Integral Alma Yöntemleri

* Önce integral alma kurallarına bakılır. Yoksa yöntemlere başvurulur.

* Yöntemler, kurallara dönüştürür.

1 Değişken Değiştirme (u- substitution)

* Bu yöntem için

$$\int f(x) \cdot f'(x)\,dx \rightarrow$$

$f'(x)$ ve dx çarpım durumunda olmalıdır.

* $\int \sqrt{\sin x} \cdot \cos x\,dx$

* $\int \frac{2x}{x^2+1}\,dx$

⋮

$$\frac{du}{dx} = \frac{df(x)}{dx}$$

$$du = f'(x) \cdot dx$$

Soru:

$$\int e^{\overbrace{x^3}^{u}} \cdot \underbrace{3x^2 dx}_{du} =$$

$$\int e^u \cdot du = \frac{e^u}{1} + c$$

$$= e^{x^3} + c$$

* $\left.\begin{aligned} u &= f(x) \\ du &= f'(x) \cdot dx \end{aligned}\right\} \quad x \longrightarrow u$

* Sonuç u'lu çıkar. u yerine $f(x)$ konur.

Soru:

$$du=(2x-5).dx$$

$$\int \underbrace{(x^2-5x+1)}_{u}{}^{10}\cdot\underbrace{(2x-5)\,dx}_{du}$$

$$\int u^{10}.\,du=\frac{u^{11}}{11}+c=\frac{(x^2-5x+1)^{11}}{11}+c$$

Soru:

$$\int \frac{2x}{\underbrace{x^2+1}_{u}}\,dx=\int \frac{\overset{\frac{1}{u}}{(x^2+1)^2}\cdot \overbrace{-2x\,dx}^{}}{\underbrace{(x^2+1)^2}_{du}\cdot\underbrace{(x^2+1)}_{u}}\;\frac{-2x}{(x^2+1)^2}$$

$x^2+1=u$

$2x\,dx=du$

$$\int\frac{1}{u}.\,du=\ln|x^2+1|+c$$

$$\int\frac{1}{u}.\,du=\int \ln|u|+c=\ln|x^2+1|+c$$

Soru:

$$\int\underbrace{\sqrt{x^3+1}}_{u}\cdot\underbrace{3x^2\,dx}_{du}=\int\frac{\cancel{2}\,\overset{u}{(x^3+1)}}{\cancel{2}\sqrt{x^3+1}}\cdot\overset{du}{3x^2\,dx}$$

$u=x^3+1$

$du=3x^2\,dx$

$$=\int\sqrt{u}\,.\,du=\frac{2u^{\frac{3}{2}}}{3}+c=\frac{2\sqrt{(x^3+1)^3}}{3}+c$$

Soru:

$$\int e^{\overset{u}{\sin x}} \cdot \underbrace{\cos x \, dx}_{du} = \int du = u + c = e^{\sin x} + c$$

$u = \sin x$

$du = \cos x \, dx$

$$\int e^u \, du = e^u + c$$

Soru: $\ln x = u$

$\frac{dx}{x} = du$

$$\int \frac{\ln x}{x} dx = \int u \, du = \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$$

$u \, du = \frac{u^2}{2} + c$

Soru:

$$\int \frac{\sin(\overset{u}{\ln x})}{x} dx = \int \sin(u) \, du = -\cos(u) + c = -\cos(\ln x) + c$$

$$\int \sin(u) \, du = -\cos(u) + c$$

Soru:

$$\int \cos(\underset{u}{x^2}) \cdot \underbrace{2x \, dx}_{du} = \int \cos(u) \cdot du = \sin(u) + c = \sin(x^2) + c$$

$\sin(u) + c$

$$\int e^{\overset{u}{x^3}} \cdot \underbrace{x^2 \cdot dx}_{\frac{du}{3}} = \int \frac{e^u}{3} \cdot du = \frac{e^u}{3} + c = \frac{e^{x^3}}{3} + c$$

$x^3 = u$

$dx \; x^2 = \frac{du}{3}$

$$\int \frac{e^u}{3} du = \frac{e^u}{3} + c$$

Soru

$$\int \frac{x}{x^2+1} \cdot dx = \int \frac{1}{2u}\, du = \frac{\ln|u|}{2} + c = \frac{\ln|x^2+1|}{2} + c$$

$x^2+1 = u$

$x\,dx = \frac{du}{2}$

Soru

$$\int \sqrt[3]{x^3+1} \cdot x^2\, dx = \int \frac{\sqrt[3]{u} \cdot du}{3} = \frac{3u^{\frac{4}{3}}}{12} + c = \frac{3(x^3+1)^{\frac{4}{3}}}{12} + c$$

$x^2 dx = \frac{du}{3}$

$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \cdot dx = \int 2\sin(u) \cdot du = -2\cos(u) + c = -2\cos(\sqrt{x}) + c$$

u ; $2du$

$x^{\frac{1}{2}} = u$

$\frac{1}{\sqrt{x}} dx = 2du$ $\quad \int 2\sin(u)\, du$

Soru $\quad -2\cos(\sqrt{x}) + c$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} \cdot dx = \frac{u^2}{2} + c = \frac{(\arcsin x)^2}{2} + c$$

u ; du

$\arcsin x = u$

$\frac{dx}{\sqrt{1-x^2}} = du$

Soru

$$\int \sin^3 x \cdot \cos x\, dx = \int u^3 \cdot du = \frac{u^4}{4} + c = \frac{\sin^4 x}{4} + c$$

Soru

$$\int \tan^3 x \cdot \sec^2 x\, dx = \int u^3 \cdot du = \frac{u^4}{4} + c$$

$\tan x = u$

$\sec^2 x\, dx = du$ $\quad \frac{u^4}{4} + c$

$u^3 \cdot du$

$$\int \frac{\cos x}{\sin x + 1} \cdot dx = \int \frac{1}{u+1} \, du = \ln|u+1| + c$$

$$\ln|\sin x + 1| + c$$

Soru:

$$\int \cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) dx = \sin(u) + c = \sin\left(\frac{1}{x}\right) + c$$

Kısmi İntegral

- **L**ogaritma ⟶ $\ln x, \ln^2 x, \ln(2x+1), \log_2 x$
- **A**rc'lılar ⟶ arcsinx, ...
- **P**olinom ⟶ $x, 2x-1, x^2, x^3+x^2+5$ ($\frac{1}{x}$ veya $\sqrt{x}$ değildir)
- **T**rigonometri ⟶ sinx, cos3x ....
- **Ü**steller ⟶ $5^x, e^x, e^{3x}, 2^x$ ....

* yalnız durumunda,
* çarpım durumunda çözer
* bölü durumunu çözmez.

Yöntem

* integral u ve dv diye iki parçaya ayrılır.
* Neye u denileceğini $\overrightarrow{LAPTÜ}$ belirler.
* Önceki u'dur.
* geriye kalan her şey dv'dir.
* x olanın türevi yazılır.
* dv'nin integrali alınır.
* c ilk başta yazılmaz.
* $u.v - \int v\,du$ = sonucu verir.

Soru

$$\int \ln x\, dx =$$

$\ln x = u \longrightarrow \frac{1}{x} dx = du$

$dv = dx \longrightarrow v = x$

$$\ln x \cdot x - \int \cancel{x} \frac{1}{\cancel{x}} dx =$$

$$x\ln x - x + c =$$

$$\int \underset{u}{x}\, \underset{dv}{e^x dx}$$

$u = x$

$du = dx$

$e^x dx = dv$

$e^x = v$

$$x.e^x - \int e^x dx$$

$$x.e^x - e^x + c$$

$u = x \mid e^x dx = dv$

$\Downarrow$

$du = 1.dx \mid \int e^x dx = \int dv$

$du = dx \mid e^x + c = v$

ilk yazılmaz.

$$x.e^x - \int e^x dx = e^x(x-1) + c$$

Soru

LAPTÜ

$$\int \underset{\text{pol.}}{x} \underset{\text{trigo}}{\cos x}\, dx = ?$$

$$u.v - \int v\,du$$

$x = u \longrightarrow dx = du$

$\cos x\, dx = dv \longrightarrow \sin x = v$

$$x.\sin x - \int \sin x\, dx = x.\sin x + \cos x + c$$

$$\frac{x}{x^2+1} \cdot e^{|x|} \cdot dx$$

$x^2 + 1 = u$

$x = \sqrt{u-1}$

$x\,dx = \frac{du}{2}$

$$\frac{1}{2u} \cdot e^{|\sqrt{u-1}|} \cdot du$$

Polinom Bölmesi ile integral

$$\int \frac{\text{Pay}}{\text{Payda}}\,dx$$

① Pay ve payda polinom olmalı

② Payın derece paydadan büyük ya da eşit olmalı.

Bu durumda bu yöntem zorunludur!

$$\int \frac{x}{x+2}\,dx \rightarrow \begin{array}{r|l} x & x+2 \\ \underline{-\;} & 1 \\ -2 & \end{array} = \int 1 + \frac{-2}{x+2}\,dx$$

$x - 2\ln|x+2| + c$

$$\int \frac{x^2}{x-1}\,dx \Rightarrow \begin{array}{r|l} x^2 & x-1 \\ \underline{-\;x^2-x} & x+1 \\ +x & \\ \underline{-\;x-1} & \\ +1 & \end{array} \Rightarrow \int \left(x+1 + \frac{1}{x-1}\right) dx$$

$\frac{x^2}{2} + x + \ln|x-1| + c$

$$\int \frac{x^3}{x^2+1}\,dx = \begin{array}{r|l} x^3 & x^2+1 \\ \underline{-\;x^3+x} & x \\ -x & \end{array} \Rightarrow \int \left(x + \frac{-x}{x^2+1}\right) dx \Rightarrow \frac{x^2}{2} - \int \frac{x}{\underbrace{x^2+1}_{u}}\,dx$$

$x^2+1 = u$

$2x\,dx = du$

$\frac{1}{2}\ln|u| + c$

Basit Kesirlere Ayırarak integral Alma

$$\int \frac{\text{Pay}}{\text{Payda}}\,dx$$

① Pay ve payda polinom

② Paydanın derecesi > Payın derecesi

③ Payda çarpanlara ayrılabilir olmalıdır.

1. Çeşit

Payda tamamen 1. derece ise

$$\int \frac{2x-5}{x^2-3x-4} \quad \Bigg| \quad \frac{2x-5}{(x-4).(x+1)} = \frac{A}{x-4} + \frac{B}{x+1} \Rightarrow \int \frac{3/5}{x-4} + \frac{7/5}{x+1}\,dx$$

$$\begin{array}{cc} x & -4 \\ x & +1 \end{array}$$

$$\frac{2x-5}{(x-4).(x+1)} = \frac{Ax+A+Bx-4B}{(x-4).(x+1)}$$

$$\begin{array}{l} A+B=2 \\ A-4B=-5 \end{array} \Rightarrow \begin{array}{l} A=3/5 \\ B=7/5 \end{array}$$

$$\frac{3}{5}\ln|x-4| + \frac{7}{5}\ln|x+1| + c$$

$$\int \frac{1}{x^2-4}\,dx$$

$(x-2).(x+2)$

$A+B=0$

$2A-2B=1$

$4A=1$

$A=1/4$

$B=-1/4$

$$\Rightarrow \int \left(\frac{1/4}{x-2} + \frac{-1/4}{x+2} \right) dx$$

$$\frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + c$$

2. Çeşit

Paydada 2. derece çarpan bulunursa

$$\int \frac{2x+1}{(x^2+1).(x-2)}\,dx = \frac{A}{x-2} + \frac{Bx+c}{x^2+1}$$

$Ax^2+Bx^2+A-2C+Cx-2Bx =$

$A+B=0$

$C-2B=2$

$A-2C=1$

$B=-1$

$C=0$

$A=1$

$$\Rightarrow \int \left(\frac{1}{x-2} + \frac{-x}{x^2+1} \right) dx$$

$x^2+1=u$

$2x\,dx=du$

$$\ln|x-2| - \frac{1}{2}\ln|u| + c$$

3. Çeşit

Paydada tamkare ifadeler bulunursa

$$\int \frac{2x+1}{(x-3)^2.(x+1)}\,dx = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$Ax^2-6Ax+9A$

$Bx^2-2Bx-3B$

$+Cx+C$

$A+B=0$

$-6A-2B+C=2$

$9A-3B+C=1$

$A=-1/16$

$B=1/16$

$C=7/4$

$$\frac{-1}{16}\ln|x+1| + \frac{1}{16}\ln|x-3| + \frac{7}{4}\int \frac{1}{(x-3)^2}\,dx$$

$x-3 = u$, $dx = du$

$$\int \frac{1}{u^2}\,du$$

$$\frac{-1}{u} + c$$

$$= -\frac{7}{4}\cdot\frac{1}{x-3} + c$$

$\int \sin^4 x \cdot \cos^2 x \cdot \sin x \cdot dx$

$\downarrow$

$(1-\cos^2 x)^2 \cdot \cos^2 x \cdot \underbrace{\sin x \cdot dx}_{-t^2\, dt}$

$\cos x = t$

$-\sin x\, dx = dt$

$-t^2(1-t^2)^2$

$\int (-t^6 + 2t^4 - t^2)\, dt$

$\ln(g(x)) = \sin x \cdot \ln(f(x))$

$g'(x) = \left(\cos x \cdot \ln(f(x)) + \sin x \cdot \frac{f'(x)}{f(x)}\right)[f(x)]^{\sin x}$

$\frac{x^3 - 3x^2 + 1}{x^3} = f(x)$

Düşey ⇒ $x = 0$

$\lim_{x \to \pm\infty} \frac{x^3 - 3x^2 + 1}{x^3} = 1$

$x^2(x^2 - 1) =$ $x = 0 \to$ tanımsız

$\left.\begin{array}{l} x = 1 \\ x = -1 \end{array}\right\}$ kritik

$-\infty$	-1	0	1	$+\infty$
$+$	$-$	$-$	$+$	

$\arctan x^2 = u$ | $x\,dx = dv$

$\frac{2x}{1+x^4}\, dx = du$ | $\frac{x^2}{2} = v$

$\frac{\arctan x^2 \cdot x^2}{2} - \int \frac{x^2}{\cancel{2}} \cdot \frac{\cancel{2}x}{1+x^4}\, dx$

$\frac{x^3}{1+x^4}\, dx$

$\int \frac{x^3}{x^4+1}\, dx$

$x^4 + 1 = u$

$4x^3\, dx = du$

$\frac{\cancel{x^3}}{4} \cdot \frac{du}{4\cancel{x^3}} = \frac{1}{4}\int \frac{1}{u}\, du$

$\frac{\ln|x^4+1|}{4}$

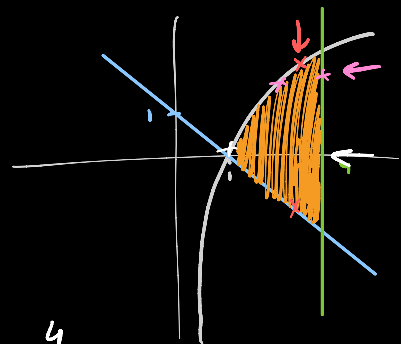
$\int_0^{\pi/4} e^{\tan x} \cdot \cancel{e^{-\ln(\cos x)}}$

$e^{\tan x} \cdot \frac{1}{\cos^2 x}$

$\tan x = u$

$\int_0^1 e^u\, du$

$e^u \Big|_0^1 = \boxed{e - 1}$



$\int_1^4 \left(\ln x - (1-x)\right) dx$

$\ln x = y$

$e^y = x$

$$\ln(\cot x)^{1/\ln x} = \ln y$$

$$\lim_{x\to 0^+} \frac{\ln \cot x}{\ln(x)} = \ln y$$

$$\frac{-\csc^2 x}{\cot} \cdot x =$$

$$\frac{-x}{s.c} \qquad -1 \cdot \frac{1}{\cancel{\cos}} \to 1 \qquad -1 = \ln y$$

$$\underline{\underline{e^{-1} =}}$$

$$\int \sqrt{1+(y')^2} = L$$

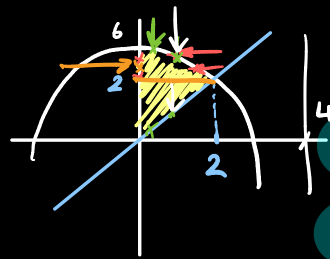
$$\frac{\cos x}{\sin x} = y'$$

$$\int_{\pi/4}^{\pi/2} \cancel{\sqrt{\csc^2}}$$

$\pi/4$ 45

$\frac{1}{\sin}$ $\pi/2$

$$\ln|\sin x| \Big|_{\pi/4}$$



$$\pi\int_2^6 (\sqrt{6-y})^2 dy + \pi\int_0^2 y^2 dy$$

$$x = \sqrt{6-y}$$

$$6 - x^2 = y$$

$$6 - x^2 = x$$

$$\pi\int_0^2 \left((6-x^2)^2 - x^2\right) dx$$

$$2\pi\int_0^2 (4-x)\cdot(6-x^2-x)$$

$$\int \cos(\ln(x))\,dx = I$$

$\cos(\ln(x)) = u$ $\qquad dx = dv$

$-\sin(\ln(x)) \cdot \frac{dx}{x} = du$ $\qquad x = v$

$$x.\cos(\ln(x)) + \int \sin(\ln(x)).\,dx$$

$\sin\ln(x) = u$ $\qquad dx = dv$

$\frac{\cos\ln(x)}{x}\,dx = du$ $\qquad x = v$

$$\underbrace{x.\sin\ln x - \int \cos\ln(x)\,dx}_{-I} = I$$

$$\frac{x.\sin\ln x + x.\cos\ln(x)}{2} + C$$

$$f'(c) = \frac{f(1) - f(0)}{1} = 0 \text{ en az bir nokta}$$

$$\frac{2 - \pi c}{\sqrt{1 - c^2}} = 0$$

$$2 = \pi c$$

$$\boxed{\frac{2}{\pi} = c}$$

$$\int_1^e \frac{dx}{x\ln x} = \lim_{h \to 1^+} \int_h^e \frac{dx}{x.\ln x}$$

$\ln x = u$

$\frac{dx}{x} = du$

$$\int_{\ln h}^{1} \frac{1}{u}\,du$$

$-\ln|\ln h|$

$$\int \frac{\cos^3 t}{\sin^3 t - \sin^2 t - 6\sin t}\,dt$$

$\sin t = u$

$\cos t.dt = du$

$1 - \cos^2 t = u^2$

$$\frac{\cos^3 t}{\sin t(\sin^2 t - \sin t - 6)}$$

$\begin{matrix}-3\\+2\end{matrix}$

$$\frac{\cos^3 t}{\sin t.(\sin t - 3).(\sin t + 2)}$$

$$\int \frac{1 - u^2}{u.(u-3).(u+2)}\,du =$$

$f'(x) = \sqrt{16-x^2} - \frac{2x^2}{\sqrt{16-x^2}}$

$-2\sqrt{2}$ $2\sqrt{2}$

$16-x^2+2x^2$

tanımsız

$\frac{16-2x^2}{\sqrt{16-x^2}}$

$x=-4,4$

-4 $-2\sqrt{2}$ $2\sqrt{2}$ 4

−	+	−
↘	↗	↘

$\int \frac{dx}{(x+1)^3\sqrt{x^2+2x-3}}$

$\int \frac{dx}{(x+1)^3 \cdot \sqrt{(x+1)^2-4}}$

$x+1 = 2\sec t \implies dx = 2\sec t \cdot \tan t \cdot dt$

$\int \frac{dt}{8\sec^2}$

$\frac{1}{8}\int \cos^2 t\, dt =$

$1+\cos 2t$

$b = 2\sqrt{a}$

3

$a=9$

$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}}$

$\frac{x^2}{\sqrt{x}}$

$x^{\frac{3}{2}}$

$\int \sec^2\sqrt{x}\cdot dx$

$\sqrt{x}=t$

$\frac{1}{2\sqrt{x}}dx = dt$

$2\int \sec^2 t \cdot t \cdot dt$

$t = u$ | $\sec^2 t\, dt = dv$

$dt = du$ | $\tan t = v$

$2\left(\tan t \cdot t - \int \tan t\, dt\right)$

$\int \frac{\sin t}{\cos t}dt$

$\cos t = x$

$-\sin t\, dt = dx$

$\int \frac{-1}{x}dx$

$-\ln|\cos t|$

$$\lim_{R\to 0^+} \int_R^1 \frac{dx}{x\cdot(1+\ln^2 x)}$$

$\ln x = u$

$\frac{1}{x}\cdot dx = du$

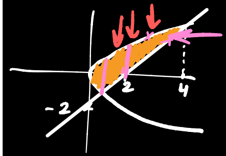
$$\frac{du}{(1+u^2)}$$

$$\arctan(\ln x)\Big|_R^1$$

$$0 - \left(\frac{-\pi}{2}\right) = \boxed{\frac{\pi}{2}}$$

$e^{\arctan x}\cdot -x^2+1$

$3x+x^3$



$y-x=-2$

$y=x-2$

$$\frac{\frac{\ln(1+x)}{x}\cdot\cancel{\frac{1}{2\sqrt{x}}}}{\cancel{\frac{1}{2\sqrt{x}}}}$$

$$\frac{\frac{1}{x+1}}{1}$$

$$\frac{2(x^2-1)^3\cdot x}{\sqrt{1+(x^2-1)^2}} = 0$$

$x=0$

$x=1,-1$

	-1		0		1	
$-$		$+$		$-$		$+$
$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

$t=\tan u$

$\frac{1}{x}\,dt = \sec^2 u \,.\, du$

$$\int_{\arctan(x^{-1})}^{\arctan 1} \frac{1}{\cos u}\,du = \ln|\cos(u)|\Big|_{\arctan(\frac{1}{x})}^{\arctan 1}$$

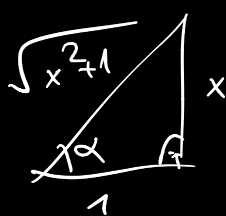
$t=\tan u$

$dt=\sec^2 u\,.\,du$

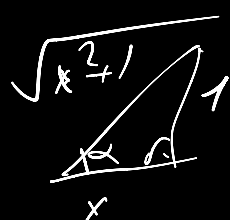
$$\int_{\arctan(1)}^{\arctan(x)} \frac{\sec u\,.\,du}{\tan.\ \cancel{\frac{1}{c}\cdot\frac{c}{s}}} \rightarrow \ln|\sin x|\Big|_{\arctan 1}^{\arctan(x)}$$

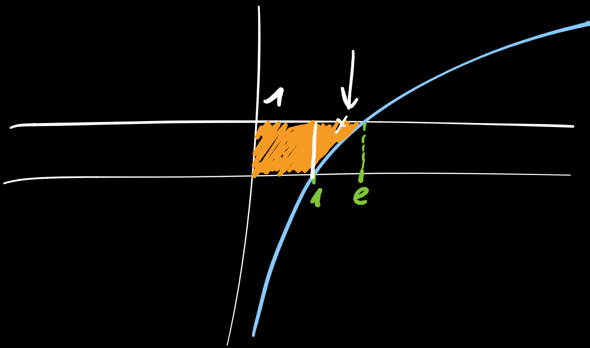
$$\cancel{\ln|\cos(\arctan 1))|}\ \cancel{-\ln|\sin(\arctan 1)}$$

$$\ln\frac{x}{\sqrt{x^2+1}}$$



x





$$\int_2^5 \sqrt{1+(y')^2}\,dx$$

$$x\sqrt{x^2-2} = y'$$

$$1 + x^2\cdot(x^2-2) = x^4-2x^2+1$$

$$\sqrt{x^4-2x^2+1}$$

$$\sqrt{(x^2-1)^2} = \int_2^5 x^2-1$$

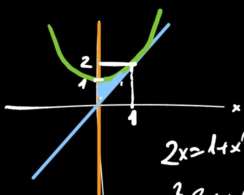
$$\frac{x^3}{3}-x$$

125−8

117/3 = 39

39−3 = 36

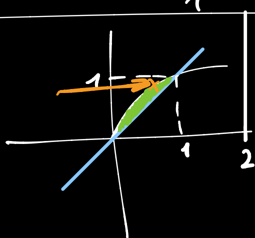
$$\int_0^1 dx + \int_1^e 1-\ln x$$



$2x = 1+x^2$

x^2-2x+1

$$2\pi\int_1^2 y\cdot\frac{y}{2} -$$



$$\pi\int_0^1 \left(\left(y\tfrac{2}{2}\right)^2-(y-2)^2\right)dy$$

$$2\pi\int_0^1 (2-x)\cdot(\sqrt{x}-x)\,dx$$

Trigonometrik Dönüşüm ile integral Alma

① Yarım Açı Yardımıyla integral Alma

$$\int \sin^2 x\, dx,\quad \int \cos^2 x\, dx,\quad \int \sin^4 5x\, dx, \ldots$$

★ Çift dereceye sahip sin ve cos'lu integrallerde

$$\cos 2x = \cos^2 x - \sin^2 x = \boxed{2\cos^2 x - 1} = \boxed{1 - 2\sin^2 x}$$

★★

$$\frac{\cos 2x + 1}{2} = \cos^2 x \qquad \sin^2 x = \frac{1 - \cos 2x}{2} \;★★$$

Soru

$$\int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2}\, dx = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$$

$$\int \cos^2 3x\, dx = \int \frac{\cos 6x + 1}{2}\, dx = \frac{1}{2}\cdot\left(x + \frac{\sin 6x}{6}\right) + c$$

$$\int \sin^4 x\, dx = \int \sin^2 x \cdot \sin^2 x\, dx$$

$$\int \frac{1 - 2\cos 2x + \cos^2 2x}{4}\, dx$$

$$\frac{1}{4}\left(x - \frac{2\sin 2x}{2}\right) + \frac{1}{4}\int \cos^2 2x\, dx$$

$$\frac{1}{4}\int \frac{1+\cos 4x}{2}\, dx = \frac{1}{8}\left(x + \frac{\sin 4x}{4}\right) + c$$

(2) $[\sin^2 x + \cos^2 x = 1]$ özdeşliği ile integral çözme

$\int \sin^3 x \, dx$, $\int \cos^5 x \, dx$ | $\int \sin^3 x . \cos^5 x \, dx$, $\int \sin^3 x . \cos^4 x$

* tek başına üssü tek olan durumlarda | * çarpım durumunda ikisi tek veya biri tek durumunda

Soru:

* değişken değiştirme

$$\int \sin^3 x \, dx = ? \quad \int \sin^2 x . \sin x . dx = \int (1 - \underbrace{\cos^2 x}_{u}) . \overbrace{(\sin x) . dx}^{-du}$$

$$= -\int (1-u^2) . du = -\cos x + \frac{\cos^3 x}{3} + c$$

Soru:

$$\int \cos^5 x \, dx = \int \cos^2 x . \cos^2 x . \cos x \, dx$$

$$\int (1 - \underbrace{\sin^2 x}_{u})^2 . \overbrace{\cos x \, dx}^{du} = u - \frac{2u^3}{3} + \frac{u^5}{5} + c = \sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$$

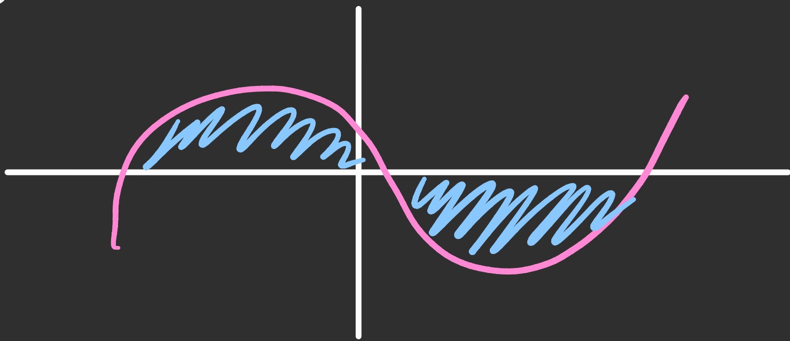
Soru:

$$\int \sin^3 x . \cos^4 x . dx = \int \sin^2 x . \sin x . \cos^4 x . dx$$

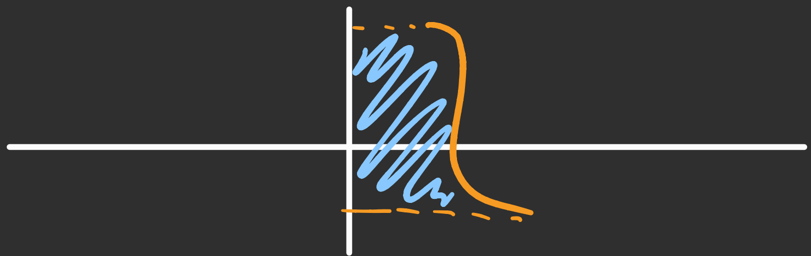
$$= \int (1 - \underbrace{\cos^2 x}_{u}) . \cos^4 x . \overbrace{\sin x \, dx}^{-du}$$

$$= -\int (1-u^2) u^4 du = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

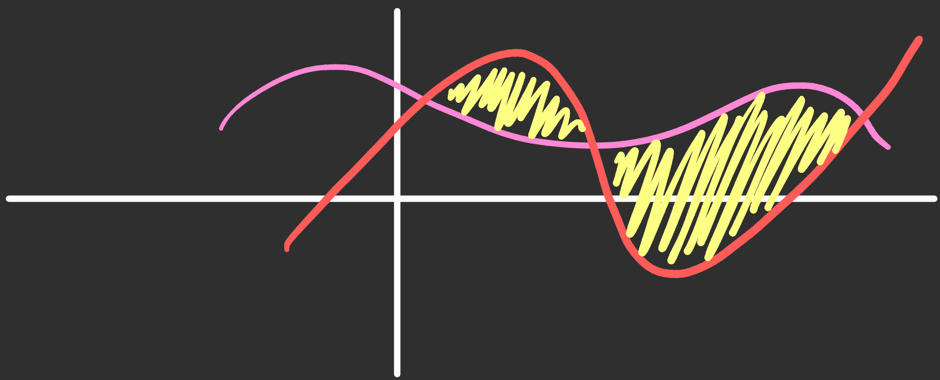
Integralde Alan Çeşitleri



x-ekseni ile arada kalan alan



y-ekseni ile arada kalan alan



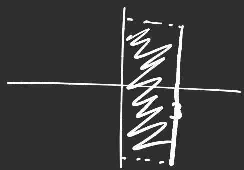
iki eğri arasında alan

Integralde Alan Hesaplarken Çizimi Bilinmesi Gereken Fonksiyonlar

1 Doğru Grafikleri

* Eksenlere dik

$x=3$

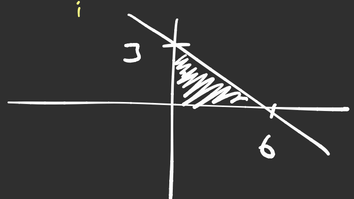


* Orijinden geçenler

* $x+2y=6$

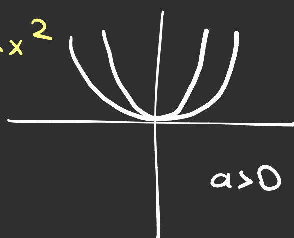
$y=3x+12$

⋮

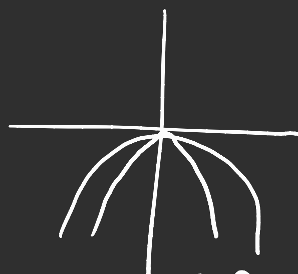


2 Parabol Grafikleri

* $y=ax^2$

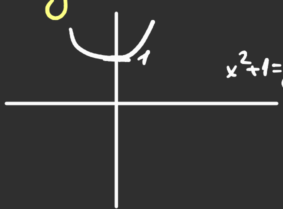


$a>0$

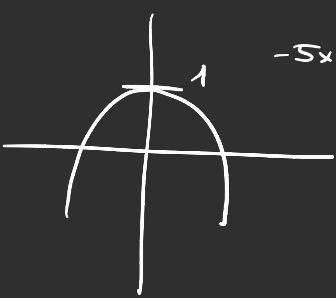


$a<0$

* $y=ax^2 \pm k$



$x^2+1=y$



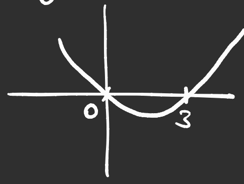
$-5x^2+1=y$

* $y=ax^2+bx+c$

$a \rightarrow$ kolların yönü

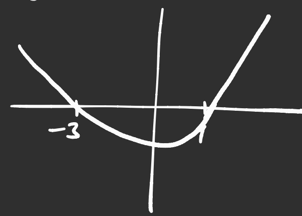
$y=0 \Rightarrow$ x-eksenini kestiği noktalar

$y=x^2-3x$



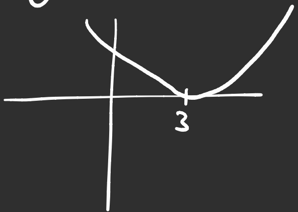
$y=x^2+2x-3$

-1
+3

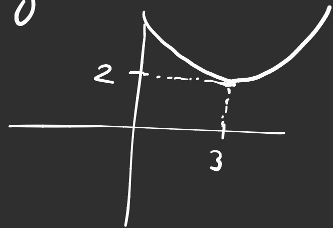


* $y=a(x-r)^2+k$ $\quad$ (r,k) = tepe noktası

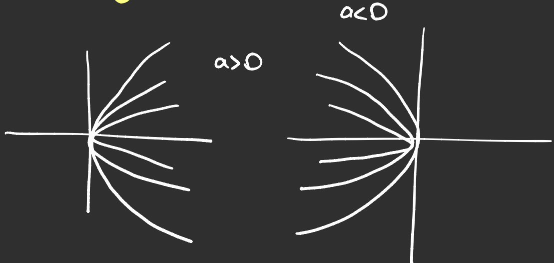
$y=(x-3)^2$



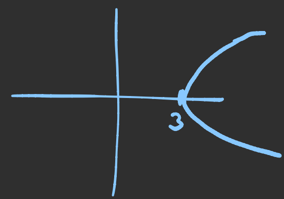
$y=(x-3)^2+2$



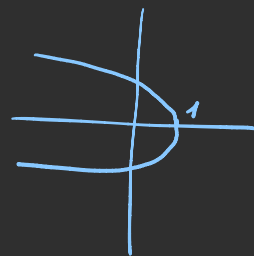
* $x=ay^2$



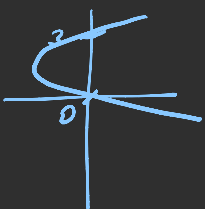
$x=y^2+3$



$x=-2y^2+1$



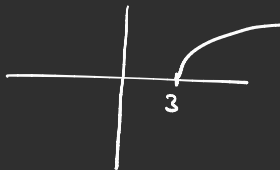
* $x=y^2-3y$



3 Özel Grafikler

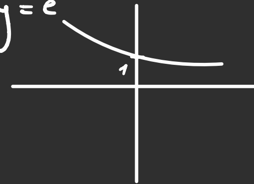
$y = \sqrt{x}$

$y = \sqrt{x-3}$

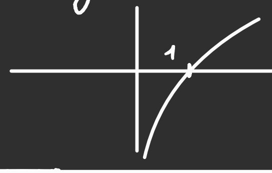


$y = e^{x}$

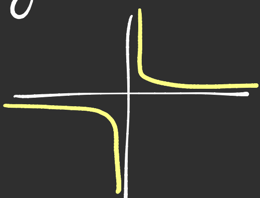
$y = e^{-x}$



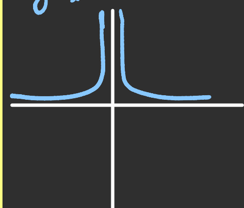
$y = \ln x$



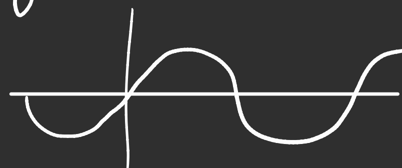
$y = \frac{1}{x}$



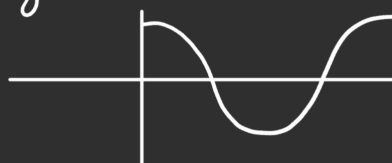
$y = \frac{1}{x^2}$



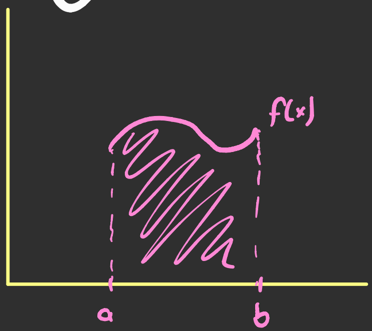
* $y = \sin x$



$y = \cos x$

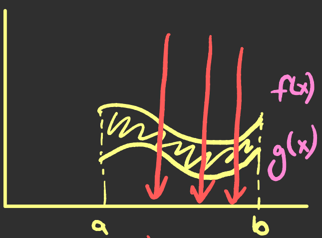


Integralde Alan



Alan = $\int_a^b f(x)\,dx$

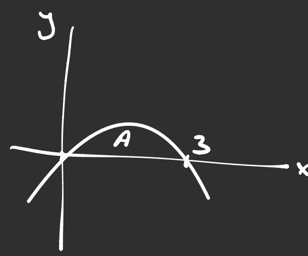
Eğriler Arasında Alan



Alan = $\int_{x=a}^{x=b} (f(x)-g(x))\,dx$

x ekseni üzerinde parabol altında kalan bölgenin alanı

$3x-x^2=f(x)$



$A=\int_0^3 (3x-x^2)\,dx$

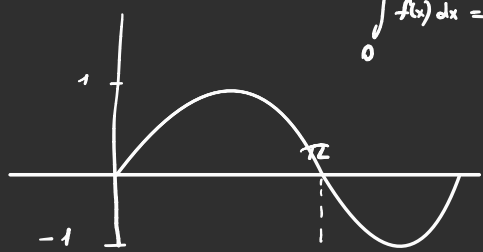
$9/2 =$

sinx = f(x) $(0,2\pi)$

$\int_0^{2\pi} f(x)\,dx = -\cos x \Big|_0^{2\pi}$

$1-1 =$

$\underline{0} =$



**

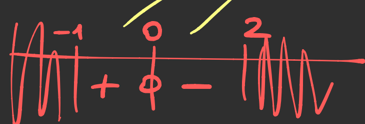
$-1\le x\le 2$ için $f(x)=x^3-x^2-2x$

- çizilemeyen fonk. için;

* türev al.

$f'(x)=3x^2-2x-2$

$x=0, 2, -1$



$\int_{-1}^{0} (x^3-x^2-2)\,dx$

$+$

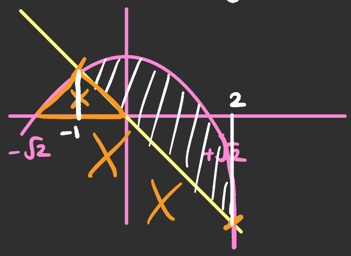
$\int_{0}^{2} (x^3-x^2-2)\,dx$

$=$

$\frac{28}{12}$

Sorular:

$y = 2 - x^2$ parabolü ve $y = -x$ ile çevrili bölge alanı = ?

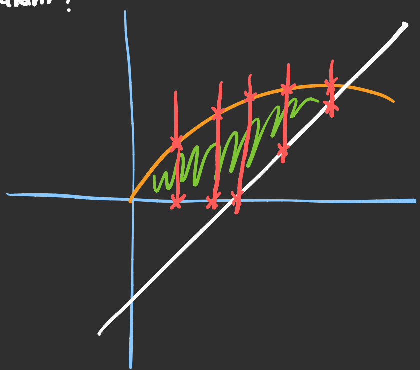


$2 - x^2 = -x$

$\boxed{x = 2, -1}$

$$\int_{-1}^{2} \left[(2-x^2) - (-x)\right] dx =$$

Birinci dörtte birlik bölgede üstten $y = \sqrt{x}$ alttan x ekseni ve $y = x-2$ ile sınırlı bölgenin alanı ?



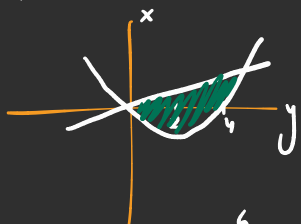
$\Rightarrow$

$$A = \int_0^2 (\sqrt{x} - 0)\,dx + \int_2^4 (\sqrt{x} - (x-2))\,dx =$$

$10/3 =$

Soru:

$y^2 - 4y$, $2y$ fonk alan



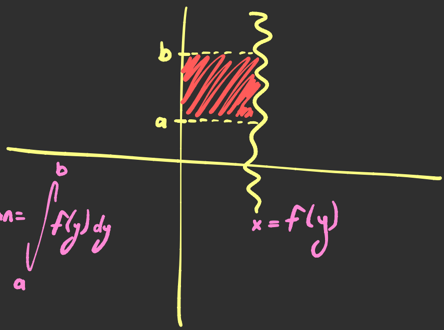
$y^2 - 4y = 2y$

$\boxed{y = 0, 6}$

$$\int_0^6 6y - y^2 = \underbrace{3y^2 - \frac{y^3}{3} \Big|_0^6}_{\boxed{+36}}$$

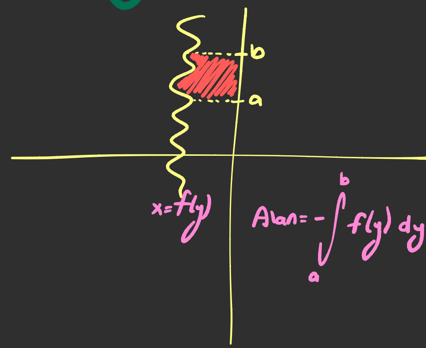
y-ekseni ile Alan Hesaplama

y-ekseninin sağında



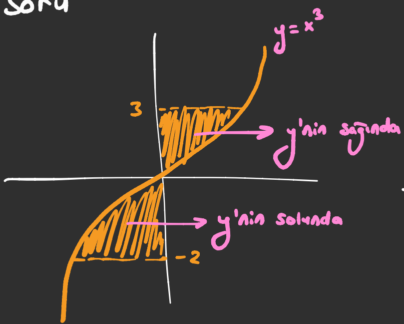
$$\text{Alan} = \int_a^b f(y)\,dy$$

y ekseninin solunda



$$\text{Alan} = -\int_a^b f(y)\,dy$$

Soru



$$\sqrt[3]{y} = x$$
$$\downarrow$$
$$f(y) = \sqrt[3]{y}$$

$$-\int_{-2}^{0} y^{\frac{1}{3}} = \frac{-3y^{\frac{4}{3}}}{4}\Bigg|_{-2}^{0}$$

$$\boxed{+\frac{3\sqrt[3]{16}}{4} + \frac{3\sqrt[3]{81}}{4}}$$

Soru:

$x=y^2-4$ ile y ekseni arası kalan alan=?

$$y^2-4=0$$
$$y=\mp 2$$

$$-\int_{-2}^{0}(y^2-4)\,dy = -\frac{y^3}{3} + 4y\Bigg|_{-2}^{0}$$

$$16 \quad \frac{-16}{3} \quad \boxed{\frac{32}{3}}$$

iki grafik arasında kalan alan

Soru

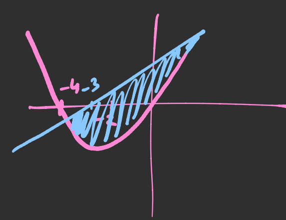
$$x^2 = x$$

$$\boxed{x=0,1}$$

$$\int_{1}^{2}(x^2-x)\,dx = \frac{x^3}{3} - \frac{x^2}{2}\Bigg|_{1}^{2} = \boxed{\frac{-1}{6}}$$

$\frac{+1}{6}$ alan

$$x^2+\cancel{4}x = \cancel{2}x+3$$
$$x^2+2x-3=0$$
$$\begin{matrix} +3 \\ -1 \end{matrix}$$
$$x=-3,1$$

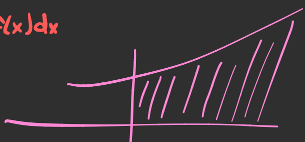


$$\int_{-3}^{1}(x^2+2x-3)\,dx = \frac{-32}{3} \;//\; \frac{+32}{3}$$

Genelleştirilmiş integral (Improper Integral)

1 Sınırlarında sonsuz içeren

* $\int_a^{\infty} f(x)\,dx$
* $\int_{-\infty}^{\infty} f(x)\,dx$
* $\int_{-\infty}^{a} f(x)\,dx$



$+\infty, -\infty$? ⟶ Iraksak

Sayı ? ⟶ Yakınsak

2 Sınırlarında tanımsızlık içeren integraller

$\int_1^{3} \frac{1}{x-3}$ $\quad$ $\int_1^{3} \frac{1}{x-2}$

---

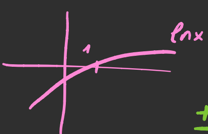
1 Sınırlarında Sonsuz İçeren

* $\int_a^{+\infty} f(x)\,dx = \lim_{h\to\infty} \int_a^{h} f(x)\,dx$

Soru: $\int_2^{+\infty} \frac{1}{x}\,dx = \lim_{t\to+\infty} \int_2^{t} \frac{1}{x}\,dx$

$= \lim_{t\to+\infty} \ln x \Big|_2^{t} = \lim_{t\to+\infty} \overset{\infty}{\overline{\ln t}} - \ln 2$

$= +\infty$



* Sonuç ⟶ $\pm\infty$ Iraksak
* Sonuç ⟶ Sayı Yakınsak

* $\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^{a} f(x)\,dx$

$\int_{-\infty}^{1} e^x\,dx = \lim_{t\to-\infty} \int_t^{1} e^x\,dx$

$= \lim_{t\to-\infty} (\overset{e}{e} - \underset{0}{e^t}) = e$

---

* $\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^{+\infty} f(x)\,dx$

$= \lim_{t\to-\infty} \int_t^{0} f(x)\,dx + \lim_{t\to+\infty} \int_0^{t} f(x)\,dx$

Soru

$$\int_{-\infty}^{+\infty}\left(\frac{1}{1+x^2}\right)dx = ?$$

$$\lim_{t\to-\infty}\int_{t}^{0}\frac{1}{1+x^2}\,dx + \lim_{t\to+\infty}\int_{0}^{t}\frac{1}{1+x^2}\,dx$$

$$\arctan x \Big|_{-\infty}^{0} \qquad\qquad \frac{\pi}{2}$$

$$0-\underbrace{\arctan(-\infty)}_{\left(\frac{-\pi}{2}\right)} = \frac{\pi}{2} \quad + \quad = \pi$$

---

2) Sınırlarında tanımsızlık içeren

* Üst sınırın tanımsız yapması
* Alt sınırın tanımsız yapması
* Sınırları arasındaki bir değeri tanımsız yapması

*

$$\int_{1}^{3}\frac{1}{x-3}\,dx \longrightarrow \lim_{x\to3^-}\int_{1}^{t}\frac{1}{x-3}\,dx$$

* üst sınırda limite soldan yaklaşılır.

$$\lim_{x\to3^-}\left(\ln|x-3|\right)\Big|_{1}^{t}$$

$$\lim_{x\to3^-}\ln\left|\frac{t-3}{2}\right|$$

$$= \ln|0^-| = \ln 0^+$$

$$= -\infty$$

*

$$\int_{1}^{3}\frac{1}{x-1}\,dx = \lim_{t\to1^+}\int_{t}^{3}\frac{1}{x-1}\,dx$$

$$\ln|x-1|\Big|_{t}^{3}$$

$$\ln\left|\frac{2}{t-1}\right| = \ln\infty = \infty$$

*

$$\int_{1}^{3}\frac{1}{x-2}\,dx$$

$$\lim_{t\to2^-}\int_{1}^{t}\frac{1}{x-2}\,dx + \lim_{t\to2^+}\int_{t}^{3}\frac{1}{x-2}\,dx$$

Integralde Hacim Hesaplama

∗ Disk Metodu

x-ekseni etrafında (eksene temas varken)
y-ekseni etrafında

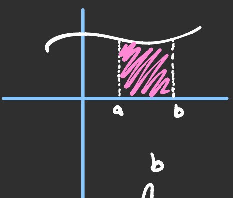
iki grafik arasında kalan alanın
x veya y ekseni etrafında döndürülmesi

∗ Shell Metodu

x-ekseni etrafında döndürme
y-ekseni " "
x=a veya y=b " "

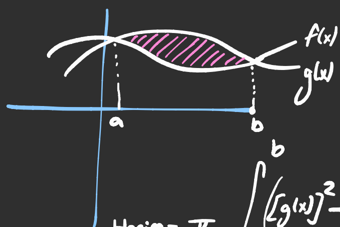
Disk Metodu

∗ x-ekseni etrafında



$$\text{Hacim} = \pi \int_a^b [f(x)]^2 \, dx$$

veya

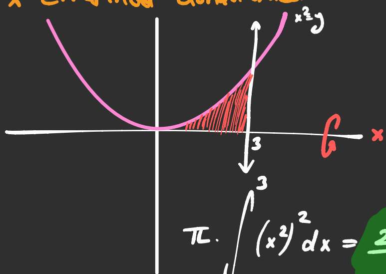


$$\text{Hacim} = \pi \cdot \int_a^b \left([g(x)]^2 - [f(x)]^2\right) dx$$

WASHER METODU !

Soru

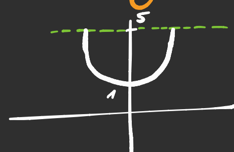
$y = x^2$, $x = 3$ ve x arasında x etrafında döndürülmesi



$$\pi \cdot \int_0^3 (x^2)^2 \, dx = \frac{243\pi}{5}$$

Soru

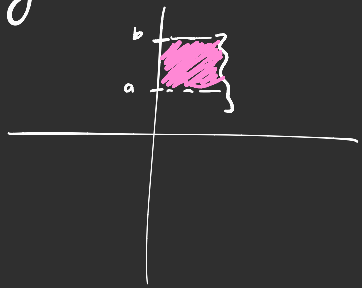
$y = x^2 + 1$, $y = 5$ ve y ekseni



$$\pi \int \left(25 - (x^2+1)^2\right) dx$$

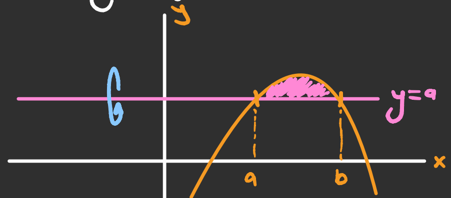
Disk Metodu

* y-etrafında



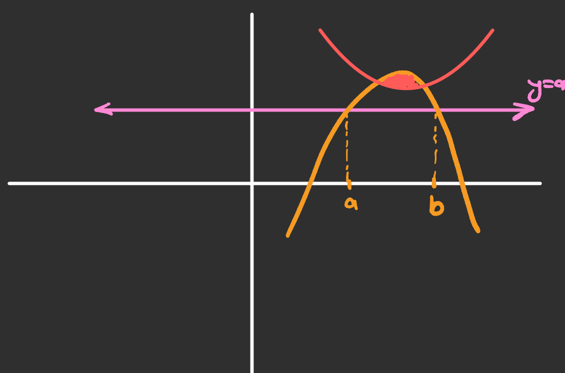
$$\text{Hacim} = \pi \int_a^b [f(y)]^2 \, dy \quad / \quad \text{Hacim} = \pi \cdot \int_a^b \left([g(y)]^2 - [f(y)]^2\right) dy$$

* Yatay doğru etrafında döndürme



$$V = \pi \cdot \int_a^b (f(x) - a)^2 \, dx$$

WASHER METODU !



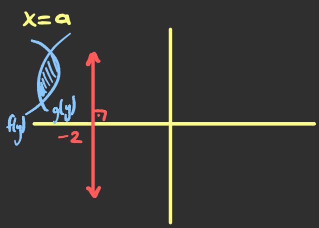
Soru

$y = x^2$, $y = 4$ ve y ekseni arasında $y=4$ doğrusu etrafında

$$V = \pi \int_0^2 (x^2 - 4)^2 \, dx$$

$$= \frac{256\pi}{15}$$

* Düşey doğru etrafında döndürme

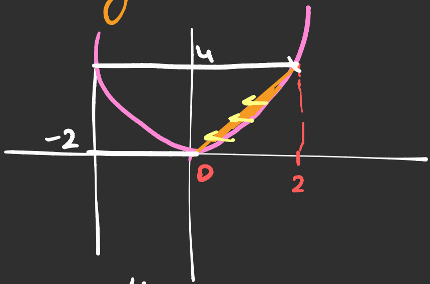


$$V = \pi \int_a^b \left[(g(y) - a)^2 - (f(y) - a)^2 \right] dy \quad \text{(Washer)}$$

$$V = \pi \int_a^b \left[f(y) \right]^2$$

Soru:

$y = x^2$, $y = 2x$, $x = -2$ etrafında döndürünüz.



$x^2 = 2x$

$$\boxed{\begin{matrix} x=0 \\ x=2 \end{matrix}}$$

$$V = \pi \int_0^4 \left[(\sqrt{y} + 2)^2 - \left(\frac{y}{2} + 2\right)^2 \right] dy$$

$V = 8\pi$

Shell Metodu y ekseni için

$$* \quad V = 2\pi \int_a^b r \cdot h \cdot dx$$

* Sınırlar x ekseninden gelir.

$a \le x \le b$

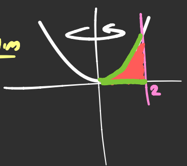
r: shell yarıçapı ⇒ $r = x$

h: shell yüksekliği ⇒ $h = \begin{bmatrix}\text{alanın} \\ \text{üst} \\ \text{sınır} \\ \text{fonksiyonu}\end{bmatrix} - \begin{bmatrix}\text{alanın} \\ \text{alt} \\ \text{sınır} \\ \text{fonksiyonu}\end{bmatrix}$

Soru

$y = x^2$, $x = 2$ ve x ekseni y ekseni etrafında döndürülürse hacim = ?

1.adım



2.adım

$$V = 2\pi \int_0^2 r \cdot h \cdot dx$$

$$V = 2\pi \int_0^2 x \cdot (x^2 - 0) \cdot dx$$

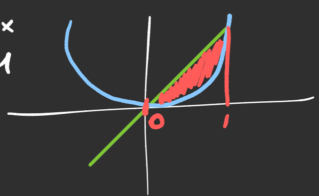
$$2\pi \cdot \left(\frac{x^4}{4} \Big|_0^2\right) = \underline{\underline{8\pi}}$$

Soru

$y = x^2$, $y = x$ arasında kalan alan (y ekseni),

$x^2 = x$

$x = 0, 1$



$$2\pi \int_0^1 x \cdot (x - x^2)\, dx$$

$$x^2 - x^3$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1\right)$$

$$2\pi \cdot \frac{1}{12} = \boxed{\frac{\pi}{6}}$$

Shell Metodu x ekseni için

* $V = 2\pi \int_a^b r \cdot h \cdot dy$

* Sınırlar y ekseninden gelir. $a \le y \le b$

r: her zaman $r = y$

h: [Alanın sağındaki fonk] − [Alanın solundaki fonksiyon]

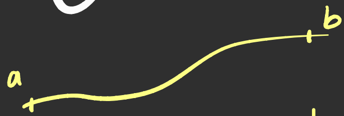
Soru:

$y = x^2$, $x = 2$ ve x etrafında

$$= 2\pi \int_0^4 y \cdot (2 - \sqrt{y})\, dy$$

$$2\pi \left(y^2 - \frac{2y^{\frac{5}{2}}}{5} \Big|_0^4 \right) = \frac{32\pi}{5}$$

Yay Uzunluğu



$f(x), \quad a \leq x \leq b$

$$\mathcal{L} = \int_a^b \left(\sqrt{1+(y')^2}\right) dx$$

$$\mathcal{L} = \int_c^d \left(\sqrt{1+(x')^2}\right) dy$$

$c \leq y \leq d$

Soru

$y = \left(\frac{x}{2}\right)^{\frac{2}{3}} \quad 0 \leq x \leq 2$

$$\cancel{\int_0^2 \left(\sqrt{1+(y')^2}\right) dx} =$$

$$\int_0^1 \sqrt{1+(x')^2}\, dy = \int_0^1 \sqrt{1+9y}\; dy = \int_1^{10} \frac{\sqrt{u}}{9}\, du$$

$1+9y = u$

$9dy = du$

$$\frac{1}{9} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_1^{10} = \boxed{10\sqrt{10}-1}$$

$f(x), \quad a \leq x \leq b$

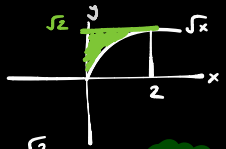
x-ekseni etrafında

dönen yüzey alanı

$$S = \int_a^b \left(2\pi \cdot y \sqrt{1+(y')^2} \right) dx$$

$x = y^2$

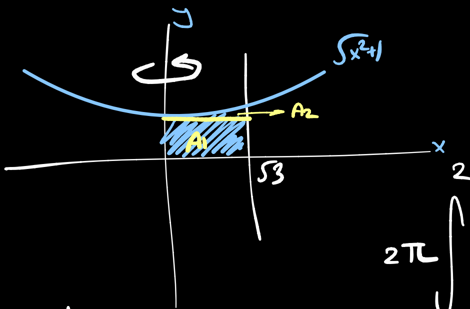
$\sqrt{x} = f(x)$



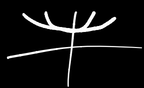
$$\pi \int_0^{\sqrt{2}} y^4$$

$$\boxed{\frac{4\sqrt{2}\,\pi}{5}}$$

$$2\pi \int_0^2 x\left(\sqrt{2} - \sqrt{x}\right) =$$

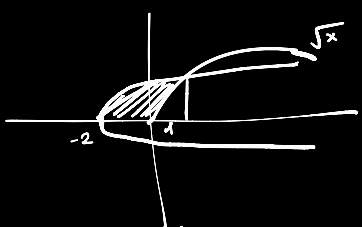


$\sqrt{x^2+1} = y$



$$2\pi \int_0 x \cdot \sqrt{x^2+1}$$

$$\pi \int$$



$$2\pi \int_{-2}^{1} y \cdot y^2 - \cancel{3y^2} + 2$$

$$-2y^3 + 2y \, dy$$

$$\boxed{3\pi}$$

$$\int \frac{dx}{\sqrt{4x-x^2}} =$$

$$-(x^2-4x+4)+4$$

$$\int \frac{dx}{\sqrt{4-(x-2)^2}}$$

$x-2=u$

$$\int \frac{du}{\sqrt{4-u^2}} = \arcsin \frac{x-2}{2} + c$$

*

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a}$$

$$\int \sqrt{1+e^x}\,dx \quad \Big\| \quad 1+e^x = u^2, \quad e^x dx = 2u\,du$$

$$\int |u| \cdot \frac{2u \cdot du}{e^x} = \frac{2u^2\,du}{(u^2-1)}$$

$$\begin{array}{r|l} 2u^2 & u^2-1 \\ \cancel{-2u^2-2} & 2 \\ +2 & \end{array} = \int \left(2 + \frac{2}{u^2-1}\right) du$$

$$2\int \left(1 + \frac{1}{u^2-1}\right) du$$

$$2u + \qquad \frac{1}{(u-1)\cdot(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$$

$A+B=0$

$A-B=1$

$A=1/2$

$B=-1/2$

$$\cancel{2}\left(\frac{1}{\cancel{2}}\ln|u-1| - \frac{1}{\cancel{2}}\ln|u+1|\right)$$

$$2u + \ln|u-1| - \ln|u+1|$$

$$\int \frac{dx}{\sqrt{x^2-2x+4}} = \int \frac{dx}{\sqrt{(x-1)^2+3}}$$

$x-1=u$

$u=\sqrt{3}\cdot\tan t$

$du=\sqrt{3}\cdot\sec^2 t \cdot dt$

$$= \int \frac{dx}{\sqrt{u^2+3}}$$

$$= \int \frac{\sqrt{3}\sec^2 t\,dt}{\sqrt{3+3\tan^2 t}} = \int \sec t\,dt$$

$$= \ln|\sec t + \tan t| + c$$

$$= \ln\left|\frac{\sqrt{3+u^2}}{\sqrt{3}} + \frac{u}{\sqrt{3}}\right| + c$$

---

* Karekoklerde

tam kare

tan, sec, sin dönüşümü

$$\int \frac{dx}{2x(1-\sqrt{x})^2} \qquad 1 = \cancel{1+u^2} - 2u + Bu - \cancel{Bu^2} + u$$

$\sqrt{x}=u$

$$\frac{dx}{2\sqrt{x}} = du$$

$$\frac{1}{u\cdot(1-u)^2} = \frac{A}{u} + \frac{B}{1-u} + \frac{C}{(1+u)^2}$$

$$\boxed{\begin{array}{l} A=1 \\ C=1 \\ B=1 \end{array}}$$

$$\int \left[\frac{1}{u} + \frac{1}{1-u} + \frac{1}{(1-u)^2}\right] du$$

$$\int \left(\frac{1}{\sqrt{x}} + \frac{1}{1-\sqrt{x}} + \frac{1}{(1-\sqrt{x})^2}\right) \cdot \frac{dx}{2\sqrt{x}}$$

$$\int_0^\infty \frac{dx}{1+e^x} = \lim_{x\to\infty} \int_0^R \frac{dx}{1+e^x}$$

$\ln|$

$e^{kx} = u$

$e^{kx}.k.dx = du$

$t = \arctan(u)$

dt

$$\frac{1}{k}\int \frac{\arctan(u)}{1+u^2}\cdot\frac{du}{u}$$

$t \to 1$

$\frac{1}{\tan t} dt \to \ln|\tan t|$

$$\frac{1}{k}\int \frac{t.\,dt}{\tan t}$$

$$\frac{1}{k}\cdot\left[t.\ln|\tan t| - \int \ln|\sin t|\,dt\right.$$

$\ln|\sin t| = u \to \frac{\sin t}{\cos t} dt = du$

$dt = dv$

$t = v$

$$\int \frac{x^2+5}{x(x^2+2x+5)}\,dx =$$

$$\frac{x^2+5}{x.(x^2+2x+5)} = \frac{A}{x} + \frac{B}{x^2+2x+5}$$

$$Ax^2+(2A+B)x+5A$$

$A=1$

$B=-2$

$$\int \frac{1}{x} + \frac{-2}{x^2+2x+5}$$

$$\ln|x| + \int \frac{-2}{(x+1)^2+4}\,dx$$

$x+1=u$

$dx=du$

$$\frac{-\cancel{2}}{\cancel{2}}\arctan\frac{x+1}{2}$$

$$\ln|x| - \arctan\frac{x+1}{2} + c$$

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx =$$

Not: $\sqrt{1+u^2} \to \tan t$

$\sqrt{u^2-1} \to \sec t$

$\sqrt{1-u^2} \to \sin t$

$x = \sqrt{3}\sin t$

$dx = \sqrt{3}\cos t.\,dt$

$$\frac{\sin^2 t}{\underbrace{(1-\sin^2 t)}_{\cos^2 t}^{\frac{3}{2}}}\cdot \cos t.\,dt$$

$$\frac{\sin^2 t}{\cancel{\cos^3 t}}\cdot \cancel{\cos t}\,dt = \int \frac{\sin^2 t}{\cos^2 t}\,dt =$$

$\sec^2 t - 1$

$\tan t - t + c$

$$\lim_{x\to 0^+} \frac{e^{f(x)}-1}{f(x)}$$

$$\frac{e^{f(x)}\cdot \cancel{f'(x)}}{\cancel{f'(x)}} = 1$$

$a^2 = 1$

$\underline{a = \mp 1}$

$F(2) - F(0) = \frac{1}{4}$

$x^2 = u$

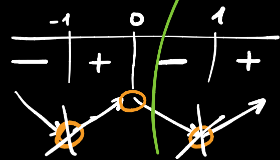
$x\,dx = \frac{du}{2}$

$$\frac{1}{2}\int_1^4 f(2u)\cdot du \rightarrow \frac{F(8)-F(2)}{\cancel{4}} = \frac{15}{\cancel{16}\ 4} + \frac{1}{}$$

$\boxed{\frac{4}{2} = 2}$

$$F'(x) = \frac{(x^2-1)^3}{\sqrt{1+(x^2-1)^2}}\cdot 2x = 0$$

$x = 0, -1, +1$



0

$$f(t)dt = \frac{t}{\sqrt{t^2+1}}dt = \frac{1}{2\sqrt{u+1}}\cdot du$$

$t^2 = u$

$2t\,dt = du$

$\sqrt{u+1}\,\Big|_a^x$

$\sqrt{t^2+1}\,\Big|_a^x$

$\cancel{\left(\sqrt{a^2+1}\right)^2 = (-1)^2}$

$a^2 + 1 = 1$

$\boxed{a=0}$

$$G^{-1}\left(\int_1^x \frac{1+t^2}{\sqrt{3+t^2}}dt\right) = x$$

$$(G^{-1})'\left(\int_1^x \frac{1+t^2}{\sqrt{3+t^2}}dt\right)\left(\frac{x^2+1}{\sqrt{x^2+3}}\right) = 1$$

$\frac{\cancel{2}}{\cancel{2}} = 1$

$t = \tan u$

$dt = \sec^2 u \cdot du$

$\int \sec u \cdot du$

$\int \operatorname{cosec} \cdot du$

$t = \tan u$

$dt = \sec^2 u \cdot du$

$$\int_1^x \frac{\sec u \cdot du}{\tan u}$$

$\frac{1}{\cancel{c}}\cdot\frac{\cancel{c}}{s} = \frac{1}{s}$

$$\frac{f(x) + f(-x)}{x^2+1} = \frac{2}{x^2+1} = G'(x)$$

$\boxed{=1}$

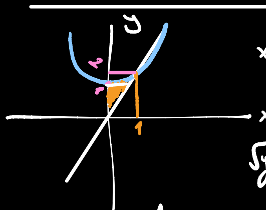
$f(-x) = -f(x)$

$$\lim_{x\to 0^+} \frac{\ln(x+1)}{x} = \frac{\frac{1}{x+1}}{1} = \boxed{1}$$

$$\frac{1}{\cancel{2}}\cdot\Big[\underbrace{F'(6-x^2) + \ln(x^2-5)^2}_{F'(2)+0}\Big]\cdot \cancel{4} = F'(x)$$

$2F'(2) = F'(2)$

$F'(2) = 0$

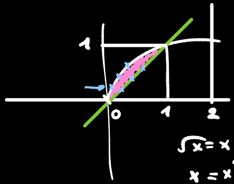


$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$\sqrt{y-1} = x$

$$2\pi\int y\cdot\left(\frac{y}{2} - \sqrt{y-1}\right)$$



$\sqrt{x} = x$

$x = x^2$

$x = 0, 1$

$y^2 = x$

$y = x$

$$\pi \int_0^1 \left((y^2-2)^2 - (y-2)^2 \right) dy$$

$y^4 - 4y^2 \cancel{+4} - y^2 + 4y \cancel{-4}$

$$\int_0^1 \pi \left(y^4 - 5y^2 + 4y \right)$$

$\left((x^2-2)^{1/2} \cdot x \right)^2 = y'$

$\sqrt{x^2-2} \cdot x$

$$\int_2^5 \sqrt{1+(y')^2}\, dy$$

$\left(\sqrt{x^4-2x^2}\right)^2$

$$\int_2^5 \sqrt{x^4-2x^2+1}\, dx$$

$$\frac{\sqrt{x^2-2} \cdot x}{\sqrt{x^4-2x^2+1}}$$

$96 -$

$2, \; f(x)|^{\frac{1}{2}}$

$16-7$

18

$$\frac{2(x^4-2x^2+1)^{\frac{3}{2}}}{1440 \, 3.(4x^3-4x)} \Bigg|_2^5$$

$x(x^2-1)$

$\cdot 24$

$y^2 = a$

$2y\,dy = da$

$\sqrt{a^2-2a+1}\; da$

$$\frac{e^{-x}\, dx}{e^{-x}+1}$$

$e^{-x} = u$

$+e^{-x}\, dx = -du$

$$\lim_{R \to \infty} \int_0^R \frac{-1}{u+1}\, du$$

$-\ln|u+1| \Big|_1^{e^{-R}}$

$$\lim_{R \to \infty} \int_0^R \frac{dx}{1+e^x}$$

$-\ln(e^x+1) + x$

$(x^2-2)^{\frac{3}{2}}$

$\left(x\sqrt{x^2-2}\right)^2$

$$\frac{(x^4-2x^2+1)^{\frac{3}{2}}}{6x(x-1).(x+1)}$$

$$\int (x^4 - 2x^2+1)^{\frac{1}{2}}\, dx$$

$x^2 = u$

$2x\,dx = du$

$\sqrt{u^2-2u+1}$

$t = \tan u$

$dt = \sec^2 u \, du$

$$\int \sec u \, du \quad - \quad \operatorname{cosec} u \, du$$

$$\lim_{c \to -1^+} \int_c^0 \frac{dx}{(x+1)^{3/5}}$$

$x+1 = u$

$dx = du$

$$\int_{c+1}^{1} \frac{1}{u^{3/5}} \, du = u^{-\frac{3}{5}} du$$

$$\frac{5u^{\frac{2}{5}}}{2} \Big|_{c+1}^{1}$$

$$\lim_{x \to -1^+} \frac{5}{2}\left(1 - (c+1)^{2/5}\right) = 5/2$$

$$\sum_{i=1}^{n} f(x_i) . \Delta x$$

$\Delta x = \frac{2}{n}$

$$\frac{8i^3}{n^3} + \frac{2i}{n}$$

$x_i = \frac{2i}{n}$

$$\frac{2}{n}\left(\sum_{i=1}^{n} \frac{8i^3}{n^3} + \frac{2i}{n}\right)$$

$$\frac{8}{n^3} \sum_{i=1}^{n} i^3 + n+1$$

$$\lim_{n \to \infty} \left(\frac{2(n+1)^2}{n} + n+1\right)$$

$$\frac{6n^2 +}{n^2} = \boxed{6}$$

$$\frac{\int_{-2}^{1} -|x| \, dx}{3} =$$

$$\int_{-2}^{0} x \, dx = -2 + \int_0^1 -x \, dx = -\frac{1}{2}$$

$$\frac{-x^2}{2}$$

$$\frac{\frac{-5}{2}}{3}$$

$$\frac{x^2+5}{x.(x^2+2x+5)} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+5}$$

$$x^2+5 = (A+B)x^2 + (2A+C)x + 5A$$

$A = 1$

$B = 0$

$C = -2$

$$\int \left(\frac{1}{x} + \frac{-2}{x^2+2x+5}\right) dx$$

$$\frac{-2}{(x+1)^2+4}$$

$x+1 = u$

$$\ln|x| +$$

$$\ln|x| - \arctan\left(\frac{x+1}{2}\right) + C$$

$x = \sqrt{3} \sin t$

$dx = \sqrt{3} \cos t . dt$

$$\frac{3\sin^2 t}{(3(\underbrace{1-\sin^2 t}_{\cos^2 t}))^{\frac{3}{2}}} . \sqrt{3} . \cos t . dt$$

$$\frac{\sin^2 t}{\cos^2 t} . dt$$

$$\int \underbrace{(\sec^2 t - 1) \, dt}_{\tan t - t + C}$$

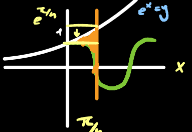
$x = \sin \theta$

$dx = \cos \theta . d\theta$

$$\int_0^{\frac{\pi}{6}} \frac{\sin^2 \theta .}{\cos^4 \theta} \, d\theta$$

$$\int_0^{\pi/6} \tan^2 \theta . \sec^2 \theta . d\theta$$

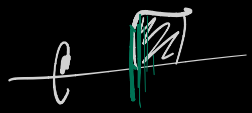
$$\frac{\tan^3 \theta}{3} \Big|_0^{\pi/6} = \left(\frac{1}{\sqrt{3}}\right)^3 . \frac{1}{3}$$



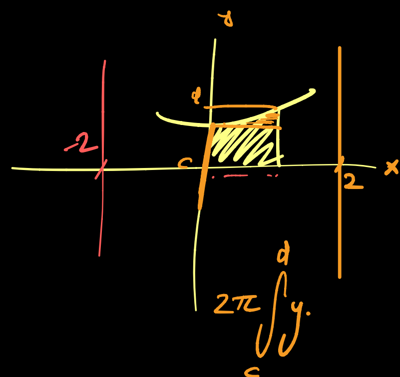
$$\pi\int_0^{\pi/4}\left(e^{2x}-\cos^2 2x\right)dx$$

$$2\pi\int (x+\pi)\cdot\left(e^x-\cos 2x\right)dx$$

$$\pi\int\left(\left(e^x-\frac{\pi}{4}\right)^2-\left(\cos 2x-\frac{\pi}{4}\right)^2\right)dx$$



	x-ekseninde	y-ekseninde
Disk	$\pi\int_a^b (f(x))^2\,dx$	$\pi\int_c^d f(y)^2\,dy$
Pul (Washer)	$\pi\int_a^b \left(f^2(x)-g^2(x)\right)dx$	$\pi\int_c^d \left((f(y))^2-(g(y))^2\right)dy$
Kabuk (Shell)	$2\pi\int_c^d y\cdot \underbrace{h}_{\text{sağ-sol}}\cdot dy$	$2\pi\int_a^b x\cdot \underbrace{h}_{\text{üst-alt}}\cdot dx$



$$2\pi\int_c^d y.$$

	x-etraf	y-etraf	x=2 etrafında
Disk	$\pi\int_a^b f(x)^2\,dx$	$\pi\int_c^d f(y)^2\,dy$	$\pi\int_c^d \left(f(y)-2\right)^2 dy$
Kabuk	$2\pi\int_c^d y.(\text{sağ}-\text{sol}).dy$	$2\pi\int_a^b x(\text{üst}-\text{alt})\,dx$	$2\pi\int_a^b (2-y).(\text{üst}-\text{alt})$