45^\circ} V_i \quad \therefore \quad v_o(t) = 3.54 \cos(53,222.56t - 45^\circ) \text{ V}$$

$$[\mathbf{d}] \quad H(j5000) = \frac{j5000(6250)}{50,000^2 - 5000^2 + j(5000)(6250)} = 0.0126 \underline{/89.3^\circ}$$

$$V_o = 0.0126 \underline{/89.3^\circ} V_i \quad \therefore \quad v_o(t) = 63.1 \cos(5000t + 89.3^\circ) \text{ mV}$$

$$[\mathbf{e}] \quad H(j500,000) = \frac{j500,000(6250)}{50,000^2 - 500,000^2 + j(500,000)(6250)} = 0.0126 \underline/{-89.3^\circ}$$

$$V_o = 0.0126 \underline/{-89.3^\circ} V_i \quad \therefore \quad v_o(t) = 63.1 \cos(500,000t - 89.3^\circ) \text{ mV}$$

$$\text{P 14.23} \quad H(s) = 1 - \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \frac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$$

$$H(j\omega) = \frac{50,000^2 - \omega^2}{50,000^2 - \omega^2 + j\omega(6250)}$$

$$[\mathbf{a}] \quad H(j50,000) = \frac{50,000^2 - 50,000^2}{50,000^2 - 50,000^2 + j(50,000)(6250)} = 0$$

$$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0 \text{ mV}$$

$$[\mathbf{b}] \quad H(j46,972.56) = \frac{50,000^2 - 46,972.56^2}{50,000^2 - 46,972.56^2 + j(46,972.56)(6250)} = \frac{1}{\sqrt{2}} \underline/{-45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}} \underline/{-45^\circ} V_i \quad \therefore \quad v_o(t) = 3.54 \cos(46,972.56t - 45^\circ) \text{ V}$$

$$[\mathbf{c}] \quad H(j53,222.56) = \frac{50,000^2 - 53,222.56^2}{50,000^2 - 53,222.56^2 + j(53,222.56)(6250)} = \frac{1}{\sqrt{2}} \underline{/45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}} \underline{/45^\circ} V_i \quad \therefore \quad v_o(t) = 3.54 \cos(53,222.56t + 45^\circ) \text{ V}$$

$$[\mathbf{d}] \quad H(j5000) = \frac{50,000^2 - 5000^2}{50,000^2 - 5000^2 + j(5000)(6250)} = 0.9999 \underline/{-0.72^\circ}$$

$$V_o = 0.9999 \underline/{-0.72^\circ} V_i \quad \therefore \quad v_o(t) = 4.9996 \cos(5000t - 0.72^\circ) \text{ V}$$

$$[\text{e}] \quad H(j500,000) = \frac{50,000^2 - 500,000^2}{50,000^2 - 500,000^2 + j(500,000)(6250)} = 0.9999/\underline{0.72^\circ}$$

$$V_o = 0.9999/\underline{0.72^\circ} V_i \quad \therefore \quad v_o(t) = 4.9996 \cos(500,000t + 0.72^\circ) \text{ V}$$

P 14.24 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\text{P 14.25 [a]} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(50 \times 10^{-9})(20 \times 10^3)^2} = 50 \text{ mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(20 \times 10^3)(50 \times 10^{-9})} = 5 \text{ k}\Omega$$

$$[\text{b}] \quad \omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 20,000 \left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$$

$$= 22.10 \text{ krad/s} \quad \therefore \quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52 \text{ kHz}$$

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 20,000 \left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$$

$$= 18.10 \text{ krad/s} \quad \therefore \quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88 \text{ kHz}$$

$$[\text{c}] \quad \beta = \frac{\omega_o}{Q} = \frac{20,000}{5} = 4000 \text{ rad/s} \quad \text{or} \quad 636.62 \text{ Hz}$$

P 14.26 [a] We need $\omega_c = 20,000$ rad/s. There are several possible approaches – this one starts by choosing $L = 1$ mH. Then,

$$C = \frac{1}{20,000^2(0.001)} = 2.5 \mu\text{F}$$

Use the closest value from Appendix H, which is $2.2 \mu\text{F}$ to give

$$\omega_c = \sqrt{\frac{1}{(0.001)(2.2 \times 10^{-6})}} = 21,320 \text{ rad/s}$$

$$\text{Then, } R = \frac{Q}{\omega_o C} = \frac{5}{(21320)(2.2 \times 10^{-6})} = 106.6 \Omega$$

Use the closest value from Appendix H, which is 100Ω to give

$$Q = 100(21,320)(2.2 \times 10^{-6}) = 4.69$$

$$[\text{b}] \text{ \% error in } \omega_c = \frac{21,320 - 20,000}{20,000}(100) = 6.6\%$$

$$\text{\% error in } Q = \frac{4.69 - 5}{5}(100) = -6.2\%$$

$$\text{P 14.27 [a]} \omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16 \text{ mH}$$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99 \text{ k}\Omega$$

$$[\text{b}] f_{c1} = 8000 \left[-\frac{1}{4} + \sqrt{1 + \frac{1}{16}} \right] = 6.25 \text{ kHz}$$

$$[\text{c}] f_{c2} = 8000 \left[\frac{1}{4} + \sqrt{1 + \frac{1}{16}} \right] = 10.25 \text{ kHz}$$

$$[\text{d}] \beta = f_{c2} - f_{c1} = 4 \text{ kHz}$$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4 \text{ kHz}$$

P 14.28 [a] We need ω_c close to $2\pi(8000) = 50,265.48$ rad/s. There are several possible approaches – this one starts by choosing $L = 10$ mH. Then,

$$C = \frac{1}{[2\pi(8000)]^2(0.01)} = 39.58 \text{ nF}$$

Use the closest value from Appendix H, which is $0.047 \mu\text{F}$ to give

$$\omega_c = \sqrt{\frac{1}{(0.01)(47 \times 10^{-9})}} = 46,126.56 \text{ rad/s} \quad \text{or} \quad f_c = 7341.27 \text{ Hz}$$

$$\text{Then, } R = \frac{\omega_o L}{Q} = \frac{(46,126.56)(0.01)}{2} = 230 \Omega$$

Use the closest value from Appendix H, which is 220Ω to give

$$Q = \frac{(46,126.56)(0.01)}{220} = 2.1$$

$$[\mathbf{b}] \text{ \% error in } f_c = \frac{7341.27 - 8000}{8000}(100) = -8.23\%$$

$$\text{\% error in } Q = \frac{2.1 - 2}{2}(100) = 5\%$$

$$\text{P 14.29 [a]} \omega_o^2 = \frac{1}{LC} = \frac{1}{(40 \times 10^{-3})(40 \times 10^{-9})} = 625 \times 10^6$$

$$\omega_o = 25 \times 10^3 \text{ rad/s} = 25 \text{ krad/s}$$

$$f_o = \frac{25,000}{2\pi} = 3978.87 \text{ Hz}$$

$$[\mathbf{b}] Q = \frac{\omega_o L}{R + R_i} = \frac{(25 \times 10^3)(40 \times 10^{-3})}{200} = 5$$

$$[\mathbf{c}] \omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 25,000 \left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$$

$$= 22.625 \text{ krad/s or } 3.60 \text{ kHz}$$

$$[\mathbf{d}] \omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 25,000 \left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$$

$$= 27.625 \text{ krad/s or } 4.4 \text{ kHz}$$

$$[\mathbf{e}] \beta = \omega_{c2} - \omega_{c1} = 27.62 - 22.62 = 5 \text{ krad/s}$$

or

$$\beta = \frac{\omega_o}{Q} = \frac{25,000}{5} = 5 \text{ krad/s or } 795.77 \text{ Hz}$$

$$\text{P 14.30 [a]} H(s) = \frac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$$

For the numerical values in Problem 14.29 we have

$$H(s) = \frac{4500s}{s^2 + 5000s + 625 \times 10^6}$$

$$\therefore H(j\omega) = \frac{4500j\omega}{(625 \times 10^6 - \omega^2) + j5000\omega}$$

$$H(j\omega_o) = \frac{j4500(25 \times 10^3)}{j5000(25 \times 10^3)} = 0.9/\underline{0^\circ}$$

$$\therefore v_o(t) = 10(0.9) \cos 25,000t = 9 \cos 25,000t \text{ V}$$

[b] From the solution to Problem 14.29,

$$\omega_{c1} = 22.625 \text{ krad/s}$$

$$H(j22.625 \text{ k}) = \frac{j4500(22.625 \times 10^3)}{(113.12 + j113.12) \times 10^6} = 0.6364/\underline{45^\circ}$$

$$\therefore v_o(t) = 10(0.6364) \cos(22,625t + 45^\circ) = 6.364 \cos(22,625t + 45^\circ) \text{ V}$$

[c] From the solution to Problem 14.29,

$$\omega_{c2} = 27.625 \text{ krad/s}$$

$$H(j27.625 \text{ k}) = \frac{j4500(27.625 \times 10^3)}{(-138.12 + j138.12) \times 10^6} = 0.6364/\underline{-45^\circ}$$

$$\therefore v_o(t) = 10(0.6364) \cos(27,625t - 45^\circ) = 6.364 \cos(27,625t - 45^\circ) \text{ V}$$

$$\text{P 14.31 [a]} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{(5 \times 10^{-3})(200 \times 10^{-12})} = 10^{12}$$

$$\omega_o = 1 \text{ Mrad/s}$$

$$\text{[b]} \quad \beta = \frac{R + R_L}{R_L} \cdot \frac{1}{RC} = \left(\frac{500 \times 10^3}{400 \times 10^3} \right) \left(\frac{1}{(100 \times 10^3)(200 \times 10^{-12})} \right) = 62.5 \text{ krad/s}$$

$$\text{[c]} \quad Q = \frac{\omega_o}{\beta} = \frac{10^6}{62.5 \times 10^3} = 16$$

$$\text{[d]} \quad H(j\omega_o) = \frac{R_L}{R + R_L} = 0.8/\underline{0^\circ}$$

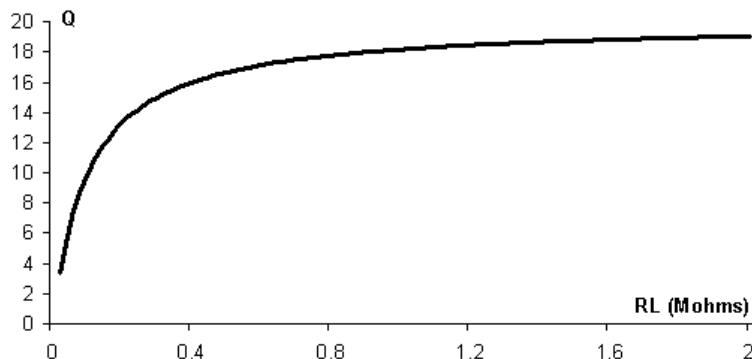
$$\therefore v_o(t) = 250(0.8) \cos(10^6 t) = 200 \cos 10^6 t \text{ mV}$$

$$\text{[e]} \quad \beta = \left(1 + \frac{R}{R_L} \right) \frac{1}{RC} = \left(1 + \frac{100}{R_L} \right) (50 \times 10^3) \text{ rad/s}$$

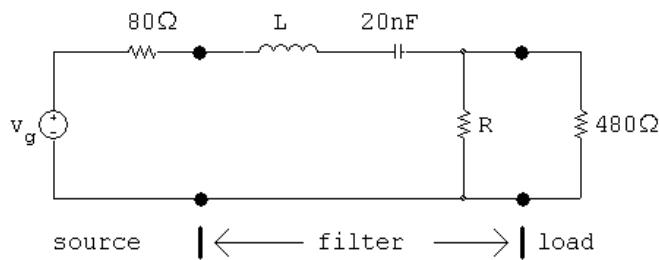
$$\omega_o = 10^6 \text{ rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{20}{1 + (100/R_L)} \quad \text{where } R_L \text{ is in kilohms}$$

[f]



P 14.32 [a]



$$[b] \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(50 \times 10^3)^2 (20 \times 10^{-4})} = 20 \text{ mH}$$

$$R = \frac{\omega_o L}{Q} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{6.25} = 160 \Omega$$

$$[c] \quad R_e = 160 \parallel 480 = 120 \Omega$$

$$R_e + R_i = 120 + 80 = 200 \Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{200} = 5$$

$$[d] \quad \beta_{\text{system}} = \frac{\omega_o}{Q_{\text{system}}} = \frac{50 \times 10^3}{5} = 10 \text{ krad/s}$$

$$\beta_{\text{system}}(\text{Hz}) = \frac{10,000}{2\pi} = 1591.55 \text{ Hz}$$

P 14.33 [a] $\frac{V_o}{V_i} = \frac{Z}{Z + R}$ where $Z = \frac{1}{Y}$

$$\text{and } Y = sC + \frac{1}{sL} + \frac{1}{R_L} = \frac{LCR_L s^2 + sL + R_L}{R_L s}$$

$$\begin{aligned}
H(s) &= \frac{R_L L s}{R_L R L C s^2 + (R + R_L) L s + R R_L} \\
&= \frac{(1/R C) s}{s^2 + \left[\left(\frac{R+R_L}{R_L} \right) \left(\frac{1}{R C} \right) \right] s + \frac{1}{L C}} \\
&= \frac{\left(\frac{R_L}{R+R_L} \right) \left(\frac{R+R_L}{R_L} \right) \left(\frac{1}{R C} \right) s}{s^2 + \left[\left(\frac{R+R_L}{R_L} \right) \left(\frac{1}{R C} \right) \right] s + \frac{1}{L C}} \\
&= \frac{K \beta s}{s^2 + \beta s + \omega_o^2}, \quad K = \frac{R_L}{R + R_L}, \quad \beta = \frac{1}{(R \| R_L) C}
\end{aligned}$$

[b] $\beta = \left(\frac{R + R_L}{R_L} \right) \frac{1}{R C}$

[c] $\beta_U = \frac{1}{R C}$

$$\therefore \beta_L = \left(\frac{R + R_L}{R_L} \right) \beta_U = \left(1 + \frac{R}{R_L} \right) \beta_U$$

[d] $Q = \frac{\omega_o}{\beta} = \frac{\omega_o R C}{\left(\frac{R+R_L}{R_L} \right)}$

[e] $Q_U = \omega_o R C$

$$\therefore Q_L = \left(\frac{R_L}{R + R_L} \right) Q_U = \frac{1}{[1 + (R/R_L)]} Q_U$$

[f] $H(j\omega) = \frac{K j \omega \beta}{\omega_o^2 - \omega^2 + j \omega \beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K \omega_c \beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2 \beta^2}}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{\omega_c \beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2 \beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2 \beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm \omega_c \beta$$

$$\therefore \omega_c^2 \pm \omega_c \beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp \frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \left(1 + \frac{R}{R_L}\right) \frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

$$\text{P 14.34 } \omega_o^2 = \frac{1}{LC} = \frac{1}{(2 \times 10^{-6})(50 \times 10^{-12})} = 10^{16}$$

$$\omega_o = 100 \text{ Mrad/s}$$

$$Q_U = \omega_o RC = (100 \times 10^6)(2.4 \times 10^3)(50 \times 10^{-12}) = 12$$

$$\therefore \left(\frac{R_L}{R+R_L}\right) 12 = 7.5; \quad \therefore R_L = \frac{7.5}{4.5} R = 4 \text{ k}\Omega$$

$$\text{P 14.35 } H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$[\text{a}] \quad \omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.15625)(100 \times 10^{-9})}} = 8000 \text{ rad/s}$$

$$[\text{b}] \quad f_o = \frac{\omega_o}{2\pi} = 1273.24 \text{ Hz}$$

$$[\text{c}] \quad Q = \sqrt{\frac{L}{R^2 C}} = \sqrt{\frac{0.15625}{(1875)^2(100 \times 10^{-9})}} = \frac{2}{3}$$

$$[\text{d}] \quad \beta = \frac{R}{L} = \frac{1875}{0.15625} = 12,000 \text{ rad/s}$$

$$\beta \text{ (Hertz)} = \frac{12,000}{2\pi} = 1909.86 \text{ Hz}$$

$$[\text{e}] \quad \omega_{c1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$= \frac{-12,000}{2} + \sqrt{\left(\frac{12,000}{2}\right)^2 + 8000^2} = 4000 \text{ rad/s}$$

$$[\text{f}] \quad f_{c1} = \frac{4000}{2\pi} = 636.62 \text{ Hz}$$

$$[\text{g}] \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$= \frac{12,000}{2} + \sqrt{\left(\frac{12,000}{2}\right)^2 + 8000^2} = 16,000 \text{ rad/s}$$

$$[\text{h}] \quad f_{c2} = \frac{16,000}{2\pi} = 2546.48 \text{ Hz}$$

P 14.36 [a] $H(j\omega) = \frac{\frac{1}{LC} - \omega^2}{\frac{1}{LC} - \omega^2 + j\frac{R}{L}\omega} = \frac{8000^2 - \omega^2}{8000^2 - \omega^2 + j12,000\omega}$

$\omega_o = 8000 \text{ rad/s}$:

$$H(j\omega_o) = \frac{8000^2 - 8000^2}{8000^2 - 8000^2 + j12,000(8000)} = 0$$

$\omega_{c1} = 4000 \text{ rad/s}$:

$$H(j\omega_{c1}) = \frac{8000^2 - 4000^2}{8000^2 - 4000^2 + j12,000(4000)} = 0.7071/\underline{-45^\circ}$$

$\omega_{c2} = 16,000 \text{ rad/s}$:

$$H(j\omega_{c2}) = \frac{8000^2 - 16,000^2}{8000^2 - 16,000^2 + j12,000(16,000)} = 0.7071/\underline{45^\circ}$$

$0.1\omega_o = 800 \text{ rad/s}$:

$$H(j0.1\omega_o) = \frac{8000^2 - 800^2}{8000^2 - 800^2 + j12,000(800)} = 0.9887/\underline{-8.62^\circ}$$

$10\omega_o = 80,000 \text{ rad/s}$:

$$H(j10\omega_o) = \frac{8000^2 - 80,000^2}{8000^2 - 80,000^2 + j12,000(80,000)} = 0.9887/\underline{8.62^\circ}$$

[b] $\omega = \omega_o = 8000 \text{ rad/s}$: $V_o = H(j8000)V_i = 0V_i$

$$v_o(t) = 0$$

$\omega = \omega_{c1} = 4000 \text{ rad/s}$:

$$V_o = H(j4000)V_i = (0.7071/\underline{-45^\circ})(80) = 56.57/\underline{-45^\circ}$$

$$v_o(t) = 56.57 \cos(4000t - 45^\circ) \text{ V}$$

$\omega = \omega_{c2} = 16,000 \text{ rad/s}$:

$$V_o = H(j16,000)V_i = (0.7071/\underline{45^\circ})(80) = 56.57/\underline{45^\circ}$$

$$v_o(t) = 56.57 \cos(16,000t + 45^\circ) \text{ V}$$

$\omega = 0.1\omega_o = 800 \text{ rad/s}$:

$$V_o = H(j800)V_i = (0.9887/\underline{-8.62^\circ})(80) = 79.1/\underline{-8.62^\circ}$$

$$v_o(t) = 79.1 \cos(800t - 8.62^\circ) \text{ V}$$

$\omega = 10\omega_o = 80,000 \text{ rad/s}$:

$$V_o = H(j80,000)V_i = (0.9887/\underline{8.62^\circ})(80) = 79.1/\underline{8.62^\circ}$$

$$v_o(t) = 79.1 \cos(80,000t + 8.62^\circ) \text{ V}$$

P 14.37 [a] In analyzing the circuit qualitatively we visualize v_i as a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o .

At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.

At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.

At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.

At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.

Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C , thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)} \\ &= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)} \end{aligned}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$.

$$\therefore \omega_o = \frac{1}{\sqrt{LC}}$$

$$[\mathbf{d}] |H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \text{ when } \omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$$

or $\pm \omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c1} = \omega_o \left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

[e] It follows from the equations derived in part (d) that

$$\beta = \omega_{c2} - \omega_{c1} = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$.

P 14.38 [a] $\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(62.5 \times 10^{-9})} = 64 \times 10^8$

$$\therefore \omega_o = 80 \text{ krad/s}$$

[b] $f_o = \frac{\omega_o}{2\pi} = 12.73 \text{ kHz}$

[c] $Q = \omega_o RC = (80,000)(3000)(62.5 \times 10^{-9}) = 15$

[d] $\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 80,000 \left[-\frac{1}{30} + \sqrt{1 + \frac{1}{900}} \right]$

$$= 77,377.77 \text{ rad/s}$$

[e] $f_{c1} = \frac{\omega_{c1}}{2\pi} = 12,315.05 \text{ Hz}$

[f] $\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 80,000 \left[\frac{1}{30} + \sqrt{1 + \frac{1}{900}} \right]$

$$= 82,711.1 \text{ rad/s}$$

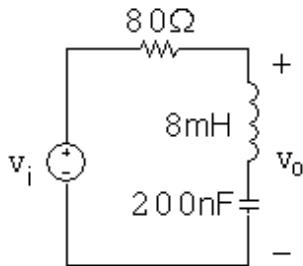
[g] $f_{c2} = \frac{\omega_{c1}}{2\pi} = 13,163.88 \text{ Hz}$

[h] $\beta = f_{c2} - f_{c1} = 848.8 \text{ Hz}$

P 14.39 [a] $\omega_o = \sqrt{1/LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{1}{(25,000)^2 (200 \times 10^{-9})} = 8 \text{ mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{25,000}{2.5} = 10,000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (8 \times 10^{-3})(10,000) = 80 \Omega$$



[b] From part (a), $\beta = 10,000 \text{ rad/s}$.

$$\begin{aligned} \omega_{c1,2} &= \pm \frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm \frac{10,000}{2} + \sqrt{\left(\frac{10,000}{2}\right)^2 + 25,000^2} \\ &= \pm 5000 + 25,495.1 \end{aligned}$$

$$\omega_{c1} = 20,495.1 \text{ rad/s} \quad \omega_{c2} = 30,495.1 \text{ rad/s}$$

P 14.40 $H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \frac{25,000^2 - \omega^2}{25,000^2 - \omega^2 + j\omega(10,000)}$

[a] $H(j25,000) = \frac{25,000^2 - 25,000^2}{25,000^2 - 25,000^2 + j(25,000)(10,000)} = 0$

$$V_o = (0)V_i \quad \therefore v_o(t) = 0 \text{ mV}$$

[b] $H(j20,495.1) = \frac{25,000^2 - 20,495.1^2}{25,000^2 - 20,495.1^2 + j(20,495.1)(10,000)} = \frac{1}{\sqrt{2}}/-45^\circ$

$$V_o = \frac{1}{\sqrt{2}}/-45^\circ V_i \quad \therefore v_o(t) = 7.071 \cos(20,495.1t - 45^\circ) \text{ V}$$

[c] $H(j30,495.1) = \frac{25,000^2 - 30,495.1^2}{25,000^2 - 30,495.1^2 + j(30,495.1)(10,000)} = \frac{1}{\sqrt{2}}/45^\circ$

$$V_o = \frac{1}{\sqrt{2}}/45^\circ V_i \quad \therefore v_o(t) = 7.071 \cos(30,495.1t + 45^\circ) \text{ V}$$

$$[\mathbf{d}] \quad H(j3125) = \frac{25,000^2 - 3125^2}{25,000^2 - 3125^2 + j(3125)(10,000)} = 0.9987/\underline{-2.91^\circ}$$

$$V_o = 0.9987/\underline{-2.91^\circ} V_i \quad \therefore \quad v_o(t) = 9.987 \cos(3125t - 2.91^\circ) \text{ mV}$$

$$[\mathbf{e}] \quad H(j200,000) = \frac{25,000^2 - 200,000^2}{25,000^2 - 200,000^2 + j(200,000)(10,000)} = 0.9987/\underline{2.91^\circ}$$

$$V_o = 0.9987/\underline{2.91^\circ} V_i \quad \therefore \quad v_o(t) = 9.987 \cos(200,000t + 2.91^\circ) \text{ mV}$$

$$\text{P 14.41} \quad H(j\omega) = \frac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \frac{j\omega(10,000)}{25,000^2 - \omega^2 + j\omega(10,000)}$$

$$[\mathbf{a}] \quad H(j25,000) = \frac{j(25,000)(10,000)}{25,000^2 - 25,000^2 + j(25,000)(10,000)} = 1$$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 10 \cos 25,000t \text{ V}$$

$$[\mathbf{b}] \quad H(j20,495.1) = \frac{j(20,495.1)(10,000)}{25,000^2 - 20,495.1^2 + j(20,495.1)(10,000)} = \frac{1}{\sqrt{2}}/\underline{45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}}/\underline{45^\circ} V_i \quad \therefore \quad v_o(t) = 7.071 \cos(20,495.1t + 45^\circ) \text{ V}$$

$$[\mathbf{c}] \quad H(j30,495.1) = \frac{j(30,495.1)(10,000)}{25,000^2 - 30,495.1^2 + j(30,495.1)(10,000)} = \frac{1}{\sqrt{2}}/\underline{-45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}}/\underline{-45^\circ} V_i \quad \therefore \quad v_o(t) = 7.071 \cos(30,495.1t - 45^\circ) \text{ V}$$

$$[\mathbf{d}] \quad H(j3125) = \frac{j(3125)(10,000)}{25,000^2 - 3125^2 + j(3125)(10,000)} = 0.051/\underline{87.09^\circ}$$

$$V_o = 0.051/\underline{87.09^\circ} V_i \quad \therefore \quad v_o(t) = 0.51 \cos(3125t + 87.09^\circ) \text{ V}$$

$$[\mathbf{e}] \quad H(j200,000) = \frac{j(200,000)(10,000)}{25,000^2 - 200,000^2 + j(200,000)(10,000)} = 0.051/\underline{-87.09^\circ}$$

$$V_o = 0.051/\underline{-87.09^\circ} V_i \quad \therefore \quad v_o(t) = 0.51 \cos(200,000t - 87.09^\circ) \text{ V}$$

$$\text{P 14.42} \quad [\mathbf{a}] \quad \omega_o = 2\pi f_o = 8\pi \text{ krad/s}$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2 (0.5 \times 10^{-6})} = 3.17 \text{ mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5 \times 10^{-6})} = 397.89 \Omega$$

$$\begin{aligned}
 [\mathbf{b}] \quad f_{c2} &= f_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 4000 \left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right] \\
 &= 4.42 \text{ kHz} \\
 f_{c1} &= f_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 4000 \left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right] \\
 &= 3.62 \text{ kHz}
 \end{aligned}$$

$$[\mathbf{c}] \quad \beta = f_{c2} - f_{c1} = 800 \text{ Hz}$$

or

$$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800 \text{ Hz}$$

$$\text{P 14.43 [a]} \quad R_e = 397.89 \| 1000 = 284.63 \Omega$$

$$Q = \omega_o R_e C = (8000\pi)(284.63)(0.5 \times 10^{-6}) = 3.58$$

$$[\mathbf{b}] \quad \beta = \frac{f_o}{Q} = \frac{4000}{3.58} = 1.12 \text{ kHz}$$

$$[\mathbf{c}] \quad f_{c2} = 4000 \left[\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}} \right] = 4.60 \text{ kHz}$$

$$[\mathbf{d}] \quad f_{c1} = 4000 \left[-\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}} \right] = 3.48 \text{ kHz}$$

$$\text{P 14.44 [a]} \quad \text{We need } \omega_c = 2\pi(4000) = 25,132.74 \text{ rad/s. There are several possible approaches – this one starts by choosing } L = 100 \mu\text{H. Then,}$$

$$C = \frac{1}{[2\pi(4000)]^2(100 \times 10^{-6})} = 15.83 \mu\text{F}$$

Use the closest value from Appendix H, which is $22 \mu\text{F}$, to give

$$\omega_c = \sqrt{\frac{1}{100 \times 10^{-6}(22 \times 10^{-6})}} = 21,320.07 \text{ rad/s} \quad \text{so} \quad f_c = 3393.19 \text{ Hz}$$

$$\text{Then, } R = \frac{Q}{\omega_o C} = \frac{5}{(21320.07)(22 \times 10^{-6})} = 10.66 \Omega$$

Use the closest value from Appendix H, which is 10Ω , to give

$$Q = 10(21,320.07)(22 \times 10^{-6}) = 4.69$$

$$[\mathbf{b}] \quad \% \text{ error in } f_c = \frac{3393.19 - 4000}{4000}(100) = -15.2\%$$

$$\% \text{ error in } Q = \frac{4.69 - 5}{5}(100) = -6.2\%$$

P 14.45 [a] Let $Z = \frac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

$$\text{Then } H(s) = \frac{V_o}{V_i} = \frac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$$

Therefore

$$\begin{aligned} H(s) &= \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]} \\ &= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2} \end{aligned}$$

$$\text{where } K = \frac{R_L}{R + R_L}; \quad \omega_o^2 = \frac{1}{LC}; \quad \beta = \left(\frac{RR_L}{R + R_L}\right) \frac{1}{L}$$

[b] $\omega_o = \frac{1}{\sqrt{LC}}$

[c] $\beta = \left(\frac{RR_L}{R + R_L}\right) \frac{1}{L}$

[d] $Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \frac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j\omega_o) = 0$$

[f] $H(j0) = \frac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \frac{K \left[(\omega_o/\omega)^2 - 1 \right]}{\left\{ \left[(\omega_o/\omega)^2 - 1 \right] + j\beta/\omega \right\}}$

$$H(j\infty) = \frac{-K}{-1} = K$$

[h] $H(j\omega) = \frac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j0) = H(j\infty) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2 \beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2 \beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm \omega_c \beta$$

$$\therefore \omega_c^2 \pm \omega_c \beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp \frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \frac{RR_L}{R+R_L} \cdot \frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

$$\text{P 14.46 [a]} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{(10^{-6})(4 \times 10^{-12})} = 0.25 \times 10^{18} = 25 \times 10^{16}$$

$$\omega_o = 5 \times 10^8 = 500 \text{ Mrad/s}$$

$$\beta = \frac{RR_L}{R+R_L} \cdot \frac{1}{L} = \frac{(30)(150)}{180} \cdot \frac{1}{10^{-6}} = 25 \text{ Mrad/s} = 3.98 \text{ MHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{500 \text{ M}}{25 \text{ M}} = 20$$

$$\text{[b]} \quad H(j0) = \frac{R_L}{R+R_L} = \frac{150}{180} = 0.8333$$

$$H(j\infty) = \frac{R_L}{R+R_L} = 0.8333$$

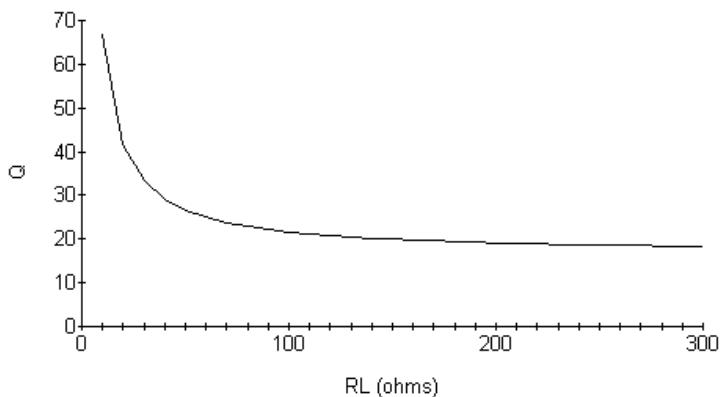
$$\text{[c]} \quad f_{c2} = \frac{250}{\pi} \left[\frac{1}{40} + \sqrt{1 + \frac{1}{1600}} \right] = 81.59 \text{ MHz}$$

$$f_{c1} = \frac{250}{\pi} \left[-\frac{1}{40} + \sqrt{1 + \frac{1}{1600}} \right] = 77.61 \text{ MHz}$$

$$\text{Check: } \beta = f_{c2} - f_{c1} = 3.98 \text{ MHz.}$$

$$\begin{aligned}
 [\text{d}] \quad Q &= \frac{\omega_o}{\beta} = \frac{500 \times 10^6}{\frac{RR_L}{R+R_L} \cdot \frac{1}{L}} \\
 &= \frac{500(R + R_L)}{RR_L} = \frac{50}{3} \left(1 + \frac{30}{R_L}\right) \\
 \text{where } R_L &\text{ is in ohms.}
 \end{aligned}$$

[e]



$$\text{P 14.47 [a]} \quad \omega_o^2 = \frac{1}{LC} = 625 \times 10^6$$

$$\therefore L = \frac{1}{(625 \times 10^6)(25 \times 10^{-9})} = 64 \text{ mH}$$

$$\frac{R_L}{R + R_L} = 0.9; \quad \therefore 0.1R_L = 0.9R$$

$$\therefore R_L = 9R \quad \therefore R = \frac{500}{9} = 55.6 \Omega$$

$$[\text{b}] \quad \beta = \left(\frac{R_L}{R + R_L}\right) R \cdot \frac{1}{L} = 781.25 \text{ rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{25,000}{781.25} = 32$$

$$\text{P 14.48 [a]} \quad |H(j\omega)| = \frac{25 \times 10^6}{\sqrt{(25 \times 10^6 - \omega^2)^2 + (1000\omega)^2}} = 1$$

$$\begin{aligned}
 \therefore 0 &= 625 \times 10^{12} = (25 \times 10^6 - \omega^2)^2 + (1000\omega)^2 \\
 &= -50 \times 10^6 \omega^2 + \omega^4 + 10^6 \omega^2
 \end{aligned}$$

From the above equation it is obvious that $\omega = 0$ is one solution. The other is as follows:

$$\omega^2 = 49 \times 10^6 \quad \text{so} \quad \omega = 7000 \text{ rad/s}$$

- [b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency for which

$$(25 \times 10^6 - \omega^2)^2 = 0 \quad \text{so} \quad \omega = \sqrt{25 \times 10^6} = 5000 \text{ rad/s}$$

[c] $|H(j5000)| = \frac{25 \times 10^6}{\sqrt{(25 \times 10^6 - 5000^2)^2 + [1000(5000)]^2}} = 5$

P 14.49 [a] $H(s) = \frac{sL}{R + sL + \frac{1}{sC}} = \frac{s^2LC}{RsC + s^2LC + 1} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

- [b] When $s = j\omega$ is very small (think of ω approaching 0),

$$H(s) \approx \frac{s^2}{\frac{1}{LC}} = 0$$

- [c] When $s = j\omega$ is very large (think of ω approaching ∞),

$$H(s) \approx \frac{s^2}{s^2} = 1$$

- [d] The magnitude of $H(s)$ approaches 0 as the frequency approaches 0, and approaches 1 as the frequency approaches ∞ . Therefore, this circuit is behaving like a high pass filter when the output is the voltage across the inductor.

[e] $|H(j\omega_c)| = \frac{\omega_c^2}{\sqrt{(10^6 - \omega_c^2)^2 + (1500\omega_c)^2}} = \frac{1}{\sqrt{2}}$

$$2\omega_c^4 = (10^6 - \omega_c^2)^2 + (1500\omega_c)^2 = 10^{12} - 2 \times 10^6\omega_c^2 + \omega_c^4 + 225 \times 10^4\omega_c^2$$

$$\text{Simplifying, } \omega_c^4 - 25 \times 10^4\omega_c^2 - 10^{12} = 0$$

Solve for ω_c^2 and then ω_c :

$$\omega_c^2 = 1,132,782.22 \quad \text{so} \quad \omega_c = 1064.322 \text{ rad/s}$$

$$f = 169.4 \text{ Hz}$$

P 14.50 [a] $H(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{1}{RsC + s^2LC + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

- [b] When $s = j\omega$ is very small (think of ω approaching 0),

$$H(s) \approx \frac{\frac{1}{LC}}{\frac{1}{LC}} = 1$$

- [c] When $s = j\omega$ is very large (think of ω approaching ∞),

$$H(s) \approx \frac{\frac{1}{LC}}{s^2} = 0$$

- [d] The magnitude of $H(s)$ approaches 1 as the frequency approaches 0, and approaches 0 as the frequency approaches ∞ . Therefore, this circuit is behaving like a low pass filter when the output is the voltage across the capacitor.

$$[e] |H(j\omega_c)| = \frac{10^6}{\sqrt{(10^6 - \omega_c^2)^2 + (1500\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$2 \times 10^{12} = (10^6 - \omega_c^2)^2 + (1500\omega_c)^2 = 10^{12} - 2 \times 10^6 \omega_c^2 + \omega_c^4 + 225 \times 10^4 \omega_c^2$$

$$\text{Simplifying, } \omega_c^4 + (1500^2 - 2 \times 10^6)\omega_c^2 - 10^{12} = 0$$

Solve for ω_c^2 and then ω_c :

$$\omega_c^2 = 882,782.22 \quad \text{so} \quad \omega_c = 939.565 \text{ rad/s}$$

$$f = 149.54 \text{ Hz}$$

P 14.51 [a] Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \quad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

$$\text{Thus } \beta = \omega_{c2} - \omega_{c1} = 1533.10 \text{ rad/s}$$

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39 \text{ H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10 \mu\text{F}$$

- [b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|V_{697\text{Hz}}| = |V_{941\text{Hz}}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}| \frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\text{Hz}}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852\text{Hz}}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}} \\ = 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this same property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

- [c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\text{Hz}}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}} \\ = 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.52 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596 \text{ rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26 \text{ krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664 \text{ rad/s}$$

Telephone circuits always have $R = 600 \Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225 \text{ H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057 \mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}| \frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$\begin{aligned} |V_\omega| &= \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}} \\ &= 0.344 |V_{\text{peak}}| \end{aligned}$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

- P 14.53 From Problem 14.51 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$\begin{aligned} |V_{20\text{Hz}}| &= \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}} \\ &= 0.00744|V_{\text{peak}}| \\ \therefore \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} &= \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5 \\ \therefore |V_{20\text{Hz}}| &= 63.7|V_{770\text{Hz}}| \end{aligned}$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.

15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1 \text{ rad/s}; \quad R_1 = 1 \Omega, \quad \therefore C = 1 \text{ F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore R_2 = R_1 = 1 \Omega$$

$$\therefore H_{\text{prototype}}(s) = \frac{-s}{s + 1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20,000}{s + 5000}$$

$$\frac{1}{R_1C} = 20,000; \quad C = 5 \mu\text{F}$$

$$\therefore R_1 = \frac{1}{(20,000)(5 \times 10^{-6})} = 10 \Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40 \Omega$$

AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20,000\pi \text{ rad/s}$$

$$\therefore k_f = 20,000\pi = 62,831.85$$

$$C' = \frac{C}{k_f k_m} \quad \therefore \quad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore k_m = \frac{1}{(0.5 \times 10^{-6})(62,831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2 C}\right)s + \left(\frac{1}{R_1 R_2 C^2}\right)}$$

Equate the transfer functions. For $C = 1\text{F}$,

$$\frac{2}{R_2 C} = \sqrt{2}, \quad \therefore R_2 = \sqrt{2} = 1.414 \Omega$$

$$\frac{1}{R_1 R_2 C^2} = 1, \quad \therefore R_1 = \frac{1}{\sqrt{2}} = 0.707 \Omega$$

AP 15.5

$$Q = 8, K = 5, \omega_o = 1000 \text{ rad/s}, C = 1 \mu\text{F}$$

For the circuit in Fig 15.26

$$\begin{aligned} H(s) &= \frac{-\left(\frac{1}{R_1 C}\right)s}{s^2 + \left(\frac{2}{R_3 C}\right)s + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)} \\ &= \frac{K \beta s}{s^2 + \beta s + \omega_o^2} \end{aligned}$$

$$\beta = \frac{2}{R_3 C}, \quad \therefore R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125 \text{ rad/s}$$

$$\therefore R_3 = \frac{2 \times 10^6}{(125)(1)} = 16 \text{ k}\Omega$$

$$K\beta = \frac{1}{R_1 C}$$

$$\therefore R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6 \text{ k}\Omega$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16,000)(10^{-6})^2}$$

Solving for R_2 ,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \quad 246R_2 = 16,000, \quad R_2 = 65.04 \Omega$$

AP 15.6

$$\omega_o = 1000 \text{ rad/s}; \quad Q = 4;$$

$$C = 2 \mu\text{F}$$

$$\begin{aligned} H(s) &= \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\frac{4(1-\sigma)}{RC}\right]s + \left(\frac{1}{R^2C^2}\right)} \\ &= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \quad \omega_o = \frac{1}{RC}; \quad \beta = \frac{4(1-\sigma)}{RC} \end{aligned}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500 \Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \sigma = 0.9375$$

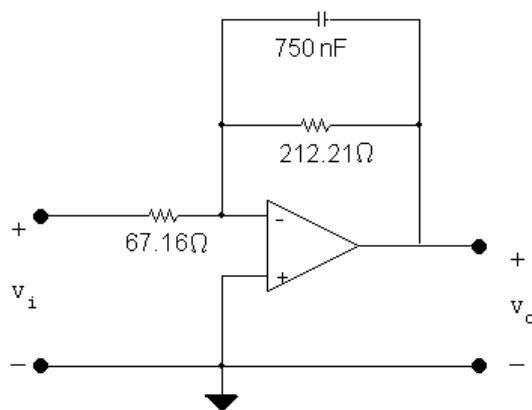
Problems

P 15.1 [a] $K = 10^{(10/20)} = 3.16 = \frac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21 \Omega$$

$$R_1 = \frac{R_2}{K} = \frac{212.21}{3.16} = 67.16 \Omega$$

[b]



P 15.2 [a] $\frac{1}{RC} = 2\pi(1000)$ so $RC = 1.5915 \times 10^{-4}$

There are several possible approaches. Here, choose $R_f = 150 \Omega$. Then

$$C = \frac{1.5915 \times 10^{-4}}{150} = 1.06 \times 10^{-6}$$

Choose $C = 1 \mu\text{F}$. This gives

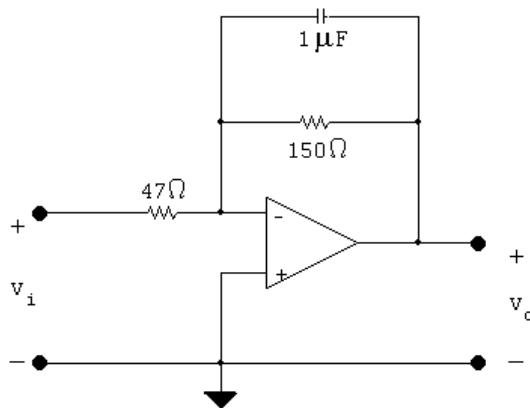
$$\omega_c = \frac{1}{(150)(10^{-6})} = 6.67 \times 10^3 \text{ rad/s} \quad \text{so} \quad f_c = 1061 \text{ Hz}$$

To get a passband gain of 10 dB, choose

$$R_i = \frac{R_f}{3.16} = \frac{150}{3.16} = 47.47 \Omega$$

Choose $R_i = 47 \Omega$ to give $K = 20 \log_{10}(150/47) = 10.08 \text{ dB}$.

The resulting circuit is

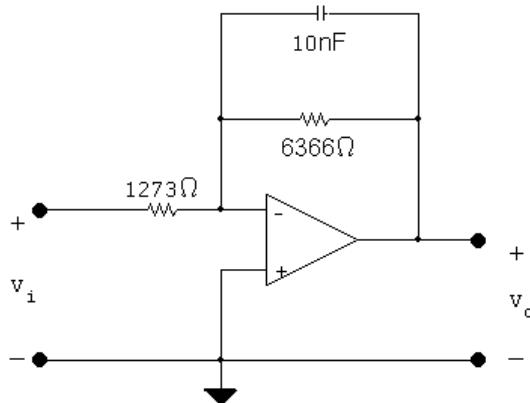


$$[b] \text{ % error in } f_c = \frac{1061 - 1000}{1000} (100) = 6.1\%$$

$$\text{ % error in passband gain} = \frac{10.08 - 10}{10} (100) = 0.8\%$$

P 15.3 [a] $\omega_c = \frac{1}{R_2 C}$ so $R_2 = \frac{1}{\omega_c C} = \frac{1}{2\pi(2500)(10 \times 10^{-9})} = 6366 \Omega$

$$K = \frac{R_2}{R_1} \text{ so } R_1 = \frac{R_2}{K} = \frac{6366}{5} = 1273 \Omega$$



[b] Both the cutoff frequency and the passband gain are changed.

P 15.4 [a] $5(3.5) = 17.5 \text{ V}$ so $V_{cc} \geq 17.5 \text{ V}$

$$[b] H(j\omega) = \frac{-5(2\pi)(2500)}{j\omega + 2\pi(2500)}$$

$$H(j5000\pi) = \frac{-5(5000\pi)}{5000\pi + j5000\pi} = -2.5 + j2.5 = \frac{5}{\sqrt{2}}/135^\circ$$

$$V_o = \frac{17.5}{\sqrt{2}}/135^\circ V_i \text{ so } v_o(t) = 12.37 \cos(5000\pi t + 135^\circ) \text{ V}$$

$$[\text{c}] \quad H(j625\pi) = \frac{-5(5000\pi)}{5000\pi + j625\pi} = 4.96/\underline{172.9^\circ}$$

$$V_o = 17.36/\underline{172.9^\circ} V_i \quad \text{so} \quad v_o(t) = 17.36 \cos(625\pi t + 172.9^\circ) \text{ V}$$

$$[\text{d}] \quad H(j40,000\pi) = \frac{-5(5000\pi)}{5000\pi + j40,000\pi} = 0.62/\underline{97.1^\circ}$$

$$V_o = 2.2/\underline{97.1^\circ} V_i \quad \text{so} \quad v_o(t) = 2.2 \cos(40,000\pi t + 97.1^\circ) \text{ V}$$

P 15.5 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

$$\begin{aligned} \text{P 15.6} \quad [\text{a}] \quad Z_f &= \frac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \frac{R_2}{R_2 C_2 s + 1} \\ &= \frac{(1/C_2)}{s + (1/R_2 C_2)} \end{aligned}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1 C_1)}$$

$$\begin{aligned} \therefore H(s) &= \frac{-(1/C_2)[s + (1/R_1 C_1)]}{[s + (1/R_2 C_2)][1/C_1]} \\ &= -\frac{C_1[s + (1/R_1 C_1)]}{C_2[s + (1/R_2 C_2)]} \end{aligned}$$

$$[\text{b}] \quad H(j\omega) = \frac{-C_1}{C_2} \left[\frac{j\omega + (1/R_1 C_1)}{j\omega + (1/R_2 C_2)} \right]$$

$$H(j0) = \frac{-C_1}{C_2} \left(\frac{R_2 C_2}{R_1 C_1} \right) = \frac{-R_2}{R_1}$$

$$[\text{c}] \quad H(j\infty) = -\frac{C_1}{C_2} \left(\frac{j}{j} \right) = \frac{-C_1}{C_2}$$

- [d] As $\omega \rightarrow 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.

As $\omega \rightarrow \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \rightarrow v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$.

Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

$$\text{P 15.7 [a]} \quad Z_f = \frac{(1/C_2)}{s + (1/R_2 C_2)}$$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1 C_1)]$$

$$\begin{aligned} H(s) &= -\frac{(1/C_2)}{[s + (1/R_2 C_2)]} \cdot \frac{s}{R_1[s + (1/R_1 C_1)]} \\ &= -\frac{1}{R_1 C_2} \frac{s}{[s + (1/R_1 C_1)][s + (1/R_2 C_2)]} \end{aligned}$$

$$\text{[b]} \quad H(j\omega) = -\frac{1}{R_1 C_2} \frac{j\omega}{\left(j\omega + \frac{1}{R_1 C_1}\right) \left(j\omega + \frac{1}{R_2 C_2}\right)}$$

$$H(j0) = 0$$

$$\text{[c]} \quad H(j\infty) = 0$$

- [d] As $\omega \rightarrow 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.

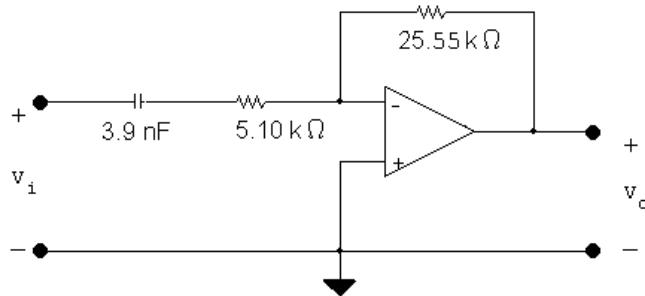
As $\omega \rightarrow \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

$$\text{P 15.8 [a]} \quad R_1 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(8 \times 10^3)(3.9 \times 10^{-9})} = 5.10 \text{ k}\Omega$$

$$K = 10^{(14/20)} = 5.01 = \frac{R_2}{R_1}$$

$$\therefore R_2 = 5.01R_1 = 25.55 \text{ k}\Omega$$

[b]



$$P 15.9 \quad [a] \quad \frac{1}{RC} = 2\pi(8000) \quad \text{so} \quad RC = 19.89 \times 10^{-6}$$

There are several possible approaches. Here, choose $C = 0.047 \mu\text{F}$. Then

$$R_i = \frac{19.89 \times 10^{-6}}{0.047 \times 10^{-6}} = 423$$

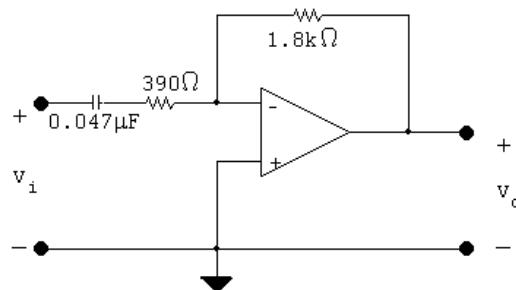
Choose $R_i = 390 \Omega$. This gives

$$\omega_c = \frac{1}{(0.047 \times 10^{-6})(390)} = 54.56 \text{ krad/s} \quad \text{so} \quad f_c = 8.68 \text{ kHz}$$

To get a passband gain of 14 dB, choose

$$R_f = 5R_i = 5(390) = 1950 \Omega$$

Choose $R_f = 1.8 \text{ k}\Omega$ to give a passband gain of $20 \log_{10}(1800/390) = 13.3$ dB. The resulting circuit is

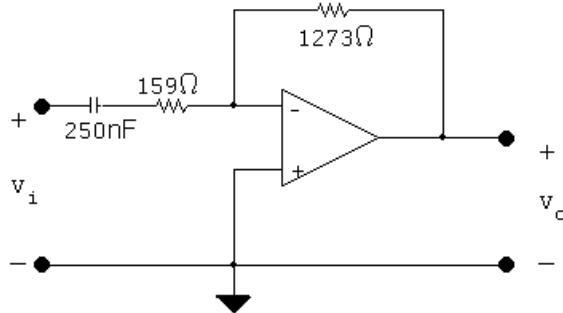


$$[b] \quad \% \text{ error in } f_c = \frac{8683.76 - 8000}{8000} (100) = 8.5\%$$

$$\% \text{ error in passband gain} = \frac{13.3 - 14}{14} (100) = -5.1\%$$

$$P 15.10 \quad [a] \quad \omega_c = \frac{1}{R_1 C} \quad \text{so} \quad R_1 = \frac{1}{\omega_c C} = \frac{1}{2\pi(4000)(250 \times 10^{-9})} = 159 \Omega$$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (8)(159) = 1273 \Omega$$



[b] The passband gain changes but the cutoff frequency is unchanged.

P 15.11 [a] $8(2.5) = 20 \text{ V}$ so $V_{cc} \geq 20 \text{ V}$

[b] $H(j\omega) = \frac{-8j\omega}{j\omega + 8000\pi}$

$$H(j8000\pi) = \frac{-8(j8000\pi)}{8000\pi + j8000\pi} = \frac{8}{\sqrt{2}} / -135^\circ$$

$$V_o = 14.14 / -135^\circ V_i \quad \text{so} \quad v_o(t) = 14.14 \cos(8000\pi t - 135^\circ) \text{ V}$$

[c] $H(j1000\pi) = \frac{-8(j1000\pi)}{8000\pi + j1000\pi} = 0.99 / -97.1^\circ$

$$V_o = 2.48 / -97.1^\circ V_i \quad \text{so} \quad v_o(t) = 2.48 \cos(1000\pi t - 97.1^\circ) \text{ mV}$$

[d] $H(j64,000\pi) = \frac{-8(j64,000\pi)}{8000\pi + j64,000\pi} = 7.94 / -172.9^\circ$

$$V_o = 19.85 / -172.9^\circ V_i \quad \text{so} \quad v_o(t) = 19.85 \cos(64,000\pi t - 172.9^\circ) \text{ V}$$

P 15.12 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \quad C' = \frac{C}{k_m k_f}$$

$$\therefore R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \quad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \quad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left(\frac{R}{L} \right) = k_f$$

$$H'(s) = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.13 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \quad C' = \frac{C}{k_m k_f}$$

$$\therefore R'C' = k_m R \frac{C}{k_m k_f} = \frac{1}{k_f} RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

$$\text{For the RL circuit} \quad H(s) = \frac{R/L}{s + R/L} \quad \text{so}$$

$$R' = k_m R; \quad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L} \right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

$$\text{P 15.14 } H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.

For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

$$\text{where } R' = k_m R; \ L' = \frac{k_m}{k_f} L; \ \text{and } C' = \frac{C}{k_f k_m}$$

$$\therefore \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L} \right) = k_f \beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f} L C} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega'_o}{\beta'} = \frac{k_f \omega_o}{k_f \beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f \beta$.

$$\therefore H'(s) = \frac{\left(\frac{k_f}{Q}\right) s}{s^2 + \left(\frac{k_f}{Q}\right) s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right) \left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q} \left(\frac{s}{k_f}\right) + 1\right]}$$

$$\text{P 15.15 [a]} \quad L = 1 \text{ H}; \quad C = 1 \text{ F}$$

$$R = \frac{1}{Q} = \frac{1}{20} = 0.05 \Omega$$

$$[\mathbf{b}] \quad k_f = \frac{\omega'_o}{\omega_o} = 40,000; \quad k_m = \frac{R'}{R} = \frac{5000}{0.05} = 100,000$$

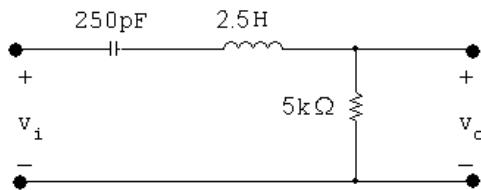
Thus,

$$R' = k_m R = (0.05)(100,000) = 5 \text{ k}\Omega$$

$$L' = \frac{k_m}{k_f} L = \frac{100,000}{40,000}(1) = 2.5 \text{ H}$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(40,000)(100,000)} = 250 \text{ pF}$$

[c]



P 15.16 [a] Since $\omega_o^2 = 1/LC$ and $\omega_o = 1 \text{ rad/s}$,

$$C = \frac{1}{L} = \frac{1}{Q} \text{ F}$$

[b] $H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1 \Omega; \quad L = 16 \text{ H}; \quad C = \frac{1}{L} = 0.0625 \text{ F}$$

$$\therefore k_m = \frac{R'}{R} = 10,000; \quad k_f = \frac{\omega'_o}{\omega_o} = 25,000$$

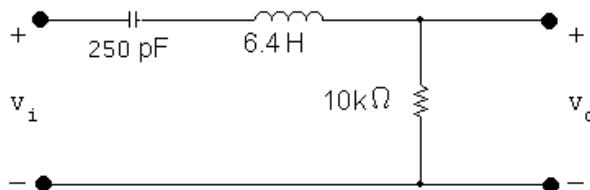
Thus

$$R' = k_m R = 10 \text{ k}\Omega$$

$$L' = \frac{k_m}{k_f} L = \frac{10,000}{25,000} (16) = 6.4 \text{ H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10,000)(25,000)} = 250 \text{ pF}$$

[d]



[e] $H'(s) = \frac{\frac{1}{16} \left(\frac{s}{25,000} \right)}{\left(\frac{s}{25,000} \right)^2 + \frac{1}{16} \left(\frac{s}{25,000} \right) + 1}$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

P 15.17 [a] Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \quad C = 1 \text{ F}; \quad L = 1 \text{ H}; \quad R = 25 \Omega$$

$$k_m = \frac{R'}{R} = \frac{40,000}{25} = 1600; \quad k_f = \frac{\omega'}{\omega_o} = \frac{1600}{50,000} = 50,000$$

Thus,

$$R' = k_m R = 40 \text{ k}\Omega; \quad L' = \frac{k_m}{k_f} L = \frac{1600}{50,000} (1) = 32 \text{ mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50,000)} = 12.5 \text{ nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \quad C = 25 \text{ F}$$

$$L = \frac{1}{25} = 40 \text{ mH}; \quad R = 1 \Omega$$

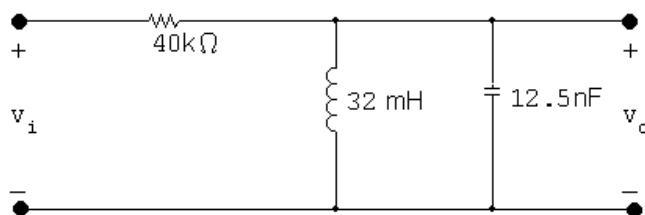
$$k_m = \frac{R'}{R} = 40,000; \quad k_f = \frac{\omega'}{\omega_o} = 50,000$$

Thus,

$$R' = k_m R = 40 \text{ k}\Omega; \quad L' = \frac{k_m}{k_f} L = \frac{40,000}{50,000} (0.04) = 32 \text{ mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40,000)(50,000)} = 12.5 \text{ nF}$$

[b]



P 15.18 [a] For the circuit in Fig. P15.18(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \frac{1}{s}}{\frac{1}{Q} + s + \frac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.18(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2 + 1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

$$\begin{aligned} [\text{b}] \quad H'(s) &= \frac{\left(\frac{s}{8000}\right)^2 + 1}{\left(\frac{s}{8000}\right)^2 + \frac{1}{10}\left(\frac{s}{8000}\right) + 1} \\ &= \frac{s^2 + 64 \times 10^6}{s^2 + 800s + 64 \times 10^6} \end{aligned}$$

P 15.19 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f} L; \quad C' = \frac{C}{k_m k_f}$$

$$\therefore \frac{1}{L'C'} = \frac{k_f^2}{LC}; \quad R' = k_m R$$

$$\therefore \frac{R'}{L'} = k_f \left(\frac{R}{L}\right)$$

It follows then that

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]} \\ &= H(s)|_{s=s/k_f} \end{aligned}$$

P 15.20 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

$$\text{where } R' = k_m R; \quad L' = \frac{k_m}{k_f} L;$$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right)s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}} \\ &= H(s)|_{s=s/k_f} \end{aligned}$$

P 15.21 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s+\frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2 + 1)}{Q(s^2 + 1) + s} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For prototype circuit (b):

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}} \\ &= \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1} \end{aligned}$$

$$\text{P 15.22 [a]} \quad k_m = \frac{80}{200} = 0.4$$

$$\frac{k_m}{k_f}(0.4) = 0.02 \quad \text{so} \quad k_f = \frac{(0.4)(0.4)}{0.02} = 8$$

$$C' = \frac{20 \times 10^{-6}}{(0.4)(8)} = 6.25 \mu\text{F}$$

$$\begin{aligned} \text{[b]} \quad Z_{ab} &= \frac{1}{j\omega C} + R\|j\omega L = \frac{1}{j\omega C} + \frac{j\omega RL}{R + j\omega L} \\ &= \frac{(R + j\omega L - \omega^2 RLC)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)} \end{aligned}$$

The denominator is purely real, so set the imaginary part of the numerator equal to 0 and solve for ω :

$$-\omega R^2 C - \omega^3 L^2 C_\omega^3 R^2 L C^2 = 0$$

$$\therefore \omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{(80)^2}{(80)^2(0.02(6.25 \times 10^{-6}) - (0.02)^2)} = 16 \times 10^6$$

Thus,

$$\omega = 4000 \text{ rad/s}$$

- [c]** In the original, unscaled circuit, the frequency at which the impedance Z_{ab} is purely real is

$$\omega_{us}^2 = \frac{(200)^2}{(200)^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250,000$$

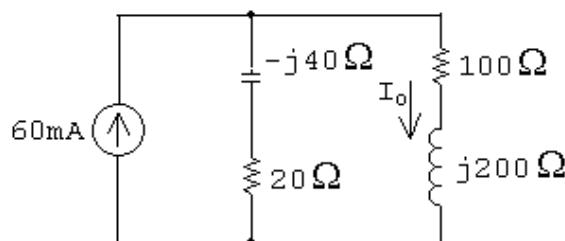
Thus, $\omega_{us} = 500 \text{ rad/s}$

$$\frac{\omega_{scaled}}{\omega_{us}} = \frac{4000}{500} = 8 = k_f$$

$$\text{P 15.23 [a]} \quad k_f = \frac{10,000}{250} = 40$$

$$C' = \frac{100 \times 10^{-6}}{40} = 2.5 \mu\text{F}$$

$$L' = \frac{0.8}{40} = 20 \text{ mH}$$



$$[\mathbf{b}] \quad I_o = \frac{20 - j40}{120 + j160} (0.06) = 13.42 / -116.57^\circ \text{ mA}$$

$$i_o = 13.42 \cos(10,000t - 116.57^\circ) \text{ mA}$$

The magnitude and phase angle of the output current are the same as in the unscaled circuit.

- P 15.24 From the solution to Problem 14.18, $\omega_o = 100 \text{ krad/s}$ and $\beta = 12.5 \text{ krad/s}$. Compute the two scale factors:

$$k_f = \frac{\omega'_o}{\omega_o} = \frac{2\pi(25 \times 10^3)}{100 \times 10^3} = \frac{\pi}{2}$$

$$L' = \frac{k_m L}{k_f} \quad \text{so} \quad k_m = \frac{L'}{L} k_f = \frac{25 \times 10^{-6}}{10 \times 10^{-3}} \left(\frac{\pi}{2}\right) = \frac{\pi}{800}$$

Thus,

$$R' = k_m R = \frac{8000\pi}{800} = 31.4 \Omega \quad C' = \frac{C}{k_m k_f} = \frac{10 \times 10^{-9}}{(\pi/800)(\pi/2)} = 1.62 \mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega'_{c1} = k_f \omega_{c1} = \frac{\pi}{2}(93.95 \times 10^3) = 147.576 \text{ krad/s}$$

$$\omega'_{c2} = k_f \omega_{c2} = \frac{\pi}{2}(106.45 \times 10^3) = 167.211 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega'_{c2} - \omega'_{c1} = 19.6 \text{ krad/s} = (\pi/2)\beta \text{ (checks!)}$$

- P 15.25 From the solution to Problem 14.38, $\omega_o = 80 \text{ krad/s}$ and $\beta = 5.33 \text{ krad/s}$. Calculate the scale factors:

$$k_f = \frac{\omega'_o}{\omega_o} = \frac{16 \times 10^3}{80 \times 10^3} = 0.2$$

$$k_m = \frac{C}{C' k_f} = \frac{62.5 \times 10^{-9}}{(50 \times 10^{-9})(0.2)} = 6.25$$

Thus,

$$R' = k_m R = (6.25)(3000) = 18.75 \text{ k}\Omega$$

$$L' = \frac{k_m L}{k_f} = \frac{(6.25)(2.5 \times 10^{-3})}{0.2} = 78.125 \text{ mH}$$

Calculate the bandwidth:

$$\beta' = k_f \beta = (0.2)[5333.33] = 1066.67 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{80,000}{5333.33} = 15$$

$$Q' = \frac{\omega'_o}{\beta'} = \frac{16,000}{1066.67} = 15 \text{ (checks)}$$

P 15.26 [a] From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

$$\text{where } K = \frac{R_2}{R_1}, \quad \omega_c = \frac{1}{R_2 C}$$

$$\therefore H'(s) = \frac{-K'\omega'_c}{s + \omega'_c}$$

$$\text{where } K' = \frac{R'_2}{R'_1} \quad \omega'_c = \frac{1}{R'_2 C'}$$

By hypothesis $R'_1 = k_m R_1$; $R'_2 = k_m R_2$,

and $C' = \frac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega'_c = k_f \omega_c$, therefore

$$H'(s) = \frac{-K k_f \omega_c}{s + k_f \omega_c} = \frac{-K \omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

$$[b] \quad H(s) = \frac{-K}{s + 1}$$

$$[c] \quad H'(s) = \frac{-K}{\left(\frac{s}{k_f}\right) + 1} = \frac{-K k_f}{s + k_f}$$

P 15.27 [a] From Eq. 15.4

$$H(s) = \frac{-Ks}{s + \omega_c} \text{ where } K = \frac{R_2}{R_1} \text{ and}$$

$$\omega_c = \frac{1}{R_1 C}$$

$$\therefore H'(s) = \frac{-K's}{s + \omega'_c} \text{ where } K' = \frac{R'_2}{R'_1}$$

$$\text{and } \omega'_c = \frac{1}{R'_1 C'}$$

By hypothesis

$$R'_1 = k_m R_1; \quad R'_2 = k_m R_2; \quad C' = \frac{C}{k_m k_f}$$

It follows that

$$K' = K \text{ and } \omega'_c = k_f \omega_c$$

$$\therefore H'(s) = \frac{-Ks}{s + k_f \omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right) + \omega_c}$$

$$[\mathbf{b}] \quad H(s) = \frac{-Ks}{s + 1}$$

$$[\mathbf{c}] \quad H'(s) = \frac{-K(s/k_f)}{\left(\frac{s}{k_f} + 1\right)} = \frac{-Ks}{s + k_f}$$

$$\text{P 15.28 [a]} \quad H_{\text{hp}} = \frac{s}{s + 1}; \quad k_f = \frac{\omega'_o}{\omega} = \frac{200(2\pi)}{1} = 400\pi$$

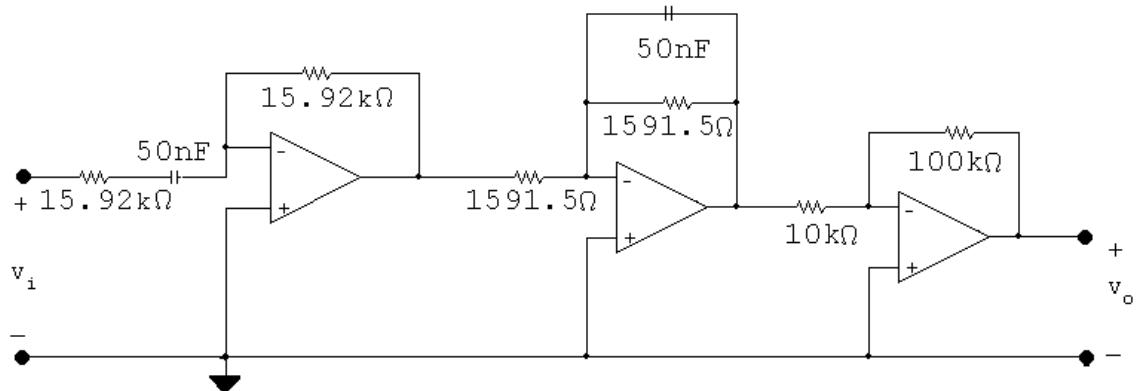
$$\therefore H'_{\text{hp}} = \frac{s}{s + 400\pi}$$

$$\frac{1}{R_H C_H} = 400\pi; \quad \therefore R_H = \frac{1}{(400\pi)(50 \times 10^{-9})} = 15.915 \text{ k}\Omega$$

$$H_{\text{lp}} = \frac{1}{s + 1}; \quad k_f = \frac{\omega'_o}{\omega} = \frac{2000(2\pi)}{1} = 4000\pi$$

$$\therefore H'_{\text{lp}} = \frac{4000\pi}{s + 4000\pi}$$

$$\frac{1}{R_L C_L} = 4000\pi; \quad \therefore R_L = \frac{1}{(4000\pi)(50 \times 10^{-9})} = 1591.5 \Omega$$



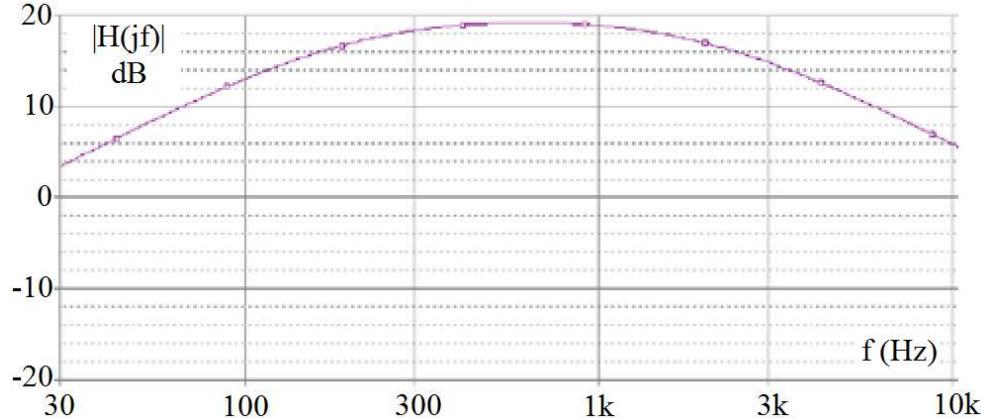
$$\begin{aligned}
 [b] \quad H'(s) &= \frac{s}{s + 400\pi} \cdot \frac{4000\pi}{s + 4000\pi} \cdot 10 \\
 &= \frac{40,000\pi s}{(s + 400\pi)(s + 4000\pi)}
 \end{aligned}$$

$$[c] \quad \omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(400\pi)(4000\pi)} = 3973.8 \text{ rad/s}$$

$$\begin{aligned}
 H'(j\omega_o) &= \frac{(40,000\pi)(j3973.8)}{(400\pi + j3973.8)(4000\pi + j3973.8)} \\
 &= 9.1/\underline{0^\circ}
 \end{aligned}$$

$$[d] \quad G = 20 \log_{10}(9.1) = 19.2 \text{ dB}$$

[e]



P 15.29 [a] For the high-pass section:

$$k_f = \frac{\omega'_o}{\omega} = \frac{800(2\pi)}{1} = 1600\pi$$

$$H'(s) = \frac{s}{s + 1600\pi}$$

$$\therefore \frac{1}{R_1(2 \times 10^{-6})} = 1600\pi; \quad R_1 = 99.5 \Omega \quad \therefore \quad R_2 = 99.5 \Omega$$

For the low-pass section:

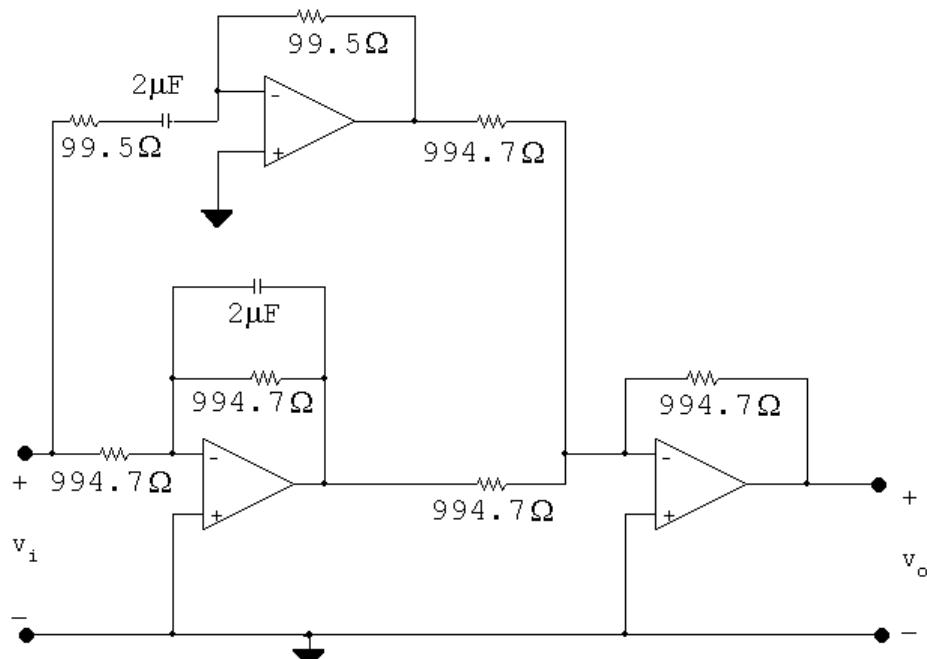
$$k_f = \frac{\omega_o'}{\omega} = \frac{80(2\pi)}{1} = 160\pi$$

$$H'(s) = \frac{160\pi}{s + 160\pi}$$

$$\therefore \frac{1}{R_2(2 \times 10^{-6})} = 160\pi; \quad R_2 = 994.7\Omega \quad \therefore R_1 = 994.7\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose R_f and R_i so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 994.7\Omega$.

[b]

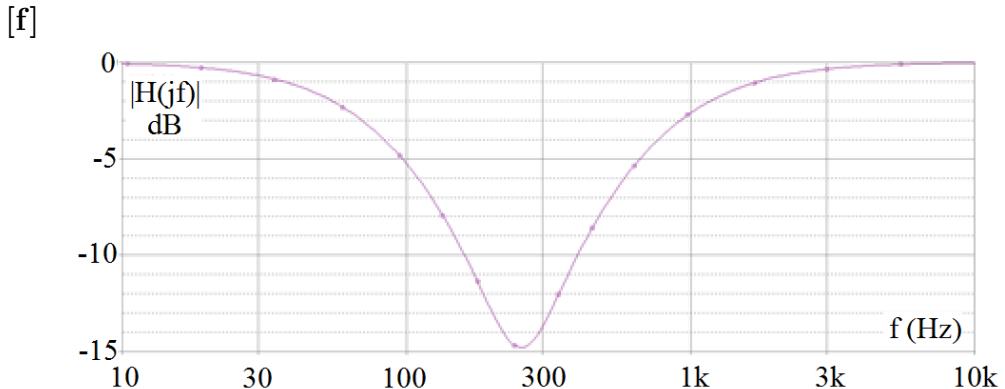


$$\begin{aligned} [c] H'(s) &= \frac{s}{s + 1600\pi} + \frac{160\pi}{s + 160\pi} \\ &= \frac{s^2 + 320\pi s + 256 \times 10^3 \pi^2}{(s + 160\pi)(s + 1600\pi)} \end{aligned}$$

$$[d] \omega_o = \sqrt{(1600\pi)(160\pi)} = 1589.5$$

$$\begin{aligned} H'(j1589.5) &= \frac{-(1589.5)^2 + 320\pi(j1589.5) + 256 \times 10^3 \pi^2}{(160\pi + j1589.5)(1600\pi + j1589.5)} \\ &= 0.1818/\underline{0^\circ} \end{aligned}$$

$$[e] G = 20 \log_{10} 0.1818 = -14.81 \text{ dB}$$



$$P\ 15.30 \quad \omega_o = 2\pi f_o = 400\pi \text{ rad/s}$$

$$\beta = 2\pi(1000) = 2000\pi \text{ rad/s}$$

$$\therefore \omega_{c_2} - \omega_{c_1} = 2000\pi$$

$$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 400\pi$$

Solve for the cutoff frequencies:

$$\omega_{c_1}\omega_{c_2} = 16 \times 10^4\pi^2$$

$$\omega_{c_2} = \frac{16 \times 10^4\pi^2}{\omega_{c_1}}$$

$$\therefore \frac{16 \times 10^4\pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

$$\text{or } \omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16 \times 10^4\pi^2 = 0$$

$$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16 \times 10^6\pi^2}$$

$$\omega_{c_1} = 1000\pi(-1 \pm \sqrt{1.16}) = 242.01 \text{ rad/s}$$

$$\therefore \omega_{c_2} = 2000\pi + 242.01 = 6525.19 \text{ rad/s}$$

$$\text{Thus, } f_{c1} = 38.52 \text{ Hz} \quad \text{and} \quad f_{c2} = 1038.52 \text{ Hz}$$

$$\text{Check: } \beta = f_{c2} - f_{c1} = 1000\text{Hz}$$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65 \Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43 \Omega$$

P 15.31 $\omega_o = 1000 \text{ rad/s}$; GAIN = 6

$$\beta = 4000 \text{ rad/s}; C = 0.2 \mu\text{F}$$

$$\beta = \omega_{c2} - \omega_{c1} = 4000$$

$$\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = 1000$$

Solve for the cutoff frequencies:

$$\therefore \omega_{c1}^2 + 4000\omega_{c1} - 10^6 = 0$$

$$\omega_{c1} = -2000 \pm 1000\sqrt{5} = 236.07 \text{ rad/s}$$

$$\omega_{c2} = 4000 + \omega_{c1} = 4236.07 \text{ rad/s}$$

Check: $\beta = \omega_{c2} - \omega_{c1} = 4000 \text{ rad/s}$

$$\omega_{c1} = \frac{1}{R_L C_L}$$

$$\therefore R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.18 \text{ k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18 \text{ k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

$$\text{If } R_i = 1 \text{ k}\Omega \quad R_f = 6R_i = 6 \text{ k}\Omega$$

$$\text{P 15.32 } H(s) = \frac{V_o}{V_i} = \frac{-Z_f}{Z_i}$$

$$Z_f = \frac{1}{sC_2} \| R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \quad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore H(s) = \frac{\frac{-1/C_2}{s + (1/R_2C_2)}}{\frac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$[\text{a}] \quad H(s) = \frac{-250s}{(s+50)(s+20)} = \frac{-250s}{s^2 + 70s + 1000} = \frac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$$

$$\omega_o = \sqrt{1000} = 31.6 \text{ rad/s}$$

$$\beta = 70 \text{ rad/s}$$

$$K = -3.57$$

$$[\text{b}] \quad Q = \frac{\omega_o}{\beta} = 0.45$$

$$\omega_{c1,2} = \pm \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$$\omega_{c1} = 12.17 \text{ rad/s} \quad \omega_{c2} = 82.17 \text{ rad/s}$$

$$\text{P 15.33 } [\text{a}] \quad H(s) = \frac{(1/sC)}{R + (1/sC)} = \frac{(1/RC)}{s + (1/RC)}$$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)^2}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal.

Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2 L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2 L^2}{R_1^2 L^2 C^2 \omega^4 + (L^2 + R_1^2 R_2^2 C^2 - 2R_1^2 LC)\omega^2 + (R_1 + R_2)^2}$$

$$= \frac{\omega^2 L^2}{R_1^2 L^2 C^2 \omega^4 + (L^2 + R_1^2 R_2^2 C^2 - 2R_1^2 LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_a sC + V_a sC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore V_a = \frac{R_2 V_{in}}{2R_1 R_2 C s + R_1 + R_2}$$

$$\text{and } V_a = -\frac{V_o}{2R_3 C s}$$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2 R_3 C s}{2R_1 R_2 C s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1 + R_2) + j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1 + R_2)^2 + \omega^24R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1 + R_2)^2 + 4R_1^2R_2^2C^2\omega^2}$$

P 15.34 [a] $n \cong \frac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$$\therefore n = 3$$

[b] Gain $= 20 \log_{10} \frac{1}{\sqrt{1 + (7000/2000)^6}} = -32.65$ dB

P 15.35 For the scaled circuit

$$H'(s) = \frac{1/(R')^2C'_1C'_2}{s^2 + \frac{2}{R'C'_1}s + \frac{1}{(R')^2C'_1C'_2}}$$

where

$$R' = k_m R; \quad C'_1 = C_1/k_f k_m; \quad C'_2 = C_2/k_f k_m$$

It follows that

$$\frac{1}{(R')^2C'_1C'_2} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C'_1} = \frac{2k_f}{RC_1}$$

$$\begin{aligned} \therefore H'(s) &= \frac{k_f^2/RC_1C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2C_1C_2}} \\ &= \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2C_1C_2}} \end{aligned}$$

P 15.36 [a] $H(s) = \frac{1}{(s+1)(s^2+s+1)}$

[b] $f_c = 2000 \text{ Hz}; \quad \omega_c = 4000\pi \text{ rad/s}; \quad k_f = 4000\pi$

$$\begin{aligned} H'(s) &= \frac{1}{(\frac{s}{k_f} + 1)[(\frac{s}{k_f})^2 + \frac{s}{k_f} + 1]} \\ &= \frac{k_f^3}{(s + k_f)(s^2 + k_f s + k_f^2)} \\ &= \frac{(4000\pi)^3}{(s + 4000\pi)[s^2 + 4000\pi s + (4000\pi)^2]} \end{aligned}$$

[c] $H'(j14,000\pi) = \frac{64}{(4 + j14)(-180 + j52)}$
 $= 0.02332/-236.77^\circ$

Gain $= 20 \log_{10}(0.02332) = -32.65 \text{ dB}$

P 15.37 [a] In the first-order circuit $R = 1\Omega$ and $C = 1 \text{ F}$.

$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \quad k_f = \frac{\omega'_o}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

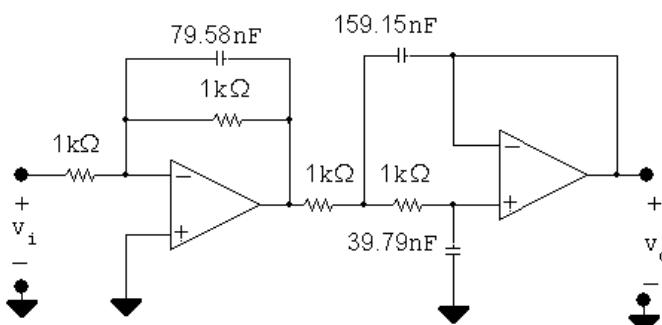
$$R' = k_m R = 1000 \Omega; \quad C' = \frac{C}{k_m k_f} = \frac{1}{(1000)(4000\pi)} = 79.58 \text{ nF}$$

In the second-order circuit $R = 1\Omega$, $2/C_1 = 1$ so $C_1 = 2 \text{ F}$, and $C_2 = 1/C_1 = 0.5 \text{ F}$. Therefore in the scaled second-order circuit

$$R' = k_m R = 1000 \Omega; \quad C'_1 = \frac{C_1}{k_m k_f} = \frac{2}{(1000)(4000\pi)} = 159.15 \text{ nF}$$

$$C'_2 = \frac{C_2}{k_m k_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79 \text{ nF}$$

[b]



P 15.38 [a] $y = 20 \log_{10} \frac{1}{\sqrt{1 + \omega^{2n}}} = -10 \log_{10}(1 + \omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10} \right) \ln(1 + \omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10} \right) \frac{2n\omega^{2n-1}}{(1 + \omega^{2n})}$$

$$x = \log_{10} \omega = \frac{\ln \omega}{\ln 10}$$

$$\therefore \ln \omega = x \ln 10$$

$$\frac{1}{\omega} \frac{d\omega}{dx} = \ln 10, \quad \frac{d\omega}{dx} = \omega \ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega} \right) \left(\frac{d\omega}{dx} \right) = \frac{-20n\omega^{2n}}{1 + \omega^{2n}} \text{ dB/decade}$$

at $\omega = \omega_c = 1 \text{ rad/s}$

$$\frac{dy}{dx} = -10n \text{ dB/decade.}$$

$$[\mathbf{b}] \quad y = 20 \log_{10} \frac{1}{[\sqrt{1 + \omega^2}]^n} = -10n \log_{10}(1 + \omega^2)$$

$$= \frac{-10n}{\ln 10} \ln(1 + \omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10n}{\ln 10} \left(\frac{1}{1 + \omega^2} \right) 2\omega = \frac{-20n\omega}{(\ln 10)(1 + \omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \quad \therefore \frac{dy}{dx} = \frac{-20n\omega^2}{(1 + \omega^2)}$$

$$\text{At the corner } \omega_c = \sqrt{2^{1/n} - 1} \quad \therefore \omega_c^2 = 2^{1/n} - 1$$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n} - 1]}{2^{1/n}} \text{ dB/decade.}$$

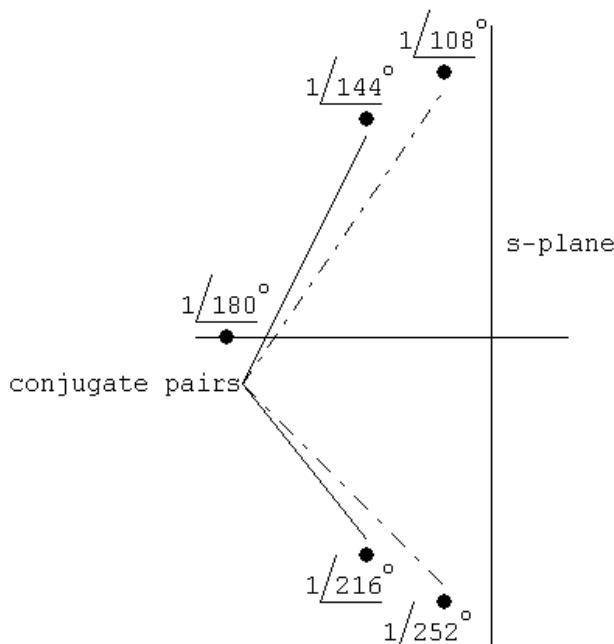
[c] For the Butterworth Filter		For the cascade of identical sections	
n	dy/dx (dB/decade)	n	dy/dx (dB/decade)
1	-10	1	-10
2	-20	2	-11.72
3	-30	3	-12.38
4	-40	4	-12.73
∞	$-\infty$	∞	-13.86

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.

Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

$$P 15.39 \quad n = 5: 1 + (-1)^5 s^{10} = 0; \quad s^{10} = 1$$

$$s^{10} = 1/(0 + 360k)^\circ \quad \text{so} \quad s = 1/36k^\circ$$



k	s_{k+1}	k	s_{k+1}
0	$1/0^\circ$	5	$1/180^\circ$
1	$1/36^\circ$	6	$1/216^\circ$
2	$1/72^\circ$	7	$1/252^\circ$
3	$1/108^\circ$	8	$1/288^\circ$
4	$1/144^\circ$	9	$1/324^\circ$

Group by conjugate pairs to form denominator polynomial.

$$(s + 1)[s - (\cos 108^\circ + j \sin 108^\circ)][(s - (\cos 252^\circ + j \sin 252^\circ)]$$

$$\cdot [(s - (\cos 144^\circ + j \sin 144^\circ)][(s - (\cos 216^\circ + j \sin 216^\circ)]$$

$$= (s + 1)(s + 0.309 - j0.951)(s + 0.309 + j0.951) \cdot$$

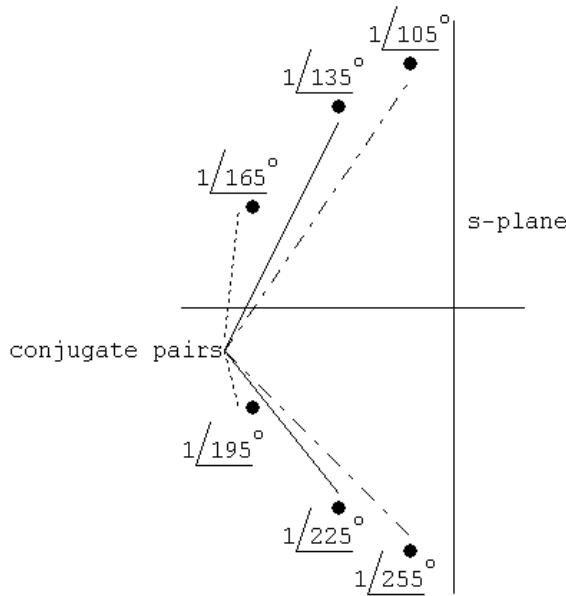
$$(s + 0.809 - j0.588)(s + 0.809 + j0.588)$$

which reduces to

$$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$$

$$n = 6: 1 + (-1)^6 s^{12} = 0 \quad s^{12} = -1$$

$$s^{12} = 1/\underline{180^\circ + 360k}$$



$$k \ s_{k+1}$$

$$0 \ 1/\underline{15^\circ}$$

$$1 \ 1/\underline{45^\circ}$$

$$2 \ 1/\underline{75^\circ}$$

$$3 \ 1/\underline{105^\circ}$$

$$4 \ 1/\underline{135^\circ}$$

$$5 \ 1/\underline{165^\circ}$$

$$k \ s_{k+1}$$

$$6 \ 1/\underline{195^\circ}$$

$$7 \ 1/\underline{225^\circ}$$

$$8 \ 1/\underline{255^\circ}$$

$$9 \ 1/\underline{285^\circ}$$

$$10 \ 1/\underline{315^\circ}$$

$$11 \ 1/\underline{345^\circ}$$

Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659) \times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071) \times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

$$\text{or } (s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9318s + 1)$$

$$\text{P 15.40 } H'(s) = \frac{s^2}{s^2 + \frac{2}{k_m R_2(C/k_m k_f)} s + \frac{1}{k_m R_1 k_m R_2(C^2/k_m^2 k_f^2)}}$$

$$\begin{aligned} H'(s) &= \frac{s^2}{s^2 + \frac{2k_f}{R_2 C} s + \frac{k_f^2}{R_1 R_2 C^2}} \\ &= \frac{(s/k_f)^2}{(s/k_f)^2 + \frac{2}{R_2 C} \left(\frac{s}{k_f}\right) + \frac{1}{R_1 R_2 C^2}} \end{aligned}$$

$$\text{P 15.41 [a]} n = \frac{(-0.05)(-55)}{\log_{10}(2500/500)} = 3.93 \quad \therefore \quad n = 4$$

From Table 15.1 the transfer function is

$$H(s) = \frac{s^2}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \quad C_1 = 2.61 \text{ F}$$

$$C_2 = \frac{1}{C_1} = 0.38 \text{ F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \quad C_1 = 1.08 \text{ F}$$

$$C_2 = \frac{1}{C_1} = 0.92 \text{ F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 8000; \quad k_f = \frac{\omega'_o}{\omega_o} = 5000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 8000 \Omega$$

$$C_1 = \frac{2.61}{(5000\pi)(8000)} = 20.8 \text{ nF}$$

$$C_2 = \frac{0.38}{(5000\pi)(8000)} = 3.04 \text{ nF}$$

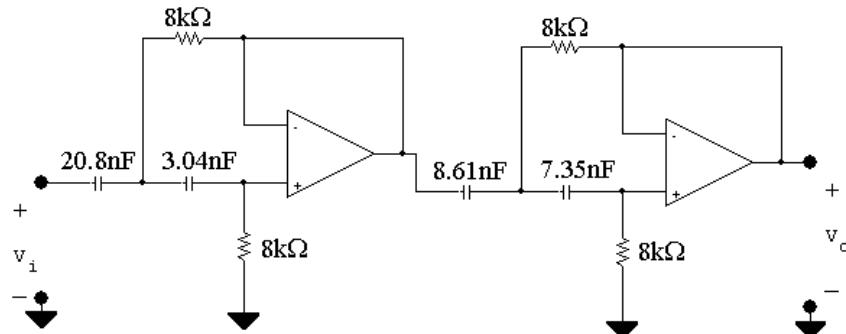
The scaled values for the second stage are

$$R_1 = R_2 = R = 8000 \Omega$$

$$C_1 = \frac{1.08}{(5000\pi)(8000)} = 8.61 \text{ nF}$$

$$C_2 = \frac{0.92}{(5000\pi)(8000)} = 7.35 \text{ nF}$$

[b]



$$\text{P 15.42 [a]} \quad n = \frac{(-0.05)(-55)}{\log_{10}(200/40)} = 3.93 \quad \therefore \quad n = 4$$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{1}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \quad R_2 = 2.61 \Omega; \quad R_1 = \frac{1}{R_2} = 0.383 \Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{1}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \quad R_2 = 1.082 \Omega; \quad R_1 = \frac{1}{R_2} = 0.9240 \Omega$$

The scaling factors are:

$$k_f = \frac{\omega'_o}{\omega_o} = \frac{2\pi(40,000)}{1} = 80,000\pi$$

$$C' = \frac{C}{k_m k_f} \quad \therefore \quad 250 \times 10^{-9} = \frac{1}{80,000\pi k_m}$$

$$\therefore k_m = \frac{1}{80,000\pi(250 \times 10^{-9})} = 15.9$$

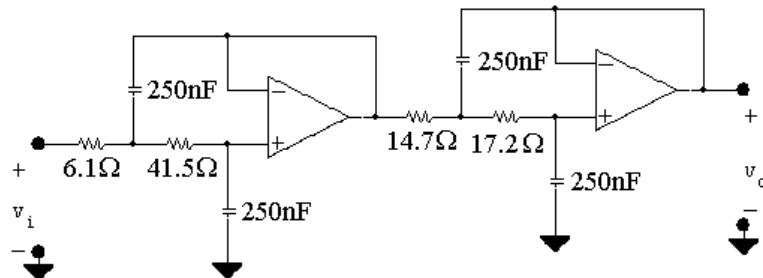
Therefore in the first section

$$R'_1 = k_m R_1 = 6.1 \Omega; \quad R'_2 = k_m R_2 = 41.5 \Omega$$

In the second section

$$R'_1 = k_m R_1 = 14.7 \Omega; \quad R'_2 = k_m R_2 = 17.2 \Omega$$

[b]



P 15.43 [a] The cascade connection is a bandpass filter.

[b] The cutoff frequencies are 2.5 kHz and 40 kHz.

$$\text{The center frequency is } \sqrt{(2.5)(40)} = 10 \text{ kHz.}$$

$$\text{The } Q \text{ is } 10/(40 - 2.5) = 0.267$$

[c] For the high pass section $k_f = 5000\pi$. The prototype transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore H'_{hp}(s) = \frac{(s/5000\pi)^4}{[(s/5000\pi)^2 + 0.765(s/5000\pi) + 1]}$$

$$\cdot \frac{1}{[(s/5000\pi)^2 + 1.848(s/5000\pi) + 1]}$$

$$= \frac{s^4}{(s^2 + 3825\pi s + 25 \times 10^6\pi^2)(s^2 + 9240\pi s + 25 \times 10^6\pi^2)}$$

For the low pass section $k_f = 80,000\pi$

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$$\therefore H'_{lp}(s) = \frac{1}{[(s/80,000\pi)^2 + 0.765(s/80,000\pi) + 1]}$$

$$\cdot \frac{1}{[(s/80,000\pi)^2 + 1.848(s/80,000\pi) + 1]}$$

$$= \frac{(80,000\pi)^4}{[(s^2 + 61,200\pi s + (80,000\pi)^2)][s^2 + 147,840\pi s + (80,000\pi)^2]}$$

The cascaded transfer function is

$$H'(s) = H'_{hp}(s)H'_{lp}(s)$$

For convenience let

$$D_1 = s^2 + 3825\pi s + 25 \times 10^6 \pi^2$$

$$D_2 = s^2 + 9240\pi s + 25 \times 10^6 \pi^2$$

$$D_3 = s^2 + 61,200\pi s + 6400 \times 10^6 \pi^2$$

$$D_4 = s^2 + 147,840\pi s + 6400 \times 10^6 \pi^2$$

Then

$$H'(s) = \frac{4096 \times 10^{16} \pi^4 s^4}{D_1 D_2 D_3 D_4}$$

[d] $\omega_o = 2\pi(10,000) = 20,000\pi \text{ rad/s}$

$$s = j20,000\pi$$

$$s^4 = 16 \times 10^{16} \pi^4$$

$$\begin{aligned} D_1 &= (25 \times 10^6 \pi^2 - 400 \times 10^6 \pi^2) + j(20,000\pi)(3825\pi) \\ &= 10^6 \pi^2(-375 - j76.5) = 10^6 \pi^2(382.72/-168.47^\circ) \end{aligned}$$

$$\begin{aligned} D_2 &= (25 \times 10^6 \pi^2 - 400 \times 10^6 \pi^2) + j(20,000\pi)(9240\pi) \\ &= 10^6 \pi^2(-375 + j184.8) = 10^6 \pi^2(418.06/153.77^\circ) \end{aligned}$$

$$\begin{aligned} D_3 &= (6400 \times 10^6 \pi^2 - 400 \times 10^6 \pi^2) + j(20,000\pi)(61,200\pi) \\ &= 10^6 \pi^2(6000 + j1224) = 10^6 \pi^2(6123.58/11.53^\circ) \end{aligned}$$

$$\begin{aligned} D_4 &= (6400 \times 10^6 \pi^2 - 400 \times 10^6 \pi^2) + j(20,000\pi)(147,840\pi) \\ &= 10^6 \pi^2(6000 + j2956.8) = 10^6 \pi^2(6689/26.23^\circ) \end{aligned}$$

$$\begin{aligned} |H'(j\omega_o)| &= \frac{(4096)(16)\pi^8 \times 10^{32}}{(\pi^8 \times 10^{24})[(382.72)(418.06)(6123.58)(6689)]} \\ &= 0.999 \end{aligned}$$

- P 15.44 [a] First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 200 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-40)}{\log_{10}(200/40)} = 2.86$$

$$\therefore n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1 \Omega, \quad C = 1 \text{ F}$$

For the prototype second-order section

$$R_1 = 0.5 \Omega, \quad R_2 = 2 \Omega, \quad C = 1 \text{ F}$$

The scaling factors are

$$k_f = \frac{\omega'_o}{\omega_o} = 2\pi(200) = 400\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10^{-6})(400\pi)} = 795.77$$

In the scaled first-order section

$$R'_1 = R'_2 = k_m R_1 = 795.77 \Omega$$

$$C' = 1 \mu\text{F}$$

In the scaled second-order section

$$R'_1 = 0.5k_m = 397.9 \Omega$$

$$R'_2 = 2k_m = 1591.5 \Omega$$

$$C' = 1 \mu\text{F}$$

For the low-pass section the cut-off frequency is 2500 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-40)}{\log_{10}(12,500/2500)} = 2.86; \quad \therefore n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1 \Omega, \quad C = 1 \text{ F}$$

For the prototype second-order section

$$R_1 = R_2 = 1 \Omega; \quad C_1 = 2 \text{ F}; \quad C_2 = 0.5 \text{ F}$$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 2500; \quad k_f = \frac{\omega'_o}{\omega_o} = (2500)(2\pi) = 5000\pi$$

For the scaled first-order section

$$R'_1 = R'_2 = 2.5 \text{ k}\Omega; \quad C' = \frac{C}{k_f k_m} = \frac{1}{(5000\pi)(2500)} = 25.46 \text{ nF}$$

For the scaled second-order section

$$R'_1 = R'_2 = 2.5 \text{ k}\Omega$$

$$C'_1 = \frac{C_1}{k_f k_m} = \frac{2}{(5000\pi)(2500)} = 50.93 \text{ nF}$$

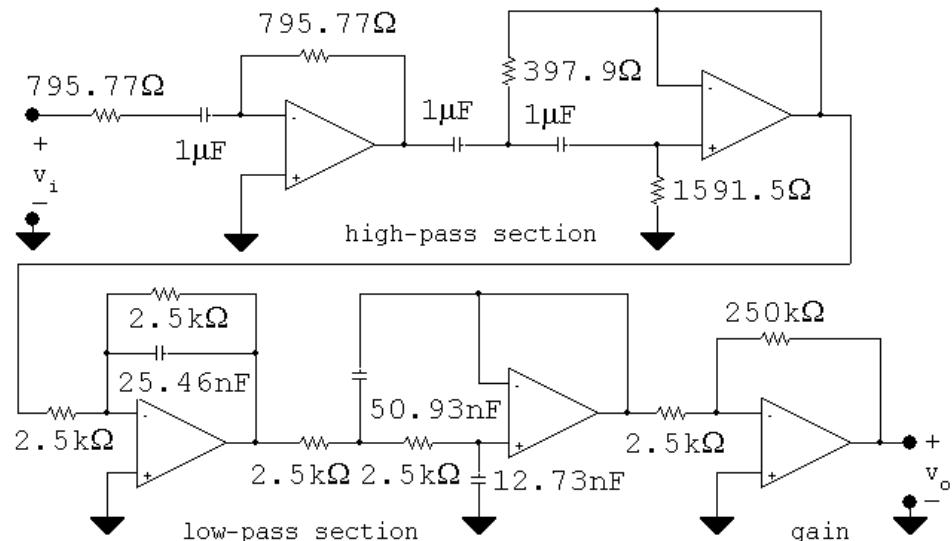
$$C'_2 = \frac{C_2}{k_f k_m} = \frac{0.5}{(5000\pi)(2500)} = 12.73 \text{ nF}$$

GAIN AMPLIFIER

$$20 \log_{10} K = 40 \text{ dB}, \quad \therefore K = 100$$

Since we are using $2.5 \text{ k}\Omega$ resistors in the low-pass stage, we will use $R_f = 250 \text{ k}\Omega$ and $R_i = 2.5 \text{ k}\Omega$ in the inverting amplifier stage.

[b]



P 15.45 [a] Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega'_o/\omega_o) = 400\pi$. Therefore the scaled transfer function is

$$\begin{aligned} H'_{hp}(s) &= \frac{(s/400\pi)^3}{\left(\frac{s}{400\pi} + 1\right) \left[\left(\frac{s}{400\pi}\right)^2 + \frac{s}{400\pi} + 1\right]} \\ &= \frac{s^3}{(s+400\pi)[s^2 + 400\pi s + 160,000\pi^2]} \end{aligned}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega'_o/\omega_o) = 5000\pi$. Therefore the scaled transfer function is

$$\begin{aligned} H'_{lp}(s) &= \frac{1}{\left(\frac{s}{5000\pi} + 1\right) \left[\left(\frac{s}{5000\pi}\right)^2 + \left(\frac{s}{5000\pi}\right) + 1 \right]} \\ &= \frac{(5000\pi)^3}{(s + 5000\pi)(s^2 + 5000\pi s + 25 \times 10^6\pi^2)} \end{aligned}$$

Thus the transfer function for the filter is

$$H'(s) = 100H'_{hp}(s)H'_{lp}(s) = \frac{125 \times 10^{11}\pi^3 s^3}{D_1 D_2 D_3 D_4}$$

where

$$D_1 = s + 400\pi$$

$$D_2 = s + 5000\pi$$

$$D_3 = s^2 + 400\pi s + 160,000\pi^2$$

$$D_4 = s^2 + 5000\pi s + 25 \times 10^6\pi^2$$

[b] At 40 Hz $\omega = 80\pi$ rad/s

$$D_1(j80\pi) = \pi(400 + j80)$$

$$D_2(j80\pi) = \pi(5000 + j80)$$

$$D_3(j80\pi) = \pi^2(153,600 + j32,000)$$

$$D_4(j80\pi) = \pi^2(24,993,600 + j400,000)$$

Therefore

$$D_1 D_2 D_3 D_4(j80\pi) = \pi^6 10^{18} (8/24.9^\circ)$$

$$H'(j80\pi) = \frac{(125\pi^3 \times 10^{11})(512 \times 10^3\pi^3) \angle -90^\circ}{\pi^6 \times 10^{18} (8/24.9^\circ)}$$

$$= 0.8 \angle -114.9^\circ$$

$$\therefore 20 \log_{10} |H'(j80\pi)| = 20 \log_{10}(0.8) = -1.94 \text{ dB}$$

At $f = 1000$ Hz, $\omega = 2000\pi$ rad/s

Then

$$D_1(j2000\pi) = \pi(400 + j2000)$$

$$D_2(j2000\pi) = \pi(5000 + j2000)$$

$$D_3(j2000\pi) = \pi^2(-3,840,000 + j800,000)$$

$$\begin{aligned}
 D_4(j2000\pi) &= \pi^2(21,000,000 + j10^7) \\
 H'(j2000\pi) &= \frac{(125 \times \pi^3 \times 10^{11})(8 \times 10^9 \pi^3) / -90^\circ}{10.02 \times 10^{20} \pi^6 / -65.8^\circ} \\
 &= 99.8 / -24.2^\circ \\
 \therefore 20 \log_{10} |H'(j2000\pi)| &= 39.98 \text{ dB}
 \end{aligned}$$

[c] From the transfer function the gain is down $39.98 + 1.94$ or 41.9 dB at 40 Hz. Because the upper cut-off frequency is 25 times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 40 Hz. For a 3rd order Butterworth,

$$\text{GAIN} = 20 \log_{10} \frac{1}{\sqrt{1 + (200/40)^6}} = -41.9 \text{ dB.}$$

1000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 1000 Hz to nearly equal 40 dB as specified in Problem 15.44. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.46 [a] From Table 15.1

$$\begin{aligned}
 H_{lp}(s) &= \frac{1}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)} \\
 H_{hp}(s) &= \frac{1}{\left(\frac{1}{s} + 1\right) \left(\frac{1}{s^2} + 0.618 \left(\frac{1}{s}\right) + 1\right) \left(\frac{1}{s^2} + 1.618 \left(\frac{1}{s}\right) + 1\right)} \\
 H_{hp}(s) &= \frac{s^5}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)}
 \end{aligned}$$

P 15.47 [a] $k_f = 800$

$$\begin{aligned}
 H'_{hp}(s) &= \frac{(s/800)^5}{[(s/800) + 1]} \\
 &\cdot \frac{1}{[(s/800)^2 + 0.618s/800 + 1][(s/800)^2 + 1.618s/800 + 1]} \\
 &= \frac{s^5}{(s+800)(s^2 + 494.4s + 64 \times 10^4)} \\
 &\cdot \frac{1}{(s^2 + 1294.4s + 64 \times 10^4)}
 \end{aligned}$$

$$\begin{aligned}
 [\mathbf{b}] \quad H'(j800) &= \frac{(j800)^5}{[800 + j800][494.4(j800)][1294.4(j800)]} \\
 &= \frac{j(800)^2}{-(1+j1)(494.4)(1294.4)} \\
 &= 0.707 / -135^\circ
 \end{aligned}$$

$$20 \log_{10} |H'(j800)| = -3.01 \text{ dB}$$

P 15.48 From Eq 15.56 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1 + R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1 + R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \quad \frac{R_1 + R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

$$\text{and } K = \frac{R_3}{2R_1}$$

By hypothesis $C = 1 \text{ F}$ and $\omega_o = 1 \text{ rad/s}$

$$\therefore \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore R_2 = \frac{Q}{2Q^2 - K}$$

P 15.49 [a] From the statement of the problem, $K = 10$ ($= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6 \Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502} \Omega$$

$$R_3 = 2Q = 32 \Omega$$

The scaling factors are

$$k_f = \frac{\omega'_o}{\omega_o} = 2\pi(6400) = 12,800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(20 \times 10^{-9})(12,800\pi)} = 1243.40$$

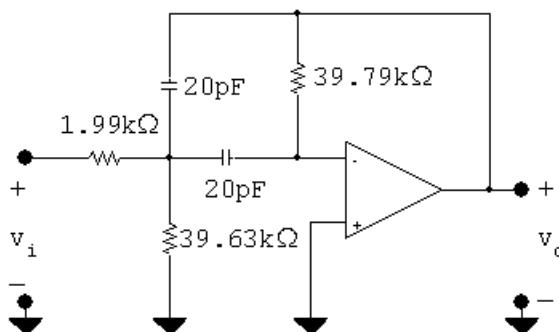
Therefore,

$$R'_1 = k_m R_1 = (1.6)(1243.30) = 1.99 \text{ k}\Omega$$

$$R'_2 = k_m R_2 = (16/502)(1243.40) = 39.63 \Omega$$

$$R'_3 = k_m R_3 = 32(1243.40) = 39.79 \text{ k}\Omega$$

[b]



P 15.50 [a] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1 . At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1 , R_2 , R_3 , and C_2 join be denoted as a , then

$$(V_a - V_i)G_1 + V_a s C_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o s C_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3} V_o$$

Solving for V_o/V_i yields

$$\begin{aligned} H(s) &= \frac{-G_1 G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2 G_3} \\ &= \frac{-G_1 G_3}{s^2 C_1 C_2 + (G_1 + G_2 + G_3)C_1 s + G_2 G_3} \\ &= \frac{-G_1 G_3 / C_1 C_2}{s^2 + \left[\frac{(G_1 + G_2 + G_3)}{C_2} \right] s + \frac{G_2 G_3}{C_1 C_2}} \\ &= \frac{-\frac{G_1 G_2 G_3}{G_2 C_1 C_2}}{s^2 + \left[\frac{(G_1 + G_2 + G_3)}{C_2} \right] s + \frac{G_2 G_3}{C_1 C_2}} \\ &= \frac{-K b_o}{s^2 + b_1 s + b_o} \end{aligned}$$

where $K = \frac{G_1}{G_2}$; $b_o = \frac{G_2 G_3}{C_1 C_2}$

and $b_1 = \frac{G_1 + G_2 + G_3}{C_2}$

[c] Rearranging we see that

$$G_1 = K G_2$$

$$G_3 = \frac{b_o C_1 C_2}{G_2} = \frac{b_o C_1}{G_2}$$

since by hypothesis $C_2 = 1 \text{ F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore b_1 = K G_2 + G_2 + \frac{b_o C_1}{G_2}$$

$$b_1 = G_2(1 + K) + \frac{b_o C_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$\begin{aligned} G_2 &= \frac{b_1}{2(1 + K)} \pm \sqrt{\frac{b_1^2 - b_o C_1 4(1 + K)}{4(1 + K)^2}} \\ &= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1 + K)C_1}}{2(1 + K)} \end{aligned}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1 + K)}$$

[d] 1. Select $C_2 = 1 \text{ F}$

$$2. \text{ Select } C_1 \text{ such that } C_1 < \frac{b_1^2}{4b_o(1+K)}$$

3. Calculate $G_2 (R_2)$

4. Calculate $G_1 (R_1); G_1 = KG_2$

5. Calculate $G_3 (R_3); G_3 = b_o C_1 / G_2$

P 15.51 [a] In the second order section of a third order Butterworth filter $b_o = b_1 = 1$
Therefore,

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05 \text{ F}$$

$\therefore C_1 = 0.05 \text{ F}$ (limiting value)

$$[b] G_2 = \frac{1}{2(1+4)} = 0.1 \text{ S}$$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5 \text{ S}$$

$$G_1 = 4(0.1) = 0.4 \text{ S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5 \Omega; \quad R_2 = \frac{1}{G_2} = 10 \Omega; \quad R_3 = \frac{1}{G_3} = 2 \Omega$$

$$[c] k_f = \frac{\omega'_o}{\omega_o} = 2\pi(2500) = 5000\pi$$

$$k_m = \frac{C_2}{C'_2 k_f} = \frac{1}{(10 \times 10^{-9}) k_f} = 6366.2$$

$$C'_1 = \frac{0.05}{k_f k_m} = 0.5 \times 10^{-9} = 500 \text{ pF}$$

$$R'_1 = (2.5)(6366.2) = 15.92 \text{ k}\Omega$$

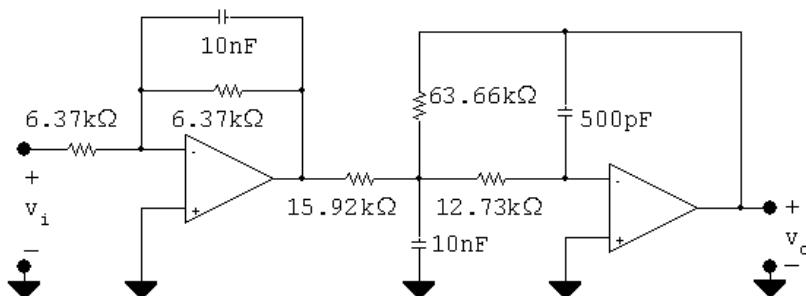
$$R'_2 = (10)(6366.2) = 63.66 \text{ k}\Omega$$

$$R'_3 = (2)(6366.2) = 12.73 \text{ k}\Omega$$

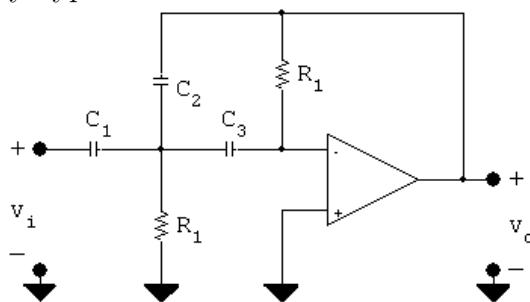
$$[d] R'_1 = R'_2 = (6366.2)(1) = 6.37 \text{ k}\Omega$$

$$C' = \frac{C}{k_f k_m} = \frac{1}{10^8} = 10 \text{ nF}$$

[e]



P 15.52 [a] By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore \$v_o\$ is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor \$C_2\$ dominating the feedback path. Hence the gain of the amplifier approaches \$(1/j\omega C_2)/(1/j\omega C_1)\$ or \$C_1/C_2\$. Therefore the circuit behaves like a high-pass filter with a passband gain of \$C_1/C_2\$.

[b] Summing the currents away from the upper terminal of \$R_2\$ yields

$$V_a G_2 + (V_a - V_i) s C_1 + (V_a - V_o) s C_2 + V_a s C_3 = 0$$

or

$$V_a [G_2 + s(C_1 + C_2 + C_3)] - V_o s C_2 = s C_1 V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a) s C_3 + (0 - V_o) G_1 = 0$$

or

$$s C_3 V_a = -G_1 V_o; \quad V_a = \frac{-G_1 V_o}{s C_3}$$

Therefore we can write

$$\frac{-G_1 V_o}{s C_3} [G_2 + s(C_1 + C_2 + C_3)] - s C_2 V_o = s C_1 V_i$$

Solving for V_o/V_i gives

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{-C_1 C_3 s^2}{C_2 C_3 s^2 + G_1(C_1 + C_2 + C_3)s + G_1 G_2} \\ &= \frac{\frac{-C_1}{C_2} s^2}{\left[s^2 + \frac{G_1}{C_2 C_3} (C_1 + C_2 + C_3)s + \frac{G_1 G_2}{C_2 C_3} \right]} \\ &= \frac{-K s^2}{s^2 + b_1 s + b_o} \end{aligned}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2 .

[c] $C_1 = K$:

$$\begin{aligned} b_1 &= \frac{G_1}{(1)(1)}(K+2) = G_1(K+2) \\ \therefore G_1 &= \frac{b_1}{K+2}; \quad R_1 = \left(\frac{K+2}{b_1} \right) \\ b_o &= \frac{G_1 G_2}{(1)(1)} = G_1 G_2 \\ \therefore G_2 &= \frac{b_o}{G_1} = \frac{b_o}{b_1}(K+2) \\ \therefore R_2 &= \frac{b_1}{b_o(K+2)} \end{aligned}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{K s^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8 \text{ F}$$

$$R_1 = \frac{8+2}{1} = 10 \Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.1 \Omega$$

P 15.53 [a] Low-pass filter:

$$n = \frac{(-0.05)(-24)}{\log_{10}(2000/1000)} = 3.99; \quad \therefore n = 4$$

In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1 \text{ F}$

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} \leq \frac{(0.765)^2}{(4)(2)} \leq 0.0732$$

choose $C_1 = 0.03 \text{ F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for G_2 , then

$$G_2 = 0.338 \text{ S}; \quad \therefore \quad R_2 = \frac{1}{G_2} = 2.96 \Omega$$

$$G_1 = KG_2 = 0.338 \text{ S}; \quad \therefore \quad R_1 = \frac{1}{G_1} = 2.96 \Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \quad R_3 = 1/G_3 = 11.3 \Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96 \Omega; \quad R_3 = 11.3 \Omega$$

$$C_1 = 0.03 \text{ F}; \quad C_2 = 1 \text{ F}$$

In the second second-order prototype circuit:

$$b_1 = 1.848, \quad b_o = 1, \quad C_2 = 1 \text{ F}$$

$$\therefore C_1 \leq \frac{(1.848)^2}{8} \leq 0.427$$

choose $C_1 = 0.30 \text{ F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139 \text{ S}; \quad \therefore \quad R_2 = \frac{1}{G_2} = 1.4008 \Omega$$

$$G_1 = KG_2 = 0.7139 \text{ S}; \quad \therefore \quad R_1 = \frac{1}{G_1} = 1.4008 \Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202 \text{ S} \quad \therefore \quad R_3 = 1/G_3 = 2.3796 \Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega'_o}{\omega_o} = 2\pi(1000) = 2000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(25 \times 10^{-9})k_f} = \frac{20,000}{\pi}$$

Therefore in the first scaled second-order section

$$R'_1 = R'_2 = 2.96k_m = 18.84 \text{ k}\Omega$$

$$R'_3 = 11.3k_m = 71.94 \text{ k}\Omega$$

$$C'_1 = \frac{0.03}{k_f k_m} = 750 \text{ pF}$$

$$C'_2 = 25 \text{ nF}$$

In the second scaled second-order section

$$R'_1 = R'_2 = 1.4008k_m = 8.92 \text{ k}\Omega$$

$$R'_3 = 2.38k_m = 15.15 \text{ k}\Omega$$

$$C'_1 = \frac{0.3}{k_f k_m} = 7.5 \text{ nF}$$

$$C'_2 = 25 \text{ nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-24)}{\log_{10}(8000/4000)} = 3.99; \quad n = 4$$

In the first prototype second-order section:

$$b_1 = 0.765; \quad b_o = 1; \quad C_2 = C_3 = 1 \text{ F}$$

$$C_1 = K = 1 \text{ F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92 \Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255 \Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$;
 $C_2 = C_3 = 1 \text{ F}$

$$C_1 = K = 1 \text{ F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623 \Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616 \Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega'_o}{\omega_o} = 2\pi(8000) = 16,000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(25 \times 10^{-9})(16,000\pi)} = \frac{2500}{\pi}$$

In the first scaled second-order section

$$R'_1 = 3.92k_m = 3119.4\Omega$$

$$R'_2 = 0.255k_m = 202.9\Omega$$

$$C'_1 = C'_2 = C'_3 = 25\text{nF}$$

In the second scaled second-order section

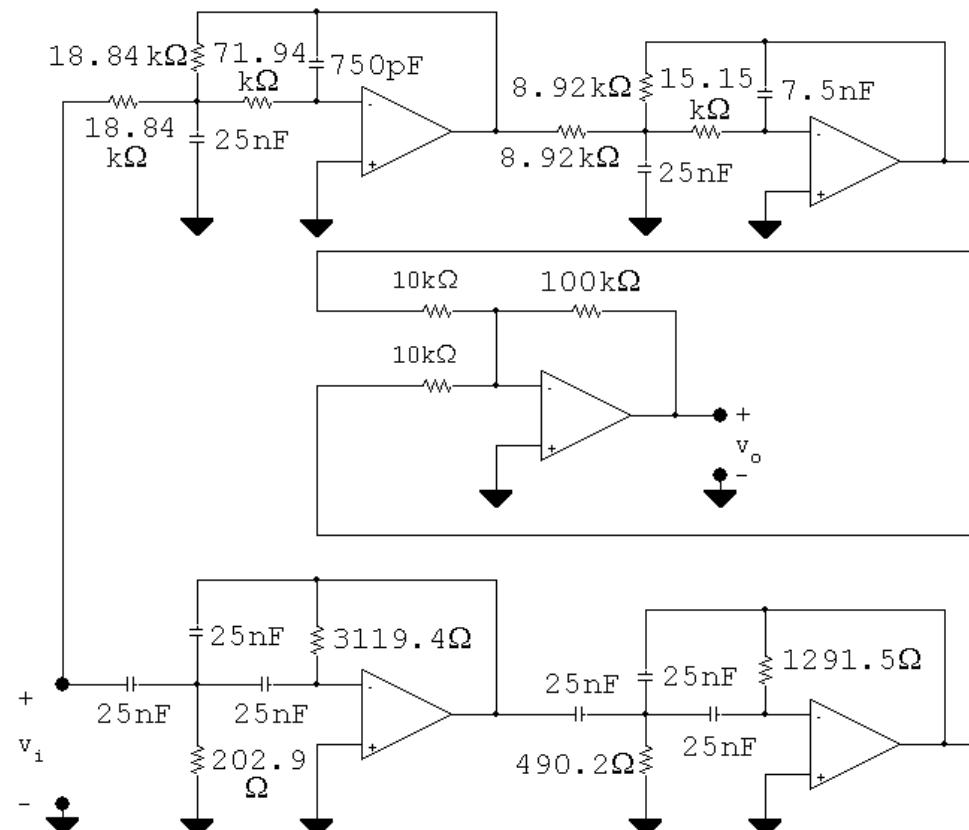
$$R'_1 = 1.623k_m = 1291.5\Omega$$

$$R'_2 = 0.616k_m = 490.2\Omega$$

$$C'_1 = C'_2 = C'_3 = 25\text{nF}$$

In the gain section, let $R_i = 10\text{k}\Omega$ and $R_f = 100\text{k}\Omega$.

[b]



P 15.54 [a] The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(1000) = 2000\pi$$

The scaled transfer function for the low-pass filter is

$$\begin{aligned} H'_{lp}(s) &= \frac{1}{\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{0.765s}{2000\pi} + 1\right] \left[\left(\frac{s}{2000\pi}\right)^2 + \frac{1.848s}{2000\pi} + 1\right]} \\ &= \frac{16 \times 10^{12}\pi^4}{[s^2 + 1530\pi s + (2000\pi)^2][s^2 + 3696\pi s + (2000\pi)^2]} \end{aligned}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(8000) = 16,000\pi$$

The scaled transfer function for the high-pass filter is

$$\begin{aligned} H'_{hp}(s) &= \frac{(s/16,000\pi)^4}{\left[\left(\frac{s}{16,000\pi}\right)^2 + \frac{0.765s}{16,000\pi} + 1\right] \left[\left(\frac{s}{16,000\pi}\right)^2 + \frac{1.848s}{16,000\pi} + 1\right]} \\ &= \frac{s^4}{[s^2 + 12,240\pi s + (16,000\pi)^2][s^2 + 29,568\pi s + (16,000\pi)^2]} \end{aligned}$$

The transfer function for the filter is

$$H'(s) = [H'_{lp}(s) + H'_{hp}(s)]$$

$$[b] f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{(1000)(8000)} = 2000\sqrt{2} \text{ Hz}$$

$$\omega_o = 2\pi f_o = 4000\sqrt{2}\pi \text{ rad/s}$$

$$(j\omega_o)^2 = -32 \times 10^6 \pi^2$$

$$(j\omega_o)^4 = 1024 \times 10^{12} \pi^4$$

$$\begin{aligned} H'_{lp}(j\omega_o) &= \frac{16 \times 10^{12}\pi^4}{[-28 \times 10^6\pi^2 + j1530(4000\sqrt{2}\pi^2)]} \times \\ &\quad \frac{1}{[-28 \times 10^6\pi^2 + j3696(4000\sqrt{2}\pi^2)]} \end{aligned}$$

$$= 0.0156/\underline{53.9^\circ}$$

$$\begin{aligned} H'_{hp}(j\omega_o) &= \frac{1024 \times 10^{12} \pi^4}{[224 \times 10^6\pi^2 + j12,240(4000\sqrt{2}\pi^2)]} \\ &\quad \frac{1}{[224 \times 10^6\pi^2 + j29,568(4000\sqrt{2}\pi^2)]} \end{aligned}$$

$$= 0.0156/\underline{-53.9^\circ}$$

$$\begin{aligned}\therefore H'(j\omega_o) &= 0.0156(1/\underline{53.9^\circ} + 1/\underline{-53.9^\circ}) \\ &= 0.0156(1.18/\underline{0^\circ}) = 0.0184/\underline{0^\circ}\end{aligned}$$

$$G = 20 \log_{10} |H'(j\omega_o)| = 20 \log_{10}(0.0184) = -34.7 \text{ dB}$$

- P 15.55 [a] At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.
- [b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1 . Observe this will also be the voltage at the left-hand terminal of R_2 . The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \left[(G_1 + sC_1) \frac{(G_2 + sC_2)}{G_2} - sC_1 \right] V_o = G_1V_i$$

$$\therefore \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

- [c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

$$[d] b_1 = \frac{G_1}{C_1} \therefore G_1 = b_1C_1$$

$$b_o = \frac{G_1G_2}{C_1C_2} \therefore G_2 = \frac{b_o}{b_1}C_2$$

- [e] No, all physically realizable capacitors will yield physically realizable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, b_1 = 0.765$

$$\therefore G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307 \Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$$R_2 = 1/G_2 = 0.765 \Omega$$

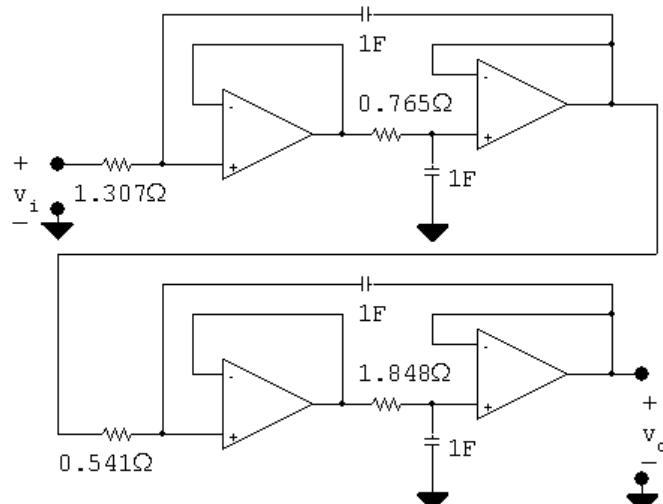
In the second section $b_o = 1, b_1 = 1.848$

$$\therefore G_1 = 1.848 \text{ S}$$

$$R_1 = 1/G_1 = 0.541 \Omega$$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541 \text{ S}$$

$$R_2 = 1/G_2 = 1.848 \Omega$$



$$P 15.56 [a] k_f = \frac{\omega'_o}{\omega_o} = 2\pi(3000) = 6000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$

In the first section

$$R'_1 = 1.307k_m = 14.75 \text{ k}\Omega$$

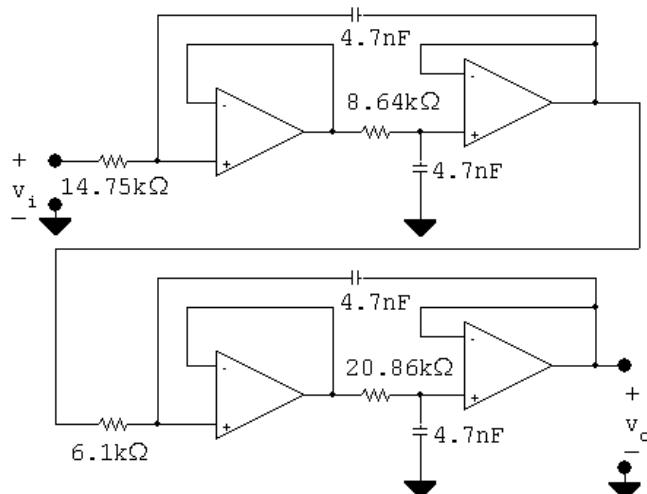
$$R'_2 = 0.765k_m = 8.64\text{ k}\Omega$$

In the second section

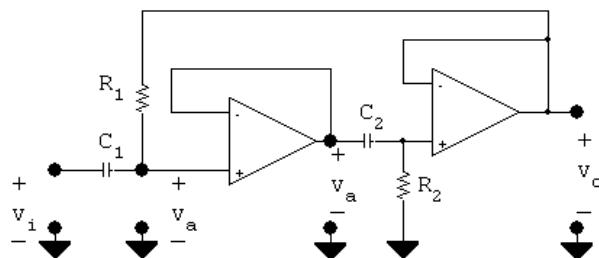
$$R'_1 = 0.541k_m = 6.1\text{ k}\Omega$$

$$R'_2 = 1.848k_m = 20.86\text{ k}\Omega$$

[b]



P 15.57 [a] Interchanging the R_s and C_s yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s -domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

$$\text{and } V_a = \frac{(G_2 + sC_2)V_o}{sC_2}$$

Thus

$$\left\{ \left[\frac{(G_2 + sC_2)}{sC_2} \right] (G_1 + sC_1) - G_1 \right\} V_o = sC_1 V_i$$

$$V_o \{ s^2 C_1 C_2 + s C_1 G_2 + G_1 G_2 \} = s^2 C_1 C_2 V_i$$

$$\begin{aligned} H(s) &= \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2} s + \frac{G_1 G_2}{C_1 C_2} \right)} \\ &= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o} \end{aligned}$$

[c] There are 4 circuit components: R_1 , R_2 , C_1 and C_2 .

There are two transfer function constraints: b_1 and b_o .

Therefore there are two free choices.

$$[d] \quad b_o = \frac{G_1 G_2}{C_1 C_2}; \quad b_1 = \frac{G_2}{C_2}$$

$$\therefore G_2 = b_1 C_2; \quad R_2 = \frac{1}{b_1 C_2}$$

$$G_1 = \frac{b_o}{b_1} C_1 \quad \therefore R_1 = \frac{b_1}{b_o C_1}$$

[e] No, all realizable capacitors will produce realizable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1 \Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1 \Omega.$$

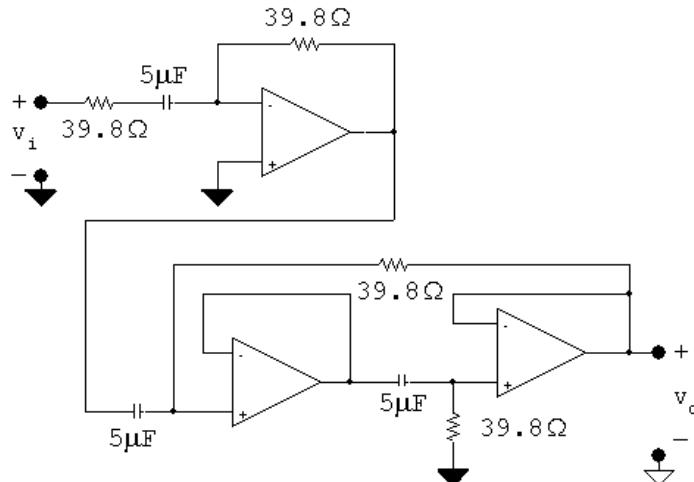
$$P 15.58 \quad [a] \quad k_f = \frac{\omega'_o}{\omega_o} = 1600\pi$$

$$k_m = \frac{C}{C' k_f} = \frac{1}{(5 \times 10^{-6})(1600\pi)} = 39.8$$

$$C'_1 = C'_2 = 5 \mu F; \quad R'_1 = R'_2 = k_m R = 39.8 \Omega$$

$$[b] \quad R = 39.8 \Omega; \quad C = 5 \mu F$$

[c]



$$[d] \quad H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

$$H'_{hp}(s) = \frac{(s/1600\pi)^3}{[(s/1600\pi)+1][(s/1600\pi)^2+(s/1600\pi)+1]}$$

$$= \frac{s^3}{(s+1600\pi)(s^2+1600\pi s+256 \times 10^4\pi^2)}$$

$$[e] \quad H'_{hp}(j1600\pi) = \frac{(j1600\pi)^3}{(j1600\pi+1600\pi)[(j1600\pi)^2+1600\pi(j1600\pi)+256 \times 10^4\pi^2]}$$

$$= 0.7071/\underline{135^\circ}$$

$$\therefore |H'_{hp}| = 0.7071 = -3.01 \text{ dB}$$

P 15.59 [a] It follows directly from Eqs 15.64 and 15.65 that

$$H(s) = \frac{s^2 + 1}{s^2 + 4(1-\sigma)s + 1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2 + 1}{s^2 + \frac{1}{Q}s + 1}$$

[b] For Example 15.13 $\omega_o = 5000 \text{ rad/s}$ and $Q = 5$. Therefore $k_f = 5000$ and

$$\begin{aligned} H'(s) &= \frac{(s/5000)^2 + 1}{(s/5000)^2 + \frac{1}{5} \left(\frac{s}{5000} \right) + 1} \\ &= \frac{s^2 + 25 \times 10^6}{s^2 + 1000s + 25 \times 10^6} \end{aligned}$$

P 15.60 [a] $\omega_o = 8000\pi$ rad/s

$$\therefore k_f = \frac{\omega'_o}{\omega_o} = 8000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(0.5 \times 10^{-6})(8000\pi)} = \frac{250}{\pi}$$

$$R' = k_m R = 79.6 \Omega \quad \text{so} \quad R'/2 = 39.8 \Omega$$

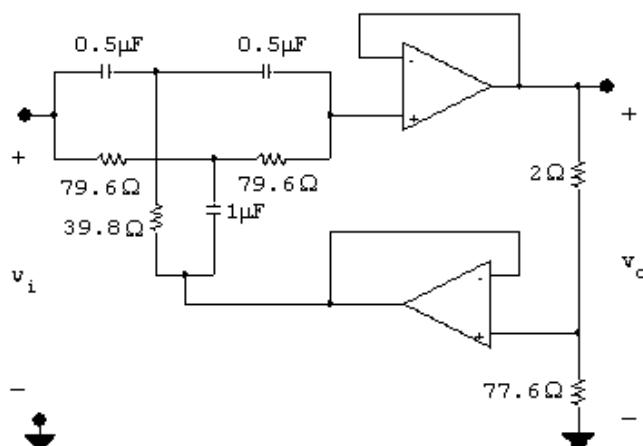
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(10)} = 0.975$$

$$\sigma R' = 77.6 \Omega; \quad (1 - \sigma)R' = 2 \Omega$$

$$C' = 0.5 \mu F$$

$$2C' = 1 \mu F$$

[b]



[c] $k_f = 8000\pi$

$$\begin{aligned} H(s) &= \frac{(s/8000\pi)^2 + 1}{(s/8000\pi)^2 + \frac{1}{10}(s/8000\pi) + 1} \\ &= \frac{s^2 + 64 \times 10^6 \pi^2}{s^2 + 800\pi s + 64 \times 10^6 \pi^2} \end{aligned}$$

P 15.61 To satisfy the gain specification of 14 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 5 \quad \text{or} \quad R_2 = 4R_1$$

Use the specified resistor of $10 \text{ k}\Omega$ for R_1 and a $50 \text{ k}\Omega$ potentiometer for R_2 . Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(50)(50,000)} = 63.66 \text{ nF}$$

Choose a capacitor value of 64 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20 \log_{10} \left(\frac{50}{10} \right)_{\alpha=1} = 13.98 \text{ dB}$$

When $C_1 = 64$ nF the frequency $1/R_2 C_1$ is

$$\frac{1}{R_2 C_1} = \frac{10^9}{50,000(64)} = 312.5 \text{ rad/s} = 49.7 \text{ Hz}$$

The magnitude of the transfer function at 312.5 rad/s is

$$|H(j312.5)|_{\alpha=1} = \left| \frac{50 \times 10^3 + j312.5(10)(50)(64)10^{-3}}{10 \times 10^3 + j312.5(10)(50)(64)10^{-3}} \right| = 3.61$$

Therefore the gain at 49.7 Hz is

$$20 \log_{10}(3.61)_{\alpha=1} = 11.1 \text{ dB}$$

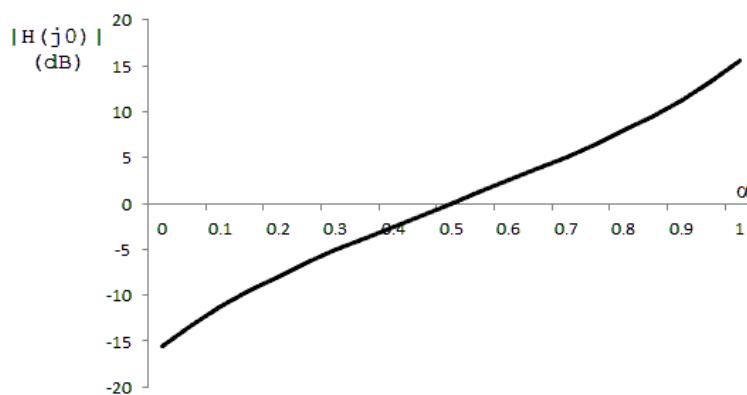
$$\text{P 15.62 } 20 \log_{10} \left(\frac{R_1 + R_2}{R_1} \right) = 20$$

$$\therefore \frac{R_1 + R_2}{R_1} = 10; \quad \therefore R_2 = 9R_1$$

Choose $R_1 = 100 \text{ k}\Omega$. Then $R_2 = 900 \text{ k}\Omega$

$$\frac{1}{R_2 C_1} = 150\pi \text{ rad/s}; \quad \therefore C_1 = \frac{1}{(150\pi)(900 \times 10^3)} = 2.36 \text{ nF}$$

$$\text{P 15.63 } |H(j0)| = \frac{R_1 + \alpha R_2}{R_1 + (1 - \alpha)R_2} = \frac{10 + \alpha(50)}{10 + (1 - \alpha)50}$$

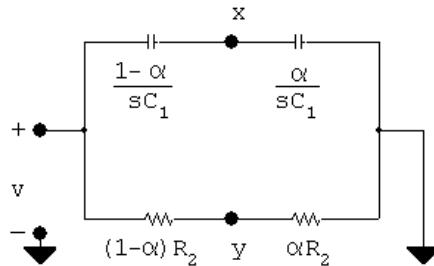


- P 15.64 [a] Combine the impedances of the capacitors in series in Fig. P15.64(b) to get

$$\frac{1}{sC_{\text{eq}}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.60(a).

[b]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1} V = \alpha V$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha V = V_x$$

- [c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

- [d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s , as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. P15.64(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1 - \alpha)/sC_1$ and $(1 - \alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2 :

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1} \cdot (1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}} \\ = \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1} \cdot \alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}} \\ = \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

- P 15.65 As $\omega \rightarrow 0$

$$|H(j\omega)| \rightarrow \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \rightarrow \infty$

$$|H(j\omega)| \rightarrow \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \approx \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or “boost”. When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_o)}{R_o(R_4 + R_3)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \approx \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or “cut”.

Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.

Also note the amplification and attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

$$\text{P 15.66 [a]} |H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)} = \frac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$$

\therefore maximum boost $= 20 \log_{10} 9.99 = 19.99$ dB

$$\text{[b]} |H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$$

\therefore maximum cut $= -19.99$ dB

$$\text{[c]} R_4 = 500 \text{ k}\Omega; \quad R_o = R_1 + R_3 + 2R_2 = 65.9 \text{ k}\Omega$$

$$\therefore R_4 = 7.59R_o$$

Yes, R_4 is significantly greater than R_o .

$$\begin{aligned} \text{[d]} |H(j/R_3C_2)|_{\beta=1} &= \left| \frac{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}{(2R_3 + R_4) + j(R_4 + R_o)} \right| \\ &= \left| \frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9} \right| \\ &= 7.44 \end{aligned}$$

$$20 \log_{10} |H(j/R_3C_2)|_{\beta=1} = 20 \log_{10} 7.44 = 17.43 \text{ dB}$$

[e] When $\beta = 0$

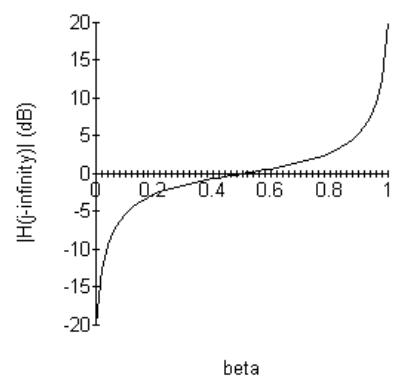
$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3 + R_4) + j(R_4 + R_o)}{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}$$

Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$$\therefore 20 \log_{10} |H(j/R_3C_2)|_{\beta=0} = -17.43 \text{ dB}$$

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

$$\begin{aligned} \text{P 15.67 } |H(j\infty)| &= \frac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta)R_4 + R_3][\beta R_4 + R_o]} \\ &= \frac{[(1-\beta)500 + 65.9][\beta 500 + 5.9]}{[(1-\beta)500 + 5.9][\beta 500 + 65.9]} \end{aligned}$$



16

Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T} \int_0^{2T/3} V_m dt + \frac{1}{T} \int_{2T/3}^T \left(\frac{V_m}{3} \right) dt = \frac{7}{9} V_m = 7\pi \text{ V}$$

$$a_k = \frac{2}{T} \left[\int_0^{2T/3} V_m \cos k\omega_0 t dt + \int_{2T/3}^T \left(\frac{V_m}{3} \right) \cos k\omega_0 t dt \right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T} \right) \sin \left(\frac{4k\pi}{3} \right) = \left(\frac{6}{k} \right) \sin \left(\frac{4k\pi}{3} \right)$$

$$b_k = \frac{2}{T} \left[\int_0^{2T/3} V_m \sin k\omega_0 t dt + \int_{2T/3}^T \left(\frac{V_m}{3} \right) \sin k\omega_0 t dt \right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T} \right) \left[1 - \cos \left(\frac{4k\pi}{3} \right) \right] = \left(\frac{6}{k} \right) \left[1 - \cos \left(\frac{4k\pi}{3} \right) \right]$$

AP 16.2 [a] $a_v = 7\pi = 21.99 \text{ V}$

[b] $a_1 = -5.196 \quad a_2 = 2.598 \quad a_3 = 0 \quad a_4 = -1.299 \quad a_5 = 1.039$

$$b_1 = 9 \quad b_2 = 4.5 \quad b_3 = 0 \quad b_4 = 2.25 \quad b_5 = 1.8$$

[c] $w_0 = \left(\frac{2\pi}{T} \right) = 50 \text{ rad/s}$

[d] $f_3 = 3f_0 = 23.87 \text{ Hz}$

[e] $v(t) = 21.99 - 5.2 \cos 50t + 9 \sin 50t + 2.6 \sin 100t + 4.5 \cos 100t$

$$-1.3 \sin 200t + 2.25 \cos 200t + 1.04 \sin 250t + 1.8 \cos 250t + \dots \text{ V}$$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T} \right) t, \quad 0 \leq t \leq T/6; \quad a_v = 0, \quad a_k = 0 \quad \text{for all } k$$

$$b_k = 0 \quad \text{for } k \text{ even}$$

$$\begin{aligned} b_k &= \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_0 t \, dt, \quad k \text{ odd} \\ &= \frac{8}{T} \int_0^{T/6} \left(\frac{6V_m}{T} \right) t \sin k\omega_0 t \, dt + \frac{8}{T} \int_{T/6}^{T/4} V_m \sin k\omega_0 t \, dt \\ &= \left(\frac{12V_m}{k^2\pi^2} \right) \sin \left(\frac{k\pi}{3} \right) \end{aligned}$$

$$v_g(t) = \frac{12V_m}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{3} \sin n\omega_0 t \text{ V}$$

$$\text{AP 16.4 [a]} \quad A_1 = -5.2 - j9 = 10.4 \angle -120^\circ; \quad A_2 = 2.6 - j4.5 = 5.2 \angle -60^\circ$$

$$A_3 = 0; \quad A_4 = -1.3 - j2.25 = 2.6 \angle -120^\circ$$

$$A_5 = 1.04 - j1.8 = 2.1 \angle -60^\circ$$

$$\theta_1 = -120^\circ; \quad \theta_2 = -60^\circ; \quad \theta_3 \text{ not defined};$$

$$\theta_4 = -120^\circ; \quad \theta_5 = -60^\circ$$

$$\begin{aligned} \text{[b]} \quad v(t) &= 21.99 + 10.4 \cos(50t - 120^\circ) + 5.2 \cos(100t - 60^\circ) \\ &\quad + 2.6 \cos(200t - 120^\circ) + 2.1 \cos(250t - 60^\circ) + \dots \text{ V} \end{aligned}$$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{n^2} \sin \frac{n\pi}{2} \right) \sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{n^2} \sin^2 \frac{n\pi}{2} \right) \cos n\omega_0 t$$

$$= \frac{8A}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250 \text{ mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi} \times 10^3 = 10$$

$$\therefore v_i = 2250 \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos 10nt \text{ mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega} \mathbf{V}_i = \frac{100}{100 + j\omega} \mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250/0^\circ \text{ mV}; \quad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}/0^\circ = 250/0^\circ \text{ mV}; \quad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}/0^\circ = 90/0^\circ \text{ mV}; \quad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250/0^\circ) = 2238.83/-5.71^\circ \text{ mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250/0^\circ) = 239.46/-16.70^\circ \text{ mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90/0^\circ) = 80.50/-26.57^\circ \text{ mV}$$

$$\therefore v_o = 2238.83 \cos(10t - 5.71^\circ) + 239.46 \cos(30t - 16.70^\circ)$$

$$+ 80.50 \cos(50t - 26.57^\circ) + \dots \text{ mV}$$

$$\text{AP 16.6 [a]} \quad \omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi}(10^3) = 10^4 \text{ rad/s}$$

$$v_g(t) = 840 \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n10,000t \text{ V}$$

$$= 840 \cos 10,000t - 280 \cos 30,000t + 168 \cos 50,000t \\ - 120 \cos 70,000t + \dots \text{ V}$$

$$\mathbf{V}_{g1} = 840/0^\circ \text{ V}; \quad \mathbf{V}_{g3} = 280/180^\circ \text{ V}$$

$$\mathbf{V}_{g5} = 168/0^\circ \text{ V}; \quad \mathbf{V}_{g7} = 120/180^\circ \text{ V}$$

$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02/\underline{88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09/\underline{84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1/\underline{0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14/\underline{-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50/\underline{88.81^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14/\underline{-95.36^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168/\underline{0^\circ} \text{ V}$$

$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32/\underline{98.30^\circ} \text{ V}$$

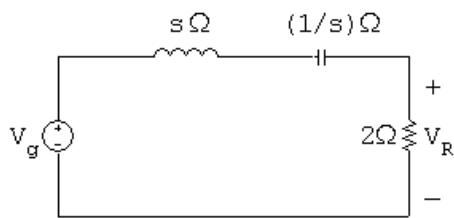
$$v_o = 17.50 \cos(10,000t + 88.81^\circ) + 26.14 \cos(30,000t - 95.36^\circ)$$

$$+ 168 \cos(50,000t) + 17.32 \cos(70,000t + 98.30^\circ) + \dots \text{ V}$$

- [b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3 \text{ rad/s}$$



$$j\omega_0 k = j3k$$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g} \right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g_1} = 25.98 \sin \omega_0 t \text{ V}; \quad V_{g_1} = 25.98/\underline{0^\circ} \text{ V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6/\underline{-53.13^\circ}; \quad V_{R_1} = 15.588/\underline{-53.13^\circ} \text{ V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75 \text{ W}$$

$$v_{g_3} = 0, \quad \text{therefore } P_3 = 0 \text{ W}$$

$$v_{g_5} = -1.04 \sin 5\omega_0 t \text{ V}; \quad V_{g_5} = 1.04/\underline{180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327/\underline{-82.37^\circ}$$

$$V_{R_5} = (1.04/\underline{180^\circ})(0.1327/\underline{-82.37^\circ}) = 138/\underline{97.63^\circ} \text{ mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76 \text{ mW}; \quad \text{therefore } P \cong P_1 \cong 60.75 \text{ W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k , $b_k = 0$ for k even; for k odd we have

$$\begin{aligned} b_k &= \frac{8}{T} \int_0^{T/8} 2 \sin k\omega_0 t dt + \frac{8}{T} \int_{T/8}^{T/4} 8 \sin k\omega_0 t dt \\ &= \left(\frac{8}{\pi k} \right) \left[1 + 3 \cos \left(\frac{k\pi}{4} \right) \right], \quad k \text{ odd} \end{aligned}$$

$$\text{Therefore } C_n = \left(\frac{-j4}{n\pi} \right) \left[1 + 3 \cos \left(\frac{n\pi}{4} \right) \right], \quad n \text{ odd}$$

$$\text{AP 16.9 [a]} \quad I_{\text{rms}} = \sqrt{\frac{2}{T} \left[(2)^2 \left(\frac{T}{8} \right) (2) + (8)^2 \left(\frac{3T}{8} - \frac{T}{8} \right) \right]} = \sqrt{34} = 5.7683 \text{ A}$$

$$[b] \quad C_1 = \frac{-j12.5}{\pi}; \quad C_3 = \frac{j1.5}{\pi}; \quad C_5 = \frac{j0.9}{\pi};$$

$$C_7 = \frac{-j1.8}{\pi}; \quad C_9 = \frac{-j1.4}{\pi}; \quad C_{11} = \frac{j0.4}{\pi}$$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2 \sum_{n=1,3,5,\dots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2} (12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2 + 0.4^2)}$$

$$\cong 5.777 \text{ A}$$

$$[c] \quad \% \text{ Error} = \frac{5.777 - 5.831}{5.831} \times 100 = -0.93\%$$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2 \sum_{n=1,3,5,\dots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2} (12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2)}$$

$$\cong 5.774 \text{ A}$$

$$\% \text{ Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32 \text{ ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$\begin{aligned} i &= \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j \frac{4}{n\pi} \left(1 + 3 \cos \frac{n\pi}{4} \right) e^{jn\omega_0(t-T/4)} \\ &= \frac{4}{\pi} \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n} \left(1 + 3 \cos \frac{n\pi}{4} \right) e^{-j(n+1)(\pi/2)} e^{jn\omega_0 t} \end{aligned}$$

Problems

P 16.1 [a] $\omega_{\text{oa}} = \frac{2\pi}{8 \times 10^{-3}} = 785.4 \text{ rad/s}$

$$\omega_{\text{ob}} = \frac{2\pi}{80 \times 10^{-3}} = 78.54 \text{ krad/s}$$

[b] $f_{\text{oa}} = \frac{1}{T} = \frac{1}{8 \times 10^{-3}} = 125 \text{ Hz}; \quad f_{\text{ob}} = \frac{1}{80 \times 10^{-3}} = 12.5 \text{ Hz}$

[c] $a_{\text{va}} = \frac{50(4 \times 10^{-3})}{8 \times 10^{-3}} = 25 \text{ V}; \quad a_{\text{vb}} = 0$

[d] The periodic function in Fig. P16.1(a):

$$a_{\text{va}} = 25 \text{ V}$$

$$a_{\text{ka}} = \frac{2}{T} \int_{-T/4}^{T/4} 50 \cos \frac{2\pi kt}{T} dt$$

$$= \frac{100}{T} \frac{T}{2\pi k} \sin \frac{2\pi k}{T} t \Big|_{-T/4}^{T/4}$$

$$= \frac{100}{\pi k} \sin \frac{\pi k}{2}$$

$$b_{\text{ka}} = \frac{2}{T} \int_{-T/4}^{T/4} 50 \sin \frac{2\pi kt}{T} dt$$

$$= \frac{-100}{T} \frac{T}{2\pi k} \cos \frac{2\pi k}{T} t \Big|_{-T/4}^{T/4}$$

$$= 0$$

The periodic function in Fig. P16.1(b):

$$a_{\text{vb}} = 0$$

$$a_{\text{kb}} = \frac{2}{T} \left[\int_0^{T/4} 90 \cos \frac{2\pi kt}{T} dt + \int_{T/4}^{T/2} 30 \cos \frac{2\pi kt}{T} dt \right]$$

$$- \frac{2}{T} \left[\int_{T/2}^{3T/4} 90 \cos \frac{2\pi kt}{T} dt + \int_{3T/4}^T 30 \cos \frac{2\pi kt}{T} dt \right]$$

$$= \frac{60}{T} \frac{T}{2\pi k} \left[3 \sin \frac{2\pi kt}{T} \Big|_0^{T/4} + \sin \frac{2\pi kt}{T} \Big|_{T/4}^{T/2} \right]$$

$$- \frac{60}{T} \frac{T}{2\pi k} \left[3 \sin \frac{2\pi kt}{T} \Big|_{T/2}^{3T/4} + \sin \frac{2\pi kt}{T} \Big|_{3T/4}^T \right]$$

$$= \frac{30}{\pi k} \left[2 \sin \frac{\pi k}{2} - 2 \sin \frac{3\pi k}{2} \right] = \frac{120}{\pi k} \sin \frac{\pi k}{2}$$

Note that a_{kb} is 0 for even values of k .

$$\begin{aligned} b_{kb} &= \frac{2}{T} \left[\int_0^{T/4} 90 \sin \frac{2\pi kt}{T} dt + \int_{T/4}^{T/2} 30 \sin \frac{2\pi kt}{T} dt \right] \\ &\quad - \frac{2}{T} \left[\int_{T/2}^{3T/4} 90 \sin \frac{2\pi kt}{T} dt + \int_{3T/4}^T 30 \sin \frac{2\pi kt}{T} dt \right] \\ &= \frac{-60}{T} \frac{T}{2\pi k} \left[3 \cos \frac{2\pi kt}{T} \Big|_0^{T/4} + \cos \frac{2\pi kt}{T} \Big|_{T/4}^{T/2} \right] \\ &\quad - \frac{-60}{T} \frac{T}{2\pi k} \left[3 \cos \frac{2\pi kt}{T} \Big|_{T/2}^{3T/4} + \cos \frac{2\pi kt}{T} \Big|_{3T/4}^T \right] \\ &= \frac{120}{\pi k} [1 - \cos(k\pi)] \end{aligned}$$

Note that b_{kb} is 0 for even values of k and equal to $120(2)/k\pi$ for odd values of k .

- [e] For the periodic function in Fig. P16.1(a),

$$v(t) = 25 + \frac{100}{\pi} \sum_{n=1}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \cos n\omega_o t \right) V$$

For the periodic function in Fig. P16.1(b),

$$v(t) = \frac{120}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \left(\sin \frac{n\pi}{2} \cos n\omega_o t + 2 \sin n\omega_o t \right) V$$

- P 16.2 [a] Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k , $b_k = 0$ for even k ; for k odd we have

$$b_k = \frac{8}{T} \int_0^{T/4} V_m \sin k\omega_0 t dt = \frac{4V_m}{k\pi}, \quad k \text{ odd}$$

$$\text{and } v(t) = \frac{4V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin n\omega_0 t V$$

- [b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T} \int_0^{T/2} V_m \sin \frac{\pi}{T} t dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T} \int_0^{T/2} V_m \sin \frac{\pi}{T} t \cos k\omega_0 t dt = \frac{2V_m}{\pi} \left(\frac{1}{1-2k} + \frac{1}{1+2k} \right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

$$\text{and } v(t) = \frac{2V_m}{\pi} \left[1 + 2 \sum_{n=1}^{\infty} \frac{1}{1-4n^2} \cos n\omega_0 t \right] V$$

$$[\text{c}] \quad a_v = \frac{1}{T} \int_0^{T/2} V_m \sin \left(\frac{2\pi}{T} t \right) dt = \frac{V_m}{\pi}$$

$$a_k = \frac{2}{T} \int_0^{T/2} V_m \sin \frac{2\pi}{T} t \cos k\omega_0 t dt = \frac{V_m}{\pi} \left(\frac{1 + \cos k\pi}{1 - k^2} \right)$$

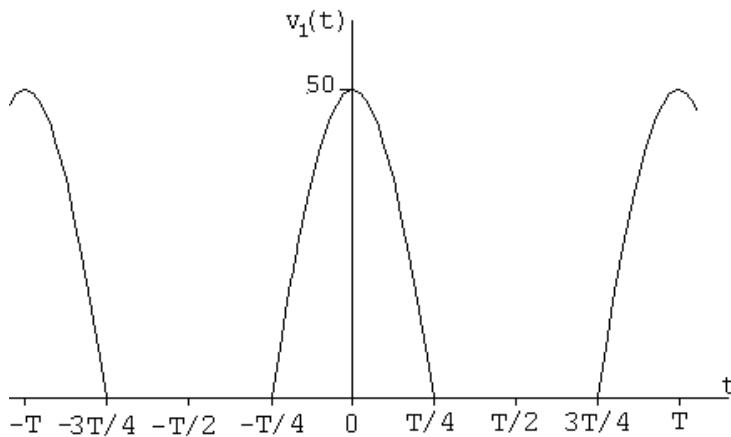
Note: $a_k = 0$ for k -odd, $a_k = \frac{2V_m}{\pi(1-k^2)}$ for k even,

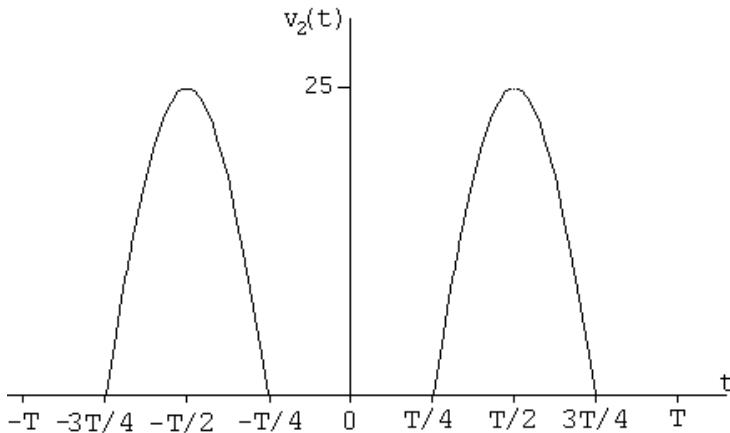
$$b_k = \frac{2}{T} \int_0^{T/2} V_m \sin \frac{2\pi}{T} t \sin k\omega_0 t dt = 0 \quad \text{for } k = 2, 3, 4, \dots$$

For $k = 1$, we have $b_1 = \frac{V_m}{2}$; therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2} \sin \omega_0 t + \frac{2V_m}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{1}{1-n^2} \cos n\omega_0 t V$$

- P 16.3 In studying the periodic function in Fig. P16.3 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.2(c).





In using the previously derived Fourier series for the half-wave rectified sine wave we note $v_1(t)$ has been shifted $T/4$ units to the left and $v_2(t)$ has been shifted $T/4$ units to the right. Thus,

$$v_1(t) = \frac{50}{\pi} + 25 \sin \omega_o(t + T/4) - \frac{100}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o(t + T/4)}{(1 - n^2)} V$$

Now observe the following:

$$\sin \omega_o(t + T/4) = \sin(\omega_o t + \pi/2) = \cos \omega_o t$$

$$\cos n\omega_o(t + T/4) = \cos(n\omega_o t + n\pi/2) = \cos \frac{n\pi}{2} \cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore v_1(t) = \frac{50}{\pi} + 25 \cos \omega_o t - \frac{100}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2) \cos(n\omega_o t)}{(1 - n^2)} V$$

Also,

$$v_2(t) = \frac{25}{\pi} + 12.5 \sin \omega_o(t - T/4) - \frac{50}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o(t - T/4)}{(1 - n^2)} V$$

Again, observe the following:

$$\sin(\omega_o t - \pi/2) = -\cos \omega_o t$$

$$\cos(n\omega_o t - n\pi/2) = \cos(n\pi/2) \cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore v_2(t) = \frac{25}{\pi} - 12.5 \cos \omega_o t - \frac{50}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2) \cos(n\omega_o t)}{(1 - n^2)} V$$

Thus: $v = v_1 + v_2$

$$\therefore v(t) = \frac{75}{\pi} + 12.5 \cos \omega_o t - \frac{150}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2) \cos(n\omega_o t)}{(1 - n^2)} V$$

$$\text{P 16.4} \quad a_v = \frac{1}{T} \int_0^{T/4} V_m dt + \frac{1}{T} \int_{T/4}^T \frac{V_m}{2} dt = \frac{5}{8} V_m = 62.5\pi \text{ V}$$

$$a_k = \frac{2}{T} \left[\int_0^{T/4} V_m \cos k\omega_0 t dt + \int_{T/4}^T \frac{V_m}{2} \cos k\omega_0 t dt \right]$$

$$= \frac{V_m}{k\omega_0 T} \sin \frac{k\pi}{2} = \frac{50}{k} \sin \frac{k\pi}{2}$$

$$b_k = \frac{2}{T} \left[\int_0^{T/4} V_m \sin k\omega_0 t dt + \int_{T/4}^T \frac{V_m}{2} \sin k\omega_0 t dt \right]$$

$$= \frac{V_m}{k\omega_0 T} \left[1 - \cos \frac{k\pi}{2} \right] = \frac{50}{k} \left[1 - \cos \frac{k\pi}{2} \right]$$

$$\begin{aligned} \text{P 16.5 [a]} \quad I_6 &= \int_{t_o}^{t_o+T} \sin m\omega_0 t dt = -\frac{1}{m\omega_0} \cos m\omega_0 t \Big|_{t_o}^{t_o+T} \\ &= \frac{-1}{m\omega_0} [\cos m\omega_0(t_o + T) - \cos m\omega_0 t_o] \\ &= \frac{-1}{m\omega_0} [\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o] \\ &= \frac{-1}{m\omega_0} [\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m, \end{aligned}$$

$$\begin{aligned} I_7 &= \int_{t_o}^{t_o+T} \cos m\omega_0 t_o dt = \frac{1}{m\omega_0} [\sin m\omega_0 t] \Big|_{t_o}^{t_o+T} \\ &= \frac{1}{m\omega_0} [\sin m\omega_0(t_o + T) - \sin m\omega_0 t_o] \\ &= \frac{1}{m\omega_0} [\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m \end{aligned}$$

$$\text{[b]} \quad I_8 = \int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t dt = \frac{1}{2} \int_{t_o}^{t_o+T} [\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t] dt$$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n .

$$\text{[c]} \quad I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t dt = \frac{1}{2} \int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t] dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2} \int_{t_o}^{t_o+T} dt - \frac{1}{2} \int_{t_o}^{t_o+T} \cos 2m\omega_0 t dt = \frac{T}{2} - 0 = \frac{T}{2}$$

$$\begin{aligned}
 [\text{d}] \quad I_{10} &= \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t \, dt \\
 &= \frac{1}{2} \int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t] \, dt
 \end{aligned}$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2} \int_{t_o}^{t_o+T} dt + \frac{1}{2} \int_{t_o}^{t_o+T} \cos 2m\omega_0 t \, dt = \frac{T}{2} + 0 = \frac{T}{2}$$

$$\text{P 16.6} \quad f(t) \sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t) \sin k\omega_0 t \, dt = 0 + 0 + b_k \left(\frac{T}{2} \right) \quad \text{or} \quad b_k = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \sin k\omega_0 t \, dt$$

$$\text{P 16.7} \quad a_v = \frac{1}{T} \int_{t_o}^{t_o+T} f(t) \, dt = \frac{1}{T} \left\{ \int_{-T/2}^0 f(t) \, dt + \int_0^{T/2} f(t) \, dt \right\}$$

$$\text{Let } t = -x, \quad dt = -dx, \quad x = \frac{T}{2} \quad \text{when } t = \frac{-T}{2}$$

$$\text{and } x = 0 \quad \text{when } t = 0$$

$$\text{Therefore } \frac{1}{T} \int_{-T/2}^0 f(t) \, dt = \frac{1}{T} \int_{T/2}^0 f(-x)(-dx) = -\frac{1}{T} \int_0^{T/2} f(x) \, dx$$

$$\text{Therefore } a_v = -\frac{1}{T} \int_0^{T/2} f(t) \, dt + \frac{1}{T} \int_0^{T/2} f(t) \, dt = 0$$

$$a_k = \frac{2}{T} \int_{-T/2}^0 f(t) \cos k\omega_0 t \, dt + \frac{2}{T} \int_0^{T/2} f(t) \cos k\omega_0 t \, dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T} \int_{-T/2}^0 f(t) \cos k\omega_0 t \, dt = -\frac{2}{T} \int_0^{T/2} f(x) \cos k\omega_0 x \, dx$$

Therefore $a_k = 0$ for all k .

$$b_k = \frac{2}{T} \int_{-T/2}^0 f(t) \sin k\omega_0 t \, dt + \frac{2}{T} \int_0^{T/2} f(t) \sin k\omega_0 t \, dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T} \int_0^{T/2} f(x) \sin k\omega_0 x \, dx$$

Therefore we have $b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin k\omega_0 t \, dt$

P 16.8 $b_k = \frac{2}{T} \int_{-T/2}^0 f(t) \sin k\omega_0 t \, dt + \frac{2}{T} \int_0^{T/2} f(t) \sin k\omega_0 t \, dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also

$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x \cos k\pi$. Therefore

$$\frac{2}{T} \int_{-T/2}^0 f(t) \sin k\omega_0 t \, dt = -\frac{2}{T} \int_0^{T/2} f(x) \sin k\omega_0 x \cos k\pi \, dx \quad \text{and}$$

$$b_k = \frac{2}{T} (1 - \cos k\pi) \int_0^{T/2} f(x) \sin k\omega_0 t \, dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin k\omega_0 t \, dt \quad \text{when } k \text{ is odd}$$

P 16.9 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T} \int_0^{T/2} f(t) \cos k\omega_0 t \, dt, \quad k \text{ odd}$$

The function also has quarter-wave symmetry; therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$; thus we write

$$a_k = \frac{4}{T} \int_0^{T/4} f(t) \cos k\omega_0 t \, dt + \frac{4}{T} \int_{T/4}^{T/2} f(t) \cos k\omega_0 t \, dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T} \int_{T/4}^{T/2} f(t) \cos k\omega_0 t \, dt = -\frac{4}{T} \int_0^{T/4} f(x) \cos k\pi \cos k\omega_0 x \, dx$$

Therefore we have

$$a_k = \frac{4}{T} (1 - \cos k\pi) \int_0^{T/4} f(t) \cos k\omega_0 t dt$$

But k is odd, hence

$$a_k = \frac{8}{T} \int_0^{T/4} f(t) \cos k\omega_0 t dt, \quad k \text{ odd}$$

- P 16.10 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k , and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin k\omega_0 t dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$. Thus we have

$$b_k = \frac{4}{T} \int_0^{T/4} f(t) \sin k\omega_0 t dt + \frac{4}{T} \int_{T/4}^{T/2} f(t) \sin k\omega_0 t dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T} \int_{T/4}^{T/2} f(t) \sin k\omega_0 t dt = -\frac{4}{T} \cos k\pi \int_0^{T/4} f(x) (\sin k\omega_0 x) dx$$

But k is odd, therefore the expression for b_k becomes

$$b_k = \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_0 t dt$$

- P 16.11 [a] $\omega_o = \frac{2\pi}{T} = \pi$ rad/s

[b] yes

[c] no

[d] yes

- P 16.12 [a] $f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25$ Hz

[b] no

[c] yes

[d] yes

[e] yes

[f] $a_v = 0$, function is odd

$a_k = 0$, for all k ; the function is odd

$b_k = 0$, for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_o t dt, \quad k \text{ odd}$$

$$= \frac{8}{T} \left\{ \int_0^{T/8} -8t \sin k\omega_o t dt + \int_{T/8}^{T/4} -0.04 \sin k\omega_o t dt \right\}$$

$$= \frac{8}{T} \{ \text{Int1} + \text{Int2} \}$$

$$\text{Int1} = -8 \int_0^{T/8} t \sin k\omega_o t dt$$

$$= -8 \left[\frac{1}{k^2 \omega_o^2} \sin k\omega_o t - \frac{t}{k\omega_o} \cos k\omega_o t \Big|_0^{T/8} \right]$$

$$= \frac{-8}{k^2 \omega_o^2} \sin \frac{k\pi}{4} + \frac{T}{k\omega_o} \cos \frac{k\pi}{4}$$

$$\text{Int2} = -0.04 \int_{T/8}^{T/4} \sin k\omega_o t dt = \frac{0.04}{k\omega_o} \cos k\omega_o t \Big|_{T/8}^{T/4} = \frac{-0.04}{k\omega_o} \cos \frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{-8}{k^2 \omega_o^2} \sin \frac{k\pi}{4} + \left(\frac{-0.04}{k\omega_o} + \frac{T}{k\omega_o} \right) \cos \frac{k\pi}{4}$$

$$T = 0.04 \text{ s}$$

$$\therefore \text{Int1} + \text{Int2} = \frac{-8}{k^2 \omega_o^2} \sin \frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T} \cdot \frac{-8}{4\pi^2 k^2} \cdot T^2 \right] \sin \frac{k\pi}{4} = \frac{-0.64}{\pi^2 k^2} \sin \frac{k\pi}{4}, \quad k \text{ odd}$$

$$i(t) = \frac{-640}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\pi/4)}{n^2} \sin n\omega_o t \text{ mA}$$

P 16.13 [a] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k , $a_k = 0$ for k -even; for odd k we have

$$a_k = \frac{8}{T} \int_0^{T/4} V_m \cos k\omega_0 t dt = \frac{4V_m}{\pi k} \sin \left(\frac{k\pi}{2} \right)$$

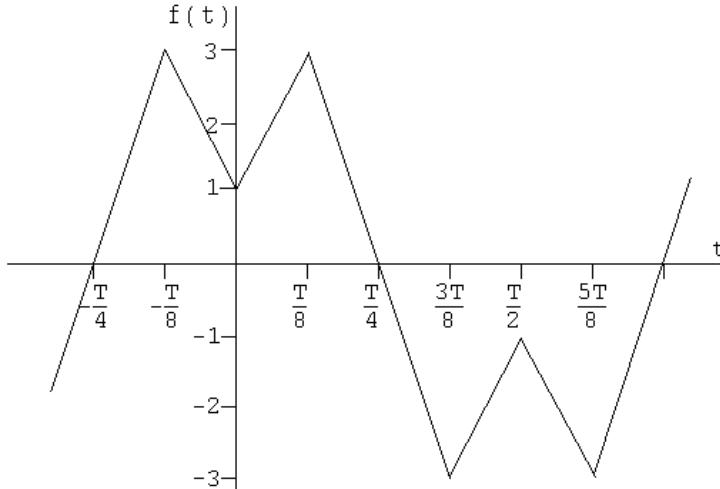
$$v(t) = \frac{4V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{1}{n} \sin \frac{n\pi}{2} \right] \cos n\omega_0 t \text{ V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k -even, $a_k = 0$ for all k ; for k -odd we have

$$a_k = \frac{8}{T} \int_0^{T/4} \left(\frac{4V_p}{T}t - V_p \right) \cos k\omega_0 t dt = \frac{-8V_p}{\pi^2 k^2}$$

$$\text{Therefore } v(t) = \frac{-8V_p}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos n\omega_0 t V$$

P 16.14 [a]



[b] $a_v = 0$; $a_k = 0$ for all k even; $b_k = 0$ for all k

$$\text{For } k \text{ odd, } a_k = \frac{8}{T} \int_0^{T/4} f(t) \cos k\omega_o t dt$$

$$a_k = \frac{8}{T} \int_0^{T/8} \left(1 + \frac{16t}{T} \right) \cos k\omega_o t dt + \frac{8}{T} \int_{T/8}^{T/4} \left(6 - \frac{24t}{T} \right) \cos k\omega_o t dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{8}{T} \int_0^{T/8} \cos k\omega_o t dt + \frac{128}{T^2} \int_0^{T/8} t \cos k\omega_o t dt$$

$$= \frac{8}{T} \frac{\sin k\omega_o t}{k\omega_o} \Big|_0^{T/8} + \frac{128}{T^2} \left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o} \sin k\omega_o t \right]_0^{T/8}$$

$$k\omega_o T = 2k\pi; \quad (k\omega_o T)^2 = 4k^2\pi^2$$

$$\text{Int1} = \frac{12}{k\pi} \sin \frac{k\pi}{4} + \frac{32}{k^2\pi^2} \left[\cos \left(\frac{k\pi}{4} \right) - 1 \right] \quad k \text{ odd}$$

$$\text{Int2} = \frac{48}{T} \int_{T/8}^{T/4} \cos k\omega_o t dt - \frac{192}{T^2} \int_{T/8}^{T/4} t \cos k\omega_o t dt$$

$$= \frac{48}{T} \frac{\sin k\omega_o t}{k\omega_o} \Big|_{T/8}^{T/4} - \frac{192}{T^2} \left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o} \sin k\omega_o t \right]_{T/8}^{T/4}$$

$$\text{Int2} = \frac{-12}{k\pi} \sin \frac{k\pi}{4} + \frac{48}{k^2\pi^2} \cos \frac{k\pi}{4} \quad k \text{ odd}$$

$$\begin{aligned} a_k &= \text{Int1} + \text{Int2} \\ &= \frac{80}{k^2\pi^2} \cos \frac{k\pi}{4} - \frac{32}{k^2\pi^2} \end{aligned}$$

[c] $a_1 = \frac{80}{\pi^2} \cos \frac{\pi}{4} - \frac{32}{\pi^2} = 2.489$

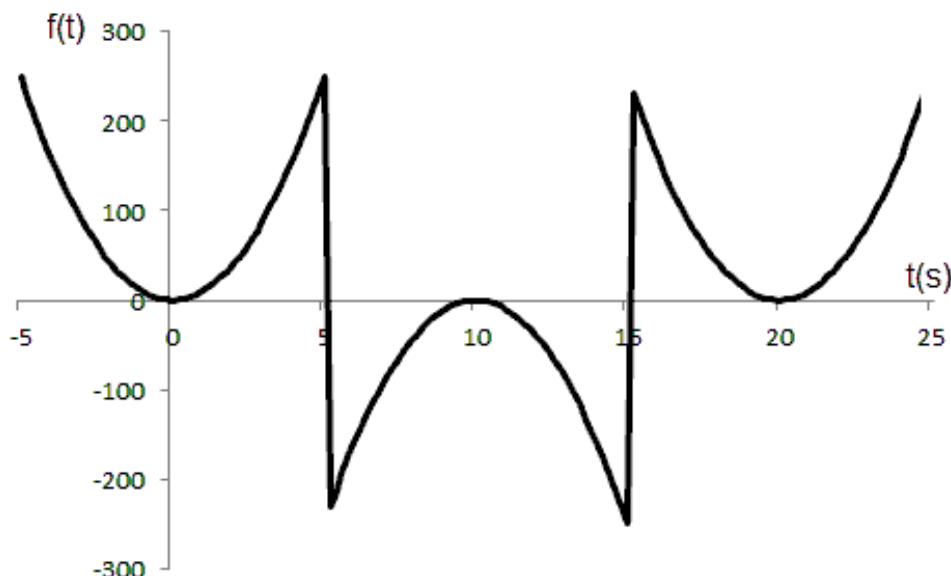
$$a_3 = \frac{80}{9\pi^2} \cos \frac{3\pi}{4} - \frac{32}{9\pi^2} = -0.9971$$

$$a_5 = \frac{80}{25\pi^2} \cos \frac{5\pi}{4} - \frac{32}{25\pi^2} = -0.359$$

$$f(t) = 2.489 \cos \omega_o t - 0.9971 \cos 3\omega_o t - 0.359 \cos 5\omega_o t - \dots$$

[d] $f(T/8) = 2.489 \cos(\pi/4) - 0.9971 \cos(3\pi/4) - 0.359 \cos(5\pi/4) = 2.719$

P 16.15 [a]



[b] Even, since $f(t) = f(-t)$

[c] Yes, since $f(t) = -f(T/2 - t)$ in the interval $0 < t < 10$.

[d] $a_v = 0, \quad a_k = 0, \quad \text{for } k \text{ even}$ (half-wave symmetry)

$b_k = 0, \quad \text{for all } k$ (function is even)

Because of the quarter-wave symmetry, the expression for a_k is

$$a_k = \frac{8}{T} \int_0^{T/4} f(t) \cos k\omega_0 t dt, \quad k \text{ odd}$$

$$= \frac{8}{20} \int_0^5 10t^2 \cos k\omega_0 t dt = 4 \left[\frac{2t}{k^2\omega_0^2} \cos k\omega_0 t + \frac{k^2\omega_0^2 t^2 - 2}{k^3\omega_0^3} \sin k\omega_0 t \right]_0^5$$

$$k\omega_0(5) = k \left(\frac{2\pi}{20} \right) (5) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0$, since k is odd

$$\therefore a_k = \frac{2}{5} \left[0 + \frac{(k^2\pi^2/4) - 2}{k^3\omega_0^3} \sin(k\pi/2) \right] = \frac{k^2\omega_0^2 - 8}{10k^3\omega_0^3} \sin(k\pi/2)$$

$$\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}; \quad \omega_0^2 = \frac{\pi^2}{100}; \quad \omega_0^3 = \frac{\pi^3}{1000}$$

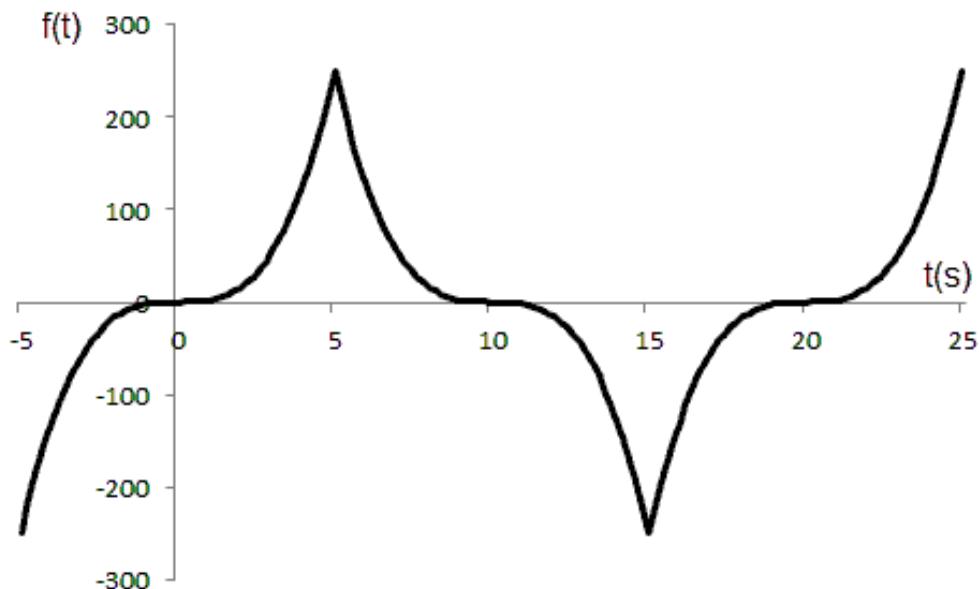
$$a_k = \left(\frac{k^2\pi^2 - 800}{k^3\pi^3} \right) \sin(k\pi/2)$$

$$f(t) = \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{n^2\pi^2 - 800}{\pi^3 n^3} \right] \sin(n\pi/2) \cos(n\omega_0 t)$$

[e] $\cos n\omega_0(t - 5) = \cos(n\omega_0 t - n\pi/2) = \sin(n\pi/2) \sin n\omega_0 t$

$$\begin{aligned} f(t) &= \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{n^2\pi^2 - 800}{\pi^3 n^3} \right] \sin^2(n\pi/2) \sin(n\omega_0 t) \\ &= \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{n^2\pi^2 - 800}{\pi^3 n^3} \right] \sin(n\omega_0 t) \end{aligned}$$

P 16.16 [a]



[b] Odd, since $f(-t) = -f(t)$

[c] $f(t)$ has quarter-wave symmetry, since $f(T/2 - t) = f(t)$ in the interval $0 < t < 4$.

[d] $a_v = 0$, (half-wave symmetry); $a_k = 0$, for all k (function is odd)

$b_k = 0$, for k even (half-wave symmetry)

$$b_k = \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_0 t dt, \quad k \text{ odd}$$

$$= \frac{16}{20} \int_0^5 t^3 \sin k\omega_0 t dt$$

$$= \frac{4}{5} \left[\frac{3t^2}{k^2\omega_0^2} \sin k\omega_0 t - \frac{6}{k^4\omega_0^4} \sin k\omega_0 t - \frac{t^3}{k\omega_0} \cos k\omega_0 t + \frac{6t}{k^3\omega_0^3} \cos k\omega_0 t \right]_0^5$$

$$k\omega_0(5) = k \left(\frac{2\pi}{20} \right) (5) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0$, since k is odd

$$\therefore b_k = \left[\frac{60}{k^2\omega_0^2} \sin(k\pi/2) - \frac{4.8}{k^4\omega_0^4} \sin(k\pi/2) \right]$$

$$k\omega_0 = k \left(\frac{2\pi}{20} \right) = \frac{k\pi}{10}; \quad k^2\omega_0^2 = \frac{k^2\pi^2}{100}; \quad k^4\omega_0^4 = \frac{k^4\pi^4}{10,000}$$

$$\therefore b_k = \frac{6000}{\pi^2 k^2} \left[1 - \frac{8}{\pi^2 k^2} \right] \sin(k\pi/2), \quad k \text{ odd}$$

$$f(t) = \frac{6000}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{1}{n^2} \left(1 - \frac{8}{\pi^2 n^2} \right) \sin(n\pi/2) \right] \sin n\omega_0 t$$

[e] $\sin n\omega_0(t - 2) = \sin(n\omega_0 t - n\pi/2) = -\cos n\omega_0 t \sin(n\pi/2)$

$$f(t) = \frac{-6000}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{1}{n^2} \left(1 - \frac{8}{\pi^2 n^2} \right) \right] \cos n\omega_0 t$$

P 16.17 [a] $i(t)$ is odd, therefore $a_v = 0$ and $b_k = 0$ for all k .

$$f(t) = i(t) = I_m - \frac{2I_m}{T}t, \quad 0 \leq t \leq T$$

$$b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin k\omega_o t dt$$

$$= \frac{4}{T} \int_0^{T/2} \left(I_m - \frac{2I_m}{T}t \right) \sin k\omega_o t dt$$

$$= \frac{4I_m}{T} \left[\int_0^{T/2} \sin k\omega_o t dt - \frac{2}{T} \int_0^{T/2} t \sin k\omega_o t dt \right]$$

$$\begin{aligned}
&= \frac{4I_m}{T} \left[\frac{-\cos k\omega_0 t}{k\omega_0} \Big|_0^{T/2} - \frac{2}{T} \left(\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t \cos k\omega_0 t}{k\omega_0} \right) \Big|_0^{T/2} \right] \\
&= \frac{4I_m}{T} \left[\frac{1 - \cos k\pi}{k\omega_0} + \frac{\cos k\pi}{k\omega_0} \right] \\
&= \frac{4I_m}{k\omega_0 T} = \frac{2I_m}{k\pi} \\
\therefore i(t) &= \frac{2I_m}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin n\omega_o t \\
[\text{b}] \quad i(t) &= \frac{2I_m}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin n\omega_o (t + T/2) \\
&= \frac{2I_m}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi}{n} \sin n\omega_o t
\end{aligned}$$

P 16.18 $v_2(t + T/8)$ is even, so $b_k = 0$ for all k .

$$\begin{aligned}
a_v &= \frac{(V_m/2)(T/4)}{T} = \frac{V_m}{8} \\
a_k &= \frac{4}{T} \int_0^{T/8} \frac{V_m}{2} \cos k\omega_0 t dt = \frac{V_m}{k\pi} \sin \frac{k\pi}{4} \\
\text{Therefore, } v_2(t + T/8) &= \frac{V_m}{8} + \frac{V_m}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{4} \cos n\omega_0 t \\
\text{so } v_2(t) &= \frac{V_m}{8} + \frac{V_m}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{4} \cos n\omega_0 (t - T/8) \\
\therefore v(t) &= \frac{V_m}{2} + \frac{V_m}{8} + \frac{V_m}{\pi} \sum_{n=1}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{4} \cos \frac{n\pi}{4} \right) \cos n\omega_0 t + \left(\frac{1}{n} \sin^2 \frac{n\pi}{4} \right) \sin n\omega_0 t \\
&= \frac{5V_m}{8} + \frac{V_m}{2\pi} \sum_{n=1}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \right) \cos n\omega_0 t + \left(1 - \cos \frac{n\pi}{2} \right) \sin n\omega_0 t \text{ V}
\end{aligned}$$

Thus, since $a_v = 5V_m/8 = 37.5\pi$ V,

$$a_k = \frac{V_m}{2\pi k} \sin \frac{k\pi}{2} = \frac{30}{k} \sin \frac{k\pi}{2}$$

and

$$b_k = \frac{V_m}{2\pi k} \left[1 - \cos \frac{k\pi}{2} \right] = \frac{30}{k} \left[1 - \cos \frac{k\pi}{2} \right]$$

These equations match the equations for a_v , a_k , and b_k derived in Problem 16.4.

P 16.19 From Problem 16.1(a),

$$a_v = 25 \text{ V} = A_0$$

$$a_n = \frac{100}{n\pi} \sin \frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{100}{n\pi} \sin \frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

$$\text{Thus, } v(t) = 25 + \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n\omega_o t \text{ V}$$

For the periodic function in Fig. P16.1(b):

$$a_n = \frac{120}{\pi n} \sin \frac{\pi n}{2} \quad \text{and} \quad b_n = \frac{240}{\pi n} \quad \text{for } n \text{ odd.}$$

$$A_n / -\theta_n = a_n - jb_n = \frac{120}{\pi n} \sin \frac{\pi n}{2} - j \frac{240}{\pi n}, \quad n \text{ odd}$$

Therefore,

$$A_n = \frac{120\sqrt{5}}{n\pi}, \quad n \text{ odd}$$

and

$$\theta_n = \tan^{-1}(-240/120) = -63.43^\circ, \quad n = 1, 5, 9, \dots$$

and

$$\theta_n = \tan^{-1}(-240/-120) = 63.43^\circ, \quad n = 3, 7, 11, \dots$$

$$\text{Thus, } v(t) = \frac{120\sqrt{5}}{\pi} \sum_{n=1,5,9,\dots}^{\infty} \frac{1}{n} \cos(n\omega_o t - 63.43^\circ)$$

$$+ \frac{120\sqrt{5}}{\pi} \sum_{n=3,7,11,\dots}^{\infty} \frac{1}{n} \cos(n\omega_o t + 63.43^\circ) \text{ V}$$

P 16.20 The periodic function in Problem 16.12 is odd, so $a_v = 0$ and $a_k = 0$ for all k . Thus,

$$A_n / -\theta_n = a_n - jb_n = 0 - jb_n = b_n / -90^\circ$$

From Problem 16.12,

$$b_k = \frac{-0.64}{\pi^2 k^2} \sin \frac{k\pi}{4}, \quad k \text{ odd}$$

Therefore,

$$A_n = \frac{-0.64}{\pi^2 k^2} \sin \frac{k\pi}{4}, \quad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \quad n \text{ odd}$$

$$\text{Thus, } i(t) = \frac{640}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\pi/4)}{n^2} \cos(n\omega_o t + 90^\circ) \text{ mA}$$

P 16.21 The periodic function in Problem 16.15 is even, so $b_k = 0$ for all k . Thus,

$$A_n / -\theta_n = a_n - jb_n = a_n = a_n / 0^\circ$$

From Problem 16.15,

$$a_v = 0 = A_0$$

$$a_n = \frac{n^2 \pi^2 - 800}{\pi^3 n^3} \sin \frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{n^2 \pi^2 - 800}{\pi^3 n^3} \sin \frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

$$\text{Thus, } f(t) = \frac{1}{\pi^3} \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{n^2 \pi^2 - 800}{n^3} \right) \sin \frac{n\pi}{2} \cos n\omega_o t$$

P 16.22 [a] The current has half-wave symmetry. Therefore,

$$a_v = 0; \quad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$\begin{aligned} a_k &= \frac{4}{T} \int_0^{T/2} \left(I_m - \frac{2I_m}{T} t \right) \cos k\omega_o t \, dt \\ &= \frac{4}{T} \int_0^{T/2} I_m \cos k\omega_0 t \, dt - \frac{8I_m}{T^2} \int_0^{T/2} t \cos k\omega_0 t \, dt \\ &= \frac{4I_m}{T} \frac{\sin k\omega_0 t}{k\omega_0} \Big|_0^{T/2} - \frac{8I_m}{T^2} \left[\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0} \sin k\omega_0 t \right]_0^{T/2} \\ &= 0 - \frac{8I_m}{T^2} \left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2} \right] \\ &= \left(\frac{8I_m}{T^2} \right) \left(\frac{1}{k^2\omega_0^2} \right) (1 - \cos k\pi) \\ &= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd} \\ b_k &= \frac{4}{T} \int_0^{T/2} \left(I_m - \frac{2I_m}{T} t \right) \sin k\omega_o t \, dt \\ &= \frac{4I_m}{T} \int_0^{T/2} \sin k\omega_0 t \, dt - \frac{8I_m}{T^2} \int_0^{T/2} t \sin k\omega_0 t \, dt \\ &= \frac{4I_m}{T} \left[\frac{-\cos k\omega_0 t}{k\omega_0} \right]_0^{T/2} - \frac{8I_m}{T^2} \left[\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0} \cos k\omega_0 t \right]_0^{T/2} \\ &= \frac{4I_m}{T} \left[\frac{1 - \cos k\pi}{k\omega_0} \right] - \frac{8I_m}{T^2} \left[\frac{-T \cos k\pi}{2k\omega_0} \right] \\ &= \frac{8I_m}{k\omega_0 T} \left[\frac{1}{2} \right] \\ &= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd} \\ a_k - jb_k &= \frac{20}{k^2} - j \frac{10\pi}{k} = \frac{10}{k} \left(\frac{2}{k} - j\pi \right) = \frac{10}{k^2} \sqrt{\pi^2 k^2 + 4} / \underline{-\theta_k} \end{aligned}$$

$$\text{where } \tan \theta_k = \frac{\pi k}{2}$$

$$i(t) = 10 \sum_{n=1,3,5,\dots}^{\infty} \frac{\sqrt{(n\pi)^2 + 4}}{n^2} \cos(n\omega_0 t - \theta_n), \quad \theta_n = \tan^{-1} \frac{n\pi}{2}$$

$$[\text{b}] \quad A_1 = 10\sqrt{4 + \pi^2} \cong 37.24 \text{ A} \quad \tan \theta_1 = \frac{\pi}{2} \quad \theta_1 \cong 57.52^\circ$$

$$A_3 = \frac{10}{9}\sqrt{4 + 9\pi^2} \cong 10.71 \text{ A} \quad \tan \theta_3 = \frac{3\pi}{2} \quad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25}\sqrt{4 + 25\pi^2} \cong 6.33 \text{ A} \quad \tan \theta_5 = \frac{5\pi}{2} \quad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49}\sqrt{4 + 49\pi^2} \cong 4.51 \text{ A} \quad \tan \theta_7 = \frac{7\pi}{2} \quad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81}\sqrt{4 + 81\pi^2} \cong 3.50 \text{ A} \quad \tan \theta_9 = \frac{9\pi}{2} \quad \theta_9 \cong 85.95^\circ$$

$$i(t) \cong 37.24 \cos(\omega_o t - 57.52^\circ) + 10.71 \cos(3\omega_o t - 78.02^\circ)$$

$$+ 6.33 \cos(5\omega_o t - 82.74^\circ) + 4.51 \cos(7\omega_o t - 84.80^\circ)$$

$$+ 3.50 \cos(9\omega_o t - 85.95^\circ) + \dots$$

$$i(T/4) \cong 37.24 \cos(90 - 57.52^\circ) + 10.71 \cos(270 - 78.02^\circ)$$

$$+ 6.33 \cos(450 - 82.74^\circ) + 4.51 \cos(630 - 84.80^\circ)$$

$$+ 3.50 \cos(810 - 85.95^\circ) \cong 26.23 \text{ A}$$

Actual value:

$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67 \text{ A}$$

P 16.23 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k -even, $a_v = 0$; for k -odd

$$a_k = \frac{4}{T} \int_0^{T/2} V_m \cos k\omega_0 t dt - \frac{8V_m}{\rho T} \int_0^{T/2} e^{-t/RC} \cos k\omega_0 t dt$$

$$\text{where } \rho = [1 + e^{-T/2RC}].$$

Upon integrating we get

$$\begin{aligned} a_k &= \frac{4V_m}{T} \frac{\sin k\omega_0 t}{k\omega_0} \Big|_0^{T/2} \\ &\quad - \frac{8V_m}{\rho T} \cdot \left\{ \frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2} \cdot \left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0 \sin k\omega_0 t \right] \Big|_0^{T/2} \right\} \\ &= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]} \end{aligned}$$

$$\begin{aligned}
b_k &= \frac{4}{T} \int_0^{T/2} V_m \sin k\omega_0 t dt - \frac{8V_m}{\rho T} \int_0^{T/2} e^{-t/RC} \sin k\omega_0 t dt \\
&= -\frac{4V_m}{T} \frac{\cos k\omega_0 t}{k\omega_0} \Big|_0^{T/2} \\
&\quad - \frac{8V_m}{\rho T} \cdot \left\{ \frac{-e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2} \cdot \left[\frac{\sin k\omega_0 t}{RC} + k\omega_0 \cos k\omega_0 t \right] \Big|_0^{T/2} \right\} \\
&= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2 C^2}{T[1 + (k\omega_0 RC)^2]}
\end{aligned}$$

P 16.24 [a] $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 R C a_k \right)^2$

$$= a_k^2 [1 + (k\omega_0 RC)^2] + \frac{8V_m}{\pi k} \left[\frac{2V_m}{\pi k} + k\omega_0 R C a_k \right]$$

But $a_k = \left\{ \frac{-8V_m R C}{T[1 + (k\omega_0 RC)^2]} \right\}$

Therefore $a_k^2 = \left\{ \frac{64V_m^2 R^2 C^2}{T^2[1 + (k\omega_0 RC)^2]^2} \right\}$, thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2 R^2 C^2}{T^2[1 + (k\omega_0 RC)^2]^2} + \frac{16V_m^2}{\pi^2 k^2} - \frac{64V_m^2 k\omega_0 R^2 C^2}{\pi k T [1 + (k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_m^2/\pi^2 k^2 (1 + \alpha^2)$. It follows that

$$\sqrt{a_k^2 + b_k^2} = \frac{4V_m}{\pi k \sqrt{1 + (k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 R C a_k + \frac{4V_m}{\pi k}$

Thus $\frac{b_k}{a_k} = k\omega_0 RC + \frac{4V_m}{\pi k a_k} = \alpha - \frac{1 + \alpha^2}{\alpha} = -\frac{1}{\alpha}$

Therefore $\frac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.25 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5,\dots}^{\infty} \frac{4V_m}{n\pi} \frac{1}{\sqrt{1 + (n\omega_0 RC)^2}} \cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan \theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.57, we have $\tan \beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan \beta_k = -a_k/b_k$ or $\tan \theta_n = -\cot \beta_n$. Therefore $\theta_n = 90^\circ + \beta_n$ and

$\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90^\circ) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.26 [a] $e^{-x} \cong 1 - x$ for small x ; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o \cong V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right) \left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$\cong \left(\frac{V_m}{RC}\right) \left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right) t - \frac{V_m T}{4RC} \quad \text{for } 0 \leq t \leq \frac{T}{2}$$

$$[b] a_k = \left(\frac{-8}{\pi^2 k^2}\right) V_p = \left(\frac{-8}{\pi^2 k^2}\right) \left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RC k^2}$$

P 16.27 [a] Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the k th harmonic phase current is

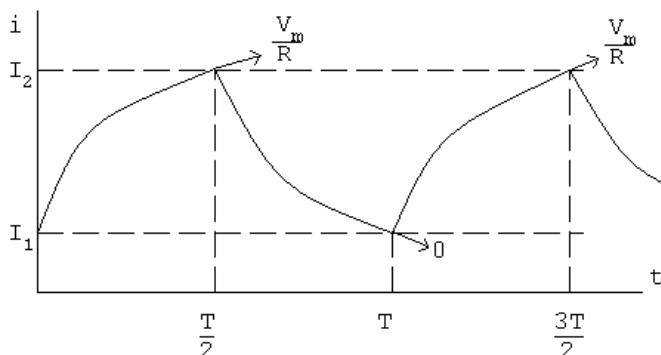
$$I_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}} / -\theta_k$$

$$\text{where } \theta_k = \tan^{-1} \left(\frac{k\omega_0 L}{R}\right)$$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}} A$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R) , we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R} \right) e^{-t/\tau} \quad \text{for } 0 \leq t \leq \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \leq t \leq T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R} \right) e^{-T/2\tau}$$

These two equations are now solved for I_1 . Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1 + e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1 + e^{-x})} \right] e^{-t/\tau} \quad \text{for } 0 \leq t \leq \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1 + e^{-x})} \right] e^{-[t-(T/2)]/\tau} \quad \text{for } \frac{T}{2} \leq t \leq T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$\begin{aligned} I_{\text{avg}} &= \frac{1}{T} \left\{ \int_0^{T/2} \left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R} \right) e^{-t/\tau} \right] dt + \int_{T/2}^T I_2 e^{-[t-(T/2)]/\tau} dt \right\} \\ &= \frac{1}{T} \left\{ \frac{V_m T}{2R} + \tau(1 - e^{-x}) \left(I_1 - \frac{V_m}{R} + I_2 \right) \right\} \\ &= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R} \end{aligned}$$

- P 16.28 [a] From the solution to Problem 16.13(a) the Fourier series for the input voltage is

$$v_g = \frac{4V_m}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{1}{n} \sin \frac{n\pi}{2} \right] \cos n\omega_0 t \text{ V}$$

Since $V_m = 0.5\pi$ V and $T = 10\pi$ ms, we can write the input voltage as

$$\begin{aligned} v_g &= 2 \sum_{n=1,3,5,\dots}^{\infty} \left[\frac{1}{n} \sin \left(\frac{n\pi}{2} \right) \right] \cos 200nt \text{ V} \\ &= 2 \cos 200t - \frac{2}{3} \cos 600t + \frac{2}{5} \cos 1000t - \frac{2}{7} \cos 1400t + \dots \end{aligned}$$

We can phasor transform this Fourier series to get

$$\mathbf{V}_{g1} = 2 \angle 0^\circ \quad \omega_0 = 200 \text{ rad/s}$$

$$\mathbf{V}_{g3} = 0.667/\underline{180^\circ} \quad 3\omega_0 = 600 \text{ rad/s}$$

$$\mathbf{V}_{g5} = 0.4/\underline{0^\circ} \quad 5\omega_0 = 1000 \text{ rad/s}$$

$$\mathbf{V}_{g7} = 0.286/\underline{180^\circ} \quad 7\omega_0 = 1400 \text{ rad/s}$$

From the circuit in Fig. P16.28 we have

$$\frac{V_o}{R} + \frac{V_o - V_g}{sL} + (V_o - V_g)sC = 0$$

$$\therefore \frac{V_o}{V_g} = H(s) = \frac{s^2 + 1/LC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values gives

$$H(s) = \frac{s^2 + 10^6}{s^2 + 40s + 10^6}$$

$$H(j200) = \frac{96}{96 + j0.8} = 0.99997/\underline{-0.48^\circ}$$

$$H(j600) = \frac{64}{64 + j2.4} = 0.9993/\underline{-2.15^\circ}$$

$$H(j1000) = 0$$

$$H(j1400) = \frac{-96}{-96 + j5.6} = 0.9983/\underline{3.34^\circ}$$

$$\mathbf{V}_{o1} = (2/\underline{0^\circ})(0.99997/\underline{-0.48^\circ}) = 1.9999/\underline{-0.48^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = (0.667/\underline{180^\circ})(0.9993/\underline{-2.15^\circ}) = 0.6662/\underline{177.85^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = 0 \text{ V}$$

$$\mathbf{V}_{o7} = (0.286/\underline{180^\circ})(0.9983/\underline{3.34^\circ}) = 0.286/\underline{-176.66^\circ} \text{ V}$$

$$v_o = 1.9999 \cos(200t - 0.48^\circ) + 0.6662 \cos(600t + 177.85^\circ)$$

$$+ 0.286 \cos(1400t - 176.66^\circ) + \dots \text{ V}$$

- [b] The 5th harmonic at the frequency $\sqrt{1/LC} = 1000 \text{ rad/s}$ has been eliminated from the output voltage by the circuit, which is a band reject filter with a center frequency of 1000 rad/s.

$$\text{P 16.29 } v_i = \frac{4A}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin n\omega_0(t + T/4)$$

$$= \frac{4A}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \right) \cos n\omega_0 t$$

$$\omega_0 = \frac{2\pi}{4\pi} \times 10^6 = 10,000 \text{ rad/s}; \quad \frac{4A}{\pi} = 120$$

$$v_i = 120 \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \right) \cos 10,000nt \text{ V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + j\omega L} \cdot j\omega L = \frac{j\omega}{R/L + j\omega} \mathbf{V}_i = \frac{j\omega}{30,000 + j\omega} \mathbf{V}_i$$

$$\mathbf{V}_{i1} = 120/0^\circ \text{ V}; \quad \omega = 10,000 \text{ rad/s}$$

$$\mathbf{V}_{i3} = -40/0^\circ = 40/180^\circ \text{ V}; \quad 3\omega = 30,000 \text{ rad/s}$$

$$\mathbf{V}_{i5} = 24/0^\circ \text{ V}; \quad 5\omega = 50,000 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{j10,000}{30,000 + j10,000} (120/0^\circ) = 37.95/71.57^\circ \text{ V}$$

$$\mathbf{V}_{o3} = \frac{j30,000}{30,000 + j30,000} (40/180^\circ) = 28.28/-135^\circ \text{ V}$$

$$\mathbf{V}_{o5} = \frac{j50,000}{30,000 + j50,000} (24/0^\circ) = 20.58/30.96^\circ \text{ V}$$

$$\therefore v_o = 37.95 \cos(10,000t + 71.57^\circ) + 28.28 \cos(30,000t - 135^\circ)$$

$$+ 20.58 \cos(50,000t + 30.96^\circ) + \dots \text{ V}$$

$$\text{P 16.30 [a]} \quad \frac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \frac{V_0}{1000} = 0$$

$$V_0 \left[\frac{1}{16s} + 12.6 \times 10^{-6}s + \frac{1}{1000} \right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \quad \omega_0^2 = 57,600\pi^2; \quad 80\omega_0 = 19,200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57,600\pi^2n^2) + j19,200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6} / -173.89^\circ$$

$$H(j2\omega_0) = 2.20 \times 10^{-6} / -176.96^\circ$$

$$H(j3\omega_0) = 9.78 \times 10^{-7} / -177.97^\circ$$

$$H(j4\omega_0) = 5.5 \times 10^{-7} / -178.48^\circ$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi} \left[\frac{1}{3} \cos \omega_0 t + \frac{1}{15} \cos 2\omega_0 t + \frac{1}{35} \cos 3\omega_0 t + \frac{1}{63} \cos 4\omega_0 t + \dots \right]$$

$$i_0 = \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi} (8.82 \times 10^{-6}) \cos(\omega_0 t - 173.89^\circ)$$

$$- \frac{1360}{15\pi} (2.20 \times 10^{-6}) \cos(2\omega_0 t - 176.96^\circ)$$

$$- \frac{1360}{35\pi} (9.78 \times 10^{-7}) \cos(3\omega_0 t - 177.97^\circ)$$

$$- \frac{1360}{63\pi} (5.5 \times 10^{-7}) \cos(4\omega_0 t - 178.48^\circ) - \dots$$

$$= 216.45 \times 10^{-3} + 1.27 \times 10^{-3} \cos(240\pi t + 6.11^\circ)$$

$$+ 6.35 \times 10^{-5} \cos(480\pi t + 3.04^\circ)$$

$$+ 1.21 \times 10^{-5} \cos(720\pi t + 2.03^\circ)$$

$$+ 3.8 \times 10^{-6} \cos(960\pi t + 1.11^\circ) - \dots$$

$$i_0 \cong 216.45 + 1.27 \cos(240\pi t + 6.11^\circ) \text{ mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$$i_0 \cong 216.45 \text{ mA} \quad (\text{a dc current})$$

- [b] The circuit is a low pass filter, so the harmonic terms are greatly reduced in the output.

P 16.31 The function is odd with half-wave and quarter-wave symmetry. Therefore,

$$a_k = 0, \quad \text{for all } k; \text{ the function is odd}$$

$$b_k = 0, \quad \text{for } k \text{ even, the function has half-wave symmetry}$$

$$\begin{aligned} b_k &= \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_o t dt, \quad k \text{ odd} \\ &= \frac{8}{T} \left\{ \int_0^{T/10} 100t \sin k\omega_o t dt + \int_{T/10}^{T/4} 5 \sin k\omega_o t dt \right\} \\ &= \frac{8}{T} \{ \text{Int1} + \text{Int2} \} \end{aligned}$$

$$\begin{aligned} \text{Int1} &= 100 \int_0^{T/10} t \sin k\omega_o t dt \\ &= 100 \left[\frac{1}{k^2 \omega_o^2} \sin k\omega_o t - \frac{t}{k\omega_o} \cos k\omega_o t \Big|_0^{T/10} \right] \\ &= \frac{100}{k^2 \omega_o^2} \sin \frac{k\pi}{5} - \frac{10T}{k\omega_o} \cos \frac{k\pi}{5} \end{aligned}$$

$$\text{Int2} = \int_{T/10}^{T/4} 5 \sin k\omega_o t dt = \frac{-5}{k\omega_o} \cos k\omega_o t \Big|_{T/10}^{T/4} = \frac{5}{k\omega_o} \cos \frac{k\pi}{5}$$

$$\text{Int1} + \text{Int2} = \frac{100}{k^2 \omega_o^2} \sin \frac{k\pi}{5} + \left(\frac{5}{k\omega_o} - \frac{10T}{k\omega_o} \right) \cos \frac{k\pi}{5}$$

$$10T = 10(0.5) = 5$$

$$\therefore \text{Int1} + \text{Int2} = \frac{100}{k^2 \omega_o^2} \sin \frac{k\pi}{5}$$

$$b_k = \left[\frac{8}{T} \cdot \frac{100}{4\pi^2 k^2} \cdot T^2 \right] \sin \frac{k\pi}{5} = \frac{100}{\pi^2 k^2} \sin \frac{k\pi}{5}, \quad k \text{ odd}$$

$$i(t) = \frac{100}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\pi/5)}{n^2} \sin n\omega_o t \text{ A}$$

From the circuit,

$$H(s) = \frac{V_o}{I_g} = Z_{\text{eq}}$$

$$Y_{\text{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + sL} + sC$$

$$Z_{\text{eq}} = \frac{1/C(s + R_2/L)}{s^2 + s(R_1R_2C + L)/R_1LC + (R_1 + R_2)/R_1LC}$$

Therefore,

$$H(s) = \frac{20,000(s + 400)}{s^2 + 10,400s + 450 \times 10^4}$$

We want the output for the third harmonic:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi; \quad 3\omega_0 = 12\pi$$

$$I_{g3} = \frac{100}{\pi^2} \frac{1}{9} \sin \frac{3\pi}{5} = 1.07/0^\circ$$

$$H(j12\pi) = \frac{20,000(j12\pi + 400)}{(j12\pi)^2 + 10,400(j12\pi) + 450 \times 10^4} = 1.78/0.403^\circ$$

Therefore,

$$V_{o3} = H(j12\pi)I_{g3} = (1.78/0.403^\circ)(1.07/0^\circ) = 1.9/0.403^\circ \text{ V}$$

$$v_{o3} = 1.9 \sin(12\pi t + 0.403^\circ) \text{ V}$$

$$\text{P 16.32 } \omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 200 \text{ krad/s}$$

$$\therefore n = \frac{3 \times 10^6}{0.2 \times 10^6} = 15; \quad n = \frac{5 \times 10^6}{0.2 \times 10^6} = 25$$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{RC} = \frac{10^{12}}{(250 \times 10^3)(4)} = 10^6; \quad \frac{1}{LC} = \frac{(10^3)(10^{12})}{10)(4)} = 25 \times 10^{12}$$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 25 \times 10^{12}}$$

$$H(j\omega) = \frac{j\omega \times 10^6}{(25 \times 10^{12} - \omega^2) + j10^6\omega}$$

15th harmonic input:

$$v_{g15} = (150)(1/15) \sin(15\pi/2) \cos 15\omega_o t = -10 \cos 3 \times 10^6 t \text{ V}$$

$$\therefore \mathbf{V}_{g15} = 10/\underline{-180^\circ} \text{ V}$$

$$H(j3 \times 10^6) = \frac{j3}{16 + j3} = 0.1843/\underline{79.38^\circ}$$

$$\mathbf{V}_{o15} = (10)(0.1843)/\underline{-100.62^\circ} \text{ V}$$

$$v_{o15} = 1.84 \cos(3 \times 10^6 t - 100.62^\circ) \text{ V}$$

25th harmonic input:

$$v_{g25} = (150)(1/25) \sin(25\pi/2) \cos 5 \times 10^6 t = 6 \cos 5 \times 10^6 t \text{ V}$$

$$\therefore \mathbf{V}_{g25} = 6/\underline{0^\circ} \text{ V}$$

$$H(j5 \times 10^6) = \frac{j5}{0 + j5} = 1/\underline{0^\circ}$$

$$\mathbf{V}_{o25} = 6/\underline{0^\circ} \text{ V}$$

$$v_{o25} = 6 \cos 5 \times 10^6 t \text{ V}$$

$$\text{P 16.33 [a]} \quad a_v = \frac{1}{T} \left[\frac{1}{2} \left(\frac{T}{2} \right) I_m + \frac{T}{2} I_m \right] = \frac{3I_m}{4}$$

$$i(t) = \frac{2I_m}{T}t, \quad 0 \leq t \leq T/2$$

$$i(t) = I_m, \quad T/2 \leq t \leq T$$

$$a_k = \frac{2}{T} \int_0^{T/2} \frac{2I_m}{T} t \cos k\omega_o t dt + \frac{2}{T} \int_{T/2}^T I_m \cos k\omega_o t dt$$

$$= \frac{I_m}{\pi^2 k^2} (\cos k\pi - 1)$$

$$b_k = \frac{2}{T} \int_0^{T/2} \frac{2I_m}{T} t \sin k\omega_o t dt + \frac{2}{T} \int_{T/2}^T I_m \sin k\omega_o t dt$$

$$= -\frac{I_m}{\pi k}$$

$$a_v = \frac{3I_m}{4}, \quad a_1 = \frac{-2I_m}{\pi^2}, \quad a_2 = 0,$$

$$b_1 = -\frac{I_m}{\pi}, \quad b_2 = -\frac{I_m}{2\pi}$$

$$\therefore I_{\text{rms}} = I_m \sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040I_m$$

$$I_{\text{rms}} = 4.02 \text{ A}$$

$$P = (4.02)^2(2500) = 40.4 \text{ kW}$$

[b] Area under i^2 :

$$\begin{aligned} A &= \int_0^{T/2} \frac{4I_m^2}{T^2} t^2 dt + I_m^2 \frac{T}{2} \\ &= \frac{4I_m^2}{T^2} \frac{t^3}{3} \Big|_0^{T/2} + I_m^2 \frac{T}{2} \\ &= I_m^2 T \left[\frac{1}{6} + \frac{3}{6} \right] = \frac{2}{3} T I_m^2 \end{aligned}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \cdot \frac{2}{3} T I_m^2} = \sqrt{\frac{2}{3}} I_m = 4.0825 \text{ A}$$

$$P = (4.0825)^2(2500) = 41.67 \text{ kW}$$

$$[\text{c}] \text{ Error } = \left(\frac{40.4}{41.67} - 1 \right) 100 = -3.05\%$$

$$\text{P 16.34 [a]} \quad a_v = \frac{2 \left(\frac{1}{2} \frac{T}{4} V_m \right)}{T} = \frac{V_m}{4}$$

$$\begin{aligned} a_k &= \frac{4}{T} \int_0^{T/4} \left[V_m - \frac{4V_m}{T} t \right] \cos k\omega_o t dt \\ &= \frac{4V_m}{\pi^2 k^2} \left[1 - \cos \frac{k\pi}{2} \right] \end{aligned}$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{200}{4} = 50 \text{ V}$$

$$a_1 = \frac{800}{\pi^2}$$

$$a_2 = \frac{800}{4\pi^2} (1 - \cos \pi) = \frac{400}{\pi^2}$$

$$V_{\text{rms}} = \sqrt{(50)^2 + \frac{1}{2} \left[\left(\frac{800}{\pi^2} \right)^2 + \left(\frac{400}{\pi^2} \right)^2 \right]} = 81.28 \text{ V}$$

$$P = \frac{(81.28)^2}{400} = 16.516 \text{ W}$$

[b] Area under v^2 ; $0 \leq t \leq T/4$

$$v^2 = 40,000 - \frac{320,000}{T}t + \frac{640,000}{T^2}t^2$$

$$A = 2 \int_0^{T/4} \left[40,000 - \frac{320,000}{T}t + \frac{640,000}{T^2}t^2 \right] dt = 6666.67T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} 6666.67T} = \sqrt{6666.67} = 81.65 \text{ V}$$

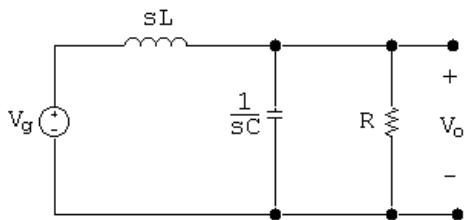
$$P = \sqrt{6666.67^2}/400 = 16.667 \text{ W}$$

[c] Error = $\left(\frac{16.516}{16.667} - 1 \right) 100 = -0.904\%$

P 16.35 $v_g = 10 - \frac{80}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos n\omega_o t \text{ V}$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} \times 10^3 = 500 \text{ rad/s}$$

$$v_g = 10 - \frac{80}{\pi^2} \cos 500t - \frac{80}{9\pi^2} \cos 1500t + \dots$$



$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701 / -43.31^\circ$$

$$H(j1500) = 0.4061 / -120.51^\circ$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701) \cos(500t - 43.31^\circ)$$

$$+ \frac{80}{9\pi^2}(0.4061) \cos(1500t - 120.51^\circ) + \dots$$

$$v_o = 10 + 7.86 \cos(500t - 43.31^\circ) + 0.3658 \cos(1500t - 120.51^\circ) + \dots$$

$$V_{\text{rms}} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44 \text{ V}$$

$$P \cong \frac{V_{\text{rms}}^2}{50\sqrt{2}} = 1.85 \text{ W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580) \left(\frac{80}{25\pi^2}\right) \cos(2500t - 146.04^\circ) = 0.0512 \cos(2500t - 146.04^\circ) \text{ V}$$

$$\text{P 16.36 [a]} \quad v = 30 + 60 \cos 2000t + 20 \cos(8000t - 90^\circ) \text{ V}$$

$$i = 3 + 4 \cos(2000t - 25^\circ) + \cos(8000t - 45^\circ) \text{ A}$$

$$P = (30)(3) + \frac{1}{2}(60)(4) \cos(25^\circ) + \frac{1}{2}(20)(1) \cos(-45^\circ) = 205.83 \text{ W}$$

$$[\mathbf{b}] \quad V_{\text{rms}} = \sqrt{(30)^2 + \left(\frac{60}{\sqrt{2}}\right)^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 53.85 \text{ V}$$

$$[\mathbf{c}] \quad I_{\text{rms}} = \sqrt{(3)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 4.18 \text{ A}$$

P 16.37 [a] Area under $v^2 = A = 4 \int_0^{T/6} \frac{36V_m^2}{T^2} t^2 dt + 2V_m^2 \left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T} \left(\frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3} \right)} = V_m \sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356 \text{ V}$

[b] From Assessment Problem 16.3,

$$v_g = 105.30 \sin \omega_0 t - 4.21 \sin 5\omega_0 t + 2.15 \sin 7\omega_0 t + \dots \text{ V}$$

Therefore $V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306 \text{ V}$

P 16.38 [a] v_g has half-wave symmetry, quarter-wave symmetry, and is odd

$$\therefore a_v = 0, \quad a_k = 0 \text{ all } k, \quad b_k = 0 \text{ } k\text{-even}$$

$$\begin{aligned} b_k &= \frac{8}{T} \int_0^{T/4} f(t) \sin k\omega_o t dt, \quad k\text{-odd} \\ &= \frac{8}{T} \left\{ \int_0^{T/8} V_m \sin k\omega_o t dt + \int_{T/8}^{T/4} \frac{V_m}{2} \sin k\omega_o t dt \right\} \\ &= \frac{8V_m}{T} \left[-\frac{\cos k\omega_o t}{k\omega_o} \right]_0^{T/8} + \frac{8V_m}{2T} \left[-\frac{\cos k\omega_o t}{k\omega_o} \right]_{T/8}^{T/4} \\ &= \frac{8V_m}{k\omega_o T} \left[1 - \cos \frac{k\pi}{4} \right] + \frac{8V_m}{2Tk\omega_o} \left[\cos \frac{k\pi}{4} - 0 \right] \\ &= \frac{8V_m}{k\omega_o T} \left\{ 1 - \cos \frac{k\pi}{4} + \frac{1}{2} \cos \frac{k\pi}{4} \right\} \\ &= \frac{4V_m}{\pi k} \left\{ 1 - 0.5 \cos \frac{k\pi}{4} \right\} = \frac{1}{k} [16 - 8 \cos(k\pi/4)] \end{aligned}$$

$$b_1 = 16 - 8 \cos(\pi/4) = 10.34$$

$$b_3 = \frac{1}{3}[16 - 8 \cos(3\pi/4)] = 7.22$$

$$b_5 = \frac{1}{5}[16 - 8 \cos(5\pi/4)] = 4.33$$

$$V_g(\text{rms}) \approx \mathbf{V}_m \sqrt{\frac{10.34^2 + 7.22^2 + 4.33^2}{2}} = 9.43$$

$$[\mathbf{b}] \text{ Area} = 2 \left[2(4\pi)^2 \left(\frac{T}{8} \right) + (2\pi)^2 \left(\frac{T}{4} \right) \right] = 10\pi^2 T$$

$$V_g(\text{rms}) = \sqrt{\frac{1}{T}(10\pi^2)T} = \sqrt{10}\pi = 9.935$$

$$[\mathbf{c}] \% \text{ Error} = \left(\frac{9.43}{9.935} - 1 \right) (100) = -5.08\%$$

$$\text{P 16.39 } [\mathbf{a}] v(t) = \frac{480}{\pi} \left\{ \sin \omega_o t + \frac{1}{3} \sin 3\omega_o t + \frac{1}{5} \sin 5\omega_o t + \frac{1}{7} \sin 7\omega_o t + \frac{1}{9} \sin 9\omega_o t + \dots \right.$$

$$\begin{aligned} V_{\text{rms}} &= \frac{480}{\pi} \sqrt{\left(\frac{1}{\sqrt{2}} \right)^2 + \left(\frac{1}{3\sqrt{2}} \right)^2 + \left(\frac{1}{5\sqrt{2}} \right)^2 + \left(\frac{1}{7\sqrt{2}} \right)^2 + \left(\frac{1}{9\sqrt{2}} \right)^2} \\ &= \frac{480}{\pi\sqrt{2}} \sqrt{1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}} \\ &= 117.55 \text{ V} \end{aligned}$$

$$[\mathbf{b}] \% \text{ error} = \left(\frac{117.55}{120} - 1 \right) (100) = -2.04\%$$

$$\begin{aligned} [\mathbf{c}] v(t) &= \frac{960}{\pi^2} \left\{ \sin \omega_o t + \frac{1}{9} \sin 3\omega_o t + \frac{1}{25} \sin 5\omega_o t \right. \\ &\quad \left. - \frac{1}{49} \sin 7\omega_o t + \frac{1}{81} \sin 9\omega_o t - \dots \right\} \end{aligned}$$

$$V_{\text{rms}} \cong \frac{960}{\pi^2 \sqrt{2}} \sqrt{1 + \frac{1}{81} + \frac{1}{625} + \frac{1}{2401} + \frac{1}{6561}}$$

$$\cong 69.2765 \text{ V}$$

$$V_{\text{rms}} = \frac{120}{\sqrt{3}} = 69.2820 \text{ V}$$

$$\% \text{ error} = \left(\frac{69.2765}{69.2820} - 1 \right) (100) = -0.0081\%$$

$$\text{P 16.40 [a]} \quad v(t) \approx \frac{340}{\pi} - \frac{680}{\pi} \left\{ \frac{1}{3} \cos \omega_o t + \frac{1}{15} \cos 2\omega_o t + \dots \right\}$$

$$\begin{aligned} V_{\text{rms}} &\approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2 \left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]} \\ &= \frac{340}{\pi} \sqrt{1 + 4 \left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819 \text{ V} \end{aligned}$$

$$\text{[b]} \quad V_{\text{rms}} = \frac{170}{\sqrt{2}} = 120.2082$$

$$\% \text{ error} = \left(\frac{120.0819}{120.2082} - 1 \right) (100) = -0.11\%$$

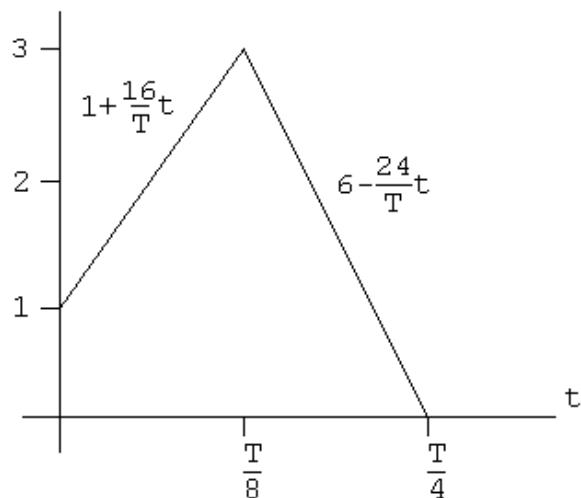
$$\text{[c]} \quad v(t) \approx \frac{170}{\pi} + 85 \sin \omega_o t - \frac{340}{3\pi} \cos 2\omega_o t$$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021 \text{ V}$$

$$V_{\text{rms}} = \frac{170}{2} = 85 \text{ V}$$

$$\% \text{ error} = -0.23\%$$

P 16.41 [a]



Area under i^2 :

$$\begin{aligned} A &= 4 \left[\int_0^{T/8} \left(1 + \frac{16}{T}t\right)^2 dt + \int_{T/8}^{T/4} \left(6 - \frac{24}{T}t\right)^2 dt \right] \\ &= 4 \left[\frac{T}{8} + \frac{T}{4} + \frac{T}{6} + 9T - 4.5T - 9T + 2.25T + 3T - 0.0375T \right] \\ &= \frac{11T}{3} \end{aligned}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \left(\frac{11T}{3} \right)} = \sqrt{\frac{11}{3}} = 1.915$$

[b] $P = I_{\text{rms}}^2 (100) = 366.7 \text{ W}$

[c] From Problem 16.14:

$$a_1 = 2.489 \text{ A}$$

$$i_g \approx 2.489 \cos \omega_o t \text{ A}$$

$$P = \left(\frac{2.489}{\sqrt{2}} \right)^2 (100) = 309.76 \text{ W}$$

[d] % error = $\left(\frac{308.76}{366.7} - 1 \right) = -15.52\%$

P 16.42 [a] Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T} \int_0^{T/4} \frac{4I_m}{T} t \cos k\omega_0 t dt = \frac{16I_m}{T^2} \int_0^{T/4} t \cos k\omega_0 t dt$$

$$= \frac{16I_m}{T^2} \left\{ \frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0} \sin k\omega_0 t \Big|_0^{T/4} \right\}$$

$$= \frac{16I_m}{T^2} \left\{ 0 + \frac{T}{4k\omega_0} \sin \frac{k\pi}{2} - \frac{1}{k^2\omega_0^2} \right\}$$

$$a_k = \frac{2I_m}{\pi k} \left[\sin \left(\frac{k\pi}{2} \right) - \frac{2}{\pi k} \right], \quad k \text{---odd}$$

$$b_k = \frac{4}{T} \int_0^{T/4} \frac{4I_m}{T} t \sin k\omega_0 t dt = \frac{16I_m}{T^2} \int_0^{T/4} t \sin k\omega_0 t dt$$

$$= \frac{16I_m}{T^2} \left\{ \frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0} \cos k\omega_0 t \Big|_0^{T/4} \right\} = \frac{4I_m}{\pi^2 k^2} \sin \left(\frac{k\pi}{2} \right)$$

[b] $a_k - jb_k = \frac{2I_m}{\pi k} \left\{ \left[\sin \left(\frac{k\pi}{2} \right) - \frac{2}{\pi k} \right] - \left[j \frac{2}{\pi k} \sin \left(\frac{k\pi}{2} \right) \right] \right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi} \left\{ \left(1 - \frac{2}{\pi} \right) - j \frac{2}{\pi} \right\} = 0.47I_m / \underline{-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi} \left\{ \left(-1 - \frac{2}{3\pi} \right) + j \left(\frac{2}{3\pi} \right) \right\} = 0.26I_m / \underline{170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi} \left\{ \left(1 - \frac{2}{5\pi} \right) - j \left(\frac{2}{5\pi} \right) \right\} = 0.11I_m / \underline{-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi} \left\{ \left(-1 - \frac{2}{7\pi} \right) + j \left(\frac{2}{7\pi} \right) \right\} = 0.10I_m / \underline{175.23^\circ}$$

$$i_g = 0.47I_m \cos(\omega_0 t - 60.28^\circ) + 0.26I_m \cos(3\omega_0 t + 170.07^\circ) + 0.11I_m \cos(5\omega_0 t - 8.30^\circ) + 0.10I_m \cos(7\omega_0 t + 175.23^\circ) + \dots$$

[c] $I_g = \sqrt{\sum_{n=1,3,5,\dots}^{\infty} \left(\frac{A_n^2}{2}\right)} \cong I_m \sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$

[d] Area $= 2 \int_0^{T/4} \left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right) \left(\frac{t^3}{3}\right) \Big|_0^{T/4} = \frac{I_m^2 T}{6}$

$$I_g = \sqrt{\frac{1}{T} \left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] % error $= \left(\frac{\text{estimated}}{\text{exact}} - 1\right) 100 = \left(\frac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right) 100 = -3.8\%$

P 16.43 Figure P16.43(b): $t_a = 0.2s$; $t_b = 0.6s$

$$v = 50t \quad 0 \leq t \leq 0.2$$

$$v = -50t + 20 \quad 0.2 \leq t \leq 0.6$$

$$v = 25t - 25 \quad 0.6 \leq t \leq 1.0$$

$$\text{Area 1} = A_1 = \int_0^{0.2} 2500t^2 dt = \frac{20}{3}$$

$$\text{Area 2} = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2) dt = \frac{40}{3}$$

$$\text{Area 3} = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1) dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1} \left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}} \text{ V.}$$

Figure P16.43(c): $t_a = t_b = 0.4s$

$$v(t) = 25t \quad 0 \leq t \leq 0.4$$

$$v(t) = \frac{50}{3}(t-1) \quad 0.4 \leq t \leq 1$$

$$A_1 = \int_0^{0.4} 625t^2 dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1) dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}} \text{ V.}$$

Figure P16.43(d): $t_a = t_b = 1$

$$v = 10t \quad 0 \leq t \leq 1$$

$$A_1 = \int_0^1 100t^2 dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}} \text{ V.}$$

$$\text{P 16.44 } C_o = A_v = \frac{V_m T}{2} \cdot \frac{1}{T} = \frac{V_m}{2}$$

$$\begin{aligned} C_n &= \frac{1}{T} \int_0^T \frac{V_m}{T} t e^{-jn\omega_0 t} dt \\ &= \frac{V_m}{T^2} \left[\frac{e^{-jn\omega_0 t}}{-n^2\omega_0^2} (-jn\omega_0 t - 1) \right]_0^T \\ &= \frac{V_m}{T^2} \left[\frac{e^{-jn2\pi T/T}}{-n^2\omega_0^2} \left(-jn\frac{2\pi}{T} T - 1 \right) - \frac{1}{-n^2\omega_0^2} (-1) \right] \\ &= \frac{V_m}{T^2} \left[\frac{1}{n^2\omega_0^2} (1 + jn2\pi) - \frac{1}{n^2\omega_0^2} \right] \\ &= j \frac{V_m}{2n\pi}, \quad n = \pm 1, \pm 2, \pm 3, \dots \end{aligned}$$

$$\begin{aligned}
 \text{P 16.45 [a]} \quad V_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T v^2 dt} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_m}{T}\right)^2 t^2 dt} \\
 &= \sqrt{\frac{V_m^2 t^3}{T^3} \frac{1}{3} \Big|_0^T} \\
 &= \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}} \\
 P &= \frac{(150/\sqrt{3})^2}{25} = 300 \text{ W}
 \end{aligned}$$

[b] From the solution to Problem 16.44

$$\begin{aligned}
 C_0 &= \frac{150}{2} = 75 \text{ V}; \\
 C_1 &= j \frac{150}{2\pi} = j \frac{75}{\pi}; & C_2 &= j \frac{150}{4\pi} = j \frac{37.5}{\pi} \\
 C_3 &= j \frac{150}{6\pi} = j \frac{25}{\pi}; & C_4 &= j \frac{150}{8\pi} = j \frac{18.75}{\pi} \\
 V_{\text{rms}} &= \sqrt{C_0^2 + 2 \sum_{n=1}^{\infty} |C_n|^2} \\
 &= \sqrt{75^2 + \frac{2}{\pi^2} (75^2 + 37.5^2 + 25^2 + 18.75^2)} \\
 &= 85.13 \text{ V}
 \end{aligned}$$

$$\text{[c]} \quad P = \frac{(85.13)^2}{25} = 289.88 \text{ W}$$

$$\% \text{ error} = \left(\frac{289.88}{300} - 1 \right) (100) = -3.37\%$$

$$\begin{aligned}
 \text{P 16.46} \quad C_n &= \frac{1}{T} \int_0^{T/4} V_m e^{-jn\omega_o t} dt = \frac{V_m}{T} \left[\frac{e^{-jn\omega_o t}}{-jn\omega_o} \Big|_0^{T/4} \right] \\
 &= \frac{V_m}{T n \omega_o} [j(e^{-jn\pi/2} - 1)] = \frac{V_m}{2\pi n} \sin \frac{n\pi}{2} + j \frac{V_m}{2\pi n} \left(\cos \frac{n\pi}{2} - 1 \right) \\
 &= \frac{V_m}{2\pi n} \left[\sin \frac{n\pi}{2} - j \left(1 - \cos \frac{n\pi}{2} \right) \right]
 \end{aligned}$$

$$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t}$$

$$C_o = A_v = \frac{1}{T} \int_0^{T/4} V_m dt = \frac{V_m}{4}$$

or

$$\begin{aligned} C_o &= \frac{V_m}{2\pi} \lim_{n \rightarrow 0} \left[\frac{\sin(n\pi/2)}{n} - j \frac{1 - \cos(n\pi/2)}{n} \right] \\ &= \frac{V_m}{2\pi} \lim_{n \rightarrow 0} \left[\frac{(\pi/2) \cos(n\pi/2)}{1} - j \frac{(\pi/2) \sin(n\pi/2)}{1} \right] \\ &= \frac{V_m}{2\pi} \left[\frac{\pi}{2} - j0 \right] = \frac{V_m}{4} \end{aligned}$$

Note it is much easier to use $C_o = A_v$ than to use L'Hopital's rule to find the limit of 0/0.

$$\text{P 16.47 [a]} \quad C_o = a_v = \frac{(1/2)(T/2)V_m}{T} = \frac{V_m}{4}$$

$$\begin{aligned} C_n &= \frac{1}{T} \int_0^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t} dt \\ &= \frac{2V_m}{T^2} \left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2} (-jn\omega_o t - 1) \right]_0^{T/2} \\ &= \frac{V_m}{2n^2\pi^2} [e^{-jn\pi} (-jn\pi + 1) - 1] \end{aligned}$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2} (\cos n\pi - 1) + j \frac{V_m}{2n\pi} \cos n\pi$$

$$\text{[b]} \quad C_o = \frac{54}{4} = 13.5 \text{ V}$$

$$C_{-1} = \frac{-54}{\pi^2} + j \frac{27}{\pi} = 10.19/\underline{122.48^\circ} \text{ V}$$

$$C_1 = 10.19/\underline{-122.48^\circ} \text{ V}$$

$$C_{-2} = -j \frac{13.5}{\pi} = 4.30/\underline{-90^\circ} \text{ V}$$

$$C_2 = 4.30/\underline{90^\circ} \text{ V}$$

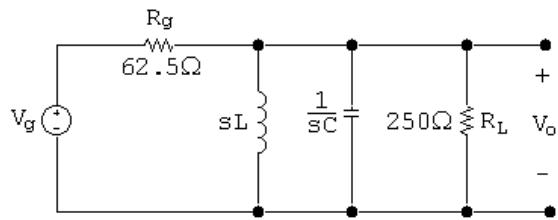
$$C_{-3} = \frac{-6}{\pi^2} + j \frac{9}{\pi} = 2.93/\underline{101.98^\circ} \text{ V}$$

$$C_3 = 2.93/\underline{-101.98^\circ} \text{ V}$$

$$C_{-4} = -j \frac{6.75}{\pi} = 2.15/\underline{-90^\circ} \text{ V}$$

$$C_4 = 2.15/\underline{90^\circ} \text{ V}$$

[c]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o s C + \frac{V_o - V_g}{62.5} = 0$$

$$\therefore (250LCs^2 + 5sL + 250)V_o = 4sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(4/250C)s}{s^2 + 1/50C + 1/LC}$$

$$H(s) = \frac{16,000s}{s^2 + 2 \times 10^4 s + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{j8k}{100(1 - k^2) + j10k}$$

Therefore,

$$H_{-1} = 0.8/\underline{0^\circ}; \quad H_1 = 0.8/\underline{0^\circ}$$

$$H_{-2} = \frac{-j16}{-300 - j20} = 0.0532/\underline{86.19^\circ}; \quad H_2 = 0.0532/\underline{-86.19^\circ}$$

$$H_{-3} = \frac{-j24}{-800 - j30} = 0.0300/\underline{87.85^\circ}; \quad H_2 = 0.0300/\underline{-87.85^\circ}$$

$$H_{-4} = \frac{-j32}{-1500 - j40} = 0.0213/\underline{88.47^\circ}; \quad H_2 = 0.0213/\underline{-88.47^\circ}$$

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19/\underline{122.48^\circ})(0.8/\underline{0^\circ}) = 8.15/\underline{122.48^\circ} \text{ V}$$

$$C_1 = 8.15/\underline{-122.48^\circ} \text{ V}$$

$$C_{-2} = (4.30/\underline{-90^\circ})(0.05/\underline{86.19^\circ}) = 0.2287/\underline{-3.81^\circ} \text{ V}$$

$$C_2 = 0.2287/\underline{3.81^\circ} \text{ V}$$

$$C_{-3} = (2.93/\underline{101.98^\circ})(0.03/\underline{87.85^\circ}) = 0.0878/\underline{-170.17^\circ} \text{ V}$$

$$C_3 = 0.0878/\underline{170.17^\circ} \text{ V}$$

$$C_{-4} = (2.15/\underline{-90^\circ})(0.02/\underline{88.47^\circ}) = 0.0458/\underline{-1.53^\circ} \text{ V}$$

$$C_4 = 0.0458/\underline{1.53^\circ} \text{ V}$$

$$\begin{aligned} [\mathbf{d}] \quad V_{\text{rms}} &\cong \sqrt{C_o^2 + 2 \sum_{n=1}^4 |C_n|^2} \cong \sqrt{2 \sum_{n=1}^4 |C_n|^2} \\ &\cong \sqrt{2(8.15^2 + 0.2287^2 + 0.0878^2 + 0.0458^2)} \cong 11.53 \text{ V} \\ P &= \frac{(11.53)^2}{250} = 531.95 \text{ mW} \end{aligned}$$

$$\begin{aligned} \text{P 16.48 [a]} \quad V_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^{T/2} \left(\frac{2V_m}{T} t \right)^2 dt} \\ &= \sqrt{\frac{1}{T} \left[\frac{4V_m^2}{T^2} \frac{t^3}{3} \right]_0^{T/2}} \\ &= \sqrt{\frac{4V_m^2}{(3)(8)}} = \frac{V_m}{\sqrt{6}} \end{aligned}$$

$$V_{\text{rms}} = \frac{54}{\sqrt{6}} = 22.05 \text{ V}$$

[b] From the solution to Problem 16.47

$$C_0 = 13.5; \quad |C_3| = 2.93$$

$$|C_1| = 10.19; \quad |C_4| = 2.15$$

$$|C_2| = 4.30$$

$$V_g(\text{rms}) \cong \sqrt{13.5^2 + 2(10.19^2 + 4.30^2 + 2.93^2 + 2.15^2)} \cong 21.29 \text{ V}$$

$$[\mathbf{c}] \quad \% \text{ Error} = \left(\frac{21.29}{22.05} - 1 \right) (100) = -3.44\%$$

P 16.49 [a] From Example 16.3 we have:

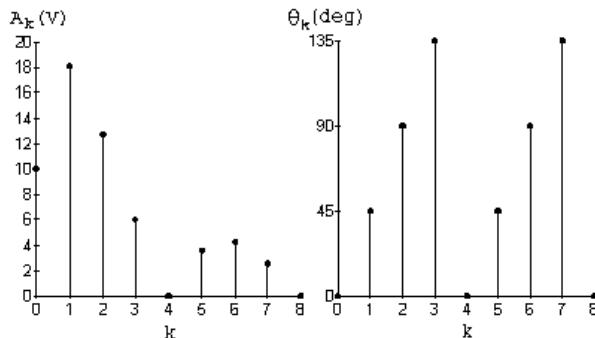
$$a_v = \frac{40}{4} = 10 \text{ V}, \quad a_k = \frac{40}{\pi k} \sin \left(\frac{k\pi}{2} \right)$$

$$b_k = \frac{40}{\pi k} \left[1 - \cos \left(\frac{k\pi}{2} \right) \right], \quad A_k/\underline{-\theta_k^\circ} = a_k - jb_k$$

$$A_1 = 18.01 \text{ V} \quad \theta_1 = 45^\circ, \quad A_2 = 12.73 \text{ V}, \quad \theta_2 = 90^\circ$$

$$A_3 = 6 \text{ V}, \quad \theta_3 = 135^\circ, \quad A_4 = 0, \quad A_5 = 3.6 \text{ V}, \quad \theta_5 = 45^\circ$$

$$A_6 = 4.24 \text{ V}, \quad \theta_6 = 90^\circ, \quad A_7 = 2.57 \text{ V}, \quad \theta_7 = 135^\circ; \quad A_8 = 0$$



[b] $C_n = \frac{a_n - jb_n}{2}, \quad C_{-n} = \frac{a_n + jb_n}{2} = C_n^*$

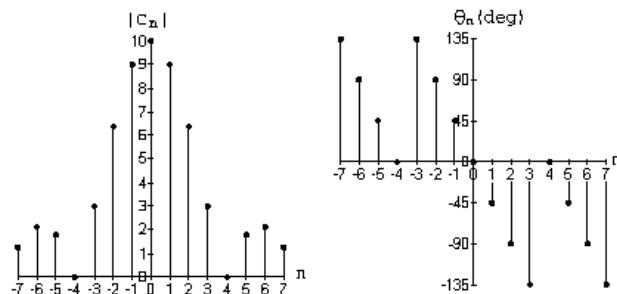
$$C_0 = a_v = 10 \text{ V} \quad C_3 = 3/\underline{135^\circ} \text{ V} \quad C_6 = 2.12/\underline{90^\circ} \text{ V}$$

$$C_1 = 9/\underline{45^\circ} \text{ V} \quad C_{-3} = 3/\underline{-135^\circ} \text{ V} \quad C_{-6} = 2.12/\underline{-90^\circ} \text{ V}$$

$$C_{-1} = 9/\underline{-45^\circ} \text{ V} \quad C_4 = C_{-4} = 0 \quad C_7 = 1.29/\underline{135^\circ} \text{ V}$$

$$C_2 = 6.37/\underline{90^\circ} \text{ V} \quad C_5 = 1.8/\underline{45^\circ} \text{ V} \quad C_{-7} = 1.29/\underline{-135^\circ} \text{ V}$$

$$C_{-2} = 6.37/\underline{-90^\circ} \text{ V} \quad C_{-5} = 1.8/\underline{-45^\circ} \text{ V}$$



P 16.50 [a] From the solution to Problem 16.33 we have

$$A_k = a_k - jb_k = \frac{I_m}{\pi^2 k^2} (\cos k\pi - 1) + j \frac{I_m}{\pi k}$$

$$A_0 = 0.75I_m = 3.75 \text{ A}$$

$$A_1 = \frac{5}{\pi^2} (-2) + j \frac{5}{\pi} = 1.89/\underline{122.48^\circ} \text{ A}$$

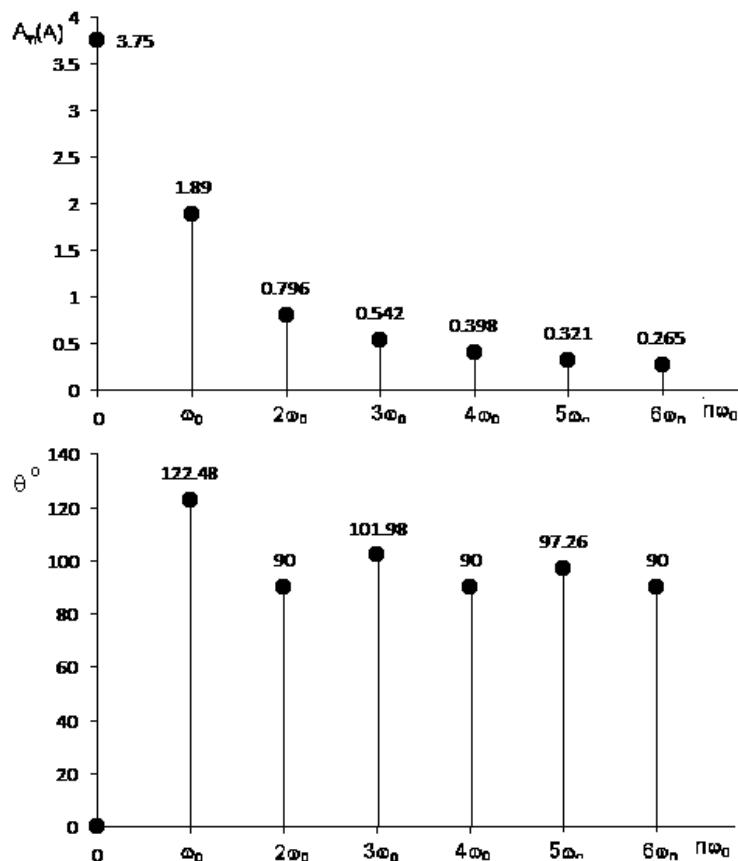
$$A_2 = j \frac{5}{2\pi} = 0.796/\underline{90^\circ} \text{ A}$$

$$A_3 = \frac{5}{9\pi^2}(-2) + j\frac{5}{3\pi} = 0.542/\underline{101.98^\circ} \text{ A}$$

$$A_4 = j\frac{5}{4\pi} = 0.398/\underline{90^\circ} \text{ A}$$

$$A_5 = \frac{5}{25\pi^2}(-2) + j\frac{5}{5\pi} = 0.321/\underline{97.26^\circ} \text{ A}$$

$$A_6 = j\frac{5}{6\pi} = 0.265/\underline{90^\circ} \text{ A}$$



[b] $C_0 = A_0 = 3.75 \text{ mA}$

$$C_1 = \frac{1}{2}A_1/\underline{\theta_1} = 0.945/\underline{122.48^\circ} \text{ A}$$

$$C_{-1} = 0.945/\underline{-122.48^\circ} \text{ A}$$

$$C_2 = \frac{1}{2}A_2/\underline{\theta_2} = 0.398/\underline{90^\circ} \text{ A}$$

$$C_{-2} = 0.398/\underline{-90^\circ} \text{ A}$$

$$C_3 = \frac{1}{2}A_3/\underline{\theta_3} = 0.271/\underline{101.98^\circ} \text{ A}$$

$$C_{-3} = 0.271 / -101.98^\circ \text{ A}$$

$$C_4 = \frac{1}{2} A_4 / \theta_4 = 0.199 / 90^\circ \text{ A}$$

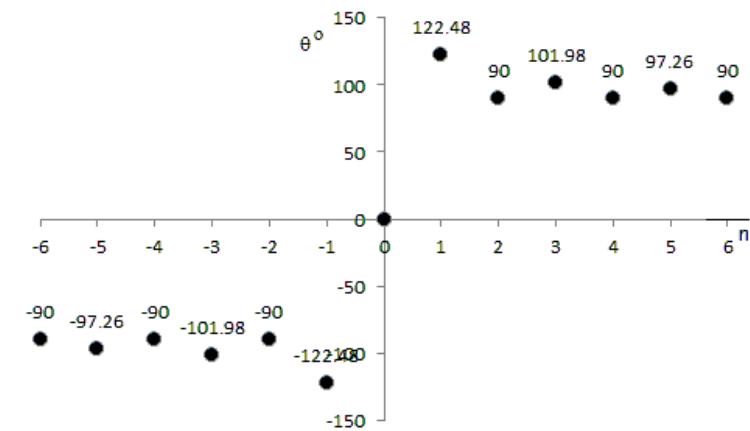
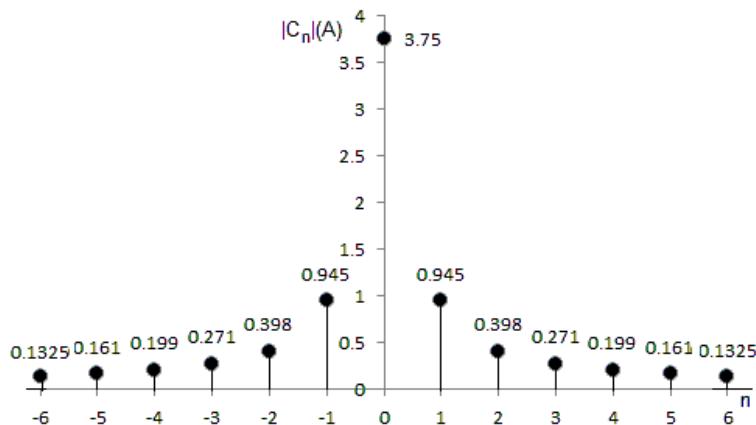
$$C_{-4} = 0.199 / -90^\circ \text{ A}$$

$$C_5 = \frac{1}{2} A_5 / \theta_5 = 0.161 / 97.26^\circ \text{ A}$$

$$C_{-5} = 0.161 / -97.26^\circ \text{ A}$$

$$C_6 = \frac{1}{2} A_6 / \theta_6 = 0.1325 / 90^\circ \text{ A}$$

$$C_{-6} = 0.1325 / -90^\circ \text{ A}$$



P 16.51 [a] $v = A_1 \cos(\omega_o t - 90^\circ) + A_3 \cos(3\omega_o t + 90^\circ)$

$$+ A_5 \cos(5\omega_o t - 90^\circ) + A_7 \cos(7\omega_o t + 90^\circ)$$

$$v = -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$$

[b] $v(-t) = -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$

$\therefore v(-t) = -v(t); \quad \text{odd function}$

$$\begin{aligned}
 [\mathbf{c}] \quad v(t - T/2) &= A_1 \sin(\omega_o t - \pi) - A_3 \sin(3\omega_o t - 3\pi) \\
 &\quad + A_5 \sin(5\omega_o t - 5\pi) - A_7 \sin(7\omega_o t - 7\pi) \\
 &= -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t
 \end{aligned}$$

$\therefore v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$$f(T/2 - t) = f(t)$$

$$\begin{aligned}
 f(T/2 - t) &= A_1 \sin(\pi - \omega_o t) - A_3 \sin(3\pi - 3\omega_o t) \\
 &\quad + A_5 \sin(5\pi - 5\omega_o t) - A_7 \sin(7\pi - 7\omega_o t) \\
 &= A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t
 \end{aligned}$$

$\therefore f(T/2 - t) = f(t)$ and the voltage has quarter-wave symmetry

$$\begin{aligned}
 \text{P 16.52 [a]} \quad i &= 8.82 \cos(250t + 90^\circ) + 0.98 \cos(500t - 90^\circ) + 0.353 \cos(750t + 90^\circ) \\
 &\quad + 0.18 \cos(1000t - 90^\circ) \text{ A} \\
 &= -8.82 \sin 250t + 0.98 \sin 500t - 0.353 \sin 750t + 0.18 \sin 1000t \text{ A}
 \end{aligned}$$

[b] $i(t) = -i(-t)$, Function is odd

[c] Yes, $A_0 = 0$, $A_n = 0$ for n even

$$[\mathbf{d}] \quad I_{\text{rms}} = \sqrt{\frac{8.82^2 + 0.98^2 + 0.353^2 + 0.18^2}{2}} = 6.28 \text{ A}$$

$$[\mathbf{e}] \quad C_{-1} = 4.41/\underline{-90^\circ} \text{ A}; \quad C_1 = 4.41/\underline{90^\circ} \text{ A}$$

$$C_{-3} = 0.49/\underline{90^\circ} \text{ A}; \quad C_3 = 0.49/\underline{-90^\circ} \text{ A}$$

$$C_{-5} = 0.177/\underline{-90^\circ} \text{ A}; \quad C_5 = 0.177/\underline{90^\circ} \text{ A}$$

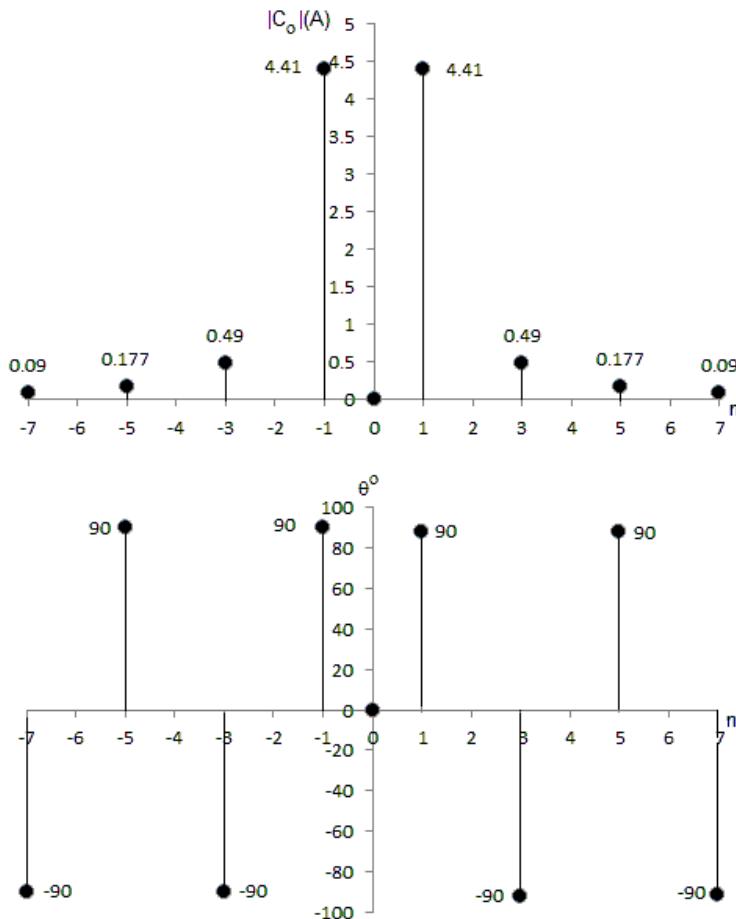
$$C_{-7} = 0.09/\underline{90^\circ} \text{ A}; \quad C_7 = 0.09/\underline{-90^\circ} \text{ A}$$

$$i = j0.09e^{-j1000t} - j0.177e^{-j750t} + j0.49e^{-j500t}$$

$$-j4.41e^{-j250t} + j4.41e^{j250t} - j0.49e^{j500t}$$

$$+j0.177e^{j750t} - j0.09e^{j1000t} \text{ A}$$

[f]



P 16.53 From Table 15.1 we have

$$H(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{s^3}{(s+2500)(s^2+2500s+625 \times 10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{400\pi} \times 10^6 = 5000 \text{ rad/s}$$

$$\therefore H'(jn\omega_o) = \frac{-j8n^3}{(1+j2n)[(1-4n^2)+j2n]}$$

It follows that

$$H(j0) = 0$$

$$H(j\omega_o) = \frac{-j8}{(1+j2)(-3+j2)} = 0.992/\underline{60.255^\circ}$$

$$H(j2\omega_o) = \frac{-j64}{(1+j4)(-15+j4)} = 0.9999/\underline{28.97^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2} \sin \omega_o t - \frac{2A}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o t}{n^2 - 1}$$

$$= 270 + 135\pi \sin \omega_o t - 180 \cos 2\omega_o t - \dots \text{ V}$$

$$\therefore v_o = 0 + 420.84 \sin(5000t + 60.255^\circ) - 179.98 \cos(10,000t + 28.97^\circ) - \dots \text{ V}$$

P 16.54 Using the technique outlined in Problem 16.18 we can derive the Fourier series for $v_g(t)$. We get

$$v_g(t) = 100 + \frac{800}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \cos n\omega_o t$$

The transfer function of the prototype second-order low pass Butterworth filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}, \quad \text{where } \omega_c = 1 \text{ rad/s}$$

Now frequency scale using $k_f = 2000$ to get $\omega_c = 2 \text{ krad/s}$:

$$H(s) = \frac{4 \times 10^6}{s^2 + 2000\sqrt{2}s + 4 \times 10^6}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{4 \times 10^6}{(j5000)^2 + 2000\sqrt{2}(j5000)^2 + 4 \times 10^6} = 0.1580/\underline{-146.04^\circ}$$

$$H(j15,000) = \frac{4 \times 10^6}{(j15,000)^2 + 2000\sqrt{2}(j15,000)^2 + 4 \times 10^6} = 0.0178/\underline{-169.13^\circ}$$

$$\mathbf{V}_{dc} = 100 \text{ V}$$

$$\mathbf{V}_{g1} = \frac{800}{n^2} \underline{0^\circ} \text{ V}$$

$$\mathbf{V}_{g3} = \frac{800}{9\pi^2} \underline{0^\circ} \text{ V}$$

$$V_{odc} = 100(1) = 100 \text{ V}$$

$$\mathbf{V}_{o1} = \frac{800}{\pi^2}(0.1580/-146.04^\circ) = 12.81/-146.04^\circ \text{ V}$$

$$\mathbf{V}_{o3} = \frac{800}{9\pi^2}(0.0178/-169.13^\circ) = 0.16/-169.13^\circ \text{ V}$$

$$v_o(t) = 100 + 12.81 \cos(5000t - 146.04^\circ) \\ + 0.16 \cos(15,000t - 169.13^\circ) + \dots \text{ V}$$

$$\text{P 16.55 } v_g = \frac{2(2.5\pi)}{\pi} - \frac{4(2.5\pi)}{\pi} \frac{\cos 5000t}{4-1} = 5 - (10/3) \cos 5000t - \dots \text{ V}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25 \times 10^6) + j5\sqrt{2} \times 10^6} = 0.04/-163.58^\circ$$

$$\therefore v_o(t) = 5 - 0.1332 \cos(5000t - 163.58^\circ) - \dots \text{ V}$$

P 16.56 [a] Let V_a represent the node voltage across R_2 , then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a s C_2 + (V_a - V_o) s C_1 = 0$$

$$(0 - V_a) s C_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1 C_1} s}{s^2 + \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) s + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}$$

$$\beta = \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$K_o = \frac{R_3}{R_1} \left(\frac{C_2}{C_1 + C_2} \right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1} \left(\frac{C_2}{C_1 + C_2} \right) \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) s}{s^2 + \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) s + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2} \right)}$$

[b] For the given values of R_1, R_2, R_3, C_1 , and C_2 we have

$$\begin{aligned} H(s) &= \frac{-400s}{s^2 + 400s + 10^8} \\ v_g &= \frac{(8)(2.25\pi^2)}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \cos n\omega_o t \\ &= 18 \left[\cos \omega_o t + \frac{1}{9} \cos 3\omega_o t + \frac{1}{25} \cos 5\omega_o t + \dots \right] \text{ mV} \\ &= [18 \cos \omega_o t + 2 \cos 3\omega_o t + 0.72 \cos 5\omega_o t + \dots] \text{ mV} \end{aligned}$$

$$\omega_o = \frac{2\pi}{0.2\pi} \times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^210^8 + j400k10^4} = \frac{-jk}{25(1 - k^2) + jk}$$

$$H_1 = -1 = 1/\underline{180^\circ}$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015/\underline{90.86^\circ}$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083/\underline{90.48^\circ}$$

$$v_o = -18 \cos \omega_o t + 0.03 \cos(3\omega_o t + 90.86^\circ)$$

$$+ 0.006 \cos(5\omega_o t + 90.48^\circ) + \dots \text{ mV}$$

- [c] The fundamental frequency component dominates the output, so we expect the quality factor Q to be quite high.
- [d] $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10,000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

P 16.57 [a] Using the equations derived in Problem 16.56(a),

$$K_o = \frac{R_3}{R_1} \left(\frac{C_2}{C_1 + C_2} \right) = \frac{400}{313}$$

$$\beta = \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = 2000 \text{ rad/s}$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2} = 16 \times 10^8$$

$$\begin{aligned}
 [\mathbf{b}] \quad H(jn\omega_o) &= \frac{-(400/313)(2000)jn\omega_o}{16 \times 10^8 - n^2\omega_o^2 + j2000n\omega_o} \\
 &= \frac{-j(20/313)n}{(1 - n^2) + j0.05n} \\
 H(j\omega_o) &= \frac{-j(20/313)}{j(0.050)} = -\frac{400}{313} = -1.28 \\
 H(j3\omega_o) &= \frac{-j(20/313)(3)}{-8 + j0.15} = 0.0240/91.07^\circ \\
 H(j5\omega_o) &= \frac{-j(100/313)}{-24 + j0.25} = 0.0133/\underline{90.60^\circ}
 \end{aligned}$$

$$v_g(t) = \frac{4A}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin(n\pi/2) \cos n\omega_o t$$

$$A = 15.65\pi \text{ V}$$

$$v_g(t) = 62.60 \cos \omega_o t - 20.87 \cos 3\omega_o t + 12.52 \cos 5\omega_o t - \dots$$

$$\begin{aligned}
 v_o(t) &= -80 \cos \omega_o t - 0.50 \cos(3\omega_o t + 91.07^\circ) \\
 &\quad + 0.17 \cos(5\omega_o t + 90.60^\circ) - \dots \text{ V}
 \end{aligned}$$

17

The Fourier Transform

Assessment Problems

$$\text{AP 17.1 [a]} \quad F(\omega) = \int_{-\tau/2}^0 (-Ae^{-j\omega t}) dt + \int_0^{\tau/2} Ae^{-j\omega t} dt$$

$$\begin{aligned} &= \frac{A}{j\omega} [2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}] \\ &= \frac{2A}{j\omega} \left[1 - \frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} \right] \\ &= \frac{-j2A}{\omega} [1 - \cos(\omega\tau/2)] \end{aligned}$$

$$[\mathbf{b}] \quad F(\omega) = \int_0^\infty te^{-at} e^{-j\omega t} dt = \int_0^\infty te^{-(a+j\omega)t} dt = \frac{1}{(a+j\omega)^2}$$

AP 17.2

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \left\{ \int_{-3}^{-2} 4e^{jt\omega} d\omega + \int_{-2}^2 e^{jt\omega} d\omega + \int_2^3 4e^{jt\omega} d\omega \right\} \\ &= \frac{1}{j2\pi t} \{ 4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t} \} \\ &= \frac{1}{\pi t} \left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2} \right] \\ &= \frac{1}{\pi t} (4 \sin 3t - 3 \sin 2t) \end{aligned}$$

$$\text{AP 17.3 [a]} \quad F(\omega) = F(s) |_{s=j\omega} = \mathcal{L}\{e^{-at} \sin \omega_0 t\}_{s=j\omega}$$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2} \Big|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

$$[\mathbf{b}] \quad F(\omega) = \mathcal{L}\{f^-(t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2} \right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$$

$$[\mathbf{c}] \quad f^+(t) = te^{-at}, \quad f^-(t) = -te^{-at}$$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(t)\} = \frac{-1}{(s+a)^2}$$

$$\text{Therefore } F(\omega) = \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2} = \frac{-j4a\omega}{(a^2+\omega^2)^2}$$

$$\text{AP 17.4 [a]} \quad f'(t) = \frac{2A}{\tau}, \quad -\frac{\tau}{2} < t < 0; \quad f'(t) = \frac{-2A}{\tau}, \quad 0 < t < \frac{\tau}{2}$$

$$\begin{aligned} \therefore f'(t) &= \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)] \\ &= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2) \end{aligned}$$

$$\therefore f''(t) = \frac{2A}{\tau}\delta\left(t + \frac{\tau}{2}\right) - \frac{4A}{\tau} + \frac{2A}{\tau}\delta\left(t - \frac{\tau}{2}\right)$$

$$\begin{aligned} [\mathbf{b}] \quad \mathcal{F}\{f''(t)\} &= \left[\frac{2A}{\tau}e^{j\omega\tau/2} - \frac{4A}{\tau} + \frac{2A}{\tau}e^{-j\omega\tau/2} \right] \\ &= \frac{4A}{\tau} \left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1 \right] = \frac{4A}{\tau} \left[\cos\left(\frac{\omega\tau}{2}\right) - 1 \right] \end{aligned}$$

$$[\mathbf{c}] \quad \mathcal{F}\{f''(t)\} = (j\omega)^2 F(\omega) = -\omega^2 F(\omega); \quad \text{therefore } F(\omega) = -\frac{1}{\omega^2} \mathcal{F}\{f''(t)\}$$

$$\text{Thus we have } F(\omega) = -\frac{1}{\omega^2} \left\{ \frac{4A}{\tau} \left[\cos\left(\frac{\omega\tau}{2}\right) - 1 \right] \right\}$$

AP 17.5

$$v(t) = V_m \left[u\left(t + \frac{\tau}{2}\right) - u\left(t - \frac{\tau}{2}\right) \right]$$

$$\mathcal{F}\left\{u\left(t + \frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t - \frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] e^{-j\omega\tau/2}$$

$$\begin{aligned} \text{Therefore } V(\omega) &= V_m \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] \left[e^{j\omega\tau/2} - e^{-j\omega\tau/2} \right] \\ &= j2V_m\pi\delta(\omega) \sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega} \sin\left(\frac{\omega\tau}{2}\right) \\ &= \frac{(V_m\tau) \sin(\omega\tau/2)}{\omega\tau/2} \end{aligned}$$

AP 17.6 [a] $I_g(\omega) = \mathcal{F}\{10\text{sgn }t\} = \frac{20}{j\omega}$

[b] $H(s) = \frac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \quad H(j\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(j\omega) \cdot I_g(\omega) = \left(\frac{j4\omega}{5+j\omega}\right) \left(\frac{20}{j\omega}\right) = \frac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)$ V

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2 \text{ A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18 \text{ A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8 \text{ A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8 \text{ A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0 \text{ V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80 \text{ V}$

AP 17.7 [a] $V_g(\omega) = \frac{1}{1-j\omega} + \pi\delta(\omega) + \frac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)\}}{1+0.5\|(1/s)\|} = \frac{1}{s+3}, \quad H(j\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned} V_a(\omega) &= H(j\omega)V_g(j\omega) \\ &= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \end{aligned}$$

Therefore $v_a(t) = \left[\frac{1}{4}e^t u(-t) + \frac{1}{6}\text{sgn }t - \frac{1}{12}e^{-3t}u(t) + \frac{1}{6}\right] \text{ V}$

$$[\mathbf{b}] \quad v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4} \text{ V}$$

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4} \text{ V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3} \text{ V}$$

AP 17.8

$$v(t) = 4te^{-t}u(t); \quad V(\omega) = \frac{4}{(1+j\omega)^2}$$

$$\text{Therefore } |V(\omega)| = \frac{4}{1+\omega^2}$$

$$\begin{aligned} W_{1\Omega} &= \frac{1}{\pi} \int_0^{\sqrt{3}} \left[\frac{4}{(1+\omega^2)} \right]^2 d\omega \\ &= \frac{16}{\pi} \left\{ \frac{1}{2} \left[\frac{\omega}{\omega^2+1} + \tan^{-1} \frac{\omega}{1} \right]_0^{\sqrt{3}} \right\} \\ &= 16 \left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6} \right] = 3.769 \text{ J} \end{aligned}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi} \left[\frac{\omega}{\omega^2+1} + \tan^{-1} \frac{\omega}{1} \right]_0^\infty = \frac{8}{\pi} \left[0 + \frac{\pi}{2} \right] = 4 \text{ J}$$

$$\text{Therefore } \% = \frac{3.769}{4}(100) = 94.23\%$$

AP 17.9

$$|V(\omega)| = 6 - \left(\frac{6}{2000\pi} \right) \omega, \quad 0 \leq \omega \leq 2000\pi$$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi} \right) \omega + \left(\frac{36}{4\pi^2 \times 10^6} \right) \omega^2$$

$$W_{1\Omega} = \frac{1}{\pi} \int_0^{2000\pi} \left[36 - \frac{72\omega}{2000\pi} + \frac{36 \times 10^{-6}}{4\pi^2} \omega^2 \right] d\omega$$

$$= \frac{1}{\pi} \left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36 \times 10^{-6}\omega^3}{12\pi^2} \right]_0^{2000\pi}$$

$$= \frac{1}{\pi} \left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36 \times 10^{-6}(2000\pi)^3}{12\pi^2} \right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24 \text{ kJ}$$

$$W_{6\text{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4 \text{ J}$$

Problems

P 17.1 [a] $F(\omega) = \int_{-\tau/2}^{\tau/2} \frac{2A}{\tau} t e^{-j\omega t} dt$

$$= \frac{2A}{\tau} \left[\frac{e^{-j\omega t}}{-\omega^2} (-j\omega t - 1) \right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{\omega^2 \tau} \left[e^{-j\omega \tau/2} \left(\frac{j\omega \tau}{2} + 1 \right) - e^{j\omega \tau/2} \left(\frac{-j\omega \tau}{2} + 1 \right) \right]$$

$$F(\omega) = \frac{2A}{\omega^2 \tau} \left[e^{-j\omega \tau/2} - e^{j\omega \tau/2} + j \frac{\omega \tau}{2} (e^{-j\omega \tau/2} + e^{j\omega \tau/2}) \right]$$

$$F(\omega) = j \frac{2A}{\tau} \left[\frac{\omega \tau \cos(\omega \tau/2) - 2 \sin(\omega \tau/2)}{\omega^2} \right]$$

[b] Using L'Hopital's rule,

$$F(0) = \lim_{\omega \rightarrow 0} 2A \left[\frac{\omega \tau (\tau/2) [-\sin(\omega \tau/2)] + \tau \cos(\omega \tau/2) - 2(\tau/2) \cos(\omega \tau/2)}{2\omega \tau} \right]$$

$$= \lim_{\omega \rightarrow 0} 2A \left[\frac{-\omega \tau (\tau/2) \sin(\omega \tau/2)}{2\omega \tau} \right]$$

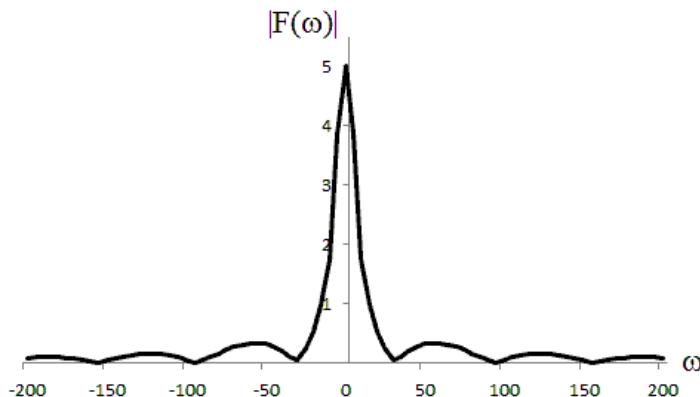
$$= \lim_{\omega \rightarrow 0} 2A \left[\frac{-\tau \sin(\omega \tau/2)}{4} \right] = 0$$

$$\therefore F(0) = 0$$

[c] When $A = 10$ and $\tau = 0.1$

$$F(\omega) = j200 \left[\frac{0.1\omega \cos(\omega/20) - 2 \sin(\omega/20)}{\omega^2} \right]$$

$$|F(\omega)| = \left| \frac{20\omega \cos(\omega/20) - 400 \sin(\omega/20)}{\omega^2} \right|$$



$$\text{P 17.2 [a]} \quad F(\omega) = A + \frac{2A}{\omega_o} \omega, \quad -\omega_o/2 \leq \omega \leq 0$$

$$F(\omega) = A - \frac{2A}{\omega_o} \omega, \quad 0 \leq \omega \leq \omega_o/2$$

$$F(\omega) = 0 \quad \text{elsewhere}$$

$$f(t) = \frac{1}{2\pi} \int_{-\omega_o/2}^0 \left(A + \frac{2A}{\omega_o} \omega \right) e^{j\omega t} d\omega$$

$$+ \frac{1}{2\pi} \int_0^{\omega_o/2} \left(A - \frac{2A}{\omega_o} \omega \right) e^{j\omega t} d\omega$$

$$f(t) = \frac{1}{2\pi} \left[\int_{-\omega_o/2}^0 A e^{j\omega t} d\omega + \int_{-\omega_o/2}^0 \frac{2A}{\omega_o} \omega e^{j\omega t} d\omega \right]$$

$$+ \int_0^{\omega_o/2} A e^{j\omega t} d\omega - \int_0^{\omega_o/2} \frac{2A}{\omega_o} \omega e^{j\omega t} d\omega \right] \\ = \frac{1}{2\pi} [\text{Int1} + \text{Int2} + \text{Int3} - \text{Int4}]$$

$$\text{Int1} = \int_{-\omega_o/2}^0 A e^{j\omega t} d\omega = \frac{A}{jt} (1 - e^{-j\omega_o t/2})$$

$$\text{Int2} = \int_{-\omega_o/2}^0 \frac{2A}{\omega_o} \omega e^{j\omega t} d\omega = \frac{2A}{\omega_o t^2} \left(1 - j \frac{\omega_o}{2} e^{-j\omega_o t/2} - e^{-j\omega_o t/2} \right)$$

$$\text{Int3} = \int_0^{\omega_o/2} A e^{j\omega t} d\omega = \frac{A}{jt} (e^{j\omega_o t/2} - 1)$$

$$\text{Int4} = \int_0^{\omega_o/2} \frac{2A}{\omega_o} \omega e^{j\omega t} d\omega = \frac{2A}{\omega_o t^2} \left(-j \frac{\omega_o}{2} e^{j\omega_o t/2} + e^{j\omega_o t/2} - 1 \right)$$

$$\text{Int1} + \text{Int3} = \frac{2A}{t} \sin(\omega_o t/2)$$

$$\text{Int2} - \text{Int4} = \frac{4A}{\omega_o t^2} [1 - \cos(\omega_o t/2)] - \frac{2A}{t} \sin(\omega_o t/2)$$

$$\therefore f(t) = \frac{1}{2\pi} \left[\frac{4A}{\omega_o t^2} (1 - \cos(\omega_o t/2)) \right]$$

$$= \frac{2A}{\pi \omega_o t^2} [2 \sin^2(\omega_o t/4)]$$

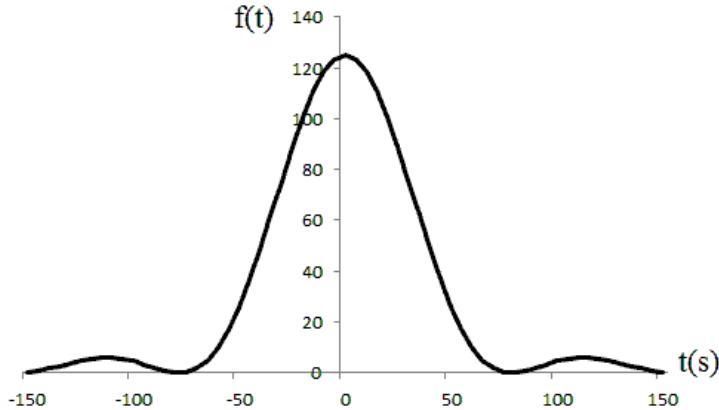
$$= \frac{4\omega_o A}{\pi \omega_o^2 t^2} \sin^2(\omega_o t/4)$$

$$= \frac{\omega_o A}{4\pi} \left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)} \right]^2$$

[b] $f(0) = \frac{\omega_o A}{4\pi} (1)^2 = 79.58 \times 10^{-3} \omega_o A$

[c] $A = 5\pi; \quad \omega_o = 100 \text{ rad/s}$

$$f(t) = 125 \left[\frac{\sin(t/25)}{(t/25)} \right]^2$$



P 17.3 [a] $F(\omega) = \int_{-2}^2 \left[A \sin\left(\frac{\pi}{2}\right) t \right] e^{-j\omega t} dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2} \sin 2\omega$

[b] $F(\omega) = \int_{-\tau/2}^0 \left(\frac{2A}{\tau} t + A \right) e^{-j\omega t} dt + \int_0^{\tau/2} \left(\frac{-2A}{\tau} t + A \right) e^{-j\omega t} dt$

$$= \frac{4A}{\omega^2 \tau} \left[1 - \cos\left(\frac{\omega\tau}{2}\right) \right]$$

P 17.4 $\mathcal{F}\{\sin \omega_0 t\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\frac{e^{-j\omega_0 t}}{2j}\right\}$

$$= \frac{1}{2j} [2\pi\delta(\omega - \omega_0) - 2\pi\delta(\omega + \omega_0)]$$

$$= j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

P 17.5 [a] $F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$

$$F(\omega) = F(s) \Big|_{s=j\omega} + F(s) \Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2} \right] + \left[\frac{1}{(a-j\omega)^2} \right]$$

$$= \frac{2(a^2 - \omega^2)}{(a^2 - \omega^2)^2 + 4a^2\omega^2} = \frac{2(a^2 - \omega^2)}{(a^2 + \omega^2)^2}$$

$$[\mathbf{b}] \quad F(s) = \mathcal{L}\{t^3 e^{-at}\} = \frac{6}{(s+a)^4}$$

$$F(\omega) = F(s) \Big|_{s=j\omega} - F(s) \Big|_{s=-j\omega}$$

$$F(\omega) = \frac{6}{(a+j\omega)^4} + \frac{6}{(a-j\omega)^4} = -j48a\omega \frac{a^2 - \omega^2}{(a^2 + \omega^2)^4}$$

$$[\mathbf{c}] \quad F(s) = \mathcal{L}\{e^{-at} \cos \omega_0 t\} = \frac{s+a}{(s+a)^2 + \omega_0^2} = \frac{0.5}{(s+a) - j\omega_0} + \frac{0.5}{(s+a) + j\omega_0}$$

$$F(\omega) = F(s) \Big|_{s=j\omega} + F(s) \Big|_{s=-j\omega}$$

$$\begin{aligned} F(\omega) &= \frac{0.5}{(a+j\omega) - j\omega_0} + \frac{0.5}{(a+j\omega) + j\omega_0} \\ &\quad + \frac{0.5}{(a-j\omega) - j\omega_0} + \frac{0.5}{(a-j\omega) + j\omega_0} \\ &= \frac{a}{a^2 + (\omega - \omega_0)^2} + \frac{a}{a^2 + (\omega + \omega_0)^2} \end{aligned}$$

$$[\mathbf{d}] \quad F(s) = \mathcal{L}\{e^{-at} \sin \omega_0 t\} = \frac{\omega_0}{(s+a)^2 + \omega_0^2} = \frac{-j0.5}{(s+a) - j\omega_0} + \frac{j0.5}{(s+a) + j\omega_0}$$

$$F(\omega) = F(s) \Big|_{s=j\omega} - F(s) \Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-ja}{a^2 + (\omega - \omega_0)^2} + \frac{ja}{a^2 + (\omega + \omega_0)^2}$$

$$[\mathbf{e}] \quad F(\omega) = \int_{-\infty}^{\infty} \delta(t - t_o) e^{-j\omega t} dt = e^{-j\omega t_o}$$

(Use the sifting property of the Dirac delta function.)

$$\begin{aligned} \text{P 17.6} \quad f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} [A(\omega) + jB(\omega)][\cos t\omega + j \sin t\omega] d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} [A(\omega) \cos t\omega - B(\omega) \sin t\omega] d\omega \\ &\quad + \frac{j}{2\pi} \int_{-\infty}^{\infty} [A(\omega) \sin t\omega + B(\omega) \cos t\omega] d\omega \end{aligned}$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [A(\omega) \cos t\omega + B(\omega) \sin t\omega] d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega) \cos t\omega d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi} \int_{-\infty}^{\infty} B(\omega) \sin t\omega d\omega$$

P 17.8 $F(\omega) = \frac{-j2}{\omega}$; therefore $B(\omega) = \frac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{-2}{\omega} \right) \sin t\omega d\omega = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin t\omega}{\omega} d\omega$$

But $\frac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin t\omega}{\omega} d\omega$

Therefore,

$$\left. \begin{aligned} f(t) &= \frac{2}{\pi} \cdot \frac{\pi}{2} = 1 & t > 0 \\ f(t) &= \frac{2}{\pi} \cdot \left(\frac{-\pi}{2} \right) = -1 & t < 0 \end{aligned} \right\} \text{from a table of definite integrals}$$

Therefore $f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.5[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \rightarrow 0$, $F(\omega) \rightarrow 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \rightarrow \infty$ at $\omega = \pm\omega_0$ as $\epsilon \rightarrow 0$. The area under each bell-shaped curve is independent of ϵ , that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \rightarrow 0$, $F(\omega) \rightarrow \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^0 f(t) \cos \omega t dt + \int_0^{\infty} f(t) \cos \omega t dt = 0$

since $f(t) \cos \omega t$ is an odd function.

$$B(\omega) = -2 \int_0^{\infty} f(t) \sin \omega t dt, \quad \text{since } f(t) \sin \omega t \text{ is an even function.}$$

$$\begin{aligned}
 \text{P 17.11} \quad A(\omega) &= \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt \\
 &= \int_{-\infty}^0 f(t) \cos \omega t \, dt + \int_0^{\infty} f(t) \cos \omega t \, dt \\
 &= 2 \int_0^{\infty} f(t) \cos \omega t \, dt, \quad \text{since } f(t) \cos \omega t \text{ is also even.}
 \end{aligned}$$

$B(\omega) = 0$, since $f(t) \sin \omega t$ is an odd function and

$$\int_{-\infty}^0 f(t) \sin \omega t \, dt = - \int_0^{\infty} f(t) \sin \omega t \, dt$$

$$\text{P 17.12 [a]} \quad \mathcal{F} \left\{ \frac{df(t)}{dt} \right\} = \int_{-\infty}^{\infty} \frac{df(t)}{dt} e^{-j\omega t} \, dt$$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t} \, dt$; let $dv = [df(t)/dt] \, dt$, then $v = f(t)$.

$$\begin{aligned}
 \text{Therefore} \quad \mathcal{F} \left\{ \frac{df(t)}{dt} \right\} &= f(t) e^{-j\omega t} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t) [-j\omega e^{-j\omega t} \, dt] \\
 &= 0 + j\omega F(\omega)
 \end{aligned}$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

$$\text{[c] To find } \mathcal{F} \left\{ \frac{d^2 f(t)}{dt^2} \right\}, \quad \text{let } g(t) = \frac{df(t)}{dt}$$

$$\text{Then } \mathcal{F} \left\{ \frac{d^2 f(t)}{dt^2} \right\} = \mathcal{F} \left\{ \frac{dg(t)}{dt} \right\} = j\omega G(\omega)$$

$$\text{But } G(\omega) = \mathcal{F} \left\{ \frac{df(t)}{dt} \right\} = j\omega F(\omega)$$

$$\text{Therefore we have } \mathcal{F} \left\{ \frac{d^2 f(t)}{dt^2} \right\} = (j\omega)^2 F(\omega)$$

Repeated application of this thought process gives

$$\mathcal{F} \left\{ \frac{d^n f(t)}{dt^n} \right\} = (j\omega)^n F(\omega)$$

$$\text{P 17.13 [a]} \quad \mathcal{F} \left\{ \int_{-\infty}^t f(x) \, dx \right\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^t f(x) \, dx \right] e^{-j\omega t} \, dt$$

$$\text{Now let } u = \int_{-\infty}^t f(x) \, dx, \quad \text{then } du = f(t) \, dt$$

$$\text{Let } dv = e^{-j\omega t} \, dt, \quad \text{then } v = \frac{e^{-j\omega t}}{-j\omega}$$

Therefore,

$$\begin{aligned}\mathcal{F} \left\{ \int_{-\infty}^t f(x) dx \right\} &= \frac{e^{-j\omega t}}{-j\omega} \int_{-\infty}^t f(x) dx \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left[\frac{e^{-j\omega t}}{-j\omega} \right] f(t) dt \\ &= 0 + \frac{F(\omega)}{j\omega}\end{aligned}$$

[b] We require $\int_{-\infty}^{\infty} f(x) dx = 0$

[c] No, because $\int_{-\infty}^{\infty} e^{-ax} u(x) dx = \frac{1}{a} \neq 0$

P 17.14 [a] $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t} dt$

Let $u = at$, $du = adt$, $u = \pm\infty$ when $t = \pm\infty$

Therefore,

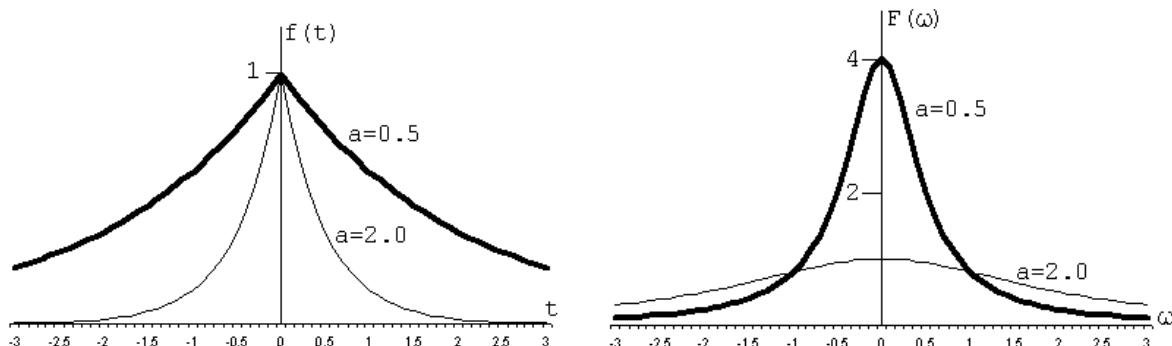
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a} \left(\frac{du}{a} \right) = \frac{1}{a} F\left(\frac{\omega}{a}\right), \quad a > 0$$

[b] $\mathcal{F}\{e^{-|t|}\} = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$

Therefore $\mathcal{F}\{e^{-a|t|}\} = \frac{(1/a)2}{(\omega/a)^2 + 1}$

Therefore $\mathcal{F}\{e^{-0.5|t|}\} = \frac{4}{4\omega^2 + 1}, \quad \mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2 + 1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2 + 1]$, yes as “ a ” increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.



$$\text{P 17.15 [a]} \quad \mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t} dt$$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$.

Therefore,

$$\begin{aligned} \mathcal{F}\{f(t-a)\} &= \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)} du \\ &= e^{-j\omega a} \int_{-\infty}^{\infty} f(u)e^{-j\omega u} du = e^{-j\omega a} F(\omega) \end{aligned}$$

$$\text{[b]} \quad \mathcal{F}\{e^{j\omega_0 t} f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t} dt = F(\omega - \omega_0)$$

$$\begin{aligned} \text{[c]} \quad \mathcal{F}\{f(t) \cos \omega_0 t\} &= \mathcal{F}\left\{f(t) \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]\right\} \\ &= \frac{1}{2}F(\omega - \omega_0) + \frac{1}{2}F(\omega + \omega_0) \end{aligned}$$

$$\text{P 17.16} \quad Y(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda \right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(t-\lambda) e^{-j\omega t} dt \right] d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

$$\begin{aligned} \text{Therefore } Y(\omega) &= \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(u) e^{-j\omega(u+\lambda)} du \right] d\lambda \\ &= \int_{-\infty}^{\infty} x(\lambda) \left[e^{-j\omega\lambda} \int_{-\infty}^{\infty} h(u) e^{-j\omega u} du \right] d\lambda \\ &= \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega\lambda} H(\omega) d\lambda = H(\omega) X(\omega) \end{aligned}$$

$$\begin{aligned} \text{P 17.17} \quad \mathcal{F}\{f_1(t)f_2(t)\} &= \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) e^{jtu} du \right] f_2(t) e^{-j\omega t} dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F_1(u) f_2(t) e^{-j\omega t} e^{jtu} du \right] dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[F_1(u) \int_{-\infty}^{\infty} f_2(t) e^{-j(\omega-u)t} dt \right] du \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u) du \end{aligned}$$

$$\text{P 17.18 [a]} \quad F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty} \frac{d}{d\omega} [f(t) e^{-j\omega t} dt] = -j \int_{-\infty}^{\infty} t f(t) e^{-j\omega t} dt = -j \mathcal{F}\{t f(t)\}$$

Therefore $j \frac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty} (-jt)(-jt)f(t)e^{-j\omega t} dt = (-j)^2 \mathcal{F}\{t^2 f(t)\}$$

Note that $(-j)^n = \frac{1}{j^n}$

Thus we have $j^n \left[\frac{d^n F(\omega)}{d\omega^n} \right] = \mathcal{F}\{t^n f(t)\}$

$$[\mathbf{b}] \quad (\text{i}) \quad \mathcal{F}\{e^{-at} u(t)\} = \frac{1}{a + j\omega} = F(\omega); \quad \frac{dF(\omega)}{d\omega} = \frac{-j}{(a + j\omega)^2}$$

$$\text{Therefore } j \left[\frac{dF(\omega)}{d\omega} \right] = \frac{1}{(a + j\omega)^2}$$

$$\text{Therefore } \mathcal{F}\{te^{-at} u(t)\} = \frac{1}{(a + j\omega)^2}$$

$$(\text{ii}) \quad \mathcal{F}\{|t| e^{-a|t|}\} = \mathcal{F}\{te^{-at} u(t)\} - \mathcal{F}\{te^{at} u(-t)\}$$

$$\begin{aligned} &= \frac{1}{(a + j\omega)^2} - j \frac{d}{d\omega} \left(\frac{1}{a - j\omega} \right) \\ &= \frac{1}{(a + j\omega)^2} + \frac{1}{(a - j\omega)^2} \end{aligned}$$

$$(\text{iii}) \quad \mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at} u(t)\} + \mathcal{F}\{te^{at} u(-t)\}$$

$$\begin{aligned} &= \frac{1}{(a + j\omega)^2} + j \frac{d}{d\omega} \left(\frac{1}{a - j\omega} \right) \\ &= \frac{1}{(a + j\omega)^2} - \frac{1}{(a - j\omega)^2} \end{aligned}$$

$$\text{P 17.19 [a]} \quad f_1(t) = \cos \omega_0 t, \quad F_1(u) = \pi[\delta(u + \omega_0) + \delta(u - \omega_0)]$$

$$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \quad \text{and } f_2(t) = 0 \text{ elsewhere}$$

$$\text{Thus } F_2(u) = \frac{\tau \sin(u\tau/2)}{u\tau/2}$$

Using convolution,

$$\begin{aligned}
 F(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u) du \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \pi[\delta(u + \omega_0) + \delta(u - \omega_0)] \tau \frac{\sin[(\omega - u)\tau/2]}{(\omega - u)(\tau/2)} du \\
 &= \frac{\tau}{2} \int_{-\infty}^{\infty} \delta(u + \omega_0) \frac{\sin[(\omega - u)\tau/2]}{(\omega - u)(\tau/2)} du \\
 &\quad + \frac{\tau}{2} \int_{-\infty}^{\infty} \delta(u - \omega_0) \frac{\sin[(\omega - u)\tau/2]}{(\omega - u)(\tau/2)} du \\
 &= \frac{\tau}{2} \cdot \frac{\sin[(\omega + \omega_0)\tau/2]}{(\omega + \omega_0)(\tau/2)} + \frac{\tau}{2} \cdot \frac{\sin[(\omega - \omega_0)\tau/2]}{(\omega - \omega_0)(\tau/2)}
 \end{aligned}$$

- [b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the width of the frequency band of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$.

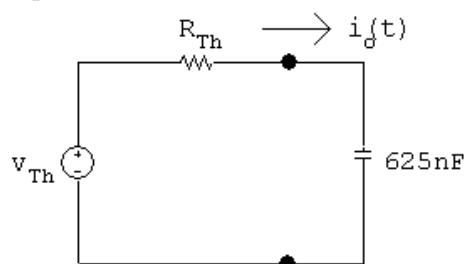
The area under the $[\sin x]/x$ function is independent of τ , that is

$$\frac{\tau}{2} \int_{-\infty}^{\infty} \frac{\sin[(\omega - \omega_0)(\tau/2)]}{(\omega - \omega_0)(\tau/2)} d\omega = \int_{-\infty}^{\infty} \frac{\sin[(\omega - \omega_0)(\tau/2)]}{(\omega - \omega_0)(\tau/2)} [(\tau/2) d\omega] = \pi$$

Therefore as $t \rightarrow \infty$,

$$f_1(t)f_2(t) \rightarrow \cos \omega_0 t \quad \text{and} \quad F(\omega) \rightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

- P 17.20 [a] Find the Thévenin equivalent with respect to the terminals of the capacitor:



$$v_{\text{Th}} = \frac{5}{6}v_g; \quad R_{\text{Th}} = 480 \parallel 2400 = 400 \text{ k}\Omega$$

$$I_o = \frac{V_{\text{Th}}}{400 + 10^9/625s} = \frac{625sV_{\text{Th}}}{250,000s + 10^9}$$

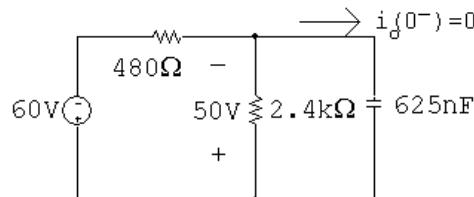
$$H(s) = \frac{I_o}{V_{\text{Th}}} = \frac{25 \times 10^{-4}s}{s + 4000}; \quad H(j\omega) = \frac{25 \times 10^{-4}j\omega}{j\omega + 4000}$$

$$v_{\text{Th}} = \frac{5}{6}v_g = 50 \text{sgn}(t); \quad V_{\text{Th}} = \frac{100}{j\omega}$$

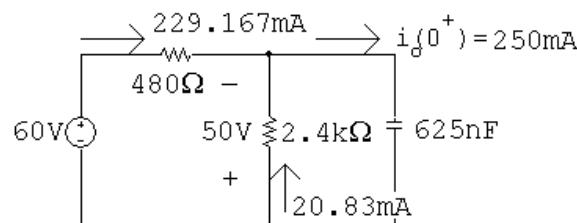
$$I_o = H(j\omega)V_{Th}(j\omega) = \left(\frac{100}{j\omega}\right)\left(\frac{25 \times 10^{-4}j\omega}{j\omega + 4000}\right) = \frac{0.25}{j\omega + 4000}$$

$$i_o(t) = 250e^{-4000t}u(t) \text{ mA}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



$$i_g(0^+) = \frac{60 + 50}{480} = 229.167 \text{ mA}$$

$$i_{2.4k}(0^+) = \frac{50}{2400} = 20.83 \text{ mA}$$

$$i_o(0^+) = 229.167 + 20.83 = 250 \text{ mA}$$

which agrees with our solution.

We also know $i_o(\infty) = 0$, which agrees with our solution.

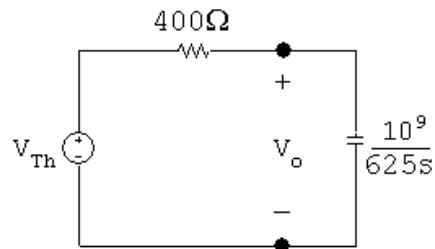
The time constant with respect to the terminals of the capacitor is $R_{Th}C$. Thus,

$$\tau = (400)(625 \times 10^{-9}) = 0.25 \text{ ms}; \quad \therefore \quad \frac{1}{\tau} = 4000,$$

which also agrees with our solution.

Thus our solution makes sense in terms of known circuit behavior.

P 17.21 [a] From the solution of Problem 17.20 we have



$$V_o = \frac{V_{Th}}{400 + (10^9/625s)} \cdot \frac{10^9}{625s}$$

$$H(s) = \frac{V_o}{V_{\text{Th}}} = \frac{4000}{s + 4000}$$

$$H(j\omega) = \frac{4000}{j\omega + 4000}$$

$$V_{\text{Th}}(\omega) = \frac{100}{j\omega}$$

$$\begin{aligned} V_o(\omega) &= H(j\omega)V_{\text{Th}}(\omega) = \left(\frac{100}{j\omega}\right) \frac{4000}{j\omega + 4000} \\ &= \frac{400,000}{(j\omega)(j\omega + 4000)} = \frac{100}{j\omega} - \frac{100}{j\omega + 4000} \end{aligned}$$

$$v_o(t) = 50\text{sgn}(t) - 100e^{-4000t}u(t) \text{ V}$$

[b] $v_o(0^-) = -50 \text{ V}$

$$v_o(0^+) = 50 - 100 = -50 \text{ V}$$

This makes sense because there cannot be an instantaneous change in the voltage across a capacitor.

$$v_o(\infty) = 50 \text{ V}$$

This agrees with $v_{\text{Th}}(\infty) = 50 \text{ V}$.

As in Problem 17.21 we know the time constant is 0.25 ms.

P 17.22 [a] $v_g = 50u(t)$

$$V_g(\omega) = 50 \left[\pi\delta(\omega) + \frac{1}{j\omega} \right]$$

$$H(s) = \frac{400}{2s + 400} = \frac{200}{s + 200}$$

$$H(\omega) = \frac{200}{j\omega + 200}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{10,000\pi\delta(\omega)}{j\omega + 200} + \frac{10,000}{j\omega(j\omega + 200)}$$

$$= V_1(\omega) + V_2(\omega)$$

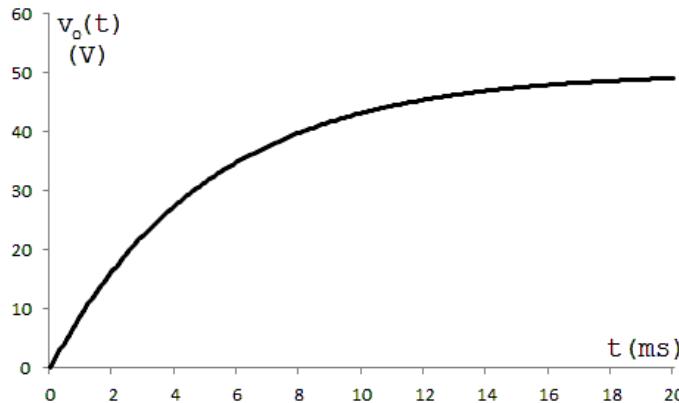
$$v_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{10,000\pi e^{j\omega t}}{j\omega + 200} \delta(\omega) d\omega = \frac{1}{2\pi} \left(\frac{10,000\pi}{200} \right) = 25 \text{ (sifting property)}$$

$$V_2(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} = \frac{50}{j\omega} - \frac{50}{j\omega + 200}$$

$$v_2(t) = 25\text{sgn}(t) - 50e^{-200t}u(t)$$

$$\begin{aligned}
 v_o(t) &= v_1(t) + v_2(t) = 25 + 25\text{sgn}(t) - 50e^{-200t}u(t) \\
 &= 50u(t) - 50e^{-200t}u(t) \\
 v_o(t) &= 50(1 - e^{-200t})u(t) \text{ V}
 \end{aligned}$$

[b]



P 17.23 [a] From the solution to Problem 17.22

$$H(\omega) = \frac{200}{j\omega + 200}$$

Now,

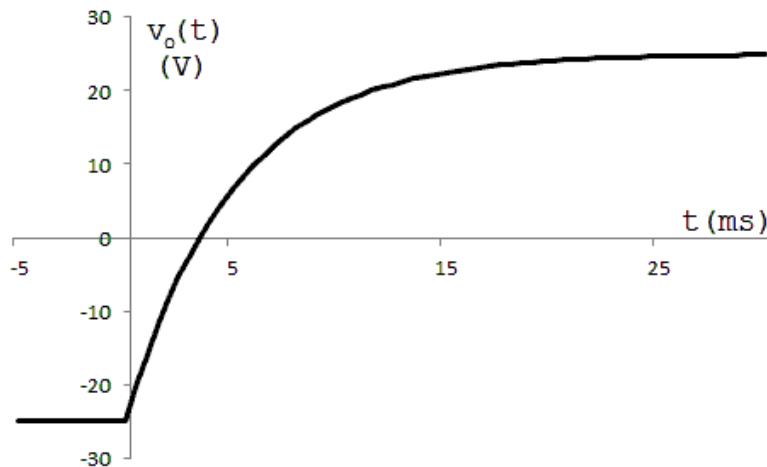
$$V_g(\omega) = \frac{50}{j\omega}$$

Then,

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{10,000}{j\omega(j\omega + 200)} = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} = \frac{50}{j\omega} - \frac{50}{j\omega + 200}$$

$$\therefore v_o(t) = 25\text{sgn}(t) - 50e^{-200t}u(t) \text{ V}$$

[b]



$$\text{P 17.24 [a]} \quad I_o = \frac{I_g R}{R + 1/sC} = \frac{RCsI_g}{RCs + 1}; \quad H(s) = \frac{I_o}{I_g} = \frac{s}{s + 1/RC}$$

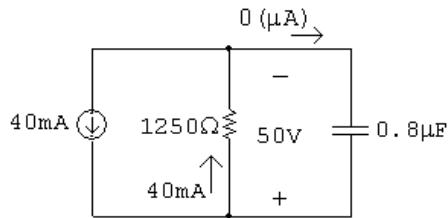
$$\frac{1}{RC} = 1000; \quad H(j\omega) = \frac{j\omega}{j\omega + 1000}$$

$$i_g = 40\text{sgn}(t) \text{ mA}; \quad I_g = (40 \times 10^{-3}) \left(\frac{2}{j\omega} \right) = \frac{80 \times 10^{-3}}{j\omega}$$

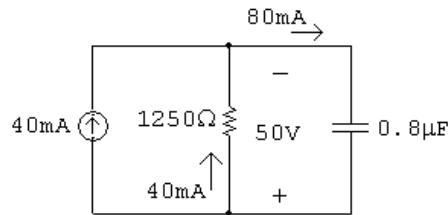
$$I_o = I_g[H(j\omega)] = \frac{80 \times 10^{-3}}{j\omega} \cdot \frac{j\omega}{j\omega + 1000} = \frac{80 \times 10^{-3}}{j\omega + 1000}$$

$$i_o(t) = 80e^{-1000t}u(t) \text{ mA}$$

- [b] Yes, at the time the source current jumps from -40 mA to $+40 \text{ mA}$ the capacitor is charged to $(1250)(0.04) = 50 \text{ V}$, positive at the lower terminal. The circuit at $t = 0^-$ is



At $t = 0^+$ the circuit is



The time constant is $(1250)(0.8 \times 10^{-3}) = 1 \text{ ms}$.

$$\therefore \frac{1}{\tau} = 1000 \quad \therefore \quad \text{for } t > 0, \quad i_o = 80e^{-1000t} \text{ mA}$$

$$\text{P 17.25 [a]} \quad V_o = \frac{I_g R (1/sC)}{R + (1/sC)} = \frac{I_g R}{RCs + 1}$$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + (1/RC)} = \frac{125 \times 10^4}{s + 1000}$$

$$H(j\omega) = \frac{125 \times 10^4}{1000 + j\omega}; \quad I_g(\omega) = \frac{80 \times 10^{-3}}{j\omega}$$

$$V_o(\omega) = H(j\omega)I_g(\omega) = \left(\frac{80 \times 10^{-3}}{j\omega} \right) \left(\frac{125 \times 10^4}{1000 + j\omega} \right)$$

$$= \frac{10^5}{j\omega(1000 + j\omega)} = \frac{100}{j\omega} - \frac{100}{1000 + j\omega}$$

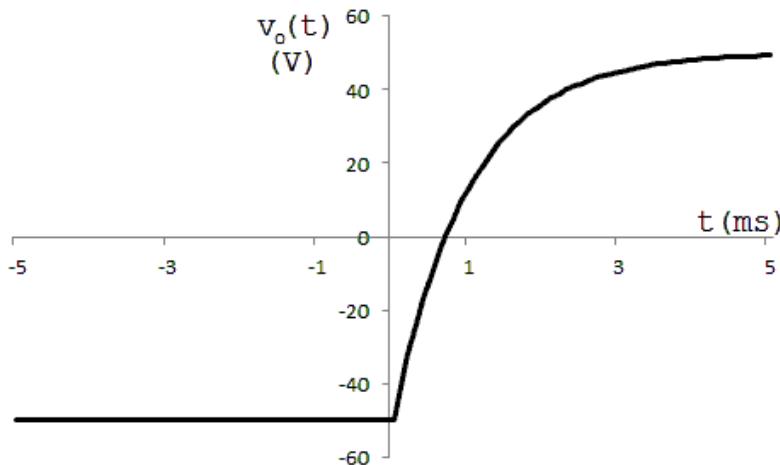
$$v_o(t) = 50\text{sgn}(t) - 100e^{-1000t}u(t) \text{ V}$$

- [b] Yes, at the time the current source jumps from -40 to $+40$ mA the capacitor is charged to -50 V. That is, at $t = 0^-$,
- $$v_o(0^-) = (1250)(-40 \times 10^{-3}) = -50 \text{ V}.$$

At $t = \infty$ the capacitor will be charged to $+50$ V. That is,

$$v_o(\infty) = (1250)(40 \times 10^{-3}) = 50 \text{ V}$$

The time constant of the circuit is $(1250)(0.8 \times 10^{-3}) = 1$ ms, so $1/\tau = 1000$. The function $v_o(t)$ is plotted below:



$$\text{P 17.26 [a]} \quad \frac{V_o}{V_g} = H(s) = \frac{16,000/s}{100 + 0.1s + 16,000/s}$$

$$H(s) = \frac{160,000}{s^2 + 1000s + 160,000} = \frac{160,000}{(s + 200)(s + 800)}$$

$$H(j\omega) = \frac{160,000}{(j\omega + 200)(j\omega + 800)}$$

$$V_g(\omega) = \frac{16}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{256 \times 10^4}{j\omega(j\omega + 200)(j\omega + 800)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 200} + \frac{K_3}{j\omega + 800}$$

$$K_1 = \frac{256 \times 10^4}{16 \times 10^4} = 16; \quad K_2 = \frac{256 \times 10^4}{(-200)(600)} = -21.33$$

$$K_3 = \frac{256 \times 10^4}{(-800)(-600)} = 5.33$$

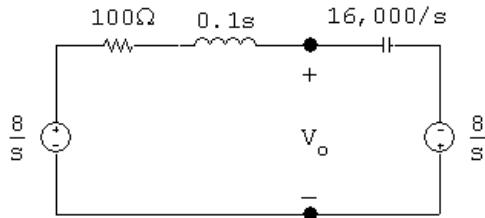
$$V_o(\omega) = \frac{16}{j\omega} - \frac{21.33}{j\omega + 200} + \frac{5.33}{j\omega + 800}$$

$$v_o(t) = 8\text{sgn}(t) - 21.33e^{-200t}u(t) + 5.33e^{-800t}u(t) \text{ V}$$

[b] $v_o(0^-) = -8 \text{ V}$

[c] $v_o(0^+) = 8 - 21.33 + 5.33 = -8 \text{ V}$

[d] For $t \geq 0^+$:



$$\frac{V_o - 8/s}{100 + 0.1s} + \frac{(V_o + 8/s)s}{16,000} = 0$$

$$V_o \left[\frac{10}{s + 1000} + \frac{s}{16,000} \right] = \frac{80}{s(s + 1000)} - \frac{8}{16,000}$$

$$V_o = \frac{-8s^2 - 8000s + 80(16,000)}{s(s + 200)(s + 800)} = \frac{K_1}{s} + \frac{K_2}{s + 200} + \frac{K_3}{s + 800}$$

$$K_1 = \frac{80(16,000)}{(200)(800)} = 8; \quad K_2 = \frac{80(16,000) - 320,000 + 16 \times 10^5}{(-200)(600)} = -21.33$$

$$K_3 = \frac{80(16,000) - 512 \times 10^4 + 64 \times 10^5}{(-800)(-600)} = 5.33$$

$$v_o(t) = (8 - 21.33e^{-200t} + 5.33e^{-800t})u(t) \text{ V}$$

[e] Yes.

P 17.27 [a] $I_o = \frac{V_g}{100 + 0.1s + 16,000/s}$

$$H(s) = \frac{I_o}{V_g} = \frac{10s}{s^2 + 1000s + 160,000} = \frac{10s}{(s + 200)(s + 800)}$$

$$H(j\omega) = \frac{10(j\omega)}{(j\omega + 200)(j\omega + 800)}$$

$$V_g(\omega) = \frac{16}{j\omega}$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{160}{(j\omega + 200)(j\omega + 800)}$$

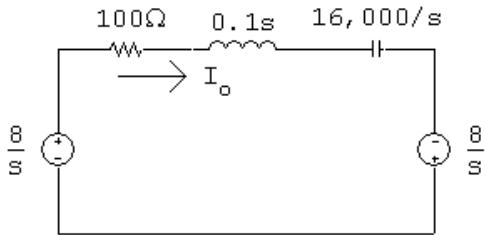
$$= \frac{0.267}{j\omega + 200} - \frac{0.267}{j\omega + 800}$$

$$i_o(t) = (0.267e^{-200t} - 0.267e^{-800t})u(t) \text{ A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 0$

[d]



$$I_o = \frac{16/s}{100 + 0.1s + 16,000/s} = \frac{160}{s^2 + 1000s + 160,000}$$

$$= \frac{160}{(s+200)(s+800)} = \frac{0.267}{s+200} - \frac{0.267}{s+800}$$

$$i_o(t) = (0.267e^{-200t} - 0.267e^{-800t})u(t) \text{ A}$$

[e] Yes.

P 17.28 [a] $i_g = 2e^{-100|t|}$

$$\therefore I_g(\omega) = \frac{2}{j\omega + 100} + \frac{2}{-j\omega + 100} = \frac{400}{(j\omega + 100)(-j\omega + 100)}$$

$$\frac{V_o}{500} + 10^{-4}sV_o = I_g$$

$$\therefore \frac{V_o}{I_g} = H(s) = \frac{10^4}{s+20}; \quad H(\omega) = \frac{10^4}{j\omega + 20}$$

$$V_o(\omega) = I_g(\omega)H(\omega) = \frac{4 \times 10^6}{(j\omega + 20)(j\omega + 100)(-j\omega + 100)}$$

$$= \frac{K_1}{j\omega + 20} + \frac{K_2}{j\omega + 100} + \frac{K_3}{-j\omega + 100}$$

$$K_1 = \frac{4 \times 10^6}{(120)(80)} = 416.67$$

$$K_2 = \frac{4 \times 10^6}{(-80)(200)} = -250$$

$$K_3 = \frac{4 \times 10^6}{(120)(200)} = 166.67$$

$$V_o(\omega) = \frac{416.67}{j\omega + 20} - \frac{250}{j\omega + 100} + \frac{166.67}{-j\omega + 100}$$

$$v_o(t) = [416.67e^{-20t} - 250e^{-100t}]u(t) + 166.67e^{100t}u(-t) \text{ V}$$

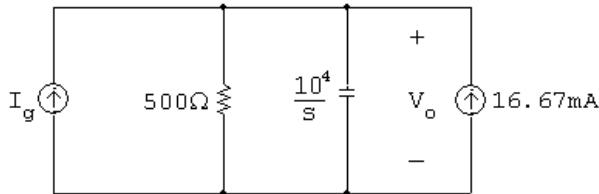
[b] $v_o(0^-) = 166.67 \text{ V}$

[c] $v_o(0^+) = 416.67 - 250 = 166.67 \text{ V}$

[d] $i_g = 2e^{-100t}u(t), \quad t \geq 0^+$

$$I_g = \frac{2}{s + 100}; \quad H(s) = \frac{10^4}{s + 20}$$

$$v_o(0^+) = 166.67 \text{ V}; \quad \gamma C = 0.0167$$



$$\frac{V_o}{500} + \frac{V_o s}{10^4} = I_g + 0.0167$$

$$\frac{V_o(s+20)}{10^4} = \frac{2}{s+100} + 0.0167$$

$$V_o = \frac{36,666.67 + 166.67s}{(s+20)(s+100)} = \frac{416.687}{s+20} - \frac{250}{s+100}$$

$$\therefore v_o(t) = (416.67e^{-20t} - 250e^{-100t})u(t) \text{ V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (416.67e^{-20t} - 250e^{-100t})u(t) \text{ V}$$

P 17.29 [a] $I_o = \frac{500}{500 + 10^4/s} I_g = \frac{500s}{500s + 10^4} I_g = \frac{s}{s + 20} I_g$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + 20}$$

$$H(j\omega) = \frac{j\omega}{j\omega + 20}$$

$$I_g(\omega) = \frac{400}{(j\omega + 100)(-j\omega + 100)} = \frac{2}{-j\omega + 100} + \frac{2}{j\omega + 100}$$

$$I_o(\omega) = H(j\omega)I_g(j\omega) = \frac{j\omega}{j\omega + 20} \left[\frac{2}{-j\omega + 100} + \frac{2}{j\omega + 100} \right]$$

$$= \frac{2j\omega}{(j\omega + 20)(-j\omega + 100)} + \frac{2j\omega}{(j\omega + 20)(j\omega + 100)}$$

$$= \frac{K_1}{j\omega + 20} + \frac{K_2}{-j\omega + 100} + \frac{K_3}{j\omega + 20} + \frac{K_4}{j\omega + 100}$$

$$K_1 = \frac{2(-20)}{120} = -0.33; \quad K_2 = \frac{2(100)}{120} = 1.67$$

$$K_3 = \frac{2(-20)}{80} = -0.5; \quad K_4 = \frac{2(-100)}{-80} = 2.5$$

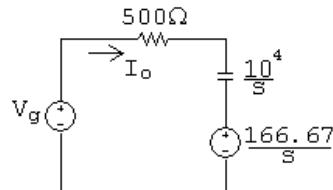
$$\therefore I_o(\omega) = \frac{-0.833}{j\omega + 20} + \frac{1.67}{-j\omega + 100} + \frac{2.5}{j\omega + 100}$$

$$i_o(t) = 1.67e^{100t}u(-t) + [-0.833e^{-20t} + 2.5e^{-100t}]u(t) \text{ A}$$

[b] $i_o(0^-) = 1.67 \text{ V}$

[c] $i_o(0^+) = 1.67 \text{ V}$

[d] Note – since $i_o(0^+) = 1.67 \text{ A}$, $v_o(0^+) = 1000 - 833.33 = 166.67 \text{ V}$.



$$I_o = \frac{V_g - (166.67/s)}{500 + (10^4/s)} = \frac{sV_g - 166.67}{500s + 10^4}; \quad V_g = \frac{1000}{s + 100}$$

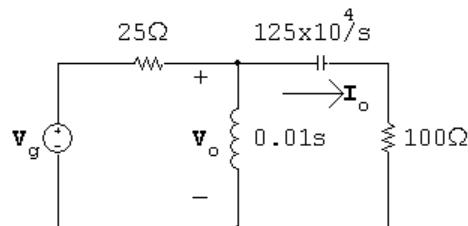
$$\therefore I_o = \frac{1.67s - 33.33}{(s + 20)(s + 100)} = \frac{-0.833}{s + 20} + \frac{2.5}{s + 100}$$

$$i_o(t) = (-0.833e^{-20t} + 2.5e^{-100t})u(t) \text{ A}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$i_o(t) = (-0.833e^{-20t} + 2.5e^{-100t})u(t) \text{ A}$$

P 17.30



$$\frac{V_o - V_g}{2s} + \frac{100V_o}{s} + \frac{V_o s}{100s + 125 \times 10^4} = 0$$

$$\therefore V_o = \frac{s(100s + 125 \times 10^4)V_g}{125(s^2 + 12,000s + 25 \times 10^6)}$$

$$I_o = \frac{sV_o}{100s + 125 \times 10^4}$$

$$H(s) = \frac{I_o}{V_g} = \frac{s^2}{125(s^2 + 12,000s + 25 \times 10^6)}$$

$$H(j\omega) = \frac{-8 \times 10^{-3}\omega^2}{(25 \times 10^6 - \omega^2) + j12,000\omega}$$

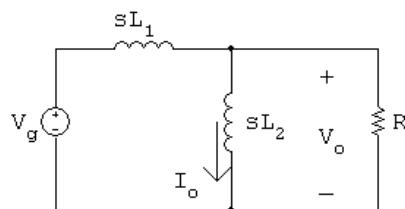
$$V_g(\omega) = 300\pi[\delta(\omega + 5000) + \delta(\omega - 5000)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{-2.4\pi\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12,000\omega}$$

$$\begin{aligned} i_o(t) &= \frac{-2.4\pi}{2\pi} \int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12,000\omega} e^{j\omega t} d\omega \\ &= -1.2 \left\{ \frac{25 \times 10^6 e^{-j5000t}}{-j(12,000)(5000)} + \frac{25 \times 10^6 e^{j5000t}}{j(12,000)(5000)} \right\} \\ &= \frac{6}{12} \left\{ \frac{e^{-j5000t}}{-j} + \frac{e^{j5000t}}{j} \right\} \\ &= 0.5[e^{-j(5000t+90^\circ)} + e^{j(5000t+90^\circ)}] \end{aligned}$$

$$i_o(t) = 1 \cos(5000t + 90^\circ) \text{ A}$$

P 17.31 [a]



$$\frac{V_o - V_g}{sL_1} + \frac{V_o}{sL_2} + \frac{V_o}{R} = 0$$

$$\therefore V_o = \frac{RV_g}{L_1 \left[s + R \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \right]}$$

$$I_o = \frac{V_o}{sL_2}$$

$$\therefore \frac{I_o}{V_g} = H(s) = \frac{R/L_1 L_2}{s(s + R[(1/L_1) + (1/L_2)])}$$

$$\frac{R}{L_1 L_2} = 12 \times 10^5$$

$$R \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = 3 \times 10^4$$

$$\therefore H(s) = \frac{12 \times 10^5}{s(s + 3 \times 10^4)}$$

$$H(j\omega) = \frac{12 \times 10^5}{j\omega(j\omega + 3 \times 10^4)}$$

$$V_g(\omega) = 125\pi[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]$$

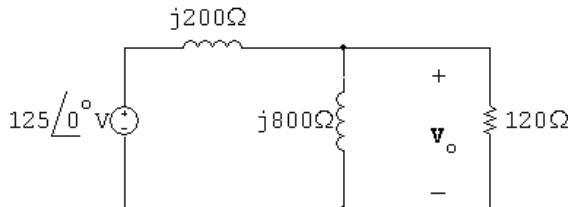
$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{1500\pi \times 10^5[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]}{j\omega(j\omega + 3 \times 10^4)}$$

$$i_o(t) = \frac{1500\pi \times 10^5}{2\pi} \int_{-\infty}^{\infty} \frac{[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]e^{j\omega t}}{j\omega(j\omega + 3 \times 10^4)} d\omega$$

$$\begin{aligned} i_o(t) &= 750 \times 10^5 \left\{ \frac{e^{-j40,000t}}{-j40,000(30,000 - j40,000)} \right. \\ &\quad \left. + \frac{e^{j40,000t}}{j40,000(30,000 + j40,000)} \right\} \\ &= \frac{75 \times 10^6}{4 \times 10^8} \left\{ \frac{e^{-j40,000t}}{-j(3 + j4)} + \frac{e^{j40,000t}}{j(3 + j4)} \right\} \\ &= \frac{75}{400} \left\{ \frac{e^{-j40,000t}}{5/-143.13^\circ} + \frac{e^{j40,000t}}{5/143.13^\circ} \right\} \\ &= 0.075 \cos(40,000t - 143.13^\circ) \text{ A} \end{aligned}$$

$$i_o(t) = 75 \cos(40,000t - 143.13^\circ) \text{ mA}$$

[b] In the phasor domain:



$$\frac{\mathbf{V}_o - 125}{j200} + \frac{\mathbf{V}_o}{j800} + \frac{\mathbf{V}_o}{120} = 0$$

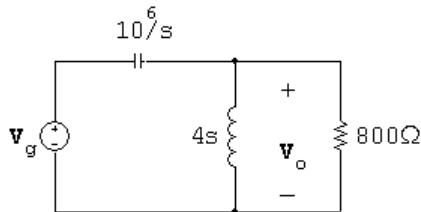
$$12\mathbf{V}_o - 1500 + 3\mathbf{V}_o + j20\mathbf{V}_o = 0$$

$$\mathbf{V}_o = \frac{1500}{15 + j20} = 60/-53.13^\circ \text{ V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{j800} = 75 \times 10^{-3} / -143.13^\circ \text{ A}$$

$$i_o(t) = 75 \cos(40,000t - 143.13^\circ) \text{ mA}$$

P 17.32 [a]



$$\frac{(V_o - V_g)s}{10^6} + \frac{V_o}{4s} + \frac{V_o}{800} = 0$$

$$\therefore V_o = \frac{s^2 V_g}{s^2 + 1250s + 25 \times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s + 250)(s + 1000)}$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega + 250)(j\omega + 1000)}$$

$$v_g = 45e^{-500|t|}; \quad V_g(\omega) = \frac{45,000}{(j\omega + 500)(-j\omega + 500)}$$

$$\therefore V_o(\omega) = H(j\omega)V_g(\omega) = \frac{45,000(j\omega)^2}{(j\omega + 250)(j\omega + 500)(j\omega + 1000)(-j\omega + 500)}$$

$$= \frac{K_1}{j\omega + 250} + \frac{K_2}{j\omega + 500} + \frac{K_3}{j\omega + 1000} + \frac{K_4}{-j\omega + 500}$$

$$K_1 = \frac{45,000(-250)^2}{(250)(750)(750)} = 20$$

$$K_2 = \frac{45,000(-500)^2}{(-250)(500)(1000)} = -90$$

$$K_3 = \frac{45,000(-1000)^2}{(-750)(-500)(1500)} = 80$$

$$K_4 = \frac{45,000(500)^2}{(750)(1000)(1500)} = 10$$

$$\therefore v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) + 10e^{500t}u(-t) \text{ V}$$

[b] $v_o(0^-) = 10 \text{ V}; \quad V_o(0^+) = 20 - 90 + 80 = 10 \text{ V}$

$$v_o(\infty) = 0 \text{ V}$$

$$\begin{aligned}
 [c] \quad I_L &= \frac{V_o}{4s} = \frac{0.25sV_g}{(s + 250)(s + 1000)} \\
 H(s) &= \frac{I_L}{V_o} = \frac{0.25s}{(s + 250)(s + 1000)} \\
 H(j\omega) &= \frac{0.25(j\omega)}{(j\omega + 250)(j\omega + 1000)} \\
 I_L(\omega) &= \frac{0.25(j\omega)(45,000)}{(j\omega + 250)(j\omega + 500)(j\omega + 1000)(-j\omega + 500)} \\
 &= \frac{K_1}{j\omega + 250} + \frac{K_2}{j\omega + 500} + \frac{K_3}{j\omega + 1000} + \frac{K_4}{-j\omega + 500} \\
 K_4 &= \frac{(0.25)(500)(45,000)}{(750)(1000)(1500)} = 5 \text{ mA} \\
 i_L(t) &= 5e^{500t}u(-t); \quad \therefore i_L(0^-) = 5 \text{ mA} \\
 K_1 &= \frac{(0.25)(-250)(45,000)}{(250)(750)(750)} = -20 \text{ mA} \\
 K_2 &= \frac{(0.25)(-500)(45,000)}{(-250)(500)(1000)} = 45 \text{ mA} \\
 K_3 &= \frac{(0.25)(-1000)(45,000)}{(-750)(-500)(1500)} = -20 \text{ mA} \\
 \therefore i_L(0^+) &= K_1 + K_2 + K_3 = -20 + 45 - 20 = 5 \text{ mA}
 \end{aligned}$$

Checks, i.e., $i_L(0^+) = i_L(0^-) = 5 \text{ mA}$

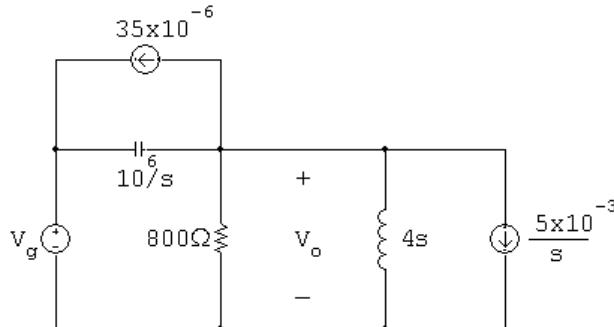
At $t = 0^-$:

$$v_C(0^-) = 45 - 10 = 35 \text{ V}$$

At $t = 0^+$:

$$v_C(0^+) = 45 - 10 = 35 \text{ V}$$

- [d] We can check the correctness of our solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes

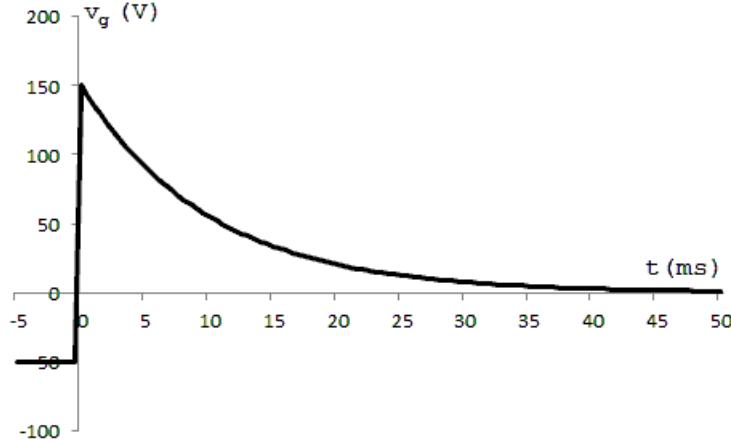


$$\frac{V_o}{800} + \frac{V_o}{4s} + \frac{(V_o - V_g)s}{10^6} + 35 \times 10^{-6} + \frac{5 \times 10^{-3}}{s} = 0$$

$$\begin{aligned} \therefore (s^2 + 1250s + 24 \times 10^4)V_o &= s^2V_g - (35s + 5000) \\ v_g(t) = 45e^{-500t}u(t) \text{ V}; \quad V_g &= \frac{45}{s+500} \\ \therefore (s+250)(s+1000)V_o &= \frac{45s^2 - (35s + 5000)(s+500)}{(s+500)} \\ \therefore V_o &= \frac{10s^2 - 22,500s - 250 \times 10^4}{(s+250)(s+500)(s+1000)} \\ &= \frac{20}{s+250} - \frac{90}{s+500} + \frac{80}{s+1000} \\ \therefore v_o(t) &= [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) \text{ V} \end{aligned}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.33 [a]



From the plot of v_g note that v_g is -50 V for an infinitely long time before $t = 0$. Therefore

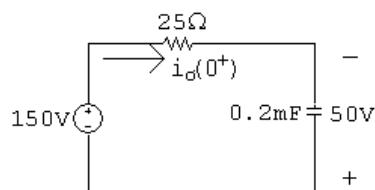
$$\therefore v_o(0^-) = -50 \text{ V}$$

There cannot be an instantaneous change in the voltage across a capacitor, so

$$v_o(0^+) = -50 \text{ V}$$

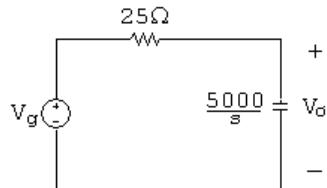
[b] $i_o(0^-) = 0 \text{ A}$

At $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{150 - (-50)}{25} = \frac{200}{25} = 8 \text{ A}$$

[c] The s -domain circuit is



$$V_o = \left[\frac{V_g}{25 + (5000/s)} \right] \left(\frac{5000}{s} \right) = \frac{200V_g}{s + 200}$$

$$\frac{V_o}{V_g} = H(s) = \frac{200}{s + 200}$$

$$H(j\omega) = \frac{200}{j\omega + 200}$$

$$V_g(\omega) = 25 \left(\frac{2}{j\omega} \right) - 25[2\pi\delta(\omega)] + \frac{150}{j\omega + 100} = \frac{50}{j\omega} - 50\pi\delta(\omega) + \frac{150}{j\omega + 100}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{200}{j\omega + 200} \left[\frac{50}{j\omega} - 50\pi\delta(\omega) + \frac{150}{j\omega + 100} \right]$$

$$= \frac{10,000}{j\omega(j\omega + 200)} - \frac{10,000\pi\delta(\omega)}{j\omega + 200} + \frac{30,000}{(j\omega + 200)(j\omega + 100)}$$

$$= \frac{K_0}{j\omega} + \frac{K_1}{j\omega + 200} + \frac{K_2}{j\omega + 100} + \frac{K_3}{j\omega + 200} - \frac{10,000\pi\delta(\omega)}{j\omega + 200}$$

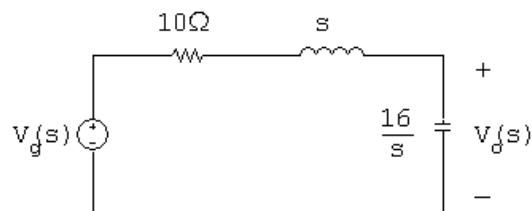
$$K_0 = \frac{10,000}{200} = 50; \quad K_1 = \frac{10,000}{-200} = -50;$$

$$K_2 = \frac{30,000}{-100} = -300; \quad K_3 = \frac{30,000}{100} = 300$$

$$V_o(\omega) = \frac{50}{j\omega} - \frac{350}{j\omega + 200} + \frac{300}{j\omega + 100} - \frac{10,000\pi\delta(\omega)}{j\omega + 200}$$

$$v_o(t) = 25\text{sgn}(t) + [300e^{-100t} - 350e^{-200t}]u(t) - 25 \text{ V}$$

P 17.34 [a]



$$V_g(\omega) = \frac{36}{4 - j\omega} - \frac{36}{4 + j\omega} = \frac{72j\omega}{(4 - j\omega)(4 + j\omega)}$$

$$V_o(s) = \frac{(16/s)}{10 + s + (16/s)} V_g(s)$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{16}{s^2 + 10s + 16} = \frac{16}{(s+2)(s+8)}$$

$$H(j\omega) = \frac{16}{(j\omega+2)(j\omega+8)}$$

$$V_o(j\omega) = H(j\omega) \cdot V_g(\omega) = \frac{1152j\omega}{(4-j\omega)(4+j\omega)(2+j\omega)(8+j\omega)}$$

$$= \frac{K_1}{4-j\omega} + \frac{K_2}{4+j\omega} + \frac{K_3}{2+j\omega} + \frac{K_4}{8+j\omega}$$

$$K_1 = \frac{1152(4)}{(8)(6)(12)} = 8$$

$$K_2 = \frac{1152(-4)}{(8)(-2)(4)} = 72$$

$$K_3 = \frac{1152(-2)}{(6)(2)(6)} = -32$$

$$K_4 = \frac{1152(-8)}{(12)(-4)(-6)} = -32$$

$$\therefore V_o(j\omega) = \frac{8}{4-j\omega} + \frac{72}{4+j\omega} - \frac{32}{2+j\omega} - \frac{32}{8+j\omega}$$

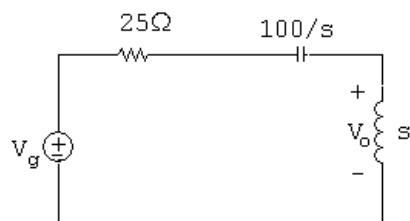
$$\therefore v_o(t) = 8e^{4t}u(-t) + [72e^{-4t} - 32e^{-2t} - 32e^{-8t}]u(t)V$$

[b] $v_o(0^-) = 8V$

[c] $v_o(0^+) = 72 - 32 - 32 = 8V$

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

P 17.35 [a]



$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \quad H(j\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = 450e^{10t}u(-t) - 450e^{-10t}u(t) \text{ V}$$

$$V_g = \frac{450}{-j\omega + 10} - \frac{450}{j\omega + 10}$$

$$V_o(\omega) = H(j\omega)V_g = \frac{450(j\omega)^2}{(-j\omega + 10)(j\omega + 5)(j\omega + 20)}$$

$$+ \frac{-450(j\omega)^2}{(j\omega + 10)(j\omega + 5)(j\omega + 20)}$$

$$= \frac{K_1}{-j\omega + 10} + \frac{K_2}{j\omega + 5} + \frac{K_3}{j\omega + 20} + \frac{K_4}{j\omega + 5} + \frac{K_5}{j\omega + 10} + \frac{K_6}{j\omega + 20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = 100 \quad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = 50 \quad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = -400 \quad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

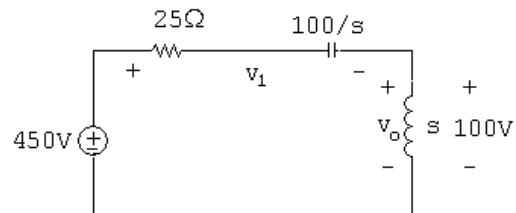
$$V_o(\omega) = \frac{100}{-j\omega + 10} + \frac{-100}{j\omega + 5} + \frac{-1600}{j\omega + 20} + \frac{900}{j\omega + 10}$$

$$v_o = 100e^{10t}u(-t) + [900e^{-10t} - 100e^{-5t} - 1600e^{-20t}]u(t) \text{ V}$$

[b] $v_o(0^-) = 100 \text{ V}$

[c] $v_o(0^+) = 900 - 100 - 1600 = -800 \text{ V}$

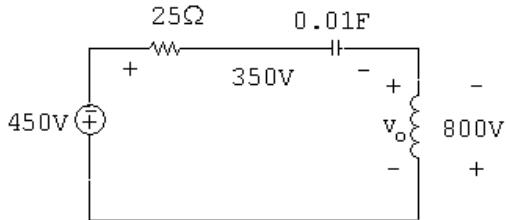
[d] At $t = 0^-$ the circuit is



Therefore, the solution predicts $v_1(0^-)$ will be 350 V.

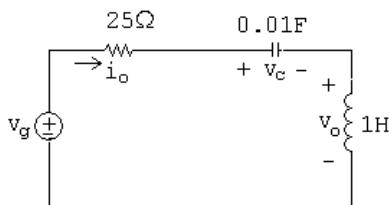
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the 25Ω resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.

At $t = 0^+$ the circuit is



From the circuit at $t = 0^+$ we see that v_o must be -800 V, which is consistent with the solution for v_o obtained in part (a).

It is informative to solve for either the current in the circuit or the voltage across the capacitor and note the solutions for i_o and v_C are consistent with the solution for v_o



The solutions are

$$i_o = 10e^{10t}u(-t) + [20e^{-5t} + 80e^{-20t} - 90e^{-10t}]u(t) \text{ A}$$

$$v_C = 100e^{10t}u(-t) + [900e^{-10t} - 400e^{-5t} - 400e^{-20t}]u(t) \text{ V}$$

$$\text{P 17.36 } V_o(s) = \frac{40}{s} + \frac{60}{s+100} - \frac{100}{s+300} = \frac{24,000(s+50)}{s(s+100)(s+300)}$$

$$V_o(s) = H(s) \cdot \frac{20}{s}$$

$$\therefore H(s) = \frac{1200(s+50)}{(s+100)(s+300)}$$

$$\therefore H(\omega) = \frac{1200(j\omega + 50)}{(j\omega + 100)(j\omega + 300)}$$

$$\therefore V_o(\omega) = \frac{40}{j\omega} \cdot \frac{1200(j\omega + 50)}{(j\omega + 100)(j\omega + 300)} = \frac{48,000(j\omega + 50)}{j\omega(j\omega + 100)(j\omega + 300)}$$

$$V_o(\omega) = \frac{80}{j\omega} + \frac{120}{j\omega + 100} - \frac{200}{j\omega + 300}$$

$$v_o(t) = 160\text{sgn}(t) + [120e^{-100t} - 200e^{-300t}]u(t) \text{ V}$$

$$\text{P 17.37 [a]} \quad f(t) = \frac{1}{2\pi} \left\{ \int_{-\infty}^0 e^\omega e^{jt\omega} d\omega + \int_0^\infty e^{-\omega} e^{jt\omega} d\omega \right\} = \frac{1/\pi}{1+t^2}$$

$$\text{[b]} \quad W = 2 \int_0^\infty \frac{(1/\pi)^2}{(1+t^2)^2} dt = \frac{2}{\pi^2} \int_0^\infty \frac{dt}{(1+t^2)^2} = \frac{1}{2\pi} \text{ J}$$

$$\text{[c]} \quad W = \frac{1}{\pi} \int_0^\infty e^{-2\omega} d\omega = \frac{1}{\pi} \frac{e^{-2\omega}}{-2} \Big|_0^\infty = \frac{1}{2\pi} \text{ J}$$

$$\text{[d]} \quad \frac{1}{\pi} \int_0^{\omega_1} e^{-2\omega} d\omega = \frac{0.9}{2\pi}, \quad 1 - e^{-2\omega_1} = 0.9, \quad e^{2\omega_1} = 10$$

$$\omega_1 = (1/2) \ln 10 \cong 1.15 \text{ rad/s}$$

$$\text{P 17.38} \quad I_o = \frac{0.5sI_g}{0.5s + 25} = \frac{sI_g}{s + 50}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + 50}$$

$$H(j\omega) = \frac{j\omega}{j\omega + 50}$$

$$I(\omega) = \frac{12}{j\omega + 10}$$

$$I_o(\omega) = H(j\omega)I(\omega) = \frac{12(j\omega)}{(j\omega + 10)(j\omega + 50)}$$

$$|I_o(\omega)| = \frac{12\omega}{\sqrt{(\omega^2 + 100)(\omega^2 + 2500)}}$$

$$\begin{aligned} |I_o(\omega)|^2 &= \frac{144\omega^2}{(\omega^2 + 100)(\omega^2 + 2500)} \\ &= \frac{-6}{\omega^2 + 100} + \frac{150}{\omega^2 + 2500} \end{aligned}$$

$$\begin{aligned} W_o(\text{total}) &= \frac{1}{\pi} \int_0^\infty \frac{150d\omega}{\omega^2 + 2500} - \frac{1}{\pi} \int_0^\infty \frac{6d\omega}{\omega^2 + 100} \\ &= \frac{3}{\pi} \tan^{-1} \left(\frac{\omega}{50} \Big|_0^\infty \right) - \frac{0.6}{\pi} \tan^{-1} \left(\frac{\omega}{10} \Big|_0^\infty \right) \\ &= 1.5 - 0.3 = 1.2 \text{ J} \end{aligned}$$

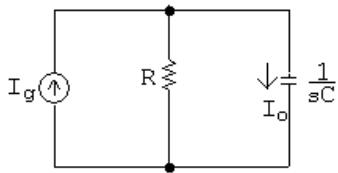
$$W_o(0 - 100 \text{ rad/s}) = \frac{3}{\pi} \tan^{-1}(2) - \frac{0.6}{\pi} \tan^{-1}(10)$$

$$= 1.06 - 0.28 = 0.78 \text{ J}$$

Therefore, the percent between 0 and 100 rad/s is

$$\frac{0.78}{1.2}(100) = 64.69\%$$

P 17.39



$$I_o = \frac{I_g R}{R + (1/sC)} = \frac{RCsI_g}{RCs + 1}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + (1/RC)}$$

$$RC = (2000)(2.5 \times 10^{-6}) = 0.005; \quad \frac{1}{RC} = \frac{1}{0.005} = 200$$

$$H(s) = \frac{s}{s + 200}; \quad H(j\omega) = \frac{j\omega}{j\omega + 200}$$

$$I_g(\omega) = \frac{0.01}{j\omega + 50}$$

$$I_o(\omega) = H(j\omega)I_g(\omega) = \frac{0.01j\omega}{(j\omega + 50)(j\omega + 200)}$$

$$|I_o(\omega)| = \frac{\omega(0.01)}{(\sqrt{\omega^2 + 50^2})(\sqrt{\omega^2 + 200^2})}$$

$$|I_o(\omega)|^2 = \frac{10^{-4}\omega^2}{(\omega^2 + 50^2)(\omega^2 + 200^2)} = \frac{K_1}{\omega^2 + 2500} + \frac{K_2}{\omega^2 + 4 \times 10^4}$$

$$K_1 = \frac{(10^{-4})(-2500)}{(37,500)} = -6.67 \times 10^{-6}$$

$$K_2 = \frac{(10^{-4})(-4 \times 10^4)}{(-37,500)} = 106.67 \times 10^{-6}$$

$$|I_o(\omega)|^2 = \frac{106.67 \times 10^{-6}}{\omega^2 + 4 \times 10^4} - \frac{6.67 \times 10^{-6}}{\omega^2 + 2500}$$

$$\begin{aligned} W_{1\Omega} &= \frac{1}{\pi} \int_0^\infty |I_o(\omega)|^2 d\omega = \frac{106.67 \times 10^{-6}}{\pi} \int_0^\infty \frac{d\omega}{\omega^2 + 4 \times 10^4} - \frac{6.67 \times 10^{-6}}{\pi} \int_0^\infty \frac{d\omega}{\omega^2 + 2500} \\ &= \frac{0.533 \times 10^{-6}}{\pi} \tan^{-1} \frac{\omega}{200} \Big|_0^\infty - \frac{0.133 \times 10^{-6}}{\pi} \tan^{-1} \frac{\omega}{50} \Big|_0^\infty \\ &= \left(\frac{0.533}{\pi} \cdot \frac{\pi}{2} - \frac{0.133}{\pi} \cdot \frac{\pi}{2} \right) \times 10^{-6} = 0.2 \times 10^{-6} = 200 \text{ nJ} \end{aligned}$$

Between 0 and 100 rad/s

$$W_{1\Omega} = \left[\frac{0.533}{\pi} \tan^{-1} \frac{1}{2} - \frac{0.133}{\pi} \tan^{-1} 2 \right] \times 10^{-6} = 31.79 \text{ nJ}$$

$$\% = \frac{31.79}{200} (100) = 15.9\%$$

$$\text{P 17.40 [a]} \quad V_g(\omega) = \frac{60}{(j\omega + 1)(-j\omega + 1)}$$

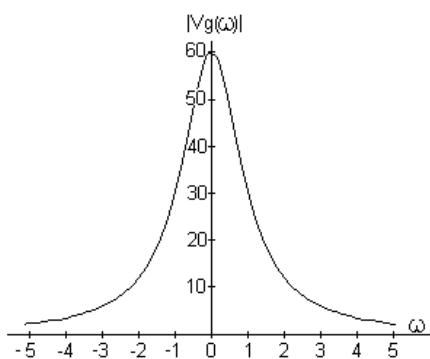
$$H(s) = \frac{V_o}{V_g} = \frac{0.4}{s + 0.5}; \quad H(\omega) = \frac{0.4}{(j\omega + 0.5)}$$

$$V_o(\omega) = \frac{24}{(j\omega + 1)(j\omega + 0.5)(-j\omega + 1)}$$

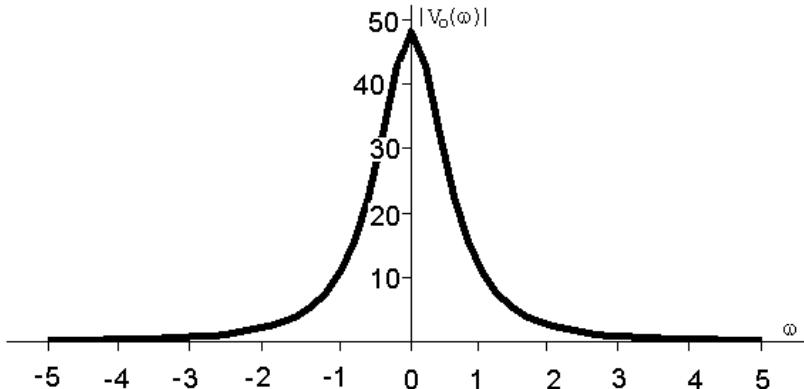
$$V_o(\omega) = \frac{-24}{j\omega + 1} + \frac{32}{j\omega + 0.5} + \frac{8}{-j\omega + 1}$$

$$v_o(t) = [-24e^{-t} + 32e^{-t/2}]u(t) + 8e^t u(-t) \text{ V}$$

$$[\text{b}] \quad |V_g(\omega)| = \frac{60}{(\omega^2 + 1)}$$



[c] $|V_o(\omega)| = \frac{24}{(\omega^2 + 1)\sqrt{\omega^2 + 0.25}}$



[d] $W_i = 2 \int_0^\infty 900e^{-2t} dt = 1800 \left[\frac{e^{-2t}}{-2} \right]_0^\infty = 900 \text{ J}$

[e] $W_o = \int_{-\infty}^0 64e^{2t} dt + \int_0^\infty (-24e^{-t} + 32e^{-t/2})^2 dt$

$$= 32 + \int_0^\infty [576e^{-2t} - 1536e^{-3t/2} + 1024e^{-t}] dt$$

$$= 32 + 288 - 1024 + 1024 = 320 \text{ J}$$

[f] $|V_g(\omega)| = \frac{60}{\omega^2 + 1}, \quad |V_g^2(\omega)| = \frac{3600}{(\omega^2 + 1)^2}$

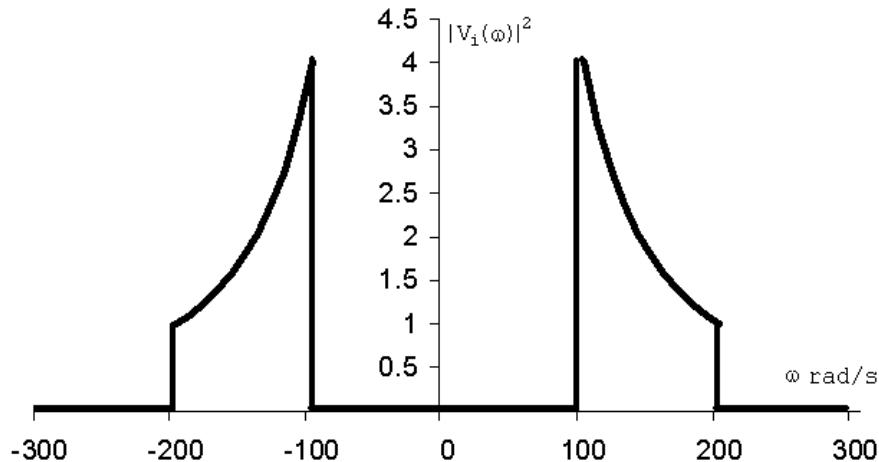
$$\begin{aligned} W_g &= \frac{3600}{\pi} \int_0^2 \frac{d\omega}{(\omega^2 + 1)^2} \\ &= \frac{3600}{\pi} \left\{ \frac{1}{2} \left(\frac{\omega}{\omega^2 + 1} + \tan^{-1} \omega \right) \Big|_0^2 \right\} \\ &= \frac{1800}{\pi} \left(\frac{2}{5} + \tan^{-1} 2 \right) = 863.53 \text{ J} \end{aligned}$$

$$\therefore \% = \left(\frac{863.53}{900} \right) \times 100 = 95.95\%$$

$$\begin{aligned}
 [\mathbf{g}] \quad |V_o(\omega)|^2 &= \frac{576}{(\omega^2 + 1)^2(\omega^2 + 0.25)} \\
 &= \frac{1024}{\omega^2 + 0.25} - \frac{768}{(\omega^2 + 1)^2} - \frac{1024}{(\omega^2 + 1)} \\
 W_o &= \frac{1}{\pi} \left\{ 1024 \cdot 2 \cdot \tan^{-1} 2\omega \Big|_0^2 - 768 \left(\frac{1}{2} \right) \left(\frac{\omega}{\omega^2 + 1} + \tan^{-1} \omega \right)_0^2 \right. \\
 &\quad \left. - 1024 \tan^{-1} \omega \Big|_0^2 \right\} \\
 &= \frac{2048}{\pi} \tan^{-1} 4 - \frac{384}{\pi} \left(\frac{2}{5} + \tan^{-1} 2 \right) - \frac{1024}{\pi} \tan^{-1} 2 \\
 &= 319.2 \text{ J}
 \end{aligned}$$

$$\% = \frac{319.2}{320} \times 100 = 99.75\%$$

$$\text{P 17.41 [a]} \quad |V_i(\omega)|^2 = \frac{4 \times 10^4}{\omega^2}; \quad |V_i(100)|^2 = \frac{4 \times 10^4}{100^2} = 4; \quad |V_i(200)|^2 = \frac{4 \times 10^4}{200^2} = 1$$



$$[\mathbf{b}] \quad V_o = \frac{V_i R}{R + (1/sC)} = \frac{s R C V_i}{R C s + 1}$$

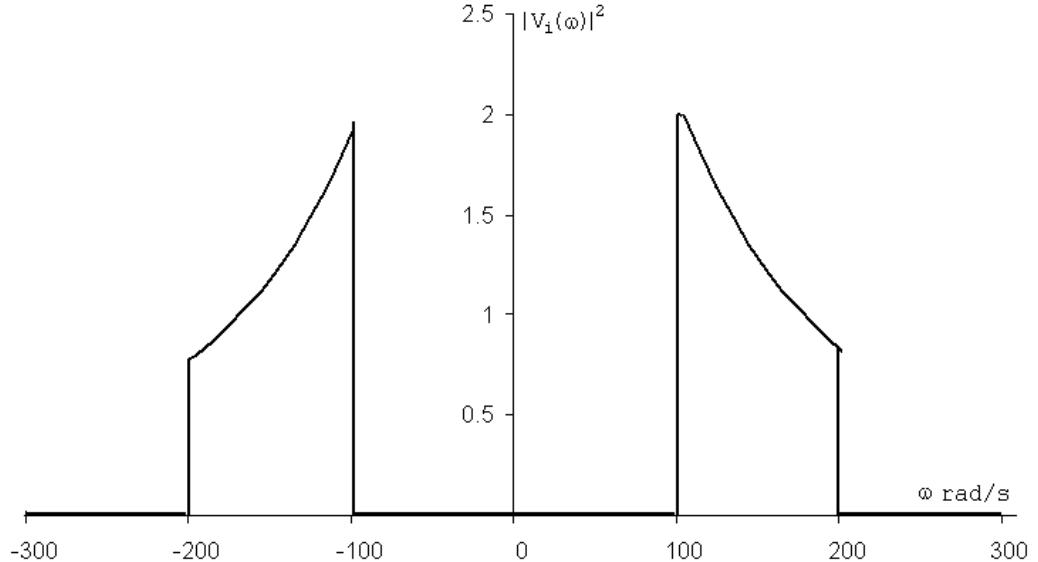
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}; \quad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(j\omega) = \frac{j\omega}{j\omega + 100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|} \cdot \frac{|\omega|}{\sqrt{\omega^2 + 10^4}} = \frac{200}{\sqrt{\omega^2 + 10^4}}$$

$$|V_o(\omega)|^2 = \frac{4 \times 10^4}{\omega^2 + 10^4}, \quad 100 \leq \omega \leq 200 \text{ rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4 \times 10^4}{10^4 + 10^4} = 2; \quad |V_o(200)|^2 = \frac{4 \times 10^4}{5 \times 10^4} = 0.8$$



$$[c] \quad W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2} d\omega = \frac{4 \times 10^4}{\pi} \left[-\frac{1}{\omega} \right]_{100}^{200}$$

$$= \frac{4 \times 10^4}{\pi} \left[\frac{1}{100} - \frac{1}{200} \right] = \frac{200}{\pi} \cong 63.66 \text{ J}$$

$$[d] \quad W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2 + 10^4} d\omega = \frac{4 \times 10^4}{\pi} \cdot \tan^{-1} \frac{\omega}{100} \Big|_{100}^{200}$$

$$= \frac{400}{\pi} [\tan^{-1} 2 - \tan^{-1} 1] \cong 40.97 \text{ J}$$

P 17.42 [a] $V_i(\omega) = \frac{A}{a + j\omega}; \quad |V_i(\omega)| = \frac{A}{\sqrt{a^2 + \omega^2}}$

$$H(s) = \frac{s}{s + \alpha}; \quad H(j\omega) = \frac{j\omega}{\alpha + j\omega}; \quad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2 + \omega^2}}$$

$$\text{Therefore } |V_o(\omega)| = \frac{\omega A}{\sqrt{(a^2 + \omega^2)(\alpha^2 + \omega^2)}}$$

$$\text{Therefore } |V_o(\omega)|^2 = \frac{\omega^2 A^2}{(a^2 + \omega^2)(\alpha^2 + \omega^2)}$$

$$W_{\text{IN}} = \int_0^\infty A^2 e^{-2at} dt = \frac{A^2}{2a}; \quad \text{when } \alpha = a \text{ we have}$$

$$W_{\text{OUT}} = \frac{A^2}{\pi} \int_0^a \frac{\omega^2 d\omega}{(\omega^2 + a^2)^2} = \frac{A^2}{\pi} \left\{ \int_0^a \frac{d\omega}{a^2 + \omega^2} - \int_0^a \frac{a^2 d\omega}{(a^2 + \omega^2)^2} \right\}$$

$$= \frac{A^2}{4a\pi} \left(\frac{\pi}{2} - 1 \right)$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi} \int_0^\infty \left[\frac{\omega^2}{(a^2 + \omega^2)^2} \right] d\omega = \frac{A^2}{4a}$$

$$\text{Therefore } \frac{W_{\text{OUT}}(a)}{W_{\text{OUT}}(\text{total})} = 0.5 - \frac{1}{\pi} = 0.1817 \quad \text{or} \quad 18.17\%$$

[b] When $\alpha \neq a$ we have

$$\begin{aligned} W_{\text{OUT}}(\alpha) &= \frac{1}{\pi} \int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2 + \omega^2)(\alpha^2 + \omega^2)} \\ &= \frac{A^2}{\pi} \left\{ \int_0^\alpha \left[\frac{K_1}{a^2 + \omega^2} + \frac{K_2}{\alpha^2 + \omega^2} \right] d\omega \right\} \\ \text{where } K_1 &= \frac{a^2}{a^2 - \alpha^2} \quad \text{and} \quad K_2 = \frac{-\alpha^2}{a^2 - \alpha^2} \end{aligned}$$

Therefore

$$W_{\text{OUT}}(\alpha) = \frac{A^2}{\pi(a^2 - \alpha^2)} \left[a \tan^{-1} \left(\frac{\alpha}{a} \right) - \frac{\alpha\pi}{4} \right]$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi(a^2 - \alpha^2)} \left[a \frac{\pi}{2} - \alpha \frac{\pi}{2} \right] = \frac{A^2}{2(a + \alpha)}$$

$$\text{Therefore } \frac{W_{\text{OUT}}(\alpha)}{W_{\text{OUT}}(\text{total})} = \frac{2}{\pi(a - \alpha)} \cdot \left[a \tan^{-1} \left(\frac{\alpha}{a} \right) - \frac{\alpha\pi}{4} \right]$$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

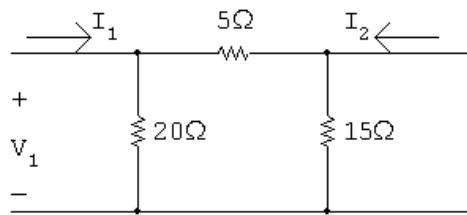
[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.

18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



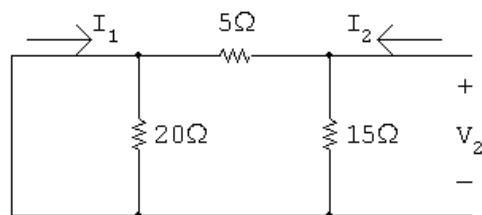
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \quad \frac{I_1}{V_1} = y_{11} = 0.25 \text{ S}; \quad I_2 = \left(\frac{-20}{25} \right) I_1 = -0.8I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2 \text{ S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \quad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15} \right) \text{ S}$$

$$I_1 = \left(\frac{-15}{20} \right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

Therefore $y_{12} = (-0.75) \frac{4}{15} = -0.2 \text{ S}$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1} \right)_{V_2=0} = 20 \| 5 = 4 \Omega$$

$$h_{21} = \left(\frac{I_2}{I_1} \right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2} \right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2} \right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75} \text{ S}$$

$$g_{11} = \left(\frac{I_1}{V_1} \right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1 \text{ S}$$

$$g_{21} = \left(\frac{V_2}{V_1} \right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2} \right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2} \right)_{V_1=0} = 15 \| 5 = \frac{75}{20} = 3.75 \Omega$$

AP 18.3

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} = \frac{5 \times 10^{-6}}{50 \times 10^{-3}} = 0.1 \text{ mS}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0} = \frac{200 \times 10^{-3}}{50 \times 10^{-3}} = 4$$

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0} = \frac{2 \times 10^{-6}}{0.5 \times 10^{-6}} = 4$$

$$g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0} = \frac{10 \times 10^{-3}}{0.5 \times 10^{-6}} = 20 \text{ k}\Omega$$

AP 18.4 First calculate the b -parameters:

$$b_{11} = \frac{V_2}{V_1} \Big|_{I_1=0} = \frac{15}{10} = 1.5 \Omega; \quad b_{21} = \frac{I_2}{V_1} \Big|_{I_1=0} = \frac{30}{10} = 3 \text{ S}$$

$$b_{12} = \frac{-V_2}{I_1} \Big|_{V_1=0} = \frac{-10}{-5} = 2 \Omega; \quad b_{22} = \frac{-I_2}{I_1} \Big|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z -parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15} \Omega; \quad z_{12} = \frac{1}{b_{21}} = \frac{1}{3} \Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6 \Omega; \quad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2} \Omega$$

AP 18.5

$$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$$

$$\text{Therefore, } z_{11} = z_{22} = 95/5 = 19 \Omega$$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right)z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17 \Omega$.

$$\begin{aligned} \text{AP 18.6 [a]} \quad I_2 &= \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g} \\ &= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)} \\ &= \frac{-50 \times 10^{-3}}{10} = -5 \text{ mA} \end{aligned}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5 \text{ mW}$$

$$\begin{aligned} \text{[b]} \quad Z_{\text{Th}} &= \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)} \\ &= \frac{7}{6 \times 10^{-4}} = \frac{70}{6} \text{ k}\Omega \end{aligned}$$

$$[\mathbf{c}] \quad V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{50 \times 10^{-3}}{6 \times 10^{-4}} = \frac{500}{6} \text{ V}$$

$$\text{Therefore } V_2 = \frac{250}{6} \text{ V}; \quad P_{\max} = \frac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4 \text{ mW}$$

AP 18.7 [a] For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \quad a'_{21} = \frac{1}{20} \text{ S}, \quad a'_{12} = 11.25 \Omega$$

The a -parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125 \Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125 \text{ S}$$

$$a_{22} = a_{11} = 2.125, \quad R_{\text{Th}} = (45.125/3.125) = 14.44 \Omega$$

$$[\mathbf{b}] \quad V_t = \frac{100}{3.125} = 32 \text{ V}; \quad \text{therefore } V_2 = 16 \text{ V}$$

$$[\mathbf{c}] \quad P = \frac{16^2}{14.44} = 17.73 \text{ W}$$

Problems

$$\text{P 18.1} \quad h_{11} = \left(\frac{V_1}{I_1} \right)_{V_2=0} = 20\|5 = 4\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1} \right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2} \right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2} \right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75} \Omega$$

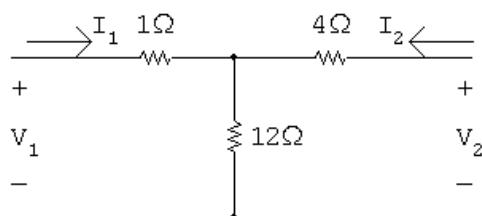
$$g_{11} = \left(\frac{I_1}{V_1} \right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1 \Omega$$

$$g_{21} = \left(\frac{V_2}{V_1} \right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2} \right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2} \right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75 \Omega$$

P 18.2



$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = 1 + 12 = 13 \Omega$$

$$z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = 12 \Omega$$

$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = 4 + 12 = 16 \Omega$$

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = 12 \Omega$$

$$P\ 18.3 \quad \Delta z = (13)(16) - (12)(12) = 64$$

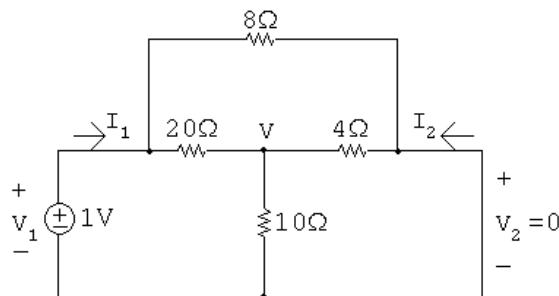
$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{16}{64} = 0.25 \text{ S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-12}{64} = -0.1875 \text{ S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-12}{64} = -0.1875 \text{ S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{13}{64} = 0.203125 \text{ S}$$

$$P\ 18.4 \quad y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}; \quad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

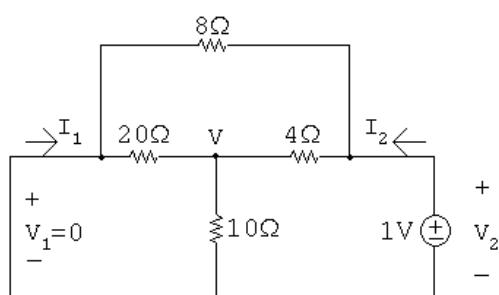


$$\frac{V-1}{20} + \frac{V}{10} + \frac{V}{4} = 0; \quad \text{so} \quad V = 0.125 \text{ V}$$

$$\therefore I_1 = \frac{1-0.125}{20} + \frac{1-0}{8} = 168.75 \text{ mA}; \quad I_2 = \frac{0-0.125}{4} + \frac{0-1}{8} = -156.25 \text{ mA}$$

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = 168.75 \text{ mS}; \quad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = -156.25 \text{ mS}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}; \quad y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$



$$\frac{V}{20} + \frac{V}{10} + \frac{V-1}{4} = 0; \quad \text{so} \quad V = 0.625 \text{ V}$$

$$\therefore I_1 = \frac{0 - 0.625}{20} + \frac{0 - 1}{8} = -156.25 \text{ mA};$$

$$I_2 = \frac{1 - 0.625}{4} + \frac{1 - 0}{8} = 218.75 \text{ mA}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = -156.25 \text{ mS}; \quad y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = 218.75 \text{ mS}$$

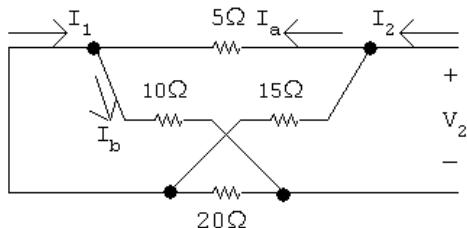
Summary:

$$y_{11} = 168.75 \text{ mS} \quad y_{12} = -156.25 \text{ mS} \quad y_{21} = -156.25 \text{ mS} \quad y_{22} = 218.75 \text{ mS}$$

$$\text{P 18.5} \quad V_2 = b_{11}V_1 - b_{12}I_1$$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{12} = \left. \frac{-V_2}{I_1} \right|_{V_1=0}; \quad b_{22} = \left. \frac{-I_2}{I_1} \right|_{V_1=0}$$



$$5\parallel 15 = (15/4) \Omega; \quad 10\parallel 20 = (20/3) \Omega$$

$$I_2 = \frac{V_2}{(15/4) + (20/3)} = \frac{12V_2}{125}; \quad I_1 = I_b - I_a$$

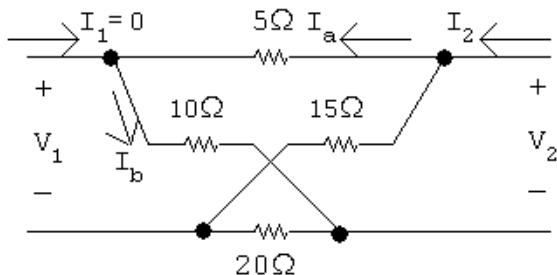
$$I_a = \frac{15}{20}I_2; \quad I_b = \frac{20}{30}I_2$$

$$I_1 = \left(\frac{20}{30} - \frac{15}{20} \right) I_2 = \frac{-5}{60}I_2 = \frac{-1}{12}I_2$$

$$b_{22} = \frac{-I_2}{I_1} = 12$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2} \left(\frac{I_2}{I_1} \right) = \frac{125}{12}(12) = 125 \Omega$$

$$b_{11} = \frac{V_2}{V_1} \Big|_{I_1=0}; \quad b_{21} = \frac{I_2}{V_1} \Big|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{10}{15}V_2; \quad V_b = \frac{20}{35}V_2$$

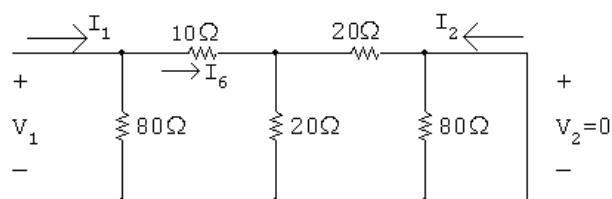
$$V_1 = \frac{10}{15}V_2 - \frac{20}{35}V_2 = \frac{2}{21}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{21}{2} = 10.5$$

$$V_2 = (10 + 5)\parallel(20 + 15)I_2 = 10.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right) \left(\frac{V_2}{V_1}\right) = \left(\frac{1}{10.5}\right) (10.5) = 1 \text{ S}$$

P 18.6 $h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}; \quad h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$

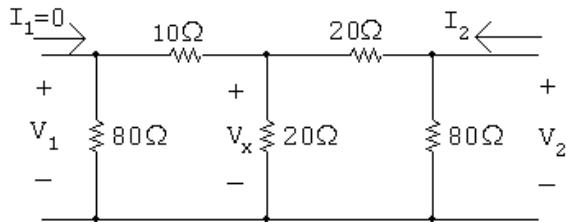


$$\frac{V_1}{I_1} = 80\parallel[10 + 20\parallel 20] = 80\parallel 20 = 16 \Omega \quad \therefore h_{11} = 16 \Omega$$

$$I_6 = \frac{80}{80 + 20}I_1 = 0.8I_1$$

$$I_2 = \frac{-20}{20 + 20}I_6 = -0.5I_6 = -0.5(0.8)I_1 = -0.4I_1 \quad \therefore h_{21} = -0.4$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0}; \quad h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80 \parallel [20 + 20 \parallel 90] = 25 \Omega \quad \therefore h_{22} = \frac{1}{25} = 40 \text{ mS}$$

$$V_x = \frac{20 \parallel 90}{20 + 20 \parallel 90} V_2$$

$$V_1 = \frac{80}{80 + 10} V_x = \frac{80(20 \parallel 90)}{90(20 + 20 \parallel 90)} V_2 = 0.4 V_2$$

$$\therefore h_{12} = 0.4$$

Summary:

$$h_{11} = 16 \Omega; \quad h_{12} = 0.4; \quad h_{21} = -0.4; \quad h_{22} = 40 \text{ mS}$$

$$\text{P 18.7} \quad h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = R_1 \parallel R_2 = 4 \quad \therefore \quad \frac{R_1 R_2}{R_1 + R_2} = 4$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} = \frac{-R_2}{R_1 + R_2} = -0.8$$

$$\therefore R_2 = 0.8R_1 + 0.8R_2 \quad \text{so} \quad R_1 = \frac{R_2}{4}$$

Substituting,

$$\frac{(R_2/4)R_2}{(R_2/4) + R_2} = 4 \quad \text{so} \quad R_2 = 20 \Omega \quad \text{and} \quad R_1 = 5 \Omega$$

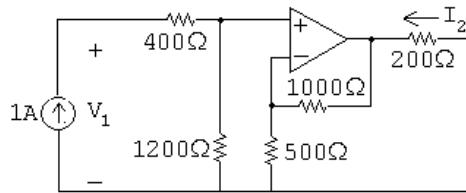
$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{1}{R_3 \parallel (R_1 + R_2)} = \frac{1}{R_3 \parallel 25} = 0.14$$

$$\therefore R_3 = 10$$

Summary:

$$R_1 = 5 \Omega; \quad R_2 = 20 \Omega; \quad R_3 = 10 \Omega$$

P 18.8 For $V_2 = 0$:



$$V_1 = (400 + 1200)I_1$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = \frac{1600}{1} = 1600 \Omega$$

$$V_p = 1200(1 \text{ A}) = 1200 \text{ V} = V_n$$

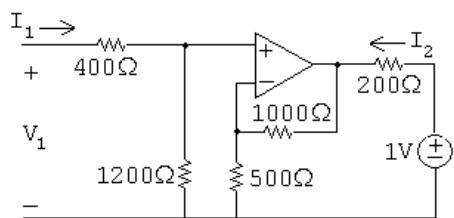
At V_n ,

$$\frac{1200}{500} + \frac{1200 - V_o}{1000} = 0 \quad \text{so} \quad V_o = 3600 \text{ V}$$

$$\therefore I_2 = -\frac{3600}{200} = -18 \text{ A}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{I_1=0} = \frac{-18}{1} = -18$$

For $I_1 = 0$:



$$V_1 = 0$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = \frac{0}{1} = 0$$

At V_n ,

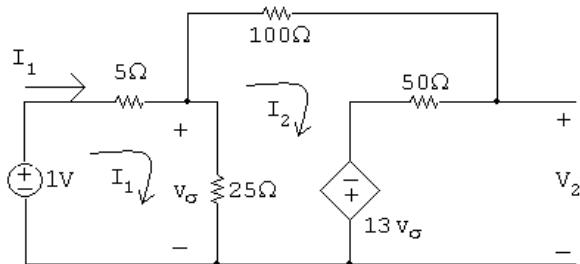
$$\frac{V_n}{500} + \frac{V_n - V_o}{100} = 0$$

But $V_n = V_p = 0$ so $V_o = 0$; therefore,

$$I_2 = \frac{1\text{V}}{200\Omega} = 5\text{mS}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{5\text{m}}{1} = 5\text{mS}$$

P 18.9 For $I_2 = 0$:



$$30I_1 - 25I_2 = 1$$

$$-25I_1 + 175I_2 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = 0$$

Solving,

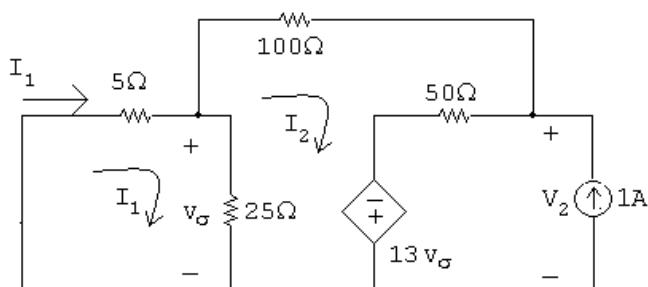
$$I_1 = 80\text{mA}; \quad I_2 = 56\text{mA}$$

$$V_2 = 50I_2 - 13(25)(I_1 - I_2) = -5\text{V}$$

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} = \frac{80\text{m}}{1} = 80\text{mS}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0} = \frac{-5}{1} = -5$$

For $V_1 = 0$:



$$30I_1 - 25I_2 = 0$$

$$-25I_1 + 175I_2 + 50 - 13(25)(I_1 - I_2) = 0 \quad \text{so} \quad -350I_1 + 500I_2 = -50$$

Solving,

$$I_1 = -200 \text{ mA}; \quad I_2 = -240 \text{ mA}$$

$$V_2 = 50(I_2 + 1) - 13(25)(I_1 - I_2) = 25 \text{ V}$$

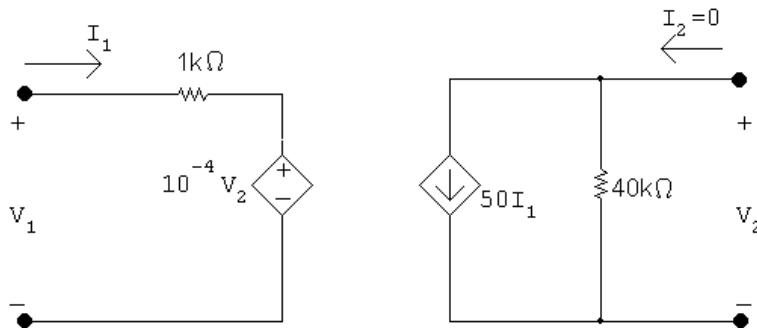
$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0} = \frac{-200 \text{ m}}{1} = -0.2$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} = \frac{25}{1} = 25 \Omega$$

$$\text{P 18.10 } V_1 = a_{11}V_2 - a_{12}I_2$$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11} = \frac{V_1}{V_2} \Big|_{I_2=0}; \quad a_{21} = \frac{I_1}{V_2} \Big|_{I_2=0}$$

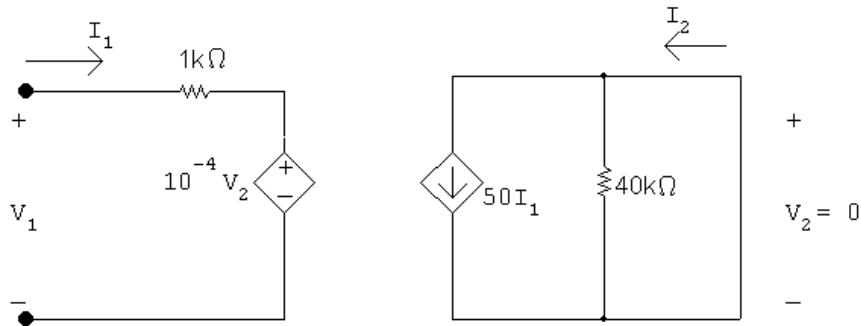


$$V_1 = 10^3 I_1 + 10^{-4} V_2 = 10^3(-0.5 \times 10^{-6})V_2 + 10^{-4}V_2$$

$$\therefore a_{11} = -5 \times 10^{-4} + 10^{-4} = -4 \times 10^{-4}$$

$$V_2 = -(50I_1)(40 \times 10^3); \quad \therefore a_{21} = -\frac{1}{2 \times 10^6} = -0.5 \mu\text{S}$$

$$a_{12} = \frac{-V_1}{I_2} \Big|_{V_2=0}; \quad a_{22} = \frac{-I_1}{I_2} \Big|_{V_2=0}$$



$$I_2 = 50I_1; \quad \therefore a_{22} = -\frac{I_1}{I_2} = -\frac{1}{50}$$

$$V_1 = 1000I_1; \quad \therefore a_{12} = -\frac{V_1}{I_2} = -\frac{V_1}{I_1} \frac{I_1}{I_2} = -(1000)(1/50) = -20 \Omega$$

Summary

$$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -20 \Omega; \quad a_{21} = -0.5 \mu S; \quad a_{22} = -0.02$$

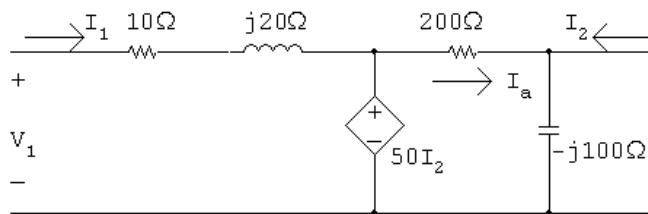
$$\text{P 18.11 } g_{11} = \frac{a_{21}}{a_{11}} = \frac{-0.5 \times 10^{-6}}{-4 \times 10^{-4}} = 1.25 \text{ mS}$$

$$g_{12} = \frac{-\Delta a}{a_{11}} = \frac{-(-4 \times 10^{-4})(-1/50) - (-0.5 \times 10^{-6})(-20)}{-4 \times 10^{-4}} = -0.005$$

$$g_{21} = \frac{1}{a_{11}} = \frac{1}{-4 \times 10^{-4}} = -2500$$

$$g_{22} = \frac{a_{12}}{a_{11}} = \frac{(-20)}{-400 \times 10^{-6}} = 5 \times 10^4 \Omega$$

P 18.12 For $V_2 = 0$:

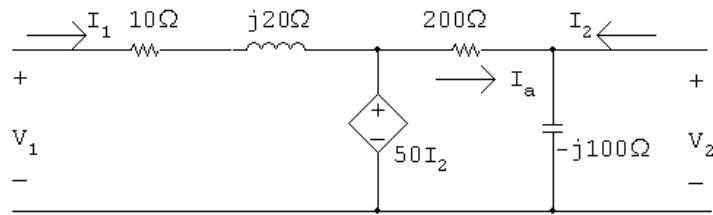


$$I_a = \frac{50I_2}{200} = \frac{1}{4}I_2 = -I_2; \quad \therefore I_2 = 0$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = 0$$

$$V_1 = (10 + j20)I_1 \quad \therefore \quad h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = 10 + j20 \Omega$$

For $I_1 = 0$:



$$V_1 = 50I_2; \quad I_2 = \frac{V_2}{-j100} + \frac{V_2 - 50I_2}{200}$$

$$200I_2 = j2V_2 + V_2 - 50I_2$$

$$250I_2 = V_2(1 + j2)$$

$$50I_2 = V_2 \left(\frac{1+j2}{5} \right) = (0.2 + j0.4)V_2$$

$$\therefore V_1 = (0.2 + j0.4)V_2$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = 0.2 + j0.4$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{1+j2}{250} = 4 + j8 \text{ mS}$$

Summary:

$$h_{11} = 10 + j20 \Omega; \quad h_{12} = 0.2 + j0.4; \quad h_{21} = 0; \quad h_{22} = 4 + j8 \text{ mS}$$

$$\text{P 18.13 } I_1 = g_{11}V_1 + g_{12}I_2; \quad V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0} = \frac{0.25 \times 10^{-6}}{20 \times 10^{-3}} = 12.5 \times 10^{-6} = 12.5 \mu\text{S}$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} = \frac{-5}{20} \times 10^3 = -250$$

$$0 = -250(10) + g_{22}(50 \times 10^{-6})$$

$$g_{22} = \frac{2500}{50 \times 10^{-6}} = 50 \text{ M}\Omega$$

$$200 \times 10^{-6} = 12.5 \times 10^{-6}(10) + g_{12}(50 \times 10^{-6})$$

$$(200 - 125)10^{-6} = g_{12}(50 \times 10^{-6})$$

$$g_{12} = \frac{75}{50} = 1.5$$

P 18.14 [a] $I_1 = y_{11}V_1 + y_{12}V_2; \quad I_2 = y_{21}V_1 + y_{22}V_2$

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{50 \times 10^{-6}}{10} = 5 \mu\text{S}$$

$$0 = y_{21}(20 \times 10^{-3}) + y_{22}(-5)$$

$$\therefore y_{22} = \frac{1}{5}y_{21}(20 \times 10^{-3}) = 20 \text{ nS}$$

$$200 \times 10^{-6} = y_{11}(10) \quad \text{so} \quad y_{11} = 20 \mu\text{S}$$

$$0.25 \times 10^{-6} = 20 \times 10^{-6}(20 \times 10^{-3}) + y_{12}(-5)$$

$$y_{12} = \frac{0.25 \times 10^{-6} - 0.4 \times 10^{-6}}{-5} = 30 \text{ nS}$$

Summary:

$$y_{11} = 20 \mu\text{S}; \quad y_{12} = 30 \text{ nS}; \quad y_{21} = 5 \mu\text{S}; \quad y_{22} = 20 \text{ nS}$$

[b] $y_{11} = \frac{\Delta g}{g_{22}}; \quad y_{12} = \frac{g_{12}}{g_{22}}; \quad y_{21} = \frac{-g_{21}}{g_{22}}; \quad y_{22} = \frac{1}{g_{22}}$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = (12.5 \times 10^{-6})(50 \times 10^6) - 1.5(-250)$$

$$= 625 + 375 = 1000$$

$$y_{11} = \frac{1000}{50 \times 10^6} = 20 \mu\text{S}; \quad y_{21} = \frac{250}{5 \times 10^6} = 5 \mu\text{S}$$

$$y_{12} = \frac{1.5}{50 \times 10^6} = 30 \text{ nS}; \quad y_{22} = \frac{1}{5 \times 10^6} = 20 \text{ nS}$$

These values are the same as those in part (a).

P 18.15 $V_1 = h_{11}I_1 + h_{12}V_2; \quad I_2 = h_{21}I_1 + h_{22}V_2$

Solve the first equation for I_1 and the second equation for V_2 :

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2; \quad V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1$$

Work with the I_1 equation, substituting in the expression for V_2 :

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} \left[\frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1 \right]$$

$$I_1 \left[1 - \frac{h_{12}h_{21}}{h_{11}h_{22}} \right] = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}h_{22}} I_2$$

Thus,

$$g_{11} = \frac{1/h_{11}}{1 - (h_{12}h_{21})/(h_{11}h_{22})} = \frac{h_{22}}{h_{11}h_{22} - h_{12}h_{21}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = \frac{-h_{12}/(h_{11}h_{22})}{1 - (h_{12}h_{21})/(h_{11}h_{22})} = \frac{-h_{12}}{h_{11}h_{22} - h_{12}h_{21}} = \frac{-h_{12}}{\Delta h}$$

Now work with the V_2 equation, substituting in the expression for I_1 :

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}} \left[\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2 \right]$$

$$V_2 \left[1 - \frac{h_{12}h_{21}}{h_{11}h_{22}} \right] = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{11}h_{22}} V_1$$

Thus,

$$g_{21} = \frac{-h_{21}/(h_{11}h_{22})}{1 - (h_{12}h_{21})/(h_{11}h_{22})} = \frac{-h_{21}}{h_{11}h_{22} - h_{12}h_{21}} = \frac{-h_{21}}{\Delta h}$$

$$g_{22} = \frac{1/h_{22}}{1 - (h_{12}h_{21})/(h_{11}h_{22})} = \frac{h_{11}}{h_{11}h_{22} - h_{12}h_{21}} = \frac{h_{11}}{\Delta h}$$

$$\text{P 18.16 } V_1 = a_{11}V_2 - a_{12}I_2; \quad I_1 = a_{21}V_2 - a_{22}I_2$$

$$I_2 = \frac{a_{21}}{a_{22}}V_2 - \frac{1}{a_{22}}I_1$$

$$\therefore \quad h_{21} = \frac{-1}{a_{22}} \quad \text{and} \quad h_{22} = \frac{a_{21}}{a_{22}}$$

$$V_1 = a_{11}V_2 - a_{12} \left(\frac{a_{21}}{a_{22}}V_2 - \frac{1}{a_{22}}I_1 \right) = \left(a_{11} - \frac{a_{12}a_{21}}{a_{22}} \right) V_2 + \frac{a_{12}}{a_{22}}I_1$$

$$\therefore \quad h_{11} = \frac{a_{12}}{a_{22}} \quad \text{and} \quad h_{12} = a_{11} - \frac{a_{12}a_{21}}{a_{22}} = \frac{\Delta a}{a_{22}}$$

$$\text{P 18.17 } I_1 = g_{11}V_1 + g_{12}I_2; \quad V_2 = g_{21}V_1 + g_{22}I_2$$

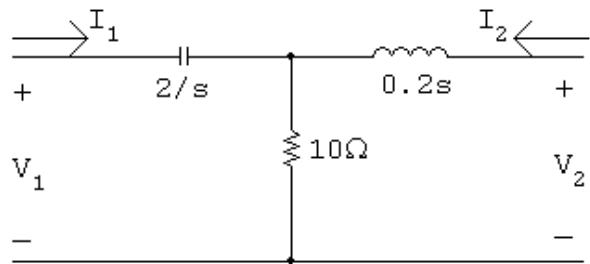
$$I_2 = \frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1$$

$$\therefore \quad y_{21} = \frac{-g_{21}}{g_{22}} \quad \text{and} \quad y_{22} = \frac{1}{g_{22}}$$

$$I_1 = g_{11}V_1 + g_{12} \left(\frac{1}{g_{22}}V_2 - \frac{g_{21}}{g_{22}}V_1 \right) = \left(g_{11} - \frac{g_{12}g_{21}}{g_{22}} \right) V_1 + \frac{g_{12}}{g_{22}}V_2$$

$$\therefore \quad y_{11} = g_{11} - \frac{g_{12}g_{21}}{g_{22}} = \frac{\Delta g}{g_{22}} \quad \text{and} \quad y_{12} = \frac{g_{12}}{g_{22}}$$

P 18.18



For $I_2 = 0$:

$$V_1 = \left(\frac{2}{s} + 10 \right) I_1 \quad \text{so} \quad z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{10s + 2}{s} \Omega$$

$$V_2 = 10I_1 \quad \text{so} \quad z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = 10 \Omega$$

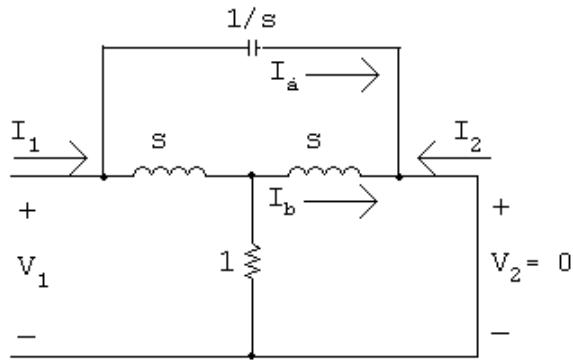
For $I_1 = 0$:

$$V_1 = 10I_2 \quad \text{so} \quad z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = 10 \Omega$$

$$V_2 = (0.2s + 10)I_2 \quad \text{so} \quad z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = 0.2s + 10 \Omega$$

$$\text{P 18.19 } I_1 = y_{11}V_1 + y_{12}V_2; \quad I_2 = y_{21}V_1 + y_{22}V_2$$

Since the circuit is symmetric and reciprocal, $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{(1/s)} + \frac{V_1}{s + (s/s + 1)}$$

$$\frac{I_1}{V_1} = s + \frac{s+1}{s^2 + 2s} = \frac{s^3 + 2s^2 + s + 1}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^3 + 2s^2 + s + 1}{s(s+2)}$$

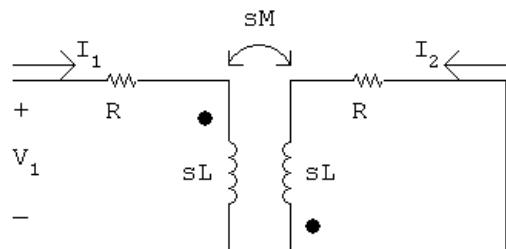
$$I_a = V_1 s; \quad I_b = \frac{V_1(s+1)}{s(s+2)} \cdot \frac{1}{(s+1)} = \frac{V_1}{s(s+2)}$$

$$I_2 = -(I_a + I_b) = -\left[V_1 s + \frac{V_1}{s(s+2)}\right]$$

$$\frac{I_2}{V_1} = -\left[s + \frac{1}{s(s+2)}\right] = -\frac{s^3 + 2s^2 + 1}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{s^3 + 2s^2 + 1}{s(s+2)}$$

P 18.20 [a] $a_{12} = \frac{-V_1}{I_2} \Big|_{V_2=0}; \quad a_{22} = \frac{-I_1}{I_2} \Big|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

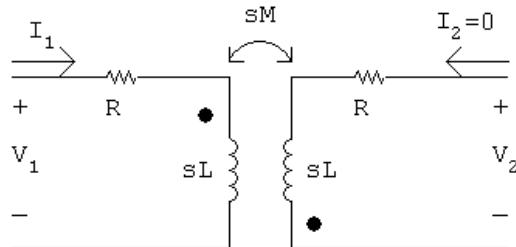
$$\Delta = \begin{vmatrix} (R + sL) & -sM \\ -sM & (R + sL) \end{vmatrix} = (R + sL)^2 - s^2 M^2$$

$$N_2 = \begin{vmatrix} R + sL & V_1 \\ -sM & 0 \end{vmatrix} = sMV_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{sMV_1}{(R + sL)^2 - s^2 M^2}; \quad a_{12} = -\frac{V_1}{I_2} = \frac{(sM)^2 - (R + sL)^2}{sM}$$

$$0 = -sMI_1 + (R + sL)I_2; \quad \therefore a_{22} = \frac{-I_1}{I_2} = \frac{-(R + sL)}{sM}$$

$$a_{11} = \frac{V_1}{V_2} \Big|_{I_2=0}; \quad a_{21} = \frac{I_1}{V_2} \Big|_{I_2=0}$$



$$V_2 = -sMI_1; \quad I_1 = \frac{V_1}{R + sL}$$

$$V_2 = \frac{-sMV_1}{R + sL}; \quad a_{11} = \frac{V_1}{V_2} = \frac{R + sL}{-sM}$$

$$a_{21} = \frac{I_1}{V_2} = \frac{-1}{sM}$$

[b] $a_{11} = a_{22}$ (symmetric, reciprocal)

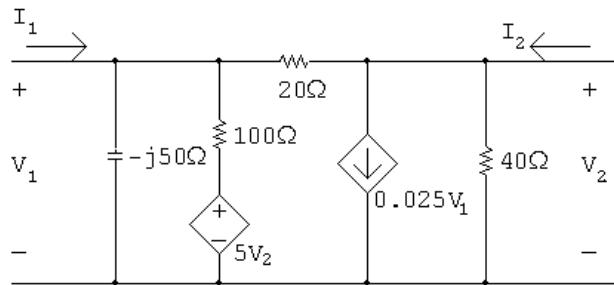
$$a_{11}a_{22} - a_{12}a_{21} = 1 \quad (\text{reciprocal})$$

$$a_{11} = \frac{R + sL}{-sM}; \quad a_{22} = \frac{R + sL}{-sM} \quad (\text{checks})$$

$$a_{11}a_{22} - a_{12}a_{21} = \frac{(R + sL)^2}{(sM)^2} + \frac{(sM)^2 - (R + sL)^2}{sM} \cdot \frac{1}{sM}$$

$$= \frac{(R + sL)^2 + (sM)^2 - (R + sL)^2}{(sM)^2} = 1 \quad (\text{checks})$$

P 18.21 For $I_2 = 0$:



$$\frac{V_2 - V_1}{20} + 0.025V_1 + \frac{V_2}{40} = 0$$

$$2V_2 - 2V_1 + V_1 + V_2 = 0 \quad \text{so} \quad 3V_2 = V_1$$

$$\therefore a_{11} = \left. \frac{V_1}{V_2} \right|_{I_2=0} = 3$$

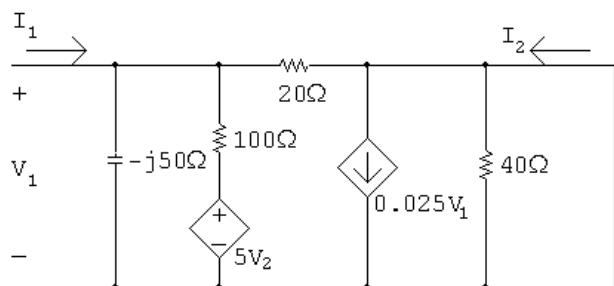
$$\begin{aligned} I_1 &= \frac{V_1}{-j50} + \frac{V_1 - 5V_2}{100} + \frac{V_1 - V_2}{20} \\ &= V_1 \left[\frac{j}{50} + \frac{1}{100} + \frac{1}{20} \right] - V_2 \left[\frac{5}{100} + \frac{1}{20} \right] \\ &= V_1 \left[\frac{6+j2}{100} \right] - V_2 \left[\frac{1}{10} \right] \end{aligned}$$

But $V_1 = 3V_2$ so

$$I_1 = \left[\frac{18+j6-10}{100} \right] V_2 = (0.08 + j0.06)V_2$$

$$a_{21} = \left. \frac{I_1}{V_2} \right|_{I_2=0} = 0.08 + j0.06 \text{ S} = 80 + j60 \text{ mS}$$

For $V_2 = 0$:



$$I_1 = \frac{V_1}{-j50} + \frac{V_1}{100} + \frac{V_1}{20} = V_1 \frac{(6+j2)}{100}$$

$$I_2 = 0.025V_1 - \frac{V_1}{20} = -0.025V_1$$

$$a_{12} = -\frac{V_1}{I_2} \Big|_{V_2=0} = \frac{1}{0.025} = 40 \Omega$$

$$a_{22} = -\frac{I_1}{I_2} \Big|_{V_2=0} = \frac{-2V_1(3+j1)}{100(-0.025)V_1} = 2.4 + j0.8$$

Summary:

$$a_{11} = 3; \quad a_{12} = 40 \Omega; \quad a_{21} = 80 + j60 \text{ mS}; \quad a_{22} = 2.4 + j0.8$$

$$\text{P 18.22 } h_{11} = \frac{a_{12}}{a_{22}} = \frac{40}{(0.8)(3+j1)} = 15 - j5 \Omega$$

$$h_{12} = \frac{\Delta a}{a_{22}}$$

$$\Delta a = 3(2.4 + j0.8) - 40(0.08 + j0.06) = 7.2 + j2.4 - 3.2 - j2.4 = 4$$

$$h_{12} = \frac{4}{(0.8)(3+j1)} = 1.5 - j0.50$$

$$h_{21} = -\frac{1}{a_{22}} = \frac{-1}{(0.8)(3+j1)} = -0.375 + j0.125$$

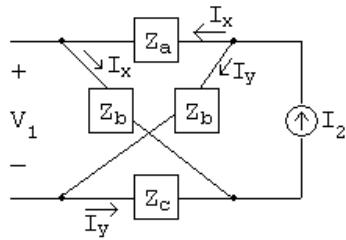
$$h_{22} = \frac{a_{21}}{a_{22}} = \frac{0.08 + j0.06}{(0.8)(3+j1)} = 0.0375 + j0.0125 = 37.5 + j12.5 \text{ mS}$$

P 18.23 First we note that

$$z_{11} = \frac{(Z_b + Z_c)(Z_a + Z_b)}{Z_a + 2Z_b + Z_c} \quad \text{and} \quad z_{22} = \frac{(Z_a + Z_b)(Z_b + Z_c)}{Z_a + 2Z_b + Z_c}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}; \quad \text{Use the circuit below:}$$

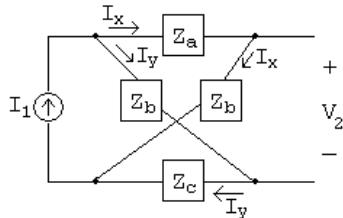


$$V_1 = Z_b I_x - Z_c I_y = Z_b I_x - Z_c (I_2 - I_x) = (Z_b + Z_c) I_x - Z_c I_2$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_2 \quad \text{so} \quad V_1 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_2 - Z_c I_2$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c}$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}; \quad \text{Use the circuit below:}$$



$$V_2 = Z_b I_x - Z_c I_y = Z_b I_x - Z_c (I_1 - I_x) = (Z_b + Z_c) I_x - Z_c I_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_1 \quad \text{so} \quad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_1 - Z_c I_1$$

$$\therefore z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

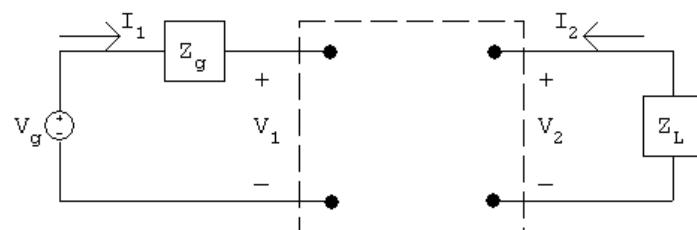
$$\text{P 18.24 } V_1 = a_{11}V_2 - a_{12}I_2; \quad V_1 = V_g - Z_g I_1$$

$$I_1 = a_{21}V_2 - a_{22}I_2; \quad V_2 = -Z_L I_2$$

$$V_1 = a_{11}V_2 - a_{12} \frac{V_2}{-Z_L} = \left(a_{11} + \frac{a_{12}}{Z_L} \right) V_2 = \frac{a_{11}Z_L + a_{12}}{Z_L} V_2$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{12} + a_{11}Z_L}$$

$$\text{P 18.25}$$



$$I_1 = y_{11}V_1 + y_{12}V_2; \quad V_1 = V_g - I_1 Z_g$$

$$I_2 = y_{21}V_1 + y_{22}V_2; \quad V_2 = -Z_L I_2$$

$$V_2 = -Z_L(y_{21}V_1 + y_{22}V_2)$$

$$V_2(1 + Z_L y_{22}) = -Z_L y_{21} V_1$$

$$V_2 = \frac{-Z_L y_{21}}{1 + Z_L y_{22}} V_1$$

$$I_1 = y_{11}V_1 - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}} V_1$$

$$\frac{I_1}{V_1} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + Z_L y_{22}}$$

$$\text{P 18.26 } V_1 = h_{11}I_1 + h_{12}V_2; \quad V_1 = V_g - I_1 Z_g$$

$$I_2 = h_{21}I_1 + h_{22}V_2; \quad V_2 = -Z_L I_2$$

$$I_2 = h_{21}I_1 + h_{22}(-Z_L I_2)$$

$$(1 + h_{22}Z_L)I_2 = h_{21}I_1$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\text{P 18.27 } I_1 = g_{11}V_1 + g_{12}I_2; \quad V_1 = V_g - I_1 Z_g$$

$$V_2 = g_{21}V_1 + g_{22}I_2; \quad V_2 = -Z_L I_2$$

$$V_{\text{Th}} = V_2 \Big|_{I_2=0} :$$

$$V_2 = g_{21}V_1; \quad V_1 = \frac{I_1}{g_{11}} = \frac{V_g - V_1}{Z_g g_{11}}$$

$$\therefore V_1(1 + Z_g g_{11}) = V_g \quad \text{so} \quad V_1 = \frac{V_g}{1 + Z_g g_{11}}$$

Thus,

$$V_2 = V_{\text{Th}} = \frac{g_{21}V_g}{1 + Z_g g_{11}}$$

$$Z_{\text{Th}} = \frac{V_2}{I_2} \Big|_{V_g=0} :$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$V_1 = \frac{I_1 - g_{12}I_2}{g_{11}} = \frac{-(V_1/Z_g) - g_{12}I_2}{g_{11}} = \frac{-V_1 - g_{12}Z_g I_2}{Z_g g_{11}}$$

$$(1 + Z_g g_{11})V_1 = -g_{12}Z_g I_2$$

$$\therefore V_2 = g_{21} \left(\frac{-g_{12}Z_g I_2}{1 + Z_g g_{11}} \right) + g_{22}I_2 = \left(g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_g g_{11}} \right) I_2$$

Thus,

$$Z_{\text{Th}} = \frac{V_2}{I_2} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + Z_g g_{11}}$$

$$\text{P 18.28 } V_2 = b_{11}V_1 - b_{12}I_1; \quad V_1 = V_g - I_1 Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \quad V_2 = -Z_L I_2$$

$$V_g - Z_g I_1 = \frac{1}{b_{11}}V_2 + \frac{b_{12}}{b_{11}}I_1$$

$$V_g = \frac{1}{b_{11}}V_2 + \left(\frac{b_{12}}{b_{11}} + Z_g \right) I_1$$

$$I_1 = \frac{V_g - V_2/b_{11}}{Z_g + (b_{12}/b_{11})} = \frac{b_{11}V_g - V_2}{Z_g b_{11} + b_{12}}$$

$$\frac{-V_2}{Z_L} = b_{21}V_1 - b_{22}I_1 = b_{21}(V_g - Z_g I_1) - b_{22}I_1$$

$$= b_{21}V_g - (Z_g b_{21} + b_{22})I_1 = b_{21}V_g - (Z_g b_{21} + b_{22}) \left[\frac{b_{11}V_g - V_2}{Z_g b_{11} + b_{12}} \right]$$

$$V_2 \left(-\frac{1}{Z_L} - \frac{Z_g b_{21} + b_{22}}{Z_g b_{11} + b_{12}} \right) = \left(b_{21} - \frac{(Z_g b_{21} + b_{22})b_{11}}{Z_g b_{11} + b_{12}} \right) V_g$$

$$V_2 \left(\frac{Z_g b_{11} + b_{12} + Z_g Z_L b_{21} + b_{22}Z_L}{Z_L(Z_g b_{11} + b_{12})} \right) = \left(\frac{Z_g b_{11}b_{21} + b_{22}b_{11} - Z_g b_{11}b_{21} - b_{12}b_{21}}{Z_g b_{11} + b_{12}} \right) V_g$$

$$\frac{V_2}{V_g} = \frac{Z_L(b_{11}b_{22} - b_{12}b_{21})}{b_{12} + Z_g b_{11} + Z_L b_{22} + Z_g Z_L b_{21}} = \frac{Z_L \Delta b}{b_{12} + Z_g b_{11} + Z_L b_{22} + Z_g Z_L b_{21}}$$

$$P\ 18.29 \quad I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

From the first measurement:

$$g_{11} = \frac{I_1}{V_1} = \frac{100 \times 10^{-6}}{0.1} = 1 \text{ mS}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{200}{0.1} = 2000$$

From the second measurement:

$$g_{12} = \frac{I_1}{I_2} = \frac{-25 \times 10^{-6}}{5 \times 10^{-3}} = -0.005$$

$$g_{22} = \frac{V_2}{I_2} = \frac{200}{5 \times 10^{-3}} = 40 \text{ k}\Omega$$

Summary:

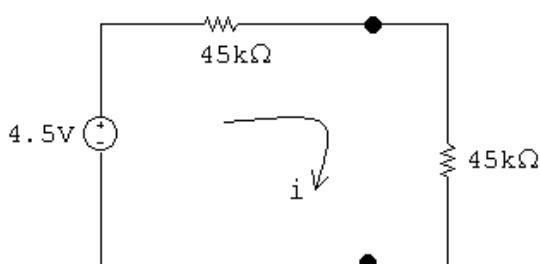
$$g_{11} = 1 \text{ mS}; \quad g_{12} = -0.005; \quad g_{21} = 2000; \quad g_{22} = 40 \text{ k}\Omega$$

From the circuit,

$$Z_g = 1 \text{ k}\Omega; \quad V_g = 4.5 \text{ mV}$$

$$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g} = 40,000 + \frac{10(1000)}{1 + 1} = 45,000$$

$$V_{\text{Th}} = \frac{g_{21}V_g}{1 + g_{11}Z_g} = \frac{2000(0.0045)}{1 + 1} = 4.5 \text{ V}$$



$$i = \frac{4.5}{90,000} = 50 \mu\text{A}$$

$$P = (50 \times 10^{-6})^2(45,000) = 112.5 \mu\text{W}$$

$$\text{P 18.30 [a]} \quad Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right) \left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3 \Omega$$

$$\therefore Z_L = 2.1 - j1.3 \Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right) (2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2} (42/\underline{0^\circ}) = 5 - j6 \text{ V(rms)} = 7.81/\underline{-50.19^\circ} \text{ V(rms)}$$

The rms value of V_2 is 7.81 V.

$$\text{[b]} \quad \mathbf{I}_2 = \frac{-\mathbf{V}_2}{Z_L} = \frac{-5 + j6}{2.1 - j1.3} = -3 + j1 \text{ A(rms)}$$

$$P = |\mathbf{I}_2|^2 (2.1) = 21 \text{ W}$$

$$\text{[c]} \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right) \left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right) \left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right) (2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

$$\begin{aligned}\frac{\mathbf{I}_2}{\mathbf{I}_1} &= \frac{-(1/2) - j(1/2)}{0.8 - j0.9} \\ \therefore \mathbf{I}_1 &= \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1} \right) \mathbf{I}_2 \\ &= (-1.7 + j0.1)(-3 + j1) = 5 - j2 \text{ A (rms)}\end{aligned}$$

$$\therefore P_g(\text{developed}) = (42)(5) = 210 \text{ W}$$

$$\% \text{ delivered} = \frac{21}{210}(100) = 10\%$$

P 18.31 [a] $\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(70,000) = -35$$

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(70 \times 10^3) = -2.5$$

$$y_{21}Z_L = (100 \times 10^{-3})(70 \times 10^3) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35 - (6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80) \times 10^{-3} = -28 \text{ V(rms)}$$

$$V_2 = 28 \text{ V(rms)}$$

[b] $P = \frac{|\mathbf{V}_2|^2}{70,000} = 11.2 \times 10^{-3} = 11.20 \text{ mW}$

[c] $\mathbf{I}_2 = \frac{-28/180^\circ}{70,000} = -0.4 \times 10^{-3}/180^\circ = 400/0^\circ \mu\text{A}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\begin{aligned}\Delta y &= (2 \times 10^{-3})(-50 \times 10^{-6}) - (-2 \times 10^{-6})(100 \times 10^{-3}) \\ &= 100 \times 10^{-9}\end{aligned}$$

$$\Delta y Z_L = (100)(70) \times 10^3 \times 10^{-9} = 7 \times 10^{-3}$$

$$y_{11} + \Delta y Z_L = 2 \times 10^{-3} + 7 \times 10^{-3} = 9 \times 10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{9 \times 10^{-3}} = \frac{100}{9}$$

$$\therefore 100\mathbf{I}_1 = 9\mathbf{I}_2; \quad \mathbf{I}_1 = \frac{9(400 \times 10^{-6})}{100} = 36 \mu\text{A(rms)}$$

$$P_g = (80)10^{-3}(36) \times 10^{-6} = 2.88 \mu\text{W}$$

$$P\ 18.32 \ [a] \ Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

From the solution to Problem 18.31

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{Th} = \frac{6}{200} \times 10^6 = 30,000 \Omega$$

$$Z_L = Z_{Th}^* = 30,000 \Omega$$

$$[b] \ y_{21}Z_L = (100 \times 10^{-3})(30,000) = 3000$$

$$y_{12}y_{21}Z_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30,000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20/\underline{180^\circ} \text{ V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30,000} = \frac{400}{30} \times 10^{-3} = 13.33 \text{ mW}$$

$$[c] \ I_2 = \frac{-\mathbf{V}_2}{30,000} = \frac{20/\underline{0^\circ}}{30,000} = \frac{2}{3} \text{ mA}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30,000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30} \text{ mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3}) \left(\frac{1}{30} \times 10^{-3} \right) = \frac{8}{3} \mu\text{W}$$

$$P\ 18.33 \ \frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = (25)(-40) - (1000)(-1.25) = 250$$

$$\therefore \frac{V_2}{V_g} = \frac{250(100)}{1000 + 25(20) - 40(100) - 1.25(2000)} = -5$$

$$V_2 = -5(120/\underline{0^\circ}) = 600/\underline{180^\circ} \text{ V(rms)}$$

$$I_2 = \frac{-V_2}{100} = \frac{-600/180^\circ}{100} = 6 \text{ A(rms)}$$

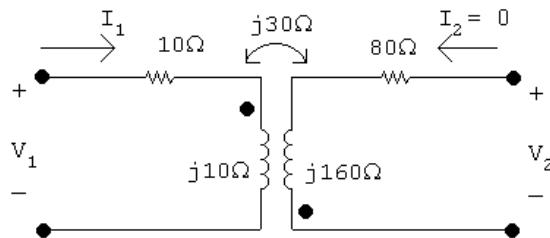
$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L} = \frac{-250}{25 - 1.25(100)} = 2.5$$

$$\therefore I_1 = \frac{I_2}{2.5} = \frac{6}{2.5} = 2.4 \text{ A(rms)}$$

$$\therefore P_g = (120)(2.4) = 288 \text{ W}; \quad P_o = 36(100) = 3600 \text{ W}$$

$$\therefore \frac{P_o}{P_g} = \frac{3600}{288} = 12.5$$

P 18.34 [a] For $I_2 = 0$:

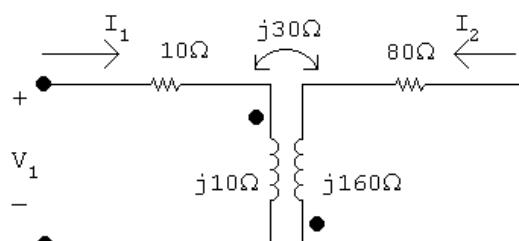


$$V_2 = -j30I_1 = -j30 \frac{V_1}{10 + j10} = \frac{-j3V_1}{1 + j1}$$

$$a_{11} = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{1 + j1}{-j3} = \frac{-1 + j1}{3}$$

$$a_{21} = \frac{I_1}{V_2} \Big|_{I_2=0} = \frac{1}{-j30} = \frac{j}{30} \text{ S}$$

For $V_2 = 0$:



$$V_1 = (10 + j10)I_1 - j30I_2$$

$$0 = -j30I_1 + (80 + j160)I_2$$

$$\Delta = \begin{vmatrix} 10 + j10 & -j30 \\ -j30 & 80 + j160 \end{vmatrix} = 100(1 + j24)$$

$$N_2 = \begin{vmatrix} 10 + j10 & V_1 \\ -j30 & 0 \end{vmatrix} = j30V_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{j30V_1}{100(1 + j24)}$$

$$a_{12} = \frac{-V_1}{I_2} \Big|_{V_2=0} = -80 + j\frac{10}{3} \Omega$$

$$j30I_1 = (80 + j160)I_2$$

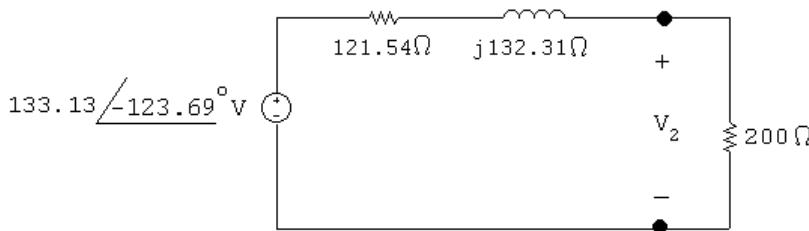
$$a_{22} = -\frac{I_1}{I_2} \Big|_{V_2=0} = -\frac{8}{3}(2 - j1)$$

$$[b] \quad V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{80/0^\circ}{(-1 + j1)/3 + j5/30} = \frac{(80/0^\circ)30}{-10 + j10 + j5} = \frac{2400/0^\circ}{-10 + j15}$$

$$= 133.13/-123.69^\circ \text{ V}$$

$$Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{[-(10/3)(24 - j1)] + [(-8/3)(2 - j1)(5)]}{[(-1 + j1)/3] + [(j/30)(5)]}$$

$$= 121.54 + j132.31 \Omega$$



$$[c] \quad V_2 = \frac{200}{121.54 + j132.31} (133.13/-123.69^\circ) = 132.87/-124.16^\circ$$

$$v_2(t) = 132.87 \cos(400t - 124.16^\circ) \text{ V}$$

P 18.35 When $V_2 = 0$

$$V_1 = 20 \text{ V}, \quad I_1 = 1 \text{ A}, \quad I_2 = -1 \text{ A}$$

When $I_1 = 0$

$$V_2 = 80 \text{ V}, \quad V_1 = 400 \text{ V}, \quad I_2 = 3 \text{ A}$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = \frac{20}{1} = 20 \Omega$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = \frac{400}{80} = 5$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} = \frac{-1}{1} = -1$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{3}{80} = 37.5 \text{ mS}$$

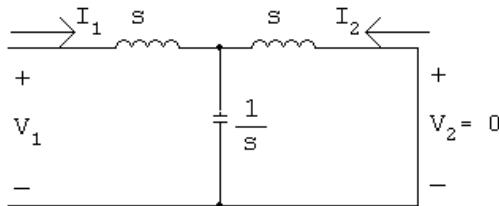
$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h} = 10 \Omega$$

Source-transform the current source and parallel resistance to get $V_g = 240$ V. Then,

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L} = -1.5 \text{ A}$$

$$P = (-1.5)^2(10) = 22.5 \text{ W}$$

P 18.36 [a] $y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}; \quad y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$



$$V_1 = \left[s + \left(\frac{1}{s} \| s \right) \right] I_1 = \frac{s(s^2 + 1) + s}{s^2 + 1} I_1$$

$$\therefore y_{11} = \frac{I_1}{V_1} = \frac{s^2 + 1}{s(s^2 + 2)}$$

$$I_2 = \frac{-(1/s)}{s + (1/s)} I_1 = \frac{-1}{s^2 + 1} \cdot \frac{s^2 + 1}{s(s^2 + 2)} V_1 = \frac{-1}{s(s^2 + 2)} V_1$$

$$\therefore y_{21} = \frac{-1}{s(s^2 + 2)}$$

Because the two-port circuit is symmetric,

$$y_{12} = y_{21} = \frac{-1}{s(s^2 + 2)} \quad \text{and} \quad y_{22} = y_{11} = \frac{s^2 + 1}{s(s^2 + 2)}$$

$$\begin{aligned}
 [\text{b}] \quad & \frac{V_2}{V_g} = \frac{y_{21}Z_g}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)} \\
 &= \frac{y_{21}}{y_{12}y_{21} - (1 + y_{11})(1 + y_{22})} \\
 &= \frac{-1}{\frac{s(s^2 + 2)}{\frac{1}{s^2(s^2 + 2)^2} - \left(1 + \frac{s^2 + 1}{s(s^2 + 2)}\right)\left(1 + \frac{s^2 + 1}{s(s^2 + 2)}\right)}} \\
 &= \frac{-s(s^2 + 2)}{1 - (s^3 + s^2 + 2s + 1)^2} \\
 &= \frac{1}{s^3 + 2s^2 + 3s + 2} \\
 &= \frac{1}{(s + 1)(s^2 + s + 2)} \\
 \therefore \quad & V_2 = \frac{10}{(s + 1)(s + 2)(s^2 + s + 2)}
 \end{aligned}$$

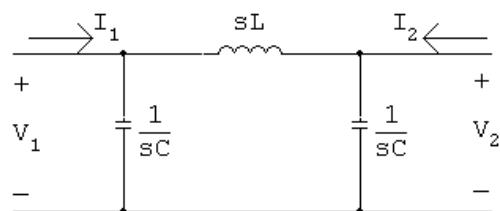
$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s + \frac{1}{2} - j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s + \frac{1}{2} + j\frac{\sqrt{7}}{2}}$$

$$K_1 = 5; \quad K_2 = -2.5; \quad K_3 = 1.34/159.3^\circ$$

$$\therefore v_2(t) = [5e^{-t} - 2.5e^{-2t} + 2.67e^{-0.5t} \cos(1.32t + 159.3^\circ)]u(t) \text{ V}$$

$$\text{P 18.37 [a]} \quad g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0}; \quad g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0}$$

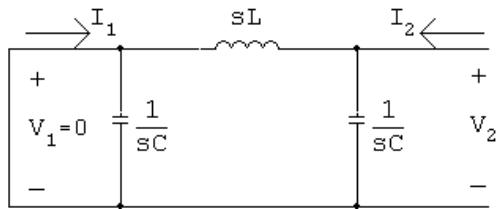


$$\begin{aligned}
 \frac{V_1}{I_1} &= \frac{1}{sC} \parallel \left(sL + \frac{1}{sC} \right) = \frac{[sL + (1/sC)](1/sC)}{sL + (2/sC)} \\
 &= \frac{sL + (1/sC)}{s^2LC + 2} = \frac{s^2LC + 1}{sC(s^2LC + 2)} = \frac{(1/C)[s^2 + (1/LC)]}{s[s^2 + (2/LC)]} \\
 \therefore \quad & g_{11} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}
 \end{aligned}$$

$$V_2 = \frac{(1/sC)}{sL + (1/sC)} V_1 \quad \text{so} \quad \frac{V_2}{V_1} = \frac{(1/sC)}{sL + (1/sC)} = \frac{1}{s^2 LC + 1} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$\therefore g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0}; \quad g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0}$$



$$I_1 = \frac{-(1/sC)}{sL + (1/sC)} I_2 \quad \text{so} \quad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{22} = sL/(1/sC) = \frac{sL/sC}{sL + (1/sC)} = \frac{sL}{s^2 LC + 1} = \frac{(1/C)s}{s^2 + (1/LC)}$$

Summary:

$$g_{11} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}; \quad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}; \quad g_{22} = \frac{(1/C)s}{s^2 + (1/LC)}$$

$$[\text{b}] \quad \frac{1}{LC} = \frac{10^9}{(0.05)(32)} = 625 \times 10^6$$

$$g_{11} = \frac{32 \times 10^{-9}s(s^2 + 1250 \times 10^6)}{s^2 + 625 \times 10^6}$$

$$g_{12} = \frac{-625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{21} = \frac{625 \times 10^6}{s^2 + 625 \times 10^6}$$

$$g_{22} = \frac{3125 \times 10^4 s}{s^2 + 625 \times 10^6}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L} = \frac{\left(\frac{625 \times 10^6}{s^2 + 625 \times 10^6}\right) 500}{\frac{3125 \times 10^4 s}{(s^2 + 625 \times 10^6)} + 500}$$

$$\frac{V_2}{V_1} = \frac{625 \times 10^6}{s^2 + 62,500s + 625 \times 10^6} = \frac{625 \times 10^6}{(s + 12,500)(s + 50,000)}$$

$$V_1 = \frac{4}{s}$$

$$V_2 = \frac{2500 \times 10^6}{s(s+12,500)(s+50,000)} = \frac{4}{s} - \frac{5.33}{s+12,500} + \frac{1.33}{s+50,000}$$

$$v_2 = [4 - 5.33e^{-12,500t} + 1.33e^{-50,000t}]u(t) \text{ V}$$

$$\text{P 18.38 } a'_{11} = \frac{z_{11}}{z_{21}} = \frac{35/3}{4000/3} = 8.75 \times 10^{-3} \Omega$$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{25 \times 10^4 / 3}{4000 / 3} = 62.5 \Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{4000/3} = 0.75 \times 10^{-3} \Omega$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{10,000/3}{4000/3} = 2.5 \Omega$$

$$a''_{11} = \frac{-y_{22}}{y_{21}} = \frac{-40 \times 10^{-6}}{-800 \times 10^{-6}} = 0.05 \text{ S}$$

$$a''_{12} = \frac{-1}{y_{21}} = \frac{-1}{-800 \times 10^{-6}} = 1250 \text{ S}$$

$$a''_{21} = \frac{-\Delta y}{y_{21}} = \frac{-4 \times 10^{-8}}{-800 \times 10^{-6}} = 50 \times 10^{-6} \text{ S}$$

$$a''_{22} = \frac{-y_{11}}{y_{21}} = \frac{-200 \times 10^{-6}}{-800 \times 10^{-6}} = 0.25 \text{ S}$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (8.75 \times 10^{-3})(0.05) + (62.5)(50 \times 10^{-6}) = 3.5625 \times 10^{-3}$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (8.75 \times 10^{-3})(1250) + (62.5)(0.25) = 26.5625$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (0.75 \times 10^{-3})(0.05) + (2.5)(50 \times 10^{-6}) = 162.5 \times 10^{-6}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (0.75 \times 10^{-3})(1250) + (2.5)(0.25) = 1.5625$$

$$\begin{aligned} V_2 &= \frac{Z_L V_g}{(a_{11} + a_{21} Z_g) Z_L + a_{12} + a_{22} Z_g} \\ &= \frac{(15,000)(0.03)}{[3.5625 \times 10^{-3} + (162.5 \times 10^{-6})(10)](15,000) + 26.5625 + (1.5625)(10)} = 3.75 \text{ V} \end{aligned}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{-5 \times 10^{-3}}{40} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25 \Omega$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-25}{40} \times 10^{-6} = -625 \times 10^{-9} \text{ S}$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

$$\text{or } a''_{11} = 1.25; \quad a''_{12} = 54 \text{ k}\Omega; \quad a''_{21} = \frac{1}{96} \text{ mS}; \quad a''_{22} = 1.25$$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(10^{-3}/96) = \frac{-10^{-2}}{24}$$

$$a_{12} = -125 \times 10^{-6}(54 \times 10^3) + (-25)(1.25) = -38 \Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(10^{-3}/96) = \frac{-10^{-4}}{96} \text{ S}$$

$$a_{22} = -625 \times 10^{-9}(54 \times 10^3) + (-25 \times 10^{-3})(1.25) = -65 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = \frac{-10^{-4}}{96}(800) = \frac{-10^{-2}}{12}$$

$$a_{11} + a_{21}Z_g = \frac{-10^{-2}}{24} + \frac{-10^{-2}}{12} = \frac{-10^{-2}}{8}$$

$$(a_{11} + a_{21}Z_g)Z_L = \frac{-10^{-2}}{8}(72,000) = -90$$

$$a_{22}Z_g = -65 \times 10^{-3}(800) = -52$$

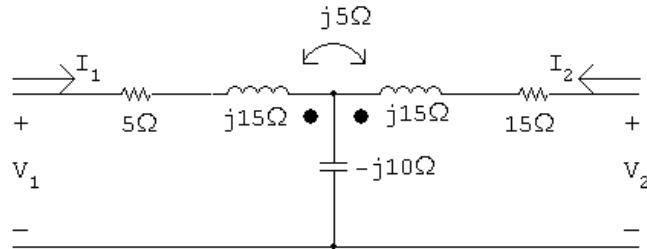
$$\frac{V_o}{V_g} = \frac{72,000}{-90 - 38 - 52} = -400$$

$$v_o = V_o = -400V_g = -3.6 \text{ V}$$

P 18.40 [a] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \quad \therefore \quad 5^2 - 24a'_{21} = 1, \quad a'_{21} = 1 \text{ S}$$

For network B



$$a''_{11} = \left. \frac{\mathbf{V}_1}{\mathbf{V}_2} \right|_{I_2=0}$$

$$\mathbf{V}_1 = (5 + j15 - j10)\mathbf{I}_1 = (5 + j5)\mathbf{I}_1$$

$$\mathbf{V}_2 = (-j10 + j5)\mathbf{I}_1 = -j5\mathbf{I}_1$$

$$a''_{11} = \frac{5 + j5}{-j5} = -1 + j1$$

$$a''_{21} = \left. \frac{\mathbf{I}_1}{\mathbf{V}_2} \right|_{I_2=0} = \frac{1}{-j5} = j0.2 \text{ S}$$

$$a''_{22} = a''_{11} = -1 + j1$$

$$\Delta a'' = 1 = (-1 + j1)(-1 + j1) - j0.2a''_{12}$$

$$\therefore a''_{12} = -10 + j5$$

Summary:

$$a'_{11} = 5 \quad a''_{11} = -1 + j1$$

$$a'_{12} = 24 \Omega \quad a''_{12} = -10 + j5 \Omega$$

$$a'_{21} = 1 \text{ S} \quad a''_{21} = j0.2 \text{ S}$$

$$a'_{22} = 5 \quad a''_{22} = -1 + j1$$

$$[b] \quad a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = -5 + j9.8$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = -74 + j49 \Omega$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = -1 + j2 \text{ S}$$

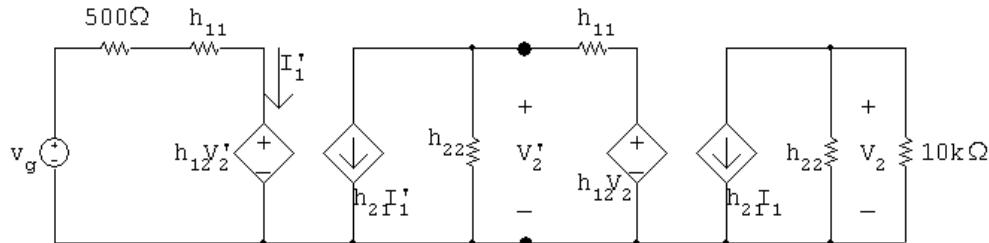
$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = -15 + j10$$

$$\mathbf{I}_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L} = 0.295 + j0.279 \text{ A}$$

$$\mathbf{V}_2 = -10\mathbf{I}_2 = -2.95 - j2.79 \text{ V}$$

P 18.41 [a] At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$

At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



$$[b] \frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$$

$$\text{therefore } I_1 = -2 \times 10^{-6}V_2$$

$$V'_2 = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$$

$$100I'_1 + 10^{-4}V'_2 + (-2 \times 10^{-6})V_2 = 0$$

$$\text{therefore } I'_1 = 205 \times 10^{-10}V_2$$

$$V_g = 1500I'_1 + 15 \times 10^{-4}V'_2 = 3000 \times 10^{-8}V_2$$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33,333$$

P 18.42 [a] $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$$

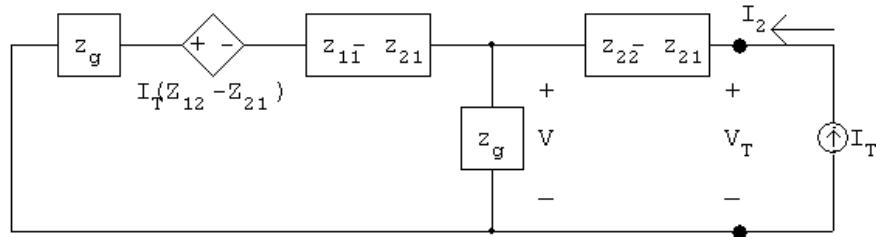
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}} \right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

$$\text{Thus } \frac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\frac{z_{12}z_{21}}{Z_g + z_{11}} \right)$$



For V_{Th} note that $V_{oc} = \frac{z_{21}}{Z_g + z_{11}}V_g$ since $I_2 = 0$.

$$P 18.43 [a] V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$$

$$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$$

[b] With port 2 terminated in an impedance Z_L , the two mesh equations are

$$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$$

$$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$$

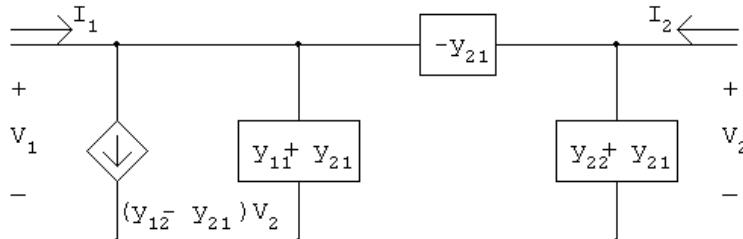
Solving for I_1 :

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

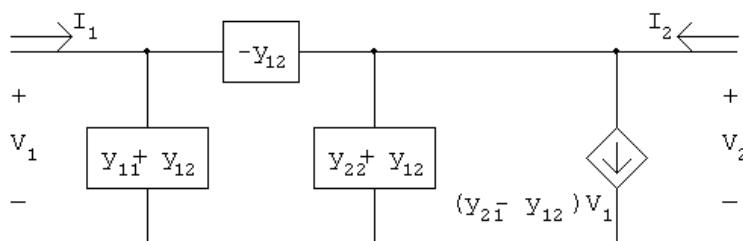
Therefore

$$Z_{in} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

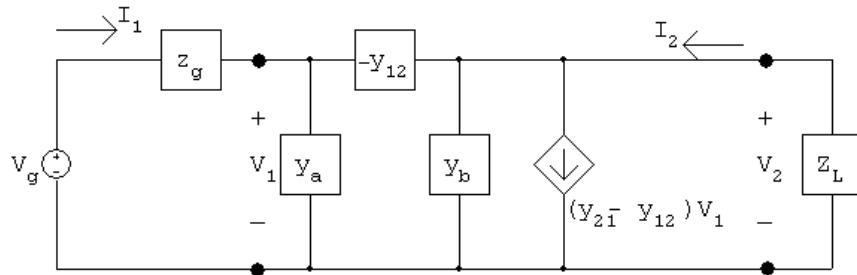
$$P 18.44 [a] I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \quad I_2 = y_{21}V_1 + y_{22}V_2$$



$$I_1 = y_{11}V_1 + y_{12}V_2; \quad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$$



[b] Using the second circuit derived in part [a], we have



$$\text{where } y_a = (y_{11} + y_{12}) \quad \text{and} \quad y_b = (y_{22} + y_{12})$$

At the input port we have

$$I_1 = y_a V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_b V_2 - y_{12}(V_2 - V_1) = 0$$

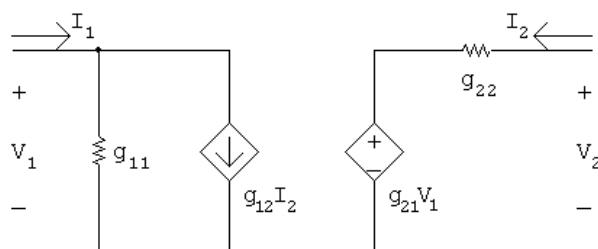
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L} \right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

- P 18.45 [a] The g -parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

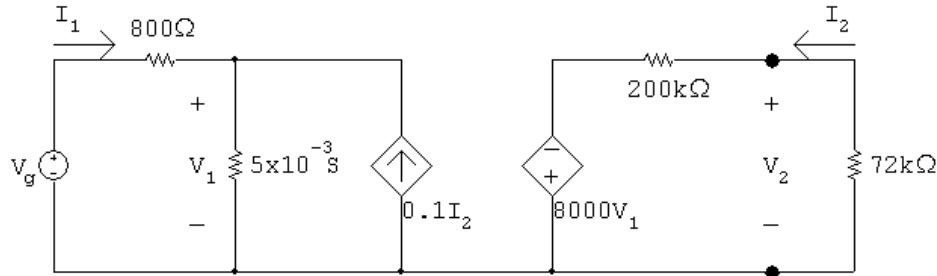
$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{25 \times 10^{-6}}{5 \times 10^{-3}} = 5 \times 10^{-3} \text{ S}$$

$$g_{12} = \frac{-h_{12}}{\Delta h} = \frac{-5 \times 10^{-4}}{5 \times 10^{-3}} = -0.10$$

$$g_{21} = \frac{-h_{21}}{\Delta h} = \frac{-40}{5 \times 10^{-3}} = -8000$$

$$g_{22} = \frac{h_{11}}{\Delta h} = \frac{1000}{5 \times 10^{-3}} = 200 \text{ k}\Omega$$

From Problem 3.67 $R_{ef} = 72 \text{ k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{-8000V_1(72)}{272}$$

$$I_2 = \frac{-V_2}{72,000} = \frac{8V_1}{272}$$

$$v_g = 9 \text{ mV}$$

$$\therefore \frac{V_1 - 9 \times 10^{-3}}{800} + V_1(5 \times 10^{-3}) - 0.1 \frac{8V_1}{272} = 0$$

$$V_1 - 9 \times 10^{-3} + 4V_1 - \frac{80V_1}{34} = 0$$

$$\therefore V_1 = 3.4 \times 10^{-3}$$

$$V_2 = \frac{-8000(72)}{272} \times 3.4 \times 10^{-3} = -7.2 \text{ V}$$

From Problem 3.67

$$\frac{V_o}{V_2} = 0.5; \quad \therefore V_o = -3.6 \text{ V}$$

This result matches the solution to Problem 18.39.

- P 18.46 [a] To determine b_{11} and b_{21} create an open circuit at port 1. Apply a voltage at port 2 and measure the voltage at port 1 and the current at port 2. To determine b_{12} and b_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.
 [b] The equivalent b -parameters for the black-box amplifier can be calculated as follows:

$$b_{11} = \frac{1}{h_{12}} = \frac{1}{10^{-3}} = 1000$$

$$b_{12} = \frac{h_{11}}{h_{12}} = \frac{500}{10^{-3}} = 500 \text{ k}\Omega$$

$$b_{21} = \frac{h_{22}}{h_{12}} = \frac{0.05}{10^{-3}} = 50 \text{ S}$$

$$b_{22} = \frac{\Delta h}{h_{12}} = \frac{23.5}{10^{-3}} = 23,500$$

Create an open circuit at port 1. Apply 1 V at port 2. Then,

$$b_{11} = \frac{V_2}{V_1} \Big|_{I_1=0} = \frac{1}{V_1} = 1000 \quad \text{so} \quad V_1 = 1 \text{ mV measured}$$

$$b_{21} = \frac{I_2}{V_1} \Big|_{I_1=0} = \frac{I_2}{10^{-3}} = 50 \text{ S} \quad \text{so} \quad I_2 = 50 \text{ mA measured}$$

Create a short circuit at port 1. Apply 1 V at port 2. Then,

$$b_{12} = -\frac{V_2}{I_1} \Big|_{V_1=0} = \frac{-1}{I_1} = 500 \text{ k}\Omega \quad \text{so} \quad I_1 = -2 \mu\text{A measured}$$

$$b_{22} = -\frac{I_2}{I_1} \Big|_{V_1=0} = \frac{-I_2}{-2 \times 10^{-6}} = 23,500 \quad \text{so} \quad I_2 = 47 \text{ mA measured}$$

- P 18.47 [a] To determine y_{11} and y_{21} create a short circuit at port 2. Apply a voltage at port 1 and measure the currents at ports 1 and 2. To determine y_{12} and y_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.
- [b] The equivalent y -parameters for the black-box amplifier can be calculated as follows:

$$y_{11} = \frac{1}{h_{11}} = \frac{1}{500} = 2 \text{ mS}$$

$$y_{12} = \frac{-h_{12}}{h_{11}} = \frac{-10^{-3}}{500} = -2 \mu\text{S}$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{1500}{500} = 3 \text{ S}$$

$$y_{22} = \frac{\Delta h}{h_{11}} = \frac{23.5}{500} = 47 \text{ mS}$$

Create a short circuit at port 2. Apply 1 V at port 1. Then,

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{I_1}{1} = 2 \text{ mS} \quad \text{so} \quad I_1 = 2 \text{ mA measured}$$

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{I_2}{1} = 3 \text{ S} \quad \text{so} \quad I_2 = 3 \text{ A measured}$$

Create a short circuit at port 1. Apply 1 V at port 2. Then,

$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{I_1}{1} = -2 \mu\text{S} \quad \text{so} \quad I_1 = -2 \mu\text{A measured}$$

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{I_2}{1} = 47 \text{ mS} \quad \text{so} \quad I_2 = 47 \text{ mA measured}$$