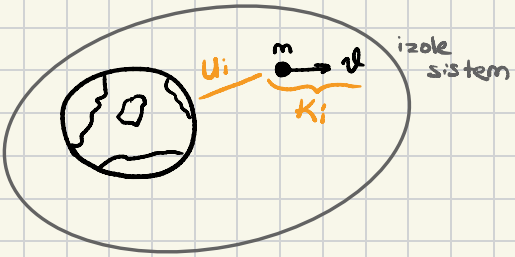
Enerji Korunumu

İzole Olmuş Sistem

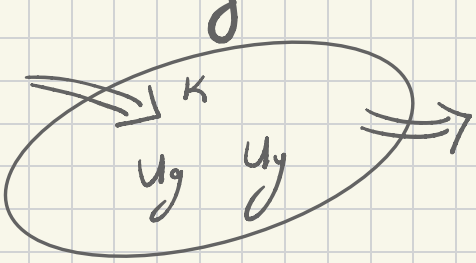


$$\Sigma E_i = \Sigma E_s$$

$$K_i + U_i = K_s + U_s$$

$$\Delta K + \Delta U_g + \Delta U_y = 0$$

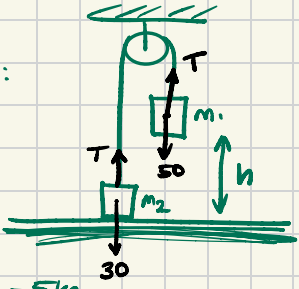
İzole Olmayan Sistem



$$W = \int \Sigma \vec{F} \cdot d\vec{r}$$

$$W_s = -\int f\,ds$$

Soru:



$m_1 = 5kg$

$m_2 = 3kg$

$h = 4m$

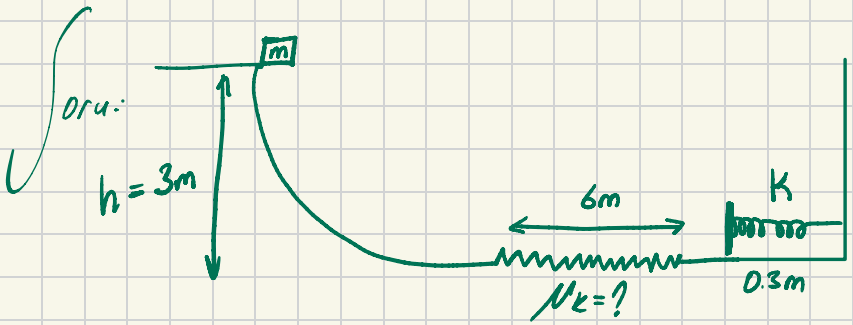
m_1, m_2 aynı yükseklikte hızları ne olur?

$E_1 = E_s = 50 \cdot 4 = 200$ $\quad V_1 = V_2 = V$

$$\cancel{200} = \cancel{80 \cdot 2} + \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2}$$

$$40 = 4V^2$$

$$\sqrt{10} = V$$



$m = 10kg$

$K = 2250\ N/m$

$E = 300J$

$E_{sürtünme} = 600 \cdot \mu_k$

$$(U_{gs} - U_{gi}) + (U_{ys} - U_{yi}) + (K_s - K_i) = 0$$

$$=$$

$$300 - 600\mu_k = \underbrace{\frac{1}{2} k \cdot x^2}_{101.25}$$

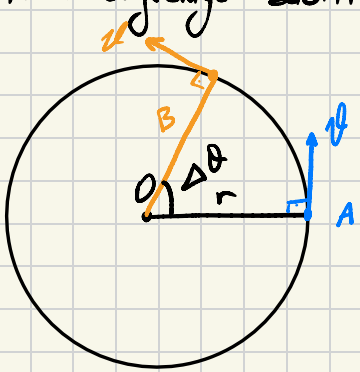
$\mu_k = 0.33$

$$300 - 600\mu_k = \frac{1}{2} 225 \cdot \frac{9}{10}$$

$\mu_k = 0.33$

Düzgün Dairesel Hareket

- Hızın büyüklüğü sabittir.



$$|\vec{v_A}| = |\vec{v_B}| = v$$

$$|\vec{r_A}| = |\vec{r_B}| = r$$

$$\frac{|\Delta\vec{r}|}{r} = \frac{|\Delta\vec{v}|}{v}$$

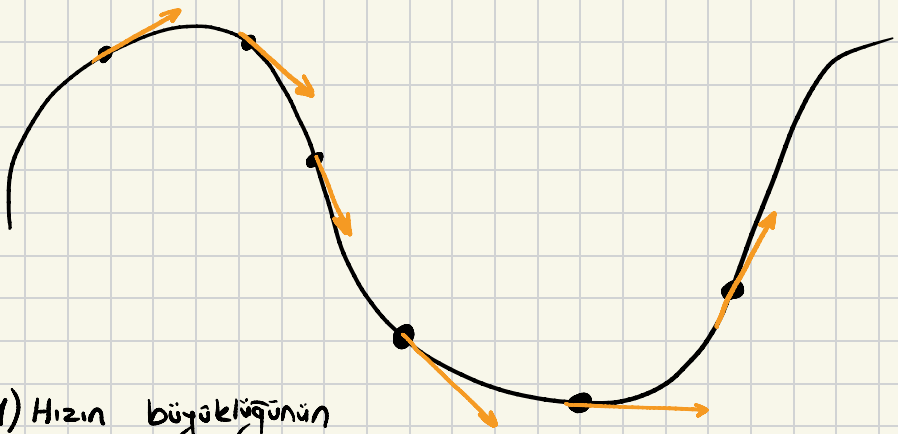
$$|\Delta\vec{v}| = \frac{v \cdot |\Delta\vec{r}|}{r}$$

$$|\vec{a_{ort}}| = \frac{\Delta\vec{v}}{\Delta t}$$

$$|\vec{a_{ort}}| = \frac{v}{r} \cdot \frac{|\Delta\vec{r}|}{\Delta t}$$

$$a = \frac{v^2}{r}$$

merkezcil ivme büyüklüğü



1) Hızın büyüklüğünün değişiminden dolayı sahip olduğu ivme

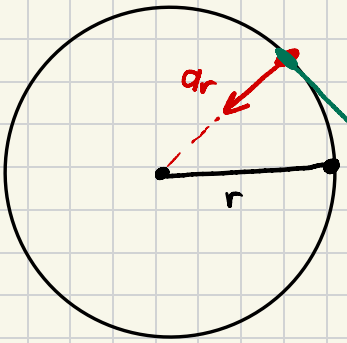
$$a_1 = \frac{dv}{dt}$$

2) Hızın yönünün değişmesinden dolayı sahip olduğu ivme

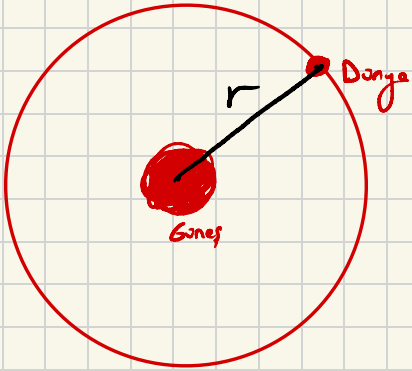
$$a_2 = \frac{v^2}{r}$$

$$a_{toplam} = \vec{a_1} + \vec{a_2}$$

Dairesel Hareket



Dünyanın Güneş etrafında dönmesi sonucu sahip olduğu merkezcil ivme nedir?

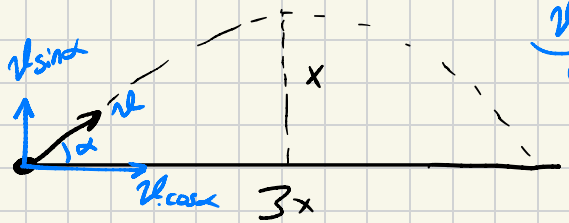


$$a = \frac{v^2}{r}$$

$$2\pi r = v_t$$

$$\frac{\left(\frac{2\pi r}{t}\right)^2}{r} = \frac{4\pi^2 r}{t^2} = a$$

Bir taş yerden belli bir açıyla fırlatılınca



$\underbrace{v\sin\alpha}_{10}\cdot t - \frac{gt^2}{2} = x$

$v\sin\alpha = 10t$

$\frac{10t^2 - 5t^2}{5t^2} = x$

$v\cos x \cdot 2 = 15t$

$v\cos x = \frac{15}{2}t$

$\tan\alpha = \frac{10}{2}\cdot\frac{2}{3}$

$\tan\alpha = \frac{4}{3}$

$\alpha = 53^\circ$

Başlangıçta bir parçacığın $\vec{a} = 3\hat{j}\ m/s^2$

$v_0 = 5\hat{i}$ m/s olarak veriliyor.

x konum vektörü = ?

hız vektörü = ?

t=2 için koordinat = ?

t=2 için sürat = ?

$x = 0 + 5\hat{i}t + \frac{3\hat{j}\cdot t^2}{2}$

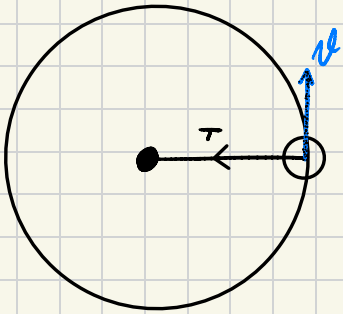
$x = 5t\,\hat{i} + \frac{3t^2}{2}\hat{j}$

$v = 5\hat{i} + 3t\hat{j}$

(10,6)

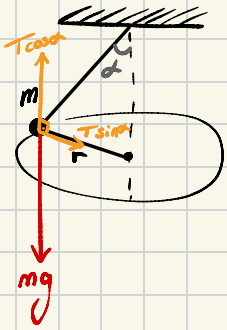
$(5\hat{i} + 6\hat{j}) = \text{hız}$

$25+36 = \sqrt{71} = \text{sürat}$



$$T = \frac{m \cdot v^2}{r}$$

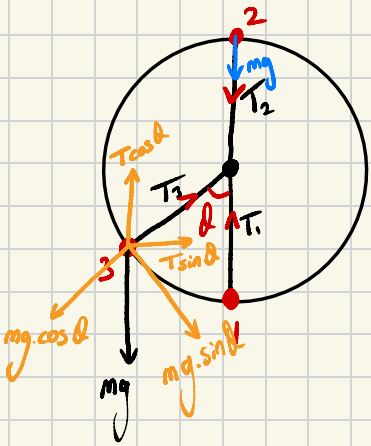
$$F_r = m \cdot a_r = \frac{m \cdot v^2}{r}$$



$$T\sin\alpha = \frac{m \cdot v^2}{r}$$

$$T\cos\alpha = mg$$

$$\Rightarrow v = \sqrt{r \cdot g \cdot \tan\alpha}$$

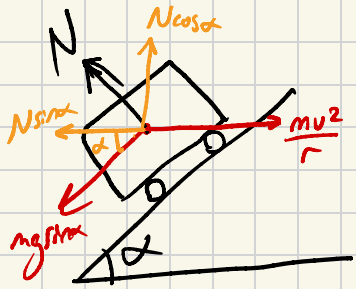


3.) $T - mg\cos\theta = \frac{mv^2}{r}$

$$T_3 = mg\left(\cos\theta + \frac{v^2}{rg}\right)$$

$$T_1 = mg\left(1 + \frac{v^2}{rg}\right) = mg + \frac{mv^2}{r}$$

$$T_2 = \left(\frac{mv^2}{r} - mg\right)$$



$v = 13.4\ m/s$

$r = 35m$

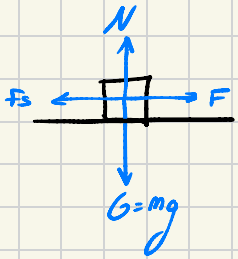
$$N\sin\alpha = \frac{mv^2}{r}$$

$$N\cos\alpha = mg$$

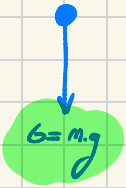
$$\tan\alpha = \frac{v^2}{r \cdot g}$$

Serbest Cisim Diyagramı

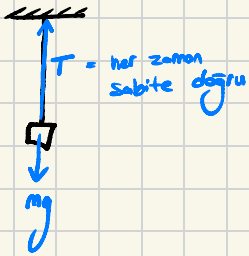
Üstünde cisme etki eden bütün kuvvetler ölçeksiz gösterilir.



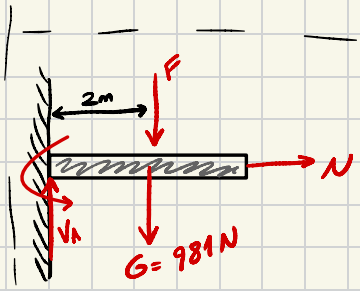
Taş binanın tepesinden bırakılıyor. Taşa etkiyen kuvvet diyagramı=?

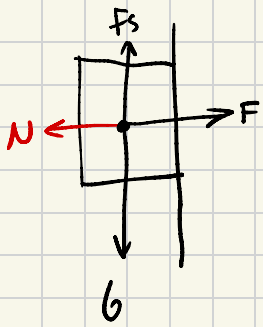


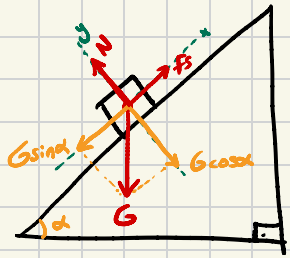
Tavandaki avize serbest cisim diyagramı

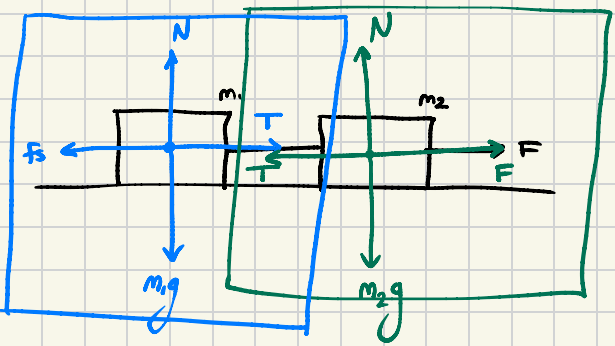


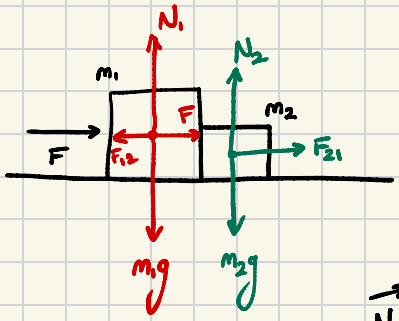
İtilen kutu diyagramı

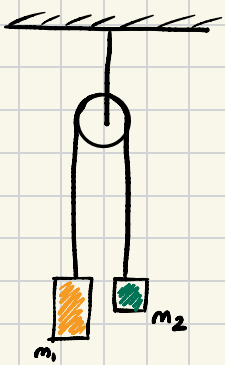


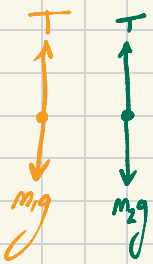


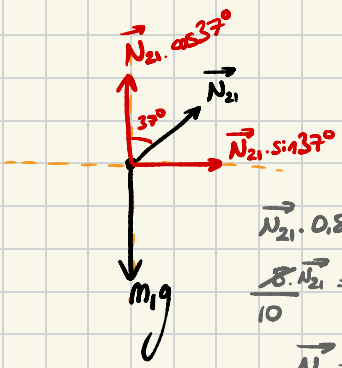


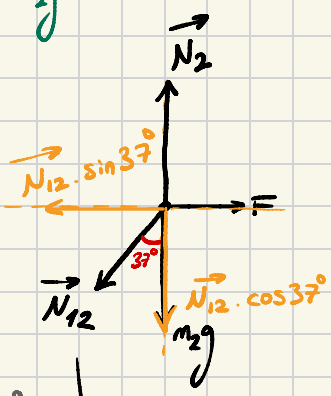












$\vec{N}_{21} \cdot 0{,}8 = m_1 \cdot g$

$\frac{8 \cdot \vec{N}_{21}}{10} = \frac{24}{3}$

$\vec{N}_{21} = 30\,N$

$\frac{3\cancel{0} \cdot \vec{N}_{21} \cdot 6}{1\cancel{0}} = 2{,}4 \cdot a$

$\frac{30}{4} = \frac{3}{0{,}4} = \frac{18}{2{,}4} = a$

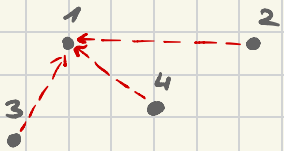
$F - \vec{N}_{12} \cdot \frac{6}{10} = \frac{4 \cdot 30}{4}$

$-\frac{30 \cdot 6}{10} = 30$

-18

$F = 48$

Lineer Momentum ve Korunumu



$$\sum \vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$
$$\sum \vec{F}_2 = \vec{F}_{32} + \vec{F}_{12} + \vec{F}_{42}$$

+

$$\sum \vec{F}_1 + \sum \vec{F}_2 + \sum \vec{F}_3 = 0$$

$$m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 = 0$$

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} = 0$$

$$\frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right) = 0$$

$$m.\vec{v} = \vec{P} \rightarrow \vec{P}_{top} = sbt$$

* Bir kuvvetsiz düzlemde kütlelerin birbirine uyguladığı toplam kuvvet "0" dır.

* $a = \frac{dv}{dt}$, $\alpha = \frac{d\omega}{dt}$

Kuvvetsiz sistemde $P_i = P_s$

* Dışarıdan kuvvet uygulanmıyorsa lineer momentum muhakkak eşittir.

Soru: Sürtünmesiz düzlemde 50kg okçu 0,5kg kütleli bir ok fırlatıyor. Okun hızı 50m/s ise okçunun hızı = ?

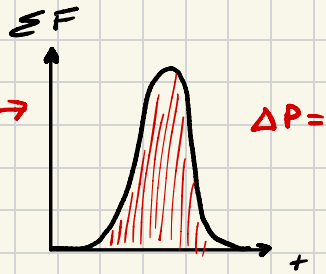
$$50.x \longleftrightarrow 0,5.50 = 25\ kg.m/s$$
$$kg.m/s$$

$$x = -0,5\ m/s$$

$$\sum \vec{F} = m.\frac{d\vec{v}}{dt}$$

$$\sum \vec{F} = \frac{d\vec{P}}{dt}$$

$$\int_{\vec{P}_i}^{\vec{P}_s} d\vec{P} = \int_0^{+} \sum F.\, dt$$



ΔP = itme

$$\Delta P = F_{ort}.\Delta t$$

$$F_{ort} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \sum F\, dt$$

İtme (İmpuls) = anlık momentum değişimi

1500kg otomobil duvara çarptırılıyor. Otomobilin çarpışmadan önceki ve sonraki hızları

$\vec{v}_i = -15\hat{i}$ m/s

$\vec{v}_s = 2.6\hat{i}$ m/s

Eğer çarpışma 0,15 sn sürüyorsa çarpışma itmeyi ve otomobile etki eden ortalama kuvveti bulunuz.

2) $26500 = \frac{15}{100}.F_{ort}$ 1)

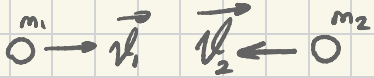
$176000\ N = F_{ort}$

$-22500\hat{i}$ kg.m/s ⟵⟶ $3900\hat{i}$ kg.m/s

1500. − 15 =

$\Delta P = 26400\hat{i}$

Çarpışmalar



$$\sum \vec{P}_i = \sum \vec{P}_s$$

* Dışarıdan kuvvet bulunmayan tüm lineer durumlarda

$$m_1 v_{1i}\hat{i} + m_2 v_{2i}\hat{i} \ldots = m_1 v_{1s} + m_2 v_{2s} \ldots$$

eğer esnekse kinetik enerji korunacağından:

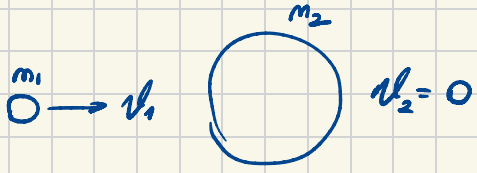
$$\frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} + \ldots = \frac{m_1 v_{1s}^2}{2} + \frac{m_2 v_{2s}^2}{2} + \ldots$$

$$\vec{v}_{1i} + \vec{v}_{1s} = \vec{v}_{2i} + \vec{v}_{2s} = \ldots$$

$$v_{1s} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2s} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) v_{2i} + \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

1. Durum



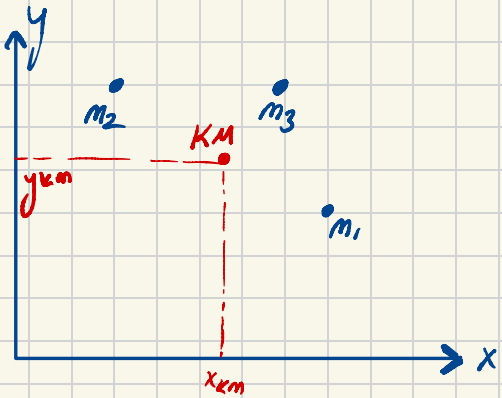
* m_1 ihmal edilebilecek kadar küçük

$$v_{1s} = -v_{1i}$$

$$v_{1s} = -v_{1i}$$

$$v_{2s} \simeq 0$$

Kütle Merkezi



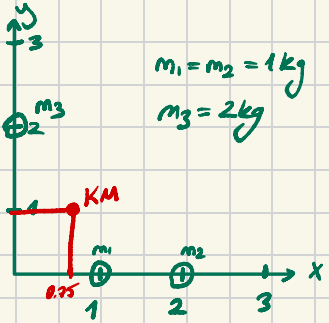
$$x_{KM} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + m_3 \dots}$$

$$x_{KM} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$

$$y_{KM} = \frac{1}{M} \cdot \sum_{i=1}^{n} m_i y_i$$

$$\overrightarrow{r_{KM}} = x_{KM}\hat{i} + y_{KM}\hat{j} + z_{KM}\hat{k}$$

Soru:



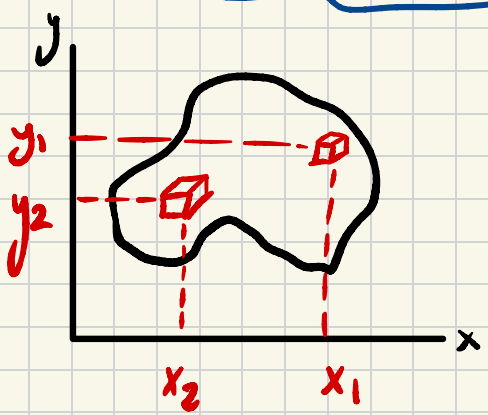
$m_1 = m_2 = 1\,kg$

$m_3 = 2\,kg$

$$x_{KM} = \frac{3}{4} = 0{,}75\,m$$

$$y_{KM} = \frac{4}{4} = 1\,m$$

Sürekli Kütle Dağılımına Sahip Cisimlerin Kütle Merkezleri



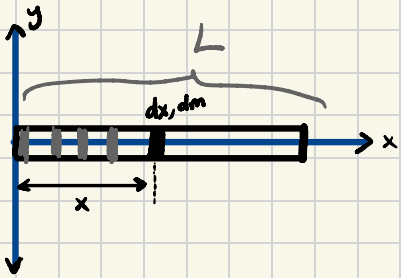
$$x_{KM} \simeq \frac{1}{M} \sum_{i=1}^{n} \Delta m_i x_i$$

$$\downarrow$$

$$x_{KM} \simeq \frac{1}{M} \cdot \lim_{\Delta m_i \to 0} \sum_{i=1}^{\infty} \Delta m_i x_i$$

$$\downarrow$$

$$x_{KM} = \frac{1}{M} \int y \, dm$$



$$\begin{matrix} L & M \\ dx & dm \end{matrix}$$

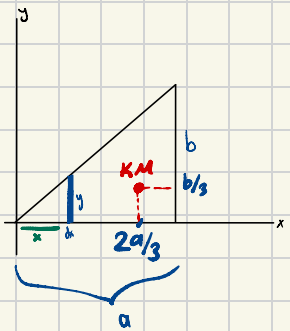
$$dm = \frac{M}{L}\,dx \quad \Big| \quad dm = \lambda\,dx$$

λ: çizgisel kütle yoğunluğu

$$X_{KM} = \frac{1}{M}\int x\,dm$$

$$X_{KM} = \frac{1}{M}\int_0^L x\,\lambda\,dx$$

$$X_{KM} = \frac{\lambda}{M}\cdot\frac{L^2}{2} = \frac{1}{2}L$$



$$dm = \sigma\,dA$$

$$\sigma = \frac{M}{A};\quad dA = y\,dx$$

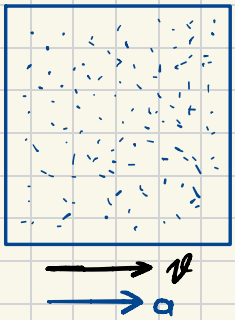
$$dm = \sigma\,y\,dx$$

$$X_{KM} = \frac{1}{M}\int x\,dm$$

$$x_{KM} = \frac{\sigma}{M}\int_0^a x\,y\,dx$$

$$X_{KM} = \frac{2a}{3}$$

$$y_{KM} = \frac{b}{3}$$



$$\vec{r}_{km} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

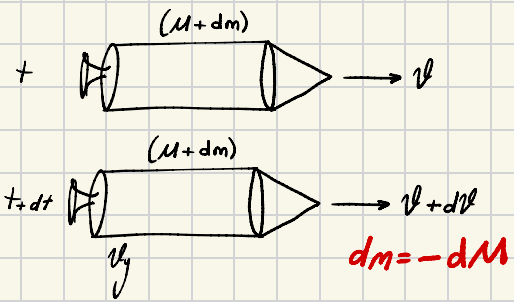
$$\vec{v}_{km} = \frac{d\vec{r}_{km}}{dt} = \frac{d}{dt}\left[\frac{1}{M}\sum_i m_i \vec{r}_i\right]$$

$$M\vec{v}_{km} = \sum_i m_i . \vec{v}_i$$

$$\vec{a}_{km} = \frac{d\vec{v}_{km}}{dt} = \frac{d}{dt}\left[\frac{1}{M}\sum_i m_i \vec{v}_i\right]$$

$$M.\vec{a} = \sum_i m_i \vec{a}_i \rightarrow \vec{F} = \sum_i \vec{F}_i$$

Roket Hareketi



$dm = -dM$

+ anında

$$(M+dm)v = M(v+dv) + dm(v - v_y)$$

$$v_s = v_i + v_y \ln\left(\frac{M_i}{M_s}\right)$$

$$T = M.\frac{dv}{dt} \rightarrow T = \left| v_y \frac{dM}{dt} \right|$$

Roket hızı $3.10^3 m/s$

Bu anda roket ateşleniyor.

Roketin yakıtı $5.10^3 m/s$'lik hızla terk ediyor.

Roketin kütlesi yarıya düşerse yere göre hız = ?

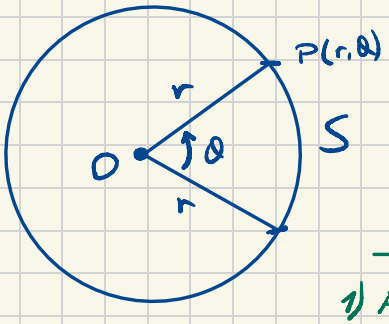
$$v_s = v_i + v_y \ln\left(\frac{M_i}{0.5M_i}\right)$$

$$(3\times10^3) + (5\times10^3).\ln 2 = 6.5\times10^3 m/s$$

Roket yakıtı 50 kg/s oranında yanıyorsa itme kuvveti = ?

$$\left| v_y \frac{dm}{dt} \right| = T = 50.5.10^3 = 25.10^4 N$$

Katı Bir Cismin Sabit Eksende Dönmesi



Tam bir tur için

$2\pi r = r\,\theta$

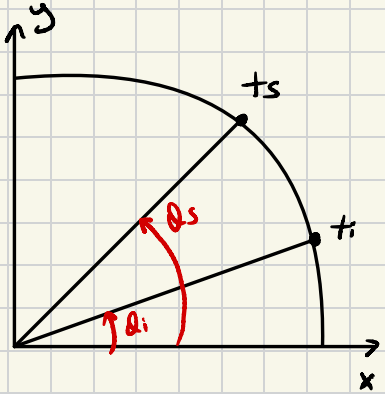
$2\pi = \theta$ (radyan)

$S = r \cdot \theta$

θ birimi = radyan

1) Açısal Yer Değiştirme

$$\Delta\theta = \theta_s - \theta_i$$



2) Ortalama Açısal Hız

$$w_{ort} = \frac{\Delta\theta}{\Delta t} \quad (rad/s) \quad (s^{-1})$$

3) Ani Açısal Hız

$$w = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta(t)}{dt}$$

türev operatörü

4) Ortalama Açısal İvme

$$\alpha_{ort} = \frac{\Delta w}{\Delta t} \quad \left(rad/s^2\right)$$

5) Ani Açısal İvme

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw(t)}{dt}$$

Sabit Açısal İvmeli Dönme Hareketi

Dönme Hareketi	Öteleme Hareketi
$\omega_s = \omega_0 + \alpha t$	$v_s = v_i + at$
$\theta_s = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2}$	$x_s = x_i + v_i t + \frac{at^2}{2}$
$\omega_s^2 = \omega_0^2 + 2\alpha(\theta_s - \theta_i)$	$v_s^2 = v_0^2 + 2a(x - x_0)$
$(\alpha = sbt)$	$(a = sbt)$
değilse;	değilse;
$\omega = \frac{d\theta}{dt}$	$v = \frac{dx}{dt}$
$\alpha = \frac{d\omega}{dt}$	$a = \frac{dv}{dt}$

Soru: $\alpha = 3{,}5\ rad/s^2$

$t_0 = 0 \Rightarrow \omega_0 = 2\ rad/s$

a) $t = 2sn$ 'deki $\Delta\theta = ?$

$\theta_s = \theta_i + \omega_0 t + \frac{\alpha t^2}{2}$

$\Delta\theta = 4 + \frac{3{,}5 \cdot 4}{2} = 11\ rad$

b) $t = 2sn$, kaç tur atar?

$\frac{11}{2\pi}$ derece yani $\frac{180 \cdot (11)}{\pi} = 630°$

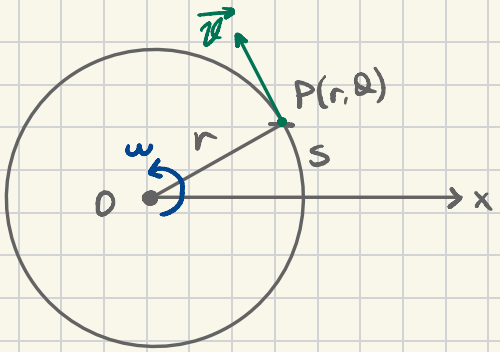
1.75 tur

c) $t = 2sn$ açısal hız $= ?$

$\omega_s = \omega_0 + \alpha t$

$\omega_s = 2 + 7$

$\omega_s = 9\ rad/s$



$$S = r\,\theta$$

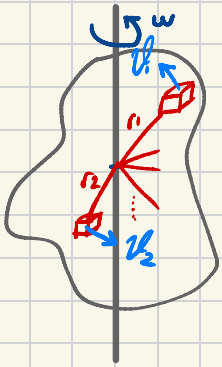
$$\frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$v = r \cdot \omega$$

$$\frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

$$a = \alpha r$$

Dönme Kinetik Enerjisi



$$K \simeq \frac{1}{2}\Delta m_1 v_1^2 + \frac{1}{2}\Delta m_2 v_2^2 + \dots$$

$$K \simeq \sum_i^N \frac{1}{2}\Delta m_i v_i^2$$

$$K \simeq \frac{1}{2}\left(\sum_i^N \Delta m_i r_i^2\right)\omega^2$$

$$K = \frac{1}{2}\left(\lim_{\Delta m_i \to 0} \sum_{i=1}^{\infty} \Delta m_i r_i^2\right)\cdot \omega^2$$

$$\int r^2\,dm = I$$

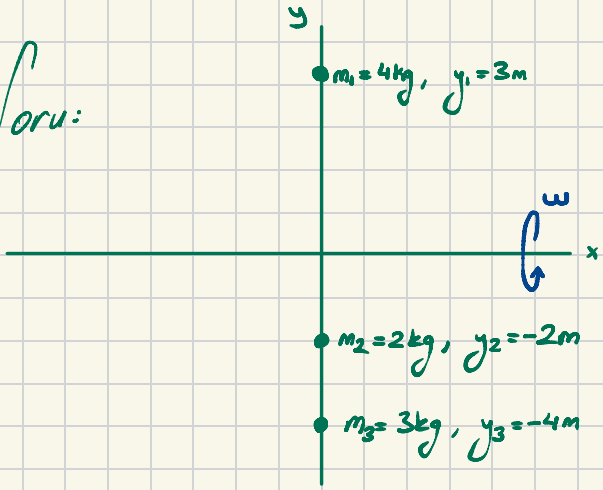
- eylemsizlik momenti

$$\sum_i m_i r_i^2 = I$$

$$K = \frac{1}{2} I \omega^2$$

- Sadece "ω" hepsinde aynı.

Soru:



$\omega = 2\ rad/sn$

Sistemin kinetik enerjisi kaç J = ?

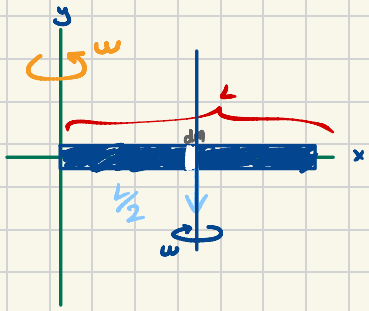
$$K = \frac{1}{2} I \omega^2$$

$$I = \sum_i m_i r_i^2 \text{ ise,}$$

$$\frac{4.9}{36} + \frac{2.4}{8} + \frac{3.16}{48} = 92\ kg.m^2$$

$$\frac{1}{2} 92.2^2 = 184\ J$$

Soru:



$$I = \int x^2\, dm$$

eylemsizlik momenti nedir?

$$dm = \frac{M}{L} \cdot dx$$

$$I = \int_0^L x^2 \frac{M}{L}\, dx$$

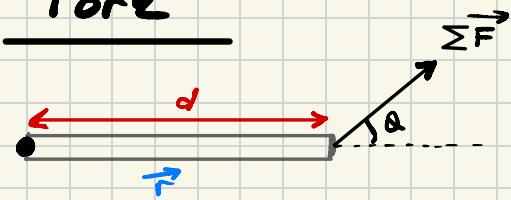
$$\frac{x^3}{3}\Big|_0^L = \frac{L^3}{3} \cdot \frac{M}{L} = \frac{ML^2}{3}$$

$$I = \int x^2\, dm$$

$$I = \lambda \cdot \frac{L^3}{3}$$

$$I = \frac{1}{3} ML^2$$

Tork

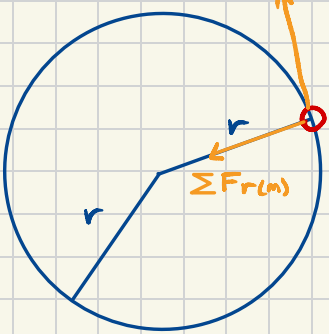


$$\tau = (\Sigma F . \sin\theta) . d$$

$$\tau = \vec{r} \times \Sigma \vec{F}$$

$\Sigma F_t = m . \alpha_t$

merkezcil kuvvet tork üretmez.



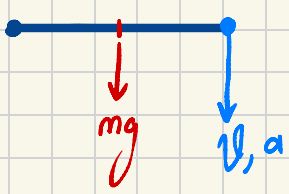
$\Sigma F_t = m . \alpha_t$

$r\alpha = a_t$

$$\Sigma \tau = (m . r^2)\,\alpha$$

$$\Sigma \tau = I\,\alpha$$

Soru: m kütleli çubuk yatayla paralelken serbest bırakılıyor. Anlık açısal ivmeyi ve çubuğun en ucunun çizgisel ivmesini bulunuz.



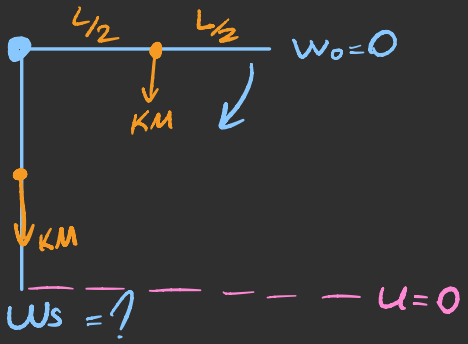
$$I = \frac{1}{3} m L^2$$

$$\tau = I . \alpha = mg . \frac{L}{2}$$

$$\frac{1}{3} \cancel{mL^2} . \alpha = \cancel{mg} . \frac{\cancel{L}}{2\cancel{L}}\,{}^{3}$$

$$\alpha = \frac{3g}{2L}$$

$$\frac{3g}{2\cancel{L}} . \cancel{L} = \frac{3g}{2}$$



$\Delta E = 0$

$$mgL = K + mg\frac{L}{2}$$

$$\frac{mgL}{2} = K$$

$$\frac{mgL}{2} = \frac{I w_s^2}{2}$$

$$\frac{3\,mgL}{mL^2} = w_s^2$$

$$\sqrt{\frac{3g}{L}} = w_s$$

$r \cdot w = v$

$$\frac{L}{2}\sqrt{\frac{3g}{L}} = v_{KM}$$

$$\sqrt{3gL} = v_p$$

Dönen katı cismin üzerindeki herhangi bir noktanın bir sabit eksenle yaptığı açının zamanla değişimi

$\theta = (5 + 10t + 2t^2)$ rad

$t = 3$ sn için

açısal konum, hız, ivme

$w = 10 + 4t \longrightarrow 22$

$\alpha = 4 \longrightarrow \alpha = 4$

$\theta = 53$ rad

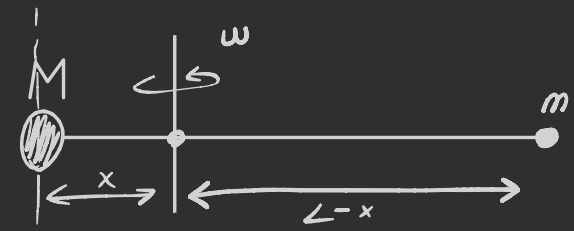
$\alpha = (10 + 6t)$ rad/s²

$10 + 6t = \frac{dw}{dt}$

$w(t) = (10t + 3t^2)$ rad/s

$\theta(t) = (5 + t^2 + t^3)$ rad

$\theta(4) = 144$ rad



$$I = \sum_i m_i r_i^2$$

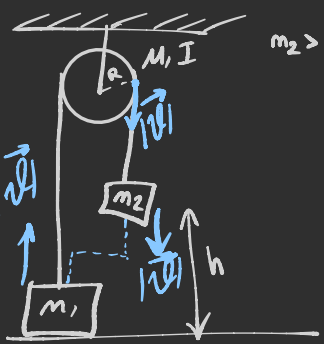
$$I = Mx^2 + m(L-x)^2$$

$$\frac{dI}{dx} = 2Mx + 2m(L-x)(-1)$$

$$I = \mu L^2 \qquad \mu = \frac{Mm}{m+M}$$

$$Mx = mL - mx$$

$$x = \left(\frac{m}{m+M}\right) L$$



$m_2 > m_1$

$$m_2 gh = \frac{m_2 gh}{2} + \frac{m_1 gh}{2} + \frac{m_1 v^2}{2} + \frac{m_2 v^2}{2} + \frac{I\omega^2}{2}$$

$$r \cdot \omega = v$$

$$\sqrt{\frac{(m_2 - m_1) gh}{m_1 + m_2 + I/R^2}} = v$$

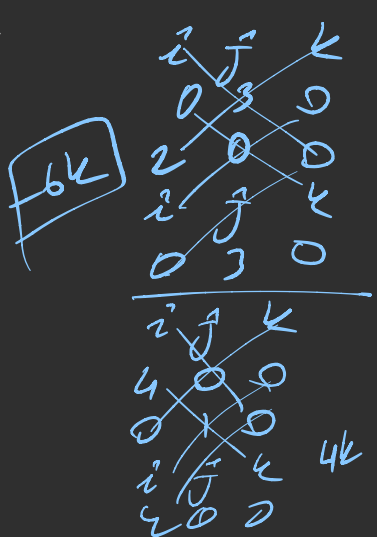
$8 \cdot 40 = 320\ J$ $\quad \frac{20 \cdot 9}{2}$

$320 + 180 = \frac{20}{2} \cdot \frac{40 \cdot v^2}{2}$

$v = 5$

$40 \cdot 3 = 48 \cdot v$

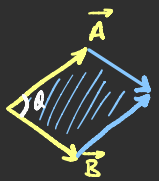
$\vec{F_1} \times \vec{r_1} =$



Açısal Momentum

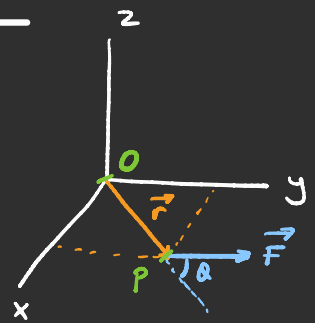
$\vec{C} = \vec{A} \times \vec{B}$

$C = A.B.\sin\theta$



$*\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

$* (\vec{A} \times \vec{B})\frac{d}{dt} = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$



$\sum \vec{\tau} = \vec{r} \times \vec{F}$

$\sum \vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{P})$

$\Downarrow$

$\vec{L} \equiv \vec{r} \times \vec{P}$

$\Downarrow$

$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$L_x = (yP_z - zP_y)$

$L_y = (zP_x - xP_z)$

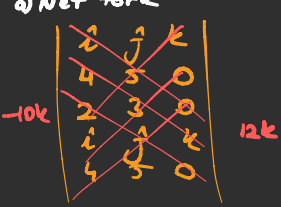
$L_z = (xP_y - yP_x)$

Soru

$\vec{F} = (2\hat{i} + 3\hat{j})\ N$

$\vec{r} = (4\hat{i} + 5\hat{j})\ m$

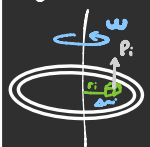
a) Net tork



Noktasal Parçacıklar ise

$\vec{L}_{top} = \sum_i \vec{L}_i$

Rijit Cisim Açısal Momentumu



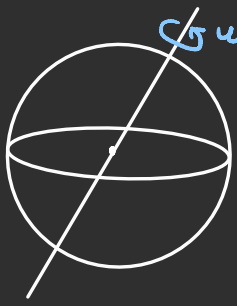
$\vec{L}_i = \vec{r}_i \times \vec{P}_i$

$L_i = r_i.\Delta m_i.v_i = r_i\,\Delta m_i.r_i.\omega$

$L_{top} \simeq \left(\sum_i r_i \Delta m_i\right)\omega$

$L_{top} = \left(\lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i\right)\omega$

$L_{top} = \left(\int r^2 dm\right).\omega \Rightarrow L = I.\omega$



$\omega = 10$ devir/sn $\longrightarrow$ $10\left(\frac{\text{devir}}{s}\right)\cdot\left(\frac{2\pi\ rad}{1\ devir}\right) = \frac{20\pi\ rad}{s}$

$m = 7kg$

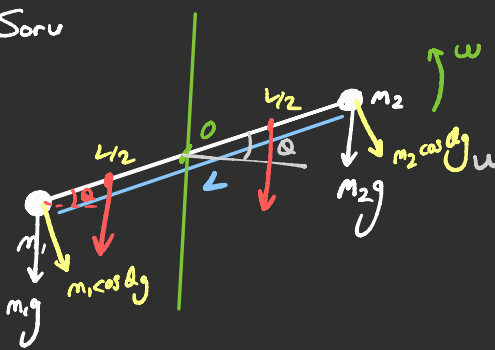
$R = 0.12\ m$

$L = I.\omega$

$\frac{2}{5}.7.(0,12)^2.\ 20\pi =$

$2.53\ kg.m^2/s =$

Soru



Açısal mom korunur.

$L = I.\omega$

$\omega.\left(m_1\frac{L^2}{4} + m_2\frac{L^2}{4} + \frac{ML^2}{12}\right) = \frac{3(m_1+m_2)L^2 + ML^2}{12}.\omega = L$

Q açısı anında sistem açısal ivmeyi bul.

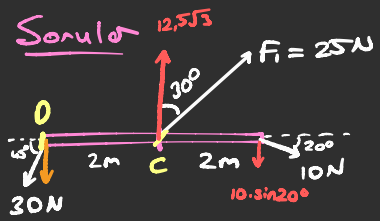
$\tau = I.\alpha$

$|m_1 \cos\theta g - m_2 \cos\theta g| = \frac{3(m_1+m_2)L^2 + ML^2}{12}.\alpha$

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$\sum \vec{\tau} = 0$ ise açısal momentum sabit

Sorular



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xy düzleminde hareket eden
1,5kg kütleli cismin hız vektörü
$(4,2\hat{i} - 3,6\hat{j}) = \vec{v}$

Orijinden çizilen konum vek = $(1,3\hat{i} + 2,2\hat{j})$

- Açısal Momentum

$L = I.w$

$L = \vec{r} \times \vec{p}$

$\vec{p} = (6,3\hat{i} - 5,4\hat{j})$

$\vec{r} = (1,5\hat{i} + 2,2\hat{j})$

$\vec{L} = -20,43\hat{k}$

$L = I.w = \vec{r} \times \vec{p}$

$mv = (m+M)\, v_{kop}$

$\frac{mv}{m+M} = v_{kop}$

$(m+M)\, v_{kop} . l$

$rw = v$

$w = \frac{v}{r}$

Statik Denge ve Esneklik

* Tüm kuvvetlerin bileşkesi sıfır ise cisim dengededir.

$\sum F = 0$ → durağan / sabit hızlı

*** Statik = Durgun**

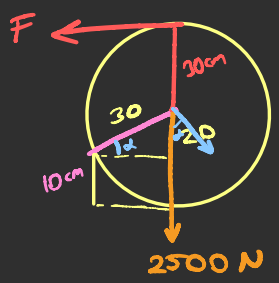
Cisim
- noktasal → $\sum F = 0$ yeterlidir.
- rijit (gerçek) → $\sum F = 0$ ve cismin herhangi bir noktaya göre torku sıfır olmalıdır.

$\vec{\tau} = \vec{F} . d \ (F \perp d) = \vec{F} \times \vec{d}$

ötteleme dengesi

$\vec{\tau} = \vec{F} . d . \sin\alpha$

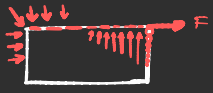
dönme dengesi



$\sum \tau_F =$

Esneklik

* Gerçekte bütün cisimler bozunmaya uğrar.



* Her bir rijit cisim, dış kuvvetlere karşı belirli bir direnme ve rijitliğini koruma sınırı vardır. Direnme noktaları değişmez ise kırılma bölgesine uğrar.

Esneklik Modülü

* Young modülü: Katının uzunluğudaki bir değişime karşı gösterdiği direnç oranı
* Kesme sabiti: Katının, atomik düzeylerde birbiri üzerinde kayması -esine kesme- ortaya çıkan direnç
* Hacim modülü: Hacim değişikliklerinde gelebilecek direnç ölçütü

$F_s = -k \cdot x$

$x = A \cdot \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega}$

$a_{max} = A\omega^2$ → maks. genlik

$v_{max} = A\omega$

$v(t) = -A \cdot \omega \cdot \sin(\omega t + \phi)$

$a(t) = -A \cdot \omega^2 \cdot \cos(\omega t + \phi)$

---

yay-kütle

$2\pi\sqrt{\frac{m}{k}} = T = \frac{2\pi}{\omega}$

anlık kuvvet uygulanırsa açısal momentum korunur.

ilerleyen cismin kaybedeceği yörünge

$f = 0$

Gizgisel

$mv_0 = mv + mwr$

$= mwd + Mwr$

$=$

Açısal

$L_i = L_s \quad L = r \times P$

$mv_0 d = I_0 \cdot w$

$mv_0 = \frac{\left(\frac{1}{3}ML^2 + md^2\right) \cdot w}{d}$

$r = \frac{L}{2}$

$\frac{\left(\frac{1}{3}ML^2 + md^2\right)w}{d} = w(md + Mr)$

$\frac{1}{3d}ML^2 + md = md + \frac{ML}{2}$

$\frac{1M}{3d} = \frac{ML}{2}$

$d = \frac{2L}{3}$