

Counting

$$P(A) = \frac{|A|}{|S|}$$

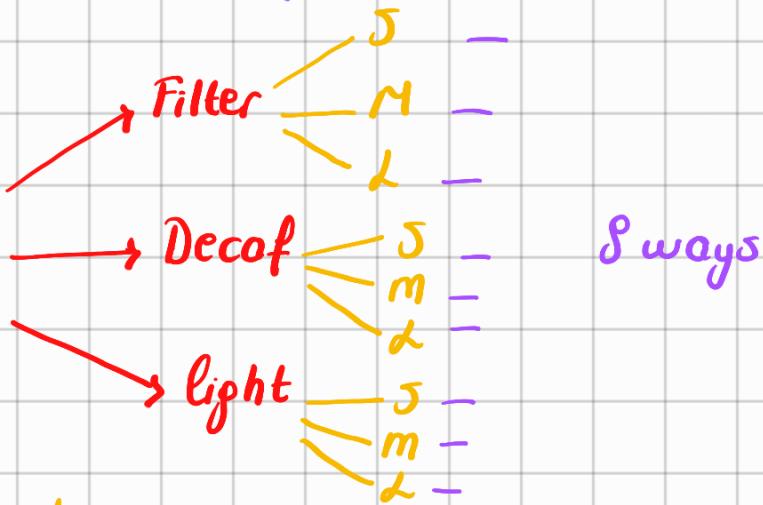
} equally likely outcome

the number "||" (cardinality)

Örneğin bir zar oturumunda reyo yaşı two oturumda bu olayların eşit derecede olası sonuçları olduğunu söylüyoruz

Coffee example S M L

elementlerin sayısını sayarak buluyoruz



in how many ways can you order your coffee?

127 drinks, 4 sizes, _____,

(ama mesela seçenekleri böyle verseydi ne yapardık
yükarda sayarak hallettik ama bunu
sayamayız)

$$3 \text{ types} \times 3 \text{ size} = 3^3$$

Bu multiplication principle'dir

(Bu yüzden multiplication principle kullanırız)

Multiplication principle

Please note that we perform n experiments

Suppose that we perform r experiments.
there are n possible outcomes for each experiments

$1 \leq k \leq r$

$1 \rightarrow n_1$
 $2 \rightarrow n_2$
 \vdots
 $k \rightarrow n_k$

There are a total of $n_1 \times n_2 \times \dots \times n_r$ possible outcomes

coffee \rightarrow Kohre örneğinde burda 2 experiment-mız var

1 coffee type $\rightarrow n_1 = 3$
2 size $\rightarrow n_2 = 3$

$n_1 \times n_2$
 $3 \times 3 = 9$

\hookrightarrow multiplication principle

Example: Choosing a password

\rightarrow begin with 2 lowercase letters followed by

\rightarrow 1 capital letter followed by

\rightarrow 4 digits

of total passwords?

a e G 4 3 7 7 ✓

A e g 4 0 3 7 x

Multiplication principle

e e C d d d d

$$= 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^3 \cdot 10^4$$

Sampling

Sampling from a set: randomly choosing an element from a set

Samplingde 2 yol vardır her örneklemeden sonra

① with replacement i put the sampled element back

her gizimden sonra
her nesneyi geri
takarız

$\rightarrow A = \{1, 2, 3, 4\}$ 2 2
(mevcut burda 2 yi seçtim sonra bunu
tekrar seçtiğimde 2 oluyor)

2) without replacement : repetition is not allowed

Burda ögeyi
peri toymuyoruz

Do not put the sampled
obj. back

② → {1, 3, 4}

2'yi aldik
mesela 2-
me artik
2 icermez

* Ordering = if ordering matters

(order is very important)

$(a_1, a_2, a_3) \neq (a_3, a_2, a_1)$

yani nesnelerin
siralomasi yani
olmali

not same in ordered sampling

$(a_1, a_2, a_3) = (a_1, a_2, a_3)$

* Unordered sampling = order is not important

$(a_1, a_2, a_3) = (a_3, a_1, a_2)$

unordered samplingte
sircları önemli değil

types of
Sampling

Sampling

Ordered

Unordered

with replacement

①

with out
replacement

②

with
replacement

④

with out
replacem-
ent

③

1 → Ordered sampling with replacement

example: $A = \{1, 2, 3\}$ draw $k=2$ samples

in how many ways the sampling can be done?

1 - (1, 2)



2 - (1, 3)

(with replace-
ment)

Yes, ordering

3 - (1, 1)

? ✓

$(1, 2) \neq (2, 1)$

4 - (2, 1)

? ✓

Ordered samples
Bu ikisi farklı outcome lar
dir. Order is important

5 - (2, 2) (with replacement)

6 - (2, 3)

9 different ways

7- (3.1)

8- (3.2)

9- (3.3)

What is the number
of possible lists?

n^k where k is number
of sampling, n is
the number of out-
comes

$\{1, 2, 3\}$ Burdon 1'i
oldum oldikton

Jonra tetrar yerine
koyuyorum Gündük with rep-
lacement demis.

Number of subsets?

A = {---} n elements. what is the number of
subsets for A?

(Bunu kullanabilir misiniz)

→ Multiplication principle?

A = {1, 2} → $\emptyset, \{1\}, \{2\}, \{1, 2\}$

↳ 4 different
subsets

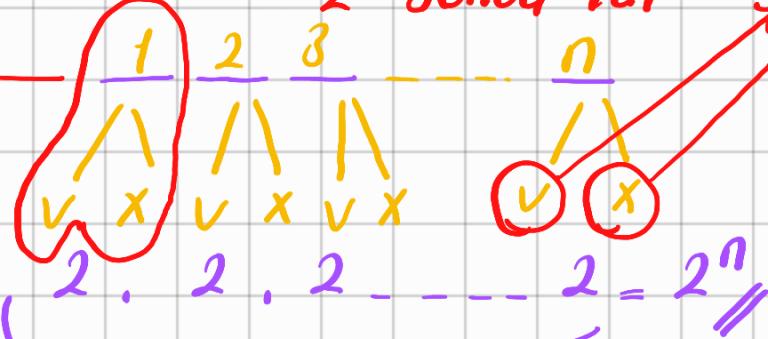
2^n

→ pördüpünüz gibi her eleman için
2 sonuc var

2 tone outcome:

var yok olt kümeye
yok ya do yok

Ben burda
multiplication
principle
kullanıyorum



n^k

n

② Ordered sampling without replacement

* No repetition

ex: $A = \{1, 2, 3\} \rightarrow k=2 \rightarrow$ ordered sampling without replacement

~~(1,1)~~ → {1, 2, 3} $k=1$ 1 $k=2$ $\begin{matrix} 2 \\ 3 \end{matrix}$ (without replacement)

(1, 2) → ✓

(1, 3) → ✓

(2, 1) → ordered (1, 2) ✓ (2, 1) ?

~~(2, 2)~~ → ? without replacement

(2, 3) → ✓

(3, 1) → ✓

(3, 2) → ✓

~~(3, 3)~~

$(a_1, a_2) \neq (a_2, a_1)$

Ordered
(order is matter)

Bunların ikisi-
de farklı

6 possible ways

neden 1 tane olsalılkı günde bunlardan
biri 20ten birinci sıradadır
 k draws n elements mi?

$$\frac{n \times (n-1) \times (n-2) \times \dots \times (n-k+1)}{1 \text{ draw} \quad 2 \text{ draw} \quad 3 \text{ draw} \quad \dots \quad k \text{ draws}} = \frac{n!}{(n-k)!}$$

k permuto-
 tion of
 elements

Example : k people in a class. what is the probability that at least two of them have the same birthday?

$n = 365$ days

minimum
yani

$$1 \text{ person} = \frac{1}{365}$$

A - event that 2 people have same birthday

A^c - No two people have the same birthday

Biz bunu hesaplaysak sonuçca ulaşırız

$$P(A) = 1 - \frac{|A^c|}{|S|} \rightarrow 365^k$$

ikinci kişi birinci kişiyle aynı gün doğmamalı

$$\frac{1}{365} \quad \frac{2}{364} \quad \frac{3}{363} \quad \dots \quad \frac{k}{365-k+1} \quad \frac{n!}{(n-k)!}$$

(yukarıda bir kişiyle aynı doğum gününe sahip olsun istemiyorum)

$$P(A) = 1 - \frac{|A^C|}{n} \rightarrow \frac{n!}{(n-k)! n^k}$$

Solution

Bunu garanti edebilirsin

$k > 365$? at least 2 of them
are born at same
day

k 365'ten büyükse
bu ne olurken gelir

peopenhole
principle

$$k=10 \quad \text{10 kişi yaradı}\quad \text{sinifta}\quad 365!$$
$$\frac{365!}{355!} = 365^{10}$$

③ k -Combination : Unordered Sampling without replacement

$$\binom{n}{k} = \frac{n!}{(n-k)! k!}$$

④ Unordered Sampling with replacement

$$A = \{1, 2, 3\}$$

(1,1) with replace

$$(1,2) = (2,1)$$

Bunların ikisi de
aynı şey

'ment

(1, 2)

6 possible lists

(1, 3)

$$(1, 2, 3) = X_1 + X_2 + X_3 = 2 + 0 + 0$$

~~(2, 1)~~

unordered sampling

(2, 2)

Bunlar

(2, 3)

secilen öge
sayısını gösteren
değerleridir

~~(3, 1)~~

~~(3, 2)~~

\rightarrow (2, 3) secildi çünkü

(3, 3)

X_1	X_2	X_3
1	1	0
1	0	1
0	1	1
0	1	0
0	0	1
0	0	2

Unordered without replacement

$$\binom{n}{k}$$

unordered with sum
replacement

k-1 kez
geri koyma-

Bu kaç defa
secileceğini
gösterir

Burda olsı liste-

Bu depistir-lerin sayısını bulu-
digim elemeler yorum
iğin

secdiğim ele-
man
sayısı

$$\binom{n+k-1}{k}$$

(yukardan farklı olarak burda tetraf
seciliyor)

Unordered Sampling with replacement

$$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

Bu itisi esit olur

Example : ten passengers get on on airport shuttle, the shuttle route includes 5 hotels, each passenger gets off his hotel

The driver records how many passengers leave the shuttle at each hotel. how many different possible lists?

$$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

$$k+n-1 = k+n-1$$

$$\binom{8}{5} = \binom{8}{3}$$

Bunların toplamı 8'e eşit
o yüzden eşitler

$$x_1 \ x_2 \ x_3 \ x_4 \ x_5$$

$$\begin{cases} 10 & 0 & 0 & 0 & 0 \end{cases} \rightarrow 10$$

{ 9 1 0 0 0 → 10

9 0 1 0 0 → 10

Unordered Sampling
with replacement

Bu depistirilcek
kisi sayisini
goosterir

($5+10-1$) / 10 → Solution

otel sayisi

