

Terminology

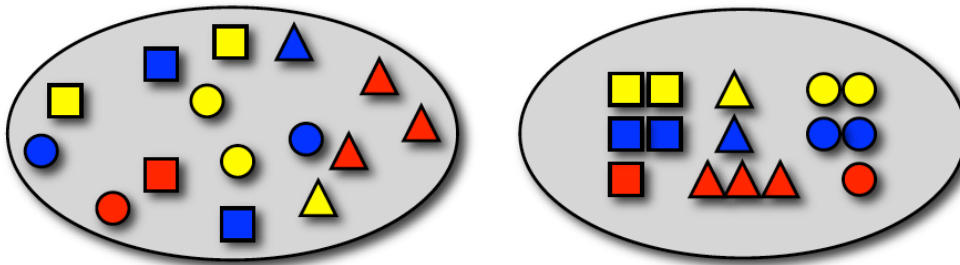
Trial: e.g. picking a shape

Sample space Ω : the set of all possible outcomes (e.g. all kinds of shapes)

Event $\omega \subseteq \Omega$: an actual outcome of a trial (a subset of Ω)

Exercise

What is the probability of...?



$$\begin{array}{lll}
 P(\text{blue square}) = 2/15 & P(\text{red square}) = 1/15 & P(\text{red square or blue triangle}) = 2/15 \\
 P(\text{blue}) = 5/15 & P(\text{red}) = 5/15 & P(\text{blue triangle | red}) = 3/5 \\
 P(\text{blue | square}) = 2/5 & P(\text{square}) = 5/15 &
 \end{array}$$

Other exercises on probability theory:

Define a Bernoulli distribution.

Why is this a good distribution for modeling a coin toss?

Can a Bernoulli distribution be used for modeling a biased coin?

How would you modify it to model a dice?

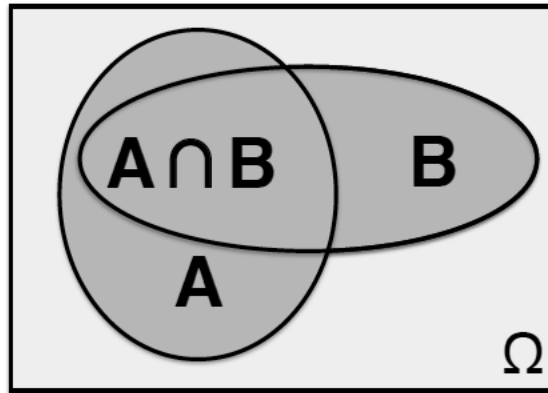
What is a Binomial distribution? Define it and state the connection to a Bernoulli distribution.

State the connection between the Binomial distribution and a Gaussian distribution.

What is the difference between a categorical and a multinomial distribution?

What is a random variable? Is it actually a variable in the algebraic sense?

Review: Laws of probability



$$P(\Omega) = 1$$

$$\forall A \subseteq \Omega: 0 \leq P(A) \leq 1$$

$$\forall A, B \subseteq \Omega: P(A \cap B) \leq P(A)$$

$$\forall A, B \subseteq \Omega: P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

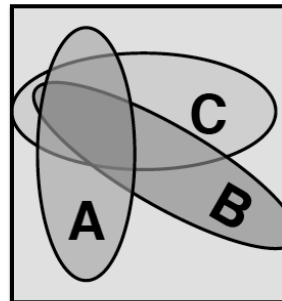
Conditional Probability:

Probability of A given B:

$$P(A | B)$$

Probability of A and B given C:

$$P(A, B | C)$$

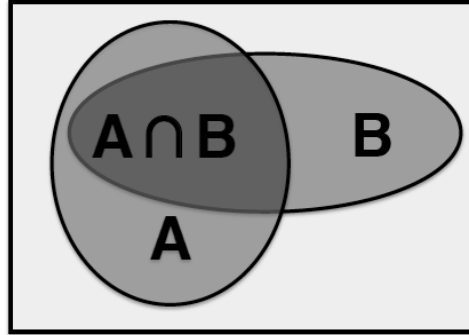


Probability of A given B and C:

$$P(A | B, C)$$

Bayes rule:

$$P(A | B) = \frac{P(A, B)}{P(B)}$$



Product rule

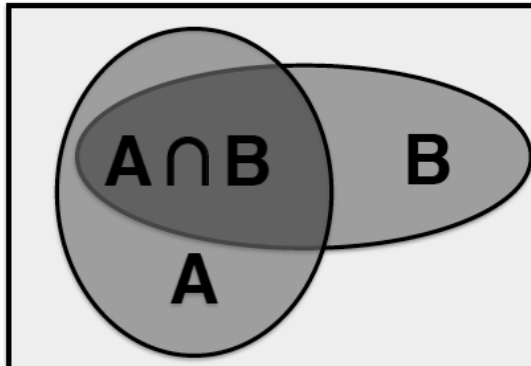
$$P(A, B) = P(A | B)P(B)$$

Joint Probability

$$P(A \cap B) = P(A, B)$$

If A and B are boolean variables:

$$P(A, B) = P(A \wedge B)$$



STUDY GUIDE: PROBABILITY

Let's work through some quantitative examples

		Weather			
		Sunny	Cloudy	Rainy	Snowy
Have Fun?	Yes	0.25	0.15	0.05	0.13
	No	0.05	0.1	0.25	0.02

$$P(\text{Rainy} \mid \neg \text{Sunny})$$

$$= \frac{P(\text{Rainy} \wedge \neg \text{Sunny})}{P(\neg \text{Sunny})}$$

$$= 0.3 / 0.7 \approx 0.43$$

$$P(\text{Fun} \mid \text{Sunny})$$

$$= \frac{P(\text{Fun} \wedge \text{Sunny})}{P(\text{Sunny})}$$

$$= 0.25 / 0.3 \approx 0.83$$

		Weather			
		Sunny	Cloudy	Rainy	Snowy
Have Fun?	Yes	0.25	0.15	0.05	0.13
	No	0.05	0.1	0.25	0.02

Do I prefer Sun or Snow?

$$P(\text{Fun} \mid \text{Snowy})$$

$$= 0.13 / 0.15 \approx 0.87 \quad \text{So I prefer snow}$$